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Collusion Under Monitoring of Sales

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Comments

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Collusion under Monitoring of Sales*

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Abstract

Collusion under imperfect monitoring is explored when firms' prices are private information and their quantities are public information; an information structure consistent with several recent price-fixing cartels such as those in lysine and vitamins. For a class of symmetric duopoly games, it is shown that symmetric equilibrium punishments cannot sustain any collusion. An asymmetric punishment is characterized which does sustain collusion and it has the firm with sales exceeding its quota compensating the firm with sales below its quota. In practice, cartels have performed such transfers through sales among the cartel members.

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... if I'm assured that I'm gonna get 67,000 tons [of lysine sales] by the year's end, we're gonna sell it at the prices we agreed to and I frankly don't care what you sell it for. [Terrance Wilson of Archer Daniels Midland from the March 10, 1994 meeting of the lysine cartel.]

And that total for us for the year, calendar year is 68,000; 68,334. 68,334 and our target was 67,000 plus alpha. Almost on target. [Mark Whitacre of Archer Daniels Midland from the January 18, 1995 meeting of the lysine cartel.][1]

1 Introduction

Many if not most price-fixing cartels involve firms selling to industrial buyers, with the lysine cartel being a notable example. As price can be settled through private negotiation, it is not typically observable. In such cases, compliance with the collusive agreement is often based on firms' sales. Indeed, cartels can go to great lengths to ensure that sales are public information among the cartel members. In the citric acid cartel, for example, firms hired an international accounting firm to independently audit sales reports (Connor, 2001). The objective of this paper is to explore collusion in an imperfect monitoring setting in which prices are private information and firms' quantities are public information.

In spite of such a monitoring environment being applicable to many market settings, there is relatively little work with such a structure even though, interestingly enough, it was the one described by Stigler (1964) when he originally raised the issue of imperfect monitoring. There are, of course, many papers using the classical monitoring setting of Green and Porter (1984) in which firms' quantities are private information and the market price is publicly observed. In the context of repeated auctions, Blume and Heidhues (2003) and Skrzypacz and Hopenhayn (2004) assume price is private information, while who won the auction is known. But the assumption of one unit per period makes the model unsuitable for many markets and, pertinent to the issue at hand, constrains the monitoring of collusion through sales (a point we elaborate upon later). Tirole (1988) and Bagwell and Wolinsky (2002) allow for multi-unit demand in the context of Bertrand price model. But assuming firm demand is discontinuous is obviously an extreme assumption and, furthermore, plays an important role in sustaining collusion. Our model is the first to consider collusion when prices are private information and monitoring occurs with respect to sales, while making standard and fairly general demand assumptions: demand is multi-unit and expected firm demand is everywhere continuous.

Our first main finding is a surprising impossibility result. For a general class of symmetric demand structures with inelastic market demand, no collusion can be sustained by symmetric equilibria (in the sense of strongly symmetric perfect public equilibria; see Section 3 for detailed definitions). By way of example, one such demand structure is when the probability distribution of demand depends only on the difference in firms' prices, as is true with the discrete choice model. The rough

[1] These quotes are from the video transcript of "The International Lysine Cartel at Work, 3/28/00" provided by the U.S. Department of Justice, Antitrust Division.

intuition for our result is as follows. One would expect punishment to occur when market shares are sufficiently skewed. Suppose, for example, punishment occurs when market share of one of the firms exceeds 70%. A firm that considers charging a price below the collusive price raises the probability that its market share exceeds 70% - which makes punishment more likely - but lowers the probability that the other firm's market share exceeds 70% - which makes punishment less likely. What we show is that for small price cuts, these two effects exactly offset each other which implies that a firm's continuation payoff is unaffected by its price. Therefore, an equilibrium price for the infinite horizon game must be the same as that for the stage game. Though shown for the extreme case of fixed market demand, robustness prevails when market demand is stochastic and sensitive to firms' prices. Specifically, if market demand is very insensitive to firms' prices then collusive prices are very close to non-collusive prices.

The conclusion we draw from this result is not that firms cannot collude but rather of the importance of treating apparent deviators differently from apparent non-deviators. The second main result is showing that collusion can be sustained with asymmetric punishments involving transfers in which the firm having sold too much compensates the other firm. In fact, some price-fixing cartels, such as those in citric acid (Arbault, 2002) and sodium gluconate (European Commission, 2002), did indeed deploy asymmetric punishments through the use of inter-firm sales which can act as transfers. The main message of this paper is that if we are to understand the actual practices of some cartels, it is essential that we take account of imperfect monitoring with respect to prices and the role of asymmetric punishments which condition on sales.

2 Model

Consider an infinitely repeated duopoly game in which firms make simultaneous price decisions. Cost functions are common and linear and, without loss of generality, cost is zero. Demand is fixed at m units and discrete.[2] We often refer to it as having m customers (with unit demands). Though total demand is fixed, firm demand is stochastic. Let

$$\phi : \{0, 1, \ldots, m\} \times \Re^2 \rightarrow [0, 1]$$

be the probability function on firm 1's demand. $\phi(b; p_1, p_2)$ is the probability that firm 1 sells b units given its price is p_1 and its rival's price is p_2, where $p_1, p_2 \in \Re$. As total demand is fixed at m units, $\phi(m - b; p_1, p_2)$ is the probability that firm 2's demand is b. One can either imagine that products are differentiated or that they are homogeneous but buyer-specific shocks, which may be independent or correlated, influence their demand in each period. We describe some examples below.

We make three assumptions on the probability distribution on firm demand.

A1 ϕ is continuously differentiable with respect to p_1 and p_2.

[2] See Section 4.2 for a generalization to when m is variable.

A2 $\phi(b; p', p'') = \phi(m-b; p'', p') \; \forall b \in \{0, 1, \ldots, m\}, \forall (p', p'') \in \Re^2$.

A3 $\frac{\partial \phi(b;p,p)}{\partial p_1} + \frac{\partial \phi(b;p,p)}{\partial p_2} = 0 \; \forall b \in \{0, 1, \ldots, m\}, \forall p \in \Re$.

A1 is standard and A2 imposes symmetry. A3 is the key restriction though is satisfied in many models. A3 implies that if we start at equal prices then the distribution of demand remains unchanged if firms make small identical price changes. It holds, for example, when ϕ depends only on the price difference, $p_1 - p_2$. Suppose $\exists \xi : \{0, 1, \ldots, m\} \times \Re \to [0, 1]$ such that

$$\phi(b; p_1, p_2) = \xi(b; \Delta) \, \forall b \in \{0, 1, \ldots, m\}, \forall (p_1, p_2) \in \Re^2,$$

where $\Delta \equiv p_1 - p_2$. Then

$$\frac{\partial \phi(b; p, p)}{\partial p_1} + \frac{\partial \phi(b; p, p)}{\partial p_2} = \frac{\partial \xi(b; 0)}{\partial \Delta} - \frac{\partial \xi(b; 0)}{\partial \Delta} = 0,$$

so that A3 holds.

