10-2002

Competition and Outsourcing with Scale Economies

Gerard. P. Cachon
University of Pennsylvania

Patrick T. Harker

Recommended Citation

Cachon, G. P., & Harker, P. T. (2002). Competition and Outsourcing with Scale Economies. *Management Science, 48* (10), 1314-1333. http://dx.doi.org/10.1287/mnsc.48.10.1314.271

Competition and Outsourcing with Scale Economies

Abstract

Scale economies are commonplace in operations, yet because of analytical challenges, relatively little is known about how firms should compete in their presence. This paper presents a model of competition between two firms that face scale economies; (i.e., each firm's cost per unit of demand is decreasing in demand). A general framework is used, which incorporates competition between two service providers with price- and time-sensitive demand (a queuing game), and competition between two retailers with fixed-ordering costs and pricesensitive consumers (an Economic Order Quantity game). Reasonably general conditions are provided under which there exists at most one equilibrium, with both firms participating in the market. We demonstrate, in the context of the queuing game, that the lower cost firm in equilibrium may have a higher market share and a higher price, an enviable situation. We also allow each firm to outsource their production process to a supplier. Even if the supplier's technology is no better than the firms' technology and the supplier is required to establish dedicated capacity (so the supplier's scale can be no greater than either firm's scale), we show that the firms strictly prefer to outsource. We conclude that scale economies provide a strong motivation for outsourcing that has not previously been identified in the literature.

Keywords

service operations, nash equilibrium, coproduction, economies of scale, ECQ, queuing

Disciplines

Operations and Supply Chain Management | Other Economics

Competition and Outsourcing with Scale Economies*

Gérard P. Cachon
cachon@wharton.upenn.edu
http://opim.wharton.upenn.edu/~cachon

Patrick T. Harker
harker@opim.wharton.upenn.edu
http://opim.wharton.upenn.edu/~harker

The Wharton School · University of Pennsylvania · Philadelphia PA 19104

November 2001

Abstract

Scale economies are commonplace in operations, yet, due to analytical challenges, relatively little is known about how firms should compete in their presence. This paper presents a model of competition between two firms that face scale economies; i.e., each firm's cost per unit of demand is decreasing in demand. A general framework is used, which incorporates competition between two service providers with price and time sensitive demand (a queuing game) and competition between two retailers with fixed ordering costs and price sensitive consumers (an EOQ game). Reasonably general conditions are provided under which there exists at most one equilibrium with both firms participating in the market. We demonstrate, in the context of the queuing game, that the lower cost firm in equilibrium may have higher market share and a higher price, an enviable situation. We also allow each firm to outsource their production process to a supplier or to their customers (e.g., co-production). Even if the supplier's technology is no better than the firms' technology and the supplier is required to establish dedicated capacity (so the supplier's scale can be no greater than either firm's scale), we show that the firms strictly prefer to outsource. We conclude that scale economies provide a strong motivation for outsourcing that has not previously been identified in the literature.

*Thanks is extended to the seminar participants at the following universities: the Department of Operations Research, University of North Carolina; the Department of Industrial and Operations Engineering, Univeristy of Michigan; the Graduate School of Business, Stanford University; the Anderson School of Business, Univeristy of California at Los Angeles; the 1999 MIT Summer Camp, Sloan School of Business, MIT; the Operations Management Department, University of Michigan; and the Management Department, University of Texas at Austin. Thanks is also extended to Philip Afeche, Frances Frei, Noah Gans, Martin Lariviere and Erica Plambeck for their many helpful comments. The previous version of this paper was titled "Service Competition, Outsourcing and Co-Production in a Queuing Game." An electronic copy is available from the first author's web page.

Scales economies are commonplace in operations. But while there is a considerable operations management literature that identifies scale economies and develops strategies to exploit them, relatively little is known about how firms should compete in their presence. Even the economics literature on competition among firms generally assumes constant or decreasing returns to scale, so as to avoid the significant analytical complications scale economies create (Vives, 1999). Nevertheless, research is needed on this challenge.

This paper studies competition between two firms that face scale economies; i.e., cost per unit of demand is decreasing in demand. A general framework is employed: it includes, among others, competition between service providers (i.e., a queuing game) and competition between two retailers with fixed ordering costs (i.e., an Economic Order Quantity game). Firms compete for demand with two instruments: the explicit prices they charge consumers and the operational performance levels they deliver. An example of the latter in the context of the queuing game is the firm's expected service time, where faster service means better operational performance.

Competition with scale economies is brutal for two reasons. First, a firm must capture a positive threshold of demand or else it is not profitable (i.e., small players cannot be profitable). Second, scale economies increase price competition: a price cut increases demand, which lowers the average cost per unit of demand. As a result, an equilibrium may not exist, even with symmetric firms (i.e., firms with the same cost and demand). However, when an equilibrium exists in which both firms have positive demand, then it is unique, under reasonable conditions. Hence, competition in this setting does have some structure. We show that the low cost firm always has a higher market share in equilibrium, which is not surprising. What is unexpected is that the low cost firm can also have the higher price, which is certainly an enviable position: the firm uses its lower cost to dominate with operational performance, which allows the firm to charge a premium and capture more demand than its rival. As an added bonus, the higher demand also allows the firm to operate more efficiently than its rival. Furthermore, in low margin conditions a small cost advantage can yield an enormous profit advantage even if it does not result in a large market share difference.

In this environment, firms could benefit from any strategy that mitigates price competitiveness. We show that outsourcing is one such strategy. We suppose that there exists a supplier with the same technology as the firms. This supplier is able to manage either firm's operations and charges a constant fee per unit of demand for that service. The supplier establishes dedicated capacity for each firm that outsources, so the supplier is unable to pool demand across firms to gain efficiency. In other words, the supplier is operationally no more efficient than either firm. Yet, we show that there are contracts that yield the supplier a positive profit and yield a higher profit to either firm than if they insourced (i.e., did not outsource with the supplier). Hence, all firms are better off with outsourcing. In this setting, the firms do not outsource because the supplier is cheaper (by assumption either firm is able to generate exactly the same cost as the supplier without paying the supplier's margin). Instead, they outsource because outsourcing dampens price competition. It is also possible that a firm can benefit from a unilateral move to outsource, i.e., a firm may find outsourcing profitable even if its competitor does not outsource. These results do not occur with a constant returns to scale technology. Hence, we conclude that in the presence of scale economies firms can benefit from outsourcing even if their supplier is unable to gain any scale advantages.

Outsourcing to another firm is not the only way to change the nature of the production process. If the firm is offering a service, then the firm may be able to outsource some of the production process onto its customers; i.e., the firms can make its customers co-producers. Again, we show that firms may use co-production even if it increases a firm's cost; i.e., the price discount the firm must give consumers to compensate for their co-production is greater than the cost the firm would incur if the firm did the service itself.

The next section reviews literature relevant to this work. §2 details our model. §3 analyzes equilibrium behavior between two firms. §4 considers the impact of outsourcing. The final section concludes.

1 Literature review

The body of research related to this work can be divided into three broad sets. The first includes papers that use queuing theory to study the delivery of services. The second set studies competition between firms that set inventory policies. The third is the literature on outsourcing and vertical integration in operations management, marketing and economics.

As mentioned in the introduction, competing queues is one of the games that falls into our framework. There are many papers that investigate competition when customers are sensitive to time: Armory and Haviv (1998), Chayet and Hopp (1999), Davidson (1988), De Vany (1976), De Vany and Saving (1983), Gans (2000), Gilbert and Weng (1997), Kalai, Kamien and Rubinovitch (1992), Lederer and Li (1997), Li (1992), Li and Lee (1994), and Loch (1994). In most of these models firms compete either with prices or with processing rates, but not both.[1] Those authors recognized that allowing for both decisions creates significant analytical complications; in particular, the firms' profit functions are not well behaved (unimodal). Further, qualitative statements regarding competition in that setting are not possible since pure strategy equilibria do not exist. A second distinction is that in many of those models customers wait in a single queue.[2] In our model, the firms maintain separate queues and customers are not able to jockey between. Further, with a single queue framework total market demand is constant (i.e., all customers join the queue and are eventually served). We allow for demand functions in which total market demand may decrease.

Deneckere and Peck (1995) and Reitman (1991) do consider a model in which firms simultaneously choose prices and processing rates, and customers choose firms based on expected utility maximization. However, there are no scale economies in their production processes, which is the main focus of this paper.

Gans (2000,2002) and Hall and Porteus (2000) consider competition between firms when customer chooses between firms based on their past service encounters. In our model,

---

[1] Li and Lee (1992) analyze a model with fixed processing rates and then discuss how the model could be expanded to allow the firms to choose prices as well. However, they emphasize that the lack of pure strategy equilibria in that game imposes a significant challenge to the analysis of the expanded game. In Lederer and Li (1997), the firms have fixed overall production capacity, but they decide how to allocate that capacity across multiple customer classes. In the single class version of their model, the firms only compete on price.

[2] Gilbert and Weng (1997) do consider a model with separate queues, however the arrival process to each queue is set so that each firm has the same expected waiting time.

customers correctly anticipate the expected operational performance of each firm and so demand is not determined by past actions.

Chase (1978) and Karmarkar and Pitbladdo (1995) recognized that an important design decision for a service firm is the degree to which the firm outsources delivery of the service to customers; i.e., the amount of co-production. Ha (1998) considers the interaction between pricing and co-production. In his model a service operation's workload is decreasing in the amount of effort customers exert in preprocessing, which the firm can influence via its price schedule. We assume the amount of co-production is fixed and the firm need only provide a fixed compensation to customers.

Several papers consider pricing and capacity decisions for a single server: Dewan and Mendelson (1990), Stidham (1992), Stidham and Rump (1998), and So and Song (1998). (The first three papers seek to maximize system value, while the last maximizes a firm's profit.) In fact, the queueing game in this paper is a competitive extension of Stidham's model; when there is a single firm in the queueing game, that firm faces the same problem that a monopolist would face in Stidham (1992). (See Cachon and Harker 1999 for details.) Stidham (1992) and Stidham and Rump (1998) also provide an extensive discussion on the stability of the firm's pricing and capacity decisions. Given the formulation of our queueing game, equilibrium stability is not an issue.

Many papers investigate queue joining behavior in which customers compete for fast service, but the service provider is not a game participant: Bell and Stidham (1983), Kulkarni (1983), Lippman and Stidham (1977), Mendelson (1985) and Naor (1969). Afèche and Mendelson (2001) extend this work considerably by incorporating generalized delay cost structures (i.e., a customer's delay cost could be proportional to a customer's valuation of the service) and priority auctions.

We now turn to models of inventory competition. Bernstein and Federgruen (1999) study a two echelon supply chain with one supplier and multiple competing retailers. Each retailers demand rate is deterministic, but a function of the firms' prices. Further, each retailer incurs fixed ordering costs. Hence, our EOQ game is functionally equivalent to their decentralized game (i.e., the game with simple wholesale price contracts.) However, their focus is on channel coordination, which we do not consider, they do not consider outsourcing and they allow for competition among more than two firms. Bernstein and Federgruen (2001) study price and operational performance competition among multiple firms that choose base stock policies, where a firm's operational performance is its fill rate. However, they work with multiplicative demand shocks, so their model has constant returns to scale.

There are a number of papers that study competing firms with demand spillovers; i.e., a portion of the unsatisfied demand at one firm (due to stockouts) transfers to the other firm: Palar (1988), Lippman and McCardle (1995), Karjalainen (1992) , Anupindi and Bassok (1999). Our model does not have demand spillovers.

Finally, there is an extensive literature on outsourcing and vertical integration. In operations management the focus is on when outsourcing reduces costs (see McMillan, 1990; Venkatesan, 1992; van Miegham, 1999). Those papers do not consider the impact of outsourcing on equilibrium prices. In economics the focus is on the location of the firm boundary; i.e., what assets does the firm own. Transaction cost theory suggests this decision hinges on asset specificity, i.e., if the asset's next best use has significantly lower value, then a firm will own the asset (e.g., Williamson, 1979). Grossman and Hart (1986) propose the firm boundary depends on contract incompleteness: if a firm cannot specify all possible future uses for an asset in a contract then the firm will seek ownership if control is sufficiently important. A third, and more recent approach, suggests that asset ownership influences relational contracts, which are unwritten agreements between parties that are support only in repeated games (i.e., if one party breaks a relational contract the other party can punish through future actions). (See Baker, Gibbons and Murphy, 2001.) Our theory of outsourcing is different. We explicitly assume away asset specificity and contract incompleteness, and our single choice model does not allow for future punishment. In our model outsourcing creates value by changing a firm's competitive behavior. In particular, the firm becomes less price competitive.

