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Beliefs-Driven Price Association

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Abstract

In addition to being a function of traditional fundamentals such as cash-flow persistence and the discount rate, the equilibrium association between a security price and a value-relevant statistic can simply be a function of what rational investors believe the association will be. We refer to this phenomenon as beliefs-driven price association (BPA). By explicitly considering the phenomenon of BPA, we show that the price response to information releases can vary over time even if the risk-free interest rate and investor preferences are static and the earnings/cash flow generating process is stable. This observation suggests, for example, that price-to-earnings associations and price volatility can vary over time even if a stable pattern of economic fundamentals suggests otherwise. The possibility of BPA suggests that measures of the cost of capital, information content, and growth prospects inferred from observed market prices will be confounded. While we do not predict when periods of BPA will arise, we provide empirically testable predictions about how prices should behave during periods of BPA. In particular, we predict that, during sufficiently long periods of high (positive or negative) BPA, price volatility, price levels, and expected returns will be higher than would be implied by a fundamental valuation framework. Finally, while BPA in the pricing of one security does not cause BPA in the pricing of other securities, the price levels of those other securities will be affected if the securities with BPA are sufficiently large relative to the market as a whole.

Keywords

higher-order-beliefs, earnings response, price volatility, pricing bubbles

Disciplines

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Beliefs-driven Price Association

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Abstract

In addition to being a function of traditional fundamentals such as cash-flow persistence and the discount rate, the equilibrium association between a security price and a value-relevant statistic can simply be a function of what rational investors believe the association will be. We refer to this phenomenon as beliefs-driven price association (BPA). By explicitly considering the phenomenon of BPA, we show that the price response to information releases can vary over time even if the risk-free interest rate and investor preferences are static and the earnings/cash flow generating process is stable. This observation suggests, for example, that price-to-earnings associations and price volatility can vary over time even if a stable pattern of economic fundamentals suggests otherwise. The possibility of BPA suggests that measures of the cost of capital, information content, and growth prospects inferred from observed market prices will be confounded. While we do not predict when periods of BPA will arise, we provide empirically testable predictions about how prices should behave during periods of BPA. In particular, we predict that, during sufficiently long periods of high (positive or negative) BPA, price volatility, price levels, and expected returns will be higher than would be implied by a fundamental valuation framework. Finally, while BPA in the pricing of one security does not cause BPA in the pricing of other securities, the price levels of those other securities will be affected if the securities with BPA are sufficiently large relative to the market as a whole.

Keywords: Higher-Order-Beliefs; Earnings Response; Price Volatility; Pricing Bubbles

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1. Introduction

The association between earnings and prices in traditional equity valuation models is a function of discount rates as well as the growth and persistence of earnings.[1] We posit another determinant of the association between earnings, or some other value-relevant statistic, and price: higher order beliefs or beliefs-about-beliefs. Our analysis stems from Keynes' observation that investors will attempt to predict future beliefs about a firm's equity value, as opposed to predicting future cash flows, because equity price is determined by beliefs. As a consequence, beliefs-about-beliefs – and not beliefs about cash flows – may drive firm share prices. Various theoretical analyses have demonstrated that beliefs-about-beliefs can foster deviations of price from fundamental value, where fundamental value is the risk-adjusted present value of expected future cash flows. In particular, higher order beliefs can lead to pricing bubbles in which share prices rise temporarily above fundamental value.[2] We extend those prior analyses by considering a setting in which higher order beliefs about the association between earnings and price can support an equilibrium association that differs from the association predicted by a traditional equity valuation framework. We call this phenomenon "beliefs-driven price association."

To illustrate the role of higher order beliefs in determining the association between prices and value-relevant statistics such as earnings, we employ a simple overlapping generations (OLG) model.[3] Identical investors with constant absolute risk aversion (CARA) utility functions live for two periods. These investors are savers (buyers) in the first period of life and consumers (sellers) in the second period of life. The investment opportunity set includes a risky asset that generates stochastic earnings each period. We assume earnings are paid out as dividends and follow a simple, one-period, auto-regressive time series process. Within the context of this model, the only news that arrives each period is earnings information.

To establish a benchmark, we characterize a steady-state linear equilibrium where the intercept and coefficient on earnings are the same at each point in time. This equilibrium characterization, which is common in the literature, is consistent with a fundamental valuation in that price equals the risk adjusted present value of future cash flows. Furthermore, the fact that the intercept and coefficient are stable over time is consistent with the stable earnings process that determines the valuation.

We depart from the literature by allowing the possibility of linear equilibria where the coefficient on earnings varies over time. To do so, we consider how current period investors price the risky asset if they believe that investors in subsequent periods will place too much (or too little) emphasis on subsequent

[1] See, for example, Kormendi and Lipe (1987).
[2] See, for example, Abreu and Brunnermeier (2003), Azariadis (1981), and Tirole (1985).
[3] The OLG model originated with Samuelson (1958), which considered a model of production, consumption, saving, and interest rates. The OLG model is very common in the literature on stock markets. For examples, see Banerjee (2011), Spiegel (1998), Tirole (1985), and Watanabe (2008).

period's earnings when pricing the risky asset. We find that these beliefs cause current period investors to rationally place a greater (lesser) emphasis on current earnings when pricing the risky asset, which implies a beliefs-driven price association (BPA). As a consequence, rational expectations equilibrium price paths can exhibit greater (or lower) associations between a value-relevant statistic and price than is implied by a fundamental valuation.

Our central contribution is to establish that higher order beliefs directly determine the association between disclosed information and price. As a consequence, BPA equilibria can arise where the price association deviates from the association predicted by a fundamental valuation model. Our analysis of BPA pricing paths suggests that the extent of BPA can vary over time, with periods of, say, high BPA followed by periods of lower or no BPA. During periods of high BPA, (i) the cost of capital inferred from the relation between price and earnings would appear to be low even though the cost of capital is high; (ii) price would appear to be more informed by earnings news, even though that news conveys the same information about future cash flows; (iii) growth prospects inferred from a price multiple would be high even though growth prospects are, say, average; and (iv) price will be more volatile than a fundamental valuation framework would suggest. Finally, if the variability associated with BPA is priced, higher expected returns are predicted for periods of BPA than for periods in which prices reflect steady-state fundamental valuations.

There is a long history of accounting literature regarding the notion that economic agents place undue, or disproportionate, emphasis on accounting information (see, for example, Ashton, 1976; Hand, 1990; Ijiri et al., 1966; Sloan, 1996). Much of this literature has alluded to bounds on cognitive capabilities, such as limited attention. For example, Bloomfield (2002) and Hirshleifer and Teoh (2003) discuss how limited attention and the nature of disclosure can jointly cause prices to overweight some statistics and underweight others. In a somewhat similar vein, Huddart et al. (2009) provides evidence that some investors fixate on firms whose prices have departed from a past trading range. Our analysis suggests that seemingly excessive (or insufficient) price associations with earnings or other value-relevant statistics can arise as an equilibrium phenomenon even when investors are not cognitively constrained.

Because we consider a setting with rational deviations from a fundamental valuation, our study relates to the literature on rational asset-pricing bubbles (for example, Tirole, 1985). More specifically, in a simple rational pricing bubble, an asset trades above its fundamental value in period t because investors at time t believe the asset will trade above fundamental value in period $t+1$. In equilibrium, the overvaluation increases over time to guarantee a sufficient rate of return on the "overpayment" at any point in time. BPA differs from a simple rational pricing bubble because it pertains to the price response to information about fundamental value as opposed to a predictable deviation from fundamental value.[4]

Our study also relates to the literature on sunspot equilibria.[5] In sunspot equilibria, prices are determined in part by stochastic events unrelated to economic fundamentals (so-called “sunspots”) because investors *believe* security prices will be a function of sunspots. As a consequence, like BPA, sunspots create volatility that is unrelated to underlying fundamental volatility.[6] Unlike sunspot models, however, BPA is based on an economic fundamental as opposed to being entirely spurious.

Finally, the seeming overemphasis of a value-relevant statistic that we characterize as an equilibrium phenomenon relates to the observation that prices overweight public information and underweight private information when investors have a short trading horizon (see, for example, Morris and Shin, 2002; Allen, Morris, and Shin, 2006; Gao, 2008). The overweighting of public information occurs because the public information directly determines the exit price anticipated by investors, which causes it to exert greater influence on their demands. In contrast, in our setting without private information, prices “overweight” current public information because future prices “overweight” future public information.

The remainder of the paper proceeds as follows. In the next section, we describe our model and characterize the benchmark steady-state equilibrium. In Section 3, we identify linear equilibria characterized by BPA and demonstrate how equilibrium pricing paths can exhibit time-varying degrees of BPA. In Section 4, we extend the model to consider BPA in a multi-asset economy. In Section 5, we provide empirical implications for periods of BPA pricing. Section 6 concludes.

2. Model

Consider an overlapping generations model where a continuum of investors can invest in shares of an infinitely lived risky asset and can lend or borrow at a risk free rate, $r > 0$. At the beginning of each period, the risky asset’s earnings are disclosed and paid out as a dividend, and last period’s risk free principal and interest are paid. After the dividend, principal, and interest payments, the investment market opens and investors form new portfolios.

There is one risky asset share per-capita in each generation, which yields earnings per share ε_t. The

[4] In a subsequent study, Fischer et al. (2014) embed our theory of BPA into a particular rational pricing-bubble framework to explain the observation that equity price multiples are higher for firms that are on meet-or-beat streaks. More specifically, while our study introduces BPA and its implications, Fischer et al. conjecture that streaks can make a BPA pricing path focal and that the end of a streak is associated with an abrupt reversion to a fundamental pricing path.

[5] See, for example, Azariadis (1981), Cass and Shell (1983), Jackson (1994), Jackson and Peck (1991), or Peck (1988).

[6] Noise trade could also be another determinant of prices and volatility. See, for example, Abreu and Brunnermeier (2002) and (2003), Allen and Gale (1994), Bhushan et al. (1997), Delong et al. (1990a) and (1990b), Spiegel (1998), and Watanabe (2008).

earnings per share follows a time series process of the form

$$\tilde{\varepsilon}_{t+1} = \lambda \varepsilon_t + \tilde{\eta}_{t+1}, \tag{1}$$

where $\lambda \in (0, 1)$ is a persistence parameter and $\tilde{\eta}_{t+1}$ is the earnings innovation in period $t + 1$. The innovations are independent normally distributed random variables with a common mean and variance of 0 and v, respectively.

Investors live for two periods and have wealth w to invest in the first period of life. In the final period of life, investors liquidate their investments and consume their wealth. Investor preferences are characterized by a negative exponential utility function with coefficient of risk aversion ρ, where $\rho = 0$ corresponds to the case of risk neutrality. The single risky asset in our economy can be interpreted as the market portfolio and the variance of its cash flow as systematic risk. We analyze an economy with multiple risky assets in the section below on spillover effects.

At time t, an investor in the first period of life chooses the quantity of shares, q, to maximize the expectation of

$$U_t = \frac{1}{\rho}\left(1 - \exp\left[-\rho\left(q(\tilde{\varepsilon}_{t+1} + \tilde{P}_{t+1}) + (1 + r)(w - qP_t)\right)\right]\right), \tag{2}$$

where P_t and $\tilde{P}_{t+1}$ are the time t and $t + 1$ share prices, "~" combined with a time $t + 1$ subscript denotes a random variable from the perspective of an investor at time t, and the absence of "~" denotes either a realization of that variable or a fixed parameter.

Before analyzing the model, we discuss seven of the model's assumptions. First, we assume that each generation of investors has a two-period life, which should be interpreted as the period where the investor is active in the market for the asset. The assumption facilitates a simple and intuitive characterization of equilibrium but is not necessary for establishing the existence of equilibrium exhibiting BPA. For example, the same equilibrium characterization results in a model where each generation of investors lives for any finite number of periods and can trade in each and every period of their life. Alternatively, BPA equilibria can arise when investors have uncertain lives in the market and each investor has a fixed probability of having to exit the market (for example, due to a liquidity shock or an alternative investment opportunity). What is critical to the existence of equilibria exhibiting BPA is that investors are interested in predicting an asset's future price in addition to predicting the asset's future cash flows (i.e., dividends).

Second, similar to the standard CAPM, we assume that all investors are homogeneous. This implies that investors share the same utility function and have the same beliefs regarding the asset's future cash flows, which determine future prices. Relaxing this assumption does not affect our results.

Third, we assume that the innovations to earnings are normally distributed and that investor preferences are characterized by a negative exponential utility function with a common risk aversion parameter. The combination of normally distributed innovations coupled with a negative exponential utility function yields price characterizations that are simple and intuitively appealing linear functions of means and variances. These assumptions are common to the literature, and we employ them, in part, because doing so highlights that our findings are not attributable to some less orthodox assumptions. We do note, however, that our results hold for any distributional assumption for the innovations in earnings if we assume risk neutral preferences. In addition, for a simple mean/variance expected utility specification, our findings hold with any symmetric distribution of the earnings innovations. Finally, the assumption that all investors have the same risk aversion parameter also facilitates a simple equilibrium characterization but is without loss of generality. Specifically, the exact same equilibrium characterizations involving BPA can be attained if we assume that investors have differing risk aversion parameters. The pricing function in such a setting reflects the average investor risk aversion parameter as opposed to a common risk aversion parameter.

Fourth, we assume that earnings/cash flows follow a simple AR(1) process, which is similar to Ohlson (1995). The AR(1) process is helpful because it implies a highly stable earnings/cash flow process, which serves to highlight our observation that price associations can vary over time even if valuation fundamentals are highly stable. An implication of this, however, is that given sufficiently negative innovations in the earnings process, prices become negative. While this is an unrealistic outcome, we can make the probability of negative prices arbitrarily small ex ante by imposing a large initial value, ε_0, incorporating a large fixed component into the earnings process, and/or assuming a positive drift (mean) in the innovations.

