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Improving Money's Worth Ratio Calculations: The Case of Singapore's Pension Annuities

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Keywords
annuities, money's worth ratio, risk measures

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Featured Article

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1 Introduction

With the growing maturity of defined-contribution pension systems in many countries, the policy challenges of the payout or decumulation phase have attracted much attention. Increasingly, regulators and plan administrators are exploring ways to encourage retiree participation in life annuities, which facilitate the conversion of accumulated account balances in defined-contribution plans into lifetime income streams. Yet from the individual's perspective, buying a payout annuity is an important – and typically irreversible – decision that can shape financial well-being for many decades to come.

A popular measure of determining whether annuities deliver an adequate value-for-money is the "money's worth ratio (MWR)". The MWR metric is defined as the expected return to the annuity purchaser per premium dollar invested and was first used in the late 1980s in the context of life annuities by Warshawsky (1988). Since then, the money's worth methodology has been extensively adopted by researchers to quantify the financial return of a range of annuity products offered across various countries including the United States (Mitchell et al. 1999), the United Kingdom (Finkelstein and Poterba 2002, 2004; Cannon and Tonks 2002), Germany (Kaschützke and Maurer 2011), Singapore (Fong, Mitchell, and Koh 2011; Doyle, Mitchell, and Piggott 2004), Chile (Thorburn, Rocha, and Morales 2005), and others.

While extensive, the existing money's worth literature has focused almost exclusively on the expected value (or weighted mean) of the distribution of total benefits owed to the annuity purchaser. Nonetheless, across annuitants, substantial uncertainty may exist regarding future annuity income received due to differences in mortality profiles. In particular, for many retirees, the fear of dying at an early age and losing a large fraction of the annuity investment is very real; a Society of Actuaries (2004) survey reports that about half of the defined-contribution plan participants interviewed rate "protecting against the loss of value from a pension or annuity investment should they die earlier than expected" as very important. Loss aversion and the fear of illiquidity could thus deter people from annuitizing (Mitchell and Utkus 2004; Hu and Scott 2007).

In this paper, we emphasize another dimension of the annuity investment decision that the existing literature on money's worth has yet to consider: risk and volatility. We argue that it is necessary to consider the entire weighted distribution of annuity benefits, instead of focusing exclusively on its expected value (the numerator of the MWR metric). For instance, if the weighted discounted benefits are spread over a large range of values, the overall financial attractiveness of annuities may be less than what the MWR indicates. We demonstrate how quantifying the risk attributes of a life annuity – in addition to its expected return – will augment the usefulness of the widely-cited money's worth valuation framework. In addition, we discuss how some of these risk measures explicitly address annuitants' concerns about dying early after annuitization and losing a large part of the initial investment.

The pension annuities offered in Singapore provide an interesting setting for our analysis for several reasons. First, Singapore is one of the few countries where the annuitization of retirement assets is mandated on a national scale. Thus, it is particularly important for Singaporean retirees and workers to assess the risk and return aspects across annuity products so as to ensure that the financial decision is a well-informed one. Second, the range of pension annuities available in

Singapore uniquely spans both private and public providers. This follows a rather unconventional move by the Central Provident Fund (CPF) Board to enter the insurance market as a provider for these annuities in 2009. Accordingly, it is valuable to evaluate whether the new government-provided CPF annuities offer better *risk-adjusted* expected returns than the commercial ones.

The rest of the paper is structured as follows. Section 2 presents a brief overview of the Singapore national retirement scheme. Section 3 describes the data and outlines the money's worth valuation model. In particular, we specify an alternative algorithm for annuity valuation and demonstrate its compatibility with the standard MWR methodology. Section 4 discusses the key results which include standard risk measures, as well as supplementary risk measures that might better appeal to less financially sophisticated retirees. Section 5 concludes.

2 Background

The primary social security tool in Singapore is the CPF. The CPF is a national, mandatory, defined-contribution scheme, premised on the principle of self-reliance, and is fully funded by employers and employees since its inception in 1955. Half a century later, the CPF has evolved into a comprehensive social security system covering 3.5 million CPF members, of whom 1.8 million were active as of March 2013. Both employees and employers are required to make monthly contributions (up to an earnings cap) to the employee's individual CPF account. The current contribution rate for employees below age 50 is 20% of the employee's gross salary for employees and 15% for employers. The contributions are distributed into three accounts: Ordinary, Special, and Medisave. Although a certain portion of the saving is earmarked for old-age protection, the CPF scheme allows withdrawals (within prescribed limits) prior to retirement, for purposes such as healthcare, home ownership, insurance purchase, education, and investment.

Demographically, the Singaporean population is aging much faster than in most other affluent countries. The 2013 life expectancy estimate of Singaporeans is 84.07 years, trailing only Japan among countries with a population exceeding four million (CIA 2013). The fertility rate is only 0.79, lowest in the world, well below the replacement rate of 2.1. To address the challenges of a rapidly aging population and help ensure that retirees have an adequate standard of living throughout old age, the government of Singapore announced significant reforms to the CPF scheme in 2008. First, starting in 2013, annuitization was made mandatory for all CPF members turning 55 with at least S$40,000 in their CPF retirement accounts (CPF 2009a). This replaces the previous default 20-year phased drawdown mechanism. Second, the government required the CPF Board to operate a new

national longevity insurance scheme to provide pension annuities to its members. This scheme is named the CPF LIFE (Lifelong Income scheme For the Elderly) and serves as the default annuitization option for CPF members unless they have an existing private annuity policy that provides a higher payout.

Under the CPF LIFE scheme, the premium paid by the CPF member at age 55 is split into a term component and an annuity component (CPF 2008). The term component covers payouts from age 65 to a vesting age (which differs across the plans offered); any unused balance from the term component and interest is fully refundable to beneficiaries. The interest earned can be substantial: about 4% compounded annually, with a statutory additional 1% paid on the first S $60,000. The annuity component then finances payouts from the vesting age to death where unused balances may, or may not, be passed on to the beneficiary depending on the plan design.[1] Interest earned on the annuity component is pooled away to fund the scheme. CPF LIFE monies are invested in special Government bonds and earn an interest pegged to the yield of the 10-year Singapore Government Securities plus 1% to reflect the long-term nature of the contract.

A pilot phase of the CPF LIFE scheme was launched in September 2009 on a voluntary opt-in basis. It is reported that more than 60,000 members signed up for CPF LIFE under this pilot phase and committed a total of S$3.6 billion to the scheme (CPF 2011). In what follows, we summarize the payout features of these CPF LIFE annuities offered as at 2009 and compare them against the commercial pension annuities that were available to retirees pre-reform in 2007. The valuation exercise herein will focus on these two sets of single-premium deferred annuities offered pre- and post-reform.

3 Data and methods

3.1 Pension annuities in Singapore

The pension annuities considered for analysis are those which were available to members of the Singaporean CPF scheme as at 2007 and 2009.

