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An Analysis of Several New Product Performance Metrics

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An Analysis of Several New Product Performance Metrics

Abstract
For most firms, new product development is the engine for growth and profitability. A firm's new product success depends on its ability to manage the product development process in a way that employs scarce resources to achieve the goal of the firm as well as the specific project's objectives. Simple and measurable performance metrics have been proposed and applied to monitor and compensate the development teams. In this paper, we develop a modeling framework to analyze the implications of setting managerial priorities for three commonly used new product performance metrics: (1) time-to-market, (2) product performance, and (3) total development cost. We model new product development as a “product performance production” process that requires scarce development resources. Setting a target for development teams for each of these performance metrics can constrain this performance production process and, thereby, affect the other performance metrics. We model the constrained process as a restricted case of a general process that does not have such constraints. We benchmark each constrained process against the optimal, unrestricted process with respect to the level of the resource intensity employed during the development process, the time-to-market, and the performance level of the new product at launch. We show that an overly ambitious time-to-market target leads to an upward bias in resource intensity usage and a downward bias in product performance (i.e., evolutionary product innovation). In addition, our results suggest that the target time-to-market approach may ignore the effect of cannibalization and, thus, can perform suboptimally if a significant degree of cannibalization in the existing product market is expected. Given a target product performance, we show that the coordination between marketing and R&D is easier because the resulting development resource intensity and time-to-market decisions becomes separable. However, an overly ambitious product performance target leads to an upward bias in the development resource intensity and a delayed product launch that misses the window of opportunity. Finally, we show that the target development cost approach can lead a downward bias in product performance and a premature product launch. The above analyses are performed for a monopolistic firm, and they are extended to passive and active competitive environment.

Keywords
new product development, product performance, time-to-market, development costs, performance metrics

Disciplines
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An Analysis of Several New Product Performance Metrics

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We benchmark each constrained process against the optimal, unrestricted process with respect to the level of the resource intensity employed during the development process, the time-to-market, and the performance level of the new product at launch. We show that an overly ambitious time-to-market target leads to an upward bias in resource intensity usage and a downward bias in product performance (i.e., evolutionary product innovation.) In addition, our results suggest that the target time-to-market approach may ignore the effect of cannibalization and thus can perform suboptimally if a significant degree of cannibalization in the existing product market is expected. Given a target product performance, we show that the coordination between marketing and R&D is easier because the resulting development resource intensity and time to market decisions becomes separable. However, an overly ambitious product performance target leads to an upward bias in the development resource intensity and a delayed product launch that misses the window of opportunity. Finally, we show that the target development cost approach can lead a downward bias in product performance and a premature product launch. The above analyses are performed for a monopolistic firm and they are extended to passive and active competitive environment.

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1 Introduction

For most firms, new product development is strategic because it can significantly affect their competitive position in the marketplace. A firm’s new product success depends on its ability to employ scarce development resources to deliver high-performance products in a timely fashion. Effective management of the development process is difficult due to its underlying complexity and the wide range of product performance criteria that it can influence. Three critical determinants of new product success that are directly affected by the management of timing and resources are: 1) time-to-market, 2) product performance, and 3) total development cost (Clark and Fujimoto, 1991). These determinants of new product success, however, are interrelated and they may conflict. Firms must consider potential tradeoffs among them. In Cohen, Eliashberg, and Ho (1996), we introduce a model for studying the tradeoff between time-to-market and product performance. In this paper, we extend our previous work by allowing the level of development resource to vary with time and we develop a more general modeling framework that simultaneously considers the potential tradeoffs among all three determinants of new product success.

Indeed, Clark and Fujimoto (1991) empirically show that the three crucial determinants of new product success are time to market, product performance and development resources. (For a comprehensive review of other determinants of new product success, see Krishnan and Ulrich (2000)). Previous research has considered only tradeoffs among a subset of these determinants. The economics R&D race literature assumes that the level of product performance is fixed and examines the tradeoff between time-to-market and development resources (Kamien and Schwartz, 1982; Reinganum, 1989). This tradeoff exists because ‘crashing’ a project costs money (Mansfield, 1988). Time-to-market has also been an active area of research in marketing (see, for example, Mahajan, Muller, and Kerin, 1984; Wilson and Norton, 1989; Lilien and Yoon, 1990). This stream of research, however, often does not consider explicitly product development-related issues. For example, it is often assumed that the development cycle and investment are negligible. Our model aims to integrate these separate streams of literature. We conceptualize new product development as a ‘product performance production’ process that requires scarce development resources. Under a control-theoretic formulation, the level of development resource intensity and time-to-market are control variables that can significantly influence the product performance (the state variable), which in turn affects the market share.
and life-cycle profits.

In practice, many firms focus their attention primarily on one or two of the success determinants; few place equal emphasis on all three. In mature industries, for example, managers tend to focus primarily on the total development cost. Start-up firms with limited development budget may also want to focus on the total development cost. In high-tech industries, on the other hand, new product evaluation is more likely to be based on product performance and the time-to-market (e.g., computer equipment). Most firms set strategic targets for one of these metrics in order to control the new product management process. We use our model to study the implications of setting such managerial targets for each of the three determinants of success.

Setting a target on time-to-market has become a commonplace strategy because of the increasingly compressed product life-cycles in many industries. For example, in response to competitive pressures, Ingersoll-Rand recently set the time-to-market for all of its new products under development in its industrial tools division to be one year (Cohen and Ho, 1996). We believe that the time factor must be put into context and that a short time-to-market must be weighed against its associated costs and potential impact on product performance. In industries where there are natural product introduction times (model year and season), firms may have less freedom to choose time-to-market. Examples include automobiles (beginning of the year), toys (Christmas season), and apparel (fashion season). In these cases, multiple targets can be set and studied using the proposed model.

A product performance target is often derived from a market share target. An ambitious product performance target can shape the development process in a way that leads to a revolutionary product introduction. For instance, Eastman Kodak sets product performance target to ensure market leadership by requiring their new product development teams to deliver a fixed increment of superiority relative to the best product in the market (Ho, 1993). The same approach can be also used by followers in a product market who wish to catch up with the leader. This approach may also be relevant in situations where firms race to overcome a certain technology barrier in order to develop a better new product (e.g., a new drug for a certain disease). It is certainly a chief metric in situations where product safety and liability are at stake (e.g., drugs, airplanes).
Setting a target for total development cost can be a result of an internal budget allocation process. In situations where development funds are limited, new product development teams may be directed to manage the development process in a manner that will not exceed the development cost budget. This approach is particularly common when the primary input to the development process is engineering labor input and the required engineering know-how is so specialized that it is impossible to hire additional people with the required know-how. In such cases, the development cost is constrained by the availability of the engineering personnel. It is also observed often in developing entertainment products such as movies.

The implications of setting specific managerial priorities on these critical determinants are unclear. We study the effect of setting a target on each of the determinants by modeling the resulting development process (with the targets) as a restricted case of a more general unconstrained development process. We benchmark the constrained development process against the globally optimal (unrestricted) process with respect to the level of resource intensity employed during the development process, the time-to-market, and the performance of new product at launch. We employ profitability as the common goal. Systematic deviations from the unconstrained process are highlighted so that firms who use the constrained processes can become aware of their potential impact.

The paper is organized as follows. In Section 2, we introduce the model formulation and validate the underlying model assumptions. Section 3 provides conditions to characterize the optimal policy under unconstrained development process and the three constrained development processes. Section 4 discusses the results and outlines future research directions.

2 Model Formulation

Figure 1 illustrates the basic problem scenarios (see Cohen, Eliashberg, and Ho, 1996 for a detailed justification and applicability of such scenarios). There is a fixed window of opportunity, $T$, beyond which the product has no value. This window of opportunity is divided into two broadly defined stages: the Development stage and the Marketing stage. The firm is assumed to have an existing product with performance $A_0$. At time $T_P$, a new product with performance $A(T_P)$ is launched. It is assumed that the new product completely replaces the existing prod-
uct. The firm’s objective is to maximize profit from existing and new products over the time window $T$. There are two strategic decisions, 1) choice of launching time, $T_p$, and 2) selection of a level of development resources over time, $X(t)$, to achieve an appropriate level of product performance, $A(T_p)$. These decisions are linked, since a high level of development resources will give rise to a faster development process. Under the setup illustrated here, a day spent in the Development stage means a delay of one day in the Marketing stage for the new product.