Fifth, different from Ohlson (1995), we assume that earnings are completely paid out as a dividend each period, which ignores the potential that the firm reinvests (a fraction of) earnings.[7] All of our results, however, can be generated in a model where earnings are a function of book value, a normal return on book value, and a stochastic residual income, and where the dividend payout is not necessarily equal to earnings. In the appendix, we show that Proposition 1 continues to hold in such a model.

Sixth, we have assumed that investors can take any position in the available risky assets and that the risk free asset has a perfectly elastic supply. If some measure of investors do not participate in the market for the risky asset (or the market for one of the risky assets when we consider multiple assets), that does not affect the possibility of BPA arising as an equilibrium phenomenon. Equilibrium prices will be affected, however, because risks, including those associated with BPA, will be less efficiently shared across investors.

[7] The assumption that all cash flow is paid out as dividends is standard in the OLG literature (see, for example, Banerjee, 2011).

Furthermore, if we instead assume that the risk free asset asset supply is fixed as opposed to having a perfectly elastic supply, the existence of BPA can still arise as an equilibrium phenomenon.

While all of the assumptions above are not integral to the existence of BPA equilibria, a seventh assumption is. Specifically, we assume that the risky asset has an infinite life, and we do not impose a terminal (or transversality) condition. These assumptions imply that backward induction arguments, which could be used to eliminate BPA equilibria, do not apply.[8] While the assumption that the risky asset has an infinite life is not unreasonable if we interpret the asset as the whole market, it is less reasonable if we interpret the risky asset as the equity of a single firm. BPA equilibria, however, can exist even if the risky asset has a probability of being terminated each period. While all investors know that *eventually* the risky asset will be terminated in such a setting, in any period in which the risky asset exists there is always a positive probability that it will continue to exist beyond the subsequent period. That probability of continuation, in turn, negates backward induction arguments and allows BPA to still be sustained in equilibrium.[9]

2.1. Linear Equilibria

An equilibrium in our model must satisfy the following three conditions: (i) each investor chooses his demand by maximizing his expected utility conditional on his expectations; (ii) expectations are rational and met in equilibrium; and (iii) markets clear. We initially focus on equilibria in which price can be written as a linear function of earnings, where the constant and coefficient on earnings may vary over time:

$$P_t = \alpha_t + \beta_t \varepsilon_t. \tag{3}$$

In any equilibrium where price is a linear function of earnings, a new investor's expected utility maximizing demand is $q_t = \frac{E[\tilde{\varepsilon}_{t+1} + \tilde{P}_{t+1}] - (1+r)P_t}{\rho Variance[\tilde{\varepsilon}_{t+1} + \tilde{P}_{t+1}]} = \frac{\lambda\varepsilon_t + \alpha_{t+1} + \beta_{t+1}\lambda\varepsilon_t - (1+r)P_t}{\rho v \left(1+\beta_{t+1}\right)^2}$. Because there is one share of the risky asset per investor, market clearing for period t requires that $q_t = 1$ for all investors, which implies an equilibrium price of

$$P_t = \frac{\lambda\varepsilon_t + \alpha_{t+1} + \beta_{t+1}\lambda\varepsilon_t - \rho v \left(1 + \beta_{t+1}\right)^2}{1+r}. \tag{4}$$

[8] Alternatively, consider a model where the firm pays a terminal dividend and ceases to exist at date T with certainty. The trading of rational investors investors at date $T-1$ would lead price at date $T-1$ to equal fundamental value (discounted future dividends). Given that the equilibrium price for $T-1$ is the fundamental value, the same arbitrage activities would lead the equilibrium price at date $T-2$ to equal fundamental value. This line of reasoning can be employed repeatedly to demonstrate that the only equilibrium linear pricing function is one in which price equals fundamental value at each trading date. Such reasoning, however, cannot be applied in a model without a certain terminal date.

[9] In models with infinitely lived assets, one approach for ruling out all but the steady state equilibrium is to impose a strong transversality condition, which, in essence, forces price to converge to a steady-state price as time passes. We discuss the implications of potential transversality conditions more in Appendix C.

The pricing condition implies that the price at time t equals the discounted expected value of next period's dividend, $\lambda\varepsilon_t$, plus next period's price, $\alpha_{t+1} + \beta_{t+1}\lambda\varepsilon_t$, minus a risk premium, $\rho v \left(1 + \beta_{t+1}\right)^2$. The risk premium arises because next period's dividend and price are both determined by next period's earnings, $\tilde{\varepsilon}_{t+1}$, which are uncertain. The equilibrium pricing condition, eqn. (4), implies that a linear equilibrium is defined by any α_t and β_t that satisfy the following two conditions for all t:

$$\beta_t = \frac{\left(1 + \beta_{t+1}\right)\lambda}{1 + r} \quad \text{and} \tag{5}$$

$$\alpha_t = \frac{\alpha_{t+1} - \rho v \left(1 + \beta_{t+1}\right)^2}{1 + r}. \tag{6}$$

2.2. Steady-State Equilibria

While we generally allow the intercepts and slope coefficients, α_t and β_t, to vary over time, we first establish a benchmark by characterizing a commonly studied steady-state equilibrium where they are constant.[10] Specifically, we restrict attention to an equilibrium of the form $P_t^S(\varepsilon_t) = \alpha + \beta\varepsilon_t$, where the superscript S indicates the steady state. Substituting $\alpha_t = \alpha$ and $\beta_t = \beta$ in eqns. (5) and (6) and solving those two equations yields Observation 1.

Observation 1. *There exists a unique equilibrium of the form $P_t^S(\varepsilon_t) = \alpha + \beta\varepsilon_t$, where $\beta = \frac{\lambda}{1+r-\lambda}$ and $\alpha = -\frac{\rho v(1+r)^2}{r(1+r-\lambda)^2}$.*

The steady-state pricing function is consistent with a fundamental valuation in the sense that price equals the risk adjusted present value of future cash flows. The present value of future cash flows at date t, discounted at the risk free rate, is given by $\sum_{i=1}^{\infty} \frac{E[\tilde{\varepsilon}_{t+1}|\Omega_t]}{(1+r)^i} = \sum_{i=1}^{\infty} \frac{\lambda^i \varepsilon_t}{(1+r)^i} = \beta\varepsilon_t$, where Ω_t denotes the information available at t. The coefficient on earnings, β, implies that the response to earnings is increasing in the persistence of earnings, λ, and decreasing in the discount rate, r. The adjustment for risk is captured in a price haircut equal to $\alpha = -\frac{\rho v(1+r)^2}{r(1+r-\lambda)^2}$, which is greater in magnitude when the variation in earnings, v, and the degree of risk aversion, ρ, are greater. The risk adjustment is also larger in magnitude (i.e., more negative) when earnings are more persistent or the discount rate is lower because the uncertain earnings innovation has a greater impact on future price when earnings are more persistent or the discount rate is lower.

The steady-state pricing function in Observation 1 maps closely to the dividend discount model that serves as a common framework for empirical analyses (see, for example, Kormendi and Lipe, 1987; Ohlson, 1995). That framework assumes that price equals the discounted present value of future dividends, where

[10] Examples of studies that focus on a steady-state equilibrium include Allen and Gale (1997), Bloomfield and Fischer (2011), and Spiegel (1998).

the discount rate is assumed to reflect a risk free rate plus a risk premium. Given our model's underlying assumptions regarding fundamentals, the dividend discount framework would assert that price satisfies

$$P_t^S(\varepsilon_t) = \frac{1}{1 + r_A - \lambda} E[\tilde{\varepsilon}_{t+1}|\Omega_t], \tag{7}$$

where r_A is the risk adjusted interest rate and $E[\tilde{\varepsilon}_{t+1}|\Omega_t] = \lambda\varepsilon_t$ is the expected dividend at date $t+1$ conditioned upon the information available at date t. Note that the main difference between our model and the dividend discount model is that we have decomposed the impact of risk adjusted rate, r_A, on price into two elements, the risk free rate, r, and a pricing adjustment for risk, $-\frac{\rho v(1+r)^2}{r(1+r-\lambda)^2}$. Thus, when investors are risk neutral and r_A equals the risk-free rate r, our steady state equilibrium pricing function above is identical to the dividend discount framework. When investors are risk averse, the haircut for risk and the risk free rate do not enter our pricing function through a single denominator effect but endogenously reflect the same conceptual notion as a risk adjusted rate.[11]

3. BPA Equilibria

The steady-state linear equilibrium is simple and intuitive; it relies on the assumption that each generation believes that price equals fundamental value (i.e., the risk adjusted present value of future cash flows). When we allow for other beliefs, however, we open up the possibility of equilibria with time-varying associations between price and earnings. We initially consider a class of equilibria where current investors exhibit a constant degree of beliefs driven price association (BPA) and then use the results of this analysis to characterize a broader class of equilibria with time-varying BPA.

3.1. Constant Degree of BPA

BPA describes the phenomenon where investors in one period believe that investors in subsequent periods will place more or less emphasis on a value-relevant statistic, earnings in our model, than the emphasis implied by fundamental valuation. The intuition underlying how BPA is sustained is as follows. Given that earnings exhibit some persistence, if period-$t+1$ investors place more (less) emphasis on $t+1$ earnings, a period-t investor's expectation of future price places more (less) emphasis on ε_t. This causes period-t investors' demand to place more (less) emphasis on ε_t, which, in turn, implies period-t price will also place more (less) emphasis on ε_t. We build off of this partial-equilibrium intuition to characterize equilibria where the price association with, or emphasis on, earnings grows or declines each generation. In other words, each

[11] More generally, dynamic models with risk averse investors do not yield equilibrium pricing functions in which price is a simple discounted sum of expected future cash flows. See, for example, Lucas (1978) or Spiegel (1998).

generation's emphasis on earnings becomes a rational response to the expected emphasis on earnings of subsequent generations.

To formally demonstrate BPA as an equilibrium phenomenon, we characterize a set of equilibrium pricing functions characterized by an exogenous parameter δ.

Lemma 1. *For any $\delta \in \Re$, there exists an equilibrium pricing function*

$$P_t = P_t^S(\varepsilon_t) + \beta_t^\delta \varepsilon_t + \alpha_t, \tag{8}$$

with the following coefficient and intercept for all $t \geq 0$:

$$\beta_t^\delta = \left(\frac{1+r}{\lambda}\right)^t \delta \text{ and} \tag{9}$$

$$\alpha_t = \rho v \delta \left(2J_t + K_t \delta\right), \tag{10}$$

where $P_t^S(\varepsilon_t) = \alpha + \beta\varepsilon_t$, $\beta = \frac{\lambda}{1+r-\lambda}$, and $\alpha = -\frac{\rho v(1+r)^2}{r(1+r-\lambda)^2}$ are the price, coefficient, and intercept in the steady-state equilibrium and where $J_t = \frac{1-\lambda^t}{1-\lambda}\left(\frac{1+r}{\lambda}\right)^{t+1}\frac{\lambda}{1+r-\lambda}$ and $K_t = \frac{(1+r)^t-\lambda^{2t}}{1+r-\lambda^2}\frac{(1+r)^{t+1}}{\lambda^{2t}}$.

The value for δ, which we call the degree of BPA, determines the deviation price from the steady-state price, where the steady-state price corresponds to $\delta = 0$. In an equilibrium with a constant positive (negative) degree of BPA, $\delta > 0$ ($\delta < 0$), investors place increasingly more (less) emphasis on earnings because the price response to earnings, β_t, increases (decreases) over time. Finally, note that a time-varying intercept term, α_t, could also include a term corresponding to a rational bubble as characterized, for example, in Tirole (1985). Since the focus of this paper is on belief-driven price association, we suppress a rational bubble term for much of our analysis.

To understand the nature of the equilibrium pricing functions characterized in Lemma 1, it is useful to decompose and highlight the critical attributes of those functions. The price function consists of three components. The first component in eqn. (8) is the steady-state price, P_t^S. The second component, $\beta_t^\delta = \left(\frac{1+r}{\lambda}\right)^t \delta$, reflects BPA, which is an increased reaction to earnings. Finally, the third component, $\alpha_t = \rho v \delta \left(2J_t + K_t\delta\right)$, is a BPA related risk premium.

Steady-state price: As discussed previously, the steady-state price, $P_t^S = \alpha + \beta\varepsilon_t$, equals the risk adjusted present value of expected future cash flows. Specifically, $\beta\varepsilon_t = \frac{\lambda}{1+r-\lambda}\varepsilon_t$ equals expected future cash flows discounted at the risk free rate, and $\alpha = -\frac{\rho v(1+r)^2}{r(1+r-\lambda)^2}$ reflects the risk adjustment attributed to the cash-flow risk. In addition to the determinants of fundamental value, Lemma 1 suggests that prices can also be determined by two other components that relate to BPA.