An example from the literature that conforms to this specification is the following m-buyer generalization of Cabral and Riordan (1994). Let the probability that firm 1 sells to a particular buyer equal $F(p_2 - p_1)$ where $F : \Re \to [0, 1]$ is continuously differentiable and non-decreasing and F' is symmetric around zero. Assume also that buyers' decisions as to whom to buy from are *iid*. That implies that a firm's demand is binomially distributed,

$$\phi(b; p_1, p_2) = \frac{m!}{b!(m-b)!} F(p_2 - p_1)^b (1 - F(p_2 - p_1))^{m-b},$$

and thus depends only on the price difference. A discrete choice model in which consumer indirect utility is linear in price will also work. In that case, the utility to consumer j from buying the product of firm i is $U_i^j - p_i$ so that firm 1's product is bought iff:

$$U_1^j - p_1 > U_2^j - p_2 \Leftrightarrow U_1^j - U_2^j > p_1 - p_2.$$

More generally, note that we can represent $\phi(b; p_1, p_2)$ by $\widehat{\phi}\left(b; f^b(p_1, p_2)\right) : \{0, 1, \ldots, m\} \times \Re \to [0, 1]$, where $f^b(\cdot)$ is allowed to vary with b. It follows that A3 holds when

$$\frac{\partial f^b(p, p)}{\partial p_1} + \frac{\partial f^b(p, p)}{\partial p_2} = 0 \; \forall b,$$

and, furthermore, for any smooth transformation $g\left(f^b(\cdot)\right)$ or $f^b(g(p_1), g(p_2))$. For example, start with $f^b(p_1, p_2) = p_1 - p_2$ and use the transformation: $g(p) = \ln(p)$. We then have $f^b(p_1, p_2) = \ln(p_1) - \ln(p_2) = \ln(p_1/p_2)$. Performing another transformation using $g(p) = \exp f^b$ gives us $f^b(p_1, p_2) = p_1/p_2$. Thus, if the probability distribution depends only on the ratio of prices then it satisfies A3.

Note that our assumptions thus far do not require that demand be decreasing in price. In our stochastic formulation of demand, the natural way in which to encompass that property is to assume that a lower price implies a first-order stochastic dominance shift in a firm's probability distribution over its demand. A4 will only be needed for some results.

A4 $\sum_{b=0}^{k} \frac{\partial \phi(b;p_1,p_2)}{\partial p_1} > 0 \ \forall k \in \{0,1,\ldots,m-1\}$.

There is an infinite horizon and each firm's payoff is the expected present value of its profit stream where the common discount factor is $\delta \in (0,1)$. The information structure is one of imperfect monitoring as firms' price decisions are private information. This conforms to the industrial buyer case in which price is negotiated between a seller and a buyer and thus is not publicly posted.[3] Given that it is common knowledge that market demand is fixed and each firm observes its demand then realized quantities are common knowledge. It is sufficient to think of a public history at the start of period t, denoted h^{t-1}, to be a sequence of quantities sold by firm 1. Denote by H^{t-1} the set of all possible histories h^{t-1}. A firm's (public) strategy is then an infinite sequence of price functions, $\{\rho_i^t(\cdot)\}_{t=1}^{\infty}$, where $\rho_i^t : H^{t-1} \rightarrow \Re$. We restrict attention to perfect public equilibria so that firms do not condition their prices on their own past prices, just on the realized quantities.[4] One final assumption is that first-order conditions are sufficient for defining an equilibrium.

The imperfect monitoring structure we consider obviously differs from the classical formulation of Green and Porter (1984) in which firms' quantities are private information and the market price is publicly observed. Assuming firms' prices are private information and monitoring is based on sales appears to conform better with many price-fixing cases. There is a limited amount of work which considers monitoring with respect to sales when prices are private information. Blume and Heidhues (2003) and Skrzypacz and Hopenhayn (2004) consider collusion in repeated single-unit auctions. The limitation to one unit per period is restrictive and, as a result, their models are not applicable to many markets. Tirole (1988) and Bagwell and Wolinsky (2002) consider the Bertrand price model with uncertain aggregate demand in which firms' prices and quantities are private information.[5] The standard Bertrand assumption of infinite elasticity of firm demand is clearly an extreme (though common) assumption, especially as even arbitrarily small deviations lead to a discontinuous change in the distribution of the monitoring variable. Our model is then unique in the imperfect monitoring literature in allowing for the following compelling features: price is private information, monitoring occurs with respect to sales, multi-unit demand, and expected firm demand is everywhere continuous. Though we do assume total market demand is fixed, robustness is established with respect to that assumption. All of these features - including highly inelastic market demand - fit well with many markets including lysine and vitamins.

There is one assumption that warrants discussion before moving on. Though buyers are discrete, we restrict a firm to charging the same price to all buyers. When it comes to deviating from a collusive arrangement, a firm may want to undercut its competitor's price on only a subset of consumers so as to make detection less likely. (Of course, to a limited extent, it can do that by not undercutting its competitor's price as much.) The first point to make is that our impossibility result (Theorem 1)

[3] Though list prices may be posted, they are often unrelated to transaction prices.

[4] For equilibria in pure strategies focusing on public strategies is without loss of generality.

[5] In our setting, firms' quantities can be private or public information. Since total demand is fixed, a firm's knowledge of its own quantity reveals the quantity of its rival.

is robust to this generalization because it shows that even if a deviator is constrained to charging the same price for all consumers, collusion is unsustainable. Where this restriction may be a concern is with regard to the result that collusion is sustainable with asymmetric punishments. Though we conjecture the sustainability of collusion is robust to non-uniform pricing, we do not have a proof at this time so let us instead offer a motivation for the assumption of uniform pricing. In most price-fixing cartels, collusion is among high-level managers rather than sales representatives (that is, those who actually deal with customers). In that it might be difficult or even suspicious for those managers to communicate different prices to different sales people (or different prices to the same sales person), colluding managers may feel constrained to charging a common price to all buyers.

3 An Impossibility Result

With single-unit demand per period ($m = 1$), symmetric equilibria[6] are trivially ineffective at supporting collusion if the players only observe sales and not actual prices.[7] The reason is prosaic: regardless of firms' prices, the customer buys from one of the sellers and this means that continuation play has to treat symmetrically the "winner" and the "loser." After either outcome we would have to end up in a punishment or non-punishment regime and hence there can be no symmetric punishment for secret price cutting.

However, with more than one customer per period, one might expect to be able to sustain collusion even with symmetric punishments. Considering the case of two customers, two natural outcomes emerge: the sellers split the market or one of the sellers serves the whole market. If the collusive scheme recommends that they set a common high price, then a market split would seem less likely if one of the players deviates by charging a lower price. If so, then a punishment can be conditioned on market shares being skewed. This intuition is confirmed if we model the market as a continuum of independent customers as, by the law of large numbers, demand is then non-stochastic which means deviations can be detected precisely. However, as we show in this section, that intuition is not correct in a large class of markets. If there are a finite number of customers then no symmetric equilibrium can achieve prices above the competitive level.

In exploring collusion in a symmetric setting, it is common (and one might suppose natural) to first consider equilibria that take full advantage of this symmetry. For a particular strategy profile, let

$$v_i^t(\cdot) : \{0, 1, \ldots, m\}^{t-1} \rightarrow \Re$$

denote the continuation payoff starting at t as a function of the public history (recall that we use as the history the sequence of quantities sold by firm 1). A set of *symmetric histories* consists of the initial null history and if m is even also of histories

[6] In the sense of strongly symmetric equilibria, as we define below.

[7] This was first noted in Blume and Heidhues (2003) and Skrzypacz and Hopenhayn (2004) who explore collusion in repeated auctions. Their work will be discussed later.

in which each firm had demand of $m/2$ in every period. A *symmetric Nash equilibrium* is a Nash equilibrium in which the strategy profile calls for identical prices when the history is symmetric. This implies that continuation payoffs are identical after such histories.

A more restrictive but commonly imposed property is that of strong symmetry.[8] A *strongly symmetric Nash equilibrium* is a Nash equilibrium in which strategies are symmetric for all histories. That implies the continuation payoffs are also symmetric after all histories:

$$v_1^t\left(h^{t-1}\right) = v_2^t\left(h^{t-1}\right)\ \ \forall h^{t-1} \in H^{t-1},\ \forall t.$$

Let this common continuation payoff be denoted $v^t(\cdot)$.

Our first main finding is an impossibility result. Under strong symmetry collusion is not sustainable *regardless* of the discount factor.

