Reforming German Civil Servant Pensions: Funding Policy, Investment Strategy, and Intertemporal Risk Budgeting

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Keywords
German, Funding, Intertemporal, Risk, Budgeting

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We analyze the risks and rewards of moving from an unfunded defined benefit pension system to a funded plan for civil servants in Germany, allowing for alternative portfolio mixes using a Monte Carlo framework and a Conditional Value at Risk metric. First, we estimate contributions as a percent of salary that would fully fund future benefit promises for active employees. Second, we identify an investment strategy for plan assets that will minimize worst-case pension costs; this turns out to be 22% in equities, 47% in bonds, and 30% in real estate. Third, we explore the time path of pension fund asset shortfalls and the chances of contribution holidays for current and future generations. We show that moving toward a funded pension system for German civil servants can be beneficial to both taxpayers and civil servants.

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Throughout the developed world, public sector employees have traditionally been promised a pay-as-you-go (PAYGO) defined benefit (DB) pension plan. In such a system, current pensions are paid through taxes or contributions made by the working generation. These systems, however, face increasing financial difficulties, since a shrinking working-age group has to support more and more retirees. If these developments continue and the systems remain unaltered, civil servants pension benefits sooner or later will have to be reduced or contributions increased, in either case requiring unpopular political decisions. At the same time, it is often argued that moving public employee pension plans toward funded systems may offer a resort to the deteriorating financial situation of these plans. The rationale behind this argument is that accumulating assets and investing them in the capital markets will strengthen the rights of plan participants, increase transparency, and might generate enhanced returns, which in turn help to reduce civil servants’ pension costs. This chapter explores the feasibility of implementing a funded pension system for German civil servants who have been promised an unfunded defined benefit (DB) plan which faces future shortfalls.

In some countries, civil servant pension plans are well funded, as in the United States or the Netherlands (ABP 2006, Mitchell et al. 2001). But German civil servant DB plans are promised benefits related to final salary and service years, yet few of these promises are backed by assets. As political decisionmakers have grown more conscious of the economic costs of public pensions, some action has already been taken. The German state of Rhineland-Palatinate was the first to introduce a fully funded pension scheme for newly recruited civil servants in 1996, which is currently endowed with 20-30 percent of the salaries of those covered by the plan. The state of Saxony followed along these lines and introduced a comparable scheme in 2005, which fully covers all employees that joined civil service since
1997. Both states essentially restrict their funds’ investment universe to government bonds, and thereby forego the opportunity to improve the funds’ financial situation by earning higher returns in equity markets. This is in sharp contrast to empirical evidence on international public pension plans’ investment strategies. For instance, Dutch-based ABP, the pension fund for those employed by the government and in education, only invests around 40 percent of plan assets into fixed-income securities, including a substantial fraction of corporate bonds (ABP 2007). Similar results are reported for the United States, where state pension plans on average only invest about one third of their assets in bonds and other debt instruments (Wilshire 2007).

As German civil servants pensions are far from being fully funded, and since in those cases where plans have at least some assets, investment policies are particularly conservative, more efforts need to be made to provide political decisionmakers with reliable information on the opportunities and risks associated with moving toward a funded pension system for civil servants. To this end, this chapter studies the implications of partially prefunding the civil servants pension plan in the German state of Hesse. We introduce a hypothetical additional tax-sponsored pension fund for currently active civil servants, similar to those already introduced in Rhineland-Palatinate and Saxony. Contributions paid into the fund are invested in the capital markets and investment returns are used to alleviate the burden of increasing pension liabilities. Based on stochastic simulations of future pension plan asset development, we estimate the expectation as well as the Conditional Value at Risk (CVaR) of pension costs. These are then evaluated in an effort to determine the optimal asset allocation that controls worst-case risks while still offering relief with respect to expected economic costs of providing the promised pensions.

This study extends prior work by Maurer, Mitchell, and Rogalla (2008) in several ways. First, we give a more detailed overview on future structural changes in the civil service population, which will contribute to a further deterioration of the public pension plan’s
financial situation. Second, we introduce a more sophisticated stochastic asset model of the vector autoregression variety which includes stocks, bonds, and real estate as an alternative asset class available to the plan manager. Finally, we study the intertemporal risk and return patterns of the suggested investment policy for current and future taxpayers.

In what follows, we first offer a concise description of the characteristics of the German civil service pension plan. Next we evaluate future public plan obligations for taxpayers in a non-stochastic context and derive the payroll related deterministic contribution rate that is able to finance accruing pension benefits in the long run. Drawing on these results, we take a plan manager’s perspective to determine reasonable investment strategies for accumulating plan assets within a stochastic asset/liability framework. The final section summarizes findings and their implications for managing funded public sector pension plans in Germany.

**German Civil Service Pension Plan Design**

Public sector employees constitute about 14 percent of the German workforce, classified into two groups: public employees and civil servants. The legal status of the roughly three million public employees is based on private sector law, while that of the 1.4 million civil servants is codified in public law. Initially, the rights and duties of civil servants were codified in the 1792 Prussian General Code, and with some modifications, the basic characteristics of this system are still in force and manifested after WWII in the German constitution (Gillis, 1968). Key components include the fact that civil servants commit to work for public sector tasks for life, they have no right to strike, and they are subject to special disciplinary rules. In exchange for this commitment, the government provides them with an appropriate salary depending on specific career paths, offers particular pre-entry training, and supplies lifelong health care, disability, and pension benefits. In contrast to the
United States, the legal status, the salary packages, and the retirement benefits for German civil servants are quite homogenous at the federal, state, and local levels.

At retirement, German civil servants receive a noncontributory, tax-sponsored, and cost-of-living-adjusted defined benefit (DB) type lifetime annuity which depends on final salary, the number of pensionable years of service in the public sector, and the retirement age. The noncontributory plan for civil servants comes at the price of significantly lower gross salaries compared to other public sector workers with equivalent qualifications. German civil servants are neither offered complementary occupational pension plans nor covered by the national social security system. Hence, their retirement benefits are higher than those of private sector workers who may be eligible for social security as well as supplementary occupational pension benefits (Heubeck and Rürup 2000).

Some argue that the generosity of civil servant pensions serves as partial compensation for their lack of portability, since accrued pension benefits are substantially if the worker were to leave public employ. Naturally, this substantially reduces turnover, particularly among older civil servants with long tenure. On the other hand, if a civil servant were to change jobs within the public sector, he would be permitted to remain in the same pension plan (even when moving from one state to another). From the plan sponsor’s perspective, the relatively generous but nonportable DB pension scheme serves as a useful instrument for attracting, recruiting, and retaining a highly skilled and stable workforce.

Of late, however, German public pension plan generosity has been substantially reduced. In 2003, a new pension benefit formula was introduced that reduced the retirement benefit formula from 1.875 percent of final salary per year of service down to 1.79375

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1 To be precise, the benefits of retired civil servants are adjusted according to the general salary increase of active civil servants.
2 Civil servants are exempt from unemployment insurance and the state covers a certain fraction of health care expenses for civil servants and their families. These fractions range from 50% - 85%, depending on family status, number of children, and state. See Börsch-Supan and Wilke (2003).
3 If, for example, a civil servant were to quit service and take a job in the private sector, he would sacrifice about 50 percent of his accrued pension claims. In this case, the state pays to the national social security system the employee’s foregone employer contributions.
percent.\footnote{To compensate for this cut in pension benefits, civil servants are allowed to (voluntarily) invest up to 4% of their salary (with a ceiling of 2,100 p.a.) into tax sponsored personal retirement account also known as “Riester accounts;” for details see Maurer and Schlag (2003).} After a maximum of 40 pensionable service years, a retiring civil servant is promised a maximum replacement rate of 71.75 percent. A surviving spouse receives survivorship benefits of 55 percent (formerly 60 percent) of the deceased civil servant’s pension. Orphans receive 20 percent and half-orphans 12 percent.

