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Constraints on energies of ^{15}F(g.s.), ^{15}O($1/2^+$, T = 3/2), and ^{16}F(0^+, T = 2)

H. Terry Fortune
University of Pennsylvania, fortune@physics.upenn.edu

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Constraints on energies of ^{15}F(g.s.), ^{15}O($\frac{1}{2}^+$, $T = \frac{3}{2}$), and ^{16}F(0^+, $T = 2$)

H. T. Fortune
Department of Physics and Astronomy, University of Pennsylvania, Philadelphia, Pennsylvania 19104, USA
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Coulomb energy calculations for the lowest 0^+, $T = 2$ state in $A = 16$ nuclei allow tight constraints on the masses of the lowest $1/2^+$, $T = 3/2$ state in ^{15}O and ^{15}F, and of the 0^+, $T = 2$ state in ^{16}F.

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I. INTRODUCTION

It is well known that the "energy" of a broad resonance depends on the details of the definition. For example, for the ground state (g.s.) of ^{15}F, two definitions that are widely used provide values that are different by about 150–200 keV [1] (theoretically) and 160–180 keV [2,3] (experimentally). Tests of the isobaric multiplet mass equation (IMME) seek to determine the absence or presence of a d term in the mass equation

$$M(T_z) = a + bT_z + cT_z^2\left(+dT_z^3\right). \quad (1)$$

This d term should be absent if the charge-dependent Hamiltonian contains only isoscalar, isovector, and isotensor terms. For a $T = 3/2$ quartet, as involves ^{15}F, the mass of the most proton-rich member enters the expression for d with a coefficient of 1/6, i.e., changing the mass of ^{15}F by 160 keV would change the value of a possible d coefficient by 26 keV—larger than most values of d heretofore extracted [4]. Of the completely measured isospin quartets, only in $A = 9$ is there convincing evidence of the need for a nonzero value of $d : \chi^2/n = 10.2$ for $d = 0$, and the fit with $d \neq 0$ gives $d = 5.5(18)$ [4]. The possibility [5] of a large nonzero value of d for $A = 33$ appears to have gone away [6]. So, does the IMME favor one definition of resonance energy over another? Can we use the properties of nearby narrow(er) states to better determine the mass of ^{15}F? We have previously used the known masses [7,8] of the 0^+, $T = 2$ states in ^{16}C, ^{16}N, ^{16}O, and ^{16}Ne (the state is not known in ^{16}F) and a simple model [9] to estimate the s^2/d^2 ratio in those states, assuming isospin invariance. This procedure uses as input the energies [7,10] of the relevant $1/2^+$ and $5/2^+$, $T = 3/2$ core states with $A = 15$. At that time, the ^{15}O $1/2^+$, $T = 3/2$ state was not known and the ^{15}F (g.s.) energy was poorly determined.

Three definitions of the position of a resonance are in common use. They are (a) the energy at which the appropriate cross section peaks, (b) the energy at which the nuclear phase shift has the value $\delta = \pi/2$, and (c) the energy at which the magnitude of the internal wave function or the derivative of the phase shift $d\delta/dE$ is a maximum. Peters *et al.* [2] give results for all three definitions for both $1/2^+$ and $5/2^+$ (Table I), Goldberg, *et al.* [3] quote results of two definitions for the $1/2^+$ resonance and state that the $5/2^+$ does not depend on the definition (also borne out in the numbers of Ref. [2]). Reference [2] states that the uncertainty in the $5/2^+$ energy is mostly due to uncertainty in the absolute calibration. They do not give an uncertainty in the width. Reference [3] states that the uncertainty in $\Gamma(5/2^+)$ is mainly due to the fact that it is large and interferes with the $1/2^+$ state. Their value of $\Gamma(1/2^+) = 0.7$ MeV, with no uncertainty, comes not from the experiment, but from the behavior of the wave function calculated in a potential well. The FWHM of their $1/2^+$ cross section is 1.2 MeV, which they state arises from interference with Coulomb and potential scattering. They quote an absolute energy calibration uncertainty of ± 15 keV. Their quoted resolution is about 25 keV near the $5/2^+$ resonance and about 75 keV just below the $1/2^+$. Both papers compare the measured cross section with cross sections calculated in a potential model, with spectroscopic factors of unity. We know $S < 1$ for both from ^{14}C(d, p) [11] and theoretical considerations.

