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Abstract—**Plane wave propagation in two kinds of Faraday chiral media, where Faraday rotation is combined with optical activity, is studied to examine methods of controlling chirality. The two types of media studied are magnetically biased chiroplasmas and chiroferrites. For propagation along the biasing magnetic field, four wavenumbers and two wave impedances are found which are dependent on the strength of the biasing field. Dispersion diagrams for the chiroplasma case are plotted. Propagation at the plasma frequency of the chiroplasma is also investigated.**

I. Introduction

Many of the previous reports on chiral media, which are also known as optically active media in the optical regime, have been devoted to the properties of isotropic media. However, once such an isotropic chiral material is created, there is very little control over the degree of chirality. Therefore, it has now become important to develop methods of achieving chirality control. One such method may be realized by introducing certain forms of anisotropy. With this goal in mind, this paper investigates the concept of chirality control in a Faraday chiral medium combining Faraday rotation with optical activity.[1] Two candidate models of this medium immediately come to mind: 1) a chiroplasma consisting of chiral objects embedded in a magnetically biased plasma, and 2) a chiroferrite made from chiral objects immersed in a magnetically biased ferrite. The main emphasis of this paper will be directed towards understanding the properties of the chiroplasma. However, several key analogous results for the chiroferrite will be presented in order to show the extensive similarities that exist between the two cases.

The behavior of electromagnetic waves in a simple isotropic chiral medium has been a topic of interest since the beginning of the nineteenth century. The early researchers, Arago [1], Biot [2], Pasteur [3] and Fresnel [4], focused their attention on the optical activity displayed by solid and liquid chiral media. They discovered that these media are capable of rotating the polarization ellipses of light because of the media's polarization birefringence. Their work also established that the handedness of the uniformly distributed and randomly oriented chiral objects, which compose the chiral medium, is responsible for the observed optical activity.

It was not until one century after the initial research on chiral media that research into wave propagation in a magnetostatically biased plasma began. Appleton [5] and Hartree [6], pioneers in the area, investigated the propagation of electromagnetic waves at arbitrary directions with respect to the static magnetic field. Their work revealed that, similar to chiral media, magnetostatically biased plasmas also exhibit polarization rotation. However, unlike chiral media, the biased plasmas are anisotropic and nonreciprocal.

Work in general bianisotropic media has been done by Post [7], Kong [8], and Chawla and Unz [9] among others. In contradistinction to these general considerations, here we examine the physical and mathematical properties of two cases of special interest and applications.

In subsequent sections, we investigate the marriage of chiral and Faraday rotation with the goal of controlling the effect of chirality. We outline the properties of waves in Faraday chiral media and examine their properties through their dispersion relations.

II. Problem Statement

An electromagnetic description of Faraday chiral media may be obtained by making a tensor generalization of the scalar constitutive relations for an ordinary isotropic chiral medium. The latter are $\mathbf{D} = \epsilon_c \mathbf{E} + i\xi_c \mathbf{B}$ and $\mathbf{H} = i\xi_c \mathbf{E} + \mathbf{B}/\mu_c$, where ξ_c is the chirality admittance,[2] ϵ_c is the permittivity, and μ_c is the permeability. For a chiroplasma, from a phenomenological point of view, we select

$$\mathbf{D} = \underline{\underline{\epsilon}} \cdot \mathbf{E} + i\xi_c \mathbf{B} \tag{1}$$

$$\mathbf{H} = i\xi_c \mathbf{E} + \mathbf{B}/\mu \tag{2}$$

as the appropriate form of the constitutive relations, where

$$\underline{\underline{\epsilon}} = \begin{bmatrix} \hat{\epsilon} & -i\hat{g} & 0 \\ i\hat{g} & \hat{\epsilon} & 0 \\ 0 & 0 & \hat{\epsilon}_z \end{bmatrix} \tag{3}$$

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[1] Chiral media exhibit optical activity, which refers to the rotation of the plane of polarization of light, while Faraday media exhibit Faraday rotation. The first is reciprocal whereas the second is not.

[2] When the handedness of the medium changes, ξ_c changes sign.

