Bias and the Commitment to Disclosure

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Keywords
committed disclosure, earnings guidance, bias, multiple firms

Disciplines
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Bias and the Commitment to Disclosure

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Abstract

This paper studies the propensity of firms to commit to disclose information that is subsequently biased, in the presence of other firms also issuing potentially biased information. An important aspect of such an analysis is the fact that firms can choose whether to disclose or withhold information. We show that allowing the number of disclosed reports to be endogenous introduces a countervailing force to some of the empirical predictions from the prior literature. For example, we find that as more firms issue a report or as the correlation across firms’ cash flows increases, the firm biases its report less. However, when we treat firms’ disclosure choices as endogenous, we show that the number of firms that commit to disclose decreases as the correlation across these cash flows increases, and this, in turn, offsets the direct effect of the correlation on bias.

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1 Introduction

The purpose of this paper is to contribute to our understanding of firms’ commitment to disclose information. Specifically, we combine following aspects of firms’ disclosure decisions. First, firms bias the information they disclose to change prices in a favorable way. Second, the extent to which firms bias their disclosure depends on the information disclosed by other firms. Third, firms are more likely to commit to disclose information when there is a larger value to managing prices in the future. That is, we investigate the propensity of a firm to commit to disclose information in conjunction with subsequently biasing the disclosure, in the presence of other firms also issuing potentially biased reports. Our main contribution comes from investigating a setting with multiple firms that choose whether to commit to disclose. By treating the number of firms that commit to disclose as endogenous, we derive predictions on how various exogenous variables, such as cash flow uncertainty and the quality of firms’ private information, affect the number of reports provided by the firms in an industry.

In our model, we extend Fischer and Verrecchia (2000) in that we allow multiple firms to commit to disclosing information ex ante but have discretion ex post about the exact information that is disclosed. That is, a firm voluntarily commits to disclose information prior to receiving it and independent of its content, but can bias the disclosed information. While our setting seems to be descriptive of firm’s commitment to voluntary disclosure, it extends to certain kinds of mandatory disclosures. For example, firms choose their exposure to mandatory disclosure regimes when they choose whether and where to list, or whether to adopt IFRS. Prior analytical literature commonly assumes a commitment to disclose; furthermore,

1 This is in contrast to ex post disclosure, which is disclosed conditional on its information content.
empirical evidence suggests that this is consistent with various voluntary disclosure choices.²

We summarize the predictions from our model as follows. With an *exogenous* number of firms that disclose, the extent of bias in disclosure is lower when more reports are disclosed and/or when firms’ cash flows are more highly correlated. The reason for these results is straightforward: the more information that is available to investors, the less they rely on a firm’s own report to estimate the firm’s future prospects. As the weight that investors assign to a firm’s report in its own price decreases, the benefit of introducing bias decreases, and hence expected bias decreases.

With an *endogenous* number of firms that disclose, we find that fewer firms commit to disclose when the prior uncertainty about firms’ cash flows decreases, or firms’ cash flows have a higher correlation. In our model, prior uncertainty about cash flows captures the market’s demand about information and the quality of firms’ private information. A lower demand for information naturally leads to fewer firms issuing reports. The result concerning the correlation across firms’ cash flows can be explained as follows. An increase in correlation, *ceteris paribus*, increases the amount of information investors have about a firm’s cash flow, and thus reduces the extent to which firms are able to influence prices by issuing a biased report. Hence, while the costs of issuing a report remain constant, the benefits decrease and fewer firms commit to disclose. This also leads us to predict that in a given industry, the firms whose reports are least informative about the industry are the most likely firms to issue one. Finally, allowing the number of firms that disclose to be endogenous introduces a countervailing force to some of the results from the prior literature: for example, the extent

² For analytical literature, see, e.g., Goex and Wagenhofer (2009), Cheynel and Levine (2012), Gao and Liang (2013), and Michaeli (2014). For empirical literature, see, e.g., Chen, Matsumoto, and Rajgopal (2011) and Tang (2012). Li, Wasley, and Zimmerman (2012) estimates that 63% of management forecasts in their sample are “released to lower the firm’s cost of capital” and thus purposefully ex ante.
of bias in disclosed information is constant over changes in the uncertainty about firms’ cash flows.

For the main part, the theory-based literature on reporting bias has limited itself to studying bias when only a single firm issues a report.\(^3\) Most related is Fischer and Verrecchia (2000) where a firm discloses a report but investors are uncertain about the firm’s incentive to manage earnings. The firm is therefore able to “fool the market” as investors cannot perfectly anticipate the firm’s bias.\(^4\) This gives rise to a value of disclosure to firms \textit{ex ante} because the firm anticipates that it can successfully react to its future preferences by managing share prices.

While the firm in Fischer and Verrecchia (2000) always discloses, Korn (2004), Kwon, Newman, and Zang (2009), and Einhorn and Ziv (2012) study settings where a single firm decides whether to disclose potentially biased information. In these settings, the firm withholds sufficiently bad news and, conditional on disclosure, the market can perfectly back out any bias because it is informed about the firm’s incentives. Korn (2004) finds that as the cost of biasing becomes very low (very high) a no-disclosure equilibrium arises (a truthful, full-disclosure equilibrium arises). Because firm values are drawn from a finite interval, a partly separating equilibrium exists for some costs of misreporting where firms with low firm values do not disclose, firms with intermediate values disclose biased reports, and firms with high values disclose the upper threshold. Einhorn and Ziv (2012) allows for non-linear equilibria and shows that the amount of bias increases in the privately observed information.

Similarly, Beyer and Guttman (2012) assumes that a firm privately observes the pro-

\(^3\) Stocken (2012) and Bertomeu (2013), respectively, provide a review and an overview of the literature.

\(^4\) Other reasons to prevent perfect unraveling of bias include strategic pooling (e.g., Guttman, Kadan, and Kandel, 2006), probabilistic earnings management (e.g., Gao, 2013 or Laux and Stocken, 2012), and incentives to manage earnings over time (e.g., Kirschenheiter and Melumad, 2002).
ductivity of an investment opportunity, then chooses the firm’s investment level, and finally
decides whether to (truthfully) publish the level of investment. The firm overinvests (real
manipulation) in the attempt to make investors believe that the investment productivity is
higher. The analysis shows that while firms with low and high productivity invest in prof-
itable new investments, firms with intermediate forego the investment opportunity due to
the interaction between disclosure and investment.