BPA: To isolate and highlight the direct effect of BPA, which is captured in the second component of price,

$\beta_t^\delta = \left(\frac{1+r}{\lambda}\right)^t \delta$, consider a setting in which there is no risk aversion, $\rho = 0$. In this case, $\alpha = \alpha_t = 0$, and the pricing function collapses to

$$P_t = \beta\varepsilon_t + \beta_t^\delta\varepsilon_t = P_t^S + \left(\frac{1+r}{\lambda}\right)^t \delta\varepsilon_t. \tag{11}$$

Without risk aversion, the steady-state pricing function does not contain a haircut for risk, $\alpha = 0$. A constant degree of BPA, $\delta \neq 0$, however, causes the total coefficient on earnings, $\beta_t = \beta + \beta_t^\delta = \beta + \left(\frac{1+r}{\lambda}\right)^t \delta$, to deviate from the steady-state coefficient, β, by a BPA-related term, β_t^δ. Hence, the coefficient on earnings is not only determined in the usual manner by fundamentals via the steady-state term, $\beta = \frac{\lambda}{1+r-\lambda}$, but is also determined by investor beliefs about the trajectory of the coefficient itself via the BPA term, $\beta_t = \left(\frac{1+r}{\lambda}\right)^t \delta$. Furthermore, note that, in an equilibrium with a positive degree of BPA, $\delta > 0$, investors place increasingly greater emphasis on earnings because the price response to earnings, β_t, increases over time. The increasing emphasis is necessary to sustain the BPA equilibrium. For example, suppose that the overemphasis in period $t+1$ is given by $\delta > 0$ (i.e., $\beta_{t+1} = \beta + \delta$). A period-$t$ investor who impounds the $t+1$ price movement in his period-t demand expects that a fraction λ of the period-t earnings surprise to persist. Therefore, in period t, the discounted value of the asset that pertains to the projected overemphasis is given by $\frac{\lambda}{1+r}\delta\varepsilon_t$. Similarly, with a constant negative degree of BPA, $\delta < 0$, the emphasis on earnings decreases every period and eventually becomes negative (i.e., for any $\delta < 0$, $\beta + \left(\frac{1+r}{\lambda}\right)^t \delta$ is decreasing in t and is negative for sufficiently high t).

A particular example of a pricing series for a case with no risk aversion and positive BPA is highlighted in Figure 1 and juxtaposed against the steady-state or fundamental-value pricing function. The BPA pricing path bounces around the steady-state path, and, consistent with the increase in β_t over time, the movements get larger as time passes. Finally, note that a small initial degree of BPA, δ, leads to large swings in price within a relatively short number of periods. Hence a little BPA can eventually generate substantial volatility.

BPA drift: Having highlighted the primary effect of BPA, we turn next to a secondary effect, which we call BPA drift. BPA drift arises because BPA leads to greater price uncertainty, which alters the equilibrium discount for risk. Specifically, BPA alters the price responsiveness to uncertain earnings innovations, which alters the variation in prices. When BPA-related increases (decreases) in price uncertainty are priced, which occurs when investors are strictly risk averse, BPA also leads to an upward (downward) drift in prices, which appropriately compensates investors for the increase (decreased) risk assumed. More formally, when we allow for strictly positive risk aversion, $\rho > 0$, the pricing function is

$$P_t = P_t^S(\varepsilon_t) + \left(\frac{1+r}{\lambda}\right)^t \delta\varepsilon_t + \rho v\delta\left(2J_t + K_t\delta\right), \tag{12}$$

where J_t and K_t are functions that are both increasing in t. When investors are risk averse, the pricing function again includes the traditional discount for risk, $\alpha = -\frac{\rho v(1+r)^2}{r(1+r-\lambda)^2}$, which is embedded into the steady-state price, P_t^S. With BPA present, that discount is offset with the term $\rho v\delta\left(2J_t + K_t\delta\right)$, reflecting BPA drift.

With a positive degree of BPA, $\delta > 0$, the drift term is strictly positive and increasing for all t. To understand the intuition underlying the increasingly positive drift term, note that, in equilibrium, risk averse investors are compensated for any variation in payoffs with a higher expected return (i.e., a risk premium). Since BPA increases the variance of an investor's payoffs, the equilibrium price path has to provide an additional premium to investors. This additional premium is provided by the positive upward drift in prices attributable to BPA. The provision of the risk premium due to BPA via an upward drift in price stands in contrast to the provision of the risk premium due to fundamental volatility via the discount in steady-state price. While the mechanism providing the risk premium differs, the intuition underlying the mechanism is the same. Furthermore, note that the upward drift is increasing at an increasing rate over time (i.e., $2J_t + K_t\delta$ is increasing and convex in t), which implies that the BPA-related risk premium is increasing over time. The reason is directly attributable to the fact that the price response to earnings is increasing at an increasing rate over time (i.e., $\beta_t^\delta = \left(\frac{1+r}{\lambda}\right)^t \delta$ is increasing and convex in t), which implies that the variance of prices is increasing at an increasing rate over time. Consequently, the risk premium to compensate investors for BPA risk must be increasing at an increasing rate over time.

With a negative degree of BPA, $\delta < 0$, the BPA drift term is negative at first and then, like the case of positive BPA, becomes positive and increasing at an increasing rate. The intuition for the change in direction of the drift is attributable to the observation that, when $\delta < 0$, the price response to earnings is initially positive and lower than the steady-state price, which reduces the variation in prices and thus the risk that investors bear. As a consequence, the BPA drift term is negative. The response to earnings eventually becomes negative and increasingly so, which results in greater variation in prices than in the steady state. As a consequence, investors eventually bear more risk, which leads to a positive BPA drift term. Hence, regardless of whether BPA drift is positive or negative, in the long run, prices have a positive drift for any $\delta \neq 0$.

An example of a pricing series for a case with risk aversion and positive BPA is also highlighted in Figure 1 and juxtaposed against the steady-state or fundamental-value pricing function. As in the case where investors are risk neutral, the BPA pricing function exhibits more bounce than the steady-state pricing function when investors are risk averse. Unlike the case of risk neutrality, however, the BPA price drifts above the steady-state pricing function due to the price drift required to compensate investors for the additional risk they must assume. Finally, note that both pricing functions initially lie below the pricing functions with risk neutrality because of the haircut for risk. While the fundamental value pricing function with risk averse investors continues to lie below the risk neutral fundamental value pricing function due to the haircut for risk, the BPA pricing function eventually exceeds the risk neutral pricing functions due to the BPA-related drift.

3.2. Time-Varying Price Association

The existence of linear equilibria with a constant degree of BPA introduces the theoretical plausibility of the BPA phenomenon in a manner that highlights the intuition underlying BPA and its consequences. The price paths exhibiting a constant degree of BPA, however, have some intuitively unappealing properties, namely that the earnings to price association and the associated price volatility explode over time and that, when the constant degree of BPA is negative, the earnings to price association ultimately becomes negative and increasingly so. In this section, we broaden the set of equilibria to consider pricing paths that exhibit time varying degrees of BPA, which permits us to consider pricing paths that do not have the unappealing properties exhibited by the paths exhibiting a constant degree of BPA. For example, a pricing path could initially exhibit a negative degree of BPA ($\delta < 0$), then revert to a steady-state earnings price association ($\delta = 0$), then exhibit a positive degree of BPA ($\delta > 0$), and then revert again to the steady-state association.

The idea behind time varying associations is that, when two hypothetical price paths with a constant degree of BPA have the same price at time $t = \tau$, then it is feasible for the equilibrium price path to switch from one degree of BPA to the other. To facilitate switching between two specific constant degree of BPA paths, it is helpful to allow the intercept term α_t to contain a parameter $\Delta \neq 0$ such that $\alpha_t = \alpha_t^{\Delta} = (1+r)^t \Delta + \rho v \delta (2J_t + K_t \delta)$.[12] The Δ-related part of the price drift is completely deterministic and follows the intuition for rational bubbles in prior work: investors are willing to pay a higher price if they expect next period's investors to pay an even higher price.[13]

To illustrate that there exist equilibria in which the degree of BPA varies over time, conjecture an equilibrium with a single switch in the degree of BPA:

$$P_t = \begin{cases} P(\varepsilon_t; \delta_1, \Delta_1) & \text{for all } t < \tau \\ P(\varepsilon_t; \delta_2, \Delta_2(\varepsilon_\tau)) & \text{for all } t \geq \tau, \end{cases} \tag{13}$$

[12] Note that, if investors are risk averse, allowing $\Delta \neq 0$ is not necessary for us to establish that there can be time-varying price association. In addition, note that, if $\Delta < 0$, the effect of Δ causes price to drift downward. When investors are risk averse and exhibit BPA, the upward price drift we discussed previously still occurs in the long run because the BPA drift grows faster over time than the first moment effect of $\Delta < 0$.

[13] See, for example, Abreu and Brunnermeier (2003), Azariadis (1981), and Tirole (1985).

where $P(\varepsilon_t; \delta_i, \Delta_i) = P_t^S(\varepsilon_t) + \alpha_t^{\Delta_i} + \beta_t^{\delta_i}\varepsilon_t$, and $\Delta_2(\varepsilon_\tau)$ satisfies

$$P(\varepsilon_\tau; \delta_2, \Delta_2(\varepsilon_\tau)) = P(\varepsilon_\tau; \delta_1, \Delta_1) \tag{14}$$

for all realizations ε_τ.[14] The conjectured equilibrium exhibits a switch at time τ from one BPA pricing function characterized by a degree of sensitivity δ_1, $P(\varepsilon_t; \delta_1, \Delta_1)$, to another BPA pricing function characterized by degree of sensitivity δ_2, $P(\varepsilon_t; \delta_2, \Delta_2(\varepsilon_\tau))$. However, the complete parameterization of the pricing function at time τ is not known until ε_τ is realized because the initial value of the constant term, $\Delta_2(\varepsilon_\tau)$, is a function of ε_τ. Nonetheless, $\Delta_2(\varepsilon_\tau)$ is well defined and, for any earnings realization ε_τ, there is a pricing function that exhibits the degree of BPA δ_2 yielding the same price as $P(\varepsilon_t; \delta_1, \Delta_1)$ at $t = \tau$, i.e., $P(\varepsilon_\tau; \delta_2, \Delta_2(\varepsilon_\tau)) = P(\varepsilon_\tau; \delta_1, \Delta_1)$. Because of this feature, the pricing function at time τ that is anticipated by young investors at time $\tau - 1$ is still a simple linear function of the realization of earnings at time τ (i.e., $P(\varepsilon_\tau; \delta_1, \Delta_1)$). To summarize, the change in the degree of BPA from δ_1 to δ_2 is facilitated by changing the value of the constant term associated with the new constant BPA pricing function being tracked, $P(\varepsilon_\tau; \delta_2, \Delta_2(\varepsilon_\tau))$. The value of constant term is contingent on the realization ε_τ to guarantee that the price under the initial constant BPA pricing function, $P(\varepsilon_t; \delta_1, \Delta_1)$, and the new constant BPA pricing function, $P(\varepsilon_t; \delta_2, \Delta_2(\varepsilon_\tau))$, yield the same price at $t = \tau$.

While the price function in the example is linear between any time $t-1$ and t, the slope on the time t earnings changes. We therefore term pricing functions that can exhibit a change in δ "piecewise-linear equilibria." Specifically, conditional upon some history of earnings realizations, any piecewise-linear pricing function must only track a constant BPA pricing function from period t to $t+1$. As a consequence, the pricing function at any time can be characterized by a degree of BPA, δ, as well as an associated Δ. The constant BPA pricing function that is tracked for a period of time, however, does not have to be the same for all periods, which opens up the possibility that the degree of BPA can vary over time.

The observation that the degree of BPA can vary over time implies that it is possible for a price path to exhibit negative BPA, $\delta < 0$, for a period of time without ever exhibiting a somewhat implausible negative association between price and earnings. That is, finite episodes of negative BPA can be sustained even under the equilibrium refinement that the association between price and earnings can never be negative.

Proposition 1 follows naturally from the definition of piecewise-linear equilibria and the fact that we have characterized an equilibrium exhibiting one degree of BPA initially followed by another degree of BPA.

Proposition 1. *There exists a piecewise-linear equilibrium in which the extent of BPA and the associated*

[14] This implies that $\Delta_2(\varepsilon_\tau) = \Delta_1 + \frac{\rho v}{(1+r)^\tau}(\delta_1(2J_\tau + K_\tau\delta_1) - \delta_2(2J_\tau + K_\tau\delta_2)) + \frac{1}{\lambda^\tau}(\delta_1 - \delta_2)\varepsilon_\tau$.

price response to earnings and price variability change over time.

Proposition 1 has the important implication that equilibrium price paths exist where the degree of BPA varies over time, which implies that, say, the price response to earnings need not necessarily monotonically increase (or decrease) if investors exhibit BPA for a finite period. Furthermore, we can use the same logic employed to derive the equilibrium price path with a single switch in the degree of BPA to construct paths with any number $n \in \{1, 2, 3, ...\}$ of changes in the degree of BPA. As a consequence, there exist equilibria in which episodes of BPA may be followed by steady-state responses to earnings.

4. Multi-Asset Economies

To this point, we have employed a single risky-asset framework to illustrate BPA. A common interpretation of the single risky-asset case is that it represents the market portfolio, and, in this case, BPA pertains to economy-wide (or aggregate) earnings disclosures, as opposed to, say, a single firm's disclosure. While the single risky-asset framework provides a parsimonious illustration of the main points of this study, a drawback is that it does not provide insights into how BPA affects prices of individual risky assets as opposed to the market. Here we attempt to provide additional insights by extending the model to include two risky assets, which is sufficient to provide the relevant BPA related insights for markets involving n risky assets.

Like our single risky-asset framework, investors are homogeneous, which means they are endowed with the same preferences and information and that they can form portfolios involving all assets. As a consequence, in equilibrium investors will take the same position in all assets, those that exhibit BPA and those that do not. Crucial here is that they correctly anticipate which assets have price paths exhibiting BPA and which do not, that is, investors have rational expectations about the equilibrium price paths. Our insights regarding BPA continue to hold if investors have heterogeneous preferences or beliefs about the assets' cash flows, or if some or all investors are constrained to invest in a subset of the available assets.