The paper with the most similar finding to our outsourcing result is from the marketing literature, McGuire and Staelin (1983). They show that competing suppliers prefer to outsource the retailing function to independent retailers rather than to perform their own retailing when demand is sufficiently price competitive. Outsourcing benefits the suppliers when retail price competition is high because double marginalization between the supplier and the retailer mitigates price competition between the two suppliers. In our setting, outsourcing mitigates price competition for different reasons. In our model, price competition derives in part from the need to increase demand to reduces costs, which is not present in McGuire and Staelin (1983) because they consider a constant returns to scale production process. Indeed, if there were constant returns to scale in our production process, then outsourcing would provide no benefit. Further, we consider the outsourcing of the production function and not the retailing function and both firms are better off with outsourcing for all levels of price competition.

There is also work in economics on divisionalization. Baye, Crocker and Ju (1996) show that in a competitive environment a firm may divide itself into multiple competing divisions even if divisionalization is costly because divisionalization mitigates price competition. As with divisionalization, outsourcing divides a firm into multiple pieces (a supplier and the firm). But there are three key differences between divisionalization and outsourcing. First, with divisionalization the parent firm sums its profits across divisions whereas with outsourcing there is no aggregation of profits. Second, with divisionalization all divisions compete for consumers whereas with outsourcing the supplier does not compete for customers. (With divisionalization a process is replicated, with outsourcing it is divided.) Third, even though firms choose to divisionalize, in equilibrium they are worse off after dividing, whereas with outsourcing firms are better off.

2 Model definition

Two firms, firm i and firm j, compete in a market based on their full prices. Unless otherwise noted, rules, parameters and functions that are defined for firm i apply analogously for firm j. Let f_i be firm i's full price. It includes two components: $f_i = p_i + g_i$. The first is the explicit fee, $p_i \geq 0$, firm i charges customers per transaction (e.g., a service occasion or a product purchase). The second, $g_i \geq 0$, is the firm's expected operational performance, where better performance means a lower g_i. For example, in a service context, g_i could be a customer's disutility for the expected time to complete the firm's service.

Firm i's expected demand rate is $d_i(f_i, f_j) \geq 0$ and firm j's is $d_j(f_j, f_i) \geq 0$. For notational parsimony, we often write the demand functions without arguments; e.g. d_i, with the understanding that d_i is always a function of the full prices. Several points are worth emphasizing regarding this demand structure. First, demand depends on *expected* operational performance. In other words, consumers do not have, or are unable to act upon, information that suggests either firm's operational performance will deviate from the expected performance: e.g., in the service context, consumers do not observe the firms' queue lengths before choosing firms (which would suggest either an above or below average service time). Second, a firm's demand depends only its full price and not on the composition of that full price: a high priced firm with fast service has the same demand rate as a low priced firm with slow service if their full prices are equal. Third, a firm's demand does not depend on the variability of its operational performance, which would create significant analytical complications. Finally, there is no ex-post reallocation of demand. For example, poor realized service at firm i does generate additional demand at firm j.

The prices, $\{p_i, p_j\}$, and the operational performance levels, $\{g_i, g_j\}$, are the firms' only actions. Allowing each firm to choose its price requires no justification. To justify that each firm commits to its operational performance, consider the natural alternative: each firm commits to an explicit operational decision; e.g., the firm's capacity. Operational performance depends on that operational decision and the firm's demand rate; e.g., for a fixed demand rate the waiting time in queue decreases as service capacity is added, and for a fixed capacity waiting time increases with the demand rate. Hence, to evaluate a firm's expected operational performance, a consumer must observe a firm's operational decision, forecast the firm's demand and understand the relationship between them. But because demand depends on operational performance, the poor consumer must solve for an equilibrium: what demand rate generates an operational performance that leads to that demand rate? This surely imposes a high computational burden on consumers. Our construction is gentler. Because a firm commits to its operational performance, the consumer does not need to forecast the firm's demand: the realized demand rate has no impact on the consumer's choice. However, the firm must have the ability to adjust its operational decisions in response to changes in the demand rate so that its operational performance commitment is indeed credible. In the short run, this may be possible for small deviations in the demand rate, but probably not possible for large deviations. Over a long horizon, this assumption is not onerous: the firm solves for the demand-rate-operational-performance equilibrium (and not consumers) and then chooses the operational decisions to generate that equilibrium.

Firms simultaneously choose their actions and then demand occurs over an infinite horizon.[3] Both firms are risk neutral and seek to maximize their expected profit rate. For fixed f_i and f_j, and hence for fixed demand rates, we assume there exists a unique optimal operational performance for each firm. Furthermore, conditional that optimal operational performances are chosen, firm i's profit function has the following form

$$\pi_i(f_i, f_j) = (f_i - c_i)d_i(f_i, f_j) - \phi_i d_i(f_i, f_j)^{\gamma_i}, \tag{1}$$

where $c_i > 0$, $\phi_i \geq 0$ and $0 \leq \gamma_i < 1$ are constants. Firm j's profit function, $\pi_j(f_j, f_i)$, is analogous. As with the demand functions, we often write the profit functions without arguments; e.g. π_i and π_j. In (1), $f_i d_i$ resembles the standard revenue function, with the distinction being that actual revenue depends on p_i and not f_i. The second term, $c_i d_i$, is the standard linear cost function. The third term, $\phi_i d_i^{\gamma_i}$, generates the firm's scale economies: the cost per unit of demand, $c_i + \phi_i d_i^{\gamma_i - 1}$, is decreasing in d_i. If $\gamma_i = 0$ then there is a fixed cost independent of demand. Given the profit functions (1), this game can be analyzed as a game in which each firm decides on a single action, its full price.

Some additional reasonable restrictions are needed on the demand functions. Demand is never negative, and for any finite $f_j \geq 0$, there exists a finite f_i such that $d_i = 0$. Define $\tilde{f}_i(f_j)$ to be the smallest of those full prices; i.e., firm i can always price itself out of the market.[4] We assume $\tilde{f}_i(f_j) - f_j$ is decreasing in f_j, i.e., firm i's price premium to exit the market is decreasing in firm j's price. For all $f_i < \tilde{f}_i(f_j)$, $d_i(f)$ is differentiable, $\partial d_i/\partial f_i < 0$, $\partial d_i/\partial f_j > 0$ and $-\partial d_i/\partial f_i \geq \partial d_i/\partial f_j$. The latter implies firm i's demand is more sensitive to firm i's full price than to firm j's full price. Furthermore, $d_i(0,0) > 0$ (i.e., firm i can have positive demand for a sufficiently low price), which implies that $\tilde{f}_i(f_j) > 0$. Finally, there exists some f_i such that $\pi_i(f_i, \tilde{f}_j(f_i)) > 0$; i.e., demand is sufficiently large that firm i can earn a positive profit if firm j exits the market.

To summarize, the firms play a simultaneous single move game with full prices as their

[3] We do not consider sequential choice games: e.g., firms choose $\{g_i, g_j\}$ and then after observing those choices they choose $\{p_i, p_j\}$, or firm i choses $\{p_i, g_i\}$ and then firm j chooses $\{p_j, g_j\}$. Bernstein and Federgruen (2001) consider the former type of sequential choice and Chayet and Hopp (1999) consider the latter.

[4] While it is possible to relax this assumption, it is cumbersome to also include the case $\tilde{f}_i(f_j) = \infty$.

strategies and (1) as their profit functions. It remains to identify specific models that conform to this structure. Two such model are detailed next.

2.1 A queuing game

Suppose each firm provides a service. Let g_i be the expected amount of time a customer spends at firm i, including time in queue and time in service. Suppose customer inter-arrival times at firm i are exponentially distributed with mean $1/d_i$. Customers wait in a single first-come-first serve queue to receive service at firm i and there is no balking. The processing times at firm i are exponentially distributed with rate μ_i. The expected time a customer spends at firm i is

$$g_i = (\mu_i - d_i)^{-1}, \tag{2}$$

assuming $\mu_i > d_i$. The steady state distribution of the number of customers at either firm is the same as the number of units in an $M/M/1$ queue.

Let k_i be firm i's capacity cost rate per unit of capacity, $k_i > 0$. From (2), firm i's expected capacity cost per unit time is $k_i \left(d_i + g_i^{-1}\right)$. Naturally, firm i incurs a higher capacity cost when it lowers its customers' service time.

Firm i's profit rate is

$$\pi_i(f_i, g_i, f_j) = (f_i - g_i - k_i)\, d_i - k_i g_i^{-1},$$

where recall $p_i = f_i - g_i$. For fixed f, the above is strictly concave in g_i and the optimal operational performance, $g_i^*(f)$, is $g_i^*(f) = \sqrt{k_i/d_i(f)}$. Given the above, $\pi_i(f_i, g_i^*(f), f_j) = \pi_i(f_i, f_j)$ and

$$\pi_i(f_i, f_j) = (f_i - k_i)\, d_i - 2\sqrt{k_i d_i},$$

which conforms to (1) when $c_i = k_i$, $\phi_i = 2\sqrt{k_i}$, and $\gamma_i = 1/2$.

2.2 An EOQ inventory game

Suppose each firm sells a product. Demand is deterministic with rate d_i. The firm pays a wholesale price w_i per unit purchased, incurs a fixed cost k_i for each replenishment, which arrives immediately, and incurs h_i per unit of inventory per unit of time. Neither firm backorders demand, so from a customer's perspective the firms have identical operational performance: let $g_i = g_j = 0$. In this game there is an industry standard regarding operational performance (i.e., no backorders) so competition between the firms occurs only with their explicit prices. Nevertheless, a firm's profit depends on the cost of delivering that performance, which depends on demand.

Firm i's profit rate is

$$\pi_i(f_i, f_j) = (f_i - w_i)\, d_i - \left(k_i d_i q_i^{-1} + h_i q_i/2\right),$$

where $f_i = p_i$, and q_i is the firm's order quantity, i.e., its operational decision. The latter part of the firm's cost corresponds to the cost function of the well known economic order quantity (EOQ) problem. The cost minimizing order quantity is $q_i^* = (2k_i d_i/h_i)^{-1/2}$. The firm's expected profit rate is then

$$\pi_i(f_i, f_j) = (f_i - w_i)\, d_i - (2h_i k_i d_i)^{1/2}$$

which conforms to (1) when $c_i = w_i$, $\phi_i = \sqrt{2h_i k_i}$ and $\gamma_i = 1/2$.

3 Analysis of equilibrium

A Nash equilibrium in this game is a pair of full prices, $\{f_i^*, f_j^*\}$, such that neither firm has a profitable unilateral deviation. In this game analysis of equilibrium is complex because the firms' profit functions are not unimodal. Hence, standard theorems for demonstrating existence and uniqueness cannot be applied. Nevertheless, we present conditions under which each firm's profit function has a single interior local maximum. That provides enough structure to obtain some results on existence and uniqueness of equilibrium.

Define firm i's reaction correspondence

$$r_i(f_j) = \{f_i \geq 0 : f_i \in \arg\max_{f_i} \pi_i(f_i, f_j)\}.$$

A pair of full prices, $\{f_i^*, f_j^*\}$ is a Nash equilibrium if $f_i^* \in r_i(f_j^*)$ and $f_j^* \in r_j(f_i^*)$. Define $f_i^*(f_j)$ as the smallest solution to firm i's first-order condition:

$$f_i^*(f_j) = \min\left\{0 \leq f_i < \tilde{f}_i(f_j) : \frac{\partial \pi_i}{\partial f_i} = 0\right\},$$

where $f_i^*(f_j) = \varnothing$ if there is no solution to the first-order condition. Due to scale economies, there may exist multiple solutions to the first-order condition or there may be no solution. The problem is that π_i is negative and convex if f_i is too close to $\tilde{f}_i(f_j)$; i.e., if demand is too low. However, according to the next theorem, under reasonable conditions $r_i(f_j)$ contains only one element if there exists some full price that generates positive profits for firm i. The condition in the following theorem is assumed throughout.