In 2007, CPF members turning 55 with at least S$99,600 in retirement balances could voluntarily annuitize a portion of their retirement balances by purchasing annuities from private insurers (in lieu of the default phased withdrawal approach). The nine annuity products which they can choose from are

1 Details on the term and annuity component of the LIFE plans are discussed extensively in the Report by the National Longevity Insurance Committee (CPF 2008).

listed in Table 1. Note that the lump-sum annuity premium of S$99,600 is stipulated by the CPF Board[2]; the participating insurers are free to determine the quantum of monthly annuity payouts and aspects of the annuity contract such as the guarantee amount and bonus payments (if any). This is also typical of pension annuities made available to CPF members in prior years. Column 1 of Table 1 shows that monthly payouts range from about S$494–595 for males and S$441–562 for females. These fixed nominal payouts, which start at age 62, are slightly lower for females due to their longer average life expectancies.

By and large, most of the 2007 annuities were similar in terms of payout attributes except for those sold by HSBC Insurance and NTUC Income. The HSBC and NTUC Income annuities had an added participating feature, where annuitants could enjoy non-guaranteed annual bonus payments depending on the insurer's performance. Specifically, NTUC Income's annual bonus rate was

Table 1: Illustrative payouts and premiums for nominal life annuities offered by private and public insurers under the CPF scheme

Company/annuity	Pre-reform 2007		Post-reform 2009	
	Male	Female	Male	Female
Commercial annuities				
Actual premium	*S$99,600*	*S$99,600*	*S$117,000*	*S$117,000*
Asia Life Assurance	505.47	454.47		
American International Assurance (AIA)	530.87	513.94		
Aviva	559.00	507.00		
Great Eastern Life	535.35	484.30		
Great Eastern Life (with long-term care benefit)	494.26	440.73		
Overseas Assurance Corporation (OAC)	535.35	494.26		
Prudential assurance	518.44	449.87		
HSBC Insurance[#]	545.50	529.50		
NTUC Income Co-op[#]	595.00	561.75	831.13	722.20
Average	**535.47**	**492.87**		

(*continued*)

2 The lump-sum annuity premium varies from year to year and is based on the so-called prevailing "Minimum Sum" which is the minimum amount that must be set aside in a CPF member's retirement account before any CPF balances can be withdrawn at age 55. For instance, the Minimum Sum is S$70,000 in 2000, S$99,600 in 2008, S$117,000 in 2009, and S$148,000 in 2013.

Table 1: (*Continued*)

Company/annuity	Pre-reform 2007		Post-reform 2009	
	Male	Female	Male	Female
CPF LIFE annuities				
Illustrative premium	–	–	*S$67,000*	*S$67,000*
LIFE Basic			524	500
LIFE Balanced			561	520
LIFE Plus			594	535
LIFE Income			636	553
Average			**578.75**	**527.00**

Source: CPF LIFE payout data obtained from CPF website (CPF 2008, 2009b); payout data for commercial annuities from CPF (2007).
Notes: In 2007, nine commercial annuities were offered. The single premium was S$99,600 paid at age 55; monthly payouts started at age 62. In 2009, only one commercial annuity was offered. The single premium was S$117,000 paid at age 55; monthly payouts started at age 65. Four CPF LIFE plans were offered in 2009. Monthly payouts shown in the Table are for a CPF participant who joins the LIFE plan at age 55 with a premium of S$67,000, and starts receiving payouts at age 65. In particular, the parameter inputs generating this set of illustrative payouts in the CPF web calculator assume the Singaporean worker was born June 1958 (so he is age 55 in 2013); his Annual Value = "More than S$11,000"; his Assessable Income = "More than S $54,000".[3]# These are participating annuities; see text.

around 1–3.5% historically (NTUC 2009). Accordingly, we take into account potential future bonus payments when modeling the future cashflows attributable from these participating annuities.[4]

In 2009, after the government's plans to launch the CPF LIFE scheme were announced, all the commercial insurers exited the CPF platform except for NTUC Income. For details on this crowd-out effect, see Fong, Mitchell, and Koh (2011). The pension annuity that NTUC Income offered in 2009 was similar to its 2007 offering except for significant adjustments: the CPF-stipulated premium is now S $117,000 and the first payout starts at age 65 (instead of age 62 previously).

3 "Annual Value" is the property value used to calculate the property tax of the CPF member's home. "Assessable Income" is the full income in a calendar year (sum of employment, dividends, interest, and other income) less allowable expenses, applicable capital allowances, charitable donations, and any loss incurred in trade and business.

4 We use a projected 2% bonus with a projected rate of return of 5.25% per annum. This is in line with the benefit illustrations in NTUC Income prospectus (NTUC 2009). NTUC Income is a home-grown co-operative formed in 1970 and is one of the Singapore's leading composite insurers offering life, health, general insurance, and investment-linked products.

Monthly level payouts for the NTUC Income annuity were set at S$831 (male) and S$722 (female).

In contrast to the commercial pension annuities with pre-specified premiums, CPF LIFE allows any retiree with a retirement balance above S$40,000 to annuitize. Thus, the payout received depends on the amount annuitized (maximum S$117,000 in 2009). As at 2009, CPF LIFE scheme members could pick from one of the four plans: LIFE Basic, LIFE Balanced, LIFE Plus, and LIFE Income. Each plan offers a different combination of trade-off between monthly payout and bequest: LIFE Basic offers the lowest payouts with highest bequest, whereas LIFE Plus offers higher payouts with lesser bequests. Unlike the other three plans, LIFE Income does not allow any opportunity for bequests but compensates by offering the most attractive payouts. LIFE Balanced is the default option for CPF LIFE members who do not pick a plan. Column 2 of Table 1 reports the monthly payouts for an illustrative premium of S$67,000. Overall, the difference in monthly payouts between the plan with the highest payout (LIFE Income) and that with the lowest payout (LIFE Basic) is not substantial: only S$50 for females and S$100 for males.

A distinctive feature of pension annuities in Singapore is their money-back guarantees.[5] All the commercial and CPF LIFE annuities examined in this study have dollar guarantees where – in the event of the insured's death – named beneficiaries will be refunded an amount at least equal to the (positive) difference between the premium and any annuity payments already made. As noted in Fong, Mitchell, and Koh (2011), some insurers are more generous than others in specifying a guarantee amount which includes not only the premium but also 0.75–2.5% of accrued interest (accumulated between age 55 and first payout age). This death benefit feature essentially provides an element of capital protection, which would positively impact the financial return and risk attributes of the life annuity investment. We explore the extent of this positive impact attributable to dollar guarantees later in the paper.

