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Abstract
This paper introduces a utility function that nests three classes of utility functions: (1) time-separable utility functions; (2) "catching up with the Joneses" utility functions that depend on the consumer's level of consumption relative to the lagged cross-sectional average level of consumption; and (3) utility functions that display habit formation. Closed-form solutions for equilibrium asset prices are derived under the assumption that consumption growth is i.i.d. The equity premia under catching up with the Joneses and under habit formation are, for some parameter values, as large as the historically observed equity premium in the United States.

Disciplines
Economics | Finance | Finance and Financial Management

Asset Prices under Habit Formation and Catching up with the Joneses

By ANDREW B. ABEL*

This paper introduces a utility function that nests three classes of utility functions: 1) time-separable utility functions; 2) "catching up with the Joneses" utility functions that depend on the consumer's level of consumption relative to the lagged cross-sectional average level of consumption; and 3) utility functions that display habit formation. Incorporating this utility function into a Lucas (1978) asset pricing model allows calculation of closed-form solutions for the prices of stocks, bills and consols under the assumption that consumption growth is i.i.d. Then equilibrium asset prices are used to examine the equity premium puzzle.

I. The Utility Function

At time t, each consumer chooses the level of consumption, c_t, to maximize $E_t(U_t)$ where $E_t\{\ \}$ is the conditional expectation operator at time t and the utility function is given by

$$(1) \qquad U_t \equiv \sum_{j=0}^{\infty} \beta^j u(c_{t+j}, \nu_{t+j})$$

where ν_{t+j} is a preference parameter. Suppose that the preference parameter ν_t is specified as

$$(2) \quad \nu_t \equiv \left[c_{t-1}^{D} C_{t-1}^{1-D}\right]^{\gamma} \quad \gamma \geq 0 \quad \text{and} \quad D \geq 0$$

where c_{t-1} is the consumer's own consumption in period $t-1$ and C_{t-1} is aggregate consumption per capita in period $t-1$. If $\gamma = 0$, then $\nu_t \equiv 1$ and the utility function in (1) is time separable. If $\gamma > 0$ and $D = 0$, the parameter ν_t depends only on the lagged level of aggregate consumption per capita. This formation is the relative consumption model or "catching up with the Joneses."[1] Finally, if $\gamma > 0$ and $D = 1$, the parameter ν_t depends only on the consumer's own past consumption. This formulation is the habit formation model.

Consider the effects on utility of a change in an individual's consumption at date t, holding aggregate consumption unchanged. Substituting (2) into (1) and then differentiating with respect to c_t yields

$$(3) \quad \partial U_t / \partial c_t = u_c(c_t, \nu_t) + \beta u_\nu(c_{t+1}, \nu_{t+1})\, \gamma D \nu_{t+1} / c_t.$$

Suppose that the period utility function $u(c_t, \nu_t)$ has the following isoelastic form

$$(4) \quad u(c_t, \nu_t) = [c_t/\nu_t]^{1-\alpha}/(1-\alpha), \quad \alpha > 0.$$

When $\gamma = 0$, the utility function in (4) is the standard constant relative risk-aversion utility function and α is the coefficient of relative risk aversion. More generally, utility depends on the level of consumption relative

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[1] The phrase "catching up with the Joneses," rather than "keeping up with the Joneses," reflects the assumption that consumers care about the *lagged* value of aggregate consumption. The April 1989 version (p. 10) of Jordi Gali (1989), but not the September revision, examines the utility function $u(c_t, C_t) = [1/(2-\beta-\gamma)]c_t^{1-\beta}(c_t/C_t)^{-(1-\gamma)}$ and shows that when $\beta = 1$, asset pricing will be equivalent to an economy without consumption externalities and with log utility.

to some endogenous time-varying benchmark ν_t.[2] Under the isoelastic utility function in (4), the expression for $\partial U_t/\partial c_t$ in (3) becomes

$$(5)\quad \partial U_t/\partial c_t = \left[1-\beta\gamma D(c_{t+1}/c_t)^{1-\alpha}(\nu_t/\nu_{t+1})^{1-\alpha}\right] \times (c_t/\nu_t)^{1-\alpha}(1/c_t).$$