[ INSERT FIGURE 1 HERE ]

The level of development resources is measured in dollars. It is the strategic development decision. During the development stage, the development team improves the performance of the product. Enhancements in performance are achieved by climbing a “performance ladder.” Let the performance of the product at time $t$ be $A(t)$. The key relationship in our model framework is the speed of performance improvement. Specifically, we define speed of increment for performance as follows:

$$\frac{dA(t)}{dt} = \dot{A}(t) = K \cdot X(t)^\alpha,$$

where

- $X(t)$ = level of development resource at time $t$,
- $\alpha$ = resource productivity parameter,
- $K$ = constant of proportionality for speed of performance improvement. It is proportional to the level of capital investment in the development technology.

There are diminishing returns to resource input, and thus $0 < \alpha < 1$. Equation (2.1) is inspired by previous models assuming that research productivity is measured by rate of research output (e.g., number of patents per year) and is driven by resource intensity (see for example, Griliches, 1994). The performance improvement function is of the Cobb-Douglas form. In Ho (1993) and Cohen, Eliashberg and Ho (1997), we provide empirical evidence for such functional form, drawn from automobile, packaged software, and packaged goods industries. It suggests that the Cobb-Douglas form is a reasonable approximation for relating the speed of performance improvement to variation in the rate of resource input (in particular engineering labor input).
Some research suggests that selecting a target for the development team (either time to market, product performance, or total development cost) may generate the following psychological benefits (Foster et al. 1985a, 1985b, Cooper, 1994, Cooper and Tanaka, 1997):

1. Setting a target may lead to higher awareness, peer-pressure, visibility, and motivation so that the members of the development team become more productive.

2. Selecting a target may lead to a higher ‘effective’ resource intensity because members of the development teams work harder via over-time in order to meet the target.

Capturing these psychological benefits would entail a good specification of how the resource productivity parameter changes over time (i.e., $\alpha$ becomes $\alpha(t)$). We are not aware of an empirically well-grounded specification. Consequently, we leave this for further research.

Based on (2.1), the performance of the new product at time $t$ is:

$$A(t) = A_0 + \int_0^t K \cdot X(s) \, ds.$$  \hspace{1cm} (2.2)

Assuming for a moment that the level of resource is fixed ($X(t) = X$), equation (2.2) implies that the speed of performance improvement is invariant with time. That is, performance of the new product at its launching time, $A(T_P)$, increases linearly with time to market, $T_P$:

$$A(T_P) = A_0 + K \cdot X^\alpha \cdot T_P.$$  \hspace{1cm} (2.3)

In order to investigate the empirical validity of (2.3), four project managers at a company site were asked to provide estimates for the expected time to market under a number of hypothetical product performance levels given a fixed development resource level (for details on data collection, see Ho, 1993). Table 1 provides the managers’ estimates. Data have been normalized to

---

1 A reviewer has pointed out that the effects of the metrics may actually be felt over time so one should consider multiple new product generations in order to fully capture their benefits. Clearly, our model captures only the ‘steady-state’ or ‘equilibrium’ behavior.

2 In the analyses of the optimal policies (section 3), we prove formally that such a stationary policy of employing the level of resource is indeed optimal.

3 The questionnaire was constructed after several rounds of structured interviews with the project members. Each project manager was asked to provide six estimates for the revised time to market given the same level of resources. To obtain more accurate estimates, project managers were probed about the reduced or additional development associated with the revised new product performance level. Our methodology is similar to that of Mansfield (1988).
the values of current targets.\textsuperscript{4}

Figure 2 shows the results of regressing $T_P$ against $A(T_P)$. The statistical significance of the parameter estimates is indicated by asterisk and the standard error is given in the parentheses. Given the statistically significant parameter estimates for $T_P$ and the high adjusted R-squared values across all projects (from 0.83-0.98), it appears that product performance and time-to-market are linearly related, at least in terms of managers’ expectations, for the range or product performance of interests. In summary, the results of this anecdotal study provide some empirical support for performance improvement equation (2.3).

The firm’s market share is a function of both its own product performance and the product performance of its competition. A reasonable market share function, frequently used in the marketing literature, is the attraction model (Bell, Keeney, and Little, 1975). The attraction model has been employed extensively in marketing and has received empirical support (See Cooper and Nakanishi, 1988 for a good review). The net revenue rate at time $t$ for the firm that develops and introduces the new product is the product of the product category demand rate, the profit margin, and and the firm’s market share:

$$R(t) = \begin{cases} 
R_0 \cdot \frac{A_0}{A_0 + A_c}, & 0 \leq t < T_P, \\
R_1 \cdot \frac{A(T_P)}{A(T_P) + A_c}, & T_P \leq t < T, 
\end{cases} \quad (2.4)$$

where

\textsuperscript{4}This helps to ensure confidentiality of the data. Moreover, the project managers appeared more comfortable at providing data that used current targets as reference points.
\[ R(t) = \text{net revenue rate at } t \text{ for the firm}, \]
\[ R_0 = \text{product category net revenue rate for the existing product}, \]
\[ R_1 = \text{product category net revenue rate for the new product}, \]
\[ A_0 = \text{performance level of the existing product}, \]
\[ A(T_P) = \text{performance level of the new product}, \]
\[ A_c = \text{competitive product performance level during the time horizon}. \]

The cumulative development costs of the new product at time \( t \) are:
\[ TC(t) = \int_{0}^{t} X(s) ds. \tag{2.5} \]

The firm’s cumulative profit at time \( t \) is determined as follows,
\[ T\Pi(t) = TR(t) - TC(t), \tag{2.6} \]
where \( TR(t) \) and \( TC(t) \) are total revenues and costs at time \( t \), respectively. The total revenue function is given by:
\[ TR(t) = \int_{0}^{t} R(s) ds. \tag{2.7} \]

where \( R(\cdot) \) is given in (2.4). The firm’s decision set is \( \Delta = \{ X(t), T_P \} \). We define the cumulative profit function, \( T\Pi(\delta, T) \), as the total profit, by end of the window of opportunity, with decision \( \delta \in \Delta \). The firm’s decision problem can be stated as
\[ \max_{\delta \in \Delta} T\Pi(\delta, T) = TR(\delta^*, T) - TC(\delta^*, T) = T\Pi^*(\delta^*, T). \tag{2.8} \]

The combination of equations (2.1) through (2.7) generates an explicit representation of the firm’s cumulative profit by the end of time horizon. This substitution yields:

\[ T\Pi^*(\delta^*, T) = \max_{\delta \in \Delta} \left[ R_0 \cdot \frac{A_0}{A_0 + A_c} \cdot T_P \right. \]
\[ + R_1 \cdot \frac{A_0 + \int_{0}^{T_P} K \cdot X(s)^{\gamma} ds}{A_0 + \int_{0}^{T_P} K \cdot X(s)^{\gamma} ds + A_c} \cdot (T - T_P) - \int_{0}^{T_P} X(s) ds \] \]
The optimization problem \([G1]\) can be reformulated as a an optimal control problem with state variable, \(A(t)\), and two control decisions, \(X(t)\) and \(T_P\) (Kamien and Schwartz, 1982). In optimal control terminology, the salvage term, \(\Phi(T_P, A(T_P))\) is defined as follows:

\[
\Phi(T_P, A(T_P)) \overset{\text{def}}{=} R_0 \frac{A_0}{A_0 + A_c} \cdot T_P + R_1 \cdot \frac{A(T_P)}{A(T_P) + A_c} \cdot (T - T_P).
\] (2.10)

The optimization problem \([G1]\) becomes:

\([G2]\)

\[
\max T \Pi(X(t), T_P) = \int_0^{T_P} -X(t)dt + \Phi(T_P, A(T_P))
\] (2.11)

subject to

\[
\dot{A}(t) = K \cdot X(t)^\alpha, \quad (2.12)
\]

\[A(0) = A_0, \quad \text{fixed}, \quad (2.13)\]

\[T_P, \ A(T_P), \ \text{are free}. \quad (2.14)\]

3 Analyses of Optimal New Product Development Policies

3.1 The Unconstrained New Product Development Process

The first proposition concerns the structure of the optimal level of resource intensity in an unconstrained development process (i.e., no restrictions on controls).

Proposition 1 : The optimal level of resource intensity is time invariant, i.e. \(X^*(t) = X^*, \forall t\).