Current pensioners, who retired under the old formula with pension benefits worth 75 percent of their final salaries, will also be affected by the benefit cut. For several years, their post-retirement benefit increases will be marginally reduced, until their replacement rate will be cut to the same 71.75 percent. The nominal pension paid to a retired civil servant will nonetheless increase over time.

In the past, civil servants’ standard retirement age has been 65, though they may retire as young as age 63 with a reduction of 0.3 percentage points per month. Special provisions for public safety workers with physically demanding jobs like police officers or fire fighters allow for retirement at earlier ages without a benefit cut. In mid 2007, however, several states as well as the federal government have followed Germany’s social security system in moving gradually to 67 as the normal retirement age.

**Deterministic Valuation of Future Public Pension Obligations**

Next we analyze the actuarial status of the civil servants’ pension plan in the state of Hesse.\footnote{Being part of former West Germany, Hesse’s civil service population appears to be rather representative of the approximately 1.5 million active (which is about 4.5% of the German workforce) and 900,000 retired civil servants in Germany as a whole. This section draws on Maurer, Mitchell, and Rogalla (2008).} Our prior research has found that already-accrued public pension liabilities for the state are on the order of 150 percent of current explicit state debt (Mitchell, Maurer, and Rogalla 2008); this analysis assumes that these claims already accumulated will be financed from other sources. In this section, we conduct a deterministic actuarial valuation of pension liabilities that will accrue in the future to existing employees and new hires over the next 50
years. We draw on a datafile provided by the Hessian Statistical Office which contains demographic and economic information on more than 100,000 active and retired civil servants in Hesse as of the beginning of 2004, including their age, sex, marital status, line of service (for active civil servants), and salary/pension payments. On average, 45 percent of the active workers are female, the average salary (in 2004) is EUR 39,000, and it is a relatively old group, averaging age 45.

Figure 1 depicts the age distribution of the sample of active employees. This distribution peaks for employees in their late 40s and early 50s. Thus, in 15 to 20 years’ time, a significant group of civil servants will retire in a concentrated fashion, and it will result in a jump in required pension payments. At the same time, there are relatively few active civil servants in their late 50s or early 60s, a pattern attributable to generous early benefits in the past.

**Figure 1 here**

**Demographic Assumptions.** In what follows, we project pension accruals of future generations of employees. Our approach is to project the time path of age and salary for all civil servants through time (we assume that the marital status remains constant). When a position becomes vacant, a new civil servant is assumed to be recruited (with equal probability of being male or female); the new worker’s age is assumed to be the average age of entering civil service, accounting for average time spent on position-related education or other types of public service that will be credited as pensionable years in civil service. The salary of the newly hired civil servant is assumed to be in line with the age-related remuneration for the position; the marital status is assumed to be that of the previous position holder. Since turnover other than retirement is virtually nil we assume no employee turnover prior to retirement; hence we do not account for early retirement, disability benefits, or dependents’ benefits due to death in service. In terms of mortality projections, we use those

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6 This time horizon could be easily extended, but after 50 years, all active workers will be fully included in the new funded system.
derived by Maurer, Mitchell, and Rogalla (2008) who have prepared mortality tables specific to retired German civil servants based on a dataset for the state of Hesse covering the period 1994 to 2004. They show that retired civil servants tend to enjoy lower mortality than the overall population. Throughout this study we also employ these tables, accounting for decreasing future mortality rates according to the trend functions published by the German Association of Actuaries (see DAV 2004). We also assume that the pension reforms are fully implemented, that is, maximum benefits only amount to 71.75 percent of final salary and the retirement age is 67.

**Economic Assumptions.** Three interrelated economic factors significantly influence the valuation of pension plan liabilities: anticipated inflation, expected salary growth rates, and investment returns on plan assets (c.f. Hustead and Mitchell 2001). While Germany has experienced only moderate inflation over the last decades, it remains an important factor for the valuation of future pension cash flows. For this reason, and because salaries as well as pensions tend to be maintained in real terms, this study therefore uses real financial values and investment returns throughout.

An issue that looms large in the public pension plan arena is what discount rate one should in valuing future promised benefits (Waring, 2008). Naturally, the discount rate selected directly influences both the reported pension liability and the contribution rate required to fund the promises. The current debate coalesces around whether public plans should use an actuarial versus an economic concept of liabilities. Many actuaries select a discount rate which reflects projected (or historical) asset returns; accordingly, if a portion of the pension fund is held in equities, the selected discount rate will include an *ex ante* risk premium which may not, in fact, be realized *ex post*. This approach also tends to downweight future liabilities and upweight the benefits of investing in stock. By contrast, if returns are lower than expected, future generations of taxpayers may end up bearing the investment risk,

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7 See Blake (2006), Gold (2003), and also Waring (2008).
if actual returns fall below the expected rates. This strategy is intended to smooth contribution rates required over time.

By contrast, many economists contend that a public plan should use a (nearly) risk-free rate on government bonds to compute liabilities, as this reflects the state’s financing costs. We argue that the riskless interest rate must be used for reporting the actuarial present value of pension promises for accounting purposes and for solvency planning, as well as for setting the contribution rates. Our simulation assumes that this real risk free interest rate is three percent for the base case; we also evaluate an alternative set of results with a real interest rate of 1.5 percent. Using a risk-free government bond rate is consistent with the often-recommended practice of nearly fully matching public plan assets and liabilities. Nevertheless, this does not mean that the public entity must, of necessity, automatically invest entirely in government bonds. Instead, it might be appropriate to invest at least part of the pension portfolio in more risky equities, depending on the plan sponsor’s risk preferences.

Projected Future Benefits for Current and Future Civil Servants

In order to move the public DB pension plan toward funding, assets need to be built up and invested in the capital markets to back the accruing liabilities. Consequently, the plan sponsor’s foremost task is to assess what contributions are required to finance the benefits based on pension liability patterns specific to the plan. As pension benefits for Hessian civil servants are calculated as a percentage of final salary times years of service, the normal cost of the plan (i.e. the cost accrued in each year supposing actuarial assumptions are realized) is determined according to the aggregate level percentage of payroll method. Total projected pension plan costs are stated as a percentage of active members’ overall payroll (McGill et al. 2005); we derive the actuarial present value of future pension benefit obligations (PBO) based

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8 The difference between the average nominal par yield of long term German government bonds and the average inflation rate for the post-WWII period is about 4 percent. Inflation protected bonds in the Eurozone currently yield about 2 percent. This market is currently not well developed for government bonds (especially those with long durations) which supports the assumption of a real interest rate of 3%.
on future salaries and service years over the next 50 years (2004-53), evolving our initial population through time in line with the dynamics discussed above. We determine the value of future pension benefits for active and future civil servants based on the projected benefit obligation (PBO) formula:

\[
PBO = \sum_i \frac{1.79375 \cdot \tau_i \cdot S_{67,i} \cdot \overline{a}_{67,i}}{(1 + r)^{\tau_i - Age_i}},
\]

where (for each civil servant \(i\) of \(Age_i\)) \(\tau_i\) is the number of service years as of retirement, \(S_{67,i}\) is the (expected) salary at retirement age 67, \(\overline{a}_{67,i}\) is the immediate pension annuity factor, and \(r\) is the discount rate. After 50 years, we assume that the plan is terminated and conduct a discontinuance valuation.

