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Bundling With Customer Self-Selection: A Simple Approach to Bundling Low-Marginal-Cost Goods

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Abstract

With declining costs of distributing digital products comes renewed interest in strategies for pricing goods with low marginal costs. In this paper, we evaluate customized bundling, a pricing strategy that gives consumers the right to choose up to a quantity M of goods drawn from a larger pool of N different goods for a fixed price. We show that the complex mixed-bundle problem can be reduced to the customized-bundle problem under some commonly used assumptions. We also show that, for a monopoly seller of low marginal cost goods, this strategy outperforms individual selling (*M = 1*) and pure bundling (*M = N*) when goods have a positive marginal cost or when customers have heterogeneous preferences over goods. Comparative statics results also show that the optimal bundle size for customized bundling decreases in both heterogeneity of consumer preferences over different goods and marginal costs of production. We further explore how the customized-bundle solution is affected by factors such as the nature of distribution functions in which valuations are drawn, the correlations of values across goods, and the complementarity or substitutability among products. Altogether, our results suggest that customized bundling has a number of advantages—both in theory and practice—over other bundling strategies in many relevant settings.

Keywords

nformation goods, digital goods, pricing, bundling, self-selection, internet

Disciplines

Marketing | Other Economics | Sales and Merchandising

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Abstract

The reduction in distribution costs of digital products has renewed interest in strategies for pricing goods with low marginal costs. In this paper, we evaluate the concept of *customized bundling* in which consumers can choose up to a quantity M of goods drawn from a larger pool of N different goods (N>M) for a fixed price. We show that the complex mixed bundle problem can be reduced to the customized bundle problem under some commonly used assumptions. We also show that, for a monopoly seller of low marginal cost goods, this mechanism outperforms individual selling (M=1) and pure bundling (M=N) when goods have a positive (even small) marginal cost, when users face costs of evaluating goods in the bundle, or when customers have heterogeneous preferences over goods. Comparative statics results also show that the optimal bundle size for customized bundling decreases in both heterogeneity of consumer preferences over goods and marginal costs of production. Altogether, our results suggest that customized bundling provides an efficient and mathematically tractable pricing scheme for selling information and other low-marginal cost goods, especially when consumers have heterogeneous preferences over goods or are budget or attention constrained.

1. Introduction

The emergence of the Internet as a low-cost, mass distribution medium has renewed interest in pricing structures for information and other digital goods (Shapiro and Varian, 1998; Choi, Stahl and Whinston, 1998). One common setting, faced by publishers, software producers, music distributors, cable television operators and a wide variety of other firms, is when a provider of a large number of information goods seeks to sell to a group of heterogeneous consumers, who may place different values on the individual goods. While it is now possible to efficiently sell individual goods separately for even small payments (Metcalfe, 1996), firms may be able to generate greater profits by engaging in bundling, where large numbers of goods are sold as a unit.

In theory, for N goods firms could offer up to (2^N-1) possible bundles, each at a different price. However, this problem is known to be computationally intractable and difficult to solve in closed form except for small numbers of goods (Hanson and Martin, 1990). Recent work (Bakos and Brynjolfsson, 1999) has shown that when the marginal costs of goods are sufficiently low and customers share a common probability distribution for valuation of different goods, pure bundling (offering all goods for a fixed price) is optimal, greatly simplifying the bundling and pricing problem. However, less is known about situations where it may be optimal to bundle large numbers of goods, yet marginal costs and consumer preferences are such that pure bundling is inefficient. These types of situations arise when goods have a small but non-negligible marginal cost (e.g., digital video distribution on a congested network), when consumers value only a subset of all available goods (e.g., music, movies, engineering software modules, past news articles), or when consumers face costs of evaluating goods that increase in the number of goods offered in a bundle.

In this paper, we analyze a pricing approach which generalizes existing results on information pricing while preserving simplicity and analytical tractability, which we term *customized bundling*. A customized bundle is the right for a consumer to buy her choice of up to *M* goods from a larger set N, for a fixed price p.[1] Note that non-trivial customized bundling is only relevant when marginal cost for each good is small and the goods share similar cost structure otherwise, it will be more profitable to sell individual units. When consumer demand can be characterized in a specific way, which is consistent with the assumptions used in some previous work on information goods pricing, we show that the mixed bundling problem can be reduced to a simple problem of non-linear pricing. This allows the application of known results to solve otherwise very complicated bundling problems such as when individual sale can coexist with pure bundling, when more than one bundle will be offered, how marginal cost affects the optimal size of bundles, and the welfare effects of bundling. In addition, because customized bundling contains unit sale and pure bundling as extreme cases, our analytical results can be compared to and extend previous work on information goods bundling.

2. Previous Literature

The literature on bundling has a long history beginning with the observation by Stigler (1963) that bundling can increase sellers' profits when consumers' reservation prices for two goods are negatively correlated. In the two-goods case, offering both a two-good bundle as well as the individual items (mixed bundling) is typically optimal (Adams and Yellen, 1976; McAfee, McMillan and Whinston, 1989). This is because bundling reduces heterogeneity in consumer valuations, enabling a monopolist to better price discriminate (Schmalensee, 1984; Salinger, 1995), while still capturing residual demand through unit sale. While the insight that bundling reduces heterogeneity in valuations is quite general, other aspects of these solutions often do not generalize beyond the two-good case.

Other work has extended the bundling literature to consider multiple goods as well as multiple types. Spence (1980) generalized the principles of single product pricing problem to the case of several products using a linear programming formulation, and shows some cases where the

[1] Our definition is almost identical to "generalized bundling" studied by MacKie-Mason, Riveros and Gazzale (1999). This mechanism was also mentioned as a possible strategy for discriminating between customers with different willingness to pay (Bakos and Brynjolfsson, 1997; Shapiro and Varian, 1998), but this suggestion was not completely explored in previous work. The earliest reference to this approach of which we are aware is Chen (1997), who studied the properties of customized bundling using mixed integer programming, although variants of this scheme have been used in practice in software and music distribution for many years.

problem can be solved in closed form. Other tractable analytical solutions have been found for a variety of special cases such as linear utility (McAfee and McMillan, 1988) or when valuations across different consumers can be ordered in specific ways or satisfy certain separability conditions (Armstrong, 1996; Sibley and Srinagesh, 1997; Armstrong and Rochet, 1999). These papers have found additional general results such as the observation that it is usually optimal to leave some consumers unserved in order to extract more revenue from the other higher value consumers (Armstrong, 1996) and that it is sometimes optimal to induce a degree of 'bunching', so that consumers with different tastes are forced to choose the same bundle of products (Rochet and Chone, 1998). These papers provide a general structure for solving quite complicated bundling problems in closed form, although the complexity increases dramatically as more goods are considered, making it difficult to extend them to large numbers bundling problems.

However, when marginal costs are very low, it is often optimal to bundle all goods together (Bakos and Brynjolfsson, 1999) which leads to a dramatic simplification of the bundling problem. These "pure bundling" results are robust to any set of consumer preferences generated by a common distribution function for the value of each good across all consumers. However, when pure bundles are not optimal, such as when consumers are budget or attention constrained or marginal costs are significant, this mechanism provides little guidance since limiting the size of a bundle of this form creates substantial deadweight loss when customers are heterogeneous.[2]

There have been several studies that have considered large numbers bundling problems in specific contexts related to information goods pricing. These studies generally find that engaging in a form of mixed bundling where a certain large bundle is offered along side individual sale dominates either strategy alone (Chuang and Sirbu, 1999; Fishburn, Odlyzko and Siders, 1997). In addition, these studies introduce the idea that allowing customers to self-

[2] The insight behind this shortcoming is straightforward. Suppose there are a large number of consumers that have valuation for each of 10 goods drawn from the same distribution function. It is clear that if we offer a pure bundle, all consumers will obtain their most preferred goods, although not all will agree which ones they are. However if we are constrained such that we can only sell, say, a 5 good bundle, there are now 252 possible bundles that a consumer might want if they can only have 5 goods. If any single bundle among the 252 possible bundles is offered, as few as 1/252 of the customers will receive their highest valued goods, creating substantial deadweight loss. Only by offering every possible combination that consumers desire would this deadweight loss disappear.

select the goods in the bundle (rather than having them predesignated) can often improve outcomes while maintaining simplicity in the pricing mechanism (Chen, 1997; Chuang and Sirbu, 1999; Mackie-Mason, Riveros and Gazzale, 1999).

3. Model

3.1 Introduction

We will examine the optimal bundling and pricing problem for a monopolist that distributes N different goods to consumers. We are interested in examining the profitability of customized bundles for a monopolist, in which a consumer is allowed to choose up to M goods ($M \le N$) for a single price p. In general, a monopolist may want to offer more than one customized bundle when facing heterogeneous customers. For notational simplicity we will use $m \in [0, 1/N, 2/N, ..., 1]$ to represent a fraction of the total number of goods available and let $p(m)$ represent the price for a customized bundle of size $M = mN$. In addition, for a function $f(m)$ we define the notation $f'(m)$ as $f(m) - f(m - \frac{1}{N})$ for $m > \frac{1}{N}$ to be consistent with the discrete nature of *m*.

