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Periodic Chiral Structures

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Abstract

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Comments

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Periodic Chiral Structures

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Abstract—**The electromagnetic properties of a structure that is both chiral and periodic are investigated using coupled-mode equations. The chirality is characterized by the constitutive relations $\mathbf{D} = \epsilon\mathbf{E} + i\xi_c\mathbf{B}$ and $\mathbf{H} = i\xi_c\mathbf{E} + \mathbf{B}/\mu$, where ξ_c is the chiral admittance. The periodicity is described by a sinusoidal perturbation of the permittivity, permeability and chiral admittance. The coupled-mode equations are derived from physical considerations. The coupled-mode equations are used to examine bandgap structure and reflected and transmitted fields. Chirality is observed predominantly in transmission while periodicity is present in both reflection and transmission.**

I. Introduction

THE STUDY OF WAVE propagation in periodic structures has attracted attention since the pioneering work of Rayleigh [1] in 1887. In the 1950's, Brillouin [2] examined electromagnetic wave propagation in periodic media using the mathematical theory developed by Floquet. This rigorous theory gives rise to an infinite number of space harmonics which need to be appropriately truncated or approximated by a finite number of terms. Alternatively, the physically motivated coupled-mode approach, first introduced by Pierce [3] for transmission lines, has also been applied to the problem of waves in periodic structures. This approximate but highly accurate approach stresses the concept of wave coupling by periodic perturbations and places in evidence the resonant space harmonics [4]. Both approaches have been used in a variety of fields such as solid-state physics, antenna theory, and more recently, integrated optics.

The fundamental techniques, both exact and approximate, used for waves in simple periodic structures have been extended to more complex periodic geometries. Here, we note applications to almost periodic structures [5], periodic anisotropic materials [6], periodic plasmas [7] and periodic guided-wave structures [8] as representative examples.

The original investigations of optical activity, or the effect of chirality on the polarization of light, also date back to the nineteenth century. Arago [9], Biot [10], Pasteur [11] and Fresnel [12] all examined optical activity in solid and liquid chiral media. A simple chiral medium can be modeled by a collection of uniformly distributed, randomly oriented chiral objects. Objects are defined as *chiral* if they cannot be brought into congruence with their mirror image, or *enantiomorph*, by translation and rotation. An example of a chiral object, the Möbius strip, and its enantiomorph are shown in Fig. 1. Note that these objects exhibit handedness and lack bilateral symmetry.

Recently, there has been revived interest in such media. A set of simple constitutive relations describing their properties has been found based on simple macroscopic models [13]. This has opened the door to new research topics such as simple and distributed sources in chiral media [14]–[16] and wave propagation through chiral-achiral interfaces [17], [18]. Additional work includes fundamental theorems on bianisotropic media [19], [20] and light scattering from bounded chiral structures [21], [22].

In this paper, we blend effects of simple periodicity and simple chirality by considering structures which are both periodic and chiral. The problem of electromagnetic wave propagation in such structures is investigated, and a set of coupled-mode equations is derived. These equations describe the coupling between forward and backward propagating eigenmodes with appropriate polarizations. The bandgap structure and the reflected and transmitted fields are then obtained from the coupled-mode equations. Physical meanings of these couplings are examined and the effects of chirality and periodicity are shown to be discernable in the transmitted and reflected waves, respectively.

II. Problem Formulation

It has been shown that a chiral medium is characterized by the constitutive relations [13]:

$$\mathbf{D} = \epsilon(z)\mathbf{E} + i\xi_c(z)\mathbf{B} \tag{1}$$

$$\mathbf{H} = i\xi_c(z)\mathbf{E} + \mathbf{B}/\mu(z). \tag{2}$$

In these relations, the z dependence of the parameters that contain the periodicity of the medium, is written explicitly. The sign of the chirality admittance $\xi_c(z)$ indicates the handedness of the medium, where a positive (negative) value of $\xi_c(z)$ corresponds to a right-handed (left-handed) structure.[1] For the achiral case, $\xi_c(z)$ is equal to zero. Here, $\epsilon(z)$ is the permittivity and $\mu(z)$ is the permeability.