The relative amount of the present values of pension liabilities to salary payments represents the deterministic annual contribution rate as a percentage of the payroll required to fund future pension promises. In our non-stochastic analysis, we presume that these contributions are paid into the pension plan at the beginning of each year. Plan assets are invested in the capital markets and earn a fixed (i.e. non-stochastic) return equal to the rate at which plan liabilities are discounted for valuation purposes. Table 1 summarizes the results for our base case with a real discount rate of 3 percent (Column 1) as well as for our alternative set-up, i.e. a discount rate of 1.5 percent (Column 2). The present values of current workers’ projected pension liabilities and salaries are reported along with the ratio of the present value of pension costs to salaries and, therefore, the notional contribution rate required to finance the pension promises.

\textit{Table 1 here}

In our benchmark case with the 3 percent discount rate, the present value of future pension liabilities comes to € 20.8 billion (Row 1, Column 1), whereas salary payments have

\[9\] As noted above, we set aside pension benefits of current retirees as well as those already accumulated by currently active civil servants and assume that these will be covered by some other financing arrangement. Thus, only future benefit accruals by active civil servants will be covered by this scheme.
a present value of € 111.5 billion (Row 2, Column 1). The ratio of present values representing
the average required contribution rate is 18.7 percent of salaries for each future year (Row 3,
Column 1). This comes close to the contribution rates for the civil servants’ pension plan of
Rhineland-Palatinate, which range from 20 to 30 percent depending on service level. It comes
at no surprise that these results are highly sensitive to the discount rate applied. A lower
discount rate increases both the present value of pension liabilities as well as the present value
of salary payments. However, as pension liabilities have a longer duration than salary
payments, contribution rates increase with falling discount rates. In our alternative setting
with a real discount rate of 1.5 percent, the present value of pension liabilities more than
doubles to € 44.8 billion while discounted salary payments only increase by less than 50
percent to € 149.3 billion (Rows 1 and 2, Column 2). Hence, the contribution rate rises to 30
percent (Row 3, Column 2).

**Pension Plan Management in a Stochastic Environment**

Uncertain capital market returns on pension plan assets are of major concern to DB
pension plan sponsors. While market gains may reduce required contributions and therefore
overall plan costs, excessive investment losses can also require a plan sponsor to make
supplementary contributions in an effort to recover from funding deficits. Selecting an
adequate asset allocation for plan funds is therefore of utmost importance to the plan manager.
Therefore in this section we evaluate the public plan sponsor’s decision-making process, to
identify a reasonable plan asset allocation in a world with uncertain investment returns. This
requires formulating an intertemporal objective function guiding tradeoffs between capital
market risk and returns, as well as between supplementary contributions and cost savings.

**Plan Design, Pension Manager Objectives, and Asset/Liability Modeling.** We follow the
approach of Maurer, Mitchell, and Rogalla (2008) who minimize the worst-case total cost of
running plan over a future long-term time horizon. The funded pension scheme we model is
designed as follows: at the beginning of every period $t$, regular contributions $RC_t$ are paid into the pension plan by the plan sponsor. These contributions are determined by a fixed contribution rate $CR$ of 18.7 percent of the current payroll for all civil servants participating in the plan, as derived in the previous section. Plan funds are used to pay for pension payments due at time $t$, while the remaining assets are invested in the capital markets.

At the end of every period, the plan manager has to analyze the plan’s funding situation. Depending on the funding ratio, defined as the fraction of the current projected benefit obligation that is covered by current plan assets, solvency rules might require additional funds to be paid into the plan to recover funding deficits. By contrast, substantial overfunding might allow future contribution rates to be reduced. Specifically, in case the funding ratio in any period drops below 90 percent, immediate supplementary contributions $SC_t$ are required to reestablish a funding ratio of 100 percent. If, on the other hand, fund assets exceed fund liabilities by more than 20 percent, $CR$ will be cut by 50 percent. In case the funding ratio even rises above 150 percent, no further regular contributions will be required from the plan sponsor until the funding level decreases again. At the end of our projection horizon, we assume the plan is frozen and all liabilities are transferred to a private insurer together with assets to fund them.

The plan manager’s investment policy aims at generating sufficient returns in order to reduce overall pension plan costs. At the same time, he tries to keep capital market fluctuations and thereby worst-case plan costs under control. Hence, the plan sponsor is interested in identifying the optimal allocation of pension funds across three broad asset classes: an equity index fund, a government bond index fund, and a real estate index fund. Specifically, we assume that the plan sponsor seeks to minimize the worst-case cost of running the plan, specified by the Conditional Value at Risk at the 5 percent level of the

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10 We assume investments in index funds to prevent the state from systematically influencing asset prices.
stochastic present value of total pension costs (TPC). The distribution of total discounted pension costs is derived from running a 10,000 iteration Monte Carlo simulation. Based on this, we identify the optimal asset allocation \( x \) fixed at the beginning of the projection horizon.

Total pension costs are the sum of regular contributions \( RC \) and supplementary contributions \( SC \) made by the plan sponsor. All payments by the plan sponsor are discounted at the fixed real interest rate \( r \), which reflects the government’s financing cost. Thus, the optimization problem with respect to the vector of investment weights \( x \) (i.e. the fraction of assets invested in bonds, stocks, and real estate) is specified by:

\[
\min \frac{CVaR}{3\%} \left\{ \sum_{t=0}^{T} \frac{RC_t + SC_t (1 + \xi)}{1 + r} \right\}
\]

(2)

The 5%-Conditional Value at Risk (CVaR) is defined as the expected present value of total pension cost under the condition that its realization is greater than the Value at Risk (VaR) for that level, i.e.:

\[
CVaR_{3\%}(TPC) = E(TPC \mid TPC > VaR_{3\%}(TPC)).
\]

(3)

The CVaR framework as a measure of risk is in many ways superior to the commonly-used VaR measure, defined as \( P(TPC > VaR_\alpha) = \alpha \), i.e. the costs that will not be exceeded with a given probability of \((1 - \alpha)\) percent. In particular, the CVaR focuses not only on a given percentile of a loss distribution, but also accounts for the magnitude of losses in the distributional tails beyond this percentile.\(^{13}\)

\(^{11}\) For a comparable objective function using the Value at Risk see Albrecht et al. (2006).

\(^{12}\) We deliberately do not dynamically optimize investment weights and contribution rates over time. While this might be appealing from a theoretical perspective, political decision makers will most likely be unable to implement this in practice. Moreover, empirical evidence on pension plan asset allocation suggests that investment weights are rather constant in real-world pension schemes (see Haberman et al. 2003).

