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Joseph E. Harrington Jr
University of Pennsylvania

Joe Chen

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Joseph E.Harrington, Jr.
Johns Hopkins University

Joe Chen
University of Tokyo

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Joseph E. Harrington, Jr.
Department of Economics
Johns Hopkins University
Baltimore, MD 21218 USA
410-516-7615, -7600 (Fax)
joe.harrington@jhu.edu

Joe Chen
Faculty of Economics
University of Tokyo
7-3-1 Hongo, Bunkyo-ku
Tokyo, 113-0033 Japan
joechen@e.u-tokyo.ac.jp

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Abstract

This paper characterizes collusive pricing patterns when buyers may detect the presence of a cartel. Buyers are assumed to become suspicious when observed prices are anomalous. We find that the cartel price path is comprised of two phases. During the transitional phase, price is generally rising and relatively unresponsive to cost shocks. During the stationary phase, price responds to cost but is much less sensitive than under non-collusion or simple monopoly; a low price variance may then be a collusive marker. Compared to when firms do not collude, cost shocks take a longer time to pass-through to price.

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1 Introduction

From an antitrust perspective, the two central tasks for a theory of price-fixing are identifying conditions that facilitate collusion and, towards discerning the presence of a cartel, characterizing the properties of collusive pricing. Though there is a large theoretical literature addressing these issues, work has generally failed to take account of an important dimension to this problem. In light of the illegality of collusion, firms don't just want to achieve prices that raise profit and are internally stable; they also want to avoid creating suspicions that a cartel has formed. Given that if such suspicions emerge they could initiate a process that ultimately means the collapse of the cartel and the levying of substantial financial penalties, avoiding detection is as crucial as deterring deviations by cartel members.

Towards developing a richer model of cartel pricing, this paper constructs a dynamic computational model of cartel pricing which endogenizes detection. Recognizing that the antitrust authorities do not generally detect collusion, the focus is on buyers which, in many if not most price-fixing cases, are industrial buyers such as with the vitamins, lysine, and graphite electrodes cartels. In deciding whether or not to form a cartel and, if they do, what prices to set, the cartel takes into account how their prices influence the likelihood of triggering detection of collusion by buyers. A modelling challenge arises in that it is highly problematic to presume that buyers are consciously engaging in detection or that they know what to look for as regards collusion. What strikes us as the most plausible specification is that buyers become suspicious when they observe anomalous pricing; that is, a price path that is unusual or inexplicable. We pursue this idea and develop a novel theory of belief formation. Buyers are more likely to be suspicious when the likelihood attached to recent prices is sufficiently small where this likelihood is based on buyers' beliefs about price changes based on the empirical history of prices. The cartel's problem is then set up as a dynamic programming problem with an endogenous terminal date determined by the buyer's belief formation process. Associated with this terminal date is a payoff based on the future profit stream after being caught colluding less any penalties where these penalties depend on the prices and costs over the time of the cartel.

Several systematic properties emerge. The cartel price path is comprised of a transition phase - in which price moves largely irrespective of cost - and a stationary phase - in which price is responsive to cost. In the transition phase, price initially rises though, for some parameter specifications, the price path overshoots so that it converges from above. While price is sensitive to cost in the stationary phase, it is much less volatile than either the non-collusive price or the simple monopoly price path. Furthermore, collusion results in cost shocks taking a longer time to pass-through to price. In industries with less cost variability, the transition phase tends to be longer, price doesn't rise as fast, and there is more overshooting. Though the analysis is clearly intended to be exploratory, the ensuing price paths are encouraging in that they look much more like actual cartel price paths than what has thus far been produced by the theory of collusive pricing.[1]

Related Work Previous work has explored optimal cartel pricing under the constraint of possible detection though using static formulations or restricted dynamic models. There are three classes of models. First are static models which use a reduced form approach to modelling detection or prosecution. The earliest paper is Block, Nold, and Sidak (1981) which assumes the probability of detection is increasing in the price-cost margin. A second class continues with an exogenous modelling of detection but considers a dynamic setting. Cyrenne (1999) modifies Green and Porter (1984) by assuming that a price war, and the ensuing raising of price after the war, results in detection for sure. Spagnolo (2000) and Motta and Polo (2003) explore the effects of leniency programs on the incentives to collude when the probability of detection and penalties are both fixed. Though considering collusive behavior in a dynamic setting with antitrust laws, these papers exclude the sources of dynamics that are the foci of the current analysis; specifically, they do not allow detection and penalties to be sensitive to firms' current and past pricing behavior. More closely related is recent work by one of the authors (Harrington, 2003, 2004a, 2005). In those papers, a dynamic theory of cartel pricing is developed in which price influences the likelihood of detection but, contrary to the current paper, a reduced form approach is used to model how prices influence detection. For example, the probability of detection is assumed to be increasing in the extent of price changes or in the price level. In the current paper, there is an explicit model of buyers' beliefs which has the virtue of generating a richer set of dynamics and being able to endogenously derive how the properties of the price path depend on industry traits such as cost variability. The third class of models are static but endogenize detection by modelling those who are engaging in it. The original work was Besanko and Spulber (1989, 1990) who use a game of incomplete information so that buyers or the antitrust authority are uncertain about some relevant parameter which makes them uncertain about whether a cartel has formed. Further work using this approach includes LaCasse (1995), Souam (2001), and Schinkel and Tuinstra (2002). In comparison, our model has multiple periods - and thus can derive results on pricing

[1] For some actual cartel price paths, see Levenstein and Suslow (2001).

dynamics - and it endogenizes detection using non-equilibrium beliefs for buyers which we believe is more plausible.

2 Model

2.1 Market Conditions

Consider a symmetric oligopoly with a linear market demand function:

$$D(P) = a - bP,$$

where $a, b > 0$. Firms have a common constant marginal cost of production. While the demand function is assumed to be fixed over time, marginal cost is allowed to vary stochastically. Letting c^t be unit cost in t, industry profit is

$$\pi\left(P, c^t\right) \equiv \left(P - c^t\right)(a - bP).$$

As our analysis will focus on characterizing the joint profit maximizing price, industry profit is all that matters. As we'll see, the analysis is quite rich even without taking account of incentive compatibility constraints which we plan to tackle in the next step of this research.

Subject to some boundary conditions, $c^t = c^{t-1} + \varepsilon^t$ with ε^t being normally distributed and *iid* over time. For future reference, let $f\left(\cdot; \mu, \sigma^2\right)$ denote the density function for the normal distribution with mean and variance $\left(\mu, \sigma^2\right)$; the density function on ε^t is then $f\left(\cdot; \mu_\varepsilon, \sigma_\varepsilon^2\right)$. Unit cost has support $[\underline{c}, \overline{c}]$ and assume $0 \leq \underline{c} < \overline{c} < a$ with the last inequality ensuring that, for all cost realizations, there exists a common price such that firm profit is positive. The stochastic process on cost is then

$$c^t = \begin{cases} \underline{c} & \text{if } c^{t-1} + \varepsilon^t < \underline{c} \\ c^{t-1} + \varepsilon^t & \text{if } \underline{c} \leq c^{t-1} + \varepsilon^t \leq \overline{c} \\ \overline{c} & \text{if } \overline{c} < c^{t-1} + \varepsilon^t \end{cases}$$

or, equivalently,

$$c^t = \upsilon\left(c^{t-1} + \varepsilon^t\right) \equiv \max\left\{\underline{c}, \min\left\{c^{t-1} + \varepsilon^t, \overline{c}\right\}\right\}.$$

Firms commonly observe the current period's cost prior to choosing price. For future reference, the joint profit-maximizing price is

$$P^m\left(c^t\right) \equiv \frac{a + bc^t}{2b}.$$

In the absence of forming a cartel, firms achieve a non-collusive solution which is characterized by a common price $\widehat{P}(c^t)$ with industry profit of

$$\widehat{\pi}\left(c^t\right) \equiv \left(\widehat{P}\left(c^t\right) - c^t\right)\left(a - b\widehat{P}\left(c^t\right)\right).$$

Given linear demand and cost functions, the analysis will focus on linear non-collusive (equilibrium) pricing rules,

$$\widehat{P}\left(c^t\right) = w_0 + w_1 c^t,$$

where $(w_0, w_1) \in [0, a/2b) \times (1/2, 1]$. This class of solutions includes the Nash equilibrium to the price game with homogeneous goods, $(w_0, w_1) = (0, 1)$, and the Nash equilibrium to the quantity game with homogeneous goods, $(w_0, w_1) = (a/b\,(n+1), n/(n+1))$, as well as the Nash equilibrium to the price game for some formulations of symmetrically differentiated products.

With an infinite horizon, each firm's payoff is the discounted sum of expected profits where the common discount factor is $\delta \in (0, 1)$. When firms are not colluding, the expected present value of the industry's profit stream at t (after the period's cost is realized) is denoted $W(c^t)$ and is defined recursively by:

$$W\left(c^t\right) = \widehat{\pi}\left(c^t\right) + \delta \int W\left(\upsilon\left(c^t + \varepsilon\right)\right) f\left(\varepsilon; \mu_\varepsilon, \sigma_\varepsilon^2\right) d\varepsilon.$$

2.2 Cartel Detection

If firms form a cartel, it is detected with some probability and firms incur penalties in that event. In practice, detection occurs from a variety of sources; some of which are related to price - such as customer complaints - and some of which are unrelated to price - such as internal whistleblowers. Hay and Kelley (1974) find that detection was attributed to a complaint by a customer or a local, state, or federal agency in 13 of 49 price-fixing cases and, more recently, cases were initiated in graphite electrodes (Levenstein and Suslow, 2001) and stainless steel (Levenstein, Suslow, and Oswald, 2004) by buyers' complaints. Anomalous pricing may cause customers to become suspicious and pursue legal action or share their suspicions with the antitrust authorities.[2] But, as a matter of practice, the antitrust authorities do not engage in detection:

> As a general rule, the [Antitrust] Division follows leads generated by disgruntled employees, unhappy customers, or witnesses from ongoing investigations.

[2] The Nasdaq case is one in which truly anomalous pricing resulted in suspicions about collusion. It was scholars rather than market participants who observed that dealers avoided odd-eighth quotes and ultimately explained it as a form of collusive behavior (Christie and Schultz, 1994).

> As such, it is very much a reactive agency with respect to the search for criminal antitrust violations. ... Customers, especially federal, state, and local procurement agencies, play a role in identifying suspicious pricing, bid, or shipment patterns. [McAnney, 1991, pp. 529, 530]

In practice, it is buyers - and, in most cartel cases, they are industrial buyers - who are part of the first line of detection.

In previous work on this topic by one of the authors (Harrington, 2003, 2004a, 2005), a reduced from approach was taken to modelling detection. Various specifications were considered including having the probability of detection be increasing in the price level and the absolute value of the change in price. In this paper, we take on the more difficult task of endogenizing detection by explicitly modelling buyers' beliefs and how they come to suspect that a cartel is present. To be clear, we are trying to model what leads buyers to think that firms may have cartelized. There are thousands of industries, yet cartels are suspected in only a few. Our goal is to model how firm behavior triggers suspicions among buyers. The implicit assumption is that once suspicions emerge, an investigation reveals evidence of collusion if indeed there is a cartel. Admittedly, this is only the start of the process in that suspicions will typically be followed with a preliminary investigation - by the potential plaintiffs or the antitrust authorities - to determine whether the case is worth pursuing. And, if it is, there is still the process by which conviction or a settlement is achieved. For the sake of parsimony, we focus on modelling the first stage - the creation of suspicions about collusion - and presume that an investigation will reveal the truth.[3]

The classical game-theoretic equilibrium approach to endogenizing buyers' beliefs about a cartel having formed is to model it as a game of incomplete information where buyers do not know some firm trait relevant to cartel formation; for example, a common cost which is privately known to the firms. A Bayes-Nash equilibrium is characterized for a setting in which buyers observe price and then Bayesian update over the two possible events - a cartel has formed and a cartel has not formed - using their prior beliefs over the unknown firm trait and their knowledge of the collusive and non-collusive pricing func-

[3] Though detection leads to conviction for sure, results would almost certainly go through if the probability of conviction is only required to be positive. The more restrictive aspect of this specification is that the probability of firms paying penalties, conditional on an investigation, is independent of prices. However, this is probably not a bad assumption. Though a cartel may be detected because of suspicious pricing, price data is typically not central to achieving a conviction or a guilty plea; rather, it is "smoking gun" evidence such as memos, meetings, and witnesses that are of primary importance. Prices are, however, important in determining penalties and this our model allows for.

tions. This approach is used in Besanko and Spulber (1989, 1990) for the static setting. What makes this approach problematic, in our opinion, is that it rests on the assumption that *buyers know how a cartel prices.* It is the objective of this research project to characterize how a cartel prices and to presume that buyers (or even the antitrust authorities) already know the answer is simply denying reality. To begin, figuring out how a cartel prices meaning solving a hard problem - how cartels price over time when they want to avoid detection in the midst of endogenous penalties. It is a problem for which the experts - academic economists and consultants - do not currently have an answer. Yet, taking the vitamins case as an example, an equilibrium approach requires us to suppose that a mid-level employee of Tyson Foods is going to have the sophistication to address a question that a PhD Economist might not be able to solve. At this point, we often resort to the argument that an economic agent doesn't need to solve for another agent's strategy but can learn it from experience. But if this is the typical industry then there is no documented history of collusion among input suppliers and thereby no experience from which the employee can draw. Furthermore, even if employees had the tools and data to address this difficult question, we contend it would not be optimal for them to do so. They have limited time and resources and using them to develop a strategy for detecting collusion, given its empirical frequency in the economy is low (as measured by detected cartels), would not pass a cost-benefit analysis. Their time would be better spent scoping out new suppliers, managing inventories, working out new contractual arrangements, controlling waste, and the like rather than focusing on the unlikely outcome of there being collusion among input suppliers.

