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Examining Insurer Allocation Decisions Between Reinsurance Purchases and CAT Bond Issuances

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Keywords

CAT bonds

Disciplines

Business

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1 Introduction

The catastrophe bond market is a relatively new class of securities which developed in the wake of Hurricane Andrew in 1992, which resulted in the largest economic cost from a hurricane at that time. These securities arose out of a need to expand catastrophe insurance capacity, with the first experimental transactions being completed in the mid-1990s through AIG, Hannover Re, St. Paul Re, and USAA (Artemis 2015).

Catastrophe bonds are instruments which allow insurance and reinsurance firms to transfer catastrophe-related risks to the capital markets, and hence lower the probability of default in the event of unpredictable, large scale catastrophes. These securities are also attractive from an investor perspective, as they are an effective diversification tool, being largely uncorrelated with market movements. Since its inception in 1992, the catastrophe bond market has since expanded from $1 billion outstanding to more than $19 billion outstanding in 2013. Over the past decade, it has enjoyed an annual growth rate of approximately 25%, as compared to the 10% growth for the rest of the insurance sector (Artemis 2015).

Given the growth of the catastrophe bond market as a new method of risk transference, it is worthwhile to analyze how this market interacts with the traditional insurance and reinsurance markets. To this end, this paper attempts to discover the extent to which catastrophe bonds can be viewed as a substitute to traditional reinsurance and in particular, the main determinants which lead insurers to substitute towards CAT bond issuances rather than purchasing traditional reinsurance from established firms.

2 The Market for Catastrophe Bonds

Catastrophe bond origination generally begins with the formation of a special purpose vehicle (SPV), generally sponsored by an insurance or reinsurance company. The sponsor then enters into a reinsurance contract with the SPV, whereby the sponsor agrees to pay premiums in exchange for receiving the coverage from principal payments in the event of a catastrophe (which is defined through a set of trigger conditions). The SPV issues the bonds to investors and receives principal payments, which are then invested in highly liquid,

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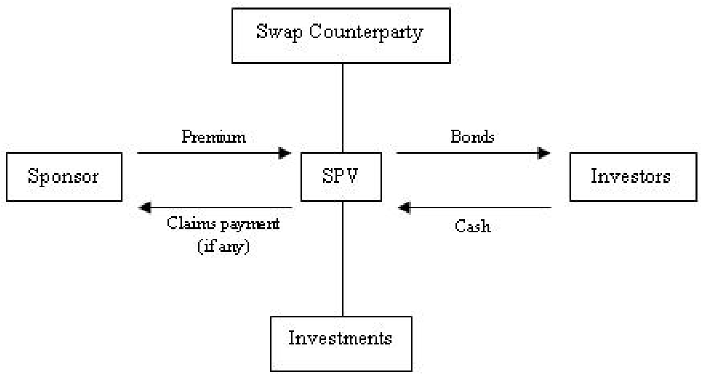
investment grade funds.

The investors receive coupon payments from their investments, which are comprised of the return from the highly rated funds as well as the premiums paid by the sponsor. If there is some natural occurrence which satisfies the trigger conditions, the principal payments made by investors are liquidated (all or in part, depending on the terms of the contract), and are used by the sponsor to cover the necessary claims payments. If such an event does not occur before the bond reaches maturity, the principal payments are returned to investors.

The structure of a catastrophe bond as described above can be seen in the diagram below:



At present, catastrophe bonds are exclusively used by institutional investors, and are not open to retail investors.

3 Literature Review

There has been extensive research in the area of insurance linked securities, which can be roughly categorized into two categories: asset pricing and assessing the impact and management of these assets.

3.1 Asset Pricing

A predominant portion of the research regarding insurance linked securities (ILS) has focused on the appropriate pricing model to be applied to these instruments. These range from theoretical pricing models which aim to price the bonds based on their risk exposures to empirically based models which largely aim to explain the premium in returns commanded by CAT bonds as compared to similarly rated debt.

Theoretical models most often make use of various diffusion processes in analyzing asset dynamics— Cummins and Geman (1995) priced CAT futures given deterministic interest rates and specific loss processes; Litzenberger, Beaglehole, and Reynolds (1996) price a CAT bond as a one year zero coupon bond with an embedded CAT call option spread. We adopt the Poisson arrival process used in Cummins and Geman's analysis, as it allows for a fully representative model of loss frequency with minimal added complexity. Finally, the decomposition of a multi-period CAT bond into simpler financial instruments in Litzenberger et. al leads us to consider a simplified analysis of a single period CAT bond, so as to focus on incentives and behaviors of the insurer rather than more complex term structure considerations in a multi-period model.