TABLE I
COMPARISON OF SEVERAL KEY RESULTS FOR THE CHIROFERRITE CASE WITH THOSE FOR THE CHIROPLASMA

	Chiroplasma	Chiroferrite
Constitutive Relations	$\mathbf{D} = \underline{\underline{\epsilon}} \cdot \mathbf{E} + i\xi_c \mathbf{B}$ $\mathbf{H} = i\xi_c \mathbf{E} + \mathbf{B}/\mu$	$\mathbf{D} = \epsilon \mathbf{E} + i\xi_c \mathbf{B}$ $\mathbf{H} = i\xi_c \mathbf{E} + \underline{\underline{\mu}}^{-1} \cdot \mathbf{B}$
Anisotropy Tensor	$\underline{\underline{\epsilon}} = \begin{bmatrix} \hat{\epsilon} & -i\hat{g} & 0 \\ i\hat{g} & \hat{\epsilon} & 0 \\ 0 & 0 & \hat{\epsilon}_z \end{bmatrix}$	$\underline{\underline{\mu}} = \begin{bmatrix} \hat{\mu} & -i\hat{\kappa} & 0 \\ i\hat{\kappa} & \hat{\mu} & 0 \\ 0 & 0 & \hat{\mu}_z \end{bmatrix}$
Helmholtz Equation	$\nabla \times \nabla \times \mathbf{E} - 2\omega\mu\xi_c \nabla \times \mathbf{E} - \omega^2\mu\underline{\underline{\epsilon}} \cdot \mathbf{E} = i\omega\mu\mathbf{J}$	$\nabla \times \nabla \times (\underline{\underline{\mu}}^{-1} \cdot \mathbf{B}) - 2\omega\xi_c \nabla \times \mathbf{B} - \omega^2\epsilon\mathbf{B} = \nabla \times \mathbf{J}$
Wavenumbers*		
(Parallel)	$k_{p\pm} = \pm\omega\mu\xi_c + \sqrt{\omega^2\mu^2\xi_c^2 + \omega^2\mu(\hat{\epsilon} \pm \hat{g})}$	$k_{p\pm} = \pm\omega(\hat{\mu} \pm \hat{\kappa})\xi_c + \sqrt{\omega^2(\hat{\mu} \pm \hat{\kappa})^2\xi_c^2 + \omega^2(\hat{\mu} \pm \hat{\kappa})\epsilon}$
(Antiparallel)	$k_{a\pm} = \mp\omega\mu\xi_c + \sqrt{\omega^2\mu^2\xi_c^2 + \omega^2\mu(\hat{\epsilon} \mp \hat{g})}$	$k_{a\pm} = \mp\omega(\hat{\mu} \mp \hat{\kappa})\xi_c - \sqrt{\omega^2(\hat{\mu} \mp \hat{\kappa})^2\xi_c^2 + \omega^2(\hat{\mu} \mp \hat{\kappa})\epsilon}$
Impedances*		
(Positive Helicity)	$\eta_1 = \dfrac{1}{\sqrt{\xi_c^2 + \dfrac{(\hat{\epsilon} + \hat{g})}{\mu}}}$	$\eta_1 = \dfrac{1}{\sqrt{\xi_c^2 + \dfrac{\epsilon}{(\hat{\mu} + \hat{\kappa})}}}$
(Negative Helicity)	$\eta_2 = \dfrac{1}{\sqrt{\xi_c^2 + \dfrac{(\hat{\epsilon} - \hat{g})}{\mu}}}$	$\eta_2 = \dfrac{1}{\sqrt{\xi_c^2 + \dfrac{\epsilon}{(\hat{\mu} - \hat{\kappa})}}}$

*These are for propagation along the biasing magnetic field.

is the modified dielectric tensor of a magnetically biased plasma, which takes into account contributions due to chirality. The biasing magnetostatic field $\mathbf{B}_0$ is chosen to be along the positive z-axis. Here μ is the scalar permeability which includes effects of chirality. The quantities $\hat{\epsilon}$, $\hat{\epsilon}_z$, and $\hat{g}$ are modified forms of those of a collisionless biased plasma and are given by [10]

$$\hat{\epsilon} = \hat{\epsilon}_\infty \left(1 - \frac{\omega_p^2}{\omega^2 - \omega_g^2}\right) \tag{4}$$

$$\hat{\epsilon}_z = \hat{\epsilon}_\infty \left(1 - \frac{\omega_p^2}{\omega^2}\right) \tag{5}$$

$$\hat{g} = \hat{\epsilon}_\infty \frac{\omega_p^2 \omega_g}{\omega(\omega^2 - \omega_g^2)}. \tag{6}$$

Here $\hat{\epsilon}_\infty$ is the high frequency permittivity of the medium when the chirality admittance ξ_c goes to zero, taking into account all contributions to the permittivity due to chirality. Furthermore ω is the wave frequency, ω_p is the plasma frequency, and ω_g ($\sim \mathbf{B}_0$) is the electron gyrofrequency.

We note that it is possible to have a medium where permittivity is anisotropic and chirality and permeability are isotropic. Likewise, it is possible to have a medium with an anisotropic permeability and isotropic chirality and permittivity. We turn now to the second material which we call chiroferrite.