3.2 The MWR framework

A vast literature focuses on measuring the money's worth of annuities in various developed and emerging markets. Formally, the MWR of a payout annuity is defined as the ratio of the expected present discounted value (EPDV) of annuity payments to the initial premium (Mitchell et al. 1999). With few exceptions, the

5 Prior to 2000, some of the commercial pension annuities offer period-certain guarantees instead of dollar guarantees.

EPDV formula used in prior studies to value immediate single-life annuities is as follows:

$$\text{EPDV} = \sum_{t=0}^{\infty} {}_tp_a \cdot \left(\prod_{m=0}^{t} v_m \right) \cdot A_a, \qquad [1a]$$

where ${}_tp_a$ is the probability that the annuitant survives t months after the annuity purchase, v_m is the discount factor for month m based on a term structure of interest rates, and A_a is the dollar amount of level annuity payout received. In essence, the EPDV reflects the theoretical (fair) price of a life annuity, involving assumptions regarding both mortality and interest rates. The EPDV is also known as the "actuarial present value" of a life annuity.

The EPDV formula for valuing capital-protected deferred annuities, such as those offered in Singapore or the United Kingdom, is slightly more complex.[6] Specifically, eq. [1b] shows the formula derived by Fong, Mitchell, and Koh (2011) in computing the MWR for the Singaporean pension annuities:

$$\text{EPDV} = \sum_{t=0}^{\infty} {}_tp_a \cdot q_{a+t} \cdot \left(\prod_{m=1}^{t+1} v_m \right) \cdot G_{t+1} + \sum_{t=D}^{\infty} {}_tp_a \cdot \left(\prod_{m=1}^{t} v_m \right) \cdot A_a. \qquad [1b]$$

This formulation comprises two summations: the first captures the death benefit arising from the money-back guarantee, while the second captures the periodic annuity payouts received after the deferral period. Taxes are not included in the valuation expression, since income from annuities bought with the CPF savings is tax-exempt. The additional terms in eq. [1b] include: D, the deferred period in months; q_{a+t}, the monthly probability to die between ages $a+t$ and $a+t+1$; and G_{t+1}, the death benefit paid.[7]

The MWR valuation framework has gained much popularity among researchers and practitioners, because it is intuitive and simple to understand. Specifically, the MWR metric compares the actuarial present value of a life annuity (or EPDV) with its actual market price (the premium): the higher the MWR, the better the value from the life annuity. If MWR $= 1$, retirees get back from the insurer the exact amount that they invested. Economists, in particular, have used the MWR framework to quantify the impact of adverse selection on the return from life annuities (e.g. Mitchell et al. 1999; Finkelstein and Poterba

6 In the United Kingdom, capital-protected or value-protected annuities feature a partial money-back option where the taxable lump-sum death benefit is permitted only up to age 75 (Boardman 2006).

7 The death benefit G_{t+1} is equal to $\max\{0, \text{premium} + \text{accrued interest from guarantee} - \text{total annuity payouts received prior to time } t+1\}$.

2002; Fong, Mitchell, and Koh 2011). To do so, they compare the MWR metric derived under two separate sets of survival probabilities, that of the annuitant group, and that of the general population.

Yet the existing money's worth literature has emphasized only the return aspect of the life annuity investment, providing no indication of its riskiness or volatility. Consider two life annuity products each with a high MWR of 0.95. One has a weighted distribution of benefits that is spread over a large range of values, but the other has a weighted distribution of benefits that is spread over a smaller range of values. Clearly, the latter is more financially attractive from a risk-adjusted perspective. If the MWR framework is to inform prospective annuitants about the financial value of annuities, it must be extended to reflect both the return and the risk attributes involved in the annuity investment. One way to do so is to report the standard deviation (SD) of the distribution, alongside the MWR metric. This would allow prospective annuitants to differentiate among products that may appear seemingly uniform in terms of money's worth, but vary in terms of their risk attributes. Associated risk measures such as the coefficient of variation (CV) and skewness of the distribution may also provide useful insights.

There are at least other two reasons why the quantification of risk measures under the MWR framework is of considerable interest. Firstly, certain annuity products include an investment component. Variable annuities, for example, are investment products which combine longevity insurance with a mutual fund (or collection of funds). Financial risk measures would be pertinent in such contexts to capture not only the uncertainty in mortality, but also market-related risks.

Secondly, prior research has shown that many individuals tend to adopt a "narrow frame of reference" when faced with the annuity decision (cf Hu and Scott 2007; Duxbury et al. 2013). This means that individuals perceive the annuity purchase as a *gamble* which increases risk rather than longevity insurance and are particularly concerned about whether they will live long enough to make back the initial annuity investment. As noted in earlier studies, the primary way to address this issue is through consumer education; for example, annuity marketers and financial advisers can better frame the annuity as longevity insurance drawing attention to the insurance value of annuities (i.e. annuities are generally welfare-enhancing because they eliminate longevity risk; see Davidoff, Brown, and Diamond 2005). However, consumer education is arguably a long-drawn process and is often easier said than done. At the same time, there is abundant evidence that many older adults above the age of 50 are financially illiterate (Lusardi and Mitchell 2008; Christelis, Jappelli, and Padula 2010). Thus rather than purely insist that people switch their frame of reference, a parallel approach proposed in this study is that plan administrators and

policymakers can do more to directly address individuals' base concerns about annuitization. In other words, quantify the individual's odds of winning the longevity gamble and put it in perspective. For example, if the retiree realizes that the odds of winning the gamble is 1:1 (50% chance of winning) on average rather than 1:9 (10% chance of winning), he might actually be more inclined to annuitize because this knowledge alleviates his fear of not living long enough to make back the initial annuity investment.

3.3 Methods

To compute risk measures associated with a life annuity investment, we need to specify an alternative algorithm for the EPDV. A drawback of eqs [1a] and [1b] presented earlier is that they do not facilitate the computation of higher-order moments. Hence while adequate for simple MWR calculations, they are not suitable for our purpose. In this study, we employ a more intuitive and elegant expression for the EPDV of life annuities:

$$\text{EPDV} = \sum_{t=0}^{\infty} {}_{t|}q_a \cdot Y_t. \qquad [2]$$

Eq. [2] presents the EPDV as a function of two key components: a random variable Y, and a set of weights ${}_{t|}q_a$. Specifically, Y is a random variable taking a discrete set of values Y_t equal to the present value of total benefits received from start of payouts up to the month of death. ${}_{t|}q_a$ is the probability of dying between months t and $t+1$ given survival up to month t. Taken together, the EPDV reflects the cumulative annuity benefits received at each possible age of death.

One main advantage of this algorithm is that it readily allows the computation of second-order and third-order moments which are necessary for the derivation of the SD, variance, and skewness of the distribution. Another advantage of this setup is that it can be flexibly applied to a range of annuity products, including the present sample of capital-protected deferred annuities considered for analysis. In the Appendix, we show that eq. [2] is compatible with both eqs [1a] and [1b].