There are few exceptions to the single-firm assumption. These papers either assume that
disclosure of information is truthful or that it is mandatory. Dye and Sridhar (1995) considers
truthful ex post disclosure by multiple firms where the information endowment by firms is
unknown to the public and the receipt of information is positively correlated across firms. In
this setting, disclosure herding arises; when one firm discloses information, the probability
that other firms disclose increases. Similar to our results, Dye and Sridhar predict that a
firm is less likely to disclose when more other firms with correlated cash flows disclose. Our
model, as well as Dye and Sridhar (1995), assumes that firms simultaneously decide whether
to disclose a report. Jorgensen and Kirschenheiter (2012) studies truthful ex post disclosure
where two firms sequentially choose whether to disclose. Jorgensen and Kirschenheiter find
that the firms’ propensity to issue reports depends on the extent and sign of their correlation.
Bagnoli and Watts (2010) and Fischer and Verrecchia (2004) analyze bias in settings with
multiple firms who engage in product market competition. Strobl (2013) investigates cost
of capital implications from firms’ biasing behavior in a setting where a given number of
firms have to disclose earnings. Finally, while Caskey, Nagar, and Petacchi (2010) does not
investigate multiple firms, it allows for multiple actors by investigating a setting where a
biased report is analyzed, modified, and potentially biased by an audit committee.
2 The Model

We consider a one-period disclosure-bias game with $N$ risk-neutral, homogeneous firms out of which $M$ firms commit to disclose potentially biased information in a perfectly competitive market with risk-neutral investors. Each of these firms yields a terminal value of $\tilde{v}_i$, $i = 1, ..., N$, where the common priors for $\tilde{v}_i$ are that each $\tilde{v}_i$ has a normal distribution with mean 0 and variance $\sigma^2$. In addition, we assume that the covariance between any $\tilde{v}_i$ and any $\tilde{v}_j$, $i \neq j$, is $\rho \sigma^2$. During the period, firms privately observe a noisy measure of earnings, $\tilde{e}_i = \tilde{v}_i + \tilde{n}_i$, where it is common knowledge that the $\tilde{n}_i$ are independent and identically distributed, each from a normal distribution with mean 0 and variance $\eta$. After observing earnings, $M$ firms disclose information. Unlike firm $i$, the market and the other firms do not observe the realization of $\tilde{e}_i$. Consequently, the market price of firm $i$ is a function of the market’s prior beliefs, as well as firms’ reports. As firms’ cash flows are correlated, investors use all available reports to determine the price of firm $i$. Consistent with the discussion in the introduction, we assume that firms commit either to disclose or withhold the information before they observe the realization of $\tilde{e}_i$. This implies that whether a firm issues a report does not, by itself, change investors’ expectations about any firm’s future cash flow.

Let $P_i$ represent the market price of firm $i$ and let $\bar{y} = \{y_1, y_2, ..., y_M\}$ be the set of firms’ reports. Because we assume that the market is perfectly competitive and risk-neutral, the price of firm $i$ is the rational expectation of its terminal value, $\tilde{v}_i$, conditioned upon the set of reports, $\bar{y}$:

$$P_i = E[\tilde{v}_i | \bar{y}] .$$  

(1)

We assume that firm $i$ has some discretion over the accounting for the report and can use
that discretion to disclose the observed earnings, \( \tilde{e}_i \), or to report some other number. We interpret the difference between the observed earnings and the number actually disclosed as "bias" in the report. Formally, conditional on firm \( i \) observing earnings of \( \tilde{e}_i = e_i \), the disclosed report, \( y_i \), equals \( e_i + b_i \) where \( b_i \) is the bias firm \( i \) introduces.

Following Fischer and Verrecchia (2000), we assume that in choosing a level of bias, firm \( i \) attempts to maximize its objective function, which we characterize by the expression:

\[
U_i = x_i P_i - \frac{c}{2} b_i^2 - K, \tag{2}
\]

where \( \tilde{x}_i = x_i \) is the realization of a random event that firm \( i \) alone observes, \( P_i \) is the market price for the firm, \( c \) is some known positive parameter, \( \frac{c}{2} b_i^2 \) represents the known cost of bias to firm \( i \), and \( K \) is a fixed cost the firm bears when disclosing information. Similar to the broad disclosure literature, we interpret \( K \) as proprietary costs or verification costs.\(^5\) As noted above, the utility function in eqn. (2) reflects a firm’s desire to manage its price. We refer to the firm here as a representation of the board and/or managers who make the actual disclosure decisions. That \( \tilde{x}_i \) is a random variable reflects the idea that price preferences can vary over time.

We assume that it is common knowledge that the variables \( \tilde{x}_i \) are identically distributed with a normal distribution with mean \( \mu \) and variance \( \theta \), and that they are independent of \( \tilde{n}_i \) and \( \tilde{v}_i \). Given its inability to discern the firm’s precise preferences, the market can only conjecture the extent to which the firm has incentives to inflate or deflate expectations.

Note that as we assume \( \tilde{x}_i \) has a normal distribution, its realization can be either positive

or negative, where the latter captures situations in which firms prefer to deflate prices.

We summarize the element of time in our model as follows. At $t = 0$ each firm commits to either issuing a report or not. At $t = 1$ each firm receives a private earnings signal, $M$ firms that (at $t = 0$) committed to disclose a (potentially biased) report do so and prices are set. Finally, at $t = 2$ uncertainty unravels. We solve the model by backward induction, starting with the equilibrium to the disclosure-bias game when $M$ firms commit to disclose.

3 A Linear Equilibrium

3.1 The Equilibrium at $t = 1$

In this section we construct an equilibrium to our disclosure-bias game. We restrict our analysis to linear equilibria (i.e., prices that are linear in $\bar{y}$ and bias strategies that are linear in $e_i$ and $x_i$) because they are easily characterized and yield compelling intuition. An equilibrium at $t = 1$ consists of a bias function for each of the firms, $b_i(e_i, x_i)$, and $M$ pricing functions for the market, $P_i$, such that three conditions are satisfied. First, firm $i$’s choice of bias for each realization $\{e_i, x_i\}$, $b_i(e_i, x_i)$, solves its optimization problem given its conjecture as to the market response. Second, firm $i$’s market price equals the expected firm value, $\bar{v}_i$, based on all reports $\bar{y}$, and a conjecture about the bias strategy of each firm type. Third, expectations are met in equilibrium. To simplify the analysis, we assume that a firm’s commitment to either disclose or withhold a report is observable both by the market.
and the other firms. Thus, we conjecture an equilibrium of the form:

\[
    b_i(e_i, x_i) = \lambda_{iy} + \lambda_{ix}e_i + \lambda_{ix}x_i, \ \text{and}
\]

\[
    P_i(y) = \alpha + \sum_{j=1}^{M} \beta_{ij}y_j.
\]

We use firm \( i \)'s optimization problem and the market-pricing function for firm \( i \) to prove that there exists a unique linear equilibrium.

**A Firm’s Problem at \( t = 1 \):** We briefly derive the biasing strategies and the market pricing functions as they mainly follow Fischer and Verrecchia (2000). To begin, suppose that firm \( i \) conjectures that the price of his firm based on all reports, \( \bar{y} \), is of the form given by eqn. (4) with conjectured values of \( \hat{\alpha} \) and \( \hat{\beta}_{ij} \forall i, j \in \{1, 2, \ldots, M\} \). The linear conjecture about the pricing function, coupled with the objective function in eqn. (2), implies that firm \( i \)'s objective is strictly concave in \( b_i \). Thus, the firm’s optimal bias is given by the first-order condition, which yields:

\[
    b_i(e_i, x_i) = \frac{\hat{\beta}_{ii}}{c} x_i
\]

for all \( \{e_i, x_i\} \). This implies that \( \lambda_{iy} = \lambda_{ix} = 0 \) and \( \lambda_{ix} = \hat{\beta}_{ii}/c \). Different from Fischer and Verrecchia (2000), we allow multiple firms to disclose and bias. However, as we assume that prices are a linear function of all disclosed reports, the difference between two reports is irrelevant for investors and firms. That is, firm \( i \) ignores the reports of all other firms and the level of its own price when introducing bias. This indicates that if \( \hat{\beta}_{ii} = \hat{\beta}_{jj} \), firms \( i \) and \( j \) react in the same way to their individual observations of \( \tilde{x}_i \) and \( \tilde{x}_j \). Note that because a firm privately observes its earnings signal, it has an information advantage over investors.
about all firms’ terminal values. However, as the bias chosen by firm \( i \) is independent of the earnings signal, no firm has an information advantage about the bias chosen by any other firm.

