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Costly Portfolio Adjustment and the Delegation of Money Management

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1 Introduction

Managing one's money can be a daunting task for people who are not deeply involved with financial markets day in and day out, or who suffer from financial illiteracy [Lusardi and Mitchell (2007)]. The fact that defined contribution pensions have become so widespread exacerbates this problem, since employees are increasingly required to manage their own retirement accruals. The reality is

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that many individuals appear to do a very poor job of managing their own finances [c.f. Tang, Mitchell, Mottola and Utkus (2010)], indicating a probable need for professional advisors.

The goal of this paper is to develop a lifecycle model to evaluate the role of financial advisors in helping employees to manage their financial portfolios. I incorporate human capital accumulation and inefficiency evidence from the finance literature in a standard lifecycle model. The first innovation is to allow for portfolio adjustment costs which many people must bear when managing their own financial wealth. This has a particular impact if the employee must accumulate job-specific human capital through learning by doing; in this instance, spending time on one's own financial management imposes an opportunity cost in terms of current and future job-related human capital accumulation. I also model an age-related time efficiency pattern for financial decision making, in keeping with observed empirical evidence. These two factors are likely to make it costly for individuals to manage their own portfolios in ways that are consistent with observed low levels of trading in workers' 401(k) accounts [Brunnermeier and Nagel (2008), Mitchell et al. (2006)].

I examine the role of financial advisors in terms of time cost minimizers. Previous efforts in household finance have focused on optimal portfolio allocation patterns for a rational forward-looking consumer who must decide on his own how to allocate his accruals between stocks and bonds [c.f. Cocco et al. (2005); Horneff et al. (2009)]. However, investors can also delegate portfolio management to financial advisors, which may be a more appealing option when there are high costs for managing their finances. In this regard, the need for a study on financial advisors in household finance has been highlighted [Campbell (2006)]. I incorporate financial advisors as one of the possible portfolio management schemes that investors can choose. When investors choose the delegation option, they can update their portfolios without sacrificing time, but do pay some portion of their wealth to financial advisors in management fees.

In this model, introducing a forgone opportunity to accumulate human capital generates a U-shaped and left-skewed pattern of portfolio inertia over ages when no delegation option exists. Young investors are most inactive and middle-aged investors are most active in managing their own money. Since young employees have a low level of human capital accumulation and also have the longest usage horizon, their cost for financial adjustment will be higher than that of middle-aged employees who have accumulated a more significant level of human capital. A different level of portfolio adjustment cost across all age groups results in a different pattern of portfolio management across age groups. The introduction of a delegation option has a significant impact on all age groups replacing the portfolio inertia, but there is still a divergent pattern of portfolio management across ages.

This paper is related to the literature of portfolio allocation with flexible labor supply [Bodie et al. (1992); Gomes and Michaelides (2003); Gomes et al. (2008); Chai et al. (2009); Horneff et al. (2009)]. The model uses a discrete dynamic choice technique as in Adda and Cooper (2000) and Bonaparte and Cooper (2009). We draw the pattern of cost for financial decision making from previous empirical findings that individual financial deficiency is a sizable component in the households' financial management [Lusardi and Mitchell (2007); Sumit Agarwal and Laibson (2009)].

The main contribution of this paper is to solve a lifecycle model of consumption, labor supply and portfolio choice with a financial management cost, which allows us to predict the demand for a delegation option over ages and measure the welfare gains of available advisory services. The calibrated model predicts that the delegation option can bring 19.5% welfare gains in terms of certainty equivalent consumption stream. This paper is also the first investigation of the impact of a time cost on investors' portfolio choice in the context of endogenous human capital accumulation in a lifecycle setting.

In what follows, section 2 describes the specification of investors' problem of portfolio choice. The model first defines a management scheme of portfolio inertia and shows a sufficiency condition for investors to choose portfolio inertia. Next I introduce the option of hiring financial advisors. Section 4 presents a numerical solution of the model. I conclude with a discussion of the implications of this paper's findings for the financial advisory industry, retirement plan sponsors and policy makers for retirement pension plans.

2 Specification of Dynamic Portfolio Choice Model

The model incorporates a dynamic choice of the equity share of the portfolio, labor supply and human capital accumulation, which influence an employee's current and future labor income and financial wealth.

2.1 Assumption on Time Budget and the Inefficiency Pattern of Financial Decision Making over Lifecycle

I assume that an investor is endowed with a normalized amount of time of 1 at each period and that he can allocate this time to working(l_t) or consuming leisure(L_t). The time can be interpreted as a physical time of 24 hours or the mental capacity that we allocate to various activities in daily life.

Managing financial assets encompasses various activities from opening a brokerage account (but not limited) to analyzing various financial products. When the main task of an investor's job is not involved in financial work, which is the case for most DC retirement plan participants, self-management of financial asset will inevitably eat into a worker's time or mental resources, because searching and processing information is costly to them. Because workers are compensated according to their job-specific skills (or human capital) and because these job-specific skills are accumulated mostly through work experience, they will incur an opportunity cost as a result of the time spent managing their financial assets. [1].

In this model, the explicit opportunity cost for adjusting one's portfolio is captured by the time efficiency (ϕ_t) of financial decision making. I assume an investor is not well informed regarding the task of financial management, so he should allocate some portion of his available time to acquire and process various information related to portfolio management[2]. Therefore, an investor faces the time constraint as follows

$$l_t + L_t + \phi_t \mathbf{1}_{\{a_t=1\}} = 1$$

where $a_t = 1$ is an indicator for active portfolio management. This time constraint condition implies that an investor should incur a time cost when making his own choices to implement an optimal portfolio[3]. Sumit Agarwal and Laibson (2009) documented younger and older people are likely to make more mistakes when it comes to financial decisions. Aguiar and Hurst (2007) also documented the time cost of the consumer's choice regarding various non-financial products. This empirical evidence shows that making an efficient financial decision depends on age and that middle-aged people tend to make fewer mistakes in controlling their wealth levels. I have implemented this age-related efficiency pattern of financial decision making as the amount of time they need for their financial decisions; a low ϕ_t implies that the investor can do so efficiently, and thus quickly implement his new portfolio choice. I have adopted the empirical evidence of an age-related pattern of efficient financial decision making with a U-shaped ϕ_t over the lifecycle in this model. Note that

[1] There might be a group of people that enjoys self-financial management or even believes that they have a good skill to outperform the market or professional investors. However, the proportion of these people is observed to be very low among investors according to the literature of retirement pension management. Moreover, their performance generally has not been superior to that of the market [Lusardi and Mitchell (2007); Mitchell et al. (2009)].

[2] Tasks related to the portfolio management may include (but are not limited to) opening accounts, tracking past market condition, monitoring market, finding the optimal portfolio level and executing the order of transaction.

[3] After investors decide how to allocate their wealth between risk-free and risky assets, they also have to spend time to implement their new choices. For example, if they are implementing their choices by purchasing mutual funds, they have to read and compare many mutual fund companies' prospectuses and execute trading orders. If they cannot find a single mutual fund that implements their choice, they need to form a portfolio of various mutual funds to achieve their desired level of equity share.

this time cost does not depend on the amount of the adjusted portfolio share[4]. In this model, investors should incur a new time cost(ϕ_t) in each period, because they should solve their lifecycle model and implement the new choice again.

2.2 Assumption on Human Capital Accumulation Process

I assume job-specific human capital is accumulated through "learning by doing" [Arrow (1962)]. I denote H_t and l_t as the job-specific human capital and working time, respectively, at time t. The law of motion of job specific human capital is

$$H_{t+1} = (1 - \delta_t) H_t + F_t (H_t, l_t)$$

where $F_t(\cdot, \cdot)$ is an experience formulation function and δ_t is a depreciation rate[5] of job specific human capital.

An important feature of this formulation is the dynamic property of labor supply. The current working time(l_t) not only increases current labor income, but also can increase the stock of future human capital, which will lead to higher labor income in the future[6]. Much previous research involving the labor supply model, including Bodie et al. (1992) and Cocco et al. (2005), incorporated wage income as an important source of wealth, but the working decision only affected the current income level. Thus, they implicitly assumed that working time is a substitute for current leisure time and that the price of leisure was the current hourly wage. In this paper's model, however, the investor should consider future human capital accumulation, an age-related efficiency pattern of financial decision making and the current level of leisure, when he decides how much time to allocate to working.

2.3 Assumption on Labor Income and Asset Return

I assume labor income is determined by an employee's job-specific human capital level(H_t) and wage shock(Y_t).

$$\text{labor income}_t = l_t H_t Y_t$$

[4] This inefficiency cost comes technically from the complexity that a normal worker faces when implementing his choice from the dynamic programming problem [see e.g. Johnson et al. (1987)].

[5] This can also be interpreted as 'obsolete rate' of skills. Some set of knowledge can be outdated by the advent of new technology.

[6] This can be also interpreted as a reputation effect in a job market. With higher level of human capital accumulated by more working time in the current period, the worker will be rewarded higher by the labor market or the current firm in the next and future periods.

where l_t represents working hours. The accumulated human capital H_t is comparable with the age-specific, deterministic wage trend in the lifecycle model literature [see e.g., Cocco et al. (2005), Gomes et al. (2008)]. In this model, however, H_t is endogenously accumulated over time by a worker's labor supply as in section 2.2.

The wage shock ($y_t \equiv \log Y_t$) follows an AR(1) process and is influenced by an idiosyncratic shock(ϵ_t).

$$y_t = \eta + \rho y_{t-1} + \epsilon_t^y$$

where $\epsilon_t^y \sim iid\, N\left(0, \sigma_y\right)$.

I consider two asset classes:a stock and a risk-less bond. The stock return(R_t) is assumed to be i.i.d log normally distributed[7] over the years.

$$\log R_t \sim iid\, N\left(\zeta,\, \sigma_\zeta\right)$$

The stock return shock and wage shock can be correlated:

$$cov_t\left(y_{t,}\ \log R_t\right) = \sigma_{\epsilon\zeta}$$

The riskless bond has return $\overline{R}$ at all periods. I do not consider the inflation rate. Thus, the model captures the inflation-adjusted phenomenon of a portfolio decision problem. I denote R_{t+1} as the stock return from t to $t+1$, so the decision time horizon is that π_{t+1} is determined at period t and the return is realized at period $t+1$.

2.4 Assumptions on Portfolio Choice and Wealth Dynamics

At time t, the investor chooses the equity portion (π_{t+1}) in his portfolio and the portfolio will have return

$$R_{t+1}^p = \left(1 - \pi_{t+1}\right)\overline{R} + \pi_{t+1} R_{t+1}$$

Note that R_{t+1}^p is a random variable at time t and is realized at time $t+1$.