To value the Singaporean pension annuities, two additional inputs are required. First is a term structure of interest rates relevant for the year of valuation. Consistent with prior studies which examined the money's worth of annuities in the Singapore marketplace (e.g. Fong 2002; Fong, Mitchell, and Koh 2011), we use the risk-free Singaporean Treasury bond rates for discounting purposes. Data are obtained from the Monetary Authority of Singapore; the

long-term rate applied from year 20 onwards is about 3.4%.[8] Second is a set of mortality estimates. To value the commercial annuities offered in 2007, we apply cohortized proxy annuitant mortality rates derived from the UK a(1990) Ultimate Tables with a 5-year setback, as in Fong, Mitchell, and Koh (2011).[9] These rates are also consistent with those required by the Monetary Authority of Singapore for annuity product reserves and liability valuations. Annuitant mortality is appropriate for our purposes here since annuitization was voluntary prior to the 2008 CPF reform. By contrast, to value the government-offered annuities (and a single commercial annuity) offered in 2009, we apply cohortized mortality rates for the Singapore general population. Specifically, we use the 2009 Singapore Resident Population life tables (SDOS 2013) which is appropriate, since a vast majority of retirees is expected to purchase these CPF LIFE annuities under the annuitization mandate. When a critical mass of participants in the pension system is required to buy an annuity, the administrator is able to pool the longevity risk of both the long-lived and the short-lived, and thus, is presumed to face average population mortality risk.

4 Results

4.1 Standard risk measures

Table 2 reports the return and risk attributes of the Singaporean pension annuities before and after the 2008 CPF reform. Prior to the reform, we see that a male retiree who purchased a life annuity from a commercial insurer would have obtained a MWR of 0.948 on average from his purchase, or about 95 cents per dollar premium invested. The corresponding expected financial return for a female annuitant is very similar: about 96 cents per dollar (a MWR of 0.957). These findings are largely consistent with those reported by Fong, Mitchell, and Koh (2011), who valued the same basket of 2007 annuities.[10] Aside from

8 The first year rate is derived from the 1-year Singapore Government Treasury bill; thereafter, the 2-, 5-, 7-, 10-, 15- and 20-year Treasury bond rates as of 2007 are used to estimate the riskless spot rates (MAS 2008). The 20-year rate is applied for periods after year 20; annual spot rates range from 1.4% to 3.44%.

9 The a(1990) tables are constructed based on UK annuitants' mortality experience from 1967–70 with mortality improvements projected to 1990. Note that annuitant life tables are not available in Singapore due to limited annuitant mortality experience.

10 Some MWR values may not be exactly identical because of different assumptions regarding the potential bonus payouts from the participating annuities and minor corrections.

Table 2: Statistical measures for benefits received from nominal life annuities offered by private insurers and the CPF Board under the CPF scheme (males only)

Company/annuity	Males				Females			
	MWR (SD)	Interval within 1 SD	CV	Skew	MWR (SD)	Interval within 1 SD	CV	Skew
2007								
Commercial annuities								
Asia Life Assurance	0.896 (0.172)	[0.724, 1.068]	0.192	−0.128	0.885 (0.145)	[0.740, 1.029]	0.163	−0.524
AIA	0.934 (0.188)	[0.746, 1.122]	0.201	−0.158	0.990 (0.177)	[0.813, 1.167]	0.178	−0.626
Aviva	0.985 (0.195)	[0.790, 1.180]	0.198	−0.127	0.982 (0.165)	[0.817, 1.148]	0.168	−0.546
Great Eastern Life	0.947 (0.183)	[0.764, 1.130]	0.193	−0.117	0.941 (0.155)	[0.786, 1.096]	0.165	−0.526
Great Eastern Life (with long-term care)	0.884 (0.161)	[0.723, 1.045]	0.182	−0.085	0.863 (0.134)	[0.729, 0.997]	0.155	−0.456
OAC	0.947 (0.183)	[0.764, 1.130]	0.193	−0.117	0.959 (0.160)	[0.798, 1.119]	0.167	−0.543
Prudential assurance	0.915 (0.180)	[0.735, 1.095]	0.197	−0.143	0.876 (0.142)	[0.734, 1.019]	0.162	−0.514
HSBC Insurance	0.975 (0.176)	[0.799, 1.150]	0.180	−0.052	1.028 (0.170)	[0.858, 1.197]	0.165	−0.512
NTUC Income	1.053 (0.201)	[0.852, 1.254]	0.191	−0.072	1.087 (0.184)	[0.904, 1.271]	0.169	−0.525
Average	**0.948 (0.182)**		**0.192**	**−0.111**	**0.957 (0.159)**		**0.166**	**−0.530**
2009								
Commercial annuities								
NTUC Income Co-op	1.025 (0.204)	[0.821, 1.228]	0.199	0.109	0.980 (0.170)	[0.810, 1.151]	0.174	−0.144
Government-offered annuities								
LIFE Basic	1.241 (0.129)	[1.112, 1.37]	0.104	1.022	1.259 (0.142)	[1.117, 1.402]	0.113	0.609
LIFE Balanced	1.259 (0.226)	[1.033, 1.485]	0.179	0.423	1.278 (0.220)	[1.058, 1.498]	0.172	0.046
LIFE Plus	1.330 (0.367)	[0.963, 1.697]	0.276	−0.208	1.355 (0.321)	[1.034, 1.675]	0.237	−0.526
LIFE Income	1.317 (0.559)	[0.758, 1.875]	0.424	−0.917	1.341 (0.447)	[0.894, 1.787]	0.333	−1.315
Average	**1.287 (0.320)**		**0.246**	**0.080**	**1.308 (0.282)**		**0.214**	**−0.296**

Notes: For annuities in 2007, MWR values are computed using 2007 annuitant mortality. For annuities in 2009, MWR values are computed using 2009 population mortality. See text for details.

expected returns, the associated risk profiles of the annuity products are reported. Table 2 shows three risk statistics: namely, the SD, the CV (SD/MWR), and the skewness coefficient of the weighted distribution. In particular, the CV is a normalized measure of risk and is generally perceived as useful in guiding investment decision-making because it combines both the return and the risk attributes of the investment.

Focusing first on the SD statistic, two interesting observations emerge. Firstly, we see that the risk and volatility in annuity returns arising from uncertain mortality is non-trivial. On average, moving one SD away from the MWR (or weighted mean) would increase or decrease expected returns by 0.182 and 0.159 for male and female annuitants, respectively. This works out to approximately a 17–19% change in expected returns. Also, evaluating across products, there appears to be a risk–return trade-off. The product with the highest expected financial return per the MWR metric – the NTUC Income annuity – is also accompanied by a higher-than-average SD.

Secondly, the average SD is higher for males (0.182) than females (0.159). To investigate this difference, we examine components of the weighted discounted payouts distribution by sex. For this illustration, we focus on the OAC annuity since its monthly payouts are most representative of the average payouts across commercial products in Table 1. Figure 1A shows the mortality weights ($_{t|}q_a$) by age for a male and female annuitant in 2007. Note that the curve for males (solid) peaks at age 96 (most probable age of death) whereas the curve for females (dotted) is more left-skewed, with weights being concentrated at later ages due to longer life expectancies. Using the same age intervals, Figure 1B then outlines the extent of the (scaled) deviation of Y_t from its mean. We see here that the cumulative discounted benefits random variable (Y_t) is inherently more volatile for males than for females throughout most ages, and this is mainly attributable to the higher payouts enjoyed by male annuitants. We conclude that the imbedded risk involved in a life annuity investment is determined by the interaction between the death probabilities and the cumulative discounted benefits at each possible age of death. Higher chances of death at points where the cumulative discounted benefit is far from its mean result in higher SD values. Consequently, the higher SD observed for Singaporean male retirees results from them facing higher death probabilities than their female counterparts during the time just after the deferral period when Y_t values are far from their mean (around ages 65–85).

