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# Sedimentary Bed Evolution as a Mean-Reverting Random Walk: Implications for Tracer Statistics

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**Abstract**

Sediment tracers are increasingly employed to estimate bed load transport and landscape evolution rates. Tracer trajectories are dominated by periods of immobility ("waiting times") as they are buried and reexcavated in the stochastically evolving river bed. Here we model bed evolution as a random walk with mean-reverting tendency (Ornstein-Uhlenbeck process) originating from the restoring effect of erosion and deposition. The Ornstein-Uhlenbeck model contains two parameters, *a* and *b*, related to the particle feed rate and range of bed elevation fluctuations, respectively. Observations of bed evolution in flume experiments agree with model predictions; in particular, the model reproduces the asymptotic $t$–1 tail in the tracer waiting time exceedance probability distribution. This waiting time distribution is similar to that inferred for tracers in natural gravel streams and avalanching rice piles, indicating applicability of the Ornstein-Uhlenbeck mean-reverting model to many disordered transport systems with tracer burial and excavation.

**Keywords**

sediment transport, bed evolution, waiting times, sediment tracers, stochastic processes, Ornstein-Uhlenbeck

**Disciplines**

Earth Sciences | Environmental Sciences | Geomorphology | Hydrology | Physical Sciences and Mathematics | Sedimentology

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**Key Points:**

- Sediment tracer waiting times are directly related to bed surface evolution
- Bed surface dynamics can be described as a mean-reverting random walk
- We predict the distribution and limiting behavior of particle waiting times

**Supporting Information:**

- Text S1
- Readme

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# Sedimentary bed evolution as a mean-reverting random walk: Implications for tracer statistics

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**Abstract** Sediment tracers are increasingly employed to estimate bed load transport and landscape evolution rates. Tracer trajectories are dominated by periods of immobility ("waiting times") as they are buried and reexcavated in the stochastically evolving river bed. Here we model bed evolution as a random walk with mean-reverting tendency (Ornstein-Uhlenbeck process) originating from the restoring effect of erosion and deposition. The Ornstein-Uhlenbeck model contains two parameters, *a* and *b*, related to the particle feed rate and range of bed elevation fluctuations, respectively. Observations of bed evolution in flume experiments agree with model predictions; in particular, the model reproduces the asymptotic $t^{-1}$ tail in the tracer waiting time exceedance probability distribution. This waiting time distribution is similar to that inferred for tracers in natural gravel streams and avalanching rice piles, indicating applicability of the Ornstein-Uhlenbeck mean-reverting model to many disordered transport systems with tracer burial and excavation.

## 1. Introduction

Understanding the evolution of landscapes depends on determining long-term rates of sediment particle transport. Due to logistical challenges in directly measuring sediment transport rates, tracers are commonly deployed to estimate sediment flux. Tracer techniques have been applied to a variety of problems, ranging from determination of contaminant dispersion [*Packman et al.*, 2004; *Buffington and Tonina*, 2009] to estimation of deposition or dunadation rates [*Anderson et al.*, 1996; *Repka et al.*, 1997]. Tracer particles interact with sedimentary surfaces that evolve stochastically [e.g., *Schumer and Jerolmack*, 2009]. If performed properly, tracer deployments sample over the full range of stochastic surface evolution and therefore perform an ensemble average that is perhaps more reliable than point sampling of transport rates.

In particular, fluxes of bed load, coarse (sand and larger) particles transported in close proximity to a river bed, are extremely difficult to measure, so their determination often depends on deployment of tagged tracers [*Hassan et al.*, 1991; *Habersack*, 2001; *Ferguson et al.*, 2002; *Nikora et al.*, 2002]. Statistical mechanical treatments [*Ancey et al.*, 2008; *Furbish et al.*, 2012] relate statistics of individual tracer motions to bulk transport behavior. Recent experimental [*Hill et al.*, 2010; *Martin et al.*, 2012] and field [*Nikora et al.*, 2002; *Bradley et al.*, 2010; *Phillips et al.*, 2013] observations indicate bed load particle dispersion that deviates from expectations of simple (Fickian) diffusion underlying many landscape evolution models [e.g., *Hanks*, 2000]. To address non-Fickian behavior of bed load, "anomalous" diffusion models have been developed [e.g., *Schumer et al.*, 2009; *Ganti et al.*, 2010; *Zhang et al.*, 2012]. Central to anomalous transport models are distributions of particle transport times [*Hill et al.*, 2010] and waiting times between transport events [*Martin et al.*, 2012], which characterize the start-and-stop motion typical of tracers [*Einstein*, 1937; *Lajeunesse et al.*, 2010]. Anomalous diffusion arises due to power law tails in particle transport and/or waiting time distributions [*Nikora et al.*, 2002; *Schumer et al.*, 2009; *Ganti et al.*, 2010].