Theorem 1 *Assuming A1-A3, the set of strongly symmetric Nash equilibrium outcomes for the infinite horizon game coincides with the set of symmetric Nash equilibrium outcomes for the stage game.*

Proof. Consider a strongly symmetric Nash equilibrium which, given the current history, calls on both firms to charge prices (p^*, p^*) and gives both firms a continuation payoff of $v(b) \equiv v^{t+1}\left(h^{t-1}, b\right)$ if the current period demand for firm 1 is b. Firm 1's expected payoff from pricing at p_1^t is then

$$\sum_{b=0}^{m} \phi\left(b; p_1^t, p^*\right) \left[p_1^t b + \delta v(b)\right].$$

By A1, a necessary condition for p^* to be an equilibrium price is:

$$\sum_{b=0}^{m} \left(\frac{\partial \phi(b; p^*, p^*)}{\partial p_1^t}\right) \left[p^* b + \delta v(b)\right] + \sum_{b=0}^{m} \phi(b; p^*, p^*)\, b = 0, \tag{1}$$

which we will rearrange to

$$\sum_{b=0}^{m} \left(\frac{\partial \phi(b; p^*, p^*)}{\partial p_1^t}\right) p^* b + \sum_{b=0}^{m} \phi(b; p^*, p^*)\, b + \sum_{b=0}^{m} \left(\frac{\partial \phi(b; p^*, p^*)}{\partial p_1^t}\right) \delta v(b) = 0. \tag{2}$$

Our method of proof is to show that the third term is zero for if that is the case then p^* must satisfy

$$\sum_{b=0}^{m} \left(\frac{\partial \phi(b; p^*, p^*)}{\partial p_1^t}\right) p^* b + \sum_{b=0}^{m} \phi(b; p^*, p^*)\, b = 0 \tag{3}$$

[8] This is assumed, for example, in Abreu (1986).

which is the condition defining a symmetric Nash equilibrium for the stage game.[9] We then want to show that

$$\sum_{b=0}^{m}\left(\frac{\partial\phi\left(b;p,p\right)}{\partial p_1}\right)v\left(b\right)=0, \tag{4}$$

where we've dropped some extraneous notation.

Note that an implication of A2 is:

$$\frac{\partial\phi\left(b;p,p\right)}{\partial p_2}=\frac{\partial\phi\left(m-b;p,p\right)}{\partial p_1}. \tag{5}$$

It follows from A3 that

$$\frac{\partial\phi\left(b;p,p\right)}{\partial p_2}=-\frac{\partial\phi\left(b;p,p\right)}{\partial p_1}. \tag{6}$$

Substituting (6) into (5) yields $\forall b$

$$\frac{\partial\phi\left(b;p,p\right)}{\partial p_1}+\frac{\partial\phi\left(m-b;p,p\right)}{\partial p_1}=0. \tag{7}$$

If m is even, it implies $\frac{\partial\phi\left(\frac{m}{2};p,p\right)}{\partial p_1}=0$. Condition (7) is the core of the proof and states that a small price cut increases the probability of a high market share, $b\left(>\frac{m}{2}\right)$, by the same amount that it decreases the probability of a low market share, $m-b$.

From the preceding steps, when m is even, (4) can be presented using (7) as

$$\sum_{b=0}^{\frac{m}{2}-1}\left(\frac{\partial\phi\left(b;p,p\right)}{\partial p_1}\right)\left[v\left(b\right)-v\left(m-b\right)\right]+\underbrace{\frac{\partial\phi\left(\frac{m}{2};p,p\right)}{\partial p_1}}_{0}v\left(\frac{m}{2}\right)=0.$$

Analogously, one can derive when m is odd,

$$\sum_{b=0}^{\frac{m-1}{2}}\left(\frac{\partial\phi\left(b;p,p\right)}{\partial p_1}\right)\left[v\left(b\right)-v\left(m-b\right)\right]=0.$$

A sufficient condition for our claim is therefore $v\left(b\right)=v\left(m-b\right)$ for all histories, which is natural to expect in a strongly symmetric equilibrium (and, in addition, it always holds when $m=2$). But that condition on the continuation payoffs is not necessary. Consider the first-order condition for firm 2,

$$\sum_{b=0}^{m}\left(\frac{\partial\phi\left(m-b;p^*,p^*\right)}{\partial p_2^t}\right)\left[p^*b+\delta v\left(m-b\right)\right]+\sum_{b=0}^{m}\phi\left(m-b;p^*,p^*\right)b=0, \tag{8}$$

[9] Let us remind the reader that we are assuming the first-order condition is both necessary and sufficient for equilibrium. If it is not sufficient then Theorem 1 as stated may not be true. Though the first-order conditions for the stage game and the infinitely repeated game still coincide, the second-order conditions need not. What is true, however, is that the set of strongly symmetric Nash equilibrium prices for the infinitely repeated game is a subset of the set of solutions to the first-order condition for the stage game.

and subtract (8) from (1) to obtain:

$$\begin{aligned}&\sum_{b=0}^{m}\left(\frac{\partial\phi\left(b;p,p\right)}{\partial p_1}-\frac{\partial\phi\left(m-b;p,p\right)}{\partial p_2}\right)pb\\&+\delta\sum_{b=0}^{m}\left[\left(\frac{\partial\phi\left(b;p,p\right)}{\partial p_1}\right)v\left(b\right)-\left(\frac{\partial\phi\left(m-b;p,p\right)}{\partial p_2}\right)v\left(m-b\right)\right]\\&+\textstyle\sum_{b=0}^{m}\left[\phi\left(b;p,p\right)-\phi\left(m-b;p,p\right)\right]b\quad=0\end{aligned}$$

The first summation is zero by (5) and the third summation is zero by A2 and that the probabilities add up to 1. Using (5) in the second summation, we derive:

$$\sum_{b=0}^{m}\frac{\partial\phi\left(b;p,p\right)}{\partial p_1}\left[v\left(b\right)-v\left(m-b\right)\right]=0$$

This can be used to complete the proof; for example, if m is odd, using again (7) it can be re-written as:

$$\sum_{b=0}^{\frac{m-1}{2}}\frac{\partial\phi\left(b;p,p\right)}{\partial p_1}\left[v\left(b\right)-v\left(m-b\right)\right]=0.$$

which establishes the claim. ■

In thinking about punishment for perceived non-compliance in this setting, one would expect it to occur when market shares are sufficiently skewed; either firm 1's sales are too high or too low. The former is consistent with firm 1 having undercut the collusive price and the latter with firm 2 having done so. Strong symmetry implies that the punishment entails identical behavior in the form of a price war. In such a situation, Theorem 1 shows that no collusion can be sustained.[10]

This result hinges on the fact that when firm 1 sets a price marginally below the collusive price, it reduces the likelihood of having a low demand (say, $b' < m/2$) and, at the same time, raises the probability of having a high demand (say, $m - b'$). The proof shows that the ensuing reduction in the probability of b' is exactly equal to the rise in the probability of $m - b'$ so that the probability of b' or $m - b'$ remains constant for a marginal change in price - see condition (7). This is true for all b'. Now suppose $v\left(b'\right) = v\left(m - b'\right) \forall b'$ so that the continuation payoffs depend only on the distribution of market share. It follows that the probability distribution over the continuation payoff is then unaffected by firm 1's price. Hence, if p^* is to be an equilibrium price, it must maximize expected current profit since, at the margin, price has no effect on the expected continuation payoff. This implies the equilibrium price must be the same as that for a Nash equilibrium for the stage game. Though strongly symmetric equilibrium does not necessarily imply $v\left(b\right) = v\left(m - b\right)$, it does

[10] As the proof of Theorem 1 only used the symmetry of the continuation payoffs and not their level, results would not change if we allowed for "money burning" activities that arbitrarily lowered $v\left(\cdot\right)$.

imply this argument holds on average since (when m is odd):

$$\sum_{b=0}^{\frac{m-1}{2}} \frac{\partial \phi\left(b; p, p\right)}{\partial p_1} v\left(b\right) = \sum_{b=0}^{\frac{m-1}{2}} \frac{\partial \phi\left(m-b; p, p\right)}{\partial p_1} v\left(m-b\right),$$

and thus the marginal effect of price on the expected continuation payoff is zero.