Formally, consider our economy with two risky assets, 1 and 2, as opposed to one. Similar to the single risky-asset setting, the earnings for asset $i \in \{1, 2\}$ for period t are

$$\varepsilon_{it} = \lambda_i \varepsilon_{it-1} + \eta_{it}. \tag{15}$$

We assume the innovation to earnings, η_{it}, is normally distributed with mean 0 and variance v_i for all $i \in \{1, 2\}$ and t, the covariance between η_{1t} and η_{2t} is c for all t, and η_{it} is independent of $\eta_{j\tau}$ for all $i, j \in \{1, 2\}$ and $t \neq \tau$. As the following analysis shows, the covariance of cash flow innovations, c, is an important determinant of the BPA price effect. Finally, we assume that there are γ shares of asset 1 per capita and $1 - \gamma$ shares of asset 2 per capita such that, in aggregate, the total number of risky asset shares per-capita is 1. This extension of the model reflects at least two scenarios: one where each asset can be thought of as reflecting a large component of the market portfolio and one where one asset is an infinitely small component of a large market and the other is the remainder of the market portfolio. The former setting, which is captured by γ assuming a strictly interior value, characterizes the relation between two industries. The latter setting, which is captured by γ approaching 0 or 1, characterizes the relation between a single risky asset as an arbitrarily small component of a large market portfolio and the market portfolio itself.

We again restrict attention to the set of linear equilibria where the end-of-period price for the asset i is a linear function of its earnings:

$$P_{it} = \alpha_{it} + \beta_{it}\varepsilon_{it}. \tag{16}$$

We rule out equilibria where asset i's price is a function of j's earnings, which would essentially be a sunspot equilibrium for firm i. In particular, in the presence of asset i's earnings, asset j's earnings have no incremental information content for i's future cash flows. Market clearing requires that $q_{1t} = \gamma$ and $q_{2t} = 1 - \gamma$, which implies that following conditions have to hold in equilibrium:

$$\beta_{it} = \frac{\lambda_i \left(1 + \beta_{it+1}\right)}{1+r} \text{ for } i \in \{1, 2\}, \tag{17}$$

$$\alpha_{1t} = \frac{\alpha_{1t+1}}{1+r} - \rho \frac{v_1 \gamma \left(1 + \beta_{1t+1}\right)^2 + c\left(1-\gamma\right)\left(1+\beta_{1t+1}\right)\left(1+\beta_{2t+1}\right)}{1+r}, \text{ and} \tag{18}$$

$$\alpha_{2t} = \frac{\alpha_{2t+1}}{1+r} - \rho \frac{v_2 \left(1-\gamma\right) \left(1 + \beta_{2t+1}\right)^2 + c\gamma\left(1+\beta_{1t+1}\right)\left(1+\beta_{2t+1}\right)}{1+r}. \tag{19}$$

Eqn. (17) is identical in structure to eqn. (5), the equilibrium condition for β_t in the single risky-asset setting. This implies that we can apply results about BPA-coefficient behavior derived in the single-asset setting to either asset's pricing function in the dual-asset setting. Furthermore, the fact that the two coefficients on earnings do not interact with one another implies that each risky asset coefficient can exhibit differing degrees of BPA. Therefore, in the absence of additional equilibrium refinements that rule out pricing paths where the two assets exhibit differing degrees of BPA, the price path of asset 1 can exhibit positive BPA, while asset 2 is priced without (or with negative) BPA. This logic extends to an economy with N assets.[15] Finally, with any set of coefficients for earnings, we can characterize the intercepts, eqns. (18) and (19), in the same manner as in the single risky-asset setting, with the caveat that two, as opposed to one, abnormal-earnings-asset

[15] Our analysis leaves open the theoretical possibility that, within the context of a large market such as the US stock market, a fundamental valuation framework holds for all assets except for one. A more plausible conjecture, perhaps, is that BPA price paths arise simultaneously for subset of assets, such as a particular industry, which are followed by a common set of active investors and subject to similar economic shocks.

coefficients determine the value each period. Hence we can derive dual-asset variations of Observation 1, Lemma 1, and Proposition 1.

Given these dual-asset variations, we discuss the spillover effects of a change in the degree of BPA in the pricing behavior of one asset to the pricing behavior of the other asset. Due to the fact that the coefficients on earnings do not interact, an increase in the degree of BPA for one asset, say asset 1, increases asset 1's price response to its earnings, β_{1t}, but has no effect on asset 2's price response to asset 2's earnings, β_{2t}, regardless of the degree of covariation in the innovations to earnings. Hence changes in the price volatility for asset 1 that are not due to changes in the volatility of fundamentals need not be associated with changes in the price volatility for asset 2.

Inspection of eqns. (18) and (19), however, reveals a relation between the price level of asset 2 and the degree of BPA for asset 1. In particular, increasing the degree of BPA for asset 1 influences the price level of asset 2 through the intercept in the pricing function, α_{2t}. The effect on the intercept depends critically on the covariance of the earnings/dividend flows. In particular, for any linear equilibrium and any two degrees of BPA, δ_1 and δ_2, the equilibrium α_{2t} satisfies:

$$\begin{aligned}\alpha_{2t} &= \alpha_2 + \rho v_2 (1-\gamma) \delta_2 (2J_t + K_t\delta_2) \\ &\quad + \rho c\gamma \left(\frac{(1+r)^{t+1}(1-\lambda^t)}{(1+r-\lambda)\lambda^t(1-\lambda)} (\delta_1+\delta_2) + (1+r)^{t+1} \frac{(1+r)^t - \lambda^{2t}}{(1+r-\lambda^2)\lambda^{2t}} \delta_1\delta_2 \right),\end{aligned} \tag{20}$$

where $\alpha_2 = -\rho\left(\frac{1+r}{1+r-\lambda}\right)^2 \frac{v_2(1-\gamma)+c\gamma}{r}$ is the steady-state intercept.

If the covariance between the earnings/dividends of the two risky assets is negative and we limit attention to the cases where BPA is nonnegative, increases in the degree of BPA for asset 1 cause the expected price level for asset 2 to be lower each period than would otherwise be the case. The reasoning behind this price-level effect stems directly from the fact that asset 2 serves as a hedge of asset 1 due to the assumed negative covariance between the two earnings flows. In particular, as asset 1 becomes more sensitive to its earnings due to a higher degree of BPA, the variance of asset 1's payoffs increases. Because the covariance between the two earnings is negative, investments in asset 2 serve as an increasingly important hedge against the increasingly volatile asset 1, which causes asset 2's price levels and associated expected price changes to decline. If, on the other hand, the covariance between the earnings of the two risky assets is positive, the opposite effect of asset 1's BPA arises: the expected price level increases over time to compensate investors for the greater degree of undiversifiable risk caused by asset 1's BPA.

Similarly, when BPA for asset 1 is negative and the covariance is positive, the expected price level for asset 2 decreases. While a positive covariance implies that holding both assets increases the risks borne by an investor, negative BPA decreases the amount of risk. Eventually, for a negative total price association, asset 2 becomes a hedge of asset 1, and investors can reduce their total risk by holding both assets.

Implicit in the discussion above is the notion that both assets represent a large component of the market (e.g., each asset represents a large sector of the economy). Our next goal is to extend the discussion of two assets to a Capital Asset Pricing Model (CAPM) perspective along the lines of Lambert, Leuz, and Verrecchia (2007). Specifically, consider the case where asset 1 represents the equity market portfolio and asset 2 represents a single equity security in a large economy with many risky assets; that is, $\gamma \rightarrow 1$. Consistent with the CAPM, the variance of asset 2 has no effect on its price. Instead, asset 2's price is determined by how its earnings covary with market earnings, asset 1. In other words, idiosyncratic risk remains unpriced even in periods of BPA. This can be seen by substituting $\gamma = 1$ into eqn. (20):

$$\begin{aligned}\alpha_{2t}\left(\gamma=1\right) &= -\rho\left(\frac{1+r}{1+r-\lambda}\right)^{2}\frac{c}{r} \\ &+\rho c\left(\frac{\left(1+r\right)^{t+1}\left(1-\lambda^{t}\right)}{\left(1+r-\lambda\right)\lambda^{t}\left(1-\lambda\right)}\left(\delta_1+\delta_2\right)+\left(1+r\right)^{t+1}\frac{\left(1+r\right)^{t}-\lambda^{2t}}{\left(1+r-\lambda^{2}\right)\lambda^{2t}}\delta_1\delta_2\right). \qquad (21)\end{aligned}$$

This shows that v_2 has no impact on α_{2t} when asset 2 in a single security in a large economy, i.e., $\gamma = 1$. Hence, consistent with our prior discussion, the effect of BPA on the price of asset 2 (an individual firm) operates through the covariance of its earnings with asset 1 (the market portfolio). This implies that BPA with respect to asset 1 increases asset 2's cost of capital by imposing more systematic risk on asset 2. Alternatively, BPA with respect to an individual firm (asset 2) has no effect on the market portfolio (asset 1) because asset 2 is an arbitrarily small element of a large economy. Note, however, that BPA with respect to asset 2 has an effect on its own price level (α_{2t}) because the increased sensitivity to its earnings increases the exposure of the asset's price to systematic risk.

Observation 2 summarizes the main insights generated by considering two risky assets.

Observation 2. *Allow for the possibility of two risky assets and restrict attention to piecewise-linear pricing functions. One asset's degree of BPA at time t has no direct effect on the price association or price volatility of the other asset at time t. One asset's degree of BPA at time t, however, affects the price level and expected change in price of the other asset if and only if the asset that exhibits BPA is large and the assets' earnings have a nonzero covariance.*

Observation 2 implies that the price association of earnings in one asset can be driven by beliefs without an effect on the price association and price levels of other assets. While our model more closely resembles single assets, this result extends to industries. For example, it may be that investors rationally expect BPA in one individual industry and thus react more (or less) strongly to all news relevant to this industry. Investors would, however, respond to news about the rest of the economy as implied by a fundamental valuation framework. The reason is that, fundamentally, BPA is driven by investors' beliefs about future prices. We only require these beliefs to be rational but impose no further constraints. Assume (somewhat outside the model) that investors specialize in subsets of assets instead of perfectly diversifying their portfolio. In such a setting, it could be that one type of investor causes a BPA path in a subset of assets. Note, however, that even investors that do not "cause" the BPA path will optimally react to news according to the BPA equilibrium.

5. A brief discussion of BPA and arbitrage

As discussed above, backward induction cannot be employed to rule out equilibria exhibiting BPA. That is, whenever a fraction of investors is interested in the next period's price (as opposed to terminal value), then backward induction fails to eliminate a BPA price path. However, would rational investors not naturally gravitate towards an equilibrium in which price reflects fundamental value? Here we discuss whether arbitrageurs would eliminate BPA price paths in the single and multi-asset economies.

Within the context of our model with atomistic investors who do not coordinate their trades, no rational investor would choose to sell if price were above fundamental value and buy if price were below fundamental value if all other investors were expected to behave in a manner consistent with a BPA equilibrium. Doing so would only generate expected trading losses because the expected price next period would not equal fundamental value. Instead, rational investors make an optimal investment decision given the correct beliefs that price will exhibit BPA next period. This behavior, in turn, allows BPA to be sustained as an equilibrium phenomenon. In other words, when an investor believes that the future price association of earnings is twice as high as it currently is, then this investor has an incentive to attach a higher multiple to the current earnings. This strategy maximizes the investor's expected utility and no investor can increase their expected utility by doing otherwise.

If we step outside of the model and allow some subset of investors, say sophisticated investors, to take a coordinated action to alter a given equilibrium price path, it is far from clear that these investors would choose terminate a BPA price path as opposed to riding it. In particular, the expected utility of investors entering at date t increases in the future stock price volatility. This implies that these investors prefer not to end a price path with positive BPA at date t but instead prefer to increase the price association of earnings.[16] The realized utility of investors exiting at date t, however, increases in the extent of BPA only when the

[16] That is, $E_t\left[U_t\left(\varepsilon_{t+1}\right)\right] \propto \frac{\rho v}{2}\left(1+\beta+\left(\frac{1+r}{\lambda}\right)^{t+1}\delta\right)^2+(1+r)\,w$, which implies that the expected utility is increasing in the variance of price. This occurs because, in equilibrium, investors are compensated with higher expected payoffs if they take on more risk.

date t news is positive. Hence informed sophisticated investors who want to unwind their trading position might try to terminate a positive (or to induce a negative) BPA path if date t earnings is negative and do the converse if date t earnings is positive.

The multi-asset economy also allows us to discuss how a BPA pricing path is sustained when seemingly perfect arbitrage opportunities exist. Specifically, consider an extreme example where asset 1 is arbitrarily small, $\gamma \to 0$, and redundant in the sense that its underlying cash flows covary perfectly with those of asset 2, $c = v$. In this setting, it might seem impossible for asset 1 to exhibit BPA if asset 2 is priced at fundamental value because a perfect cash flow arbitrage exists. For example, if asset 1 trades above fundamental value, then asset 1's exact future underlying cash flows can be acquired at a lower cost by buying asset 2. Hence it would seem that all rational traders should sell asset 1, which would drive asset 1's price towards fundamental value. Within the context of the model where the asset is infinitely lived and investors live for two periods, this logic falls short because, in any given period, investors are only interested in the asset's underlying cash flows in the next period *plus* the asset's price next period. They do not concern themselves with the asset's underlying cash flows after the next period. Hence, as long as investors believe the asset will follow the pricing path exhibiting BPA, the BPA pricing path can be sustained as an equilibrium pricing path.

6. Empirical Implications

The analysis to this point has been focused on our main contribution: to establish that the price association of earnings can be driven by beliefs in a setting with rational investors. In establishing the theoretical plausibility of BPA, however, we have identified a plethora of equilibria. Unfortunately, models with multiple equilibria cannot offer simple empirical predictions based upon standard comparative statics analysis because it is not clear which equilibrium should be perturbed. As a consequence, in Appendix B, we undertake an analysis of three common forms of equilibrium refinements in an attempt to eliminate all but one equilibrium. Because we never assume that the asset is liquidated at a known terminal date and we never impose a strong transversality or terminal assumption, however, the refinements fail to narrow our attention to a single focal equilibrium.