Proof: See Appendix.
The intuition behind this result is based on two observations. First, the performance production process exhibits diminishing return to scale (i.e., the speed of performance improvement is strictly concave in the resource intensity). Thus, the average of the speeds of performance improvement at any two resource intensity levels is strictly smaller than the speed of performance improvement at the average of the two resource intensity levels. Second, the performance of the new product before launch has no implication on the life-cycle profits. That is, for a given level of product performance at launch, the evolution of the product performance during development does not matter as long as it begins with the same initial product performance. These two observations explain the above structural (stationarity) result.

The same result can be generalized to a new product development process that has multiple stages as long as the performance improvement over stages are additive in nature. Here, it can be shown that the level of resource intensity at each of the development stages should be constant across time if it is diminishing return to scale but different stages can have different levels of resource intensity (see Ho, 1993).

Having established formally that the optimal level of resource intensity is time invariant simplifies the mathematical derivations greatly. Since resource intensity is stationary (i.e., \( X(t) = X \)), we have \( A(T_P) = \int_0^{T_P} K \cdot X(t)^\alpha dt = A_0 + K \cdot X^\alpha \cdot T_P \). Consequently, we can express the salvage term \( \Phi(T_P, A(T_P)) \) in terms of \( X \) and \( T_P \) explicitly. Substituting \( A(T_P) = A_0 + K \cdot X^\alpha \cdot T_P \) into equation (2.10), we have:

\[
\Phi(T_P, X) = R_0 \frac{A_0}{A_0 + A_c} \cdot T_P + R_1 \cdot \frac{A_0 + K \cdot X^\alpha \cdot T_P}{A_0 + K \cdot X^\alpha \cdot T_P + A_c} \cdot (T - T_P). 
\]

The second proposition characterizes the globally optimal policies.

**Proposition 2**: Under the unconstrained development process, the optimal level of resource intensity \( (X^*) \) and time to market \( (T_P^*) \) are jointly characterized by following optimality conditions:

\[
\frac{R_1 \cdot A_c}{A_0 + K \cdot (X^*)^\alpha \cdot T_P^* + A_c} \cdot (T - T_P^*) = \frac{1}{\alpha} \cdot \frac{A_0 + K \cdot (X^*)^\alpha \cdot T_P^* + A_c}{K \cdot (X^*)^\alpha \cdot T_P^*} \cdot [X^* \cdot T_P^*],
\]
\begin{equation}
R_1 \cdot \frac{A_0 + K \cdot (X^* \cdot T^*_P)}{A_0 + K \cdot (X^* \cdot T^*_P + A_c)} - R_0 \cdot \frac{A_0}{A_0 + A_c} = X^* \cdot \frac{1 - \alpha}{\alpha}.
\end{equation}

\textit{Proof: See Appendix.}

The above optimality conditions can be interpreted as follows. The first optimality condition \(3.2\) specifies a condition relating to the competitive’s cumulative net revenue after product launch. It states that the resource intensity and time to market should be chosen such that the competitive’s cumulative net revenue after product launch (during the time window \(T - T^*\)) (the left-hand-side) is equal to a factor times the total development cost \((X^* \cdot T^*_P)\) (the last term of the right-hand-side). This factor is the ratio of the total product performance in the market after launch \((A_0 + K \cdot (X^*)^\alpha \cdot T^*_P + A_c)\) to the product of the increase in the firm’s product performance \((K \cdot (X^*)^\alpha \cdot T^*_P)\) and its resource productivity parameter \(\alpha\). This optimality condition suggests that a compression of product life-cycle (i.e., a decrease in \(T\)) will be accompanied by either a lower total development cost (i.e., a smaller \(X^* \cdot T^*_P\)) or a lower level of product performance improvement (i.e., a smaller \(K \cdot (X^*)^\alpha \cdot T^*_P\)). For a fixed level of resource intensity, this means an early and more evolutionary product innovation.

The second optimality condition \(3.3\) relates the increase in the firm’s net revenue rate after product launch (the left-hand-side) to the optimal resource intensity. It states that the former is a factor times the latter and the factor is the ratio of one minus the resource productivity parameter and the resource productivity parameter. Consequently, a firm that has an attractive existing product (high \(A_0\)) will have a lower optimal resource intensity and a longer time to market. In other words, an attractive existing product reduces the need for the firm to rush to market by employing a higher level of resource intensity.

Another way to interpret the two optimality conditions is to apply Dorfman-Steiner (1954) theory on static monopolistic optimization. Recalling the definition of the salvage term in equation \(3.1\), the elasticities of the net revenue with respect to the decision variables (time to market \((T_P)\) and resource intensity \((X)\)) are given by \(e_{T_P} = \frac{\delta \Phi}{\delta T_P}\) and \(e_X = \frac{\delta \Phi}{\delta X}\), respectively. At optimality, \(\frac{\delta \Phi}{\delta T_P} = X^*\) and \(\frac{\delta \Phi}{\delta X} = T^*_P\). Thus, the two elasticities are identical and given by \(\frac{X^* \cdot T^*_P}{\Phi(T^*_P, X^*)}\). That is, at optimality, the elasticities are both equal to the total development cost per unit of net revenue.
3.2 The Target Time-to-Market ($\hat{T}_P$) Development Process

The target time-to-market development process is equivalent to setting $T_P$ equal to or less than a particular value $\hat{T}_P$, in problem G2. We consider the interesting case where the firm sets an ambitious (accelerated) target for time-to-market (i.e., $\hat{T}_P < T_P^*$).

Proposition 3 characterizes the optimal level of the development resource under the target timing development process. It is denoted as $X^T$.

**Proposition 3**: Under the target time-to-market development process, the optimal level of development resource ($X^T$) is characterized by the following optimality condition:

$$
\frac{R_1 \cdot A_c}{A_0 + K \cdot (X^T)^{\alpha} \cdot T_P + A_c} \cdot (T - \hat{T}_P) = \frac{1}{\alpha} \cdot \left[ \frac{A_0 + K \cdot (X^T)^{\alpha} \cdot \hat{T}_P + A_c}{K \cdot (X^T)^{\alpha} \cdot T_P} \right] \cdot [X^T \cdot \hat{T}_P] 
$$

(3.4)

**Proof**: See Appendix.

Unlike the unconstrained development process, the target timing development process has only one optimality condition. This optimality condition is similar to the optimality condition (3.2) except we have the term $\hat{T}_P$ instead of $T_P^*$. If the firm sets an ambitious target for time to market (which is common in practice) so that $\hat{T}_P < T_P^*$, it can be shown that $X^T > X^*$ and $A(T_P^*) < A(T_P^*)$. Thus we expect to see a systematic upward bias in the development resource intensity and a more evolutionary product performance under the target timing development process.

Comparing $X^T$ with $X^*$, we note that $X^T$ is not a function of $R_0$ and $A_0$, whereas $X^*$ is. This suggests that the target time-to-market development process can be seriously flawed in situations when the firm has an existing product that has a high net revenue rate. That is, the target time-to-market development process fails to account for the cannibalistic effect of new product on the existing product.

\footnote{It follows from the fact that the left-hand-side is decreasing in $\hat{T}_P$ and increasing in $X^D$ and the right-hand-side is increasing in $\hat{T}_P$ and decreasing in $X^D$.}
Indeed, Cohen and Ho (1996) investigated strategies of several new product launches by an industrial equipment firm. They observed that a short time to market target across all product launches can be problematic. This is so because the approach fails to account for differences in market characteristics especially the degree of success of the existing products. Consequently, the new product launches did not meet the firm’s expectation of success.

Similarly, we can interpret the above optimality condition using the concept of elasticity. The elasticity of the net revenue with respect to resource intensity \((X)\) is given by \(e_X = \frac{\delta \Phi}{\delta X} = \frac{X^T \cdot T_P}{\Phi(T_P, X^T)}\). At optimality, \(\frac{\delta \Phi}{\delta X} = T_P\) and hence the elasticity is given by 

\[
3.3 \text{ The Target Performance (\(\hat{A}\)) Development Process}
\]

The target performance development process is equivalent to constraining the state variable \(A(t)\) to be equal to or greater than a fixed value, \(\hat{A}\), in problem G2. We consider the interesting case of \(\hat{A} \geq A(T_P^*)\) where the firm sets an ambitious performance target.