For these various reasons, we have chosen not to assume buyers know the collusive pricing function, nor that they consciously engage in detection. This does not imply, however, that detection is exogenous, nor that buyers are oblivious to the possibility of collusion. Our working assumption is that when something strange happens - like a sudden price increase or an unusual pattern in price changes - the possibility of collusion enters into the mind of buyers. The observation of an "unlikely" price series may trigger buyers to reevaluate their implicit model of how prices are determined and thereby put into question their maintained hypothesis of competition. This could result in a variety of alternative hypotheses with collusion being one of them. In the remainder of this subsection, we put forth a theoretical model of buyers' beliefs and follow it with a description of how we implement it computationally.

2.2.1 Modelling Anomalous Events - Theoretical Specification

The task before us is to model what it means for buyers to observe an anomalous event. It is such events that provide the epiphany to re-evaluate one's presumptions about how price is determined and which allow the possibility of collusion to move to the forefront of a buyer's mind. The spirit of the approach we take is based on the idea of hypothesis testing.[4] Buyers have a null hypothesis about the pricing process and become "suspicious" when the observed price series is sufficiently unlikely under the null. Thus, one can think of the discrete event of "becoming suspicious" as being associated with rejecting the null hypothesis. More specifically, buyers' null beliefs are based on price data when firms had been competing. Then, unbeknownst to the buyers, the firms have cartelized. The issue is whether buyers will pick up the structural break.

Given the true underlying stochastic process on firms' common unit cost and that the non-collusive price is an affine function of cost, it is natural to presume that buyers believe the price-generating process to be:

$$P^t = P^{t-1} + \zeta^t,$$

where ζ^t is normally distributed. However, they do not know the mean and variance of the distribution on price changes. One motivation for this specification is that buyers have the maintained hypotheses that price is an affine function of cost and cost changes are normally distributed but do not know the coefficients to the pricing function or the moments of the cost distribution.[5] To derive moments to their beliefs, buyers use observed prices. Buyers have a memory of k periods so that, coming into period t, their data set is comprised of the k most recent price changes, $\left\{\Delta P^{t-k}, \ldots, \Delta P^{t-1}\right\}$, where $\Delta P^{\tau} \equiv P^{\tau} - P^{\tau-1}$. The i^{th} moment of the sampling distribution coming into t is

$$m_i^{t-1} \equiv \left(\frac{1}{k}\right) \sum_{\tau=t-k}^{t-1} \left(\Delta P^{\tau}\right)^i .$$

Buyers' beliefs over the period t price change are assumed to have a normal distribution based on the sampling moments, $N\left(m_1^{t-1}, m_2^{t-1} - \left(m_1^{t-1}\right)^2\right)$.

Now consider buyers "testing" a sequence of the $z < k$ most recent observations, as of the end of t. The idea is not that they test all these price changes at once but are, in

[4]For the use of hypothesis testing in learning in games, see Foster and Young (2003).

[5]One might object at this point that there is an inconsistency in this formulation in that buyers presume the stochastic price process is fixed when, in fact, it can change because of cartel formation. But then that is really the heart of the problem. Will buyers pick up the "break" in the price process associated with cartelization?

a sense, testing each price change as it occurs. Recall that buyers are not assumed to be consciously engaging in this process. The likelihood of the z most recent price changes is specified to be

$$l^t \equiv \Pi_{\tau=t+1-z}^{t} f\left(\Delta P^\tau; m_1^{\tau-1}, m_2^{\tau-1} - \left(m_1^{\tau-1}\right)^2\right).$$

This is a moving likelihood in that the density function is updated along the way. It is as if buyers remember how surprised they were in the past. Allowing $z \geq 2$ can capture the compound unlikeliness of having consecutive periods of price increases when such price increases are atypical.[6] The maximum likelihood is the highest likelihood one could assign to price changes over the preceding z periods given what buyers knew at the time the price change occurred. Letting ml^t denote the maximum likelihood then

$$\begin{aligned} ml^t &\equiv \Pi_{\tau=t+1-z}^{t} \max_{y^\tau} f\left(y^\tau; m_1^{\tau-1}, m_2^{\tau-1} - \left(m_1^{\tau-1}\right)^2\right) \\ &= \Pi_{\tau=t+1-z}^{t} f\left(m_1^{\tau-1}; m_1^{\tau-1}, m_2^{\tau-1} - \left(m_1^{\tau-1}\right)^2\right), \end{aligned}$$

where the second equality follows by f having its mode equal to the first moment. Suspicions depend on realized likelihood relative to maximum likelihood: $L^t \equiv l^t/ml^t$. The probability of detection is assumed to be a decreasing function of the relative likelihood and, more specifically, takes the form:

$$\phi\left(L^t\right) \equiv \alpha_0 + \alpha_1 \left(1 - L^t\right)^{\alpha_2},$$

where $\alpha_0 \geq 0$ and $\alpha_1, \alpha_2 > 0$. In that α_0 is independent of prices, it captures sources of detection unrelated to price such as an internal whistleblower or incidental discovery through an unrelated legal case.

A key assumption of the model is that $\phi\left(L^t\right)$ is the probability firms assign to "being caught" which means paying penalties and discontinuing collusion. What we really imagine is that $\phi\left(L^t\right)$ is the probability of suspicions emerging which is seen as a discrete event - buyers have an epiphany that firms may not be competing. Of course, suspicions do not immediately translate into conviction. Buyers must decide to inform the authorities and/or bring a private case themselves, and if there is public or private case then firms must either plead guilty or be convicted in order for penalties to be levied. This is a rich process which we have summarized in a single probability, $\phi\left(L^t\right)$, for reasons of

[6] Though it needs to be emphasized that buyers are not engaging in pattern recognition. If buyers' beliefs over price changes have a zero first moment - and thereby are symmetric around zero - then a series of price changes - all with the same absolute value - will have the same likelihood. This means, for example, that three consecutive price increases of size $\varepsilon > 0$ is just as likely as price changes of ε, $-\varepsilon$, and ε.

tractability. We hope that future work will open up that black box and enrich this class of models further.

2.2.2 Modelling Anomalous Events - Computational Specification

There are several changes that need to be made to numerically implement this model. One immediate problem is that the buyers have at least k state variables. In that we are going to use dynamic programming to solve the cartel's problem and buyers' beliefs are part of it, the dimensionality of the state space can be a serious obstacle to numerical analysis. Rather than limit k, we pursue an approach approximating what was described above.

Consider the following manipulation of the moments:

$$\begin{aligned} m_i^t &= \left(\frac{1}{k}\right) \sum_{\tau=t-k+1}^{t} \left(\Delta P^\tau\right)^i \\ &= \left(\frac{1}{k}\right) \sum_{\tau=t-k}^{t-1} \left(\Delta P^\tau\right)^i + \left(\frac{1}{k}\right) \left[\left(\Delta P^t\right)^i - \left(\Delta P^{t-k}\right)^i\right] \\ &= m_i^{t-1} + \left(\frac{1}{k}\right) \left[\left(\Delta P^t\right)^i - \left(\Delta P^{t-k}\right)^i\right]. \end{aligned}$$

Thus, in updating moment i in response to the observed price change at t, a weight of $1/k$ is transferred from the $t - k^{th}$ observation to the t^{th} observation. Now consider instead transferring weight from all past observations - not just the $t - k^{th}$ observation - and assigning it to the new observation,

$$m_i^t = \left(\frac{k-1}{k}\right) m_i^{t-1} + \left(\frac{1}{k}\right) \left(\Delta P^t\right)^i.$$

Generalizing this equation of motion, we have

$$m_i^t = \lambda_i m_i^{t-1} + (1 - \lambda_i) \left(\Delta P^t\right)^i.$$

With this specification, the two state variables (m_1^t, m_2^t) help take the place of the data set of k past price changes. Note that we can capture a "bigger" data set (that is, a higher value for k) by setting a higher value for λ_i.

An analogous procedure can be done with the likelihood function. First, perform the following steps:

$$
\begin{aligned}
l^t &= \Pi_{\tau=t+1-z}^{t} f\left(\Delta P^\tau; m_1^{\tau-1}, m_2^{\tau-1} - \left(m_1^{\tau-1}\right)^2\right) \\
&= \Pi_{\tau=t-z}^{t-1} f\left(\Delta P^\tau; m_1^{\tau-1}, m_2^{\tau-1} - \left(m_1^{\tau-1}\right)^2\right)\left(\frac{f\left(\Delta P^t; m_1^{t-1}, m_2^{t-1} - \left(m_1^{t-1}\right)^2\right)}{f\left(\Delta P^{t-z}; m_1^{t-z-1}, m_2^{t-z-1} - \left(m_1^{t-z-1}\right)^2\right)}\right) \\
&= l^{t-1}\left(\frac{f\left(\Delta P^t; m_1^{t-1}, m_2^{t-1} - \left(m_1^{t-1}\right)^2\right)}{f\left(\Delta P^{t-z}; m_1^{t-z-1}, m_2^{t-z-1} - \left(m_1^{t-z-1}\right)^2\right)}\right) \\
&= \left[\frac{l^{t-1}}{f\left(\Delta P^{t-z}; m_1^{t-z-1}, m_2^{t-z-1} - \left(m_1^{t-z-1}\right)^2\right)}\right] f\left(\Delta P^t; m_1^{t-1}, m_2^{t-1} - \left(m_1^{t-1}\right)^2\right).
\end{aligned}
$$

The approximation entails replacing

$$
f\left(\Delta P^{t-z}; m_1^{t-z-1}, m_2^{t-z-1} - \left(m_1^{t-z-1}\right)^2\right)
$$

with the geometric average density for the previous z periods which is $\left(l^{t-1}\right)^{1/z}$. The approximation is then

$$
\begin{aligned}
l^t &= \left[\frac{l^{t-1}}{\left(l^{t-1}\right)^{1/z}}\right] f\left(\Delta P^t; m_1^{t-1}, m_2^{t-1} - \left(m_1^{t-1}\right)^2\right) \\
&= \left(l^{t-1}\right)^{(z-1)/z} f\left(\Delta P^t; m_1^{t-1}, m_2^{t-1} - \left(m_1^{t-1}\right)^2\right).
\end{aligned}
$$

Similarly, one can go through these steps for the maximum likelihood to derive the equation of motion for the relative likelihood:

$$
L^t = \left(L^{t-1}\right)^\xi \left[\frac{f\left(\Delta P^t; m_1^{t-1}, m_2^{t-1} - \left(m_1^{t-1}\right)^2\right)}{\max_y f\left(y; m_1^{t-1}, m_2^{t-1} - \left(m_1^{t-1}\right)^2\right)}\right]
$$

where $\xi \in (0,1)$. A higher value for ξ corresponds with buyers using more price changes in their "test." The state variables defining buyers' beliefs are then reduced from the k most recent price changes to (m_1^t, m_2^t, L^t).

Two additional simplifications are made. First, for numerical purposes, the set of price changes, denoted Φ, is assumed to be finite. Taking account of this property, buyers' beliefs over price changes are a discrete analogue to the normal distribution. So, $f\left(\cdot; \mu, \sigma^2\right)$ is replaced with $h\left(\cdot; \mu, \sigma^2\right)$ where:

$$
h\left(\eta'; \mu, \sigma^2\right) \equiv \begin{cases} \frac{f\left(\eta'; \mu, \sigma^2\right)}{\sum_{\eta \in \Phi} f\left(\eta; \mu, \sigma^2\right)} & \text{if } \eta' \in \Phi \\ \\ 0 & \text{otherwise} \end{cases}.
$$

In that the formulation rests on whether the buyers will pick up the break in the pricing function as it goes from competition to collusion, buyers will enter the collusive regime with beliefs based on non-collusive pricing data. Recognizing this fact and in order to reduce the number of state variables, we will delete the state variable m_2^t by assuming the variance of buyers' beliefs over price changes is fixed at the variance of price changes for the non-collusive case, $w_1^2\sigma_\varepsilon^2$. This is a good approximation for what buyers would have entering the cartel formation phase. What this rules out is allowing the variance of price changes to adjust in response to firms' prices. If such a response was allowed then the sensitivity of the relative likelihood to the price series could evolve over time. We consider this a second-order effect. More important is, given a particular relative likelihood function, how the cartel can influence the likelihood that buyers assign to the observed price series.