The variability in cumulative discounted benefits over the lifetime of the annuitant depends on aspects of the annuity contract. This includes the payout quantum, payout structure (e.g. level or escalating), the presence of guarantee features, and so on. It is therefore important to bear in mind that the

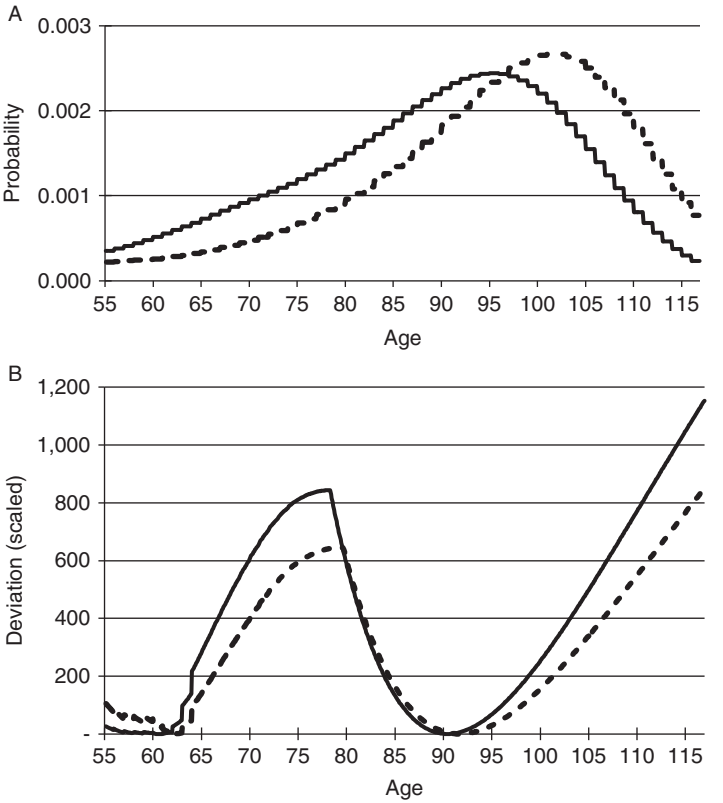

Figure 1: Components of the weighted discounted payouts distribution (2007; OAC annuity; annuitant mortality). (A) Cohortized conditional death probabilities, by gender. (B) Scaled diagram of the deviation of Y_t from $\bar{Y}$, by gender

Note: On the *y*-axis, the deviation is computed by $(Y_t - \bar{Y})^2$, and then scaled by a factor of 1 million. Plots for males are shown in solid lines and those for females are shown in dotted lines.

Singaporean pension annuities are capital-protected annuities with death benefit guarantees. Non-capital-protected products are likely to exhibit higher variability in the discounted benefits structure. To explore the impact of removing the guarantee on the risk profile of an annuity, we recomputed the SD for the illustrative OAC annuity for a male annuitant assuming no death benefit: the SD almost doubles from 0.183 to 0.328, while the MWR falls from 0.947 to 0.856. This suggests that the element of capital protection in the Singaporean pension annuities is extremely valuable; the money-back guarantee substantially reduces the uncertainty associated with the annuity investment. Annuities offered in other countries are likely to be more volatile.

Next, we turn attention to the CV statistic. A smaller CV is preferred because it implies that the product's risk is lower per unit of annuity benefit achieved. Interestingly, the ranking of annuities based on the CV metric is quite different from that based on the MWR metric. For instance, the NTUC Income product, with the highest MWR at 1.053, has a CV of 0.191, higher than the HSBC annuity (with the lowest CV of 0.180, but also a lower MWR). Nonetheless, Table 2 shows that the variation in CV values is not large across products by sex. We also note that all the weighted payout distributions for the 2007 annuities are negatively skewed. This is not surprising since time-until-death is a very skewed variable; few people die at early ages; the bulk of deaths is concentrated at later ages.

After the 2008 CPF reform, only NTUC Income continued to offer life annuity policies to CPF members. The rest of the commercial insurers were crowded out by the launch of the new CPF LIFE scheme operated by the CPF Board. Table 2 shows that a set of pension annuities as at 2009 exhibit the same broad sex-specific differences in terms of risk attributes, that is, higher variance and less negative skewness for male annuitants. The NTUC Income annuity continues to offer retirees very competitive returns: its MWR is close to unity (1.025 for males and 0.957 for females).

Across the four CPF LIFE annuities, MWR estimates are rather uniform. Money's worth ranges between 1.241 and 1.330 for males and 1.259 and 1.355 for females. If a retiree simply selected a product based on the MWR value, then the LIFE Plus plan (highest payouts, least bequest) would be the most attractive. Strikingly, however, risk characteristics vary widely across these four products. For male purchasers, the SD is four times higher under LIFE Income (SD = 0.559) than under LIFE Basic (SD = 0.129). The CV values range from as low as 0.104 to as high as 0.424. Overall, the LIFE Basic and Balanced are less risky options than the other two plans because of the substantial death benefit refundable to beneficiaries during the early years of the investment. This also results in their payout distributions being positively skewed since more annuity benefits are derived earlier in life. It is therefore not surprising that the CPF Board has selected the LIFE Balanced plan to be the default vehicle for CPF members under the annuitization mandate, since it offers retirees a moderate mix of benefits and risk. Nonetheless, if a retiree is to choose a plan based on risk–return characteristics, then the LIFE Basic plan (lowest payouts, highest bequest) is the most appealing given its lowest CV value for both males and females.

Also noteworthy is the fact that the LIFE Income plan risk–return attributes are dominated by the Plus plan. The MWR estimate for LIFE Plus is 1.330 (as compared to 1.317 for Income plan); its CV is 35% lower. Is the Income plan suitably designed for retirees? On the one hand, it contributes to diversity

among the LIFE plans and might be appropriate for individuals who do not mind forfeiting bequests for higher monthly payouts, for example those unmarried or without dependents. On the other hand, a retiree can get better money's worth from the Plus plan with lower risk. Moreover, under the Income plan, the bulk of the benefit flows to the annuitant only later in life. The Income plan is so inherently risky that its CV value (0.559) is more than quadruple that of the Basic plan (0.129) and is more than double that of the commercial annuities (0.182).

While the standard risk measures are meaningful to individuals with some fundamental level of financial knowledge, they may still be too difficult to comprehend for older adults who are financially illiterate. As discussed earlier, many people tend to perceive an annuity purchase as a longevity gamble and an important question for these individuals is: Will I live long enough to make back my initial investment in this annuity? (Hu and Scott 2007). In what follows, we utilize the valuation framework to directly address this base concern; we quantify the individual's odds of winning the longevity gamble.

