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Quantifying the Significance of Abrasion and Selective Transport for Downstream Fluvial Grain Size Evolution

Kimberly Litwin Miller
University of Pennsylvania

Tímea Szabó

Douglas J. Jerolmack
University of Pennsylvania, sediment@sas.upenn.edu

Gabor Domokos

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Abstract

It is well known that pebble diameter systematically decreases downstream in rivers. The contribution of abrasion is uncertain, in part because (1) diameter is insufficient to characterize pebble mass loss due to abrasion and (2) abrasion rates measured in laboratory experiments cannot be easily extrapolated to the field. A recent geometric theory describes abrasion as a curvature-dependent process that produces a two-phase evolution: in Phase I, initially blocky pebbles round to smooth, convex shapes with little reduction in axis dimensions; then, in Phase II, smooth, convex pebbles slowly reduce their axis dimensions. Here we provide strong evidence that two-phase abrasion occurs in a natural setting, by examining downstream evolution of shape and size of thousands of pebbles over ~10 km in a tropical montane stream. The geometric theory is verified in this river system using a variety of manual and image-based shape parameters, providing a generalizable method for quantifying the significance of abrasion. Phase I occurs over ~1 km, in upstream bedrock reaches where abrasion is dominant and sediment storage is limited. In downstream alluvial reaches, where Phase II occurs, we observe the expected exponential decline in pebble diameter. Using a discretized abrasion model (the so-called “box equations”) with deposition, we deduce that abrasion removes more than one third of the mass of a pebble but that size-selective sorting dominates downstream changes in pebble diameter. Overall, abrasion is the dominant process in the downstream diminution of pebble mass (but not diameter) in the studied river, with important implications for pebble mobility and the production of fine sediments.

Keywords

abrasion, selective transport, sorting, diminution, box equations

Disciplines

Earth Sciences | Environmental Sciences | Hydrology | Physical Sciences and Mathematics | Sedimentology

RESEARCH ARTICLE

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Kimberly Litwin Miller and Tímea Szabó contributed equally to this work.

Key Points:

- Pebble abrasion occurs in two distinct phases
- Abrasion controls the reduction in mass; sorting controls decrease in diameter
- Tracking downstream evolution of pebble shape provides field proof for abrasion

Correspondence to:

K. L. Miller,
klitwin@sas.upenn.edu

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Quantifying the significance of abrasion and selective transport for downstream fluvial grain size evolution

Kimberly Litwin Miller¹, Tímea Szabó², Douglas J. Jerolmack¹, and Gábor Domokos²
¹Department of Earth and Environmental Science, University of Pennsylvania, Philadelphia, Pennsylvania, USA, ²Department of Mechanics, Materials and Structures, Budapest University of Technology and Economics, Budapest, Hungary

Abstract It is well known that pebble diameter systematically decreases downstream in rivers. The contribution of abrasion is uncertain, in part because (1) diameter is insufficient to characterize pebble mass loss due to abrasion and (2) abrasion rates measured in laboratory experiments cannot be easily extrapolated to the field. A recent geometric theory describes abrasion as a curvature-dependent process that produces a two-phase evolution: in Phase I, initially blocky pebbles round to smooth, convex shapes with little reduction in axis dimensions; then, in Phase II, smooth, convex pebbles slowly reduce their axis dimensions. Here we provide strong evidence that two-phase abrasion occurs in a natural setting, by examining downstream evolution of shape and size of thousands of pebbles over ~10 km in a tropical montane stream. The geometric theory is verified in this river system using a variety of manual and image-based shape parameters, providing a generalizable method for quantifying the significance of abrasion. Phase I occurs over ~1 km, in upstream bedrock reaches where abrasion is dominant and sediment storage is limited. In downstream alluvial reaches, where Phase II occurs, we observe the expected exponential decline in pebble diameter. Using a discretized abrasion model (the so-called “box equations”) with deposition, we deduce that abrasion removes more than one third of the mass of a pebble but that size-selective sorting dominates downstream changes in pebble diameter. Overall, abrasion is the dominant process in the downstream diminution of pebble mass (but not diameter) in the studied river, with important implications for pebble mobility and the production of fine sediments.