2.3 Cartel's Problem

Suppose firms decide in period 1 to form a cartel. We can think of detection as the end of the horizon with a terminal payoff of $W\left(c^t\right) - X^t - F$ where X^t is the cartel's damages in the event it is detected and F is any (fixed) fines. Though it is assumed that, once caught, firms do not collude thereafter, we conjecture results are robust to allowing them to restart collusion after some specified number of periods. The cartel's damages are assumed to evolve in the following manner:

$$X^t = \beta X^{t-1} + \gamma x\left(P^t, c^t\right) \text{ where } \beta \in [0,1), \gamma \geq 0,$$

where P^t is the cartel price at t. As time progresses, damages incurred in previous periods become increasingly difficult to document and $1-\beta$ measures the rate of deterioration. $x\left(P^t, c^t\right)$ is the level of damages incurred in the current period where γ is the multiple of damages that a firm can expect to pay if found caught colluding. Damages depend on both the price the cartel set and on c^t because the latter determines the competitive benchmark. We will specify the formula consistent with U.S. antitrust practice:

$$x\left(P^t, c^t\right) = \left(P^t - \widehat{P}\left(c^t\right)\right)\left(a - bP^t\right).$$

$\widehat{P}\left(c^t\right)$ is referred to as the "but for" price and $P^t - \widehat{P}\left(c^t\right)$ as the "overcharge." It is worth mentioning that government penalties have, in recent years, been sensitive to cartel behavior and are no longer trivial compared to damages. While the exact formula used to calculate government fines is unclear - formally, there is a formula but the "fudge" factor is big enough to leave the true formula unclear- it appears to be related to the same types of variables as used in $x(P^t, c^t)$ though may give weight to the revenue involved, independent of the overcharge.[7]

In specifying the cartel's problem, incentive compatibility constraints ensuring the self-enforcing nature of the collusive arrangement are ignored. It seems natural to first characterize the unconstrained joint profit-maximizing case which is challenging enough in itself. When the cartel goes to choose price in period t, the state variables are $\left(P^{t-1}, X^{t-1}, c^t, m_1^{t-1}, L^{t-1}\right)$ where P^{t-1} is the lagged (common) price, X^{t-1} is accumulated damages, c^t is current cost, m_1^{t-1} is the first moment of buyers' beliefs on price changes, and L^{t-1} is the relative likelihood that buyers attach to recent prices. Letting η^t denote the price change in period t, the equations of motion are

$$P^t = P^{t-1} + \eta^t$$

$$c^{t+1} = \upsilon\left(c^t + \varepsilon^{t+1}\right)$$

$$X^t = \beta X^{t-1} + \gamma x\left(P^{t-1} + \eta^t, c^t\right)$$

$$m_1^t = \lambda m_1^{t-1} + (1-\lambda)\eta^t$$

$$L^t = \left(L^{t-1}\right)^{\xi} \varphi\left(\eta^t, m_1^{t-1}\right),$$

where

$$\varphi\left(\eta^t, m_1^{t-1}\right) \equiv \left[\frac{h\left(\eta^t; m_1^{t-1}, w_1^2\sigma_\varepsilon^2\right)}{h\left(m_1^{t-1}; m_1^{t-1}, w_1^2\sigma_\varepsilon^2\right)}\right].$$

The set of price changes is slightly modified,

$$\Phi\left(P^{t-1}\right) \equiv \left\{\eta \in \Phi : P^{t-1} + \eta \in \left[\underline{c}, \overline{P}\right]\right\},$$

to ensure that price remains in the set $\left[\underline{c}, \overline{P}\right]$ where $\overline{P} \geq P^m(\overline{c})$.

[7] A key implicit assumption is that the cartel anticipates the plaintiffs and/or antitrust authorities being able to successfully identify the true but for price and successfully arguing in court as to its value. In practice, this is a major source of contention. For an analysis of how a standard method for estimating the but for price may be biased, see Harrington (2004b).

The cartels' value function is defined recursively:

$$
\begin{aligned}
& V\left(P^{t-1}, X^{t-1}, c^t, m_1^{t-1}, L^{t-1}\right) \\
= & \max_{\eta^t \in \Phi\left(P^{t-1}\right)} \pi\left(P^{t-1}+\eta^t, c^t\right)+\delta \phi\left(\left(L^{t-1}\right)^{\xi} \varphi\left(\eta^t, m_1^{t-1}\right)\right) \times \\
& \left[\int W\left(\upsilon\left(c^t+\varepsilon\right)\right) f\left(\varepsilon ; \mu_{\varepsilon}, \sigma_{\varepsilon}^2\right) d\varepsilon-\beta X^{t-1}-\gamma x\left(P^{t-1}+\eta^t, c^t\right)-F\right] \\
& +\delta\left[1-\phi\left(\left(L^{t-1}\right)^{\xi} \varphi\left(\eta^t, m_1^{t-1}\right)\right)\right] \times \\
& \int V\left(P^{t-1}+\eta^t, \beta X^{t-1}+\gamma x\left(P^{t-1}+\eta^t, c^t\right), \upsilon\left(c^t+\varepsilon\right), \lambda m_1^{t-1}+(1-\lambda) \eta^t,\right. \\
& \left.\left(L^{t-1}\right)^{\xi} \varphi\left(\eta^t, m_1^{t-1}\right)\right) f\left(\varepsilon ; \mu_{\varepsilon}, \sigma_{\varepsilon}^2\right) d\varepsilon .
\end{aligned}
$$

The cartel earns current profit of $\pi\left(P^{t-1}+\eta^t, c^t\right)$ by making a price change of η^t and, with probability $\phi\left(\left(L^{t-1}\right)^{\xi} \varphi\left(\eta^t, m_1^{t-1}\right)\right)$, the cartel is detected. In that event, firms receive non-collusive profits thereafter and pay penalties. With the complementary probability, the cartel is not detected in which case the future value is that attached to colluding given the new values to the state variables.

To summarize, the parameters in the model are:

Parameters

Demand	$a, b>0$
Cost levels	$\underline{c} \in[0, a), \bar{c} \in(\underline{c}, a)$
Cost shocks	$\mu_{\varepsilon} \in \Re$, $\sigma_{\varepsilon}^2>0$
Non-collusive solution	$w_0 \in[0, a / 2)$, $w_1 \in(1 / 2,1]$
Penalty	$\gamma, F \geq 0$, $\beta \in(0,1)$
Detection	$\alpha_0 \in[0,1)$, $\alpha_1 \in\left(0,1-\alpha_0\right]$, $\alpha_2>0$
Time preferences	$\delta \in(0,1)$
Updating	$\xi, \lambda \in(0,1)$

If a cartel is formed, it occurs in period 1 so that the accumulated damages entering into period 1 is zero and the inherited price is the non-collusive price. The initial conditions are then

$$
\left(P^0, X^0, c^1, m_1^0, L^0\right)=\left(w_0+w_1 c^0, 0, c^0+\varepsilon^1, m_1^0, L^0\right) .
$$

Thus, the system is initialized with $\left(c^0, m_1^0, L^0\right)$ and requires a randomly selected sequence of cost shocks, $\left\{\varepsilon^t\right\}_{t=1}^T$, where T is the length of the simulation.

In concluding, this model is unorthodox in that firms are "smarter" than buyers in that firms know the process by which buyers detect collusion but buyers do not know how firms choose prices. While some economists may be uncomfortable with this departure from the prevalent paradigm, it strikes us as eminently plausible if not compelling. Pricing is a first-order consideration for firms, while detecting collusion is far down the list of things to do for a buyer, particularly in light of the low empirical frequency of cartels.[8]

3 Cartel Pricing Patterns

Our analysis proceeds by first considering a benchmark case under a variety of initial conditions. The benchmark parameter configuration is

$$\begin{aligned} a &= 100, b = 1, \underline{c} = 20, \overline{c} = 40, \gamma = 1.5, F = 0, \beta = .75, \delta = .75, \xi = .5, \\ \lambda &= .75, \mu_\varepsilon = 0, \sigma_\varepsilon^2 = 2, \alpha_0 = .05, \alpha_1 = .45, \alpha_2 = 2, w_0 = 25, w_1 = .75. \end{aligned}$$

The probability of detection is a quadratic in the relative likelihood with a range of $[.05, .5]$. As $\xi = (z-1)/z$ then $\xi = .5$ is equivalent to buyers using the two most recent price changes in deciding on whether the price series is "suspicious." The discount factor is set at the relatively low value of .75 because convergence is very slow for high values of δ (and also high values of β). $(w_0, w_1) = (25, .75)$ corresponds to the Nash equilibrium price for the quantity game when there are three firms. We suspect that the particular demand parameters and range of cost levels are unimportant for results and thus will not experiment with them but instead focus precious CPU time on other parameters. Note that the number of firms is not a parameter in the model. Given symmetry and that we ignore incentive compatibility constraints, the cartel solution is independent of the number of firms (though it implicitly matters through the non-collusive solution).

To ascertain robustness, we will also consider the following 12 modifications to the benchmark case:

$$\begin{aligned} \sigma_\varepsilon^2 &\in \{1, 3, 4\}, \gamma \in \{2.25, 3\}, (w_0, w_1) = (0, 1), \delta = .9, \\ \beta &= .9, (\xi, \lambda) \in \{(.25, .25), (.75, .9)\}, \alpha_1 = .2, \alpha_2 = 3. \end{aligned}$$

A description of the numerical methods is provided in the Appendix.

3.1 General Analysis

The initial task is to identify how a cartel price path compares to that for a non-collusive industry. The following procedure is used to produce a collection of price paths for this purpose.

[8] But let us state the caveat that buyers may consciously look for collusion if the industry has a history of past collusion. The model is then relevant to those industries ;acking such a history.

Step 1 Randomly select initial conditions for cost and buyers' beliefs. c^0 is selected according to a uniform distribution with support $[25, 35]$, m_1^0 is selected according to a uniform distribution with support $[-1, 1]$, and L is selected according to a uniform distribution with support $[.25, .75]$ with $L^0 = L^{\frac{1}{1-\xi}}$.[9]

Step 2 Randomly select a sequence of 120 cost shocks.

Step 3 Run the non-collusive model for periods 1-40 and then run the collusive model for periods 41-120 (that is, a cartel is formed in period 41). Continue to run the non-collusive model for periods 41-120 for purposes of comparison.

Though the conditions at period 1 are random, the initial conditions when the cartel is formed are generated by non-collusive pricing in the preceding 40 periods. This serves to provide a more accurate simulation of what a newly formed cartel would face with respect to what buyers believe are "typical" price changes. In analyzing these price paths, keep in mind that the non-collusive price path is an affine function of cost, $25 + .75c^t$, so that movements in it can be used to track cost (cost movements are then 33% larger than the reported non-collusive price movements). In that any individual simulation is dependent on the particular initial conditions and sequence of cost shocks, we've conducted more than 100 simulations and report representative results here.

For the benchmark parameter configuration, Figure 1 presents eight randomly selected simulated price paths. The first observation is that there are two clearly identifiable phases to collusion. The initial "transition" phase involves a steady rise in price which appears largely unrelated to cost shocks (at a minimum, the direction of price is unrelated to cost shocks). For example, in Figure 1g, the transition phase runs from period 41 to about period 65. In some instances, this steady rise in price is followed by a fall in price. For example, in Figure 1a, the transition phase runs until about period 70, with pricing rising until period 60 and then falling thereafter. We'll return to this point later. The ensuing "stationary" phase has price move with cost though, in comparison to the non-collusive price path, the collusive price is much less sensitive to cost shocks. This two-phase structure was found to hold for all parameter configurations and we provide some simulated price paths for $\sigma_\varepsilon^2 = 4$ (Figure 2), $(\xi, \lambda) = (.25, .25)$ (Figure 3), and $\delta = .9$ (Figure 4).

[9] Note that if $L^0 = L^{\frac{1}{1-\xi}}$ then $L^1 = L$ if the relative likelihood of the period 1 observation is L. We specify $L^0 = L^{\frac{1}{1-\xi}}$ so that the intial conditions are not changed as ξ is changed.

Result #1 The cartel price path has a transition phase - in which price moves largely independent of cost - and a stationary phase - in which price is responsive to cost. The transition phase involves a steady rise in price though may conclude with a modest decline in price.