A similar impossibility result can be obtained in environments such as are modeled in Blume and Heidhues (2003) and Skrzypacz and Hopenhayn (2004).[11] These papers consider tacit collusion in auctions, where the bidders submit bids and the auctioneer chooses the best bid, announcing the winner but not the bids. One can think about those auctions as having one customer per period that performs closed-door price negotiations with the two potential sellers. In such a model, strongly symmetric equilibria also cannot support any collusion. The reason is more prosaic than in our model; at any point of the game, there are only two possible outcomes: firm 1 sells or firm 2 sells. That makes it impossible to detect a deviation if firms follow the same pricing strategy. In our model, however, asymmetric market shares can be used to detect deviations; it is just that the tests are too weak for small deviations. We will elaborate on this point later.

It is also interesting to ask why symmetric equilibria can be used to sustain collusion in Green and Porter (1984) but not in our model. As we show in the next section, it is not per se the difference in strategic variables (quantity versus price). It is instead the quality of information contained in the market signals. In Green and Porter (1984) a deviation has a first-order effect on the probability of punishment. In our model, due to A1 and A3, a deviation to a lower price has no first-order effect on the probability of going to a punishment. That intuition becomes clearer in the next section as we present an example which violates A1 and A3.

4 Robustness of the Impossibility Result

Here we explore the robustness of this impossibility result. In Section 4.1, departures from assumptions A1-A3 are considered, while market demand is allowed to respond to firms' prices in Section 4.2.

4.1 Non-Differentiability of Firm Demand

The proof of Theorem 1 is based on the property that the probability of a particular distribution of market shares is locally independent of a firm's price when firms charge a common price. Thus, skewness in market share is not made more likely when a firm undercuts the collusive price. One might imagine it is essential that ϕ is continuously differentiable at $p_1 = p_2$ so that small price changes have small effects on the probability distribution over sales. That, however, proves not to be the case.[12]

[11] In Blume and Heidhues (2003) and Skrzypacz and Hopenhayn (2004), the bidders have private shocks that affect the efficient allocation of the object. This feature is not shared by our model, but it does not affect the result.

[12] We thank Phil Reny for conjecturing that differentiability is unnecessary.

Here, we present a simple model which assumes ϕ is discontinuous at the point where firms' have identical prices and show that collusion through symmetric punishments still need not be sustainable; we also show when it is sustainable.

Consider the following modest modification of a discrete version of the standard Bertrand price game with homogeneous goods. (By discrete, we mean that we are retaining our assumption of m units.) Assume that when $p_1 \neq p_2$, all buyers go to the firm with the lower price for sure. When $p_1 = p_2$, the probability distribution on firm demand is symmetric and let $q \in (0,1)$ denote the probability that both firms have positive demand; that is, $b \in \{1, \ldots, m-1\}$.

This model is related to that in Tirole (1988) and Bagwell and Wolinsky (2002) in their specification of discontinuous expected residual demand when firms' prices are identical. Tirole (1988) assumes prices and quantities are private information. With homogeneous goods, market demand is stochastic and takes two possible states: it is positive (and non-increasing in price) or it is zero for all prices. The inference problem is that if a firm has zero sales, it isn't sure whether market demand is low (that is, zero) or market demand is positive and its rival cheated. Collusion is shown to be sustainable if the discount factor is sufficiently high. Whereas Tirole (1988) and Bagwell and Wolinsky (2002) allow market demand to be stochastic and firm demand to be deterministic (conditional on market demand), here we fix market demand and allow firm demand to be stochastic. This distinction proves unimportant as results are qualitatively similar.

Consider a strongly symmetric strategy profile that has both firms price at the collusive price p in period 1 and do so in period t as long as both firms' sales have always been positive: $b^\tau \in \{1, \ldots, m-1\}\ \forall \tau < t$. Otherwise, firms go to the static Nash equilibrium price of zero forever. Denoting the (rescaled) collusive payoff to be V, it is defined recursively by:

$$V = (1-\delta)(m/2)\, p + \delta q V.$$

From this we get:

$$V = \left(\frac{1-\delta}{1-\delta q}\right) p\,(m/2)$$

The equilibrium condition is

$$V \geq (1-\delta)\, mp \Leftrightarrow \left(\frac{1-\delta}{1-\delta q}\right) p\,(m/2) \geq (1-\delta)\, mp \Leftrightarrow q \geq \frac{1}{2\delta}.$$

Thus, if $q \geq \frac{1}{2\delta}$ then any collusive price (up to the maximum price that consumers are willing to pay) can be sustained by this strategy profile. If $q < \frac{1}{2\delta}$ then only the static Nash equilibrium price is sustainable: since the strategy profile contains the worst equilibrium punishment.[13] Hence, regardless of the discount factor, collusion is not sustainable using symmetric punishments when $q < \frac{1}{2}$.

[13] It can be verified that because the stage-game Nash equilibrium produces the minimax payoff and that only totally skewed market shares are possible when there is a deviation, our condition is both necessary and sufficient for collusion.

The key issue here is whether firms can statistically detect deviations in the sense that the distribution of market shares under deviation and no deviation are different. Under the assumptions of Theorem 1, the likelihood of skewness in market share is unaffected when a firm marginally undercuts the collusive price. Thus, no statistical detection is possible. For the example of this section, this property doesn't hold as the probability of a maximally skewed market share is one when a firm deviates and $1-q$ when it doesn't. But that isn't sufficient for collusion. Though statistical detection follows from $q>0$, collusion is sustainable only when $q \geq \frac{1}{2\delta}$. The reason is that the probability of a false positive (that is, going to a punishment even though no firm deviated) is $1-q$ and if is too high then the continuation payoff from colluding is too small which makes it hard to provide incentives. In order for collusion to be sustainable, the statistical test must be sufficiently precise so that punishment is sufficiently less likely when a firm colludes than when it cheats.

To see this more clearly, add some more structure by supposing that, when firms charge equal prices, each buyer randomly chooses between the two firms and their decisions are *iid*. It follows that

$$q = 1 - \left(\frac{1}{2}\right)^{m-1}.$$

Since then $q \to 1$ as $m \to \infty$, collusion can be sustained with *iid* buyers as long as there are sufficiently many of them and $\delta > \frac{1}{2}$. The probability of a false positive is $\left(\frac{1}{2}\right)^{m-1}$ so the statistical test is very precise when there are many buyers. This reduces the likelihood of wrongly triggering a punishment which enhances the collusive payoff. On the other hand, with $m=2$ collusion is not sustainable for any discount factor.[14]

4.2 Elastic Market Demand

Theorem 1 was proven under the extreme assumption that market demand is fixed and insensitive to firms' prices. Robustness is established by showing that very little collusion can be sustained when market demand is very inelastic.

Assume there is an upper bound on market demand of M units. A stochastic realization is comprised of total demand and an allocation of that demand, which can be represented as an element of

$$\Omega \equiv \{(m,b) : m \in \{0,1,...,M\}, b \in \{0,1,...,m\}\};$$

m is total sales and b is firm 1's sales. Letting $\xi : \Omega \times \Re^2 \to [0,1]$ denote the probability function on (m,b) given prices, the expected continuation payoff is

$$\sum_{m=0}^{M} \sum_{b=0}^{m} \xi\left(m, b | p_1, p_2\right) V\left(m, b\right),$$

[14] In the Appendix we also present a modified version of the Hotelling model which makes similar points to those made in this sub-section. It entails a smooth expected demand function but where the probability distribution on firm demand has a point of non-differentiability (though is continuous everywhere). Symmetric punishments are still not capable of sustaining collusion when the kink is sufficiently small but collusion can be sustained when the kink is sufficiently large.

where $V : \Omega \to \Re$. Defining $\rho(m|p_1, p_2)$ as the marginal probability function on m and $\phi(b|p_1, p_2, m)$ as the conditional probability function on b then

$$\xi(m, b|p_1, p_2) = \rho(m|p_1, p_2)\,\phi(b|p_1, p_2, m).$$

The expected continuation payoff can then be represented by

$$\sum_{m=0}^{M} \rho(m|p_1, p_2) \sum_{b=0}^{m} \phi(b|p_1, p_2, m)\, V(m, b).$$

Assume $\rho(\cdot|p_1, p_2)$ is differentiable in (p_1, p_2) and is symmetric with respect to the firms in that $\frac{\partial \rho(m|p,p)}{\partial p_1} = \frac{\partial \rho(m|p,p)}{\partial p_2} \forall p$. Finally, assume $\phi(\cdot|p_1, p_2, m)$ satisfies A1-A3, $\forall m \in \{1, ..., M\}$.