**Proposition 4**: Under the target performance development process, the optimal level of resource intensity \((X_{Perf})\) and time to market \((T_{Perf}^P)\) are given by the following closed-form expressions:

\[
X_{Perf} = \frac{R_1 \cdot \frac{\hat{A}}{A + A_c} - R_0 \cdot \frac{A}{A_0 + A_c}}{(\frac{1}{\alpha} - 1)}, \quad (3.5)
\]

\[
T_{Perf}^P = \frac{(\hat{A} - A_0) \cdot (\frac{1}{\alpha} - 1)^\alpha}{K \cdot \left[R_1 \cdot \frac{\hat{A}}{A + A_c} - R_0 \cdot \frac{A}{A_0 + A_c}\right]^\alpha}. \quad (3.6)
\]

In addition, total development cost, \(TC\), as a function of the optimal time to market \(T_{Perf}^P\) when the size of the development team is chosen optimally, is given as follows:

\[
TC = \frac{[A - A_0]^{\frac{1}{\alpha}}}{[T_{Perf}^P]^{\frac{1}{\alpha}}}. \quad (3.7)
\]

**Proof**: See Appendix.
The structure of the optimal new product development policy is interesting. Given a product performance target \( \hat{A} \), the strategic decisions \( X^{Post} \) and \( T^{Post} \) become separable. In other words, \( X^{Post} \) is not a function of \( T^{Post} \) and vice versa. This suggests that once a strategic target level of performance (\( \hat{A} \)) is chosen, the timing and level of resource intensity decisions can be decentralized. In this respect, the target performance development process has an edge over other development processes because it requires less coordination between the marketing and R&D functions.

If \( \hat{A} > A(T^*_p) \), then \( X^{Post} > X^* \). Thus there is systematic upward bias in the development resource intensity under the target performance development process. Note that the optimal resource intensity under the target performance development process \( X^{Post} \) is not a function of \( K \), whereas it is under the unconstrained development process (compare equations (3.2) and (3.5)). This suggests that the development resource intensity under the target performance development process can be seriously biased when the level of capital investment in the development technology changes rapidly. From equation (3.6), it can be easily shown that \( T^{Post} > T^*_p \) under most combinations of parameters. Thus, the target performance development process may lead to delayed product launches that miss the window of opportunity.

Proposition 4 has several implications that can be obtained via standard comparative statics analyses. It implies that the optimal resource intensity should be larger if the target level of product performance (\( \hat{A} \)) is high. Put differently, a revolutionary new product should be accompanied by a more intense development resource strategy. The optimal resource intensity increases with the product category net revenue rate of the new product market (\( R_1 \)). Also, it increases with the labor productivity parameter (\( \alpha \)). It decreases, however, with the product category net revenue rate of the existing product \( R_0 \), and the performance level of the existing product (\( A_0 \)). The optimal time-to-market, \( T^{Post} \), decreases with the product category net revenue rate of the new product market (\( R_1 \)), the labor productivity parameter (\( \alpha \)), and the constant of proportionality for speed of performance improvement (\( K \)). It increases with product category net revenue rate of the existing product \( R_0 \).

---

\(^6\)This follows directly from equation (3.5). Note that \( X^{Post} = X^* \) if \( \hat{A} = A(T^*_p) \).
Since \( 0 < \alpha < 1 \), total development cost, \( TC \) in (3.7), is a decreasing convex function of time to market. This result links our work with the economics/R&D race and PERT/CPM literatures. The R&D race literature assumes that firms pursue a fixed performance target and that development cost is convex in time to market. The PERT/CPM literature shows that for a given R&D project complexity, and if each separate project activity has a linear time-cost tradeoff, then total development cost is a convex function of time to market (Fulkerson, 1961; Rosenbloom, 1964). Proposition 4 provides an analytical basis for this convex cost relationship.

The target performance is frequently employed by companies where product liability is crucial to the company’s reputation and survival (e.g., pharmaceutical, aircraft manufacturing). This extreme emphasis on performance involves numerous and rigorous product testing that lead to extended time to market. This often leads to less than optimal profitability. The gap between actual and optimal profitability represents the cost of insurance the firm bears to protect itself from a product liability suit.

### 3.4 The Target Development Cost (\( \hat{TC} \)) Process

The approach is similar to the globally optimal procedure [G2] except that the term \( \int_0^{T_P} X(t)dt \) is constrained to be equal to or less than \( \hat{TC} \). We consider the interesting case of a limited budget where \( \hat{TC} < X^* \cdot T_P \) (i.e., under spending). Indeed, Ming and Eliashberg (2000) observe under-spending behavior on product development in the pharmaceutical industry.

**Proposition 5:** Under the target development cost process, the optimal level of resource intensity \( (X^D) \) and time to market \( (T^D_P) \) are characterized by the following optimality conditions:

\[
R_1 \cdot \frac{A_0 + K \cdot (\hat{TC})^\alpha \cdot (T^D_P)^{1-\alpha}}{A_0 + K \cdot (TC)^\alpha \cdot (T^D_P)^{1-\alpha} + A_c} - R_0 \cdot \frac{A_0}{A_0 + A_c} = (1 - \alpha) \cdot \left[ \frac{K \cdot (\hat{TC})^\alpha (T^D_P)^{-\alpha} \cdot (T - T^D_P)}{A_0 + K \cdot (TC)^\alpha \cdot (T^D_P)^{1-\alpha} + A_c} \right] \cdot \left[ \frac{R_1 \cdot A_c}{A_0 + K \cdot (TC)^\alpha \cdot (T^D_P)^{1-\alpha} + A_c} \right],
\]

\[
X^D = \frac{\hat{TC}}{T^D_P}.
\]

**Proof:** See Appendix.
The optimality condition (3.8) relates the increase in the firm’s net revenue rate (the left-hand-side) to that of the competitive’s net revenue rate (the last term of the right-hand-side) after product launch. The former is a factor of the latter and the factor is the product of one minus resource productivity parameter and the ratio of maximal allowable increase in the firm’s product performance (i.e., $K \cdot (\bar{T}C)^\alpha \cdot (T^D - T_P) = K \cdot (X^D)^\alpha \cdot (T - T_P)$) and the total product performance in the market after launch.

If limited budget is available ($\bar{T}C < X^*_D \cdot T^*_P$), then either $X^D < X^*$ or $T^D_P < T^*_P$ must be true. If the former is true ($X^D < X^*$), then $A(T^D_P) < A(T^*_P)$ is true for most combinations of parameters. However, if $T^D_P < T^*_P$, it is necessary that $A(T^D_P) < A(T^*_P)$. Thus, the new product launch under the target development cost process tends to be premature with evolutionary product innovations.

4 Competitive Scenarios

A limitation of the analyses discussed so far is that we do not explicitly capture competition. There are two ways to model competition under our modeling set up. The first way is to capture an increasingly competitive environment by replacing $A_c$ with $A_c(t)$ where $A_c(t)$ is increasing in $t$. This is the case where the underlying firm is not explicitly competing with any particular firm in an increasing performance norm industry. The second way is to model the new product launch as a truly competitive game. Here we focus on the leading firm. We analyze how the firm will have to take into account her action on a follower. Both scenarios are discussed next.

4.1 Passive Competitive Scenarios

We model a non-stationary environment where the industry performance norm gradually increases over time such that $A_c(t) = A_c + \beta \cdot t$. Under this scenario, the firm’s profit function becomes for decision vector $\delta = \{T_P, X\}$ is:

$$
T\Pi(\delta, T) = \int_0^{T_P} R_0 \cdot \frac{A_0}{A_0 + A_c + \beta \cdot t} dt + \int_0^{T} R_1 \cdot \frac{A_0 + K \cdot X^\alpha \cdot T_P}{A_0 + K \cdot X^\alpha \cdot T_P + A_c + \beta \cdot t} dt - X \cdot T_P
$$
The next proposition characterizes the globally optimal policies.

**Proposition 6**: Under the unconstrained development process and a non-stationary competitive environment such that \( A_c(t) = A_{c0} + \beta \cdot t \), and if \( \beta \) is small enough, the optimal level of resource intensity \((X^*)\) and time to market \((T_P^*)\) are jointly characterized by following optimality conditions:

\[
\frac{R_1 \cdot (A_{c0} + \beta \cdot T)}{A_0 + K \cdot (X^*)^\alpha \cdot T_P^* + A_{c0} + \beta \cdot T} \cdot (T - T_P) = \frac{\beta^2 \cdot (T - T_P)^2}{2(A_0 + K \cdot (X^*)^\alpha \cdot T_P^* + A_{c0} + \beta \cdot T_P^*)}
\]

\[
= \frac{1}{\alpha} \cdot \left[ \frac{A_0 + K \cdot (X^*)^\alpha \cdot T_P^* + A_{c0} + \beta \cdot T_P^*}{K \cdot (X^*)^\alpha \cdot T_P^*} \right] \cdot [X^* \cdot T_P^*],
\]

\[
R_1 \cdot \frac{A_0 + K \cdot (X^*)^\alpha \cdot T_P^*}{A_0 + K \cdot (X^*)^\alpha \cdot T_P^* + A_{c0} + \beta \cdot T_P^*} - R_0 \cdot \frac{A_0}{A_0 + A_{c0} + \beta \cdot T_P^*} = x^* \cdot \frac{1 - \alpha}{\alpha}.
\]

**Proof**: See Appendix.

Comparing the above optimality conditions with those given in equations (3.2)-(3.3) for the stationary competitive environment, we note the following similarities and differences:

1. The two sets of optimality conditions are similar except for an extra term associated with the increasingly competitive case in the first optimality condition.