Theorem 1 *If*

$$-d_i \left(\frac{\partial d_i}{\partial f_i} \right)^{-1}$$

is decreasing and strictly convex in f_i *for* $f_i \leq \tilde{f}_i(f_j)$, *then*

$$r_i(f_j) = \begin{cases} \{f_i : f_i \geq \tilde{f}_i(f_j)\} & f_i^*(f_j) = \varnothing \text{ or } \pi_i(f_i^*(f_j), f_j) < 0 \\ \{f_i : f_i \geq \tilde{f}_i(f_j)\} \cup f_i^*(f_j) & \pi_i(f_i^*(f_j), f_j) = 0 \\ f_i^*(f_j) & \pi_i(f_i^*(f_j), f_j) > 0 \end{cases}.$$

Proof. Differentiate and rearrange terms:

$$\begin{aligned} \frac{\partial \pi_i}{\partial f_i} &= d_i + \left(f_i - c_i - \gamma_i \phi_i d_i^{\gamma_i - 1} \right) \frac{\partial d_i}{\partial f_i} \\ &= \left(-\frac{\partial d_i}{\partial f_i} \right) \left(-(f_i - c_i) + \left[-d_i \left(\frac{\partial d_i}{\partial f_i} \right)^{-1} + \gamma_i \phi_i d_i^{\gamma_i - 1} \right] \right). \end{aligned} \tag{3}$$

Since $\partial d_i / \partial f_i < 0$ and $d_i > 0$ for $f_i < \tilde{f}_i(f_j)$, it follows that $\partial \pi_i / \partial f_i > 0$ for $f_i = 0$. Furthermore, $\partial \pi_i / \partial f_i \to \infty$ as $f_i \to \tilde{f}_i(f_j)$. Thus, it is optimal for firm i to either price itself out of the market, $f_i \geq \tilde{f}_i(f_j)$, or to choose some interior $0 < f_i < \tilde{f}_i(f_j)$ that satisfies the first order condition. Recall that $f_i^*(f_j)$ is the smallest f_i that satisfies the first order condition. It is the unique interior optimal f_i if $\pi_i(f_i^*(f_j), f_j) > 0$ and if it can be shown there exists a unique pair (f_i', f_i''), $0 < f_i' \leq f_i'' < \tilde{f}_i(f_j)$, such that $\partial \pi_i / \partial f_i$ is positive for $0 \leq f_i \leq f_i'$, negative for $f_i' \leq f_i \leq f_i''$ and positive for $f_i'' \leq f_i \leq \tilde{f}_i(f_j)$. If that holds and $f_i' < f_i''$, then $f_i^*(f_j) = f_i'$ is a local maximum and f_i'' is a local minimum. If $f_i' = f_i''$ then $\pi_i(f_i', f_j) < 0$.

From (3), $\partial \pi_i / \partial f_i < 0$ when

$$(f_i - c_i) > \left[-d_i \left(\frac{\partial d_i}{\partial f_i} \right)^{-1} + \gamma_i \phi_i d_i^{\gamma_i - 1} \right]. \tag{4}$$

(4) neither holds for $f_i = 0$ (because $d_i(0, f_j) > 0$) nor for $f_i = \tilde{f}_i(f_j)$ (because then $d_i(f) = 0$). The left hand side is positive and linearly increasing in f_i. The right hand side is positive. Therefore, the $\{f_i', f_i''\}$ pair exists if the right hand side is strictly convex for $f_i \leq \tilde{f}_i(f_j)$. (Note that $f_i' = f_i''$ is possible.) The second term on the right hand side of (4), $\gamma_i \phi_i d_i^{\gamma_i - 1}$, is strictly convex in f_i if $-d_i \left(\partial d_i / \partial f_i \right)^{-1}$ is decreasing. Thus, the right hand side of (4) is strictly convex if $-d_i \left(\partial d_i / \partial f_i \right)^{-1}$ is also strictly convex.□

The following demand functions satisfy the above requirement: linear demand,

$$d_i(f_i, f_j) = a_i - b_i f_i + \beta_i f_j$$

with $a_i > 0$, $b_i > 0$ and $b_i > \beta_i > 0$; and truncated logit demand,

$$d_i(f_i, f_j) = \left[m \frac{a_i e^{bf_i}}{a_i e^{bf_i} + a_j e^{bf_j}} - \varepsilon \right]^+$$

with $a_i > 0$, $b < 0$ and $m > 2\varepsilon > 0$.[5] Note that d_i may be convex in f_i, but not too convex.[6]

The next theorem further characterizes each firm's optimal response. In particular, it demonstrates that there is a single discontinuity in $r_i(f_j)$ (at $\mathring{f}_j$) and $r_i(f_j)$ is a function for all $f_j > \mathring{f}_j$.

Theorem 2 *There exists an $\mathring{f}_j \geq 0$ such that $\pi_i(f_i^*(\mathring{f}_j), \mathring{f}_j) = 0$ and $\pi_i(f_i^*(f_j), f_j) > 0$ for all $f_j > \mathring{f}_j$.*

Proof. By assumption, $\pi_i(f_i^*(f_j), f_j) > 0$ for some f_j. From the envelope theorem:

$$\begin{aligned}
\frac{d\pi_i(f_i^*(f_j), f_j)}{df_j} &= \frac{\partial \pi_i(f_i^*(f_j), f_j)}{\partial f_i} \frac{\partial f_i^*(f_j)}{\partial f_i} + \frac{\partial \pi_i(f_i^*(f_j), f_j)}{\partial f_j} \\
&= \left(f_i^*(f_j) - c_i - \gamma_i \phi_i d_i^{\gamma_i - 1} \right) \frac{\partial d_i}{\partial f_j} \\
&= -d_i \frac{\partial d_i}{\partial f_j} \left(\frac{\partial d_i}{\partial f_i} \right)^{-1} > 0
\end{aligned}$$

because $\partial \pi_i(f_i^*(f_j), f_j)/\partial f_i = 0$ when $\pi_i(f_i^*(f_j), f_j) \geq 0$. Thus, when $f_i^*(f_j)$ exists, π_i is strictly increasing in f_j. (When $f_i^*(f_j)$ does not exist, π_i is strictly increasing in f_i and so $\tilde{f}_i(f_j)$ is optimal for firm i.) Hence, there exists some $\mathring{f}_j$ such that $\pi_i(f_i^*(\mathring{f}_j), \mathring{f}_j) = 0$ and $\pi_i(f_i^*(f_j), f_j) > 0$ for all $f_j > \mathring{f}_j$.□

Due to the discontinuity in $r_i(f_j)$, existence of a Nash equilibrium is not assured.[7] Al-

[5] The b constant must be the same for firm i and firm j due to the $-\partial d_i(f)/\partial f_i \geq \partial d_i(f)/\partial f_j$ requirement. $\varepsilon > 0$ ensures that a finite $\tilde{f}_i(f_j)$ exists. $m > 2\varepsilon$ ensures that $d_i(0,0) > 0$.

[6] Convex $1/d_i(f)$ is the most general condition for quasi-concave payoff functions when $\gamma \geq 1$ (i.e., costs are convex and increasing in demand), which is equivalent to the condition that the slope of $-d_i(f)(\partial d_i(f)/\partial f_i)^{-1}$ is less than 1. Thus, the condition in Theorem 1 is more restrictive. However, it is not a necessary condition.

[7] Discountinuities in the reaction correspondence do not automatically rule out the ex-

ternatively, there may be multiple equilibria. However, it is possible to provide conditions under which there is at most one Nash equilibrium in which both firms have positive demand. (In other words, if there are multiple equilibria under those conditions, then in all but one of them at least one of the firm exits the market.) We refer to any equilibrium in which both firms have positive demand as a *full-participation* equilibrium.

Theorem 3 *Define*

$$z_i(f_i, f_j) = 1 + \gamma_i \phi_i (1-\gamma_i) d_i^{\gamma_i - 2} \frac{\partial d_i}{\partial f_i}.$$

If for both firms

$$d_i \frac{\partial^2 d_i}{\partial f_i^2} + \left| d_i \frac{\partial^2 d_i}{\partial f_i \partial f_j} - z_i(f_i, f_j) \frac{\partial d_i}{\partial f_i} \frac{\partial d_i}{\partial f_j} \right| < \left(\frac{\partial d_i}{\partial f_i} \right)^2 (1 + z_i(f_i, f_j)) \tag{5}$$

holds for all $\{f_i^*(f_j), f_j\}$ *when* $\pi_i(f_i^*(f_j), f_j) \geq 0$, *then there exists at most one full-participation equilibrium; i.e., an equilibrium in which both firms have positive demand.*

Proof. The first step is to show if $|r_i'(f_j)| < 1$ for all $f_j \geq \mathring{f}_j$ and the same for firm j, then there is at most one equilibrium with positive demand for both firms. (This is less restrictive than showing that the best-reply mapping is a contraction, which it is not.) The second step shows (5) implies those conditions. For the first step proof is by contradiction. Suppose there are two equilibria, $\{f_i^*, f_j^*\}$ and $\{f_i^{**}, f_j^{**}\}$ with $f_j^* < f_j^{**}$. Since both firms have positive demand, $\mathring{f}_i \leq f_i^*$, $\mathring{f}_i \leq f_i^{**}$ and $\mathring{f}_j \leq f_j^*$, i.e., the reaction functions are continuous between the two equilibria. $|r_i'(f_j)| < 1$ implies $\left| r_i(f_j^{**}) - r_i(f_j^*) \right| < f_j^{**} - f_j^*$ and $\left| r_j'(f_i) \right| < 1$ implies $|f_i^{**} - f_i^*| > f_j^{**} - f_j^*$. But $|f_i^{**} - f_i^*| > \left| r_i(f_j^{**}) - r_i(f_j^*) \right| = |f_i^{**} - f_i^*|$: a contradiction. For the second step, assuming $\pi_i(f_i^*(f_j), f_j) \geq 0$, the implicit function theorem provides

$$\frac{\partial r_i(f_j)}{\partial f_j} = -\frac{\partial^2 \pi_i}{\partial f_i \partial f_j} \left(\frac{\partial^2 \pi_i}{\partial f_i^2} \right)^{-1}$$

Using the first-order condition, the above derivatives can be written as

$$\begin{aligned} \frac{\partial^2 \pi_i}{\partial f_i \partial f_j} &= \frac{\partial d_i}{\partial f_j} z_i(f_i, f_j) + d_i \left(-\frac{\partial d_i}{\partial f_i} \right)^{-1} \frac{\partial^2 d_i}{\partial f_i \partial f_j} \\ \frac{\partial^2 \pi_i}{\partial f_i^2} &= \frac{\partial d_i}{\partial f_i} (1 + z_i(f_i, f_j)) + d_i \left(-\frac{\partial d_i}{\partial f_i} \right)^{-1} \frac{\partial^2 d_i}{\partial f_i^2}. \end{aligned}$$

istence of Nash equilibrium. For example there exists a Nash equilibrium if $r_i(f_i)$ is everywhere decreasing (see Vives 1999). But that condition does not hold in this game. The theory of supermodular games (see Topkis, 1998) applies even if there are discontinuities, but this game is neither supermodular nor log-supermodular.

Note that substitution of the first-order condition into the positive-profit condition, $f_i - c_i - \phi_i d_i^{\gamma_i - 1} \geq 0$, yields

$$1 + (1-\gamma_i)\phi_i d_i^{\gamma_i - 2} \frac{\partial d_i}{\partial f_i} \geq 0.$$

Therefore $z_i(f_i, f_j) \geq 1 - \gamma_i > 0$. Hence $\partial r_i(f_j)/\partial f_j < 1$ holds if

$$d_i \frac{\partial^2 d_i}{\partial f_i^2} + \left[d_i \frac{\partial^2 d_i}{\partial f_i \partial f_j} - z_i(f_i, f_j) \frac{\partial d_i}{\partial f_i} \frac{\partial d_i}{\partial f_j} \right] < \left(\frac{\partial d_i}{\partial f_i} \right)^2 (1 + z_i(f_i, f_j)). \tag{6}$$

Further, $\partial r_i(f_j)/\partial f_j > -1$ holds if

$$d_i \frac{\partial^2 d_i}{\partial f_i^2} - \left[d_i \frac{\partial^2 d_i}{\partial f_i \partial f_j} - z_i(f_i, f_j) \frac{\partial d_i}{\partial f_i} \frac{\partial d_i}{\partial f_j} \right] < \left(\frac{\partial d_i}{\partial f_i} \right)^2 (1 + z_i(f_i, f_j)). \tag{7}$$

Since $-d_i(\partial d_i/\partial f_i)^{-1}$ is decreasing it follows that $(\partial d_i/\partial f_i)^2 > d_i \partial^2 d_i/\partial f_i^2$. Hence, combining (6) with (7) yields (5).□

Since $z_i(f_i, f_j) > 0$ for all $\{f_i^*(f_j), f_j\}$, the condition in Theorem 3 can be written in a simpler, albeit more restrictive form:

$$\frac{\partial^2 d_i}{\partial f_i^2} + \left| \frac{\partial^2 d_i}{\partial f_i \partial f_j} \right| < \frac{1}{d_i} \left(\frac{\partial d_i}{\partial f_i} \right)^2 \tag{8}$$

The above clearly holds for linear demand. (In fact, with linear demand it holds for all $\{f_i, f_j\}$.) But (8) does not hold for logit demand. Fortunately, the more cumbersome condition (5) does hold for logit demand when $\gamma \leq 1/2$. (Recall that $\gamma = 1/2$ in both the queuing and inventory games.)[8]

While Theorems 3 provides conditions under which there is at most one equilibrium with both firms participating in the market, it does not guarantee the existence of an equilibrium. In fact, as is shown by example later, a Nash equilibrium may not even exist in a symmetric game (a game in which the firms' parameters are identical). Nevertheless, the next theorem provides a condition for the existence of a Nash equilibrium.