The third and most recent elastic experiment [12] used R-matrix analysis. Even though their data are shifted by about 150–200 keV from the earlier work, their derived $5/2^+$ energy is nearly identical to the others. Another experiment [13] used the transfer reaction ^{16}O(^{14}N,^{15}C)^{15}F. Their cross section is considerably larger (factor of 3–4) for the $5/2^+$ than for the $1/2^+$ state. The larger cross section and the narrower peak provide smaller uncertainties for the $5/2^+$ state than for the g.s. Nevertheless, within these uncertainties, the $1/2^+$ parameters agree with those from elastic scattering. The relevant experimental results [2,3,12–15] are summarized in Table II. We look first at the $5/2^+$ state, for which the various definitions of resonance energy (Table I) virtually agree. Then, we consider the $1/2^+$ state.

II. ANALYSIS

A. $A = 15$, $T = 3/2$

Isospin mixing can be important for $T = 3/2$ states in $T_z = \pm 1/2$ nuclei. As we outline in the Appendix, we conclude that some mixing is present here, but not of sufficient magnitude to bother our analysis. So, we make no further mention of it.

B. $5/2^+$

The mass excesses of the lowest $5/2^+ T = 3/2$ states for $A = 15$ are listed in the fourth column of Table III. The

TABLE I. Energies (MeV) in ^{15}F, relative to ^{14}O+p, for different definitions of resonance energy.

Definition:		$\sigma = \max$	$\delta = \pi/2$	$\Psi = \max$
Goldberg *et al.*	$1/2^+$	—	$1.45^{+.16}_{-.10}$	$1.29^{+.08}_{-.06}$
Peters *et al.*	$1/2^+$	1.51	1.47	1.29
	$5/2^+$	2.853	2.87	2.85
Calculated (Ref. [1])	$1/2^+$	—	1.39–1.51	1.19–1.36[a]

[a]Maximum of $d\delta/dE$.

known masses of this state in ^{15}C, ^{15}N, and ^{15}O allow the computation of the mass of ^{15}F ($5/2^+$) if we assume $d = 0$. The result is a mass excess of 18.081(46) MeV [$E_p = 2.785(46)$ MeV], where the uncertainty comes from propagating the uncertainties in the other masses. The difference of experimental minus IMME ($d = 0$) results are listed in the last column of Table II for the various ^{15}F experiments. The first number in parentheses is the quoted experimental uncertainty, and the second number is the uncertainty in the energy computed with the IMME. All of the experiments agree with the IMME and with each other, frequently to much better than the uncertainties. It would thus appear that we are far from any serious test of IMME with the $5/2^+$ quartet. We do, however, note an oddity concerning the widths.

The $5/2^+$ widths and spectroscopic factors for all four nuclei are also summarized in Table III. Table IV compares the earliest and most recent widths for ^{15}F ($5/2^+$). Table III also lists the single-particle widths for the unbound cases. The last column is the spectroscopic factor from ^{14}C(d, p) [11] for ^{15}C and from the expression $C^2S = \Gamma_{\rm exp}/\Gamma_{\rm sp}$ for the others, where $C^2 = 1/3, 2/3$, and 1 for ^{15}N, ^{15}O, and ^{15}F, respectively. A similar comparison is made in Table IV.

As stated above, Peters *et al.* do not quote an uncertainty on the $5/2^+$ width, but it is probably similar to those of the other two elastic experiments, both of which are 60 keV. A simple average of the width from the three elastic experiments is 322(35) keV (Table IV), a value that presents a problem, as we now discuss. In a much earlier experiment, using the reaction ^{20}Ne(^{3}He,^{8}Li), Benenson *et al.* [14] quote a $5/2^+$ width of 240(30) keV, which they state is the result of a Gaussian fit after correcting for an experimental resolution of 210(20) keV. If we understand them correctly, the implication is that the total measured width was 320(10) keV We find that fitting with the convolution of a Gaussian resolution function and a Breit-Wigner shape—as required for a state with natural width—would then have resulted in a Breit-Wigner width of 180(30) keV, which is even smaller than their published value, which was already significantly smaller than the widths from elastic scattering [2,3,12]. The difference between this width and the average from elastic scattering is 140(46) keV—roughly a 3σ difference. This feature is relevant because of the resulting spectroscopic factors, summarized in Tables III and IV. Benenson *et al.* state that their width of 240(30) keV gives a spectroscopic factor of 0.92(12), so that a width of 180(30) keV would correspond to $S = 0.69(10)$, not very different from the value of 0.69 from ^{14}C(d, p)—which we used in our earlier analysis [1]. With our sp width of 250 keV, this 180-keV "measured" width gives $S = 0.72(10)$. For the $5/2^+$, $T = 3/2$ states in ^{15}N and ^{15}O, the measured widths [10,16] (Table III) also correspond to similar values of S. All the widths determined from elastic scattering (and the resulting S's) are significantly larger. If we assign a 20% uncertainty to the S value from (d, p), then the average for the first three nuclei is 0.73(6), which is more than 4σ from the average of 1.34(12) for ^{15}F. This S cannot exceed unity, on very firm grounds, but we see that it does in ^{15}F with the quoted widths. Is it possible that the resolution width has not been properly accounted for in those experiments? It is unlikely that the problem lies with the other nuclei for two reasons: (1) they all agree on the value of S, and (2) it is the ^{15}F value that exceeds the sum-rule limit. We note that the width of 135(15 keV) [14] for ^{15}O($5/2^+$, $T = 3/2$) from ^{17}O(p, t) is also from a Gaussian fit, where, of course, a Breit-Wigner shape is more appropriate. A Gaussian of width 135 keV plus some background closely resembles a Breit-Wigner shape of width 150 keV.

