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Abstract

We develop a model of customer choice in response to random variation in quality. The choice model yields closed-form expressions which reflect the effect of competing suppliers' service quality on the long-run fraction of purchases a customer makes at the various competitors. We then use the expressions as the basis of simple normative models for suppliers seeking to maximize their long-run average profits. The results provide insight into the effect of switching behavior on the service levels offered by competing suppliers.

Keywords

Customer loyalty, quality competition, bayesian bandit

Disciplines

Operations and Supply Chain Management

Customer Loyalty and Supplier Strategies for Quality Competition II: Supplier Quality Strategies *

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Abstract

We wish to understand how companies should set the service-levels provided by their operations. To this end, we use the measures of customer loyalty obtained in [5] to develop simple, normative models for suppliers seeking to maximize long-run average profits. The results provide insight to suppliers on the level of service they should offer.

1 Introduction

What should the line item fill rate be? How should one set the expected delay in queue? These are common questions that managers must answer as they set the service level in their operating systems. A common response is for companies to match the competition. Benchmarking exercises and data drawn from trade associations allow managers to compare their operations to competitors. Still, managers may wonder whether they have converged on the "right" service level, whether there is something to be gained from deviating from the industry standard.

Traditional normative models in operations management are not of much help. Typically, they use exogenously-defined "good will" costs that depend on the magnitude of the service failure: the number of stockouts, the average time spent in queue. There is wide recognition,

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however, that the use of these cost models does not adequately account for the damage done by quality failures. Repeated failures, such as stockouts or delays in queue, may have a cumulative, negative effect on an individual customer's satisfaction which these cost functions do not take into account.

In a companion paper [5] we develop a detailed model of consumers' long-term responses to quality failures that is intended to help identify what the proper quality level should be. In our model, a consumer repeatedly chooses among a set of suppliers, and the outcome of each visit to a supplier is some (instantaneous) utility. The utility offered by each supplier is, in fact, a random variable that reflects the quality of that supplier's offering: whether or not there was a stockout, how long the wait in queue was, how fresh the fish was that day. The consumer uses a crude form of Bayesian revision to keep track of which of the suppliers she prefers. Each time she enters the market, she myopically chooses the supplier that she thinks is most likely to be best.

In this paper, we use the expressions developed in [5] to develop profit models for suppliers that explicitly integrate the revenue and cost consequences of their quality choices. The models reflect the fact that poor service at a supplier leads to customer switching and that, when aggregated, the switching behavior of individual customers obtains market shares for suppliers that are dynamic and stochastic, rather than fixed.

Given this stochastic view of market shares, we formulate suppliers' objectives as the maximization of long-run average profit, and we analyze the results of oligopoly competition in two contexts: a generic cost model in which unit costs do not exhibit economies of scale, and a more specific model of competing M/M/1 queues for which there are economies of scale. In both models unit prices are fixed, and suppliers compete on the basis of the quality levels that they set.

In the first case, in which costs are simply convex and increasing in the overall level of quality offered, we find the following:

- Given symmetric competitors, there is a unique pure-strategy Nash equilibrium, and it is symmetric.

- Given asymmetric competitors equilibria are always asymmetric. In a duopoly, the competitor with lower costs will invest its advantage to increase quality, capture market share, and earn higher long-run average profits.

- As the number of competitors m increases, competitive pressure drives quality levels to increase. Still, consumers' lack of quality information about providers allows suppliers to earn positive profits from current customers even as $m \to \infty$.

In the second case M/M/1 queues complete for the patronage of customers who are risk-neutral with respect to system sojourn times. Here the set of equilibria is more difficult to characterize. Nevertheless, we are able to say a great deal about the nature of competitive equilibria, including the following:

- In a symmetric duopoly there is a unique, pure-strategy equilibrium that is profitable for all suppliers, and it is symmetric.

- For $m > 2$ symmetric suppliers we have not been able to rule out the possibility of asymmetric equilibria. We do show, however, that for any m there is exactly one symmetric equilibrium in which suppliers earn positive average profits.

- A closed-form expression for the quality level obtained in a symmetric equilibrium describes it as a function of: 1) the underlying profitability of the market, excluding congestion-related costs to suppliers; 2) the number of competitors in the market; and 3) customer expectations regarding congestion.

- As the number of competitors m increases, loss of economies of scale drives suppliers' quality levels to decrease. This behavior stands in direct contrast to the case in which there are no economies of scale.

Thus the results demonstrate that, given our underlying assumptions, there are natural forces that drive competitors to adopt an industry quality standard, and they show how this industry standard changes with the number of suppliers serving the market. Quality may increase or decrease, depending on the nature of the industry's cost structure.

At the same time, the optimization models used to derive these results can be used by a supplier to verify whether or not the current standard in its industry is consistent with the long-run competitive equilibrium. To the extent that it is not, a supplier can use the models to see better understand what its own quality strategy should be.

The remainder of the paper is organized as follows. In §3, we develop our model of customer reaction to service quality. Then §4 presents the analysis for industries that do not exhibit economies of scale, and §5 presents that for competing M/M/1 queues, which do enjoy returns to scale. Finally §6 discusses the results, as well as directions for future work.

2 Literature Review

The underlying model of consumer behavior is developed in a companion paper [5]. In that paper we review literature that addresses customer responses to variation in product and service quality and, more generally, the decision-making strategies people use when making sequential choices under uncertainty.

In this paper, we use the model of consumer behavior to investigate suppliers' decisions regarding how best to set quality standards, and we highlight two recent streams of literature that investigate the nature of quality competition and are closely related to this paper. The first group looks at quality competition more generally and represents the supplier's cost of service in a more highly stylized fashion. Recent examples of these papers include Karmarkar and Pitbladdo [9], Tsay and Agrawal [21], and the references therein. The second group analyzes queueing systems, and in them delay in queue or system sojourn time is typically defined to be the measure of quality, the shorter the better. Examples that analyze the case of a monopolist service provider include papers by Mendelson and coworkers [16, 17, 4], Stidham [20], and van Mieghem [22]. Mandelbaum and Shimkin [14] characterize equilibrium behavior of customer abandonments from a monopolist. Other work, such as that of Li and Lee [12], Lederer and Li [11], Ho and Zheng [8], Cachon and Harker [1], and Chayet and Hopp [2] perform competitive analyses in which a supplier competes within an oligopoly.

The analyses in both groups assume that, in equilibrium, consumers are well-informed about (or have beliefs that are consistent with) the service level offered by a supplier: each customer knows sufficient statistics concerning the service quality at the various suppliers. In [12] customers continually monitor queue lengths and jockey among suppliers; they exhibit no loyalty towards suppliers. In the remaining work, customers know the expected quality level offered by all suppliers in equilibrium, and they frequent only the supplier that maximizes expected utility. They do not modify their choice based on the history of the level of service they actually receive.

Our model falls somewhere in between these two extremes. Consumers are not well-informed about quality levels, and they do not continually jockey among suppliers. In the short run they remain "loyal" and stay with one supplier. But they do respond to the history of the service they actually receive, and the resulting equilibria are ones in which consumers continue to switch among suppliers.

The paper closest in spirit to ours is Hall and Porteus [7], which develops models in which inventory and queueing systems compete on the basis of service quality. (For an earlier, related paper see also Smallwood and Conlisk [19].) They show that when one firm has an advantage of more loyal customers – with lower probabilities of switching upon a service failure – then it is optimal for the firm to offer a lower level of quality than its competitor.

The paper also explicitly models customers switching among suppliers, but it differs from this paper in two respects. First, it considers dynamic policies in which suppliers may change service quality in response to short-term changes in market share. We consider only static quality policies in which each supplier decides on a quality level and maintains it. Our stationary quality policies are simple to implement, and they are intended to be consistent with industry practice, which typically sets stationary targets for fill rates and queueing delays. Nevertheless, they clearly may not perform as well as more dynamic policies. Second, in the models of [7] customer switching behavior is a function of both service quality and an exogenously defined loyalty factor. In our underlying model of consumer behavior, loyalty is completely (endogenously) driven by supplier quality.

3 Model of Customer Loyalty and Supplier Quality

Suppose there are m suppliers indexed $i \in \{1, \ldots, m\}$ that serve a market with n consumers, and at regular time intervals $t = 1, 2, \ldots$ each consumer patronizes one of the suppliers. Because there is inherent uncertainty in the process of delivering the product or service, the utility of the good provided by supplier i to consumer j at t is a random variable, $U_t^{i,j}$.

3.1 The Relationship between Quality and Utility

While the quality of each item or service encounter is uncertain, each supplier *can* control the overall level of utility it provides, and for each supplier this choice of a "quality level" manifests itself in the choice of a distribution for $U_t^{i,j}$. In this paper we make two fundamental assumptions that limit the nature of the variation:

(A1) All customers derive the same utility from the same physical experience.

(A2) The sequence of utilities that any customer obtains from a supplier is *i.i.d.*.

The first assumption allows us to translate any physical measure of the distribution of quality, such as the probability of a stockout or the delay in queue, directly into a distribution of utility. The second implies that each supplier's choice of a quality distribution is a strategic decision. Supplier i must choose its particular quality distribution once, before consumers enter the market, and then live with the consequences.

Together the assumptions allow us to describe a supplier's quality choice as a single distribution of utility U^i with mean $\mu_i \triangleq \mathsf{E}[U^i]$. We may think of physical measures of system performance and of the resulting customer utility in similar terms, although for consumers that are risk averse with respect to the physical measure of service quality the two need not be identical. For example, suppose that a physical measure of system quality is normally distributed but that the customer is risk averse. Then two service providers that offered the same mean level of the physical quantity, but different levels of variability, would have different – and not necessarily normally distributed – U^i's. Furthermore, the supplier with the lower variance would provide a higher expected utility, μ_i.