3.2. Multiproduct Nonlinear Pricing for Heterogeneous Consumers

We begin by defining a structure for the standard bundling problem in which customers demand at most one unit of each good. Consumers purchase a bundle of goods $\mathbf{x} = < x_1, .., x_j, .., x_N >$ (where the elements of $\mathbf{x}$ are binary variables, $x_j \in \{0,1\}$, $j = 1..N$ representing the purchase or non-purchase of a bundle component) over all **N** goods available. Consumers derive benefits from these goods which leads to a willingness to pay (WTP) $W(\mathbf{x})$, a weakly increasing function in all components of $\mathbf{x}$ with $W(\mathbf{0})=0$. In general there will be more than one set of consumer preferences over these goods. Let there be I distinct types of consumers indexed $i \in [1, 2, ..I]$ with a unique willingness to pay function $W^i(\mathbf{x})$. The proportion of each consumer type in the population is denoted by α^i (where $\sum_{i=1}^{I} \alpha^i = 1$). If the price of a set of goods is $p(\mathbf{x})$, we could write the net utility a consumer i obtains from consuming this bundle as: $U^i(\mathbf{x}, p(\mathbf{x})) = W^i(\mathbf{x}) - p(\mathbf{x})$.[3] We denote the cost of providing a vector of goods $\mathbf{x}$ as $C(\mathbf{x})$ which is weakly increasing in all components of $\mathbf{x}$. Using this notation, the general bundling problem the monopolist faces is the determination of the set of bundles offered $\{\mathbf{x}\}$ and a set of prices $p(\mathbf{x})$ solving the well known mixed bundle pricing problem with heterogeneous consumers (Spence, 1980):

$$\begin{aligned}
&\max \sum_{i=1}^{I} \alpha^i [p(\mathbf{x}^i) - C(\mathbf{x}^i)] \; s.t. \\
&\text{IR: } W^i(\mathbf{x}^i) - p(\mathbf{x}^i) \geq 0 \; \forall i \qquad\qquad (1)\\
&\text{IC: } W^i(\mathbf{x}^i) - p(\mathbf{x}^i) \geq W^i(\mathbf{x}^j) - p(\mathbf{x}^j) \; \forall i, j \neq i
\end{aligned}$$

The first set of constraints, individual rationality (IR), guarantees that if a consumer chooses to purchase a bundle, it provides non-negative surplus. In other words, the monopolist cannot force a consumer to purchase the bundle. The second set of constraints, incentive compatibility (IC), guarantees that a consumer segment receives at least as much surplus for purchasing the bundle intended for them than they would for choosing another bundle. Implicit in this assumption is that the monopolist cannot price discriminate by group – it must be in the consumer's self-interest to purchase their intended bundle. This formulation treats the problem as a direct revelation mechanism where consumers reveal their "type" through their choice of product, which will yield the profit maximizing solution for the monopolist, if such a solution exists (Myerson, 1979). Note from this formulation that for I consumer groups and N products, the monopolist must determine the optimal set of I bundle compositions and prices out of $2^N - 1$ possibilities.

3 3. Customized Bundling

Our initial interest is in determining the conditions under which the full bundling problem can be reduced to the much simpler customized bundling problem. Following the literature on information goods pricing, we will assume that the cost structure of providing goods to

[3] The assumptions on W guarantee that U obeys the normal properties of utility functions.

consumers depends only on the number and not on which goods provided, thus $C(\mathbf{x}) = C(m)$ where $m = \frac{1}{N}\mathbf{x}\bullet\mathbf{1}$ (where $\bullet$ denotes a vector dot product, and $\mathbf{1}$ is a vector of all 1's). We further assume that C is weakly increasing, with decreasing differences in m (that is, $C'(m) \geq 0$ and $C''(m) \leq 0$). Note also that $C(0) = 0$, consistent with an additional assumption that the monopolist has already sunk any fixed cost necessary to produce these goods. Given this cost structure, we now examine what types of consumer preferences (utility or willingness-to-pay) will yield an equivalence between the optimal bundling problem and the optimal customized bundling problem.

Before establishing these results, it is useful to introduce some additional notation. Let $w^i(m)$ represent the most a consumer of type i is willing to pay for mN goods (formally, $w^i(m) = \max_{\mathbf{x}} W^i(\mathbf{x}) \quad s.t. \sum_{k=1}^{N} x_k \leq mN$). Note that this implies that $w^i(0) = 0,\ w^i{}'(m) \geq 0$ and $w^i{}''(m) \leq 0\ \forall i$. Although there can exist as many as I such functions,[4] in general there will be less than I because different preferences $W(\mathbf{x})$ can yield the same expression for $w(m)$. We can now formulate customized bundling problem as:

$$\begin{aligned}
&\max \sum_{i=1}^{I} \alpha^i [p(m^i) - C(m^i)]\ s.t. \\
&\text{IR: } \mathrm{w}^i(m^i) - p(m^i) \geq 0\ \forall i \qquad (2) \\
&\text{IC: } \mathrm{w}^i(m^i) - p(m^i) \geq \mathrm{w}^i(m^j) - p(m^i)\ \forall i, j \neq i
\end{aligned}$$

This problem is the well-known non-linear quantity discrimination pricing problem with heterogeneous consumers (also known as second-degree price discrimination; see Tirole, 1988, p. 148-154). In addition to the mathematical formulation being identical, customized bundling is also intuitively similar to second-degree price discrimination as it accomplishes discrimination among different groups through customer self-selection from a menu of offerings. The key distinction is that the non-linear pricing problem generally refers to different quantities of an identical good, while customized bundling refers to heterogeneous goods with

[4] In the context of information goods, it is reasonable to assume that I, number of consumer types, is much smaller than N, i.e., number of information goods offered.

similar valuations. Note also that this problem is significantly simpler than the full mixed bundling problem (given in (1)) as it only requires a selection of a maximum of *I* prices from a total space of *N* possible customized bundles.

An interesting question to explore is when we can reduce the complex mixed bundling pricing problem (1) to the much simpler customized bundling pricing problem (2). Clearly, the two problems are equivalent if they yield the same optimal pricing solution. The conditions for this to hold are presented in Result 1 (all proofs are shown in the Appendix).

<u>Result 1</u>: The optimal bundle price schedule $\{\mathbf{x}^i, p(\mathbf{x}^i)\}\ \forall i$, is equivalent to a customized bundling solution $p(m)\ \forall\ m$ iff for every pair of optimal bundles $\mathbf{x}^i, \mathbf{x}^j$ where $\mathbf{x}^i \bullet \mathbf{1} = \mathbf{x}^j \bullet \mathbf{1}$, $w^i(m) = w^j(m)$

This result shows for every optimal bundling solution there is an equivalent customized bundling solution as long as the willingness to pay function over customized bundles is the same for all customers whose optimal regular bundle is the same size. While this condition seems both abstract and rather restrictive, a surprising number of assumptions on consumer preferences meet these conditions. Specifically, this will hold if all preferences over regular bundles ($W(\mathbf{x})$) have a common customized bundle representation (*w(m)*).

The simplest example of this condition is when heterogeneous preferences map to a single willingness to pay in customized bundles. For instance, if there are two consumer groups (i={1,2}) and three goods (N=3) and the marginal cost per good is 0.25, the following two sets of values for each of the three goods satisfy this property: {0.1,0.4,1} for consumer 1 and {1,0.4,0.1} for consumer 2. This yields a WTP over customized bundles for all consumers of {1.5, 1.4, 1,0} for m={1, 2/3, 1/3,0} respectively, and the monopolist optimally offers a 2-good customized bundle at a price of 1.4, yielding a profit of 0.9. Chuang and Sirbu (1999) and Fay and MacKie-Mason (2001) considered a more general form of this relationship, representing consumer WTP by a linear function over their rank ordered preferences that satisfies this condition. We examine in detail a minor generalization of their formulation in Section 3.5.

Interestingly, when there are a large number of consumers whose valuation is drawn randomly from the same distribution function, as is common in marketing choice models (e.g,, McFadden, 1974) and previous work on information good pricing (e.g., Bakos and Brynjolfsson, 1999), the resulting distribution of preferences over goods $W(\mathbf{x})$ yields a common distribution of preferences over customized bundles *w(m)*. As shown in Result 2, the equivalence between regular bundling and customized bundling holds in this setting quite generally, only requiring that valuations of goods can be described by a common cumulative distribution function where the expected absolute value of each good is finite, a common assumption in previous work.