TABLE II. Energies and widths (MeV) of lowest $5/2^+$ and $1/2^+$ states in ^{15}F.

	Reference	$1/2^+$		$5/2^+$		
		E_p[a]	Γ	E_p[a]	Γ	Exp–IMME (keV)[b]
Exp	14	1.6(2)	< 0.9	2.8(2)	0.24(3)	15(200(46))
	15	1.37(18)	0.8(3)	2.67(10)	0.5(2)	−115(100(46))
	2	1.51(15)	1.2	2.853(45)	0.34	70(45(46))
	13	1.56(13)	$0.6^{+.8}_{-.4}$	2.80(5)	0.38(10)	15(50(46))
	3	$1.45^{+0.16}_{-0.10}$	0.7	2.795(45)	0.32(6)	10(45(46))
	12	1.23(5)	0.50–0.84	2.810(20)[c]	0.30(6)	25(20(46))
Calc	1	1.39–1.51	~0.80	2.785(46)[d]	See text	

[a]Energy relative to ^{14}O+p. (E_p = mass excess – 15.296 MeV).
[b]First number in parentheses is experimental uncertainty, and second one is uncertainty in IMME calculation (arising from uncertainties in the other masses).
[c]But curve is about 150–200 keV lower than data of Peters *et al.*
[d]This is the IMME value, with $d = 0$ (present).

TABLE III. Mass excesses (ME) and widths (both in keV) and spectroscopic factors of lowest $5/2^+$, $T = 3/2$ states in $A = 15$ nuclei.

Nucleus	ME (g.s.)[a]	$E_x(5/2^+)$[b]	ME ($5/2^+$)	Width		S[d]
				Exp[b]	sp[c]	
^{15}C	9873.1(8)	740.0(15)	10613.1(17)	–	–	0.69[e]
^{15}N	101.4380(7)	12522(8)	12623(8)	58(4)	230	0.76(5)
^{15}O	2855.6(5)	12255(13)	15111(13)	135(15)	270	0.75(8)
^{15}F	$E_p = 2795$–2853[f]		$E_p + 15296$[f]	320(35)[g]	250	1.28(14)
				335(30)[h]		1.34(12)

[a]Reference [7].
[b]Reference [10].
[c]Single-particle width calculated in potential model ($r_0, a = 1.25, 0.65$ fm) with depth adjusted to put resonance at its experimental energy (see Table II).
[d]From $\Gamma_{exp} = C^2 S \Gamma_{sp}$, where $C^2 = 1/3, 2/3$, and 1 for ^{15}N, ^{15}O, and ^{15}F, respectively.
[e]From ^{14}C(d, p) [11].
[f]See text and Table II.
[g]From Refs. [2,3,12].
[h]Average of four most recent values (Table II).

We expect a Breit-Wigner fit to the (p, t) data would result in a width of 150 keV, but that is only one σ from the published value.