3.2 Individual Consumer Response

We next define the choice behavior of an arbitrary consumer. She enters the market at time 0 with an initial level of *satisfaction* for each of the suppliers, S_0^i, $i = 1, \ldots, m$. At time $t = 1$ she patronizes the supplier with which she is most satisfied: $i = \arg\max\{S_{t-1}^j : j = 1, \ldots, m\}$ and obtains utility according to a realization of U^i. Given this realization, at $t = 1$ she updates her satisfaction with the m suppliers as follows,

$$S_t^i \;=\; S_{t-1}^i + U^i - \mu^* \qquad \text{and} \qquad S_t^j \;=\; S_{t-1}^j \text{ for all } j \neq i\,, \tag{1}$$

where μ^* is the level of utility at which she is "satisfied" with the supplier's performance. (To be consistent with A1, we assume that all consumers have the same μ^*.) If $U^i > \mu^*$, then the consumer's satisfaction with i increases, while if $U^i < \mu^*$, then her satisfaction with i decreases. Again at $t = 2$ – and at each t thereafter – the consumer chooses supplier $i = \arg\max\{S_{t-1}^j : j = 1, \ldots, m\}$ and updates her satisfaction with the suppliers using (1). Note that while all customers have the same preferences, differences in initial beliefs and random variation in actual experiences ultimately make each customer's sequence of choices unique.

Remark This model of customer choice is consistent with the model of consumer learning developed in Gans [5]. It is also a direct analogue to a version of "Cumulative Utility Consumer Theory" presented in Gilboa and Pazgal [6]. In [5], the customer tests the simple hypothesis that a supplier has a "good" quality distribution against the simple alternative that is has a "bad" one, and her subjective probability estimate that a supplier i is good, rather than bad, corresponds to her satisfaction S_t^i in this paper. The normalizing factor μ^* is sometimes called the "aspiration level" of the customer, and in [5] it corresponds to the average quality-level at which the consumer cannot statistically distinguish whether the supplier is good or bad. For the purposes of this paper, μ^* is not strictly needed, since without loss of generality we might have shifted the distributions U^i by μ^*. We maintain it for expositional convenience. $\square$

3.3 Two Characterizations of Customer Loyalty

In the short run, a consumer may stay with her current supplier for several periods. As long as her satisfaction for supplier i remains above that of the other suppliers, she will continue to patronize supplier i. As soon her satisfaction with i drops below that of another, however, she will switch to the competitor.

Suppose a customer most prefers supplier i at time t. Then $S_t^i = max\{S_t^j : j = 1, \ldots, m\}$. Let $k = \arg\max\{S_t^j : j = 1, \ldots, m; j \neq i\}$ be the satisfaction of the "next-best" supplier and $b_t^i = S_t^i - S_t^k$ be the balance of "good will" that the customer has for supplier i at time t. In turn, define $\tau_t^i = \inf\{s : S_{t+s}^i < S_t^k\}$ to be the number of periods after which the customer switches from i to k. We call τ_t^i the duration of the customer's remaining *sojourn* at supplier i, as well as the duration of her remaining *short-term loyalty*, and we note that it is a stopping time for a random walk with increments $(U^i - \mu^*)$ and stopping boundary b_t^i. Using Wald's equation, the following result is immediate:

Lemma 1 (Gans [5]) *Given assumptions A1 and A2, suppose* $\mathrm{var}(U^i) < \infty$. *Then* $\mathsf{E}[\tau_t^i] < \infty$ *if and only if* $\mu_i < \mu^*$. *Furthermore, when* $\mu_i < \mu^*$

$$\mathsf{E}[\tau_t^i] \quad = \quad \frac{b_t^i \, + \, \mathsf{E}[S_{\tau_t^i} - b_t^i]}{\mu^* \, - \, \mu_i} \quad \geq \quad \frac{b_t^i}{\mu^* \, - \, \mu_i} \, . \tag{2}$$

After the consumer leaves supplier i she may spend many periods patronizing i's competitors. At some future time, however, she may return to i. Indeed, if switching among suppliers is frequent, then supplier i may be interested in the long-run relative frequency with which the customer chooses i.

Consider the long-run switching behavior of an arbitrary customer. Let $\pi(s)$ be the index of the supplier chosen by the consumer in period s and $f_i \triangleq \lim_{t\to\infty} \frac{1}{t} \sum_{s=1}^{t} \mathbf{1}\{\pi(s) = i\}$ denote the limiting relative frequency with which the consumer chooses supplier i.

Lemma 2 (Gilboa and Pazgal [6] and Gans [5]) *Given assumptions A1 and A2, suppose* $-\infty < \mu_i < \mu^*$ *and* $\mathrm{var}(U^i) < \infty$ *for* $i = 1, \ldots, m$. *Then with probability one* f_i *exist for* $i = 1, \ldots, m$, *and*

$$f_i(\mu_i, \mu_{-i}) \quad = \quad \frac{1/(\mu^* - \mu_i)}{\sum_{j=1}^{m} 1/(\mu^* - \mu_j)} \quad = \quad \frac{1}{1 + (\mu^* - \mu_i)\Delta}\,, \tag{3}$$

where $\mu_{-i} = \{\mu_j, j \neq i\}$ *and* $\Delta = \sum_{j \neq i} (\mu^* - \mu_j)^{-1}$. *Furthermore,*

$$\frac{\partial f_i}{\partial \mu_i} = \frac{f_i\,(1 - f_i)}{(\mu^* - \mu_i)} \qquad \text{and} \qquad \frac{\partial^2 f_i}{\partial \mu_i^2} = \frac{2\, f_i\,(1 - f_i)^2}{(\mu^* - \mu_i)^2}\,. \tag{4}$$

Equation (3) describes the long-run share of a customer's purchases as a function of her aspiration level and the suppliers' quality levels. In the oligopoly games developed in §4 and §5, this customer share will drive supplier i's revenues and costs. Equation (4), which will be used to verify optimality conditions, shows that supplier i's share is convex and increasing in μ_i.

Observe that μ^* plays a special role in both Lemma 1 and 2. It is the level of quality at which a supplier can guarantee the loyalty of a consumer. In Lemma 1 μ^* is the quality level at which the expected duration of a customer's sojourn at a supplier becomes infinite. This limit is independent of the consumer's initial level satisfaction. Similarly, in Lemma 2 it is the quality level at which the long-run "shares" of customer no longer exists. If there exists a $\mu_i \geq \mu^*$, then once a customer begins patronizing supplier i she is expected to remain loyal forever. For a more general discussion and interpretation of these results, see [5].

Remark In the analysis that follows, we use the lower bound of (2) as if it were an equality. In [5] we offer evidence that the behavior of $\mathsf{E}[\tau]$ itself is roughly that of the bound: $\mathsf{E}[\tau] = \mathrm{O}(1/(\mu^* - \mu_i))$. Furthermore, we note that the long-run characterization of f_i is the composition of $\mathrm{O}(1/(\mu^* - \mu_i))$ terms for the various suppliers. As we shall see in §4, there is a close connection between these short and long-run views of customer loyalty. □

3.4 Supplier Revenues

We assume that prices are fixed and that each customer visit yields a supplier \$$r$ of revenue. Therefor, to determine a supplier's revenues we need only derive the number of customers that patronize a supplier.

Let $\pi^j(t)$ be customer j's choice of supplier in period t, and let n_t^i be the number of customers that patronize supplier i in period t. Then given assumptions A1 and A2, we can use Lemma 2 to derive the long-run average number of customers that patronize supplier i:

Corollary 1 *Given assumptions A1 and A2, suppose* $-\infty < \mu_i < \mu^*$ *and* $\mathrm{var}(U^i) < \infty$ *for* $i = 1, \ldots, m$. *Then*

$$\lim_{t\to\infty} \frac{1}{t} \sum_{s=1}^{t} n_s^i \;=\; \lim_{t\to\infty} \frac{1}{t} \sum_{s=1}^{t} \sum_{j=1}^{n} \mathbf{1}\{\pi^j(s) = i\} \;=\; \sum_{j=1}^{n} \lim_{t\to\infty} \frac{1}{t} \sum_{s=1}^{t} \mathbf{1}\{\pi^j(s) = i\} \;=\; n\, f(\mu_i, \mu_{-i})\,. \quad (5)$$

Thus, with identical customers long-run average market share is a multiple of the long-run share of an arbitrary customer. Note that the interchange of limit and summation is justified by the fact that $0 \le \sum_{j=1}^{n} \mathbf{1}\{\pi^j(s) = i\} \le n$ is bounded for every s.

Remark While we will not require the result in this paper, one may also be interested in the limiting distribution, $\lim_{t\to\infty} \frac{1}{t} \sum_{s=1}^{t} \mathbf{1}\{n_s^i = x\}$. For example, if we were to assume that in any period t the distribution of utility obtained by one customer is independent of that of the next, then it should not be difficult to show that the limiting random variable is binomially distributed with parameters n and $f_i(\mu_i, \mu_{-i})$. $\square$

3.5 Supplier Costs and Information Requirements

The assumption that supplier i's quality distribution U^i is fixed allows us to derive the long-run average of its market share. This assumption also has important implications for supplier i's actions. First, it limits the supplier to consider only stationary quality policies. As we noted in the literature review, we do not consider policies, such as those investigated by Hall and Porteus [7], that dynamically change the service level in response to market conditions at time t.