<u>Result 2</u>: If each of a large number of individual consumer's willingness to pay for a vector of goods $\mathbf{x} \in [0,1]^N$ is given by a vector $\mathbf{v} \in R^N$ drawn independently from a common distribution function with cdf $F(\mathbf{v})$ with finite expected absolute value for all goods, there exists a willingness to pay function for customized bundles common across consumers. This function is given by $w(m) = E[\sum_{k=(1-m)N}^{N} X_{k:N}]$ where $X_{i:N}$ is the i$^{\text{th}}$ order statistic from $F(\mathbf{v})$

Result 2 shows that to calculate consumers' willingness to pay across customized bundles for random distributions, one need only calculate $w(m) = E[\sum_{k=(1-m)N}^{N} X_{k:N}]$. The expression inside the expectation, a linear combination of order statistics, is a special case of a general class of functions called L-estimates (see a survey in Rychlik, 1998). The fact will prove useful in later results we derive for random valuations.

3.4 General Solutions

We now consider the general solution to the generalized bundling problem (2). To solve this problem, it is common to impose some additional structure on the variation across consumers known as the Spence-Mirrlees single crossing property. This assumption enables closed form solutions and straightforward comparative statics results, and is commonly used in theoretical work on non-linear pricing. Let *a* and *b* represent bundle sizes (different values of m), let consumers be indexed by *i* and *j*, the single crossing property (SCP) holds if there exists an ordering of consumers such that:

$$w^i(a) > w^j(a)$$
$$w^i(a) - w^i(b) > w^j(a) - w^j(b) \quad \forall a > b, i > j \ .$$

This implies that a "higher type" consumer (meaning higher value of *i* in this condition) places a greater value on any given bundle than "lower type" consumers. For all subsequent discussion assume that customer types *i* are ordered to satisfy this condition. In addition, given any two bundles, there is a greater difference in valuation due to size for higher type consumers (or in other words, increasing differences in type and bundle size). While this appears to be restrictive, it is a common assumption in most models of this type, and simply rules out cases where the orderings of consumer value change as the bundle size changes. Moreover, it is necessary for any general characterization of the solution to this problem.

Let $\{m^{i*}, p^{i*}\}$ denote the optimal offering of the monopolist when there are multiple customer types, and $\{\hat{m}^i, \hat{p}^i\}$ represent the bundle that would be offered to consumer type (i) if there were no incentive compatibility constraints (that is, if they were the only consumer type being served). Using standard results and proof techniques from the theory of non-linear pricing we can show the following Result.

<u>Result 3</u>: A monopolist will offer a set of customized bundles that have the following properties:

a) The lowest-type customer that is served is priced at their willingness to pay: $p^{i*} = w^i(m^{i*})$

b) The prices for all other bundles are determined to satisfy IC, and leave all consumers except the lowest type with positive surplus (let $i_{\min}$ be the lowest type that is served):

$$p^{i*} = p^{i-1*} + w^i(m^{i*}) - w^i(m^{i-1*}) < w^i(m^{i*}) \quad \forall i > i_{\min}$$

c) The highest type customer is always served at the size they would have received if they were the only customer segment: $m^{I*} = \hat{m}^I$

d) All other customers receive bundles smaller than the bundle size they would have received if they were the only customer segment. These sizes are the greatest values of $m \in [0, 1/N, 2/N, ..., 1]$ that satisfy:

$$(\sum_{j=i}^{I}\alpha^{j})w^{i}{}'(m^{i*})-(\sum_{j=i+1}^{I}\alpha^{j})w^{i+1}{}'(m^{i*})\geq(\sum_{j=i}^{I}\alpha^{j})C'(m^{i*})\quad\forall i<I$$

e) There, in general, may be a customer segment such that all customers below that segment are not served (that is, $\exists i_{\min}>0\ s.t.\ m^{i*}=0\ for\ i<i_{\min}$)

f) The optimal size of the customized bundle is weakly decreasing in marginal cost

Result 3 replicates some of the well known results on non-linear pricing with a small modification to account for the discrete nature of *m*. First, there is generally one optimal bundle per type of consumer if that segment is served at all. Second, not all consumers are served with since under single crossing it is more profitable to extract additional surplus form the "higher" types than have them cannibalized by bundles targeted to the lower types. Third, only the highest type consumer is served at their socially optimal bundle size, the rest being weakly lower to discourage high types consumers from consuming bundles targeted at the lower types. Fourth, because the monopolist cannot perfectly price discriminate, all consumers except the lowest types that are served earn some surplus, an information rent due to their hidden type. Finally, a more subtle observation is that without extremely restrictive assumptions on cost and preferences, prices will not be linear in bundle size (with or without a "fixed fee" component) suggesting that in general the optimal solution will outperform a two-part tariff (a formal proof of this is available from the authors). All of these are a direct consequence of customized bundling being a well-behaved nonlinear pricing problem under our assumptions.

There are also some additional insights these results bring that are unique to the customized bundling problem. First, if cost and willingness to pay are known for each customer segment (whether it is deterministic or the expectation of a random valuation), it is a simple calculation of complexity O(I) to determine the optimal price and bundle sizes that will be offered. This contrasts with the intractable mixed bundling problem of a large number of goods. Second, in this formulation Result 3f provides a simple but powerful result on the relationship between customized bundling, single good selling and pure bundling – as the marginal cost per good increases, there is a monotonic shift between in optimal bundling policy from pure bundling to customized bundling to unit sale.

Customized bundling becomes increasingly desirable relative to pure bundling when consumers bear additional marginal costs of consuming larger bundles, for instance, when consumer attention is scarce and larger bundles require greater consumer attention to evaluate and identify their preferred goods. Let $Z^i(m)$ represent the cost a consumer of type i faces in evaluating a bundle of size m. Assume that $Z^i(m)$ is positive and weakly increasing in m with weakly increasing differences ($Z^i{}'(m) \geq 0$ and $Z^i{}''(m) \geq 0$). We find that under some mild assumptions, the optimal bundle sizes are further reduced, making it more likely that customized bundling dominates pure bundling as shown in Corollary 1 below:

<u>Corollary 1</u>: Under the assumptions above, if consumers face an additional (private) cost of evaluating a bundle then bundle size is weakly decreased and prices are strictly decreased compared to the situation of no evaluation costs if $Z^i{}'(m) \leq Z^{i+1}{}'(m) < (1 + \frac{\alpha^i}{\sum_{j=i+1}^{I} \alpha^j}) Z^i{}'(m) \; \forall i < I$. Moreover, this condition will always be satisfied if $Z^i(m) = Z(m) \; \forall i$.

In other words, as long as evaluation costs are enough to matter in the optimal solution, and are the same across consumers or at least do not increase too fast in consumer type, evaluation costs will tend to yield a smaller optimal customized bundle (the weak inequality due to the discrete nature of *m* requiring a certain level of evaluation cost before bundle size is affected).

Overall, we are able to use our formulation and slight modifications of standard results to derive some interesting insights into large numbers bundling problems regarding the number of optimal bundles, the relationship between pure bundling and customized bundling, the response to marginal cost changes, welfare effects, and the tractability of the pricing algorithm. However, these general results do not say much how customized bundling is affected by the nature of consumer preferences over different goods, the existence of attention limits (a maximum number of goods desired), or budget constraints. In the next two sections, we make some specific assumptions about cost and preferences to enable us to study these relationships.

3.5. Bundling Under a Two-Parameter Preference Function

For this section, we build on results by Chuang and Sirbu (1999) (CS) by considering a problem where different consumers can be described by a willingness to pay function that depends on two parameters: an overall budget constraint or total willingness to pay (*b*) and the number of goods they value positively (*K*). We denote *k* as the fraction of goods that consumers' value positively (that is $k = \frac{K}{N}$). Consumers are assumed to have similar utility functions over a rank ordering of goods, which in the CS model is assumed to be linear in the rank order of their preferences. We generalize this case to allow other relationships by assuming $w(m) = b \bullet y(\frac{m}{k})$, where *y(.)* captures customers' relative valuations, or degree of preference, for different goods.[5] Therefore we can write:

$$u(m, p_m) = \begin{cases} by(\frac{m}{k}) - p_m & if \quad m \le k \\ b - p_m & if \quad m > k \end{cases} \tag{3}$$

where y is such that $y(0) = 0, y(1) = 1, y' > 0, y'' \le 0$ over the domain $[0,1]$ (recall that this is consistent with $w'(m) \ge 0$ and $w''(m) \le 0$ given our definition of *w(m)* and that *y'* refers to a difference, not a derivative, to account for the discrete nature of *m*).

For any common function *y(·)* and any set of parameters $\{b^i, k^i, \alpha^i\}$ characterizing consumer preferences that satisfies single crossing we can directly apply Result 3 to obtain the optimal bundling solution.

