



3-2010

# Competing Manufacturers in a Retail Supply Chain: On Contractual Form and Coordination

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## Recommended Citation

Cachon, G. P., & Kok, A. (2010). Competing Manufacturers in a Retail Supply Chain: On Contractual Form and Coordination. *Management Science*, 56 (3), 571-589. <http://dx.doi.org/10.1287/mnsc.1090.1122>

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# Competing Manufacturers in a Retail Supply Chain: On Contractual Form and Coordination

## **Abstract**

It is common for a retailer to sell products from competing manufacturers. How then should the firms manage their contract negotiations? The supply chain coordination literature focuses either on a single manufacturer selling to a single retailer or one manufacturer selling to many (possibly competing) retailers. We find that some key conclusions from those market structures do not apply in our setting, where multiple manufacturers sell through a single retailer. We allow the manufacturers to compete for the retailer's business using one of three types of contracts: a wholesale-price contract, a quantity-discount contract, or a two-part tariff. It is well known that the latter two, more sophisticated contracts enable the manufacturer to coordinate the supply chain, thereby maximizing the profits available to the firms. More importantly, they allow the manufacturer to extract rents from the retailer, in theory allowing the manufacturer to leave the retailer with only her reservation profit. However, we show that in our market structure these two sophisticated contracts force the manufacturers to compete more aggressively relative to when they only offer wholesale-price contracts, and this may leave them worse off and the retailer substantially better off. In other words, although in a serial supply chain a retailer may have just cause to fear quantity discounts and two-part tariffs, a retailer may actually prefer those contracts when offered by competing manufacturers. We conclude that the properties a contractual form exhibits in a one-manufacturer supply chain may not carry over to the realistic setting in which multiple manufacturers must compete to sell their goods through the same retailer.

## **Keywords**

contracting, competition, retailing, wholesale-price contract, quantity discount, two-part tariff

## **Disciplines**

Operations and Supply Chain Management | Other Business

# Competing Manufacturers in a Retail Supply Chain: On Contractual Form and Coordination<sup>1</sup>

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January 18, 2009

## Abstract

It is common for a retailer to sell products from competing manufacturers. How then should the firms manage their contract negotiations? The supply chain coordination literature focuses either on a single manufacturer selling to a single retailer or one manufacturer selling to many (possibly competing) retailers. We find that some key conclusions from those market structures do not apply in our setting. We allow the manufacturers to compete for the retailer's business using one of three types of contracts, a wholesale-price contract, a quantity-discount contract or a two-part tariff. It is well known that there are two reasons why a monopolist manufacturer prefers either of the latter two, more sophisticated, contracts relative to the wholesale-price contract. First, they can be used to coordinate the supply chain, meaning that they induce the retailer to sell more because they reduce the double marginalization caused by wholesale-price contracts. Second, they can be used to extract rents from the retailer, in theory allowing the manufacturer to leave the retailer only with her reservation profit. However, we show that in our market structure these two sophisticated contracts force the manufacturers to compete more aggressively than when they only offer wholesale-price contracts, and this may leave them worse off and the retailer substantially better off. In other words, although in a serial supply chain a retailer may have just cause to fear quantity discounts and two-part tariffs, a retailer may actually prefer those contracts when offered by competing manufacturers. We conclude that the properties a contractual form exhibits in a one-manufacturer supply chain may not carry over to the realistic setting in which multiple manufacturers must compete to sell their goods through a single retailer.

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<sup>1</sup>The authors would like to thank Awi Federgruen, Leslie Marx and Richard Staelin for helpful discussions.

# 1. Introduction

The literature on supply chain coordination has studied several contractual forms in settings with a single manufacturer and one or more retailers. One of the key results from this literature is that wholesale-price contracts lead to suboptimal decisions for the supply chain (i.e., double marginalization) and more sophisticated contracts (like quantity discounts or two-part tariffs) can be employed to achieve both channel coordination (i.e., maximize the supply chain's profit) and rent extraction (i.e., the ability to allocate a high share of the profits to the manufacturer). Our objective in this paper is to test this conclusion in a setting in which multiple manufacturers compete to sell their products through a single retailer.

In our model, two manufacturers simultaneously offer to the retailer one of three types of contracts: a wholesale-price contract, a quantity-discount contract (i.e., a decreasing per-unit price in the quantity purchased) or a two-part tariff (i.e., a per unit price and a fixed fee). The retailer's prices determine the products' demand rates, and she sets her prices to maximize her total profit given the offered contracts and her inventory costs, which may exhibit economies of scale. The model setting represents one of the most common supply chain structures, as many manufacturers sell their products through retailers that also sell products of other manufacturers. A typical example is Procter&Gamble's Crest versus Colgate in the toothpaste category at a supermarket.

As with all supply chain structures, the firms are indirectly interested in the total profit in the supply chain, and more directly interested in their share of that profit. In supply chains with a monopolist manufacturer it has been shown that a properly designed quantity-discount or two-part tariff allows the manufacturer to maximize his product's total profit (i.e., the two firms' combined profit). Furthermore, the manufacturer can extract the largest possible share of that profit, thereby leaving the retailer with her reservation profit, which is the profit the retailer can earn if the retailer rejects the contract. In a serial supply chain the retailer's reservation profit is assumed to be an exogenous constant that reflects the retailer's bargaining power - an increase in the retailer's bargaining power is modeled by increasing the retailer's reservation profit. However, in our structure with two manufacturers, the retailer's reservation profit is endogenous - the profit the retailer can earn if it were to reject manufacturer A's offer depends on what manufacturer B offers, and vice-versa. This distinction is significant and, as we demonstrate, important for our

findings.

We show that, holding the other manufacturer's contract offer fixed, a manufacturer can increase its profit by using a more sophisticated contract relative to a wholesale-price contract. Furthermore, in equilibrium (i.e., a pair of contracts such that neither manufacturer has an incentive to offer a different contract given the other manufacturer's contract offer), we show that the more sophisticated contracts can increase the supply chain's total profit, again relative to the wholesale-price contract. These results are analogous to those found in models with a monopolist manufacturer. However, in sharp contrast to the results with one manufacturer, we also find that in equilibrium, if the products are close substitutes, the manufacturers may earn substantially less when sophisticated contracts are offered and the retailer may earn substantially more.

In a one-manufacturer setting sophisticated contracts are advantageous to the manufacturer because they allow the manufacturer to increase and extract rents. These abilities are present even with multiple manufacturers (holding the other manufacturer's contract fixed), but now there is an additional effect - the sophisticated contracts also yield more aggressive competition between the manufacturers. This additional effect may dominate the others - quantity discounts and two-part tariffs effectively increase the retailer's bargaining power relative to wholesale-price contracts, so much so that the manufacturers can be worse off with them relative to an equilibrium in which they both offer wholesale-price contracts. To explain further, in our multiple-manufacturer model the retailer's reservation profit is endogenous and competition between the manufacturers serves to raise it, in particular when sophisticated contracts are used.

The comparison of equilibrium profits in this setting with two competing manufacturers and a common retailer leads to additional insights on contracting that are different from those in a serial supply chain and those in two competing supply chains with dedicated retailers (as in McGuire and Staelin, 1983). In those settings, two-part tariff and quantity-discount contracts are equivalent from the perspective of the manufacturers and the retailer, and the retailer always prefers the wholesale-price contracts. In our setting, the manufacturers always prefer two-part tariffs to quantity discounts. The retailer prefers aggressive quantity discounts to two-part tariffs, and prefers the wholesale-price contract to both when the products are close substitutes. While the retailer's contract preferences are the reverse of the manufacturers' for low and high substitutability cases, all firms prefer the more sophisticated contracts to wholesale-price contracts for a range of medium

substitutability because the increase in supply chain profit dominates the effects of competition and rent extraction.

The rest of the paper is organized as follows. Section 2 reviews the relevant literature. Section 3 describes the model. Section 4 present our analysis of the retailer’s problem, Section 5 the analysis and comparison of the wholesale-price, quantity-discount and two-part tariff games, and Section 6 the numerical study. Section 7 concludes the paper. All proofs are presented in the Appendix.

## 2. Relation to Literature

The present paper is foremost a commentary on the supply chain coordination literature (see Cachon 2003 for a review). As already discussed, this literature focuses on either relationships with bilateral monopoly or models with one manufacturer and multiple retailers. Wholesale-price contracts are nearly always found to be inefficient and more sophisticated contracts can be used to eliminate that inefficiency and reallocate rents arbitrarily between the parties in the supply chain.<sup>2</sup> There is an extensive literature on supply chain coordination with quantity-discount contracts and price-dependent demand (e.g., Jeuland and Shugan 1983, Moorthy 1987, Ingene and Perry 1995) and on lot-size coordination with fixed demand (e.g., Monahan 1984, Corbett and de Groote 2000), but they consider only one manufacturer.

Choi (1991), Trivedi (1998), Lee and Staelin (1997), and Martinez de Albeniz and Roels (2007) study systems with multiple manufacturers and a common retailer, but they only consider wholesale-price contracts. The literature on vertical separation in economics is also relevant as they study models with multiple manufacturers and retailers that could sell more than one product. Mathewson and Winter (1987) explore whether or not exclusive dealing arrangements lead to foreclosure of rivals and the implications for anti-trust laws considering only wholesale-price contracts. Bernheim and Whinston (1998) and O’Brien and Shaffer (1993, 1997) explore similar issues with two-part tariffs. Other variations of the basic model include manufacturers that invest in the retailers to reduce the marginal selling cost (Besanko and Perry, 1994) and manufacturers

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<sup>2</sup>A wholesale-price contract can maximize profits in a system with one manufacturer and multiple quantity competing retailers. However, it provides only one allocation of the system’s rents and it isn’t even the manufacturer’s optimal wholesale price (see Cachon and Lariviere 2005). If retailers compete on price and quantity, then the wholesale price no longer guarantees coordination (see Bernstein and Federgruen 2003, 2005).

that assign exclusive territories to retailers to reduce competition (Rey and Stiglitz, 1995). The general result is that the manufacturers may prefer exclusive dealing due to reduced competition at the retailer level even though societal welfare and industry profits may be higher with common agency. This is essentially a comparison of different supply chain structures, whereas we are concerned with the effect of different contract types in a given supply chain structure. Also related to two-part tariffs, there is a literature on slotting fees (which are essentially two-part tariffs with negative payments to the manufacturer, see, for example, Marx and Shaffer 2004). Kuksov and Pazgal (2007) show that slotting fees do not occur in a setting with simultaneous manufacturer competition and a single retailer, and the same result applies in our model.

The literature on strategic decentralization in marketing is highly relevant. McGuire and Staelin (1983) study two competing supply chains under two structural forms: in each supply chain either the manufacturer sells to a dedicated retailer via a wholesale-price contract or the manufacturer vertically integrates into retailing. In either structure the products of the two manufacturers are sold to consumers from different firms, whereas in our model the manufacturers' products are sold through a single independent retailer. Nevertheless, there are some similarities in our results. McGuire and Staelin (1983) find that the manufacturers may prefer to sell via wholesale-price contracts, despite the fact that they do not coordinate the channel nor allow the manufacturer to extract all rents, because they dampen retail competition between the two products relative to the vertically integrated structure. In our model, competition to consumers is held constant, because we have a single retailer, so what changes is the retailer's bargaining power relative to the manufacturers. Coughlan (1985) confirms the McGuire and Staelin findings in an empirical study of the international semiconductor industry. In the context of the McGuire and Staelin model, Moorthy (1988) provides conditions on the characteristics of the game and the demand function for decentralization to be an equilibrium strategy and Bonanno and Vickers (1988) show that decentralization is always the equilibrium when both manufacturers employ two-part tariffs. In a different context with strategic consumers timing their purchase of a product, Desai et al. (2004) and Arya and Mittendorf (2006) consider a monopolist manufacturer and show that decentralization through wholesale-price contracts can yield higher profits for the channel.

### 3. The Model

There are two products in the market supplied by two different manufacturers. The products are partial substitutes and are sold through a common retailer. In the first stage of the game, the manufacturers simultaneously announce the payment schemes for their products. In the second stage, the retailer chooses prices, which determine the products' demand rates, to maximize her profit. In addition to the payments to the manufacturer, the retailer incurs operating costs that depend on the average volume sold of each product. The manufacturers incur constant marginal production costs.

The retailer faces price sensitive customers. The revenue from product  $i$  is

$$R_i(d) = p_i(d)d_i,$$

where  $d_i$  is the demand rate of product  $i$ ,  $d$  is the pair of demand rates, and the inverse demand function is

$$p_i(d) = \theta_i - \beta_i d_i - \gamma_i d_j, \text{ and } \beta_i > \gamma_j > 0 \text{ for all } i, j.$$

We elect to work with the inverse demand function for expositional simplicity. The formulation with the demand function is equivalent to the above.

Let  $G_i(d_i)$  be the retailer's inventory related operational costs of product  $i$ ,

$$G_i(d_i) = K_i d_i^\lambda, \quad K_i \geq 0, \quad 0 < \lambda < 1,$$

where  $K_i$  and  $\lambda$  are exogenous constants. This functional form, which exhibits economies of scale, is a general representation of the inventory costs that arise in common inventory replenishment models such as a base-stock model<sup>3</sup> or an economic order quantity model<sup>4</sup>.

Let  $\pi_i$  denote the retailer's profit from product  $i$  and  $\pi = \pi_i + \pi_j$ , the retailer's total profit. It follows that

$$\pi_i = R_i(d) - G_i(d_i) - T_i(d_i),$$

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<sup>3</sup>In a periodic review model where demand follows a Normal distribution with mean  $d_i$  and standard deviation  $\sigma_i d_i^\lambda$ , the total inventory related costs with the optimal base-stock level is given by  $(b + h) \phi(z^*) \sigma_i d_i^\lambda$ , where  $b$  is the backlog penalty per unit, and  $h$  is the inventory holding cost per period. Defining  $K_i = (b + h) \phi(z^*) \sigma_i$  leads to the  $G_i$  function.