Theorem 4 *In a symmetric game, i.e.,* $a_i = a_j$, $c_i = c_j$, $\phi_i = \phi_j$, $\gamma_i = \gamma_j$, *and* $d_i(f_1, f_2) = d_j(f_1, f_2)$ *for any* f_1 *and* f_2, *there exists a unique Nash equilibrium and both firms have positive demand in equilibrium if the conditions in Theorem 3 hold and* $f_i^*(\mathring{f}_j) \geq \mathring{f}_j$.

[8] $\partial^2 d_i/\partial f_i + |\partial^2 d_i/\partial f_i \partial f_j| < 0$ for all f_i and f_j is often presented as a uniqueness condition in economics (see Vives, 1999). That condition is even more restrictive than (8) for two reasons: the right hand side constant is positive in (8); and (8) need only be satisfied on the reactions functions.

Proof. From Theorem 3 $r_i(f_j) < 1$. Hence, there exists a *full-participation* equilibrium, $\{f_i^*, f_j^*\}$, with $f_i^* = f_j^* \geq \mathring{f}_j$ if $f_i^*(\mathring{f}_j) \geq \mathring{f}_j$. In words, because the slope of firm i's reaction function is less than 1, the reaction function must intersect $f_i = f_j$ if it starts above that line. Given that $\tilde{f}_i(f_j) - f_j$ is decreasing in f_j (by assumption) it follows that $f_i^*(f_j) \geq f_j$ for all $f_j \leq \mathring{f}_j$. Therefore, there is no equilibrium with $f_j < \mathring{f}_j$.$\square$

To explore the condition in Theorem 4 further, define

$$\mathring{d}_i = d_i(f_i^*(\mathring{f}_j), \mathring{f}_j),$$

i.e., $\mathring{d}_i$ is firm i's positive demand when firm i's optimal profit is zero. In a symmetric game with linear demand

$$\mathring{d}_i = ((1-\gamma)\phi b)^{\frac{1}{2-\gamma}}.$$

Thus, after some algebra, if $\mathring{f}_j > 0$, then $f_i^*(\mathring{f}_j) \geq \mathring{f}_j$ simplifies to

$$\mathring{d}_i \left(\frac{2-\gamma-\beta/b}{1-\gamma} \right) \leq a - (b-\beta)c = d_i(c,c)$$

The above is more likely to hold as a, β or γ increase and as b, ϕ or c decrease, i.e., the existence of equilibrium becomes more likely as base demand increases, scale effects decrease (ϕ decreases or γ increases), as cost decreases and as the market becomes less price sensitive ($b - \beta$ decreases).

To illustrate the possible equilibrium configurations, consider the queueing game with logit demand: $a = -b = m = 1$; $\varepsilon = \rho = 1\text{E-}5$. Figure 1 displays each firm's reaction function in a symmetric game with low capacity cost, $c_i = c_j = 0.1$. In this situation each firm always participates in the market and there is a unique equilibrium. Figure 2 shows that either firm may choose to not participate in the market if costs are higher, $c_i = c_j = 0.4$, and the other firm chooses a low full price. Yet, there still is a unique equilibrium and both firms participate in the market. If costs are increased substantially, $c_i = c_j = 3.75$, there may not exist an equilibrium, as is shown in Figure 3, even in a symmetric game. If costs are further increased, $c_i = c_j = 4.75$, then two equilibria emerge, as shown in Figure 4. With either equilibrium only one firm participates in the market. Figure 5 demonstrates that with asymmetric costs, $c_i = 4.75$ and $c_j = 0.4$, there may exist a single equilibrium in which only one firm participates in the market (in this case it is firm j).[9]

[9] In fact, there is a continuum of equilibria in this case, where any $\{\tilde{f}_i(f_j^*) > f_j^*, f_j^*\}$ is an

From a predictive point of view it is heartening that there exists at most one full-participation equilibrium. But if there is no equilibrium then, by definition the game is not stable, and we are unable to say much more with this model.

To move away from the issue of existence, consider the characteristics of a full-participation equilibrium. The first result is expected.

Theorem 5 *Consider two games that are identical except with respect to two parameters: one game has c_i^l and ϕ_i^l whereas the other has c_i^h and ϕ_i^h where $c_i^l \leq c_i^h$, $\phi_i^l \leq \phi_i^h$ and at least ones of those inequalities is strict. Suppose a full-participation equilibrium exists in both games. Then $f_i^l < f_i^h$, where f_i^l is firm i's equilibrium full price in the first game and f_i^h is firm i's equilibrium full price in the second game.*

Proof. Given that firm j's parameters are held constant, $r_j(f_j)$ is unchanged across these two treatments. The result follows if $r_i^l(f_j) < r_i^h(f_j)$ where the former is firm i's reaction function with $\{c_i^l, \phi_i^l\}$ and the latter is with $\{c_i^h, \phi_i^h\}$. From the implicit function theorem

$$\frac{\partial r_i(f_j)}{\partial c_i} = -\frac{\partial \pi_i(f)}{\partial f_i \partial c_i}\left(\frac{\partial^2 \pi_i(f)}{\partial f_i^2}\right)^{-1}.$$

Since

$$\frac{\partial \pi_i(f)}{\partial f_i \partial c_i} = -c_i \frac{\partial d_i(f)}{\partial f_i} > 0,$$

it follows that $\partial r_i(f_j)/\partial c_i > 0$. The analogous process demonstrates the needed result for the ϕ_i parameter.□

From Theorem 5 it follows that if the game is symmetric with respect to parameters and demand with the exception that one firm has a lower cost than the other, then the low cost firm has a higher market share. But Theorem 5 makes no claim regarding the firms' explicit prices. In fact, it is quite possible that the low cost firm has a higher market share and a higher explicit price; a highly enviable position from a manager's perspective.[10] To illustrate, suppose $c_i = 0.1$, $c_j = 0.4$ and all other parameters are as defined in Figures 1 and 2. In that case $f_i^* = 2.65$, $f_j^* = 2.76$, $p_i^* = 2.21$ and $p_j^* = 1.84$. Firm i can have a higher price and a higher market share because firm i serves its customers more quickly, thereby allowing it to charge a premium.

---

equilibrium.

[10] In the inventory game, a firm's full price equals its explicit price, so in that case the theorem states the low cost firm has the lower explicit price as well.

To explore further when the low cost firm has a higher explicit price we study a particular game that is amenable to analysis. Consider the queuing game with the following symmetric linear demand

$$d_i(f_i, f_j) = a - b(f_i - f_j). \tag{9}$$

Firm i's profit function is $\pi_i = (f_i - c_i)\, d_i - 2\sqrt{c_i d_i}$ where recall that $p_i = f_i - \sqrt{c_i/d_i}$. If in addition the firms have symmetric costs, $c_i = c_j = c$, then there exists a unique full-participation equilibrium, $\{f_i^*, f_j^*\}$,

$$f_i^* = c + \left(\frac{c}{a}\right)^{1/2} + \frac{a}{b} \tag{10}$$

$$\pi_i(f_i^*, f_j^*) = \frac{a^2}{b}(1 - \theta) \tag{11}$$

where θ is defined as

$$\theta = \frac{bc^{1/2}}{a^{3/2}}$$

and $\theta \in (0, 1)$ to ensure positive profits.

Now suppose firm j's cost is increased slightly. The next theorem provides the conditions for which $p_i > p_j$ in the new equilibrium (assuming it exists). In other words, when (13) holds a slight increase in firm j's cost increases firm i's price in equilibrium more than firm j's price.

Theorem 6 *If a full-participation equilibrium exists in the symmetric queuing game, i.e., $c_i = c_j = c$ and demand is given by (9) then*

$$\frac{\partial p_i^*(c, c)}{\partial c_j} > \frac{\partial p_j^*(c, c)}{\partial c_j} \tag{12}$$

when

$$1 > \sqrt{ac}\,(1 - \theta)\,, \tag{13}$$

where $p_i^*(c_i, c_j)$ is firm i's explicit price in the full-participation equilibrium.

Proof. Define $f_i^*(c_i, c_j)$ as firm i's equilibrium full price. From differentiation,

$$\frac{\partial p_i^*}{\partial c_j} = \frac{\partial f_i^*}{\partial c_j} + (1/2) c_i^{1/2} d_i^{-3/2} \left(-b\frac{\partial f_i^*}{\partial c_j} + b\frac{\partial f_j^*}{\partial c_j} \right)$$

$$\frac{\partial p_j^*}{\partial c_j} = \frac{\partial f_j^*}{\partial c_j} + (1/2) c_j^{1/2} d_j^{-3/2} \left(b\frac{\partial f_i^*}{\partial c_j} - b\frac{\partial f_j^*}{\partial c_j} - \frac{d_j}{c_j} \right)$$

where the arguments for $f_i^*(c_i, c_j)$ and $p_i^*(c_i, c_j)$ have been dropped for notational clarity. From the implicit function theorem and Cramer's rule

$$\frac{\partial f_i^*}{\partial c_j} = \frac{|J_{f_i}|}{|J|}; \quad \frac{\partial f_j^*}{\partial c_j} = \frac{|J_{f_j}|}{|J|}$$

where, $|J|$, $|J_{f_i}|$ and $|J_{f_j}|$ are evaluated at the symmetric equilibrium and

$$
\begin{aligned}
|J| &= \begin{vmatrix} \frac{\partial^2 \pi_i}{\partial f_i^2} & \frac{\partial^2 \pi_i}{\partial f_i \partial f_j} \\ \frac{\partial^2 \pi_j}{\partial f_i \partial f_j} & \frac{\partial^2 \pi_j}{\partial f_j^2} \end{vmatrix} = b^2 (3-\theta) \\
|J_{f_i}| &= \begin{vmatrix} -\frac{\partial^2 \pi_i}{\partial f_i \partial c_j} & \frac{\partial^2 \pi_i}{\partial f_i \partial f_j} \\ -\frac{\partial^2 \pi_j}{\partial f_j \partial c_j} & \frac{\partial^2 \pi_j}{\partial f_j^2} \end{vmatrix} = b^2 \left(1 + \frac{1}{2\sqrt{ca}}\right)\left(1 - \frac{1}{2}\theta\right) \\
|J_{f_j}| &= \begin{vmatrix} \frac{\partial^2 \pi_i}{\partial f_i^2} & -\frac{\partial^2 \pi_i}{\partial f_i \partial c_j} \\ \frac{\partial^2 \pi_j}{\partial f_i \partial f_j} & -\frac{\partial^2 \pi_j}{\partial f_j \partial c_j} \end{vmatrix} = b^2 \left(1 + \frac{1}{2}\frac{1}{\sqrt{ca}}\right)\left(2 - \frac{1}{2}\theta\right)
\end{aligned}
$$

Given that $\theta < 1$ (12) can be simplified to (13).□

Given $\theta < 1$, (13) fails to hold only if

$$\frac{1}{a} < c < \frac{a^3}{b}.$$

Hence, in markets with low demand, $a < 1$, (13) always holds (because $c < a$). In markets with greater demand, (13) is more likely as the market becomes more price sensitive, i.e, as b increases.