Furthermore, to execute a stationary quality policy, a supplier must be able to dynamically change its capacity in response to changes in the size of its customer base, n_t^i. For example, in a queueing system, as the number of customers patronizing supplier i changes from period t to period $t + 1$, so will the arrival rate of customers. To maintain the same sojourn-time distribution across periods, supplier i must be able to change its rate of service.

The ability to maintain a stationary quality distribution also has important implications for the information required by a supplier to operate. If the supplier is to match capacity to demand in each period, then it must know what demand will be, and we assume that in each period each supplier knows how many consumers in the market are current customers. This implies that each period a supplier knows the numbers of new customers that arrive and defecting customers that leave – and it knows these numbers early enough to be able to adjust its capacity accordingly.

We define supplier i's cost of delivering the product or service to an arriving customer as a function $\$c_i(\cdot)$ that is determined at least in part by the overall level of quality chosen, μ_i. If there are no economies of scale in production, then $c_i(\cdot)$ does not vary with n_t^i, and we will assume $c_i(\mu_i)$ is increasing and convex in μ_i. In this case a fairly general analysis is possible. (See Section 4.)

With economies of scale, however, there are two complications that make the behavior of $c_i(\cdot)$ more complex. First, given fixed μ_i, period-to-period changes in n_t^i drive changes in unit costs. Second and more fundamentally, an increase in μ_i affects average cost in at least two ways: it has a direct effect that drives up unit costs; and it has an indirect effect, through $f_i(\mu_i, \mu_{-i})$, that drives up long-run average market share and lowers unit costs. The complexity of this relationship makes a general analysis of the case "economies of scale" impossible, and we are reduced to analyzing specific functional forms for $c_i(\cdot)$. In Section 5 we therefor analyze one specific case that is of special interest to us, that of competing $M/M/1$ queues.

4 No Economies of Scale: General Model

In this section we analyze systems in which costs are convex and increasing in the average level of quality offered and which do not exhibit economies of scale, and we consider profit-maximization under two different information regimes. In the first, an individual supplier knows little about its competitors, but it knows the nature of an individual consumer's response to its unilateral changes in quality (2), and it seeks to set quality to maximize the expected profits from each customer sojourn. In the second, the supplier knows much more. In particular, it knows its competitors' cost functions, as well as the form of (3).

In both settings, we can derive the fundamentals of a supplier's quality strategy. In the case of unilateral quality changes, the supplier's profit function is pseudo-concave in the quality level chosen. If in addition costs are strictly convex, then there is a unique profit-maximizing quality level that a supplier should choose. For the case of oligopoly competition, we obtain analogous equilibrium results. Each competitor's reaction function is pseudo-concave, so there exists at least one pure strategy Nash equilibrium. This structure provides the foundation for the results summarized in the introduction.

4.1 When a Supplier Sets Quality Unilaterally

Consider a supplier that operates in the absence of competitive information. It knows its own cost function, as well as (current and potential future) consumers' short-term responses to its quality, the lower bound of (2). Suppose the supplier wishes to maximize the expected profits from the sojourn of each customer that becomes a patron.

Recall that a customer pays \$$r$ per unit of time it remains with the supplier and that the cost of providing service to a customer is \$ $c_i(\mu_i)$ per customer per unit of time and is independent of the number of customers being served. Then for a customer with initial balance of good will b_t^i, the supplier maximizes expected profits of the sojourn by choosing its quality level as follows:

$$\max_{\mu_i} \left\{ \Pi_i(\mu_i) \triangleq (r - c_i(\mu_i)) \times b_t^i \,/\, (\mu^* - \mu_i) \right\} . \tag{6}$$

If $c_i(\mu_i)$ is a convex function of μ_i, then $r - c_i(\mu_i)$ is concave, and $(r - c_i(\mu_i))\, b_t^i \,/ (\mu^* - \mu_i)$ is pseudo-concave in μ_i (see Mangasarian [15]). This implies (in the unconstrained case) that the set of quality levels that maximize profits is contiguous and satisfies the first order optimality conditions. In turn, for all μ_i above the set of profit maximizers, ${\Pi_i}'(\mu_i) < 0$, and for all μ_i below the set of profit maximizers, ${\Pi_i}'(\mu_i) > 0$.

The first order conditions are satisfied when the profit due to an increase in the expected sojourn time equals the marginal cost required to increase the level of service over the same period,

$$\Pi_i{}'(\mu_i) \quad = \quad (r - c_i(\mu_i))\,\frac{b_t^i}{(\mu^* - \mu_i)^2} \;-\; c'(\mu_i)\,\frac{b_t^i}{(\mu^* - \mu_i)} \quad = \quad 0\,, \tag{7}$$

or, equivalently, when

$$\frac{r - c_i(\mu_i)}{\mu^* - \mu_i} \;-\; c'(\mu_i) \quad = \quad 0\,. \tag{8}$$

Observe that the first order conditions (8) are independent of b_t^i. Therefor, the quality strategy for the supplier is quite simple. Because all customers have the same aspiration level μ^*, the same quality level simultaneously maximizes expected profits from all customers' sojourns, no matter what their prior satisfactions are for the supplier.

If in addition $c_i(\cdot)$ is strictly convex, there is a unique profit-maximizing level of quality:

Proposition 1 *Suppose $c_i(\mu_i)$ is strictly convex and increasing in μ_i. If there exists a minimum average quality level, $-\infty < \underline{\mu} < \mu^*$, offered by the supplier and $r - c_i(\underline{\mu}) > 0$, then there exists a unique quality level, $\widehat{\mu}$, that maximizes expected profits from every customer sojourn. Furthermore: i) if $r - c_i(\mu^*) > 0$ then $\widehat{\mu} = \mu^*$; and ii) if $r - c_i(\mu^*) \leq 0$ then $\widehat{\mu} \in [\underline{\mu}, \mu^*)$.*

The proofs of most propositions can be found in the appendix.

Note that no matter what the form of the cost function is, when $r - c_i(\mu^*) > 0$ the objective function (6) is unbounded, and our model breaks down. Formally, the expected duration of customer loyalty is infinite, and the supplier earns infinite profits. Ultimately, this is due to the simplifying assumption underlying (2) that customers live forever.

In broader terms, if a supplier can afford to offer a quality level μ^*, then quality is no longer the basis of competition. By offering μ^*, a supplier locks in customer loyalty. In this case all suppliers may offer μ^*, and the focus of competition would shift to other arenas, such as efforts to attract new customers or product and service differentiation.

4.2 Quality Competition in an Oligopoly

We now turn to an analogous model of oligopoly competition. We assume that m firms compete for the patronage of n identical consumers.

Suppose, again, that each customer pays $\$\, r$ per period to the appropriate supplier and that the cost of providing service to a customer is $\$\, c_i(\mu_i)$ per customer per period and is independent of the number of customers being served. Then a supplier's long-run average costs are directly proportional to its average customer base, which from (5) is $n\, f_i(\mu_i, \mu_{-i})$.

Given fixed quality levels for competitors, μ_{-i}, supplier i solves

$$\max_{\mu_i} \left\{ \Pi_i^{\mathrm{m}}(\mu_i) \ \stackrel{\triangle}{=}\ (r - c_i(\mu_i)) \ \times\ n\, f_i(\mu_i, \mu_{-i}) \right\} \tag{9}$$

to maximize its long-run average profit. Here Π_i^{m} denotes supplier i's profit when m suppliers compete in an oligopoly.

Again, when μ_{-i} is fixed and $c_i(\mu_i)$ is convex in μ_i, then it is not difficult to show that $\Pi_i^{\mathrm{m}}(\mu_i)$ is pseudo-concave in μ_i. The unit profit, $r - c_i(\mu_i)$ is concave, as before, and if we let $g_i = 1/f_i = 1 + (\mu^* - \mu_i)\Delta$, then g_i is linear and decreasing in μ_i. In turn, for $\mu_i < \mu^*$ the profit, $(r - c_i(\mu_i))\, n \,/\, g_i$, is pseudo-concave in μ_i (see Mangasarian [15]). Therefore,

Proposition 2 *Suppose the $c_i(\cdot)$ are convex and increasing and that there exist $-\infty < \underline{\mu}_i < \overline{\mu}_i < \mu^*$ such that $\mu_i \in [\underline{\mu}_i, \overline{\mu}_i]$, for $i = 1, \ldots, m$. Then there exists a pure-strategy, Nash equilibrium to the quality competition game.*

Proof The strategy spaces of the suppliers are nonempty, compact, convex subsets of the real line, and each supplier's response function is quasi-concave in its quality level. Therefore from Debreu [3] the result follows. □

The assumption that there exists a $\underline{\mu}_i > -\infty$ is mild. A reasonable assumption for $\overline{\mu}_i$ would be $\overline{\mu}_i = \{\mu : r - c_i(\mu) = 0\}$. This is consistent with the assumptions $r - c_i(\underline{\mu}_i) > 0$, $r - c_i(\mu^*) < 0$, and $c(\cdot)$ increasing.

The first order conditions are satisfied when the marginal increase in profits due to an increase in market share equals the marginal increase in cost over that share. Using (4) we have:

$$\Pi_i^{\mathrm{m}\prime}(\mu_i) \quad = \quad (r - c_i(\mu_i))\, n\, \frac{f_i\,(1 - f_i)}{\mu^* - \mu_i} \ - \ c_i'(\mu_i)\, n\, f_i \quad = \quad 0\,, \tag{10}$$

for all suppliers, $i = 1, \ldots, m$.