Lee and Yu (2002) are among the first to incorporate a stochastic interest rate process in the valuation model, and assume the general lognormal diffusion process for the insurer's asset value. By contrast, Vaugirard (2003) prices CAT bonds through an arbitrage approach based on an underlying risk index. Vaugirard's analysis provides valuable insight into how market prices are formed; in our analysis, it is taken as given that

CAT bonds are fairly priced (arbitrage free), and we adopt the same stochastic processes used in Lee and Yu to model insurer and reinsurer asset, as this method of analysis will allow for simulation of firm assets under different assumptions (e.g. varying frequencies of catastrophic occurrences).

With regards to empirically based research, Lane and Mahul (2008) used data from 250 catastrophe bonds to investigate the possible risk factors which affect how catastrophe risks are priced, revealing that the underlying peril, expected loss, market cycles, as well as the risk profile of the transactions form the primary factors relevant to pricing. In the analysis presented in this paper, two of these major factors—the underlying peril and expected loss is incorporated into the model of loss distributions; we chose to omit other empirical factors which had a less direct causal effect on loss figures, such as market cycles and risk profiles as they would have contributed to an excessively complex model without being directly relevant to the analysis of insurer incentives or behaviors.

Bantwal and Kunreuther (2000) noted the empirical premium on cat bond returns as compared to debt securities of comparable ratings. Their work provides several reasons rooted in behavioral economics to explain the premium, including factors such as risk aversion, ambiguity aversion, and many additional considerations. Among others, Dieckmann (2010) and Froot (2001) have attempted to explain the excess return earned by CAT bonds through concepts such as consumption framework analyses (catastrophes are small probability, high impact economic shocks that could bring investors close to their subsistence level) as well as supply restrictions of CAT bonds associated with capital market imperfections, as well as the high market power enjoyed by traditional reinsurers.

3.2 The Impact and Management of Catastrophe Bonds

CAT bonds remain a relatively nascent asset class, and much research has been done in exploring the potential benefits of CAT bonds to both investors and sponsors, as well as optimal ways in which they may be used to diversify risk at minimal cost.

In particular, Canabarro, Finkemeier, Anderson, and Bendimerad (2000) show that a relatively small allocation dedicated to insurance linked securities within a fixed income portfolio can serve the dual purpose of increasing expected return and decreasing risk exposures, without significant alterations to higher moments of the return distribution. However, the core of their analysis is conducted from the perspective of a CAT bond investor, so that there is no consideration of risk management on the part of an insurer, or CAT bond issuer.

Croson and Kunreuther (2000) suggest six principles of system design for CAT risk transfer, which details how combinations of insurance, reinsurance, CAT bonds, and government reinsurance can meet objectives for different stakeholders associated with risk management, such as homeowners, insurers, investors, and the government. These principles include logistical considerations such as expediting settlements for catastrophic claims, as well as security structuring isues, such as customizing risk transfer instruments to address basis risk.

Doherty (1997) argues that the existence of high reinsurance costs presents an opportunity for competitive hedging instruments, though the emergence of these new instruments present insurance companies and other hedgers with the challenge of managing different types of risk—moral hazard, credit risk, as well as basis risk. The CAT bond market offers increased opportunity in terms of designing options which can reduce moral hazard and credit risk while providing an affordable source of reinsurance, but exposes the risk managers to consider tradeoffs involving more complex sources of risk.

4 Model Design and Setup

4.1 Assumptions

Diversified Market: this assumes that the impact of any single insurance firm on the overall reinsurance and CAT bond market will be minimal. More specifically, this assumes that any additional demand in traditional reinsurance at market prices is readily available and that any additional supply of CAT bonds from the insurer is readily absorbed. In short, this assumes that no single insurer can influence market supply or demand of both reinsurance and/or CAT bonds.

Indemnity Trigger: this assumption is made so as to ensure that the payout profile from a CAT bond and a reinsurance contract is as similar as possible. Traditionally, reinsurance contracts generally cover a specified range within a particular layer of insurance. An indemnity trigger serves a similar function, in that the associated CAT bond compensates the insurer a fixed sum above a pre-specified loss threshold.

By contrast, a CAT bond equipped with a parametric trigger will issue a payout based upon various parametric readings (e.g. wind speed at the time of the incident, etc.), which results in basis risk and introduces a further complication differentiating between reinsurance and CAT bonds.

Linearity of CAT Bond Issuances: We assume that the per unit cost of issuing CAT bonds remains constant over time (i.e. the total costs of issuance increases linearly with size). There are two ways in which this assumption is violated in practical settings.

Firstly, we recognize that the indemnity trigger assumption above may introduce moral hazard considerations. However, as a simplifying assumption, we will not consider moral hazard as a significant factor affecting an investor's demand for CAT bonds, or an important factor in driving the decision making process for an insurer deciding between reinsurance or CAT bonds.