C. ^{15}F ($1/2^+$) and ^{15}O($1/2^+$, $T = 3/2$)

Mass excesses of the lowest $1/2^+ T = 3/2$ states in $A = 15$ nuclei are listed in Table V, along with widths and spectroscopic factors. Even if we assume $d = 0$, we cannot compute the mass of ^{15}F(g.s.) from the IMME because the $1/2^+$, $T = 3/2$ state in ^{15}O is not known. An early candidate [16] at $E_x = 10.938(3)$ MeV is almost certainly $T = 1/2$. It has a large decay branch ($\Gamma = 39$ keV) to the g.s. of ^{14}N, a decay that is isospin forbidden for a $T = 3/2$ state. Furthermore, the width of 60 keV for (isospin-allowed) decay to the 0^+, $T = 1$ state of ^{14}N is a small fraction ($\leqslant$10%) of the width expected for that decay (see table and text below). However, we can use the IMME to derive the connection between the masses of ^{15}O($1/2^+$, $T = 3/2$) and ^{15}F (g.s.) for any value of d. This relationship is plotted in Fig. 1 for $d = 0$ and 10 keV. Also plotted there as horizontal lines are the two "experimental" values from Goldberg *et al.* (values of Peters *et al.* are nearly identical)—the solid line for $\delta = \pi/2$, the dashed line for the peak in the wave function. We see that without further information, the IMME does not allow a preference for one definition over the other. We return to this point later.

TABLE IV. Widths (keV) and spectroscopic factors for ^{15}F($5/2^+$).

Source	Width	$S = \Gamma_{exp}/\Gamma_{sp}$
Benenson *et al.* Gaussian[a]	240(30)	0.92(12)[e]
Benenson *et al.* BW[b]	180(30)	0.69(10)[f] or 0.72(10)[g]
elastic ^{14}O+p[c]	322(35)	1.29(14)
single particle[d]	250	1.0[h]

[a]From the reaction ^{20}Ne(^{3}He,^{8}Li) (Gaussian fit).
[b]Our convolution of Breit-Wigner and Gaussian (see text).
[c]Average of three experiments.
[d]Computed at $E_p = 2.8$ MeV, in a Woods-Saxon potential with $r_0, a = 1.25, 0.65$ fm.
[e]Quoted in Ref. [14].
[f]Using Γ_{sp} from Ref. [14].
[g]Using our Γ_{sp}.
[h]Theoretical (rigorous) upper limit.

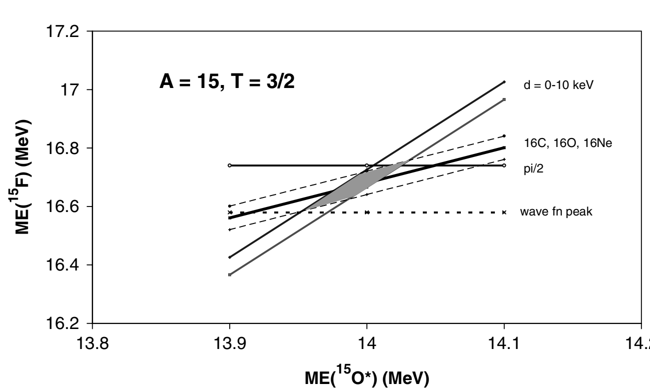

FIG. 1. (Color online) Steeper upward sloping lines exhibit the IMME relationship between masses of ^{15}F(g.s.) and ^{15}O($1/2^+$, $T = 3/2$) for $d = 0$ and $d = 10$ keV. Horizontal lines are at the ^{15}F values of Ref. [3] for $\delta = \pi/2$ (solid) and wave-function maximum (dashed). Shallower sloping solid line gives the relationship between the ^{15}F and ^{15}O* masses if ^{16}C, ^{16}O*, and ^{16}Ne all have the same s^2 fraction. Dashed lines with this slope represent a 40 keV model uncertainty.

TABLE V. Mass excesses (ME) (keV) and widths (MeV) and spectroscopic factors of lowest $1/2^+$, $T = 3/2$ states in $A = 15$ nuclei.

Nucleus	ME (g.s.)[a]	$E_x(1/2^+)$[b]	ME $(1/2^+)$	Width		S[d]
				Exp[b]	sp[c]	
^{15}C	9873.1(8)	0	9873.1(8)	–	–	0.88[e]
^{15}N	101.4380(7)	11615(4)	11716(4)	0.401(6)	~1.6	~0.75
^{15}O	2855.6(5)	unknown	—	unknown	~1.2	—
^{15}F	—	—	see text and Table II	0.2-1.4[f]	~0.8	See text

[a]Reference [7].
[b]Reference [10].
[c]Calculated in potential model ($r_0, a = 1.25, 0.65$ fm).
[d]From $\Gamma_{exp} = C^2 S \Gamma_{sp}$, where $C^2 = 1/3, 2/3$, and 1 for ^{15}N, ^{15}O, and ^{15}F, respectively.
[e]From ^{14}C(d, p) [11].
[f]From Refs. [2,3,12,13].