The maximization problem of firm 1 is:

$$\max_{p_1} \pi_1(p_1, p_2) + \delta \sum_{m=0}^{M} \left(\rho(m|p_1, p_2) \sum_{b=0}^{m} \left(\phi(b|p_1, p_2, m)\, V(m, b)\right) \right)$$

where

$$\pi_1(p_1, p_2) \equiv \sum_{m=0}^{M} \rho(m|p_1, p_2) \sum_{b=0}^{m} \phi(b|p_1, p_2, m)\, p_1 b.$$

The necessary first-order condition at a strongly symmetric Nash equilibrium is then

$$\begin{aligned} 0 \quad = \quad & \frac{\partial \pi_1(p,p)}{\partial p_1} + \delta \sum_{m=0}^{M} \left(\frac{\partial \rho(m|p,p)}{\partial p_1} \sum_{b=0}^{m} \phi(b|p,p,m)\, V(m,b) \right) \\ & + \delta \sum_{m=0}^{M} \left(\rho(m|p,p) \sum_{b=0}^{m} \frac{\partial \phi(b|p,p,m)}{\partial p_1} V(m,b) \right). \end{aligned} \tag{9}$$

By the method used in the proof of Theorem 1, it follows that:

$$\sum_{m=0}^{M} \left(\rho(m|p,p) \sum_{b=0}^{m} \frac{\partial \phi(b|p,p,m)}{\partial p_1} V(m,b) \right) = 0.$$

To elaborate on this point, note that this expression is equal to*:*

$$\sum_{m=0}^{M} \left(\rho(m|p,p) \sum_{b=0}^{k} \frac{\partial \phi(b|p,p,m)}{\partial p_1} \left[V(m,b) - V(m, m-b)\right] \right). \tag{10}$$

(where $k = m/2$ for m even and $(m-1)/2$ for m odd). Using the assumption that $\frac{\partial \rho(m|p,p)}{\partial p_1} = \frac{\partial \rho(m|p,p)}{\partial p_2}$, (10) can be shown to be zero by subtracting the first-order conditions for the two firms (at equal prices).[15] Thus, (9) becomes

$$\frac{\partial \pi_1(p,p)}{\partial p_1} + \delta \sum_{m=0}^{M} \left(\frac{\partial \rho(m|p,p)}{\partial p_1} \sum_{b=0}^{m} \phi(b|p,p,m)\, V(m,b) \right) = 0. \tag{11}$$

[15] An alternative to assuming $\frac{\partial \rho(m|p,p)}{\partial p_1} = \frac{\partial \rho(m|p,p)}{\partial p_2}$ is to suppose $V(m,b) = V(m, m-b)$ for every history.

We conclude that a necessary condition to sustain collusion is that the second term in (11) is non-zero.

First note that if $\frac{\partial\rho(m|p,p)}{\partial p_1} = 0$, so that the distribution on market demand is independent of prices, (11) then becomes the condition for a stage game equilibrium. Hence, collusion cannot be sustained as long as market demand is insensitive to prices, regardless of whether or not it is stochastic. When $\frac{\partial\rho(m|p,p)}{\partial p_1}$ is close to zero then the set of values of p that satisfy (11) are, generically, close to the set of stage game symmetric equilibrium prices. We conclude that the collusive price is close to a stage game equilibrium price when market demand is sufficiently insensitive to firms' prices. In that sense, Theorem 1 is robust with respect to market demand being variable and sensitive to firms' prices.

An assumption of highly inelastic market demand is plausible for many of the price-fixing cartels mentioned including those that arose in the markets for vitamins, lysine, and citric acid. These products are largely being purchased by other firms as inputs; for example, vitamins and lysine are mixed with animal feed in the food processing industry. As they make up a very small fraction of the unit cost of these purchasers, their demand is likely to be insensitive for a wide range of prices. Of course, the cartel members could set price high enough so as to induce a non-negligible fall in market demand but the size of the price increase required to make that happen may be of such magnitude as to create suspicions among buyers that the input suppliers are colluding. As a result, the cartel would want to avoid such large price increases.[16] This may argue to the assumption that, over the *relevant* range of prices, market demand is highly inelastic.

Finally, note that if we define

$$V(m) \equiv \sum_{b=0}^{m} \phi(b|p,p,m) V(m,b),$$

then (11) can be rewritten as

$$\frac{\partial \pi_1(p,p)}{\partial p_1} + \delta \sum_{m=0}^{M} \frac{\partial \rho(m|p,p)}{\partial p_1} V(m) = 0.$$

$V(m)$ is the expected continuation payoff conditional on the total market size and ignoring the division of market shares. This suggests that collusion may be supportable by conditioning on the size of market demand, and not on the allocation of that demand across firms. An exploration of that conjecture we leave to future work. However, if $V(m)$ is constant - so firms expect the same continuation payoff regardless of the realized total market size - then it follows from $\sum_{m=0}^{M} \frac{\partial\rho(m|p,p)}{\partial p_1} = 0$ (which holds as the probabilities always sum up to 1) that again no collusion is sustainable.

[16] For studies that model the effect of the prospect of detection on the cartel price path, see Harrington (2004, 2005) and Harrington and Chen (2004).

5 Collusion with Asymmetric Punishments

In this section, asymmetric punishments with side payments are characterized which sustain collusion. We will need to suppose that firm demand is declining in its price as specified by A4.

Consider the following symmetric strategy profile which allows for side payments between firms. Recall that b^t is the number of units sold by firm 1 in period t. It is assumed that k is a positive integer and $\frac{m}{2} < k \leq m$.

- If in the *collusive* state in period t then set $p_i^t = p^*$ and

  if $b^t \in \{m-k, m-k+1, \ldots, k-1, k\}$ then remain in the *collusive* state in period $t+1$

  if $b^t > k$ then go to the *type 1 punishment* state in period $t+1$

  if $b^t < m-k$ then go to the *type 2 punishment* state in period $t+1$.

- If in the *type i punishment* state in period t then firm i pays z to firm $j(\neq i)$ and

  if firm i pays z to firm j then switch to the *collusive* state in period t

  if firm i does not pay z to firm j then play the static Nash equilibrium forever.

Note that unlike in symmetric equilibria, the *type i punishment* state does not call for a price war, just for a transfer from firm i to firm j and immediate continuation of the collusive state.