\(^{13}\) For a detailed discussion of the advantages of the CVaR over the more widely acknowledged VaR see, e.g., Artzner et al. (1997, 1999) and Rockafellar and Uryasev (2002).
We argue that pension benefits as a rule should be covered by regular plan contributions. Hence, supplementary contributions ought to be required only as a last resort. In case a plan sponsor is often asked to make supplementary contributions, regular contribution rates are likely to be insufficient. To discourage making too few regular contributions, we include a penalty factor $\xi$ for supplementary contributions. Thus, if one unit of supplementary contributions is required to recover a funding deficit, then $(1 + \xi)$ units are accounted for as plan costs. This penalty can also be interpreted as the additional costs in excess of the risk free rate of financing the required supplementary contributions, countering the notion that public monies paid into public pension plans are “free” money.

At the same time, measures need to be taken to discourage overfunding the plan significantly. The sponsor might find it appealing to excessively short government bonds and invest the proceeds into the pension plan in an effort to “cash in” on the equity premium. To this end, we disallow funds being physically transferred out of the plan; the minimum contribution rate in any single period is zero. In case plan assets exceed plan liabilities after plan termination, these funds are lost from the perspective of the plan manager as they are not accounted for as revenues in his objective function. Later we relax this assumption.

**Stochastic Asset Model.** We model the long run stochastic dynamics of future returns on assets accumulated in the pension plan using a first-order vector autoregressive (VAR) model, which is widely used by practitioners as well as in the academic literature (Hoevenaars, Molenaar, and Steenkamp 2003; Campbell and Viceira 2002). The pension plan’s investment universe comprises broadly diversified portfolios of equities, bonds and real estate investments. Our asset model draws on the specification employed by Hoevenaars et al. (2008), who extend the models in Campbell, Chan, and Viceira (2003) as well as in Campbell and Viceira (2005) by including additional asset classes, in particular alternative investments like real estate, commodities, and hedge funds. Following the notation of Hoevenaars et al. (2008), let $z_t$ be the vector
that contains the real money market log return at time $t$ ($r_{m,t}$), the vector $x_{1,t}$, which includes the excess returns of equities and bonds relative to $r_{m,t}$ (i.e. $x_{1,t} = r_{i,t} - r_{m,t}$), the vector $x_{2,t}$, which includes the excess return of real estate relative to $r_{m,t}$, and a vector $s_t$ describing state variables that predict $r_{m,t}$, $x_{1,t}$, and $x_{2,t}$. We include the nominal 3-months interest rate ($r_{nom}$), the dividend-price ratio ($dp$), and the term spread ($spr$) as predicting variables.\textsuperscript{14}

While historical return data are easily available for traditional asset classes, this does not hold for alternative investments, like real estate in our case. Typically, return time series for these asset classes are comparably short. This imposes difficulties when trying to calibrate the model. The large number of parameters to be estimated can lead to these estimates being unreliable as data availability is insufficient. To resolve this problem, restrictions are being imposed on the VAR with respect to $x_{2,t}$. In particular, we assume that $x_{2,t}$ has no dynamic feedback on the other variables. In other words, real estate returns are influenced by the returns on traditional asset classes and the predictor variables, while these in turn do not depend on the development of real estate returns. To this end, let $y_t$ be the vector

$$y_t = \begin{pmatrix} r_{m,t} \\ s_t \\ x_{1,t} \\ x_{2,t} \end{pmatrix}$$

The dynamics of $y_t$ are assumed to follow an unrestricted VAR(1) according to

$$y_{t+1} = a + By_t + \varepsilon_{t+1}$$

with $\varepsilon_{t+1} \sim N(0, \Sigma_{\varepsilon\varepsilon})$. The return on real estate investments are modeled according to

$$x_{2,t+1} = c + D_0 \cdot y_{t+1} + D_1 \cdot y_t + H \cdot x_{2,t} + \eta_{t+1},$$

\textsuperscript{14} The state variables included here are commonly used in the strategic asset allocation literature (see e.g. Campbell and Shiller (1988, 1991); Fama and French (1989); Campbell, Chan and Viceira (2003); Campbell and Viceira (2005); Cochrane (2005); Brandt and Santa-Clara (2006)).
with $\eta_{t+1} \sim N(0, \sigma_{re})$. The innovations $\varepsilon_{t+1}$ and $\eta_{t+1}$ are assumed to be uncorrelated, as contemporaneous interrelations are captured by $D_0$. Based on this set-up and following Stambaugh (1997), we can then optimally exploit available data by estimating the unrestricted VAR (Equation 6) over the complete data sample and by using the smaller sample only for estimating the parameters in Equation 7.

The unrestricted VAR model is calibrated to quarterly logarithmic return series starting in 1973:I and ending in 2007:I. The real money market return is the difference between the nominal log 3-months Euribor and inflation (Fibor is used for the time before Euribor was available). Log returns on equities and log dividend-price ratios draw on time series data for the DAX 30 – an index portfolio of German blue chips - provided by DataStream. We use the approach in Campbell and Viceira (2002) to derive return series for diversified bond portfolios. The bond return series $r_{n,t+1}$ is constructed according to

$$r_{n,t+1} = \frac{1}{4} y_{n-1,t+1} - D_{n,t}(y_{n-1,t+1} - y_{n,t}),$$

employing 10 year constant maturity yields on German bonds, where $y_{n,t} = \ln(1+Y_{n,t})$ is the $n$-period maturity bond yield at time $t$. $D_{n,t}$ is the duration, which can be approximated by

$$D_{n,t} = \frac{1 - (1 + Y_{n,t})^{-n}}{1 - (1 + Y_{n,t})^{-1}}.$$  

We approximate $y_{n-1,t+1}$ by $y_{n,t+1}$ assuming that the term structure is flat between maturities $n-1$ and $n$. As for equities, excess returns are calculated by subtracting the log money market return, $x_{b,t} = r_{n,t} - r_{m,t}$. The yield spread is computed as the difference between the log 10-year zeros yield on German government bonds and the log 3-months Euribor, both provided by Deutsche Bundesbank.
Deriving reliable return time series for real estate as an asset class is difficult due to the peculiarities of property investments.\textsuperscript{15} In contrast to equity and bond indices, inhomogeneity, illiquidity, and infrequent trading in individual properties result in transaction-based real estate indices not being able to adequately describe the returns generated in these markets. Moreover, such price indices do not account for rental income, which constitutes a significant source of return on real estate investments. By contrast, it is comparably easy to construct indices that try to approximate the income on direct real estate investments by using the return on investing indirectly through traded property companies like real estate investment trusts (REITs). However, empirical evidence on these forms of indirect real estate investments suggests that they exhibit a more equity-like behavior.\textsuperscript{16} These indices are therefore a much less than perfect proxy for direct real estate investments (see Hoesli and MacGregor 2000).

Appraisal-based indices, like the one this study draws on, are the most widely used representatives for real estate investments in the academic literature as well as among practitioners. These indices account for easy to sample continuous rental income as well as for returns from changes in property values, which are estimated through periodic appraisals by real estate experts. As individual properties’ values are usually estimated only once a year and due to the fact that there is no single valuation date for all properties, not every return observation in the index can be substantiated with a new and observation date consistent appraisal of the overall property portfolio underlying the index. Moreover, annual appraisals often draw significantly on prior valuations. Consequently, returns derived from appraisal-based indices exhibit substantial serial correlation and low short term volatilities that understate the true volatility of real estate returns. Different methodologies have been suggested to reduce undue smoothing in real estate return time series, which subsequently will

\textsuperscript{15} For an extensive discussion of design and characteristics of real estate indices we refer to – among others – Albrecht and Maurer (2005, Ch. 14) and Hoesli and MacGregor (2000, Ch. 4).