Table 1: Equilibrium results with symmetric linear demand: $a = 1.25$, $\beta = b$, $c_i = (\theta/b)^2 a^3$

θ	b	c_j/c_i	$d_j^*/(2a)$	p_j^*/p_i^*	g_i^*/g_j^*	$(\pi_j^* - \pi_i^*)/\pi_j^*$
0.5	0.20	0.99	0.50	0.999	1.01	0.06
0.5	0.20	0.95	0.52	0.997	1.07	0.28
0.5	0.20	0.90	0.54	0.993	1.15	0.49
0.9	0.20	0.99	0.52	1.000	1.04	0.67
0.9	0.20	0.95	0.58	1.001	1.21	1.30
0.9	0.20	0.90	0.67	1.004	1.49	1.37
0.5	0.75	0.99	0.50	1.001	1.01	0.03
0.5	0.75	0.95	0.51	1.003	1.04	0.12
0.5	0.75	0.90	0.52	1.007	1.09	0.23
0.9	0.75	0.99	0.51	1.001	1.02	0.30
0.9	0.75	0.95	0.53	1.007	1.08	0.88
0.9	0.75	0.90	0.55	1.013	1.17	1.14

Table 1 provides some data on the impact of a cost advantage. In those scenarios firm j's cost is either 1%, 5% or 10% lower than firm i's cost ($c_j/c_i = 0.99$, 0.95 and 0.90 respectively). This cost advantage gives firm j a modest market share advantage ($d_j^*/(2a)$). Firm j may have lower equilibrium price than firm i when demand is not price sensitive ($b = 0.2$), and always has a higher equilibrium price when demand is price sensitive ($b = 0.75$). However, the price difference between the firms across all scenarios is small (p_j^*/p_i^*). What is not small is firm j's operational performance advantage (g_i^*/g_j^*, where recall a higher ratio means worse performance for firm i). In these scenarios, rather than beating its competitor on price, firm j exploits its cost advantage to offer customers better operational performance. The result is a substantial profit bonus for firm j.

4 Outsource to a supplier

This section explores the motivation for outsourcing. Suppose now there exists a third firm, called the supplier. The supplier does not (or cannot) sell directly to consumers, but the supplier has the ability to perform the firms' operations. (van Mieghem 1999 takes the same approach to subcontracting). For example, the operation in question may be a call center, which could be owned and managed by a firm, or, the firm could outsource that function to the supplier.

We model outsourcing with a two stage game. In the first stage, called the negotiation stage, both firms attempt to negotiate an outsourcing contract with the supplier. The contract has two parameters, w_s and g_s : w_s is the amount the supplier charges the firm per customer the supplier serves for the firm, and g_s is the operational performance the supplier guarantees. For example, in a call center context the contract could specify a fee for each call processed (w_s) and a guaranteed average waiting time (g_s). We assume that it is easy to monitor the supplier's operational performance and so ensuring compliance with contractual terms is not an issue. In addition, we rule out any renegotiate of contractual terms after they are set. For notational convenience, we will often define the contract in terms of c_s and g_s, where $c_s = g_s + w_s$. We do not explicitly model this negotiation process (e.g., which firm makes the first offer or the process by which the firms converge to a signed contract). Instead, we will focus on identifying the set of contracts that leave both parties at least as well off as they would be if no contract were signed.[11]

[11] Much of the supply chain contracting literature assumes one of the firms makes a take-it-or-leave-it offer to the other firm, thereby implicitly assigning all bargaining power to the offering firm. We could adopt that approach, but then the outcome of the analysis

In the second stage, called the competitive stage, the firms compete for customers as in §2. For analytical tractability we assume in the second stage the firms play the queuing game, $c_i = c_j = c$, and demand has the linear form given by (9),

$$d_i(f_i, f_j) = a - b(f_i - f_j)$$

The negotiations in the first stage do not necessarily lead to signed outsourcing agreements. The supplier, being a rational player, will sign a contract only if she expects to earn a non-negative profit. The firms, also acting rationally, will sign contracts only if they expect to earn at least as much with the contract as they would without an outsourcing agreement, i.e., each firm has the option to "insource" and compete in the second stage with complete control of his operations. To be specific, if negotiations in the first stage fail to reach an agreement (i.e., the firm insources), then the firm, as in §2, has two decisions in the second stage, his explicit price and his operational performance, and incurs a cost c per unit of capacity installed. But if a firm has a signed outsourcing agreement with the supplier, then in the second stage the firm only chooses its explicit price, since his operational performance is specified by the outsourcing agreement, and incurs a w_s cost per unit of demand.

One would expect to observe outsourcing agreements if the supplier is able to offer the firms a good deal because the supplier has lower costs than the firms: e.g., the supplier has better technology, lower labor costs (e.g., due to the absence of unions) or greater scale. The latter is possible if the supplier is able to combine the demands of multiple firms. While the "low cost" explanation for outsourcing is plausible, it does not appear to be suitable for all cases. For example, there are cases observed in practice in which outsourcing occurs between a firm and a supplier that establishes a dedicated facility for the firm (e.g., a factory that produces output only for the firm or a call center that process calls only from the firm's customers) and the supplier's technology is arguably no better than her clients' technology. Thus, we seek an alternative explanation for outsourcing. To control for the low cost hypothesis, we assume the supplier does not have better technology or lower costs, i.e., all outsourcing agreements involve dedicated operations (the supplier cannot pool demand

would be a single contract, the one that leaves the receiving firm indifferent between accepting it or not and assigns all incremental gains from the contract to the offering firm. It is unlikely that outsourcing contracts are managed in that way in practice.

across both firms) and the supplier's cost is identical to either firm's. To be specific, for any operational performance level and demand rate, the supplier's cost with an outsourcing agreement is identical to what the firm's cost would be if the firm choose instead to insource: i.e., the supplier incurs a cost c per unit of capacity that must be installed to generate the promised operational performance given the anticipated demand rate.

Since the supplier is unable to offer lower costs to the firms, it is not at all clear that there even exists an outsourcing contract that the parties can agree to in the first stage. If for any operational performance level and demand rate the firm can achieve the same cost as the supplier without having to pay the supplier's margin, then why would a firm agree to any contract that gives the supplier a positive margin? But there is a flaw in that argument: it does not account for how the equilibrium in the competitive stage depends on the outcome of the negotiation stage. In other words, a firm that has an outsourcing agreement behaves differently in the competitive stage than one that does not, and this difference is significant.

4.1 Both firms outsource

In this section we first demonstrate the firms prefer that they both outsource rather than they both insource. But just as the two players in a Prisoners' Dilemma game prefer that they both cooperate over they both defect, this does not mean the outcome will be both firms outsourcing. Several conditions are necessary for that to happen: a firm must prefer to outsource if the other firm outsources (which does not happen in the Prisoners' Dilemma, defect is optimal if the other cooperates) ; a firm must prefer to outsource if the other firm insources (which also does not happen in the Prisoners' Dilemma, defect is optimal if the other defects); and the supplier must earn a non-negative profit with both contracts.

Lets begin with the scenario that both firms insource (i.e., they both fail or refuse to negotiate a deal with the supplier in stage one). This scenario is evaluated in section 3: the equilibrium full price is (10), and the equilibrium profit is (11), repeated here for convenience,

$$\pi_i(f_i^*, f_j^*) = \frac{a^2}{b}(1-\theta), \tag{14}$$

where $\theta = bc^{1/2}a^{-3/2}$ and $\theta \in (0,1)$ ensures positive profits.

The next scenario to consider in stage two has both firms outsourcing. In this case each firm in the competition stage faces linear demand and a constant marginal cost. This is the classical differentiated Bertrand competition game with constant marginal cost. It is well studied in economics (Vives, 1999) and it is known to have a unique closed form equilibrium. For simplicity, assume the outsourcing agreement, $\{w_s, g_s\}$, is the same for the two firms, which has several justifications: the firms are *a priori* identical, so it is not clear why one of them would be able to negotiate a better deal; antitrust regulations generally require suppliers to treat their customers equally unless it can be shown that there are differences in costs to serve customers (which do not exist in this case by assumption); and it is less likely that both firms outsource if one firm's contract is less favorable than the other firm's (because that firm is then more likely to prefer insourcing). In the competition stage firm i's profit is $\pi_i(f_i, f_j) = (f_i - c_s)d_i(f_i, f_j)$, where recall $c_s = w_s + g_s$ and $p_i = f_i - g_s$. The equilibrium full price is $f_i^* = (a/b) + c_s + g_s$ and each firm's profit is

$$\pi_i(f_i^*, f_j^*) = \frac{a^2}{b}. \tag{15}$$

A quick comparison of (15) with (14) reveals that each firm's profit is higher when the firms both outsource than when they both insource. Remarkably, the result is independent of the outsourcing terms. The reason follows from two observations: (1) when both firms outsource they set their price, $(a/b + c_s)$, equal to a fixed markup over c_s, and (2) neither firm's demand decreases in its full price as long as the firms choose the same full price (i.e., there is a constant market size and prices only function to allocate that market between the firms). Hence, the sum of the firms' costs is independent of the full prices as long as the firms choose the same full price.

Now that we have established that both firms prefer the competitive stage with both firms outsourcing rather than both insourcing, we need to confirm they will indeed make that choice and the supplier can earn a non-negative profit. Let's begin with the supplier. The supplier's profit from her contract with firm i is

$$\pi_s(c_s, g_s) = (c_s - g_s - c)d_i - \frac{c}{g_s}$$

where d_i is firm i's demand rate in the stage two equilibrium, $c(d_i + g_s^{-1})$ is the supplier's capacity cost rate and recall $w_s = c_s - g_s$. To know whether a non-negative expected profit will be earned with this contract, the supplier must anticipate what d_i will be. Clearly it depends on firm i's profit function if firm i signs the outsourcing contract:

$$\pi_i(f_i, f_j) = (f_i - c_s)d_i(f_i, f_j)$$

where note $f_i - c_s = p_i - w_s$. The above tells us that the equilibrium f_i, which will determine d_i, depends only on c_s and not on how c_s is divided between w_s and g_s. As a result, if c_s is fixed, then d_i is fixed (i.e., independent of g_s), $\pi_s(c_s, g_s)$ is strictly concave in g_s, the supplier's optimal operational performance is

$$g_s = (c/d_i)^{1/2} \tag{16}$$

and the supplier's profit is

$$\pi_s(c_s) = (c_s - c)d_i - 2\,(cd_i)^{1/2}\,. \tag{17}$$

The supplier can then accept any outsourcing contract as long as $\pi_s(c_s) \geq 0$. From (16) and (17), the set of such contracts, parameterized by ρ, is

$$\{c_s, g_s : c_s = c + 2\rho(c/d_i)^{1/2}, g_s = (c/d_i)^{1/2}, \rho \geq 1\}, \tag{18}$$

where d_i is what the supplier anticipates the competitive stage demand rate for the firm will be. (Note that $c_s > g_s$, which ensures a non-negative w_s.)

Recall that our main objective is to determine if there exists a set of outsourcing contracts that all three firms can agree to sign. Suppose the supplier anticipates that the firm signing the contract will have a competitive stage equilibrium demand rate $d_i = a$. In that case, from (18), the set of acceptable contracts is

$$\{c_s, g_s : c_s = c + 2\rho(c/a)^{1/2}, g_s = (c/a)^{1/2}, \rho \geq 1\}. \tag{19}$$

We next explore whether (19) is acceptable to the firms. To do so we must explore what would happen if only one firm made an outsourcing agreement.

Suppose firm i does not accept an outsourcing contract, but firm j does. The firms' profit functions are then

$$\begin{aligned}
\pi_i(f_i, f_j) &= (f_i - c)\,d_i(f_i, f_j) - 2\sqrt{cd_i(f_i, f_j)} \\
\pi_j(f_i, f_j) &= (f_j - c_s)\,d_j(f_j, f_i)
\end{aligned}$$

where recall $p_j = f_j - g_s$. The next theorem details what happens in the competitive stage with a subset of the contracts in (19). (A full participation competitive stage equilibrium does not exist with higher ρ.)

Theorem 7 *Suppose firm i insources but firm j signs an outsourcing contract from (19) with $1 \leq \rho < 3/(2\theta) + 8^{-1/2}$. Define*

$$\begin{aligned} m &= d_i(f)/a \\ \delta(m,\rho) &= m + \frac{1}{3}\theta m^{-1/2} - \frac{2}{3}\theta\rho \\ \lambda(m) &= m^2 - \theta m^{1/2}. \end{aligned}$$

where recall $\theta = bc^{1/2}a^{-3/2}$ and $\theta \in (0,1)$. In the competition stage there exists a unique equilibrium; firm i's demand is $d_i^* = am^*$, where m^* is the largest solution to

$$\delta(m,\rho) = 1;$$

$2 > m^* > 1$; firm i's demand is greater than firm j's demand; firm j's profit is $(a^2/b)(2-m^*)^2$; firm i's profit is $(a^2/b)\lambda(m^*)$; and firm's profit is greater than firm j's profit.

Proof. Both firms exiting the market cannot be an equilibrium because total demand is constant at $2a$. Now rule out that firm j exits the market; i.e., chooses $f_j = (a/b) + f_i$. Firm j's profit is concave in f_j, so that full price is not optimal if $\partial\pi_j(f_i, f_j)/\partial f_j$ evaluated at $f_j = (a/b) + f_i$ is negative; i.e., if

$$-b\left(\frac{a}{b} + f_i - c - 2\rho(c/a)^{1/2}\right) < 0$$

Substitute firm i's first-order condition into the above and simplify yields $\rho < 3/(2\theta) + 8^{-1/2}$. Similarly, it can be shown that if firm j anticipates firm i exits the market, then there exists an f_i such that firm i earns positive profit; i.e., firm i exiting the market is also not an equilibrium. We now show there exists a unique interior equilibrium.