A chiroferrite is described by a set of tensor constitutive relations:

$$\mathbf{D} = \epsilon \mathbf{E} + i\xi_c \mathbf{B} \tag{7}$$

$$\mathbf{H} = i\xi_c \mathbf{E} + \underline{\underline{\mu}}^{-1} \cdot \mathbf{B} \tag{8}$$

where

$$\underline{\underline{\mu}} = \begin{bmatrix} \hat{\mu} & -i\hat{\kappa} & 0 \\ i\hat{\kappa} & \hat{\mu} & 0 \\ 0 & 0 & \hat{\mu}_z \end{bmatrix} \tag{9}$$

is the permeability tensor. Relationships exist for $\hat{\mu}$, $\hat{\mu}_z$, and $\hat{\kappa}$ which are similar to those for $\hat{\epsilon}$, $\hat{\epsilon}_z$, and $\hat{g}$. However, these are not of interest here, since in the remainder of the paper, we will discuss almost exclusively the chiroplasma case. Nevertheless, several key results comparing the two cases can be found in Table I. These results will be derived in subsequent parts of this paper.

From the chiroplasma constitutive relations, (1) and (2), and the time-harmonic Maxwell equations with $e^{-i\omega t}$ excitation, the Helmholtz equation with current excitation $\mathbf{J}$ is found to be [11]

$$\nabla \times \nabla \times \mathbf{E} - 2\omega\mu\xi_c \nabla \times \mathbf{E} - \omega^2\mu\boldsymbol{\epsilon} \cdot \mathbf{E} = i\omega\mu\mathbf{J}. \tag{10}$$

It should be noted that if the anisotropy were extended to μ or ξ_c, the Helmholtz equation would no longer be of the same form as (10). This is evident for the chiroferrite case, where one must solve for $\mathbf{B}$ instead of $\mathbf{E}$ in order to obtain a useful relation (see Table I).

Plane wave propagation in the chiroplasma can be examined by setting $\mathbf{J}$ equal to zero and assuming waves of the form $\mathbf{E}_0 e^{i(\mathbf{k}\cdot\mathbf{r} - \omega t)}$, where $\mathbf{k}$ is the wave vector. Under these conditions, the electric field must satisfy

$$\underline{\underline{\mathbf{W}}} \cdot \mathbf{E} = 0 \tag{11}$$

with

$$\underline{\underline{\mathbf{W}}} = \begin{bmatrix} k^2 - \omega^2\mu\hat{\epsilon} - k_x^2 & -k_x k_y + 2i\omega\mu\xi_c k_z + i\omega^2\mu\hat{g} & -k_x k_z - 2i\omega\mu\xi_c k_y \\ -k_x k_y - 2i\omega\mu\xi_c k_z - i\omega^2\mu\hat{g} & k^2 - \omega^2\mu\hat{\epsilon} - k_y^2 & -k_y k_z + 2i\omega\mu\xi_c k_x \\ -k_x k_z + 2i\omega\mu\xi_c k_y & -k_y k_z - 2i\omega\mu\xi_c k_x & k^2 - \omega^2\mu\hat{\epsilon}_z - k_z^2 \end{bmatrix}. \tag{12}$$

Here, k_x, k_y, and k_z represent the three Cartesian components of the wave vector $\mathbf{k}$. Equation (11) only has nontrivial solutions if the determinant of the wave matrix $\underline{\underline{\mathbf{W}}}$ is equal to zero. In general, the polynomial expression obtained for k is cumbersome to solve. However, certain special cases, which provide much insight into the physical properties of the medium, can be solved. The two cases which will be examined here are: 1) propagation along the biasing magnetic field and 2) propagation at the plasma frequency.

III. Propagation Along the Biasing Magnetic Field of a Chiroplasma

If waves are confined to propagate along the z-axis, it is possible to solve $\det \underline{\underline{\mathbf{W}}} = 0$ in a straightforward manner in order to find the wavenumbers supported by the medium. Thus, the following wavenumbers are found when k_x and k_y are set equal to zero:

$$\begin{aligned} k_{p\pm} &= \pm\omega\mu\xi_c + \sqrt{\omega^2\mu^2\xi_c^2 + \omega^2\mu(\hat{\epsilon} \pm \hat{g})} \\ &= \pm\omega\mu\xi_c + \sqrt{\omega^2\mu^2\xi_c^2 + \omega^2\mu\hat{\epsilon}_\infty\left(1 - \frac{\omega_p^2}{\omega(\omega \pm \omega_g)}\right)} \end{aligned} \tag{13}$$

$$\begin{aligned} k_{a\mp} &= \mp\omega\mu\xi_c - \sqrt{\omega^2\mu^2\xi_c^2 + \omega^2\mu(\hat{\epsilon} \mp \hat{g})} \\ &= \mp\omega\mu\xi_c - \sqrt{\omega^2\mu^2\xi_c^2 + \omega^2\mu\hat{\epsilon}_\infty\left(1 - \frac{\omega_p^2}{\omega(\omega \mp \omega_g)}\right)}. \end{aligned} \tag{14}$$