<sup>4</sup>In the economic order quantity (EOQ) model, the retailer incurs a fixed cost  $k_i$  per order and a holding cost  $h_i$  per unit of inventory held for one period. The well known EOQ formula suggests ordering every  $\sqrt{2k_i/h_i d_i}$  periods. The resulting optimal inventory and ordering costs is given by  $\sqrt{2k_i d_i h_i}$ . Defining  $K_i = \sqrt{2k_i h_i}$  leads to the  $G_i$  function with  $\lambda = 1/2$ .



and  $T_i(d_i)$  is the payment made to the manufacturer based on the retailer's demand rate and their agreed upon contract.<sup>5</sup> Manufacturer  $i$ 's profit is

$$\Pi_i = T_i(d_i) - c_i d_i, \quad (1)$$

where  $c_i$  is the manufacturer's cost per unit.

We consider three different types of contracts. With a wholesale-price contract, the payment function is

$$T_i(d_i) = w_i d_i,$$

where  $w_i$  is the wholesale price chosen by manufacturer  $i$ .

We include the following set of quantity-discount contracts:

$$T_i(d_i) = \begin{cases} w_i d_i - v_i d_i^2 / 2, & \text{if } d_i \leq (w_i - c_i) / v_i \\ T_i((w_i - c_i) / v_i) + c_i (d_i - (w_i - c_i) / v_i), & \text{otherwise.} \end{cases} \quad (2)$$

where  $w_i$  is a flexible parameter chosen by the manufacturer,  $c_i$  is the manufacturer's marginal cost per unit, and  $v_i$  is an exogenous constant, where  $v_i \in [0, \bar{v})$  and  $\bar{v} = 2\beta_i - (\gamma_i + \gamma_j)$ . We have intentionally designed this set of quantity-discount contracts such that (1) they have a single parameter, just like the wholesale-price contract,  $w_i$ , and (2) wholesale-price contracts are a subset of our quantity-discount contracts - a quantity discount with  $v_i = 0$  is a wholesale-price contract.

Although we only consider a subset of possible quantity discounts, this is not overly restrictive. Our quantity discounts are continuous, differentiable, concave, and the manufacturer does not sell even the marginal unit for less than its production cost. The bound on  $v_i$  implies that the quantity-discount is not too "aggressive" in the sense that the marginal price paid does not fall too rapidly as the purchase volume increases, i.e.,  $T_i''(d_i) \geq -v_i$ . In fact, it can be shown that our quantity discounts are optimal for the manufacturer (holding the other manufacturer's contract offer fixed) among all concave and increasing payment functions given the  $T_i''(d_i) \geq -v_i$  constraint (see Proposition 2 in the Appendix). Furthermore,  $T_i''(d_i) \geq -v_i$  ensures that the retailer's profit function excluding the operational costs is strictly concave in  $(d_1, d_2)$ . This naturally raises the question of whether the manufacturer could do better by offering an even more aggressive

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<sup>5</sup>The retailer's payment  $T_i(d_i)$  to a manufacturer can be interpreted as a yearly (average) payment based on yearly (average) volume. Many manufacturer-retailer purchasing contracts are based on the yearly volume rather than on the volume of individual shipments.

quantity discount. In a single-product environment, the contract with  $v_i$  arbitrarily close to  $\bar{v}$  is the optimal contract, which achieves coordination and full rent extraction. In a two-product environment, the answer is not clear because retail profits are no longer strictly concave under more aggressive quantity discounts (i.e.,  $v_i \geq \bar{v}$ ) and we have found that an equilibrium in the manufacturer's contract offer game may not exist<sup>6</sup>. However, a two-part tariff can be interpreted as the most aggressive quantity discount<sup>7</sup>, and we do have results for those contracts. Thus, although we cannot characterize the equilibrium dynamics for all quantity discounts, we have results for quantity-discounts that are more aggressive than wholesale-price contracts and results for the most aggressive contract, the two-part tariff.

As already mentioned, our third contract form is the two-part tariff, which is characterized by a fixed fee  $F_i$  and a marginal cost  $w_i$  :

$$T_i(d_i) = F_i 1_{\{d_i > 0\}} + w_i d_i, \quad (3)$$

where indicator function  $1_{\{d_i > 0\}} = \{1, \text{ if } d_i > 0; 0, \text{ otherwise}\}$ .

A symmetric game across manufacturers means that the data for the two products are identical, i.e.,  $c_i, \theta_i, \beta_i, \gamma_i, K_i, v_i$  are the same for any  $i$ . The subscript  $i$  will be dropped in those cases. In a symmetric solution, the decisions ( $d_i$  at the retail level,  $w_i$  or  $T_i$  at the manufacturer level) are identical across products.

In the following sections, we solve the problem using backward induction. We analyze the retailer's decision first and then the game between the manufacturers.

## 4. The Retailer's Problem

In the first stage of the game, the manufacturers simultaneously announce their contract offers. We assume that the particular contractual form offered is established before the game begins, but we later discuss what happens when the manufacturers choose their contractual form and parameters simultaneously. In the second stage, given functions  $T_1$  and  $T_2$ , the retailer chooses the demand rates to maximize her profit.

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<sup>6</sup>We provide a numerical example in which there exists no equilibrium. In the example  $\theta = 20, \beta = 2, \gamma = 1, c = 1$ , and  $v = 2(\beta - \gamma)$ , we get cycling, because manufacturer 1's best response is  $w_1(w_2) = \{w_2 - \varepsilon, \text{ if } w_2 \geq 7.75; 8.10, \text{ otherwise}\}$  where  $\varepsilon$  is a small positive constant.

<sup>7</sup>The marginal cost under a quantity discount contract (2) is given by  $T_i(d_i) = \{w_i - v_i d_i, \text{ if } d_i \leq (w_i - c_i)/v_i; c_i, \text{ otherwise}\}$ . As  $v_i \rightarrow \infty$ ,  $T_i(d_i) = \{w_i, \text{ if } d_i = 0; c_i, \text{ if } d_i > 0\}$ , which has the same shape as the marginal cost under a two-part tariff contract.

We present the analysis assuming that the manufacturers offer the retailer quantity discounts, which, as discussed earlier, are wholesale-price contracts when  $v_i = 0$ . The analysis of the retailer's problem under two-part tariffs is similar to that under the wholesale-price contracts. Define

$$d_i(d_j) = \arg \max_{d_i} \{\pi_i(d_i, d_j)\}, \quad (4)$$

$$(\tilde{d}_i, \tilde{d}_j) = \arg \max_{d_i, d_j} \{\pi(d_i, d_j) : d_i d_j > 0\}, \quad (5)$$

$$\tilde{\pi} = \pi(\tilde{d}_i, \tilde{d}_j) \quad (6)$$

The retailer's optimization problem can now be written as

$$\max\{\tilde{\pi}, \pi_1(d_1(0), 0), \pi_2(0, d_2(0))\}. \quad (7)$$

Hence, the optimal solution to the retailer's problem is

$$(d_1^*, d_2^*) \in \{(\tilde{d}_1, \tilde{d}_2), (d_1(0), 0), (0, d_2(0))\}.$$

For expositional simplicity, we assume the retailer breaks ties in favor of carrying a full product line over a single product.

Consider the problem of maximizing total supply chain profit. It is equivalent to the retailer's problem if the manufacturers charge only their production cost. Define  $\hat{\pi}_1 = \pi_1(d_1(0), 0)$ ,  $\hat{\pi}_2 = \pi_2(0, d_2(0))$ , and  $\hat{\pi}_{12} = \pi(\tilde{d}_1, \tilde{d}_2)$  when  $T_i(d_i) = c_i d_i$  for  $i = 1, 2$ . These profit levels are respectively the maximum profit the system earns, if it were to carry only product 1, only product 2 and both products. We assume

$$\hat{\pi}_{12} > \hat{\pi}_i > 0, \text{ for } i = 1, 2, \quad (8)$$

which implies that it is always optimal for the system to carry both products.

Now return to the system with independent manufacturers and a retailer. Let  $H_i$  denote the first derivative of  $\pi$  with respect to  $d_i$ . We have

$$H_i = \partial\pi/\partial d_i = \theta_i - 2\beta_i d_i - (\gamma_i + \gamma_j) d_j - G'_i(d_i) - T'_i(d_i), \quad j \neq i. \quad (9)$$

Consider first the case with no economies of scale (i.e.,  $K_1 = K_2 = 0$ ). The retailer's profit function  $\pi$  is jointly concave in  $(d_1, d_2)$ , so the unique solution to  $\{H_i = 0, i = 1, 2\}$  is the unique optimal solution. If an interior solution does not exist, then the optimal solution is either  $(d_1(0), 0)$  or  $(0, d_2(0))$ , where  $d_i(0)$  is the unique solution to  $H_i = 0$  with  $d_j = 0$ . Because  $\pi$  is well behaved, the

optimal solution  $(d_1^*, d_2^*)$  can be easily characterized and it is a continuous, differentiable function of the problem inputs such as the parameters of the manufacturer contracts.

The case with economies of scale, however, is rather complicated. Observe that  $\partial^2\pi/\partial d_i^2 = -2\beta_i - G_i''(d_i) - T_i''(d_i)$  is positive at  $d_i = 0$  and then decreasing in  $d_i$ . Thus,  $\pi$  is convex-concave in  $d_i$  for fixed  $d_j$ . There are up to two solutions to  $H_i = 0$  and the larger of the two solutions is a local maximum.  $d_i(0)$  is either the larger solution or zero. Evidently,  $\pi$  is not jointly concave in  $(d_i, d_j)$ . As a result, there may be multiple solutions to  $\{H_i = 0, i = 1, 2\}$  and we do not know which one could be the interior optimal solution. Furthermore, the global optimal solution may be at one of the boundaries  $d_i = 0$ . A final technical note is that the profit function is not necessarily unimodal (in one or two dimensions).

The next theorem shows that there can be at most one interior local maximum and that the optimal solution is either that interior solution or at one of the boundary lines. That is, there are at most three candidate optimal solutions and each is characterized by a set of first order conditions. Furthermore, in a symmetric problem, the unique interior maximum is a symmetric solution.

**Theorem 1** *The retailer's optimal solution  $(d_1^*, d_2^*) \in \{(\tilde{d}_1, \tilde{d}_2), (d_1(0), 0), (0, d_2(0))\}$ .  $(\tilde{d}_1, \tilde{d}_2)$  is the unique interior optimal solution to  $\{H_i = 0, i = 1, 2\}$  and  $d_i(0)$  is given by the larger of the two solutions to  $\{H_i = 0, s.t. d_j = 0\}$ . In a symmetric problem,  $(\tilde{d}_1, \tilde{d}_2) = (\tilde{d}, \tilde{d})$ , where  $\tilde{d}$  is the larger of the two solutions to*

$$\theta - 2(\beta + \gamma)d - G'(d) - T'(d) = 0.$$

In summary, there are at most three local maxima for a retailer's problem: one interior solution in which the retailer carries both products, and two boundary solutions in which the retailer carries only one product.

While the retailer's profit function is generally complex in the presence of economies of scale, the following conditions ensure that it is jointly concave in  $(d_1, d_2)$ . We state and prove this result in Lemma 1 in the Appendix.

$$\beta_1 = \beta_2 = \beta, \quad \gamma_1 = \gamma_2 = \gamma, \tag{10}$$

$$T_i''(d_i) \geq -v_i, \quad \text{where } 0 \leq v_i < \bar{v} = \beta - \gamma, \tag{11}$$

$$2R_i/G_i \geq |\varepsilon_{ii}|, \quad \text{for all } i, \quad \text{where } \varepsilon_{ii} = -\frac{\beta_j}{\beta_i\beta_j - \gamma_i\gamma_j} \frac{p_i}{d_i} \tag{12}$$

The condition (10) requires the own- and cross-price coefficients to be symmetric. This is not a very restrictive assumption, because we allow nonidentical  $\theta_i$ , which implies different demand rates and price elasticities for the products. The condition (11) is stricter than our earlier assumption: it requires the quantity discount to be less concave to guarantee the concavity of the retailer's profit function. The condition (12) stipulates that the own-price elasticity ( $\varepsilon_{ii}$ ) is less than two times the revenues-to-average inventory costs ratio of the product. (It is similar to the conditions Bernstein and Federgruen (2003) developed for decentralized retailers and as they point out the conditions hold for most retailers based on the industry data in Dun and Bradstreet (2006) and Tellis (1988)).

## 5. Manufacturers' Problem

In this section, we analyze the game between the manufacturers which offer quantity discounts (possibly with  $v_i = 0$ ) or two-part tariffs to the retailer. First, we define the game and make some initial observations. We then characterize the equilibria under quantity discounts and two-part tariffs in Sections 5.1 and 5.2, respectively, and compare the equilibria in Section 5.3 for a special case of the problem.

Each manufacturer chooses its own best response  $T_i(T_j)$  given the other manufacturer's contract offer  $T_j$ .

$$T_i(T_j) = \arg \max_{T_i(d)} \Pi_i(d^*) \text{ for all } i, \text{ where } d^* = \arg \max \pi. \quad (13)$$

An equilibrium of the game is a pair of contracts  $(T_i^*, T_j^*)$  such that neither manufacturer has an incentive to offer a different contract.

The following remark demonstrates how the contracting problem with multiple manufacturers is different from that with a single manufacturer.