Letting V denote the (rescaled) collusive payoff, the payoff faced by firm 1 in the collusive state is

$$(1-\delta)\left[\sum_{b=0}^{m} p_1 b\phi\left(b; p_1, p^*\right) + \sum_{b=0}^{m-k-1} \delta z\phi\left(b; p_1, p^*\right) - \sum_{b=k+1}^{m} \delta z\phi\left(b; p_1, p^*\right)\right] + \delta V. \quad (12)$$

Assuming the first-order condition is sufficient, p^* is then defined by:

$$\sum_{b=0}^{m}\left[\phi\left(b; p^*, p^*\right) + p^* \frac{\partial\phi\left(b; p^*, p^*\right)}{\partial p_1}\right] b + \delta z\left[\sum_{b=0}^{m-k-1} \frac{\partial\phi\left(b; p^*, p^*\right)}{\partial p_1} - \sum_{b=k+1}^{m} \frac{\partial\phi\left(b; p^*, p^*\right)}{\partial p_1}\right] = 0.$$

Re-arranging this expression,

$$p^* = \left(\frac{\sum_{b=0}^{m} \phi\left(b; p^*, p^*\right) b}{-\sum_{b=0}^{m} \frac{\partial\phi(b;p^*,p^*)}{\partial p_1} b}\right) + \delta z\left(\frac{\sum_{b=0}^{m-k-1} \frac{\partial\phi(b;p^*,p^*)}{\partial p_1} - \sum_{b=k+1}^{m} \frac{\partial\phi(b;p^*,p^*)}{\partial p_1}}{-\sum_{b=0}^{m} \frac{\partial\phi(b;p^*,p^*)}{\partial p_1} b}\right). \quad (13)$$

Let us represent (13) as follows:

$$p^* = \psi_1\left(p^*\right) + \delta z \psi_2\left(p^*\right).$$

Denoting by $\widehat{p}$ a stage-game symmetric Nash equilibrium price, it follows from (3) that:

$$\widehat{p} = \psi_1 (\widehat{p}).$$

We next want to show that $\psi_2 (p) > 0 \ \forall p$; that is,

$$\frac{\sum_{b=0}^{m-k-1} \frac{\partial \phi(b;p,p)}{\partial p_1} - \sum_{b=k+1}^{m} \frac{\partial \phi(b;p,p)}{\partial p_1}}{-\sum_{b=0}^{m} \frac{\partial \phi(b;p,p)}{\partial p_1} b} > 0.$$

The denominator is positive if expected demand is decreasing in a firm's price which is indeed an implication of A4. This leaves having to prove that

$$\sum_{b=0}^{m-k-1} \frac{\partial \phi (b; p, p)}{\partial p_1} - \sum_{b=k+1}^{m} \frac{\partial \phi (b; p, p)}{\partial p_1} > 0. \tag{14}$$

By A2,

$$\frac{\partial \phi (b; p, p)}{\partial p_1} = \frac{\partial \phi (m - b; p, p)}{\partial p_2}$$

so that (14) is equivalent to:

$$\sum_{b=0}^{m-k-1} \left[\frac{\partial \phi (b; p, p)}{\partial p_1} - \frac{\partial \phi (b; p, p)}{\partial p_2} \right] > 0. \tag{15}$$

By A3,

$$\frac{\partial \phi (b; p, p)}{\partial p_1} = -\frac{\partial \phi (b; p, p)}{\partial p_2},$$

so that (15) is equivalent to:

$$2 \sum_{b=0}^{m-k-1} \frac{\partial \phi (b; p, p)}{\partial p_1} > 0,$$

which holds by A4.[17]

If ψ_1 has multiple fixed points then let $\widehat{p}$ be the smallest one. It follows from the preceding steps that:

$$\psi_2 (p) > 0 \ \forall p \Rightarrow \psi_1 (p) + \delta z \psi_2 (p) > 0 \ \forall p \leq \widehat{p}.$$

As we've assumed that $\psi_1 + \delta z \psi_2$ has a fixed point, it must then exceed $\widehat{p}$. This proves that $p^* > \widehat{p}$.

By the definition of p^* in (13), each firm finds it optimal to price at p^* in the collusive state given the other firm is anticipated to do so. The last step is to ensure that it is optimal for firm i to pay z to firm j in punishment state i. This is true iff:

$$V - (1 - \delta) z \geq W, \tag{16}$$

[17] By setting $k = m - 1$, A4 can be weakened to $\frac{\partial \phi(0;p_1,p_2)}{\partial p_1} > 0$.

where W is the (rescaled) payoff from the punishment of infinite reversion to the static Nash equilibrium. We can calculate V as it is defined recursively by:

$$V = (1-\delta)\left[\sum_{b=0}^{m} p^* b\phi\left(b; p^*, p^*\right) + \sum_{b=0}^{m-k-1} \delta z\phi\left(b; p^*, p^*\right) - \sum_{b=k+1}^{m} \delta z\phi\left(b; p^*, p^*\right)\right] + \delta V,$$

which yields:

$$V = \sum_{b=0}^{m} p^* b\phi\left(b; p^*, p^*\right) = p^*\left(\frac{m}{2}\right).$$

The punishment payoff is

$$W = \widehat{p}\left(\frac{m}{2}\right).$$

Condition (16) takes hence the explicit form:

$$p^*\left(\frac{m}{2}\right) - (1-\delta)\, z \geq \widehat{p}\left(\frac{m}{2}\right) \Leftrightarrow \left(p^* - \widehat{p}\right)\left(\frac{m}{2}\right) \geq (1-\delta)\, z. \tag{17}$$

Next note that that $p^* - \widehat{p}$ is bounded above zero because $\psi_2(p)$ is bounded above zero. Therefore, (17) holds as $\delta \to 1$. Note that an arbitrarily high collusive price can be achieved by raising z. Thus, even if there are other stage game Nash equilibria (with prices necessarily higher than $\widehat{p}$), there are equilibrium collusive prices which are sure to exceed them.

Suppose we were to put additional structure on the problem:

A5 $\exists \widehat{\phi} : \{0, 1, \ldots, m\} \times \Re \to [0, 1]$ such that $\phi(b; p_1, p_2) = \widehat{\phi}(b; p_1 - p_2)\ \forall b \in \{0, 1, \ldots, m\}$, $\forall (p_1, p_2) \in \Re^2$.

Since $\phi(b; p, p)$ is independent of p by A3 then $\frac{\partial \phi(b;p,p)}{\partial p_1}$ is independent of p by A5. It follows that (13) becomes

$$p^* = \widehat{p} + \delta z\theta,$$

where θ is a positive constant. p^* and $\widehat{p}$ are then uniquely defined and $p^* > \widehat{p}$. The condition for it to be optimal to pay z is:

$$\begin{aligned}
p^*\left(\frac{m}{2}\right) - (1-\delta)\, z &\geq \widehat{p}\left(\frac{m}{2}\right) \Leftrightarrow \\
\left(\widehat{p} + \delta z\theta\right)\left(\frac{m}{2}\right) - (1-\delta)\, z &\geq \widehat{p}\left(\frac{m}{2}\right) \Leftrightarrow \\
\delta z\theta\left(\frac{m}{2}\right) - (1-\delta)\, z &\geq 0 \Leftrightarrow \delta \geq \frac{2}{2 + \theta m}.
\end{aligned}$$

Since $\theta m > 0$, we conclude this is a subgame perfect equilibrium when δ is sufficiently close to one.

In sum, collusion can be sustained by a punishment strategy in which a firm with above-normal sales compensates the other firm. This is, of course, an asymmetric punishment and is sustainable as long as firms are sufficiently patient. In practice, the transfer z can be implemented by having the firm with excess sales buy output from the other firm at an inflated price. Several recent price-fixing cartels engaged in various forms of side payments including the citric acid cartel of 1991-95 (Connor, 2001), the graphite electrodes cartel of 1992-97 (Levenstein, Suslow, and Oswald, 2004), and the vitamins cartel, in particular vitamins A and E over 1989-99 (European Commission, 2003).

6 Concluding Remarks

A common perception of collusive schemes is built around the idea of price wars: the cartel members are recommended to set high prices and if deviation is detected (actual or perceived through a noisy signal) then the firms punish each other by setting low prices. The actual practice of many well-documented price-fixing cartels tells a very different story. It is quite common to employ more complicated schemes involving history-dependent transfers among members. Our analysis suggests that imperfect monitoring in those markets may be the key reason why they did so. For a natural class of multi-unit demand functions, symmetric price wars are incapable of sustaining any collusion regardless of how patient firms are. It may then be necessary for cartels to deploy punishments that discriminate between the firms that sold too much and those that sold too little.