Note that the set of Nash equilibria includes

$$\{(\mu_1, \ldots, \mu_m) : \Pi_i^{\mathrm{m}\prime}(\mu_i) = 0 \ \forall i = 1, \ldots, m\}\,, \tag{11}$$

the set of quality vectors for which the first order conditions (10) hold for all i. It also may include vectors with some elements on the boundary of the action space: $\{\mu_i = \underline{\mu}_i \ \cap \ \Pi_i^{m\prime}(\underline{\mu}_i) \ \leq \ 0\}$ or $\{\mu_i = \overline{\mu}_i \ \cap \ \Pi_i^{m\prime}(\overline{\mu}_i) \ \geq \ 0\}$.

One general problem with using the search for Nash equilibria as a method of strategic analysis is that set of possible equilibrium points may be large. In this case one must justify how competitors may converge on one of many possible equilibria.

Given our problem structure, we are able to show that the set of Nash equilibria is well structured, however. In particular, when the suppliers' technologies are identical, we can use the first order conditions (10) to prove the following.

Proposition 3 *For $i = 1, \ldots, m$, suppose that $c_i(\cdot) \equiv c(\cdot)$ is convex and increasing, that there exist $-\infty < \underline{\mu} < \overline{\mu} < \mu^*$ such that $\mu_i \in [\underline{\mu}, \overline{\mu}]$, and that $r - c(\overline{\mu}) = 0$. Then there exists a unique pure strategy Nash equilibrium and it is symmetric.*

Just as symmetric costs drive symmetric equilibria, asymmetric costs strongly drive equilibria to be asymmetric. As the following proposition shows, when one supplier has a (percentage) cost advantage over another, the two never choose the same quality level in equilibrium.

Proposition 4 *Let $c_i(\cdot) \equiv \alpha_i c(\cdot)$ for $i = 1, \ldots, m$, where $\alpha_i \in (0, \infty)$ and $c(\cdot)$ is positive, convex, and increasing. For each i suppose there exist $\underline{\mu}_i < \overline{\mu}_i < \mu^*$ such that $\mu_i \in [\underline{\mu}_i, \overline{\mu}_i]$, and that $r - \alpha_i c(\overline{\mu}_i) = 0$.*

i) For any m, consider arbitrary suppliers j and k. If $\alpha_j < \alpha_k$, then there is no equilibrium with $\mu_j = \mu_k$ such that $\Pi_j^{\mathrm{m}\prime}(\mu_j) = \Pi_k^{\mathrm{m}\prime}(\mu_k) = 0$.

ii) When $m = 2$, if $\alpha_1 < \alpha_2$ then any pure strategy equilibrium with $\Pi_1^{\mathrm{m}\prime}(\mu_1) = \Pi_2^{\mathrm{m}\prime}(\mu_2) = 0$ has $\mu_1 > \mu_2$. Furthermore $\Pi_1^{\mathrm{m}}(\mu_1) > \Pi_2^{\mathrm{m}}(\mu_2)$.

Part (i) of the proposition shows that asymmetric equilibria will generally result from asymmetric costs. Without more structure, however, it is difficult to say more about the nature of the equilibria. Part (ii) adds the restriction that $m = 2$. In this case we can also say that for any positive, convex, and increasing cost function, the supplier with a cost advantage will offer higher quality *and* earn higher profits.

Remark Proposition 4 is an analogue to results in Lederer and Li [11] and a complement to results in Hall and Porteus [7]. In our case a lower cost structure allows a supplier to increase quality, customer loyalty, and average profits. In [7] the authors show that, given customers are *a priori* more loyal, a supplier can decrease quality and improve profits. $\square$

4.3 Relationship Between Oligopoly and Unilateral Solutions

A natural relationship exists between the model in which the supplier sets quality unilaterally and the one in which it offers the equilibrium-level of quality obtained in an oligopoly. As the following proposition shows, the first is a natural limit of the second:

Proposition 5 *Suppose that: a) there exist $-\infty < \underline{\mu} < \overline{\mu} < \mu^*$ such that for any m, $\underline{\mu} \leq \underline{\mu}_i < \overline{\mu}_i \leq \overline{\mu}$ and $\mu_i \in [\underline{\mu}_i, \overline{\mu}_i]$ for $i = 1, \ldots, m$; and b) for any i, $c_i(\cdot)$ is positive, convex and increasing and $r - c_i(\overline{\mu}_i) = 0$. Let $\widehat{\mu}_i$ be profit-maximizing when the supplier acts unilaterally and μ_i^{m} be a quality level offered by supplier i in an m-player oligopoly equilibrium.*

i) For any fixed m: if $\underline{\mu}_i < \widehat{\mu}_i$, then $\mu_i^{\mathrm{m}} < \widehat{\mu}_i$; if $\widehat{\mu}_i = \underline{\mu}_i$ then $\mu_i^{\mathrm{m}} = \underline{\mu}_i$ as well.

ii) Given the case $\mu_i^{\mathrm{m}} < \widehat{\mu}_i$, consider a sequence of equilibrium solutions for supplier i in oligopolies with $m = 2, 3, \ldots$ suppliers. Then $\lim_{m\to\infty} \mu_i^m = \widehat{\mu}_i$.

iii) If in case (ii) the suppliers' costs are symmetric, then the equilibrium quality of service offered in an oligopoly is pointwise increasing in m: $\Pi_i^{\mathrm{m}\prime}(\mu_i) = 0 \Rightarrow \Pi_i^{\mathrm{m+1}\prime}(\mu_i) > 0$.

Part (i) of the proposition implies that optimal quality levels that arise from unilateral action strongly dominate those in symmetric oligopoly. The pseudo-concavity of (6) implies that the set of μ_i's that maximize (6) is a contiguous set. Therefor, for any m the entire set of unilateral profit maximizers must lie above any μ_i^m that is obtained as a part of an oligopoly equilibrium.

Part (ii) shows that as the number of competitors grows, the set of equilibrium quality levels for supplier i converges to that which maximizes expected profits from customer sojourns. Furthermore, when the suppliers are symmetric there is a single, symmetric equilibrium and all μ_i^m's are identical; the "set" of equilibria is a point. Part (iii) shows that in this case the equilibrium point can be seen as "rising" to the limit.

This limiting result has an intuitively appealing interpretation. When it sets quality unilaterally, the supplier uses (2) to maximize the expected profit obtained from each, individual customer sojourn. Implicitly, it is as if the supplier has one chance to serve a customer; once she defects, she never returns. Oligopoly competition does not maximize the expected profits from each customer sojourn. Rather, it maximizes profits in terms of long-run customer share, f_i, and implicitly recognizes that each customer comes back. As the number of competitors m increases, however, share of customer decreases, and in the limit each supplier has zero customer share. That is, in the limit the supplier again has just one chance to serve the customer.

Note that increased competition does not drive quality up so that $r = c_i(\widehat{\mu}_i)$. Rather, the customer's lack of information concerning i's quality allows $r - c_i(\widehat{\mu}_i) > 0$, and each customer sojourn with supplier i earns positive profits. For fixed n, as $m \to \infty$ long-run average supplier profits do vanish, however, since i's market share drops to zero. In the limit, the expected time between customer visits to i becomes infinite.

The proposition also shows that there is a clear incentive for a set of suppliers to coordinate with each other by sharing operating information. For example, suppliers that do not recognize (3) or are very small relative to the overall market effectively maximize with (2) and offer a higher level of service. If suppliers recognize (3) to be the determinant of their revenues, however, they can all lower their quality standards and increase profits. Note that this implicit coordination falls short of outright collusion; the cooperative solution to the symmetric game is $\mu_i = \underline{\mu}_i$ for all i.

4.4 The Oligopoly Model with Fixed Costs

If we include fixed costs in the basic model, then the supplier's objective becomes

$$\max_{\mu_i} \left\{ \Pi_i^{\mathrm{m}}(\mu_i) \triangleq (r - c_i(\mu_i)) \times n\, f_i(\mu_i, \mu_{-i}) \; - \; F \right\} . \tag{12}$$

where $c_i(\mu_i)$ is the original unit cost function that is convex and increasing in μ_i, and F is some positive constant that represents a fixed, per-period expense paid by the supplier.

As (12) shows, the addition of these fixed costs should not change the behavior of existing suppliers. Neither $\Pi_i^{\mathrm{m}\prime}(\mu_i)$ nor $\Pi_i^{\mathrm{m}\prime\prime}(\mu_i)$ depend on F, and expressions for both the first and second order conditions have nothing to do with fixed costs. Thus, for suppliers that are committed to competing in the market, fixed costs mean little, and previous results hold: symmetric costs produce symmetric equilibria; asymmetric costs produce asymmetric equilibria; and quality levels increase with the number of competitors.

Instead, the impact of fixed costs is felt on the number of firms that can profitably enter the market. Given the assumptions of Propositions 3 and 4, a firm that has already entered the market can always make a positive profit. As $m \to \infty$, supplier profits drop to zero because market share, rather than unit profit, drops to zero. With a fixed cost per period F, however, there is a ceiling to the number of firms that can profitably survive. Proposition 5 implies that, with symmetric competitors, each additional entrant drives up quality, drives down market share, and lowers each supplier's profits. In turn, there exists an m beyond which profits before fixed costs are not large enough to cover F.