2. The optimality conditions for the increasingly competitive case help to clarify those for the stationary case. We note that the denominator in the LHS of the first optimality condition is the total industry product performance at the end of the life cycle while the denominator in the LHS of the second optimality condition is the total industry product performance immediately right after the product launch. (Since they are identical in the stationary case, there is no way to make this distinction).
3. We shall focus on optimality condition (3.2). The term in the LHS is decreasing in \( T^*_P \) and \( X^* \) and the term in the right hand side in increasing in \( T^*_P \) and \( X^* \). Since the extra term is strictly positive in optimality condition (4.2), the effect of an increasingly competitive environment is to make both sides of the optimality condition take a lower value. This can be accomplished by either a lower \( X^* \) or \( T^*_P \). The optimality conditions (3.3) and (4.3) suggest that a lower \( T^*_P \) must be accomplished by a lower \( X^* \) (since the LHS is increasing in \( X^* \) and \( T^*_P \) and the RHS side is in increasing in \( X^* \) and is independent of \( T^*_P \)). Thus we expect either a

earlier product launch accompanied by a lower investment

in resource intensity or simply a lower investment in resource intensity in an increasingly competitive environment.

### 4.2 Active Competitive Scenarios

Active competitive scenarios incorporating incumbents and new entrants (e.g., Eliashberg and Jeuland, 1986; Roy, et al., 1994; Purohit, 1994) have typically employed the Stackelberg game set up (Stackelberg, 1934). The competitive environment of interest is one in which new product development is undertaken by a leader \( L \) and a follower \( F \). The leader believes that the follower will react to the leader’s choice of time to market in a best response fashion. Knowing this, the leader then chooses a time to market that maximizes her profit. The leader and the follower may have different levels of initial product performance \((A^L_0, A^F_0)\) and performance improvement functions \((\dot{A}^L, \dot{A}^F)\). Below, we assume that both firms have exogenously given performance improvement functions (i.e., a fixed resource intensity \( X \) so that the decision set \( \Delta = \{T_P\} \)) and that the leader has a higher speed of product performance than the follower (i.e., \( \dot{A}^L > \dot{A}^F \)). Figure 3 shows the total industry product performance over time under this competitive environment. Note that we now have two discrete jumps in the total industry performance (rather than one discrete jump in the total industry performance).

[ INSERT FIGURE 3 HERE ]

#### 4.2.1 Leader Facing a Prepared Follower

The follower is taken to be ‘prepared’ in the sense that it starts product development at exactly the same time as the leader. Let the time to market of firms \( L \) and \( F \) be \( T^*_P \) and \( T^*_F \) respectively.
With the usual notations, leader and follower’s life-cycle profits are:

\[
TP_L(T^L_T, T^F_F) = \frac{R_0 \cdot A^L_0}{A^L_0 + A^F_0} \cdot T^L_T + \frac{R_1 \cdot (A^L_0 + \hat{A}_L \cdot T^L_T)}{A^L_0 + A^L \cdot T^L_T + A^F_0} \cdot (T^F_F - T^L_T) \\
+ \frac{R_1 \cdot (A^L_0 + \hat{A}_L \cdot T^L_T)}{A^L_0 + A^L \cdot T^L_T + A^F_0} \cdot (T - T^F_F) - X^L \cdot T^L_T, \tag{4.4}
\]

\[
TP_F(T^L_T, T^F_F) = \frac{R_0 \cdot A^F_0}{A^L_0 + A^F_0} \cdot T^L_T + \frac{R_1 \cdot A^F_0}{A^L_0 + A^L \cdot T^L_T + A^F_0} \cdot (T^F_F - T^L_T) \\
+ \frac{R_1 \cdot (A^F_0 + \hat{A}_F \cdot T^F_F)}{A^L_0 + A^L \cdot T^L_T + A^F_0 + A^F \cdot T^F_F} \cdot (T - T^F_F) - X^F \cdot T^F_F. \tag{4.5}
\]

The following lemma characterizes the prepared follower’s optimal best response given a leader’s choice of \(T^L_T\).

**Lemma 1**: The optimal time to market for the follower given a \(T^L_T\) is:

\[
T^F_F(T^L_T) = \sqrt{\frac{R_1(A^L_0 + A^L \cdot T^L_T)(A^F_0 + \hat{A}_F \cdot T + A^F_0 + A^L \cdot T^L_T)}{R_1 + X^F \cdot A^L_0 + A^F_0 + A^L \cdot T^L_T} - (A^L_0 + \hat{A}_L \cdot T^L_T)} \tag{4.6}
\]

**Proof**: See Appendix.

The following proposition characterizes the optimal time to market for the leader under certain conditions:

**Proposition 7**: Define \(\rho = \frac{\hat{A}_L}{\hat{A}_F}\) be the relative development capability of the leader. If \(A^L_0 = A^L = 0\) and \(R_1 \gg \max\{X^F, X^L\}\), then the optimal time to market for the leader is:

\[
T^L_T^* = \left[\frac{-(4\rho + 1) + (2\rho + 1)\sqrt{4\rho + 1}}{2(4\rho + 1)^2}\right]T \tag{4.7}
\]

**Proof**: See Appendix.

It can be easily shown that \(T^L_T^*\) is decreasing in \(\rho\) so that the stronger the advantage of the leader in product development capability, the sooner is her optimal time to market. Note that the advantage in product development capability can come from resource allocated to product development (i.e., \(X\)) or the level of productivity in resource utilization (i.e., \(\alpha\)).
4.2.2 Leader Facing a Surprised Follower

The follower is taken to be ‘surprised’ in the sense that it starts product development only after the leader has launched its new product. Let \( T_F^L \) be the follower’s time to market measured from \( T_F^L \). The leader and follower’s life-cycle profits are:

\[
\begin{align*}
\Pi_L(T_F^L, T_F^F) & = R_0 \cdot A_0^L \cdot T_F^L + \frac{R_1 \cdot (A_0^L + \Delta_L \cdot T_F^L)}{A_0^L + A_L \cdot T_F^L + A_0^F} \cdot T_F^F \\
& + \frac{R_1 \cdot (A_0^L + \Delta_L \cdot T_F^L)}{A_0^L + A_L \cdot T_F^L + A_0^F + A_F} \cdot (T - T_F^F - T_F^L) - X_L \cdot T_F^L \\
\Pi_F(T_F^L, T_F^F) & = R_0 \cdot A_0^F \cdot T_F^F + \frac{R_1 \cdot A_0^F}{A_0^F + A_F \cdot T_F^F + A_0^F + A_F} \cdot T_F^F \\
& + \frac{R_1 \cdot (A_0^F + \Delta_F \cdot T_F^F)}{A_0^F + A_L \cdot T_F^F + A_0^F + A_F} \cdot (T - T_F^F - T_F^L) - X_F \cdot T_F^F
\end{align*}
\]

The following lemma characterizes the prepared follower’s optimal best response given a leader’s choice of \( T_F^L \).

**Lemma 2**: The optimal time to market for the follower given a \( T_F^L \) is:

\[
T_F^*(T_F^L) = \sqrt{\frac{R_1 \cdot (A_0^F + \Delta_F \cdot T_F^L)(A_0^F + A_F \cdot T_F^L + A_0^L + A_L \cdot T_F^L)}{R_1 + X_F - \frac{R_0 \cdot A_0^F}{A_0^F + A_0^L + A_L \cdot T_F^L}} - (A_0^F + A_0^L + \Delta_L \cdot T_F^L)}
\]

**Proof**: See Appendix.

The following proposition characterizes the optimal time to market for the leader under certain conditions:

**Proposition 8**: Define \( \rho = \frac{\Delta_L}{\Delta_F} \). If \( A_0^F = A_0^L = 0 \) and \( R_1 \gg \max\{X_F, X_L\} \), then the optimal time to market for the leader is:

\[
T_F^L = \frac{-4 + \sqrt{16 + 4(4\rho - 1)}}{2(4\rho - 1)}|T|
\]

**Proof**: See Appendix.
Again, it can be easily shown that $T_{p^*}^{L_\rho}$ is decreasing in $\rho$ so that the stronger the advantage of the leader, the smaller is her optimal to market.