The subscripts p and a refer to the parallel and antiparallel directions of energy propagation, that is the direction of the real part of the Poynting's vector, with respect to the static magnetic field, while the plus and minus subscripts denote right-circular polarized (RCP) and left-circular polarized (LCP) forward propagating waves,[3] respectively. Fig. 1 depicts these behaviors of the wavenumbers as a function of $\hat{g}$. To understand these results, note that when the biasing magnetostatic field is not present the wavenumbers become equivalent to those of a simple chiral medium [11]:

$$\begin{aligned} k_{p+}(\hat{g} = 0) &= -k_{a+}(\hat{g} = 0) = k_+ \\ &= \omega\mu\xi_c + \sqrt{\omega^2\mu^2\xi_c^2 + k_e^2} \end{aligned} \tag{15}$$

$$\begin{aligned} k_{p-}(\hat{g} = 0) &= -k_{a-}(\hat{g} = 0) = k_- \\ &= -\omega\mu\xi_c + \sqrt{\omega^2\mu^2\xi_c^2 + k_e^2} \end{aligned} \tag{16}$$

where $k_e = \omega\sqrt{\mu\hat{\epsilon}}$ is the wavenumber in the absence of biasing.

The helicity and polarization state corresponding to each of the wavenumbers can be found by substituting (13) and (14) into (11). The results obtained by this substitution are summarized in Table II. Note that positive (negative) helicity is defined as right (left) handed with respect to the positive z-axis and left (right) handed with respect to the negative z-axis. Therefore, k_{p+} and k_{a-} are of positive helicity while k_{p-} and k_{a+} are of negative helicity. It should be noted that for this special case of propagation along $\mathbf{B}_0$, the propagating field vectors $\mathbf{E}$, $\mathbf{D}$, $\mathbf{B}$ and $\mathbf{H}$ are all perpendicular to $\mathbf{k}$. Also, $\mathbf{E}$ is parallel to $\mathbf{D}$ and $\mathbf{B}$ is parallel to $\mathbf{H}$. This is not true for propagation in an arbitrary direction, as will be evident in the next section.

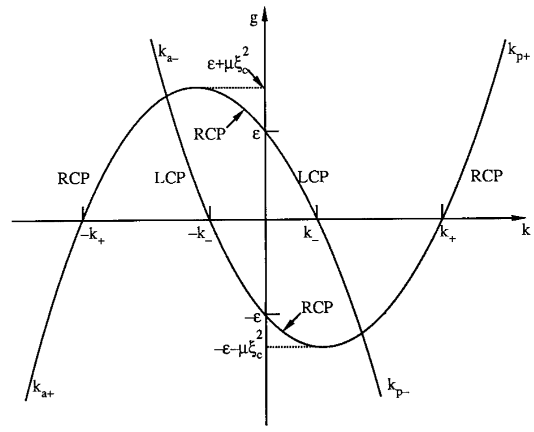


Fig. 1. A graph of the $\hat{g}$ dependence of the four wavenumbers present along the magnetostatic field of a Faraday chiral medium. Note that the transition from k_{p+} to k_{a-} (k_{p-} to k_{a+}) occurs at the minimum (maximum) of the appropriate parabola.

TABLE II
The Helicities, Polarization States, and Signs of the Four Wavenumbers in Three Regions of Interest are Shown; the Characters in Italics Represent Regimes of Backward Wave Propagation

	$\hat{g} < -\hat{\epsilon}$			$-\hat{\epsilon} < \hat{g} < \hat{\epsilon}$			$\hat{g} > \hat{\epsilon}$		
	HEL	SIGN	POL	HEL	SIGN	POL	HEL	SIGN	POL
k_{p+}	POS	+	RCP	POS	+	RCP	POS	+	RCP
k_{p-}	NEG	+	LCP	NEG	+	LCP	*NEG*	−	*RCP*
k_{a+}	NEG	−	RCP	NEG	−	RCP	NEG	−	RCP
k_{a-}	*POS*	+	*RCP*	POS	−	LCP	POS	−	LCP