5 Economies of Scale: Competing M/M/1 Queues

When unit costs exhibit economies of scale, it becomes much more difficult to analyze a general formulation of the supplier's problem. While a marginal increase in quality may increase unit costs, it also will lead to an increase in market share, which may decrease unit costs. Neither of these forces necessarily dominates the other, and it becomes difficult to characterize the effect of economies of scale without making specific assumptions regarding the form of the supplier's cost function.

For example, the introduction of fixed costs into the model analyzed in §4 generates economies of scale: unit costs in period t become $c_i(\mu_i) + F/n_t^i$, which decrease with n_t^i. Yet in this case we saw that, given m suppliers are already competing in the market, economies of scale have no effect on the nature of competitive equilibria. In other cases, however, this is not true.

In particular, in the model of competing $M/M/1$ queues analyzed in this section, there exist fixed costs that behave slightly differently than those of §4. Given a fixed level of service μ_i, the supplier pays a fixed cost "$F(\mu_i)$." Increases in μ_i require higher fixed costs, and the magnitude of the fixed cost is outcome of competition.

Even though the structure of this problem differs only slightly from that of the fixed-cost model in §4, the results of competition differ significantly. While there is still evidence of pressure for symmetric equilibria, that pressure does not appear to be as strong as before. More importantly, we find that as m increases service levels *decrease*, due to loss of economies of scale.

5.1 Model of Competing M/M/1 Queues

We represent a supplier as operating an M/M/1 queue with arrival rate λ_t^i and service rate ξ_t^i. We assume that in period t, the arrival rate is proportional to the size of supplier i's customer base, and without loss of generality we let $\lambda_t^i \equiv n_t^i$ and $\lambda \equiv n$. The processing rate in period t is controlled by the supplier to maintain a fixed level of service.

We define the utility offered by the supplier to be a function of the sojourn time in the system. In some instances, such as some computer applications, system responsiveness is determined by sojourn time, and the measure is appropriate. In others, such as telephone call centers, time spent "on hold" is the more appropriate measure of service, and delay in queue is a better fit.

While the use of sojourn time in an M/M/1 queue is stylized, its essential feature – exponentially-distributed sojourn times – may also be thought of as an analytically tractable approximation for delay. For example, it is well known that in heavy traffic the distribution of delay in more complex, multi-server systems is also exponentially distributed (see Wolff [23]).

To satisfy assumption A1 of the model, suppliers must be able to maintain stationary quality distributions. Since sojourn time in an M/M/1 queue is exponentially distributed with mean

$(\xi_t^i - \lambda_t^i)^{-1}$, as λ_t^i changes, the supplier must change ξ_t^i to maintain a stationary exponential distribution. Thus, using sojourn time, stationary quality strategies are quite simple to characterize: maintain constant $\xi_t^i - \lambda_t^i$.

Suppose that customers are *risk-neutral* with respect to system sojourn time. That is, their disutility grows linearly with the time spent in the system. Therefore, the realization of a sojourn time is identical to the realization of the *negative* of utility, since utility decreases with the wait, and expected quality level is

$$\mu_i \;=\; -\frac{1}{\xi_t^i - \lambda_t^i}\,, \tag{13}$$

the negative of the expected sojourn time. Note that $\mu_i \,<\, 0$.

Then solving (13) for ξ_t^i we have

$$\xi_t^i \;=\; \lambda_t^i \;-\; \frac{1}{\mu_i}\,. \tag{14}$$

Again, given a fixed service level μ_i, in any period t supplier i simply sets ξ_t^i to be $-\frac{1}{\mu_i}$ units above λ_t^i. In effect, given a target service level the supplier makes a fixed investment in "excess capacity" that remains constant in the face of a changing arrival rates. As we noted previously, this differs from the fixed-cost F of §4, which was an all-or-nothing proposition. Here, investment in excess capacity $-\frac{1}{\mu_i}$ increases with service quality.

Let the revenue per arrival be \$$r$ and the cost of service capacity be \$$c_i$ per unit per period. Given fixed μ_i, average profits are straightforward to calculate using (14) and Corollary 1:

$$\Pi_i^{\mathrm{m}}(\mu_i) \;=\; \lim_{t\to\infty}\frac{1}{t}\sum_{s=1}^{t}(r\,\lambda_s^i \;-\; c_i\,\xi_s^i) \;=\; (r\;-\;c_i)\lim_{t\to\infty}\frac{1}{t}\sum_{s=1}^{t}\lambda_s^i \;+\; \frac{c_i}{\mu_i} \;=\; (r\;-\;c_i)\,\lambda\,f_i(\mu_i,\mu_{-i}) \;+\; \frac{c_i}{\mu_i}\,.$$

Then the supplier's profit maximization problem is

$$\max_{\mu_i}\left\{\Pi_i^{\mathrm{m}}(\mu_i) \;\triangleq\; (r\;-\;c_i)\,\lambda\,f_i(\mu_i,\mu_{-i}) \;+\; \frac{c_i}{\mu_i}\right\}. \tag{15}$$

Remark Risk aversion can be introduced into the model in a straightforward fashion, though it requires that we define an appropriate form of (dis)utility function for waiting. For example, suppose a customer has constant relative risk aversion with parameter α (see Kreps

[10]). Then a sojourn time s induces disutility $(-e^{\alpha s})$, and the expected utility of waiting is $\mu_i = \int_0^\infty (-e^{\alpha s})\,(\xi_t^i - \lambda_t^i)e^{-(\xi_t^i-\lambda_t^i)s}ds = -\frac{\xi_t^i-\lambda_t^i}{\xi_t^i-\lambda_t^i-\alpha}$. Solving for ξ_t^i we find $\xi_t^i = \lambda_t^i + \alpha\mu_i/(\mu_i+1)$, and the supplier's problem becomes: $\max_{\mu_i}\left\{\Pi_i^{\mathrm{m}}(\mu_i) \triangleq (r - c_i)\,\lambda\, f_i(\mu_i,\mu_{-i}) - \frac{c_i\,\alpha\mu_i}{\mu_i+1}\right\}$. $\square$

5.2 Results of Oligopoly Competition

Inspection of (15) shows that it is the sum of a positive, convex, and increasing term, $(r - c_i)\,\lambda\, f_i(\mu_i,\mu_{-i})$ and a negative, concave, and decreasing term, c_i/μ_i. Given this form, the equilibria that obtain from oligopoly competition are more difficult to characterize.

While it is not difficult to numerically find asymmetric equilibria, we have not been able to find asymmetric equilibria in which all suppliers make a positive profit. Furthermore, for the case $m = 2$, we can prove that no such (interior) equilibrium exists.

Proposition 6 *When* $m = 2$ *and* $c_1 = c_2 = c$*, there exists no asymmetric equilibrium* $\mu_1 \neq \mu_2$ *for which* $\Pi_1^{m\prime}(\mu_1) = \Pi_2^{m\prime}(\mu_2) = 0$ *and both suppliers earn strictly positive profits.*

Similarly, when the suppliers have asymmetric costs, it is possible to show that there exists no interior symmetric equilibrium. We omit a formal statement, however, due to space constraints.

For symmetric competitors with common unit costs, $c_i \equiv c$, it is not difficult to show that only one symmetric equilibrium exists. To do so, we develop an upper bound on the quality level that might reasonably be offered by the suppliers. First, note that for any fixed arrival rate λ_t^i, a supplier earns zero profit when $r\,\lambda_t^i = c\,\xi_t^i$, and

$$-\mu = \frac{1}{\xi_t^i - \lambda_t^i} = \frac{1}{(r/c - 1)\,\lambda_t^i}\,. \tag{16}$$

Second note that the right hand side of (16) is decreasing in λ_t^i. That is, the minimum expected sojourn time that can be offered on a breakeven basis decreases with the arrival rate; equivalently, the maximum possible level of quality that can be offered profitably is increasing in λ_t^i. This reflects the economies of scale in inherent in queueing systems.

Using λ as an upper bound on λ_t^i, we can define

$$-\overline{\mu} \triangleq \frac{1}{(r/c - 1)\,\lambda} \tag{17}$$

to be a reasonable upper bound on the quality level that a supplier would offer. This is the highest level of quality that could be profitably offered by a monopolist supplier. In the context of oligopoly competition, a supplier that offered quality $\overline{\mu}$ would actually lose money, since the arrival rate would be something less than λ.

Given $\overline{\mu}$, we can then characterize the symmetric equilibrium. The following proposition states this formally. Note that it drops the i subscripts.

Proposition 7 *Suppose there exist m symmetric queues and that $\overline{\mu}$ is defined as in (17). Whenever $\overline{\mu} < \mu^*$ there exists a single symmetric equilibrium that is profitable:*

$$\mu \;=\; m\,\overline{\mu}\,\left[\frac{m}{2\,(m-1)}\left(1 \;+\; \sqrt{1 \;-\; 4\left(\frac{m-1}{m^2}\right)\left(\frac{\mu^*}{\overline{\mu}}\right)}\right)\right]. \tag{18}$$

Thus for any m there exists one profitable, symmetric equilibrium. Furthermore, together Propositions 6 and 7 imply that when $m = 2$ the symmetric equilibrium is unique.

Note that the term in square brackets in (18) is greater than one and reflects the fact that $\mu < m\,\overline{\mu}$ is necessary for suppliers to be profitable. More generally, we have:

Corollary 2 *The symmetric equilibrium (18) is decreasing in m.*

Thus, as m increases the equilibrium behavior of competing M/M/1 queues is the opposite of that of the suppliers in §4: service levels decrease, rather than increase. As m increases, each supplier's $1/m^{th}$ of the market provides a smaller customer base, and the loss of volume prompts suppliers to lower fixed costs – and with them service levels.