4.2 Simpler risk measures

The supplementary risk measures discussed here usefully employ visuals in the form of net present value (NPV) plots. The NPV of the cumulative annuity benefits is defined as Y_t (present value of total benefits) minus the premium. Because the lump-sum premium constitutes a large negative cash outflow, the NPV of an annuity investment generally starts negative before gradually increasing over the annuitant's lifetime. Annuitants with a "narrow frame of reference" are ultimately most concerned about whether they win or lose the longevity gamble. To help these less financially literate retirees better understand their annuity contract, we explore answers to three related questions:

1. How long do I need to live to in order for the discounted benefits to exceed my initial outlay? (cross-over age)
2. What is the "worst time to die"?
3. On average, what is the chance that I will lose or win on the deal?

In contrast to the standard risk measures, these simpler measures of risk are obtained by examining the unweighted distribution of cumulative annuity benefits rather than the weighted distribution. As in the previous sub-section, we use the OAC annuity for illustration. Figure 2 depicts the NPV profile of the annuity investment for a male who purchased the OAC annuity for S$99,600 at

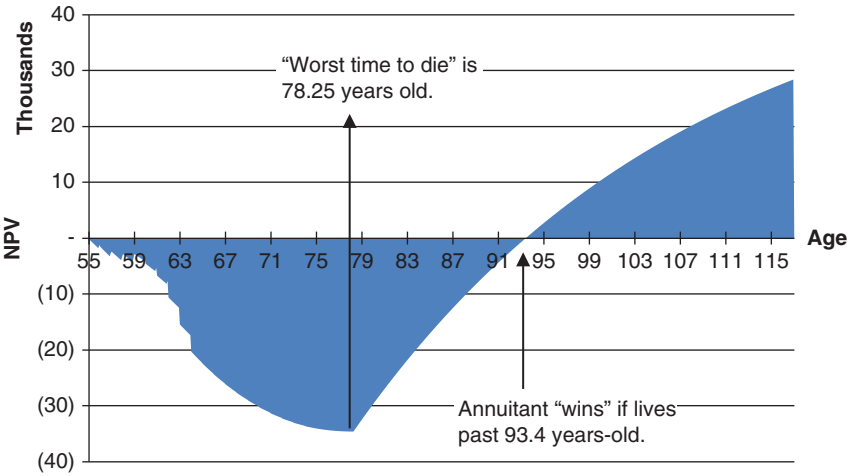

Figure 2: NPV of cumulative annuity benefits (2007; OAC annuity; male)
Note: Computations pertain to a male CPF plan participant who purchases the annuity offered by OAC under the CPF retirement scheme in 2007. His entry age is 55; premium is S$99,600; monthly payouts of S$535.35 start at age 62.

age 55 and lives to the terminal age of 117. This graph highlights two interesting points. First, we see that the presence of the capital guarantee negates potentially large negative NPV losses in the early years. This is especially so over the 7-year deferment period from age 55 to 62.[11] After the monthly payouts start at age 62, the NPV becomes more and more negative because of discounting and reduction in the remaining death benefit.[12] Nonetheless, the capital protection on the premium defers the "worst time to die" to about age 78.25, which is the minimum point of −35,000 in the NPV graph. This point coincides with the time when the capital guarantee is almost depleted. (Without the guarantee, the NPV will be −99,600 at age 55, and the "worst time to die" would have been just after handing the money over to the insurer). Second, the "cross-over age" for the commercial pension annuity apparently occurs rather late in life. The figure

11 The jagged portions between ages 55 and 62 in Figure 2 are due to stepped increases in the guarantee amount accruing from accrued interest of 0.75% per annum (up to age 62), as well as the non-monotonicity of the discount rates. Accrued interest on the guarantee does not exceed 2.5% for the rest of the annuities; three insurers do not pay such interest. Over time, the guarantee starts to wear off due to discounting and the commencement of payouts.
12 The death benefit wears down as monthly payouts accumulate.

shows that the male annuitant has to live past age 93.4 (point where the NPV curve crosses the *x*-axis) before he can break-even.

At first glance, these statistics may seem surprising if compared with population life tables. The 2007 Singapore population life tables estimate that a 55-year-old male is expected to live to only about age 80.4. Even supposing that he survives to age 62, the additional number of years he can expect to live is only 19.5 (i.e. the expected death age is 81.5). If so, then the annuities offered by private insurers can be costly for the average retiree even with the dollar guarantee feature. However, population period tables are not an appropriate benchmark for comparison, since they do not factor in mortality improvements; moreover, annuitants tend to live longer than the average population. Using our cohortized annuitant mortality estimates instead, the average age of death for a male annuitant is actually around 89.3 and his most probable age of death is between 95 and 96. This suggests that he can most probably live past the "worst time to die". The seemingly late "break-even age" is also put in perspective; in fact the probability of a 55-year-old male annuitant eventually "winning the bet" (living past 93.4 years-old) is actually about 42.8%.

A corresponding analysis for a female annuitant who purchases the OAC life annuity shows similar patterns. Her "worst time to die" is about age 79.5; because of lower monthly payouts, she has to live longer ("break-even age" of 98.6) than the male annuitant before she recoups her investment in present value terms. Her probability of "winning the bet" is 47.7%. In other words, about 52% of female retirees who buy the annuity will lose on the deal. In sum, our results show that there is generally a 50–50 chance of profiting financially from the deal for a CPF member who chose to annuitize in 2007.[13] A notable exception is the NTUC Income annuity which offered retirees substantially higher payouts, as well as a higher accrued interest on the guarantee during the deferral period. These features worked in favor of annuitants resulting in a higher assurance of "winning" at 57.0% (males) and 68.7% (females). In addition, the cross-over ages were lower at 88.25 (males) and 90.83 (females), respectively.

Figure 3 shows the NPV profiles for each of the four newly launched CPF life annuities, based on their illustrative payouts as at September 2009.[14] The payout

13 This generalization is made possible because the nine annuities (provided by eight different insurers) offered largely similar specifications: a S$99,600 premium and low variation in sex-specific payouts/benefits across insurers.

14 A recent cross-check shows that illustrative payouts (based on the CPF web calculator) have not altered much since September 2009; only small decreases in the range of S$5–10 were noted.