**Market Pricing Function:** Now we turn to the market pricing function. Assume a conjectured bias function of the form specified by eqns. (3) and (5).\(^6\) The market price of firm \( i \) is equal to the expectation of firm \( i \)’s terminal value conditional on all reports:

\[
P_i = \alpha_i + \beta_1 y_i + \beta_2 \sum_{j=1, j \neq i}^{M} y_j, \text{ where}
\]

\[
\alpha = - (\beta_1 + (M - 1) \beta_2) \tilde{\lambda}_{ix} \mu,
\]

\[
\beta_1 = \frac{\sigma^2}{Q_1} \left( \sigma^2 (1 - \rho) (1 + (M - 1) \rho) + \eta + \lambda^2_{ix} \theta \right),
\]

\[
\beta_2 = \frac{\rho \sigma^2}{Q_1} \left( \eta + \lambda^2_{ix} \theta \right), \text{ and}
\]

\[
Q_1 = (\sigma^2 (1 + (M - 1) \rho) + \eta + \lambda^2_{ix} \theta) (\sigma^2 (1 - \rho) + \eta + \lambda^2_{ix} \theta).
\]

Intuitively, eqn. (6) provides the expression for firm value that results from regressing the terminal value of firm \( i, \tilde{v}_i \), on the set of reports, \( \tilde{y} \). Note that the pricing function is indeed linear in firms’ reports. Also note that the weight a firm’s report receives in its price is the same for all firms (i.e., \( \beta_{ii} = \beta_1 \) for all \( i \)); furthermore, all report other than firm \( i \)’s receive equal weight in \( P_i \) (i.e., \( \beta_{ij} = \beta_2 \) for all \( i \neq j \)). This follows from our assumption that ex ante all firms are homogeneous (i.e., \( \sigma_i^2 = \sigma^2 \) and \( \text{Cov}[\tilde{v}_i, \tilde{v}_j] = \text{Cov}[\tilde{v}_i, \tilde{v}_k] = \rho \sigma^2 \)) such that the information about firm \( i \) provided by \( y_j \) is as valuable to investors as the information provided by \( y_k \), where \( j, k \neq i \). As \( \beta_{ii} = \beta_1 \) for all \( i \), each firm’s response to a realization of

\(^6\) Note that eqn. (5) implies that \( \lambda_{ii} = 0, \lambda_{ie} = 0, \) and \( \lambda_{ix} = \frac{\beta_{ii}}{c} \) in eqn. (3). Because the first two results hold regardless of the conjecture about the linear-pricing function, we restrict both to 0 for the remainder of the analysis.
\( \bar{x}_i \) is the same for all firms (i.e., \( \lambda_{ix} = \lambda_x \) for all \( i \)).

Similar to Fischer and Verrecchia (2000), the conjectured coefficient on the realization of \( \bar{x}_i \) in the firms’ bias function, \( \hat{\lambda}_x \), captures the conjectured extent of bias. Because biasing activities add noise to the reports, from the market’s perspective, the market sensitivity to firm \( i \)'s report in its price, \( \beta_1 \), decreases when the market believes that the firm is biasing to a greater extent. Our assumption that all firms are homogeneous, however, implies that as \( |\hat{\lambda}_x| \) becomes larger, more of the variance of all reports is attributable to \( \bar{x}_i \). At the extremes, when the market believes reports manifest no bias, \( \hat{\lambda}_x \to 0 \), the weight in price is maximized.

When the market believes that bias is unbounded, \( |\hat{\lambda}_x| \to \infty \), the firms’ reports do not affect prices.

To derive the equilibrium, we replace the conjectures in eqns. (5) and (6) with their equilibrium values and then show that the four equations have a unique solution. Note that eqns. (5), (7), and (9) imply that \( \alpha \) and \( \beta_2 \) are unique functions of \( \lambda_x \) and/or \( \beta_1 \). Furthermore, from eqn. (5), it is easy to see that \( \lambda_x \) can be written as a unique function of \( \beta_1 \). This implies that for there to be a unique solution, there must exist a unique value for \( \beta_1 \) that solves eqns. (5) and (8). Solving eqns. (3) and (5) for \( \lambda_x \) and substituting it in for \( \lambda_x \) in eqn. (8) provides the following equilibrium condition

\[
0 = F (\beta_1) = \beta_1 - \frac{c^2 \sigma^2}{Q_2} \left( c^2 \sigma^2 (1 - \rho) (1 + (M - 1) \rho) + c^2 \eta + \beta_1^2 \theta \right),
\]

with \( Q_2 = (c^2 \sigma^2 (1 + (M - 1) \rho) + c^2 \eta + \beta_1^2 \theta) \left( c^2 \sigma^2 (1 - \rho) + c^2 \eta + \beta_1^2 \theta \right) \). Note that the uncertainty about firms’ preferences is crucial for biasing activities to affect the weight of reports in price. If this was not the case, i.e. \( \theta = 0 \), the market sensitivity is identical to
that attained when firms are constrained to disclose the observed earnings signal (while firms
would still introduce bias as long as $\mu \neq 0$, this will be perfectly anticipated by the market
and thus taken out).

Finally, note that $F(\beta_1)$ has a unique positive solution for $\beta_1$ when the number of firms
that disclose is exogenous. This case essentially reflects Fischer and Verrecchia (2000) with
multiple firms. Accordingly, the characteristics of the equilibrium and the comparative
statics from Fischer and Verrecchia (2000) still apply. Specifically, $\beta_1$ is decreasing in the
uncertainty about firms’ preferences, constant in the expected value of preferences, increasing
in the quality of the earnings observed by firms and the prior uncertainty regarding terminal
value, and increasing in the marginal cost of bias.

In addition to Fischer and Verrecchia (2000), $\beta_1$ is decreasing in the correlation across
cash flows and in the number of firms that disclose.\footnote{Table 2 summarizes all relevant comparative static results for both an exogenous and an endogenous number of firms that disclose.} An increase in the correlation among firms’ cash flows increases the amount of information an investor can glean about the terminal
cash flow of a firm from any other firm’s report. This implies that the weight the market
assigns to firm $j$’s report when evaluating firm $i$ increases, and hence the weight on firm
$i$’s own report decreases. In the limit (i.e., $\rho \to 1$), the weight on both firms’ reports is
identical: that is, $\beta_1 \to \beta_2$, which can be seen by setting $\rho = 1$ in eqns. (8) and (9). This
result arises from our assumption that all firms are homogeneous, and hence the quality of,
and the bias in, reports is the same. Similarly, the more firms, $M$, that issue a report, the
more information that is available for the market to assess the value of a firm. This leads to
a lower market sensitivity.
While this discussion indicates that extending Fischer and Verrecchia to multiple firms does not change the results, it also indicates that the number of disclosing firms has an effect on the equilibrium. The main contribution of our model is to discuss the incentives that firms have to disclose information. Specifically, we assume that firms anticipate their interest in managing their stock price. Because variations in $\beta_1$ affect the extent to which firms can manage prices, these variations also affect the incentives to disclose information in the first place. The following subsection investigates firms’ decision to disclose information.

