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Consumers often purchase goods that are "hard to find" to conspicuously display their exclusivity and social status. Firms that produce such conspicuously consumed goods such as designer apparel, fashion goods, jewelry, etc., often face challenges in making optimal pricing and production decisions. Such firms are confronted with precipitous trade-off between high sales volume and high margins, because of the highly uncertain market demand, strategic consumer behavior, and the display of conspicuous consumption. In this paper, we propose a model that addresses pricing and production decisions for a firm, using the rational expectations framework. We show that, in equilibrium, firms may offer high availability of goods despite the presence of conspicuous consumption. We show that scarcity strategies are harder to adopt as demand variability increases, and we provide conditions under which scarcity strategies could be successfully adopted to improve profits. Finally, to *credibly* commit to scarcity strategy, we show that firms can adopt sourcing strategies, such as sourcing from an expensive production location/supplier or using expensive raw materials.

Keywords
strategic customer behavior, game theory, conspicuous consumption, pricing, scarcity, sourcing

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Selling to Conspicuous Consumers: Pricing, Production and Sourcing Decisions[1]

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Abstract

Consumers often purchase goods that are "hard-to-find" to conspicuously display their exclusivity and social status. Firms that produce such conspicuously consumed goods such as designer apparel, fashion goods, jewelry, mobile electronic devices, etc., often face challenges in making optimal pricing and production decisions. Marketing and retail managers of such firms are confronted with precipitous tradeoff between high sales volume and high margins, due to the highly uncertain market demand, strategic consumer behavior, and the display of conspicuous consumption. In this paper, we propose a model that addresses pricing and production decisions for a firm, using the rational expectations framework. We show that, in equilibrium, firms may offer high availability of goods despite the presence of conspicuous consumption. We also provide conditions under which scarcity or stockout strategies could be successfully adopted to improve profits. Finally, unlike prices, availability information is not easily verifiable. Therefore, to *credibly* commit to scarcity strategy, firms can adopt sourcing strategies, such as sourcing from an expensive production location/supplier, installing complex production process, or using expensive raw materials all of which may signal deeper investment in unit production costs.

Keywords: Strategic Customer Behavior, Game Theory, Conspicuous Consumption, Pricing, Scarcity, Sourcing.

1. Introduction

Consumers looking to signal their uniqueness and exclusivity, have often expressed them by consuming goods prominently to display their status. Firms that design and sell luxury products or innovative gadgets have often desired exclusivity in their looks and design. The prominent display of logos, limited availability and expensive designs are some ways through which firms have displayed their exclusivity. For instance, the "*Big Pony*" apparel designed by Ralph Lauren, have more prominent logos that could be displayed conspicuously by the wearer.[2] Many luxury watches with intricate designs, such as *Piaget*, are sold only through limited number of boutique stores and authorized retailers in the United States (*www.piaget.com*). Firms often face decisions on how to make production and pricing decisions when selling such conspicuous goods.

[1]We thank the faculty of Operations and Information Management, and Marketing departments at the Wharton School. We thank the seminar participants at the 2009 MSOM Conference at MIT, Revenue Management and Pricing Workshop at Kellogg, and the 2009 INFORMS Annual Meeting at San Diego. Our sincere acknowledgements to Fishman-Davidson Center for financial support.

[2]A comparison between Classic-Fit Polo and the more conspicuous Classic-Fit Big Pony Polo shirts on *http://www.ralphlauren.com* shows the big pony designs being sold at higher prices.

We study the decisions of a firm when there is conspicuous consumption, i.e., when some members of the population are motivated by *invidious comparison* (Bagwell and Bernheim 1996). Invidious comparison refers to situations in which a member of a customer class consumes conspicuously to distinguish himself from other members. We examine the cases when some consumers seek, purchase and consume hard-to-find products to display their distinction from the other consumers in the population. Consistent with the literature (Leibenstein 1950, Amaldoss and Jain 2005a), we term customers that are driven by such invidious comparisons as engaging in *snobbish* behavior.

With increasingly unpredictable market demand conditions, many firms face difficult tradeoffs between profits and exclusivity which puts them in a bind. Some firms adopt the strategy to compete on prices, and hope to increase revenues through sales volume. In recent times, retailers such as Nordstorm have attributed their increased revenues to slashed prices and increased inventory availability.[3] On the other hand, other firms have chosen to limit their product availability by creating scarcity, and such shortages for new products have been commonly observed (Gumbel 2007).

In general, a reduction in product availability leads to reduced sales, which may hurt firm profits. Thus, it is still unclear if the firms should use scarcity strategies in selling goods, and if they do so, when those strategies should be implemented. Thus, both from practitioner and research perspectives, it is imperative to understand how firms should make interconnected decisions such as how much of the good to produce, how to price those goods, and when to invest in innovative designs or use an expensive supplier, etc.

In this paper, we analyze a monopolist firm's decisions in a market with uncertain demand from conspicuous consumers. The firm sells a single conspicuous good to a market consisting of uncertain number of snobs and commoners. The firm has to make its pricing and the production quantity decisions, based on the forward-looking consumer behavior. When the demand in not deterministic, it is difficult to point out if scarcity occurred due to an unexpected high demand (a random realization) or due to decidedly low inventory (a strategic decision). Often, it is difficult to separate the two effects, due to the lack of full information on the production process (unobservability).[4]

[3] Nordstorm CEO David Spatz argued for cutting prices of several products to respond effectively to the market. For instance, handmade Anyi Lu designer shoes sold at less than \$400, instead of the regular retail price \$595 which was accompanied by 69% increase in store inventory (Giacobbe 2009).

[4] Over the last few years shortages have been widely observed for game consoles with significant accompanying speculation. There was widespread speculation that Nintendo was deliberately creating shortages for Wii (Dubner 2007). GameStop COO Dan DeMatteo publicly theorized that "[Nintendo] intentionally dried up supply" (Huang 2007). Nintendo argued that the shortages occurred because the demand turned out to be higher than the forecast (Kohler 2007). Similar speculation and claims were repeated for Xbox 360 shortages (Duncan 2008). In all the cases, the inventory and availability information remained fully or partially unobservable.

This is a key focal point of our approach. We show that indeed scarcity strategy could emerge in equilibrium in such markets due to the presence of demand uncertainty coupled with demand externalities.

Our model uses the rational expectations framework (Muth 1961, Stokey 1981) to analyze conspicuous consumption. This concept has been used in some recent Marketing and Operations papers (See for instance, Amaldoss and Jain (2005a), Jerath *et al* (2009), Su (2007).). Our research bridges some key marketing and operational decisions by considering an analytical model of pricing and production when customers express their propensity for exclusivity through their purchase behavior.

The scheme of our paper is as follows. We position our contributions with respect to extant literature in Section 2. We build the model of the firm and analyze its equilibrium pricing and production decisions in a homogenous market in Section 3, and heterogenous market in Section 4. In Section 4.1, we show that our structural insights hold even if the conspicuous consumers are not forward-looking. Using our structural results, we consider strategic "scarcity" decisions in Section 5, and how firms can resort to increased sourcing costs to signal their commitment to scarcity strategies in Section 6.

2. Our position in the literature

Many new products – gadgets such as Playstation portables, fashion apparel and goods (designer brands), new cellular phones, portable electronic devices such as MacBook Air – are often treated as vehicles of self-expression through which consumers exhibit their desire for exclusivity or conformity. How should firms produce and market these new products? Analyzing the impact of conspicuous consumption on firms' decision has been gaining traction in the recent literature.

Economists and Marketing Researchers have long been interested in how consumer decision-making related to a purchase could be dependent on social factors. Recently, there has also been emergent interest in the operations management literature, on how production decisions are impacted by consumers decision making behavior (within the rational framework). This paper bridges the marketing and operational decisions of a firm when it sells to consumers who involve in conspicuous consumption, and notes how operational decisions in sourcing and production investment can be employed together with marketing strategies such as scarcity strategies.

Economics Literature: Economists have pointed out how consumption could be beset with positive externalities, due to social conformity in the context of restaurant choice (Becker 1991), due to network effects in the context of technology (Katz and Shapiro 1985), due to market frenzies (DeGraba 1995), or due to herd behavior (Bikhchandani et al 1992).

However, the notion of consumers purchasing goods to be *conspicuous* dates back to Veblen (1899) who, in his *"The Theory of the leisure class"*, wrote how individuals consumed highly conspicuous goods and services in order to advertise their wealth or social status. Leibenstein (1950) emphasized the significance of social factors in consumption, and argued that price by itself might enhance utility. Bagwell and Bernheim (1996) argue that the relationship between price and demand should emerge in equilibrium, and derive conditions for such "Veblen effects" to arise in equilibrium. Corneo and Jeanne (1997) establish that conspicuous consumption might emerge as a tool to signal wealth. While economics literature has focussed on when Veblen effects may emerge in market equilibrium, the pricing and demand management decisions of a firm facing conspicuous consumers have been relatively underexplored.

Marketing Literature: In a series of papers, Amaldoss and Jain (2005a, 2005b) were the first to model the marketing decisions related to consumer conspicuous consumption behavior. Amaldoss and Jain (2005a) study the pricing decision of a firm facing *deterministic* price-dependent demand, and show that snobs may exhibit upwards sloping demand curve, only in a heterogenous market. They conduct laboratory experiments that confirm the equilibrium price derived from the model. In Amaldoss and Jain (2005b), the pricing problem related to the model is analyzed for a duopoly. Finally, Amaldoss and Jain (2007) show that addition of costly features to a product can increase profits in a market with reference group effects. Recent research on shortages of goods as a marketing strategy is also relevant to our paper. Stock and Balachander (2005) provide a signaling strategy to explain product shortages to sell 'hot' products in a market with quality uncertainty. Balachander and Stock (2009) provide strategic directions on when to offer "limited products" as a part of the product line.

In contrast, we explore a market with conspicuous consumption and *uncertain* demand. Both pricing and production decisions need to be made before a random demand is realized. In such a market, in the absence of signaling explanations, we show that scarcity strategy could emerge in the market due to the presence of demand uncertainty.

Further, it is difficult to separate if scarcity occurred due to a strategic decision or missed forecasts. Therefore, we offer a signaling explanation for high investment: A firm can credibly commit to scarcity strategy by sourcing or producing its goods in a more expensive production channel, even without reference group effects, due to demand uncertainty. Increased sourcing costs signal an ex ante commitment to exclusivity and low production volumes from the firm.

More importantly, we show such decisions are credible and consistent ex post (i.e. firm does not overproduce goods after demand realization).

Operations Literature: While operations management literature has a tradition in modeling demand uncertainties, the interest in modeling strategic consumer behavior is recent and gaining increased attention. The operational impact of forward-looking or strategic customers have been considered in variety of contexts such as seasonal goods (Aviv and Pazgal 2008), commitment in supply chain performance (Su and Zhang 2008), triggering early purchases (Liu and van Ryzin 2008), measuring salvage value (Cachon and Kok 2008), price-match guarantees (Lai *et al* 2009), and quick response strategy (Cachon and Swinney 2009). There has been some recent interest in Operations in understanding how inventory shortages (Debo and van Ryzin 2009) or long queues (Debo and Veeraraghavan 2009) may signal quality. None of the above papers in this stream of literature consider conspicuous consumption. We believe our work establishes how scarcity strategies could emerge in equilibrium in stochastic demand environments. We now detail our contributions to the extant literature.

- We build an analytical framework for a firm making joint operational and marketing decisions (viz. pricing, production quantity and sourcing strategy), when selling to a market with uncertain demand and when consumers exhibit strategic purchasing behavior and/or conspicuous consumption. Our equilibrium results hold under general conditions of demand uncertainty.

- While it has been shown that scarcity can be a strategy to signal quality (Stock and Balachander 2005), it is unclear if scarcity can lead to improved profits when there is no quality uncertainty in the market. Scarcity necessarily implies a reduction in product availability, and therefore a reduction in unit sales. We show that demand uncertainty coupled with conspicuous consumption can indeed lead to market conditions where products are scarce and the firm makes higher profits. Therefore, consumption externalities and demand uncertainty alone can drive the scarcity strategy of a firm.

- We show that scarcity strategy is beneficial to the firm when the fraction of consumers engaging in conspicuous consumption ('snobs') is neither too high nor too low. When there are too few snobs in the market, the firm decides to sell to everyone at lower prices. When there are too many snobs in the market, the attractive profit margins trigger the firm to overcommit to large production quantities, to minimize the 'lost sales'. As a result, the product would not be scarce.

- We show that when selling to markets with conspicuous consumption, due to increased margins, firms may *overproduce* goods compared to its production decision in a market without such conspicuous consumption. Therefore, surprisingly, there may be *fewer* stockouts in a market in which sufficient number of consumers prefer exclusivity. For instance, if the market is composed of snobs, it may be optimal for the firm to overproduce, even more than it would produce in a market in which all strategic consumer behavior is ignored. This finding contrasts starkly with the extant literature, which show that conspicuous consumption and strategic buying lead to a reduction in production quantities.

• Finally, firms that sell to consumers exhibiting conspicuous consumption may resort to expensive sourcing or increased production costs. In such cases, firms deliberately source the good from a more expensive location, or use a costlier supplier, and/or use more expensive raw material components in producing the good.[5] Often inventory commitment is not fully verifiable by consumers. A firm can *credibly* commit to its scarcity strategy, by marketing its sourcing strategy. If products are produced through an expensive process, it is unlikely that the firm can invest in upfront costs to produce too many units of the good. Therefore, consumers believe that the product is likely to be scarce, which drives up the valuation for snobs in the market.

3. Pricing and Production in a Homogenous Market

This paper builds on the classical newsvendor production model for a monopolist firm. Our model involves a single producer (a monopolist firm) who has to make two decisions – production quantity, Q, and the price charged per unit, p, – before a random demand, D, is realized in a market composed of non-atomistic customers. The demand is distributed with cumulative distribution function F_D, with density function f_D.[6] The firm incurs a constant marginal cost, c, per unit produced. If the firm produces more than the realized demand, it will be able to salvage the remaining leftover inventory at a lower price, s, at the end of the selling period (i.e., during the salvage period). In line with the extant models, the cost of production is higher than the salvage value, i.e., $c > s$. Let x^+ denotes $max(x, 0)$. The firm's expected profit can be written as,

$$\begin{aligned}\Pi_N(Q,p) &= E[p \cdot \min(D,Q) + s \cdot (Q-D)^+ - c \cdot Q] \\ &= (p-s) \cdot E[\min(D,Q)] - (c-s) \cdot Q \end{aligned} \tag{1}$$

[5] In Operations Literature, sourcing exclusively from a more expensive supplier has been considered an unviable strategy unless the supplier has faster delivery times or better reliability (Tomlin 2006). In those cases, an expensive supply source is sparingly used as an expeditious alternative.

[6] We assume that demand distribution F_D has increasing generalized failure rate (IGFR, Lariviere 2005). This is a mild assumption that fits many distributions including the Normal distribution, the Uniform distribution, the Gamma distribution, and the Weibull distribution. We suppress subscript D and use $F(\cdot)$ to denote $F_D(\cdot)$ when it is unambiguous. Further, let $\bar{F}(\cdot)$ denote $1 - F(\cdot)$.

We assume that all the customers have the same valuation, v, for the newly introduced product.[7] We begin with a model of a market without any strategic behavior or conspicuous consumption. If the customers are not strategic, the optimal production quantity and price are set as per newsvendor decision (see Cachon and Terwiesch 2008).

$$p_0 = v \qquad F_D(Q_0) = (p_0 - c)/(p_0 - s). \tag{2}$$

Henceforth, the p_0 and Q_0 shall be referred to as the traditional newsvendor price and quantity.

However, customers are strategic, i.e, the customers recognize that if the product remains unsold it would be available in the salvage market at price s. The decision of the strategic customer in the market is to maximize her individual surplus by choosing whether to buy the product in the selling period, or to buy the product later in the salvage period. We term such a customer as a *strategic customer* (Su and Zhang 2008).

For simplicity (and staying consistent with extant literature), we assume that there is a selling period during which the product sells at some price p. The prices are *observed* by the customers, however, the actual production quantity remains *unobserved*. Since the inventory remains unobserved, each strategic customer has to form a belief over the expectation not being able to find the product, i.e., the stockout probability ε_s, during the selling period. Based on these expectations, the customer's expected surplus if she faces an actual regular price p is $U_{strategic} = \max\{v - p, (1 - \varepsilon_s) \cdot (v - s)\}$. We apply *rational expectations* (Muth 1961) to solve for the equilibrium price and production quantity chosen by the firm in this environment.

3.1 Modeling Conspicuous Consumption

Customers, in addition to being *strategic (or forward looking)*, may also exhibit conspicuous consumption. As per Leibenstein (1950) and Amaldoss and Jain (2005a), we address these customers as *snobs*. In this section, we begin the analysis with a market composed solely of strategic customers who exhibit conspicuous consumption. We extend this assumption to include heterogeneous markets in Section 4. Snobs have a higher utility for consuming a product when they figure that other consumers are unable to consume the same product. Suppose a firm produces a good in very limited amounts. If snobs acquire the product and consume it, they will be seen as the select few members in the market who consume such a scarce good (i.e., their consumption is conspicuously observed), which in turn increases their utility for such products.

[7] To eliminate trivial outcomes, we assume that the customer will value a product more than its cost of production, i.e., $v > c$.