\textsuperscript{16} In a survey by Eichholtz (1997), correlations between common equities and property company shares range from 0.12 to 0.96.
exhibit more realistic levels of volatility.\textsuperscript{17} In this study we employ the approach developed by Blundell and Ward (1987) that suggests transforming the original (smoothed) return series according to:

\begin{equation}
    r^*_t = \frac{r_t}{1-a} - \frac{a}{1-a} r^*_{t-1},
\end{equation}

where \( r^*_t \) represents the unsmoothed return in \( t \) and \( a \) the coefficient of first-order autocorrelation in the return time series. Under this transformation, expected returns remain constant, \( E(r^*_t) = E(r_t) \), but the return standard deviation increases according to:

\begin{equation}
    STD(r^*_t) = STD(r_t) \sqrt{\frac{1-a^2}{(1-a)^2}}.
\end{equation}

We rely on an appraisal-based index for a diversified property portfolio as elaborated in Maurer, Reiner and Sebastian (2003), which provides quarterly returns on German real estate back to January 1980. The index is a value weighted index constructed from the returns on German open-end real estate funds’ units. These fund units represent portfolios of direct real estate investments and liquid assets like money market deposits or short- to medium-term government bonds.\textsuperscript{18} The return on direct property investments is then approximated by subtracting from the funds’ returns their earnings resulting from investing in liquid assets.

While our asset/liability model is run on a yearly basis, the VAR is calibrated to quarterly data, resulting in higher reliability of parameter estimates due to a higher number of available observations. Quarterly returns generated by the asset model are aggregated and parameters \( a, c, \sigma_{re} \) and \( \Sigma_{ee} \) are adapted so that the model’s simulated empirical return moments (see Table 2 and the Appendix) reflect those of annual historic returns.\textsuperscript{19}

\textsuperscript{17} Other methods to unsmooth real estate return time series have been suggested by – among others – Firstenberg, Ross, and Zisler (1988), Ross and Zisler (1991), Fisher, Geltner, and Webb (1994), Geltner (1993) and Barkham and Geltner (1994).

\textsuperscript{18} A thorough analysis of the institutional design of German open-end real estate funds, as well as their risk and return profile can be found in Maurer, Reiner, and Rogalla (2004).

\textsuperscript{19} Mean real log returns on bonds in our time series come to almost 5 percent p.a. while equities only yield an excess return of 1.5 percent. We reduce expected bond returns to 4 percent, considering this to be more appropriate in the long term.
**Optimal Asset Allocation under Stochastic Investment Returns.** Next we derive the optimal investment strategy for plan assets assuming that the rate of regular contributions, $CR$, is fixed at a given ratio of projected benefit obligation to the present value of projected future salaries. From Table 1 we know that for a real discount rate of 3 percent, a fixed contribution rate of 18.7 percent of current salaries is sufficient to finance the PBO that comes to € 20.8 billion in the deterministic case. Against this deterministic PBO and contribution rate, we benchmark our results for an environment in which investment returns are stochastic. In our base case, we will assume the same real discount rate of 3 percent and a penalty factor on supplementary contributions $\xi$ of 20 percent. A following section will investigate into the impact of varying these assumptions.

Table 3 summarizes key findings for four distinct asset allocations, the three polar cases of 100 percent equities, 100 percent bonds, and 100 percent real estate investments as well as the optimal investment strategy, which is determined endogenously by minimizing the 5%-CVaR of total pension costs. Panel 1 of Table 3 contains the portfolio weights of equities, bonds, and real estate investments assuming a static asset allocation (Rows 1 to 3), the expected present value of total pension costs (Row 4), and the 5% Conditional Value at Risk (Row 5). Expectation and 5% Conditional Value at Risk of discounted supplementary contributions are shown in Panel 2 of Table 3 (Rows 6 and 7). Figure 2 provides closer insight into the dispersion of possible total pension cost outcomes for the four asset allocations under investigation, showing box plots of various percentiles of the overall cost distributions.

When the fund is fully invested in equities, total expected pension costs for active employees come to € 21.71 billion (Row 4, Column 1) while the 5%-CVaR amounts to € 36.27 billion or about 75 percent higher than the deterministic PBO benchmark of € 20.8 billion (Row 5, Column 1). In addition to the regular pension contributions of 18.7 percent of
the payroll, taxpayers face another expected € 8.69 billion in supplementary contributions, which rise to € 21.51 billion in CVaR (Rows 6 and 7, Column 1). As one would expect, high volatility of investment returns result in high dispersion of possible cost outcomes. From Figure 2 it can be seen that overall pension costs may vary widely from € 12.6 billion (5\textsuperscript{th} percentile) to € 33 billion (95\textsuperscript{th} percentile). Although high return volatility comes with high expected returns, expected pension costs are substantial due to the capped upside potential inherent in the plan design. While the plan manager is fully liable for funding deficits resulting from capital market losses, he is not able to recover excess funds in an effort to reduce overall pension costs. Thus, there is a strong disincentive for the plan manager to overinvest plan funds into equities.

If, on the other hand, plan funds were fully invested in bonds, worst-case pension costs would only come to € 26.48 billion, while expected costs would even drop to € 18.62 billion (Rows 4 and 5, Column 2). Expected returns are moderate and therefore the cap on excess fund withdrawal is only of minor relevance. However, returns are still sufficient to earn some excess income over the discount rate, cutting expected costs down below their deterministic value. Lower volatility of investment returns results in lower dispersion of costs, ranging from € 13.5 billion (5\textsuperscript{th} percentile) to € 24.6 billion (95\textsuperscript{th} percentile). This keeps worst-case pension costs under control. On average, only € 1.56 billion in supplementary contributions are required while their 5\%-CVaR amounts to € 6.74 billion, less than one third compared to the all-equities allocation (Rows 6 and 7, Column 2).

Column 3 of Table 3 presents the results for an investment strategy that allocates all plan funds to real estate, the least risky single asset class under consideration in this study. Consequently, with an overall amount of € 25.88 billion, worst-case pension costs are the lowest compared to the other polar cases (Row 5, Column 3). This also holds for expected and worst-case supplementary contributions, which come to € 1.42 billion and € 5.05 billion, respectively (Rows 6 and 7, Column 3). Low investment risk, however, comes at the cost of
low expected returns. Real estate investments hardly outperform the fixed discount rate. Thus, there is not much of a risk premium to cash in and the upside potential is heavily limited. Expected pension costs amount to € 21.99 billion, which exceeds those in the other polar cases as well as the deterministic PBO (Row 4, Column 3).