[7] Tang et al. (2010) showed individuals generally have lower returns from managing their own portfolio. For simplicity, this paper assumes equity returns are the same for every portfolio management schemes (inertia portfolio, active management and delegation).

Denoting c_t as consumption, the dynamic budget constraint can be formulated as [8]

$$W_{t+1} = R^p_{t+1}\left(W_t + l_t H_t Y_t - c_t\right) \tag{1}$$

Total cash-in-hand in period t (W_{t+1}) consists of financial wealth(W_t) and labor income$(l_t H_t Y_t)$. After consuming c_t in period t, it is invested with return R^p_{t+1}.

2.5 Assumptions on Preferences and Time Horizon

In the manner of Gomes et al. (2008), an investor has a standard time-separable, modified Cobb-Douglas power utility function over consumption(c_t) and leisure(L_t) given by

$$U\left(c_t,\, L_t\right) = \frac{1}{1-\gamma}\left(c_t\left(L_t\right)^{\alpha}\right)^{1-\gamma}$$

where α captures an investor's preference over consuming leisure. I only consider a portfolio adjustment decision during working periods and there is no decision problem after the retirement time (T). The retirement time is exogenously fixed(T). [9].

3 Dynamic Portfolio Choice Problem with Portfolio Inertia and the Role of Financial Advisors

One important feature of individual portfolio management is the low turnover ratio or portfolio inertia [see e.g., Bilias et al. (2009); Mitchell et al. (2006)].

3.1 Portfolio Inertia

The model in this paper defines inactivity or inertia in portfolio management with respect to the opportunity cost of the portfolio adjustment and the time efficiency of financial decision making.

Definition 1. *Portfolio inertia* at period t is defined as a naive choice of the previous period's portfolio (π_t) for next period's portfolio (π_{t+1}) without incurring time cost(ϕ_t).

By simply choosing his previous portfolio (π_t) as the next period's portfolio (π_{t+1}), the investor does not have to sacrifice any portion(ϕ_t) of his available time to solve his optimization problem

[8] We can also introduce a direct transaction cost$(C\left(\pi_{t,}\, \pi_{t+1}\right))$ for portfolio adjustment. Then the wealth dynamics will be $W_{t+1} = R^p_{t+1}\left(W_t + l_t H_t Y_t - C\left(\pi_t,\, \pi_{t+1}\right)\mathbf{1}_{\{adjustment\}} - c_t\right)$ when the direct transaction cost is incorporated. However, I do not focus on direct monetary costs in this model. See Bonaparte and Cooper (2009) for estimating the direct monetary cost of portfolio adjustment.

[9] See Smetters and Chen (2009) for a discussion about the role of social security in explaining the low level of portfolio share among young workers.

and implement its solution. Therefore, if the investor behaves with portfolio inertia, his next period portfolio(π_{t+1}) share does not change from the previous level(π_t) and his time constraint is not impacted by the efficiency pattern(ϕ_t) of financial decision making. Thus, when an investor chooses portfolio inertia, he has the equity share and time constraint as follows

$$\pi_{t+1} = \pi_t$$
$$l_t + L_t = 1$$

Recall that l_t and L_t denote working time and leisure respectively.

Note that the same portfolio choice over the subsequent periods does not constitute portfolio inertia. It is possible for an investor to choose to actively manage his portfolio by incurring the time cost (ϕ_t) and end up choosing the previous portfolio(π_t) as the optimum for the next period's portfolio(π_{t+1}). In this case, the portfolio choice is not naive and the investor has to sacrifice some portion (ϕ_t) of his available time.

Since portfolio inertia allows that the previous portfolio choice can affect the current period's decision regarding a portfolio management scheme, the previous portfolio also serves as a state variable[10]. Other state variables include wealth level(W_t), accumulated human capital level(H_t) and current wage shock(y_t). In total, we have 4 choice variables: portfolio management scheme (i.e., portfolio inertia or active management), labor supply(l_t), the next period's equity share (π_{t+1}) and consumption(c_t).

An investor should solve a sequential problem as follows:

$$\max_{\{c_t, l_t, \pi_{t+1}, (a_t)\}_{t=0}^{T}} \mathbb{E}\left[\sum_{t=0}^{T} \beta^t u_t\left(c_t, L_t\right)\right]$$
$$s.t\ c_t \leq W_t + l_t H_t Y_t$$
$$W_{t+1} = R_{t+1}^{p}\left(W_t + l_t H_t Y_t - c_t\right)$$
$$H_{t+1} = \left(1-\delta_t\right) H_t + F_t\left(H_t, l_t\right)$$
$$l_t + L_t + \phi_t 1_{\{a_t=1\}} = 1$$
$$y_{t+1} = \eta + \rho y_t + \epsilon_{t+1}$$

where $\{a_t\}$ is a decision set in which $a_t = 1$ indicates a portfolio adjustment and $a_t = 0$ indicates

[10] Bonaparte and Cooper (2009) also investigates the cost of portfolio adjustment and used the previous portfolio as one of state variables.

portfolio inertia.

In the manner of Adda and Cooper (2000), I define $V_t^a(W_{t,}H_t,\pi_t,y_t)$ as the discounted lifetime utility of an investor when he chooses an 'active management' scheme. Similarly, $V_t^i(W_{t,}H_t,\pi_t,y_t)$ denotes the discounted lifetime utility of an investor when he chooses a 'portfolio inertia' scheme. I define the value function at period t as $V_t(W_{t,}H_t,\pi_t,y_t) \equiv \max\left\{V_t^a(W_{t,}H_t,\pi_t,y_t),\ V_t^i(W_{t,}H_t,\pi_t,y_t)\right\}$. The value function for each portfolio management scheme is defined as

$$
\begin{aligned}
V_t^a(W_t,H_t,\pi_t,y_t) = & \max_{\{c_t,\pi_{t+1},l_t\}} u_t(c_t,L_t) + \beta E_t\left[V_{t+1}(W_{t+1},H_{t+1},\pi_{t+1},y_{t+1})\right] \\
& s.t.\ c_t \leq W_t + l_t H_t Y_t \\
& \quad W_{t+1} = R_{t+1}^p(W_t + l_t H_t Y_t - c_t) \\
& \quad H_{t+1} = (1-\delta_t) H_t + F_t(H_t,l_t) \\
& \quad l_t + L_t + \phi_t = 1 \\
& \quad y_{t+1} = \eta + \rho y_t + \epsilon_{t+1}
\end{aligned}
$$

and the value function for choosing portfolio inertia is defined as

$$
\begin{aligned}
V_t^i(W_t,H_t,\pi_t,y_t) = & \max_{\{c_t,l_t\}} u_t(c_t,L_t) + \beta E_t\left[V_{t+1}(W_{t+1},H_{t+1},\pi_{t+1}=\pi_t,y_{t+1})\right] \\
& s.t\ c_t \leq W_t + l_t H_t Y_t \\
& \quad W_{t+1} = R_{t+1}^p(W_t + l_t H_t Y_t - c_t) \\
& \quad H_{t+1} = (1-\delta_t) H_t + F_t(H_t,l_t) \\
& \quad l_t + L_t = 1 \\
& \quad y_t = \eta + \rho y_{t-1} + \epsilon_t
\end{aligned}
$$

When $V_t^a \geq V_t^i$, the investor chooses an active management scheme ($a_t = 1$). Otherwise, he chooses portfolio inertia. The difference between the two value functions are the time constraint and the next period's portfolio choice. The benefit of portfolio inertia is the saved time but the previous portfolio may not be the optimal choice for the current period, which maximizes the lifetime utility even when considering time cost ϕ_t.

Before investigating the role of financial advisors in this setting, I will briefly discuss sufficient conditions for an investor to choose portfolio inertia. Let $\left(\hat{l}_t^a, \hat{\pi}_{t+1}^a, \hat{c}_t^a\right)$ and $\left(\hat{l}_t^i, \hat{\pi}_{t+1}^i = \pi_t, \hat{c}_t^i\right)$ be maximizers[11] of the objective functions of an active management scheme and a portfolio inertia scheme.

Proposition 2. *For any* $\left(\hat{l}_t^a, \hat{\pi}_{t+1}^a, \hat{c}_t^a\right)$ *with* $\max\left\{\left|\hat{l}_t^a - \hat{l}_t^i\right|, \left|\hat{c}_t^a - \hat{c}_t^i\right|\right\} < \left|\hat{\pi}_{t+1}^a - \pi_t\right|$, *there exists* $\delta^* > 0$ *such that* $\forall \hat{\pi}_{t+1}^a$ *with* $\left|\hat{\pi}_{t+1}^a - \pi_t\right| < \delta^*$ *implies* $V_t^i\left(W_t, H_t, \pi_t, y_t\right) > V_t^a\left(W_t, H_t, \pi_t, y_t\right)$.

Proof. See the Appendix. □

The implication of this proposition is simple and intuitive. When next period's labor and consumption levels chosen by an active portfolio management scheme are very close to those chosen by a portfolio inertia scheme, there is a 'dominating boundary of portfolio inertia' where the portfolio inertia is superior to the active management scheme. In other words, when an investor expects that he would end up choosing a similar pair of consumption level and labor supply in the next period, a small change in the portfolio will only be costly without improving his discounted lifetime utility. So it is optimal for him not to fiddle around with portfolio management.

3.2 The Role of Financial Advisors

The current model enables us to explore the role of financial advisors[12] in portfolio management and to conduct counterfactual experiments about their contributions to investors' portfolio choices. Reasons for delegating portfolio management may include time cost, the efficiency gain due to lower transaction costs and positive beliefs regarding professional managers' skills. In this paper, I focus on investors' time costs associated with human capital accumulation.

When an investor chooses to delegate a portfolio management task, he pays some portion (φ_t) of the total financial wealth (W_t) to a financial advisor as a management fee. The explicit benefit of hiring financial advisors is the saved time, which can then be used to work (and accumulate more job-specific knowledge) or enjoy leisure. If he chooses to manage his financial portfolio by himself, he does not have to pay this fee (φ_t), but should incur a time cost (ϕ_t), which is associated with his age-based efficiency pattern of financial decision making. In this paper, I consider a financial advisor, who is very involved in an investor's decision making in the sense that she not only chooses a portfolio, but also proposes an optimal consumption level and labor supply.

[11] The existence of these solutions is discussed in the following chapter.

[12] In U.S financial markets, RIAs (Registered Investment Advisors) are registered with the Securities and Exchange Commission and give advice on investing in various financial products such as stocks, bond, mutual funds, etc. They also manage portfolios of securities for their household or firm clients. This role can be technically interpreted as helping to implement the optimal portfolio choice of investors.