Flume experiments by *Martin et al.* [2012] displayed an asymptotic $t^{-1}$ power law tail for the tracer waiting time distribution cumulative density (i.e., $t^{-2}$ probability density tail), providing a possible explanation for anomalous diffusion. *Voepel et al.* [2013] showed that the waiting time distribution can be related to the "return times" of the evolving sedimentary bed, i.e., times for the bed surface to return to an initial bed elevation following aggradation. However, as in previous studies [*Yang and Sayre*, 1971; *Nakagawa and Tsujimoto*, 1980], *Voepel et al.* [2013] derived a return time distribution (and corresponding particle waiting time distribution) that contains an exponential (not power law) tail [*Voepel et al.*, 2013].

  

Because of the stochastic nature of sediment transport, we expect a sedimentary bed to fluctuate unpredictably up and down in elevation. Fluctuations should be bounded; thus, *Voepel et al.* [2013] modeled bed evolution as a random walk within reflecting upper and lower surfaces. Here we explore an alternative, more subtle model for stochastic bed evolution. Noting that topographic highs are preferentially eroded while lows are preferentially filled [*Straub et al.*, 2009], we treat the evolving bed not as reflecting off hard boundaries but as experiencing a softer mean-reverting tendency. A model for mean-reverting random walks is the Ornstein-Uhlenbeck (O-U) process. A rich theory exists for describing O-U processes [*Uhlenbeck and Ornstein*, 1930; *Gillespie*, 1992], which are observed in a variety of physical [*Doob*, 1942] and financial systems [*Vasicek*, 1977]. Given the similarity of bed surface evolution to O-U processes and the relationship between bed surface fluctuations and particle waiting times, we hypothesize that the waiting time distribution should be described by the return times of an O-U process.

Below, we develop O-U theory for sedimentary bed evolution. Then we describe idealized experiments to track the coupled behavior of tracer trajectories and bed evolution in a water-driven granular bed with bidisperse glass spheres. From these experimental results, we demonstrate the applicability of O-U theory for predicting bed evolution and particle waiting times. Finally, we show how particle waiting times predicted from O-U theory fit with existing observations of natural sediment tracer dynamics, and we offer broader implications of our model for describing sedimentary surfaces and granular tracers.

## 2. Ornstein-Uhlenbeck Theory

Consider a dimensionless random variable, $Y(t)$, with mean 0, describing the evolution of bed elevation through time, $t$. $Y(t)$ evolves through random discrete jumps of length $\xi$ related to entrainment and deposition of particles on the bed. These jumps are separated by random pausing times of duration, $\tau$, describing intervals between transport events. Assuming temporal homogeneity (jumps occur independently of $t$), $\xi$ and $\tau$ may be described by probability densities, which can in turn be approximated as continuous advection and diffusion terms, $A(y)$ and $D(y)$, respectively, in the Fokker-Planck equation (see the supporting information for a more detailed explanation) [*Gillespie*, 1992].