The optimal investment strategy given the fixed contribution rate of 18.7 percent of salaries is depicted in Column 4 of Table 3. It consists of 22.3 percent equities, 47.2 percent bonds, and 30.5 percent real estate investments (Rows 1-3). Equities acquire a significant share in the optimal portfolio, indicating that current investment policy for the few funded German pension schemes, i.e. only investing in pure bond portfolios, might not be a favorable solution. Nonetheless, optimal equity weights are considerably lower than the almost 60 percent reported for US state pension plans (Wilshire 2007). Allocating a substantial fraction of assets to real estate is in line with the results of Ziobrowski and Ziobrowski (1997) and Firstenberg, Ross, and Zisler (1988), among others. In a more recent study however, Craft (2001) argues that in an asset/liability framework allocations to private real estate investments should only range from 12 to 16 percent. This is more in line with empirical observations of real estate allocations varying between 5 and 10 percent (see e.g. Wilshire 2007, ABP 2007). To a certain extent, the relatively high allocation to real estate in this study may be attributed to the underlying pension plan design. Due to the pension plan’s up-side potential being restricted for political reasons, the plan manager will favor more stable real estate investments compared to riskier assets like equities.

Given the optimal investment strategy, expected pension costs for active employees are reduced to only € 16.09 billion (Row 4, Column 4), more than 20 percent below the € 20.8 billion required in the deterministic case. This cost reduction can directly be attributed to the considerable benefits, which can be expected from investing in diversified portfolios. From the outset, the fund is endowed with 18.7 percent of payroll, while actual pension payments are initially negligible. Expected returns well above the discount rate at which the benchmark
contribution rate was derived and moderate return volatilities enable the fund to quickly accumulate considerable assets. The possibility of being able to reduce the actual contribution rate increases through time, while the risk of having to make supplementary contributions to reduce funding deficits diminishes.

This optimal funding and investment strategy also keeps worst-case risk under control. The 5% Conditional Value at Risk of total pension costs, or the expected cost in the five percent worst cases, only amounts to € 21.02 billion (Row 5, Column 4), almost equal to the deterministic benchmark. Supplementary contributions are also low. Their present value only comes to € 500 million in expectation and even in the worst case – again defined as the five percent CVaR – they only amount to € 2.85 billion, slightly more than half the cost that was reported for the least risky pure real estate investment (Rows 6 and 7, Column 4).

The benefit of diversification can also be seen in Figure 2 with pension costs for the optimal asset allocation ranging from € 12.5 billion (5th percentile) to € 20 billion (95th percentile). This range is smaller than for pure equity or bond investments, while investing only in real estate will result in an even smaller range. However, the overall level of costs resulting from following the optimal strategy is substantially lower compared to the pure real estate investment case. Only investing in real estate will result in the 5th percentile of overall costs being only marginally lower than the 95th percentile of costs in the optimal case.

As a result, introducing an at least partially funded public pension plan that follows an optimized investment policy could be expected to substantially reduce the economic cost of providing covered pensions, while simultaneously keeping the consequences of capital market volatility under control.

Figure 3 provides deeper insight into the temporal structure of risks and rewards of following the cost minimizing investment strategy (i.e. 22% stocks, 47% bonds, 30% real estate). Panel A depicts the time path of the probability of having to make supplementary contributions due to substantial underfunding resulting from unfavorable investment returns.
(solid line). It indicates that there is a relatively low risk of additional contributions in the first decade of operations (much less than ten percent probability), and a negligible risk thereafter. The other two lines depict the probability of the regular contribution rate being reduced by 50 (dashed line) or even 100 percent (dotted line). It can be seen that the probability of enjoying partial or full contribution holidays because of overfunding rises with time. Ten years into the program, the probability of a contribution holiday is only two percent, but 35 percent after 20 years. In other words, the risk of additional contributions is front-loaded, but the potential benefits savings are back-loaded.

Figure 3 here

Panel B of Figure 3 indicates that the expected value of required supplementary contributions (solid line) is highest at 12 years, where it amounts to € 40 million. The dotted line represents expected savings due to contribution holidays. Ten years after the program is launched, the expected savings amount to € 8.3 million, and rise to € 145 (578) million in year 20 (40). The dashed line shows our estimate of the ‘worst case’ value of supplementary contributions measured by the 5%-CVaR risk metric. This suggests that, with a low probability, the plan sponsor might have to contribute substantially more during the early period: € 800 million at the 10 year mark, and € 360 million after 20 years. Reinforcing the message of Panel A, the optimal investment strategy greatly reduces the burden on future generations while controlling the risk on current contributors.

Further Results. Naturally, the results derived so far depend heavily on model calibration. To check for robustness, we have analyzed optimal pension fund investment strategies for a selection of alternative parameterizations. While it is impossible to investigate all sensitivities, the findings presented below provide a good understanding of the basic interrelations. Results are summarized in Table 4 for three alternative parameter sets. For ease of comparison, Column 1 repeats the result derived above for our base case. Alternative 1 investigates the impact of the penalty factor on supplementary contributions by redoing the analysis using a
penalty factor for supplementary contributions of $\xi=0$ (Column 2). We then study the influence of expected asset returns on the optimal asset allocation (Column 3). To this end, we analyze the plan assuming a real discount rate of 1.5 percent (instead of the 3% in our base case) together with the low return scenario from Table 2. Finally, we ease the restriction on withdrawing assets from the pension plan in an extremely overfunded situation by imposing a small cost on withdrawals. Panel 1 of Table 4 presents optimal investment weights into equities, bonds, and real estate (Rows 1-3), as well as the expectation and the 5 percent CVaR of the present value of total pension costs (Rows 4 and 5). Rows 6 and 7 in Panel 2 again present the expectation and worst-case realization of the present value of supplementary contributions. Finally, Rows 8 and 9 present the expected value as well as the 5 percent CVaR of withdrawals from the pension plan.

*Table 4 here*

In our base case, we levy a penalty of 20 percent on supplementary contributions, giving plan managers an incentive to follow a sustainable investment policy, which only relies on extra payments as a last resort. Moreover, this penalty was introduced to support the notion that such payments do not come for free but rather involve some form of financing costs. If supplementary contributions were free of extra costs, the plan manager would engage in a more risky investment strategy. Under these circumstances, low risk real estate investments would be significantly reduced by more than 6 percentage points to an overall investment weight of 24.2 percent, while the weights of equities and bonds would both increase by about 3 percent to 25.6 percent and 50.2 percent, respectively (Column 2, Rows 1 – 3). Equity exposure, however, continues to be comparably low, since the plan’s upside potential is still limited. Having to account for such a penalty increases overall pension costs. Hence, it comes at no surprise that reducing the penalty factor will automatically reduce plan costs. For a penalty factor of 0 percent, expected plan costs come to € 15.6 billion, while their worst-case value amounts to € 20.5 billion (Column 2, Rows 4 and 5). Both figures are about € 500
million below the ones reported for a penalty factor of 20 percent. Expected and worst-case supplementary contributions in Rows 6 and 7 of Column 2 are also lower than their counterparts in our base case (Column 1). Their decrease due to the reduced penalty factor, however, falls short of the 20 percent one might expect. This results from the slightly more aggressive optimal investment policy.