6 Discussion

Our model of consumer behavior provides simple, closed-form expressions which allow us to clearly characterize both the source and the effect of the revenue response to changes in a supplier's quality level. In turn, the competitive analysis presented in the paper provides insights into how suppliers should set quality levels when quality cannot be completely controlled and is inherently uncertain.

The equilibrium results generally reflect forces that drive competitors to match the competition, particularly when economies of scale have been exhausted. Furthermore, when a specific functional form for suppliers' costs is used, the analysis allows a supplier to calculate what the target level of quality should be in equilibrium.

If a supplier believes that competitors will not respond (quickly) to its change in quality, then it may also use reaction functions, such as (10) and (15) to determine an "off-equilibrium" strategy as well. (The functions can be used to maximize average profits no matter what quality levels the other suppliers offer.) The information required by i is as follows: an estimate of customer preferences, μ^*; its own cost function, $c_i(\cdot)$; it's current quality level μ_i and market share f_i; and details about the form of optimization problem (9). In particular, given a knowledge μ_i and f_i supplier i can use (3) to back out Δ, the required measure of competitive intensity, without having to know the actual quality levels of individual competitors.

More broadly, we think of the approach taken in this paper as one method of extending the traditional framework of hierarchical production planning to include quality attributes. After using a stylized model to determine the appropriate service level, a more detailed analysis can be performed that explicitly accounts for the many complexities involved in actually operating the system.

For example, inbound telephone call centers are typically operated to minimize costs, subject to a service-level constraint placed on the time calls spend waiting in on hold. Production planning in calls centers is actually carried out at several hierarchical levels – from long-term hiring and training plans, to weekly workforce scheduling plans, down to schemes for real-time routing of calls as they arrive. The service-level constraint is most often defined on an ad hoc basis, but it should not be. Stylized models, such as the model of competing M/M/1 queues presented in this paper, are tractable and often capture the essence of the economies of scale inherent in larger, more complex systems. They can be used to determine the level of service that *should* be provided.

This type of analysis can also be applied to other systems in which suppliers set static service-level targets and dynamically vary capacity to meet the target. In particular, inventory systems are another domain in which the analysis would be appropriate. In them, the service target is the line-item fill rate, and the supplier varies capacity from period to period by changing the inventory level. Because costs do not necessarily vary linearly with demand, however, the steady-state distribution of demand – rather than just the long-term average – is required to characterize long-run average costs.

The analysis may also be enriched in a number ways. For example, a supplier may use common capacity to serve multiple classes of customer, each with it's owns μ^*. In this case, the supplier must make more complex, joint capacity sizing and allocation decisions. Similarly, suppliers may ultimately manage price and quality together. Many of the papers cited in the literature review have analyzed both capacity allocation and price controls, although they have not addressed these factors in the context of customer switching behavior.

Finally, the underlying choice model of [5] may be further developed and extended. For example, switching costs are likely to systematically change customer loyalty behavior. Similarly, advertising and word of mouth also influence customer satisfaction and are likely to influence switching behavior. Furthermore, the underlying assumption that suppliers' quality levels are stationary may be revisited. In particular, in industries that enjoy rapid technological advances, supplier quality may be systematically improving, and expectations of improvement may induce customers to test alternative suppliers more frequently.

Appendix

Proof of Proposition 1

Proof We begin by differentiating (7).

$$\begin{aligned}
\Pi_i''(\mu_i) &= -\frac{c_i'(\mu_i)\, b_t^i}{(\mu^*-\mu_i)^2} + \frac{2\,(r-c_i(\mu_i))\, b_t^i}{(\mu^*-\mu_i)^3} - \frac{c_i''(\mu_i)\, b_t^i}{(\mu^*-\mu_i)} - \frac{c_i'(\mu_i)\, b_t^i}{(\mu^*-\mu_i)^2} && (19)\\
&= \frac{2}{(\mu^*-\mu_i)}\left[\frac{(r-c_i(\mu_i))\, b_t^i}{(\mu^*-\mu_i)^2} - \frac{c_i'(\mu_i)\, b_t^i}{(\mu^*-\mu_i)}\right] - \frac{c_i''(\mu_i)\, b_t^i}{(\mu^*-\mu_i)} && (20)\\
&= \frac{2}{(\mu^*-\mu_i)}\Pi_i''(\mu_i) - \frac{c_i''(\mu_i)\, b_t^i}{(\mu^*-\mu_i)}\,. && (21)
\end{aligned}$$

Suppose that an interior point $\mu_i \in (\underline{\mu}, \mu^*)$ maximizes Π. Since Π is pseudoconcave, $\Pi_i'(\mu_i) =$

0, and from (21) we see $\Pi_i''(\mu_i) < 0$, since $\frac{c_i''(\mu_i)\, b_t^i}{(\mu^* - \mu_i)} > 0$ for strictly concave $c_i(\cdot)$. In turn, $\Pi_i''(\mu_i) < 0$ means that Π is strictly concave at the maximum, so the maximum must be unique.

Otherwise there is no $\Pi_i'(\mu) = 0$ for $\mu \in (\underline{\mu}, \mu^*)$. If $r - c(\mu^*) > 0$ then it is maximum, since a supplier can earn infinite expected profits. If $r - c(\mu^*) \leq 0$, then by inspection of (7) we see $\Pi_i'(\mu^*) < 0$. Furthermore $\Pi_i'(\mu) < 0$ for all $\mu \in (\underline{\mu}, \mu^*)$, otherwise there would be an interior solution. Thus the last mutually exclusive case is $\mu_i = \underline{\mu}$. □

Proof of Proposition 3

Proof First we show that, given symmetric suppliers, there exists no asymmetric pure-strategy equilibrium. By contradiction, consider the quality levels of two suppliers, j and k, in an asymmetric equilibrium and, without loss of generality, suppose $\underline{\mu} \leq \mu_k < \mu_j$.

Letting $c_i(\cdot) \equiv c(\cdot)$, for arbitrary supplier i we note that $r - c(\overline{\mu}) = 0$ implies that $\mu_j < \overline{\mu}$, since μ_j is an equilibrium solution, and inspection of (10) shows that $\Pi_j^{m\prime}(\overline{\mu}) < 0$. Similarly, the fact that $\underline{\mu} \leq \mu_k < \mu_j$ implies that $\Pi_j^{\mathrm{m}\prime}(\mu_j) = 0$, since $\mu_j \in (\underline{\mu}, \overline{\mu})$ is an interior-point solution for the equilibrium. Thus, we have $\Pi_k^{\mathrm{m}\prime}(\mu_k) \leq \Pi_j^{\mathrm{m}\prime}(\mu_j) = 0$.

We can write the first order conditions (10) as:

$$\frac{nf_i(1-f_i)}{\mu^* - \mu_i}\left[(r - c(\mu_i)) - \frac{\mu^* - \mu_i}{1 - f_i}\, c'(\mu_i)\right] = 0 \tag{22}$$

$$\frac{nf_i(1-f_i)}{\mu^* - \mu_i}\left[(r - c(\mu_i)) - \frac{(\mu^* - \mu_i)\sum_{l=1}^{m}(\mu^* - \mu_l)^{-1}}{\sum_{l\neq i}(\mu^* - \mu_l)^{-1}}\, c'(\mu_i)\right] = 0\,. \tag{23}$$

In turn, letting $\delta \triangleq \sum_{\{i: i\neq j,k\}}(\mu^* - \mu_i)^{-1}$, we can rewrite the term in square brackets within (23) and rearrange terms to define

$$g(\mu_1, \mu_2) \;\triangleq\; (r - c(\mu_1)) - \left(\frac{\mu^* - \mu_2}{\delta\,(\mu^* - \mu_2) + 1} + (\mu^* - \mu_1)\right) c'(\mu_1) \tag{24}$$

for arbitrary suppliers 1 and 2. Note that $\Pi_k^{\mathrm{m}\prime}(\mu_k) \leq \Pi_j^{\mathrm{m}\prime}(\mu_j) = 0$ implies that $g(\mu_k, \mu_j) \leq g(\mu_j, \mu_k) = 0$ as well, since the terms outside of the square brackets in (23) are positive.

At the same time, note that for any $\mu_1 < \mu^*$, $\mu_2 < \mu^*$, and convex, increasing $c(\cdot)$, we have

$$\frac{\partial g}{\partial \mu_1} = -(\mu^* - \mu_1)\, c''(\mu_1) \;\leq\; 0 \qquad \text{and} \qquad \frac{\partial g}{\partial \mu_2} = \frac{c'(\mu_1)}{(\delta\,(\mu^* - \mu_2) + 1)^2} > 0\,. \tag{25}$$

This implies that $g(\mu_j, \mu_k) \leq g(\mu_k, \mu_k) < g(\mu_k, \mu_j)$, which contradicts $g(\mu_k, \mu_j) \leq g(\mu_j, \mu_k) = 0$. Thus, given symmetric competitors there cannot be an asymmetric equilibrium.