Let us explain these findings. When formed in period 41, the cartel inherits the non-collusive price of $25 + .75c^{40}$. Ideally, it would like price to be much higher, generally in the vicinity of the simple monopoly price of $50 + .5c^{40}$.[10] However, if it were to rapidly raise price, it would very likely create suspicions among buyers since, in light of preceding price changes, price rises of that magnitude are perceived as being highly unlikely. Hence, the cartel gradually raises price so that the series of price changes is not too unlikely in light of buyers' beliefs. Of course, buyers' beliefs are adapting so as price rises they come to expect more price increases (a point to which we'll return later). At the same time, cost is changing which alters the target price for the cartel. However, unless there are some large negative cost shocks, the target will generally remain above the cartel's price for some length of time; this implies a steadily rising price as part of the transition phase. For example, consider the path in Figure 1-d and recall that cost wanders in the interval $[20, 40]$ and that a non-collusive price of 40 corresponds to a cost of 20 and that a cost of 20 means a simple monopoly price of 60. Even though cost falls after the formation of the cartel, price steadily rises until it hits around 60 at which point the cartel shifts into the stationary phase.

During the stationary phase, the cartel is adjusting price to cost for the usual reasons but, in doing so, it wants price movements to be consistent with buyers' beliefs so as to avoiding triggering detection of collusion. This has a number of implications that we'll develop over the course of our discussion and briefly summarize here. If buyers have come to expect price increases then the cartel may need to raise price even if cost is unchanged or falls slightly. Similarly, the cartel cannot respond commensurately to large cost shocks which means that extreme cost changes may not be passed through as with a non-collusive industry. This suggests that price variability may be less under collusion and that cost shocks may take a longer time to pass through. The responsiveness of the cartel price to cost is most easily seen for large trends in cost. In Figure 1-b, cost rises over periods 60-85 and price rises with it though the hills and valleys are milder. In Figure 1-d, cost rises sharply over periods 110-120 and, though the response is lagged and more gradual, price

[10] It may want a price below the simple monopoly price in order to reduce the amount of expected damages.

eventually rises sharply as well. In Figure 1-e, cost rises then falls over periods 95-120 and the cartel price follows that movement in a lagged manner. It is clear from Figures 1-4 that the cartel price path is much less sensitive to cost than the non-collusive price path and we'll provide some more systematic evidence shortly.

The speed with which price rises during the transition phase is closely linked to the parameters influencing buyers' beliefs. Comparing the price paths for $\sigma_{\varepsilon}^2 \in \{1, 2, 3, 4\}$, the transitional phase is shorter and price rises faster when the cost variance is higher. The results for $\sigma_{\varepsilon}^2 = 2$ (Figure 1) and $\sigma_{\varepsilon}^2 = 4$ (Figure 2) are reported here. Recall that buyers' beliefs over price changes are based on the non-collusive price variance, $w_1^2 \sigma_{\varepsilon}^2$. As cost variability rises so does the variability of buyers' beliefs over price changes. This means that a series of price increases, when the expected price change is positive but small or negative, is perceived as being more likely because price is perceived as more volatile. As the probability of detection is then less sensitive to the price path, the cartel can raise price quicker without being as concerned about detection. In other words, there is less of a need to manipulate buyers' beliefs - as buyers implicitly have a stricter criterion to consider something anomalous - though it is still important to restrain price changes and have price increase gradually.

ξ and λ are the updating parameters for buyers' beliefs and should be influential in transitional pricing. A buyer's current expectation on price changes is a weighted average of the previous period's expectation, which is given weight λ, and the current price change. As λ is reduced, the expectation then becomes more sensitive to recent price changes. This means, for example, that if firms pursue a series of price increases when buyers initially expected no price changes, buyers' beliefs will respond quick to these price increases and thus buyers are less likely to find them anomalous. As ξ is reduced, the relative likelihood of the price series puts more weight on the most recent price changes; in essence, the test of the likelihood of recent price changes uses a shorter price series. Therefore, similar to a lower value for λ, the likelihood of triggering detection is less sensitive to price changes in the distant past. Moving from (ξ, λ) equal to $(.5, .75)$ (Figure 1) to $(.25, .25)$ (Figure 3), buyers' beliefs respond quicker and, as shown by the simulated price paths, the length of the transition phase shortens and price rises more rapidly. This property is confirmed for unreported results for $(\xi, \lambda) = (.75, .9)$. Buyers' beliefs adjust quicker to the price rises which not only means price rises are less likely to trigger detection but they induce the cartel to lay the groundwork for future price increases by raising price more in the current period. This highlights how the cartel price path is driven not only by the desire to have price movements appear reasonable to buyers but also how the cartel can manipulate what is perceived to be reasonable.

Result #2 The transitional phase is shorter and the transitional price path rises faster when: i) the variance of cost shocks is greater; and ii) buyers' beliefs are more sensitive to recent price changes.

To more systematically explore price variability during the stationary phase, we ran the following procedure. The first two steps are the same as before except that, in step 2, we generate a sequence of 200 cost shocks. The third step serves to produce a collection of stationary price paths.

Step 3' Run the non-collusive model and the collusive model for periods 1-200. Using data from periods 101-200, calculate the variance of price.

For each of the 13 parameter configurations, this simulation is performed ten times. Table 1 reports the price variance (averaged across those ten runs) along with the range of the price variance, under both collusion and non-collusion.[11] As the results show, the cartel price path is much less variable than the non-collusive price path. For the benchmark case, for example, the variance of the non-collusive price path has a minimum value of .7945 (over the ten runs), while the variance of the collusive price path has a maximum value of .0994. This reflects the desire of the cartel to keep suspicious price movements to a minimum. Also note that the theoretical price variance for a monopolist is on the order of $(1/2)^2 \sigma_\varepsilon^2$ so that the cartel price variance is much lower than that as well. That the the price variance is lower under collusion is consistent with the empirical finding of Abrantes-Metz, Froeb, Geweke, and Taylor (2005).[12]

Not surprisingly, as the cost variance rises, the cartel price variance rises with it; see Table 2. More interesting is that price variability rises faster than cost variability as reflected in the ratio of the price variance to σ_ε^2 falling with respect to σ_ε^2. As the cost variance rises, so does the variance of the non-collusive price. Hence, buyers' beliefs become more diffuse which makes it harder to trigger suspicions. As a result, the cartel can make price more responsive to cost without as much risk of collusion being detected.

To explore the issue about cost pass-through, we used 100 periods of data from the stationary phase and regressed the change in price on the contemporaneous change in cost

[11] If cost had an unbounded support then the theoretical non-collusive price variance would be $w_1^2 \sigma_\varepsilon^2 = .5625\sigma_\varepsilon^2$.

[12] Mixed results regarding collusion reducing the variance of price is provided in Bolotova, Connor, and Miller (2005).

and the four most recent lagged cost shocks. This exercise was performed for a randomly selected price and cost series for each of the 13 parameter configurations. Note that only the contemporaneous cost shock matters for the non-collusive and monopoly price series. As shown in Table 3, lagged cost changes are economically and statistically significant in explaining price fluctuations when firms collude. The cartel is then gradually passing cost changes to price in order to avoid anomalous price changes that arouse suspicions.

Result #3 Compared to the non-collusive price and the simple monopoly price, the variability of the cartel price is much less and cost shocks are passed through more gradually. The variance of the cartel price increases faster than the variance in cost.

In concluding this section, it is worth discussing here an assumption on the evolution of buyers' beliefs on price changes. Upon formation of the cartel, buyers' beliefs are based on the non-collusive price path. For that reason, we assumed that the variance of buyers' beliefs on price changes is $w_1\sigma_\varepsilon^2$. But we also assumed that this variance remained fixed over the life of cartel. The reason was a pragmatic one - adding the variance of price changes as a sixth state variable would impose a severe computational burden that might prevent us from solving for the value function. To what extent does this assumption of a fixed variance to buyers' beliefs bias our results? During the transitional phase, this assumption is probably fine because the issue is whether buyers will consider the cartel price path unlikely based on their beliefs prior to cartel formation. More problematic is during the stationary phase when the variance on price changes would adjust if given a chance. We conjecture that our results would be robust to that modification. Since our analysis showed the cartel price path is less variable than the non-collusive price path, buyers would reduce the estimated variance on price changes below $w_1\sigma_\varepsilon^2$ which would cause the cartel to smooth out price changes even more. Hence, the stationary cartel price path would still be much less volatile than the non-collusive or simple monopoly price paths.

3.2 Deterministic Cost Case

A challenge in identifying properties of the collusive price path is that any individual series depends on the realization of the cost shocks which makes it difficult to discern how firms are "trying" to change prices. In other words, any price path is a confluence of the cartel's target path and the change in the target path due to the change in cost. To isolate the intended trajectory for price, we use the policy function solved for the stochastic model and simulate it when cost is unchanging: $c^t = c^0 \ \forall t$. The initial conditions are:

$$\left(c^0, m_1^0, L\right) = \left(30, 0, .75^{\frac{1}{1-\xi}}\right).$$

At a cost of 30, the simple monopoly price is 65. Note that the cartel price path is now deterministic (subject to the caveat of being detected but we report the price path in the event that it is not detected).

For the benchmark parameter configuration, Figure 5 reports the time series on price, change in price, average price change as perceived by buyers (that is, first moment), probability of detection, value of the cartel, and accumulated damages. The value of forming a cartel starts around 3950 which exceeds the non-collusive value of 3680. The long-run value of the cartel is about 4370 though moves non-monotonically. This is due to a variety of forces at work. When the cartel begins, it has no damages which serves to raise the value of collusion. However, it also inherits the non-collusive price and bigger price increases bring with them a larger chance of detection. As time moves on, damages accumulate (though they also can move non-monotonically) which lowers the collusive value but this is typically more than compensated by inheriting a higher price, which not only means higher profit but it makes it easier to achieve yet higher prices. The latter results in a falling probability of detection; the probability of detection is initially around 13% and then declines to its steady-state level of 5%.

Turning to the price path, detection considerations are a severe constraining influence on price, as suggested by the stochastic price paths. Though the steady-state price is just a little below the simple monopoly price of 65, it takes many periods to get to that level. The striking property, however, is that price *overshoots* its long-run price. After several periods above the long-run price, the cartel lowers price and eventually settles down.[13] Returning to Figure 1, overshooting - that is, price rises then falls during the transition phase - occurs in cases a, c, d, e, and h. We now know this is not a feature of movements in cost but is rather the intended trajectory for price. This overshooting phenomenon is present in some but not all of the cases we've examined. Figure 6 shows the price paths for $\sigma_\varepsilon^2 \in \{1, 3\}$, Figure 7 for $(\xi, \lambda) \in \{(.25, .25), (.75, .9)\}$, and Figure 8 for $\delta = .9$ and $(w_0, w_1) = (0, 1)$ (which is when the non-collusive solution is the competitive solution). The overshooting is observed in several of these cases and, in some, even entails price temporarily exceeding the simple monopoly price of 65; see $\sigma_\varepsilon^2 = 1, (\xi, \lambda) = (.75, .9)$, and $(w_0, w_1) = (0, 1)$.

[13] Though there is a steady-state cycle, we believe this is an artifact of the discreteness of the price space and is of little importance.

What underlies these dynamics is that the cartel is trying to systematically raise price in such a manner that the buyers don't perceive the price series as anomalous. Of course, what buyers consider anomalous is itself endogenous and depends on how firms have priced in the past. What this means is that the cartel wants to raise price sufficiently gradually so as to get buyers accustomed to its price changes. But there is a problem. When the cartel price reaches its long-run steady-state level, the buyers have largely seen price increases and have come to expect such (that is, $m_1^t > 0$). Notice in Figure 8 that the steady-state price is 65 and is first reached around period 20, at which point the expected price increase is about 1.25. As buyers expect price to rise by 1.25 each period, a series of zero price changes is apt to be perceived as anomalous. The cartel instead gradually lowers the rate of price increases so as to make detection less likely. As price may not come down fast enough, the cartel may eventually have to engage in some price decreases. As shown, the cartel continues to raise price up to about 68 at which point price is brought back down and then converges to 65.

Consistent with this explanation, overshooting is noticeably more when the variance of cost shocks is less (compare $\sigma_\varepsilon^2 = 1$ with $\sigma_\varepsilon^2 = 3$), buyers' beliefs are more sluggish (compare $(\xi, \lambda) = (.75, .9)$ with $(\xi, \lambda) = (.25, .25)$), and the cartel weighs the future less (compare the benchmark case of $\delta = .75$ with $\delta = .9$). If cost shocks are less variable than buyers are more likely to become suspicious since their beliefs are less diffuse. There is then more of a need to manipulate prices in order to manipulate beliefs. If buyers beliefs are more sluggish then they respond slower which means more manipulation is required. In considering the role of the discount factor, first note that a cartel faces an intertemporal trade-off as raising price faster means higher current profit but a lower future payoff due to greater damages and a higher probability of detection. A more patient cartel then tends to raise price more gradually so they bring the price path in for a "softer landing." It is when price rises faster that buyers come to expect bigger price increases and overshooting is more significant.

Result #4 The cartel price path exhibits more overshooting when: i) the cost variance is greater; ii) buyers' beliefs respond slower; and iii) firms value the future more.