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[1] For example, in the optical regime for 1-μm wavelengths, TeO_2 exhibits a chirality admittance magnitude of 2.2×10^{-7} S. This results in a rotation of the plane of polarization, characteristic of chiral media, of 30° per mm.

Fig. 1. A sample chiral object, the Möbius strip, and its enantiomorph.

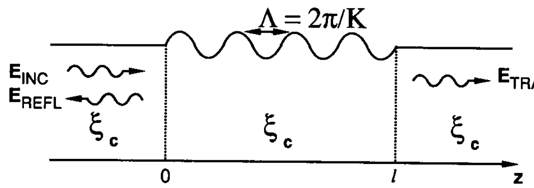

Fig. 2. A chiral medium of infinite transverse extent with a perturbation over a length l. Also shown are the incident, reflected and transmitted fields. Here Λ is the spatial period of the perturbation.

A periodic one-dimensional perturbation, of spatial frequency K, is here assumed to be

$$\begin{pmatrix} \epsilon(z) \\ \mu(z) \\ \xi_c(z) \end{pmatrix} = \begin{pmatrix} \epsilon \\ \mu \\ \xi_c \end{pmatrix} \cdot \begin{cases} \left[1 + \begin{pmatrix} \eta_\epsilon \\ \eta_\mu \\ \eta_c \end{pmatrix} \cos(Kz) \right], & 0 \le z \le l \\ 1 & z < 0 \text{ or } z > l \end{cases} \tag{3}$$

where the η represent the amplitudes of the periodicities and ϵ, μ, and ξ_c are the unperturbed values of $\epsilon(z)$, $\mu(z)$,and $\xi_c(z)$. Taken together, (1)–(3) define the periodic chiral medium under investigation. As shown in Fig. 2, the perturbation extends from $z = 0$ to $z = l$. In subsequent developments, a weak coupling approximation will be used. Therefore, the amplitudes of the perturbations are taken to be much less than unity although this condition may be relaxed somewhat.

From the constitutive relations and the sourceless time-harmonic Maxwell equations with $e^{-i\omega t}$ excitation, the inhomogeneous chiral Helmholtz equation[2] is found to be

$$\nabla \times \nabla \times \mathbf{E} - 2\omega\mu(z)\xi_c(z)\nabla \times \mathbf{E} - \omega^2\mu(z)\epsilon(z)\mathbf{E} - \nabla\mu(z) \times \nabla \times \mathbf{E}/\mu(z) - \omega\xi_c(z)\nabla\mu(z) \times \mathbf{E} = 0 \tag{4}$$

where $\mathbf{E}$ is the electric field vector. The second and fifth terms in (4) produce polarization birefringence, whose effect will become clear later, since a single curl operation distinguishes between two opposite directions of rotation of the electric field.

[2] Equation (4) is obtained from (1), (2) and

$$\nabla \times \mathbf{E} = i\omega\mathbf{B} \qquad \nabla \cdot \mathbf{B} = 0$$
$$\nabla \times \mathbf{H} = -i\omega\mathbf{D} \quad \nabla \cdot \mathbf{D} = 0.$$

The solution of the source driven chiral Helmholtz equation, using either Green's functions [14] or a direct examination of (4), reveals that two eigenmodes exist in chiral media, one right-circularly polarized (RCP) and the other left-circularly polarized (LCP). Separate wavenumbers can be associated with each of these modes and are given by

$$k_+ = \omega\mu\xi_c + \sqrt{\omega^2\mu^2\xi_c^2 + k^2} \tag{5}$$

$$k_- = -\omega\mu\xi_c + \sqrt{\omega^2\mu^2\xi_c^2 + k^2} \tag{6}$$

with k_+ corresponding to the RCP mode and k_- to the LCP mode. Note that for $\xi_c = 0$, $k_+ = k_- = \omega\sqrt{\mu\epsilon} = k$, and that for $\xi_c > 0$, k_+ is greater than k_-. In the latter case, the RCP wave will propagate slower than the LCP wave.