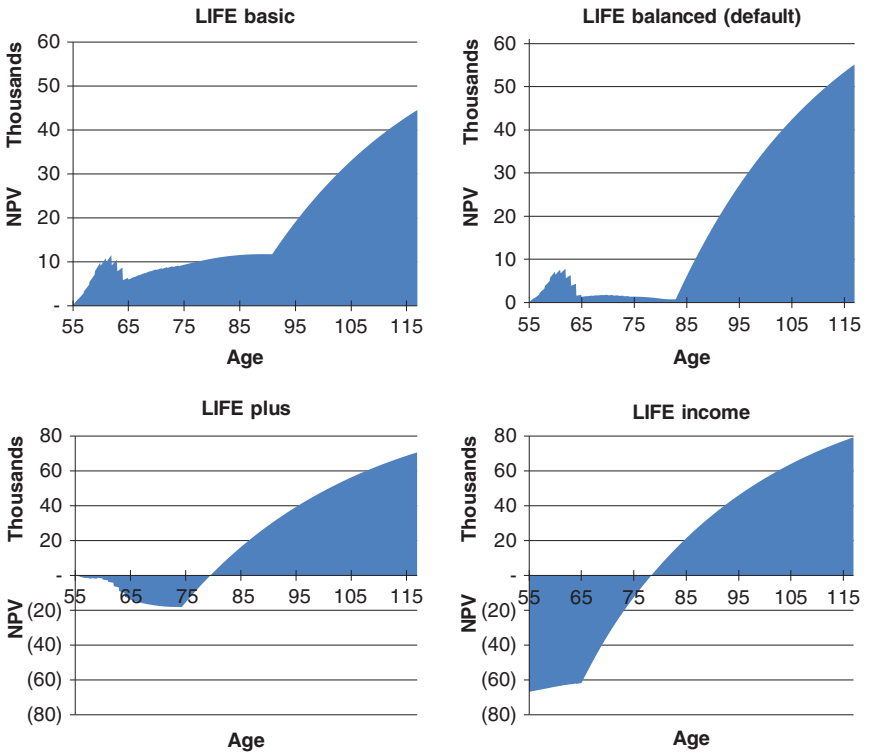

Figure 3: NPV of cumulative benefits (September 2009; CPF LIFE annuities; male)
Note: Computations pertain to a male CPF plan participant who joins the LIFE plan in September 2009. His entry age is 55; premium is S$67,000; payouts are as illustrated in Table 1 and starts at age 65.

distributions are quite varied across the four products, which may appear rather surprising especially since the monthly payouts did not differ vastly. In particular, the NPV plots for the LIFE Basic and Balanced contrast greatly with those of the LIFE Plus and Income in terms of risk and return attributes. The NPV profile of the LIFE Plus plan most resembles that of a commercial pension annuity under the CPF scheme (represented by the OAC product discussed earlier). Notably, however, the cross-over age for this government-offered product occurs much earlier at age 79.6 (compared to 93.4 for OAC). This is good news for CPF plan participants on the whole since it implies a higher probability of recouping the annuity premium while alive.

The LIFE Basic and Balanced plans have similar NPV profiles and will likely appeal to more risk-averse or conservative investors. By channeling the bulk of

the annuity premium into the term component (~91% and 70%, respectively), these plans offer relatively higher bequests since interest earned on the term component is refundable to beneficiaries upon death. This substantial death benefit protects the plans from dipping into negative NPV territory over the lifetime of the retiree. Quite strikingly, the profiles both exhibit strictly positive NPV at all ages which means that a CPF retiree who opts for either of these plans will "win" financially on the investment from day one! From this perspective, it appears that CPF Board's decision of setting LIFE Balanced as the default option under the mandatory annuitization scheme is judicious. Under the LIFE Balanced plan, a retiree will *never* experience a financial loss in NPV terms, regardless of when he dies; this insight may help mitigate potential public dissent against the annuitization mandate rolled out in 2013. Payoff prospects are also slightly better under LIFE Balanced than LIFE Basic should the annuitant live a reasonably long life, say above age 90.

Another observation is that seemingly trivial differences in monthly annuity income (on the scale of about S$50–100) become greatly magnified at advanced ages. Supposing the CPF LIFE participant lives to the terminal age of 117, she can almost double her financial gains by opting for the Income plan (NPV S$80,000) as opposed to the Basic plan (NPV S$45,000) for the same premium invested. A cross-sectional comparison at older ages shows that the LIFE Plus and Income plans offer superior cumulative discounted annuity benefits as compared to LIFE Basic and Balanced. This has important implications. While the death benefits may seem attractive if the retiree dies shortly after purchasing the annuity, they become less of a concern if the retiree is likely to live a reasonable amount of years after annuity purchase.

Based on our 2009 cohortized mortality estimates, the average age of death for a male randomly drawn from the general population is 87.2; the most probable age of death is between 93 and 94. While these estimates may seem rather high with our current understanding of mortality, this may not be the case in the next generation, after 30 years of continued improvements. Together, these mortality estimates suggest a male LIFE plan participant can likely make it past the break-even ages (approximately age 80) given in both the LIFE Plus and Income plans and could potentially take advantage of his longevity to reap large financial gains.[15] Our results suggest that by forfeiting

15 The Income plan channels 100% of the premium into the annuity component. Also, any unused balance (premium less payouts made) is non-refundable. Hence, this product offers no death benefit offering the most attractive monthly payouts as a trade-off. LIFE Plus is similar, except that any unused balance is refundable to one's estate. This explains why its NPV profile does not dip into the negative region as much as the Income plan in early years.

bequests, the annuitant can obtain a higher NPV on his investment assuming survival to older ages.

5 Conclusions

All insurance products, and annuities among them, involve an element of risk to the policyholder. The MWR, being the expected value of the discounted return per dollar invested, does not capture risk. Despite the substantial subsidy implied by annuities with high MWR values close to (or exceeding) unity, it is possible that retirees could be detracted from the purchase of an annuity by the risk of losing a substantial part of their investment in case of early death – one of the arguments often mentioned to explain the "annuity puzzle", or the empirical observation that annuity markets around the world are not very developed. In this article, we provide some insights into the imbedded risk of the pension annuities offered in Singapore. We examined the entire distribution of weighted benefits and calculated measures of dispersion such as SD, CV, and skewness.

We find that annuity products which have similar money's worth valuations can differ widely in terms of risk attributes. Specifically, the four new government-offered annuities vary widely in risk profiles despite their uniform money's worth valuations (1.24–1.33 for males and 1.26–1.36 for females). LIFE Plus, in particular, offers the highest money's worth. Yet LIFE Basic has the lowest CV for both genders and is most attractive when judged based on both risk and return characteristics. The Basic and Balanced plans exhibit rather similar NPV profiles whereby substantial death benefits protect annuitants from net losses (or negative NPV) in the early years; payout distributions are positively skewed since more annuity benefits are derived earlier in life. The riskier options, LIFE Plus and Income, promise slightly higher expected mean returns but the additional volatility in returns associated with both products results in CV values that are about double or quadruple that of the LIFE Basic. These plans may thus appeal to more risk-loving investors, or separately, long-lived annuitants who can foresee themselves living to a ripe old age to reap large financial gains. At the terminal age of 117, a male annuitant who opted for LIFE Plus or Income will achieve net gains about 1.3–1.8 times that of a LIFE Basic holder, for a given premium. Interestingly, the LIFE Income plan risk–return attributes are dominated by the Plus plan which makes it the least attractive among the two.