To make this analysis concrete, however, we focus on a single consumer type and make some functional form assumptions for preferences and costs (the multi-type case is also easily solved under SCP, but adds little insight over the general single type case). Specifically, assume that

[5] One can think *y(t)* as the proportion or fraction of total budget that a customer is willing to spend on the top *t* percent of the goods she positively values. Intuitively, y is an increasing and concave function of t.

there is a constant marginal cost (c) for all goods, $C(m) = cmN$, and that relative valuation (y) across goods can be described by a quadratic function with a parameter (a) capturing customers' preferences across different information goods. The quadratic formulation is, perhaps, the simplest functional form assumption that enables us to examine the differences between relatively uniform preferences over goods and preferences skewed toward a few high value goods. Moreover, given the properties of the y function ($y(0) = 0, y(1) = 1, y' > 0, y'' \leq 0$), quadratic functions provide a reasonably good local (and often global) approximation to arbitrary functional forms for $y(\cdot)$ while maintaining computational tractability. Thus we have:

$$y(t) = (1+a)t - at^2 \quad where\ t = \frac{m}{k} \in [0,1]\ and\ a \in [0,1] \tag{4}$$

By varying a, we can examine different conditions of valuation for a given consumer or representative consumer across different goods. If $a=1$ then we have the CS assumptions (linearly decreasing value in rank order). If $a=0$, the consumer values all goods equally.

Under this formulation we can compare the efficiency (ability to maximize social welfare) as well as the profitability of different bundling schemes. In this example, pure bundling is a trivial solution as long as the pure bundle is profitable (that is, $b - C(1) > 0$). The monopolist sets price to total value ($p=b$) and extracts all surplus, although not at minimum cost when $k < 1$ so it is not efficient (the monopolist incurs marginal cost to offer goods that are not consumed). The optimal price per good for individual sale (P_{IS}) is found by maximizing profits subject to a constraint that the marginal utility of customers gained by purchasing additional units of the good is equated with the prices paid:

$$P_{IS} = \arg\max_P PmN - C(m) \quad s.t. \quad w'(m) = PN$$

The optimal customized bundling solution is just a special case of Result 3 where there is only one type of customer. Because there is only one type, we can ignore incentive compatibility and focus entirely on individual rationality. Thus the price for a customized bundle (p_{CB}) is given by:

$$p_{CB} = \arg\max_p \; p - C(m) \quad s.t. \quad w(m) - p \geq 0$$

Note that the constraint is always binding at the optimum, so we can rewrite the objective function as:

$$m_{CB} = \arg\max_m w(m) - cNm$$

This equation is identical to the problem of maximizing social welfare and, thus, there is no deadweight loss, so the customized bundling solution is efficient.

We summarize the solutions and results to individual sale (m_{IS}, π_{IS}), pure bundling (m_{PB}, π_{PB}) and customized bundling (m_{CB}, π_{CB}) in the following two graphs (detailed derivations appear in the Appendix). For ease of comparison, we define $\overline{w} \equiv \dfrac{b}{kN}$ to be the average willingness to pay for the goods that have positive values. Figures 1a and 1b characterizes profit and bundle size for various regions of marginal cost per good (c) and customer preference parameter over goods (a). Note that since customized bundling contains both pure bundling and individual sale as extreme cases, it will always weakly dominate. However, the degree of difference depends on marginal cost and the dispersion of values across goods. As shown in Figure 1a, only when marginal cost is zero is customized bundling and pure bundling equivalent in profits. This continues up until marginal costs are equal to $k\overline{w}$ when pure bundling is no longer feasible while customized bundling is still profitable. Finally at $\overline{w}(1+a)$ customized bundling is no longer feasible. Altogether, these results suggest that the profitable region of customized bundling expands as a increases (i.e., when there is increasing difference in good valuations).

Figure 1a: Pure bundling vs. customized bundling for different marginal cost and customer preference parameter

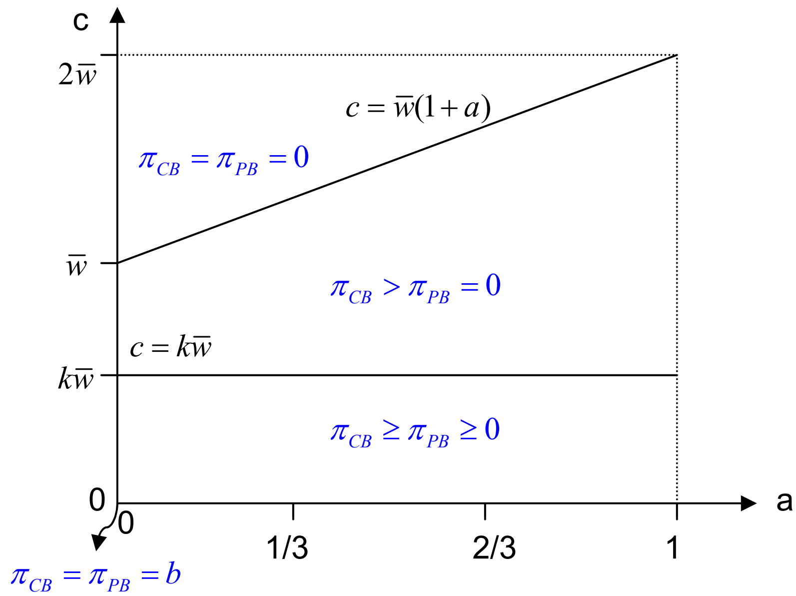
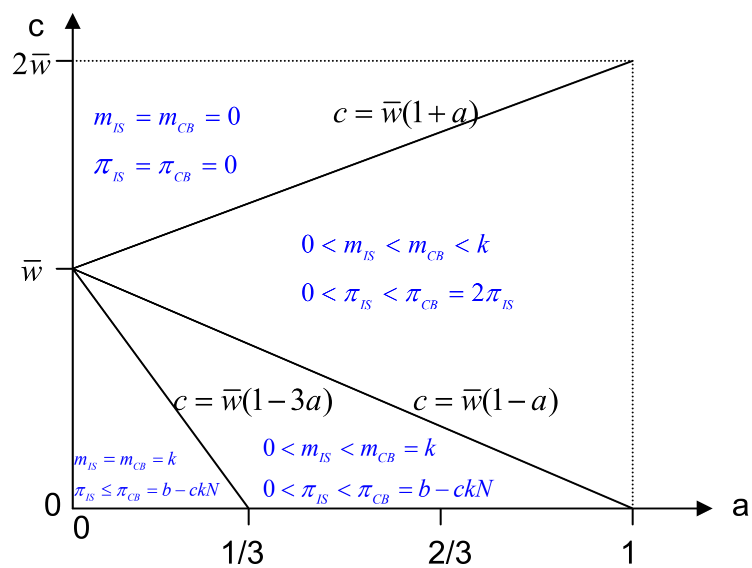


Figure 1b: Individual selling vs. customized bundling for different marginal cost and customer preference parameter



In Figure 1b, at very low marginal cost and low dispersion of valuation across goods when $C'=c<\overline{w}(1-3a)$, individual selling and customized bundling are equivalent in number of goods sold, which is the efficient solution. They also achieve the same profit level when a=0. But as a departs from zero but is smaller than $\overline{w}(1-3a)$, individual selling is efficient but not profit maximizing. Note also that market demand (goods that are positively valued) can be fully satisfied using customized bundling for *a* values three times as large as the case of individual selling for the same level of marginal cost. Finally, as marginal costs increase, the size of customized bundle decreases until marginal cost is so high that bundling is infeasible.

The key insight from this analysis is found from an examination of the "normal" case with non-zero marginal costs and consumers placing different values over different goods (a>0). In this case, customized bundling dominates the alternative approaches, even when there is only a single customer segment (except in the boundary cases where they are equivalent, $m = k$ [6] for individual sale, and $m=1$ for pure bundling). In other words, *as marginal costs (c) increase or customer heterogeneity across goods (a) becomes larger, it is increasingly attractive to consider customized bundling over the alternatives of pure bundling and individual sale.* While pure bundling is only feasible when average cost drops below average budget ($c \leq \frac{b}{N} = k\overline{w}$), customized expands the range of feasible bundling to include $k\overline{w} \leq c \leq \overline{w}(1+a)$, and customized bundling is strictly better whenever there is heterogeneity in valuations (a>0) or consumers do not value all goods (0<k<1). These analyses are illustrated in Figures 2a-2c (see remaining figures at the end) showing the profitability of the various approaches for varying levels of the parameters (c,a,b,k).

All of these results hold for a single group of customers. The contrast will only increase if we allow multiple customer segments since customized bundling can offer tailored bundles to each segment, a strategy not possible with individual sale or pure bundling without some segmentation mechanism.

3.6. Customized Bundling Under Random Valuation

[6] m represents total number of goods sold to each customer in the case of individual selling.

We earlier showed that simple customized bundling solutions may exist when consumers have valuations for multiple goods drawn from a single valuation distribution.[7] Since a number of previous papers in information goods bundling have used such distributional assumptions (especially Bakos and Brynjolfsson, 1999, referred to hereafter as BB), it may be useful to compare the customized bundling solution to the pure bundling alternative under random valuation drawn from a certain distribution. Unlike BB who considered the effects of changing the number of goods in the population, we will consider a simplified structure compared to BB in which N is fixed at the largest possible number of goods. It should be noted that once N is fixed, pure bundling is a special case of customized bundling. We will retain the BB assumptions of identically distributed v_i (the value of the i^{th} good) and their assumption of constant marginal cost per good (which may be zero). In addition, all distributions considered here are assumed to meet the conditions described in Result 2 (essentially finite expected absolute value). For the following results, it is useful to define the "Quantile" or "Inverse Distribution Function" of a distribution function *F(t)* as $Q_F(z) = \sup(t : F(t) \le z)$.