As before, we assume that the actual quantities produced by the firm for the market remains unobservable to the snobs.[8] Thus, belief on product availability is one important factor that snobs can use to exhibit their conspicuous consumption. Based on their beliefs on product availability, they seek out hard-to-find products, and derive a higher utility in their exclusiveness.

A consumer might build her belief on availability through two observations: First, she observes that the shelf space dedicated to the product at a retailer is often empty or running low. Second, she deciphers, through information accrual, that many other customers are trying to locate the product, but often facing stockouts. Together, the general non-availability of the product increases her utility for the product, although it might be equally hard for her to get the product. Mathematically, we integrate this snobbishness to her utility function based on the stockout belief ε_s as $U_{snob} = \max\{v + k \cdot \varepsilon_s - p, (1 - \varepsilon_s) \cdot (v - s)\}$, where k represents the *sensitivity to stockouts*. It measures a consumer's responsiveness to the product scarcity.[9] For a snob, the higher the value of k is, the higher the utility she gets from purchasing the product on the observation of a stock-out. There is substantial evidence from literature on how stockouts may improve a customer's utility, or enhance her preference for the product (see Lynn 1991 and references therein).

Note that the firm does not observe the exact valuation a customer possesses for its product. Therefore, the firm has to develop beliefs over the customers' reservation price for the product. We denote the firm's (seller's) belief over the reservation price as ε_r. Based on ε_r, it chooses the price optimally, and will produce the corresponding optimal quantity to maximize its profits. A customer's problem is then to decide on whether she should buy the product in the selling period, or in the salvage period. She buys in the selling period, if and only if $v + k \cdot \varepsilon_s - p \geq (1 - \varepsilon_s) \cdot (v - s)$. This leads to the snob's actual reservation price, $r = \varepsilon_s \cdot (k + v - s) + s$. We are ready to define the rational expectations equilibrium (RE equilibrium) in our model.

Definition 1. *A RE equilibrium $(p, Q, r, \varepsilon_s, \varepsilon_r)$ satisfies the following conditions: (i) $p = \varepsilon_r$, (ii) $Q = argmax_q\ \Pi_N(q, p)$, (iii) $r = \varepsilon_s \cdot (k + v - s) + s$, (iv) $\varepsilon_s = \bar{F}(Q)$, (v) $\varepsilon_r = r$.*

Conditions (i), (ii) and (iii) assert that, under expectations ε_r and ε_s, the firm and all consumers will rationally act to maximize their utilities. Condition (iv) specifies that, in equilibrium, the

[8] Many firms produce exclusive goods to sell to snobs. Sometimes, firms announce the exact quantities (Liverpool FC commemorative phones, Sung 2009). More often, firms do not. For instance, precise Xbox360 shipments remained unannounced despite widespread shortages (Morris 2005). Often being proprietary, inventory and shipment quantities are often not easily verifiable information, because the production process remains unobserved by customers.

[9] There is some evidence that even snobbish customers are price-sensitive, and wait for a good deal (Rice 2010). Firms such as *bluefly.com* engage in salvage markets for luxury goods. In any case, our results hold even if snobs are myopic, and not forward-looking in their purchase behavior (See §4.1).

stockout expectations ε_s must match with the actual probability of not being able to find the product (consistency conditions).

Consider a customer who is indifferent between buying in the first period and waiting to buy in the salvage period. Since she knows that every other customer is also strategic and snobbish, she builds a belief on availability of the product. She rationalizes that other customers who are trying to buy the product face the same stockout probabilities as she does. Since we assume homogeneous non-atomistic decision makers, the mass of remaining customers is D if the realization of the demand is D. Thus, she faces a possibility of stock-out, $P(D \geqslant Q)$, which must be consistent with her belief ε_s, as stated by (iv). Finally, condition (v) requires that the firm correctly predicts snob's reservation price.

Conditions in Definition 1 can be reduced to conditions in p and Q only: $p = \bar{F}(Q)\cdot(k+v-s)+s$ and $Q = argmax_q \Pi_N(q,p)$. With the aforementioned conditions, we are ready to describe the RE equilibrium in Proposition 1.

Proposition 1. *In the RE equilibrium all customers can buy immediately, and the firm's price and quantity choices are characterized by*

$$p_s^* = s + \sqrt{(k+v-s)(c-s)}, \qquad \bar{F}_D(Q_s^*) = \sqrt{\frac{c-s}{k+v-s}}$$

All proofs are deferred to the Appendix. We use p_s^* and Q_s^* to denote the equilibrium price and quantity decision the firm makes, (and subscript s in general), when it chooses to sell the product based on *snobs'* reservation prices. For the purposes of benchmarking, we compare the optimal production decision in the case when conspicuous consumption is present in the market to the case when it is absent. When the consumers in the market do not exhibit any sensitivity to stockouts, i.e., when there is no conspicuous consumption, we have $k = 0$. In this case, Corollary 2 indicates how the previous results on strategic customers (*cf* Su and Zhang 2008) in a market without conspicuous consumption, emerges as a special case of our problem.

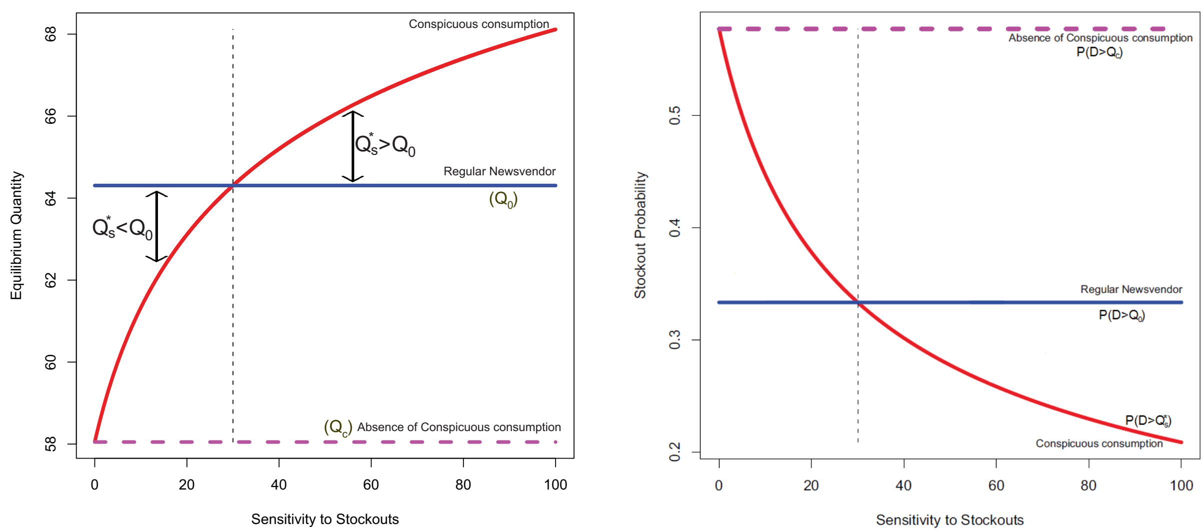
Corollary 2. *In the RE equilibrium, the firm's price and quantity choices in the absence of conspicuous consumption are characterized by*

$$p_c = s + \sqrt{(v-s)(c-s)}, \qquad \bar{F}_D(Q_c) = \sqrt{\frac{c-s}{v-s}}.$$

The comparison of the equilibrium price and quantity in our model to the results without conspicuous consumption (Su and Zhang 2008) reveals the following relationships.

- The equilibrium price when faced with snobs, p_s^*, turns out to be higher than the equilibrium price choice when faced with just strategic customers, p_c. This reaffirms our intuition.

- Intriguingly, the equilibrium production quantity, when conspicuous consumption is present, Q_s^*, is *higher* than the equilibrium production, Q_c (when there is no conspicuous consumption).[10] The firm 'overproduces' due to higher margins (underage costs). Just because consumers exhibit conspicuous consumption does *not* imply that the consequent production quantities would be low. In fact, as customers become more snobbish (i.e., their valuation increases significantly due to stockouts), the equilibrium stockout probability *falls*. This is illustrated in Figure 1(b) where equilibrium stockout probability decreases steadily with sensitivity to stockouts.



(a) (b)

Figure 1: The equilibrium quantities (left panel, a) and stock-out probabilities $P(D > Q)$ (right panel, b) are plotted with respect to the sensitivity to stockouts, k for various markets. The curved line represents the market with conspicuous consumption (i.e., Q_s^* in the left figure and $P(D > Q_s^*)$ in the right figure). The horizontal dotted line represents the market with strategic customers with no conspicuous consumption (Q_c in figure (a) and $P(D > Q_c)$ in (b)). The thick horizontal line represents the regular newsvendor model (Q_0 in figure (a) and $P(D > Q_0)$ in (b)). For illustrative purposes, the demand distribution is $N(\mu = 60, \sigma^2 = 100)$, and the parameters are $v = 20$, $c = 10$, $s = 5$.

Finally, note that production quantity when customers are just strategic is lower than the

[10]Notice that $Q_s^* > Q_c$ because $\bar{F}(Q_c) = \sqrt{\frac{c-s}{v-s}} > \sqrt{\frac{c-s}{k+v-s}} = \bar{F}(Q_s^*)$.

regular production quantity, i.e., $Q_c < Q_0$. Hence, given that $Q_s^* > Q_c$, it is unclear whether under conspicuous consumption the production quantities are lower or higher than the regular production quantity. In the sequel, this comparative analysis aids us in describing product scarcity.

A comparison of the equilibrium price and quantity choices in our model with those in the classical newsvendor model[11] reveals the dependency of the relationship on the sensitivity to stockouts due to conspicuous consumption. This is summarized in Lemma 3 (and in Figures 1(a) and 1(b)).

Lemma 3. *i) $Q_s^* < Q_0$ and $p_s^* < p_0$ when $k < \frac{(v-s)\cdot(v-c)}{c-s}$*

ii) $Q_s^* > Q_0$ and $p_s^* > p_0$ when $k > \frac{(v-s)\cdot(v-c)}{c-s}$

The firm facing snobs produces less quantity than it would in the traditional newsvendor setting if the sensitivity to stockouts, k, is low. If the sensitivity to stockouts is high, the firm produces more than it would in the traditional newsvendor setting. We define this unique threshold level of sensitivity to stockouts, $\frac{(v-s)\cdot(v-c)}{c-s}$, to discuss where scarcity strategy might be profitable to apply.

Thus, even though consumers exhibit strategic buying behavior and conspicuous consumption, we find that the firm may not necessarily produce less inventory. This result stands in contrast to the extant results which show that the production quantity in strategic customer market is always less than the regular newsvendor quantity. The higher margins that can be accrued from conspicuous consumers, make the firm 'overcommit' to higher production volume, more so than it would if those consumers were not conspicuous consumers. Thus accounting for marketing behaviors, such as pricing under conspicuous consumption, impacts other areas of the firm and leads to distinct operational decisions.

4. Heterogenous Market (Snobs and Commoners)

In this section, we address firms' strategies in a heterogenous market. The market is composed of *two* different types of customers, whom we term as *snobs* and *commoners* (*cf.* Leibenstein 1950). We use β to denote the fraction of customer population who are snobs. The rest of the population $(1-\beta)$ is composed of *commoners*. A *commoner* is distinguished from a *snob* in the following sense: A commoner does not exhibit any inclination for conspicuous consumption, but she may still be strategic in her decision-making.

Both types of customers (snobs and commoners) are willing to buy the product in the selling period as long as the firm does not charge a price higher than *their own* reservation price. Since there

[11] $p_0 = v$ and $\bar{F}(Q_0) = \frac{c-s}{v-s}$.

are two possible reservation prices[12] within the market, the firm will have two possible consistent quantity choices, and this will in turn affect the equilibrium availability and beliefs (ε_s).

Thus, there are two possible candidates for the RE equilibrium. The firm charges one of the reservation prices based on the percentage of snobs and produces an optimal quantity that will make the expectations of the customers consistent. Thereafter, the customers observe the price and decide whether to buy the product in the selling period.

Definition 2. *When the firm charges the snob's (commoner's) reservation price, an RE equilibrium* $(p, Q, r, \varepsilon_s, \varepsilon_r)$ *satisfies the following conditions:* (i) $p = \varepsilon_r$*,* (ii) $Q = argmax_q \; \Pi_N(q,p)$*,* (iii) $r = \varepsilon_s \cdot (k+v-s)+s$ $(r = \varepsilon_s \cdot (v-s)+s)$*,* (iv) $\varepsilon_s = \bar{F}_{\beta D}(Q)$ $\big(\varepsilon_s = \bar{F}_D(Q)\big)$*,* (v) $\varepsilon_r = r$*.*

The conditions imposed in Definition 2 are the same as those imposed in Definition 1 except for the conditions (iii) and (iv). Those conditions relate to the beliefs on the reservation price and product availability. The total mass of the customers who are in the market for the product will vary based on the price charged by the firm, and therefore the beliefs on stockouts and reservation prices will also change.

If the firm prices the product based on its belief of snobs' reservation price, then only snobs are present in the market to purchase the product (since the high price rules out commoners from buying the product). Thus, the random variable D is rescaled from D to βD and stockout probability becomes $P(\beta D \geqslant Q)$ or simply, $\bar{F}_{\beta D}(Q)$. The corresponding equilibrium production quantity is given by Proposition 4(1).

On the other hand, if the firm charges the commoner's reservation price, the mass of the customers in the market remains identical to the initial demand distribution, since the offered price is lower than everyone's reservation price. In this case, a possibility of stock-out stays the same as in Definition 1, $P(D \geqslant Q)$. This is indicated in Proposition 4(2).

Proposition 4.

1. ***(Limited Production)*** *In the RE equilibrium under limited production, only snobs can buy, and the firm's price and quantity choices are characterized by* $P(\beta \cdot D > Q_s^*) = \sqrt{\frac{c-s}{k+v-s}}$ *and* $p_s^* = \sqrt{(c-s)\cdot(k+v-s)} + s$*.*

2. ***(Regular Production)*** *In the RE equilibrium, all customers (snobs & commoners) can buy, and the firm's price and quantity decisions are characterized by* $P(D > Q_c) = \sqrt{\frac{c-s}{v-s}}$ *and* $p_c^* = \sqrt{(c-s)\cdot(v-s)} + s$*.*

[12] $i) r = \varepsilon_s \cdot (k+v-s)+s$ for snobs and $ii) r = \varepsilon_s \cdot (v-s)+s$ for commoners

Depending on the market parameters, the profit-maximizing firm would adopt one of the aforementioned strategies. Since consumers are rational, and can correctly form expectations about firm's strategies, the corresponding RE equilibrium would emerge. We investigate the two candidate strategies to see when limited production or regular production would be preferred by the firm. We use $\Pi^*_{N,s}$ to denote the firm's optimal profit obtained under the *Limited Production* strategy (selling only to snobs), and $\Pi^*_{N,c}$ to denote the firm's optimal profit obtained under the *Regular Production* strategy (selling to snobs and commoners).

The ensuing Lemma 5 sheds more light on when the firm chooses *Limited Production* and sells only to snobs, and when it tries to adopt the *Regular Production* strategy to cover the whole market (subject to demand uncertainty).

Lemma 5. *There exists a unique threshold of snobs, β^*, where $\Pi^*_{N,s} < \Pi^*_{N,c}$ when $\beta \leq \beta^*$, and $\Pi^*_{N,s} > \Pi^*_{N,c}$ when $\beta > \beta^*$.*[13]

Lemma 5 shows the firm may adopt different policies based on the concentration of conspicuous consumption in the market. The decision depends on the threshold fraction of snobs in the market (β^*). If the number of snobs in the market is low (i.e. $\beta \leq \beta^*$), the firm will price the product at the commoner's reservation price, and make its product available to all consumers in the market. It does not pay to exclude the commoners out of the market (by selling the product at the snob's reservation price), since the additional profit accrued from the higher price premiums can be compensated by selling to a significantly larger market at a lower price. However, if there is sufficient presence of snobs in the market ($\beta > \beta^*$), the firm can adopt the *limited production* strategy, by attempting to sell only to the snobs. On average, the firm can afford to sell to snobs at high prices, even though the volume of sales has been pushed down due to reduced market coverage. (Note that this does not imply that there would be shortages, since only snobs are in the market, and they might find the product availability to be high). Thus, equilibria with different characteristics can emerge in the market, depending on the density of snobs in the market. Proposition 6 provides the expressions for optimal prices and quantities in the market.

Proposition 6. *If $\beta \leq \beta^*$ then in the RE equilibrium, the firm's price and quantity choices are characterized by $\bar{F}(Q^*_c) = \sqrt{\frac{c-s}{v-s}}$ and $p^*_c = \sqrt{(c-s)\cdot(v-s)} + s$, and all customers can buy. However, if $\beta > \beta^*$ then in the RE equilibrium, the firm's price and quantity choices are characterized by $\bar{F}_{\beta D}(Q^*_s) = \sqrt{\frac{c-s}{k+v-s}}$ and $p^*_s = \sqrt{(c-s)\cdot(k+v-s)} + s$, and only snobs can buy.*

[13] The unique threshold level is $\beta^* = \sqrt{\frac{v-s}{k+v-s}} \cdot \int_0^{\bar{F}_D^{-1}(\sqrt{\frac{c-s}{v-s}})} u f_D(u)\, du / \int_0^{\bar{F}_D^{-1}(\sqrt{\frac{c-s}{k+v-s}})} u f_D(u)\, du$

Serving only to the snobs might also be perceived as "scarcity" strategy, since the firm chooses to sell only to a fraction of the total population. As discussed previously, we show that this may not be necessarily true.