D. $0^+ T = 2$ in $A = 16$

We turn now to the lowest 0^+, $T = 2$ state in $A = 16$ nuclei. This state is not known in ^{16}F, but with four masses known (Table VI) it is possible to investigate the possible presence of a nonzero d coefficient. The result is $d = 4.6(28)$ keV. With this value of d, the IMME value for the mass excess of the $0^+, T = 2$ state in ^{16}F is 20.789(21) MeV ($E_x = 10.109$ MeV). If, alternatively, we assume $d = 0$, the problem is overdetermined, but we can perform a fit to find the mass of ^{16}F that minimizes χ^2. The result is a mass excess of 20.761(13) MeV. As mentioned earlier, we used the known masses of $A = 16$, $T = 2$, and $A = 15$, $T = 3/2$ states to estimate the s^2 fraction (assumed independent of T_z) in the lowest 0^+, $T = 2$ state [9]. The results ranged from 0.39 in ^{16}N to 0.50 in ^{16}Ne , with a large uncertainty in the latter because of large uncertainties in the ^{15}O and ^{15}F masses. (In that paper, we used the IMME masses for ^{15}O* and ^{16}F*.) Here, we take a slightly different approach. We assume, as before, that β^2, the fraction of s^2, is the same in ^{16}C, ^{16}Ne, and the 0^+, $T = 2$ state of ^{16}O, but leave its value un-specified. The known masses of those three $A = 16$ states and the $T = 3/2$ states in ^{15}C and ^{15}N allow the derivation of another relationship between the masses (assumed unknown) of ^{15}O$(1/2^+, T = 3/2)$ and ^{15}F (g.s.). This relationship is also plotted in Fig. 1, where we have used a model uncertainty of ± 40 keV. If the model is valid, the masses of ^{15}O$(T = 3/2)$ and ^{15}F are then constrained to lie at the intersection of these lines with the lines from the IMME (if we have $d = 0$–10 keV). This area is shaded in the figure. We conclude that the values of ^{15}O* and ^{15}F mass that satisfy the IMME with $d = 0$ or 5(5) keV also meet the requirement of simultaneously fitting the ^{16}C, ^{16}O* and ^{16}Ne masses with a single value of the s^2 fraction. This value, for the $d = 0$ masses, is 0.43(4), to be compared with the values of 0.39 and 0.45 in Ref. [9] for ^{16}C, ^{16}N, and ^{16}O. The results for the midpoint of the allowed range for $d = 5(5)$ keV in Fig. 1 (with uncertainties) are

$$\mathrm{ME}(^{15}\mathrm{O}(1/2^+, T = 3/2)) = 13.992(37)\,\mathrm{MeV},$$
$$\mathrm{ME}(^{15}\mathrm{F(g.s.)}) = 16.632(45)\,\mathrm{MeV}; \quad E_p = 1.336(45)\,\mathrm{MeV}.$$

For $d = 0$, the results are 13.975(33) and 16.652(40) MeV. The shaded area almost touches the two ^{15}F (g.s.) values from the two resonance definitions.

With our potential model [1] , the g.s. mass of ^{15}C led to a ^{15}F(g.s.) $=^{14}$O$+p$ $2s1/2$ resonance energy of 1.19 or 1.39 MeV with the two definitions of resonance energy. The addition of configuration mixing increased this energy because the $2s1/2$ sp part has the smallest computed Coulomb energy. The allowed range, while still requiring a spectroscopic factor of 0.88, was (Fig. 2) 1.19–1.36 and 1.39–1.51 MeV. The results of the current analysis significantly narrow the allowed range, as can be seen in Fig. 2, where we also show the experimental results of Goldberg *et al.* Computing the sp width for the new allowed range of energies results in 0.86(8) and 0.81(8) MeV, for $d = 0$ and 5(5) keV, respectively. With a spectroscopic factor of 0.88, the expected ^{15}F(g.s.) width is then 0.76(8) or 0.71(7), where most of the uncertainty arises from the uncertainty in resonance energy.

As stated above, these results require a value of $\beta^2 = 0.43(4)$. This value of $\beta^2$and the $A = 15$ masses allow us to compute the mass excess of ^{16}F$(0^+, T = 2) = 20.742(16)$ MeV ($E_x = 10.062$ MeV). This value is

TABLE VI. Energies (keV) of lowest 0^+, $T = 2$ states in $A = 16$ nuclei.