In essence, this assumption states that the per unit cost of CAT bond issuance remains constant as the face value of the bond grows; in practical settings, this may not be the case, an insurer who chooses to reinsure a fixed layer of losses entirely through CAT bond issuance may be faced with higher coupon payments as compared to an insurer who reinsures the same fixed layer with a combination of reinsurance and CAT bond issuance. This is because investors in the first case are more likely to experience losses on their principle investment in the event that any single catastrophic event occurs; by contrast, the investors in the second case face a smaller risk of losing principle payments in the event of a catastrophe, as some losses are insured through traditional reinsurance, so the principle CAT payments may not be recalled by the insurer. This increased risk experienced by the first group of investors may lead them to demand higher coupon payments, so that the cost of issuance may increase nonlinearly with the amount issued.

Secondly, this assumption ignores any fixed costs associated with the CAT bond issuance process—specifically, the related legal expenses in forming a special purpose vehicle for the firm, the logistical costs in forming partnerships with the necessary financial institutions, etc. These fixed costs would result in decreasing per unit issuance costs, as the large initial fixed costs would be spread over more bond issuances. Further discussion about incorporating this aspect of bond issuance is discussed in section 6.2, which includes details on further research ideas.

Overall, we restrict the cost of issuance to the linear case, which reduces the optimization problem to minimizing costs over reinsurance and issuing as much CAT bonds are necessary without considering the possibility of prohibitive costs as issuance grows larger.

Interest Rate Process: the interest rate is assumed to follow the stochastic process outlined in Cox, Ingersoll, and Ross (1985). This model is chosen over the Vasicek model to avoid the possibility of negative interest rates.

4.2 Interest Rates

CAT bonds are fixed income instruments and both insurers as well as reinsurers typically have significant holdings in fixed income instruments. Hence, interest rate fluctuations are highly relevant in modelling both CAT bond dynamics as well as the dynamics of firm value and liabilities. We adopt the process exhibited in

Cox, Ingersoll, and Ross (1985), where the interest rate process can be expressed as:

$$dr_t = \kappa(m - r_t)\, dt + v\sqrt{r_t}\, dZ_t \tag{1}$$

where r_t is the instantaneous interest rate at time t, κ is the mean-reverting force, m is the long-run expected value of the interest rate, v is the standard deviation of the interest rate, and Z_t is the Wiener process (independent of W_V, t).

4.3 Insurer's Assets

We adopt the methodology used in Duan, Moreau, and Sealey (1995) in viewing the insurer's total asset value as comprised of interest rate and credit risk as two separate components:

$$\frac{dI_t}{I_t} = \mu_I\, dt + \phi_I\, dr_t + \sigma_I\, dW_{I,t} \tag{2}$$

where I_t is the value of the insurer's assets at time t, r_t is the instantaneous interest rate at time t, $W_{V,t}$ is the Wiener process governing the credit risk of the firm, σ_V is the standard deviation or volatility of the firm's credit risk, and ϕ_V the interest rate elasticity of the firm's assets.

We model the reinsurer's total asset value in the same way, letting R_t represent the value of the reinsurer's assets at time t:

$$\frac{dR_t}{R_t} = \mu_R\, dt + \phi_R\, dr_t + \sigma_R\, dW_{R,t} \tag{3}$$

4.4 Risk Neutral Valuation

In asset pricing, it is standard practice to use the risk-neutralized pricing measure, also referred to as the equilibrium measure. This pricing measure is derived from the fundamental theorem of asset pricing, which states that in a complete market (arbitrage free), the price of a derivative should not depend on an individual investor's risk preferences, and thus it is equivalent to price all assets based on a risk neutral risk preference. The dynamics for the interest rate process under the risk neutral process is as follows:

$$dr_t = \kappa^*(m^* - r_t)\, dt + v\sqrt{r_t}\, dZ_t^* \tag{4}$$

where

$$\begin{aligned}
\kappa^* &= k + \lambda_r \\
m^* &= \frac{km}{k + \lambda_r} \\
dZ_t^* &= dZ_t + \frac{\lambda_r \sqrt{r_t}}{v}\, dt
\end{aligned}$$

Asset dynamics for the insurer under the risk neutral measure is as follows:

$$\frac{dI_t}{I_t} = r_t\, dt + \phi_I v\sqrt{r_t}\, dZ_t^* + \sigma_I dW_{I,t}^*$$

The analogous transformation can be applied for the reinsurer's assets.