The firm's pricing and production decisions are dictated by two counter-acting factors. First, selling only to snobs means the average demand in the market is reduced – this means the production quantity will tend to *reduce* on average. However, selling only to snobs increases the *underage cost* or the *product margin*, since the product is now marketed to snobs at more expensive prices. This means that the production quantity will *increase*. Due to the higher underage cost, more units of the product are produced to avoid the opportunity cost of missing those high margin sales (lost sales). These two effects counteract each other. Thus, the resultant production quantity may be higher or lower than the production quantity when the firm sells to everyone in the market. We find that if the fraction of snobs in the market is below a certain threshold, the product might be scarce to find compared to the case when the firm sells the product to all consumer types, i.e., the probability that product is in-stock is lower. This is captured in Proposition 7.

Proposition 7. *There exists a unique level of percentage of snobs, β_Q, where $Q_s^* < Q_c$ when $\beta < \beta_Q$ and $Q_s^* > Q_c$ when $\beta > \beta_Q$. This threshold level is given by*

$$\beta_Q = \frac{\bar{F}_D^{-1}(\sqrt{\frac{c-s}{v-s}})}{\bar{F}_D^{-1}(\sqrt{\frac{c-s}{k+v-s}})}.$$

Proposition 7 asserts that the strategy of restricting the sales only to snobs does not always imply the product is scarce to find. In fact, the product might be commonly available even though the firm covers only the snobs in the market. Consequent to Proposition 7, the product is scarce in the market only if (i) the product is limited to snobs (Limited Production), and (ii) the production quantities are lower than the quantities firm produces when it sells to all the market (i.e. $Q_s^* < Q_c$). Thus scarcity exists only when $\beta \in (\beta^*, \beta_Q)$. We elaborate this interesting finding on scarcity further in Section 5. Before analyzing scarcity in detail, we establish the robustness of our result, by showing such "intermediate" scarcity profile continues to exist even when snobs are myopic in §4.1.

4.1 Heterogenous Market: Myopic Snobs

In this section, customers exhibit *conspicuous consumption* but they are *not* strategic. There is some evidence that a large fraction of snobs waits for the markdowns (Economist 2009). On the other hand, perhaps snobs would be willing to pay extreme prices for scarce products to distinguish themselves from others, and always buy myopically.

Conditions imposed in this market setting are the same as those imposed in Definition 2 except for the reservation price condition of snobs in (*iii*). Snobs are myopic in their decision so their reservation price will change. If the firm prices its good based on its belief of snobs' reservation price, then only snobs are present in the market to purchase the product. Then, the random variable D which stands for the demand is descaled by the scalar β to βD. This changes the actual stockout probability to $P(\beta D \geq Q)$ or simply, $\bar{F}_{\beta D}(Q)$. On the other hand, if the firm sets the commoner's reservation price, the mass of the customers remains identical to the demand distribution, since the offered price is lower than everyone's reservation price. In this case, the probability of stockout stays same as in Definition 2 ($P(D \geq Q)$).

Again, we investigate the two candidate strategies to see when each strategy would be preferred by the firm. The following Proposition 8 sheds light on when the firm adopts the limited production strategy vs. when it would produce for the entire market. Note that a threshold structure similar to that established in Section 4 holds, except that the threshold values have changed.

Proposition 8. *There exists a unique threshold of snobs,* β^*_{mySn}[14]*,*

1. *If* $\beta < \beta^*_{mySn}$*, in the RE equilibrium, the firm's price and quantity choices are characterized by* $\bar{F}(Q_c) = \sqrt{\frac{c-s}{v-s}}$ *and* $p_c = \sqrt{(c-s)\cdot(v-s)} + s$*, and all customers will try to buy.*

2. *If* $\beta > \beta^*_{mySn}$*, in the RE equilibrium, the firm's price and quantity choices are characterized by* $\bar{F}_{\beta D}(Q^*_{mySn}) = \frac{\sqrt{(v-s)^2+4k(c-s)}-(v-s)}{2k}$ *and* $p^*_{mySn} = \frac{v+s+\sqrt{(v-s)^2+4k(c-s)}}{2}$*, and only snobs will try to buy.*

In particular, the threshold for the limited production strategy with myopic snobs is smaller than the threshold when the snobs are strategic (i.e., $\beta^*_{mySn} < \beta^*$).[15] The firm begins to exclude the commoners for lower fraction of snobs, since the additional profit accrued from the higher price premiums (provided by myopic consumption of the snobs) compensates for any loss due to reduced sales (from selling the product to the snobs only). Therefore, the firm adopts its *limited production* strategy in more scenarios. Again as before, scarcity exists only in an intermediate region of β values even when snobs are myopic. The result is provided in Proposition A4 in the Appendix.

[14] $\beta^*_{mySn} = \sqrt{(c-s)(v-s)} \int_0^{\bar{F}_D^{-1}(\sqrt{\frac{c-s}{v-s}})} u f_D(u)\, du \Big/ \frac{(v-s)+\sqrt{(v-s)^2+4k(c-s)}}{2} \int_0^{\bar{F}_D^{-1}(\frac{\sqrt{(v-s)^2+4k(c-s)}-(v-s)}{2k})} u f_D(u)\, du$

[15] Comparing the expressions for β^*_{mySn} and β^* in the preceding footnotes.

5. Analysis of Scarcity Strategies

To discuss scarcity strategies in the market, we first clearly define notion of "scarcity". We define a product is *scarce*, when the total production quantity available in the market with conspicuous customers, is lower than the optimal quantity that would have been produced for the market (with an identical demand distribution) in which the customers are forward-looking but do not exhibit such conspicuous consumption behavior (i.e. $Q_s^* < Q_c$).[16]

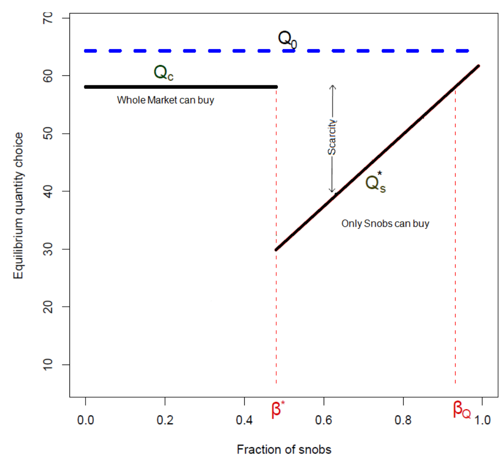


Figure 2: The equilibrium production for different (β). For representative illustration, $v = 20, c = 10, s = 5, k = 10$, and the demand distribution is $N(\mu = 60, \sigma^2 = 100)$. The dotted line above represents the production quantity when customer behavior is entirely ignored. Note that in the region between β^* and β_Q, the optimal production capacity is lower the production quantity when all customers are strategic (Q_c). This defines the scarcity region. The difference between Q_c and Q_s^*, pointed out in the plot denotes the extent or the *degree* of scarcity in the market.

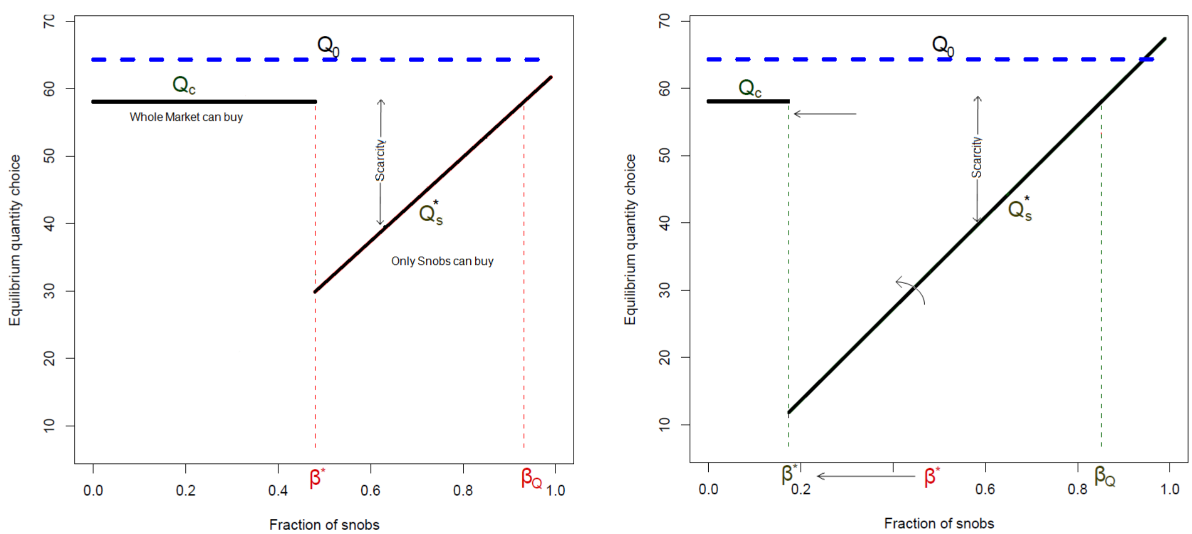
The equilibrium decisions for different market parameters are captured in Figure 2. When $\beta \leq \beta^*$, regular production is adopted to cover the market as much as possible and Q_c units are available in the market. From Lemma 5, when the fraction of snobs in the population exceeds β^*, the firm switches to selling only to snobs (i.e. Limited Production). As β increases, selling only to snobs continues to remain the optimal selling strategy. However, note that the production volume increases since the mean demand (i.e. the fraction of snobs in the market) is increasing. As a result, if the fraction of snobs in the market is higher than β_Q (from Proposition 7), the total production volume and the availability of products (in-stock probability) are both *higher* than in

[16] Figures 2 and 3 demonstrate this notion of scarcity is stricter than simply comparing the equilibrium prices and production quantity to the standard newsvendor prices and production quantity.

the case when customers are just strategic. Thus, the in-stock probability for the product is lower (i.e. the product is *scarcer* to find) in the intermediate region between β^* and β_Q. Furthermore, Figure 2 also reveals that the extent of scarcity is the strongest when the fraction of snobs is just higher than β^*.

5.1 Increased Response to stock-outs

We now study the prevalence of scarcity as the snobs' sensitivity to stockouts varies. In Figure 3, we study how scarcity decisions vary with the fraction of snobs in the market, as the sensitivity to stockouts increases (from $k = 10$ in Figure 3(a) to $k = 45$ in Figure 3(b)). We find that when the market is concentrated with snobs, who are highly sensitive to stockouts (high k), the firm might produce more quantity than the regular newsvendor quantity in equilibrium (even though the customers are strategic). Note that these results extend the observations from Lemma 3 which showed that the equilibrium production may exceed the newsvendor production quantity when the sensitivity to stockouts is high.



(a) (b)

Figure 3: The equilibrium quantity choice for each possible value of percentage of snobs (β). Parameters are the same as Figure 2. Increase in responsiveness to stockouts follows by a decrease in β^* and an increase in the slope of Q_s^*.

Furthermore, as the snobs become more sensitive to stockouts (comparing (a) and (b)), we make two key observations:

1. The threshold β^* decreases with sensitivity to stockouts. If scarcity becomes more desirable to snobs, the firm is more likely to offer *limited production*, i.e. when only snobs can buy, even when the number of snobs in the market is very low. In other words, the firm adopts the *limited production* strategy more often.

2. On the converse, the optimal equilibrium production quantity under the *limited production* strategy increases more steeply with the fraction of snobs in the market as the sensitivity to stockouts increases (i.e. slope of the line under *limited production* strategy increases). If the snobs respond strongly to stockouts, the reservation prices would be even higher, which results in higher price (and an increased underage cost). As a result, the production quantities increase steeply despite the firm adopting a strategy of selling only to snobs. This has the effect of *reducing* the degree of scarcity (fewer stockouts).

Lemma 9. *For higher k, the equilibrium production quantity Q_s^* increases more steeply in β.*

Lemma 9 demonstrates that the optimal production quantity increases faster in β as the sensitivity to stockouts are higher. As snobs become more sensitive to stockouts, the firm *increases* its production quantities even further since the margins from the sales to snobs has also increased. Even though snobs are sensitive to stockouts, their willingness to pay more for exclusivity, causes the firm to produce more goods than usual, since the opportunity cost of losing a sale to such a customer is very high. In other words, the firm is averse to losing a high margin sale (on those rare stockouts), and stocks up on inventory, even though it runs the risk of reduced exclusivity amongst the snobs. Proposition 10 summarizes the behavior of the thresholds with respect to sensitivity of snobs to stockouts.

Proposition 10. *The threshold levels, β^* and β_Q, decrease with increase in sensitivity to stock-out, k.*

Recall that the firm adopts the limited production strategy when the number of snobs in the market is more than β^*. Proposition 10 indicates that the firm would adopt the limited production strategy *more often* as the sensitivity to stockouts increases in the market for the snobbish customers. Conversely, Proposition 10 also states that β_Q decreases in k. The more sensitive the snobs are to stockouts, the more likely the strategy of selling to snobs leads to over-production (i.e. more than the equilibrium quantity produced when the good is available to the whole market). As seen in Lemma 9, the increased opportunity cost drives the firm into producing more goods. In other words, the cost of stockouts are high, when the sensitivity of stockouts for snobs is high.

As a result, the firm produces more goods, even though it is limiting its market to a fraction of customers (snobs) in the market. Aided with the results of Lemma 9 and Proposition 10, we can now analyze the region of scarcity.

It is unclear if the scarcity region that exists in the region $\beta \in (\beta^*, \beta_Q)$ is expanding as snobs become more receptive to stockouts (i.e. as k increases). Propositon 11 provides conditions under which the region of scarcity (i.e. $\beta_Q - \beta^*$) expands as snobs become more sensitive to stockouts.

Proposition 11. *Scarcity region expands if and only if generalized failure rate of the distribution is greater than a threshold, i.e.* $g(Q_s^*/\beta) \geq \frac{\beta_Q}{\beta^*} M(Q_s^*/\beta)$ *where* M *is a constant dependent on* Q_s^* *and* β.[17]

Proposition 11 provides a condition based on demand variability for the prevalence of stockout strategy. If the snobs are very sensitive to stockouts, the scarcity strategy is often in equilibrium if the distribution of the uncertain demand has a high generalized failure rate. Broadly speaking, demand variability in cohesion with conspicuous consumption plays a strong role in the stockouts as an optimal strategy.

6. Commitment to Scarcity: Signaling through Sourcing Investments

While it is true that scarcity strategies can be adopted by firms to generate more revenues, when the market conditions are favorable, it is far from certain that such shortage information is credible, especially since the production decisions are often unobservable. For instance, firms can often stock their shelves as the demand evolves, and it is clear that overall availability is higher, even though more stockouts are observed on store visits. Amaldoss and Jain (2007) correctly observe that "limited edition strategy is constrained by the firms' ability to credibly convince consumers that it will not sell a higher quantity ... (since it is ex post profitable to do so)". In this section, we study how firms may signal their exclusivity by strengthening their commitment to scarcity strategies credibly. We show that in equilibrium, the firms *may* end up with lower production volume (depending on the market structure), due to higher upfront investments in sourcing costs. In our model, the firm does not have any additional utility to produce more goods after the demand is realized, since the reservation price for the remaining consumers is reduced to salvage value s (i.e., overage cost is incurred on additional units).

[17] Lariviere (2005) defines $g(\xi) = \frac{\xi f_D(\xi)}{\bar{F}_D(\xi)}$ as the generalized failure rate of D where D is a non-negative random variable with distribution F_D.

In particular, we look at the sourcing strategies of the firm and examine how the supply-side decisions can be employed as signaling devices to indicate possible shortages to the market. Such strategies are not uncommon in the market. Many luxury apparel firms advertise their products to be handmade or Italian leather,[18] signaling higher value to the customer. Many firms that produce conspicuous products, such as Timbuk2,[19] prominently claim their expensive sourcing decisions to sell the goods at a higher premium. We argue that in some cases such information may not be themselves intrinsically valuable to snobs (i.e., snobs may have no additional utility in handmade bags or shoes made of imported Italian leather). However, such information may be processed by snobs as being indicative of the firm's cost-commitment to the product.

Consider a firm that makes a sourcing or production decision for a conspicuous good before the decisions are made on price and production quantity. The sourcing decision will distinguish the product from the functionally equivalent product sourced elsewhere. For simplicity, we assume that there are two possible production methods. The cheaper sourcing method has a marginal cost c_L, and more expensive method involves c_H, i.e., $c_H > c_L$. The more expensive source might involve a combination of factors that increase the marginal cost of production - an in-sourced supplier whose assembling wages are higher, or the utilization of more expensive raw materials, or the employment of a more-intricate and less-efficient production process.

We consider the decision of the firm and consumers in a multi-period game. In the first period, the firm makes its sourcing decisions. In the second period, firm and consumers play their strategies: pricing and quantity decisions are made by the firm before demand is known, and the consumers make their purchase decisions. This is followed by the period in which left over goods are salvaged. We derive the RE equilibrium through backward induction. In the second period, given the sourcing decision, the subgame proceeds exactly identical as analyzed in the previous sections (except that c_L or c_H replaces c). Since there are two possible production methods with different marginal costs, in each subgame, the firm decides the profit maximizing strategy given the sourcing decision. Then, in the first period, the firm compares the profits obtained from each sourcing decision, and will choose the alternative that maximizes its profit.