Second we show that, given all equilibria are symmetric, there exists at most one equilibrium that satisfies the first order conditions. Since the equilibrium is symmetric we let $f_i = 1/m$ and $\mu_i = \mu$ for all i, and we rewrite first order conditions (10) as follows:

$$\Pi^{\mathrm{m}\prime}(\mu) \quad = \quad \left(\frac{m-1}{m^2}\right)\left(\frac{n}{\mu^* - \mu}\right)\left[(r - c(\mu)) - \frac{m}{m-1}\,(\mu^* - \mu)\,c'(\mu)\right] \quad = \quad 0\,. \tag{26}$$

Thus, the first order conditions are satisfied if and only if the terms in the square brackets sum to zero, or equivalently if $g(\mu) \triangleq c(\mu) + \left(\frac{m}{m-1}\right)(\mu^* - \mu)\,c'(\mu) = r$. Differentiating $g(\mu)$ and collecting terms, we see that

$$g'(\mu) \quad = \quad \frac{1}{m}c'(\mu) \ + \left(\frac{m}{m-1}\right)(\mu^* - \mu)\,c''(\mu) \tag{27}$$

is positive for all $\mu < \mu^*$, since $c(\cdot)$ is increasing and convex. Thus there can be at most one μ for which $\Pi^{\mathrm{m}\prime}(\mu) = 0$.

Finally, recall that $r - c(\overline{\mu}) = 0$ implies that $\Pi^{\mathrm{m}\prime}(\overline{\mu}) < 0$. If there is a $\widehat{\mu} \in (\underline{\mu}, \overline{\mu})$ such that $\Pi^{\mathrm{m}\prime}(\widehat{\mu}) = 0$, then $\Pi^{\mathrm{m}\prime}(\mu) < 0$ for all $\mu \in (\widehat{\mu}, \overline{\mu}]$, $\Pi^{\mathrm{m}\prime}(\mu) > 0$ for all $\mu \in [\underline{\mu}, \widehat{\mu})$, and there is exactly one equilibrium. Otherwise, $\Pi^{\mathrm{m}\prime}(\mu) < 0$ for all $\mu \in (\underline{\mu}, \overline{\mu}]$, and $\underline{\mu}$ is the unique equilibrium. □

Proof of Proposition 4

Proof When $c_i(\cdot) = \alpha_i c(\cdot)$, we can write the first order conditions (10) as

$$(r - \alpha_i c(\mu_i))\,n\,\frac{f_i\,(1 - f_i)}{\mu^* - \mu_i} \ - \ \alpha_i c'(\mu_i)\,n\,f_i \quad = \quad 0\,. \tag{28}$$

Part (i). Proof by contradiction. Assume that there exists an equilibrium with suppliers j and k in which $\alpha_j < \alpha_k$ but $\mu_j = \mu_k$ and $\Pi_j^{\mathrm{m}\prime}(\mu_j) = \Pi_k^{\mathrm{m}\prime}(\mu_k) = 0$. Then the only difference between the first order conditions of j and k are the α terms. Furthermore, since $c(\cdot)$ is positive and increasing, all terms in (28) are positive, so it must be the case that $\Pi_j^{\mathrm{m}\prime}(\mu_j) > \Pi_k^{\mathrm{m}\prime}(\mu_k)$, a contradiction.

Part (ii). For $m = 2$, we assume $\alpha_1 < \alpha_2$ and rewrite the first order conditions (28) for arbitrary supplier i as follows:

$$n\,f_i\,\alpha_i\left[(r/\alpha_i \;-\; c(\mu_i))\,\frac{1-f_i}{\mu^*-\mu_i} \;-\; c'(\mu_i)\right] \;=\; n\,f_i\,\alpha_i\left[\frac{r/\alpha_i \;-\; c(\mu_i)}{(\mu^*-\mu_i)\;+\;(\mu^*-\mu_j)} \;-\; c'(\mu_i)\right], \quad (29)$$

since $1-f_i = f_j = \frac{(\mu^*-\mu_i)}{(\mu^*-\mu_i)+(\mu^*-\mu_j)}$. Furthermore, $\Pi_i^{\mathrm{m}\prime}(\mu_i) = 0$ if and only if the term in square brackets equals zero, or equivalently when

$$r \;=\; \alpha_i\,c(\mu_i)) \;+\; \alpha_i\,c'(\mu_i)\,(\mu^*-\mu_i \;+\; \mu^*-\mu_j)\,. \quad (30)$$

Since $c(\cdot)$ is positive and increasing, $\alpha_1 < \alpha_2$ implies that $\mu_1 > \mu_2$ is necessary to satisfy $\Pi_1^{\mathrm{m}\prime}(\mu_1) = \Pi_2^{\mathrm{m}\prime}(\mu_2) = 0$. Thus lower costs prompt supplier 1 to offer higher average quality.

Finally we show that equilibrium profits must be higher for supplier 1. By contradiction, suppose that $\alpha_1 < \alpha_2$ and $\mu_1 > \mu_2$ but that $\Pi_1^{\mathrm{m}}(\mu_1) \leq \Pi_2^{\mathrm{m}}(\mu_2)$. Noting that $\mu_1 > \mu_2$ implies $f_2 < 1/2 < f_1$, we have $\Pi_2^{\mathrm{m}}(\mu_2) \;=\; (r-\alpha_2 c(\mu_2))\,n\,f_2 \;<\; (r-\alpha_2 c(\mu_2))\,n\,\frac{1}{2} \;<\; (r-\alpha_1 c(\mu_2))\,n\,\frac{1}{2}$. Thus, supplier 1 could have earned higher profits than supplier 2 simply by lowering its quality level from μ_1 to μ_2, and μ_1 must not have been profit maximizing in the first place. □

Proof of Proposition 5

Proof We rewrite (10) as follows:

$$\Pi_i^{\mathrm{m}\prime}(\mu_i) \;=\; nf_i(1-f_i)\left[\frac{r-c_i(\mu_i)}{\mu^*-\mu_i} \;-\; \frac{c_i'(\mu_i)}{1-f_i}\right] \;=\; nf_i(1-f_i)\left[\frac{r-c_i(\mu_i)}{\mu^*-\mu_i} \;-\; c_i'(\mu_i) \;-\; \frac{f_i}{1-f_i}\,c_i'(\mu_i)\right]. \quad (31)$$

The first order conditions are satisfied if and only if the term in square brackets equals zero.

Part (i). For any fixed μ_i, the term in square brackets is strictly less than the value of the analogous expression for unilateral action (8), since $f_i \in (0,1)$ and $c_i(\cdot)$ is increasing. Note that the actual equilibrium value of f_i does not matter, as long it is positive. Thus, when $\underline{\mu}_i < \widehat{\mu}_i$, the unilateral optimum, we have $\Pi_i^{\mathrm{m}\prime}(\widehat{\mu}_i) < \Pi_i'(\widehat{\mu}_i) = 0$, so $\mu_i^{\mathrm{m}} \in [\underline{\mu}_i, \widehat{\mu}_i)$. Similarly, , when $\widehat{\mu}_i = \underline{\mu}_i$, then $\Pi_i^{\mathrm{m}\prime}(\underline{\mu}_i) < \Pi_i'(\underline{\mu}_i) \leq 0$, and $\underline{\mu}_i$ is optimal in the oligopoly equilibrium as well.

Part (ii). We recall from (3) that $f_i = 1/(1+(\mu^*-\mu_i)\Delta)$ and write the term in square brackets in (31) as

$$g(\mu_i,\Delta) \;=\; \frac{r-c_i(\mu_i)}{\mu^*-\mu_i} \;-\; c_i{}'(\mu_i) \;-\; \frac{1}{(\mu^*-\mu_i)\Delta}\,c_i{}'(\mu_i) \quad (32)$$

with partial derivative

$$\frac{\partial g}{\partial \mu_i} = \frac{r - c_i(\mu_i) \; - \; c_i{}'(\mu_i)(\mu^* - \mu_i)}{(\mu^* - \mu_i)^2} \; - \; c_i{}''(\mu_i) \; - \; \frac{c_i{}'(\mu_i)}{(\mu^* - \mu_i)^2 \Delta} \; - \; \frac{c_i{}''(\mu_i)}{(\mu^* - \mu_i)\Delta} \tag{33}$$

$$= \frac{1}{\mu^* - \mu_i}\, g(\mu_i, \Delta) \; - \; c_i{}''(\mu_i) \left(1 - \frac{1}{(\mu^* - \mu_i)\Delta} \right) \tag{34}$$

Note that $\Pi_i^{m\prime}(\mu_i, \mu_{-i}) = 0$ implies $g(\mu_i, \Delta) = 0$, which in turn implies $\frac{\partial g(\mu_i, \Delta)}{\partial \mu_i} < 0$. Therefore the inverse of the partial derivative exists at $g(\mu_i, \Delta) = 0$, and by the Implicit Function Theorem there is mapping from Δ to the μ_i that solves $g(\mu_i, \Delta) = 0$ and is continuously differentiable at Δ. Call this function $\mu(\Delta)$.