Defining $u(y)$ as the probability, given a current bed elevation, $y$, that the next jump, $\xi$, will be positive, and noting the mean-reverting tendency of the bed evolution process, we treat $u(y)$ as varying linearly with $y$ such that

$$u(y) = \frac{1}{2}\left(1 - \frac{y}{b}\right), \quad -b \le y \le b, \tag{1}$$

where $b$ is a parameter approximating the typical range of $y$ (in fact, $y$ can range outside of $\pm b$, but this linear approximation greatly simplifies the analytical derivation). We treat pausing times as occurring independently of $y$, yielding an exponential pausing time density, $p(\tau)$:

$$p(\tau) = a\exp(-a\tau), \tag{2}$$

where $a$ is a rate parameter for the distribution. Based on this discrete description for jump directions and pausing times, the continuous advection term in the Fokker-Planck equation is approximated by [*Gillespie*, 1992]

$$A(y) = au(y) - a(1 - u(y)) = -\frac{ay}{b}, \tag{3}$$

and the diffusion term is approximated by

$$D(y) = au(y) + a(1 - u(y)) = a. \tag{4}$$

These forms of $A(y)$ and $D(y)$ are described by the well-known Ornstein-Uhlenbeck process.

Treating our discrete birth-death Markov process for bed evolution as a continuous Ornstein-Uhlenbeck process based on the equations described above, we may now make useful predictions about our random variable for bed evolution, $Y(t)$. First, we expect the stationary distribution, $P_s(y)$, of bed elevation to be normal and depend on $b$:

$$P_s(y) = \frac{1}{\sqrt{b\pi}}\exp\left(-\frac{y^2}{b}\right). \tag{5}$$

We note that the standard deviation of this distribution is $\sigma = \sqrt{b/2}$.

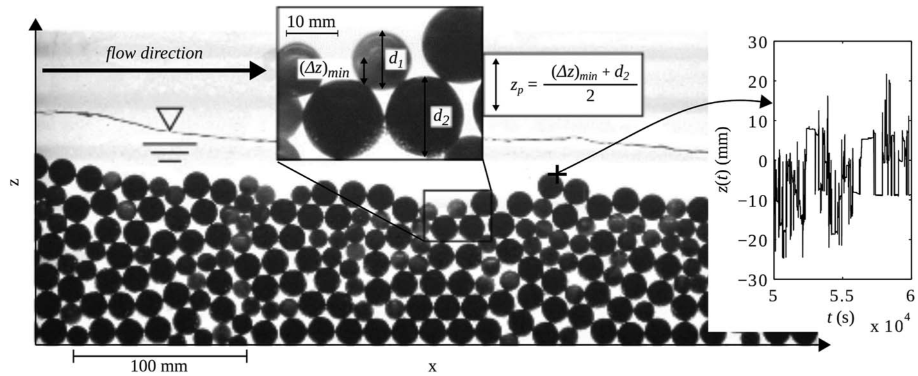


**Figure 1.** Sample image of granular bed from side of experimental flume. Flow is from left to right. Thin streak above the bed is the water surface. The bed surface elevation, $z$, is the uppermost elevation for bed particles. Inset diagram shows diameters of small and large particles ($d_1$ and $d_2$, respectively), and minimum displacement, $(\Delta z)_{\mathrm{min}}$. $z_p$ is determined as the mean of $(\Delta z)_{\mathrm{min}}$ and $d_2$. Plot on the right shows a sample time series, $z(t)$, of detrended bed elevation at the location indicated by the cross.

Second, we can make predictions about return times, denoted as $T_r$. Given an initial value of $Y = y_0$ and subsequent increase in $Y$ from this initial value, how long do we expect $Y$ to evolve until it first decreases below $y_0$? Based on determination of the mean first exit time for $Y$ outside of the interval $(y_0 - \epsilon, \infty)$, where $\epsilon$ is determined from the minimum bed displacement length (see the supporting information for complete explanation), we can estimate the expected conditional mean return time, $\langle T_r(y_0)\rangle$, as

$$\langle T_r(y_0)\rangle = \frac{\epsilon\sqrt{b\pi}}{a} \exp\left(\frac{y_0^2}{b}\right)\left[1 - \mathrm{erf}\left(\frac{y_0}{\sqrt{b}}\right)\right]. \tag{6}$$

For increasing values of $y_0$, equation (6) indicates a decrease in $\langle T_r(y_0)\rangle$, which is expected since the downward mean-reverting tendency of the O-U process grows stronger as $y_0$ increases. No analytical expression exists for the full unconditional distribution of $T_r$; we will estimate this later through Monte Carlo simulations.