Nucleus	Mass excess (g.s.)[a]	$E_x(0^+)$[b]	Mass excess (0^+)
^{16}C	13694(4)	0	13694(4)
^{16}N	5683.7(2.6)	9928(7)	15611.7(7.5)
^{16}O	-4737.0014(1)	22721(3)	17984(3)
^{16}F	10680(8)	unknown	—[c]
^{16}Ne	23996(20)	0	23996(20)

[a]Reference [7].
[b]Reference [10].
[c]IMME predicts a mass excess of 20.761(13) MeV if $d_{16} = 0$, and 20.789(20) MeV with $d_{16} = 4.6(28)$ keV.

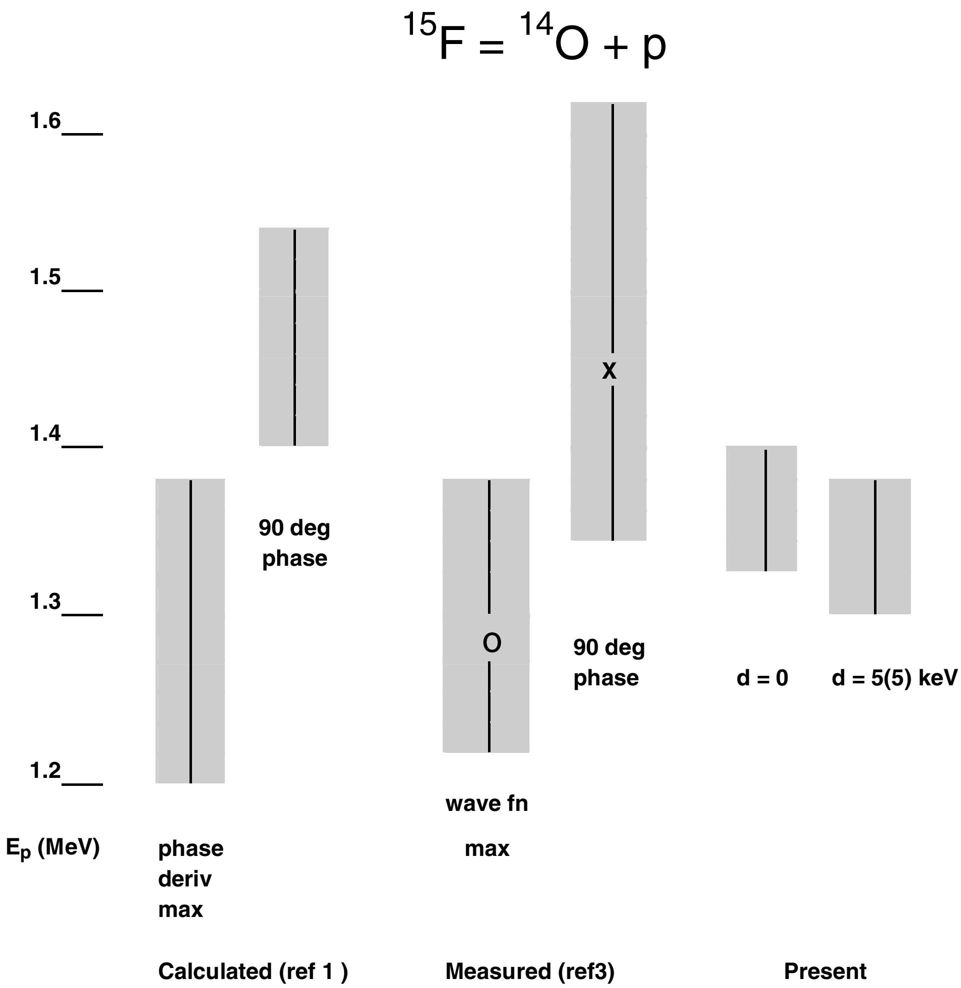

FIG. 2. Schematic representation of the energy of ^{15}F(g.s.) calculated in Ref. [1], measured in Ref. [3], and from present analysis.

TABLE VII. Expected energies and widths (both in MeV) of ^{15}O($1/2^+, T = 3/2$), ^{15}F(g.s.), and ^{16}F($0^+, T = 2$). (Requires s^2 fraction to be 0.43(4) in A = 16.)