Discounting pension liabilities with a reduced real rate of 1.5 percent increases the deterministic PBO to € 44.8 billion and the corresponding contribution rate to 30 percent of the payroll (Table 1, Column 2, Rows 2 and 3). Assuming that expected returns on assets drop by the same 1.5 percent, the optimal asset allocation will generate worst-case costs of € 44.79 billion (Row 5 Column 3), virtually equal to the deterministic PBO. Expected pension costs come to € 33.65 billion, down 25 percent compared to their non-stochastic counterpart (Row 4, Column 3). The optimal asset allocation consists of 22.5 percent equities, 47.5 percent bonds, and 30 percent real estate (Column 3, Rows 1-3). In essence, this equals the optimal allocation in our base case. The weight of real estate is marginally reduced by 0.5 percent, which are evenly distributed to equities and bonds. Thus, the interrelations between the asset classes as well as between plan assets and plan liabilities and the overall plan design determine optimal portfolio weights to a far greater extent than the absolute level of investment returns.

Finally we allow the plan manager to almost completely participate in the upside potential of investing plan assets more aggressively into equities. This alternative permits the plan manager to recover assets that exceed liabilities by more than 80 percent. To prevent the manager from treating the pension as a hedge fund, we levy a 20 percent penalty on withdrawals. Now, investing in equities becomes much more appealing to the plan manager, as he is now rewarded for accepting higher return volatility with higher expected investment

\[ TPC = \sum (RC_t + (1 + \xi_t) \cdot SC_t - (1 - \xi_t) \cdot W_t), \]

where \( W_t \) are the withdrawals in the case of a funding ratio higher than 180 percent and \( \xi_t \) is the penalty factor.

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20 Formally, we expand the total pension cost in Formula 2 to \( TPC = \sum (RC_t + (1 + \xi_t) \cdot SC_t - (1 - \xi_t) \cdot W_t) \), where \( W_t \) are the withdrawals in the case of a funding ratio higher than 180 percent and \( \xi_t \) is the penalty factor.
returns. Equity weights in the optimal portfolio rise by more than 30 percentage points to about 53 percent (Row 1, Column 4). While bond holdings remain virtually constant, assets are no longer invested into real estate due to their lack of expected return (Column 4, Rows 2 and 3). As expected investment returns significantly outperform the discount rate at which plan liabilities are valued, pension costs decrease substantially. In expectation, the plan exhibits negative pension costs of € 2.46 billion (Row 4, Column 4). This means that after initially paying contributions into the plan for some years, investment returns on accumulated plan funds are sufficient to finance ongoing pension payments and even allow withdrawals that exceed earlier contributions in present value terms. Withdrawals come to € 17.4 billion in expectation, and even in the worst case, almost € 3.5 billion can be withdrawn from the plan (Rows 8 and 9, Column 4). Worst-case risks in this scenario are also well under control. While worst-case supplementary contributions come to € 6.71 billion, more than double the amount of the base case (Row 7, Columns 1 and 4), and the five percent CVaR of total pension only amounts to € 16 billion, 20 percent less than the deterministic pension cost (Row 5, Column 4).

Conclusions

As in many countries, civil servants in Germany are promised an unfunded defined benefit pension. These benefits represent a significant liability to taxpayers, one which is currently not recognized as explicit state debt. We analyze the implications of moving Hesse’s civil servants pension plan toward funding. We focus only on future benefit accruals, assuming that pensions paid to current retirees as well as claims already accumulated by active civil servants will be financed from other sources. With a non-stochastic framework based on a real discount rate of three percent, the annual contribution rate would be around 19 percent of salary which would be sufficient to cover future benefit accruals. Drawing on these results, we scrutinize alternative asset allocation strategies within a stochastic asset/liability
framework. Here, we seek to minimize the worst-case costs of providing the promised pensions. In our base case, we find that, given the contribution rate of about 19 percent, the optimal investment policy for pension plan assets comprises 22 percent equities, 47 percent bonds, and about 31 percent real estate investments. Following this funding and investment policy will curtail worst-case pension costs to the deterministic PBO, while expected costs fall below these by almost 25 percent.

These results indicate that moving toward a funded pension system for German civil servants could be beneficial to both taxpayers as well as employees. Taxpayers can expect substantial cost reductions due to the favorable impact of earning investment returns in the capital markets, while their exposure to investment risks is limited for reasonable investment policies. Civil servants, in turn, benefit from being less exposed to discretionary pension cuts in times of tight government’s budgets. Additionally, they might enjoy greater flexibility as pension claims backed by assets are much more portable than unfunded promises. Finally, we argue that public plans that hold 60 percent or more in equities, as is true in the US public case, is likely too aggressive. Nevertheless, investing in pure bond portfolios as in the few German pension schemes that hold some assets provides stability, but can be quite expensive.
References


Table 1: Projected Benefit Liabilities and Contribution Rates: Deterministic Model

<table>
<thead>
<tr>
<th>Discount Rate</th>
<th>3%</th>
<th>1.5%</th>
</tr>
</thead>
<tbody>
<tr>
<td>(1) PV Pension Liabilities (in € bn)</td>
<td>20.8</td>
<td>44.8</td>
</tr>
<tr>
<td>(2) PV Future Salaries (in € bn)</td>
<td>111.5</td>
<td>149.3</td>
</tr>
<tr>
<td>(3) Contribution Rate: (1) / (2) (in %)</td>
<td>18.7</td>
<td>30.0</td>
</tr>
</tbody>
</table>

Notes: Authors’ calculations using data provided by the State of Hesse for 2004. Base case defined with a 3% discount rate, alternative case uses 1.5%.
Source: Derived from Maurer, Mitchell and Rogalla (2008).

Table 2: Simulated Parameters for Stochastic Asset Case

<table>
<thead>
<tr>
<th>Expected Returns</th>
<th>Standard deviations</th>
<th>Correlations</th>
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</thead>
<tbody>
<tr>
<td></td>
<td>Equities</td>
<td>Bonds</td>
</tr>
<tr>
<td>Base case scenario</td>
<td>6.57%</td>
<td>5.07%</td>
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<tr>
<td>Low return scenario</td>
<td>4.08%</td>
<td>2.58%</td>
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<tr>
<td>Real Estate</td>
<td>3.13%</td>
<td>1.63%</td>
</tr>
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</table>

Notes: *: Unsmoothed volatility following Blundell and Ward (1987). Base case scenario relates to a discount rate of 3%, low return scenario relates to a discount rate of 1.5%. See the Appendix for estimated quarterly VAR parameters which generate these moments based on 10,000 simulations.
Source: Authors’ calculations.