1. Introduction

The ubiquitous pattern of rounded river rocks has long been known to result from the smoothing action of abrasion, due to grain-grain collisions during bed load transport [Wentworth, 1919; Kuenen, 1956; Sneed and Folk, 1958; Parker, 1991; Kodama, 1994b; Lewin and Brewer, 2002]. The transformation of initially blocky and angular rocks—typical of upstream reaches of rivers—to ellipsoidal pebbles downstream implies that a significant fraction of pebble mass is lost due to abrasion [Domokos et al., 2009b; Szabó et al., 2013]. The daughter products of abrasion are not infinitesimal; sand and silt produced from pebble collisions may be an important contributor to downstream floodplains, estuaries, and beaches and may help to maintain the observed bimodality of grain size distributions of riverbeds [Jerolmack and Brzinski, 2010, and references therein]. Although it has long been recognized that shape is an important indicator of the degree of abrasion of a sedimentary particle (so-called “maturity”), surprisingly few studies have quantified the downstream evolution of pebble shape in rivers [Sneed and Folk, 1958; Bradley et al., 1972; Adams, 1979; Mikos, 1995; Szabó et al., 2013].

The most commonly measured quantity in field studies of river rocks is the middle axis length, typically called “diameter,” which is used as a proxy descriptor of particle “size” [Kodama, 1994a; Lewin and Brewer, 2002; Attal and Lavé, 2006]. A near-universal trend observed in alluvial rivers, often referred to as “Sternberg’s law” [Sternberg, 1875], is that pebble diameter (D) decreases exponentially with distance downstream (x):

$$D(x) = D_0 e^{-\gamma x}, \quad (1)$$

where D_0 is initial pebble diameter at the upstream alluvial boundary and γ is an empirically determined parameter. For decades, researchers have attempted to rationalize this relation (equation (1)) from theory

and laboratory experiments [Krumbein, 1941; Adams, 1978; Kodama, 1994b; Mikos, 1995; Lewin and Brewer, 2002; Attal and Lavé, 2009]. Although Sternberg originally proposed that this downstream decline in particle diameter was due to abrasion, there is a consensus now that the dominant effect is size-selective transport, in which larger grains are preferentially deposited and smaller particles travel farther downstream [Paola et al., 1992; Seal and Paola, 1995; Paola and Seal, 1995; Ferguson et al., 1996; Gasparini et al., 1999]. In particular, Fedele and Paola [2007] derived a simplified theory in which equation (1) arises as a consequence of size-selective sorting. What role, if any then, does abrasion play in the downstream fining of pebbles?

Many researchers have used laboratory experiments to quantify abrasion rate as a result of collisions among pebbles during bed load transport, typically through employing tumbling mills [Wentworth, 1919; Krumbein, 1941; Kodama, 1994b; Lewin and Brewer, 2002] or circular flumes [Lewin and Brewer, 2002; Attal et al., 2006]. Although measured rates vary greatly depending on the type of apparatus employed and also pebble lithology, a unifying conclusion has been that abrasion rates are generally too slow to account for observed fining trends (described by equation (1)) in rivers [Adams, 1978; Hoey and Bluck, 1999; Morris and Williams, 1999]. This conclusion is not without objection, however, mainly on two fronts: (1) most experiments do not simulate the high-energy collisions typical of steep mountain streams, and those that do produce results more consistent with expectations from the field [Kodama, 1994a; Lewin and Brewer, 2002; Attal and Lavé, 2009] and (2) experiments usually measure mass loss while field studies typically measure changes in diameter, but the two can only be directly compared if the exact shape of the pebbles are known, which they are not [Kodama, 1994b; Lewin and Brewer, 2002; Domokos et al., 2014]. Even if suitable collision energies can be generated in the laboratory, extrapolating these results to the field also requires reliable estimates of the frequency of grain-grain collisions and the transport distances between collisions [Sklar and Dietrich, 2004; Turowski et al., 2013]. Duration of tumbling mill experiments is used as a proxy for distance in the field, providing only an indirect link to abrasion rate.

A new approach has been undertaken recently, in which a generalized geometric theory for abrasion [Firey, 1974; Bloore, 1977] has been adapted to describe the evolution of pebble shape; this process can be visualized by plotting shape descriptors versus volume [Domokos et al., 2014]. The model will be described below, but its essence is that abrasion rate at any point on a pebble's surface is a function of the local curvature. Experiments involving a single initially cuboid pebble in a tumbler, designed to simulate the idealized conditions assumed in the derivation of the model, showed quantitative agreement with model predictions [Domokos et al., 2014]. Results imply that the significance of bed load abrasion in a river may be assessed by examining changes in pebble shape and volume downstream, circumventing the need to extrapolate abrasion rates from the laboratory. A major finding from the geometric theory and its companion experiment was that abrasion of an initially blocky particle occurs in two phases: Phase I, in which the pebble abrades to a convex shape *without any major change in axis dimensions*; after which it proceeds to Phase II, where the convex pebble slowly reduces its axis dimensions.