	^{15}O		^{15}F		^{16}F
	E_x	$\Gamma^{a,b}$	E_p	Γ^{a}	E_x
Fit with $d_{15} = 0$	11.119(33)	~ 0.70	1.356(40)	0.76(8)	10.062((16)
Fit with $d_{15} = 5(5)$ keV	11.136(37)	~ 0.70	1.336(45)	0.71(7)	—

[a]Calculated for S = 0.88.
[b]For proton decay to 0^+, $T = 1$ state of ^{14}N.

TABLE VIII. Candidates for isospin mixing in ^{15}N and ^{15}O (E_x in MeV, widths in keV). Information from Ref. [7] and work cited therein.

Nucl	J^π	T	E_x	Γ	Γ_n	Γ_p	Γ_α
^{15}N	$1/2^+$	1/2	11.4376(7)	41.4(11)	23-35	6.8–11	<0.3
		3/2	11.615(4)	405(6)	4.0(2)	401(6)	<0.3
	5/2+	1/2	12.493(4)	40(5)	28–37	0.3–0.5	5.5–9
		3/2	12.522(8)	58(4)	—	80	—
^{15}O	5/2+	1/2	12.129(15)	200(50)	—	200(50)	—
		3/2	12.255(13)	135(15)	—	135(15)[a]	—

[a]Decay to 0^+, $T = 1$ level of ^{14}N.

reasonably close to the prediction of $E_x = 10.08(2)$ in Ref. [15]. The old IMME result was $E_x = 10.093(13)$ MeV. Expected properties of ^{15}O($1/2^+$, $T = 3/2$) and ^{15}F (gs) are listed in Table VII.

III. CONCLUSION

Combining information from the lowest $A = 15$, $T = 3/2$ and $A = 16$, 0^+, $T = 2$ states, we have obtained simultaneous constraints on the mass excess of ^{15}O($1/2^+$, $T = 3/2$), ^{15}F (g.s.), and ^{16}F(0^+, $T = 2$). The required fraction of s^2 in the $A = 16$ 0^+ states is 0.43(4). Results are summarized in Table VII. We have also pointed out a problem with the ^{15}F($5/2^+$) width.

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APPENDIX A: EFFECTS OF POSSIBLE ISOSPIN MIXING

Before embarking on the analysis, we considered the possibility of isospin mixing and the effects such mixing might have on our results. Isospin mixing is always a concern in the “interior” nuclei of an isospin multiplet. For example, in $T_z = \pm 1/2$ nuclei, the lowest $T = 3/2$ states frequently have very near neighbors with $T = 1/2$ and the same J^π. Such is the case here. In ^{15}N, both $1/2^+$ and $5/2^+ T = 3/2$ states lie very close to $T = 1/2$ states, as do the $5/2^+$ states in ^{15}O (indicated in Table VIII). Our primary concern is the possibility of an energy shift of the $T = 3/2$ states caused by T mixing. Changes in the (allowed) proton widths of the $T = 3/2$ states will be very small here. Of course, any neutron or alpha width for the $T = 3/2$ states is an indication of isospin mixing. And, for ^{15}O, p decay of the $T = 3/2$ state to the ^{14}N(g.s.) would be a signature of T mixing. Thus, from the value of $\Gamma_n = 4$ keV for the $1/2^+$, $T = 3/2$ state in ^{15}N, the $1/2^+$ states seem clearly to be mixed. The entire profile of the $T = 1/2$ state lies fully within the envelope of the $T = 3/2$ state—the energy splitting is 177 keV and the width of the $T = 3/2$ state is 400 keV. The $5/2^+$, $T = 3/2$ state has no discernible neutron or alpha width in ^{15}N and no alpha or g.s. proton width in ^{15}O, so any mixing there is very small. Because the two $5/2^+$ states in ^{15}N are so close together [$\Delta E = 29(9)$ keV], any energy shift of the $5/2^+ T = 3/2$ state is negligible. For the $1/2^+$ states, we estimate that the mixing inferred from the neutron widths could have shifted the $T = 3/2$ state upward by 5–18 keV. With real mixing, summed strengths are preserved. If we use the values of Table 15.12 of Ref. [10] for neutron widths of the two $1/2^+$ states, viz. 4.0(2) keV for the upper state and 34.6(9) keV for the lower, then because the $T = 3/2$ state had no neutron width before mixing, the square of the mixing amplitude is given by $\varepsilon^2 = 4.0/38.6 = 0.104(6)$. This amount of mixing implies each of the two states has shifted by 18 keV. As mentioned in the Introduction, we are dealing with a 150–200 keV difference in the central value of the energy of ^{15}F(g.s.) and spreads of a few hundred keV about those central values. Furthermore, our final uncertainties in the relevant energies are in the range 33–45 keV. So, for our present purposes, we eschew any further consideration of isospin mixing. But, in the future, if combined uncertainties approach 10–20 keV, then it will be necessary to deal with such mixing in these nuclei—at least for the $1/2^+$ states.