In spite of the fact that our analysis does not rule out multiple equilibria, the insights offered by it can still be empirically relevant because multiple equilibria may actually reflect an important element of economic reality. An equilibrium is defined by a set of endogenous outcomes that fall from some intuitively appealing criteria, which in our case can be loosely stated as: individuals (1) act in their best interest, (2) anticipate the behavior of others, and (3) process information well. In essence, we constrain ourselves to focus on outcomes that make sense. It might very well be the case that actual economic agents employ similar reasoning to guide their own actions and, as a consequence, those actions will lead to one of those outcomes that makes sense. In settings where more than one outcome satisfies the equilibrium criteria, however, it is plausible that individuals will not ultimately play just one of those sensible outcomes with probability 1. Instead, individuals may interact with each other repeatedly before their interactions naturally converge to one of the equilibrium outcomes. Hence, while the equilibrium criteria do not predict exactly where an economy settles, they restrict the set of possible outcomes. An economy might then settle on an outcome in the restricted set seemingly at random. If this is true, then restricting attention to modeling frameworks that force a single equilibrium misses an element of economic reality.

The empirical insights offered by our model with multiple equilibria take one of two forms, which we refer to as cautionary empirical implications and predictive empirical implications. Cautionary empirical implications are not directly testable implications but provide a reason to exercise caution when employing some common measurement constructs employed in the empirical literature. On the other hand, our predictive empirical implications are testable predictions.

6.1. Cautionary Empirical Implications

If the BPA phenomenon that we develop in this paper is present in markets, it could have significant implications for empirical measurement constructs that rely upon market prices. Consider first a common approach for inferring discount rates, cost of capital, or expected returns (henceforth, the "market risk premium"), which relies on the assumption that price equals fundamental value (i.e., expected future cash flows discounted at the risk adjusted rate). Under this assumption, the market risk premium can be inferred from price level coupled with expectations of future earnings/cash flows and the risk free rate.[17] The possibility of BPA drift, however, calls such an inference into question. For example, assume an equilibrium pricing function with $\delta > 0$ and $\rho > 0$. The expected price level and expected return on this price path are higher than in the steady state equilibrium because of BPA drift. Applying a fundamental valuation framework in an empirical study would suggest that the discount rate is lower in the BPA equilibrium than in the steady-state equilibrium. In other words, agents within the model know the cost of capital is high, whereas a researcher would incorrectly conclude the cost of capital is low.

As another example, consider a common approach for inferring the information content of earnings or other disclosed statistics, which is to compute the association between the market price and a statistic, or the market price response to the disclosure of the statistic, and then to assume that the information content of the

[17] Empirical papes that rely on such inferred discount rates/cost of capital/expected return include, for example, Botosan and Plumlee (2005), Sadka (2007), and Vuolteenaho (2000).

statistic is greater when the magnitude of the association or response is greater.[18] Holthausen and Verrecchia (1988) provide a theoretical framework that supports this working assumption. If BPA is present, however, the association and response are determined by higher order beliefs in addition to Bayesian expectations of future cash flows and discount rates. As a consequence, a statistic with a greater price association or price response may be no more informative about future cash flows than a statistic with a lower price association or price response.[19]

As a final example, consider common approaches to infer a firm's growth opportunities from market prices: the market-to-book ratio or Tobin's q.[20] When price equals fundamental value, a higher market-to-book ratio implies higher perceived growth prospects. Whenever positive BPA is present, however, the effect of book value on market values can be higher simply because investors believe they will be higher. Similarly, when the price path exhibits negative BPA, multiples decrease even when growth prospects remain constant. This implies that the inferred growth prospects could be systematically biased.

While the above examples illustrate that BPA could cause conceptual constructs inferred from market prices to be misleading, this is not to say that BPA guarantees they will be significantly misleading. Whether they are significantly misleading depends upon the prevalence and magnitude of BPA episodes that the researcher has not taken into account. Given that we have not identified how a researcher can identify and account for a BPA episode, at best we can merely suggest that researchers exercise caution with their interpretations of conceptual constructs inferred from market prices and, to the extent possible, that they validate findings with alternative measures of those constructs.

6.2. Predictive Empirical Implications

In addition to the above cautionary implications, our model can also guide empirical researchers towards more specific empirical tests. A main condition to conduct these tests is that the researcher identifies when certain equilibria are more likely to be played. For example, one might hypothesize that investors will play a BPA equilibrium when firm management and the media focus their attention on a single performance statistic such as earnings (or sales or clicks). Given that hypothesis, our model provides predictions as to

[18] In their review of the earnings quality literature, Dechow et al. (2010) discusss the lengthy literature that employs market responses to measure information content.

[19] For example, our model provides an alternative explanation for the observed increase in market responses to earnings over time, which has been documented in Landsman and Maydew (2002) as well as Francis et al. (2002). While Landsman and Maydew (2002) suggests that changes in firm characteristics have contributed to the increased market response, the increase in association over time is significant even controlling for those characteristics. Francis et al. (2002) suggest that the amount of information disclosed concurrently with earnings has increased, leading to the finding that market response around the time of an earnings announcement has increased. In addition to the explanations offered by Landsman and Maydew (2002) and Francis et al. (2002), our model suggests the increased market response to earnings could be an equilibrium outcome even if firm characteristics and the amount of information disclosed are held constant.

[20] For example, see Collins and Kothari (1989) and Penman (1996).

the pricing behavior that would be observed during periods when management and the media emphasize a particular statistic in their disclosures.

6.2.1. Price Association and Price Variation

In periods with positive BPA, $\beta_t > 0$, the association between earnings and price, and the resulting price variation, can be significantly larger than the association and variation implied by a fundamental valuation. Furthermore, β_t can vary over time even though the relation between earnings and future cash flows is stable. In addition to providing these straightforward observations, however, our formal model also provides some additional observations regarding price associations and price variation.

Corollary 1. *Assume the equilibrium price path is characterized by the pricing function $P(\varepsilon_t; \delta, \Delta)$, where $\delta > 0$, over the time span τ to $\tau + k$. The coefficient on earnings and the variance in price is increasing each period over the time span τ to $\tau + k$. Furthermore, the growth in the coefficient on earnings and the variation in prices along that path is increasing in the risk free interest rate, r, and decreasing in the degree of persistence in abnormal earnings, λ.*

Given that an empiricist identifies a time span when positive BPA is expected to occur, Corollary 1 offers a number of empirically oriented implications that should arise over that time span. First, the price association with earnings and the variance of prices should be increasing. In addition, the growth in the coefficient and the variance of earnings should be increasing in the risk free rate and decreasing in the persistence of earnings.

We should point out that a perfectly clean analogue to Corollary 1 does not arise for episodes of negative BPA, $\delta < 0$. In particular, during such an episode, the coefficient on earnings will decrease each period and eventually become, perhaps implausibly, negative. While the variance in price will also decrease initially, it begins to increase as soon as the coefficient on earnings becomes negative. That is, in contrast to the case of positive BPA described in Corollary 1, the variance of price is U-shaped as opposed to monotonic over time.

6.2.2. Expected Prices

While an important characteristic of positive (negative) BPA is a greater (smaller) association between prices and earnings as well as greater (lesser and then, ultimately, greater) price volatility, BPA also results in price levels that differ from the steady-state price level. More interestingly, the *expected* price levels systematically differ if investors are strictly risk averse.

Corollary 2. *Assume investors are risk averse and the equilibrium price path is characterized by the pricing function $P(\varepsilon_t; \delta, \Delta)$, where $\delta > 0$, over the time span τ to $\tau + k$. The time-τ expectation of price for time-$t \in (\tau, \tau+k]$ exceeds the steady-state expected price. For $\delta < 0$ and $|\beta_t| < \beta$, the price is below the steady-state price in expectation; for $\delta < 0$ and $|\beta_t| > \beta$, the price exceeds the steady-state price in expectation.*

The first part of Corollary 2 holds because, when prices are more volatile, risk averse investors are compensated with greater expected payoffs, which is achieved in equilibrium by greater expected returns and an associated upward drift in prices. The upward drift in prices, in turn, implies higher expected price levels. The increased price volatility occurs whenever $|\beta_t| > \beta$. This implies that expected price levels will be higher for sufficiently negative BPA (or a sufficiently long $\delta < 0$ price path). However, when $\delta < 0$ leads to a sufficiently small decrease in β_t, such that $|\beta_t| < \beta$, then price volatility is reduced, and expected price levels are below those of the steady-state path.

From an empirical perspective, Corollary 2 also implies that, in a time span in which BPA is predicted to occur, price-to-earnings ratios should be higher despite the fact that economic fundamentals are unchanged. Note, that while the increased expected prices on a BPA equilibrium path seem desirable, they come at the cost of high expected returns. That is, the cost of capital, as defined by expected returns, is higher and increasing on a BPA path. If this cost of capital is an important determinant in firms' investment choices, it could lead to less investment. If, on the other hand, price levels are the more important determinant, then more investment could result.

6.2.3. Reversion to a Focal Steady-State Path

Among the piecewise-linear pricing paths, the steady-state price path ($\beta_t = \beta$ and $\alpha_t = \alpha$) might be viewed as focal due to its inherent simplicity and the stability of the pricing parameters over time. Even if the steady-state pricing path is focal, however, temporary periods of BPA, or BPA bubbles, might arise. We take as given the possibility of BPA bubbles and then assess the types of earnings realizations that must occur for the pricing function to revert back to the steady-state pricing function. To study the properties of an equilibrium in which BPA pricing bubbles arise and the steady-state path is "focal," we consider an equilibrium in which the price path begins on the steady state, characterized by pricing function $P(\varepsilon_t; 0, 0)$, then enters a period with positive BPA of degree $\delta > 0$ at time τ, and reverts back to the steady state at time $\tau + m$. In order to switch, the two paths have to meet twice, at $t = \tau$ and at $t = \tau + m$. Therefore the price path is defined by $P(\varepsilon_t; \delta, \Delta)$, where $P(\varepsilon_\tau; \delta, \Delta) = P(\varepsilon_\tau; 0, 0)$, and reverts back to $P(\varepsilon_t; 0, 0)$ at the first date $\tau + m$ in which $P(\varepsilon_{\tau+m}; 0, 0) = P(\varepsilon_{\tau+m}; \delta, \Delta)$. Corollary 3 shows that cumulative earnings between times τ and $\tau + m$ has to be negative such that a path with positive BPA and the steady-state price path can converge again.

Corollary 3. *Assume investors are strictly risk averse and, at time $t = \tau$, the price path changes from steady state to one that exhibits positive BPA, i.e., $\delta > 0$. For the equilibrium price to converge back to the steady-state price at time $t = \tau + m$, where $m > 0$, earnings at $t = \tau + m$ must equal $\varepsilon^*_{\tau+m}$ where $\varepsilon^*_{\tau+m} < \lambda^m \varepsilon_\tau$. Furthermore, $\varepsilon^*_{\tau+m}$ is decreasing in m and approaches negative infinity as m approaches* *infinity.*

The first statement in Corollary 3 implies that earnings news must be bad for the BPA and steady-state paths to meet. Hence, once investors exhibit BPA, a return to steady-state fundamental pricing must be associated with bad news. Empirically, this predicts that a large decrease in earnings response coefficients should follow surprising negative earnings. The second statement in the corollary implies that, as the period of BPA increases in duration, the worse the news needs to be to converge back to a steady state. Given that extreme realizations are less likely, this observation suggests that the "likelihood" of convergence back to steady-state prices decreases as the duration of BPA increases. Of course, given our assumption that earnings are distributed continuously whereas trading occurs in discrete intervals, the probability of precise convergence is always 0. If earnings were discrete or trading were continuous, however, the probability of precise convergence would not be 0, and we would expect the insight in Corollary 3 to hold. Finally, Corollary 3 suggests that an empiricist who identifies a time span when BPA is expected to occur should expect to see those periods come to an end if and only if earnings news is bad.

Corollary 3 only applies to episodes of positive BPA, $\delta > 0$. A simple analogue to Corollary 3 does not arise for episodes of negative BPA because, when $\delta < 0$, the direction of the earnings surprise necessary for the BPA price and steady-state price to converge changes over time. As described above, $\delta < 0$ leads to $-\beta < \beta_t < \beta$ for the initial periods of a negative BPA episode, which implies that the expected future price is lower than in the steady state because prices are less volatile (i.e., BPA drift is negative during these periods). During these periods, then, the earnings surprise has to be negative for the paths to converge. Finally, the negative BPA episode becomes so long that $\beta_t < -\beta$, and the BPA price exceeds the steady-state price in expectation because prices are more volatile (i.e., BPA drift is positive during these periods). In this range, only a positive earnings surprise will lead to convergence (i.e., a positive earnings surprise coupled with the counterintuitive negative price response along the BPA path causes the two paths to converge).

7. Conclusion

Using an overlapping-generations modeling framework, we consider the implications that arise from investors' beliefs that investors in subsequent periods will place greater or lesser emphasis on earnings information relative to fundamental valuation. We describe this phenomenon as one where current investors exhibit "beliefs-driven price association" (BPA). Our analysis demonstrates that BPA can be a self-fulfilling phenomenon and, as a consequence, can arise as an equilibrium behavior in a setting in which all investors have rational expectations. In addition, we show that there are equilibrium pricing paths exhibiting episodes of BPA followed by periods in which prices are consistent with a fundamental valuation framework.