If two-phase abrasion occurs in rivers, it suggests that most of the mass loss from abrasion goes undetected in field studies because researchers only measure diameter. It is an open question, however, whether the idealized geometric model may be applied to abrasion by bed load in natural field settings. This paper presents the first use of geometric theory to quantify the significance of curvature-driven abrasion in a natural river. First, we present the general theoretical framework, which informs our choice of parameters to characterize the size and shape of pebbles. We then introduce a field location in northeast Puerto Rico, where a river was selected that allows us to isolate the contributions of abrasion and size-selective sorting. By examining downstream trends in pebble size and shape over 10 km, we test for the qualitative patterns of curvature-driven abrasion and seek quantitative verification of the geometric model. A simplified abrasion and deposition model is then employed to determine the contribution of abrasion and size-selective sorting to downstream diminution of pebble mass and diameter. Finally, we present a generalized method for determining the contribution of abrasion in other field settings.

2. Theory of Pebble Abrasion

2.1. Kinetic Energy, Mass Loss, and Sternberg's Law

Numerous theoretical and experimental studies have demonstrated that the magnitude of mass removed from a particle undergoing collision is proportional to the kinetic energy of the impact [Anderson, 1986;

Figure 1. Sketch of shape evolution under the two terms of equation (3). (a) The Eikonal term and (b) the curvature term. Figures are adapted from Szabó et al. [2013].

Attal and Lavé, 2009; Domokos and Gibbons, 2013]. Assuming a steady rate of impacts over time, in the continuous limit the rate of mass loss for a pebble of mass M due to abrasion becomes $dM/dt = -kMu_s^2$, where u_s is the velocity of the pebble and k is a coefficient related to strength of the rock and additional transport parameters not explicitly considered. Experiments have demonstrated that, for a wide range of bed load transport conditions, particle velocity u_s is proportional to the stream fluid velocity (u_f) [see Lajeunesse et al., 2010; Martin et al., 2012]; observations of tracer cobbles in our study area are also consistent with this finding [Phillips et al., 2013; Phillips and Jerolmack, 2014]. Fluid velocity, in turn, is only a slowly varying function of discharge ($u_f \propto Q^n$ where n can range from 0.14 to 0.17) [Leopold et al., 1964; Bray, 1982; Parker et al., 2007]. To first order, one can thus consider pebble velocity constant, with two consequences: (1) the rate of mass loss of a pebble is proportional to its mass and (2) downstream distance in a river is proportional (but not equivalent) to time ($dx \propto u_s dt$). Thus, one expects that downstream changes in pebble mass due to abrasion will take the form of an exponential, $dM/M \propto -kdx \rightarrow M \propto M_0 e^{-kx}$. Cast in terms of pebble volume (assuming constant density) and neglecting coefficients of proportionality, the downstream diminution of pebble size due to abrasion takes the form

$$V = V_0 e^{-kx}. \quad (2)$$

It should be apparent that equation (2) is related to Sternberg's Law (equation (1)). Indeed, laboratory experiments that measure mass loss demonstrate the validity of equation (2) but then convert it into Sternberg's Law by assuming that $D \propto V^{1/3}$ (and hence $\gamma = k/3$) to predict the anticipated effect of abrasion on downstream fining [Kodama, 1994b; Lewin and Brewer, 2002]. Although the heuristic derivation above is rather simplistic, it demonstrates that one can rationalize Sternberg's law (equation (1)) from either size-selective sorting [Fedele and Paola, 2007] or abrasion. The assumed proportionality between pebble diameter and volume results from an assumption that pebbles abrade in a self-similar fashion. This is certainly not true for river rocks which evolve from blocky fragments to smooth ellipsoidal shapes [Krumbein, 1941; Kuenen, 1956; Lewin and Brewer, 2002; Durian et al., 2006; Domokos et al., 2014]. For initially polyhedral particles, abrasion may remove up to half of pebble mass without any reduction in D [Lewin and Brewer, 2002; Domokos et al., 2014]. Proper accounting for this geometric effect will paint a more accurate picture of the significance of abrasion.