It is interesting to compare the optimal launching times under both scenarios. The following proposition establishes the value of “surprising” the follower:

**Proposition 9**: If $A_0^F = A_0^L = 0$ and $R_1 \gg \max\{X^F, X^L\}$, then the leader that faces a surprised follower always launches a product with higher level of performance and at a later time than the one that faces a prepared follower.

*Proof: See Appendix.*

Intuitively, the leader facing a prepared follower feels a greater pressure to launch the new product than the one who manages to surprise the competitor.

## 5 Discussion

We have developed a model for examining the interplay of three determinants of new product success: 1) time to market, 2) product performance, and 3) development cost. We have applied the model to analyze the merits and shortcomings of setting a target on each of the three new product performance metrics commonly used in industry: (1) the time-to-market target, (2) the product performance target, (3) the development cost target.

Our analytical results show that:

- An overly ambitious time-to-market target leads to an upward bias in resource intensity usage and a downward bias in product performance (i.e., evolutionary product innovation.) In addition, the optimal resource intensity is not affected by $R_0, A_0$. This result suggests that the target time-to-market development process may ignore the effect of cannibalization and thus can perform suboptimally if there is a significant degree of cannibalization in the existing product market (e.g., $R_0$ is large).
• Under the target performance development process, the coordination between marketing and R&D is easier because the resulting development resource intensity and time to market decisions are separable. An overly ambitious target leads to an upward bias in the development resource intensity. In addition, the resource intensity is not affected by changes in $K$ which capture capital investments in development technology. This result suggests that the target performance development process may not fully reflect the level of development capability in its development resource intensity decision and may perform suboptimally when there is a significant change in firm’s development capability. The target performance development process also leads to delayed product launches which miss the window of opportunity.

• Under the target development cost approach, allocating a limited budget to a new product development project can lead to a downward bias in product performance and a premature product launch (i.e., an evolutionary product introduction).

• A firm facing a gradually increasing performance norm in her industry will lower the investment in resource intensity.

• A leader facing a prepared follower feels a greater time pressure to launch the new product than the one who manages to surprise the competitor.

Extensions of the work described in this paper could include explicit incorporation of risk (both for product development and in the market). In addition, the policy implications of our results suggest a number of testable hypotheses, which can be studied using cross-sectional procedures. Finally, the modeling approach, presented here, can be implemented through a decision support system in a specific company context (see Cohen, Eliashberg and Ho, 1997). Application of such a system would introduce formalism and rigor to a complex and critical management process.

6 Appendix

Proof of Proposition 1

Suppressing the time argument, the Hamiltonian of G2 is:

$$H = -X + \lambda \cdot K \cdot X^0.$$ (6.1)
The necessary conditions for optimality are (Kamien and Schwartz, 1992):

\[ H_X = 0, \quad (6.2) \]
\[ \lambda^* = -H_A, \quad (6.3) \]
\[ H(T_P^*) = -\Phi_{T_P}(T_P^*) \text{ since } T_P \text{ is free}, \quad (6.4) \]
\[ \lambda^*(T_P^*) = \Phi_A(T_P^*) \text{ since } A(T_P) \text{ is free}. \quad (6.5) \]

From (6.3), we have \( \dot{\lambda}_A^* = -H_A \). Since \( H \) is not a function of \( A \), we have \( \dot{\lambda}_A^* = 0 \). Differentiating (6.1) with respect to \( X \), we obtain:

\[ H_X = -1 + \lambda \cdot K \cdot X^{\alpha-1}. \quad (6.6) \]

Setting (6.6) to zero and solving for \( X^* \), we obtain:

\[ X^* = [\lambda^* \cdot K \cdot \alpha]^{-\frac{1}{\alpha}}. \quad (6.7) \]

Since \( \dot{\lambda}_A^* = 0 \) (i.e., \( \lambda_A^* \) is time invariant), \( X^* \) is time invariant. Q. E. D.

**Proof of Proposition 2**

From (6.1), we have the maximized Hamiltonian:

\[ H^* = -X^* + \lambda^* \cdot K \cdot (X^*)^\alpha. \quad (6.8) \]

Since \( X^* \) and \( \lambda^* \) are time invariant, \( H^* \) is also time invariant. Since \( H^* \) is independent of \( A \) for given \( \lambda^* \), the necessary optimality conditions (6.2) - (6.5) are also sufficient (Kamien and Schwartz, 1992, pp. 221-226).

From (6.5), we obtain:

\[ \lambda^* = \Phi_A(T_P^*) = R_1 \cdot \frac{A_c}{[A(T_P^*) + A_c]^2} \cdot (T - T_P^*). \quad (6.9) \]

Since \( \lambda^* \) is time invariant, it is completely determined from 0 to \( T_P^* \) by the RHS of equation (6.9). Substituting equation (6.9) into (6.7), we obtain the desired optimality condition (3.2).

From (6.4), we have:

\[ H(T_P^*) = -\Phi_{T_P}(T_P^*) = -R_0 \cdot \frac{A_0}{A_0 + A_c} + R_1 \cdot \frac{A(T_P^*)}{A(T_P^*) + A_c}. \quad (6.10) \]
$H^*$ is completely characterized from 0 to $T_p^*$ by RHS of equation (6.10) because it is time invariant. Equating (6.8) and (6.10) we obtain:

$$-X^* + \lambda^* \cdot K \cdot (X^*)^\alpha = -R_0 \cdot \frac{A_0}{A_0 + A_c} + R_1 \cdot \frac{A(T_p^*)}{A(T_p^*) + A_c}.$$

(6.11)

Simplifying terms, we obtain the desired optimality condition (3.3). Q. E. D.

**Proof of Proposition 3**

The necessary and sufficient optimality conditions are identical to problem [G2] except that condition (6.4) is no longer valid. The expression for optimal level of resource intensity (6.7) remains the same. The revised auxiliary variable ($\lambda^T$) is:

$$\lambda^T = R_1 \cdot \frac{A_c}{[A(T_p^*) + A_c]^2} \cdot (T - T_p^*).$$

(6.12)

The optimal level of resource intensity can be solved by substituting $\lambda^T$ into equation (6.7). Q. E. D.

**Proof of Proposition 4**

The necessary and sufficient optimality conditions are identical to problem [G2] except that condition (6.5) is no longer valid. The expression for optimal level of resource intensity (6.7) remains the same except that the auxiliary variable ($\lambda^{Perf}$) is now different. Instead of (6.5), we have the following transversality condition:

$$\int_{T_p^{Perf}}^{T_p} K \cdot [K \cdot \lambda^{Perf} \cdot \alpha]^{\frac{\alpha}{1-\alpha}} dt = \hat{A} - A_0$$

(6.13)
The above transversality condition requires that the total improvement in the level of performance from 0 to $T_{P_{Perf}}$ is $\hat{A} - A_0$. The revised auxiliary variable $\lambda_{Perf}$, is found to be:

$$\lambda_{A} = \frac{1}{\alpha} \left( \frac{\hat{A} - A_0}{K \cdot T_{P_{Perf}}} \right)^{\frac{1 - \alpha}{\alpha}}$$

(6.14)

From (6.4), we have:

$$H(T_{P_{Perf}}) = -\Phi_{T_{P}}(T_{P_{Perf}}) = -R_0 \cdot \frac{A_0}{A_0 + A_c} + R_1 \cdot \frac{\hat{A}}{A + A_c}.$$  

(6.15)

Equations (6.7) and (6.11) become:

$$X_{Perf} = [K \cdot \lambda_{Perf} \cdot \alpha]^{\frac{1}{1 - \alpha}},$$

(6.16)

$$X_{Perf} + R_1 \cdot \frac{\hat{A}}{A + A_c} = R_0 \cdot \frac{A_0}{A_0 + A_c} + \lambda_{Perf} \cdot K \cdot (X_{Perf})^\alpha.$$  

(6.17)