The most general result we can show for arbitrary distribution functions, including those where valuation is dependent, is that customized bundling value is bounded below by mean valuation, and above by an integral expression involving the quantile function:

<u>Result 4</u>: If the valuation for any individual good is drawn from a common but possibly dependent distribution *F(·)* with finite mean (μ) then $\int_{1-m}^{1} Q_F(z)dz \ge w(m) \ge mN\mu$

There are two interesting insights from Result 4. First, the upper bound can sometimes serve as a reasonable approximation for the value of customized bundling, as it can be interpreted as an approximate average value of the portion of the distribution that exceeds the m^{th} percentile. Simulation results on many common distributions (uniform, normal, logistic and exponential) suggest this approximation is good when the valuation of different goods is not too dependent. Second, the result shows that the expected value of a customized bundle always (weakly)

[7] This can be extended to multiple distributions as long as they generate willingness to pay functions that obey the single crossing condition. However, this formulation yields few incremental insights beyond the single type case discussed here and the general formulation in Result 3.

exceeds the mean value of the same number of goods, implying that average valuation per good of customized bundles under most circumstances exceeds the average value of per good of pure bundles. The strict lower bound only holds when *m=1* or valuations of goods are perfectly correlated.

With addition distributional assumptions we can apply the theory of L-estimates to obtain a number of additional general results. The most straightforward exact results can be obtained when we further assume independence of good valuations, that is $F(\mathbf{v})=\prod_{i=1}^{N}F(v_i)$. This assumption yields an explicit expression for $w(m)$.

Result 5: If the valuation of individual goods is independently and identically distributed with quantile function $Q_F(z)$ then $w(m)=\int_0^1 Q_F(z)\sum_{i=mN}^{N}N_{i:N}(z)dz$ where $N_{i:N}(z)=N\binom{N-1}{i-1}z^{i-1}(1-z)^{N-i}$ (the Bernstein Polynomials).

This expression can be used to numerically calculate the values of the consumer willingness to pay for arbitrary distribution functions and may be solvable in closed form for some distributions such as the uniform and the exponential. It can also be applied to make comparative statics predictions regarding distribution parameters such as mean or variance and the size of the optimal customized bundle.

Because the customized bundling profit function ($\pi(m)=w(m)-C(m)$) has a direct relationship with the quantile function, a number of useful results can be obtained restricting our attention to distributions in the location-scale family, which includes most common distributions assumed in prior work such as the exponential, normal and uniform. Location-scale distributions are such that for a distribution with two parameters (a,b) known, we can write the quantile function as $Q_F(z;a,b)=a+bQ_F(z;0,1)$ where a is referred to as the location and b as the scale. Using this definition and the usual assumption of a constant marginal cost for each good (c), it is now possible to derive a relationship between location (proportional to the mean) and scale (proportional to variance) for an arbitrary i.i.d. distribution in this family and the optimal customized bundle size (m^*).

Result 6: Let the valuation for any individual good be drawn i.i.d. from a distribution $F(x)$ with mean (μ), in the location-scale family with location a and scale b. Then:

a) m^* is weakly increasing in a.

b) For general distributions m^* is weakly increasing in b if $E[X_{\underline{M}+1:N}] < c$ where $\underline{M}$ is the lowest order statistic of the standard distribution for $F(x)$ (a=0, b=1) with non-negative expected value. m^* is weakly decreasing in b if $E[X_{\underline{M}-1:N}] > c$.

c) Profits are always increasing in m^* at optimum

For any fixed marginal cost, an increase in location simply shifts the valuation curve outward in marginal value-size space, increasing optimal bundle size (unless the optimal bundle is already the pure bundle). The intuition behind the scale is somewhat more complex. Note that as scale (variance) increases, the distribution spreads out. The highest order statistics become larger and the lowest order statistics become lower. If the optimum lies in a region where the order statistics are increasing in variance (i.e., when $c > E[X_{\underline{M}+1:N}]$, that is, when c is relatively large and the optimal bundle only includes the very highest valued goods), then increasing scale raises the size of the bundle. If the optimum lies in a region where they are decreasing in scale (that is, when c is relatively small), the optimal bundle size is decreasing in scale. The conditions in 6b guarantee the location of this optimum and that this optimum doesn't move to the opposite side of the curve as scale is changed.

This result shows an interesting relationship between the pure bundling and customized bundling. When it is feasible to have a pure bundling solution (the average value greater than marginal cost), then greater variance will decrease the performance of pure bundling relative to customized bundling because it means that the lowest valued goods in the bundle become even worse with increasing variance and thus will make customized bundling more attractive. This provides an additional reason why greater ex-ante uncertainty about consumer valuations makes pure bundling less undesirable. The BB explanation is that it slows convergence of valuation to the mean in finite samples which leaves consumers with more surplus. However, we add an additional explanation that the goods being bundled on the margin under higher variance get increasingly worse. We also show that increasing variance will decrease the size of customized bundle when marginal cost is relatively small. Interestingly, however, in the region where pure bundling is infeasible but there are still feasible customized bundles ($\mu < c < E[X_{N:N}]$) variance actually leads to larger customized bundles and greater bundling profits in contrast to the BB results for low marginal costs.

If we are willing to make stronger distribution assumptions, it is possible to relax the independence assumption slightly. While in general it is difficult to calculate order statistics from non-independent distributions, simple expressions exist for the multivariate normal with common correlation (ρ). These results are given in Result 7, using the same notation introduced in Result 6b:

<u>Result 7</u>: The optimal bundle size and total bundle profits are decreasing in the correlation among goods when $E[X_{\underline{M}+1:N}] < c$, and increasing in correlation when $E[X_{\underline{M}-1:N}] > c$ where $\underline{M}$ is the median.

This Result indicates that negative correlation acts similarly to variance, with negative correlations raising the value of the highest valued goods, but also decreasing the value of the lower valued goods. In the region where pure bundling is efficient we have another contrasting result with BB – while negative correlation improves price discrimination ability of the monopolist, it is offset by the fact that the lowest value goods are worth even less, reducing the price discrimination gains of pure bundling and favoring customized bundles.

These analyses can be straightforwardly expanded to multiple consumer types provided that the implied customized bundle valuations satisfy single crossing using Result 3 (or can be solved by mixed integer programming when that is not true). However, this analysis does not yield any additional insights beyond the individual contribution of Result 3 and the single-type results in this section.

4. Summary and Conclusion

We have analyzed an alternative bundling mechanism for low marginal cost goods that allows a consumer to choose up to M of their preferred goods from a larger set N for a fixed price *p*. Comparing to traditional second-degree price discrimination, in which firms try to solve p(**x**) over 2^{N-1} possibilities, customized bundling is much simpler in implementation when consumer preferences are such that the optimal full bundling solution has an equivalent customized bundling representation. We further show that these requirements are satisfied by assumptions used in prior bundling work, especially when consumer valuations are drawn from an identical valuation distribution across goods. Customized bundling yields a natural form of price discrimination for heterogeneous consumers by offering a price-bundle size schedule that includes individual sale and pure bundling as special cases.

Since customized bundling is simply an application of a well-behaved non-linear pricing problem, it retains the properties of these problems including one type of offering per customer segment, the "no distortion at the top" result that the highest valuation consumers receive their optimal bundle, information rents to all but the lowest type consumers, and the possibility that some of the lowest types are not served. In addition, customized bundles become optimal when marginal costs are non-zero but not so large that the solution is individual selling. We also demonstrate using specific representations of consumer preferences that customized bundling can be advantageous when consumer preferences are concentrated on a few goods, consumers have limited attention for evaluating goods in a bundle, or they have budget constraints (either attention or financial). In addition, for the case when consumer valuations are generated by common distributions we also show that uncertainty about consumers valuations (variance) makes customized smaller bundles more attractive when marginal costs are low, but the optimal customized bundle size increases in variance when marginal costs are high (in contrast to Bakos and Brynjolfsson, 1999). However, unless marginal costs are zero and customers value all goods, customized bundling will strictly dominate pure bundling.

The advantages of customized bundling are likely to be especially relevant when monopolists are selling large numbers of high value goods (e.g., movies) since it is very likely that budget constraints are binding and distribution costs are significant, at least using current technologies or when customers have heterogeneous valuations over different goods and do not positively value all goods. Other markets that have similar characteristics are high-quality digital music or modular packaged software (Office suites, e-commerce platform software, or the SAP R/3 system, for example). For smaller numbers or lower value goods, it is likely that pure bundling will prevail, at least if consumers are homogeneous in the ways considered in our model and those of our predecessors. However, even in these cases, customer heterogeneity is likely to create opportunities for offering different size of customized bundles, as it provides a greater ability to target different segments than individual pricing or pure bundling, which must rely on third-degree price discrimination to deal with residual customer differences.