### 3.2 The Equilibrium at $t = 0$

In order to investigate the number of firms that disclose reports and its effect on the results derived above, we first characterize a firm’s expected utility when disclosing a (potentially biased) report. Here we assume that (i) a firm faces additional proprietary cost of $K$ from disclosure; (ii) that the commitment to disclose is made before observing the realizations of $\tilde{e}_i$ and $\tilde{x}_i$; and (iii) that firms commit to disclose if they expect to profit from the option to manage their price. The equilibrium concept we apply is similar to the one in Grossman and Stiglitz (1980): if the expected utility of a firm that commits to disclose is higher than the expected utility of a firm that commits not to disclose, one of the latter will also commit to disclose.\(^8\) With these assumptions, using the equilibrium condition that $b_i = x_i (\beta_1/c)$, the \textit{ex ante} utility of firm $i$ when committing to issue a report is:

$$E \left[ \tilde{x}_i P_t - \frac{c}{2} \tilde{b}_i^2 - K \right] = \frac{\beta_1^2}{2c} (\theta - \mu^2) - K. \quad (12)$$

\(^8\) Similar to the investors in Grossman and Stiglitz (1980), all firms in our model are homogeneous \textit{ex ante}; thus, which specific firms commit to disclose is irrelevant.
In contrast, if firm $i$ does not disclose a report, then its \textit{ex ante} expected utility is given by:

$$E\left[ \tilde{x}_i \left( \alpha + \beta_2 \sum_{j=1}^{M} (\tilde{b}_j + \tilde{e}_j) \right) \right] = 0. \quad (13)$$

Eqn. (12) provides some further insight to our formulation of the benefits and costs of disclosing information. Committing to disclose allows a firm to bias this disclosure so that it can manage the price in response to its preferences. The ability to manage prices comes at a cost that is twofold: 1) the fixed cost $K$, and 2) the direct cost of introducing bias $\frac{1}{2}c^2b_i^2$.

As investors correct for any expected bias, $\mu$ negatively enters the expected utility, similar to the deadweight cost in the signal-jamming literature (e.g., Stein, 1989).\textsuperscript{9} However, the commitment to disclose allows the firm to manage price in response to its preferences $\tilde{x}_i$. Managing prices, in turn, becomes more valuable as the firm faces higher uncertainty about its future preferences.\textsuperscript{10}

It follows from eqns. (12) and (13) that the firm prefers disclosure whenever $\frac{2}{1-2c}c^2K > 0$, which implies that following condition has to hold:

$$\theta > \mu^2. \quad (14)$$

As Fischer and Verrecchia (2000) notes, eqn. (14) can be thought of as capturing the uncertainty about whether firms will try to inflate or deflate prices relative to the expected

\textsuperscript{9} If investors did not expect firms to introduce bias (i.e., if $\alpha = 0$), then both $\theta$ and $\mu$ would increase the benefit of disclosure to the firm.

\textsuperscript{10} See Fischer and Verrecchia (2000) for a more detailed discussion of the ex ante benefits of biasing. To strengthen the link between our model and the notion that firms attempt to “manage expectations” one could assume that the firms benefit whenever their preferences deviate from expectations, i.e., when $U_M = (\tilde{x}_i - \mu) P_i - \frac{1}{2}b_i^2$. In this case the above condition would reduce to $\beta_1^2 \frac{\theta}{2c} - K = 0$. 

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bias. Specifically, when $\theta$ is large or when $\mu^2$ is close to 0, the probability that firms inflate their reports (and the respective prices) and the probability that firms deflate their reports move closer together (i.e., each probability approaches one-half). With this interpretation, eqn. (14) suggests that when ex ante uncertainty about types is large ($\theta$ is large or $\mu$ is close to 0), firms benefit from the option to bias. On the other hand, when firms almost always desire higher or lower prices (i.e., $\theta$ is small or $\mu$ is far from 0), investors can back out almost all bias from the reports, which makes the ex ante returns to biasing behavior negative.

As the option to move price in the preferred direction is the only benefit of disclosure in our model, the uncertainty about firms’ incentives has to be sufficiently high (i.e., eqn. (14) has to hold) for a firm to provide a report: if $\theta < \mu^2$, a firm incurs negative expected utility from disclosing a report such that no firm would commit to disclosure. For the remainder of the analysis we assume $\theta > \mu^2$. Alternatively, in Dye and Sridhar (2008) the market knows the firms’ price preferences but is uncertain about the cost of manipulation. When withholding disclosure eliminates the cost of manipulation, all firms would prefer to not disclose (because a firm’s ex ante expected price is independent of the disclosure). That is, a firm prefers to withhold information when this eliminates the preference (cost) uncertainty.\textsuperscript{11} We argue that it is realistic to assume that firms have an interest in managing their prices (see, for example, Aboody and Kasznik, 2000; and Rogers and Stocken, 2005). While we provide empirical guidance under the assumption that $\theta > \mu^2$, whether the benefit of disclosure in managing stock price outweighs the cost of manipulation is an empirical question.

\textsuperscript{11} For example, assume that the uncertainty in the cost of manipulation represents uncertainty about when it is costly to not walk expectations up or down. In this case, non-disclosure could be interpreted as not managing expectations and the firm faces a cost in the non-disclosure case.
We assume that there are $N$ firms in the economy out of which $M$ firms commit to disclose. In order to avoid a trivial solution we assume that $N > M$. In equilibrium expected utility from disclosure has to equal the expected utility from not disclosing, which, following eqn. (13), equals zero:

$$\hat{\beta}_1^2 \mu^2 - \frac{\mu^2}{2c} - K = 0. \quad (15)$$

Since at the time a manager decides whether to issue a report he does not know how many firms will decide to do so, $\hat{\beta}_1$ in eqn. (15) is a function of the expectation about the number of firms that decide to disclose, $\hat{M}$. Expectations have to be met in equilibrium, such that $\hat{M} = M$ has to hold.

In the last subsection we developed the intuition for why the equilibrium in market pricing and managerial biasing at $t = 1$ can be written as an equilibrium in $\beta_1$. Combined with the requirement that $\hat{\beta}_1 = \beta_1$ and eqn. (15), the resulting equilibrium is a pair $\{\beta_1, M\}$ that solves eqns. (11) and (15). From eqn. (11) it is obvious that $\beta_1$ is a function of the exogenous parameters as well as $M$; this is the case because the extent to which investors use a firm’s report depends on the quality of information they can glean from it, and also depends on the amount of information they can glean from all other reports. The more information is available, the less weight investors place on any single report. This logically implies that $\beta_1$ decreases in $M$. Eqn. (15) then shows that the number of firms that commit to disclose will adjust such that $\beta_1 = \sqrt{K \frac{2e}{\theta - \mu^2}}$.

From eqn. (15), it is straightforward that an exogenous increase in $\beta_1$ will increase the benefit of disclosure, which, in turn, should increase $M$. On the other hand, when the
number of issued reports increases, investors can use more information and, thus, potentially decrease the weight on any specific firm’s report. These two effects introduce a tension into the model and can yield a solution where some but not all firms commit to disclose.