Any interior equilibrium, $\{f_i^*, f_j^*\}$, satisfies the first-order conditions:

$$\begin{aligned} \frac{\partial\pi_i}{\partial f_i} &= d_i^* - b\left(f_i^* - c - (c/d_i^*)^{1/2}\right) = 0 \\ \frac{\partial\pi_j}{\partial f_j} &= d_j^* - b\left(f_j^* - c_s\right) = 0 \end{aligned}$$

with $d_i^* = d_i(f_i^*, f_j^*)$. It is not feasible to obtain closed form solutions for f_i^* and f_j^*, so we express the equilibrium implicitly in terms of m, which is a proxy for firm i's market share. If d_i is the equilibrium demand rate, then from the two equations above we have

$$f_i^* = c + (c/d_i)^{1/2} + d_i^*/b \tag{20}$$

$$f_j^* = c_s + (2a - d_i^*)/b \tag{21}$$

where recall, $d_j = 2a - d_i$. If d_i^* is indeed an equilibrium, then it must be that $d_i^* = a - b(f_i^* - f_j^*)$, where f_i^* and f_j^* are given in (20) and (21). Thus, substitute (20) and (21) into $d_i^* = a - b(f_i^* - f_j^*)$ and simplify:

$$m + \frac{1}{3}\theta m^{-1/2} - \frac{1}{3}\frac{b}{a}(c_s - c) = 1.$$

Given that $c_s - c = 2\rho c^{1/2} a^{-1/2}$, the above can be written as

$$\delta(m, \rho) = 1. \tag{22}$$

For the remainder of this proof $m \geq 0$ is implied. $\delta(m, \rho)$ is convex; let $\bar{m}$ minimize $\delta(m, \rho)$, $\bar{m} = (\theta/6)^{2/3}$. It can be shown that $\delta(\bar{m}, \rho) < 1$, so there are two solutions to (22). π_i is concave for $m > (\theta/4)^{2/3}$,

$$\frac{\partial^2 \pi_i}{\partial f_i^2} = -b\left(2 - (1/2)\theta m^{-3/2}\right),$$

and $\delta((\theta/4)^{2/3}, \rho) < 1$, so the smaller solution to (22) is a local minimum for firm i and the larger solution is a local maximum. Let m^* be that larger solution to $\delta(m, \rho) = 1$. It is easy to confirm that $m^* > 1$ when $\rho \geq 1$. m^* is the unique interior equilibrium if both firms earn positive profit. Substitute firm i's first-order condition into the profit function to yield firm i's equilibrium profit in terms of equilibrium demand:

$$\pi_i(f_i^*, f_j^*) = d_i^2/b - \sqrt{cd_i^*} = (a^2/b)\lambda(m)$$

Since $\lambda(m) > 0$ for $m > 1$ it follows that firm i indeed earns a positive profit at m^*. A similar approach yields firm j's profit. The boundary condition on ρ ensures that $m^* < 2$, hence firm j also earns a positive profit. Firm j's demand is $d_j^* = 2a - d_i^* = a(2 - m^*)$, which is less than $d_i^* = am^*$ given that $m^* > 1$. Finally, we wish to show $\lambda(m^*) > (2 - m^*)^2$. Firm i's profit is increasing in ρ and firm j's is decreasing in ρ, so it is sufficient to compare profits for $\rho = 1$. Use $\delta(m^*, 1) = 1$ to solve for θ and substitute into the profit condition. That yields $8 > 3\sqrt{m^*} + 4/\sqrt{m^*}$, which simplifies to $0 > (3\sqrt{m^*} - 2)(\sqrt{m} - 2)$, which holds for $m^* \in (1, 2)$.□

According to Theorem 7, in the insource-outsource scenario (one firm insources, the other outsources) then the insource firm has a higher market share and a higher profit. Nevertheless, according to the next theorem, there exists a subset of (19) with which both firms prefer to outsource whether the other firm outsources or not. Furthermore, twith that subset of contracts the supplier earns a non-negative profit because the supplier's anticipated demand rate with each contract (a) indeed materializes in equilibrium.

Theorem 8 *Define*

$$\hat{\rho} = 1 + \frac{3}{2\theta}\left(\delta(\hat{m}, 1) - 1\right)$$

where $\hat{m}$ *is the unique solution to* $\lambda(\hat{m}) = 1$ *and* $\theta \in (0,1)$*. It holds that* $\hat{\rho} > 1$*. If both firms have the opportunity to sign an outsourcing contract chosen from (*19*) with* $1 \le \rho < \hat{\rho}$*, then each firm prefers to outsource whether the other firm outsources or insources.*

Proof. Suppose firm j outsources. We first check that firm i prefers to outsource too. If firm i outsources then it earns a^2/b. If firm i insources, then it earns, from Theorem 7, $(a^2/b)\lambda(m^*)$, where $\delta(m^*, \rho) = 1$. Hence, firm i prefers to outsource if $\lambda(m^*) < 1$. From $\delta(m^*, \rho) = 1$ solve for θ in terms of m^* :

$$\theta(m^*) = \frac{3(m^* - 1)}{2\rho - 1/\sqrt{m^*}}.$$

Substitute $\theta = \theta(m^*)$ into the condition $\lambda(m^*) < 1$ and simplify:

$$(m^* + 1)\left(2\rho\sqrt{m^*} - 1\right) < 3m^*$$

The above can be confirmed numerically for $m^* \in (1,2)$ and $\rho = 1$. Given that $\delta(m, \rho)$ is linearly decreasing in ρ, it is straightforward to show that $\delta(m^*, \hat{\rho}) = 1 = \lambda(\hat{m})$, i.e., with $\rho = \hat{\rho}$ firm i is indifferent between insourcing and outsourcing ($\lambda(m^*) = 1$).

Now suppose firm i insources and check that firm j prefers to outsource even though firm i insources. If firm j insources then it earns $(a^2/b)(1-\theta)$. If firm j outsources, then it earns, from Theorem 7, $(a^2/b)(2 - m^*)^2$, where $\delta(m^*, \rho) = 1$. Thus, firm j prefers to outsource if $(2 - m^*)^2 > 1 - \theta$. Define $\chi(m) = (2 - m)^2 + \theta$. So firm j prefers to outsource when $\chi(m^*) > 1$. Because $\chi(m)$ is decreasing and convex for $m \in (1,2)$, and $m^* < \hat{m}$ for all$\rho < \hat{\rho}$, $\chi(m^*) > 1$ if $\chi(\hat{m}) > 1$. From $\lambda(\hat{m}) = 1$ solve for θ in terms of $\hat{m}$:

$$\theta(\hat{m}) = (\hat{m}^2 - 1)/\sqrt{\hat{m}}.$$

Substitute $\theta = \theta(\hat{m})$ into the condition $\chi(\hat{m}) > 1$ and simplify:

$$(2 - \hat{m})^2 + (\hat{m}^2 - 1)/\sqrt{\hat{m}} > 1$$

The above can be confirmed numerically for $\hat{m} \in (1,2)$. Hence, both firms prefer to outsource no matter whether the other firm outsources or not.□

The firms benefit from outsourcing even though outsourcing provides no operational advantage because outsourcing mitigates price competition. In either the competitive stage equilibrium with both firms outsourcing or the competitive stage equilibrium with both firms insourcing each firm's demand equals a, and so their costs are identical in either game. But in the former their equilibrium price is $c+(a/b)+2(c/a)^{1/2}$ whereas in the latter their equilibrium price is $c+(a/b)$. Prices rise with outsourcing because with outsourcing the firms face constant returns to scale; i.e., their costs per customer are w_s no matter how many customers they have. Outsourcing eliminates the need to cut prices to increase demand to lower costs; i.e., it eliminates the additional price competition due to scale economies.

To emphasize the importance of scale economies, consider the same game except with constant returns to scale; i.e., firm i's profit function is

$$\Pi_i(f_i, f_j) = (f_i - c)d_i(f_i, f_j)$$

if it insources and

$$\Pi_i(f_i, f_j) = (f_i - w_s)d_i(f_i, f_j)$$

if it outsources, where w_s is the wholesale price the supplier charges and demand is the original linear function, $d_i(f_i, f_j) = a - bf_i + \beta f_j$. If they both outsource each firm's profit is

$$\Pi_i^*(w_s) = \frac{b((2b+\beta)a - w_s\left(2b^2-\beta^2-b\beta\right))^2}{(4b^2-\beta^2)^2}$$

and if they both insource their profit is $\Pi_i^*(c)$. Since the supplier can only offer $w_s \geq c$, and $b \geq \beta$ implies $2b^2 - \beta^2 - b\beta > 0$, it is clear that the firms do not benefit from outsourcing; i.e., $\Pi_i^*(w_s) < \Pi_i^*(c)$. (It is also possible to show that a single firm cannot benefit from outsourcing if the other firm insources.)

Table 2 presents some numerical analysis for each of the three scenarios in the competitive stage. As costs increase or as the market becomes more competitive (b increases), i.e., the θ parameter increases, the incremental gain to the firms from outsourcing increases. Even if the firms negotiate the most attractive contract for them, $\rho = 1$, a firm does not benefit from insourcing if the other firm outsources, even though the insourcing firm can gain a significant market share advantage ($d^I/2a$). In the insource-outsource scenario it is the outsourcing firm that fairs the worse, but that firm still fairs better than if it were to insource as well. Finally, it is not necessary that the supplier merely break even ($\rho = 1$). The final column in the table provides the supplier's profit with the supplier's most attractive contract, $\rho = \hat{\rho}$. But the supplier's profit gain is clearly much smaller than the firms' gains from outsourcing: even a monopoly supplier's profit potential is limited by the firms' threat to insource.

Table 2: Equilibrium results in the competitive stage under three scenarios with contracts chosen from (19)

	Insource-insource scenario	Insource-outsource scenario, $\rho = 1$				Outsource-outsource scenario, $\rho = \hat{\rho}$
		Market share		Profit		
θ	π^I/π^O	$d^I/2a$	$d^O/2a$	π^{IO}/π^O	π^{OI}/π^O	π_s/π^O
0.1	0.9	0.52	0.48	0.97	0.95	0.05
0.2	0.8	0.53	0.47	0.94	0.87	0.09
0.3	0.7	0.55	0.45	0.91	0.80	0.13
0.4	0.6	0.57	0.43	0.88	0.74	0.16
0.5	0.5	0.59	0.41	0.85	0.67	0.19
0.6	0.4	0.61	0.39	0.82	0.61	0.22
0.7	0.3	0.63	0.37	0.80	0.55	0.24
0.8	0.2	0.65	0.35	0.78	0.49	0.26
0.9	0.1	0.67	0.33	0.76	0.43	0.28
1.0	0.0	0.69	0.31	0.74	0.38	0.29

d^I = insource firm's demand
d^O = outsource firm's demand
π^I = a firm's equilibrium profit in the insource-insource scenario
π^O = a firm's equilibrium profit in the outsource-outsource scenario
π^{IO} = the insource firm's equilibrium profit in the insource-outsource scenario
π^{OI} = the outsource firm's equilibrium profit in the insource-outsource scenario

4.2 One firm outsources

Theorem 8 establishes that there is a set of outsourcing contracts that all firms are willing to sign. While those contracts earn the supplier a non-negative profit on each contract, it is essential that the competitive stage equilibrium demand rate with each contract be no less than a. Any lower demand rate could generate a negative profit for the supplier, and surely would do so if $\rho = 1$. That could occur if one firm insources: in the insource-outsource competitive stage equilibrium the insourcing firm prices aggressively to build scale, thereby leaving the outsourcing firm with less than a demand, as shown in Theorem 7. Thus, even though in our model it is not in the interest of a firm to insource (i.e., there exists outsourcing contract that make the firm better off), it is useful to explore what would happen if, for reasons that we do not model, one firm surely insources. This imposes an even higher challenge to the viability of outsourcing: the supplier needs better terms to break even because the supplier correctly anticipates that the outsourcing firm's demand rate will be less than a due to the price aggressiveness of the insourcing firm. Hence, we now consider the outsourcing game described in the previous section with one modification: in the negotiation stage only the supplier and firm j negotiate an outsourcing contract and both firms know for sure that firm i will insource.