Following Proposition 8, for each source, there is a unique threshold level of percentage of snobs in the market above which the firm always chooses limited production, i.e. the firm chooses limited production when $\beta \geq \beta^*_{c_L}$ when the sourcing cost is c_L, and when $\beta \geq \beta^*_{c_H}$ with the sourcing cost

[18]For instance, Louis Vuitton offers its *Monogram Multicolore Marilyn OR* with 33 colors all handcrafted on to the white leather canvas. Louis Vuitton uses a special hand painting or stamping process depending on the type of the product.

[19]Timbuk2 bags sold at higher premiums are handmade and customized in a (more-expensive) facility in San Francisco, rather than being sourced from the overseas supplier in China (Cachon *et al* 2007).

is c_H. Proposition 12 shows that this unique threshold level is decreasing with the marginal cost of supply.

Proposition 12. *The threshold level for limited production decreases with the marginal cost c of the supply source. Therefore $\beta^*_{c_H} < \beta^*_{c_L}$.*

Since, the threshold ($\beta^*_{c_H}$) under the more expensive supply is lower than the threshold level ($\beta^*_{c_L}$) under the cheaper supply, we note the limited production is more prevalent when the supplier is expensive. This yields three possible positions for the fraction of snobs within the population:
(*i*) $\beta < \beta^*_{c_H} < \beta^*_{c_L}$ (Low Intensity). The firm prefers to use *regular production* when using either source.
(*ii*) $\beta^*_{c_H} < \beta < \beta^*_{c_L}$ (Medium Intensity). The firm prefers to use *limited production* for the expensive source, and the *regular production* strategy for the cheaper source.
(*iii*) $\beta^*_{c_H} < \beta^*_{c_L} < \beta$ (High Intensity). The firm prefers to use *limited production* for both sources.

From Figure 2, we know the scarcity occurs when the production is limited to serve only the snobs in the market. We focus our attention on the most interesting case (Case (*iii*)), when the firm adopts *limited production* with either source. The other cases offer the same qualitative conclusions, and are omitted for the sake of brevity.

To analyze the sourcing decisions, we study the profits of the firm as function of the expensive source cost c_H (holding the cost of the cheaper source c_L constant). We show that this profit function is unimodal and attains the global maximum at $c_H = c^* \in [s, v]$ (the unique global minimum is at v). Further, at $c_H = c_{equal}$, the profits using expensive supply matches the profit using the low cost source.[20] This property of the profit function helps us to provide equilibrium results for a general demand distribution and product costs in Proposition 13. We present the equilibrium result for the High Intensity region, when the firm prefers to adopt *limited production* when using either of the two supply sources, but the results for low-intensity and medium-intensity are qualitatively similar. Proposition 13 provides conditions under which an expensive option is chosen by the firm. The specific sourcing decisions are indicated in Figure 4.

Proposition 13. *The subgame perfect equilibrium of the game when $\beta > \beta^*_{c_L} (> \beta^*_{c_H})$ depends on the following conditions.*

1. *If $c_L \geq c^*$, then the firm chooses the cheaper source, sells only to snobs by setting price p^*_s and produces Q^*_s. [Region D].*

[20]See Appendix for technical details (Proof of Proposition 13).

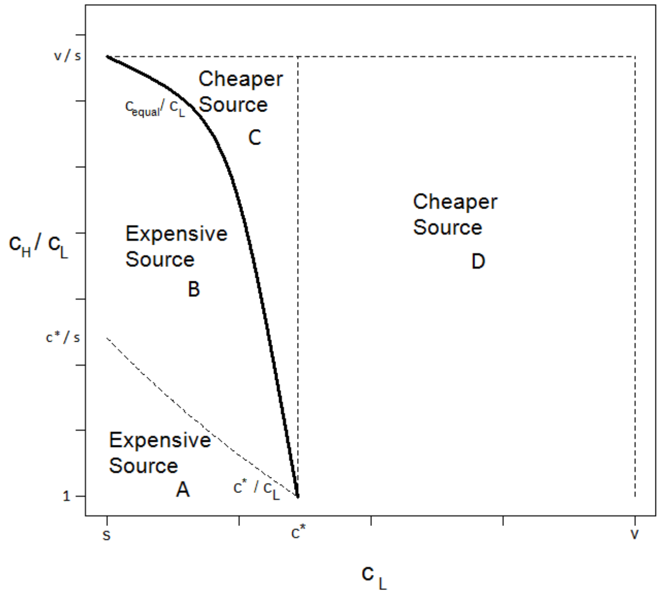


Figure 4: Equilibrium decisions of the firm on which source to use based on given variable cost of the sources. For illustrative purposes $v = 20$, $s = 5$. Note that expensive supply source is chosen when the low-cost source is cheap. The sourcing choice is employed as a signal for commitment to scarcity strategy.

2. *If $c_H \leq c^*$, (thus $c_L \leq c^*$), the firm chooses the more expensive source, and sells only to snobs by setting p_s^* and produces Q_s^*. [Region A]*

3. *If $c_L < c^* < c_H < c_{equal}$, the firm chooses the more expensive source, sets p_s^* and produces Q_s^*. [Region B].*

4. *If $c_L < c^* < c_{equal} < c_H$ the firm chooses the cheaper source, sets p_s^* and produces Q_s^*. [Region C].*

Figure 4 reveals that a more expensive source may be chosen to signal scarcity in the market. The firm decides to use the more expensive source,

(i) when the low-cost source is cheap (i.e. c_L is low), and

(ii) when the expensive source is (comparatively) not too costly (i.e. $c_h/c_L < c_{equal}/c_L$).

The latter point is intuitive. We focus on the intriguing first condition (Condition (i)). It is interesting to note that the firm avoids sourcing from the cheaper source when the source is at its cheapest cost. In other words, a sufficiently cost-efficient production process or supply source will be unused in equilibrium. This is because using a *really* low-cost supplier is perceived by the market as a signal that the firm is committing to a high volume of production (low-exclusivity) in equilibrium.

Thus, contrary to the notions of cost-reduction with sourcing and production, there might be market scenarios where a firm obtains higher profits by choosing the more expensive source. This result mirrors the higher marginal cost result of Amaldoss and Jain (2007), which shows, using reference group effects, that increased marginal costs can improve the profits of a firm. In their paper, increased costs make the product less attractive to followers, thus leaders (to differentiate themselves) adopt the product at a *lower* price (at high volume of sales). Our explanations are based on demand uncertainty. Given the firm has to make a "bet" on optimal quantity in an uncertain demand market, the firm with higher sourcing costs produces less goods, because the marginal cost of unsold goods ($c - s$) is high. This low inventory in turn increases the valuation for snobs, and hence, the equilibrium price. Thus, in equilibrium, the firm with higher costs, produces fewer quantities sold at a higher price. Also note that the *limited production* strategy and an expensive supplier need *not* be employed concurrently in our model.

Thus investment in development, sourcing, and production with higher marginal cost signals a firm's commitment to producing exclusive goods. Consumers can rationalize that given the uncertain demand environment, the firm's increased investment and production costs can only be recouped by producing a few exclusive items and selling each of those items at a high margin. Therefore, the snobs derive a higher utility because of the exclusivity of the product, and expensive sourcing acts as a signal of ex-ante commitment to exclusivity. Even if the product produced using the cheaper source is indistinguishable in terms of performance quality, a firm selling conspicuous goods may prefer to use an expensive source to produce those goods as a commitment to scarcity.

7. Conclusions

This paper attempts to fill the gap in how a firm combines marketing decisions such as pricing and scarcity strategy with operational decisions such as production and sourcing. In particular, we model the role stockouts (inventory unavailability) play in the decisions of a firm. Su and Zhang (2009) show how the cost of customers of not being able to find the product, might force firms to provide availability guarantee to allay scarcity fears. Our paper takes a different approach. Just as scarcity may be a signal of product quality (Stock and Balachander 2005), we show in markets with uncertain demand, how scarcity may also be used to influence demand and consumer valuations, especially when some consumers' decisions are affected by the desire for exclusivity.

We considered an analytical model of a firm selling to a market composed of uncertain number of snobs and commoners. We calculated the equilibrium pricing, production quantity and market coverage decisions of the firm, under very general demand uncertainties.

We showed the existence of conspicuous consumption, by itself, does not guarantee scarcity and low production volumes. In fact, if there are sufficient number of snobs in the market, the firm may be driven by high margins to produce more goods (because the cost of losing a sale is high).

We provide an explanation for why some firms limit their production before introducing the product to the market, and others do not, even in uncertain market where demand remains unobserved. Unlike extant results, our results on limited production are ex-post consistent, i.e. the commitment the firm can make to limited quantities is *credible*, and the firm cannot produce and sell more items after purchasing occurs. Using the *limited production* results, we consider when and how firms should adopt the scarcity strategy, and how it is dependent on market parameters and demand uncertainty.

We find that the scarcity strategy by itself is worthwhile to apply when the fraction of snobs in the market is neither too high nor too low. For low percentage of snobs, it is not worth excluding the commoners by charging the snob's reservation price because the number of snobs is not enough to overcome the revenues accrued from additional sales. When the fraction of snobs in the market is too high, the firm is influenced by the margins/underage costs to overproduce. Thus the scarcity of products occurs only when the fraction of snobs in the market is in the intermediate range. We provide an analytical identification of the interval of percentage of snobs where scarcity is a more profitable strategy. This scarcity region is dependent on how the uncertain demand is distributed, and on the sensitivity of snobs to stockouts.

Finally, we explore why firms adopt more expensive sourcing decisions, incurring upfront higher costs to produce a functionally equivalent good. We find that when a sequential decision is made related to sourcing, and then price and production quantity, there emerge scenarios in which the firm prefers to invest in a more-expensive source when selling to conspicuous consumers. The firm invests in a source that has higher variable cost, which in turn helps in distinguishing the firm's product in terms of exclusivity, even though the utility of the product remains unaltered. Surprisingly, we find that such a choice of a more expensive sourcing may or may not be employed in conjunction with scarcity strategies.

Finally, our model is not without its limitations. Ours is a static model in which the firm, snobs, and commoners make decisions simultaneously. However, the game could dynamic, and the firms and consumers could make product decisions periodically over time. In such a dynamic model, learning about stockouts may play a role in how snobs and commoners make their future decisions. Some future directions include testing our analytical findings using data from natural or laboratory experiments. We believe that a careful empirical analysis of the relationship between consumer characteristics and the impact of stockouts on their buying behavior (*cf* Anderson *et al* 2006) would help us understand how exclusive goods are sold and bought in a market with conspicuous consumption.

References

Amaldoss, W., S. Jain. 2005a. Conspicuous Consumption and Sophisticated Thinking. *Management Science*, **51** (10), pp. 1449–1466.

Amaldoss, W., S. Jain. 2005b. Pricing of Conspicuous goods: A comparative analysis of social effects. *J. Marketing Research*, 42(1), pp. 30–42.

Amaldoss, W., S. Jain. 2007. Trading Up: A strategic analysis of reference group effects. *Marketing Science*, **27**(5), pp. 932–942.

Anderson, E., G. Fitzsimons, and D. Simester. 2006. Measuring and Mitigating the Costs of Stockouts, *Management Science*, **52**(11), pp. 1751–1763.

Aviv, Y., A. Pazgal. 2008. Optimal pricing of seasonal products in the presence of forward-looking consumers. *Manufacturing & Service Operations Management.*, **10**(3) pp. 339–359.

Balachander, S., A. Stock. 2009. Limited edition products: When and when not to offer them. *Marketing Science*, **28** (2), pp. 336–355.

Bikhchandani, S., D. Hirshleifer and I. Welch 1992. A Theory of Fads, Fashion, Custom and Cultural change as Information Cascades. *Journal of Political Economy*, 100, pp. 992–1026.

Becker, G. 1991. A Note on Restaurant Pricing and Other Examples of Social Influences on Price. *J. Political Econom.*, **99** (5), pp. 1109–1116.

Bryson, M. C., M. M. Siddiqui. 1969. Some Criteria for Aging. *Journal of the American Statistical Association*, **64** (328), pp. 1472–1483.

Cachon, G. K. Cattani, and S. Netessine. 2007. Where in the World is Timbuk2? Outsourcing, Offshoring, and Mass Customization. Wharton Teaching Case.

Cachon, G. and G. Kok. 2007. Implementation of the newsvendor model with clearance pricing: how to (and how not to) estimate a salvage value. *Manufacturing & Service Operations Management*, **9** (3), pp. 276–290.

Cachon, G., R. Swinney. 2009. Purchasing, Pricing, and Quick Response in the Presence of

Strategic Consumers. *Management Science*, Vol. 55(3), pp. 497–511.

Cachon, G. and C. Terwiesch. 2009. Matching Supply with Demand: An Introduction to Operations Management, 2nd ed. McGraw-Hill/Irvine, New York.

Corneo, G., O. Jeanne. 1997. Conspicuous consumption, snobbism and conformism. *J. Public Economics*, **66**, pp. 55–71.

Debo, L. G., G. J. van Ryzin. 2009. Inventories and customer search behavior when product quality is uncertain. *Working Paper*.

Debo, L. G., S. Veeraraghavan. 2009. Joining Longer Queues: Information Externalities in Queue Choice. *Manufacturing & Service Operations Management*, Vol 11(4), pp. 543–562.

DeGraba, P. 1995. Buying frenzies and seller-induced excess demand. *RAND J. Economics*, **26** (2), pp. 331–342.

Dubner, S. J. 2007. We Want a Wii! (Still). New York Times. April 19, 2007. `http://freakonomics.blogs.nytimes.com/2007/04/19/we-want-a-wii-still/`

Duncan, G. 2008. Microsoft Claims Xbox 360 Shortage. *Digitial Trends*, February 14, 2008.

The Economist. 2009. A special report on the rich: Bling on a budget. April 2 2009 Issue.

Giacobbe, A. 2009. Luxury Prices: To cut or not to cut? CNN Money, December 17, 2009.

Gumbel, P. 2007. Mass vs. Class. *Fortune*, September 17, 2007. pp 83–88.

Huang, E. 2007. GameStop calls Wii shortage "intentional". *Gamepro*, March 27, 2007. `http://www.gamepro.com/article/news/107465/gamestop-calls-wii-shortage-intentional/`

Jerath, K., S. Netessine and S. Veeraraghavan. 2009. Revenue Management with Strategic Customers: Last Minute Selling and Opaque Selling. *Management Science*, Articles in Advance. DOI 10.1287/mnsc.1090.1125

Katz, M., C. Shapiro. 1985. Network externalities, competition and compatibility. *American Economic Review*, 75(3), pp. 424–440.

Kohler, C. 2007. Why you can't get your hands on Wii. *Wired Magazine*, November 28, 2007.

Lai, G., L. Debo, and K. Sycara. 2009. Buy Now and Match Later: Impact of Posterior Price Matching on Profit with Strategic Consumers, *Manufacturing & Service Operations Management*, Forthcoming.

Lariviere, M. 2005. A Note on Probability Distributions with Increasing Generalized Failure Rates. *Operations Research*, Vol. 54, No. 3, pp. 602-604.

Leibenstein, H. May 1950. Bandwagon, Snob, and Veblen effects in a theory of consumers' demand. *Quarterly Journal of Economics*, **64** (2), pp. 183–207.

Liu, Q., G. J. van Ryzin. 2008. Strategic capacity rationing to induce early purchases. *Management Science*, **54** (6), pp. 1115–1131.

Lynn, M. 1991. Scarcity effect on value: A quantitative review of the commodity theory literature.

Psych. Marketing, **8**, pp. 43–57.

Muth, J. F. 1961. Rational expectations and the theory of price movements. *Econometrica*, **29** (3), pp. 315–335.

Rice, A. 2010. What's a Dress Worth? *New York Magazine*, February 14, 2010 issue.

Stock, A., S. Balachander. 2005. The making of a hot product: A signaling explanation of marketers' scarcity strategy. *Management Science*, **51** (8), pp. 1181–1192.

Stokey, N. 1981. Rational expectations and durable good pricing, *Bell J. Economics*, pp. 112-128.

Su, X. 2007. Intertemporal Pricing with Strategic Customer Behavior. *Management Science*, **53**(5), pp. 726-741.

Su, X., F. Zhang. 2008. Strategic customer behavior, commitment, and supply chain performance. *Management Science.*, **54** (10), pp. 1759–1773.

Su, X. and F. Zhang. 2009. On the Value of Commitment and Availability Guarantees When Selling to Strategic Consumers, *Management Science*, **55**(5), pp. 713–726.

Sung, D. 2009. Liverpool FC Legends Edition mobile phone. *TechDigest*, April 24, 2009.

Tomlin, B. 2006. On the Value of Mitigation and Contingency Strategies for Managing Supply-Chain Disruption Risks. *Management Science*, 52 (5), pp. 639–657.

Veblen, T. 1899. The theory of the leisure class: An economic study of institutions. Dover Publications, New York. (Republished in 1994).