One important issue in delegating portfolio management is the possible conflict of interest between an investor (principal) and a financial advisor (agent). Because an investor maximizes his own utility over consumption and leisure, which are afforded by accumulated wealth, his optimal portfolio decision may be different from that of a financial advisor who maximizes only her total revenue from managing clients' wealth.

If an investor observes the choice of an advisor (first-best case), he can make sure his first-best outcome is achieved. Denote $c_t^{FB}(W_t, H_t, \pi_t, y_t)$, $\pi_{t+1}^{FB}(W_t, H_t, \pi_t, y_t)$ and $l_t^{FB}(W_t, H_t, \pi_t, y_t)$ for $t = 1, \ldots, T$ as the first-best policy of consumption, portfolio and labor supply for the investor. It is a solution of the following dynamic optimization problem

$$
\begin{aligned}
& V_t(W_t, H_t, \pi_t, y_t) \\
= & \max_{\{c_t, \pi_{t+1}, l_t\}} u_t(c_t, L_t) + \beta E_t[V_{t+1}(W_{t+1}, H_{t+1}, \pi_{t+1}, y_{t+1})] \\
s.t \quad & c_t \leq W_t + l_t H_t Y_t \\
& W_{t+1} = (1 - \varphi_t) R_{t+1}^p (W_t + l_t H_t Y_t - c_t) \\
& R_{t+1}^p = (1 - \pi_{t+1}) \overline{R} + \pi_{t+1} R_{t+1} \\
& H_{t+1} = (1 - \delta_t) H_t + F_t(H_t, l_t) \\
& l_t + L_t = 1 \\
& y_{t+1} = \eta + \rho y_t + \epsilon_{t+1}
\end{aligned}
$$

Note that the employee does not have to incur a time cost ϕ_t and pays a management fee φ_t out of his wealth.

However, an investor does not usually directly observe the financial advisor's choice at the beginning of time t. The advisor will choose $\{c_t, \pi_{t+1}, l_t\}$ to maximize $\varphi_{t-1}W_t + \beta E_t\left[\varphi_t R_{t+1}^p (W_t + l_t H_t Y_t - c_t)\right]$ given the above constraints. Since the financial advisor's objective function is different from that of an investor, the chosen policy function can be different from the first-best solution. Then the investor may incur additional costs to monitor the financial advisor's behavior.

In a dynamic setting, however, this information cost from a moral hazard problem can be mitigated because the financial advisor should also consider future revenue (or reputation), which will depend on the current period's outcome. One important condition for the investor to implement his first-best choice is the verifiability of the financial advisor's choice. This is possible in our model setting when we assume the return process is easily observed by the investor or other competing financial advisors. Since the return process R_t is revealed to the investor[13], he can easily discover the portfolio choice of the financial advisor in the previous period. More formally, with the knowledge of wealth level W_{t+1}, fee level φ_t, bond return $\overline{R}$, wage shock y_t and consumption-labor choice $\{c_t, l_t\}$, which are all known at period $t+1$, the investor can calculate π_t from $W_{t+1} = (1-\varphi_t) R^p_{t+1} (W_t + l_t H_t Y_t - c_t)$.

Now, consider a contract that specifies the following

1. If $\left(c_t^{FB}, \pi_t^{FB}, l_t^{FB}\right) = \left(c_t^D, \pi_t^D, l_t^D\right)$ for $t = 1, \ldots, T-1$, the investor pays a pre-determined fee φ_t.

2. If $\left(c_t^{FB}, \pi_t^{FB}, l_t^{FB}\right) \neq \left(c_t^D, \pi_t^D, l_t^D\right)$ for $t = 1, \ldots, T-1$, the investor fires the current financial advisor and hires another advisor. The original financial advisor has no outside option in the next period by assumption.

3. This contract is effective at every period.

In short, an investor can punish a financial advisor by firing her (and replacing her with another advisor) when he learns that the first-best has not been chosen at the beginning of period t. Because the outside option for a financial advisor is zero by assumption, cheating will never be superior for a financial advisor to choosing an investor's first-best choice. Thus, this contract will ensure that the financial advisor chooses the first-best.

This contract enables us to solve only one (i.e., the investor's) dynamic programming problem instead of two (the investor's and the financial advisor's). An investor's maximization problem will be implemented by a financial advisor.[14]

Therefore, the investor's problem can be summarized as

[13] We can also assume competitive market of financial advisors. This implies every advisors are paid same fee φ_t when he is hired as a financial advisor by an investor. I also assume this fee includes next-period monitoring cost . Even though the investor does not observe the financial advisor's portfolio choice at the beginning of time t, she can easily obtain information about past return process and total wealth level at the end of time $t = 1, \ldots, T$. Competitive market assumption implies the financial advisor is monitored by his competitors at the end of period T and mischievous act will be publicized by them, which will damage his reputation and lower the possibility of being hired by another investor. Therefore, the financial advisor will choose the first-best outcome of an investor's problem and the investor dose not have to consider the incentive compatibility condition of the financial advisor.

[14] See Ou-Yang (2003) for continuous-time dynamic optimization problem in a delegated portfolio management problem. He argues that a financial advisor will exactly follow an investor's optimal portfolio policy if a symmetric (i.e., reward and punishment) remuneration scheme is offered.

$$
\begin{aligned}
& V_t\left(W_t, H_t, \pi_t, y_t\right) \\
= & \max_{\left\{a_t, l_t, \pi_{t+1}, c_t\right\}} u_t\left(c_t, L_t\right)+\beta E_t\left[V_{t+1}\left(W_{t+1}, H_{t+1}, \pi_{t+1}, y_{t+1}\right)\right] \\
s.t\ & c_t \leq W_t+l_t H_t Y_t \\
& W_{t+1}=\left(1-\mathbf{1}_{\left\{a_t=2\right\}} \varphi_t\right) R_{t+1}^p\left(W_t+l_t H_t Y_t-c_t\right) \\
& R_{t+1}^p=\left(1-\pi_{t+1}\right) \overline{R}+\pi_{t+1} R_{t+1} \\
& H_{t+1}=\left(1-\delta_t\right) H_t+F_t\left(H_t, l_t\right) \\
& l_t+L_t+\phi_t \mathbf{1}_{\left\{a_t=1\right\}}=1 \\
& y_{t+1}=\eta+\rho y_t+\epsilon_{t+1} \\
& \pi_{t+1}=\pi_t \text { if } \quad a_t=0
\end{aligned}
$$

where I denote $a_t = 0$ as portfolio inertia, $a_t = 1$ as active management and $a_t = 2$ as hiring a financial advisor. And $V_t \equiv \left\{V_t^i, V_t^a, V_t^d\right\}$ where V_t^i is the value function for the portfolio inertia case, V_t^a is the value function for active management, and V_t^d is the value function for delegating portfolio management.

An important specification in this model is the job-specific human capital accumulation function $F_t\left(H_{t,}, h_t\right)$. I will specify this function [see Ben-Porath (1967)] as follows

$$
F_t\left(H_t, l_t\right)=a\left(H_t l_t\right)^\theta, \quad (\theta<1)
$$

where a is a parameter that represents the individual efficiency or the learning ability for accumulating human capital[15]. The elasticity θ of human capital accumulation is assumed to have decreasing returns to scale ($\theta \in (0,1)$).

4 Model solution

4.1 Existence of the Solution

Since an investor is not sure about the future chosen portfolio adjustment scheme, there is no simple Euler equation that links the marginal benefit of today's portfolio adjustment with the

[15] Note that I am not using exactly the same notion of human capital as Ben-Porath (1967). He interpreted human capital as something to be accumulated only by getting more education at school. In this model, human capital represents job-specific skill, knowledge or reputation in a current workplace which is accumulated by working, not education at school.

future marginal benefit [Adda and Cooper (2003)]. However, the existence of a solution can be shown by the Backward Induction and the Weierstrass Theorem.

Proposition 3. *There exist optimal sets of policies* $\{a_t, c_t, \pi_{t+1}, l_t\}_{t=1}^{T}$ *for an investor's dynamic portfolio choice problem.*

Proof. See the appendix. □

The existence of solutions is guaranteed, but deriving them is analytically intractable. Therefore, this model will be solved numerically via backward induction, polynomial approximation of the value function, Monte-Carlo integration and the Nelder-Mead simplex method.

4.2 Numerical Solution and Baseline Parameters

I will briefly describe the procedure for obtaining the numerical solution to the investor's problem[16]. In the last period T, assuming $V_{T+1} = 0$ and $a_T = 0$, the investor maximizes her utility over c_T and l_T at every pair of state variables (W_T, H_T, π_T, y_T). Thus, $V_T(W_T, H_T, \pi_T, y_T) = \max_{\{c_T, l_T\}} u(c_T, 1 - l_T)$. This maximization problem is solved by the Nelder-Mead simplex method. Then, I approximate $\hat{V}_T$ by the polynomial regression of the maximized value V_T over the pairs of state variable (W_T, H_T, π_T, y_T). In period $T-1$, I calculate V_{T-1}^i, V_{T-1}^a, V_{T-1}^d by their definition with the Monte Carlo integration of $E_{T-1}\left[\hat{V}_T(W_T, H_T, \pi_T, y_T)\right]$ and the Nelder-Mead optimization method over (l_{T-1}, π_T, c_T). Of course, $\pi_T = \pi_{T-1}$ in calculating V_{T-1}^i. Then, I get $V_{T-1}(W_{T-1}, H_{T-1}, \pi_{T-1}, y_{T-1}) = \max\left\{V_{T-1}^i, V_{T-1}^a, V_{T-1}^d\right\}$ and we know portfolio inertia is optimal when $V_{T-1}^i = \max\left\{V_{T-1}^i, V_{T-1}^a, V_{T-1}^d\right\}$. Another choice of management scheme is similarly derived. Then I approximate $\hat{V}_{T-1}$ by the polynomial regression of V_{T-1} over the pair of state variables$(W_{T-1}, H_{T-1}, \pi_{T-1}, y_{T-1})$. Iterating these steps until the first period, I get the approximated value functions $\left\{\hat{V}_t\right\}_{t=1}^{T}$ and this characterizes the solution of the investor's problem completely. Then I generate 1,000 sample paths of individual investors with the variations of the wage shock and the stock market return shock[17].

In order to describe the model's characterization of the portfolio inertia and its prediction of the impact of a financial advisory service, we need to choose a reasonable set of parameters. I set the coefficient of risk aversion γ to 2.5 and the leisure preference α to 0.9 as in Gomes et al. (2008). The discounting factor β is set to 0.95. I set the elasticity parameter θ in the experience

[16] This numerical procedure is implemented with FORTRAN90 with the GNU Gfortran compiler in the Wharton Grid system and it took approximately 20 hours.