Given these advantages while maintaining a relatively simple pricing structure, it is somewhat surprising that these mechanisms are not already widespread. However, there is increasing evidence that these types of mechanisms are being implemented or could be favourably employed. In a field experiment MacKie-Mason, Riveros and Gazzale (1999) found that librarians shifted toward purchasing journals through customized bundling (or "generalized bundling" in their terminology) when this option was offered along with other more traditional pricing schemes. Their consumption of customized bundles increased over time relative to other pricing approaches, even when it was likely that preferences over journal articles was largely unchanged. We have also identified a number of other examples used in practice. Many firms offering engineering drawing software – a moderately expensive and modular software package – offer modules in customized bundles (e.g., purchase a "10-pack" for $2000 or a 5-pack for $1250 where the consumer chooses from all available modules). There is also the well known "10 CDs for a $1" promotions by firms such as Columbia House, which represent the purchase of a customized bundle of around 14 CDs for approximately $75 (once contractual requirements are met). At least one online movie rental club (netflix.com) currently uses a customized bundling scheme – their pricing scheme allows users to choose different plans that enable them to simultaneously borrow N videos for p(N) dollars per month where multiple values of N are allowed (currently 2, 3,4, 5, and 8). The New York Times has experimented with a bundle pricing scheme for access to their article archives with four pricing options, with which the articles purchased are chosen by the consumer: 25 articles for $25.95, 10 articles for $15.95, four articles for $7.95 or a single article for $2.95.[8] This suggests that firms are exploring the use of these mechanisms, although it is likely that many of the domains in which this is most relevant have not yet seen substantial experimentation in pricing structure.

While we have specifically focused our attention to information goods, customized bundling technique is not unique to information goods, and it is especially attractive when number of goods offered is large but marginal cost and valuations are such that it is optimal for consumers to purchase more than one good. For example, some restaurants have traditionally "bundled" a selection of two or three side dishes with a meal where consumers choose from a specified set. More recently, McDonalds changed their menu from traditional second-degree price discrimination with fixed bundles ("value meals" with a sandwich, french fries and drink) to an arrangement where customers choose two side dishes form a list of items.

In addition to the potential for practical use, our customized bundling analysis provides another simplification to the general problem of mixed bundling that may be appropriate in some circumstances. Given the complexity of the general problem, there has been tremendous interest in the marketing, management, computer science, and economics communities for approaches that yield tractable analytic bundling solutions.

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[8] http://www.nytimes.com/premiumproducts/archive.html.

Appendix (Proofs):

Proof of Result 1

(sufficiency). Starting the program described by (1), we show that it can be converted to the problem stated in (2). Consider any collection of consumers {i} where the optimal solution to (1) is $\{\mathbf{x}^i\}$ s.t. $\mathbf{x}^i \cdot \mathbf{1} = mN$ for all members of this set. For every other value of m for this set of consumers, there must exist some $\mathbf{x}$, with $\mathbf{x} \bullet \mathbf{1} = mN$ such that $W^i(\mathbf{x})$ is maximized, thus $w^i(m)$ and $W^i(\mathbf{x}) = w^i(m)$ for this set of $\mathbf{x}$. By the sufficient condition, all consumers i in this set must be such that $w^i(m) = w(m)$. We now establish an equivalence $p(\mathbf{x}^i) = p(m)$ for all consumers in this set. If there is only one such element, there is a 1-1 mapping between $\mathbf{x}^i$ and m^i and thus $p(\mathbf{x}^i) = p(m^i)$ for that value of *m*. If there are two or more, then choose any two arbitrarily, labeling them $\mathbf{x}^k$ and $\mathbf{x}^l$. For both of these bundles, either IR or IC must bind, otherwise $p(\bullet)$ is unbounded. Changing variables in the willingness to pay functions from **x** to *m* and applying the assumed condition we can rewrite the constraints as: $w(m) - p(\mathbf{x}^k) \geq 0$, $w(m) - p(\mathbf{x}^l) \geq 0$ and $w(m) - p(\mathbf{x}^k) \geq w(\mathbf{x}^j \cdot \mathbf{1}/N) - p(\mathbf{x}^j)\ \forall j$ and $w(m) - p(\mathbf{x}^l) \geq w(\mathbf{x}^j \cdot \mathbf{1}/N) - p(\mathbf{x}^j)\ \forall j$. There are 4 possible cases, both IR are binding, both IC, and two cases where 1 IR and 1 IC binds. If IR is binding for both then it immediately follows that: $p(\mathbf{x}^j) = p(\mathbf{x}^l) = p(m)$. If both IC bind, then it must bind at some value j for which $w(\mathbf{x}^j \cdot \mathbf{1}/N) - p(\mathbf{x}^j)$ is maximized. This value (call it Q) does not depend on *k* or *l*. Thus if both IC constraints hold, $w(m) - p(\mathbf{x}^l) = w(m) - p(\mathbf{x}^k) = Q$ or $p(\mathbf{x}^j) = p(\mathbf{x}^l) = p(m)$. The case where 1 IC binds and 1 IR binds cannot be an optimum since it implies that *Q* must be both greater than and less than zero. This establishes that $\forall i, \mathbf{x}^i \cdot \mathbf{1} = mN$, $p(\mathbf{x}^i) = p(m)$. Substituting for $w(m^i), C(m^i), p(m^i)$ for $C(\mathbf{x}^i), p(\mathbf{x}^i)$ and deleting redundant constraints we obtain the customized bundling problem shown in (2)

(necessity). Given an optimal customized bundling schedule $p(m)\,\forall m$, suppose that there exists two bundles $\mathbf{x}^k$ and $\mathbf{x}^l$ for some m where $\mathbf{x}^i \cdot \mathbf{1} = mN$ where $p(\mathbf{x}^j) > p(\mathbf{x}^l)$. From the same type of argument shown above, if at least 1 constraint (IR, IC) must be binding for each, this implies that $W^i(\mathbf{x}^j) > W^i(\mathbf{x}^l)$ which contradicts the condition $W^k(\mathbf{x}^k) = W^l(\mathbf{x}^l) = w(m)$.

Proof of Result 2: (by construction) Proof (by construction). Let the i[th] order statistic of the valuation of N goods given by $F(\mathbf{v})$ be denoted as $X_{i:N}$ (where the largest order statistic is given by $X_{N:N}$). The random willingness to pay for any consumer is $\tilde{w}(m) = \sum_{k=mN}^{N} X_{k:N}$. This function does not depend on the consumer examined (equivalence). For any given distribution there exists a distribution for each order statistic and summation is a Borel measurable function. Therefore, there exists a cdf for $w(m)$ for each *m* (existence). Finally, we know $E[|\tilde{w}(m)|] < \infty$ since $E[|\tilde{w}(m)|] = E[\sum_{i=mN}^{N} |X_{i:N}|] \leq E[\sum_{i=1}^{N} |X_{i:N}|] < \infty$. Across each consumer, $\tilde{w}(m)$ is independent and identically distributed with finite mean (since $E[\tilde{w}(m)] \leq E[|\tilde{w}(m)|] < \infty$). Therefore, letting Q represent the number of customers, average valuation $\frac{1}{Q}\sum_{i=1}^{Q} \tilde{w}(m) \to E[\tilde{w}(m)]$ as $Q \to \infty$ by the Strong Law of Large Numbers.

Proof of Result 3:
The maximization program is given by:

$$\text{Max} \sum_{i=1}^{n} \alpha^i \cdot \left[P^i - C(m^i) \right]$$

$$\text{s.t.} \quad \begin{aligned} & IR: w^i(m^i) \geq p^i \;\forall i \\ & DIC: w^i(m^i) - p^i \geq w^j(m^i) - p^i \quad for \quad i = 2..I, and \quad \forall j < i \\ & UIC: w^i(m^i) - p^i \geq w^i(m^j) - p^j \quad for \quad i = 1..I, and \quad \forall i < j < I \\ & m^i \in [0, 1/N, 2/N, ..., 1] \;\forall i \end{aligned}$$

Assume initially that all types can be profitably served in this market by an ordering of bundles $m^{1*} \leq m^{2*} \leq \cdots \leq m^{I*}$). We can show that only adjacent DIC constraints can bind. Define the adjacent incentive compatibility constraints to be

$$w^i(m^i) - p^i \geq w^i(m^{i-1}) - p^{i-1} \quad \forall i \geq 2$$

Now we will show that if the adjacent incentive compatibility constraints hold, then all the incentive compatibility constraints hold. Consider any j<i-1;

$$w^i(m^i) - w^i(m^j) = \sum_{k=j}^{i-1} [w^i(m^{k+1}) - w^i(m^k)] \geq \sum_{k=j}^{i-1} [p^{k+1} - p^k] = p^i - p^j$$

or equivalently, $w^i(m^i) - p^i \geq w^i(m^j) - p^j$

As a result, DIC can be replaced by the adjacent incentive compatibility constraints.
After these simplifications, the original problem is equivalent to the following problem, ignoring UIC:

$$\text{Max} \sum_{i=1}^{n} \alpha^i \cdot \left[P^i - C(m^i) \right]$$

$$\text{s.t.} \quad \begin{aligned} & w^1(m^1) \geq p^1 \\ & w^i(m^i) - p^i = w^i(m^{i-1}) - p^{i-1} \quad \forall i \geq 1 \\ & m^i \in [0, 1/N, 2/N, ..., 1] \;\forall i \end{aligned}$$