While our modeling framework does not predict which price path is taken in equilibrium, it does yield a number of empirical insights. For example, during periods of positive BPA, the model predicts that prices will have a higher association with the value-relevant statistic, will exhibit greater volatility, and will appear to be high relative to a fundamental valuation framework. Furthermore, our model suggests that, during periods of BPA, common approaches that rely upon market prices to infer costs of capital, the information content of a value relevant statistic, or growth opportunities will be systemically biased. Finally, the fact that BPA can be time varying suggests that price volatility and expected returns can be time varying even if underlying fundamentals are stable.

As an extension, we consider a setting with two risky assets in order to assess how BPA in the market for one asset can spill over to affect the pricing of another asset. We show that increasing BPA with respect to one asset need not have any impact on the price volatility of the other asset, irrespective of the correlation between the assets' fundamental earnings flows. Increasing BPA with respect to one asset, however, does influence the price level and expected price change of the other asset if the former asset is large in magnitude and the correlation between the assets' fundamental flows is not zero.

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Table of Notation

ε_t	earnings/cash-flows at time-t
η_t	earnings/cash-flow innovation at time-t
λ	persistence of earnings
v	variance of earnings innovation
r	risk free return
ρ	coefficient of constant absolute risk aversion
α_t	intercept term in price at time-t
Δ	first-moment drift parameter
β_t	price association at time-t
δ	degree of BPA
q	demand in the asset's shares
γ	shares of asset 1 per capita
c	covariance between η_{1t} and η_{2t}

Appendix A – Proofs

Lemma 1. *The proof follows directly from the fact that the coefficient specifications satisfy the equilibrium conditions (5) and (6) for any t.*

Proposition 1. *To prove Proposition 1, it is useful to first adopt some notation for characterizing the linear equilibria from Lemma 1. Specifically, let* $h_t \equiv \{\varepsilon_0, \varepsilon_1, ...\varepsilon_{t-1}\}$ *denote the history of earnings realizations up to but not including time t's realization. A piecewise-linear equilibrium is defined as follows:*

> Definition 1. *A piecewise-linear equilibrium is characterized by a pricing function of the form* $P(h_t, \varepsilon_t) = P_t^S(\varepsilon_t) + \alpha_t(h_t) + \beta_t(h_t)\varepsilon_t$ *that satisfies the following condition for any period t realization of* h_{t+1}*: There exist values* $\Delta(h_{t+1})$ *and* $\delta(h_{t+1})$ *such that* $P(h_t, \varepsilon_t) = P(\varepsilon_t; \Delta(h_{t+1}), \delta(h_{t+1}))$ *given* h_{t+1} *and* $P(h_{t+1}, \varepsilon_{t+1}) = P(\varepsilon_{t+1}; \Delta(h_{t+1}), \delta(h_{t+1}))$ *for all* ε_{t+1}.

Conjecture an equilibrium pricing function such that, for all $t \in \{0, 1, 2, ...\tau\}$*, the price is* $P(\varepsilon_t; \delta_1, \Delta_1) = \alpha_{1t} + \beta_{1t}\varepsilon_t$*, where* $\beta_{1t} = \beta + \left(\frac{1+r}{\lambda}\right)^t \delta_1$*, and* $\alpha_{1t} = \alpha + (1+r)^t \Delta_1 + \rho v \delta_1 (2J_t + K_t\delta_1)$ and *for all* $t \in \{\tau, \tau+1, \tau+2, ...\}$*, the price is* $P(\varepsilon_t; \delta_2, \Delta_2) = \alpha_{2t} + \beta_{2t}\varepsilon_t$ *where* $\beta_{1t} = \beta + \left(\frac{1+r}{\lambda}\right)^t \delta_2$*,* $\delta_2 \geq 0$*,* $\delta_2 \neq \delta_1$*,* $\alpha_{2t} = \alpha + (1+r)^t \Delta_2 + \rho v \delta_2 (2J_t + K_t\delta_2)$*, and* Δ_2 *is set so that* $P(\varepsilon_t; \delta_1, \Delta_1) = \alpha_{1\tau} + \beta_{1\tau}\varepsilon_\tau = \alpha_{2\tau} + \beta_{2\tau}\varepsilon_\tau = P(\varepsilon_t; \delta_2, \Delta_2)$ *or* $\Delta_2 = \Delta_1 + \frac{1}{\lambda^\tau}(\delta_1 - \delta_2)\varepsilon_\tau + \frac{\rho v}{(1+r)^\tau}\left[2J_\tau(\delta_1 - \delta_2) + K_\tau(\delta_1^2 - \delta_2^2)\right]$*. Hence, all investors believe that at time* τ *the pricing function changes so that it is characterized by a different degree of BPA,* δ_2 *as opposed to* δ_1*, and that the constant term adjusts at time* τ *so that the time* τ *price is identical to the price that would be realized under the initial pricing function* $P(\varepsilon_t; \delta_1, \Delta_1)$*. In order for this path to constitute an equilibrium, markets have to clear at any point in time. For* $t < \tau - 1$*, the market clears at price* $P(\varepsilon_t; \delta_1, \Delta_1)$ *following Lemma 1. At* $t = \tau - 1$*, young investor demands as a function of price are the same as they would be if the pricing function for* τ *continued to be characterized by* $P(\varepsilon_t; \delta_1, \Delta_1)$ *because* $P(\varepsilon_t; \delta_1, \Delta_1) = P(\varepsilon_t; \delta_2, \Delta_2)$ *for any realization* ε_τ*. Hence the market clears a time* $\tau - 1$ *at price* $P(\varepsilon_t; \delta_1, \Delta_1)$*. Finally, for all* $t \geq \tau$*, the market clears at* $P(\varepsilon_t; \delta_2, \Delta_2)$ *following Lemma 1.*

Corollary 1. *The variance in price at time t equals* $\beta_t^2 v = \left(\beta + \left(\frac{1+r}{\lambda}\right)^t \delta\right)^2 v$*, so the growth in variation between* τ *and* $\tau + k$ *equals*

$$\left(\left(\beta + \left(\frac{1+r}{\lambda}\right)^{\tau+k} \delta\right)^2 - \left(\beta + \left(\frac{1+r}{\lambda}\right)^\tau \delta\right)^2\right) v = \delta\left(\left(\frac{1+r}{\lambda}\right)^k - 1\right)\left(\frac{1+r}{\lambda}\right)^\tau \left(2\beta + \left(\frac{1+r}{\lambda}\right)^\tau \delta\left(\left(\frac{1+r}{\lambda}\right)^k + 1\right)\right) v > 0.$$

The inequality holds since $\lambda \in (0,1)$ *and* $r > 0$*. The growth in variation between* τ *and* $\tau+k$ *is increasing in r and decreasing in* λ *since* $\frac{\partial\left(\frac{\lambda}{1+r-\lambda}\left(\left(\frac{1+r}{\lambda}\right)^k - 1\right)\right)}{\partial\lambda} = -\frac{1}{\lambda}\frac{1+r}{(1+r-\lambda)^2}\left(\lambda\left(1 - \left(\frac{1+r}{\lambda}\right)^k\right) + k\left(\frac{1}{\lambda}(r+1)\right)^{k-1}(1+r-\lambda)\right) < 0$ *and* $\frac{\partial\left(\frac{\lambda}{1+r-\lambda}\left(\left(\frac{1+r}{\lambda}\right)^k - 1\right)\right)}{\partial r} = \frac{1}{(1+r-\lambda)^2}\left(\lambda\left(1 - \left(\frac{1+r}{\lambda}\right)^k\right) + k\left(\frac{1}{\lambda}(r+1)\right)^{k-1}(1+r-\lambda)\right) > 0.$

Corollary 2. *At time* τ*, it must be the case that the price in the steady state equals the price on path*

$P(\varepsilon_t;\delta,\Delta)$:

$$\alpha+(1+r)^{\tau}(\Delta-\alpha)+\rho v\delta\left(2J_{\tau}+K_{\tau}\delta\right)+\left(\beta+\left(\frac{1+r}{\lambda}\right)^{\tau}\delta\right)\varepsilon_{\tau}=\alpha+\beta\varepsilon_{\tau}, \tag{A.1}$$

which implies

$$(1+r)^{\tau}(\Delta-\alpha)+\rho v\delta\left(2J_{\tau}+K_{\tau}\delta\right)+\left(\frac{1+r}{\lambda}\right)^{\tau}\delta\varepsilon_{\tau}=0. \tag{A.2}$$

At time-τ, the expected time-$(\tau+m)$ price along path $P(\varepsilon_t;\delta,\Delta)$, where $m\in(0,k)$, less the steady-state price is

$$\begin{aligned}&\alpha+(1+r)^{\tau+m}(\Delta-\alpha)+\rho v\delta\left(2J_{\tau+m}+K_{\tau+m}\delta\right)\\&+\left(\beta+\left(\frac{1+r}{\lambda}\right)^{\tau+m}\delta\right)\lambda^{m}\varepsilon_{\tau}-\left(\alpha+\beta\lambda^{m}\varepsilon_{\tau}\right),\end{aligned} \tag{A.3}$$

or

$$(1+r)^{\tau+m}(\Delta-\alpha)+\rho v\delta\left(2J_{\tau+m}+K_{\tau+m}\delta\right)+\left(\frac{1+r}{\lambda}\right)^{\tau+m}\delta\lambda^{m}\varepsilon_{\tau}. \tag{A.4}$$

Relying on eqn. (A.2), eqn. (A.4) can be written as:

$$2\rho v\delta\left(J_{\tau+m}-(1+r)^{m}J_{\tau}\right)-\rho v\delta^{2}\left(K_{\tau+m}-(1+r)^{m}K_{\tau}\right), \tag{A.5}$$

or

$$\begin{aligned}&2\rho v\delta\frac{(1+r)^{\tau+m+1}\left(1-\lambda^{\tau}\right)}{\lambda^{\tau+m}(1+r-\lambda)(1-\lambda)}\left(\frac{1-\lambda^{\tau+m}}{1-\lambda^{\tau}}-\lambda^{m}\right)\\&+\rho v\delta^{2}\frac{(1+r)^{\tau+m+1}}{1+r-\lambda^{2}}\frac{(1+r)^{\tau}-\lambda^{2(\tau)}}{\lambda^{2(\tau+m)}}\left(\frac{(1+r)^{\tau+m}-\lambda^{2(\tau+m)}}{(1+r)^{\tau}-\lambda^{2(\tau)}}-\lambda^{2m}\right),\end{aligned} \tag{A.6}$$

which is strictly positive if $\rho>0$ because $\lambda\in(0,1)$ implies $\frac{1-\lambda^{\tau+m}}{1-\lambda^{\tau}}-\lambda^{m}>0$ and $\frac{(1+r)^{\tau+m}-\lambda^{2(\tau+m)}}{(1+r)^{\tau}-\lambda^{2(\tau)}}-\lambda^{2m}>0$.

Corollary 3. *Assume at time $t=\tau$ investors deviate from the steady-state path and begin to exhibit BPA. Similar to eqn. (A.1), the following equation has to hold for the two price paths to meet at time t:*

$$(1+r)^{t}(\Delta-\alpha)+\rho v\delta\left(2J_{t}+K_{t}\delta\right)+\left(\frac{1+r}{\lambda}\right)^{t}\delta\varepsilon_{t}=0. \tag{A.7}$$

Additionally, in order for the steady-state price path to meet the one where investors exhibit BPA at date

$t+m$, it must be the case that

$$\begin{aligned}
(1+r)^{t+m}(\Delta-\alpha)+\rho v\delta\left(2J_{t+m}+K_{t+m}\delta\right)+\left(\frac{1+r}{\lambda}\right)^{t+m}\delta\varepsilon_{t+m} &= 0, \text{ or} \\
(1+r)^{t}(\Delta-\alpha)+\frac{\rho v\delta}{(1+r)^{m}}\left(2J_{t+m}+K_{t+m}\delta\right)+\left(\frac{1+r}{\lambda}\right)^{t}\delta\frac{\varepsilon_{t+m}}{\lambda^{m}} &= 0.
\end{aligned} \tag{A.8}$$

Using eqn. (A.7) and the fact that $\varepsilon_{t+m}=\sum_{i=1}^{m}\lambda^{m-i}\eta_{t+i}+\lambda^{m}\varepsilon_{t}$, we can re-express (A.8) as

$$\left(\frac{1+r}{\lambda}\right)^{t}\sum_{i=1}^{m}\lambda^{-i}\eta_{t+i}=-\rho v\left(\frac{2J_{t+m}+K_{t+m}\delta}{(1+r)^{m}}-\left(2J_{t}+K_{t}\delta\right)\right). \tag{A.9}$$

The proof to Corollary 2 shows that the right hand (A.9) is negative, which completes the proof.

Observation 2. *In any linear equilibrium of the form in (16), the demand of shares in asset i by a new investor in period t is*

$$q_{it}=\frac{\lambda_i\varepsilon_{it}+\alpha_{it+1}+\beta_{it+1}\lambda_i\varepsilon_{it}-\rho c\left(1+\beta_{it+1}\right)\left(1+\beta_{jt+1}\right)q_{jt}-(1+r)P_{it}}{\rho v_i\left(1+\beta_{it+1}\right)^{2}}, \tag{A.10}$$

for $i,j\in\{1,2\}$ and $i\neq j$. Market clearing yields the pricing conditions in 17–19. The proof of the observation follows directly from inspection of eqn. (20).