**Remark 1** *For any fixed contract offered by manufacturer 2 such that  $\pi_2(0, d_2(0)) > 0$ :*

1. *Consider the set of contracts such that the retailer's payment to manufacturer 1 is a non-decreasing function of  $d_1$ . There does not exist a contract in this set such that the manufacturer can extract all of the profit from his product (i.e., it is not possible to have  $\Pi_1 > 0$  and  $\pi_1 = 0$ ).*

2. The retailer accepts manufacturer 1's contract offer and stocks both products only if

$$\pi_1(\tilde{d}_i, \tilde{d}_j) \geq \pi_2(0, d_2(0)) - \pi_2(\tilde{d}_i, \tilde{d}_j) \quad (14)$$

Unlike in a serial supply chain, the first statement implies that a manufacturer must leave the retailer with some profit to induce the retailer to carry the manufacturer's product (see the Appendix for a detailed proof). In other words, the retailer's reservation profit for carrying a product is greater than zero. However, this does not mean that a single reservation profit exists. The second statement provides an intuitive condition, (14), for when the retailer is willing to carry manufacturer 1's product - the retailer must earn more with product 1 in the assortment than without product 1 in the assortment. The right hand side of (14) can be considered the retailer's reservation profit that the retailer must earn from product 1 for the retailer to be willing to carry product 1. The first term,  $\pi_2(0, d_2(0))$ , is fixed, given manufacturer 2's contract offer. However, the second term,  $\pi_2(\tilde{d}_i, \tilde{d}_j)$ , depends on manufacturer 1's contract offer and it decreases if  $\pi_1(\tilde{d}_i, \tilde{d}_j)$  is increasing, making it more difficult for the manufacturer to make its offer attractive. Therefore, even if manufacturer 2's contract offer is held fixed, there does not exist a reservation profit for product 1 that is independent of manufacturer 1's contract offer, as is assumed in models with a single manufacturer. Put another way, a serial supply chain with a fixed reservation profit for the retailer cannot replicate the dynamics of our model.

It remains to characterize the equilibrium contract offers by the manufacturers. We start with the following observation that rules out equilibria in which the retailer carries only one product in symmetric games.

**Remark 2** *In a symmetric game, there does not exist a manufacturer equilibrium where  $d_i = 0$  for some  $i$ .*

The result is due to (8), which guarantees that the inclusion of a manufacturer strictly increases system profit. Suppose there were an equilibrium in which manufacturer  $i$  is excluded. Regardless of the fraction of  $\hat{\pi}_j$  the retailer earns, manufacturer  $i$  can offer to sell to the retailer at  $c_i + \varepsilon$  for an arbitrarily small  $\varepsilon$  and then the retailer's profit increases if it carries product  $i$ . If  $j$  is excluded as a result, it will react similarly and be included.

## 5.1 Wholesale-Price and Quantity-Discount Contracts

In this section, we partially characterize the equilibrium of the game under quantity-discount and wholesale-price contracts. In the presence of economies of scale (i.e., if  $K_i > 0$  for any  $i$ ), we assume that conditions (10)-(11) hold and we restrict our attention to a region defined by (12). Define  $\underline{w}_i(w_j) = \max\{w_i : d_j^*(w_i, w_j) = 0\}$ , the maximum  $w_i$  that makes the retailer exclude product  $j$  and  $\bar{w}_i(w_j) = \min\{w_i : d_i^*(w_i, w_j) = 0\}$ , the minimum wholesale price of  $i$  that makes the retailer exclude product  $i$ . Note that  $\underline{w}_i(w_j)$  may not exist for every  $w_j$ . In that case, set  $\underline{w}_i(w_j) = c_i$ . Define  $w_i(w_j)$  as the best response of manufacturer  $i$ , which can be found via a line-search between  $[\underline{w}_i, \bar{w}_i]$ .

The following theorem characterizes the unique symmetric equilibrium of the contract offer game.

**Theorem 2** *Consider a symmetric game in which the manufacturers offer quantity discounts. There exists a unique symmetric solution to*

$$d_i^*(w_1, w_2) + (w_i - v_i - c_i) \frac{\partial d_i^*}{\partial w_i} = 0, \text{ for all } i,$$

*denoted  $(w_1^*, w_2^*)$ , which is the unique candidate to be a symmetric equilibrium.*

As can be seen in the proof of the above theorem, showing the unimodality of a manufacturer's profit in  $w_i$  requires the use of the second-order properties of the retailer's optimal solution  $(d_1^*, d_2^*)$ . The analysis of a manufacturer game that is built on the solution of a complex problem at the retailer presents technical difficulties in the presence of economies of scale, that have not been present in any other competition paper in the literature.

We cannot guarantee that the candidate point described in Theorem 2 is an equilibrium. We show that at  $(w_1^*, w_2^*)$ ,  $w_i^*$  is a local optimum for manufacturer  $i$ , and  $\Pi_i$  is concave for  $w_i > w_i^*$ . However, the optimal solution  $w_i(w_j^*)$  may be different than  $w_i^*$  in the range  $[\underline{w}_i, w_i^*]$ . If  $w_i(w_j^*) = w_i^*$ , then  $(w_1^*, w_2^*)$  is indeed an equilibrium point. If not, there there exists no symmetric equilibrium.

Now consider the situation in which there are no economies of scale. This substantially simplifies the analysis of both the retailer's demand decisions and the manufacturers' contract offer problem. For any  $(v_1, v_2)$  and asymmetric products, we can now guarantee joint concavity of the

retailer's profit and the existence and the uniqueness of the equilibrium without the symmetry assumptions. The next theorem provides the closed form solutions for the demand rates and the contract parameters by solving the first-order conditions given in Theorem 2.

**Theorem 3** *With no economies of scale (i.e.,  $K_i = 0$ ), there exists a unique equilibrium of the game in which the manufacturers offer quantity discounts. It is characterized by the following reaction functions and the optimal demand rates:*

$$d_i^* = \frac{(2\beta_j - v_j)(\theta_i - w_i) - (\gamma_i + \gamma_j)(\theta_j - w_j)}{(2\beta_i - v_i)(2\beta_j - v_j) - (\gamma_i + \gamma_j)^2},$$

$$w_i^*(w_j) = \frac{[(2\beta_j - v_j)(\theta_i) - (\gamma_i + \gamma_j)(\theta_j - w_j)](\delta + v_i(2\beta_j - v_j)) + c_i(2\beta_j - v_j)\delta}{(2\beta_j - v_j)(\delta + \delta + v_i(2\beta_j - v_j))},$$

where  $\delta \equiv (2\beta_i - v_i)(2\beta_j - v_j) - (\gamma_i + \gamma_j)^2$ .

In this section we have assumed that the manufacturers offer quantity discounts with an exogenously specified  $v_i$ . Suppose the manufacturers now simultaneously offer a  $(w_i, v_i)$  where they are free to choose any  $v_i \in [0, \hat{v}]$  for some  $\hat{v} < \bar{v}$ . In other words, they can choose to offer a wholesale-price contract ( $v_i = 0$ ) or a quantity discount ( $v_i > 0$ ). The next proposition indicates, as in supply chains with a single manufacturer, that a manufacturer prefers to offer a quantity-discount contract and, in particular, prefers more aggressive quantity discounts. Quantity discounts allow the manufacturer to increase supply chain rents (i.e., reduce double marginalization) and to extract rents, so they are the preferred contract when the other manufacturer's contract is held fixed even when the retailer can adjust its demand allocations between the two products in response.

**Proposition 1** *Manufacturer  $i$ 's profit strictly increases with  $v_i$  at the optimal  $w_i$ . That is, if a manufacturer is given the option to choose between three contractual forms with  $v_i \in \{0, a, b\}$  such that  $0 < a < b$ , then  $\Pi_i(w_i^*|v_i = 0) < \Pi_i(w_i^*|v_i = a) < \Pi_i(w_i^*|v_i = b)$ .*

The immediate implication of this proposition is that if the manufacturers endogenously choose the quadratic parameter of their quantity discount as well as the starting wholesale price, each manufacturer's best response is to choose the most aggressive contract allowed in the system. The next corollary states that any equilibrium of this game would then have the most aggressive contracts by both manufacturers.



**Corollary 1** *In the contract choice game in which the manufacturers simultaneously offer  $(w_i, v_i)$  and they are free to choose any  $v_i \in [0, \hat{v}]$  for some  $\hat{v} < \bar{v}$ , the equilibrium contracts are such that  $v_i = \hat{v}$  for both manufacturers.*

This does not mean however, that the manufacturers are better off at this equilibrium under quantity discounts than the equilibrium under wholesale-price contracts. Table 1 provides a numerical example to compare the equilibrium solution when the manufacturers offer wholesale-price and quantity-discount contracts for  $\theta_i = 20$ ,  $\beta_i = 2$ ,  $\gamma_i = 1$ ,  $c_i = 1$ ,  $K_i = 0$  for all  $i$ , and  $v_1 = v_2 = 1.6$  when quantity discounts are offered.

| Contracts offered  | $v_1$ | $v_2$ | $w_1^*$ | $w_2^*$ | $d_1^*$ | $d_2^*$ | $\pi$ | $\Pi_1$ | $\Pi_2$ |
|--------------------|-------|-------|---------|---------|---------|---------|-------|---------|---------|
| Wholesale price    | 0     | 0     | 7.33    | 7.33    | 2.11    | 2.11    | 26.74 | 13.37   | 13.37   |
| Quantity discounts | 1.6   | 1.6   | 7.59    | 7.59    | 2.82    | 2.82    | 35.02 | 12.22   | 12.22   |

Table 1: Equilibrium contract parameters, demand rate, and profits under wholesale-price and quantity-discount contracts.

Recall that a manufacturer prefers to offer a quantity discount for any fixed contract offer by the other manufacturer. However, this preference does not carry over to the equilibrium analysis. Competition between the manufacturers is different when they both offer quantity discounts than when they both offer wholesale-price contracts. Apparently, it can be a more aggressive type of competition. Note, the supply chain is better off with quantity-discounts - as in a supply chain with a monopolist manufacturer, quantity discounts improve supply chain efficiency. However, in this example, the retailer captures all of the improved efficiency and even more, leaving the manufacturers worse off. This need not always be true, as we demonstrate in Sections 5.3 and 6.

## 5.2 Two-Part Tariffs

In this section, we consider the game between manufacturers that offer two-part tariff contracts. Several papers in the industrial organization literature (e.g., Bernheim and Whinston, 1998) provide a characterization of the equilibrium with two-part tariffs. The next theorem generalizes this characterization to the case with economies of scale at the retailer.

**Theorem 4** *Given any contract by manufacturer  $j$ , the optimal contract of manufacturer  $i$  is a two-part tariff contract of the type  $(F_i, c_i)$ . The equilibrium of the two-part tariff game is given by*

$F_i^* = \hat{\pi}_{12} - \hat{\pi}_j$  for  $i = 1, 2, j = 3 - i$ . Hence,  $\Pi_i = \hat{\pi}_{12} - \hat{\pi}_j$  and  $\pi = \hat{\pi}_1 + \hat{\pi}_2 - \hat{\pi}_{12}$ .

As can be seen in the proof of the theorem, showing the optimality of a two-part tariff in the presence of economies of scale requires checking the boundary solutions of the multi-modal retail profit function so that its product does not get dropped when a manufacturer changes its contract. The first result states that the optimal contract for a manufacturer is to set the variable cost equal to the marginal production cost, thereby making a profit exclusively from the fixed fee,  $\Pi_i = F_i$ . The second part of the theorem states that in equilibrium each manufacturer charges a fixed fee that equals exactly the incremental benefit it brings to the whole system. Overall, just as we have shown that a manufacturer prefers quantity discounts to wholesale-price contracts for a fixed contract offer from the other manufacturer (Proposition 1), Theorem 4 indicates that two-part tariffs are a dominant strategy for the manufacturer over all contracts - a manufacturer prefers the more aggressive two-part tariffs over a wholesale-price contract.

When the manufacturers offer two-part tariffs, the retailer chooses system optimal pricing and ordering decisions (because the manufacturers' per unit prices equal their marginal costs). Hence, the system's profit is maximized and the allocation of that profit is determined by the fixed fees. System profits are not always maximized when the manufacturers offer wholesale-price or quantity-discount contracts.

Although the manufacturers are able to extract rent more effectively with two-part tariffs than with quantity-discounts, they cannot extract all of the profit from their product. The retailer gains considerable power when she retains stocking and pricing decision rights and the manufacturers compete. Especially when the products are close substitutes,  $\hat{\pi}_{12}$  is not much higher than  $\hat{\pi}_1$  or  $\hat{\pi}_2$ . Thus, in that case, the retailer captures a large part of the total profit and each manufacturer gets only  $\hat{\pi}_{12} - \hat{\pi}_i$ , which is relatively small.

### 5.3 Comparison of the Equilibria for a Special Case

In this section, we consider a special case, a system with symmetric products (i.e., identical  $\theta_i, \beta_i, \gamma_i, c_i, v_i$ ) and no economies of scale at the retailer (i.e.,  $K_i = 0$ ), in order to fully characterize and compare the equilibria under all three contract types. We present a numerical study for the general case in the next section, which provides results similar to the results obtained here.