Importantly, dispersion measures allow researchers using the MWR valuation framework to construct an uncertainty interval around the MWR metric. For

instance, rather than just reporting that the NTUC Income annuity offered in 2007 has a MWR of 1.05, one can now specify the upper and lower bounds of the expected annuity returns within one SD from the mean (i.e. 0.85–1.25). This not only provides greater insights into those interested in annuity pricing but also helps lessen the fixation on the weighted mean value. For example, James and Song (2004) find "surprisingly high" MWR values exceeding 0.95 or even unity in various annuity markets, including Canada, Switzerland, Australia, Israel, Chile, and Singapore. Likewise, Cannon and Tonks (2002) report high MWR values of between 0.90 and 1.10 for annuities in the United Kingdom over a 30-year period. Nonetheless, when we take into account the uncertainty interval around the expected mean (MWR), it becomes easier to put in perspective the seemingly high returns on annuity products offered by profit-driven insurers.

The risk measures quantified are also useful in facilitating the comparison of risk across lines of insurance business. The CV statistic is commonly employed by insurance researchers and industry practitioners in this regard; for instance, Bühlmann and Gisler (2005) find CV values ranging from two (for household and motor hull insurance) to around nine (for motor liability and workers' compensation insurance). A study by AON (2009) reports CV values ranging between 0.33 for private passenger auto insurance and 0.81 for property insurance in Singapore.[16] Comparatively, the CV values obtained for the Singaporean life annuities (averaging 0.19 for males and 0.17 for females) provided by commercial insurers are relatively very low. This suggests that retirees who purchased one such annuity to mitigate their longevity risk own a random variable that exhibits very little risk. In sum, the guarantee feature that characterizes Singaporean pension annuities results in a superior product, eliminating the fear that annuitants end up losing most of their investment by dying early. While annuitants may end up winning or losing some money – this is the essence of insurance – these gains and losses are not very large; there is little risk involved in the purchase of an annuity.

Finally, our results show that a 55-year-old annuitant who purchased a commercial annuity in 2007 under the Singaporean pension system had an almost 50% chance of recouping his premium even though the break-even age occurs late in life. While annuitants may lose some of their investment (negative NPV in early years), this is typical of all insurance contracts. Nonetheless, the presence of the money-back guarantee buffers the loss, limiting the maximum loss to about one-third of the premium. This "worst time to die" is estimated to occur at about age 78 for a male annuitant. After that age, the net return to the annuitant progressively increases, and he can hope to break-even on his investment about

16 This study extends to 17 countries comprising over 75% of the world's global premium; it defines risk as the CV of the loss ratio of each book of business.

past age 90. While this break-even age seems high based on our current understanding of mortality, this may not be the case after decades of continued mortality improvements. Past the break-even age, the net return to the annuitant becomes positive and may reach tens of thousands of dollars for long lives.

Overall, the Singaporean pension annuities are low-risk, high-return, financial products that should be very attractive to retirees seeking longevity insurance. This is for the most part due to the guarantee offered to retirees that, in case of early death, their contributions will be returned to beneficiaries. Not only do the annuities offer superior returns (especially the new CPF LIFE plans) but the risk retained by annuitants is low. When compared to other lines of insurance products, such as auto insurance or property insurance, the coefficients of variation for the Singaporean pension annuities are at the low end. The CPF LIFE contracts appear to be uniquely generous and sufficiently diverse and should be very popular with future retirees.

In evaluating the imbedded risk involved in a life annuity investment, the popular money's worth metric is a natural starting point. Nonetheless, it is purely a financial measure of annuity value which does not account for risk aversion among utility-maximizing individuals. Specifically, life-cycle annuity purchasers may be heterogeneous in terms of their preferences and the strength of their bequest motives, which will in turn affect the way they compare and evaluate annuity options. Extending this analysis to account for the utility implications is left for future research.

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Appendix

For immediate single-life annuities, eq. [1a] can be written as (see Bowers et al. 1997)[17]:

$$\begin{aligned}\text{EPDV} &= A_a \cdot \sum_{t=0}^{\infty} {}_tp_a \cdot \left(\prod_{m=0}^{t} v_m\right) \\ &= A_a \cdot \sum_{t=0}^{\infty} {}_tp_a \cdot q_{a+t} \cdot \ddot{a}_{\overline{t+1}|} \\ &= \sum_{t=0}^{\infty} {}_{t|}q_a \cdot Y_t\end{aligned}$$

17 Bowers et al. (1997, 143) provide proof for the following mathematical equivalence: Actuarial present value of an annuity $= \sum_{t=0}^{\infty} {}_tp_a v^t = \sum_{t=0}^{\infty} \left(\ddot{a}_{\overline{t+1}|t}p_a \cdot q_{a+t}\right)$.

where $Y_t = A_a \cdot \ddot{a}_{\overline{t+1}|}$.

For immediate single-life annuities with money-back guarantees, such as the Singaporean pension annuities, eq. [1b] can be written as:

$$\begin{aligned}\text{EPDV} &= \sum_{t=0}^{\infty} {}_tp_a \cdot q_{a+t} \cdot \left(\prod_{m=1}^{t+1} v_m\right) \cdot G_{t+1} + A_a \cdot \sum_{t=\text{D}}^{\infty} {}_tp_a \cdot \left(\prod_{m=1}^{t} v_m\right) \\ &= \sum_{t=0}^{\infty} {}_tp_a \cdot q_{a+t} \cdot \left(\prod_{m=1}^{t+1} v_m\right) \cdot G_{t+1} + A_a \cdot \sum_{t=D}^{\infty} {}_tp_a \cdot q_{a+t} \cdot \left(\prod_{m=1}^{D} v_m\right) \cdot \ddot{a}_{\overline{t-D+1}|} \\ &= \sum_{t=0}^{\infty} {}_{t|}q_a \cdot Y_t\end{aligned}$$

where

$$Y_t = \begin{cases} \left(\prod_{m=1}^{t+1} v_m\right) \cdot G_{t+1}, & \text{for } t < D \\ \left(\prod_{m=1}^{t+1} v_m\right) \cdot G_{t+1} + \left(\prod_{m=1}^{D} v_m\right) \cdot A_a \cdot \ddot{a}_{\overline{t-D+1}|}, & \text{for } t \geq D. \end{cases}$$

Definitions

- a is the age at which the annuity is purchased;
- t is the number of months beyond the annuity start date;
- D is the deferred period in months;
- ${}_tp_a$ is the probability that an individual of age a survives after t months;
- q_{a+t} is the monthly probability to die between month $a+t$ and $a+t+1$;
- v_m is the discount factor for month m based on a riskless term structure;
- G_{t+1} is the death benefit paid:
  $G_{t+1} = \max\{0, \text{premium} + \text{accrued interest} - \text{total annuity payouts received prior to time t+1}\}$,
- A_a is the monthly level annuity payout for the individual purchasing the annuity at age a,
- $\ddot{a}_{\overline{n}|}$ is the present value of an n-payment annuity-due.

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