According to the next theorem, even though the supplier is forced to operate at a lower scale than the insourcing firm and outsourcing provides no operational advantage, there may exist contracts that are acceptable to both the supplier and firm j. In other words, outsourcing may be a profitable unilateral strategy even though the outsourcing firm's scale is lower than what it would have if it insourced.

Theorem 9 *Define*

$$\tilde{\delta}(m) = m + \frac{1}{3}\theta m^{-1/2} - \frac{2}{3}\theta(2-m)^{-1/2}.$$

If $0 < \theta < 3/4$, then there exists a unique $\tilde{m}$ is the interval $[1, 2-(1-\theta)^{1/2}]$ that satisfies $\tilde{\delta}(\tilde{m}) = 1$. Furthermore, if firm i insources and firm j outsources with contract $c_s = c + 2(c/d_j^*)^{1/2}$, $g_s = (c/d_j^*)^{1/2}$, $d_j^* = 2a - d_i^*$, and $d_i^* = a\tilde{m}$, then in the competitive stage equilibrium firm j's demand is indeed $2a - d_i^*$, firm j's profit is $a^2(2-\tilde{m})^2/b$, firm j prefers to outsource than insource and the supplier earns zero profit with that outsourcing contract.

Proof. From (18), the supplier's break even contract with $\rho = 1$ and $\tilde{m} = d_i^*/a$ is

$$\begin{aligned} c_s - c &= 2c^{1/2}(2a - d_i^*)^{-1/2} \\ &= 2(c/a)^{1/2}(2-\tilde{m})^{-1/2}. \end{aligned}$$

As in Theorem 7, the first order conditions and the above contract lead to the following implicit equation for the equilibrium in terms of firm i's demand rate relative to a :

$$\tilde{\delta}(m) = m + \frac{1}{3}\theta m^{-1/2} - \frac{2}{3}\theta(2-m)^{-1/2} = 1$$

The above can have up to three solutions. The solution with $m < 1$ leads to a local minimum for firm i, so it is ruled out. If $\theta = 0$, then $\tilde{m} = 1 = 2-(1-\theta)^{1/2}$ and $\tilde{\delta}(\tilde{m}) = 1$. If $\theta = 3/4$, then $\tilde{m} = 3/2 = 2-(1-\theta)^{1/2}$. For $0 < \theta < 3/4$ it can be shown that $\tilde{\delta}(1) < 1 < \tilde{\delta}(2-(1-\theta)^{1/2})$ and $\tilde{\delta}(m)$ is increasing for $1 < m < 2-(1-\theta)^{1/2}$. Hence, there is a unique $\tilde{\delta}(\tilde{m}) = 1$ in that interval. Finally, firm j earns more by accepting the outsourcing contract than by insourcing if $a^2(2-\tilde{m})^2/b > a^2(1-\theta)/b$, which simplifies to $2-(1-\theta)^{1/2} > \tilde{m}$.$\square$

While the theorem assumes the supplier breaks even with the outsourcing contract ($\rho = 1$), if $\theta < 3/4$ then there exists some $\rho > 1$ that achieves the same outcome and yields the supplier a positive profit. For brevity, the analysis of the upper bound on ρ is omitted.

4.3 Discussion

Taken together, Theorems 8 and 9 suggest that outsourcing is a very attractive strategy in the presence of scale economies. Outsourcing mitigates downstream price competition which generates incremental rents that can be captured by all of the firms, i.e., there exists a set of contracts that result in non-negative profits for all firms. The particular contract that will be chosen depends the relative bargaining power of the firms, which could depend on a number of factors that we do not model (e.g., the number of suppliers that can provide outsourcing services, which firm makes the first offer, how long the negotiations last, etc.). Nevertheless, we feel that the key contribution of this research is to demonstrate that viable outsourcing contracts exist even if outsourcing provides no cost advantage.

It is worthwhile to discuss a number of extensions to this model. To begin, we assumed that the firms's default profit level is zero, e.g., the supplier is willing to accept any contract that yields a non-negative profit. It is not difficult conceptually to extend the results to consider a positive profit threshold (e.g., to reflect the supplier's outside opportunities if the firms fail to negotiate acceptable terms or to reflect additional coordination costs that could occur with outsourcing), but that change is cumbersome analytically and would clearly reduce the set of feasible outsourcing contracts without changing our main qualitative insight.

While we have only a single supplier, our results extend to multiple suppliers. Because the supplier establishes dedicated capacity for each customer, each contract is evaluated on its own. Hence, there is no difference between one supplier signing a $\{c_s, g_s\}$ contract with two firms and two different suppliers each signing a $\{c_s, g_s\}$ with a single firm. The presence of multiple suppliers could influence which contract is signed in the feasible set (i.e., more suppliers probably means contracts that are more favorable to the firms), but it does not influence the set of feasible contracts. In addition, it is not necessary that the firms sign the same outsourcing contract. The firm that is lucky enough to get better terms would have an advantage in the competitive stage, which makes insourcing more attractive to the other firm. But because outsourcing is strictly preferred for a wide range of parameters, it is still possible that one firm prefers to outsource even if his terms are not as good as his competitor's terms.

More restrictive is our assumption that demand has a particular linear form. We do so because that demand results in closed form solutions for two of the three scenarios in the competitive stage. We suspect that out results carry over to other demand models, but this is difficult to confirm analytically. (We have confirmed this for logit demand numerically.) But we do admit that the is a special feature in our demand model that makes outsourcing particularly attractive: total demand is independent of the firms' full prices as long as the full prices are identical. As a result of this feature, increasing industry prices does not reduce industry demand and therefore does not reduce the industry's scale. With other demand models the mitigation of price competition could lead to lower industry demand and therefore higher industry costs. That works against outsourcing, but outsourcing is still viable if demand does not decline too much, which would be typical of price competitive industries where price functions primarily to allocate share.

While we have emphasized throughout our analysis that the supplier does not have lower costs and cannot build additional scale by pooling the firms' demands, it should also be noted that the "low cost" explanation for outsourcing is not refuted by our price mitigation explanation, nor is the price mitigation explanation refuted by the low cost explanation. In other words, if the supplier were able to have lower costs by pooling demand across the two firms then both motivations for outsourcing would be in place, thereby making outsourcing even more attractive.

Finally, although we have concentrated on outsourcing to another firm, in a service context it may even be possible to outsource in part to customers; i.e., co-production. For example, in the financial service industry it is increasingly more common for customers to enter trade orders rather than brokers (Schonfeld, 1998). A key issue with co-production is how it can transform a process with scale economies to one with constant returns to scale. In the extreme co-production allows each customer to be their own server, hence, congestion effects are eliminated and the process exhibits constant returns to scale. However, in most cases a firm must compensate its customers for their additional work in the form of a lower explicit price.[12] If customers are inefficient at their tasks, then the needed price discount may be unacceptable to the firm.

As in the previous models, let p_i be firm i's explicit price. Let g_c be a co-producing customer's time and effort costs and let w_c be the firm's additional costs per customer. As before, $f_i = p_i + g_c$ is firm i's full price. Assuming co-production is a constant return to scale technology, if firm i uses co-production then its profit function is

$$\pi_i(f_i, f_j) = (f_i - c_c)d_i(f_i, f_j)$$

where $c_c = g_c + w_c$. Thus, whether a firm chooses to outsource to its customers or to a supplier is functionally equivalent: with the supplier the firm faces a constant cost per unit of demand equal to c_s whereas with co-production the firm faces a constant cost per unit of demand equal to c_c. As a result, the analysis from section 4 continues to hold: even if a firm's cost with co-production is greater than with insourcing each firm prefers that both firms use co-production, and if co-production's cost is not too excessive both firms outsource even though they could choose to insource. There are only two small distinctions between outsourcing to a supplier and co-production. First, with supplier outsourcing the firms negotiate the terms of trade whereas with co-production c_c is set exogenously: co-production may not be feasible if c_c happens to be too large. Second, a unilateral move to co-production is actually more likely than a unilateral move to supplier outsourcing: if only one firm outsources then the supplier must charge a premium to reflect the lower amount of demand served, but the cost per unit of demand is independent of demand with co-production.

It is beyond the scope of this research to delve deeper into the issue of co-production, but we do mention two promising directions for future research. First, a customer's co-production cost, c_c, could depend on a number of factors under a firm's control: e.g., the firm's design effort, and the number of tasks consumers perform. Second, co-production could still exhibit scale economies, just less so than if the firm insources.

---

[12] Co-production is a rich issue, of which we briefly discuss only one facet. See Moon and Frei (2000) for additional discussion.

5 Conclusion

The prevalence of outsourcing has surely grown in most industries. For example, five large contract manufacturers increased their revenues from \$1.7 billion in 1992 to \$53.6 billion in 2001.[13]. PC manufacturers have begun to outsource their final assembly to their distributors (Hansell, 1998). Retailers and hospitals have outsourced the inventory function to their suppliers (Cachon and Fisher, 1997; Bonneau et al., 1995). Banks have begun to outsource many of their back-office operations (Dalton, 1998). There may be many reasons for this trend, and so we surely do not claim our results provide the single answer for why outsourcing has grown in all industries. Nor do our results contradict previous theories to explain the insource/outsource decision: e.g., asset specificity (Williamson 1979), incomplete contracts (Grossman and Hart 1986), relational contracts (Baker, Gibbons and Murphy, 2001) or capacity pooling (van Mieghem 1999).

Our theory of outsourcing is novel in that we highlight how outsourcing changes the nature of downstream competition. In particular, we find that scale economies make price competition brutal, and so firms naturally can benefit from strategies to mitigate price competition. We show that outsourcing is one such strategy. Much to our surprise and keen interest, we also find a firm can benefit from a unilateral move to mitigate price competition even if that move puts the firm at a cost disadvantage. Hence, it is not required for an industry to simultaneously transition from complete insourcing to complete outsourcing. An industry may transition one firm at a time, and once the industry's structure has transitioned to outsourcing, firms do not have an incentive to revert back to insourcing. Furthermore, firms need not outsource to other firms. Some firms, in particular if they provide a service, may be able to outsource some of the production to their customers.

In a broader sense, this works provides a bridge between two large literatures; it combines fundamental models from the operations management literature (the $M/M/1$ model from queuing and the EOQ model from inventory) with a cornerstone model from oligopolistic competition in economics (differentiated Bertrand competition). Clearly there are numerous extensions worth pursuing. We await many interesting managerial insights from this melding

[13] Annual report data from Solectron, Flextronics, Celestica, SCI Systems and Jabil Circuit.

of operational detail with competitive dynamics.

References

Afeche, P. and H. Mendelson. 2001. Priority auctions vs. uniform pricing in queuing systems with a generalized delay cost structure. Northwestern University working paper.

Anupindi, R. and Y. Bassok. 1999. Centralization of stocks: retailers vs. manufacturer. *Management Science*. **45**(2). 178-91.

Armony, M. and M. Haviv. 1998. Nash Equilibrium in a Duopoly Model with Waiting Lines. Hebrew University of Jerusalem working paper.

Baker, G., R. Gibbons and K. Murphy. 2001. Bringing the market inside the firm? *American Economic Review*. **91**(2). 212-8.

Baye, M., K. Crocker, and J. Ju. 1996. Divisionalization, franchising and divestiture incentives in oligopoly. *American Economic Review*. **86**(1): 223-236.

Bell, C. E. and S. Stidham. 1983. Individual versus Social Optimization in the Allocation of Customers to Alternative Servers. *Management Sci*. **29**, 831-839.

Bernstein, F. and A. Federgruen. 1999. Pricing and replenishment strategies in a distribution system with competing retailers. Columbia University working paper.

Bernstein, F. and A. Federgruen. 2001. A general equilibrium model for decentralized supply chains with price and service competition. Fuqua School of Business working paper. Durham, NC. 27708.

Bonneau, S., H. Curtis, S. Peters, T. Verbeck and M. Weissel. 1995. Healthcare Supply Chain Management: Product Flow from the Distributor to the Hospital. Fuqua School of Business working paper, Duke University.

Cachon, G. and M. Fisher. 1997. Campbell Soup's Continuous Product Replenishment Program: evaluation and enhanced decision rules. *Production and Operations Management*. **6**(3). 266-76.

Cachon, G. and P. Harker. 1999. Service competition, outsourcing and co-production in a queuing game. University of Pennsylvania working paper. Philadelphia, PA 19104.

Chase, R. 1978. Where Does the Customer Fit in a Service Operation? *Harvard Business Review*. November-December, 138-139.

Chayet, S. and W. Hopp. 1999. Lead time competition under uncertainty. Northwestern University working paper, Evanston, IL.

Dalton, G. 1998. Online Billing Partners. *Informationweek*. 701 30.