In this case, Theorem 1 guarantees a unique equilibrium for the quantity-discount game for all

$v < 2(\beta - \gamma)$ . (For  $v_i = 2(\beta - \gamma)$ , there may exist no equilibrium.) We are not able to analytically compare the wholesale-price equilibrium (i.e.,  $v = 0$ ) with the equilibrium under a quantity discount for arbitrary  $v$ , but we can make this comparison at the limit as  $v$  goes to  $2(\beta - \gamma)$ . The limit case can be interpreted as the equilibrium outcome if the manufacturers can endogenously choose  $(w, v)$  as contract parameters, because Corollary 1 states that both manufacturers' will choose the most aggressive quantity discounts at equilibrium. As we shall see next, the equilibrium of the game with this most aggressive quantity discount achieves the supply chain optimal solution. Table 2 presents the equilibrium profit terms under the three contract types. The derivation of the profit expressions are provided in the Appendix.

| Contract  | Manufacturer ( $\Pi_i$ )   | Retailer ( $\pi$ )   | Supply Chain ( $\pi + \Pi_1 + \Pi_2$ )                                       |
|---|--|--|--|
| Wholesale price   | $\frac{(\theta-c)^2(\beta)(\beta-\gamma)}{2(\beta+\gamma)(2\beta-\gamma)^2}$ | $\frac{(\theta-c)^2\beta^2}{2(\beta+\gamma)(2\beta-\gamma)^2}$ | $\frac{(\theta-c)^2\beta(3\beta-2\gamma)}{2(\beta+\gamma)(2\beta-\gamma)^2}$ |
| Quantity discount<br>as $v \rightarrow 2(\beta - \gamma)$ | $\frac{(\theta-c)^2(\beta-\gamma)}{4(\beta+\gamma)^2}$                       | $\frac{(\theta-c)^2\gamma}{(\beta+\gamma)^2}$                  | $\frac{(\theta-c)^2}{2(\beta+\gamma)}$                                       |
| Two-part tariff   | $\frac{(\theta-c)^2(\beta-\gamma)}{4\beta(\beta+\gamma)}$                    | $\frac{(\theta-c)^2\gamma}{2\beta(\beta+\gamma)}$              | $\frac{(\theta-c)^2}{2(\beta+\gamma)}$                                       |

Table 2: Equilibrium profits under wholesale-price, quantity-discount, and two-part tariff contracts in the case with symmetric products and no economies of scale.

From the supply chain perspective, the limiting quantity-discount and two-part tariff equilibria result in supply chain optimal profits, while wholesale-price equilibrium does not. This is consistent with the results for supply chains with a single manufacturer. Other quantity-discount equilibria with  $v < 2(\beta - \gamma)$ , however, may not result in supply chain optimal profits as we shall see in the numerical study.

The next theorem compares the manufacturers' profits under the three contract types. Even though quantity discounts and two-part tariffs are the dominant strategies over the wholesale-price contracts, the manufacturers may be worse off in equilibrium with quantity-discounts and two-part tariffs relative to wholesale-price contracts:

**Theorem 5 (*Manufacturers' Perspective*)** (i) *The manufacturers profits are always higher under two-part tariffs than quantity-discount contracts.* (ii) *The manufacturers profits are higher un-*

der quantity discounts than wholesale-price contracts if and only if  $\gamma < (3 - \sqrt{7}) \beta \approx (0.354) \beta$ . (iii) The manufacturers profits are higher under two-part tariffs than wholesale-price contracts if and only if  $\gamma < (2 - \sqrt{2}) \beta \approx (0.586) \beta$ .

Manufacturer profits are always higher with two-part tariffs than quantity discounts because they both yield the same supply chain optimal price levels at equilibrium, but two-part tariffs are more effective in rent extraction. This is different from the cases of a serial supply chain and two competing supply chains with dedicated retailers, where the two contract forms are equivalent. Comparing these contracts with the wholesale-price contracts is nontrivial. If the products are highly substitutable, the manufacturers' profits are lower under two-part tariffs and quantity discounts than wholesale-price contracts because they compete more aggressively with these contracts.

Now, let us consider the problem from the retailer's perspective. The next theorem compares the retailer's equilibrium profits under the three contract types.

**Theorem 6 (*Retailer's Perspective*)** (i) *The retailer's profit is always higher under quantity-discount contracts than two-part tariffs.* (ii) *The retailer's profit is higher under quantity discounts than wholesale-price contracts if and only if  $\gamma > \frac{1}{2} (3 - \sqrt{7}) \beta \approx (0.177) \beta$ .* (iii) *The retailer's profit is higher under two-part tariffs than wholesale-price contracts if and only if  $\gamma > \frac{1}{2} (3 - \sqrt{5}) \beta \approx (0.382) \beta$ .*

The retailer always prefers quantity-discount contracts to two-part tariffs because they are equally efficient from a system perspective, but quantity discounts are less effective in rent extraction from the retailer. The second and third parts of the theorem state that the retailer's profit is lower under two-part tariff and quantity-discount contracts only if the product substitutability is very low. For a wide range of  $\gamma$ , the retailer is better off with quantity discounts and two-part tariffs. These results are different from the cases of a serial supply chain and two competing supply chains with dedicated retailers, where the two-part tariff and quantity-discount contracts are equivalent and the retailer always prefers the wholesale-price contracts to both of them. In the extreme case of independent products, i.e.,  $\gamma = 0$ , the retailer's profit is zero with a quantity discount or a two-part tariff, because there is no competition and each manufacturer is able to extract all the profit with sophisticated contracts.

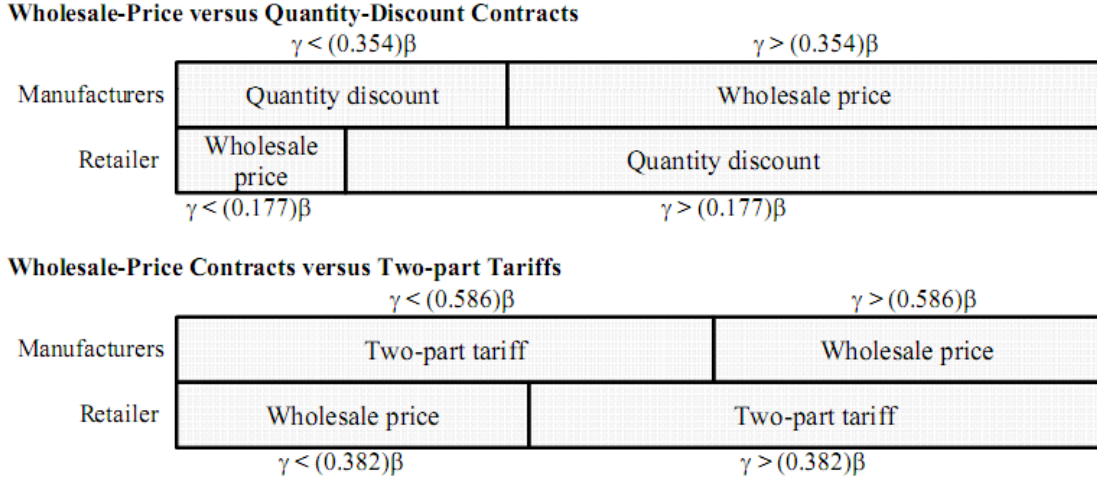


Figure 1: Contract preferences of the firms for different levels of substitutability.

These results demonstrate that the root cause of the retailer’s power is product substitutability. We do not assign the retailer the power to dictate contract terms, yet with highly substitutable products the retailer earns a larger share of the total profit due to quantity or pricing decision rights. The retailer even prefers the more aggressive contract types when products are close substitutes, because the benefits of more intense competition dominates the disadvantage of the rent extraction by the manufacturers.

Figure 1 illustrates the manufacturers’ and the retailer’s contract preferences for different levels of substitutability. While the contract preference of the retailer is the reverse of the manufacturers’ for low and high substitutability cases, for a range of medium substitutability, all players prefer the more sophisticated contracts to wholesale-price contracts, as the increase in supply chain profit dominates the effects of the competition and rent extraction. These results have implications on which contractual forms could be observed in practice, depending on who determines the contract forms in a supply chain and the level of substitutability between products.

#### 5.4 Complementary Products

Two products are complements if the demand rate of one increases with the consumption of the other. Complementary products can be included in our model by allowing the cross-elasticity parameter  $\gamma$  to be negative. In this section, we briefly describe what elements of the above analysis and results apply in the case with complementary products. The analysis of the retailer’s

problem remains the same, with the technical stipulation that the quadratic parameter of the quantity discounts  $v_i$  must be chosen from  $[0, 2(\beta + \gamma))$  for joint concavity of the retail profit function. More importantly, the nature of the game between the manufacturers changes with complementary products. The manufacturers are no longer in competition and would like the other to sell as much as possible. This leads to the decreasing best response functions  $w_i(w_j)$  as can be seen from the expression given in Theorem 3. The equilibrium solutions given in Table 2 no longer apply for the limiting quantity discount and two-part tariff cases. Evaluating the profit functions as  $v$  goes to  $2(\beta + \gamma)$  for the special case considered above, we find that the manufacturer profits are equal to the supply chain optimal,  $(\theta - c)^2 / 2(\beta + \gamma)$ , and the retailer's profit is zero. There are multiple equilibria in the two-part tariff game where the payoffs are  $(\Pi_1, \Pi_2, \pi) = (\hat{\pi}_{12} - \hat{\pi}_2, \hat{\pi}_2, 0)$  or  $(\hat{\pi}_1, \hat{\pi}_{12} - \hat{\pi}_1, 0)$ , or a linear combination of these points. For the special case,  $\hat{\pi}_{12} = (\theta - c)^2 / (\beta + \gamma)$  and  $\hat{\pi}_i = (\theta - c)^2 / 2\beta$ , and the quantity-discount equilibrium is the midpoint of the continuum of the equilibria of the two-part tariff game. These results are formally stated and proved in Theorem 7 in the Appendix. In summary, without substitutability, the retailer has no power, because it can no longer play the two manufacturers off of each other. Both the most aggressive quantity discounts and two-part tariffs achieve coordination as well as full rent extraction.

## 6. Numerical Study

This section presents numerical examples comparing the equilibrium solution when the manufacturers offer wholesale-price, quantity-discount and two-part tariff contracts. We start with exploring the effect of the quadratic parameter of the quantity-discount contracts on manufacturer profits. Recall the example in Table 1, in which the manufacturers are worse off under quantity discounts with  $v = 1.6$  than wholesale-price contracts. Figure 2(a) plots the change in the manufacturers' profit for that example as  $v$  varies from zero to  $2(\beta - \gamma)$ . As  $v$  decreases from 1.6 to 0.4, then the manufacturers are better off offering quantity discounts than wholesale-price contracts - in this case the manufacturer competition has not been intensified substantially, allowing them to increase their profit. However, if the products become more substitutable (lower  $\beta$  or higher  $\gamma$ ), then the manufacturers are worse off with quantity-discounts no matter how weak the quantity discounts are (i.e., no matter how low  $v$  is). On the other hand, with less substitutable products (lower  $\gamma$ ) the

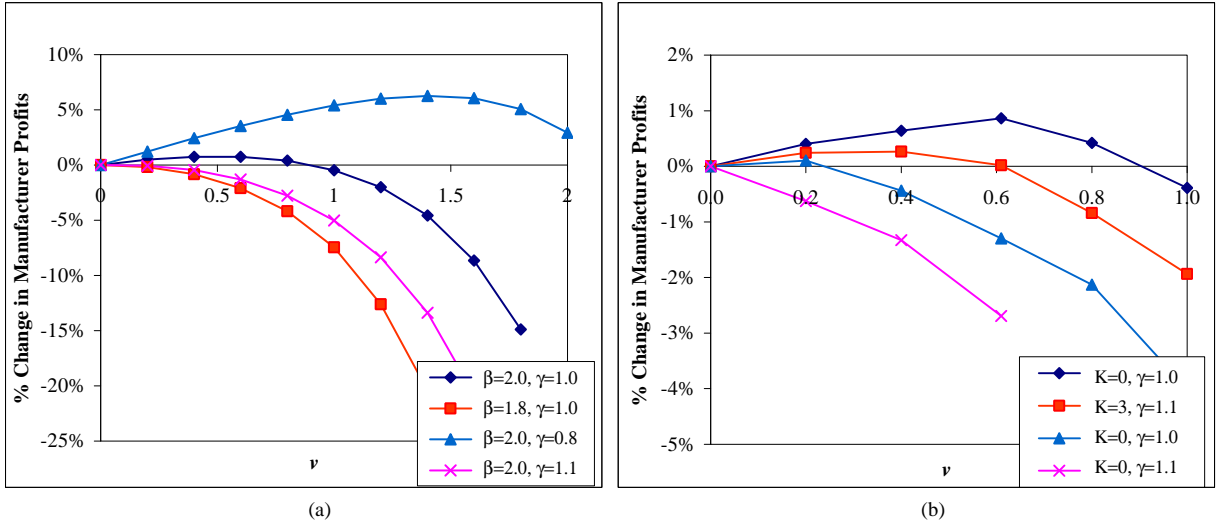


Figure 2: The effect of the concavity of quantity-discount contracts on manufacturer profits: % change in  $\Pi_1$  at equilibrium as  $v$  increases. Base case:  $\theta = 20, \beta = 2, \gamma = 1, c = 1, K = 0$ .

manufacturers benefit from quantity discounts at equilibrium. Figure 2(b) shows that including inventory costs can make the negative effects of competition with quantity discounts on the manufacturers' profits more pronounced. This is because a higher level of economies of scale makes the retailer more sensitive to the manufacturer offers in making the quantity decisions, and in effect increases the retailer's power in the system.