## 3. Experiments

The experimental setup consisted of a narrow (19 mm) downward sloping (6%) channel of length $\approx 2$ m through which bidisperse spherical glass beads ($d_1 = 12.2 \pm 0.4$ mm and $d_2 = 16.3 \pm 0.3$ mm with 1:1 number ratio) were propelled by a steady water flow of 37.9 L/min (see *Martin* [2013] for complete description). The narrowness of the channel confined particles to streamwise ($x$) and vertical ($z$) motions, though the $d_1$ particles (necessary to maintain a disordered bed configuration; see *Bohm et al.* [2004] for explanation of crystallization effect for monodisperse spheres) did experience slight transverse motions. This quasi-2-D configuration offered the simplest possible description of bed load transport, and it also allowed for accurate imaging of particle motion by time-lapse photography (using Nikon D5200 camera) of the backlit channel (Figure 1). Rates of sediment feed, $n$, for the five experiments were 12, 30, 60, 90, and 120 particles per minute. Names of experiments indicate respective particle feed rates (e.g., "S60" for 60 particles/min feed). Corresponding rates of time-lapse imaging for these experiments were 30, 6, 12, 30, and 30 photos per minute, respectively.

Flume sidewall images were processed through a thresholding technique distinguishing particle and non-particle areas of the bed. Particle centroid positions were determined (with precision $\pm 1.0$ mm) by an image "watershedding" technique distinguishing outlines of unique individual particles. For a particle centroid in a given frame, if, in the subsequent frame, no particle centroids were detected within a $0.5d$ distance (where $d$ is particle diameter) of the original centroid position, we identified this event as a particle entrainment. A converse method was employed for particle deposition. Waiting times, $T_w$, were then defined as durations between deposition and subsequent entrainment for a particle.

Bed surface $(x, z)$ coordinates were determined (with precision S12: $\pm 0.17$ mm, S30: $\pm 0.50$ mm, S60: $\pm 0.26$ mm, S90: $\pm 0.20$, S120: $\pm 0.17$ mm) as uppermost elevations of continuously contacting particles (Figure 1 inset). Time series of bed elevations, $z(t)$, refer to values detrended relative to long-term mean

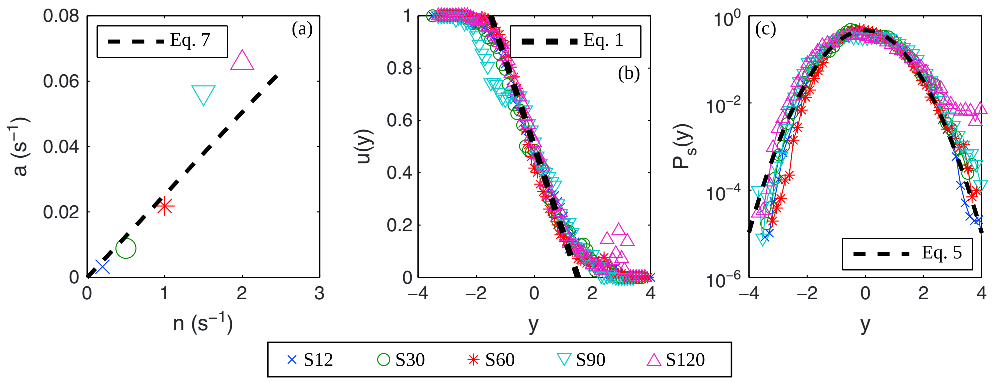


**Figure 2.** (a) Values of $a$ versus feed rates, $n$, for the experiments. Best fit (determined as mean of $a/n$ ratio) yields a value of $k = 0.025$ in $a = kn$. Values of $a$ were determined based on fitting to the exponential form of equation (2). (b) Transition probabilities, $u(y)$, for all experiments. Dashed line indicates values for $b = 1.5$ in equation (1). (c) Stationary distribution, $P_s(y)$, for all experiments, compared to prediction from equation (5) with $b = 1.5$.