To keep track of the two waves as they traverse the slab, define the circular basis unit vectors

$$\hat{\mathbf{e}}_\pm = \frac{(\hat{\mathbf{e}}_x \pm i\hat{\mathbf{e}}_y)}{\sqrt{2}}. \tag{7}$$

These vectors represent coordinate systems which are rotating with the propagating wave fields. All of the fields can then be expressed in terms of $\hat{\mathbf{e}}_+$ and $\hat{\mathbf{e}}_-$.

In the perturbation region, there is weak coupling between forward and backward propagating waves. The forward propagating waves represent the depleted incident electric field, while the backward propagating waves represent local reflections. Since the medium distinguishes between the $\hat{\mathbf{e}}_+$ and $\hat{\mathbf{e}}_-$ polarizations, we make the ansatz

$$\mathbf{E} = \mathbf{E}_+ + \mathbf{E}_- \tag{8}$$

where

$$\mathbf{E}_+ = [F_+ e^{ik_+ z} + B_- e^{-ik_- z}]\hat{\mathbf{e}}_+ \tag{9}$$

$$\mathbf{E}_- = [F_- e^{ik_- z} + B_+ e^{-ik_+ z}]\hat{\mathbf{e}}_- \tag{10}$$

and where F_+, F_-, B_+, and B_- are assumed to be slowly varying quantities of z. It is convenient to group the fields in this manner since, upon reflection, a normally incident RCP (LCP) wave will become an LCP (RCP) wave. From these physical considerations, we expect there to be no coupling between the components of $\mathbf{E}_+$ and those of $\mathbf{E}_-$. Indeed, the following calculations confirm this notion.

Based on the above form for the electric field and the chiral Helmholtz equation (4), the latter reduces to a pair of equations:

$$\left[\frac{\partial^2}{\partial z^2} \mp 2i\omega\mu\xi_c[1 + (\eta_c + \eta_\mu)\cos(Kz)]\frac{\partial}{\partial z} + k^2[1 + (\eta_\epsilon + \eta_\mu)\cos(Kz)] + K\eta_\mu \sin(Kz)\frac{\partial}{\partial z} \pm i\omega\mu\xi_c K\eta_c \sin(Kz) \right] \mathbf{E}_\pm = 0. \tag{11}$$

The upper (lower) sign corresponds to an incident RCP (LPC) wave which, upon reflection, becomes an LCP (RCP) wave.

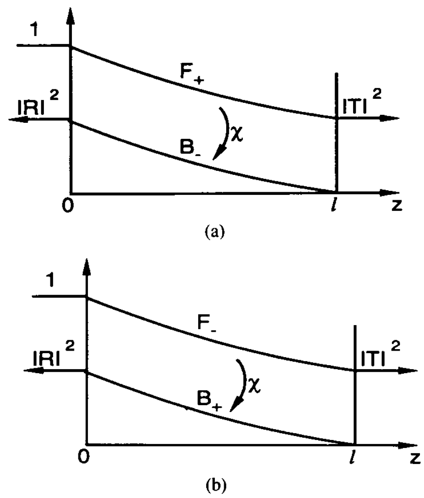

Fig. 3. Schematic of coupling between (a) modes F_+ and B_- and (b) modes F_- and B_+. Here, $|R|^2$ and $|T|^2$ represent the power reflection and transmission coefficients.

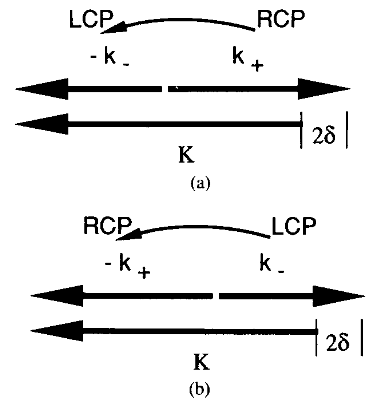

Fig. 4. The Bragg mismatch for (a) incident RCP and reflected LCP and (b) incident LCP and reflected RCP, with $\xi_c > 0$. Note k_+ is greater than k_- in magnitude for the example shown here.