We constructed a numerical study to better understand the extent of these observations. We chose parameters from the following sets, leading to 144 scenarios:  $\theta = \{20, 40\}$ ,  $\beta = \{1, 2, 4\}$ ,  $\gamma = \{0.25, 0.5, 0.75, -0.25\} \times \beta$ ,  $c = \{1, 3\}$ ,  $K = \{0, 1, 3\}$ . We evaluate each scenario under wholesale-price, quantity-discount, and two-part tariff contracts. For quantity-discount contracts, we set  $v = \{0, 0.5, 0.95, 1.8\} \times (\beta - |\gamma|)$  if  $K = 0$  and  $v = \{0, 0.5, 0.95\} \times (\beta - |\gamma|)$ , if  $K > 0$ . This leads to a total of 672 instances. In eight of the 144 scenarios, we haven't been able to find an equilibrium for at least one of the contracts (including the wholesale-price contracts in five scenarios). Investigating the best response functions in those cases reveals that the effect of the economies of scale is very strong at the retailer and the manufacturers cycle between undercutting prices to get the retailer to exclude the other manufacturer and being undercut. In those cases, there does not exist an asymmetric equilibrium either. We report the results for the remaining 136 scenarios. As a validity check, we compare the average cost per unit that the retailer pays to

the manufacturers,  $T^*(d)/d$ , with the two types of contracts: the average cost is 5% lower in the quantity discounts equilibrium than the wholesale prices when  $v = 0.5(\beta - |\gamma|)$  and 11% lower when  $v = 0.95(\beta - |\gamma|)$ , which indicates that the quantity discounts in equilibrium in these examples are modest.

Table 3 provides a summary of the comparison of profits relative to the wholesale-price equilibrium. From the supply chain perspective, the total profit increases in all cases when quantity discounts or two-part tariffs are offered. This is of course expected for two-part tariffs, because they achieve system coordination. Quantity discounts on the other hand achieve coordination only at the extreme case as  $v$  goes to the limit. The total profit under quantity-discount contracts get closer to the supply chain optimal as  $v$  increases and is almost as good as two-part tariffs when  $v = 1.8\beta$ . The percentage increase in total profit decreases as product substitutability increases, because the inefficiency of the system under wholesale-price contracts decreases with higher  $\gamma$ .

For the retailer and manufacturer profits, let us first focus on substitutable products case (i.e.,  $\gamma > 0$ ). The manufacturers' profits increase under quantity discounts and two-part tariffs relative to wholesale-price contracts when substitution is low (i.e.,  $\gamma = 0.25\beta$ ). Furthermore, the manufacturers' profit increase is greater than the supply chain's profit increase, which means that the manufacturers are able to extract a higher share of the supply chain's profit. As product substitutability increases, the manufacturers' profits are lower when they offer quantity discounts and two-part tariffs and the magnitude of the loss can be as high as 44%. The profit loss for the manufacturers increase is higher with a more aggressive quantity discount (i.e., higher  $v$ ) or a two-part tariff. In those cases the effect of more aggressive competition among the manufacturers dominates the effect of increased supply chain profits. As the level of economies of scale increases, the retailer benefits even more from the switch to sophisticated contracts and the manufacturers benefit less. With a higher level of economies of scale, because the retailer has more incentive to consolidate its demand to one product, the competition becomes more intense and the retailer gains more power. In other words, the positive effect of quantity discounts and two-part tariffs on the manufacturers' profits in the case of low substitutability diminishes with higher degree of economies of scale and the negative effect in the case of high substitutability becomes more pronounced. For instance, with  $\gamma = 0.5\beta$  and  $v = 0.95(\beta - \gamma)$  or two-part tariffs, both the percentage of cases in which the manufacturers are worse off and the average magnitude of the loss increase with  $K$ .



| $\gamma = 0.25 \times \beta$  |                         |                          |                         |                    |                         |                          |                    |                         |                          |                    |
|-------------------------------|-------------------------|--------------------------|-------------------------|--------------------|-------------------------|--------------------------|--------------------|-------------------------|--------------------------|--------------------|
| K=0                           |                         |                          |                         | K=1                |                         |                          | K=3                |                         |                          |                    |
| Change in Profit              | Quantity Discount       |                          |                         | Two-Part<br>Tariff | Quantity Discount       |                          | Two-Part<br>Tariff | Quantity Discount       |                          | Two-Part<br>Tariff |
|                               | $\nu=0.5(\beta-\gamma)$ | $\nu=0.95(\beta-\gamma)$ | $\nu=1.8(\beta-\gamma)$ |                    | $\nu=0.5(\beta-\gamma)$ | $\nu=0.95(\beta-\gamma)$ |                    | $\nu=0.5(\beta-\gamma)$ | $\nu=0.95(\beta-\gamma)$ |                    |
| Supply Chain                  | 5.7%                    | 11.2%                    | 21.3%                   | 22.5%              | 5.8%                    | 11.3%                    | 22.6%              | 6.0%                    | 11.8%                    | 22.9%              |
| Retailer                      | 3.3%                    | 6.0%                     | 14.7%                   | -23.4%             | 3.2%                    | 5.9%                     | -20.1%             | 3.3%                    | 6.1%                     | -11.2%             |
| Manufacturers                 | 7.3%                    | 14.6%                    | 25.8%                   | 53.1%              | 7.5%                    | 14.8%                    | 50.2%              | 7.7%                    | 15.2%                    | 43.6%              |
| % Cases Manuf. Loss           | 0%                      | 0%                       | 0%                      | 0%                 | 0%                      | 0%                       | 0%                 | 0%                      | 0%                       | 0%                 |
| $\gamma = 0.5 \times \beta$   |                         |                          |                         |                    |                         |                          |                    |                         |                          |                    |
| Supply Chain                  | 3.5%                    | 6.8%                     | 12.0%                   | 12.5%              | 3.6%                    | 6.9%                     | 12.4%              | 3.7%                    | 7.1%                     | 12.1%              |
| Retailer                      | 6.2%                    | 13.8%                    | 38.5%                   | 12.5%              | 6.6%                    | 14.7%                    | 16.3%              | 7.1%                    | 15.8%                    | 25.6%              |
| Manufacturers                 | 0.8%                    | -0.2%                    | -14.5%                  | 12.5%              | 0.7%                    | -0.6%                    | 8.6%               | 0.5%                    | -1.2%                    | -0.3%              |
| % Cases Manuf. Loss           | 0%                      | 100%                     | 100%                    | 0%                 | 0%                      | 100%                     | 0%                 | 0%                      | 100%                     | 41.7%              |
| $\gamma = 0.75 \times \beta$  |                         |                          |                         |                    |                         |                          |                    |                         |                          |                    |
| Supply Chain                  | 1.4%                    | 2.6%                     | 4.1%                    | 4.2%               | 1.4%                    | 2.6%                     | 3.9%               | 1.4%                    | 2.6%                     | 3.4%               |
| Retailer                      | 5.9%                    | 12.4%                    | 28.1%                   | 17.3%              | 6.1%                    | 12.9%                    | 19.5%              | 6.3%                    | 13.1%                    | 23.0%              |
| Manufacturers                 | -7.5%                   | -16.9%                   | -44.2%                  | -21.9%             | -8.2%                   | -18.6%                   | -27.9%             | -8.6%                   | -19.3%                   | -39.8%             |
| % Cases Manuf. Loss           | 100.0%                  | 100.0%                   | 100.0%                  | 100.0%             | 100.0%                  | 100.0%                   | 100.0%             | 100.0%                  | 100.0%                   | 100.0%             |
| $\gamma = -0.25 \times \beta$ |                         |                          |                         |                    |                         |                          |                    |                         |                          |                    |
| Supply Chain                  | 9.2%                    | 19.0%                    | 40.9%                   | 44.6%              | 9.3%                    | 19.3%                    | 45.1%              | 9.6%                    | 20.0%                    | 46.1%              |
| Retailer                      | -3.1%                   | -10.9%                   | -63.9%                  | -100.0%            | -3.7%                   | -12.3%                   | -100.0%            | -5.1%                   | -15.7%                   | -100.0%            |
| Manufacturers                 | 14.1%                   | 30.9%                    | 82.9%                   | 102.5%             | 14.3%                   | 31.3%                    | 100.6%             | 14.7%                   | 32.2%                    | 96.5%              |
| % Cases Manuf. Loss           | 0%                      | 0%                       | 0%                      | 0%                 | 0%                      | 0%                       | 0%                 | 0%                      | 0%                       | 0%                 |

Table 3. Average percentage change in the supply chain, retailer and manufacturer profits in equilibrium under quantity-discount and two-part tariff contracts relative to the equilibrium under wholesale-price contracts.

The retailer’s profit decreases relative to the wholesale-price contracts only with two-part tariffs and when substitutability is low. This is because manufacturers can extract rents well with two-part tariff contracts. (We would expect similar results for the quantity-discount contracts for  $\gamma$  lower than  $0.177 \times \beta$  based on Theorem 6.) Even with two-part tariffs, the average change decreases (from -23.4% to -11.2%) as the level of economies of scale increases in the system. Higher substitutability and higher economies of scale endow the retailer with more power in the system, and the switch to more aggressive contracts is beneficial for the retailer. As can be seen from low economies of scale and low substitution cases, the retailer may prefer less aggressive quantity discounts to the two-part tariffs, because quantity discounts extract rents from the retailer less effectively. For medium substitutability and low economies of scale cases, both the retailer and the manufacturers benefit from the use of more sophisticated contracts relative to the wholesale-price contracts.

Our observations from the cases with complementary products is consistent with those with substitutable products. Because the retailer power diminishes with complementary products, the retailer’s profit always decreases and the manufacturer profits always increase with quantity discounts. Two-part tariffs achieve both system coordination and full rent extraction in all cases.

In summary, the decisions for the two products are more tightly linked if they are close substitutes or the level of economies of scale is high. This increases the retailer’s endogenous reservation profit and effectively the retailer’s power in its relation with each manufacturer. Our results suggest that the manufacturers are better off employing the simple wholesale-price contracts when dealing with a powerful retailer rather than the more sophisticated quantity discounts or two-part tariffs. When quantity discounts and two-part tariffs are not very useful to the supply chain, because products are highly substitutable, they are particularly destructive for the manufacturers. If they benefit the supply chain substantially, then they are good for the manufacturers too.

## 7. Discussion

This paper is a first attempt to consider contracts other than wholesale-price contracts in systems with multiple competing manufacturers and a single common retailer. We demonstrate that, holding manufacturer B’s contract offer fixed, manufacturer A can use a quantity discount or two-part tariff to improve supply chain performance and extract rents, as is known with contracting in supply chains with a monopolist manufacturer. However, when the manufacturers compete in

the contract offers, then we show that the manufacturers may be worse off with quantity discounts or two-part tariffs relative to wholesale-price contracts even though the supply chain can be better off. When downstream competition between the manufacturers is high, i.e., when the products are close substitutes, the retailer benefits considerably when the manufacturers compete with the more aggressive contracts, and the manufacturers are worse off. If the products are not close substitutes, then the retailer may be harmed by the more aggressive contracts and the manufacturers are better off, because the manufacturers are able to extract rent more effectively. We find these effects to be more significant when the level of economies of scale at the retailer is high. In summary, when the products are close substitutes or the level of economies of scale at the retailer is high, the retailer's endogenous reservation profit (the value of the option to carry more of the competitor's product) increases and effectively the retailer's power in its relation with each manufacturer. Our results suggest that the manufacturers are better off employing the simple wholesale-price contracts in such cases rather than the more sophisticated quantity discounts or two-part tariffs. While the retailer's preference is the reverse of the manufacturers' for low and high substitutability cases, all firms prefer the more sophisticated contracts for a range of medium substitutability. Finally, unlike in serial supply chains and two competing supply chains with dedicated retailers, two-part tariffs and quantity discounts are not equivalent from the manufacturers' and the retailer's perspectives because two-part tariffs allow more rent extraction by the manufacturers at equilibrium. We conclude that it is important to study the properties of a contractual form in the presence of competition between manufacturers.

The prominence of wholesale-price contracts in practice and their widespread use in many industries, despite the fact that the manufacturers' optimal contract is usually a more sophisticated one, has been an open empirical question for researchers. One possible explanation has to do with the simplicity and lower implementation costs associated with wholesale-price contracts. Our results may provide another hypothesis to include in this discussion. The manufacturers in our model essentially face a prisoners' dilemma game - their best myopic action is always to offer the most aggressive contract, but this then leads to an equilibrium in which they are both worse off. In a repeated game setting, it is well known that the players may be able to coordinate on the good outcome (offering wholesale-price contracts when the products are close substitutes) by utilizing trigger strategies (defections are penalized by the other firm for a limited number of periods)

even though the bad outcome, offering quantity-discount or two-part tariff contracts, is the Nash equilibrium in a single-shot game. It may be argued that the manufacturers have learned not to engage in actions that will trigger punishment, and hence choose wholesale-price contracts over more sophisticated contracts that are myopically optimal, but harmful at equilibrium. It will be interesting to test this hypothesis empirically by investigating which contract types are commonly used for various product groups with different levels of substitutability.

A quantity discount essentially reduces a retailer's marginal cost. Other contracts, such as buy-back and revenue-sharing contracts, have a similar effect on the retailer. Thus, we suspect that similar results can be found for manufacturers competing with these contracts. That may require, however, retailer models that are different from ours, such as a model with inventory competition (i.e., substitution between products based on availability). While the particular retailer model we have worked with presents many technical difficulties in the presence of economies of scale, alternative models may not be less challenging. For the inventory competition model, the characterization of the best response of a manufacturer even in the simpler wholesale-price case can be achieved only under restrictive assumptions (see K ok 2003). The derivation of the optimal wholesale-price contract with the method in Lariviere and Porteus (2001) is not possible because the retailer changes the quantities of both products in response to a change in the wholesale-price of one manufacturer. Finally, extension of this discussion to a setting with multiple common retailers or considering other forms of competition between manufacturers, such as based on product quality or assortment depth, may lead to richer and more complicated dynamics.