Appendix

Proof of Proposition 1:

The RE equilibrium conditions reduce to

$$p = \bar{F}(Q_s) \cdot (k + v - s) + s \tag{A1}$$

The producer will obtain the critical fractile quantity choice as:

$$\frac{\partial \Pi_N}{\partial Q} = (p - s) \cdot P(D > Q_s^*) - (c - s) = 0$$

$$\bar{F}(Q_s^*) = \frac{c - s}{p - s} \tag{A2}$$

Solving equations (A1) and (A2) provides the equilibrium quantity:

$$\bar{F}(Q_s^*) = \frac{c - s}{p - s} = \frac{c - s}{\bar{F}(Q_s^*) \cdot (k + v - s) + s - s} = \frac{c - s}{\bar{F}(Q_s^*) \cdot (k + v - s)}$$

$$\Rightarrow (\bar{F}(Q_s^*))^2 = \tfrac{c-s}{k+v-s}$$
$$\Rightarrow \bar{F}(Q_s^*) = \sqrt{\tfrac{c-s}{k+v-s}}$$

Putting the last term back in (A1) provides the equilibrium price:

$$p_s^* = \bar{F}(Q_s^*) \cdot (k+v-s) + s = \sqrt{\frac{c-s}{k+v-s}} \cdot (k+v-s) + s = \sqrt{(c-s)\cdot(k+v-s)} + s$$

$\square$

Proof of Corollary 2:

This result follows directly from the proof of Proposition 1 by taking k to 0. The RE-conditions in Definition 1 will remain the same. Solving equations for price and stock-out probability together and applying limit $k \to 0$ lead to the desired results. $\square$

Proof of Lemma 3:

We define $\triangle Q(k) = Q_s^*(k) - Q_0 = \bar{F}^{-1}\left(\sqrt{\frac{c-s}{k+v-s}}\right) - \bar{F}^{-1}\left(\frac{c-s}{v-s}\right)$. Showing that $\triangle Q(k)$ is negative at $k=0$ and $\triangle Q(k)$ strictly increases with k, is sufficient to say that there exists a unique k^* such that $\triangle Q(k^*) = 0$:

- $\triangle\mathbf{Q(0)} < \mathbf{0}$: Note that $\sqrt{\frac{c-s}{v-s}} > \frac{c-s}{v-s}$ since $v > c > s$. Then, $\bar{F}^{-1}\left(\sqrt{\frac{c-s}{v-s}}\right) < \bar{F}^{-1}\left(\frac{c-s}{v-s}\right)$ which confirms that $\triangle Q(0) = \bar{F}^{-1}\left(\sqrt{\frac{c-s}{v-s}}\right) - \bar{F}^{-1}\left(\frac{c-s}{v-s}\right) < 0$.
- $\frac{\partial\triangle\mathbf{Q(k)}}{\partial\mathbf{k}} > \mathbf{0}$: $\frac{\partial\triangle Q(k)}{\partial k} = \frac{\partial Q_s^*(k)}{\partial k} = -\frac{1}{2}\frac{\sqrt{c-s}}{(k+v-s)^{3/2}}\frac{1}{-f(\bar{F}^{-1}(\sqrt{\frac{c-s}{k+v-s}}))} = \frac{1}{2}\frac{\sqrt{c-s}}{(k+v-s)^{3/2}}\frac{1}{f(\bar{F}^{-1}(\sqrt{\frac{c-s}{k+v-s}}))}$
  Note that $\sqrt{c-s} > 0$ and $k+v-s > 0$ since $v > c > s$ and $k > 0$. Then, $\frac{\partial\triangle Q(k)}{\partial k} > 0$.

Thus, there exists an unique k^* such that $\triangle Q(k^*) = 0$:

$$\begin{aligned} Q_s^*(k^*) - Q_0 &= 0 \\ \Rightarrow \bar{F}^{-1}\left(\sqrt{\frac{c-s}{k^*+v-s}}\right) &= \bar{F}^{-1}\left(\frac{c-s}{v-s}\right) \\ \Rightarrow \sqrt{\frac{c-s}{k^*+v-s}} &= \frac{c-s}{v-s} \\ \Rightarrow k^* &= \frac{(v-s)\cdot(v-c)}{c-s} \end{aligned}$$

$\triangle Q(0) < 0$ and $\frac{\partial\triangle Q(k)}{\partial k} > 0$ imply that $\triangle Q(k)$ changes sign only at k^* as k increases. It is easy to show that the same threshold, k^*, holds for the relation between p_s^* and p_0. This leads to the following result:

i) $Q_s^* < Q_0$ and $p_s^* < p_0$ when $k < \frac{(v-s)\cdot(v-c)}{c-s}$

ii) $Q_s^* > Q_0$ and $p_s^* > p_0$ when $k > \frac{(v-s)\cdot(v-c)}{c-s}$

$\square$

Proof of Proposition 4(1):

The producer sets the reservation price of snobs so the commoners are excluded from consideration (βD instead of β). The RE equilibrium conditions reduce to

$$p = \bar{F}_{\beta D}(Q_s^*) \cdot (k+v-s) + s = P(\beta \cdot D > Q_s^*) \cdot (k+v-s) + s \quad \text{(A3)}$$

The producer will obtain the critical fractile quantity choice as:

$$\frac{\partial \Pi_N}{\partial Q} = (p - s) \cdot P(\beta \cdot D > Q) - (c - s) = 0$$

$$P(\beta \cdot D > Q_s^*) = \frac{c - s}{p - s} \tag{A4}$$

Solving equations (A3) and (A4) provides the equilibrium quantity:

$$\bar{F}_{\beta D}(Q_s^*) = \frac{c - s}{p - s} = \frac{c - s}{\bar{F}_{\beta D}(Q_s^*) \cdot (k + v - s) + s - s} = \frac{c - s}{\bar{F}_{\beta D}(Q_s^*) \cdot (k + v - s)}$$

$$\Rightarrow (\bar{F}_{\beta D}(Q_s^*))^2 = \tfrac{c-s}{k+v-s}$$
$$\Rightarrow \bar{F}_{\beta D}(Q_s^*) = \sqrt{\tfrac{c-s}{k+v-s}}$$

Putting the last term back in (A3) provides the equilibrium price:

$$p_s^* = \bar{F}_{\beta D}(Q_s^*) \cdot (k + v - s) + s = \sqrt{\frac{c - s}{k + v - s}} \cdot (k + v - s) + s = \sqrt{(c - s) \cdot (k + v - s)} + s$$

□

Proof of Proposition 4(2):
The RE equilibrium conditions reduce to

$$p = \bar{F}_D(Q_c^*) \cdot (v - s) + s = P(D > Q_c^*) \cdot (v - s) + s \tag{A5}$$

The producer will obtain the critical fractile quantity choice as:

$$\frac{\partial \Pi_N}{\partial Q} = (p - s) \cdot P(D > Q_c^*) - (c - s) = 0$$

$$P(D > Q_c^*) = \frac{c - s}{p - s} \tag{A6}$$

Solving equations A5 and A6 provides the equilibrium quantity:

$$\bar{F}_D(Q_c^*) = \frac{c - s}{p - s} = \frac{c - s}{\bar{F}_D(Q_c^*) \cdot (v - s) + s - s} = \frac{c - s}{\bar{F}_D(Q_c^*) \cdot (v - s)}$$

$$\Rightarrow (\bar{F}_D(Q_c^*))^2 = \tfrac{c-s}{v-s}$$
$$\Rightarrow \bar{F}_D(Q_c^*) = \sqrt{\tfrac{c-s}{v-s}}$$

Putting the last term back in (A5) provides the equilibrium price:

$$p_c^* = \bar{F}_D(Q_c^*) \cdot (v - s) + s = \sqrt{\frac{c - s}{v - s}} \cdot (v - s) + s = \sqrt{(c - s) \cdot (v - s)} + s$$

□

Proof of Lemma 5:

We show that the difference between profits obtained from "Limited" and "Regular" production changes sign only at a unique threshold of snobs, β^*, as β increases. Recall the implicit formulation of Q_s^* from Proposition 4(1). This leads to the following explicit solution:

$$Q_s^* = \beta \bar{F}_D^{-1}\left(\sqrt{\frac{c-s}{k+v-s}}\right)$$

Then, the optimal profit of the producer is:

$$\begin{aligned}
\Pi_{N,s}^* &= \sqrt{(c-s)\cdot(k+v-s)}\cdot E[min\{\beta\cdot D, Q_s^*\}] - (c-s)\cdot Q_s^* \\
&= \sqrt{(c-s)\cdot(k+v-s)}\cdot\left(\int_0^{\frac{Q_s^*}{\beta}} \beta\cdot u\cdot f_D(u)\cdot du + \int_{\frac{Q_s^*}{\beta}}^{\infty} Q_s^*\cdot f_D(u)\cdot du\right) - (c-s)\cdot Q_s^* \\
&= \sqrt{(c-s)\cdot(k+v-s)}\cdot\int_0^{\bar{F}_D^{-1}\left(\sqrt{\frac{c-s}{k+v-s}}\right)} \beta\cdot u\cdot f_D(u)\cdot du
\end{aligned}$$

Recall the implicit formulation of Q_c^* from Proposition 4(2). This leads to the following explicit solution:

$$Q_c^* = \bar{F}_D^{-1}\left(\sqrt{\frac{c-s}{v-s}}\right)$$

Then the optimal profit of the producer is:

$$\begin{aligned}
\Pi_{N,c}^* &= \sqrt{(c-s)\cdot(v-s)}\cdot E[min\{D, Q_c^*\}] - (c-s)\cdot Q_c^* \\
&= \sqrt{(c-s)\cdot(v-s)}\cdot\left(\int_0^{Q_c^*} u\cdot f_D(u)\cdot du + \int_{Q_c^*}^{\infty} Q_c^*\cdot f_D(u)\cdot du\right) - (c-s)\cdot Q_c^* \\
&= \sqrt{(c-s)\cdot(v-s)}\cdot\int_0^{\bar{F}_D^{-1}\left(\sqrt{\frac{c-s}{v-s}}\right)} u\cdot f_D(u)\cdot du
\end{aligned}$$

We define $\Pi_{N,s}^*(\beta)$ to represent the producer's optimal profit function when limited production strategy is applied given that the percentage of snobs is β. We assume that $Q_s^*(\beta) > 0$ except for $\beta = 0$ and $Q_c^* > 0$ without loss of generality. Also, we define $\triangle\Pi(\beta) = \Pi_{N,s}^*(\beta) - \Pi_{N,c}^*$. Note that β is in the domain [0,1]. Then, showing that $\triangle\Pi(\beta)$ is negative at $\beta = 0$, $\triangle\Pi(\beta)$ is positive at $\beta = 1$ and $\triangle\Pi(\beta)$ strictly increases in β is sufficient to say that there exists β^* such that $\triangle\Pi(\beta^*) = 0$:

- $\mathbf{\triangle\Pi(0) < 0}$:

$$\begin{aligned}
\triangle\Pi(0) &= \Pi_{N,s}^*(0) - \Pi_{N,c}^* \\
&= -\Pi_{N,c}^* \\
&= -\sqrt{(c-s)\cdot(v-s)}\cdot\int_0^{\bar{F}_D^{-1}\left(\sqrt{\frac{c-s}{v-s}}\right)} u\cdot f_D(u)\cdot du
\end{aligned}$$

$\sqrt{(c-s)(v-s)}$ is positive since $v > c > s$. The support of D is non-negative and $Q_c^* > 0$. Then, the last term of the equality above must be non-positive.

- $\triangle\boldsymbol{\Pi}(\mathbf{1}) > \mathbf{0}$:

$$\begin{aligned}\triangle\Pi(1) &= \Pi_{N,s}^*(1) - \Pi_{N,c}^* \\ &= \sqrt{(c-s)\cdot(k+v-s)}\cdot\int_0^{\bar{F}_D^{-1}\left(\sqrt{\frac{c-s}{k+v-s}}\right)} u\cdot f_D(u)\cdot du \\ &\quad -\sqrt{(c-s)\cdot(v-s)}\cdot\int_0^{\bar{F}_D^{-1}\left(\sqrt{\frac{c-s}{v-s}}\right)} u\cdot f_D(u)\cdot du\end{aligned}$$

$\sqrt{(c-s)(v-s)} \le \sqrt{(c-s)\cdot(k+v-s)}$ since $k \ge 0$. Also, $\bar{F}_D^{-1}\left(\sqrt{\frac{c-s}{v-s}}\right) \le \bar{F}_D^{-1}\left(\sqrt{\frac{c-s}{k+v-s}}\right)$, since $\sqrt{\frac{c-s}{v-s}} \ge \sqrt{\frac{c-s}{k+v-s}}$. Then, the last equality above must be positive.

- $\frac{\partial\triangle\boldsymbol{\Pi}(\beta)}{\partial\beta} > \mathbf{0}$:

$$\begin{aligned}\frac{\partial\triangle\Pi(\beta)}{\partial\beta} &= \frac{\partial\Pi_{N,s}^*(\beta)}{\partial\beta} \\ &= \sqrt{(c-s)\cdot(k+v-s)}\cdot\int_0^{\bar{F}_D^{-1}\left(\sqrt{\frac{c-s}{k+v-s}}\right)} u\cdot f_D(u)\cdot du\end{aligned}$$

The first term of the last equality is positive since $v > c > s$ and $k \ge 0$. The support of D is non-negative and $Q_s^*(\beta)/\beta > 0$. Then, the last equality above must be positive.

Then, there exists a unique root β^* such that $\triangle\Pi(\beta^*) = 0$:

$$\begin{aligned}&\Rightarrow \Pi_{N,s}^*(\beta^*) - \Pi_{N,c}^* = 0 \\ &\Rightarrow \sqrt{c-s}(\sqrt{k+v-s}\cdot\beta^*\int_0^{\bar{F}_D^{-1}\left(\sqrt{\frac{c-s}{k+v-s}}\right)} u\cdot f_D(u)\cdot du \\ &\qquad\qquad -\sqrt{v-s}\cdot\int_0^{\bar{F}_D^{-1}\left(\sqrt{\frac{c-s}{v-s}}\right)} u\cdot f_D(u)\cdot du) = 0 \\ &\Rightarrow \beta^* = \sqrt{\frac{v-s}{k+v-s}}\cdot\frac{\int_0^{\bar{F}_D^{-1}(\sqrt{\frac{c-s}{v-s}})} u\cdot f_D(u)\,du}{\int_0^{\bar{F}_D^{-1}(\sqrt{\frac{c-s}{k+v-s}})} u\cdot f_D(u)\,du}\end{aligned}$$

$\triangle\Pi(0) < 0$, $\triangle\Pi(1) > 0$, and $\frac{\partial\triangle\Pi(\beta)}{\partial\beta} > 0$ imply that $\triangle\Pi(\beta)$ changes sign only at β^* as β increases. This leads to the following result:

- $\Pi_{N,s}^* < \Pi_{N,c}^*$ when $\beta < \beta^*$
- $\Pi_{N,s}^* > \Pi_{N,c}^*$ when $\beta > \beta^*$

$\square$

Proof of Proposition 6:

Follows from the results of Lemma 5. If $\beta < \beta^*$ then it is more profitable to apply the "Regular Production" strategy since $\Pi^*_{N,s} < \Pi^*_{N,c}$ and if $\beta > \beta^*$ then it is more profitable to apply the "Limited Production" strategy since $\Pi^*_{N,s} > \Pi^*_{N,c}$.

$\square$

Proof of Proposition 7:

We show that the difference between Q^*_s and Q^*_c changes sign only at a particular threshold level, β_Q, as β increases. We define $Q^*_s(\beta)$ as the equilibrium quantity choice under the limited production strategy when snobs allocate β percentage of the market. We assume that $Q^*_s(\beta) > 0$ except for $\beta = 0$ and $Q^*_c > 0$ without loss of generality. Also, we define $\triangle Q(\beta) = Q^*_s(\beta) - Q^*_c = \beta\bar{F}_D^{-1}\left(\sqrt{\frac{c-s}{k+v-s}}\right) - \bar{F}_D^{-1}\left(\sqrt{\frac{c-s}{v-s}}\right)$. Then, showing that $\triangle Q(0) < 0$, $\triangle Q(1) > 0$, and $\frac{\partial \triangle Q(\beta)}{\partial \beta} > 0$ is sufficient to say there exists unique β_Q such that $\triangle Q(\beta_Q) = 0$:

- $\triangle \mathbf{Q}(\mathbf{0}) < \mathbf{0}$: $\triangle Q(0) = -\bar{F}_D^{-1}\left(\sqrt{\frac{c-s}{v-s}}\right)$. $\bar{F}_D^{-1}\left(\sqrt{\frac{c-s}{v-s}}\right)$ is positive since we assume that the support of D is non-negative and $Q^*_c > 0$. Then, $\triangle Q(0) < 0$.
- $\triangle \mathbf{Q}(\mathbf{1}) > \mathbf{0}$: $\triangle Q(1) = \bar{F}_D^{-1}\left(\sqrt{\frac{c-s}{k+v-s}}\right) - \bar{F}_D^{-1}\left(\sqrt{\frac{c-s}{v-s}}\right)$. We show that $\bar{F}_D^{-1}\left(\sqrt{\frac{c-s}{k+v-s}}\right) > \bar{F}_D^{-1}\left(\sqrt{\frac{c-s}{v-s}}\right)$ within the proof of Lemma 5. Then, $\triangle Q(1) > 0$.
- $\frac{\partial \triangle \mathbf{Q}(\beta)}{\partial \beta} > \mathbf{0}$: $\frac{\partial \triangle Q(\beta)}{\partial \beta} = \frac{\partial Q^*_s}{\partial \beta} = \bar{F}_D^{-1}\left(\sqrt{\frac{c-s}{k+v-s}}\right) > 0$
  The inequality follows directly from the assumption that the support of D is non-negative and $Q^*_s(\beta)/\beta > 0$. Then, $\frac{\partial \triangle Q(\beta)}{\partial \beta} > 0$.