By substituting in all the IC constraints (which generates a recursive equation that gives all prices in terms of willingness to pay) and collecting the terms together for each m_i we can see that the IC constraints include a term for $w^i(m^i)$ for all prices in sequence above p^i and $w^{i+1}(m^i)$ for all prices in the sequence of prices above p^{i+1}. Therefore and taking derivatives for each m^i yields

$$\begin{aligned} & \alpha_i w^i{}'(m^{i*}) - \alpha_{i+1} w^{i+1}{}'(m^{i*}) = \alpha^i C'(m^{i*}) \quad \forall i < I \\ & where \quad \alpha_i = \sum_{j=i}^{I} \alpha^j \end{aligned} \tag{C1}$$

and at i=I, $\alpha^I [w^I{}'(m^{I*}) - C'(m^{I*})] = 0$, because there is no group above them to have an IC constraint. Therefore, the highest type purchases the efficient optimal bundle size. Two final issues arise – what is the lowest group served and what happens when (C1) cannot be satisfied for a positive m^i. An approach is to compute the entire sequence of m^i that arise from the program above. Whenever monotonicity is violated in the m^i, say $m^{x*} < m^{x-1*}$, a more profitable strategy is to pool type x and type x-1, that is set $m^{x*} = m^{(x-1)*}$. Note that this action does not alter any other constraints, namely IR and IC constraints we have, in particular, the relevant IC constraint to ith group, $p^{i*} = p^{i-1*} + w^i(m^{i*}) - w^i(m^{i-1*}) = p^{i-1*} + w^i(m^{i-1*}) - w^i(m^{i-1*}) = p^{i-1*}$, is still satisfied. (*) By inspecting the sequence of optimal sizes to the program, we could identify the relatively unprofitable groups, those with optimal size smaller than that of their adjacent lower types, and then we pool these relatively unprofitable types to their adjacent lower types and relabel the segments and adjust the size of each segment, note that by doing this the number of bundles (prices and sizes) we have to determine is reduced. Calculate the solution to this modified problem, if monotonicity is satisfied, then the solution is an optimal one; if not, we run (*) again until monotonicity is satisfied.

Proof of Corollary 1: Substitute $w^i(m^i) - Z^i(m^i)$ for $w^i(m^i)$ in the proof of Result 1. The quantity reduction arises from condition in Result 1d combined with the condition on Z' above which yields a relationship of marginal revenue is less than marginal cost and thus a quantity reduction. The last expression follows directly from the fact that α^i is positive.

Proof of Result 4. Using Rychlik (1999, p. 108, eq. 11) we have that the trimmed mean of a set of random variables has tightest bounds (for a general distribution): $\frac{k}{n}\int_0^{k/n} Q_F(z)dz \le \frac{1}{k+1-j} E\sum_{i=j}^{k} X_{i:n} \ge \frac{n}{n+1-j}\int_{(j-1)/n}^{1} Q_F(z)dz$. Substituting $n = N, j = N - mN + 1, k = N$ yields $\int_0^1 Q_F(z)dz \le \frac{1}{mN} E\sum_{i=mN}^{N} X_{i:n} \le \frac{N}{mN}\int_{1-m}^{1} Q_F(z)dz$. Multiplying both sizes by mN and applying the definition of expected WTP yields the result.

Proof of Result 5: For i.i.d random variables, $E\sum_{i=1}^{N} c_i X_{i:N} = \int_0^1 Q_F(z)\sum_{i=1}^{N} c_i N_{i:N}(z)dz$ where $X_{i:N}$ is the ith (highest) order statistic from a sample of size *N*. If $c_i = \begin{cases} 0 & i < mN \\ 1 & i \ge mN \end{cases}$ then the first term in the expression becomes $E\sum_{i=mN}^{N} c_i X_{i:N} = E[w(m)]$ and the right hand term becomes the expression shown above.

Proof of Result 6: Denote the solution to the monopolists problem $\max_m \pi(m) = \max_m w(m) - C(m)$ as m^*. From Result 4, $w(m) = \int_0^1 [a + bQ_F(z;0,1)]\sum_{i=mN}^{N} N_{i:N}(z)dz$. The range of possible values (a,b,m) forms a lattice ($\mathbb{R}^2 \times [0,1/N...,1]$). Therefore comparative statics on the parameters can be examined using Topkis' theorem (Topkis, 1978). Denote $m\_ = m - \frac{1}{N}$. From Topkis' theorem, we know that m^* is increasing in ζ if $\frac{\partial \pi(m)}{\partial \zeta} - \frac{\partial \pi(m\_)}{\partial \zeta} \ge 0$. Writing $w(m) - w(m\_) = \int_0^1 [a + bQ_F(z;0,1)]N_{mN:N}(z)dz$. For comparative statics on *a* we have $\frac{\partial \pi(m)}{\partial a} - \frac{\partial \pi(m\_)}{\partial a} = \frac{\partial}{\partial a}[w(m) - w(m\_) - c] = \int_0^1 N_{mN:N}(z)dz > 0$ (the Bernstein polynomials are all positive) so m^* is increasing in *a*. For comparative statics on b the derivative of the difference reduces to: $\frac{\partial}{\partial b}\int_0^1 [a + bQ_F(z;0,1)]N_{mN:N}(z)dz = \int_0^1 Q_F(z;0,1)N_{mN:N}(z)dz$. This is just a mN order statistic from a standardized distribution. Define $\underline{M}$ as the lowest order statistic of the standard distribution with non-negative expected value. The comparative statics w.r.t. b depend on whether the optimum M^* is greater or less than this size. If the optimum bundle size is less than the $\underline{M}$ order statistic will be positive – this is guaranteed if $E[X_{\underline{M}-1:N}] \le c$ (note that for symmetric distributions with an odd number of goods, this condition simplifies to $a \le c$). By the same argument, one can guarantee the opposite sign for the order statistic at optimum when $E[X_{\underline{M}+1:N}] \le c$, so M^* is decreasing in b under this condition. Part c follows directly from the observation that the marginal (lowest valued) good in the bundle must at optimum be greater than marginal cost so contributes positively to profit. Thus, optimal bundles that are larger, must have greater profits than smaller ones since profit for all goods above the marginal good are no less, and the profit contribution from the marginal good is non-negative.

Proof of Result 7: Let $R_{i:N}$ represent the ith order statistic from repeated sampling of the standard normal. Let $S_{i:N}$ be the order statistics from sampling from a equicorrelated (ρ) standard multivariate normal. Owen and Steck (1962) showed that $E[S_{i:N}] = (1-\rho)^{1/2} E[R_{i:N}]$. Using this relation and argument from the proof of Result 6 we have $w(m) = (1-\rho)^{1/2} \int_0^1 Q_F(z) \sum_{i=mN}^{N} N_{i:N}(z)dz$. Applying the same argument as we did for the scale parameter in the proof of Result 6, we have that m* is increasing when $sign \frac{\partial}{\partial \rho}(1-\rho)^{1/2} \ge 0$ and $\mu < c$. Calculating the derivative yields $\frac{-1}{2(1-\rho)^{1/2}} < 0$. Thus, the sign relationship between optimal bundle size and correlation is of the optimal bundle size is decreasing in the correlation when $\mu < c$ and increasing in the correlation otherwise.

Derivations of solutions under the two-parameter case:

Individual selling: Given a fixed unit price for each good, consumers will choose to consume additional goods until their marginal utility (willingness to pay per good) is equated with the price of the good. Therefore:

$$P = \arg\max_p (P-c)\cdot Nm \qquad s.t. \qquad w'(m) = PN$$

When the solution is interior (that is, 0<*m**<*k*) the optimum is given by:

$$P_{IS} = \frac{b(1+a)+cNk}{2Nk} = \frac{b}{Nk}(\frac{1+a}{2}) + \frac{c}{2}$$

$$m_{IS} = \frac{k}{a}\frac{b(1+a)-cNk}{4b}$$

$$p_{IS} = P_{IS}\cdot m_{IS}\cdot N = \frac{b^2(1+a)^2 - c^2k^2N^2}{8ab}$$

$$\pi_{IS} = \frac{\left[b(1+a)-cNk\right]^2}{8ab}$$

From this, we know that optimal price for each good in individual sale setting is determined by average willingness to pay for all goods the consumers positively value ($\overline{w} \equiv \frac{b}{kN}$). *Price is increasing in willingness to pay, skewness of valuation (a) and marginal cost.*

There are two possible boundary conditions to this problem that can be found by examining the behavior of *m**. First, if costs are sufficiently high then no goods are sold. This occurs when $c \geq \overline{w}(1+a)$. A second boundary solution is when the cost-benefit tradeoff is such that all goods that are positively valued are sold. This holds when $c \leq \overline{w}(1-3a)$ or $a \leq \frac{1}{3}(1-\frac{c}{\overline{w}})$. For example, when costs are below average value and all goods have the same valuation (a=0), the solution is on the boundary: $p_{IS} = b$ (or $P_{IS} = \frac{b}{Nk}$), and $m_{IS}=k$, while $\pi_{IS} = b - ckN$. As a departs from zero, this boundary condition becomes $p_{IS} = w'(k)\cdot k$ (or $P_{IS} = w'(k)/N$), $m_{IS}=k$, and $\pi_{IS} = w'(k)\cdot k - ckN$, and there exists positive consumer surplus. This implies that when we have a boundary solution, individual sale is efficient but not profit maximizing. When marginal costs are zero, we can either have a boundary condition when preferences are sufficiently uniform (a<1/3) or an interior solution. When the solution is interior, there is quantity restriction due to the consumers equating their marginal rather than total benefit with price – there is generally non-zero consumer surplus as well as some deadweight loss.