There are still many puzzles associated with the observed behavior of cartels. The asymmetric punishment scheme we characterized had transfers that were independent of how much a firm's sales exceeded its quota. However, in practice, transfers seem to depend on the difference between sales and the quota. Obviously, we could amend the scheme to allow the transfer to depend on this difference and collusion would still be sustainable, but the question is why firms choose to have the transfer depend on it. On one level, it seems natural to tailor the punishment to the crime but that is not a feature which naturally emerges. This suggests that the usual class of models is missing some crucial elements. Identifying what they are and how to encompass them is an important item on the research agenda for cartels.

7 Appendix

In Theorem 1 we have shown that, under assumptions A1-A3, collusion is not sustainable with strongly symmetric exchangeable equilibria. Section 4.1 provided an example with a Bertrand flavor in which discontinuity of firm demand is able to produce collusion in some cases. Here we provide two additional examples that shed more light on the robustness of our impossibility result. In particular, we consider a model with a smooth expected demand and a demand distribution ϕ that is continuous but has a kink when firms charge identical prices. These examples show that the kink has to be sufficiently high for collusion to be possible. The first example derives a demand distribution from a model of consumer choice. The second example is more general but starts with the distribution on demand as given.

7.1 Example A1

Consider the Hotelling line model defined on $[0,1]$ with firm 1 located at 0 and firm 2 at 1. In each period, two customers arrive so $m = 2$. Denoting the location of customer $i \in \{1,2\}$ by ε_i and assuming transportation costs are 1, customer i buys from firm 1 iff

$$\varepsilon_i \leq \frac{1}{2}\left(1 - (p_1 - p_2)\right).$$

With probability α, the two customers' locations are independently and uniformly distributed over $[0,1]$. Call that event A. With probability $(1-\alpha)$, one customer's location is uniformly distributed over $\left[0, \frac{1}{2}\right]$ and the other's is uniformly distributed over $\left[\frac{1}{2}, 1\right]$. Call that event B.[18]

Defining $\Delta \equiv p_1 - p_2$, the probability of splitting the market can be shown to be:

$$\phi(1;\Delta) = \begin{cases} \frac{1}{2}\alpha\left(1-\Delta^2\right) + (1-\alpha)(1+\Delta) & \text{if } p_1 < p_2 \\ 1 - \frac{1}{2}\alpha & \text{if } p_1 = p_2 \\ \frac{1}{2}\alpha\left(1-\Delta^2\right) + (1-\alpha)(1-\Delta) & \text{if } p_1 > p_2 \end{cases}$$

in which case the derivative is

$$\phi'(1;\Delta) = \begin{cases} 1 - \alpha - \alpha\Delta & \text{if } p_1 < p_2 \\ -1 + \alpha - \alpha\Delta & \text{if } p_1 > p_2 \end{cases} \tag{18}$$

$$\begin{aligned} \phi'\left(1;0^+\right) &= -1+\alpha \\ \phi'\left(1;0^-\right) &= 1-\alpha \end{aligned}$$

Given the lack of differentiability at $\Delta = 0$, A1 and A3 (using one-sided derivatives) are not satisfied.

The probability of firm 1 obtaining two customers is

$$\phi(2;\Delta) = \begin{cases} \frac{1}{4}\alpha(1-\Delta)^2 + (1-\alpha)(-\Delta) & \text{if } p_1 < p_2 \\ \frac{1}{4}\alpha & \text{if } p_1 = p_2 \\ \frac{1}{4}\alpha(1-\Delta)^2 & \text{if } p_1 > p_2 \end{cases}$$

[18] Under event B, one can show that the customer located in $\left[0, \frac{1}{2}\right]$ buys from firm 1 with probability one when $p_1 \leq p_2$ and probability $1 - (p_1 - p_2)$ when $p_1 > p_2$ (assuming $1 - (p_1 - p_2) \geq 0$). We offer the following motivation for this structure. Suppose, ex ante, the customer knows she prefers firm 1's product but doesn't know by how much. If $p_1 \leq p_2$ then assume she buys firm 1's product for sure since it dominates the product of firm 2 in price-product trait space. However, if $p_1 > p_2$ then she must evaluate how much better firm 1's product is. If we think of this evaluative process as being stochastic (either in reality or from the perspective of the firms), this two-stage decision process can generate this probabilistic demand structure. An analogous rationale applies to the customer located in $\left[\frac{1}{2}, 1\right]$ and firm 2.

with derivative

$$\phi'(2;\Delta) = \begin{cases} -\frac{1}{2}\alpha(1-\Delta) - (1-\alpha) & \text{if } p_1 < p_2 \\ -\frac{1}{2}\alpha(1-\Delta) & \text{if } p_1 > p_2 \end{cases}$$

$$\begin{aligned} \phi'\left(2;0^{+}\right) &= -\frac{1}{2}\alpha \\ \phi'\left(2;0^{-}\right) &= \frac{1}{2}\alpha - 1 \end{aligned}$$

For the stage game, a firm's problem is

$$\max_{p_1} p_1 D_1(p_1,p_2)$$

where $D_1(p_1,p_2) = \phi(1;p_1-p_2) + 2\phi(2;p_1-p_2)$ is firm 1 expected demand. It turns out that the demand simplifies to:

$$D_1(p_1,p_2) = 1-\Delta$$

It is interesting to notice that despite $\phi(.)$ having a kink at $\Delta = 0$, the expected demand is differentiable.

Letting $\widehat{p}$ denote a symmetric Nash equilibrium price, it is defined by the first-order condition:

$$\widehat{p}\frac{\partial D_1(\widehat{p},\widehat{p})}{\partial p_1} + D_1(\widehat{p},\widehat{p}) = 0$$

The unique symmetric static Nash equilibrium is then $\widehat{p} = 1$ with expected profits equal to 1.[19]

Now consider the following strongly symmetric exchangeable strategy profile. Firms start in the collusive phase and, in the collusive phase, both price at p^*. If they split the market, they remain in the collusive phase which means they price at p^* next period. If they do not split the market then with probability γ they switch to the static Nash equilibrium forever (that is, pricing at 1 thereafter) and with probability $(1-\gamma)$ remain in the collusive phase. As $\phi(1;0) = 1-\frac{1}{2}\alpha$, the value (rescaled by $1-\delta$) from this scheme is defined recursively by

$$V = (1-\delta)p^* + \delta\left(\left(1-\frac{1}{2}\alpha\gamma\right)V + \frac{1}{2}\alpha\gamma\right), \tag{19}$$

where recall the per period payoff to the static Nash equilibrium is 1. Solving for the collusive value,

$$V = \frac{2(1-\delta)p^* + \delta\alpha\gamma}{2-2\delta+\delta\alpha\gamma}. \tag{20}$$

The problem faced by a firm in the collusive phase is:

$$\max_{p_1}(1-\delta)p_1 D_1(p_1,p_2) + \delta\left[(1-(1-\phi(1;\Delta))\gamma)V + (1-\phi(1;\Delta))\gamma\right],$$

[19] The second-order condition clearly holds.

or equivalently

$$\max_{p_1} (1-\delta) p_1 D_1 (p_1, p_2) + \delta\gamma\phi (1; \Delta) (V-1) + \delta V - \delta\gamma (V-1).$$

p^* is an equilibrium if:[20]

$$\begin{array}{rcl} (1-\delta)(-p^*+1) - (1-\alpha)\delta\gamma (V-1) & \leq & 0 \\ (1-\delta)(-p^*+1) + (1-\alpha)\delta\gamma (V-1) & \geq & 0 \end{array}$$

or

$$1 - (1-\alpha)\left(\frac{\delta}{1-\delta}\right)\gamma (V-1) \leq p^* \leq 1 + (1-\alpha)\left(\frac{\delta}{1-\delta}\right)\gamma (V-1).$$

The highest price p^* that can be supported is then

$$p^* = 1 + (1-\alpha)\left(\frac{\delta}{1-\delta}\right)\gamma (V-1) \tag{21}$$

Substituting (21) into (20),

$$V = (1-\delta)\left(1 + (1-\alpha)\left(\frac{\delta}{1-\delta}\right)\gamma (V-1)\right) + \delta\left(\left(1 - \frac{1}{2}\alpha\gamma\right)V + \frac{1}{2}\alpha\gamma\right), \tag{22}$$

and the solution with respect to V is:

$$\begin{cases} 1 & \text{if} \quad \delta + \delta\gamma - \frac{3}{2}\alpha\delta\gamma - 1 \neq 0 \\ \mathbb{R} & \text{if} \quad \delta + \delta\gamma - \frac{3}{2}\alpha\delta\gamma - 1 = 0 \end{cases}$$

By selecting γ so that

$$\delta + \delta\gamma - \frac{3}{2}\alpha\delta\gamma - 1 = 0, \tag{23}$$

any value for V can be achieved as firms can induce any value for p^*.