II. Equilibrium

Let y_t be the amount of the perishable consumption good per capita produced by the capital stock. In equilibrium, all output is consumed in the period in which it is produced, as in Lucas. Because all consumers are identical, $c_t = C_t = y_t$ in every period. Now let $x_{t+1} \equiv y_{t+1}/y_t$ be the gross growth rate of output. Because $c_t = C_t = y_t$, it follows that $c_{t+1}/c_t = C_{t+1}/C_t = x_{t+1}$. Therefore, equation (2) implies that $\nu_{t+1}/\nu_t = x_t^{\gamma}$ which allows us to rewrite (5) as

$$(6)\qquad \partial U_t/\partial c_t = H_{t+1}\nu_t^{\alpha-1}c_t^{-\alpha}$$

where $H_{t+1} \equiv 1-\beta\gamma D x_{t+1}^{1-\alpha}x_t^{-\gamma(1-\alpha)}$.

Note that $H_{t+1} \equiv 1$ if $\gamma D = 0$, which is the case for both time-separable and relative consumption preferences.[3]

III. Asset Pricing

To calculate asset prices, let us examine a consumer who considers purchasing an asset in period t and then selling it in period $t+1$. If asset prices are in equilibrium, this pair of transactions does not affect expected discounted utility. Suppose that a consumer reduces c_t by 1 unit, purchases an asset with a gross rate of return R_{t+1}, sells the asset in period $t+1$, and increases c_{t+1} by R_{t+1} units. The equilibrium rate of return R_{t+1} must satisfy

$$(7)\quad E_t\{-(\partial U_t/\partial c_t) + \beta R_{t+1}(\partial U_{t+1}/\partial c_{t+1})\} = 0.$$

Equation (7) can be rewritten as

$$(8)\quad E_t\{\beta R_{t+1}(\partial U_{t+1}/\partial c_{t+1}) / E_t\{\partial U_t/\partial c_t\}\} = 1.$$

Equation (8) is the familiar result that the conditional expectation of the product of the intertemporal marginal rate of substitution and the gross rate of return equals one.[4] We can obtain an expression for $(\partial U_{t+1}/\partial c_{t+1})/E_t\{\partial U_t/\partial c_t\}$ using equation (6) to divide $\partial U_{t+1}/\partial c_{t+1}$ by $E_t\{\partial U_t/\partial c_t\}$ to obtain

$$(9)\quad (\partial U_{t+1}/\partial c_{t+1})/E_t\{\partial U_t/\partial c_t\} = [H_{t+2}/E_t\{H_{t+1}\}]\,x_t^{\gamma(\alpha-1)}x_{t+1}^{-\alpha}.$$

IV. The Price of Risky Capital

Let p_t^S be the exdividend price of a share of stock in period t, which is a claim to a unit of risky capital. The rate of return on stock is $R_{t+1}^S \equiv (p_{t+1}^S + y_{t+1})/p_t^S$. Let $w_t \equiv p_t^S/y_t$ be the price-dividend ratio. Therefore, $p_t^S = w_t y_t$ and $p_{t+1}^S = w_{t+1}y_{t+1}$ so that

$$(10)\qquad R_{t+1}^S = (1+w_{t+1})x_{t+1}/w_t.$$

Substituting (10) into (8) yields

$$(11)\quad w_t = \beta E_t\{(1+w_{t+1})x_{t+1} \times (\partial U_{t+1}/\partial c_{t+1})/E_t\{\partial U_t/\partial c_t\}\}.$$

[2] George Constantinides (1988), Jerome Detemple (1989), John Heaton (1989), and Suresh Sundaresan (1989) also examine asset prices in the presence of habit formation. James Nason (1988) includes a time-varying benchmark level of consumption that differs from habit formation in that it is independent of an individual consumer's own consumption.

[3] A sufficient condition for $\partial U_t/\partial c_t > 0$ when $\gamma = D = 1$ (habit formation) is $1 + \ln\beta/\ln(\max\{x\}/\min\{x\}) < \alpha < 1 + \ln\beta/\ln(\min\{x\}/\max\{x\})$. For $\beta = 0.99$ and the 2-point distribution in Table 1, the sufficient condition is $0.858 < \alpha < 1.142$.