From equation (6.16), we have $\lambda_{Perf} = \frac{(X_{Perf})^{-\alpha}}{K \cdot \alpha}$. Substituting this into equation (6.17) and simplifying, we obtain:

$$X_{Perf} = \frac{R_1 \cdot \frac{\hat{A}}{A + A_c} - R_0 \cdot \frac{A_0}{A_0 + A_c}}{(\frac{1}{\alpha} - 1)}.$$  

(6.18)

The desired expression for $T_{P_{Q}}^Q$ can be found by substituting $X^Q$ into $K \cdot (X^Q)^\alpha \cdot T_{P_{Q}}^Q = \hat{A} - A_0$. This yields:

$$T_{P_{Perf}} = \frac{(\hat{A} - A_0) \cdot (\frac{1}{\alpha} - 1)^\alpha}{K \cdot [R_1 \cdot \frac{A}{A + A_c} - R_0 \cdot \frac{A_0}{A_0 + A_c}]^\alpha}.$$  

(6.19)

Given that the firm pursues a fixed performance target $\hat{A}$ and chooses the optimal size of the development team $X_{Perf}$, the total development cost, $TC$, as a function of $T_P$ is given by:

$$TC = X_{Perf} \cdot T_P.$$  

(6.20)
Since a fixed target level of performance is pursued, we have $K \cdot (X_{perf})^\alpha \cdot T_P = (\hat{A} - A_0)$, or $X_{perf} = \left[ \frac{\hat{A} - A_0}{K \cdot T_P} \right]^\frac{1}{\alpha}$. Substitute $X_{perf}$ into (6.20), we obtain:

$$
TC = \left[ \frac{\hat{A} - A_0}{K \cdot T_P} \right]^\frac{1}{\alpha} \cdot T_P
$$

(6.21)

$$
= \left[ \frac{\hat{A} - A_0}{K} \right]^\frac{1}{\alpha} \cdot T_P^{1 - \frac{1}{\alpha}}
$$

(6.22)

To prove that $TC$ is a decreasing convex function of $T_P$, we take the first and second derivatives of $TC$ with respect to $T_P$.

$$
\frac{\partial TC}{\partial T_P} = \left[ \frac{\hat{A} - A_0}{K} \right]^\frac{1}{\alpha} \cdot (1 - \frac{1}{\alpha}) \cdot T_P^{-\frac{1}{\alpha}}
$$

(6.23)

$$
\frac{\partial^2 TC}{\partial T_P^2} = \left[ \frac{\hat{A} - A_0}{K} \right]^\frac{1}{\alpha} \cdot (1 - \frac{1}{\alpha}) \cdot (-\frac{1}{\alpha}) \cdot T_P^{-\frac{1}{\alpha} - 1}
$$

(6.24)

Note that if $\alpha < 1$, we have $\frac{\partial TC}{\partial T_P} < 0$ and $\frac{\partial^2 TC}{\partial T_P^2} > 0$. Q. E. D.

**Proof of Proposition 5**

In problem [G2], if $\int_0^{Tr} X(t) dt = -T\dot{C}$, we have a static optimization problem (since we know that $X^D(t) = X^D$ at optimal). Substitute $A(T_P^D) = K \cdot (T\dot{C})^\alpha \cdot (T_P^D)^{1-\alpha}$ into $\Phi(T_P, A(T_P))$, the objective is a function of $T_P^D$ only. Differentiating $\Phi(T_P, A(T_P))$ with respect to $T_P^D$, we have:

$$
R_0 \cdot \frac{A_0}{A_0 + A_c} - R_1 \cdot \frac{A_0 + K \cdot (T\dot{C})^\alpha \cdot (T_P^D)^{1-\alpha}}{A_0 + (T\dot{C})^\alpha \cdot (T_P^D)^{1-\alpha} + A_c} + R_1 \cdot (1 - \alpha) \frac{A_c \cdot K \cdot (T\dot{C})^\alpha \cdot (T_P^D)^{-\alpha} \cdot (T - T_P^D)}{[A_0 + (T\dot{C})^\alpha \cdot (T_P^D)^{1-\alpha} + A_c]^2}
$$

Setting the above expression to zero and simplifying we obtain the desired first-order condition.

**Proof of Proposition 6**
Differentiating $T^I$ with respect to $T_P$ and $X$ yields the following first-order conditions:

$$\frac{R_1 \cdot (A_0 + K \cdot (X^*)^\alpha \cdot T_P^*)}{\beta} \cdot \left( \frac{K \cdot (X^*)^\alpha}{A_0 + K \cdot (X^*)^\alpha \cdot T_P^* + A_{c0} + \beta \cdot T_P^*} - \frac{K \cdot (X^*)^\alpha + \beta}{A_0 + K \cdot (X^*)^\alpha \cdot T_P^* + A_{c0} + \beta \cdot T_P^*} \right) + \ln \left( \frac{A_0 + K \cdot (X^*)^\alpha + A_{c0} + \beta \cdot T_P^*}{A_0 + K \cdot (X^*)^\alpha \cdot T_P^* + A_{c0} + \beta \cdot T_P^*} \right) \cdot \frac{R_1 \cdot K \cdot (X^*)^\alpha}{\beta} + \frac{R_0 \cdot A_0}{A_0 + A_{c0} + \beta \cdot T_P^*} - X^* = 0 \quad (6.25)$$

Simplifying equation (6.26), we have:

$$\frac{R_1 \cdot (A_0 + K \cdot (X^*)^\alpha \cdot T_P^*)}{\beta} \cdot \left( \frac{K \cdot (X^*)^\alpha}{A_0 + K \cdot (X^*)^\alpha \cdot T_P^* + A_{c0} + \beta \cdot T_P^*} - \frac{K \cdot (X^*)^\alpha}{A_0 + K \cdot (X^*)^\alpha \cdot T_P^* + A_{c0} + \beta \cdot T_P^*} \right) + \ln \left( \frac{A_0 + K \cdot (X^*)^\alpha \cdot T_P^* + A_{c0} + \beta \cdot T_P^*}{A_0 + K \cdot (X^*)^\alpha \cdot T_P^* + A_{c0} + \beta \cdot T_P^*} \right) \cdot \frac{R_1 \cdot K \cdot (X^*)^\alpha}{\beta} = \frac{X^*}{\alpha} \quad (6.27)$$

Substituting the above equation into equation (6.25), we have:

$$\frac{X^*}{\alpha} - \frac{R_1 \cdot (A_0 + K \cdot (X^*)^\alpha \cdot T_P^*)}{A_0 + K \cdot (X^*)^\alpha \cdot T_P^* + A_{c0} + \beta \cdot T_P^*} + \frac{R_0 \cdot A_0}{A_0 + A_{c0} + \beta \cdot T_P^*} - X^* = 0 \quad (6.28)$$

Simplifying, we obtain the desired optimality condition (4.3). From equation (6.27), we have:

$$\frac{R_1 \cdot (A_0 + K \cdot (X^*)^\alpha \cdot T_P^*)}{\beta} \cdot \left( \frac{K \cdot (X^*)^\alpha}{A_0 + K \cdot (X^*)^\alpha \cdot T_P^* + A_{c0} + \beta \cdot T_P^*} - \frac{K \cdot (X^*)^\alpha}{A_0 + K \cdot (X^*)^\alpha \cdot T_P^* + A_{c0} + \beta \cdot T_P^*} \right) + \ln \left( \frac{A_0 + K \cdot (X^*)^\alpha \cdot T_P^* + A_{c0} + \beta \cdot T_P^*}{A_0 + K \cdot (X^*)^\alpha \cdot T_P^* + A_{c0} + \beta \cdot T_P^*} \right) \cdot \frac{R_1 \cdot K \cdot (X^*)^\alpha}{\beta} = \frac{X^*}{\alpha} \quad (6.29)$$

Simplifying the above equation, we have:

$$\frac{R_1 \cdot K \cdot (X^*)^\alpha}{\beta} \cdot \left( \frac{(A_0 + K \cdot (X^*)^\alpha \cdot T_P^*) \cdot \beta(T_P^* - T)}{(A_0 + K \cdot (X^*)^\alpha \cdot T_P^* + A_{c0} + \beta \cdot T) \cdot \beta(T_P^* - T)} \right) + \ln \left( \frac{\beta \cdot (T - T_P^*)}{A_0 + K \cdot (X^*)^\alpha \cdot T_P^* + A_{c0} + \beta \cdot T_P^*} \right) = \frac{X^*}{\alpha} \quad (6.30)$$
If \( \beta \) is small enough such that \( \frac{\beta(T - T^*_P)}{\lambda_0 + K(T - T^*_P)} \ll 1 \), we can approximate \( \ln(1 + \frac{\beta(T - T^*_P)}{\lambda_0 + K(T - T^*_P)}) \) by \( \frac{\beta(T - T^*_P)^2}{2(\lambda_0 + K(T - T^*_P))} \). Simplifying, we obtain the desired optimality condition (4.2).