[17] Variation in the stock market return implies that individual investors hold different sets of equity, so they may face different stock returns but the return distributions are still the same (IID log normal).

accumulation function to 0.209 and the accumulation rate a is set to 0.7 as in Huggett et al. (2006). Additionally, human capital H_t depreciates with rate of 1.4% per annum as in Huggett et al. (2006).

For the AR(1) process for the wage shock, the drift parameter η is set to 0.08 and the autocorrelation coefficient is set to 0.85 with a wage shock standard deviation of 0.1389. The riskless asset return $\overline{R}$ is set to 1.02 [Cocco et al. (2005)] and the risk premium is 4% with a standard deviation of 0.205 [Gomes et al. (2008)]. The portfolio management fee φ_t is set to 1.3%, which is the average management fee for U.S portfolio allocation mutual funds [MorningStar 2009] [18].

The efficiency pattern of financial decision making is assumed to be of convex form, as supported by evidence presented in Sumit Agarwal and Laibson (2009). The age group with 30 working years is assumed to be the most financially savvy, with $\phi_{22} = 0.03$ (they are assumed to sacrifice only 3% of their normalized time). Young investors are assumed to have the lowest efficiency $\phi_1 = 0.09$[19]. The functional form of decision efficiency is assumed to be $\phi_t = \frac{0.09-0.03}{30^4}(age - 30)^4 + 0.03$, where the 4th power represents a flatter efficiency pattern in middle-aged. This set of baseline parameters is summarized in Table 1.

4.3 Solution and its Implication

Figures 1 and 2 plot the average proportions of the chosen portfolio management schemes in different scenarios; one without a delegation option and one with a delegation option. Figure 1 shows that, consistent with the empirical evidence, portfolio inertia is the main portfolio management scheme implemented by most investors. Most younger workers choose portfolio inertia rather than to actively self-manage their asset allocations. Middle-aged workers are the most active group, but almost 40% of them still find it optimal to not touch their portfolio allocation. This high level of inactivity is consistent with several examples from the empirical literature [Mitchell et al. (2006); Vissing-Jorgensen (2002)]. Portfolio inertia in a young working group reflects their concern about human capital accumulation, which will be a source of their labor income in future periods. Because they have the longest horizon of human capital usage among working groups, it is optimal for them to not sacrifice their time by fiddling around with their financial portfolios, which is not in their professional area. The middle-aged group has the lowest deficiency in financial decision making, and

[18] Even though the role of a financial advisor is somewhat different from that of a mutual fund, it is known that their fee levels are similar (c.f., Investopia.com).

[19] This choice does not depend on any empirical evidence, so it needs to be estimated by this model with any relevant data. A time cost of 9% is quite high, but this will make it apparent how efficiency patterns will affect the portfolio management scheme.

their human capital is abundant compared to their younger counterparts. Thus, sacrificing a small amount of time will not necessarily hurt their life-time value much, even when their labor income is not very high. Inactivity in the older working group can similarly be explained by their deficiency in financial decision making. These people face low efficiency in their financial decision making, which means they have to sacrifice a larger fraction of their time to self-manage their portfolio. Thus, it will be costlier for them to self-manage their portfolio than it will be for the middle-aged group. However, they are still more active than young investors, because the decreasing returns to scale property of human capital accumulation makes it less costly to adjust their portfolio in terms of future labor income.

Figure 2 shows the proportions of the chosen management schemes when there is an option to hire financial advisor. First, we observe a decrease in portfolio inertia across all age groups. Approximately 85% of young workers, 20% of middle-aged workers and 7% of old workers choose portfolio inertia as their management scheme. Second, delegating portfolio management becomes the dominant portfolio management scheme across all age groups, replacing the importance of the active management scheme in the previous case. Approximately 15% of young workers, 60% of middle-aged workers and 90% of old workers want to delegate their portfolio management task to financial advisors. Third, the active management scheme is implemented mostly by middle-aged workers with working experience between 23 to 41 years (i.e., workers age 34 to 62, if we assume people enter the labor market at 21 years old). Only a small fraction (less than 1%) of young workers and 5% of old workers choose active self-management. About 15% of middle-aged workers choose active self-management as their financial management scheme. These observations show that introducing a portfolio delegation option has a substantial impact on all age groups, especially younger and older investors. The model incorporates a delegation option along with portfolio inertia and active management, so if the new option does not provide any benefit to investors, their choice should not be different from the previous one. But as the new solutions shows, the middle-aged and older groups chose the delegation option as their main management scheme.

The pattern of portfolio management scheme selection reflects the pattern of decision-making deficiency and human capital accumulation. For younger investors, their shallow pool of human capital makes it too costly for them to spend their time managing their financial assets, which does not explicitly increase their human capital or job-specific knowledge. When there is no delegation option, their best strategy was to choose 'no-touch' so that they could fully make use of their available time to work and accumulate job-specific skills. But when the delegation option is available, many of them find it optimal to pay the management fee and delegate their portfolio management rather than self-manage their money. If some of them expect their current portfolios to be near optimal in the next period, they will still choose portfolio inertia without paying a fee to a financial advisor or sacrificing their time (see Proposition 2). Older investors also find it helpful to have a financial advisor. With more accumulated financial wealth, their need to have someone to manage their assets becomes very high. Because they undergo a high level of deficiency when making their financial decisions, they would choose to pay a management fee and they do not sacrifice their time. They find it more profitable to fully make use of their time to work or enjoy leisure. It is noteworthy that the middle-aged working group remains the most active financial decision makers. They are actively participating in their portfolio reallocation by either choosing active management or hiring financial advisors. Since this group is more active in self-management, they may have more demand for a brokerage service than other age groups.

Figure 3 plots the portfolio choice in each scenario over the lifecycle. One noticeable finding is that people without a delegation option are likely to hold a lower fraction of the risky asset in comparison with those with a delegation option in their early career stages. Since the delegation option saves investors' time, they can fully make use of their time to work and accumulate more human capital, which is safer than equity. Therefore, they have more of a buffer to the potentially negative shock of equity returns and they can invest more in risky asset. With our baseline parameters, middle-aged people invest most of their wealth in equity.

Figure 4 plots the consumption level over the lifecycle in the two scenarios. We find that investors with a delegation option can consume more than those without a delegation option. There is little difference in the two scenarios (i.e., with and without a delegation option) in the early working periods, but the delegation service brings more consumption in the middle and later years. When delegating financial management is available, workers can allocate more time to their work and accumulate more job-specific knowledge, which leads to higher income and consumption. Figure 5 plots the average wealth of investors. We observe that financial advisory service can bring a higher level of wealth eventually. This is not because they bring much higher excess return in financial management but they save time and the deficiency cost which is associated with portfolio management in this model. These two figures suggest that there is a benefit introduced by a delegation option. Investors can consume more and accumulate more wealth when a delegation option is available because they can fully make use of their time to accumulate their job-specific skills. In this model, a small management fee (1.13%) is a worthwhile cost for investors, who are responsible for managing their financial assets, but are not usually very well informed about the task.

Figure 6 plots the fraction of investors' available time allocated to their own work. It has an inverse U-shape over the lifecycle, which is consistent with the macroeconomics literature. If a financial advisory service is not available, the worker must sacrifice some portion of his time, which could have been allocated to working, to self-manage his asset. In the early career stage, the delegation option enables workers to allocate more time to working and accumulating more human capital. In the later career stages, the option enables the worker to work less (and therefore enjoy more leisure), but continue to accumulate human capital by sparing time spent on financial management. Saved time can be allocated to leisure too, which will lead to higher life-time utility.

Figure 7 plots the pattern of accumulated human capital over the lifecycle. We find that investors with a delegation option can accumulate more human capital than those without a delegation option. With the delegation option available, workers can fully make use of their time to work without fiddling around with their financial wealth and thus enjoy more leisure with the same level of human capital accumulation compared to that of a self-management case.

The welfare gain is measured in terms of certainty equivalent (CE) constant consumption stream, which is standard in the related literature. It is defined as the stream of consumption that would provide the same level of expected lifetime utility as the uncertain consumption and leisure the investor expects[20].

Figure 8 plots the pattern of welfare gains over age when utilizing a financial advisory service for different levels of relative risk aversion. With the baseline parameters, we get a 19.5% increased level of annual consumption stream when the delegation option is available to young investors. This implies that investors can enjoy a 19.5% higher annual consumption stream when they have the

[20] In the manner of Chai et al. (2009), the certainty equivalent(CE) consumption (c_{CE}) is defined as

$$\begin{aligned} V_t\left(W_1, H_1, \pi_1, y_1\right) =& E\left[\sum_{i=t-1}^{T} \beta^i \frac{1}{1-\gamma}\left(c_i\left(L_i\right)^{\alpha}\right)^{1-\gamma}\right] \\ =& \sum_{i=t-1}^{T} \beta^i \frac{1}{1-\gamma}\left(c_{CE}^t\left(L_*\right)^{\alpha}\right)^{1-\gamma} \end{aligned}$$

where L_* is a fixed level of leisure and (W_1, H_1, π_1, y_1) is the initial pair of state. With some algebraic manipulation, we get

$$c_{CE}^t = \left[\frac{(1-\gamma)\, V_t}{\sum_{i=t-1}^{T}\left(L_*^{\alpha}\right)^{1-\gamma}\beta^i}\right]^{1-\gamma}$$

In calculating this measure, I set leisure amount L_* as time deducted by mean labor hours up to working year 40 because labor supply decreases significantly after that time.

option to hire financial advisors to manage their financial portfolios. This quantity is substantial compared to that of Cocco et al. (2005), which measured the welfare gains of flexible portfolio allocation at around 2% compared to the fixed equity share investment heuristic.

It also shows that the magnitude of welfare gain over age is U-shaped. Young and old workers are most benefited by the delegation option. Welfare gains get higher when the relative risk aversion gets higher. When investors have high risk aversion, the time sacrificed to accumulate more human capital will be even costlier, because their safe asset (labor income) decreases. Therefore, the option to delegate the task of financial management will be more beneficial to them compared to those with lower risk aversion.

It is noteworthy that the welfare gains introduced by a delegation option in this model are smaller than the true enhancement. This measure does not take into account the possible additional benefits that financial advisors can bring about, such as low transaction costs by economies of scale and (possible) excess returns.

4.4 Sensitivity Analysis

In this subsection, I check the robustness of the result by varying key parameters and investigate the role that the parameters play in the model's predictions.