Each case can be treated separately, since F_+ is only coupled with B_-, and conversely, F_- with B_+, as shown in Fig. 3.

Since there is no coupling between oppositely traveling waves of the same handedness, conservation of momentum[3] dictates that maximum coupling will only occur when $K = k_+ + k_-$ (the Bragg condition). Therefore, to allow for small variations from the Bragg condition and to satisfy the requirements imposed by chirality, the wavenumbers k_+ and k_- are here cast into the following form:

$$k_+ = \frac{K}{2} + \Delta + \delta \tag{12}$$

$$k_- = \frac{K}{2} - \Delta + \delta \tag{13}$$

where 2δ is a measure of the phase match per unit length from perfect Bragg matching, as shown in Fig. 4, and $\Delta = \omega\mu\xi_c$.

Substituting (9) and (10) into (11), and neglecting higher order terms,[4] we obtain

$$\begin{aligned}\mathbf{E}_+ :\ & i(k_+ + k_-)F'_+ e^{ik_+ z} - i(k_+ + k_-)B'_- e^{-ik_- z} \\ & + \frac{(\eta_\epsilon - \eta_\mu)k^2 + 2\eta_c\omega^2\mu^2\xi_c^2}{2} \\ & \cdot [F_+ e^{-i(k_- - 2\delta)z} + B_- e^{i(k_+ - 2\delta)z}] = 0\end{aligned} \tag{14}$$

$$\begin{aligned}\mathbf{E}_- :\ & i(k_+ + k_-)F'_- e^{ik_- z} - i(k_+ + k_-)B'_+ e^{-ik_+ z} \\ & + \frac{(\eta_\epsilon - \eta_\mu)k^2 + 2\eta_c\omega^2\mu^2\xi_c^2}{2} \\ & \cdot [F_- e^{-i(k_+ - 2\delta)z} + B_+ e^{i(k_- - 2\delta)z}] = 0\end{aligned} \tag{15}$$

[3] This is often denoted the condition for phase matching.

[4] Second-order derivatives are neglected in this adiabatic approximation (i.e., F_+, F_-, B_+, and B_- are slowly varying). Furthermore, higher order phases are unphysical from phase matching considerations.

where the primes denote differentiation with respect to the coordinate z. For each equation, we group together coefficients of the terms $e^{ik_\pm z}$ and $e^{-ik_\pm z}$, which correspond to the two directions of wave propagation. For both eouations to equal zero, the grouped coefficients must themselves be set equal to zero. This leads us to the coupled-mode equations:

$$F'_\pm = i\frac{(\eta_\epsilon - \eta_\mu)k^2 + 2\eta_c\omega^2\mu^2\xi_c^2}{2(k_+ + k_-)} e^{-i2\delta z} B_\mp \tag{16}$$

$$B'_\mp = -i\frac{(\eta_\epsilon - \eta_\mu)k^2 + 2\eta_c\omega^2\mu^2\xi_c^2}{2(k_+ + k_-)} e^{i2\delta z} F_\pm . \tag{17}$$

These equations can be expressed in a more familiar form by making the following substitutions:

$$\tilde{F}_\pm = e^{i\delta z} F_\pm \tag{18}$$

$$\tilde{B}_\pm = e^{-i\delta z} B_\pm \tag{19}$$

$$\begin{aligned}\chi &= \frac{(\eta_\epsilon - \eta_\mu)k^2 + 2\eta_c\omega^2\mu^2\xi_c^2}{2(k_+ + k_-)} \\ &= \frac{(\eta_\epsilon - \eta_\mu)k^2 + 2\eta_c\omega^2\mu^2\xi_c^2}{4\sqrt{\omega^2\mu^2\xi_c^2 + k^2}} .\end{aligned} \tag{20}$$

Then, (16) and (17) become:

$$\tilde{F}'_\pm - i\delta\tilde{F}_\pm = i\chi\tilde{B}_\mp \tag{21}$$

$$-\tilde{B}'_\mp - i\delta\tilde{B}_\mp = i\chi\tilde{F}_\pm \tag{22}$$

where χ denotes the coupling per unit length and δ the phase mismatch per unit length. The coupled-mode equations, for each modal pair (F_+ and B_-) and (F_- and B_+), are of the same form as those for an achiral periodic medium [4]. However, we have two sets describing the coupling between the incident RCP (LCP) and the reflected LCP (RCP).