Do we think that overshooting is a real phenomenon? The available empirical evidence on cartel pricing doesn't really speak to the issue in that no one has looked for it and it is a subtle property. Though it is a natural implication of strategic cartel pricing when buyers' beliefs are empirically based, its empirical relevance is unclear. In any case, this overshooting pattern is not ubiquitous and occurs only for some parameter configurations.

If the data doesn't reveal such a pattern, it may just be telling us that the parameter values are more consistent with, say, $\delta = .9$ than with $\delta = .75$.

4 Concluding Remarks

The analysis of this paper is very much exploratory in intent. It is an initial attempt to model cartel pricing while specifying a plausible belief structure regarding detection. On the plus side, it is able to produce cartel price paths which are starting to look like real cartel price paths. The analysis produced a number of systematic properties of which the most significant are that the cartel price path is comprised of a transition and stationary phase and that, in the stationary phase, collusion reduces the price variance and results in price changes being more responsive to lagged cost changes. These may prove valuable in the development of testable hypotheses that to identify the presence of a cartel. On the negative side, this approach has had to make a number of strong assumptions about the buyer belief formation process. Just as buyers having equilibrium beliefs is too sophisticated for this context, our approach may be too naive in that buyers are purely empirical and don't use any understanding about collusion. Pluses and minuses aside, the primary intent of this research is to begin the process of developing better models of price-fixing cartels. There is rich institutional information on cartels that we can use in our modelling and a growing set of empirical properties that we can try to explain. We hope that this paper will encourage others to develop models that can capture the richness observed in cartel behavior.

5 Appendix: Numerical Methods

The process that we used to come up with a numerical version of $W(c)$ is a straightforward application of the value function iteration method. First, discretize the state space, $[\underline{c}, \overline{c}]$, for c and denote it $\mathbb{C}$. Second, use a m-node Gaussian quadrature to approximate the density function for the cost shock $f(\varepsilon; \mu_\varepsilon, \sigma_\varepsilon^2)$. Third, use the following algorithm to derive $W(c)$:

Step I [Initialization]: Set $W^{(0)}(c) = 0$, $\forall\ c \in \mathbb{C}$. For all $c \in \mathbb{C}$, compute the non-collusive price $\widehat{P}(c)$ and the associated current period profit $[\widehat{P}(c) - c]D(\widehat{P}(c))$.

Step II [Interpolation]: Given $W^{(k)}(c)$, for all $c \in \mathbb{C}$, compute:

$$E_\varepsilon[W(\upsilon(c+\varepsilon))] \equiv \int W(\upsilon(c+\varepsilon)) f(\varepsilon; \mu_\varepsilon, \sigma_\varepsilon^2) d\varepsilon \cong \sum_{i=1}^{m} \Pr.(\varepsilon_i) W^{(k)}(\upsilon(c+\varepsilon_i)),$$

and derive $W^{(k+1)}(c) = [\widehat{P}(c) - c]D(\widehat{P}(c)) + E_\varepsilon[W(\upsilon(c+\varepsilon))]$.

Step III [Convergence check]: Compute the infinite norm for $W^{(k+1)}(c) - W^{(k)}(c)$. Stop and set $W(c) = W^{(k)}(c)$, if the value $\simeq 0$; otherwise, go back to Step II.[14]

With knowledge on $W(\cdot)$, $V(\cdot)$ is solved using the collocation method. Different from value function iteration, it starts with an initial approximation of the unknown function $V(\cdot)$ which is a linear combination of some known basis functions. By Miranda and Fackler (2002), a $k-$degree approximation for $V(\cdot)$, where $k \equiv k_P \times k_X \times k_c \times k_{m_1} \times k_L$, can be represented as:

$$\begin{aligned} V(P, X, c, m_1, L) &\cong \sum_{i_P=1}^{k_P} \sum_{i_X=1}^{k_X} \sum_{i_x=1}^{k_x} \sum_{i_{m_1}=1}^{k_{m_1}} \sum_{i_L=1}^{k_L} e_{i_P i_X i_c i_{m_1} i_L} \varphi_{i_P i_X i_c i_{m_1} i_L}(P, X, c, m_1, L) \\ &= [\mathbf{\Gamma}_L(L) \otimes \mathbf{\Gamma}_{m_1}(m_1) \otimes \mathbf{\Gamma}_c(c) \otimes \mathbf{\Gamma}_X(X) \otimes \mathbf{\Gamma}_P(P)]\mathbf{e}, \end{aligned}$$

where k_i is the degree of approximation for dimension $i \in \{P, X, c, m_1, L\}$, $\mathbf{\Gamma}_i(i)$ is a $1 \times k_i$ vector of basis functions over dimension i, and $\mathbf{e}$ is a $k \times 1$ vector with properly stacked coefficients. One can start with a initial guess of $\mathbf{e}$ and fix it by requiring the approximant to satisfy the Bellman equation at the k nodes. The Chebychev polynomial with the associated Chebychev nodes is used for the basis functions.[15]

[14] As, in general, $\upsilon(c+\varepsilon_i) \notin \mathbb{C}$, evaluation of $W^{(k)}(\upsilon(c+\varepsilon_i))$ at step II uses first-degree interpolation (table lookup).

[15] See Judd (1999) and Miranda and Fackler (2002) for the pros and cons of different interpolation methods.

The state space is defined and the $\mathbf{\Gamma}_i(\cdot)$ functions and k_i nodes are chosen for each state variable. Given the nodes, the choice set (action space) is constructed for each state.[16] For each state and action, compute $h(\cdot)$ and $h(\cdot)/\max h(\cdot)$.[17] The new state is then derived according to the equations of motion.[18] Compute the following: current period profit, $\pi(P', c)$, the probability of detection, $\phi(L')$, and the expected non-collusive payoff $E_\varepsilon[W(c')]$. Finally, we use the following algorithm to derive the numerical version of $V(\cdot)$:

Step I [Initialization]: Initialize $\mathbf{e}^{(0)}$ and, thereby, $V^{(0)}(\cdot)$.

Step II [Interpolation]: Given $V^{(k)}(\cdot)$, evaluate the expected collusive payoff $E_\varepsilon[V^{(k)}(\cdot)]$ at the resulting new state for each state and action.

Step III [Maximization]: Maximize $\pi(\cdot)+\delta\phi(\cdot)(E_\varepsilon[W(\cdot)] - X' - F)+\delta(1-\phi(\cdot))E_\varepsilon[V^{(k)}(\cdot)]$ and set the maximized value as $V^{(k+1)}$.

Step IV [Updating]: With $V^{(k)}$ and $V^{(k+1)}$, solve for $\mathbf{e}$, and update $\mathbf{e}^{(k)}$ to $\mathbf{e}^{(k+1)}$.

Step V [Convergence check]: Compute the infinite norm for $\mathbf{e}^{(k+1)} - \mathbf{e}^{(k)}$. Stop and set $V(\cdot) = V^{(k+1)}(\cdot)$, if this value $\simeq 0$; otherwise, go back to Step II.

Given the collusive value function, the simulated price paths were created by maximizing the value function for each realization of the state, given a sequence of cost shocks.[19] Given the initial values for these state variables,

$$P^0 = w_0 + w_1 c, X^0 = 0, c^1 = c, m_1^0, L^0 = L^{1/(1-\xi)},$$

this leaves three variables, (c, m_1^0, L), unspecified. To avoid interpolation beyond the specified state space, the smallest Chebchev node for the damage state variable is used. Given $(P^0,\ X^0,\ c^1,\ m_1^0,\ L^0)$, construct the choice set: $\eta \in \Phi(P^0)$.[20] For each $\eta \in \Phi(P^0)$, compute $h(\eta)$ and $h(\eta)/\max h(\eta)$, derive the new states $(P(\eta),\ X(\eta),\ m_1(\eta),\ L(\eta))$ according to the equations of motion, and calculate the current period profit $\pi(P(\eta), c^1)$,

[16] In general, the action space for price changes with $P \in [\underline{P}, \overline{P}]$ is $\Phi(P) \equiv [\underline{P} - P, \overline{P} - P]$ and we use the following discretized subset, $\{\eta \in \Phi(P)\colon\ P + \eta \in [P + \underline{m}_1, P + \overline{m}_1] \cap [0, \overline{P}]\}$.

[17] To calculate $h(\cdot)$, the normal probability density function is used instead of the Gaussian quadrature approximant.

[18] Throughout the numerical process, only two states are evaluated: the initial state as decided by the approximation method and the new state given by the equations of motion.

[19] We found this gave more stable results than approximating the policy function and using it for generating the price path.

[20] Again, we consider a discretized subset of $\Phi(P^0) : \{\eta \in \Phi(P^0)\colon\ P^0 + \eta \in [P^0 + \underline{m}_1, P^0 + \overline{m}_1] \cap [0, \overline{P}]\}$.

the probability of detection $\phi\left(L(\eta)\right)$, the expected non-collusive payoff $E_\varepsilon[W(c^1+\varepsilon)]$, and the expected collusive payoff $E_\varepsilon[V(P^1,\, X^1,\, c^1+\varepsilon,\, m_1^1,\, L^1)]$.[21] The rest of the calculation involves solving the maximization problem and thereby deriving η^1:

$$\begin{aligned}\eta^1 \quad &\equiv \quad \arg\max_{\eta\in\Phi(P^0)} \pi(P(\eta), c^1) + \delta\phi\left(L(\eta)\right)\left(E_\varepsilon[W(c^1+\varepsilon)] - X(\eta) - F\right) \\ &\qquad + \delta(1-\phi\left(L(\eta)\right))E_\varepsilon[V(P(\eta), X(\eta), c^1+\varepsilon, m_1(\eta), L(\eta))].\end{aligned}$$

After deriving η^1, set $P^1 = P^0+\eta^1$, and update (X^0, m_1^0, L^0) to (X^1, m_1^1, L^1) according to the equations of motion, while randomly selecting ε^1 and setting $c^2 = c^1+\varepsilon^1$ (for the stochastic cost case) and setting $c^2 = c^1$ (for the determistic cost case). With $(P^1, X^1, c^2, m_1^1, L^1)$, the process is repeated for at least 140 periods.

The results in this paper are based on the number of nodes as listed under Case I in the table below with the node placement determined by the Chebychev method. To test the robustness of this approximation, the value function was re-calculated for the following additional cases which vary in the number of nodes.

Number of Nodes

State variable/Case	I	II	III	IV	V	VI	VII	VIII
$P^{t-1}\in[20,80]$	6	7	7	8	7	7	8	8
$X^{t-1}\in[0,5000]$	5	4	5	4	4	5	4	5
$c^t\in[20,40]$	5	5	4	4	4	5	5	4
$m_1^{t-1}\in[-2.5,2.5]$	5	5	5	5	5	5	5	5
$L^{t-1}\in[0,1]$	5	5	5	5	5	5	5	5
Total # of states	3,750	3,500	3,500	3,200	2,800	4,375	4,000	4,000

We generally gave more nodes to the price state variable because that is the one most likely to be non-monotonic and, hence, more nodes might make a greater difference. Though some properties of the value function do change, the price paths are very robust with the exception of a few cases when $m_1^0=-1$. The range of X^{t-1} is changed to $[0,10000]$ when $\gamma=3$, $\beta=.9$, and $(w_0,w_1)=(0,1)$.

For the computation, all the value function iterations were done on a Dell Precision™ Workstation 340 with 2.0 GHz Intel® Xeon™ CPU. For the path simulations, we used a Dell OptiPlex™ SX270T with 3.0 GHz Intel® Patinum® 4 CPU. The computing time for the value function in the benchmark case was about 3.18 hours, while each price path simulation (for 200 periods) took about 11 minutes.

[21] Note that the cost shock is in the path simulations. We use the expected value operator, $E_\varepsilon[\cdot]$, for notational consistence.