bed elevation at each point (Figure 1 inset). Due to discreteness of particle entrainment/deposition processes, changes in bed elevation, $\Delta z$, primarily occurred in the range $(\Delta z)_{\min} < |\Delta z| < d_2$, where $(\Delta z)_{\min} = (d_1/2)\sqrt{1-(d_2/(d_1+d_2))^2} = 5.0$ mm is calculated based on idealized geometry of a small particle resting on two large particles (Figure 1 inset). We compute a "characteristic" bed displacement, $z_p$, as the mean value of this range; $z_p = 10.7$ mm for our experiments. For comparison to theory, we take $y = z/z_p$ as dimensionless bed elevation and $\xi = \Delta z/z_p$ as dimensionless bed displacement.

## 4. Results

To test applicability of the O-U theory, we derived values of $a$ and $b$ based on observation of bed surface evolution. First, we determined distributions of pausing times, $\tau$, for each experiment. Values of $\tau$ were calculated as time increments between successive bed displacements for which $|\xi| \geq \xi_{\min}$, where $\xi_{\min} = (\Delta z)_{\min}/z_p$. The resulting distribution of $\tau$, determined from ensemble statistics of $z(t)$ at all locations, $x$, along the bed, is roughly exponential, and this exponential distribution is independent of initial $y$. For each experiment, values of $a$ were then determined by fit to the exponential portion of the pausing time distribution described by equation (2). $a$ appears to increase linearly with $n$, the particle feed rate, for the five experiments (Figure 2a).

Upon completion of a pausing time, probability of a positive $\xi$ step depends on current bed state, $y$. Figure 2b shows that observed $u(y)$ declines monotonically with increasing $y$; the curve is roughly sigmoidal, with a linear section for $-1.5 < y < 1.5$. A similar trend is apparent for all experiments, with $b = 1.5$ (estimated by eye) providing the best fit for equation (1). Although the modeled linear function for $u(y)$ fails to capture the observed tails, it provides reasonable first-order description of the data that allows for adoption of the O-U model using equation (1).

Based on values of $a$ and $b$ derived from local, short-time dynamics of the evolving granular bed, we compare O-U theoretical predictions to direct observations of long-time bed dynamics. First, we find that the observed stationary distributions of bed elevations, $P_s(y)$, closely match the O-U prediction (equation (5)) for all experiments (Figure 2c). Second, we compare predicted mean return times (equation (6)) to those observed in Experiment S12, which had the largest data set (other experiments yielded similar results). Choosing $\epsilon = \xi_{\min}/2$ as the minimum discrete displacement for a return time, we find close correspondence between predictions and observations of conditional mean return times, $\langle T_r(y_0)\rangle$ (Figure 3a). Third, mean conditional observed waiting times, $\langle T_w(y_0)\rangle$, determined for the $d_2$ (large) particles, are similar to observed and predicted $\langle T_r(y_0)\rangle$ (Figure 3a), indicating correspondence between particle waiting and bed surface return times.

In addition to mean behavior, we are interested in distributions of return and waiting times for their effects on sediment tracer dispersion. To our knowledge, existing theory does not provide predictions for the full (unconditional) return time distribution for the O-U process. Instead, we resort to Monte Carlo methods to simulate $y(t)$ evolution by the advection and diffusion equations (equations (3) and (4)) over $10^7$ s (see the

  