Davidson, C. 1988. Equilibrium in Servicing Industries: An Economic Application of queueing Theory. *J. of Business*. **61**, 347-367.

De Vany, A. 1976. Uncertainty, Waiting Time, and Capacity Utilization: A Stochastic Theory of Product Quality. *J. of Political Economy*. **84**(3), 523-541.

De Vany, A. and T. Saving. 1983. The Economics of Quality. *J. of Political Economy*. **91**(6), 979-1000.

Deneckere, R. and J. Peck. 1995. Competition Over Price and Service Rate When Demand is Stochastic: A Strategic Analysis. *RAND Journal of Economics*. **26**(1), 148-162.

Dewan, S. and H. Mendelson. 1990. User Delay Costs and Internal Pricing for a Service Facility. *Management Sci*. **36**, 1502-1517.

Gans, N. 2000. Customer Loyalty and Supplier Strategies for Quality Competition II. Wharton School working paper.

Gans, N. 2002. Customer Loyalty and Supplier Quality Competition. *Management Sci*. **48**, 207-221.

Gilbert, S. and Z. K. Weng. 1997. Incentive Effects Favor Non-Competing Queues in a Service System: The Principal-Agent Perspective. *Management Sci*. **44**(12), 1662-1669.

Grossman, S. and O. Hart. 1986. The Costs and Benefits of Ownership: A Theory of Vertical and Lateral Integration. *J. of Political Economy*. **94**, 691-719.

Ha, A. 1998. Incentive-Compatible Pricing for a Service Facility with Joint Production and Congestion Externalities. *Management Sci*. **44**(12), 1623-1636.

Hall, J. and E. Porteus. 2000. Customer service competition in capacitated systems. *Manufacturing and Service Operations Management*. **2**(2). 144-65.

Hansell, S. 1998. Is this the factory of the future? *Wall Street Journal*. July 26.

Kalai, E., M. Kamien, and M. Rubinovitch. 1992. Optimal Service Speeds in a Competitive Environment. *Management Sci.* **38**(8), 1154-1163.

Karjalainen, R. 1992. The newsboy game. University of Pennsylvania working paper, Philadelphia, PA.

Karmarkar, U. and R. Pitbladdo. 1995. Service Markets and Competition. *J. of Operations Management.* **12**(3-4), 397-411.

Kulkarni, V. 1983. A Game Theory Model for Two Types of Customers Competing for Service. *Operations Research Letters.* **2**(3), 119-122.

Lederer, P. J. and L. Li. 1997. Pricing, Production, Scheduling, and Delivery-Time Competition. *Operations Research.* **45**(3), 407-420.

Li, L. 1992. The Role of Inventory in Delivery Time-Competition. *Management Sci.* **38**(2), 182-197.

Li, L. and Y. S. Lee. 1994. Pricing and Delivery-time Performance in a Competitive Environment. *Management Sci.* **40**(5), 633-646.

Lippman, S. and K. McCardle. 1995. The competitive newsboy. *Operations Research.* **45**.54-65.

Lippman, S. and S. Stidham. 1977. Individual Versus Social Optimization in Exponential Congestion Systems. *Operations Research.* **25** 233-247.

Loch, C. 1994. Incentive Compatible Equilibria in Markets with Time Competition. INSEAD working paper.

McGuire, T. and R. Staelin. 1983. An Industry Equilibrium Analysis of Downstream Vertical Integration. *Marketing Sci.* **2**, 161-192.

McMillan, J. 1990. Managing Suppliers: Incentive Systems in Japanese and U.S. Industry. *California Management Review.* 32(4), 38-55.

Mendelson, H. 1985. Pricing Computer Services; queueing Effects. *Comm ACM.* **28**, 312-321.

Moon, Y. and F. Frei. 2000. Exploding the self-service myth. *Harvard Business Review.* May-June.

Naor, P. 1969. On the Regulation of Queue Size by Levying Tolls. *Econometrica.* **37** 15-24.

Parlar, M. 1988. Game theoretic analysis of the substitutable product inventory problem with random demands. *Naval Research Logistics Quarterly.* **35**.397-409.

Reitman, D. Endogenous quality differentiation in congested markets. *Journal of Industrial Economics.* **39** (1991). 621-647.

Schonfeld, E. 1998. Schwab Puts it All Online. *Fortune.* **138**(11), 94-100.

So, K. and J.-S. Song. 1998. Price, Delivery Time Guarantees and Capacity Selection. *European J. of Operations Research.* **111**, 28-49.

Stidham, S. 1992. Pricing and Capacity Decisions for a Service Facility: Stability and Multiple Local Optima. *Management Sci.* **38** (8), 1121-1139.

Stidham, S. and C. Rump. 1998. Stability and Chaos in Input Pricing for a Service Facility with Adaptive Consumer Response to Congestion. *Management Sci.* **44**(2), 246-261.

Topkis, D. 1998. *Supermodularity and Complementarity.* Princeton University Press. Princeton.

van Mieghem, J. 1999. Coordinating investment, production and subcontracting. *Management Science.* **45**(7). 954-71.

Venkatesan, R. 1992. Strategic Sourcing: To Make or Not to Make. *Havard Business Review.* **70**(6), 98-107.

Vives, X. 1999. *Oligopolistic Competition: Old Ideas and New Tools.* MIT Press. Cambridge, MA.

Williamson, O. 1979. Transaction-Cost Economics: The Governance of Contractual Relations. *J. of Law and Economics.* **22**, 233-261.

Figure 1: Queuing game reaction functions with logit demand: $a = -b = m = 1$; $\varepsilon = \rho = 1E\text{-}5$; $c_i = c_j = 0.1$

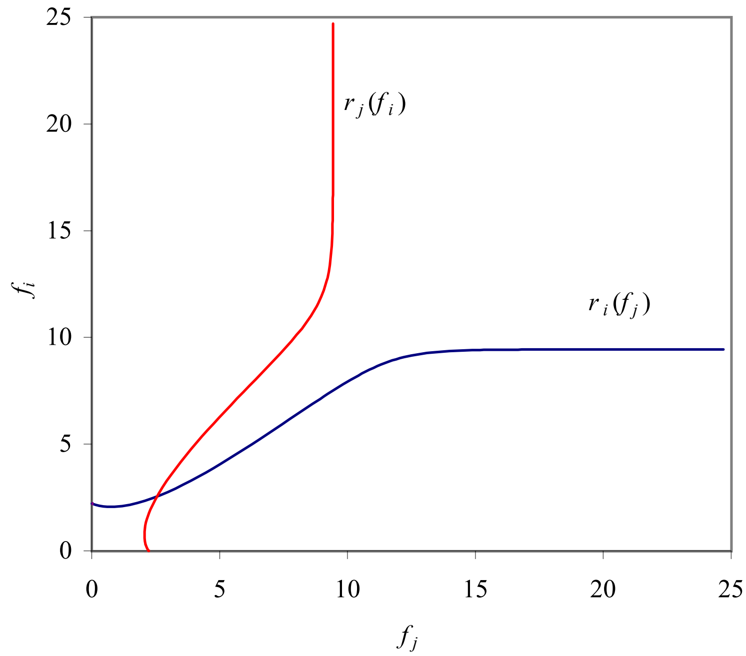

Figure 2: Queuing game reaction functions with logit demand: $a = -b = m = 1$; $\varepsilon = \rho = 1E\text{-}5$; $c_i = c_j = 0.4$

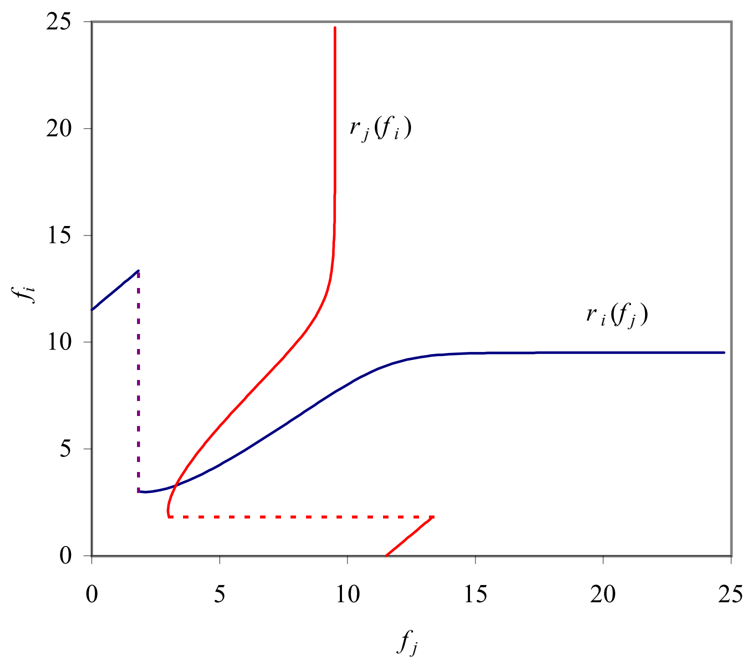

Figure 3: Queuing game reaction functions with logit demand: $a = -b = m = 1$; $\varepsilon = \rho = 1E\text{-}5$; $c_i = c_j = 3.75$

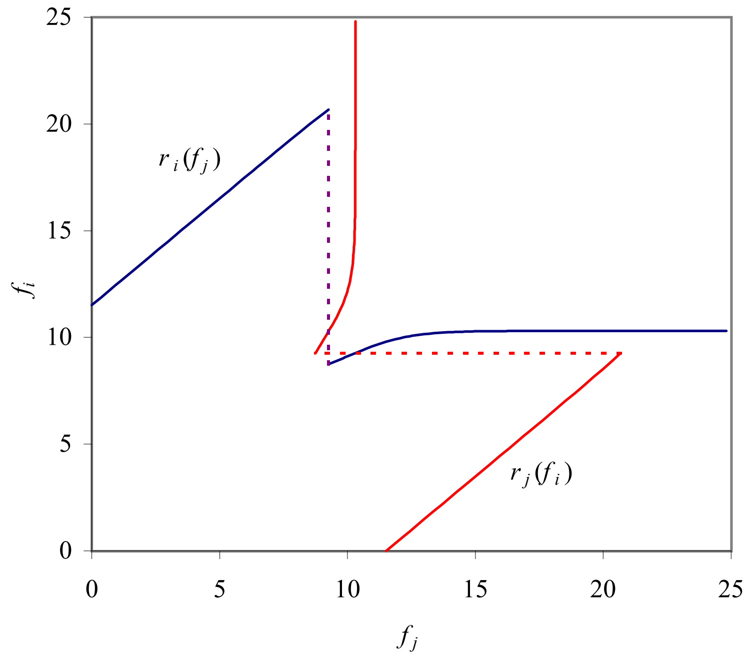

Figure 4: Queuing game reaction functions with logit demand: $a = -b = m = 1$; $\varepsilon = \rho = 1E\text{-}5$; $c_i = c_j = 4.75$

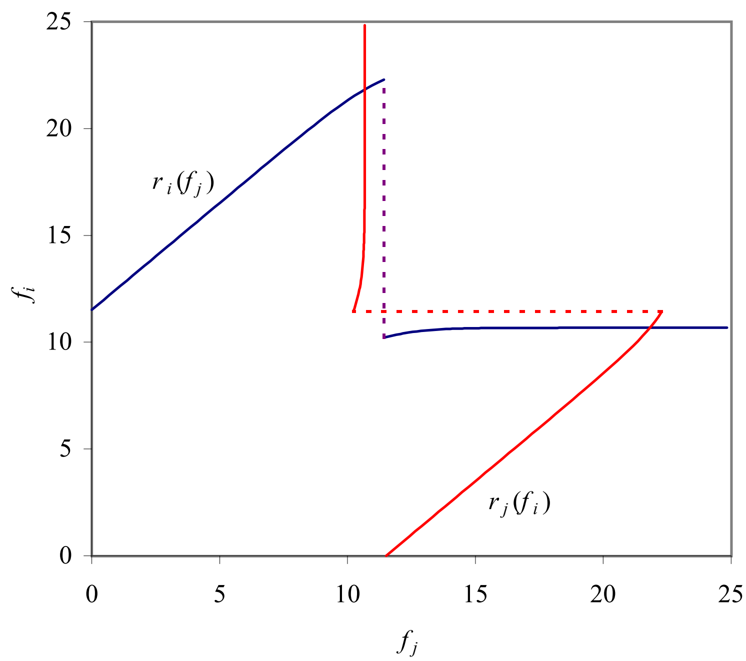

Figure 5: Queuing game reaction functions with logit demand: $a = -b = m = 1$; $\varepsilon = \rho = 1E\text{-}5$; $c_i = 4.75$; $c_j = 0.4$