4.5 Payoffs to Insurers from Reinsurance Contracts

We assume the structure of a general reinsurance contract, which exhibits an excess-of-loss policy subject to a maximum claim. This means that in the event of a loss, the insurer bears the relevant costs up to an attachment point A. The reinsurer then reimburses the insurer for an costs in excess of A, up to some maximum claims amount M. Hence, the maximum obligation that the reinsurer owes to the insurer is $M - A$, if the insurer incurs costs, C, is greater than the attachment point. The following provides an outline of the possible payout scenarios under a reinsurance contract:

$$P = \begin{cases} M - A & C > M \\ C - A & M > C > A \\ 0 & A > C \end{cases}$$

However, we must also consider the case where the reinsurance company does not have sufficient capital to pay their obligations—in short, that the asset value of the reinsurance firm, R is less than the payout they owe to the insurer. In this case, we assume that the insurer will receive a payment proportional to the total liabilities, L of the reinsurer. The payout structure of a reinsurance contract in the case of insolvency is as follows:

$$P' = \begin{cases} (\frac{M-A}{L})V & C > M \\ (\frac{C-A}{L})V & M > C > A \\ 0 & A > C \end{cases}$$

This yields a combined payout structure under a reinsurance contract, P_R, as follows:

$$P_R = \begin{cases} M - A & C > M, R > L \\ (\frac{M-A}{L})V & C > M, R < L \\ C - A & M > C > A, R > L \\ (\frac{C-A}{L})V & M > C > A, R < L \\ 0 & A > C \end{cases} \tag{5}$$

4.6 Optimal Mix Between Reinsurance and CAT Bonds

We establish the costs for purchasing reinsurance and issuing CAT bonds respectively; the optimal mix for the insurer when choosing between these two instruments will be the combination which minimizes the cumulative cost subject to a coverage constraint .

4.6.1 Cost of Reinsurance

Given the payouts P outlined above, the expected cost that an insurer should expect to pay for reinsurance should be equal to the present value of the expected payout:

$$C_R = E\left[\exp\left(\int_0^T r_s\, ds\right) \times P_R\right]$$

This is an actuarially fair price for reinsurance, though we expect there to be some loading factor charged in excess of the fair price, which is necessary to maintain a positive return on capital as well as covering any fixed costs in excess of the payout that the reinsurer must incur. Denote this loading factor as $p(P_R)$, we then expect the cost of reinsurance to be $(1 + p(P_R))C_R$. We note that the loading factor p will be an increasing function of the expected payout from the reinsurer, P_R. This reflects the increased possibility of default with higher amounts of reinsurance coverage.

4.6.2 Cost of Issuing CAT Bonds

In a similar fashion, we expect that the cost of issuing CAT bonds should also be the expected value of the payouts that an insurer should expect to receive from the CAT bond. Given the trigger level G, face value FV, and indemnity claims C_T, the payout that an insurer should expect to receive are as follows:

$$P_{CB} = \begin{cases} C_T - K & K < C_T < FV + K \\ FV & C_T \geq FV + K \end{cases}$$

The present value of the expected value of this payout for the insurer should be:

$$C_{CB} = E\left[\exp\left(\int_0^T r_s\, ds\right) \times P_{CB}\right]$$

As in the case above, we expect to see some loading factor on the expected value, to account for a spread in the CAT bond market, which has been empirically noted. Denoting this spread s, we expect the cost of issuing a CAT bond to be $(1+s)C_{CB}$. In accordance with the linearity assumptions outlined in section 4.1, the spread s is a constant, and does not increase with the expected CAT bond payout, CB.

4.6.3 Optimal Mix

We assume that the insurer faces some mandatory reinsurance requirement based on either local or federal legislature. This is similar to the concept behind of the Florida Hurricane Catastrophe Fund, which was created by the Florida Legislature in 1993 and requires insurers to enter into reinsurance contracts which cover a percentage of losses above a certain attachment point. Hence, the insurer faces an optimization problem of purchasing sufficient coverage to protect against potential losses R.

We assume that the face value and trigger levels are fixed for a CAT bond, so that the optimization problem can be quantified as follows:

$$\min_{M,A} \quad (1+p)C_R + (1+s)C_{CB}, \quad \text{s.t.}(M-A) + FV = R$$

4.7 Loss Events

The frequency of loss events will be modelled by a homogeneous Poisson process with parameter λ. Furthermore, we assume that the lognormal distribution of losses is normal with mean μ_c and variance σ_c^2. Finally, we make the simplifying assumption that all catastrophes which occur within a given year will incur the same magnitude of losses—that is, if a single catastrophe occurs in a given year, the magnitude of the loss l is randomly drawn from a lognormal distribution with mean mu_c, variance σ_c^2; if there are m occurrences within the given year, the total loss is simply ml, as opposed to the sum of separate, independently and identically distributed losses.