Then, there exists a unique root β_Q such that $\triangle Q(\beta_Q) = 0$:

$$
\begin{aligned}
&\Rightarrow Q^*_s(\beta_Q) - Q^*_c = 0 \\
&\Rightarrow \beta_Q \bar{F}_D^{-1}\left(\sqrt{\frac{c-s}{k+v-s}}\right) - \bar{F}_D^{-1}\left(\sqrt{\frac{c-s}{v-s}}\right) = 0 \\
&\Rightarrow \beta_Q = \frac{\bar{F}_D^{-1}\left(\sqrt{\frac{c-s}{v-s}}\right)}{\bar{F}_D^{-1}\left(\sqrt{\frac{c-s}{k+v-s}}\right)}
\end{aligned}
$$

$\triangle Q(0) < 0$, $\triangle Q(1) > 0$, and $\frac{\partial \triangle Q(\beta)}{\partial \beta} > 0$ imply that $\triangle Q(\beta)$ changes sign only at β_Q as β increases. This leads to the following result:

- $Q^*_s < Q^*_c$ when $\beta < \beta_Q$
- $Q^*_s > Q^*_c$ when $\beta > \beta_Q$

$\square$

Before we prove more results, we prove Proposition A1.

Proposition A1. *β_Q is larger than or equal to β^* when $\frac{x}{y} \geq \frac{\bar{F}_D(y)}{\bar{F}_D(x)}$.*

Proof of Proposition A1:

We show that β_Q is larger than or equal to β^* for given parameters (s,c,v and k) under a particular condition. We define $\varrho = \bar{F}_D^{-1}\left(\sqrt{\frac{c-s}{v-s}}\right)$ and $\omega = \bar{F}_D^{-1}\left(\sqrt{\frac{c-s}{k+v-s}}\right)$ for simplification of the system. Rewriting β^* and β_Q with the new notation leads to the following equations:

$$\beta_Q = \frac{\bar{F}_D^{-1}\left(\sqrt{\frac{c-s}{v-s}}\right)}{\bar{F}_D^{-1}\left(\sqrt{\frac{c-s}{k+v-s}}\right)} = \frac{\varrho}{\omega}$$

$$\begin{aligned}
\beta^* &= \sqrt{\frac{v-s}{k+v-s}} \cdot \frac{\int_0^{\bar{F}_D^{-1}(\sqrt{\frac{c-s}{v-s}})} u \cdot f_D(u)\,du}{\int_0^{\bar{F}_D^{-1}(\sqrt{\frac{c-s}{k+v-s}})} u \cdot f_D(u)\,du} = \frac{\sqrt{\frac{c-s}{k+v-s}}}{\sqrt{\frac{c-s}{v-s}}} \cdot \frac{\int_0^{\bar{F}_D^{-1}(\sqrt{\frac{c-s}{v-s}})} u \cdot f_D(u)\,du}{\int_0^{\bar{F}_D^{-1}(\sqrt{\frac{c-s}{k+v-s}})} u \cdot f_D(u)\,du} \\
&= \frac{\bar{F}_D\left(\bar{F}_D^{-1}\left(\sqrt{\frac{c-s}{k+v-s}}\right)\right)}{\bar{F}_D\left(\bar{F}_D^{-1}\left(\sqrt{\frac{c-s}{v-s}}\right)\right)} \cdot \frac{\int_0^{\bar{F}_D^{-1}(\sqrt{\frac{c-s}{v-s}})} u \cdot f_D(u)\,du}{\int_0^{\bar{F}_D^{-1}(\sqrt{\frac{c-s}{k+v-s}})} u \cdot f_D(u)\,du} \\
&= \frac{\bar{F}_D(\omega)}{\bar{F}_D(\varrho)} \cdot \frac{\int_0^{\varrho} u \cdot f_D(u)\,du}{\int_0^{\omega} u \cdot f_D(u)\,du}
\end{aligned}$$

The last line can be further reduced to the following equation by applying integration by parts on the numerator and the denominator of the second term:

$$\begin{aligned}
\beta^* &= \frac{\bar{F}_D(\omega)}{\bar{F}_D(\varrho)} \cdot \frac{u \cdot F_D(u)\mid_0^{\varrho} - \int_0^{\varrho} F_D(u)\,du}{u \cdot F_D(u)\mid_0^{\omega} - \int_0^{\omega} F_D(u)\,du} \\
&= \frac{\bar{F}_D(\omega)}{\bar{F}_D(\varrho)} \cdot \frac{\varrho F_D(\varrho) - \int_0^{\varrho} (1 - F_D(u))\,du}{\omega F_D(\omega) - \int_0^{\omega} (1 - F_D(u))\,du} \\
&= \frac{\bar{F}_D(\omega)}{\bar{F}_D(\varrho)} \cdot \frac{\int_0^{\varrho} \bar{F}_D(u)\,du - \varrho \bar{F}_D(\varrho)}{\int_0^{\omega} \bar{F}_D(u)\,du - \omega \bar{F}_D(\omega)}
\end{aligned}$$

Showing that $\beta_Q - \beta^* \geq 0$ is sufficient for the validity of the claim:

$$\beta_Q - \beta^* = \frac{\varrho}{\omega} - \frac{\bar{F}_D(\omega)}{\bar{F}_D(\varrho)} \cdot \frac{\int_0^{\varrho} \bar{F}_D(u)\,du - \varrho \bar{F}_D(\varrho)}{\int_0^{\omega} \bar{F}_D(u)\,du - \omega \bar{F}_D(\omega)}$$

Recall that, in the Proof of Lemma 5, we showed both the first and the second term of β^* are less than or equal to 1 and non-negative. Thus, eliminating the second term will provide us a lower bound for $\beta_Q - \beta^*$:

$$\begin{aligned}\beta_Q - \beta^* &= \frac{\varrho}{\omega} - \frac{\bar{F}_D(\omega)}{\bar{F}_D(\varrho)} \cdot \frac{\int_0^{\varrho} \bar{F}_D(u)\,du - \varrho \bar{F}_D(\varrho)}{\int_0^{\omega} \bar{F}_D(u)\,du - \omega \bar{F}_D(\omega)} \\ &\geq \frac{\varrho}{\omega} - \frac{\bar{F}_D(\omega)}{\bar{F}_D(\varrho)}\end{aligned}$$

Therefore, we have shown that β_Q is larger than or equal to β^* if $\frac{\varrho}{\omega} \geq \frac{\bar{F}_D(\omega)}{\bar{F}_D(\varrho)}$ for given parameters (s,c,v and k). □

Before we prove our main results on Myopic customers, we prove the following Proposition A2 and Lemma A3.

Proposition A2 (Limited Production)**.** *In the RE equilibrium under limited production, all snobs will try to buy in the current period, and the firm's price and quantity choices are characterized by* $\bar{F}_{\beta D}(Q^*_{s,my}) = \frac{\sqrt{(v-s)^2+4k(c-s)}-(v-s)}{2k}$ *and* $p^*_{s,my} = \frac{v+s+\sqrt{(v-s)^2+4k(c-s)}}{2}$.

Proof of Proposition A2:

The RE equilibrium conditions reduce to

$$p = v + k\bar{F}_{\beta D}(Q^*_{s,my}) \tag{A7}$$

The producer will obtain the critical fractile quantity choice as:

$$\frac{\partial \Pi_N}{\partial Q} = (p-s) \cdot P(\beta D > Q^*_{s,my}) - (c-s) = 0$$

$$\bar{F}_{\beta D}(Q^*_{s,my}) = \frac{c-s}{p-s} \tag{A8}$$

Solving equations (A7) and (A8) provides the following quadratic equation:

$$\begin{aligned}&\bar{F}_{\beta D}(Q^*_{s,my}) = \frac{c-s}{p-s} = \frac{c-s}{v + k\bar{F}_{\beta D}(Q^*_{s,my}) - s} \\ &\Rightarrow v\bar{F}_{\beta D}(Q^*_{s,my}) + k(\bar{F}_{\beta D}(Q^*_{s,my}))^2 - s\bar{F}_{\beta D}(Q^*_{s,my}) = c - s \\ &\Rightarrow k(\bar{F}_{\beta D}(Q^*_{s,my}))^2 + (v-s)\bar{F}_{\beta D}(Q^*_{s,my}) - (c-s) = 0\end{aligned}$$

Two real value solutions to this quadratic equation can be obtained easily by the quadratic formula:

$$\bar{F}_{\beta D}(Q^*_{s,my}) = \frac{-(v-s) \pm \sqrt{(v-s)^2 + 4k(c-s)}}{2k} \tag{A9}$$

Recall that $s < c < v$. Then, one of the solutions, $\bar{F}_{\beta D}(Q^*_{s,my}) = \frac{-(v-s)-\sqrt{(v-s)^2+4k(c-s)}}{2k}$, is infeasible since the survival function must always be non-negative. This leaves one solution which provides the equilibrium quantity:

$$\bar{F}_{\beta D}(Q^*_{s,my}) = \frac{-(v-s) + \sqrt{(v-s)^2 + 4k(c-s)}}{2k} \tag{A10}$$

Putting (A10) back in (A7) provides the equilibrium price:

$$\begin{aligned} p^*_{s,my} \quad &= v + k\bar{F}_{\beta D}(Q^*_{s,my}) \\ &= v + k\frac{-(v-s) + \sqrt{(v-s)^2 + 4k(c-s)}}{2k} \\ &= v + \frac{-(v-s) + \sqrt{(v-s)^2 + 4k(c-s)}}{2} \\ &= \frac{v + s + \sqrt{(v-s)^2 + 4k(c-s)}}{2} \end{aligned}$$

□

Before we proceed with more results, we prove Lemma A3. Let $\Pi^*_{N,mySn}$ denote the firm's optimal profit obtained under the Limited Production strategy, and $\Pi^*_{N,c}$ denote the firm's optimal profit obtained under the Regular Production strategy.

Lemma A3. *There exists a unique threshold of snobs, β^*_{mySn}, where $\Pi^*_{N,mySn} < \Pi^*_{N,c}$ when $\beta < \beta_{mySn}$ and $\Pi^*_{N,mySn} > \Pi^*_{N,c}$ when $\beta > \beta_{mySn}$. The unique threshold level is*

$$\beta^*_{mySn} = \frac{\sqrt{(c-s)(v-s)} \displaystyle\int_0^{\bar{F}_D^{-1}(\sqrt{\frac{c-s}{v-s}})} u f_D(u)\, du}{\frac{(v-s)+\sqrt{(v-s)^2+4k(c-s)}}{2} \displaystyle\int_0^{\bar{F}_D^{-1}(\frac{\sqrt{(v-s)^2+4k(c-s)}-(v-s)}{2k})} u f_D(u)\, du}$$

Proof of Lemma A3: We show that the difference between profits obtained from "Limited" and "Regular" production changes sign only at a unique threshold of snobs, β^*_{mySn}, as β increases. Recall the implicit formulation of $Q^*_{s,my}$ from proposition A2. This leads to the following explicit solution:

$$Q^*_{s,my} = \beta\bar{F}_D^{-1}\left(\frac{\sqrt{(v-s)^2 + 4k(c-s)} - (v-s)}{2k}\right)$$

Then, the optimal profit of the producer is:

$$\begin{aligned}
\Pi^*_{N,mySn} &= \frac{v-s+\sqrt{(v-s)^2+4k(c-s)}}{2} \cdot E[min\{\beta \cdot D, Q^*_{s,my}\}] - (c-s)\cdot Q^*_{s,my} \\
&= \frac{v-s+\sqrt{(v-s)^2+4k(c-s)}}{2} \cdot \left(\int_0^{\frac{Q^*_{s,my}}{\beta}} \beta \cdot u \cdot f_D(u) \cdot du + \int_{\frac{Q^*_{mySn}}{\beta}}^{\infty} Q^*_{s,my} \cdot f_D(u) \cdot du \right) \\
&\qquad - (c-s)\cdot Q^*_{s,my} \\
&= \frac{v-s+\sqrt{(v-s)^2+4k(c-s)}}{2} \cdot \int_0^{\bar{F}_D^{-1}\left(\frac{\sqrt{(v-s)^2+4k(c-s)}-(v-s)}{2k}\right)} \beta \cdot u \cdot f_D(u) \cdot du
\end{aligned}$$

We know the implicit solution of Q^*_c and the optimal profit of the producer under regular production strategy from the previous setting:

$$Q^*_c = \bar{F}_D^{-1}\left(\sqrt{\frac{c-s}{v-s}}\right)$$

$$\Pi^*_{N,c} = \sqrt{(c-s)\cdot(v-s)} \cdot \int_0^{\bar{F}_D^{-1}\left(\sqrt{\frac{c-s}{v-s}}\right)} u \cdot f_D(u) \cdot du$$

We define $\Pi^*_{N,mySn}(\beta)$ to represent the producer's optimal profit function when limited production strategy is applied given that the percentage of snobs is β. Again, we assume that $Q^*_{mySn}(\beta) > 0$ except for $\beta = 0$ and $Q^*_c > 0$ without loss of generality. Also, we define $\triangle\Pi(\beta) = \Pi^*_{N,mySn}(\beta) - \Pi^*_{N,c}$. Note that β is in the domain [0,1]. Then, showing that $\triangle\Pi(\beta)$ is negative at $\beta = 0$, $\triangle\Pi(\beta)$ is positive at $\beta = 1$ and $\triangle\Pi(\beta)$ strictly increases in β is sufficient to say that there exists β^*_{mySn} such that $\triangle\Pi(\beta^*) = 0$:

- $\boldsymbol{\triangle\Pi(0) < 0}$:

$$\begin{aligned}
\triangle\Pi(0) &= \Pi^*_{N,mySn}(0) - \Pi^*_{N,c} \\
&= -\Pi^*_{N,c} \\
&= -\sqrt{(c-s)\cdot(v-s)} \cdot \int_0^{\bar{F}_D^{-1}\left(\sqrt{\frac{c-s}{v-s}}\right)} u \cdot f_D(u) \cdot du
\end{aligned}$$

  $\sqrt{(c-s)(v-s)}$ is positive since $v > c > s$. The support of D is non-negative and $Q^*_c > 0$. Then, the last term of the equality above must be non-positive.

- $\triangle\mathbf{\Pi(1) > 0}$:

$$\begin{aligned}\triangle\Pi(1) &= \Pi^*_{N,mySn}(1) - \Pi^*_{N,c} \\ &= \frac{v-s+\sqrt{(v-s)^2+4k(c-s)}}{2}\cdot\int_0^{\bar{F}_D^{-1}\left(\frac{\sqrt{(v-s)^2+4k(c-s)}-(v-s)}{2k}\right)} \beta\cdot u\cdot f_D(u)\cdot du \\ &\quad -\sqrt{(c-s)\cdot(v-s)}\cdot\int_0^{\bar{F}_D^{-1}\left(\sqrt{\frac{c-s}{v-s}}\right)} u\cdot f_D(u)\cdot du\end{aligned}$$

Claim: $\frac{v-s+\sqrt{(v-s)^2+4k(c-s)}}{2} \geq \sqrt{(c-s)(v-s)}$ when $s < c < v$.
$\sqrt{\frac{(v-s)^2}{4}+k(c-s)}+\frac{v-s}{2} \overset{Take\ square}{\Rightarrow} \frac{(v-s)^2}{4}+k(c-s)+(v-s)\sqrt{\frac{(v-s)^2}{4}+k(c-s)}+\frac{(v-s)^2}{4}$

$$\begin{aligned}&\frac{(v-s)^2}{4}+k(c-s)+(v-s)\sqrt{\frac{(v-s)^2}{4}+k(c-s)}+\frac{(v-s)^2}{4} \\ &=\frac{(v-s)^2}{2}+k(c-s)+\sqrt{\frac{(v-s)^4}{4}+k(c-s)(v-s)^2} \\ &=k(c-s)+\frac{(v-s)^2}{2}+\frac{(v-s)^2}{2}\sqrt{1+4k\frac{(c-s)}{(v-s)^2}} \\ &> k(c-s)+(v-s)^2\end{aligned}$$

Taking square root of both sides, we have obtained the following inequality,

$$\sqrt{\frac{(v-s)^2}{4}+k(c-s)}+\frac{v-s}{2} > \sqrt{k(c-s)+(v-s)^2}$$

$$\sqrt{\frac{(v-s)^2}{4}+k(c-s)}+\frac{v-s}{2} > \sqrt{k(c-s)+(v-s)^2} > \sqrt{k(c-s)+(v-s)(c-s)}$$

$$\sqrt{\frac{(v-s)^2}{4}+k(c-s)}+\frac{v-s}{2} > \sqrt{(k+v-s)(c-s)} \tag{A11}$$

Hence, we have shown that the claim holds when $s < c < v$.