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## Appendix: Proofs

### Theorem 1

**Proof.** The proof holds for general quantity discount contracts that satisfy

$$T_i(0) = 0, \quad T_i'(d) \geq c_i, \quad -v_i \leq T_i''(d) \leq 0, \quad T_i'''(d) \geq 0, \quad T_i''''(d) \leq 0. \quad (15)$$

The quantity discount contract given by (2) satisfies these conditions. The proof consists of two steps. The first step proves that the retailer's problem would not admit more than one local maximum in  $\{(d_i, d_j) : d_i > 0 \text{ for } i = 1, 2\}$ . The second step characterizes each of the solutions  $(\tilde{d}_1, \tilde{d}_2), (\tilde{d}_1(0), 0), (0, \tilde{d}_2(0))$ . The proof of the first step is by contradiction. Suppose that there are two interior local maxima:  $(x', y')$  and  $(x'', y'')$ . The line that connects  $(x', y')$  and  $(x'', y'')$  can be characterized by  $(x' + \alpha t, y' + \delta t)$ , where  $\alpha = x'' - x', \delta = y'' - y', \alpha, \delta \in \Re, t \in [0, 1]$  represents the line segment between the two points, and  $t \in \Re$  represents the whole line. Define  $\Delta(t)$  as the value of  $\pi$  on that line,

$$\Delta(t) = \pi(x' + \alpha t, y' + \delta t)$$

We have,

$$\Delta'(t) = \frac{\partial \pi}{\partial d_i} \frac{\partial d_i}{\partial t} + \frac{\partial \pi}{\partial d_j} \frac{\partial d_j}{\partial t} = \alpha \frac{\partial \pi}{\partial d_i} + \delta \frac{\partial \pi}{\partial d_j}.$$

Because  $\Delta(t)$  achieves local maxima at the points  $(x', y')$  and  $(y', x')$ , we have  $\Delta'(t) = 0$  and  $\Delta''(t) < 0$  at these points. We now derive higher order derivatives:

$$\begin{aligned} \Delta'''(t) &= -\alpha^3 G_i'''(x' + \alpha t) - \delta^3 G_j'''(y' + \delta t) - \alpha^3 T_i'''(x' + \alpha t) - \delta^3 T_j'''(y' + \delta t) \\ \Delta''''(t) &= -\alpha^4 G_i''''(x' + \alpha t) - \delta^4 G_j''''(y' + \delta t) - \alpha^4 T_i''''(x' + \alpha t) - \delta^4 T_j''''(y' + \delta t) \end{aligned}$$

It follows that  $\Delta''''(t) > 0$  because  $G_i''''(d_i) = -\lambda(1-\lambda)(2-\lambda)(3-\lambda)K_i d_i^{\lambda-3} < 0$  and  $T_i'''' \leq 0$  by equation (2). Therefore,  $\Delta''''(t)$  is increasing.

Recall that  $\Delta'(t) = 0$  and  $\Delta''(t) < 0$  at  $t \in \{0, 1\}$ . Given that  $\Delta(t)$  is continuous in  $t$ , this can only occur if there is at least one segment in  $t \in [0, 1]$  such that  $\Delta'(t)$  is convex-concave, which requires that  $\Delta''''(t)$  is decreasing along some segment. However, we have established that  $\Delta''''(t)$  is increasing. Hence, a contradiction.

For the second step, it suffices to say that the interior optimal solution  $(\tilde{d}_1, \tilde{d}_2)$  satisfies the first order conditions  $\{H_i = 0, i = 1, 2\}$ . The solution on the boundary  $(\tilde{d}_1(0), 0)$  satisfies the first order condition  $\{H_1 = 0, \text{ s.t. } d_2 = 0\}$ . Similarly for  $(0, \tilde{d}_2(0))$ .

It is easy to see that the profit function is convex-concave along the  $d_i = 0$  lines.

$$\begin{aligned}\partial\pi(d_i, 0)/\partial d_i &= \theta_i - 2\beta_i d_i - G'_i(d_i) - T'_i(d_i), \\ \partial^2\pi(d_i, 0)/\partial d_i^2 &= -2\beta_i - G''_i(d_i) - T''_i(d_i).\end{aligned}$$

The second derivative is (i) decreasing in  $d_i$ , (ii) positive at  $d_i = 0$  (because  $G''_i(d_i) = -\lambda(1-\lambda)K_i d_i^{\lambda-2} \rightarrow -\infty$  as  $d_i \rightarrow 0^+$ ), and (iii) negative for sufficiently large  $d_i$ . Thus, there can be at most one local maximum for each problem.

For a symmetric problem, it follows from the above argument that the unique interior optimal solution is on the  $d_i = d_j$  line. (If  $(d_i, d_j)$  with  $d_i \neq d_j$  is an interior optimal solution, then there are at least two local maxima, because  $(d_j, d_i)$  is also an optimal solution by the symmetry of the profit function. This contradicts with the result above.) Similar to the boundary lines, the profit function is convex-concave on the  $d_i = d_j$  line and the solution is characterized by the first order condition

$$\partial\pi(d, d)/\partial d = \theta - 2(\beta + \gamma)d - G'(d) - T'(d) = 0.$$

■

**Lemma 1** *The the retailer function  $\pi(d_1, d_2)$  is jointly concave in the region where  $(d_1, d_2)$  satisfies (12) and  $T_i(d_i)$  satisfies (11).*

**Proof.** First consider the retailer's problem under wholesale-price contracts from both manufacturers. We show that the Hessian is a negative semi-definite matrix, which guarantees joint concavity of the profit function. Note that (12) implies that  $2R_i/G_i = 2p_i d_i/G_i > (\beta/(\beta^2 - \gamma^2))(p_i/d_i)$ . First,

$$\partial^2\pi/\partial d_i^2 = -2\beta + \lambda(1-\lambda)G_i d_i^{-2} \leq 0$$

if and only if  $(1/\beta_i)p_i/d_i \leq 2\lambda^{-1}(1-\lambda)^{-1}p_i d_i/G_i$ , which is implied by (12),  $\beta/(\beta^2 - \gamma^2) > (1/\beta)$ , and  $\lambda^{-1}(1-\lambda)^{-1} \geq 4$ . Second, we show that the Hessian is a diagonally dominant matrix.

$$|\partial^2\pi/\partial d_i^2| = 2\beta - \lambda(1-\lambda)G_i d_i^{-2} \geq |\partial^2\pi/\partial d_j \partial d_i| = 2\gamma$$

if and only if  $p_i/d_i \leq 2(\beta - \gamma)\lambda^{-1}(1-\lambda)^{-1}p_i d_i/G_i$ , which holds under (12) if  $\beta/(\beta^2 - \gamma^2) > 1/(2\beta - 2\gamma)$ , equivalently  $2\beta > \beta + \gamma$ .



The proof of the case with quantity discount contracts is similar. To show diagonal dominance, we split the right-hand-side of the second derivatives to show  $(\beta - \gamma) > -G_i''$  and  $(\beta - \gamma) > -T_i''$ . The former inequality holds by the condition (12) and the latter by (11). ■

**Remark 1**

**Proof.** Suppose manufacturer 1 offers  $T_1(d_1)$  and there exists a demand pair  $(d'_1, d'_2)$  such that  $\pi_1 = 0$ . By definition,  $\pi_2(d'_1, d'_2) = R_2(d'_1, d'_2) - T_2(d'_2) - G_2(d'_2)$ . We have  $\pi_2(d'_1, d'_2) < \pi_2(0, d'_2) \leq \pi_2(0, d_2(0))$ . The first inequality is due to  $\partial R_2'(d_1, d_2)/\partial d_1 < 0$  for all  $d_1, d_2$  and  $T_1(d_1)$  is non-decreasing: the retailer can always increase her revenue from product 2 by dropping product 1, and her payment to manufacturer 1 cannot increase. The second inequality follows by optimality of  $d_2(0)$ . Therefore  $\pi(0, \tilde{d}_2(0)) > \pi(d'_1, d'_2)$ . ■

**Proposition 1**

The proof follows from the proof of Proposition 2.

**Proposition 2** *Among all quantity discounts that satisfy  $T_i''(d_i) \leq -v_i$ , the dominant strategy for manufacturer  $i$  has the following functional form.*

$$T_i(d_i, w_i, v_i) = \begin{cases} w_i d - v_i d_i^2/2, & \text{if } d_i \leq (w_i - c_i)/v_i \\ T_i((w_i - c_i)/v_i) + c_i (d_i - (w_i - c_i)/v_i), & \text{otherwise.} \end{cases} \quad (16)$$

Thus, for given  $T_j(d_j, w_j, v_j)$  and fixed  $v_i$ , the best response of manufacturer  $i$  is given by  $T_i(d_i, w_i^*, v_i)$  for some  $w_i^* \geq [c_i, \theta_i(v_i/\beta_i + 1)]$ .

**Proof.** Without loss of generality, let  $j = 2$ , and  $i = 1$ . Take any  $T_1(d_1)$  and  $T_2(d_2)$ . Suppose that the retailer's optimal solution is an internal point  $\tilde{d} = (\tilde{d}_1, \tilde{d}_2)$ . (The proof is simpler if one of  $\tilde{d}_i$  is zero.) At the optimal solution the first order conditions are satisfied:  $\{H_1 = 0, H_2 = 0\}$ .

Define  $\tau$  such that

$$\tau = H_1(\tilde{d}) + T_1'(\tilde{d}_1) = \frac{\partial (R_1(\tilde{d}) + R_2(\tilde{d}) - G_1(\tilde{d}_1))}{\partial d_1}$$

We have  $T_1'(\tilde{d}_1) = \tau$ . The optimal discount scheme for M1 among those that generate  $\tilde{d}$  is the solution to

$$\begin{aligned} & \max T_1(\tilde{d}_1) \\ & \text{subject to } H_i(\tilde{d}) = 0, \text{ for } i = 1, 2. \end{aligned}$$

The constraints guarantee that both first order conditions are satisfied at  $(\tilde{d}_1, \tilde{d}_2)$ . The retailer function is jointly concave everywhere in the absence of economies of scale at the retailer as long as  $T_i$  satisfies  $T_i''(d_i) \leq -v_i$ . Thus  $(\tilde{d}_1, \tilde{d}_2)$  remains the optimal solution (and the unique local maximum) for the retailer. In the presence of economies of scale,  $(\tilde{d}_1, \tilde{d}_2)$  remains the unique interior optimal solution for the retailer by Theorem 1 as long as  $T_i$  satisfies (11). Because, the retailer profit is jointly concave when (10)-(12) are satisfied,  $(\tilde{d}_1, \tilde{d}_2)$  remains the optimal solution in the region defined by (12).

The manufacturer's problem can be written as  $\{\max T_1(\tilde{d}_1) : T_1'(\tilde{d}_1) = \tau\}$  because the condition implies  $H_1(\tilde{d}) = 0$  and we already have  $H_2(\tilde{d}) = 0$ . Therefore, the objective is to increase  $T_1(\tilde{d}_1)$  while keeping the marginal cost at  $\tilde{d}_1$  the same. This can be achieved by reducing  $T_1'(d_1)$  as little as possible for all  $d_1 < \tilde{d}_1$ , which can be achieved by setting  $T_1''(d_1) = v_1$ . Together with  $T_1'(\tilde{d}_1) = \tau$ , this implies that

$$T_1'(d_1) = -v_1 (d_1 - \tilde{d}_1) + \tau, \text{ for all } d_1 \in [0, \tilde{d}_1]$$

That is, the marginal cost to the retailer is decreasing as slowly as possible and equals  $\tau$  at  $\tilde{d}$ . We have  $T_1''(\tilde{d}_1) = -v_1$ .  $T_1'(d_1)$  can be specified in any way for  $d_1 > \tilde{d}_1$  as long as it satisfies (15). We use the same functional form to specify  $T_1'(d_1)$  (which implies that  $T_1''(d_1) = -v$ ) for all  $d_1 > \tilde{d}_1$  to make sure that the argument applies to all  $\tilde{d}_1$ . Another condition in (15) requires  $T_1'(d_1) \geq c_1$ .  $T_1'$  is decreasing linearly in  $d_1$  and it will reach  $c_1$  at some finite value. Rewriting  $T_1'$ , we replace  $v_1\tilde{d}_1 + \tau$  with  $w_1$  to obtain,

$$T_1'(d_1) = \begin{cases} w_1 - v_1 d_1, & \text{for } d_1 \leq (w_1 - c_1)/v_1 \\ c_1, & \text{for } d_1 > (w_1 - c_1)/v_1 \end{cases}$$

Integrating  $T'$  and recalling the boundary value  $T(0) = 0$  by (15), we obtain the quantity discount schedule

$$T_1(d_1) = \begin{cases} w_1 d_1 - v_1 d_1^2/2, & \text{if } d_1 \leq (w_1 - c_1)/v_1 \\ T_1((w_1 - c_1)/v_1) + c_1 (d_1 - (w_1 - c_1)/v_1), & \text{otherwise.} \end{cases}$$

Note that this is not the optimal discount scheme over all possible discount schemes. It is the best scheme among the ones that produce  $\tilde{d}$ . In other words, the functional form dominates other functional forms of  $T_1(d_1)$ . Thus, the optimal quantity discount scheme can be found by considering the functions of this type only. We will see in Theorem 2 that the constant part of  $T_1'(d_1)$  is not relevant. A technical note is needed here that (15) requires continuous and differentiable payment

functions, but the suggested solution has a non-differentiable point. This can be circumvented by replacing the two-piece marginal cost function with a differentiable convex decreasing function that approximates it very closely.