Table 3: Risk of Alternative Asset Allocation Patterns, Assuming Fixed Contribution Rate

<table>
<thead>
<tr>
<th>Fixed contribution rate: 18.7%</th>
<th>100%</th>
<th>100%</th>
<th>100%</th>
<th>Cost min.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Deterministic PBO: € 20.8 bn</td>
<td>Equities</td>
<td>Bonds</td>
<td>Real Estate</td>
<td>Asset Mix</td>
</tr>
<tr>
<td>Real Discount Rate: 3%</td>
<td>(1)</td>
<td>(2)</td>
<td>(3)</td>
<td>(4)</td>
</tr>
</tbody>
</table>

Panel 1

| (1) Equity Weight (%) | 100 | 0 | 0 | 22.3 |
| (2) Bond Weight (%)   | 0   | 100 | 0 | 47.2 |
| (3) Real Estate Weight (%) | 0 | 0 | 100 | 30.5 |
| (5) 5%-CVaR Pension Costs (€ bn) | 36.27 | 26.48 | 25.88 | 21.02 |

Panel 2

| (6) Exp. Suppl. Contributions (€ bn) | 6.89 | 1.56 | 1.43 | 0.50 |
| (7) 5%-CVaR Suppl. Contrib. (€ bn) | 21.51 | 6.74 | 5.05 | 2.85 |

Notes: Contribution rate in % of salaries. Supplementary contributions required in case of funding ratio (i.e. fund assets/PBO) below 90% to restore funding ratio of 100%. Contribution rate reduced by 50% (100%) in case of funding ratio above 120% (150%). Opportunity costs of supplementary contributions addressed by accounting for a penalty of $\xi = 20\%$.
Source: Authors’ calculations using data provided by the State of Hesse.
<table>
<thead>
<tr>
<th></th>
<th>Base Case</th>
<th>Alternative 1</th>
<th>Alternative 2</th>
<th>Alternative 3</th>
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</thead>
<tbody>
<tr>
<td>Fixed contribution rate (in %)</td>
<td>18.7</td>
<td>18.7</td>
<td>30</td>
<td>18.7</td>
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<tr>
<td>Deterministic PBO (in € bn)</td>
<td>20.8</td>
<td>20.8</td>
<td>44.8</td>
<td>20.8</td>
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<tr>
<td>Real Discount Rate (in %)</td>
<td>3.0</td>
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<td>1.5</td>
<td>3.0</td>
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<td>Penalty Factor on Suppl. Contributions</td>
<td>0.2</td>
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<tr>
<td>Penalty Factor on Withdrawals</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>0.2</td>
</tr>
</tbody>
</table>

**Panel 1**

(1) Equity Weight (%)  22.3  25.6  22.5  53.1  
(2) Bond Weight (%)  47.2  50.2  47.5  46.9  
(3) Real Estate Weight (%)  30.5  24.2  30.0  0.0  
(4) Expected Pension Costs (€ bn)  16.09  15.56  33.65  -2.46  
(5) 5%-CVaR Pension Costs (€ bn)  21.02  20.54  44.79  16.02  

**Panel 2**

(6) Exp. Suppl. Contributions (€ bn)  0.50  0.49  0.59  1.68  
(7) 5%-CVaR Suppl. Contrib. (€ bn)  2.85  2.63  4.79  6.71  
(8) Exp. Withdrawals (€ bn)  0.00  0.00  0.00  17.37  
(9) 5%-CVaR Withdrawals (€ bn)  0.00  0.00  0.00  3.42  

Notes: Contribution rate in % of salaries. Supplementary contributions required in case of funding ratio (i.e. fund assets/PBO) below 90% to restore funding ratio of 100%. Contribution rate reduced by 50% (100%) in case of funding ratio above 120% (150%). Withdrawal of funds exceeding 180% of pension liabilities (subject to respective penalty factor). 
Source: Authors’ calculations using data provided by the State of Hesse.
### Appendix: Quarterly VAR Parameters

<table>
<thead>
<tr>
<th>Parameter estimates</th>
<th>$r_{m,t+1}$</th>
<th>$x_{e,t+1}$</th>
<th>$x_{b,t+1}$</th>
<th>$d_{p,t+1}$</th>
<th>$s_{p,t+1}$</th>
<th>$r_{nom,t+1}$</th>
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<td>$r_{m,t+1}$</td>
<td>-0.0338</td>
<td>0.0035</td>
<td>-0.0226</td>
<td>-0.2118</td>
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<td>$x_{e,t+1}$</td>
<td>0.1267</td>
<td>0.0116</td>
<td>0.0920</td>
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<td>$x_{b,t+1}$</td>
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<td>$d_{p,t+1}$</td>
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<td>$s_{p,t+1}$</td>
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<td>0.0005</td>
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<td>$r_{nom,t+1}$</td>
<td>-0.0268</td>
<td>0.0010</td>
<td>-0.0173</td>
<td>0.0434</td>
<td>-0.0869</td>
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<tr>
<td>$D_{0}$</td>
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<td>-0.0068</td>
<td>-0.2699</td>
<td>-0.3993</td>
<td>-0.2348</td>
<td>-0.5134</td>
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<tr>
<td>$D_{1}$</td>
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<td>-0.0073</td>
<td>-0.0033</td>
<td>0.1551</td>
<td>0.3570</td>
<td>0.3802</td>
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### Error correlation matrix

<table>
<thead>
<tr>
<th></th>
<th>$r_{m,t}$</th>
<th>$x_{e,t}$</th>
<th>$x_{b,t}$</th>
<th>$d_{p,t}$</th>
<th>$s_{p,t}$</th>
<th>$r_{nom,t}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$r_{m,t}$</td>
<td>0.54</td>
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<tr>
<td>$x_{e,t}$</td>
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<tr>
<td>$x_{b,t}$</td>
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<td>-0.07</td>
<td>3.00</td>
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<td>$d_{p,t}$</td>
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<td>-0.87</td>
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<td>0.30</td>
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<tr>
<td>$s_{p,t}$</td>
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<td>0.05</td>
<td>-0.42</td>
<td>-0.10</td>
<td>0.62</td>
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<tr>
<td>$r_{nom,t}$</td>
<td>0.21</td>
<td>-0.16</td>
<td>0.12</td>
<td>0.23</td>
<td>-0.35</td>
<td>0.15</td>
</tr>
</tbody>
</table>

$H = -0.4897$

$\sigma_r = 0.0065$

Source: Authors' calculations.
**Figure 1: Age Distribution of Active Civil Servants in 2004**

![Age Distribution of Active Civil Servants in 2004](image)

Notes: Age distribution for all active civil servants, N = 104,919.  
Source: Authors’ calculations using data provided by the State of Hesse for 2004.

**Figure 2: Range of Pension Costs under Alternative Asset Allocations**

![Range of Pension Costs under Alternative Asset Allocations](image)

Notes: Total Pension Costs defined as net of Regular and Supplementary Contributions using 3% discount rate. Annotations refer to the respective percentiles of total pension cost distributions for various asset allocations.  
Source: Authors’ calculations.
Figure 3: Time Paths of Supplementary Public Pension Contributions and Cost Savings under Optimal Asset Allocation Strategy

A. Probabilities of Supplementary Contributions and Contribution Holidays Over Time

![Graph showing probabilities over time]

- **P(SC)**: probability of supplementary contributions being required in any period.
- **P(CR=50%)/P(CR=0%)**: probability of regular contribution rate being reduced to 50%/0%.

B. Magnitudes (in € 2004) of Expected Supplementary Contributions and Cost Saving due to Contribution Holidays

![Graph showing magnitudes over time]

- **Exp. SC**: expected value of supplementary contributions in any given period.
- **5%-CVaR SC**: “worst case” value of supplementary contributions in any given period.
- **Exp. Savings**: expected value of cost savings due to cuts in contribution rates in any given period.

Notes:
- **P(SC)**: probability of supplementary contributions being required in any period.
- **P(CR=50%)/P(CR=0%)**: probability of regular contribution rate being reduced to 50%/0%.
- **Exp. SC**: expected value of supplementary contributions in any given period.
- **5%-CVaR SC**: “worst case” value of supplementary contributions in any given period.
- **Exp. Savings**: expected value of cost savings due to cuts in contribution rates in any given period.

Source: Authors’ calculations.