Figure 10 plots the choice of portfolio management scheme with different levels of risk-aversion. People are likely to choose portfolio inertia more as risk-aversion increases. When a delegation option is available, it dominates the other two management schemes for most age groups. However, more people are likely to self-manage their portfolios when risk aversion decreases. This increased level of active management can be explained by the human capital accumulation process in this model. When an investor is more risk-tolerant, the cost of active management is less costly because they have more appetite for a risky asset, thus the sacrificed time to accumulate human capital, which leads to higher labor income (a safer asset than equity) becomes less costly.

Figure 11 plots the welfare gain with use of a financial advisory service for different levels of financial decision making efficiency. This figure implies that the welfare gain is higher when investors' financial management efficiency is low. Since financial advisors help to save investors' time associated with financial management inefficiency, people with low levels of financial management skill will be benefited more than those with high levels of skill. This result suggests that governments should devise a policy to make financial advisory services accessible to investors' with low financial literacy, especially younger and older investors.

5 Conclusion

This study develops a lifecycle model to solve an optimal portfolio management scheme of finitely-lived investors who face portfolio management costs and the age-dependent inefficiency of financial decision making. Since investors accumulate job-specific knowledge by working, portfolio adjustment costs can have different impacts on different age groups. Based on a reasonable set of parameters, the model replicates portfolio inertia over all age groups, especially for young and old investors. This is because investors in their early career stages have a higher rate of human capital accumulation and thus spending time in financial management, which is not closely relevant to their job-specific skills, is very costly. Middle-aged investors are the most active group in terms of managing their own portfolios but almost 50% of them choose to remain inactive. A decreased efficiency level of financial decision making induced a significant portion of old investors to also choose portfolio inertia.

The model enables us to perform counterfactual experiments about the choice of portfolio management scheme when the option of delegating portfolio management to financial advisors is available. Under the baseline parameters, the delegation option replaces portfolio inertia across all age groups. About 30% of young investors switch from portfolio inertia to portfolio management delegation. Approximately 70% of middle-aged investors hire financial advisors, but they still remain as the most active self management group. Approximately 80% of old investors delegate their portfolio management to financial advisors and less than 5% of them still manage their portfolios themselves. In general, the model predicts that old investors will be the biggest customer group of financial advisors.

The welfare gains resulting from the introduction of a delegation option are substantial as measured by the constant consumption stream of certainty equivalent (CE). With baseline parameters, the introduction of a delegation option will increase young investors' constant stream of certainty equivalent (CE) consumption by 19.5%. This means investors can enjoy a higher level of annual consumption across the lifecycle when the delegation option is available. The level of the welfare gain is substantially higher than that of Cocco et al. (2005) who measured the welfare gains of flexible portfolio management compared to fixed asset allocation. The model also shows that the magnitude of welfare gains over age is U-shaped. Young and old workers are most benefited by the delegation option. Since this model only considers the welfare gains of investors, it ignores the welfare gains for the financial advisory industry. Thus, the actual welfare gains to the economy will be greater than the calculated level.

These findings have relevant implications for retirement plan sponsors, the financial advisory industry and policy makers wishing to support diverse age groups in retirement plans. As this paper's model predicts, financial advisory services will be very appealing to younger and older investors, and the availability of such services will have a meaningful impact on these groups' portfolio management. In addition, the financial advisors to middle-aged investors should also consider the fact that some of them still want to remain active in managing their financial assets, even when a delegation option is available. These people may have demand for brokerage services to self-manage their financial wealth.

Policy makers should consider the potential positive welfare gains of improving investors' access to prudential financial advisory services. Devising a policy to secure the fiduciary role of financial advisors will assist investors in managing their financial wealth optimally.

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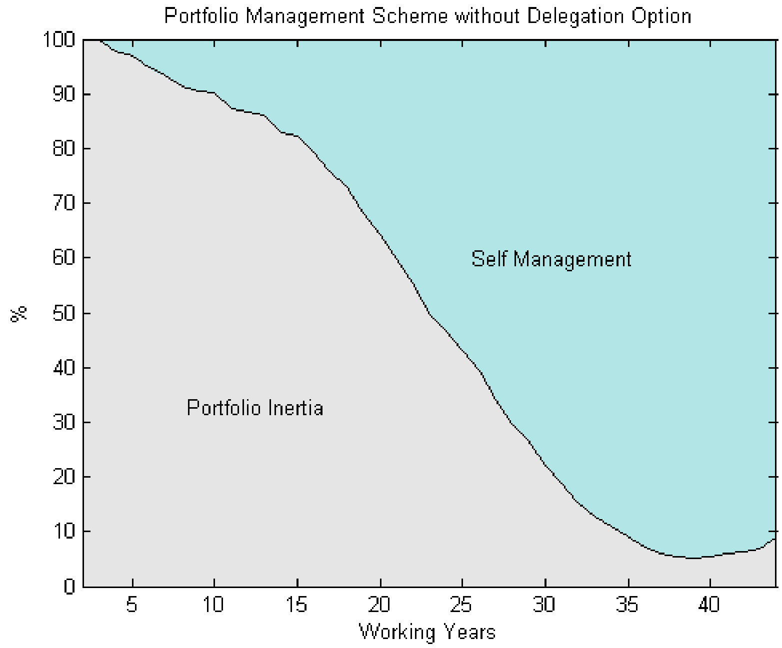
Table 1: Parameter Values for Numerical Solution

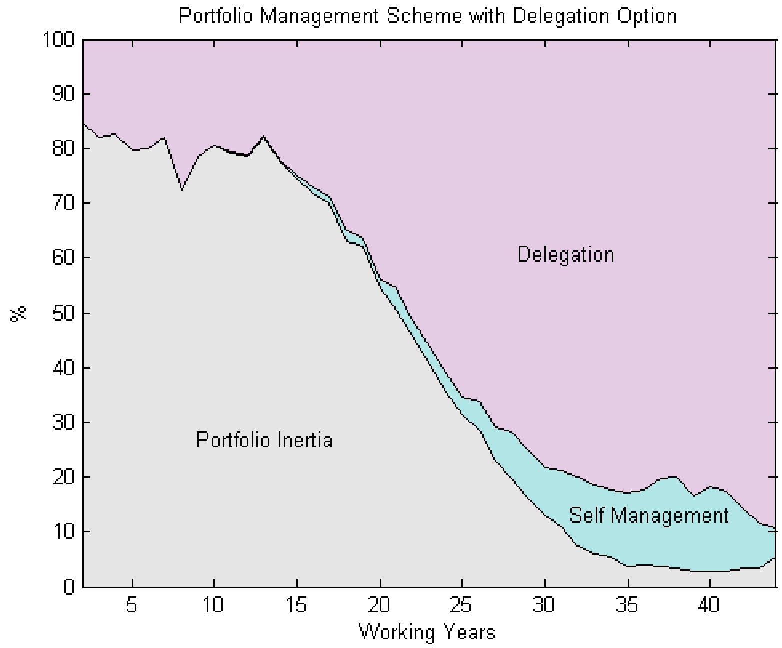
Parameter	Baseline	Source
Working periods T	45	-
Time discounting β	0.95	-
Risk aversion γ	2.5	-
Leisure preference α	0.9	Gomes et al. (2008)
Experience formulation a	0.209	Huggett et al. (2006)
Elasticity of H_t accumulation θ	0.7	Huggett et al. (2006)
Lowest inefficiency of financial decision ϕ_{low}	0.03	-
Highest inefficiency of financial decision ϕ_{high}	0.09	-
Depreciation of Human Capital δ_t	1.4% per annum	Huggett et al. (2006)
Efficiency of financial decision making ϕ_t	$\frac{0.09-0.03}{30^4}(age-30)^4+0.03$	-
Wage shock drift η	0.08	-
Wage shock auto correlation ρ	0.85	-
Std. of Wage shock σ_{wage}	0.1389	Gomes et al. (2008)
Risk premium	0.04	Gomes et al. (2008)
Std. of stock return σ_{stock}	0.205	Gomes et al. (2008)
Risk free rate $\overline{R}$	1.02	Cocco et al. (2005)
Delegation annual fee φ_t	1.3% per annum	MorningStar(2009)
Correlation between wage and stock return $\sigma_{\epsilon\zeta}$	0	Cocco et al. (2005)
Initial wealth for simulation W_0	0	-
Initial human capital for simulation H_0	10	90.48% of first year wage
Initial equity share for simulation π_0	0	Ameriks and Zeldes (2000)
Initial wage shock for simulation y_0	0.1	-

Figure 1: Portfolio Management Scheme Without Delegation Option



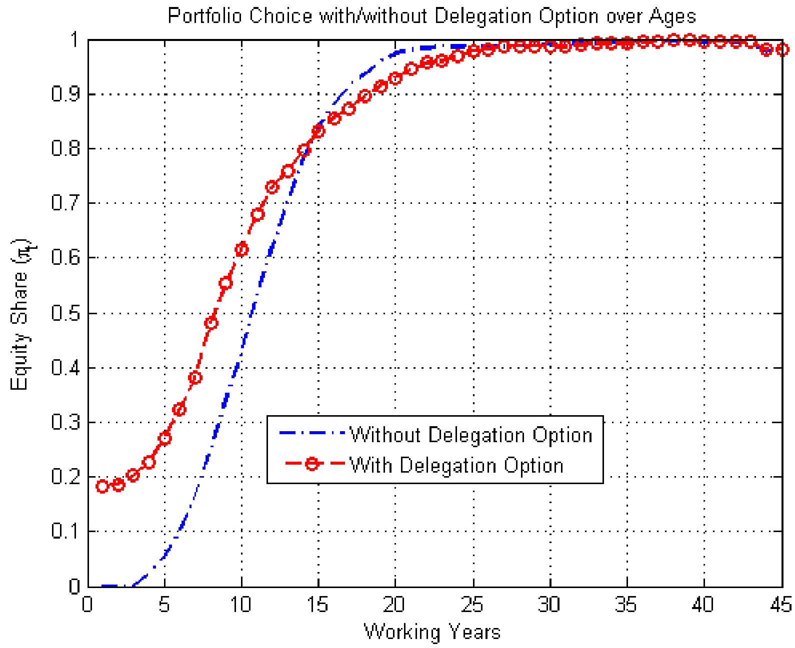
This figure shows the choice of portfolio management scheme when financial advisory service is NOT available. This figure replicates the empirical finding in the household finance literature that severe inactivity in individual portfolio management is widespread. When an investor does not have a delegation option, young investors are likely to choose 'no touch' strategy for their portfolio management. Since young workers have low levels of accumulated human capital but have long horizon to use it, their foregone opportunity to accumulate human capital might be costlier than the other ages groups with a high level of human capital accumulation. A fraction of old investors are likely to choose to be inactive because they have to incur inefficiency cost for making a sophisticated financial decision. Middle-aged people is more active in portfolio management compared to their younger and older counterparts.