It is interesting to examine the form of the coupling coefficient χ and to find the maximum coupling $\chi_{\max}$. It appears the value of the chiral admittance is limited by [13]

$$\mu^2(z)\xi_c^2(z) \le \mu(z)\epsilon(z). \tag{23}$$

Therefore, to first order in perturbation

$$[1+2\eta_c \cos(Kz)]\xi_c^2 \leq [1+(\eta_\epsilon-\eta_\mu)\cos(Kz)]\epsilon/\mu \quad (24)$$

which yields

$$(\eta_\epsilon-\eta_\mu)\epsilon/\mu - \epsilon/\mu + \xi_c^2 \leq 2\eta_c\xi_c^2 \leq (\eta_\epsilon-\eta_\mu)\epsilon/\mu + \epsilon/\mu - \xi_c^2. \quad (25)$$

Substituting (25) into (20), we obtain the maximum coupling coefficient, subject to condition (23),

$$\chi_{\max} = \frac{\left(|\eta_\epsilon-\eta_\mu| + \frac{1}{2} - \xi_c^2\frac{\mu}{2\epsilon}\right)k^2}{k_+ + k_-}. \quad (26)$$

Keeping in mind that χ is limited by $\chi_{\max}$, the coupled-mode equations (21) and (22) can be solved to examine various aspects of the propagating waves. In particular, we are interested in finding the bandgap structure and the fields at the ends of the periodicity. As can be seen from (26), due to the extra degree of freedom in chiral media, the chirality admittance ξ_c can modify the effects of perturbations in ϵ and μ and can fine tune the coupling coefficient.

The bandgap structure can be found by obtaining wave equations for $\tilde{F}_\pm$ and $\tilde{B}_\pm$ directly from the coupled-mode equations:

$$\begin{pmatrix}\tilde{F}_\pm\\ \tilde{B}_\pm\end{pmatrix}'' - D^2\begin{pmatrix}\tilde{F}_\pm\\ \tilde{B}_\pm\end{pmatrix} = 0 \quad (27)$$

where

$$D = \sqrt{(\chi^2-\delta^2)}. \quad (28)$$

The solutions become

$$\begin{pmatrix}\tilde{F}_\pm\\ \tilde{B}_\pm\end{pmatrix} \sim e^{\pm Dz} \quad (29)$$

which yield either purely decaying or purely propagating modal amplitudes $\tilde{F}_\pm$ and $\tilde{B}_\pm$. It is apparent that a bandgap exists when $\delta^2 < \chi^2$, as in the achiral case. Here, the incident mode decays as it passes through the perturbation due to modal coupling. The largest exponential decay occurs when the Bragg matching condition is satisfied, $\delta = 0$. Far from the Bragg condition, $\delta^2 > \chi^2$, the modal amplitudes are proportional to imaginary exponentials, which indicate propagating waves.

We can write the Riccati equation for the local reflection coefficient, $r_\pm = \tilde{B}_\mp/\tilde{F}_\pm$;

$$\frac{dr_\pm}{dz} = -i\chi(1+r_\pm^2) - 2i\delta r_\pm. \quad (30)$$

Solving the equation for a periodicity extending from 0 to l, we obtain the same result for the total amplitude reflection coefficient[5] R as for the achiral case [23]:

$$R = \frac{i\chi}{D\coth Dl - i\delta}. \quad (31)$$

[5] $R = r_+(0) = r_-(0)$ where (26) is integrated from $z = l$ to $z = 0$.