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Figure 1. Benchmark

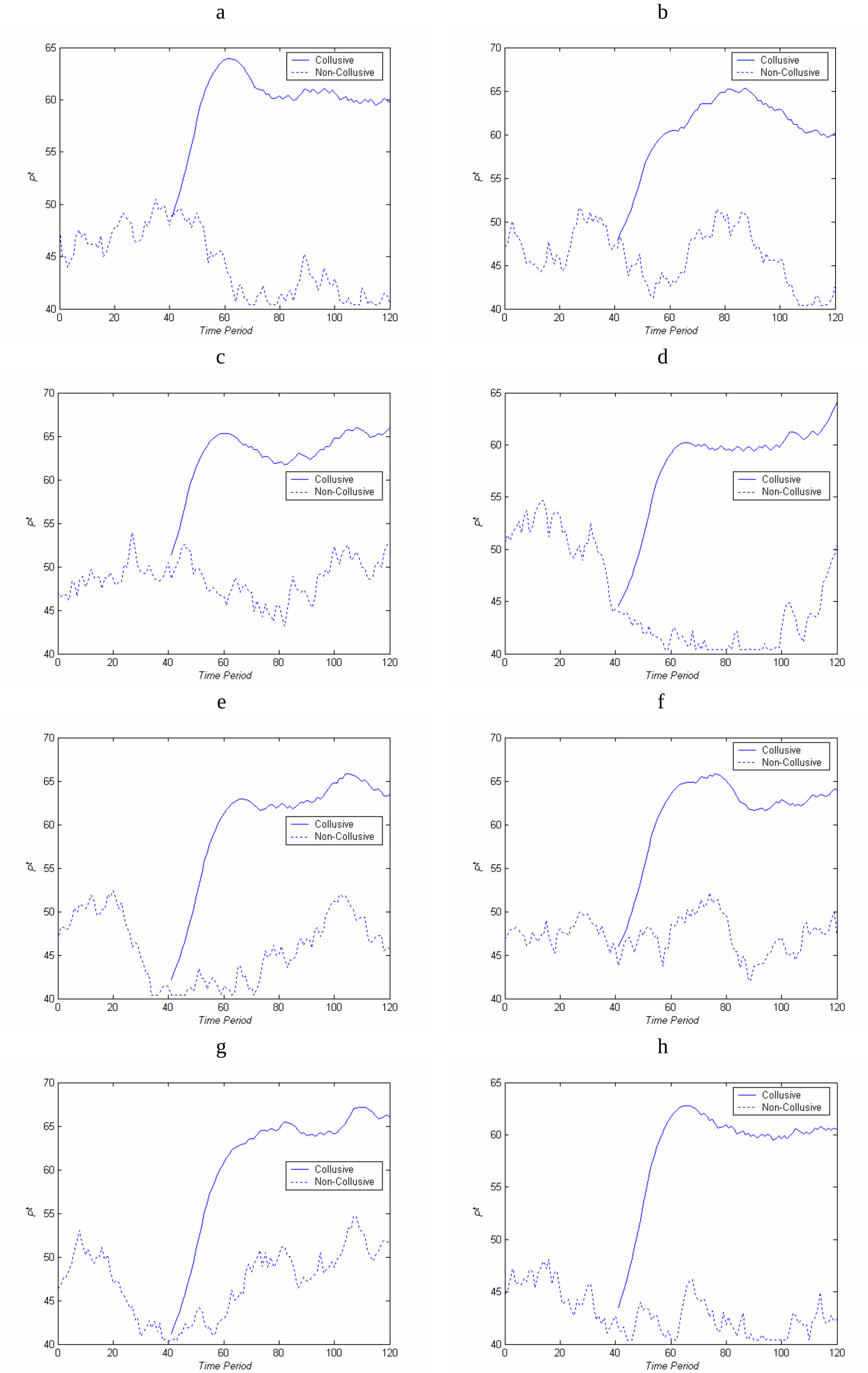


Figure 2. $\sigma_{\varepsilon}^{2} = 4$

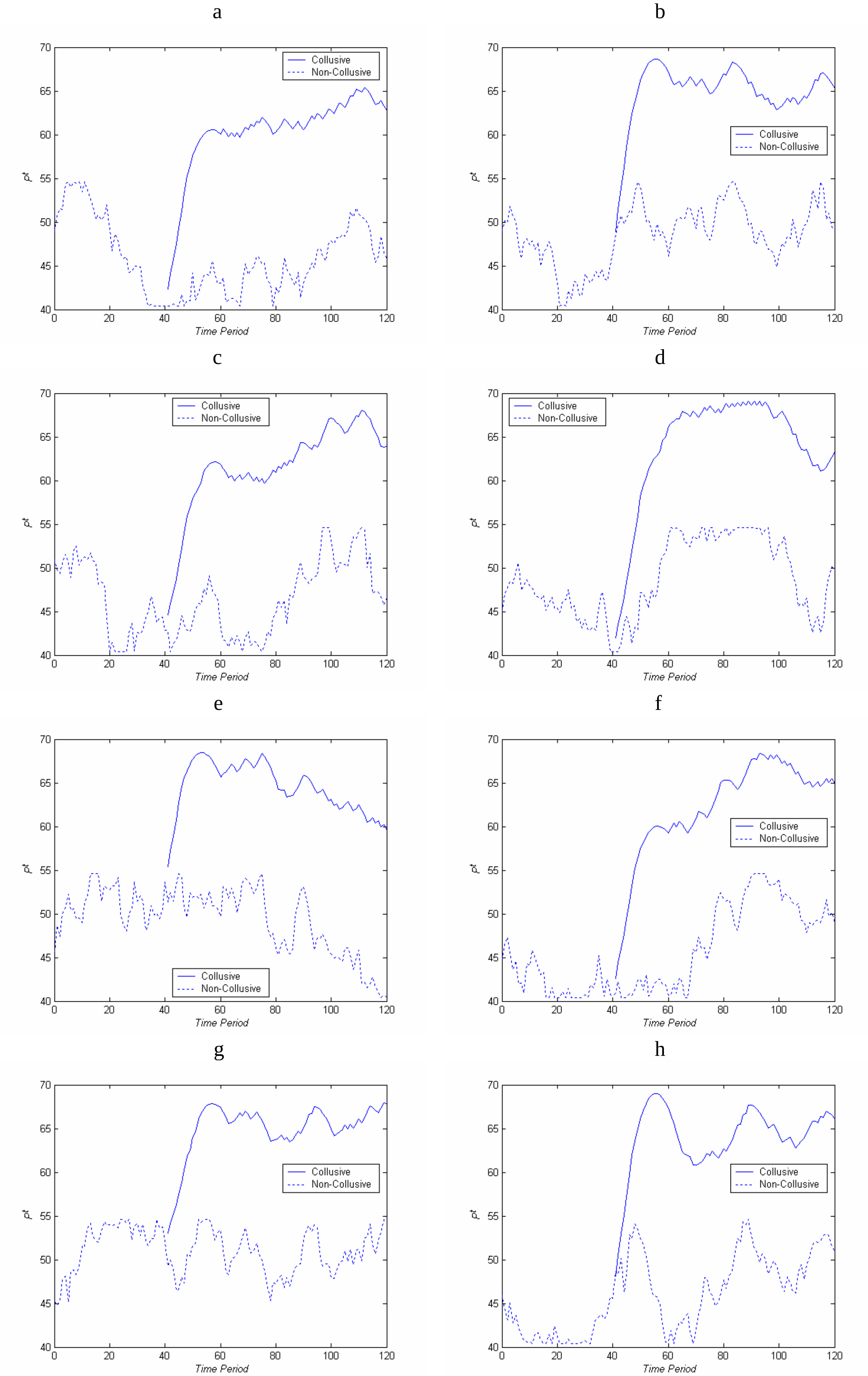
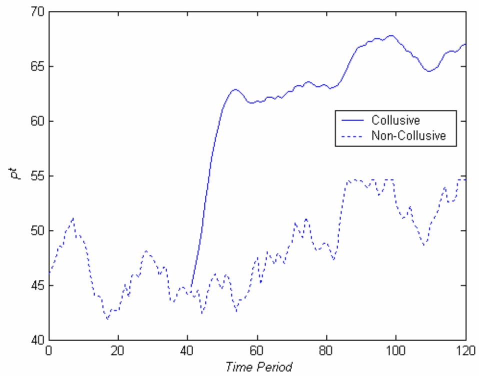


Figure 3. $(\xi, \lambda) = (.25, .25)$

a



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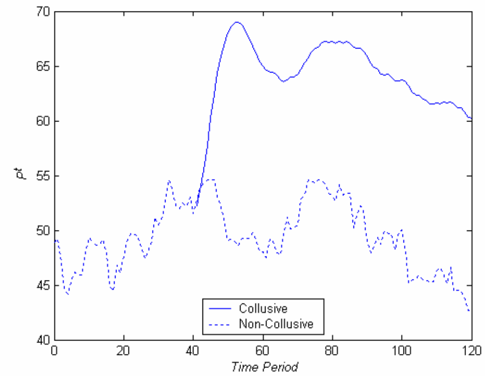

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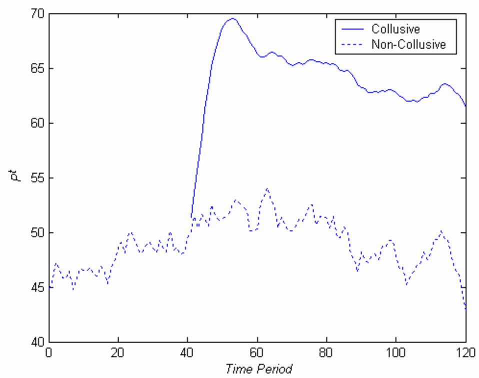

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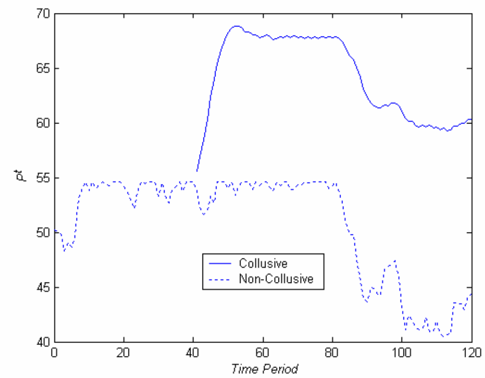

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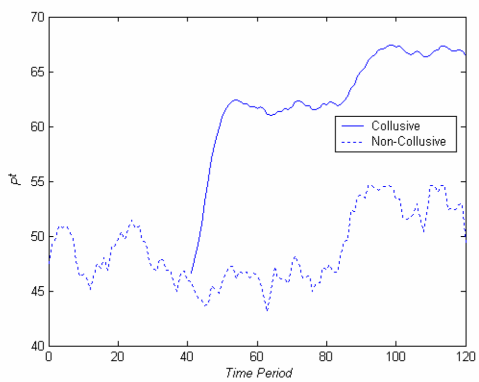

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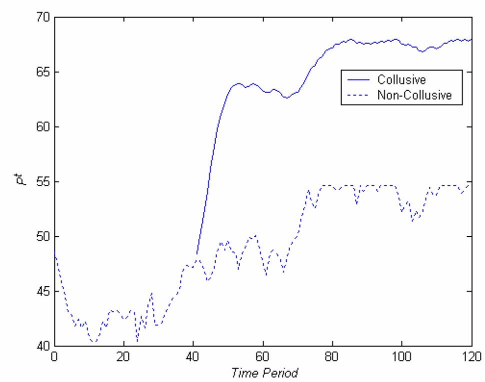

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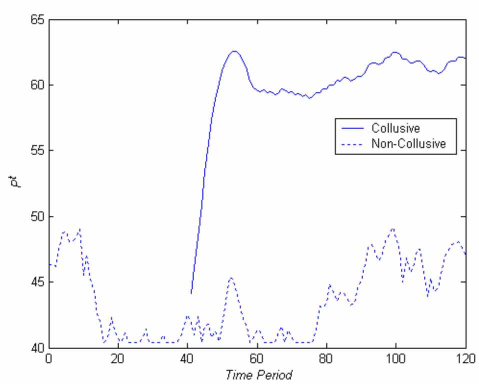

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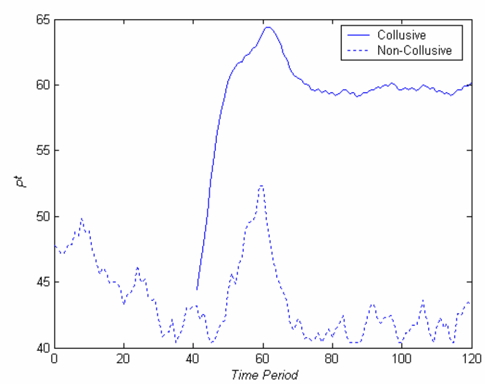