5 Numerical Simulation: Results and Analysis

We use the following base parameters for the model, based on existing benchmarks in the current literature (Lee and Yu 2006)

Initial Parameters	
Interest Rate	
Initial Rate	2%
Long Run Rate	5%
Volatility	5%
Mean Reverting Force	0.2
Assets & Liabilities	
Initial Asset/Liability Ration	1.3
Interest Rate Elasticity	-3
Volatility of Credit Risk	10%
Initial Liabilities	100
Interest Rate Elasticity	-3
Losses	
Poisson Process Parameter	1
Average Losses (of lognormal loss distribution)	1
Standard Deviation of Losses	0.5
Reinsurance and CAT Bond Costs	
CAT bond spread (premium)	0.4
Reinsurance base premium	0.4
Reinsurance incremental premium	0.04

For each simulation, 1,000 trials are run, and the average cost recorded. Through these simulations, we wish to analyze factors which affect insurer's optimal allocation of reinsurance expenditures across traditional markets and CAT bond markets; though there are many such factors, we will focus on three main factors: the relative premiums between traditional reinsurance and CAT bonds, the intensity and volatility of catastrophes, as well as the asset to liability ratio of the reinsurer in question.

5.1 Relative Premiums

This model attempts to bring together multiple disparate components (firm assets and liabilities, loss dynamics, interest rate processes, etc.), so in an initial analysis, we would like to verify that the model produces results that we would intuitively expect to be true. In particular, an obvious result to expect is that an insurer should choose to issue more CAT bonds as the premium on reinsurance increases. The figure below shows the relative cost of purchasing and issuing different amounts of reinsurance CAT bonds respectively, such that the total quantity reinsured remains constant at 100. The first column represents the face value of a CAT bond issuance (CAT FV), and the column headers represent the reinsurance premium (RP). All other values are held constant at the initial parameters outlined in the table above. The coloring of the cells is conditional on the magnitude of the cost, so that darker shading represents lower costs and lighter shading represents higher costs. For a given level of reinsurance premiums, the cost minimizing issuance size is highlighted in red:

	reinsurance premium				
CAT FV	0.40	0.60	0.80	1.00	1.20
0	13.95	13.85	13.86	16.31	15.55
10	12.96	13.46	14.41	14.82	13.29
20	14.26	14.82	15.97	15.09	14.93
30	13.14	13.68	11.73	13.43	14.57
40	13.69	**10.41**	11.00	14.31	13.65
50	**9.58**	12.75	11.45	13.91	10.89
60	10.56	11.57	**10.10**	12.99	12.66
70	12.10	12.24	11.37	13.46	12.44
80	12.58	12.91	11.72	12.77	10.99
90	12.03	13.27	12.55	12.50	10.40
100	12.35	10.90	13.90	**11.44**	**9.50**

Figure 1: Reinsurance Costs For Different Reinsurance Premia

For example, in the first column, we see the various costs reinsuring a fixed layer of losses (normalized to 100) corresponding to issuing different face value CAT bonds when the reinsurance premium is set at 0.40. In particular, the cost minimizing size of the issuance occurs at 50, so the optimal allocation across CAT bonds and reinsurance for the insurer is to purchase traditional reinsurance and issue CAT bonds in equal quantities. Moving to the second column, we see the various costs associated with reinsuring the same fixed layer of losses corresponding to different CAT bond issuances when the reinsurance premium is higher, at 0.60. In this instance, the cost minimizing option is to issue CAT bonds with face value 40, and reinsure the remaining 60 dollars of losses through traditional reinsurance.

These results lead to two general observations; firstly, it is apparent that the solution to the insurer's cost minimization problem is indeed achieved with greater CAT bond issuances as the reinsurance premium is increased relative to the spread paid on CAT bonds. This is evident as the cost minimizing issuance generally increases as the reinsurance premium grows (summarized in the table below). The substitution towards the cheaper reinsurance instrument is in accordance with our expectations of insurer behavior.

Reinsurance Premium	0.40	0.60	0.80	1.00	1.20
Optimal CAT FV Issuance	50	40	60	100	100

Secondly, we note that this pattern is by no means clear—there is no clear linear pattern in the distribution of reinsurance costs for the insurer; in short, we can see a visual trend in that lower costs are produced with higher face value bonds when the reinsurance risk premium increases (darker shaded areas trend diagonally from the upper left to the lower right corner), with higher costs roughly trending towards the upper right corner of the table, represented by regions where there is a high reinsurance premium and the insurer issues few CAT bonds. The noise in pattern can be attributed to several sources, and is likely a combination of sampling error (due to a relatively low number of sample paths simulated) as well as the volatile nature of catastrophe events—in many periods, the price of reinsurance may be much higher or lower than expected, based solely on the appearance of a single extra catastrophic occurrence, since each catastrophe often has a very high impact on the reinsurance or CAT bond payouts. In this way, the occurrence of catastrophic events introduces more noise into the reinsurance cost simulations, which prevents us from seeing particularly smooth trends.