Finally, from eqn. (15), it is easy to see that $\beta_1$ can be written as a unique function of the exogenous parameters. We complete the proof by taking the solution for $\beta_1$ from eqn. (15), substituting it in eqn. (11), and then showing that the resulting equation has a unique solution for $M$: the resulting equilibrium condition for $M$ is given by $G(M) = 0$, where

$$G(M) = \sqrt{cX} - c\sigma^2 \frac{c\eta + X\theta + c\sigma^2 (1 - \rho) (1 + (M - 1) \rho)}{(c\eta + X\theta + c\sigma^2 (1 + (M - 1) \rho)) (c\eta + X\theta + c\sigma^2 (1 - \rho))}$$  \tag{16}$$

and $X = \frac{2K}{\theta - \mu^2}$. It is straightforward to see that eqn. (16) is strictly increasing in $M$. This indicates that if $G(M = 0) < 0$ and $G(M = N) > 0$, there exists a unique solution for $M$ such that $G(M) = 0$. The lower and upper bound on the exogenous parameters indicate that if, for example, the fixed cost of disclosure, $K$, is too high (low), no firm (all firms) will commit to disclose and, thus, no interior solution exists. Proposition 1 shows the existence of a linear equilibrium.

**Proposition 1** There exists a unique linear equilibrium for the disclosure-bias game: $P(\tilde{y}) = \alpha + \beta_1 y_i + \beta_2 \sum_{j=1,j\neq i}^M y_j$ and $b_i(e_i, x_i) = \lambda_y + \lambda_e e_i + \lambda_x x_i$, where $M$ and $\beta_1$ solve $G(M) = 0$, $\beta_1^2 \frac{\sigma^2 - \mu^2}{2 \epsilon} = K$, $\alpha = - (\beta_1 + (M - 1) \beta_2) \lambda_x \mu$, $\beta_2 = \frac{\sigma^2}{Q_1} (\eta + \lambda_x^2 \theta)$, $\lambda_y = 0$, $\lambda_e = 0$, $\lambda_x = \frac{\beta_1}{\epsilon}$, and $Q_1 = (\sigma^2 (1 + (M - 1) \rho) + \eta + \lambda_x^2 \theta) (\sigma^2 (1 - \rho) + \eta + \lambda_x^2 \theta)$ if $G(M = 0) < 0$ and $G(M = N) > 0$.

Note that, similar to Admati and Pfleiderer (2000) and Jorgensen and Kirschenheiter (2012), when the firms’ cash flows are not correlated, the game reduces to a single-firm game.
because eqn. (16) is independent of $M$ when we substitute $\rho = 0$. The lower bound on the exogenous parameters indicates that when the cost of disclosure, $K$, is too high 
\[ \sqrt{\frac{2cK}{\theta - \mu^2}} > c\sigma^2 \frac{X\theta + c\eta + c\sigma^2(1-\rho)^2}{(X\theta + c\eta + c\sigma^2(1-\rho))^2}, \]
no firm commits to disclose. The upper bound suggests that when there are too few firms in the industry ($N$ is too small) or when the cost of disclosure is too small 
\[ \sqrt{\frac{2cK}{\theta - \mu^2}} < \frac{c\sigma^2(1-\rho)}{c\sigma^2(1-\rho) + c\eta + X\theta}, \]
when $N$ and $M$ approach infinity), all firms commit to disclose. In the latter situation, the analysis from Fischer and Verrecchia (2000) applies, as discussed above.

Finally, in the knife edge case of perfect correlation, $\rho = 1$, all reports are equally valuable to the investors of a specific firm such that $\beta_1 = \beta_2 = \beta$. In this situation, the condition $F(\beta)$ from eqn. (11) reduces to $\beta = \frac{\sigma^2}{M\sigma^2 + \eta + (\beta/c)^2\theta}$. This condition is similar to the one in Fischer and Verrecchia (2000), adjusted for the number of reports. That is, the more reports are available, the lower the weight on any specific report (via the term $M\sigma^2$ in the denominator).

This reduces the incentives to bias, which makes reports less noisy (via the term $(\beta/c)^2\theta$).

The entry condition from eqn. (15) remains unchanged such that the equilibrium condition from eqn. (16) reduces to $\sqrt{cX} = \frac{c\sigma^2}{\sigma^2 M c\eta + X\theta}$. We discuss comparative statics of the general case in the following section.

### 4 Empirical Implications

Treating the number of firms that disclose as endogenous provides a countervailing force to the results documented in Fischer and Verrecchia (2000). Intuitively, the countervailing force arises because changes in exogenous parameters that make it easier to bias the report also make it more appealing to disclose a report. While the direct effect (more biasing) leads
to less informative reports, the increase in the number of disclosed reports provides more
information to investors. In this section we provide empirical implications by characterizing
the solution to our model.

4.1 The Number of Firms in Equilibrium

Before providing predictions on the coefficients in a linear regression of reports on price, we
investigate the number of firms that choose to disclose in equilibrium. Corollary 1 summarizes
comparative static results for $M$ in equilibrium.

**Corollary 1** When firms can choose whether to disclose, the number of firms that choose
to do so: (i), decreases in the expected value of firms’ preferences and the proprietary cost
of disclosure; (ii) increases in the uncertainty about firms’ preferences; (iii) increases in the
quality of the earnings observed by the firms and the prior uncertainty regarding terminal
value; (iv) decreases in the correlation across cash flows; and (v) is ambiguous with respect
to the marginal cost of bias.

Note that the results in Corollary 1 are driven by changes in the expected utility from
disclosing a report. From eqn. (15) it is straightforward to see that the number of firms,$M$, itself enters the expected benefit of disclosure only through its effect on the market
response to disclosure, $\beta_1$. This implies that while more firms choose to issue a report when
the expected benefit increases (i.e., $\theta$ increases or $\mu$ decreases) or the cost, $K$, decreases,
there is also an indirect effect of changes in exogenous parameters. This indirect effect exists
because changes in parameters also lead to changes in $\beta_1$. For example, from eqn. (11) we
can infer that an increase in $\theta$ increases the noise in the disclosed reports, which decreases
the weight investors place on the report. As Corollary 1 shows, the direct effect of increasing the expected utility dominates the indirect effect of a decrease in $\beta_1$.

The 4th, 5th, and 6th comparative static results (i.e., $dM/d\eta$, $dM/d\sigma^2$, and $dM/d\rho$), however, are driven by the indirect effect on $\beta_1$. As discussed above, with an exogenous determination of $M$, $\beta_1$ decreases in $\eta$ and $\rho$ (increases in $\sigma^2$). This reduces (increases) the expected benefit of disclosure and decreases (increases) $M$. The final result shows that the effect of an increase in the marginal cost of bias, $c$, can increase or decrease the number of firms that disclose a report in equilibrium. Again, there is a direct effect (a higher $c$ decreases the expected utility from disclosure) and an indirect effect (ceteris paribus, $c$ increases $\beta_1$ as reports become less biased, thereby increasing the expected utility). Which of these dominates depends on the value of the model’s fundamental parameters.

While the effect of $c$ on $M$ is ambiguous, we know that for sufficiently small values of $c$ the number of firms that disclose increases as $c$ increases, whereas for sufficiently large values of $c$ the number of firms that disclose decreases as $c$ increases. The intuition for this is as follows, for $c = 0$ investors treat all reports as pure noise such that no firm has an incentive to disclose a report. When $c$ increases investors start to include the reports in their valuation, this provides incentives for firms to disclose. As $c$ increases further, it becomes increasingly costly for firms to manage their stock price, which reduces their incentives to disclose information.