**Proof of Lemma 1**

The optimal time to market for the follower for a given \( T^*_P \) can be determined as in Proposition 1 with \( A_c = A^L_0 + \hat{A}^L \cdot T^*_P \). Thus we have the following optimality condition:

\[
\frac{R_1 \cdot A_c}{A^F_0 + A^F \cdot T^F_* + A^L_0 + A^L \cdot T^L_P} (T - T^F_*) = \frac{1}{\alpha_F} \cdot \left[ \frac{A_0 + \hat{A}^F \cdot T^F_P + A^L_0 + \hat{A}^L \cdot T^L_P}{A^F \cdot T^F_P} \right] \cdot [X^F \cdot T^F_P]
\]

The proposition follows directly from solving the above expression.

**Proof of Proposition 7**

If \( A^L_0 = A^F_0 = 0 \) and \( R_1 \gg \max\{X^F, X^L\} \), then equation (4.6) becomes:

\[
T^F_*(T^L_P) = \sqrt{\rho \cdot T^L_P (\rho T^L_P + T)} - \rho \cdot T^L_P
\]

where \( \rho = \frac{\hat{A}^L}{\alpha_F} \). Substituting \( T^F_*(T^L_P) \) into equation (4.4) and simplifying, we obtain:

\[
TP_{II}(T^L_P) = R_1 \cdot [2\sqrt{\rho T^L_P (\rho T^L_P + T)} - 2\rho T^L_P - T^L_P] - X^L \cdot T^L_P.
\]

\( TP_{II}(T^L_P) \) is concave in \( T^L_P \) because \( \sqrt{\rho T^L_P (\rho T^L_P + T)} \) is concave in \( T^L_P \). Thus the first order condition is necessary and sufficient. Differentiating \( TP_{II} \) with respect to \( T^L_P \) and setting the first derivative to zero, we obtain the following first-order condition:

\[
\frac{2\rho^2 \cdot T^L_P + \rho \cdot T}{\sqrt{\rho \cdot T^L_P (\rho T^L_P + T)}} = 2\rho + 1 \quad \frac{X^L}{R_1} \approx 2\rho + 1
\]

Simplifying the expression, we have the following quadratic expression:

\[
(4\rho^2 + \rho)(T^L_P)^2 + (4\rho \cdot T + T)T^L_P - \rho \cdot T^2 = 0.
\]
Solving the quadratic equation we obtain the required expression for $T_P^{L*}$.

**Proof of Lemma 2**

The optimal time to market for the follower for a given $T_P^L$ can be determined as in Proposition 1 with $A_c = A_0^L + \dot{A}^L \cdot T_P^L$ and has the time window of $(T - T_P^L)$. Thus we have the following optimality condition:

$$\frac{R_1 \cdot A_c}{A_0 + \dot{A}^F \cdot T_P^L + A_0^L + \dot{A}^L \cdot T_P^L} (T - T_P^L - T_P^{F*})$$

$$= \frac{1}{\alpha_F} \cdot \left(\frac{A_0 + \dot{A}^F \cdot T_P^L + A_0^L + \dot{A}^L \cdot T_P^L}{\dot{A}^F \cdot T_P^L}\right) \cdot [X^F \cdot T_P^F] \quad (6.35)$$

The proposition follows directly from solving the above expression.

**Proof of Proposition 8**

If $A_0^L = A_0^F = 0$ and $R_1 \gg \max \{X^F, X^L\}$, then equation (4.10) becomes:

$$T_P^{F*}(T_P^L) = \sqrt{\rho \cdot T_P^L (\rho \cdot T_P^L + (T - T_P^L))} - \rho \cdot T_P^L$$

where $\rho = \frac{\dot{A}^L}{\dot{A}^P}$. Substituting $T_P^{F*}(T_P^L)$ into equation (4.8) and simplifying, we obtain:

$$T\Pi_L(T_P^L) = R_1 \cdot [2\sqrt{\rho T_P^L (\rho T_P^L + (T - T_P^L))} - 2\rho T_P^L - T_P^L] - X^L \cdot T_P^L$$

Which is concave in $T_P^L$ as long as $\rho > 1$. Thus the first order condition is necessary and sufficient. Differentiating $T\Pi_L$ with respect to $T_P^L$ and setting the first derivative to zero, we obtain the following first-order condition:

$$\frac{(2\rho^2 - 2\rho)T_P^L + \rho \cdot T}{\rho T_P^L (\rho T_P^L + T - T_P^L)} = 2\rho + \frac{X^L}{R_1} \approx 2\rho \quad (6.38)$$
Simplifying the expression, we have the following quadratic expression:

\[(4\rho - 1)(T_P^L)^2 + 4T \cdot T_P^L - T^2 = 0. \quad (6.39)\]

Solving the quadratic equation we obtain the required expression for \(T_P^{L*}\).

**Proof of Proposition 9**

If \(\rho > 1\), then

\[
\frac{-4 + \sqrt{16 + 4(4\rho - 1)}}{2(4\rho - 1)} < \frac{1(4\rho + 1) + (2\rho + 1)\sqrt{4\rho + 1}}{2(4\rho + 1)\rho}
\]

Thus, Proposition 9 follows.

**References**


Figure 1 The Firm's Product Performance in the Market Over Time
• Project *Alpha*:

\[
A(T_p) = -0.0684 + 0.8694 \ T_p
\]

(0.1287) \hspace{2cm} (0.01177)**

(Adj R-sq = 0.90)

• Project *Beta*:

\[
A(T_p) = -0.2631 + 0.8320 \ T_p
\]

(0.1361) \hspace{2cm} (0.1490)**

(Adj R-sq = 0.83)

• Project *Gamma*:

\[
A(T_p) = 0.3714 + 0.5714 \ T_p
\]

(0.0425)** \hspace{2cm} (0.0369)**

(Adj R-sq = 0.98)

• Project *Delta*:

\[
A(T_p) = 0.5121 + 0.4322 \ T_p
\]

(0.0497)** \hspace{2cm} (0.0408)**

(Adj R-sq = 0.95)

** statistically significant at 1% level

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Figure 2: The Relationship Between Product Performance and Time to Market
Figure 3: Total Industry Product Performance Over Time

Leader Launches its New Product

Follower Launches its New Product

Total Industry Product Performance

T Time
Table 1: Time to Market Estimates Under Different Levels of Product Performance for Four Projects at an Industrial Equipment Firm

<table>
<thead>
<tr>
<th>Project</th>
<th>Alpha</th>
<th>Beta</th>
<th>Gamma</th>
<th>Delta</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.7 Ax</td>
<td>0.80 Tα</td>
<td>0.50 Tβ</td>
<td>0.60 Tγ</td>
<td>0.60 Tδ</td>
</tr>
<tr>
<td>0.8 Ax</td>
<td>0.90 Tα</td>
<td>0.60 Tβ</td>
<td>0.80 Tγ</td>
<td>0.70 Tδ</td>
</tr>
<tr>
<td>0.9 Ax</td>
<td>0.95 Tα</td>
<td>0.90 Tβ</td>
<td>0.90 Tγ</td>
<td>0.80 Tδ</td>
</tr>
<tr>
<td>1.0 Ax</td>
<td>1.00 Tα</td>
<td>1.00 Tβ</td>
<td>1.00 Tγ</td>
<td>1.00 Tδ</td>
</tr>
<tr>
<td>1.1 Ax</td>
<td>1.10 Tα</td>
<td>1.00 Tβ</td>
<td>1.30 Tγ</td>
<td>1.30 Tδ</td>
</tr>
<tr>
<td>1.2 Ax</td>
<td>1.25 Tα</td>
<td>1.10 Tβ</td>
<td>1.50 Tγ</td>
<td>1.60 Tδ</td>
</tr>
<tr>
<td>1.3 Ax</td>
<td>1.50 Tα</td>
<td>1.10 Tβ</td>
<td>1.60 Tγ</td>
<td>1.90 Tδ</td>
</tr>
</tbody>
</table>

Ax, Tx are realized/expected project outcomes of project x. Project managers were asked to provide subjective estimates of what the time to market would be if product attraction was revised to a different level given that the level of development resource was held fixed.