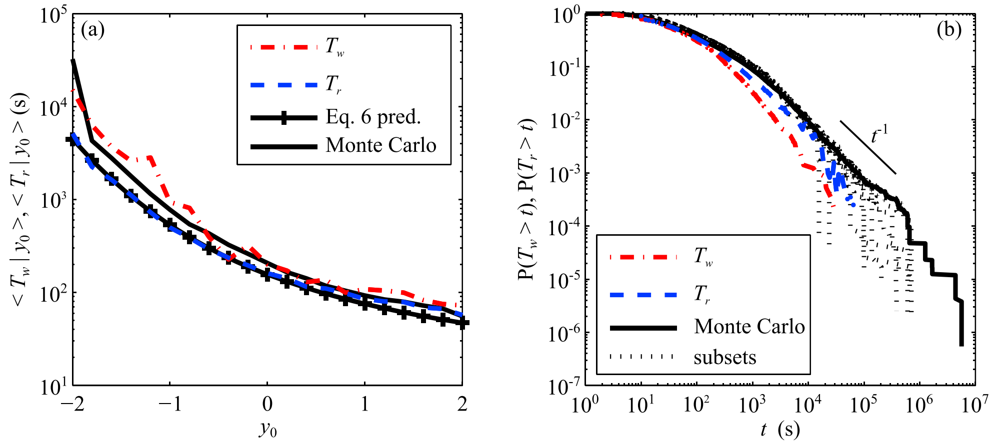


**Figure 3.** (a) Comparison of observed mean waiting times ($\langle T_w \rangle$) and mean bed surface return times ($\langle T_r \rangle$) for Experiment S12 conditioned on $y_0$, compared to $\langle T_r \rangle$ predictions from equation (6) and computed from Monte Carlo simulation. (b) Experiment S12 particle return time and waiting time exceedance probabilities ($P(T_w > t)$ and $P(T_r > t)$, respectively) versus $T_r$ distribution predicted by Monte Carlo simulation run for $10^7$ time steps of duration $\Delta t = 1$ s. In addition, return times computed from 10 subsets of the Monte Carlo simulation (each of duration $10^6$ s) are plotted alongside the main $T_r$ results. $t^{-1}$ line is shown for comparison.

supporting information for further explanation). From synthetic $y(t)$ time series, we then compute $T_r$ based on first passage times as described above. As first confirmation of the validity of the Monte Carlo simulation, we note similarity between analytical (equation (6)) and simulation predictions for $\langle T_r(y_0) \rangle$ (Figure 3a). Slight differences between simulations and predictions likely arise due to arbitrary choice of $\epsilon = \xi_{\min}/2$.

Figure 3b compares the Monte Carlo simulated return time exceedance probability distribution, $P(T_r > t)$, to observed distributions of $T_r$ and $T_w$ for Experiment S12. The O-U predicted $T_r$ distribution not only agrees with observations of $T_r$ but also predicts the distribution of $T_w$ with reasonable accuracy. Differences between observed $T_r$ and $T_w$ are apparent and may result from arbitrary choice of displacement lengths for identifying return and waiting time events. However, these differences appear primarily in the timing of transition to asymptotic tail behavior rather than the slope of this tail. Other experiments displayed similar correspondence between return and waiting time distributions but are not shown here for brevity.

All experiments show a similar waiting time distribution as Experiment S12 (Figure 4a). The major difference is the time scale at which $T_w$ curves approach limiting power law tail behavior. Multiplying $t$ by $a$ to normalize the $T_w$ distributions for the five experiments, Figure 4b indicates reasonable data collapse for all experiments. For $at > 1$, all experiments appear to converge to a power law tail with $t^{-\alpha}$, where $\alpha \approx 1$ or slightly greater than 1. While all experiments display similar asymptotic behavior, slight differences in the distributions remain following normalization by $a$, possibly resulting from the arbitrary choice of $\epsilon$ for determining waiting times or from other artifacts of the imaging methods.

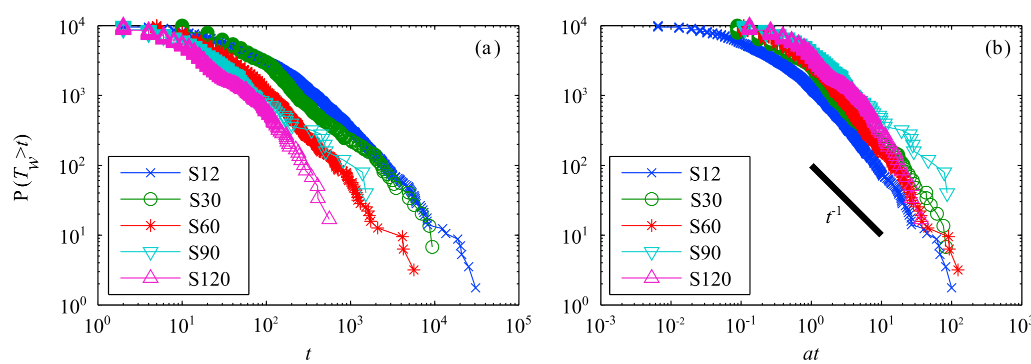


**Figure 4.** (a) Waiting time exceedance probability distributions ($P(T_w > t)$) for the five experiments versus $t$. (b) Normalizing time by bed activity parameter, $a$, produces a reasonably good collapse of the data. Limiting power law scaling is slightly steeper than the $t^{-1}$ line shown for comparison.