Note that the comparative static results in Corollary 1 are a direct result of changes in a firm’s expected utility from disclosing a report. This implies that the corollary also speaks to which firms in an industry are more likely to disclose. However, firm heterogeneity can reduce the impact of the endogenous disclosure decision. For example, assume some firms
have very low proprietary costs and the others have very high costs. In this situation, small changes in exogenous parameters have no effect on the equilibrium when all low cost but no high cost firms disclose. That is, the endogenous entry condition, eqn. (15), would not bind and local comparative statics are, similar to Fischer and Verrecchia (2000), determined by eqn. (11).

The empirical literature on the propensity of firms to issue management forecasts in the presence of proprietary costs is based on Verrecchia (1983) and finds mixed results (e.g., Ali, Klasa, and Yeung, 2014; and Li, 2010). However, consistent with our setting of a commitment to disclose, Ali et al also document that firms prefer private over public placements to raise funds when proprietary costs are higher.

In the ex post disclosure literature, Einhorn (2007) predicts that a firm can withhold information (is less likely to disclose) when investors are uncertain about whether the firm prefers to increase or decrease stock price. Corollary 1 suggests the opposite: more firms disclose when the uncertainty about firms’ preferences is higher. The crucial difference is that we study a commitment to disclose whereas Einhorn investigates discretionary disclosure. Similarly, in a model of multi period ex post disclosure, Beyer and Dye (2012) suggest that as future cash flows become more volatile, more firms will disclose contemporaneous information to develop a reputation for being “forthcoming.” The reputation provides managers with the ability to withhold more negative information (that is, managers can better manage future stock prices). In Beyer and Dye (2012) fewer managers disclose early as the average probability of being forthcoming decreases. While this is a result of different economic forces, it is similar to our result regarding the expected value of firms’ preferences.
In line with our results, Verrecchia (1990) predicts that an increase in cash flow uncertainty leads to less disclosure. Empirically, Kim, Pandit, and Wasley (2014) suggests higher cash flow uncertainty leads to a lower frequency of discretionary management earnings forecasts. Kim, et al. partly ascribe this finding to the negative impact of greater market uncertainty on the quality of firms’ private information. In our model (and in Verrecchia, 1990), the quality of privately observed information captures this aspect of the disclosure decision. Higher cash flow uncertainty itself increases investors’ interest in obtaining information, which increases the value of disclosure. This highlights the importance of controlling for the quality of firms’ information when investigating the relation between market uncertainty and the propensity to disclose.

Finally, Bonsall, Bozanic, and Fischer (2013) suggests that management forecasts contain macroeconomic information. However, Bonsall at al. do not investigate whether firms are more likely to issue forecasts when other firms’ forecasts are more informative about systematic events.

4.2 Regression of Price on Reports

4.2.1 The Weight on a Firm’s Own Report

In our model, all reports provide information about one firm’s cash flows because all firms’ reports are correlated. In such a setting, it is a standard result that the incremental information conveyed by one report decreases as more reports are available, such that an inverse relation between $M$ and $\beta_1$ exists. From eqn. (15), however, it is straightforward that a firm’s utility increases in $\beta_1$, which suggests a complementary relation. This indicates that
the comparative static results on $\beta_1$ from the prior literature might be altered when firms are allowed to choose whether to disclose or not. In what follows, we focus our discussion on comparative statics that change as a result of allowing firms to choose whether they disclose. Corollary 2 summarizes comparative static results on the slope of a firm’s report in a regression of its price on all available reports.

**Corollary 2** When firms can choose whether to disclose, the weight of a firm’s report in its price: (i) increases in the expected value of firms’ preferences and the fixed cost of disclosure; and (ii) is constant in the quality of the earnings observed by the firms, the prior uncertainty regarding terminal value, and the correlation across cash flows.

Note the difference in comparative static results in Corollary 2 to the results in Fischer and Verrecchia (2000). Specifically, while changes in exogenous parameters still have a direct effect on the weight on a firm’s report at $t = 1$, they also have an indirect effect through the number of firms that commit to disclose at $t = 0$. That is, while eqn. (11) describes the direct effect of parameters on $\beta_1$, eqn. (15), $\beta_{1}^{2} \frac{\theta - \mu^2}{2c} - K = 0$, shows that $M$ will adjust such that changes in $\eta$, $\sigma^2$, and $\rho$ will not affect $\beta_1$.

Furthermore, while increases in $K$ and $\mu$ have no direct effect on $\beta_1$, they reduce $M$ (see Corollary 1), which increases $\beta_1$. That is, $\beta_1$ increases in $K$ and $\mu$ and has an inverse relation to $M$. Finally, note that an increase in $\theta$ (or a decrease in $c$) decreases $\beta_1$ when $M$ is constant. Because $M$ increases in $\theta$ (decreases in $c$) the comparative statics from Fischer and Verrecchia (2000) with respect to $\theta$ and $c$ are amplified.
4.2.2 The Weight on Other Firms’ Reports

Different from models that focus on a single firm, we study a multi-firm game. Because firms publish reports that are informative about all firms’ cash flows, the market uses all reports when pricing firm $i$. The existence of other information suggests that if a firm is expected to introduce more bias (and hence the quality of the report decreases), investors rely less on his report and increase the weight on other information. Because we assume that all firms are homogeneous, however, an increase in the expected bias in firm $i$’s report comes with a comparable increase in the expected bias in all other firms’ reports; this reduces the weight these reports receive in the price of firm $i$. When examining the weight of other information in price, this indicates that there are two countervailing forces; these countervailing forces make comparative static results on the slope of another firm’s report in a regression generally ambiguous. To illustrate these forces, imagine a simpler setting where firms are unable to introduce bias into their reports. The only source of noise in the disclosed reports is the noise in firms’ private information. When firms receive (and disclose) perfect information, $\eta = 0$, investors only use a firm’s own report when determining price, i.e., $\beta_2 = 0$. Increasing the noise to $\eta > 0$ provides a role for other firms’ reports such that $\beta_2$ increases. However, with an infinite level of noise investors will ignore all reports such that, again, $\beta_2 = 0$.

In order to provide testable results, we investigate the ratio between the weight on another firm’s report relative to the weight firm $i$’s own report, i.e.,

$$ RW \equiv \frac{\beta_2}{\beta_1} = \frac{\rho (c\eta + X\theta)}{c\sigma^2 (1 - \rho) (1 + (M - 1) \rho) + c\eta + X\theta}, $$

(17)

where $X = \frac{2K}{\theta - \mu^2}$. Corollary 3 summarizes our results regarding the relative weights of other
firms’ reports and a firm’s own report in price when we treat $M$ as: (a) endogenous and (b) exogenous.

**Corollary 3** When firms can choose whether to disclose, the ratio of the weight on all other firms’ reports and the weight on a firm’s report in that firm’s price: (i) increases in the expected value of managers’ incentives and the fixed cost of disclosure; (ii) decreases in the uncertainty about managers’ incentives; and (iii) is ambiguous with respect to the marginal cost of bias.