Appendix B – Alternative Model

Consider an alternative overlapping generations model where earnings follow a process of the form

$$e_t=r_eBV_{t-1}+\varepsilon_t. \tag{B.1}$$

Here, e_t is period t earnings, BV_{t-1} is the book value at the beginning of period t (end of $t-1$), r_e is the normal return on equity, and ε_t is the abnormal earnings in period t. The abnormal earnings follows the same process as before,

$$\varepsilon_t=\lambda\varepsilon_{t-1}+\eta_t. \tag{B.2}$$

However, different from the model above, only a proportion ω of earnings is paid out as a dividend. We assume that clean surplus accounting holds such that the ending book value equals beginning book value plus earnings less dividends:

$$BV_t=BV_{t-1}+e_t-\omega e_t=BV_{t-1}+(1-\omega)(r_eBV_{t-1}+\varepsilon_t). \tag{B.3}$$

In order for the present value of future dividends to be finite, reinvested earnings cannot yield too high of an expected return. The necessary condition for this to be the case, which we assume holds throughout, is that $r > r_e(1-\omega)$. Investors have the same negative exponential utility functions as in the main text. Just as in the main text, we first establish the steady-state equilibrium and then move to the BPA equilibrium.

Steady state: The steady-state price is given by

$$P_t = \alpha + \beta BV_t + \gamma \varepsilon_t, \tag{B.4}$$

where

$$\alpha = -\frac{\rho v \omega^2 r (1+r)^2}{(1+r-\lambda)^2 (r - r_e (1-\omega))^2}, \tag{B.5}$$

$$\beta = \frac{\omega r_e}{r - r_e (1-\omega)}, \text{ and} \tag{B.6}$$

$$\gamma = \frac{\lambda \omega r}{(1+r-\lambda)(r - r_e(1-\omega))}. \tag{B.7}$$

Therefore, β is increasing in the equity rate of return, r_e, because a greater average return on invested capital implies greater future cash flows for investors. It is decreasing in the dividend payout ratio ω as long as the average return on invested capital exceeds the risk free rate, $r_e > r$, because the invested capital yields cash flows incremental to the opportunity cost of the investment. Finally, it is decreasing in the risk free rate, r, which effectively discounts the future cash flows to investors. The coefficient on abnormal earnings is greater if the persistence parameter, λ, is greater, because a more persistent abnormal earnings has more significant implications for future cash flows. Given that abnormal earnings are reinvested to earn r_e, the coefficient on abnormal earnings is increasing in r_e and is decreasing in ω if $r_e > r$ for the same reasons the coefficient on ending book value changes in r_e and ω. Finally, the constant term, $\alpha < 0$, captures the price haircut for risk, and it is intuitively larger in magnitude (i.e., more negative) when abnormal earnings innovations have more impact on price. Hence it is greater in magnitude if the abnormal earnings innovation is more persistent (i.e., λ is greater), if the reinvested abnormal earnings earn a greater return (i.e., r_e is greater), if the discount rate is lower (i.e., r is lower), and, assuming $r_e > r$, if the dividend payout is lower (i.e., r_e is lower). Obviously, it is also increasing in magnitude if the variance of the innovation, v, is larger or the degree of risk aversion, ρ, is larger.

BPA: For any $\delta > 0$ and initial value for the constant term in the pricing function Δ, there exists an equilibrium pricing function

$$P_t = \alpha_t + \beta_t BV_t + \gamma_t \varepsilon_t, \tag{B.8}$$

where

$$\beta_t = \beta = \frac{\omega r_e}{r - r_e(1-\omega)}, \tag{B.9}$$

$$\gamma_t = \gamma + \left(\frac{1+r}{\lambda}\right)^t \delta, \tag{B.10}$$

$$\alpha_t = \alpha + (1+r)^t(\Delta - \alpha) + \rho v \delta (J_t + K_t \delta). \tag{B.11}$$

Again, $\beta = \frac{\omega r_e}{r - r_e(1-\omega)}$, $\gamma = \frac{\lambda \omega r}{(1+r-\lambda)(r-r_e(1-\omega))}$, and $\alpha = -\frac{\rho v \omega^2 r (1+r)^2}{(1+r-\lambda)^2 (r-r_e(1-\omega))^2}$ are the steady-state coefficients and intercept, and $J_t = 2\frac{(1+r)^t(1-\lambda^t)}{(1-\lambda)\lambda^t}\left(\frac{r\omega(1+r)}{(1+r-\lambda)(r-r_e(1-\omega))}\right)$ and $K_t = \frac{(1+r)^{t+1}\left((1+r)^t - \lambda^{2t}\right)}{(1+r-\lambda^2)\lambda^{2t}}$.

Appendix C – Refinements

The set of piecewise-linear equilibria captures a plethora of equilibria, some of which might be deemed to have implausible properties such as excessive price responses to earnings or price "blowing up" to infinity as time passes. In an effort to focus attention on plausible equilibrium paths, we consider a few equilibrium refinements. We initially consider a refinement that is consistent in spirit with the imposition of a transversality condition, which is an exogenous requirement that any equilibrium must satisfy as time passes. While arguably ad hoc, transversality conditions are justified by a notion that an economy should gravitate towards a pricing path that is conceptually appealing. We term this refinement *limiting condition*. The next two refinements that we consider are tied to thought experiments regarding individual investor behavior. We term the first of these refinements *strategic dominance* as it is based on the thought experiment of how individuals might react to the strategic uncertainty induced by the possibility of multiple equilibrium price paths. In the second of the two, *local stability*, we focus on a local stability criterion, which relies on a thought experiment for how individuals would respond to small perturbations from equilibrium behaviors.

1 Limiting Condition

Within the context of our model, the steady-state price path is a sensible anchor for a transversality-like condition because the steady-state pricing function resembles a conceptually appealing fundamental valuation in which price equals the discounted expectation of future cash flows with an adjustment for risk. Accordingly, we consider a refinement characterized by a limiting condition linked to deviations from the steady-state price, which is characterized by a date $\tau > 0$ and a probability $\phi \in (0, 1]$.

Definition 2. *An piecewise-linear equilibrium price path $P(h_t, \varepsilon_t)$ satisfies a $\{\tau, \phi\}$ limiting condition if, as of date $t = 0$, $E[P(h_{\tau+1}, \varepsilon_{\tau+1}) - \alpha - \beta\varepsilon_{\tau+1} | h_{\tau+1}] = 0$ with probability ϕ.*

Intuitively, a $\{\tau, \phi\}$ refinement requires that there be a sufficiently high probability at the start of the market that investors in some future period will expect the deviation from the steady-state price to be 0.

Obviously, the steady-state price path satisfies any $\{\tau, \phi\}$ refinement. Furthermore, any $\{\tau, \phi\}$ refinement is sufficiently stringent that any path characterized by a $\{\delta, \Delta\}$ pricing function fails to satisfy the condition. Eliminating these paths from consideration is arguably appealing because the magnitude of the price response to earnings at date t, $|\beta_t|$, and the expected price at date t, $E\left[P\left(\varepsilon_t; \delta_1, \Delta_1\right)|\varepsilon_0\right]$, both approach infinity as t approaches infinity, which seems a priori implausible. The critical question remaining is whether a $\{\tau, \phi\}$ convergence refinement rules out all piecewise-linear pricing paths besides the steady-state path. In the proof to Observation 3, we demonstrate through example that, as long as $\phi < 1$, there exist piecewise-linear equilibria in which price exhibits varying degrees of BPA or negative BPA, and that the only piecewise-linear equilibrium satisfying a $\{\tau, \phi = 1\}$ refinement is the steady-state equilibrium. The last observation is attributable to the fact that the support for the innovation to earnings is unbounded.

Observation 3. *Given any $\{\tau, \phi\}$ refinement where $\phi < 1$, (1) The steady-state equilibrium satisfies the $\{\tau, \phi\}$ refinement. (2) No constant BPA equilibrium in which $\delta \neq 0$ and/or $\Delta \neq \alpha$ satisfies the $\{\tau, \phi\}$ refinement. (3) There exist piecewise-linear equilibria exhibiting time-varying degrees of BPA or negative BPA that satisfy the $\{\tau, \phi\}$ refinement. (4) Given any $\{\tau, \phi = 1\}$ refinement, the only piecewise-linear equilibrium that satisfies the refinement is the steady-state equilibrium.*

Proof *(1) Obviously, $E\left[\alpha + \beta\varepsilon_{\tau+1} - \alpha - \beta\varepsilon_{\tau+1}|h_{\tau+1}\right] = 0$ with probability 1. (2) Since $(1+r)^t(\Delta - \alpha)$ and $\rho v\delta\left(2J_t + K_t\delta\right)$ are monotone in t and do not converge, it is the case that $E\left[P\left(h_\tau, \varepsilon_\tau\right) - \alpha - \beta\varepsilon_\tau|h_\tau\right] \neq 0$ for any $\tau \neq 0$ and either $\delta \neq 0$ or $\Delta \neq 0$. (3) Assume that the price path $P\left(h_\tau, \varepsilon_\tau\right)$ has exhibited a steady degree of BPA since $t = \sigma$, δ_σ, and had an intercept term of α_σ at $t = \sigma$. At any time $t = \mu$, where $\sigma \leq \mu \leq \tau$, the price path can switch the degree of BPA to δ_μ and intercept term α_μ, provided that $P\left(h_\mu, \varepsilon_\mu\right) = P\left(\varepsilon_\mu; \alpha_\mu(\varepsilon_\mu), \delta_\mu\right)$. This path can be chosen such that $\Phi = E\left[P\left(\varepsilon_{\mu+1}; \alpha_\mu, \delta_\mu\right) - \alpha - \beta\varepsilon_{\mu+1}|\left\{h_\mu, \varepsilon_\mu\right\}\right] = 0$, which implies that, in expectation, the price path can switch to the steady-state price path at $t = \mu + 1$. This further implies that $E\left[P\left(\varepsilon_{\tau+1}; \alpha_\tau, \delta_\tau\right) - \alpha - \beta\varepsilon_{\tau+1}|\left\{h_\tau, \varepsilon_\tau\right\}\right] = 0$. The difference Φ is given by*

$$\Phi = A_\mu + B\delta_\mu + C\delta_\mu^2, \tag{C.1}$$

where

$$A_\mu = (1+r)\left((1+r)^{\mu-\sigma}\left(\alpha_\mu - \alpha\right) + \rho v\delta_\sigma\left(2J_{\sigma,\mu} + K_{\sigma,\mu}\delta_\sigma\right) + \left(\frac{1+r}{\lambda}\right)^{\mu-\sigma}\delta_\sigma\varepsilon_\mu\right), \tag{C.2}$$

$$B = 2\rho v\frac{(1+r)^2}{\lambda(1+r-\lambda)}, \text{ and} \tag{C.3}$$

$$C = \rho v\left(\frac{1+r}{\lambda}\right)^2, \tag{C.4}$$

as well as

$$J_{\sigma,\mu} = \frac{1-\lambda^{\mu-\sigma}}{1-\lambda}\left(\frac{1+r}{\lambda}\right)^{\mu-\sigma+1}\frac{\lambda}{1+r-\lambda} \text{ and} \tag{C.5}$$

$$K_{\sigma,\mu} = \frac{(1+r)^{\mu-\sigma}-\lambda^{2(\mu-\sigma)}}{1+r-\lambda^2}\frac{(1+r)^{\mu-\sigma+1}}{\lambda^{2(\mu-\sigma)}}. \tag{C.6}$$

From eqn. (C.1), it is obvious that, due to the risk-drift-premium, $\rho v\delta_\mu\left(2J_{\mu,\mu+1}+K_{\mu,\mu+1}\delta_\mu\right)$, the expected difference to the steady-state is quadratic in δ_μ. The coefficient of the quadratic term, C, is nonnegative ($\rho v\left(\frac{1+r}{\lambda}\right)^2 \geq 0$) such that the pricing function is U-shaped in the price association. Note that only A_μ depends on the time of the switch. Eqn. (C.1) further implies that, for any $B^2-4A_\mu C\geq 0$, there exist two solutions for δ_μ such that the expected difference to the steady state in $t=\mu+1$ equals zero. However, the condition $B^2-4A_\mu C\geq 0$ depends on the earnings realization in $t=\mu$, ε_μ. Specifically, assuming $\delta_\sigma>0$, a sufficiently positive earnings realization ε_μ increases $E\left[P\left(\varepsilon_{\mu+1};\alpha_\mu,\delta_\mu\right)\right]$ above $E\left[P\left(\varepsilon_{\mu+1};\alpha,\delta=0\right)\right]$ for any level of δ_μ. Therefore, for any price path, there exists a probability that at $t=\mu$ the economy can switch to a path such that $E\left[P\left(\varepsilon_{\mu+1};\alpha_\mu,\delta_\mu\right)\right]=E\left[P\left(\varepsilon_{\mu+1};\alpha,\delta=0\right)\right]$. Condition $B^2-4A_\tau C\geq 0$ can be rewritten as $\eta_\mu\leq D$, where

$$D\equiv\frac{\lambda^{\mu-\sigma}}{\delta_\sigma}\left(\frac{\rho v\left(1+r\right)^{\sigma-\mu+1}}{\left(1+r-\lambda\right)^2}-\left(\alpha_\mu-\alpha\right)\right)-\left(\frac{\lambda}{1+r}\right)^{\mu-\sigma}\rho v\left(2J_{\sigma,\mu}+K_{\sigma,\mu}\delta_\sigma\right)-\lambda\varepsilon_{\mu-1}. \tag{C.7}$$

Our refinement therefore requires that

$$\int_{-\infty}^{D} f\left(\eta_\mu\right)d\eta_\mu\geq\phi \;\text{ or }\; F\left(D\right)\geq\phi, \tag{C.8}$$

where $F\left(\eta_\mu\right)$ denotes the cumulative density function of the earnings surprise. (4) As the distribution for η is unbounded, there is always a nonzero probability that $\eta_\mu>D$.