APPENDIX B: INDEPENDENCE FROM THE IMME

Solving for the three unknown masses and the s^2 fraction, β^2, is straightforward without recourse to the IMME. Consider this four-step procedure:

(i) Given the known masses of ^{16}C(g.s.), ^{16}N(0^+, $T = 2$), and the $1/2^+$ and $5/2^+ T = 3/2$ states of ^{15}C and ^{15}N, it is a simple matter to compute β^2 (assumed to be equal in ^{16}C and ^{16}N*). As stated in Ref. [9], the result is $\beta^2 = 0.39$.
(ii) Adding the known masses of ^{16}O(0^+, $T = 2$) and ^{15}O($5/2^+$, $T = 3/2$), the unknown mass ^{15}O($1/2^+$, $T = 3/2$) can be computed as a function of β^2, and, in particular, for $\beta^2 = 0.39$ (above) or 0.43 (text).
(iii) The known masses of ^{16}Ne(g.s.) and ^{15}F($5/2^+$), in conjunction with a value of β^2, can be used to compute the mass of ^{15}F($1/2^+$) (assumed unknown for present purposes).
(iv) With all these masses and β^2, the mass of ^{16}F(0^+, $T = 2$) can then be computed.

Note that nowhere in this procedure is there any reference to the IMME. Doing the analysis this way, the arithmetic is simple, but the uncertainties compound in a way that is difficult to untangle.

Thus, we chose the route of combining these steps above, leaving the value of β^2 unspecified (but assumed equal in all five 0^+, $T = 2$ states). The result is a “best” value of $\beta^2 = 0.43(4)$ and values of the three unknown masses, with their uncertainties. The model is such that the masses will satisfy the IMME, with $d = 0$. Allowing d to differ from zero allows a slightly wider range for the masses, and hence slightly larger uncertainties. But, nowhere did we impose a requirement that d must have some specific value.

[1] H. T. Fortune and R. Sherr, Phys. Rev. C **72**, 024319 (2005).
[2] W. Peters *et al.*, Phys. Rev. C **68**, 034607 (2003).
[3] V. Z. Goldberg *et al.*, Phys. Rev. C **69**, 031302(R) (2004).
[4] M. S. Antony, A. Pape, and J. Britz, At. Data Nucl. Data Tables **66**, 1 (1997); J. Britz, A. Pape, and M. S. Antony, *ibid.* **69**, 125 (1998).
[5] F. Herfurth *et al.*, Phys. Rev. Lett. **87**, 142501 (2001).
[6] M. C. Pyle *et al.*, Phys. Rev. Lett. **88**, 122501 (2002).
[7] G. Audi and A. H. Wapstra, Nucl. Phys. **A595**, 409 (1995);

G. Audi, A. H. Wapstra, and C. Thibault, *ibid.* **A729**, 337 (2003).
[8] D. R. Tilley, H. R. Weller, and C. M. Cheves, Nucl. Phys. **A564**, 1 (1993).
[9] H. T. Fortune and R. Sherr, Phys. Rev. C **66**, 017301 (2002).
[10] F. Ajzenberg-Selove, Nucl. Phys. **A523**, 1 (1991).
[11] J. D. Goss, P. L. Jolivette, C. P. Browne, S. E. Darden, H. R. Weller, and R. A. Blue, Phys. Rev. C **12**, 1730 (1975).
[12] F. Q. Guo *et al.*, Phys. Rev. C **72**, 034312 (2005).
[13] A. Lépine-Szily *et al.*, Nucl. Phys. **A734**, 331 (2004); (private communication).
[14] W. Benenson, E. Kashy, A. G. Ledebuhr, R. C. Pardo, R. G. H. Robertson, and L. W. Robinson, Phys. Rev. C **17**, 1939 (1978).
[15] G. J. KeKelis *et al.*, Phys. Rev. C **17**, 1929 (1978).
[16] G. W. Phillips, P. Richard, D. O. Elliot, F. F. Hopkins, and A. C. Porter, Phys. Rev. C **5**, 297 (1972).