Figure 4. $\delta = .9$

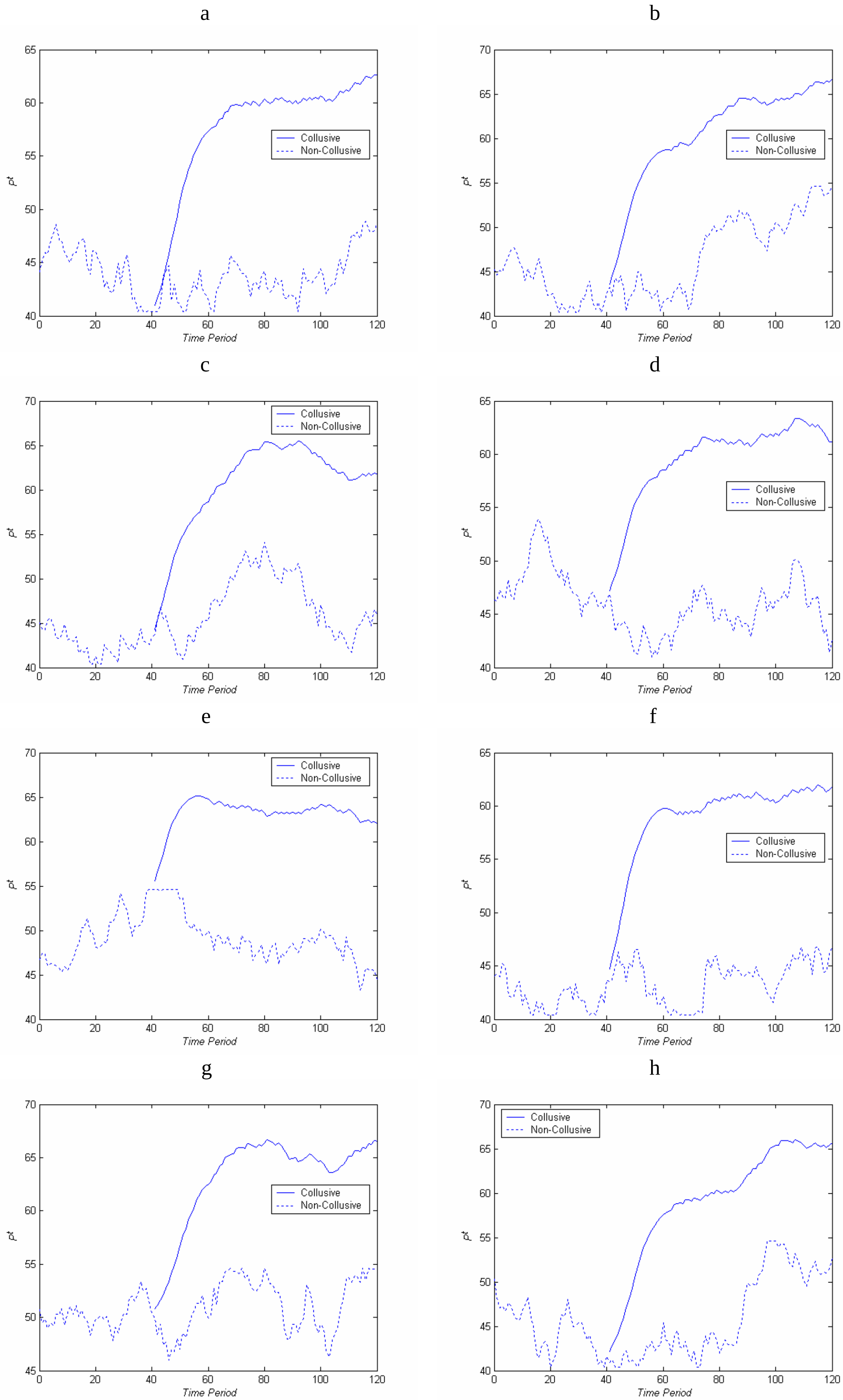


Figure 5. Benchmark: Deterministic Cost

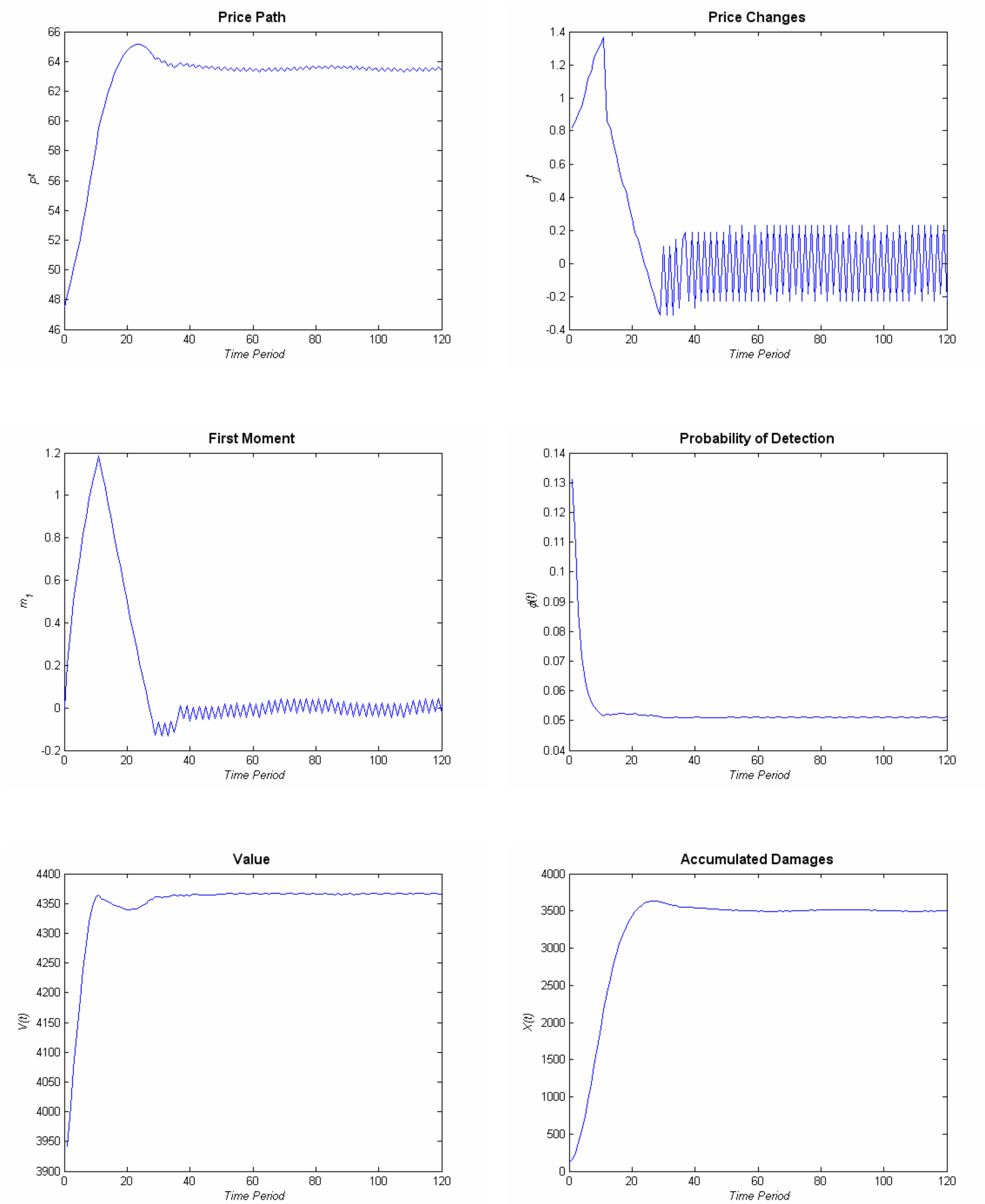
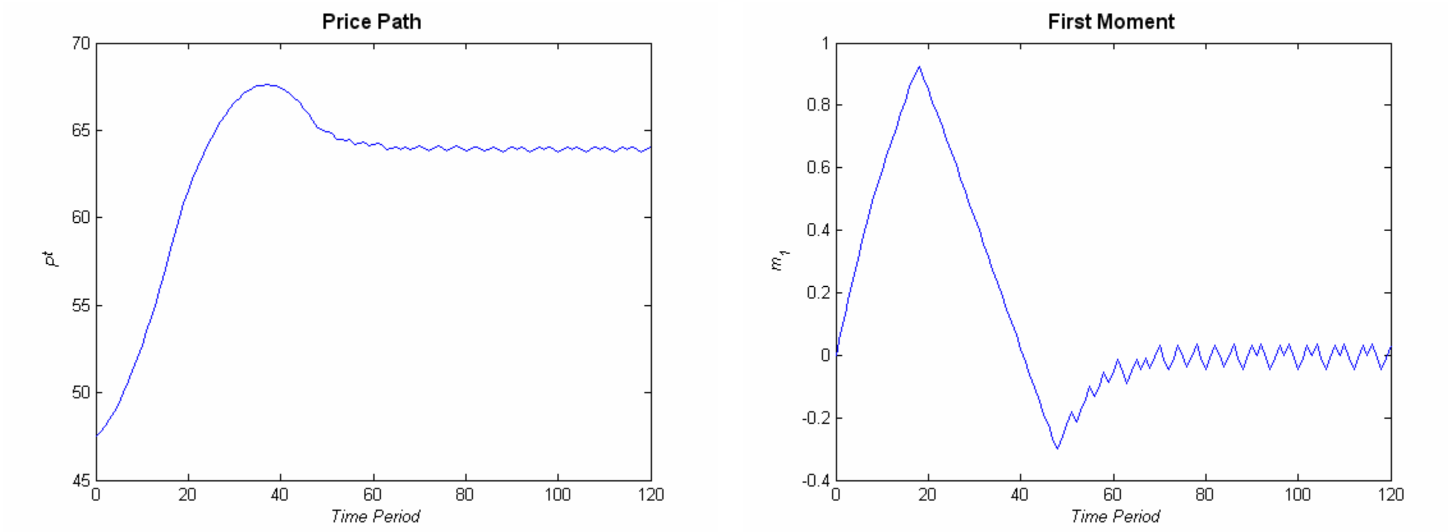


Figure 6

$\sigma_{\varepsilon}^{2}=1$, **deterministic cost**



$\sigma_{\varepsilon}^{2}=3$, **deterministic cost**

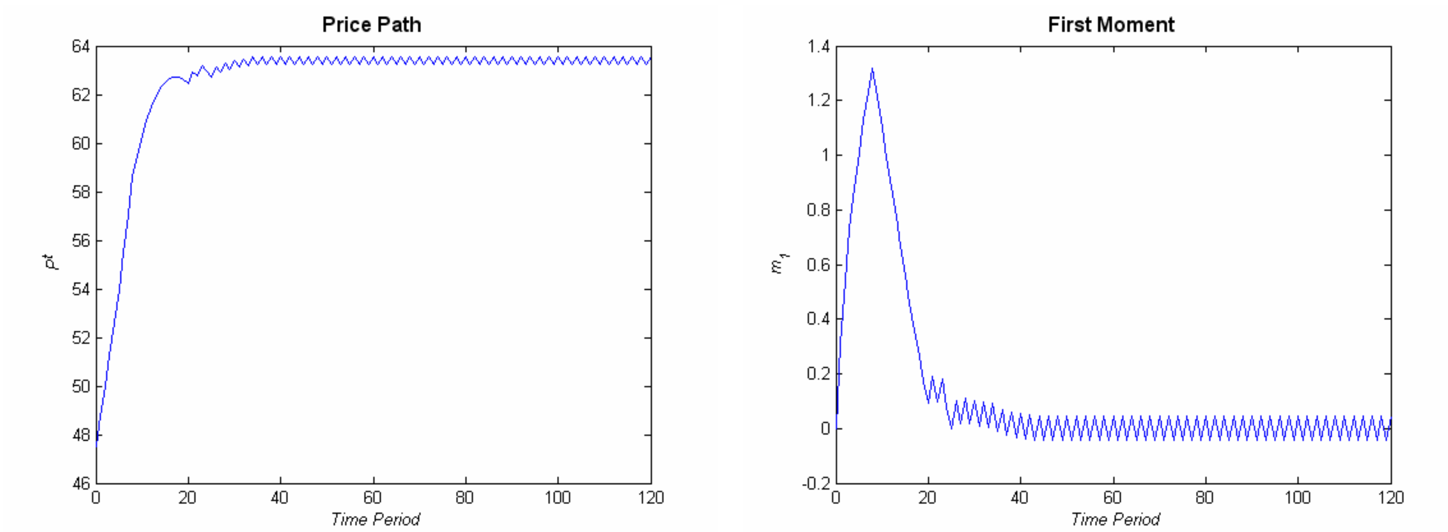
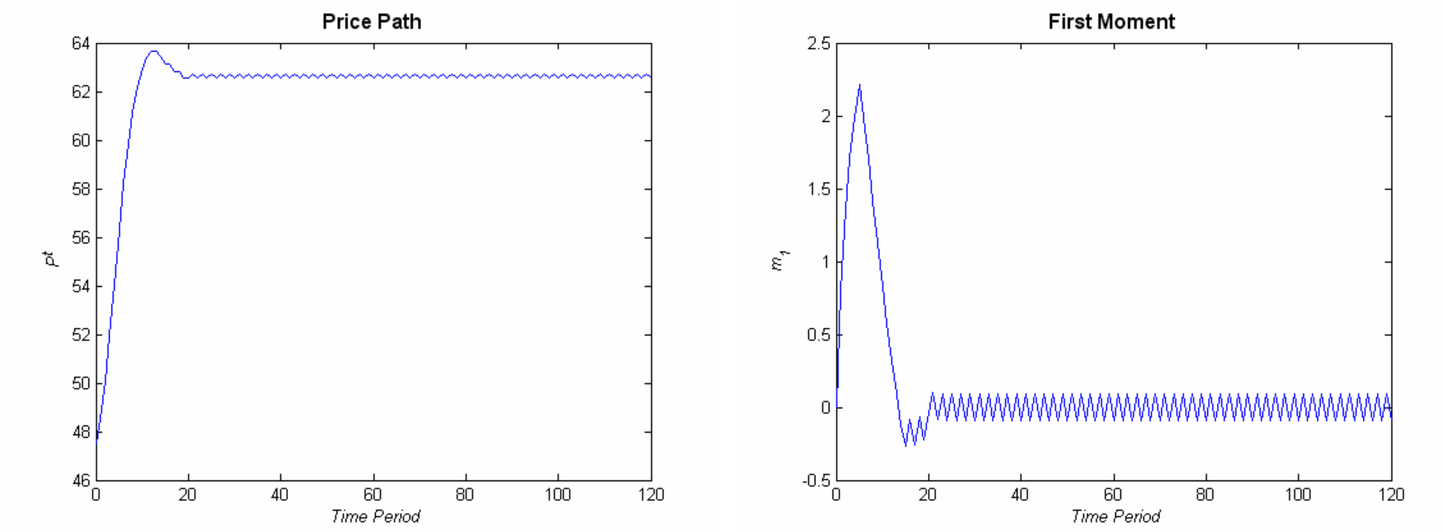