Similarly, we can find the expression for the total transmission coefficient amplitude:

$$T = \frac{D}{D\cosh Dl - i\delta\sinh Dl}. \quad (32)$$

The total transmitted and reflected[6] electric fields can be found separately for each component of the incident field in the $\hat{\mathbf{e}}_+$ and $\hat{\mathbf{e}}_-$ directions. In the $\hat{\mathbf{e}}_\pm$ bases, the incident field can be written as follows:

$$\mathbf{E}_{\mathrm{INC}_{z=0}} = a_+\hat{\mathbf{e}}_+ + a_-\hat{\mathbf{e}}_- \quad (33)$$

where a_+ and a_- are the magnitudes of the incident RCP and LCP components, respectively. Then, the reflected and transmitted fields become:

$$\mathbf{E}_{\mathrm{REFL}_{z=0}} = R(a_+\hat{\mathbf{e}}_+ + a_-\hat{\mathbf{e}}_-) \quad (34)$$

$$\mathbf{E}_{\mathrm{TRANS}_{z=l}} = T(a_+e^{ik_+l}\hat{\mathbf{e}}_+ + a_-e^{ik_-l}\hat{\mathbf{e}}_-). \quad (35)$$

Since the incident field is usually written in terms of x and y (parallel and perpendicular) polarization components rather than in terms of circular polarization components, it is convenient to express all the fields in the Cartesian coordinate system:

$$\mathbf{E}_{\mathrm{INC}_{z=0}} = a_x\hat{\mathbf{e}}_x + a_y\hat{\mathbf{e}}_y \quad (36)$$

$$\mathbf{E}_{\mathrm{REFL}_{z=0}} = [R]_{z=0}\mathbf{E}_{\mathrm{INC}_{z=0}} \quad (37)$$

$$\mathbf{E}_{\mathrm{TRANS}_{z=l}} = [T]_{z=l}\mathbf{E}_{\mathrm{INC}_{z=0}} \quad (38)$$

with

$$[R]_z = 0 = R[I] \quad (39)$$

$$[T]_{z=l} = Te^{i\left(\frac{k_++k_-}{2}\right)l}\begin{bmatrix}\cos(l\Delta) & -i\sin(l\Delta)\\ i\sin(l\Delta) & \cos(l\Delta)\end{bmatrix} \quad (40)$$

where $[I]$ is the identity matrix.

III. Discussion

As can be obtained from results of Jaggard *et al.* on reflections from homogeneous chiral slabs [13], the handedness, i.e., the sign of ξ_c, of the medium plays no role in reflection coefficient matrix (39). We also observe that the polarization ellipse of the transmitted wave has been rotated by an angle of $\phi = l\Delta = \omega\mu\xi_c l$, as in the aperiodic chiral case [17]. This rotation is a result of the two unequal wavenumbers, k_+ and k_-, corresponding to the two eigenmodes of the chiral medium.

Furthermore, the transmitted wave experiences a phase delay of $(k_+ + k_-)l/2$ with respect to the incident wave. This delay is equivalent to that of a wave propagating through a length l with the average chiral wavenumber. When ξ_c is small, the magnitudes of the reflection and transmission coefficients are

[6] Significant coupling and reflection takes place when the product χl approaches unity. This can be easily accomplished with very small perturbations, on the order of 10^{-3} in the optical frequency regime, or with somewhat larger perturbations in the microwave or millimeterwave regime.

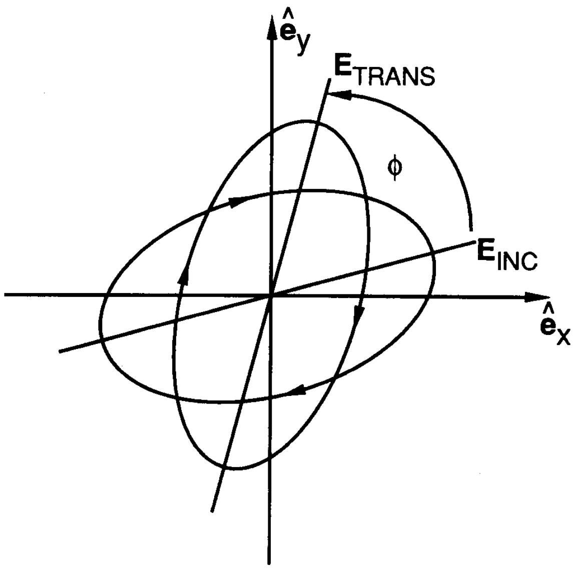

Fig. 5. The rotation of the polarization ellipse of the electric field after the wave has traversed a chiral slab of length l.