Now in three steps we show that the sequence $\{\mu_i^m, \mu_i^{m+1}, \ldots\}$ converges to $\widehat{\mu}_i$. First, since $\mu(\Delta)$ is continuous at (μ, Δ), then for any $\varepsilon > 0$ there exists a $\delta > 0$ such for all $|\Delta' - \Delta| < \delta$, $|\mu(\Delta') - \mu(\Delta)| < \varepsilon$. Second, call the equilibrium value of Δ for m suppliers Δ^m and note that

$$\frac{(m-1)}{(\mu^* - \underline{\mu})} \leq \Delta^m = \sum_{j \neq i} (\mu^* - \mu_j)^{-1} \leq \frac{(m-1)}{(\mu^* - \underline{\mu})} \,. \tag{35}$$

Therefore, for any $\delta > 0$ there exists an $m^* > 0$ such that for all $m, n \geq m^*$, $|1/\Delta^m - 1/\Delta^n| < \delta$. Together, these two facts imply that for any $\varepsilon > 0$ there exists an m^* such that for all $m, n \geq m^*$, $|\mu(\Delta^m) - \mu(\Delta^n)| < \varepsilon$. Thus, the sequence $\{\mu_i^m = \mu(\Delta^m), \mu_i^{m+1} = \mu(\Delta^{m+1}), \ldots\}$ converges. Third, the fact that the sequence converges to $\widehat{\mu}_i$ can be seen by comparing (32) to (8) and noting that: 1) $\frac{c(\mu_i)}{\mu^* - \mu_i} \in (0, r/(\mu^* - \overline{\mu}_i)]$ for all $\mu_i \in [\underline{\mu}_i, \overline{\mu}_i]$; and 2) $\lim_{m \to \infty} 1/\Delta^m = 0$.

Part (iii). From Proposition 3 we know that symmetric suppliers will find a unique, symmetric equilibrium, and we can rewrite the term within the square brackets of (31) as follows:

$$\frac{r - c_i(\mu_i)}{\mu^* - \mu_i} \; - \; c_i'(\mu_i) \; - \; \frac{1}{m-1}\, c_i'(\mu_i) \,. \tag{36}$$

Again, note that $\Pi_i^{\mathrm{m}\prime}(\mu_i) = 0$ if and only if (36) evaluates to zero. Now suppose for some m that μ_i satisfies these first order conditions and is the unique, symmetric equilibrium. Then given $m+1$ suppliers and the same μ_i, the far right term decreases from $c_i'(\mu_i)/(m-1)$ to $c_i'(\mu_i)/m$, and the entire expression becomes positive. Thus, $\Pi_i^{\mathrm{m}\prime}(\mu_i) = 0 \;\; \Rightarrow \;\; \Pi_i^{\mathrm{m+1}\prime}(\mu_i) > 0$. $\square$

Proof of Proposition 6

Proof The first order conditions for (15) are

$$(r - c_i)\,\lambda\,\frac{f_i\,(1-f_i)}{\mu^* - \mu_i} - \frac{c_i}{\mu_i^2} = 0 \tag{37}$$

$$(r/c_i - 1)\,\lambda = \frac{\mu^* - \mu_i}{\mu_i^2}\,\frac{1}{f_i\,(1-f_i)}\,. \tag{38}$$

Similarly, we can use (15) to write "profits are greater than zero" as follows:

$$(r/c_i - 1)\,\lambda \geq -\frac{1}{\mu_i\,f_i}\,. \tag{39}$$

Then together (38) and (39) imply that a necessary condition for supplier i to have positive profits in equilibrium is

$$\frac{\mu^* - \mu_i}{\mu_i^2}\,\frac{1}{f_i\,(1-f_i)} \geq -\frac{1}{\mu_i\,f_i} \iff -\frac{\mu^* - \mu_i}{\mu_i} \geq 1 - f_i = f_j\,, \tag{40}$$

since $-\mu_i > 0$.

When $m = 2$ we have $f_j = \frac{(\mu^* - \mu_i)}{(\mu^* - \mu_i) + (\mu^* - \mu_j)}$, so substituting for f_j on the right hand side and rearranging terms we obtain $\mu_j \leq 2\,\mu^*$ as the necessary condition for i to be profitable. This holds for $i = 1, 2$.

Next, using the first order conditions (38) for i and j, we can define μ_j in terms of μ_i as follows:

$$\frac{\mu^* - \mu_i}{\mu_i^2}\,\frac{1}{f_i\,(1-f_i)} = \frac{\mu^* - \mu_j}{\mu_j^2}\,\frac{1}{f_j\,(1-f_j)} \iff \frac{\mu^* - \mu_i}{\mu_i^2} = \frac{\mu^* - \mu_j}{\mu_j^2} \tag{41}$$

since for $m = 2$, the two market shares add up to one, and $f_i\,(1 - f_i) = f_j\,(1 - f_j)$. Rearranging terms, we then have the following sequence of equivalent equalities: $\mu_j^2\,(\mu^* - \mu_i) = \mu_i^2\,(\mu^* - \mu_j) \iff (\mu_j^2 - \mu_i^2)\,\mu^* = \mu_j^2\,\mu_i - \mu_i^2\,\mu_j \iff (\mu_j - \mu_i)\,(\mu_j + \mu_i)\,\mu^* = \mu_i\,\mu_j\,(\mu_j - \mu_i) \iff (\mu_j + \mu_i)\,\mu^* = \mu_i\,\mu_j \iff \mu_j = -\frac{\mu^*\,\mu_i}{\mu^* - \mu_i}$.

Finally we demonstrate that we cannot simultaneously satisfy $\mu_i \leq 2\mu^*$, $\mu_j \leq 2\mu^*$, and $\mu_j = -\mu^*\,\mu_i\,/\,(\mu^* - \mu_i)$. Suppose that $\mu_j \leq 2\mu^*$. Then $-\frac{\mu^*\,\mu_i}{\mu^* - \mu_i} \leq 2\,\mu^* \iff -\mu^*\,\mu_i \leq 2\,\mu^*\,(\mu^* - \mu_i) \iff \mu^*\,\mu_i \leq 2\,\mu^{*2} \iff \mu_i \geq 2\,\mu^*$, since $\mu^* < 0$. Thus, the necessary conditions ($\mu_j \leq 2\,\mu^*$) for both suppliers to have (strictly) positive profits cannot hold when $\mu_j \neq \mu_i$. □

Proof of Proposition 7

Proof We begin with (38). With symmetric data and equilibrium $\mu_i = \mu$ and $f_i = 1/m$ for all i, and the first order conditions become

$$\Pi_i^{\mathrm{m}\prime}(\mu) \;=\; (r-c)\,\lambda\,\frac{f_i\,(1-f_i)}{\mu^*-\mu} \;-\; \frac{c}{\mu^2} \;=\; (r-c)\,\lambda\,\frac{(m-1)/m^2}{\mu^*-\mu} \;-\; \frac{c}{\mu^2}\,. \tag{42}$$

Rearranging terms and substituting $-\overline{\mu}^{-1}$ for $(r/c-1)$, we can verify that $\Pi_i^{\mathrm{m}\prime}(\mu) = 0$ whenever $\left[-\frac{1}{\overline{\mu}}\frac{m-1}{m^2}\right]\mu^2 + \mu - \mu^* = 0$.

The roots of this quadratic equation are $\frac{\overline{\mu}\,m^2}{2\,(m-1)}\left(1 \pm \sqrt{1 - 4\left(\frac{m-1}{m^2}\right)\left(\frac{\mu^*}{\overline{\mu}}\right)}\right)$, and it is not difficult to verify that for $m \geq 2$ they are real whenever $\overline{\mu} < \mu^*$. Furthermore, when $\overline{\mu} < \mu^*$ it can be verified that the less negative root is strictly greater than $m\,\overline{\mu}$ and the more negative root is strictly less than $m\,\overline{\mu}$. Thus, the more negative root $\mu = m\,\overline{\mu}\left[\frac{m}{2\,(m-1)}\left(1 + \sqrt{1 - 4\left(\frac{m-1}{m^2}\right)\left(\frac{\mu^*}{\overline{\mu}}\right)}\right)\right]$, is the unique symmetric equilibrium that is profitable. □

Proof of Corollary 2

Proof Let $\mu(m)$ and $\mu(m+1)$ be the equilibrium quality levels offered for m and $m+1$ suppliers. Then using (18) to verify that $\mu(m+1) < \mu(m)$, we check that

$$\frac{\overline{\mu}(m+1)^2}{2\,m}\left(1 + \sqrt{1 - 4\left(\frac{m}{(m+1)^2}\right)\left(\frac{\mu^*}{\overline{\mu}}\right)}\right) \;<\; \frac{\overline{\mu}m^2}{2\,(m-1)}\left(1 + \sqrt{1 - 4\left(\frac{m-1}{m^2}\right)\left(\frac{\mu^*}{\overline{\mu}}\right)}\right)$$

if and only if

$$\left(1 + \sqrt{1 - 4\left(\frac{m}{(m+1)^2}\right)\left(\frac{\mu^*}{\overline{\mu}}\right)}\right) \;>\; \left(\frac{m^3}{(m+1)^2\,(m-1)}\right)\left(1 + \sqrt{1 - 4\left(\frac{m-1}{m^2}\right)\left(\frac{\mu^*}{\overline{\mu}}\right)}\right).$$

Then it is not difficult to verify that $\frac{m^3}{(m+1)^2\,(m-1)} < 1$ for $m \geq 2$. In turn, it is sufficient if

$$\left(1 + \sqrt{1 - 4\left(\frac{m}{(m+1)^2}\right)\left(\frac{\mu^*}{\overline{\mu}}\right)}\right) \;>\; \left(1 + \sqrt{1 - 4\left(\frac{m-1}{m^2}\right)\left(\frac{\mu^*}{\overline{\mu}}\right)}\right)$$

or that

$$1 - 4\left(\frac{m}{(m+1)^2}\right)\left(\frac{\mu^*}{\overline{\mu}}\right) \;>\; 1 - 4\left(\frac{m-1}{m^2}\right)\left(\frac{\mu^*}{\overline{\mu}}\right) \quad\Longleftrightarrow\quad \frac{m^3}{(m+1)^2\,(m-1)} < 1\,,$$

which, again is satisfied for $m \geq 2$. Thus as m increases, μ decreases. □

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