2 ϕ-strategic Dominance

We term the next refinement that we investigate "ϕ-strategic dominance," which is consistent with a variety of refinements that have been used in the literature. Our notion of strategic dominance captures the following thought experiment for how an investor would behave when faced with strategic uncertainty (i.e., uncertainty over which equilibrium path is being played). Assume a generation τ investor observes $\{h_\tau,\varepsilon_\tau\}$ and price $\hat{P}\left(h_\tau,\varepsilon_\tau\right)=\bar{P}(h_\tau,\varepsilon_\tau)$, and that the investor faces strategic uncertainty regarding which pricing path, $\hat{P}\left(h_t,\varepsilon_t\right)$ or $\bar{P}\left(h_t,\varepsilon_t\right)$, will be played going forward. The investor's decision-making in this context is characterized by a two-stage process. In the first stage, the investor commits to one of the two equilibrium price paths. In the second stage, the investor chooses an optimal quantity given the beliefs committed to in the first stage. In other words, the investor will play a strategy consistent with either $\hat{P}(h_t, \varepsilon_t)$ or with $\bar{P}(h_t, \varepsilon_t)$. The second stage is a standard approach for characterizing behavior and merely requires that the investor maximize expected utility given beliefs about the equilibrium pricing path. The first stage is, however, is nonstandard and is a means for characterizing the beliefs an investor will adopt. We assume that the investor experiences regret if he plays the wrong equilibrium. Specifically, we assume that the investor weights his expected utility from playing the correct equilibrium with ϕ and his expected utility from playing the wrong equilibrium with $1-\phi$. Denote $E[\tilde{U}_t|q(h_t, \varepsilon_t), P(h_t, \varepsilon_t)]$ as a generation-t investor's expected utility given that he chooses quantity $q(h_t, \varepsilon_t)$; the price path is given by $P(h_t, \varepsilon_t)$, where $q(h_\tau, \varepsilon_\tau)$ maximizes a generation τ investor's expected utility given equilibrium price path $P(h_t, \varepsilon_t)$ and earnings history $\{h_\tau, \varepsilon_\tau\}$. Therefore the expected utility from choosing $\hat{q}(h_\tau, \varepsilon_\tau)$ or $\bar{q}(h_\tau, \varepsilon_\tau)$ is given by

$$E[\tilde{U}_\tau|\hat{q}(h_\tau, \varepsilon_\tau)] = \phi E[\tilde{U}_\tau|\hat{q}(h_\tau, \varepsilon_\tau), \hat{P}(h_t, \varepsilon_t)] + (1-\phi) E[\tilde{U}_\tau|\hat{q}(h_\tau, \varepsilon_\tau), \bar{P}(h_t, \varepsilon_t)] \tag{C.9}$$

and

$$E[\tilde{U}_\tau|\bar{q}(h_\tau, \varepsilon_\tau)] = \phi E[\tilde{U}_\tau|\bar{q}(h_\tau, \varepsilon_\tau), \bar{P}(h_t, \varepsilon_t)] + (1-\phi) E[\tilde{U}_\tau|\bar{q}(h_\tau, \varepsilon_\tau), \hat{P}(h_t, \varepsilon_t)]. \tag{C.10}$$

Given $\{h_\tau, \varepsilon_\tau\}$ and $\hat{P}(h_\tau, \varepsilon_\tau) = \bar{P}(h_\tau, \varepsilon_\tau)$, price path $\hat{P}(h_\tau, \varepsilon_\tau)$ dominates $\bar{P}(h_\tau, \varepsilon_\tau)$ under ϕ-strategic dominance when the weighted expected utility from $\hat{q}(h_\tau, \varepsilon_\tau)$ is larger that the one from $\bar{q}(h_\tau, \varepsilon_\tau)$, i.e., whenever

$$E[\tilde{U}_\tau|\hat{q}(h_\tau, \varepsilon_\tau)] \geq E[\tilde{U}_\tau|\bar{q}(h_\tau, \varepsilon_\tau)]. \tag{C.11}$$

Depending upon the choice of ϕ, the ϕ-strategic dominance refinement criteria can collapse to other seemingly reasonable refinement criteria. For example, if $\phi = 1$, any equilibrium price path the satisfies the refinement is one that maximizes each generation's ex ante expected utility, which is consistent with Harsanyi and Selten's (1988) notion of payoff dominance. At the other extreme, if $\phi = 0$, a generation-t investor would commit to the equilibrium price path that minimizes his "loss" from conjecturing the wrong equilibrium. This calibration captures the notion of risk dominance in Harsanyi and Selten (1988).

While the single period example is useful for conveying the intuition underlying ϕ-strategic dominance, it must be extended to accommodate our multi-period model.

Definition 3. *A piecewise-linear price path $\hat{P}(h_t, \varepsilon_t)$ is a ϕ-strategic dominant price path if, for any period τ and associated $\{h_\tau, \varepsilon_t\}$, there does not exist another piecewise-linear price path $\bar{P}(h_t, \varepsilon_t)$ such that $\bar{P}(h_\tau, \varepsilon_\tau) = \hat{P}(h_\tau, \varepsilon_\tau)$ and $E[\tilde{U}_\tau|\bar{q}(h_\tau, \varepsilon_\tau)] > E[\tilde{U}_\tau|\hat{q}(h_\tau, \varepsilon_\tau)]$.*

Observation 3 identifies properties of any ϕ-strategic dominant equilibrium pricing path.

Observation 4. *If $\phi > \frac{1}{2}$, there does not exist a ϕ-strategic dominant pricing path $P(h_t, \varepsilon_t)$ such that, for any h_t, $\delta(h_t)$ is finite. If $\phi < \frac{1}{2}$, any ϕ-strategic dominant pricing path has the property that, for any date t, $\{\alpha(h_t), \delta(h_t)\}$ eliminates all uncertainty at date $t-1$. If $\phi = \frac{1}{2}$, any piecewise-linear pricing path is ϕ-strategic dominant.*

Proof. *Since the investor observes the current price, P_t, it has to be the case that $\hat{P}(h_t, \varepsilon_t) = \bar{P}(h_t, \varepsilon_t)$. Following the proof of Proposition 1, without loss of generality, we can rewrite $\hat{P}(h_t, \varepsilon_t)$ and $\bar{P}(h_t, \varepsilon_t)$ as $P(\delta_1, \Delta_1)$ and $P(\delta_2, \Delta_2)$, respectively, and assume that both $P(\delta_1, \Delta_1)$ and $P(\delta_2, \Delta_2)$ are the result of a shift; in other words, $J_t = K_t = 0$.*

Denote q_i the investor's optimal quantity for price path $P(\delta_i, \Delta_i)$, choosing q_i yields an expected utility of

$$\frac{1}{\rho}\left(1 - \phi \exp[-\rho X_{i,i}] - (1-\phi)\exp[-\rho X_{i,j}]\right),$$

for $i, j \in \{1, 2\}$ and $i \neq j$. $X_{i,j}$ denotes the investor's certainty equivalent when choosing q_i after observing P_t while P_{t+1} is determined by the path $P(\delta_j, \Delta_j)$. This implies that

$$X_{i,j} = q_i\left(\alpha_{t+1,j} + \left(1 + \beta_{t+1,j}\right)\lambda\varepsilon_t\right) + (1+r)(w - q_i P_t) - q_i^2 \frac{\rho v}{2}\left(1 + \beta_{t+1,j}\right)^2. \tag{C.12}$$

The requirement that the investor first observes P_t implies that $\hat{P}(h_t, \varepsilon_t) = \bar{P}(h_t, \varepsilon_t)$ and that the investor's optimal demand q_i is the equilibrium demand, $q_i = 1$, irrespective of the path determining P_t. This simplifies (C.12) to

$$X_{i,j} = \frac{1}{2}\rho v\left(1 + \beta_{t+1,j}\right)^2 + (1+r)w. \tag{C.13}$$

An investor will commit to q_1 whenever

$$(1-2\phi)\exp[-\frac{\rho^2 v}{2}\left(1+\beta_{t+1,1}\right)^2] \geq (1-2\phi)\exp[-\frac{\rho^2 v}{2}\left(1+\beta_{t+1,2}\right)^2]. \tag{C.14}$$

This implies that when $\phi < \frac{1}{2}$ the investor will choose the lowest absolute ($1 + \beta_{t+1} = 0$) and when $\phi > \frac{1}{2}$ the investor would prefer $\delta(h_t) \to \infty$. For $\phi = \frac{1}{2}$, both paths yield the same expected utility.

Observation 4 implies that the equilibrium selection criterion requires an infinite earnings response (either positive or negative) when $\phi > \frac{1}{2}$. The reason investors prefer the infinite response is because they bear the most risk in such an equilibrium. At first pass, this observation might seem counterintuitive because investors are risk averse. Such knee-jerk intuition is misplaced, however, because the market clearing condition implies that investors must be compensated for taking on the risk associated with the last marginal share they acquire. Because the degree of compensation for risk required to get investors to take a marginally higher stake is increasing in the quantity of shares held, investors earn rents for taking on risk associated with all of the shares held except for the last marginal share. These rents reflect the investors' consumer surplus in our model. Due to the fact that market prices adjust to clear the market and those price adjustments more than compensate investors for the total risk assumed, investors earn more rents when they assume more risk in equilibrium. As a consequence, investors always prefer an equilibrium with greater price variability, which implies they prefer equilibria exhibiting the greatest degree of BPA.

In contrast to the case when $\phi > \frac{1}{2}$, when $\phi < \frac{1}{2}$ investors are highly averse to committing to the wrong equilibrium. This aversion dominates the rents from taking on risk, and investors want to minimize price variability. As a consequence, the degree of BPA or negative BPA is set in each period to offset all risk by ensuring that the dividend payment exactly offsets the exit price. In other words, the required rate of return is "guaranteed" each period via a fixed first moment price drift.

As an alternative refinement related to our notion of ϕ-strategic dominance, one might instead consider our thought experiment where a generation τ investor commits to a set of beliefs, $\hat{P}(h_\tau, \varepsilon_\tau)$ or $\bar{P}(h_\tau, \varepsilon_\tau)$, in a first stage with the caveat that the investor believes that $\hat{P}(h_t, \varepsilon_t)$ and $\bar{P}(h_t, \varepsilon_t)$ are played with probability $\kappa \in [0, 1]$ and $1 - \kappa$, respectively. A refinement structured around this thought experiment is as follows. Consider an equilibrium price path $\hat{P}(h_t, \varepsilon_t)$. $\hat{P}(h_t, \varepsilon_t)$ is a κ-strategic dominant equilibrium if, for any $\{h_\tau, \varepsilon_\tau\}$ and any other equilibrium price path, $\bar{P}(h_t, \varepsilon_t)$ such that $\bar{P}(h_\tau, \varepsilon_\tau) = \hat{P}(h_\tau, \varepsilon_\tau)$:

$$\begin{aligned} & \kappa E[\tilde{U}_\tau | \hat{q}(h_\tau, \varepsilon_\tau), \hat{P}(h_t, \varepsilon_t)] + (1-\kappa) E[\tilde{U}_\tau | \hat{q}(h_\tau, \varepsilon_\tau), \bar{P}(h_t, \varepsilon_t)] \\ \geq \quad & \kappa E[\tilde{U}_\tau | \bar{q}(h_\tau, \varepsilon_\tau), \hat{P}(h_t, \varepsilon_t)] + (1-\kappa) E[\tilde{U}_\tau | \bar{q}(h_\tau, \varepsilon_\tau), \bar{P}(h_t, \varepsilon_t)]. \end{aligned} \tag{C.15}$$

This alternative refinement does not rule out any of our piecewise-linear equilibria.

3 Local Stability

In models involving perfect competition, an equilibrium is locally stable if small perturbations from the equilibrium naturally give rise to forces in the marketplace that move the system back towards the equilibrium. Within the context of our model, we consider the effect of small perturbations in demand for all investors at time t and assess whether their demands would naturally shift back in response to the change in the equilibrium price induced by the perturbed demand. If investors' demand shifts in the opposite direction of the perturbed demand, the equilibrium is deemed to be stable in the sense that the investor demand response keeps the price from drifting away from the equilibrium price. If, on the other hand, the investors shift demand in the same direction as the perturbed demand, the equilibrium is unstable because their response causes the price to move further from the equilibrium price. Because investor demands at any time t are not directly influenced by the behavior of other investors and are decreasing in price at time t, it is relatively easy to establish that any piecewise-linear equilibrium is locally stable.

Assume that every investors' demand at time t is perturbed away from their optimal demand by χ so their demand is given by

$$q_{pt} = \frac{\alpha_{t+1} + \left(1 + \beta_{t+1}\right) \lambda \varepsilon_t - (1 + r) P_t}{\rho v \left(1 + \beta_{t+1}\right)^2} + \chi, \tag{C.16}$$

where the h_{t+1} corresponding to α_{t+1} and β_{t+1} is suppressed. Given these perturbed demands, the market clearing price would rise to

$$P_{pt} = \frac{\alpha_{t+1} + \left(1 + \beta_{t+1}\right) \lambda \varepsilon_t - (1 - \chi) \rho v \left(1 + \beta_{t+1}\right)^2}{(1 + r)}, \tag{C.17}$$

which exceeds the equilibrium price by $\frac{\chi \rho v \left(1+\beta_{t+1}\right)^2}{(1+r)}$. At the perturbed equilibrium price, the trader's optimal demand is

$$q_t = \frac{\alpha_{t+1} + \left(1 + \beta_{t+1}\right) \lambda \varepsilon_t - (1 + r) P_{pt}}{\rho v \left(1 + \beta_{t+1}\right)^2} = 1 - \chi. \tag{C.18}$$

Hence, given the change in price induced by the perturbation in their demands, the investors naturally alter their demands in the opposite direction of the perturbation. Accordingly, we have our final observation.

Observation 5. *All piecewise-linear equilibria are locally stable.*

Figure 1: Steady State and High Volatility