Claim: $\sqrt{\frac{c-s}{v-s}} \geq \frac{\sqrt{(v-s)^2+4k(c-s)}-(v-s)}{2k}$ when $s < c < v$.
We use (A11) to show that the claim holds.

$$\sqrt{\frac{(v-s)^2}{4}+k(c-s)}+\frac{v-s}{2} > \sqrt{(k+v-s)(c-s)}$$

$$\frac{c-s}{\sqrt{\frac{(v-s)^2}{4}+k(c-s)}+\frac{v-s}{2}} < \frac{c-s}{\sqrt{(k+v-s)(c-s)}}$$

$$\frac{c-s}{\sqrt{\frac{(v-s)^2}{4}+k(c-s)}+\frac{v-s}{2}} < \sqrt{\frac{c-s}{k+v-s}} \tag{A12}$$

Thus, $\sqrt{(c-s)(v-s)} \leq \frac{v-s+\sqrt{(v-s)^2+4k(c-s)}}{2}$ by (A11). Also, $\bar{F}_D^{-1}\left(\sqrt{\frac{c-s}{v-s}}\right) \leq \bar{F}_D^{-1}\left(\frac{\sqrt{(v-s)^2+4k(c-s)}-(v-s)}{2k}\right)$, since $\sqrt{\frac{c-s}{v-s}} \geq \frac{\sqrt{(v-s)^2+4k(c-s)}-(v-s)}{2k}$ by (A12). Then, $\triangle\Pi(1)$ must be positive.

- $\frac{\partial \triangle \mathbf{\Pi}(\beta)}{\partial \beta} > \mathbf{0}$:

$$\begin{aligned}
\frac{\partial \triangle\Pi(\beta)}{\partial \beta} &= \frac{\partial \Pi^*_{N,mySn}(\beta)}{\partial \beta} \\
&= \frac{v-s+\sqrt{(v-s)^2+4k(c-s)}}{2} \cdot \int_0^{\bar{F}_D^{-1}\left(\frac{\sqrt{(v-s)^2+4k(c-s)}-(v-s)}{2k}\right)} u \cdot f_D(u) \cdot du
\end{aligned}$$

The first term of the last equality is positive since $v > c > s$ and $k \geq 0$. Also, $Q^*_{s,my}/\beta > 0$. Then, the last equality above must be positive.

Then, there exists a unique root β^*_{mySn} such that $\triangle\Pi(\beta^*_{mySn}) = 0$:

$$\begin{aligned}
&\Rightarrow \Pi^*_{N,mySn}(\beta^*_{mySn}) - \Pi^*_{N,c} = 0 \\
&\Rightarrow \frac{v-s+\sqrt{(v-s)^2+4k(c-s)}}{2}\beta^*_{mySn} \int_0^{\bar{F}_D^{-1}\left(\frac{\sqrt{(v-s)^2+4k(c-s)}-(v-s)}{2k}\right)} u \cdot f_D(u) \cdot du \\
&\qquad - \sqrt{(c-s)(v-s)} \cdot \int_0^{\bar{F}_D^{-1}\left(\sqrt{\frac{c-s}{v-s}}\right)} u \cdot f_D(u) \cdot du) = 0 \\
&\Rightarrow \beta^*_{mySn} = \frac{\sqrt{(c-s)(v-s)} \int_0^{\bar{F}_D^{-1}(\sqrt{\frac{c-s}{v-s}})} u \cdot f_D(u)\, du}{\frac{v-s+\sqrt{(v-s)^2+4k(c-s)}}{2} \int_0^{\bar{F}_D^{-1}\left(\frac{\sqrt{(v-s)^2+4k(c-s)}-(v-s)}{2k}\right)} u \cdot f_D(u) \cdot du}
\end{aligned}$$

$\triangle\Pi(0) < 0$, $\triangle\Pi(1) > 0$, and $\frac{\partial \triangle\Pi(\beta)}{\partial \beta} > 0$ imply that $\triangle\Pi(\beta)$ changes sign only at β^*_{mySn} as β increases. This leads to the following result:

- $\Pi^*_{N,mySn} < \Pi^*_{N,c}$ when $\beta < \beta^*_{mySn}$
- $\Pi^*_{N,mySn} > \Pi^*_{N,c}$ when $\beta > \beta^*_{mySn}$

□

Proof of Proposition 8:

Follows from the results of Lemma A3. If $\beta < \beta^*_{mySn}$ then it is more profitable to apply the *Regular Production* strategy since $\Pi^*_{N,mySn} < \Pi^*_{N,c}$ and if $\beta > \beta^*_{mySn}$ then it is more profitable to apply the *Limited Production* strategy since $\Pi^*_{N,mySn} > \Pi^*_{N,c}$.

□

Before we prove more results, we prove Proposition A4.

Proposition A4. *There exists another unique level of percentage of snobs, $\beta_{Q,mySn}$, where $Q^*_{mySn} < Q_c$ when $\beta < \beta_{Q,mySn}$ and $Q^*_{mySn} > Q_c$ when $\beta > \beta_{Q,mySn}$. This unique threshold level is*

$$\beta_{Q,mySn} = \frac{\bar{F}_D^{-1}\left(\sqrt{\frac{c-s}{v-s}}\right)}{\bar{F}_D^{-1}\left(\frac{\sqrt{(v-s)^2+4k(c-s)}-(v-s)}{2k}\right)}$$

We realize that $\beta_{Q,mySn} < \beta_Q$ since $\bar{F}_D^{-1}\left(\sqrt{\frac{c-s}{k+v-s}}\right) \leq \bar{F}_D^{-1}\left(\frac{\sqrt{(v-s)^2+4k(c-s)}-(v-s)}{2k}\right)$ as $\sqrt{\frac{c-s}{k+v-s}} \geq \frac{\sqrt{(v-s)^2+4k(c-s)}-(v-s)}{2k}$ by (A12).

Proof of Proposition A4:

We show that the difference between $Q^*_{s,my}$ and Q_c changes sign only at a particular threshold level, $\beta_{Q,mySn}$, as β increases. We define $Q^*_{s,my}(\beta)$ as the equilibrium quantity choice under the limited production strategy when snobs allocate β percentage of the market. We assume that $Q^*_{s,my}(\beta) > 0$ except for $\beta = 0$ and $Q_c > 0$. Also, we define $\triangle Q(\beta) = Q^*_{s,my}(\beta) - Q_c = \beta\bar{F}_D^{-1}\left(\frac{\sqrt{(v-s)^2+4k(c-s)}-(v-s)}{2k}\right) - \bar{F}_D^{-1}\left(\sqrt{\frac{c-s}{v-s}}\right)$. Then, showing that $\triangle Q(0) < 0$, $\triangle Q(1) > 0$, and $\frac{\partial \triangle Q(\beta)}{\partial \beta} > 0$ is sufficient to say there exists unique β_Q such that $\triangle Q(\beta_Q) = 0$:

- $\triangle \mathbf{Q}(\mathbf{0}) < \mathbf{0}$: $\triangle Q(0) = -\bar{F}_D^{-1}\left(\sqrt{\frac{c-s}{v-s}}\right)$. $\bar{F}_D^{-1}\left(\sqrt{\frac{c-s}{v-s}}\right)$ is positive since we assume that the support of D is non-negative and $Q^*_c > 0$. Then, $\triangle Q(0) < 0$.
- $\triangle \mathbf{Q}(\mathbf{1}) > \mathbf{0}$: $\triangle Q(1) = \bar{F}_D^{-1}\left(\frac{\sqrt{(v-s)^2+4k(c-s)}-(v-s)}{2k}\right) - \bar{F}_D^{-1}\left(\sqrt{\frac{c-s}{v-s}}\right)$. $\bar{F}_D^{-1}\left(\frac{\sqrt{(v-s)^2+4k(c-s)}-(v-s)}{2k}\right) > \bar{F}_D^{-1}\left(\sqrt{\frac{c-s}{v-s}}\right)$ since $\frac{\sqrt{(v-s)^2+4k(c-s)}-(v-s)}{2k} < \sqrt{\frac{c-s}{v-s}}$ by equation (A12) within the proof of Lemma A3. Then, $\triangle Q(1) > 0$.
- $\frac{\partial \triangle \mathbf{Q}(\beta)}{\partial \beta} > \mathbf{0}$: $\frac{\partial \triangle Q(\beta)}{\partial \beta} = \frac{\partial Q^*_{s,my}}{\partial \beta} = \bar{F}_D^{-1}\left(\frac{\sqrt{(v-s)^2+4k(c-s)}-(v-s)}{2k}\right) > 0$
  The inequality follows directly from the assumption that the support of D is non-negative and $Q^*_{s,my} > 0$. Then, $\frac{\partial \triangle Q(\beta)}{\partial \beta} > 0$.

Then, there exists a unique root $\beta_{Q,mySn}$ such that $\triangle Q(\beta_{Q,mySn}) = 0$:

$$\Rightarrow Q^*_{mySn}(\beta_{Q,mySn}) - Q_c = 0$$
$$\Rightarrow \beta_{Q,mySn}\bar{F}_D^{-1}\left(\frac{\sqrt{(v-s)^2+4k(c-s)}-(v-s)}{2k}\right) - \bar{F}_D^{-1}\left(\sqrt{\frac{c-s}{v-s}}\right) = 0$$
$$\Rightarrow \beta_{Q,mySn} = \frac{\bar{F}_D^{-1}\left(\sqrt{\frac{c-s}{v-s}}\right)}{\bar{F}_D^{-1}\left(\frac{\sqrt{(v-s)^2+4k(c-s)}-(v-s)}{2k}\right)}$$

$\triangle Q(0) < 0$, $\triangle Q(1) > 0$, and $\frac{\partial \triangle Q(\beta)}{\partial \beta} > 0$ imply that $\triangle Q(\beta)$ changes sign only at β_Q as β increases. This leads to the following result:

- $Q^*_{s,my} < Q_c$ when $\beta < \beta_Q$
- $Q^*_{s,my} > Q_c$ when $\beta > \beta_Q$

$\square$

Proof of Lemma 9:
We show that for higher k, Q^*_s increases more steeply in β. Recall $Q^*_s = \beta \bar{F}_D^{-1}\left(\sqrt{\frac{c-s}{k+v-s}}\right)$ from the proof of Lemma 5. In Proposition 7, we show that $\frac{\partial Q^*_s}{\partial \beta} = \bar{F}_D^{-1}\left(\sqrt{\frac{c-s}{k+v-s}}\right)$. What we need to show now is that the partial derivative of $\frac{\partial Q^*_s}{\partial \beta}$ with respect to k is positive:

$$\begin{aligned}\frac{\partial^2 Q^*_s}{\partial\beta\partial k} &= -\frac{1}{2}\cdot\frac{\sqrt{c-s}}{(k+v-s)^{\frac{3}{2}}}\cdot\frac{1}{-f_D(\bar{F}_D^{-1}\left(\sqrt{\frac{c-s}{k+v-s}}\right))} \\ &= \frac{1}{2}\cdot\frac{\sqrt{c-s}}{(k+v-s)^{\frac{3}{2}}}\cdot\frac{1}{f_D(\bar{F}_D^{-1}\left(\sqrt{\frac{c-s}{k+v-s}}\right))}\end{aligned}$$

The term, $\frac{\sqrt{c-s}}{(k+v-s)^{\frac{3}{2}}}$, is positive by the assumptions $v > c > s$ and $k > 0$. Then, $\frac{\partial^2 Q^*_s}{\partial\beta\partial k} > 0$. Therefore, we have shown that for higher k, Q^*_s increases more steeply in β.

$\square$

Proof of Proposition 10:
Follows directly from showing that the first derivatives of both threshold levels (β^* and β_Q) with respect to k are negative:

- $\frac{\partial \beta^*}{\partial k} = -\frac{1}{2}\sqrt{\frac{v-s}{k+v-s}}\frac{\int_0^{\bar{F}_D^{-1}(\sqrt{\frac{c-s}{v-s}})} uf_D(u)du}{\int_0^{\bar{F}_D^{-1}(\sqrt{\frac{c-s}{k+v-s}})} uf_D(u)du}\left(\frac{1}{k+v-s} + \frac{\bar{F}_D^{-1}\left(\sqrt{\frac{c-s}{k+v-s}}\right)}{\int_0^{\bar{F}_D^{-1}(\sqrt{\frac{c-s}{k+v-s}})} uf_D(u)du}\frac{\sqrt{c-s}}{(k+v-s)^{\frac{3}{2}}}\right) < 0$
  The terms, $\sqrt{\frac{v-s}{k+v-s}}$ and $\frac{\sqrt{c-s}}{(k+v-s)^{\frac{3}{2}}}$, are positive by the assumptions $v > c > s$ and $k > 0$. In Proposition 7, we show that both $\bar{F}_D^{-1}(\sqrt{\frac{c-s}{v-s}})$ and $\bar{F}_D^{-1}(\sqrt{\frac{c-s}{k+v-s}})$ are positive so both terms within the parentheses are positive as well. Then, $\frac{\partial \beta^*}{\partial k} < 0$.
- $\frac{\partial \beta_Q}{\partial k} = -\frac{1}{2}\frac{\bar{F}_D^{-1}\left(\sqrt{\frac{c-s}{v-s}}\right)}{\bar{F}_D^{-1}\left(\sqrt{\frac{c-s}{k+v-s}}\right)^2}\frac{1}{f_D(\bar{F}_D^{-1}\left(\sqrt{\frac{c-s}{k+v-s}}\right))}\frac{\sqrt{c-s}}{(k+v-s)^{\frac{3}{2}}} < 0$
  Follows directly from the proof of the previous point.

Therefore, we have shown that the threshold levels, β^* and β_Q, decrease with increase in sensitivity to stock-out, k. $\square$

Proof of Proposition 11:
We derive the conditions under which the the region of scarcity (i.e. $\beta_Q - \beta^*$) expands as snobs become more sensitive to stockouts. We define $\triangle\beta'(k) = \frac{\partial \beta^*}{\partial k} - \frac{\partial \beta_Q}{\partial k}$. We will use the same notation, ϱ and ω, which we used in the proof of Proposition A1. Showing that $\triangle\beta(k)$ is negative for all k

in $[0,\infty)$ is sufficient for proving the proposition.

$$\begin{aligned}
\triangle\beta'(k) &= -\frac{1}{2}\sqrt{\frac{v-s}{k+v-s}}\frac{\int_0^{\bar{F}_D^{-1}(\sqrt{\frac{c-s}{v-s}})} u f_D(u)du}{\int_0^{\bar{F}_D^{-1}(\sqrt{\frac{c-s}{k+v-s}})} u f_D(u)du}\left(\frac{1}{k+v-s} + \frac{\bar{F}_D^{-1}\left(\sqrt{\frac{c-s}{k+v-s}}\right)}{\int_0^{\bar{F}_D^{-1}(\sqrt{\frac{c-s}{k+v-s}})} u f_D(u)du}\frac{\sqrt{c-s}}{(k+v-s)^{\frac{3}{2}}}\right) \\
&+ \frac{1}{2}\frac{\bar{F}_D^{-1}\left(\sqrt{\frac{c-s}{v-s}}\right)}{\bar{F}_D^{-1}\left(\sqrt{\frac{c-s}{k+v-s}}\right)^2}\frac{1}{f_D(\bar{F}_D^{-1}\left(\sqrt{\frac{c-s}{k+v-s}}\right))}\frac{\sqrt{c-s}}{(k+v-s)^{\frac{3}{2}}} \\
&= -\frac{1}{2}\frac{1}{k+v-s}\left(\frac{\bar{F}_D(\omega)}{\bar{F}_D(\varrho)}\cdot\frac{\displaystyle\int_0^{\varrho} u\cdot f_D(u)\,du}{\displaystyle\int_0^{\omega} u\cdot f_D(u)\,du}\left(1+\frac{\omega}{\int_0^{\omega} u f_D(u)du}\bar{F}_D(\omega)\right) - \frac{\varrho}{\omega}\frac{1}{\omega}\frac{\bar{F}_D(\omega)}{f_D(\omega)}\right) \\
&= -\frac{1}{2}\frac{1}{k+v-s}\left(\beta^*\left(1+\frac{\omega}{\int_0^{\omega} u f_D(u)du}\bar{F}_D(\omega)\right) - \beta_Q\frac{1}{\omega}\frac{\bar{F}_D(\omega)}{f_D(\omega)}\right) \\
&= -\frac{1}{2}\frac{1}{k+v-s}\left(\beta^*\left(1+\frac{\omega\bar{F}_D(\omega)}{\int_0^{\omega}\bar{F}_D(u)\,du - \omega\bar{F}_D(\omega)}\right) - \beta_Q\frac{1}{\omega}\frac{\bar{F}_D(\omega)}{f_D(\omega)}\right) \quad (\mathit{Int.\ by\ parts}) \\
&= -\frac{1}{2}\frac{1}{k+v-s}\left(\beta^*\left(\frac{\int_0^{\omega}\bar{F}_D(u)\,du}{\int_0^{\omega}\bar{F}_D(u)\,du - \omega\bar{F}_D(\omega)}\right) - \beta_Q\frac{1}{\omega}\frac{\bar{F}_D(\omega)}{f_D(\omega)}\right)
\end{aligned}$$

The last equality is less than or equal to zero if and only if the equation in parentheses is non-negative. We know from the proofs of lemma 5 and proposition 7 that all terms within the parentheses is non-negative. Thus, we require a sufficient condition that will make the equation in parentheses non-negative:

$$\begin{aligned}
&\beta^*\left(\frac{\int_0^{\omega}\bar{F}_D(u)\,du}{\int_0^{\omega}\bar{F}_D(u)\,du - \omega\bar{F}_D(\omega)}\right) - \beta_Q\frac{1}{\omega}\frac{\bar{F}_D(\omega)}{f_D(\omega)} \geq 0 \\
&\Rightarrow \frac{\omega f_D(\omega)}{\bar{F}_D(\omega)} \geq \frac{\beta_Q}{\beta^*}\frac{\int_0^{\omega}\bar{F}_D(u)\,du - \omega\bar{F}_D(\omega)}{\int_0^{\omega}\bar{F}_D(u)\,du} \\
&\Rightarrow \omega h(\omega) \geq \frac{\beta_Q}{\beta^*}\frac{\int_0^{\omega}\bar{F}_D(u)\,du - \omega\bar{F}_D(\omega)}{\int_0^{\omega}\bar{F}_D(u)\,du} \qquad (h(\omega) = f(\omega)/\bar{F}_D(\omega); \mathit{the\ hazard\ rate\ of\ } D) \\
&\qquad\qquad = \frac{\beta_Q}{\beta^*}M(Q_s^*/\beta) \qquad\qquad (\mathit{since\ } \omega = Q_s^*/\beta)
\end{aligned}$$

We define the hazard rate of D above as $h(.)$ (see Bryson and Siddiqui 1969 for details). The second term on the right-hand side of the inequality is less than or equal to 1 since $\int_0^{\omega}\bar{F}_D(u)\,du - \omega\bar{F}_D(\omega) \leq \int_0^{\omega}\bar{F}_D(u)\,du$ and $\omega \geq 0$. The right hand side of the inequality can be more than or less than or equal to 1 depending on the relation between the first and the second term. Implicit sufficient conditions can be similarly obtained in this case.