The manufacturer profit is zero if  $w_1 \leq c_1$ . It is also zero, if  $w_1 \geq \theta_1(v_1/\beta_1 + 1)$ , because the marginal cost to the retailer  $w_1 - v_1 d_1 > w_1 - v_1 \theta_1/\beta_1 > \theta_1(v_1/\beta_1 + 1) - v_1 \theta_1/\beta_1 = \theta_1$ , which implies zero demand. The first inequality is due to  $d_1 \leq \theta_1/\beta_1$  to keep  $p_1$  nonnegative. If the retailer charges  $\theta_1$  for product 1, the demand cannot be positive, which implies that  $\Pi_i = 0$ . ■

## Theorem 2

**Proof.** Recall that  $v_i = 0$  when the manufacturers employ wholesale-price contracts. Furthermore, while evaluating the equilibrium, we need to only consider the quadratic part of the payment functions. Suppose by contradiction, that the retailer chose  $d_i$  such that  $d_i > (w_i - c_i)/v_i$ . The manufacturer can increase  $w_i$  such that  $d_i = (w_i - c_i)/v_i$  and increase its profit without affecting the retailer's or the other manufacturer's decisions.

We show in the proof that  $\Pi_i$  is concave in  $w_i$  at the symmetric solution to the first order condition stated in the theorem. Hence, the solution is (at least) a local maximum.

Using the quadratic part of  $T_i$ , we obtain the first order condition for a manufacturer as follows.

$$\frac{\partial \Pi_i}{\partial w_i} \text{ at } w_i^* = d_i^* + (w_i - c_i - v_i d_i^*) \frac{\partial d_i^*}{\partial w_i} = 0$$

Applying the Implicit Function Theorem on the retailer's first order conditions  $\{H_i = 0$ , for all  $i\}$ , we can derive the impact of the wholesale prices on the optimal demand rates. Define  $A_i = (2\beta_i + G_i'' + T_i'')$ ,  $A_i' = \partial A_i / \partial d_i = G_i''' = \lambda(1 - \lambda)(2 - \lambda)d_i^{\lambda-3} = -(2 - \lambda)d_i^{-1}G_i'' > 0$ ,  $B = (\gamma_i + \gamma_j)$ , and  $\Delta = A_i A_j - B^2$ . Note that  $\Delta > 0$  because of the diagonal dominance of the Hessian as show in Lemma 1.

$$\left[ \frac{\partial d_i^*}{\partial w_i} \right] = \frac{1}{A_i A_j - B^2} \begin{bmatrix} -A_j \\ (\gamma_i + \gamma_j) \end{bmatrix}$$

As expected,  $d_i$  decreases with  $w_i$  and increases with  $w_j$ . The second derivative of  $\Pi_i$  yields

$$2 \frac{\partial d_i^*}{\partial w_i} - v \left( \frac{\partial d_i^*}{\partial w_i} \right)^2 + (w_i - c_i - v_i d_i^*) \frac{\partial^2 d_i^*}{\partial w_i^2},$$

which is negative when  $d_1^* = d_2^*$ , because we have  $A'_1 = A'_2$  and

$$\begin{aligned}
\frac{\partial^2 d_i^*}{\partial w_i^2} &= \Delta^{-2} [-A'_j B \Delta^{-1} \Delta + A_j (A_i A'_j B \Delta^{-1} + A'_i A_j (-A_j) \Delta^{-1})] \\
&= \Delta^{-3} [-A'_j B \Delta + (A_i A_j A'_j B - A'_i A_j^3)] \\
&= \Delta^{-3} [-A'_j B A_i A_j + A'_j B^3 + (A_i A_j A'_j B - A'_i A_j^3)] \\
&= \Delta^{-3} [-A'_i A_j^3 + A'_j B^3] < 0
\end{aligned}$$

Hence, the first order condition above has a unique solution when  $d_1^* = d_2^*$ . The solution  $(w_1^*, w_2^*)$  satisfies the first and second order conditions for both manufacturers. That is,  $w_i^*$  is a local maximum of  $\Pi_i$  for fixed  $w_j$ , and vice versa. If  $w_i > w_i^*$ , then  $d_1^*$  decreases and  $d_2^*$  increases,  $A'_i$  increases,  $A_j$  increases, and  $A'_j$  decreases, implying that  $\partial^2 d_i^* / \partial w_i^2$  remains negative. Hence,  $\Pi_i$  is concave in  $w_i$  for  $w_i > w_i^*$  for fixed  $w_j^*$ . Thus, there can be no other local maximum greater than  $w_i^*$ , but a linear search in  $[w_i, w_i^*]$  is necessary to find the optimal wholesale price. ■

### Theorem 1

**Proof.** The optimal demand rates can be solved as the unique solution to first order conditions  $\{H_i = 0, i = 1, 2\}$  for given  $(w_1, w_2)$ . We have  $\alpha \equiv \partial d_i / \partial w_i = -(2\beta_j - v_j) / \delta$ ,  $\partial^2 d_i / \partial w_i^2 = 0$ , and  $\partial^2 d_i / \partial w_i \partial w_j = 0$ ,  $\partial d_i / \partial w_j = (\gamma_i + \gamma_j) / \delta$ . Substituting these in the first order conditions in Theorem 2, we verify that the profit function of manufacturer  $i$  is concave in  $w_i$ . This guarantees the existence of equilibria in the quantity discount game between the manufacturers. The best response  $w_i(w_j)$  is the explicit solution to each first order condition. Differentiating, we obtain

$$\frac{\partial w_i}{\partial w_j} = \frac{(1 - v_i \alpha) (\gamma_i + \gamma_j) / \delta}{-2\alpha + v_i \alpha^2} = \frac{(1 - v_i \alpha) (\gamma_i + \gamma_j)}{(2\beta_j - v_j) (2 - v\alpha)} > 0 \text{ and } < 1.$$

The second inequality is due to  $v_i < 2\beta_i - (\gamma_i + \gamma_j)$ . Hence, we have increasing reaction functions with a slope less than 1. This implies that there is a unique equilibrium of the game between the manufacturers. ■

### Theorem 4

**Proof.** The first part of the proof shows that for any set of contracts by the manufacturers, manufacturer  $i$  can do better by switching to a two-part tariff  $(F_i, c_i)$ . Let us focus without loss of generality on manufacturer 1. Let superscripts  $b$  and  $a$  denote the solutions before and after the contract change. Take any potential manufacturer equilibrium  $(T_1(d_1), T_2(d_2))$ , where the

retailer chooses  $(d_1^b, d_2^b)$  with  $d_1^b > 0$ . Suppose that manufacturer 1 (M1) switches to two-part tariff  $(T_1(d_1^b) - c_1 d_1^b, c_1)$ . If the retailer optimal solution  $(d_1^a, d_2^a)$  is interior or  $d_2^a = 0$ , M1's profit remains the same, i.e., equal to  $T_1(d_1^b) - c_1 d_1^b$ . It is not possible that  $d_1^a = 0$ , because  $\pi^{r,a}(d_1^a, d_2^a) \geq \pi^{r,a}(d_1^b, d_2^b) = \pi^{r,b}(d_1^b, d_2^b) > \pi^{r,b}(0, d_2^b(0)) = \pi^{r,a}(0, d_2^a(0))$ . The first inequality is due to optimization, the equality because the payment to M1 has not changed, the second inequality from the optimality of  $(d_1^b, d_2^b)$  before the contract change, and the last equality holds because the contract for product 2 has not been changed. If  $d_1^b = 0$ , then M1 profit was zero, switching the contract to two-part tariff  $(0, c_1)$  yields zero profit for any  $(d_1^a, d_2^a)$ .

The second part characterizes the equilibrium fees  $(F_1^*, F_2^*)$ . For any  $(F_1, c_1), (F_2, c_2)$ , we have  $\Pi_i = F_i 1_{\{d_i^* > 0\}}$ , and  $\pi = \max\{\hat{\pi}_1 - F_1, \hat{\pi}_2 - F_2, \hat{\pi}_{12} - F_1 - F_2, 0\}$ . Let us focus on M1. If  $F_2 \leq \hat{\pi}_{12} - \hat{\pi}_1$ , then  $\hat{\pi}_1 - F_1 \leq \hat{\pi}_{12} - F_1 - F_2$  and  $\pi = \max\{\hat{\pi}_2 - F_2, \hat{\pi}_{12} - F_1 - F_2\}$  and M1's best response to  $F_2$  is to set  $F_1$  as high as possible while ensuring that  $\hat{\pi}_{12} - F_1 - F_2 \geq \hat{\pi}_2 - F_2$  hence  $F_1^*(F_2) = \hat{\pi}_{12} - \hat{\pi}_2$ . If  $\hat{\pi}_2 \geq F_2 > \hat{\pi}_{12} - \hat{\pi}_1$ , then  $\hat{\pi}_1 - F_1 > \hat{\pi}_{12} - F_1 - F_2$  and  $\pi = \max\{\hat{\pi}_1 - F_1, \hat{\pi}_2 - F_2\}$  and M1's best response to  $F_2$  is to set  $F_1$  as high as possible while ensuring that  $\hat{\pi}_1 - F_1 \geq \hat{\pi}_2 - F_2$ . Hence  $F_1^*(F_2) = \hat{\pi}_1 - \hat{\pi}_2 + F_2 - \varepsilon$  for arbitrarily small  $\varepsilon$ . If  $F_2 > \hat{\pi}_2$ , then  $\hat{\pi}_1 - F_1 > \hat{\pi}_{12} - F_1 - F_2$  and  $\pi = \max\{\hat{\pi}_1 - F_1, 0\}$  and M1's best response is to set  $F_1 = \hat{\pi}_1$ . The best response function of manufacturer 2 is similarly obtained:  $F_2^*(F_1) = \{\hat{\pi}_{12} - \hat{\pi}_1, \text{ if } F_1 \leq \hat{\pi}_{12} - \hat{\pi}_2; \hat{\pi}_2 - \hat{\pi}_1 + F_1 - \varepsilon, \text{ otherwise}\}$ . The unique equilibrium point is  $(F_1^*, F_2^*) = (\hat{\pi}_{12} - \hat{\pi}_1, \hat{\pi}_{12} - \hat{\pi}_1)$ . At equilibrium, the retailer's profit is  $\pi = \hat{\pi}_{12} - (\hat{\pi}_{12} - \hat{\pi}_1) - (\hat{\pi}_{12} - \hat{\pi}_2) = \hat{\pi}_1 + \hat{\pi}_2 - \hat{\pi}_{12}$ . The retailer's profit is nonnegative, because  $\hat{\pi}_{12} \leq \hat{\pi}_1 + \hat{\pi}_2$  follows from the substitutability of the products. ■

## Derivation of the profit expressions in Table 2

Total profit in the centralized system:

The optimal demand rates and the profit are as follows.

$$\begin{aligned} \begin{bmatrix} d_i^* \\ d_j^* \end{bmatrix} &= \frac{1}{4(\beta^2 - \gamma^2)} \begin{bmatrix} 2\beta & -2\gamma \\ -2\gamma & 2\beta \end{bmatrix} \begin{bmatrix} \theta_i + c_i \\ \theta_j + c_j \end{bmatrix}, \\ d_i^* &= \frac{\beta(\theta_i - c_i) - \gamma(\theta_j - c_j)}{2(\beta^2 - \gamma^2)}, \quad p_i^* - c_i = \frac{\theta_i - c_i}{2}, \\ \hat{\pi}_{ij} &= \frac{\beta(\theta_i - c_i)^2 + \beta(\theta_j - c_j)^2 - 2\gamma(\theta_j - c_j)(\theta_i - c_i)}{4(\beta^2 - \gamma^2)}, \\ \text{if symmetric:} \quad \hat{\pi}_{ij} &= 2 \frac{(\theta - c)^2}{4(\beta + \gamma)}. \end{aligned}$$

If only product  $i$  were carried, then

$$p_i^* - c_i = (\theta_i - c_i) / 2, \quad d_i^* = \frac{(\theta_i - c_i)}{2\beta}, \quad \text{and} \quad \hat{\pi}_i = \frac{(\theta_i - c_i)^2}{4\beta}.$$

Equilibrium under two-part tariffs:

By Theorem 4, manufacturer  $i$ 's optimal strategy is of a  $(F_i, c_i)$  type two-part tariff.

$$\begin{aligned} \Pi_i &= F_i^* = \hat{\pi}_{ij} - \hat{\pi}_j = \frac{\beta(\theta_i - c_i)^2 + \beta(\theta_j - c_j)^2 - 2\gamma(\theta_j - c_j)(\theta_i - c_i)}{4(\beta^2 - \gamma^2)} - \frac{(\theta_j - c_j)^2}{4\beta} \\ &= \frac{\beta^2(\theta_i - c_i)^2 + \gamma^2(\theta_j - c_j)^2 - 2\gamma\beta(\theta_j - c_j)(\theta_i - c_i)}{4\beta(\beta^2 - \gamma^2)}, \end{aligned}$$

if symmetric :  $\Pi_i = \frac{(\theta - c)^2(\beta - \gamma)}{4\beta(\beta + \gamma)}.$

The retailer's profit in the symmetric case is

$$\begin{aligned} \pi &= \hat{\pi}_i + \hat{\pi}_j - \hat{\pi}_{ij} \\ &= \frac{2(\theta - c)^2}{4\beta} - 2 \frac{(\theta - c)^2}{4(\beta + \gamma)} = \frac{(\theta - c)^2 \gamma}{2\beta(\beta + \gamma)}. \end{aligned}$$

Equilibrium under quantity discount and wholesale-price contracts:

Based on the demand rates and reaction functions derived in Theorem 1, we can solve the equilibrium for the symmetric problem. Recall that  $\delta \equiv (2\beta - v)^2 - (2\gamma)^2$ . The unique symmetric equilibrium given  $v_i = v_j = v$  is characterized by

$$\begin{aligned} w &= \frac{\theta(2\beta - v - 2\gamma)(\delta + v(2\beta - v)) + c(2\beta - v)\delta}{\delta(2(2\beta - v) - 2\gamma) + v(2\beta - v)(2\beta - v - 2\gamma)} \\ &= \frac{\theta(\delta + v(2\beta - v)) + c(2\beta - v)(2\beta - v + 2\gamma)}{(2\beta - v + 2\gamma)(2(2\beta - v) - 2\gamma) + v(2\beta - v)}. \\ w - c &= \frac{(\theta - c)(2\beta - v - 2\gamma)(\delta + v(2\beta - v))}{\delta(2(2\beta - v) - 2\gamma) + v(2\beta - v)(2\beta - v - 2\gamma)} \\ &= \frac{(\theta - c)(\delta + v(2\beta - v))}{(2\beta - v + 2\gamma)(2(2\beta - v) - 2\gamma) + v(2\beta - v)}. \end{aligned}$$

Substituting  $v = 0$  yields the equilibrium wholesale prices, and taking the limit as  $v \rightarrow 2(\beta - \gamma)$  yields the equilibrium under the most aggressive quantity discounts.