Figure 2: Portfolio Management Scheme With Delegation Option



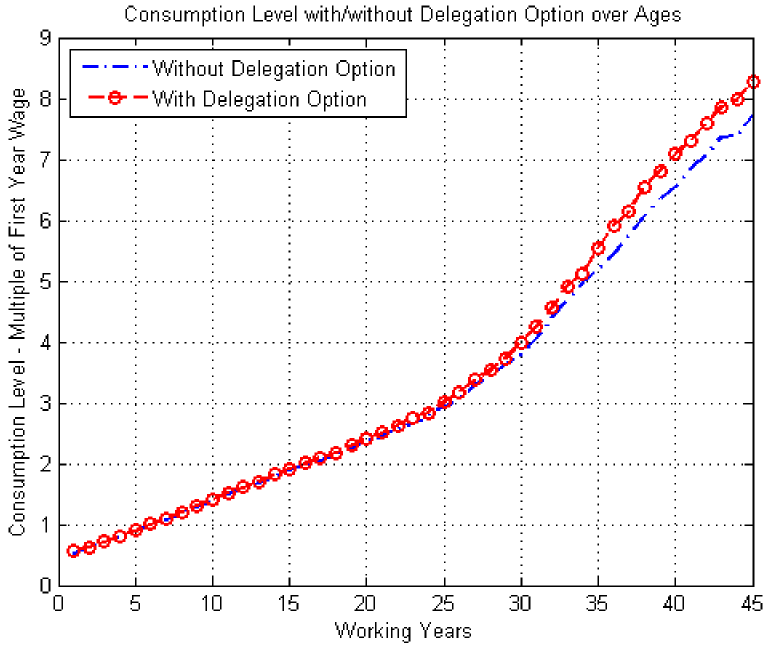
This figure shows the proportions of the chosen management schemes when there is an option to hire financial advisor. First, we observe a decrease in portfolio inertia across all age groups. Approximately 85% of young workers, 20% of middle-aged workers and 7% of old workers choose portfolio inertia as their management scheme. Second, delegating portfolio management becomes the dominant portfolio management scheme across all age groups, replacing the importance of the active management scheme in the previous case. Approximately 15% of young workers, 60% of middle-aged workers and 90% of old workers want to delegate their portfolio management task to financial advisors. Third, the active management scheme is implemented mostly by middle-aged workers with working experience between 23 to 41 years (i.e., workers age 34 to 62, if we assume people enter the labor market at 21 years old). Only a small fraction (less than 1%) of young workers and 5% of old workers choose active self-management. About 15% of middle-aged workers choose active self-management as their financial management scheme. These observations show that introducing a portfolio delegation option has a substantial impact on all age groups, especially younger and older investors.

Figure 3: Portfolio Choice over the Lifecycle



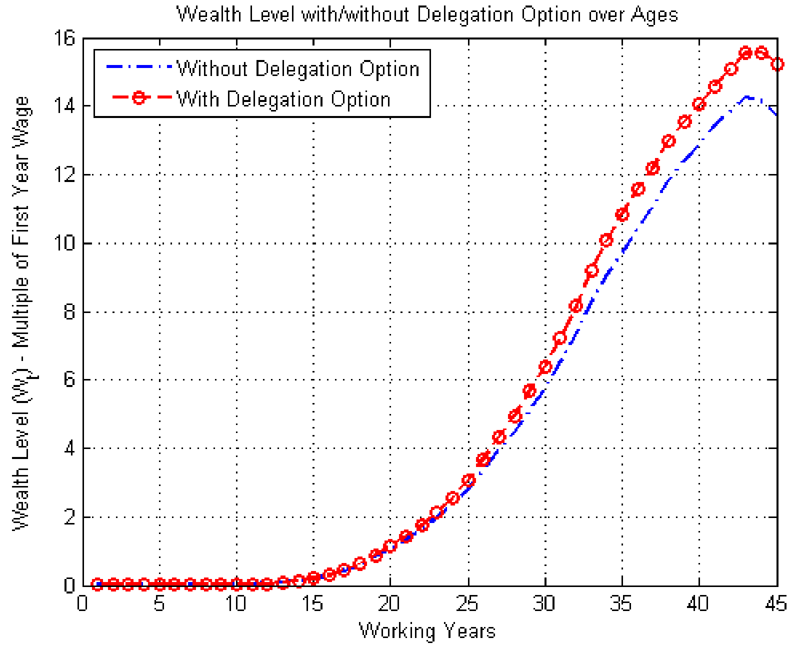
This figure plots the portfolio choice in each scenario over the lifecycle. One noticeable finding is that people without a delegation option are likely to hold a lower fraction of the risky asset in comparison with those with a delegation option in their early career stages. Since the delegation option saves investors' time, they can fully make use of their time to work and accumulate more human capital, which is safer than equity. Therefore, they have more of a buffer to the potentially negative shock of equity returns and they can invest more in risky asset. With our baseline parameters, middle-aged people invest most of their wealth in equity.

Figure 4: Consumption Level over the Lifecycle



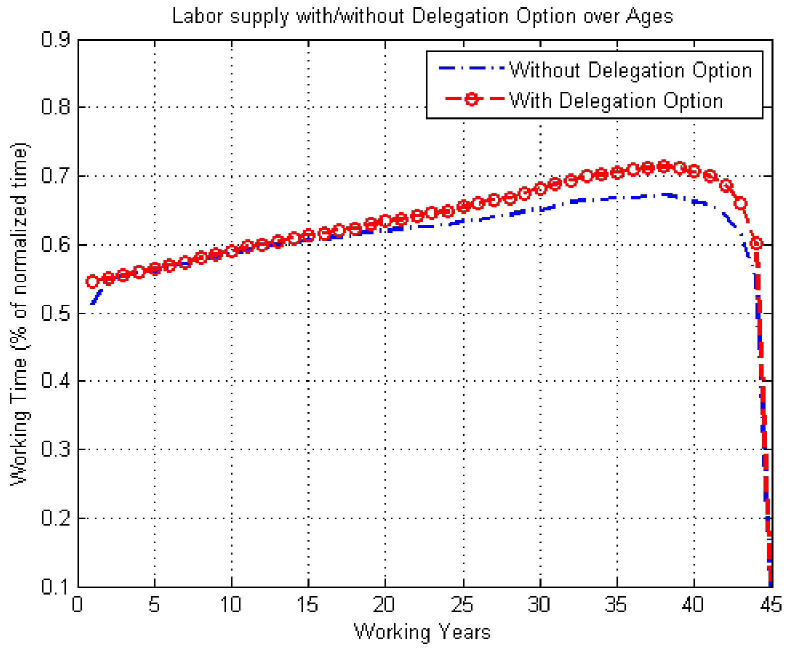
This figure plots the consumption level over the lifecycle in the two scenarios. We find that investors with a delegation option can consume more than those without a delegation option. There is little difference in the two scenarios (i.e., with and without a delegation option) in the early working periods, but the delegation service brings more consumption in the middle and later years. When delegating financial management is available, workers can allocate more time to their work and accumulate more job-specific knowledge, which leads to higher income and consumption.

Figure 5: Wealth Level over the Lifecycle



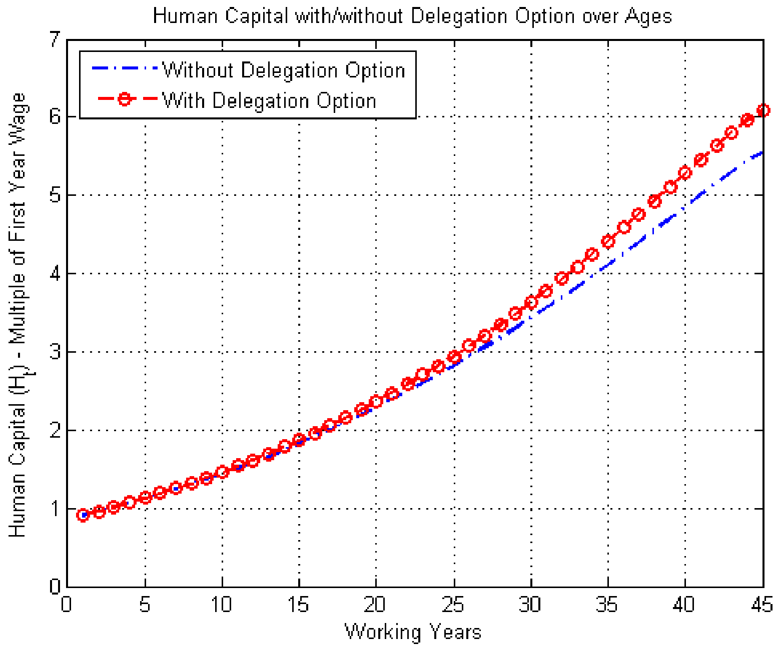
This figure plots the average wealth level of investors over the lifecycle. We observe that financial advisory service can bring a higher level of wealth eventually. This is not because they bring much higher excess return in financial management but they save time and the deficiency cost which is associated with portfolio management in this model. This is another evidence for the claim that introducing delegation or advisory service will produce significant welfare gain for investors.

Figure 6: Working Time Chosen over the Lifecycle



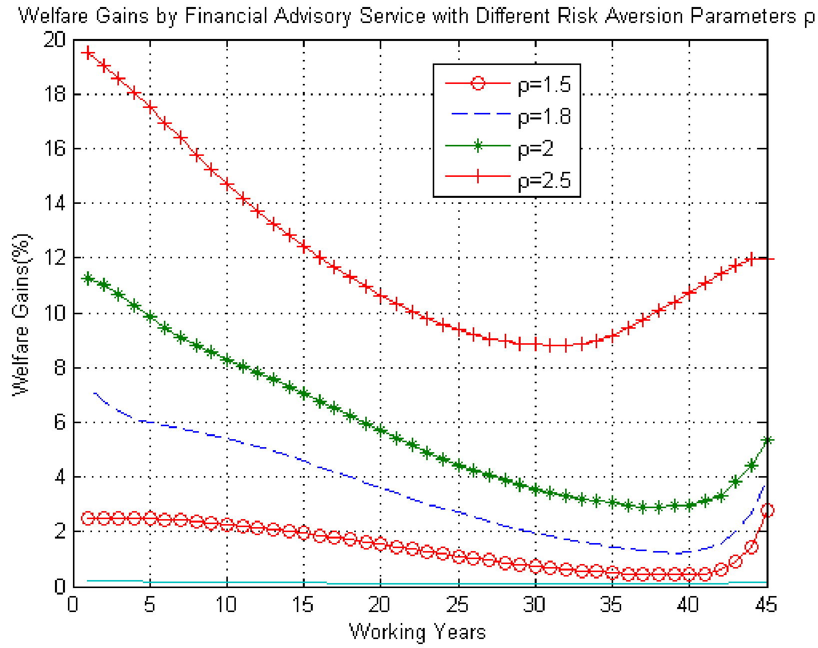
This figure plots the fraction of investors' available time allocated to their own work. It has an inverse U-shape over the lifecycle, which is consistent with the macroeconomics literature. If a financial advisory service is not available, the worker must sacrifice some portion of his time, which could have been allocated to working, to self-manage his asset. In the early career stage, the delegation option enables workers to allocate more time to working and accumulating more human capital. In the later career stages, the option enables the worker to work less (and therefore enjoy more leisure), but continue to accumulate human capital by sparing time spent on financial management. Saved time can be allocated to leisure too, which will lead to higher life-time utility.