## 5. Discussion

Treatment of sedimentary bed evolution as an Ornstein-Uhlenbeck process, or mean-reverting random walk, predicts a bed return time distribution with asymptotic power law tail, i.e., $P(T_r > t) \sim t^{-\alpha}$, with tail parameter $\alpha \approx 1$ for $at > 1$. $\alpha = 1$ is the minimum value for which the distribution has convergent mean. When $at < 1$, local slope of the $P(T_r > t)$ distribution indicates values of $\alpha < 1$; i.e., the mean is nonconvergent. This nonconvergent behavior is expected for random walks unconstrained by boundary effects [i.e., *Voepel et al.*, 2013]; therefore, the parameter $t = 1/a$ represents the time scale over which boundary effects become significant and limit bed evolution. However, whereas *Voepel et al.* [2013] modeled this boundary effect as an exponential truncation to the return time distribution, our Ornstein-Uhlenbeck model yields an asymptotic power law distribution. This difference likely results from the fact that *Voepel et al.* [2013] treated bounding bed elevations as hard reflecting surfaces (i.e., "first passage on a bounded interval" [*Redner*, 2007]), whereas the Ornstein-Uhlenbeck model exerts a "softer" boundary encoded in an ever increasing restoring force away from the mean. Interestingly, the $T_r$ distribution determined from the Monte Carlo simulation (duration $10^7$ s) shows apparent truncation for $t > 10^6$; however, when $T_r$ distributions are computed from $10^6$ s subsamples of the full simulation, these appear truncated when $t > 10^5$ (Figure 3b). In other words, truncation effects appear to result from finite sampling rather than dynamics of the bed evolution process.

Our model and observations yield bed surface return times and tracer waiting times with asymptotic tail parameter, $\alpha \approx 1$, matching flume experiments in a three-dimensional channel with natural angular sediments [*Martin et al.*, 2012]. For such power law distributions of waiting times accompanied by thin-tailed hop lengths [e.g., *Hassan et al.*, 1991; *Lajeunesse et al.*, 2010; *Martin et al.*, 2012] that are asymmetric (i.e., particles can move only one direction, as in the longitudinal profile of a river), *Weeks et al.* [1996] predict that particle displacement variance, $\sigma_x^2$, will grow with time, $t$, as $\sigma_x^2 \sim t^\gamma$, where $\gamma$, the dispersion scaling exponent, is given by $\gamma = 3 - \alpha$. Our case of $\alpha \approx 1$ yields a value of $\gamma = 2$. For comparison, *Phillips et al.* [2013] found $\gamma = 1.9$ for tracers in a natural stream, in general agreement with our direct experimental observation. Our model of bed evolution as a stochastic mean-reverting process therefore provides a plausible explanation for tracer dispersion in natural streams.

In addition to explaining the particle waiting time distribution that gives rise to observed sediment tracer dispersion behavior, the Ornstein-Uhlenbeck model may potentially explain evolution of a broad variety of sedimentary and granular surfaces. For example, steadily driven avalanching rice piles experience stochastic local bed elevation fluctuations around a mean value; as in our experiments, rice pile tracers exhibit a residence time distribution with $t^{-1}$ asymptotic tail [*Christensen et al.*, 1996]. Similarly, solutes [e.g., *Haggerty et al.*, 2002] and fine particles [e.g., *Drummond et al.*, 2014] display power law tailed residence times as they are stored and released from evolving sedimentary beds. Other examples include river deltas [*Ganti et al.*, 2011], bed forms [*Martin*, 2013], and bedrock rivers [*Finnegan et al.*, 2014] or any system where tracers interact with a fluctuating but bounded surface. Many of these systems (both in the field and laboratory) display a Gaussian stationary bed elevation distribution [*Crickmore and Lean*, 1962; *Yang and Sayre*, 1971; *Nakagawa and Tsujimoto*, 1980; *Wong et al.*, 2007; *Coleman et al.*, 2011], which is consistent with the Ornstein-Uhlenbeck model (Figure 2c).