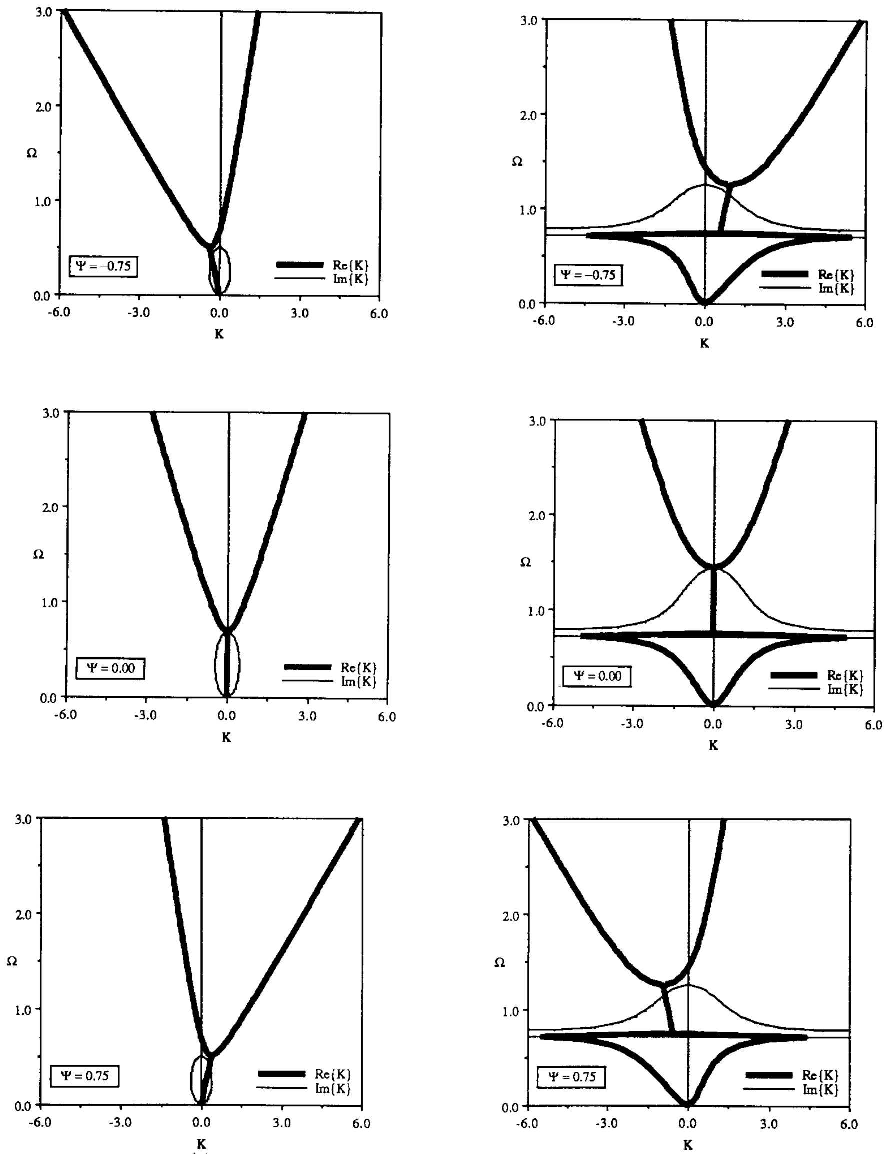
The dispersive properties of the medium are illustrated in Figs. 2 and 3. All variables are made dimensionless through the following substitutions: $\Omega = \omega/\omega_p$, $\Omega_g = \omega_g/\omega_p$, $K = k_{\alpha\beta}/\omega_p\sqrt{\mu\hat{\epsilon}_\infty}$ ($\alpha = a, p$ and $\beta = +, -$) and $\Psi = \mu\xi_c^2/\hat{\epsilon}_\infty$. Furthermore, phase velocities are shown in terms of Ω/K, but group velocities, as defined by $\partial\Omega/\partial K$, are omitted because they lack physical significance.[4] We introduce the four quantities ω_{c1}, ω_{n1}, ω_{c2} and ω_{n2}.

$$\omega_{c1} = -\frac{\omega_g}{2} + \sqrt{\left(\frac{\omega_g}{2}\right)^2 + \frac{\omega_p^2}{\left(\frac{\mu\xi_c^2}{\hat{\epsilon}_\infty} + 1\right)}} \tag{17}$$

$$\omega_{n1} = -\frac{\omega_g}{2} + \sqrt{\left(\frac{\omega_g}{2}\right)^2 + \omega_p^2} \tag{18}$$

[3] This notation is not valid for backward wave propagation, since the handedness of the wave changes when k_{p-} or k_{a-} change sign.

[4] The physical meaning usually associated with group velocity is less applicable to this medium since the energy and group velocities can be in opposite directions. Similar phenomena can occur in achiral biased plasmas [10].



(a) (b)

Fig. 2. Brillouin diagrams of waves of (a) positive and (b) negative helicities for several values of ξ_c. The right-hand branch of each graph corresponds to parallel waves while the left hand branch corresponds to antiparallel waves. At points where the two branches join, the same wavenumbers occur for both types of waves.

$$\omega_{c2} = \frac{\omega_g}{2} + \sqrt{\left(\frac{\omega_g}{2}\right)^2 + \frac{\omega_p^2}{\left(\frac{\mu \xi_c^2}{\hat{\epsilon}_\infty} + 1\right)}} \tag{19}$$

$$\omega_{n2} = \frac{\omega_g}{2} + \sqrt{\left(\frac{\omega_g}{2}\right)^2 + \omega_p^2} \tag{20}$$

where the subscripts n and c refer to null and complex. For positive values of chiral admittance,[5] different kinds of wave behavior are seen in the following frequency ranges.