Figure 7: Human Capital Accumulation Pattern over the Lifecycle



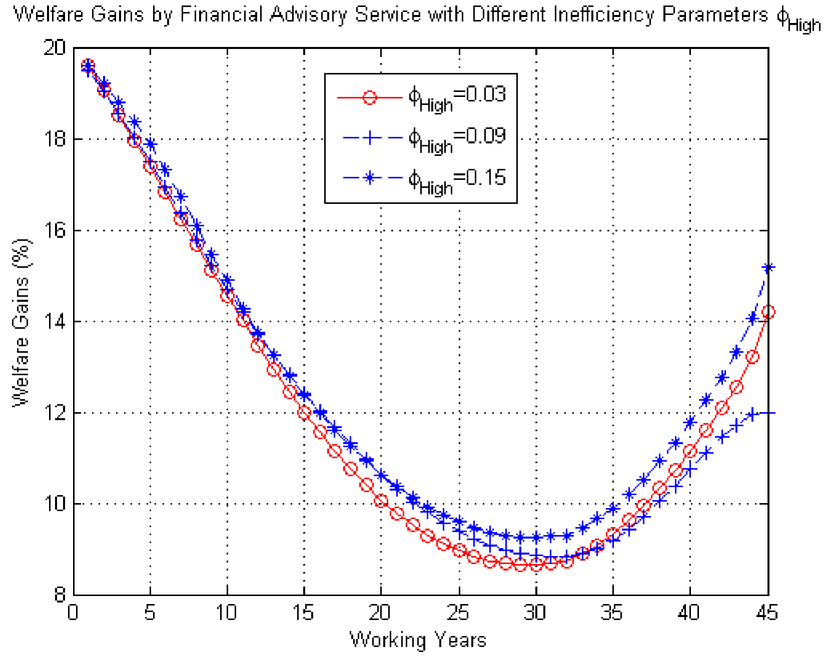
This figure plots the pattern of accumulated human capital over the lifecycle. We find that investors with a delegation option can accumulate more human capital than those without a delegation option. With the delegation option available, workers can fully make use of their time to work without fiddling around with their financial wealth and thus enjoy more leisure with the same level of human capital accumulation compared to that of a self-management case.

Figure 8: Welfare gains for Different Risk Aversion Parameters (ρ)



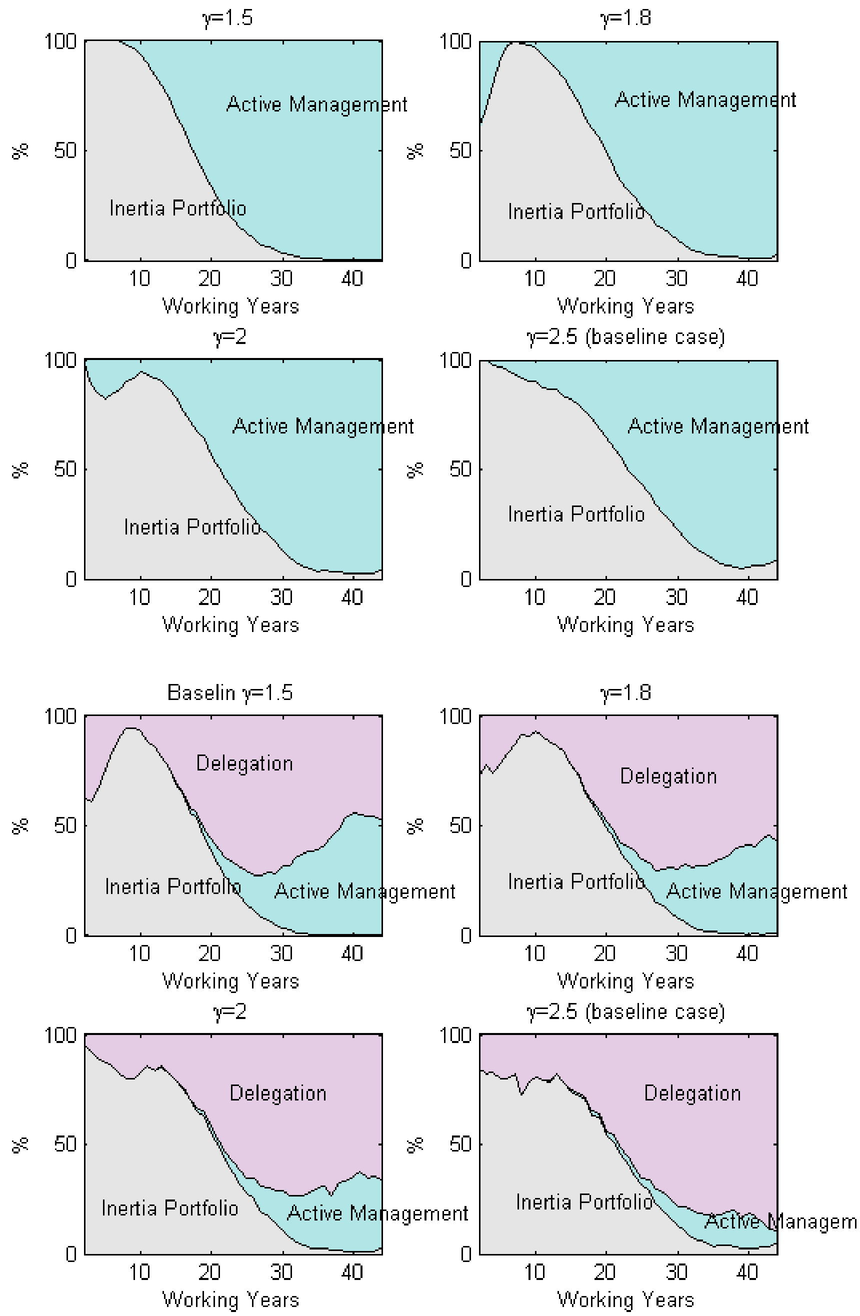
This figure plots the pattern of welfare gains over ages by a financial advisory service for different levels of relative risk aversion. It shows that the magnitude of welfare gains over ages are U-shaped. Young and old workers are most benefited by the delegation option. Welfare gains get higher when the relative risk aversion gets higher. When investors have high risk aversion, the time sacrificed to accumulate more human capital will be even costlier, because their safe asset (labor income) decreases. Therefore, the option to delegate the task of financial management will be more beneficial to them compared to those with lower risk aversion.

Figure 9: Welfare gains for Different Inefficiency Parameters (ϕ_{High})



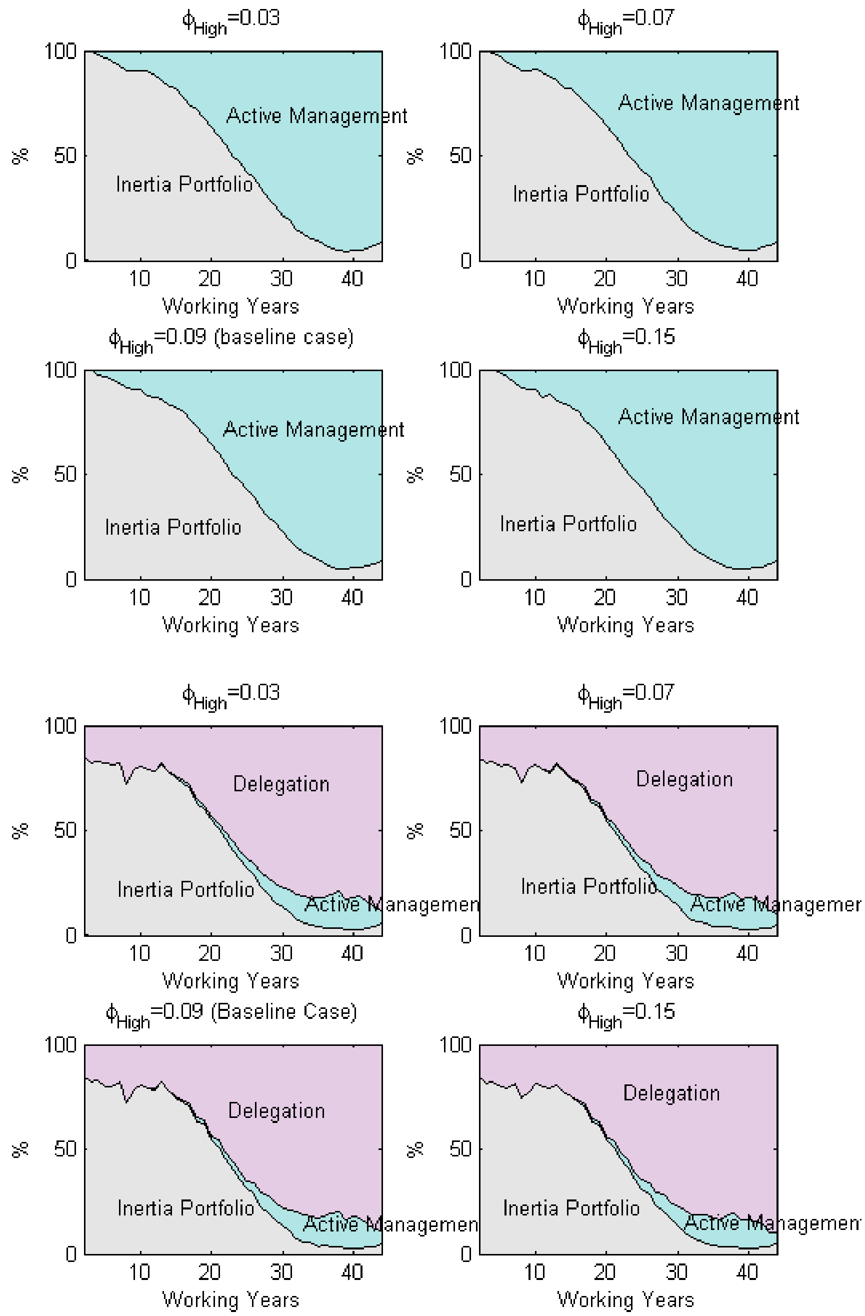
This figure plots the welfare gain with use of a financial advisory service for different levels of financial decision making efficiency. This figure implies that the welfare gain is higher when investors' financial management efficiency is low. Since financial advisors help to save investors' time associated with financial management inefficiency, people with low levels of financial management skill will be benefited more than those with high levels of skill. This result suggests that governments should devise a policy to make financial advisory services accessible to investors' with low financial literacy, especially younger and older investors.