5.2 Asset Liability Ratio of the Reinsurer: Examining the Effect of Credit Risk

The reinsurer's capital position is also an important factor to consider in the insurer's optimization problem—in particular, a weaker capital position is more likely to lead to insolvency in the event of catastrophic occurrences, which consequently leads to reduced reinsurance payouts and hence a greater financial burden on the insurer.

The capital position is measured through the asset liability ratio of the reinsurer; from historical patterns and industry standards, we take the average ratio to be 1.3 (Lee and Yu 2006). The table below shows the optimal purchasing behavior according to different asset to liability ratios:

	A/L ratio				
CAT FV	1.00	1.10	1.20	1.30	1.40
0	9.41	9.42	10.88	10.28	9.60
10	11.00	8.81	8.27	9.89	9.40
20	8.43	9.44	8.72	8.60	10.50
30	8.53	8.27	8.52	8.79	10.48
40	7.84	9.59	8.85	8.82	10.09
50	8.18	8.44	7.17	8.76	8.10
60	7.99	8.48	7.29	**7.47**	**8.35**
70	6.92	7.81	8.84	7.61	6.96
80	6.48	6.81	7.91	8.30	7.99
90	6.65	7.00	**6.29**	8.57	8.64
100	**6.10**	**5.28**	7.85	7.93	9.64

Figure 2: Reinsurance Costs For Different Reinsurance Asset/Liability Ratios

The general trends in Figure 2 are as expected, as insurers move towards more CAT bond issuances as the asset to liability ratio grows smaller, as these are instances where reinsurer insolvency is more likely, as seen in the table below. For example, given low asset/liability ratios around 1.00, where there is a high likelihood of default in the event of a catastrophic occurrence, an insurer should optimally issue only CAT bonds and purchase no reinsurance, as it is likely that the reinsurance company will default on their payments. Moving to higher asset/liability ratios where the reinsurance firm is not likely to default on payments due to a stable capital position, we see that the optimal issuance of CAT bonds shrinks to 60, reflecting the idea that the insurer should incorporate both bond issuances and traditional reinsurance when covering their fixed layer of losses.

Asset/Liability Ratio	1.00	1.10	1.20	1.30	1.40
Optimal CAT FV Issuance	100	100	90	60	60

However, we see that for a given asset to liability ratio, the cost of CAT bond issuances does not always behave in a uniform manner:

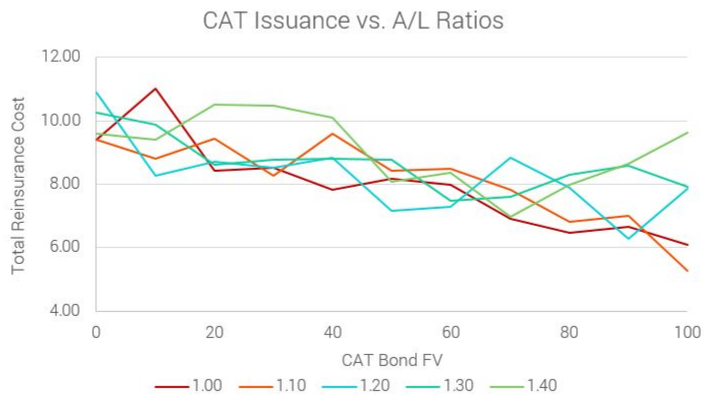


Figure 3: Reinsurance Costs For Different Reinsurance Asset/Liability Ratios

Though there is a rough downward trend, there is significant fluctuation, particularly at higher asset to liability ratios (note that for low ratios, the downward trend appears to be slightly more significant and consistent). This is likely due to the stabilizing effect of a threshold capital position, so that once a sufficiently high asset to liability ratio is reached, the impact of this metric on the cost of reinsurance is likely diminished, giving way to more noisy processes such as interest rate fluctuations or the magnitude of loss events. For example, we see that the optimal CAT bond issuance stabilizes at 60 for asset/liability ratios of 1.30 and 1.40. Hence, it is likely that at asset/liability in this range, the chance of default is sufficiently low so as to not be a determining factor in the insurer's allocation decision, which will be dominated by other processes.