The parameter $a$ for pausing times is determined by the frequency of changes in bed elevation, $z$. In the field, $a$ could be determined directly from repeat bed surveys, which are becoming increasingly accessible [e.g., *Anderson and Pitlick*, 2014]. Because particle erosion and deposition generate $z$ fluctuations, we expect the frequency of bed elevation change to be linearly related to the rate of particle movement, i.e.,

$$a = kn, \tag{7}$$

where $k$ is a proportionality constant effectively describing the fraction of passing particles that deposit or erode at a single point on the bed. Recall that the $t^{-1}$ power law tail of the $T_w$ distribution occurs for $t > 1/a$; thus, faster rates of sediment input, $n$, correspondingly decrease the time for the evolving bed to experience mean-reverting boundary effects that produce convergence toward the power law tail. Based on O-U theory, $a$ is equivalent to a length-normalized diffusivity, $D$, by equation (4). In this manner, equation (7) is reminiscent of particle diffusion in granular flows, for which particle diffusivity, $K$, is proportional to the imposed shear rate, $\dot{\gamma}$, as $K/d^2 = k\dot{\gamma}$ [*Natarajan et al.*, 1995; *Garzo*, 2002; *Utter and Behringer*, 2004; *Wandersman et al.*, 2012], where $d$ is particle diameter and $k$ is a proportionality constant (as in equation (7) above). Relating our experiments to this granular kinetic theory, $n$ is a scale parameter equivalent to $\dot{\gamma}$ while $a$ is equivalent to $K/d^2$. Following this equivalence, the fitted value for our data, $k = 0.025$ (Figure 2a), is similar in magnitude to values reported for dry granular flows at comparable shear rates [*Utter and Behringer*, 2004]. While not definitive, this suggests that bed load transport exhibits similar diffusive dynamics to granular shear flows.

The parameter $b$ in our model (equation (1)) encodes the bounding and mean-reverting effect of the surface elevation range. $u(y)$ decreases with $y$ because particles with elevations significantly above the mean are more susceptible to erosion, due to greater flow exposure and higher fluid velocities with increasing height; conversely, particles at very low elevations are prone to deposition. $b$ here is a strictly empirical value that depends on the specific configuration of our experiments. The value of $b = 1.5$ in our system indicates a relatively narrow range of fluctuations over a few particle diameters, consistent with field observations of bed load [*DeVries*, 2002]. In general, $b$ could be determined from the range of bed elevation fluctuations described by the stationary distribution in equation (5).

## 6. Conclusion

Particles moving as bed load in a sheared fluid flow experience intermittent transport, leading to dispersion of tracers within the particle population. Tracers experience a broad distribution of waiting times in their trajectories related to return times in stochastic evolution of the bed surface. Modeling bed evolution as an Ornstein-Uhlenbeck (O-U) process, i.e., a mean-reverting random walk, yields asymptotic $t^{-1}$ scaling in the return time distribution, which in turn explains the distribution of waiting times in idealized flume experiments. Our O-U bed evolution model contains two parameters, $a$ and $b$. $a$ describes frequency of bed surface changes related to particle flux, while $b$ describes bounds on bed elevation fluctuations. Both parameters could be estimated in rivers based on repeat topographic surveys, providing the basis for testing and applying O-U theory to practical field situations. More generically, our O-U model provides a testable framework for understanding tracer burial and excavation in avalanching rice piles, sheared granular flows, and any system with a stochastically fluctuating surface. Our findings provide fresh insight into the notoriously challenging problem of predicting bed load sediment transport and open up new avenues for cross-disciplinary work relating the dynamics of limited tracer populations to underlying granular processes in sedimentary systems.

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