Figure 10: Patterns of Management Scheme with Different Risk Aversions (ρ)



This figure plots the choice of portfolio management scheme with different levels of risk-aversion. People are likely to choose portfolio inertia more as risk-aversion increases. When a delegation option is available, it dominates the other two management schemes for most age groups. However, more people are likely to self-manage their portfolios when risk aversion decreases. This increased level of active management can be explained by the human capital accumulation process in this model. When an investor is more risk-tolerant, the cost of active management is less costly because they have more appetite for a risky asset, thus the sacrificed time to accumulate human capital, which leads to higher labor income (a safer asset than equity) becomes less costly.

Figure 11: Patterns of Management Scheme with Different Efficiency Parameters (ϕ_{High})



This figure shows that the choice patterns of portfolio management schemes are similar for different inefficiency parameters ϕ_{High} on financial decision making. One interesting finding is that the result is robust to the assumption on a shape of inefficiency pattern over ages. The baseline case assumes a U-shaped inefficiency pattern of financial decision making but the model with a flat inefficiency pattern over ages also generate similar results to that of the baseline case. The portion of portfolio inertia is lowest among middle-aged workers and highest among young workers. The delegation option dominates the other two alternatives and small portion of middle-aged workers are likely to self-manage their portfolio. This robustness result shows that the choice pattern of financial management schemes over ages is not driven by the shape of inefficiency pattern of financial decision making.

6 Appendix

6.1 Proof of Proposition 2

Define the excess value of choosing inertia portfolio over active management scheme as

$$\begin{aligned}\tilde{V}_t\left(l_t, \pi_{t+1}, c_t;\, W_t, H_t, \pi_t, y_t\right) \equiv & V_t^i\left(W_t, H_t, \pi_t, y_t\right) \\ & -\left\{u\left(c_t, 1-l_t-\phi_t\right)+\beta E_t\left[V_{t+1}\left(W_{t+1}, H_{t+1}, \pi_{t+1}, y_{t+1}\right)\right]\right\}\end{aligned}$$

The latter part is the objective function of active portfolio management scheme. Then,the excess value function is

$$\begin{aligned}& \tilde{V}_t\left(\hat{l}_t^i, \hat{\pi}_{t+1}^i, \hat{c}_t^i;\, W_t, H_t, \pi_t, y_t\right) \\ = & u\left(\hat{c}_t^i, 1-\hat{l}_t^i\right)-u\left(\hat{c}_t^i, 1-\phi_t-\hat{l}_t^i\right) \\ > & 0\end{aligned}$$

because utility function u is increasing in leisure time. Since $\tilde{V}_t\left(\cdot;\, W_t, H_t, \pi_t, y_t\right)$ is continuous in $\left(l_t, \pi_{t+1}, c_t\right)$, then $\exists \delta>0$ such that $\forall\left(l_t, \pi_{t+1}, c_t\right)$ with $d\left[\left(l_t, \pi_{t+1}, c_t\right),\left(\hat{l}_t^i, \pi_t, \hat{c}_t^i\right)\right]<\delta$, we have $\tilde{V}_t\left(l_t, \pi_{t+1}, c_t;\, W_t, H_t, \pi_t, y_t\right)>0$. Choose $\delta^*=\sqrt{\frac{\delta}{3}}$. By the assumption that $\max \left\{\left|\hat{l}_t^a-\hat{l}_t^i\right|,\left|\hat{c}_t^a-\hat{c}_t^i\right|\right\}<\left|\hat{\pi}_{t+1}^a-\pi_t\right|$, the condition $\left|\hat{\pi}_{t+1}^a-\pi_t\right|<\delta^*$ implies that

$$\left|\hat{l}_t^a-\hat{l}_t^i\right|^2+\left|\hat{\pi}_{t+1}^a-\pi_t\right|^2+\left|\hat{c}_t^a-\hat{c}_t^i\right|^2<3\left(\delta^*\right)^2=\delta$$

Thus, $V_t^i\left(W_t, H_t, \pi_t, y_t\right)-\left\{u\left(\hat{c}_t^a, 1-\hat{l}_t^a-\phi_t\right)+\beta E_t\left[V_{t+1}\left(W_{t+1}, H_{t+1}, \hat{\pi}_{t+1}^a, y_{t+1}\right)\right]\right\}>0$ and the latter part is now $V_t^a\left(W_t, H_t, \pi_t, y_t\right)$ because $\left(\hat{l}_t^a, \hat{\pi}_{t+1}^a, \hat{c}_t^a\right)$ is the solution of active management scheme. So we showed $V_t^i>V_t^a$ for all $\hat{\pi}_{t+1}^a$ with $\left|\hat{\pi}_{t+1}^a-\pi_t\right|<\delta^*$. QED

6.2 Proof of Proposition 3

I use backward induction to show the existence of a solution for an investor's portfolio choice problem without delegation option. The existence of solution for delegation option can be similarly proved. Using a discrete choice model, I can define a value function as

$$V_t\left(W_t, H_t, \pi_t, y_t\right)=\max \left\{V_t^a\left(W_t, H_t, \pi_t, y_t\right),\, V_t^i\left(W_t, H_t, \pi_t, y_t\right)\right\}$$

for all state vector $\{(W_{t,}H_t, \pi_t, y_t)\}_{t=1}^{T}$. The superscript a denotes the portfolio adjustment and i denotes inaction. The value functions for each decisions are defined as

$$V_t^a\left(W_t, H_t, \pi_t, y_t\right) = \max_{\{c_t, \pi_{t+1}, l_t\}} u_t\left(c_t, L_t\right) + \beta E_t\left[V_{t+1}\left(W_{t+1}, H_{t+1}, \pi_{t+1}, y_{t+1}\right)\right]$$

$$s.t\ c_t \leq W_t + l_t H_t Y_t$$

$$W_{t+1} = R_{t+1}^p\left(W_t + l_t H_t Y_t - c_t\right)$$

$$H_{t+1} = \left(1 - \delta_t\right) H_t + F_t\left(H_t, l_t\right)$$

$$l_t + L_t + \phi_t = 1$$

$$y_{t+1} = \eta + \rho y_t + \epsilon_{t+1}$$

and for the inactivity case

$$V_t^i\left(W_t, H_t, \pi_t, y_t\right) = \max_{\{c_t, l_t\}} u_t\left(c_t, L_t\right) + \beta E_t\left[V_{t+1}\left(W_{t+1}, H_{t+1}, \pi_{t+1} = \pi_t, y_{t+1}\right)\right]$$

$$s.t\ c_t \leq W_t + l_t H_t Y_t$$

$$W_{t+1} = R_{t+1}^p\left(W_t + l_t H_t Y_t - c_t\right)$$

$$H_{t+1} = \left(1 - \delta_t\right) H_t + F_t\left(H_t, l_t\right)$$

$$l_t + L_t = 1$$

$$y_t = \eta + \rho y_{t-1} + \epsilon_t$$

Now, I can use a backward induction.

1. At the retirement period T, an investor does not make any portfolio decision($\pi_{T+1} = \pi_T$) and consumes all his wealth

$$c_T = R_T^p\left(W_T + l_t^* H_T Y_T\right)$$

   where l_t^* is determined by static optimal decision between l_t and L_t with $l_t + L_t = 1$ ($\phi_t = 0$). Now, $V_T\left(W_{T,}H_T, \pi_T, y_T\right)$ for each state is well defined and we can find $V_{T-1}^a\left(\cdot\right)$ and $V_{T-1}^i\left(\cdot\right)$ by their definitions.

2. With known $V^a_{T-1}(\cdot)$ and $V^i_{T-1}(\cdot)$, I can find $V_{T-1}(\cdot)$ as

$$V_{T-1}(\cdot) = \max\left\{V^a_{T-1}(\cdot), V^i_{T-1}(\cdot)\right\}$$

   And we know there exist a solution for V^a_{T-1} and V^i_{T-1} because the constraint sets are compact and objective function is continuous [The Weierstrass Theorem].

3. Repeat step 1 and 2 until the first period.

4. After I find value functions at every periods, I can derive policy functions, especially portfolio adjustment decision at each period.

When delegation is available, an investor can fully utilize her time to work optimally. The value function at each state vector is defined as

$$V_t(W_t, H_t, \pi_t, y_t) = \max\left\{V^a_t(W_t, H_t, \pi_t, y_t), V^i_t(W_t, H_t, \pi_t, y_t), V^d_t(W_t, H_t, \pi_t, y_t)\right\}$$

for all state vector (W_t, H_t, π_t, y_t). The superscript d denotes delegating to a professional money manager. The value functions $V^a_t(W_t, H_t, \pi_t, y_t)$, $V^i_t(W_t, H_t, \pi_t, y_t)$ are defined in the same way in the previous section. With management fee(φ_t), the value function with delegation option is defined as

$$\begin{aligned}
V^d_t(W_t, H_t, \pi_t, y_t) = & \max_{c_t, \pi_{t+1}, l_t} u_t(c_t, L_t) + \beta E_t\left[V_{t+1}(W_{t+1}, y_{t+1}, \pi_{t+1}, y_{t+1})\right] \\
& s.t\ c_t \leq W_t + l_t H_t Y_t \\
& \quad W_{t+1} = (1-\varphi_t) R^p_{t+1}(W_t + l_t H_t Y_t - c_t) \\
& \quad H_{t+1} = (1-\delta_t) H_t + F_t(H_t, l_t) \\
& \quad l_t + L_t = 1 \\
& \quad y_t = \eta + \rho y_{t-1} + \epsilon_t
\end{aligned}$$

Since V_T is well defined as shown above, $\{V_t\}_{t=1}^{T-1}$ is well defined and we know the maximization problems have solutions.