Corollary 2 shows that sufficiently large changes in $\eta$, $\sigma^2$, and $\rho$ have no impact on $\beta_1$ because they affect the number of firms such that the information that can be gleaned from a specific firm’s report remains constant. However, this is not the case for the information content of other reports such that changes in these parameters affect the relative weights similar to a setting with a given number of reports. Corollary 3 shows that the endogenous number of reports does, however, affect the comparative statics with respect to $\mu$, $K$, $\theta$, and $c$. Because all reports, other than the firm’s own report, are perfect substitutes, an increase in $M$ decreases the relative weights, i.e., $\partial (\beta_2/\beta_1)/\partial M < 0$. This implies that the relative weights increase in $\mu$ and $K$ and decrease in $\theta$. As $M$ can increase or decrease in $c$, the same holds true for the relative weights.

### 4.3 Expected Bias

The bias a firm introduces into the report it provides to the market is determined by the weight the market assigns to its report when determining his firm’s market value, the cost of introducing bias, and the firm’s price-based incentives, i.e., $b_i = (\beta_1/c) x_i$. Thus, the extent
of bias in a given report crucially depends on the firm’s (unobservable) preferences. As in prior literature, however, we can make predictions on the expected bias in published reports, where

$$E[b_i] = (\beta_1/c) E[x_i] = (\beta_1/c) \mu. \quad (18)$$

Clearly, with the potential exception of the result concerning the marginal cost of introducing bias, the comparative static results from Corollary 2 will continue to hold. The sign of the results, however, depends on the sign of the firms’ expected preferences, $\mu$. For the remainder of our analysis, we assume that firms, on average, have a greater interest to inflate price (i.e., $\mu > 0$), such that expected bias is increasing in the market sensitivity. While the comparative static with respect to $c$ changes relative to Corollary 2, it remains unchanged from the result in Fischer and Verrecchia (2000). That is, expected bias decreases in $c$. Similarly, expected bias increases in the firms’ expected price preferences. Here, the endogenous number of firms that disclose amplifies the result in Fischer and Verrecchia (2000) because as $\mu$ increases, fewer firms disclose information, which implies that $d\beta_1/d\mu > 0$.

Corollary 4 summarizes comparative static results on expected bias.

**Corollary 4** Assume that the firms are more likely to inflate price (i.e., $\mu > 0$). When firms can choose whether to disclose, expected bias: (i) increases in the fixed cost of disclosure; and (ii) is constant in the quality of the earnings observed by the firms, the prior uncertainty regarding terminal value, and the correlation across cash flows.

Note that in Fischer and Verrecchia (2000), expected bias increases in the quality of privately observed earnings and the prior uncertainty. The reason is that both increase the information content of disclosure and, thus, the incentives to bias. The endogenous entry offsets this such
that expected bias is independent of the two parameter values.

The empirical evidence in Rogers and Stocken (2005) suggests that uncertainty about the firms’ preference increases expected bias. This is consistent with both our model and Fischer and Verrecchia (2000). However, we are not aware of empirical studies that examine how the variables in Corollary 4 affect the average bias in disclosure.

4.4 Price efficiency

The final results we derive consider the information content of issued reports or the degree of “price efficiency” (i.e., the extent to which prices reflect all relevant public and private information). One measure of price efficiency is the variance of terminal value conditional upon the market price, $Var[\tilde{v}_i|P_i]$, divided by the prior variance, $\sigma^2$. This measure reflects the proportion of uncertainty remaining after the disclosure. To perform the comparative static analysis for price efficiency, it is useful to focus on the proportion of variance revealed by the reports:

$$V \equiv 1 - \frac{Var[\tilde{v}_i|P_i]}{\sigma^2}. \quad (19)$$

Corollary 5 summarizes comparative static results on price efficiency.

**Corollary 5** When firms can choose whether to disclose, price efficiency: (i) decreases in the expected value of firms’ preferences and the fixed cost of disclosure; (ii) increases in the uncertainty about firms’ preferences, (iii) is constant in the correlation across cash flows, and (iv) is ambiguous with respect to the marginal cost of bias.

Corollary 1 suggests that greater disclosure about firms’ preferences (e.g., managerial incentive plans) that reduces $\theta$ may, in turn, result in fewer value-relevant reports. While
providing information about $\theta$ increases investors' understanding of the incentives to bias makes any published report more value relevant, it causes fewer firms to commit to disclose. Corollary 5 shows that the second effect dominates such that price efficiency decreases. Two similar forces are at work when the correlation of cash flows increases. With a given number of available reports, price efficiency increases with an increase in $\rho$. With an endogenous number, both effects offset each other such that price efficiency is constant at all points where condition (15) holds with equality. Between these points, price efficiency increases in $\rho$: this is similar to the effect on expected bias.

Furthermore, Corollary 5 provides insights pertaining to the value relevance of firms’ disclosures. Settings with an exogenous number of disclosing firms predict that greater enforcement of disclosure regulations, or stiffer penalties for violations of those regulations (as represented by an increase in $c$), increase the value relevance of firms’ disclosed reports. Corollary 5 shows that this is not necessarily the case when the number of disclosing firms is determined endogenously. Specifically, the comparative static can be expressed as follows

$$
\frac{dV}{dc} = \frac{1}{2\sigma^2} \left( \frac{2K}{c(\theta - \mu^2)} \theta - \eta \right) \sqrt{\frac{2K}{c(\theta - \mu^2)}}. 
$$

(20)

Starting at $c = 0$, an increase in $c$ increases price efficiency (because $\frac{2K}{c(\theta - \mu^2)} \theta - \eta > 0$ for small $c$). However, as $c$ increases the rate of increase in price efficiency declines and, eventually, price efficiency decrease in $c$. That is, when the cost of bias exceeds the threshold $c_V = \frac{2K}{\theta - \mu^2} \frac{\theta}{\eta}$, further increases in $c$ deter a sufficient number of firms from disclosing a report such that prices become less efficient. This analysis suggests that enforcement of disclosure regulations helps price efficiency only to a certain degree. Once the enforcement becomes too strong,
further increases in enforcement reduce price efficiency.

5 Conclusion

In this paper we discuss bias in firms’ disclosures in a multi-firm setting; this extends the literature on bias in single-firm settings. We assume that the market cannot observe the chosen bias and, additionally, is uncertain about a firm’s preferences as it relates to managing its stock price. Our main innovation comes from treating the number of firms that disclose as endogenous. We believe that this assumption is descriptive of many types of disclosure, given that firms even have (some) influence on their exposure to mandatory disclosure regimes.