[5] If the chiral admittance is made negative instead of positive, the comments made about parallel and antiparallel waves should be interchanged.

For k_{p+} and k_{a-}:

1) When $0 \leq \omega < \omega_{c1}$, the waves of positive helicity associated with k_{p+} and k_{a-} decay as they propagate since both wavenumbers are complex. However, the phases of each advance in the same direction and with equal velocities.
2) When $\omega_{cl} \leq \omega < \omega_{zl}$, k_{p+} and k_{a-} are purely real and positive, thus signifying that here the phases also advance in the same direction. Furthermore, as will be shown later, the antiparallel LCP wave with wavenumber k_{a-} is a backward wave because the energy velocity is opposite in direction to the phase velocity.

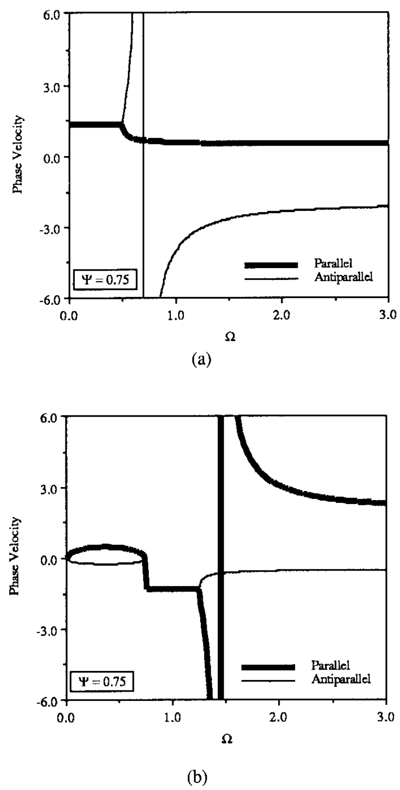


Fig. 3. Phase velocities as a function of frequency for waves of (a) positive and (b) negative helicities.

3) When $\omega \geq \omega_{zl}$, the usual plane wave propagation occurs.

For k_{p-} and k_{a+}:

4) When $0 \leq \omega < \omega_g$, the usual plane wave propagation occurs.
5) When $\omega = \omega_g$, waves of negative helicity are resonant with the gyrofrequency of the electrons.
6) When $\omega_g < \omega < \omega_{c2}$, the waves of negative helicity behave much like the waves of positive helicity in 1).
7) When $\omega_{c2} \leq \omega < \omega_{z2}$, the parallel LCP wave with wavenumber k_{p-} is a backward wave similar to that of 2).
8) When $\omega \geq \omega_{z2}$, the usual plane wave propagation occurs.

In general, waves of negative helicity display more interesting behavior than those of positive helicity because the current associated with the static magnetic field is determined by the right-hand rule. Therefore, the electrons in the plasma gyrate in a counterclockwise direction with respect to the positive z-axis and can interact resonantly with the waves of negative helicity. The roles of the two helicities can be reversed by placing $\mathbf{B}_0$ in the negative z direction.

Since each of the four propagating modes has a distinct wavenumber, the polarization ellipses experience unequal rotations per unit length in the parallel and antiparallel directions, as given by[6]

$$\phi_p/z = \left(\frac{k_{p+} - k_{p-}}{2}\right) \tag{21}$$

$$\phi_a/z = \left(\frac{k_{a+} - k_{a-}}{2}\right) \tag{22}$$

These rotations are counterclockwise when viewed in the direction of wave propagation. The frequency dependence of both ϕ_p/z and ϕ_a/z is plotted in Fig. 4. In general, it is possible for either ϕ_p/z or ϕ_a/z to equal identically zero. When this occurs, the nonzero rotation becomes equal to twice that of the simple chiral case [11]:

$$\phi_p/z(\text{when } \phi_a/z = 0) = 2\omega\mu\xi_c \tag{23}$$

$$\phi_a/z(\text{when } \phi_p/z = 0) = -2\omega\mu\xi_c. \tag{24}$$

Hence, the effective chiral length of the medium can be increased by reflection. This is not possible in the case of an ordinary chiral medium since the rotation is undone after reflection.

As mentioned earlier, at certain frequencies, the parallel and antiparallel LCP waves can be backward propagating with opposite directions of phase and energy velocities. The energy velocities' directions are the same as those of the Poynting's vectors which are found directly from the Maxwell equations and the constitutive relations:

$$\mathbf{S}_{p+} = \hat{z}\,|\mathbf{E}_0|^2/2\eta_1 \tag{25}$$

$$\mathbf{S}_{p-} = \hat{z}\,|\mathbf{E}_0|^2/2\eta_2 \tag{26}$$

$$\mathbf{S}_{a+} = -\hat{z}\,|\mathbf{E}_0|^2/2\eta_2 \tag{27}$$

$$\mathbf{S}_{a-} = -\hat{z}\,|\mathbf{E}_0|^2/2\eta_1 \tag{28}$$

where the carat denotes the unit vector and the wave impedances of the positive and negative helicities, η_1 and η_2, respectively, are found to be