As (23) is equivalent to

$$\gamma\left(1 - \frac{3}{2}\alpha\right) = \frac{1-\delta}{\delta},$$

a necessary condition is $\alpha < \frac{2}{3}$. Given $\alpha < \frac{2}{3}$ and in light of $\gamma \in (0, 1]$, it is also necessary that

$$\delta \geq \frac{1}{2 - \frac{3}{2}\alpha}.$$

To sum up, if

$$\alpha < \frac{2}{3} \text{ and } \delta \geq \frac{1}{2 - \frac{3}{2}\alpha}$$

[20] The second-order condition is satisfied as the stage profits are concave and $\phi'' (1, \Delta) = -\alpha < 0$.

then $\exists\gamma \in (0,1]$ such that (23) holds which implies that any price (up to the maximum price that consumers are willing to pay) can be sustained by a strongly symmetric exchangeable subgame perfect equilibrium.

With Theorem 1, a slight undercutting of the collusive price did not alter the probability distribution over the continuation payoff which meant that firms would set price to maximize current expected profit. This property does not hold here, however. Using (18), note that if firm 1 prices slightly below the collusive price, the marginal effect on the probability of splitting the market is $1-\alpha$ so that there is a first-order decrease in the probability of that event. This means there is a first-order increase in the probability of the extreme event of one firm selling to both buyers and thus an increase in the probability of the low punishment payoff. This allows firms to support collusion as long as α is sufficiently small - so the marginal effect of price undercutting on the probability of an extreme sales event is sufficiently large - and firms are sufficiently patient.

7.2 Example A2

The surprising feature of the previous example is that some detectability of a deviation ($\alpha < 1$) is insufficient to sustain collusion and this is regardless of the discount factor, even though, with δ close to 1, the benefits to deviation are very small compared to the threat of the loss of continuation payoffs. This contrasts with standard imperfect monitoring settings in which even a small probability of detection is often enough to provide the necessary incentives when the punishment is severe enough. The reason that mechanism does not work here is that the size of the punishment is endogenous and, in particular, depends on the probability of a false-positive punishment (that is, when no firm deviated).

Example A1 has then established that a small kink in ϕ may not be enough to sustain prices above stage game Nash equilibrium prices. To show that this example is not special, we explore more generally the relationship between the size of the kink and the scope of collusion. Our main finding is that a necessary condition for collusion is that the kink is sufficiently large.

Assume two units: $m = 2$. Denote the expected demand for firm 1 by $D(\Delta)$, and the probability of not splitting the market by $\rho(\Delta) = 1 - \phi(1;\Delta)$. Assume $D(\Delta)$ is differentiable and $\rho(\Delta)$ is continuous and symmetric, but has a possible kink at 0. By symmetry, $D(0) = 1$.

We will provide a bound on the collusive payoffs which is uniform for all δ in a strongly symmetric exchangeable equilibrium. As $m = 2$, this implies $V(0) = V(2)$ after all histories. Let V_0 denote the worst possible equilibrium payoff, which has to be between the minimax payoff and the static Nash equilibrium payoff.

Using the methods of Abreu, Pearce, and Stacchetti (1986), we can show that the highest collusive payoff is achievable by a strategy that starts with recommending a price vector (p,p) and: i) if the realized sales is $b = 1$ then the strategy restarts; and ii) if the realized sales is $b \in \{0,2\}$ then with probability γ the firms go to the worst possible punishment and with probability $(1-\gamma)$ the strategy restarts.

If the price is p and the value is V in that equilibrium then:

$$V = (1-\delta)\,\pi(p,p) + \delta\left[(1-\rho(0)\,\gamma)\,V + \rho(0)\,\gamma V_0\right],$$

where $\pi(p,p)$ is the expected profit of firm 1 at equal prices (in case there are no costs, it is simply p). That implies:

$$V - V_0 = (1-\delta)\,\frac{\pi(p,p) - V_0}{1-\delta+\delta\rho(0)\,\gamma}. \tag{24}$$

The necessary incentive compatibility constraint (ICC) that keeps a firm from undercutting its price is:

$$(1-\delta)\left(\frac{\partial\pi(p,p)}{\partial p_1}\right) + \delta\,(V - V_0)\left(-\rho'\left(0^-\right)\gamma\right) \geq 0$$

where $\rho'(0^-)$ is the left derivative of $\rho(\Delta)$ at $\Delta = 0$. For differentiable distribution of market allocations, $\rho'(0) = 0$ and hence it is never possible to sustain any p for which $\frac{\partial\pi(p,p)}{\partial p_1} < 0$ - as established in Theorem 1 (note that the above formulation allows non-linear costs as well).

Now suppose $\rho(\Delta)$ is not differentiable; that is, $\rho'(0^-) < 0$. The local ICC can be re-written using (24) :

$$\frac{\pi(p,p)}{\partial p_1} + \delta\gamma\frac{\pi(p,p) - V_0}{1-\delta+\delta\rho\gamma}\left(-\rho'\left(0^-\right)\right) \geq 0.$$

As $\delta\gamma\frac{\pi(p,p)-V_0}{1-\delta+\delta\rho\gamma}$ is increasing in δ, we get the following uniform upper bound:

$$-\frac{\frac{\partial\pi(p,p)}{\partial p_1}}{\pi(p,p) - V_0} \leq \frac{-\rho'\left(0^-\right)}{\rho(0)}. \tag{25}$$

The RHS is independent of p, so this expression gives an easy way of finding an upper bound on collusive prices as a function of fundamentals. In particular, if $-\frac{\rho'(0^-)}{\rho(0)}$ is close to 0 then $\frac{\partial\pi(p,p)}{\partial p_1}$ has to be close to zero. Assuming the profits are well-behaved, it implies that (p,p) has to be close to static best responses (that is, static Nash equilibrium prices), which in turn implies a very limited degree of collusion.

To relate this general example to Example A1, note that $\pi(p,p) = p$ and $\frac{\partial\pi(p,p)}{\partial p_1} = D'(0)\,p + D(0) = 1 - p$. Using those properties, condition (25) becomes:

$$\frac{p-1}{p-V_0} \leq \frac{-\rho'\left(0^-\right)}{\rho(0)}.$$

In that the minimax payoff is 0 (as firms can always set a price of 0), we get a bound on the highest collusive price:

$$p \leq \frac{1}{1+\frac{\rho'(0^-)}{\rho(0)}},$$

which converges to static Nash equilibrium prices as $\rho'(0^-) \to 0$. Finally, if we use as a punishment only infinite reversion to static Nash equilibrium then $V_0 = 1$ and the condition becomes:

$$1 \leq \frac{-\rho'(0^-)}{\rho(0)}$$

which corresponds to our result in Example A1 that α has to be large enough to sustain any collusion.[21]

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