The model allows us to derive novel, and potentially testable, predictions. For example, we show that when we treat the number of firms that disclose as endogenous, this number increases in the prior uncertainty about firms’ cash flows, and decreases in the correlation across these cash flows. Further, we show that several empirical implications from the setting with an exogenous number of firms do not continue to hold when the number of firms is allowed to be endogenous. This highlights that the number of firms that disclose information is an important variable to control for in empirical studies. In other words, some of the predictions from a standard model of reporting bias are different for settings where firms have to disclose and where they can choose whether to disclose.
Appendix

Table 1 - Notation

| \( \bar{v}_i \sim N(0, \sigma^2) \) | final cash flow of firm \( i \), with variance \( \sigma^2 \) |
| \( \rho \) | correlation among the cash flows of firms \( i \) and \( j \), \( i \neq j \) |
| \( e_i = \bar{v}_i + \bar{n}_i \) | report about firm \( i \)'s cash flow |
| \( \bar{n}_i \sim N(0, \eta) \) | measurement noise in report, with variance \( \eta \) |
| \( x_i \sim N(\mu, \theta) \) | firm \( i \)'s interest in its price, with variance \( \theta \) |
| \( \mu \) | expected value of firm \( i \)'s price preferences |
| \( K \) | fixed cost of disclosing information |

Table 1: Notation

Table 2 - Comparative Statics

<table>
<thead>
<tr>
<th>Number of firms, ( M^* )</th>
<th>Response coefficient, ( \beta_1 )</th>
<th>Relative weight, ( \beta_2/\beta_1 )</th>
<th>Expected bias, ( E[b_i] )</th>
<th>Price efficiency, ( V )</th>
</tr>
</thead>
<tbody>
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<td>( M )</td>
<td>( M^* )</td>
<td>( M )</td>
<td>( M^* )</td>
<td>( M )</td>
</tr>
<tr>
<td>Expected incentive, ( \mu )</td>
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<td>+</td>
<td>0</td>
</tr>
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<td>Proprietary cost, ( K )</td>
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<td>+</td>
<td>0</td>
</tr>
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<td>Incentive uncertainty, ( \theta )</td>
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<td>–</td>
<td>+</td>
</tr>
<tr>
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<td>+</td>
<td>0</td>
<td>–</td>
</tr>
<tr>
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<td>+</td>
<td>0</td>
<td>–</td>
</tr>
<tr>
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<td>+</td>
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<tr>
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<td>+</td>
<td>+</td>
<td>–</td>
</tr>
<tr>
<td>Number of firms, ( M )</td>
<td>( \emptyset )</td>
<td>–</td>
<td>( \emptyset )</td>
<td>+</td>
</tr>
</tbody>
</table>

Table 2: Comparative statics with an exogenous number of reports, \( M \), and an endogenous number of reports, \( M^* \).
Proofs

Corollary 1

- Differentiating eqn. (16) with respect to $\theta$ yields:

$$\frac{\partial G(M)}{\partial \theta} = -\frac{1}{2} \frac{\sqrt{2K}}{(\theta - \mu^2)^2} \frac{c}{\sqrt{K - \frac{\sigma^2}{\theta - \mu^2}}}$$

$$-2 \frac{K\mu^2}{c} \frac{(2K\theta + (\theta - \mu^2) c (\eta + \sigma^2 (1 - \rho) (1 + (M - 1) \rho)))^2}{Q_4}$$

$$-2 \frac{K\mu^2 + c^2 \sigma^4 \rho^2 (\theta - \mu^2)^2 (1 - \rho) (M - 1) (1 - \rho + M \rho)}{c}$$

$Q_4 = (2K\theta + c (\eta + \sigma^2 - \sigma^2 \rho) (\theta - \mu^2))^2 (2K\theta + c (\theta - \mu^2) (\eta + \sigma^2 - \sigma^2 \rho + M \sigma^2 \rho))^2$.

As $\frac{\partial G(M)}{\partial \theta} > 0$, the above implies that $M$ is increasing in $\theta$.

- Differentiating eqn. (16) with respect to $X$ yields:

$$\frac{\partial G(M)}{\partial X} = \frac{c}{2\sqrt{Xc}}$$

$$+c\theta \sigma^2 \frac{Mc\sigma^2 \rho (1 - \rho) (2X\theta + 2c\eta + 2c\sigma^2 (1 - \rho) + (M - 1) c\sigma^2 \rho)}{(c\sigma^2 (1 - \rho) + c\eta + X\theta + c\sigma^2 M\rho)^2 (c\sigma^2 (1 - \rho) + c\eta + X\theta)^2}$$

$$+c\theta \sigma^2 \frac{(X\theta + c\eta + c\sigma^2 (1 - 2\rho) (1 - \rho))}{(c\sigma^2 (1 - \rho) + c\eta + X\theta + c\sigma^2 M\rho)^2 (c\sigma^2 (1 - \rho) + c\eta + X\theta)}.$$

Thus, $M$ decreases in $K$ and $\mu$. 

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• Differentiating eqn. (16) with respect to $\eta$ yields:

$$\frac{\partial G(M)}{\partial \eta} = \frac{c^2 \sigma^2 H(M)}{(X\theta + c\eta + c\sigma^2 - c\sigma^2 \rho)^2 (X\theta + c\eta + c\sigma^2 - c\sigma^2 \rho + Mc^2 \rho)^2}, \text{ with}$$

$$H(M) = Mc^2 \rho (1 - \rho) (X\theta + c\eta - c\rho - (M - 1) \rho)$$

$$+ (X\theta + c\eta + c\sigma^2 (1 - \rho) (1 - \rho + (M - 1) \rho) (X\theta + c\eta + c\sigma^2 (1 - \rho)).$$

Thus, $M$ decreases in $\eta$.

• Differentiating eqn. (16) with respect to $\sigma^2$ yields:

$$\frac{\partial G(M)}{\partial \sigma^2} = \frac{-c (X\theta + c\eta) H(M)}{(X\theta + c\Sigma + c\eta - c\Sigma \rho)^2 (X\theta + c\Sigma + c\eta - c\Sigma \rho + Mc\Sigma \rho)^2}, \text{ with}$$

$$H(M) = Mc^2 \rho (1 - \rho) (2X\theta + 2c\sigma^2 + 2c\eta - 3c\sigma^2 \rho + Mc^2 \rho)$$

$$+ (X\theta + c\sigma^2 + c\eta - 3c\sigma^2 \rho + 2c\sigma^2 \rho^2) (X\theta + c\sigma^2 + c\eta - c\sigma^2 \rho).$$

Thus, $M$ increases in $\sigma^2$.

• Differentiating (16) with respect to $\rho$ yields:

$$\frac{\partial G(M)}{\partial \rho} = c^2 \sigma^4 \rho (M - 1) (X\theta + c\eta) (2X\theta + 2c\eta + 2c\sigma^2 - 2c\sigma^2 \rho + Mc\sigma^2 \rho)$$

$$\frac{(X\theta + c\eta + c\sigma^2 - c\sigma^2 \rho)^2 (X\theta + c\eta + c\sigma^2 - c\sigma^2 \rho + Mc\sigma^2 \rho)^2}. \text{ with}$$

Thus, $M$ decreases in $\rho$. 
Differentiating (16) with respect to $c$ yields:

\[
\frac{\partial G(M)}{\partial c} = \frac{1}{2} \sqrt{X} - \sigma^2 X \theta \frac{(X \theta + c \eta + c \sigma^2 (1 - \rho) (1 + (M - 1) \rho))^2}{(X \theta + c \eta + c \sigma^2 (1 - \rho))^2 (X \theta + c \eta + c \sigma^2 (1 + (M - 1) \rho))^2} \\
- \sigma^2 X \theta \frac{(c \sigma^2)^2 (1 - \rho) \rho^2 (M - 1) (1 + (M - 1) \rho)}{(X \theta + c \eta + c \sigma^2 (1 - \rho))^2 (X \theta + c \eta + c \sigma^2 (1 + (M - 1) \rho))^2}.
\]

It can be shown that there exist conditions under which either $\frac{\partial F(M)}{\partial c} > 0$ or $\frac{\partial F(M)}{\partial c} < 0$.

**Corollaries 2 - 5**

Corollaries 2 - 5 are straightforward derivatives of the respective variables.
References