□

Proof of Proposition 12:

We show that the threshold level for limited production decreases with the marginal cost of the supply c. Recall that $\beta^* = \sqrt{\frac{v-s}{k+v-s}} \frac{\int_0^{\bar{F}_D^{-1}\left(\sqrt{\frac{c-s}{v-s}}\right)} uf_D(u)du}{\int_0^{\bar{F}_D^{-1}\left(\sqrt{\frac{c-s}{k+v-s}}\right)} uf_D(u)du}$ is defined $\forall c \in [s, v]$ from Lemma 5. Showing that the first derivative of β^* with respect to c is non-positive will be sufficient for the argument:

$$
\begin{aligned}
\frac{\partial \beta^*}{\partial c} &= \sqrt{\frac{v-s}{k+v-s}} \frac{\bar{F}_D^{-1}(\sqrt{\frac{c-s}{v-s}}) f_D(\bar{F}_D^{-1}(\sqrt{\frac{c-s}{v-s}})) \frac{\frac{-1}{2\sqrt{(c-s)(v-s)}}}{f_D(\bar{F}_D^{-1}(\sqrt{\frac{c-s}{v-s}}))} \int_0^{\bar{F}_D^{-1}(\sqrt{\frac{c-s}{k+v-s}})} uf_D(u)du}{(\int_0^{\bar{F}_D^{-1}(\sqrt{\frac{c-s}{k+v-s}})} uf_D(u)du)^2} \\
&- \sqrt{\frac{v-s}{k+v-s}} \frac{\bar{F}_D^{-1}(\sqrt{\frac{c-s}{k+v-s}}) f_D(\bar{F}_D^{-1}(\sqrt{\frac{c-s}{k+v-s}})) \frac{\frac{-1}{2\sqrt{(c-s)(k+v-s)}}}{f_D(\bar{F}_D^{-1}(\sqrt{\frac{c-s}{k+v-s}}))} \int_0^{\bar{F}_D^{-1}(\sqrt{\frac{c-s}{v-s}})} uf_D(u)du}{(\int_0^{\bar{F}_D^{-1}(\sqrt{\frac{c-s}{k+v-s}})} uf_D(u)du)^2} \\
&= \sqrt{\frac{v-s}{k+v-s}} \frac{1}{(\int_0^{\bar{F}_D^{-1}(\sqrt{\frac{c-s}{k+v-s}})} uf_D(u)du)^2} \frac{1}{2\sqrt{c-s}} \\
&\cdot \left(\frac{\bar{F}_D^{-1}(\sqrt{\frac{c-s}{k+v-s}})}{\sqrt{k+v-s}} \int_0^{\bar{F}_D^{-1}(\sqrt{\frac{c-s}{v-s}})} uf_D(u)du - \frac{\bar{F}_D^{-1}(\sqrt{\frac{c-s}{v-s}})}{\sqrt{v-s}} \int_0^{\bar{F}_D^{-1}(\sqrt{\frac{c-s}{k+v-s}})} uf_D(u)du \right)
\end{aligned}
$$

Terms outside the parentheses are positive since $s < c < v$ and D has a non-negative support. Thus, the last equality is non-positive if and only if terms in the parentheses give non-positive value. Analysis of the terms in parentheses will provide the sufficient and necessary condition:

$$
\begin{aligned}
\frac{\bar{F}_D^{-1}(\sqrt{\frac{c-s}{k+v-s}})}{\sqrt{k+v-s}} \int_0^{\bar{F}_D^{-1}(\sqrt{\frac{c-s}{v-s}})} uf_D(u)du &- \frac{\bar{F}_D^{-1}(\sqrt{\frac{c-s}{v-s}})}{\sqrt{v-s}} \int_0^{\bar{F}_D^{-1}(\sqrt{\frac{c-s}{k+v-s}})} uf_D(u)du \le 0 \\
\frac{\bar{F}_D^{-1}(\sqrt{\frac{c-s}{k+v-s}})}{\sqrt{k+v-s}} \int_0^{\bar{F}_D^{-1}(\sqrt{\frac{c-s}{v-s}})} uf_D(u)du &\le \frac{\bar{F}_D^{-1}(\sqrt{\frac{c-s}{v-s}})}{\sqrt{v-s}} \int_0^{\bar{F}_D^{-1}(\sqrt{\frac{c-s}{k+v-s}})} uf_D(u)du \\
\sqrt{\frac{v-s}{k+v-s}} \frac{\int_0^{\bar{F}_D^{-1}(\sqrt{\frac{c-s}{v-s}})} uf_D(u)du}{\int_0^{\bar{F}_D^{-1}(\sqrt{\frac{c-s}{k+v-s}})} uf_D(u)du} &\le \frac{\bar{F}_D^{-1}(\sqrt{\frac{c-s}{v-s}})}{\bar{F}_D^{-1}(\sqrt{\frac{c-s}{k+v-s}})} \\
\beta^* &\le \beta_Q
\end{aligned}
$$

Recall that we provide one sufficient condition in Proposition A1 for the last inequality to hold. Then, this is sufficient to say that $\frac{\partial \beta^*}{\partial c} \le 0$. We have shown that the threshold level for limited production decreases with the marginal cost of the supply c. Therefore, the threshold level for the more expensive source, $\beta^*_{c_H}$, is less than the threshold level for the cheaper source, $\beta^*_{c_L}$. Simply, $\beta^*_{c_H} < \beta^*_{c_L}$ since $c_L < c_H$. □

Proof of Proposition 13:

We derive the conditions which dictate the choice of the source by the profit maximizing producer for the high intensity region. We define $v' = v + k$ without loss of generality. (Proof for the low intensity region can be obtained by setting $k = 0$ and $\beta = 1$). To generalize our results for all demand distributions, we derive the structure of the profit function for changing cost of the supplier. We show that there exists a global maximum at c^*, at least one inflection point in (c^*, v'), and a global minimum at v'. Recall $\Pi^*_{N,s}(c)$ from the proof of Lemma 5 that stands for the optimal profit of the producer experiencing high intensity of snobs under limited production strategy:

$$\Pi^*_{N,s}(c) = \sqrt{(c-s)(v'-s)}\beta \int_0^{\bar{F}_D^{-1}(\sqrt{\frac{c-s}{v'-s}})} u f_D(u) du$$

$\Pi^*_{N,s}(c)$ is a continuous function on the closed interval $[s, v']$, and differentiable on the open interval (s, v'), where $s < v'$. Note that $\Pi^*_{N,s}(s) = 0$ and $\Pi^*_{N,s}(v') = 0$. Then, there exists at least one c^* in (s, v') such that $\frac{\partial \Pi^*_{N,s}(c^*)}{\partial c} = 0$ by the mean value theorem. Now that we show there must be at least one extreme point within (s, v'), the next step is to show that there can only be one extreme point which is a global maximum in (s, v'). We check the first derivative of $\Pi^*_{N,s}(c)$ with respect to c:

$$\begin{aligned}
\frac{\partial \Pi_{N,s}(c)}{\partial c} &= \frac{\beta}{2}\sqrt{\frac{v'-s}{c-s}} \int_0^{\bar{F}_D^{-1}(\sqrt{\frac{c-s}{v'-s}})} u f_D(u) du - \frac{\beta \bar{F}_D^{-1}\left(\sqrt{\frac{c-s}{v'-s}}\right)}{2} \\
&= \frac{\beta}{2}\sqrt{\frac{v'-s}{c-s}} \left(-\bar{F}_D^{-1}(\sqrt{\frac{c-s}{v'-s}}) \bar{F}_D(\bar{F}_D^{-1}(\sqrt{\frac{c-s}{v'-s}})) + \int_0^{\bar{F}_D^{-1}(\sqrt{\frac{c-s}{v'-s}})} \bar{F}_D(u) du \right) - \frac{\beta \bar{F}_D^{-1}\left(\sqrt{\frac{c-s}{v'-s}}\right)}{2} \\
& \qquad (Int.\ by\ parts) \\
&= \frac{\beta}{2}\sqrt{\frac{v'-s}{c-s}} \int_0^{\bar{F}_D^{-1}(\sqrt{\frac{c-s}{v'-s}})} \bar{F}_D(u) du - \beta \bar{F}_D^{-1}\left(\sqrt{\frac{c-s}{v'-s}}\right)
\end{aligned}$$

Any extreme point c^* in $[s, v']$ must satisfy $\frac{\partial \Pi_{N,s}(c^*)}{\partial c} = 0$:

$$\int_0^{\bar{F}_D^{-1}(\sqrt{\frac{c^*-s}{v'-s}})} \bar{F}_D(u) du = 2\bar{F}_D^{-1}\left(\sqrt{\frac{c^*-s}{v'-s}}\right) \sqrt{\frac{c^*-s}{v'-s}} \tag{A13}$$

We check the sign of the second derivative of $\Pi^*_{N,s}(c)$ with respect to c at the possible extreme points:

$$\begin{aligned}
\frac{\partial^2 \Pi_{N,s}(c)}{\partial c^2} &= \frac{-\beta}{4} \frac{\sqrt{v'-s}}{(c-s)^{3/2}} \int_0^{\bar{F}_D^{-1}(\sqrt{\frac{c-s}{v'-s}})} \bar{F}_D(u) du \\
&+ \frac{\beta}{2}\sqrt{\frac{v'-s}{c-s}} \bar{F}_D\left(\bar{F}_D^{-1}(\sqrt{\frac{c-s}{v'-s}})\right) \frac{-1}{2} \frac{1}{\sqrt{(c-s)(v'-s)}} \frac{1}{f_D(\bar{F}_D^{-1}(\sqrt{\frac{c-s}{v'-s}}))} \\
&- \frac{\beta}{2} \frac{-1}{\sqrt{(c-s)(v'-s)}} \frac{1}{f_D(\bar{F}_D^{-1}(\sqrt{\frac{c-s}{v'-s}}))} \\
&= \frac{\beta}{4} \frac{1}{\sqrt{(c-s)(v'-s)}} \left(-\frac{v'-s}{c-s} \int_0^{\bar{F}_D^{-1}(\sqrt{\frac{c-s}{v'-s}})} \bar{F}_D(u) du + \frac{1}{f_D(\bar{F}_D^{-1}(\sqrt{\frac{c-s}{v'-s}}))} \right)
\end{aligned}$$

$$
\begin{aligned}
\frac{\partial^2 \Pi_{N,s}(c^*)}{\partial c^2} &= \frac{1}{4}\frac{\beta}{\sqrt{(c^*-s)(v'-s)}}\left(-\frac{v'-s}{c^*-s}\int_0^{\bar{F}_D^{-1}(\sqrt{\frac{c^*-s}{v'-s}})}\bar{F}_D(u)du + \frac{1}{f_D(\bar{F}_D^{-1}(\sqrt{\frac{c^*-s}{v'-s}}))}\right)\\
&= \frac{1}{4}\frac{\beta}{\sqrt{(c^*-s)(v'-s)}}\left(-\frac{v'-s}{c^*-s}2\bar{F}_D^{-1}\left(\sqrt{\frac{c^*-s}{v'-s}}\right)\sqrt{\frac{c^*-s}{v'-s}} + \frac{1}{f_D(\bar{F}_D^{-1}(\sqrt{\frac{c^*-s}{v'-s}}))}\right) \quad (by\ (A13))\\
&= \frac{1}{4}\frac{\beta}{\sqrt{(c^*-s)(v'-s)}}\left(-\frac{2\bar{F}_D^{-1}\left(\sqrt{\frac{c^*-s}{v'-s}}\right)}{\sqrt{\frac{c^*-s}{v'-s}}} + \frac{1}{f_D(\bar{F}_D^{-1}(\sqrt{\frac{c^*-s}{v'-s}}))}\right)
\end{aligned}
$$

Terms outside the parentheses are positive since $s < c < v'$. Thus, the sign of the last equality is dictated by the terms in the parentheses. We define $\bar{F}_D^{-1}\left(\sqrt{\frac{c^*-s}{v'-s}}\right) = \xi^*$. Note that $\frac{\partial \xi^*}{\partial c^*} \leq 0$. The analysis of the terms in the parentheses reveals the following result:

$$
-\frac{2\bar{F}_D^{-1}\left(\sqrt{\frac{c^*-s}{v'-s}}\right)}{\sqrt{\frac{c^*-s}{v'-s}}} + \frac{1}{f_D(\bar{F}_D^{-1}(\sqrt{\frac{c^*-s}{v'-s}}))} = -\frac{2\xi^*}{\bar{F}_D(\xi^*)} + \frac{1}{f_D(\xi^*)} = \frac{1}{f_D(\xi^*)}\left(-\frac{2f_D(\xi^*)\xi^*}{\bar{F}_D(\xi^*)} + 1\right)
$$

Hence, the structure of the function at the potential extreme point is dictated by the following conditions:

- $\frac{\partial^2 \Pi_{N,s}(c^*)}{\partial c^2} < 0$ if and only if $\frac{f_D(\xi^*)\xi^*}{\bar{F}_D(\xi^*)} > \frac{1}{2}$
- $\frac{\partial^2 \Pi_{N,s}(c^*)}{\partial c^2} > 0$ if and only if $\frac{f_D(\xi^*)\xi^*}{\bar{F}_D(\xi^*)} < \frac{1}{2}$
- $\frac{\partial^2 \Pi_{N,s}(c^*)}{\partial c^2} = 0$ if and only if $\frac{f_D(\xi^*)\xi^*}{\bar{F}_D(\xi^*)} = \frac{1}{2}$

Recall that the demand in our model has an increasing generalized failure rate (IGFR) property. Then, when we move from a potential extreme point to a higher potential extreme point, $g(\xi^*)$ must decrease since ξ^* decreases with c^*. This property eliminates the possibility of more than one combination of local maximum and local minimum in (s, v').

We know that $\Pi^*_{N,s}(s) = 0$, $\Pi^*_{N,s}(v') = 0$ and $\Pi^*_{N,s}(c) > 0$ in (s, v'). Note that $\frac{\partial \Pi_{N,s}(s)}{\partial c}$ and $\frac{\partial^2 \Pi_{N,s}(s)}{\partial c^2}$ are undefined so the function might be tangent at s since we know that the function is continuous and differentiable in (s, v'). Since $\lim_{c\to s^+} \frac{\partial^2 \Pi_{N,s}(c)}{\partial c^2} < 0$, the function can only be increasing concave after s. Having increasing concave structure $\forall c \in (s, s+\varepsilon)$ implies that the first extreme point in (s, v') can either be an inflection point or a local maximum.

It is easy to show that the first extreme point can not be an inflection point by contradiction. Having inflection point first as an extreme point would imply that there exists no local maximum since $g(\xi)$ can not attain values larger than $1/2$ anymore. This contradicts with the fact that function returns back to 0 at v'. Thus, the first extreme point must be a local maximum which we state as c^*. Since there is no possibility of more than one combination of local maximum and local minimum in (s, v'), next possible set of extreme points after c^* is a set of inflection points plus a local minimum point. In fact, it can be immediately shown that there exists at least one inflection point in (c^*, v') by the mean value theorem. Hence, the unique local minimum is v' since $\frac{\partial \Pi_{N,s}(v')}{\partial c} = 0$ and $\frac{\partial^2 \Pi_{N,s}(v')}{\partial c^2} > 0$.

We have shown that $\Pi^*_{N,s}$ reaches a global maximum at c^*, has at least one inflection point in (c^*, v'), and reaches a global minimum at v'.

Therefore,

1. If $(c_H >) c_L \geq c^*$ then $\Pi^*_{N,s}(c_L) \geq \Pi^*_{N,s}(c_H)$. [Region D]
2. If $(c_L <) c_H \leq c^*$ then $\Pi^*_{N,s}(c_L) \leq \Pi^*_{N,s}(c_H)$. [Region A]
3. If $c_L < c^* < c_H < c_{equal}$ then $\Pi^*_{N,s}(c_L) \leq \Pi^*_{N,s}(c_H)$. [Region B]
4. If $c_L < c^* < c_{equal} < c_H$ then $\Pi^*_{N,s}(c_L) \geq \Pi^*_{N,s}(c_H)$. [Region C]

where $\Pi^*_{N,s}(c_L) = \Pi^*_{N,s}(c_H)$ when $c_H = c_{equal}$.

$\square$