[4] In the conventional time-separable formulation of this problem, $\partial U_t/\partial c_t$ is known as of time t, and hence $E_t\{\partial U_t/\partial c_t\}$ on the left-hand side of (8) equals $\partial U_t/\partial c_t$.

V. Bills and Consols

A one-period riskless bill can be purchased in period t at a price of s_t; in period $t+1$, the bill is worth 1 unit of consumption. The gross rate of return on the bill is $R^B_{t+1} \equiv 1/s_t$. Substituting $1/s_t$ for the rate of return in (8) yields

$$(12) \quad s_t = \beta E_t\{(\partial U_{t+1}/\partial c_{t+1}) / E_t\{\partial U_t/\partial c_t\}\}.$$

A consol bond, that pays one unit of consumption in each period, can be purchased at an excoupon price p^C_t in period t. In period $t+1$, the consol pays a coupon worth one unit of consumption and then sells at a price of p^C_{t+1}. The one-period rate of return on the consol is $R^C_{t+1} \equiv (1+p^C_{t+1})/p^C_t$. Substituting R^C_{t+1} into (8) yields

$$(13) \quad p^C_t = \beta E_t\{(1+p^C_{t+1})(\partial U_{t+1}/\partial c_{t+1}) / E_t\{\partial U_t/\partial c_t\}\}.$$

VI. I.I.D. Consumption Growth

Suppose that consumption growth x_{t+1} is i.i.d. over time. In this case, we can obtain explicit solutions for the prices of stock, bills, and consols. The price-dividend ratio w_t is

$$(14) \quad w_t = A x^\theta_t / J_t,$$

where $\theta \equiv \gamma(\alpha - 1)$

$$A \equiv \beta E\{x^{1-\alpha}\}\left[1 - \beta\gamma D E\{x^{(1-\alpha)(1-\gamma)}\}\right] / \left[1 - \beta E\{x^{(1-\alpha)(1-\gamma)}\}\right]$$

$$J_t \equiv E_t\{H_{t+1}\} \equiv 1 - \beta\gamma D E\{x^{1-\alpha}\} x^\theta_t.$$

The price of a one-period riskless bill is

$$(15) \quad s_t = q\beta x^\theta_t / J_t,$$

where

$$q \equiv E\{x^{-\alpha}\} - \beta\gamma D E\{x^{1-\alpha}\} E\{x^{\theta-\alpha}\}$$

and the price of a consol is

$$(16) \quad p^C_t = Q x^\theta_t / J_t,$$

where $Q \equiv \beta q / \left[1 - \beta E\{x^{\theta-\alpha}\}\right]$.

Given a distribution for x, the moments of x can be calculated and the three asset prices are easily calculated. For time-separable preferences ($\gamma = 0$) and relative consumption ($\gamma > 0$; $D = 0$), we can obtain closed-form solutions (in terms of preference parameters and the moments of x) for the unconditional expected returns $E\{R^S\}$, $E\{R^B\}$ and $E\{R^C\}$:

$$(17) \quad E\{R^S\} = E\{x^{-\theta}\} \times \left[E\{x\} + A\, E\{x^{1+\theta}\}\right] / A$$

$$(18) \quad E\{R^B\} = E\{x^{-\theta}\} / \beta q$$

$$(19) \quad E\{R^C\} = E\{x^{-\theta}\}\left[1 + Q E\{x^\theta\}\right] / Q.$$

Under habit formation, unconditional expected returns can be calculated numerically using the asset prices in (14)–(16).