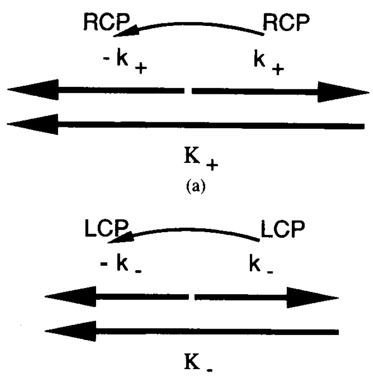

Fig. 6. Other Bragg matching possibilities. (a) $K = K_+ = k_+ + k_+$. (b) $K = K_- = k_- + k_-$.

equal, to second order, to those of the achiral case. However, the chirality of the medium is still present in the rotation of the major axis of the polarization ellipse. This effect is depicted in Fig. 5. These results indicate that the amplitudes of reflected and transmitted fields are more influenced by the periodicity than by the chirality of the medium. However, the chirality is unequally manifested in these fields. The only information about chirality conveyed by the reflection coefficient matrix is contained as a second-order term in ξ_c affecting the reflected field amplitude. On the other hand, chirality plays a larger role in the transmitted wave through rotation of the polarization ellipse, even for small ξ_c.

Analogously to the multitone case [5], where several bandgaps exist, one might here expect two extra bandgaps, since the Bragg condition can also be satisfied if either $K = K_+ = k_+ + k_+$ or $K = K_- = k_- + k_-$. See Fig. 6.

However, we have rigorously shown that these two modes of coupling do not exist in the unbounded, singly periodic case. They can also be rejected on physical grounds, since normally incident waves will change handedness upon reflection. RCP (LCP) waves, propagating with wavenumbers $k_+(k_-)$, upon normal incidence, become LCP (RCP) waves with wavenumbers $k_-(k_+)$.

IV. Conclusion

We examine electromagnetic wave propagation in simple periodic chiral structures and study the effects of combining chirality and periodicity. A set of coupled-mode equations is derived from the chiral Helmholtz equation. Reflection and transmission characteristics are investigated along with the bandgap structure. We find that the periodicity affects the magnitude of both the reflected and transmitted waves. The chirality is predominantly manifested through the polarization state of the transmitted wave. We also observe that the general characteristics of bandgap structure are similar to that of the achiral periodic case. However, the coupling and bandgap size can be tailored when chirality is present, due to the extra degree of freedom afforded by the chirality admittance.

The analysis reported in this paper can be extended to the case of periodic chiral guided-wave structures. Such structures have a variety of potential applications to integrated optical devices and systems and to their millimeter wave and microwave counterparts.

For oblique incidence, new effects are observed due to a richer coupling scheme between forward and backward modes. These richer couplings may occur, for example, in holographic gratings and wave guides. This problem is under present investigation.

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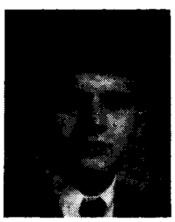
Dwight L. Jaggard (S'68-M'77-SM'86), for a photograph and biography please see page 946 of the August 1987 issue of this TRANSACTIONS.

Nader Engheta (S'80-M'82-SM'89), for a photograph and biography please see page 330 of the March 1988 issue of this TRANSACTIONS.



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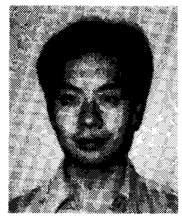

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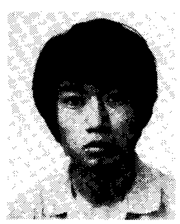

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