$$\eta_1 = \frac{1}{\sqrt{\xi_c^2 + \dfrac{(\hat{\epsilon} + \hat{g})}{\mu}}} = \frac{1}{\sqrt{\xi_c^2 + \dfrac{\hat{\epsilon}_\infty}{\mu}\left(1 - \dfrac{\omega_p^2}{\omega(\omega + \omega_g)}\right)}} \tag{29}$$

$$\eta_2 = \frac{1}{\sqrt{\xi_c^2 + \dfrac{(\hat{\epsilon} - \hat{g})}{\mu}}} = \frac{1}{\sqrt{\xi_c^2 + \dfrac{\hat{\epsilon}_\infty}{\mu}\left(1 - \dfrac{\omega_p^2}{\omega(\omega - \omega_g)}\right)}}. \tag{30}$$

Therefore, we see that the directions of the Poynting's vectors are frequency independent whereas the phase velocities can change sign (see Fig. 3) as a function of frequency so as to be opposite to the direction of power flow. This type of backward wave propagation is also observed in backward wave oscillators (BWO) [12], [13].

[6] These same relationships hold for the polarization rotation in a chiroferrite.

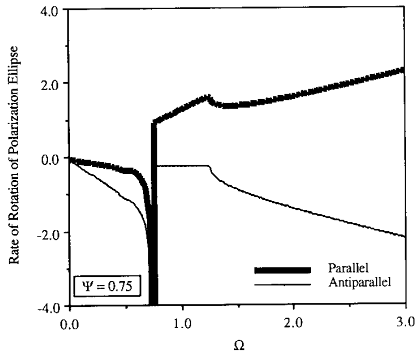


Fig. 4. The normalized rotation per unit length of the polarization ellipses in the parallel and antiparallel directions. The normalization is provided by dividing ϕ_p/z and ϕ_a/z by $\omega_p\sqrt{\mu\hat{\epsilon}_\infty}$.

IV. Wave Propagation at the Plasma Frequency of a Chiroplasma

When the wave frequency is equal to the plasma frequency, the dispersion relation obtained from (11) has two solutions off the z-axis:

$$k_{1(\omega=\omega_p)} = 0 \tag{31}$$

$$k_{2(\omega=\omega_p)} = \sqrt{4\omega_p^2\mu^2\xi_c^2 + \omega_p^2\mu\frac{(\hat{\epsilon}^2 - \hat{g}^2)}{\hat{\epsilon}}} = \sqrt{4\omega_p^2\mu^2\xi_c^2 + \omega_p^2\mu\hat{\epsilon}_\infty}. \tag{32}$$

These wavenumbers are valid for all directions of propagation except for propagation along the z-axis. In this case, a limiting approach must be used so as to avoid automatically setting the determinant of the wave matrix equal to zero. Following such an approach, we again find wavenumbers k_{p+}, k_{p-}, k_{a+} and k_{a-} with $\omega = \omega_p$.

At this point, it is worthwhile to examine the wave matrix evaluated at $k_{1(\omega=\omega_p)}$ and $k_{2(\omega=\omega_p)}$. For simplicity, the y-z plane is defined to be the plane containing the static magnetic field and the wave vector **k**, thus making k_x equal to zero. Furthermore, the angle θ is defined as the clockwise angle between the z-axis and **k**, as shown in Fig. 5. Substituting $k_{1(\omega=\omega_p)}$ into (11) we obtain

$$\begin{bmatrix} -\omega_p^2\mu\hat{\epsilon} & i\omega_p^2\mu\hat{g} & 0 \\ -i\omega_p^2\mu\hat{g} & -\omega_p^2\mu\hat{\epsilon} & 0 \\ 0 & 0 & 0 \end{bmatrix}\begin{bmatrix} E_x \\ E_y \\ E_z \end{bmatrix} = 0 \tag{33}$$

and similarly for $k_{2(\omega=\omega_p)}$

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{12}^* & a_{22} & a_{23} \\ a_{13}^* & a_{23}^* & a_{33} \end{bmatrix}\begin{bmatrix} E_x \\ E_y \\ E_z \end{bmatrix} = 0 \tag{34}$$

with the asterisk representing the complex conjugate and

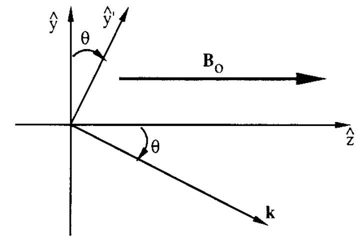


Fig. 5. A sketch of the coordinate system used in determining the modes supported by the medium at the plasma frequency.