VII. The Equity Premium

Rajnish Mehra and Edward Prescott (1985) report that from 1889 to 1978 in the United States, the average annual real rate of return on short-term bills was 0.80 percent and the average annual real rate of return on stocks was 6.98 percent. Thus the average equity premium was 618 basis points. They calibrated an asset pricing model with time-separable isoelastic utility to see whether the model could deliver unconditional rates of return close to the historical average rates of return on stocks and bills. They used a 2-point Markov process for consumption growth with $E\{x_t\} = 1.018$, $\mathrm{Var}\{x_t\} = (0.036)^2$, and correlation $(x_t, x_{t-1}) = -0.14$. For values of the preference parameters that Mehra and Prescott deemed reasonable, the model could not produce more than a 35 basis point equity premium ($E\{R^S\} - E\{R^B\}$) when the expected riskless rate, $E\{R^B\}$, was less than or equal to 4

TABLE 1—UNCONDITIONAL EXPECTED RETURNS
$\beta = 0.99$; $E\{x\} = 1.018$; $\text{VAR}\{x\} = (0.036)^2$

α	Stocks	Bills	Consols
A. Time-separable preferences ($\gamma = 0$)			
0.5	1.93	1.87	1.87
	[1.93]	[1.87]	[1.87]
1.0	2.83	2.70	2.70
	[2.83]	[2.70]	[2.70]
6.0	10.34	9.52	9.52
	[10.33]	[9.51]	[9.51]
10.0	14.22	12.85	12.85
	[14.13]	[12.72]	[12.72]
B. Relative consumption ($\gamma = 1$; $D = 0$)			
0.5	2.80	2.76	2.73
	[2.80]	[2.76]	[2.73]
1.0	2.83	2.70	2.70
	[2.83]	[2.70]	[2.70]
6.0	6.70	2.07	5.84
	[6.72]	[2.06]	[5.86]
10.0	14.73	1.59	13.16
	[14.95]	[1.55]	[13.32]
C. Habit formation ($\gamma = 1$; $D = 1$)			
0.86	33.56	4.53	35.25
0.94	6.83	3.48	7.44
1.00	2.83	2.70	2.70
1.06	8.43	1.93	7.40
1.14	38.28	0.93	35.16

percent per year. This result is the equity premium puzzle.

Table 1 reports the unconditional expected rates of return on stocks, bills, and consols under the assumption that x_t is i.i.d., $E\{x\} = 1.018$ and $\text{Var}\{x\} = (0.036)^2$. For time-separable and relative consumption preferences, two unconditional expected returns are reported in each cell: the first is calculated under a 2-point i.i.d. distribution; the second, shown in brackets, is calculated under a lognormal distribution for x.

Panel A of Table 1, which reports the unconditional expected rates of return under time-separable preferences, displays the equity premium puzzle. Although $E\{R^S\}$ increases as α increases from 0.5 to 10.0, $E\{R^B\}$ also increases. The equity premium, $E\{R^S\} - E\{R^B\}$, does not come anywhere close to the 600-point historical average. Incidentally, the unconditional expected rates of return of bills and consols are exactly equal under time-separable preferences.

Panel B reports the unconditional expected rates of return in the relative consumption model. For $\alpha = 6$, the equity premium is 463 basis points and the unconditional riskless rate is 2.07 percent per year. Although the unconditional expected returns on stocks and bills are much closer to their historical averages, the conditional expected rates of return (not reported in the table) vary too much. For the 2-point distribution for x, the standard deviation of $E_t\{R^B_{t+1}\}$ is 17.87 percent when $\alpha = 6$. This unrealistic implication of the model poses a challenge for future research.

Panels A and B report unconditional rates of return for a lognormal distribution with $E\{x\} = 1.018$ and $\text{Var}\{x\} = (0.036)^2$. For the parameter values reported, it makes no substantial difference for expected returns whether the growth rate is lognormal or has a 2-point distribution.

Panel C presents the unconditional expected rates of return under habit formation. The expected rates of return on both long-lived assets (stocks and consols) are extremely sensitive to the value of α. Under logarithmic utility ($\alpha = 1$), the expected rates of return are the same as under time-separable preferences and relative consumption. However, with $\alpha = 1.14$, the expected rates of return on stocks and consols are both greater than 35 percent.

Further research using the utility function introduced in this paper will explore the implications of other settings for the parameters γ and D. For instance, if D is between zero and one, the utility function would contain elements of both catching up with the Joneses as well as habit formation. Also the assumption of i.i.d. consumption growth rates can be relaxed, and asset prices can then be analyzed numerically.

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