Customized bundling: Under customized bundling, the firm's optimization problem is the same, but the consumer individual rationality (IR) constraint is changed. Instead of the price being determined by the marginal good, total bundle price (which is average unit price multiplied by quantity) is determined by overall willingness to pay. Thus the firm's problem becomes:

$$p_{CB} = \arg\max_p p - cNm \qquad s.t. \quad (IR) \quad w(m) - p \geq 0$$

IR is always binding to achieve profit maximization, so we can rewrite the objective function as:

$$m_{CB} = \arg\max_m w(m) - cNm$$

This equation is identical to the problem of maximizing social welfare and, thus, there is no deadweight loss.

The optimal solution of this program when m_{CB} is interior is:

$$m_{CB} = \frac{k}{a} \cdot \frac{b(1+a) - kNc}{2b}$$

with

$$\pi_{CB} = \frac{[b(1+a) - kNc]^2}{4ab}$$

$$p_{CB} = \frac{(1+a)^2 b^2 - c^2 k^2 N^2}{4ab}$$

As stated before, there is no deadweight loss, but because the monopolist can perfectly price discriminate, there is no consumer surplus either.

Again, it is interesting to explore the boundary conditions for this solution. Positive quantities are sold whenever: $c \geq \overline{w}(1+a)$, which is the same condition as with individual sale. This is intuitive since the smallest customized bundle is an individual unit. However, customized bundling attains its upper bound at a much higher level of the cost and preference heterogeneity parameters than individual selling. The optimal customized bundle is the largest necessary to serve all consumers fully ($m_{CB}=k$) whenever $c \leq \overline{w}(1-a)$ or $a \leq (1-\frac{c}{\overline{w}})$. In other words, the entire market demand can be fully satisfied using customized bundling for a values three times as large as the case of individual selling. And even when all goods positively valued are sold in both strategies (customized bundling and individual sale), customized bundling will yield higher profits as long as a is greater than zero. This implies that customized bundling becomes more effective as a strategy when consumers' valuations of goods show greater heterogeneity.

Summary of results:

	Pure Bundling[9]	**Individual Selling**	**Customized Bundling**
Quantity (m)	1 if $b-cN \geq 0$	$m_{IS}=\min(\frac{k}{a}\frac{b(1+a)-cNk}{4b},k)$ if $c \leq \bar{w}(1+a)$, 0 otherwise	$m_{CB}=\min(\frac{k}{a}\frac{b(1+a)-cNk}{2b},k)$ if $c \leq \bar{w}(1+a)$, 0 otherwise
Total sales or bundle price (p)	b if $b-cN \geq 0$	if nonzero quantity then $p_{IS}=\frac{(1+a)^2 b^2-(ckN)^2}{8ab}$ if $c \geq \bar{w}(1-3a)$, b otherwise	if nonzero quantity then $p_{CB}=\frac{(1+a)^2 b^2-(ckN)^2}{4ab}=2\,p_{IS}$ if $c \geq \bar{w}(1-a)$ b otherwise
Profits	b-cN if $b-cN \geq 0$	if 0< m_{IS} <k then $\pi_{IS}=\frac{[b(1+a)-ckN]^2}{8ab}$ otherwise b-ckN if m_{IS} >k	if 0< m_{CB} <k then $\pi_{CB}=\frac{[b(1+a)-ckN]^2}{4ab}=2\pi_{IS}$ otherwise b-ckN if m_{CB} >k

[9] We do not consider situations where a random pure bundle of size smaller than 1 is offered. Calculation of the value of this type of bundle is a complex combinatorial problem and depends strongly on assumptions regarding the distribution of values over each potential good combination. We know from the example given in footnote 2 however, that this type of bundle will always create deadweight loss unless consumer preferences over each good are identical.

Figure 2: Profitability comparison of the three approaches

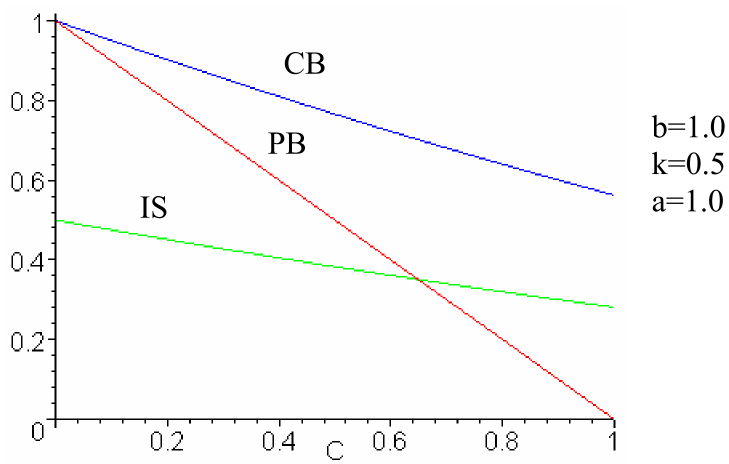

Figure 2a: Profitability of alternative bundling strategies under different marginal costs.

Results: CB strictly dominates the other two extreme strategies when c>0. PB is much more sensitive to marginal cost than IS.

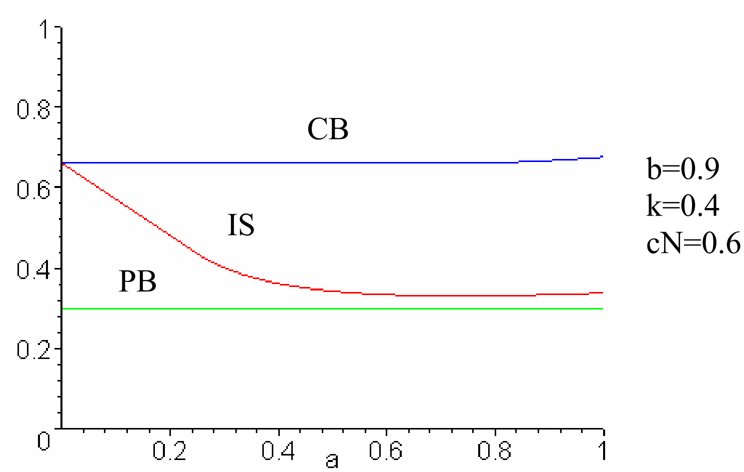

Figure 2b: Profitability of alternative bundling varying consumer preferences across goods

Results: CB strictly dominates the other two extreme strategies when a>0. The relative performance of the other two strategies depends on parameter setting. For some parameter settings, IS is never worse than PB).

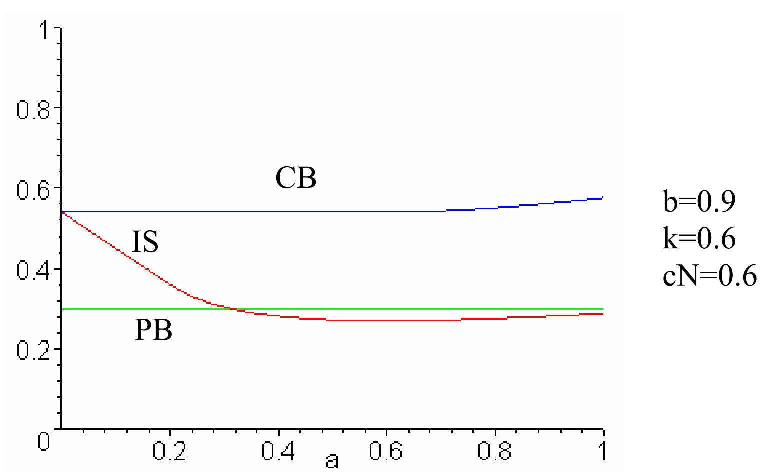

Figure 2c: Profitability of alternative bundling varying consumer preferences across goods.

Results: CB strictly dominates the other two extreme strategies when a>0. The relative performance of the other two strategies depends on parameter setting. For some parameter settings, IS can be worse than PB.

CB: Customized Bundling
PB: Pure Bundling
IS: Individual Selling

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