5.3 Volatility and Intensity of Catastrophe Events

We begin by examining the effects of various volatilities in the loss distribution—this is interpreted as the standard deviation of the lognormal distribution of losses. It is evident in Figure 4 below that there does not appear to be any discernable pattern in costs as the volatility of catastrophes increases.

	volatility				
CAT FV	0.5	1	1.5	2	2.5
0	8.86	12.82	16.90	24.49	26.76
10	**8.93**	13.13	19.21	23.88	**30.26**
20	7.58	12.66	18.78	**23.57**	30.14
30	9.83	14.53	16.65	26.28	30.36
40	11.00	**11.77**	19.69	27.22	32.12
50	10.63	14.01	19.48	25.45	32.38
60	8.90	15.08	**15.84**	27.71	31.45
70	8.23	14.72	20.56	25.27	33.28
80	9.26	14.97	19.94	26.54	30.61
90	9.06	15.06	19.81	25.07	32.00
100	9.21	14.79	22.01	25.25	33.69

Figure 4: Reinsurance Costs For Different Loss Distribution Volatilities

Though there may be some underlying fundamental process which governs how costs should respond to increasing volatilities, the apparent randomness in the results is likely due to the noise generated by the increased variation in losses, which drowns out any systematic effect that volatility may have on cost. This noise could potentially be reduced by significantly increasing the number of simulated periods; however, the model under consideration in this analysis is subject to some computing constraints, so we restrict our analysis to 1,000 iterations of simulated periods.

In short, it is likely that the random noise naturally inherent in higher volatility distributions is obscuring any consistent, systematic effect they may have on costs. At present, the current model and simulation process does not allow for any meaningful conclusions on the impact of volatility on an insurer's optimization decisions between traditional reinsurance and CAT bonds.

Moving to the effect of increased intensity of catastrophe events, a clear trend emerges in the increased issuance of CAT bonds. In this context, the 'intensity' refers to the parameter λ, which drives the Poisson process behind the frequency of occurrences. A higher λ thus results in a greater frequency of expected event occurrences, though the mean and standard deviation of the resulting losses remain constant. From Figure 4 below, the optimal issuance of CAT bonds increases rapidly and maxes out very quickly, as a minor increase in intensity greatly increases the magnitude of losses—given the presence of a lognormal distribution, a marginal increase in the number of occurrences can greatly amplify the total loss borne by the insurer, increasing the event of reinsurer default.

	intensity				
CAT FV	1	1.25	1.5	1.75	2
0	8.94	14.35	16.49	19.08	22.07
10	**7.42**	13.05	17.66	18.61	19.75
20	9.13	11.92	17.94	18.59	20.48
30	7.93	13.98	14.47	19.17	17.81
40	9.79	12.58	14.17	18.50	22.42
50	9.11	12.15	15.69	17.39	20.14
60	8.68	11.24	16.15	19.75	21.23
70	8.09	11.50	16.55	18.62	20.31
80	9.89	12.52	14.33	15.58	22.24
90	9.10	**10.95**	13.84	17.35	20.37
100	8.44	11.03	**11.93**	**15.35**	**16.93**

Figure 5: Reinsurance Costs For Different Event Intensities

These credit events become the dominant driver of costs as the intensity increases, as reinsurer insolvency becomes a virtual certainty in regions where $\lambda \geq 1.75$. In these cases, the reinsurance coverage purchased is often not paid out in full. Consider the following observations, which shows a two single instances of 1,000 simulated runs with the associated CAT bond, reinsurance, and total payouts:

Total Loss	CAT Payout	After CAT Losses	RI Coverage	RI $A_T - L_T$	RI Payout	**Total Payout**
1324.96	0.00	1324.96	100.00	30.08	64.83	**64.83**
5965.09	50.00	5915.09	50.00	30.85	43.55	**93.55**

Both observations were conducted with $\lambda = 2$, at the top of the range of simulated intensities. The first observation is one instance where the insurer purchases only traditional reinsurance. The second observation is an instance where the insurer allocates evenly across traditional reinsurance and CAT bonds, issuing bonds of face value 50 and purchasing the remaining 50 from a reinsurance firm. The insurer purchases a fixed amount of reinsurance (100) in each case, though due insolvency in the first case, the reinsurance payout is only a fraction of the coverage purchased (64.83/100), as opposed to the higher payout ratio achieved in the second observation of 93.55/100. In the second instance, the insurer purchased reinsurance coverage worth 50, and despite the reinsurer default (RI payout was 43.55/50), the total payout (reinsurance and CAT bond combined) received by the insurer was very nearly the full amount reinsured (100), as the CAT bond payout is default-free, paying out the full amount that was issued (50).

The virtual certainty of reinsurer default imposes a higher cost on insurers, in the form of bearing more loss payments and hence a higher chance that the firm itself will be insolvent, as they must cover the shortfall suffered due to insufficient reinsurance. The greater costs of bearing this risk is reflected in the heavy preference towards CAT bond issuance as λ grows large. The sharp growth in issuance shows that there is likely some upper bound λ beyond which the certainty of reinsurer default makes it optimal to issue only CAT bonds and avoid any reinsurance purchases.