Figure 7

$(\xi, \lambda) = (.25, .25)$, **deterministic cost**



$(\xi, \lambda) = (.75, .9)$, **deterministic cost**

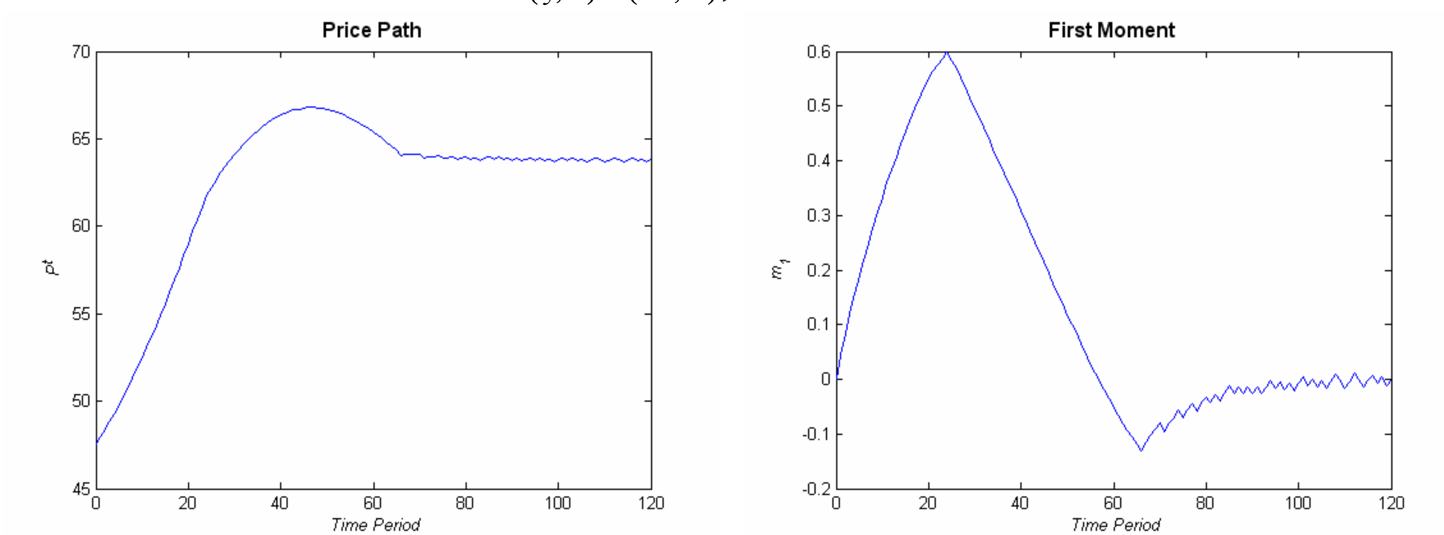
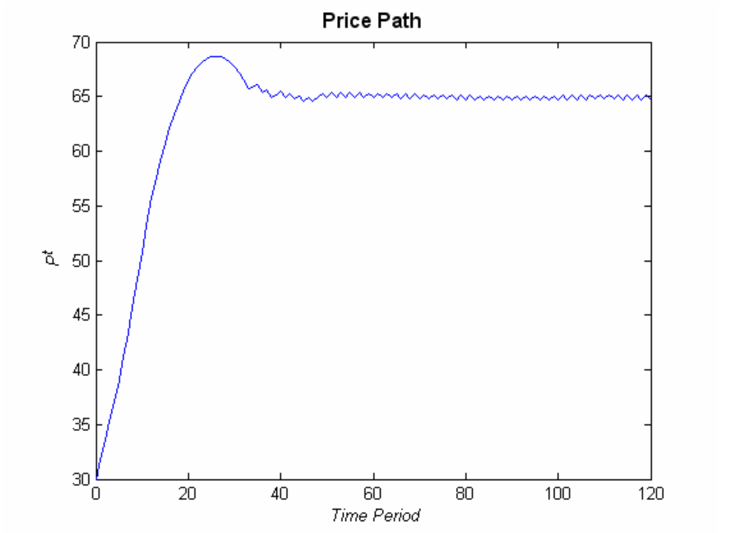
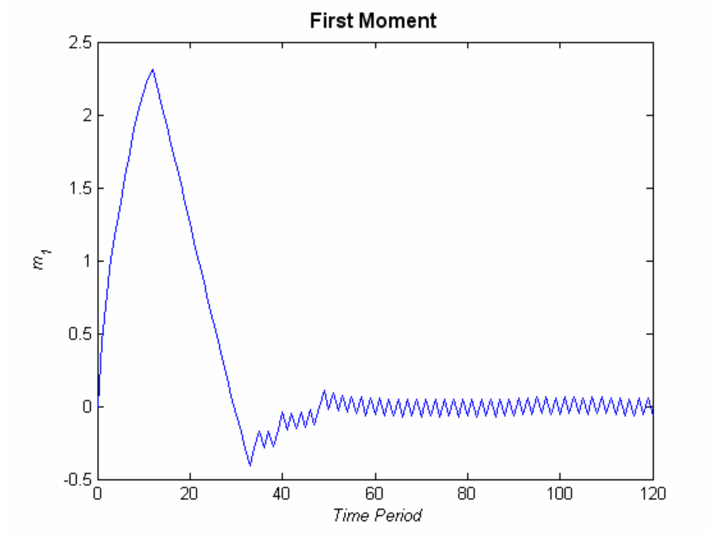


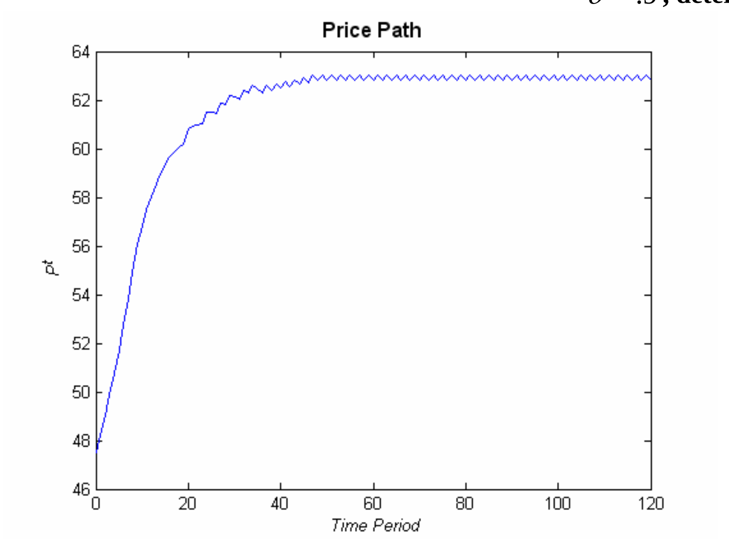
Figure 8

$(w_0, w_1) = (0, 1)$, **deterministic cost**





$\delta = .9$, **deterministic cost**



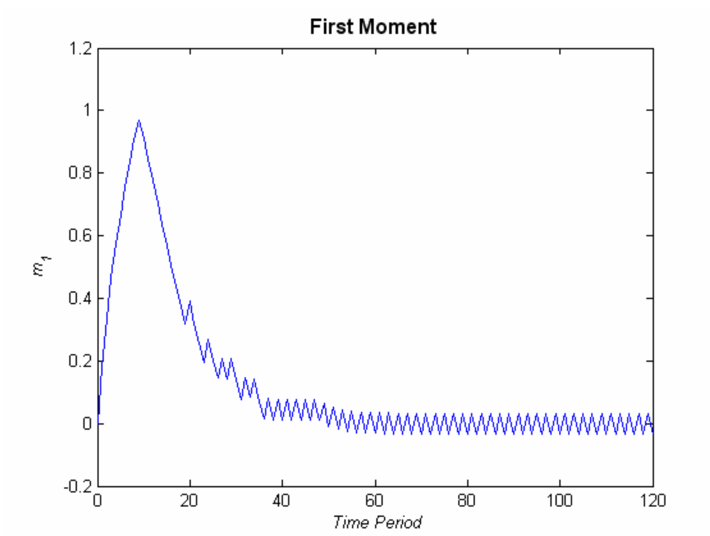


Table 1: Variance of Price

Parameters	Collusion			Non Collusion		
	Min	Max	Average	Min	Max	Average
Benchmark	0.0616	0.0994	0.078	0.7945	1.1958	0.9667
$\sigma_\varepsilon^2 = 1$	0.0225	0.0374	0.0289	0.3338	0.6685	0.4854
$\sigma_\varepsilon^2 = 3$	0.1265	0.1668	0.1441	1.3022	1.8679	1.5755
$\sigma_\varepsilon^2 = 4$	0.2199	0.2828	0.2546	1.1591	3.4468	1.9802
$\gamma = 3$	0.0633	0.0853	0.0753	0.7264	1.3753	1.0152
$\gamma = 2.25$	0.0597	0.1035	0.0774	0.6175	1.2552	0.955
$(\xi, \lambda) = (0.25, 0.25)$	0.0517	0.0898	0.0633	0.7073	1.2636	0.9404
$(\xi, \lambda) = (0.75, 0.9)$	0.0246	0.0366	0.0314	0.7473	1.367	1.053
$(\omega_0, \omega_1) = (0, 1)$	0.1857	0.2195	0.2066	1.4383	2.2561	1.818
$(\alpha_0, \alpha_1, \alpha_1) = (0.05, 0.2, 2)$	0.0659	0.1102	0.0826	0.7537	1.2683	1.0087
$(\alpha_0, \alpha_1, \alpha_1) = (0.05, 0.45, 3)$	0.098	0.1208	0.1091	0.6048	1.1991	0.9343
$\beta = 0.9$	0.0623	0.1418	0.0862	0.8099	1.3123	1.0152
$\delta = 0.9$	0.0591	0.0794	0.0702	0.8814	1.1546	1.0007

Table 2

σ_ε^2	Collusive Price Variance	Ratio of σ_ε^2 to collusive price variance
1	0.029	34
2	0.078	26
3	0.144	21
4	0.255	16

Table 3. OLS Regression Results: Price Differences on Cost Differences

	diff_cost		diff_lag_1_cost		diff_lag_2_cost		diff_lag_3_cost		R-bar
	est.	t-stat.	est.	t-stat.	est.	t-stat.	est.	t-stat.	squared
Benchmark	0.14	(9.17)	0.09	(6.04)	0.03	(2.23)	0.05	(3.17)	0.5355
$\sigma_{\varepsilon}^{2} = 1$	0.07	(5.09)	0.07	(5.06)	0.07	(4.91)	0.05	(3.87)	0.5169
$\sigma_{\varepsilon}^{2} = 3$	0.12	(7.18)	0.08	(4.63)	0.04	(2.62)	0.07	(4.23)	0.5481
$\sigma_{\varepsilon}^{2} = 4$	0.17	(9.41)	0.11	(6.26)	0.02	(1.10)	0.05	(2.45)	0.5826
$\gamma = 3$	0.13	(8.53)	0.09	(5.81)	0.06	(4.06)	0.04	(2.44)	0.5243
$\gamma = 2.25$	0.12	(8.14)	0.05	(3.24)	0.06	(3.81)	0.02	(1.25)	0.4277
$(\xi, \lambda) = (.25,.25)$	0.10	(6.42)	0.08	(5.41)	0.07	(4.22)	0.06	(3.88)	0.4935
$(\xi, \lambda) = (.75,.9)$	0.06	(4.86)	0.05	(4.10)	0.04	(2.96)	0.03	(2.75)	0.3699
$(w_0, w_1) = (0,1)$	0.19	(7.39)	0.05	(2.05)	0.04	(1.62)	0.06	(2.46)	0.4355
$\alpha_1 = .2$	0.14	(10.79)	0.10	(7.70)	0.06	(4.21)	0.05	(3.48)	0.6585
$\alpha_2 = 3$	0.18	(9.64)	0.06	(3.23)	0.05	(2.83)	0.01	(0.39)	0.5011
$\beta = .9$	0.09	(5.88)	0.08	(5.73)	0.07	(4.97)	0.06	(3.74)	0.5021
$\delta = .9$	0.10	(6.31)	0.03	(1.53)	0.03	(1.70)	0.05	(3.03)	0.4101