At  $v = 0$  :

$$w = \frac{\theta(\beta - \gamma) + c(\beta)}{(2\beta - \gamma)}, \quad w - c = \frac{(\theta - c)(\beta - \gamma)}{(2\beta - \gamma)}, \quad \theta - w = \frac{(\theta - c)\beta}{(2\beta - \gamma)}.$$

As  $v \rightarrow 2(\beta - \gamma)$  :

$$w = \frac{\theta(\beta - \gamma) + c(2\gamma)}{(\beta + \gamma)}, \quad w - c = \frac{(\theta - c)(\beta - \gamma)}{(\beta + \gamma)}, \quad \theta - w = \frac{(\theta - c)(2\gamma)}{(\beta + \gamma)}.$$

Similarly, the demand rates at the wholesale and the quantity discount rates are given by

$$\begin{aligned}
d^* &= \frac{(2\beta - v - 2\gamma)(\theta - w)}{(2\beta - v)^2 - (2\gamma)^2} = \frac{\theta - w}{2\beta - v + 2\gamma} \\
&= \frac{\left( \theta \left( \begin{array}{c} \delta(2(2\beta - v) - 2\gamma) + v(2\beta - v)(2\beta - v - 2\gamma) \\ - (2\beta - v - 2\gamma)(\delta + v(2\beta - v)) \end{array} \right) - c(2\beta - v)\delta \right)}{\delta(2(2\beta - v) - 2\gamma) + v(2\beta - v)(2\beta - v - 2\gamma)} \frac{1}{2\beta - v + 2\gamma} \\
&= \frac{(\theta - c)\delta(2\beta - v)}{\delta(2(2\beta - v) - 2\gamma) + v(2\beta - v)(2\beta - v - 2\gamma)} \frac{1}{2\beta - v + 2\gamma} \\
&= \frac{(\theta - c)(2\beta - v)(2\beta - v)}{\delta(2(2\beta - v) - 2\gamma) + v(2\beta - v)(2\beta - v - 2\gamma)} \\
&= \frac{(\theta - c)(2\beta - v)}{(2\beta - v + 2\gamma)(2(2\beta - v) - 2\gamma) + v(2\beta - v)}.
\end{aligned}$$

$$\text{At } v = 0 : d^* = \frac{(\theta - c)\beta}{(2\beta + 2\gamma)(2\beta - \gamma)},$$

$$\text{As } v \rightarrow 2(\beta - \gamma) : d^* = \frac{(\theta - c)}{2(\beta + \gamma)}.$$

$$p^* - w = \theta - (\beta + \gamma) \frac{\theta - w}{2\beta - v + 2\gamma} - w = \frac{(\theta - w)(\beta - v + \gamma)}{2\beta - v + 2\gamma}.$$

The retailer's profit at equilibrium is

$$\begin{aligned}
\pi^* &= 2(pd - wd + vd^2/2) = \frac{2(\theta - w)(\beta - v + \gamma)}{2\beta - v + 2\gamma}d + vd^2 \\
&= 2 \frac{(\theta - c)^2(2\beta - v + 2\gamma)(2\beta - v)^2}{((2\beta - v + 2\gamma)(2(2\beta - v) - 2\gamma) + v(2\beta - v))^2} \frac{(\beta - v + \gamma)}{2\beta - v + 2\gamma} \\
&\quad + v \frac{(\theta - c)^2(2\beta - v)^2}{((2\beta - v + 2\gamma)(2(2\beta - v) - 2\gamma) + v(2\beta - v))^2} \\
\pi^* &= \frac{(\theta - c)^2(2\beta - v)^2(2\beta - v + 2\gamma)}{((2\beta - v + 2\gamma)(2(2\beta - v) - 2\gamma) + v(2\beta - v))^2} \tag{17}
\end{aligned}$$

$$\text{At } v = 0 : \pi^* = \frac{(\theta - c)^2\beta^2}{(2\beta + 2\gamma)(2\beta - \gamma)^2}.$$

$$\text{As } v \rightarrow 2(\beta - \gamma) :$$

$$\begin{aligned}
\pi^* &= \frac{2(\theta - w)(\beta - v + \gamma)}{2\beta - v + 2\gamma} + vd^2 \\
&= \frac{2 \frac{(\theta - c)(2\gamma)}{(\beta + \gamma)}(\beta - v + \gamma)}{2\beta - v + 2\gamma} \frac{(\theta - c)}{2(\beta + \gamma)} + v \frac{(\theta - c)^2}{4(\beta + \gamma)^2} \\
&= \frac{2(2\gamma)(\beta - v + \gamma)}{2\beta - v + 2\gamma} \frac{(\theta - c)^2}{2(\beta + \gamma)^2} + v \frac{(\theta - c)^2}{4(\beta + \gamma)^2}
\end{aligned}$$

$$\pi^* = \frac{(\theta - c)^2\gamma}{(\beta + \gamma)^2}.$$

Manufacturer's profit at equilibrium is

$$\begin{aligned}
\Pi &= (w - c)d - vd^2/2 = ((w - c) - vd/2)d = \\
&= \left( \begin{array}{l} (\theta - c)(2\beta - v - 2\gamma)(\delta + v(2\beta - v)) \\ -(v/2)(\theta - c)(2\beta - v - 2\gamma)(2\beta - v) \end{array} \right) \frac{(\theta - c)(2\beta - v - 2\gamma)(2\beta - v)}{(\delta(2(2\beta - v) - 2\gamma) + v(2\beta - v)(2\beta - v - 2\gamma))^2} \\
&= \frac{(\theta - c)^2(2\beta - v - 2\gamma)^2(2\beta - v)(\delta + (v/2)(2\beta - v))}{(\delta(2(2\beta - v) - 2\gamma) + v(2\beta - v)(2\beta - v - 2\gamma))^2} \\
&= \frac{(\theta - c)^2(2\beta - v)(\delta + (v/2)(2\beta - v))}{((2\beta - v + 2\gamma)(2(2\beta - v) - 2\gamma) + v(2\beta - v))^2}. \tag{18}
\end{aligned}$$

$$\text{At } v = 0 : \Pi = \frac{(\theta - c)^2(2\beta)(\delta)}{((2\beta + 2\gamma)(2(2\beta) - 2\gamma))^2} = \frac{(\theta - c)^2(\beta)(\beta - \gamma)}{2(\beta + \gamma)(2\beta - \gamma)^2}.$$

$$\text{As } v \rightarrow 2(\beta - \gamma) : \Pi = \frac{(\theta - c)^2(2\gamma)((\beta - \gamma)(2\gamma))}{((4\gamma)(2\gamma) + 2(\beta - \gamma)(2\gamma))^2} = \frac{(\theta - c)^2(\beta - \gamma)}{4(\beta + \gamma)^2}.$$

### Theorem 5

**Proof.** Comparing the manufacturers' profit expressions given in Table 2, we see that profits under quantity discounts are always lower than those under two-part tariffs because  $\beta < (\beta + \gamma)$ . Profit under wholesale-price contracts is less than that under quantity discounts if and only if  $2\beta(\beta + \gamma) < (2\beta - \gamma)^2$ , which is equivalent to  $0 < 2\beta^2 - 6\beta\gamma + \gamma^2$ . The right-hand-side is equal to  $2\beta^2$  at  $\gamma = 0$  and  $-3\beta^2$  at  $\gamma = \beta$ , and it is decreasing in between, crossing zero only once at  $\beta(3 - \sqrt{7})$ . Similarly, profit under wholesale-price contracts is less than that under two-part tariffs if and only if  $(2\beta^2) < (2\beta - \gamma)^2$ , which is equivalent to  $0 < 2\beta^2 - 4\beta\gamma + \gamma^2$ . The right-hand-side is equal to  $2\beta^2$  at  $\gamma = 0$  and  $-\beta^2$  at  $\gamma = \beta$ , and it is decreasing in between, crossing zero only once at  $\beta(2 - \sqrt{2})$ . ■

### Theorem 6

**Proof.** Comparing the retailer's profit expressions given in Table 2, we see that profits under quantity discounts are always higher than those under two-part tariffs because  $2\beta > (\beta + \gamma)$ . Profit under wholesale-price contracts is less than that under quantity discounts if and only if  $\beta^2(\beta + \gamma) - 2\gamma(2\beta - \gamma)^2 < 0$ . The left-hand-side is positive for all  $\gamma \leq 0$ , it is equal to 0 at  $\gamma = \beta$ , and it is convex in between, implying at most a single crossing point in  $(0, \beta)$ , which would be the smallest root of this third-degree polynomial, i.e.,  $\frac{1}{2}\beta(3 - \sqrt{7})$ . Profit under wholesale-price



contracts is less than that under two-part tariffs if and only if  $\beta^3 - \gamma(2\beta - \gamma)^2 < 0$ . The left-hand-side is positive for all  $\gamma \leq 0$ , is equal to 0 at  $\gamma = \beta$ , and it is convex in between, implying at most a single crossing point in  $(0, \beta)$ , which would be the smallest root, i.e.,  $\frac{1}{2}\beta(3 - \sqrt{5})$ . ■

**Theorem 7** *Consider the case of symmetric complementary products and no economies of scale.*

*Under quantity discounts as  $v$  goes to  $2(\beta + \gamma)$ , the equilibrium profit of a manufacturer is  $(\theta - c)^2 / 2(\beta + \gamma)$ .*

*Under two-part tariffs, there are multiple equilibria where the manufacturer profits are given by  $(\hat{\pi}_{12} - \hat{\pi}_2, \hat{\pi}_2)$  or  $(\hat{\pi}_1, \hat{\pi}_{12} - \hat{\pi}_1)$ , or a linear combination of these points. The retailer profit is zero under both contracts.*

**Proof.** For the quantity discount contract, substituting  $v = 2(\beta + \gamma)$  in equations (18) and (17) gives the profits as stated. For the two-part tariff equilibrium, the equilibrium analysis of Theorem 4 needs to be revisited because with complementary products we have  $\hat{\pi}_{12} > \hat{\pi}_1 + \hat{\pi}_2$ . For any  $(F_1, c_1), (F_2, c_2)$ , we have  $\Pi_i = F_i 1_{\{d_i^* > 0\}}$ , and  $\pi = \max\{\hat{\pi}_1 - F_1, \hat{\pi}_2 - F_2, \hat{\pi}_{12} - F_1 - F_2, 0\}$ . Let us focus on the response of manufacturer 1 (M1). If  $F_2 \leq \hat{\pi}_2$ , then  $\hat{\pi}_1 - F_1 \leq \hat{\pi}_{12} - F_1 - F_2$  and  $\pi = \max\{\hat{\pi}_2 - F_2, \hat{\pi}_{12} - F_1 - F_2\}$  and M1's best response to  $F_2$  is to set  $F_1$  as high as possible while ensuring that  $\hat{\pi}_{12} - F_1 - F_2 \geq \hat{\pi}_2 - F_2$  hence  $F_1^*(F_2) = \hat{\pi}_{12} - \hat{\pi}_2$ . If  $\hat{\pi}_{12} - \hat{\pi}_1 \geq F_2 = \hat{\pi}_2 + \delta$  for  $\delta \in [0, \hat{\pi}_{12} - \hat{\pi}_1 - \hat{\pi}_2]$ , then  $\pi = \max\{\hat{\pi}_{12} - F_1 - \hat{\pi}_2 - \delta, 0\}$  and M1's best response is  $F_1^* = \hat{\pi}_{12} - F_2$ . If  $F_2 > \hat{\pi}_{12} - \hat{\pi}_1$  then  $\pi = \max\{\hat{\pi}_1 - F_1, 0\}$  and M1's best response is  $F_1^* = \hat{\pi}_1$ . The best response function of manufacturer 2 is similar:  $F_2^*(F_1) = \{\hat{\pi}_{12} - \hat{\pi}_1, \text{ if } F_1 \leq \hat{\pi}_1; \hat{\pi}_{12} - F_1, \text{ if } \hat{\pi}_{12} - \hat{\pi}_2 \geq F_2 > \hat{\pi}_1; \hat{\pi}_2, \text{ otherwise}\}$ . The equilibrium points are  $(F_1^*, F_2^*) = (\hat{\pi}_{12} - \hat{\pi}_2, \hat{\pi}_2)$ ,  $(\hat{\pi}_1, \hat{\pi}_{12} - \hat{\pi}_1)$ , or any linear combination of these two points. The retailer's profit is zero at all equilibria. In the symmetric products case,  $\hat{\pi}_i = (\theta - c)^2 / 4\beta$  and  $\hat{\pi}_{12} = (\theta - c)^2 / 2(\beta + \gamma)$ , leading to  $(F_1^*, F_2^*) = ((\theta - c)^2 / 4\beta, (\theta - c)^2 (\beta - \gamma) / 4\beta (\beta + \gamma))$  or the reverse. ■