6 Conclusions and Further Research

6.1 Summary of Findings

We began by stipulating a theoretical model for firm assets, loss dynamics, as well as cost structures for both reinsurance and CAT bond instruments. Using this model, numerical procedures are run to simulate potential outcomes under a given set of parameters (asset to liability ratio, reinsurance premium, etc.). By varying these parameters, we can examine the effect of these parameters on the insurer's cost minimization problem in allocating reinsurance purchases between across traditional markets and CAT bond markets.

In particular, we found that the results largely agreed with a rational expectation of insurer behavior—in short, that higher reinsurance premiums, lower asset/liability ratios, and greater frequency of catastrophe events should encourage increased CAT bond issuances. However, it was noted that varying the volatility of catastrophic events had no discernable effect on reinsurance costs—though this is likely due to flaws within the model which obscured the fundamental connection between increased volatility in the loss distribution and the cost of obtaining reinsurance.

In particular, we note that the impact of reinsurer credit risk has a significant impact on insurer costs and hence allocation decisions—the financial burden resulting from reinsurer insolvency was the driving force in insurer costs when examining the initial capital position, as well as the intensity (frequency) of catastrophic events. Hence, we can conclude that aside from standard considerations such as relative reinsurance and CAT bond premiums, the credit worthiness of the reinsurer is a major factor in reinsurance purchase allocations.

6.2 Further Research

This project focused on a narrow set of parameters and their effects on how insurers might optimally allocate reinsurance purchasing decisions. There are many ways in which to expand upon the ideas examined in this paper—I will focus on two general categories of possible extensions.

6.2.1 Technical Considerations

The first category of possible extensions arises from a more careful consideration of the technical underpinnings of the model—the most obvious example of this is incorporating the presence if *iid* distributed losses during each period, rather than assuming that all losses within a given period are of the same size. Other considerations such as decreasing interval size when evaluating the effect of a parameter (e.g. rather than varying the reinsurance premium from 0.40 to 1.20 in increments of 0.20, using increments of 0.10 or 0.010) or increasing the number of simulations are also important technical improvements on the model which will lead to more accurate, precise results. Both these attributes are important in building an effective model—*accuracy* is important in deriving results that we believe to be fundamentally correct, based on the given parameters; *precision* guarantees consistency, in ensuring white noise signals can be sufficiently managed so as to avoid obscuring the underlying patterns, allowing accurate results to be obtained throughout multiple iterations. In short, there are a number of technical improvements that can be made upon the model given in this paper, which may enable us to state the relevant conclusions with increasing confidence.

6.2.2 Conceptual Extensions

Technical considerations aside, it is also important to consider the underlying ideas expressed in the model—a technically sound model built on faulty intuition would yield incorrect results. The main finding of this paper is in regards to the significant impact of reinsurer credit risk on the insurer's cost minimization problem. A further extension on this topic could incorporate an in-depth examination of the credit risk posed by individual reinsurers through more representative measures such as credit ratings or liquidity measures, rather than asset to liability ratios, which may not be entirely representative of a reinsurer's current liquidity and ability to honor debt obligations. Furthermore, credit modelling is a field of study in its own right, and a detailed analysis on the impact of reinsurer credit risk could consider incorporating such a credit model for a more comprehensive analysis.

Additionally, we return to the idea of linear issuance costs as discussed in the model assumptions. In particular, it would be valuable to consider the incorporation of fixed costs in the insurer's allocation decision process. The high fixed costs could encourage more bond issuances, since spreading the initial fixed cost over a greater volume of bond issuances reduces the per unit cost of issuance. A simple assessment of the situation may lead us to believe that this would generally result in increased bond issuances as compared to the levels seen in the analysis conducted in this paper; however, we must also contend with moral hazard considerations, which act on prices in the opposite direction and may increase the per unit costs of issuance as the face value of the bond increases. In this way, the interplay of these two effects make the net effect difficult to discern without explicit assumptions.

Finally, the analysis conducted in this paper is predicated on a fixed layer of losses that an insurer must cover through either purchase of reinsurance or CAT bond issuances. In actuality, this layer may not be fixed, and the optimal decision making on the part of the insurer could fluctuate based on the amount of reinsurance and CAT bonds they wish to purchase—in short, allocation decisions are not necessarily dependently solely on the ratio of reinsurance and CAT bond issuances, but may depend on the absolute value of the insured layers. An examination of optimal insurer decisions where the condition of fixed loss layers is relaxed would yield a richer discussion of how CAT bonds and traditional reinsurance could potentially act as both substitutes and complements in different environments.

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