where

$$\begin{aligned} a_{11} &= 4\omega_p^2\mu^2\xi_c^2 - \omega_p^2\mu\hat{g}^2/\hat{\epsilon} \\ a_{12} &= 2i\omega_p\mu\xi_c\sqrt{4\omega_p^2\mu^2\xi_c^2 + \omega_p^2\mu\hat{\epsilon}_\infty}\cos\theta + i\omega_p^2\mu\hat{g} \\ a_{13} &= 2i\omega_p\mu\xi_c\sqrt{4\omega_p^2\mu^2\xi_c^2 + \omega_p^2\mu\hat{\epsilon}_\infty}\sin\theta \\ a_{22} &= \left(4\omega_p^2\mu^2\xi_c^2 + \omega_p^2\mu\hat{\epsilon}_\infty\right)\cos^2\theta - \omega_p^2\mu\hat{\epsilon} \\ a_{23} &= \left(4\omega_p^2\mu^2\xi_c^2 + \omega_p^2\mu\hat{\epsilon}_\infty\right)\sin\theta\cos\theta \\ a_{33} &= \left(4\omega_p^2\mu^2\xi_c^2 + \omega_p^2\mu\hat{\epsilon}_\infty\right)\sin^2\theta. \end{aligned}$$

If (33) is to hold in general, either $\hat{\epsilon} = \hat{g}$ or $E_x = E_y = 0$. The first case can only be true if the biasing magnetic field is not present such that $\hat{\epsilon} = \hat{g} = 0$, which corresponds to the cut-off condition in an isotropic plasma. The second case represents a quasi-static field since it is possible for an electric field with zero wavenumber $k_{1(\omega=\omega_p)}$ to exist in the z direction. The rest of field vectors in the quasi-static field are found directly from the Maxwell equations and the constitutive relations:

$$\mathbf{B} = \frac{\mathbf{k}_{1(\omega=\omega_p)} \times \mathbf{E}}{\omega_p} = 0 \tag{35}$$

$$\mathbf{D} = -\frac{\mathbf{k}_{1(\omega=\omega_p)} \times \mathbf{H}}{\omega_p} = 0 \tag{36}$$

$$H_x = H_y = 0 \tag{37}$$

$$H_z = i\xi_c E_z. \tag{38}$$

From (38), we observe that the chirality forces an **H** field in phase quadrature with **E**. We also note that, when the medium is achiral, the quasi-static field is purely electric.

On the other hand, the solution of (34) yields a wave with elliptical polarization in the plane transverse to the direction of propagation. This wave is peculiar because the wavenumber is direction-independent whereas the polarization in direction-dependent. Note that when the plasma becomes isotropic and chirality is removed, both (33) and (34) can be satisfied. However, from physical considerations, the only wavenumber that can exist must be $k_{1(\omega=\omega_p)}$.

V. Conclusion

We have examined the problem of electromagnetic wave propagation in two types of Faraday chiral media, chiroplasmas and chiroferrites, with the goal of examining potential methods for chirality control. We stress the properties of the chiroplasmas since those of the chiroferrites are quite similar in nature (see Table I). It has been found that, for propagation along the magnetostatic field, four circularly polarized eigenmodes, possessing differing wavenumbers, are present. Two of these correspond to wave propagation parallel to the biasing magnetic field and two others correspond to propagation antiparallel to it. All these wavenumbers can be altered by varying the strength of the biasing magnetic field. Furthermore, their frequency dependence leads to a set of dispersion diagrams which provide insight into wave propagation in this complex material. We have also examined the behavior of the chiroplasmas when the frequency of the propagating wave is equal to the plasma frequency. In this case, two direction-independent wavenumbers were found to exist. The blending of chirality with anisotropy, that we have presented here, may be the first step in understanding chirality control. Such control may have potential applications in chirowaveguides [14], in controlling the radar cross section of coated targets [15], and the control of radiation and polarization properties of antennas and arrays in chiral media [16], [17]. For instance, if chiroferrite material is used as a substrate or superstrate of printed-circuit antennas, the radiation properties of such radiators can be controlled by varying the biasing magnetic field. Of particular interest, is the polarization diversity afforded by chirality of these substrates. Likewise for guided-wave structures, we anticipate control of mode configuration and coupling within such guides. Finally, for microwave and millimeter-wave coatings, one can control the reflection and polarization properties of chiroferrite coatings using concepts developed here. Work in these areas is in progress.

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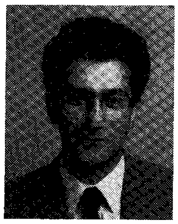

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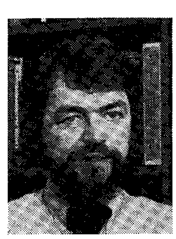

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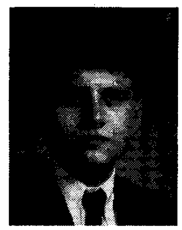

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