2.2. Shape Evolution and Two-Phase Abrasion

The geometric modeling of pebble abrasion dates back to Bloore [1977], who described the shape evolution of a single pebble under collisional abrasion. The 2-D equivalent of Bloore's equation can be formulated as

$$v = 1 + c\kappa \quad (3)$$

where v is the attrition speed in the inward normal direction, c is the (average) perimeter of the abrading particles in the environment [Várkonyi and Domokos, 2011], and κ is the local curvature of the evolving 2-D curve. In this description of abrasion there are two competing terms. If the abrading particles are small then c is also small and the first (so-called Eikonal) term ($v = 1$) dominates the process. This causes shapes to develop sharp edges and flat areas (Figure 1a) such as in the case of sandblasting [Knight, 2008; Domokos et al., 2009a]. The second, curvature term ($v = c\kappa$) dominates if the abrading particles are much larger, i.e., c is also large (Figure 1b).

Figure 2. Conceptual figure of two-phase abrasion on a rectangle, and its expected behavior along a river profile. In the energetic upper reaches of a river, Phase I occurs where corners are abraded without any change in axis lengths, while the numbers of both stable and unstable equilibrium points decrease. In lower gradient reaches Phase II occurs, where the axis ratio S/L (in 3-D, S/L and I/L) increases, indicating that the pebble approaches a circle (in 2-D, a sphere), while the number of equilibrium points remains constant.

We will call this second case curvature-driven abrasion. In three dimensions, κ is replaced by the linear combination of the so-called mean and Gaussian curvatures; however, in case of very large abraders the Gaussian term dominates. In the field, curvature-driven abrasion can be interpreted as a saltating pebble undergoing abrasion by collision with a substrate composed of very large particles (boulders or bedrock). The curvature-driven abrasion theory treats surface abrasion of a pebble as a diffusion process—akin to hillslope diffusion [Culling, 1960; Hirano, 1968; Roering *et al.*, 1999]—and it predicts that arbitrary initial shapes converge asymptotically to a sphere (in 2-D, a circle, Figure 1b) [Firey, 1974; Andrews, 1999]. (For a more elaborate description of geometrical abrasion theory, see Várkonyi and Domokos [2011] and Szabó *et al.* [2013].)

Recently, Domokos *et al.* [2014] performed laboratory experiments simulating curvature-driven abrasion in which a single cuboid was abraded in a rotating drum that can be thought of as a very large abradar. They demonstrated that curvature-driven abrasion commences in two phases, both in numerical simulation and in experiments: in Phase I, sharp edges with high curvatures rapidly round off without major changes in the global axis dimensions until the original blocky particle evolves to an ellipsoid-like shape; subsequently, in Phase II, axis dimensions start to decrease slowly and the pebble becomes more spherical in shape (Figure 2). While shape evolution occurred in two phases, they found that the rate of mass loss was continuous through both phases and depended only on pebble mass; in other words, the volumetric diminution described by equation (2) applies to abrasion in all phases, regardless of shape. However, the diameter diminution described by equation (1) (Sternberg's law) does not apply to Phase I abrasion, where diameter is almost constant. These authors thus suggested that equation (2) is a more applicable "Generalized Sternberg's law," relevant for abrasion. Although theory and experiment were for the idealized case of a single particle colliding with an infinitely large abradar, Domokos *et al.* [2014] suggested that this assumption might be relaxed such that the theory could apply to like-sized colliders in bed load transport. There are qualitative indications from a reexamination of classic experiments by Krumbein [1941] and Kuenen [1956] that this is indeed the case. This idea is explicitly tested in this study with field data.

Domokos *et al.* [2014] tracked several shape descriptors in both laboratory experiments and the corresponding numerical models. The simplest shape descriptors are the axis ratios S/L and I/L , where $L > I > S$ denote the three axis lengths of the bounding box of the pebble (Figure 3). Axis ratios S/L and I/L remained approximately constant during Phase I and increased in Phase II as the particle evolved toward a spherical shape. Another shape descriptor tracked in their work was the convexity defined as $\text{Conv}_{3-D} = S_C/S_H$, where S_C is the surface area of the convex regions and S_H is the total surface area of the convex hull of the particle. As the area of the intact surfaces of the abraded particle decreased during Phase I, convexity increased in their experiments and numerical simulation until reaching $\text{Conv}_{3-D} = 1$, and this value stayed constant in the subsequent Phase II.

Numerous other shape parameters have been proposed in the literature to quantify the morphology of pebbles [Blott and Pye, 2008]. Many of these parameters are defined on 2-D projections of pebbles. The advantage of the latter is that one may take photographs of pebbles in the field and compute the shape descriptors automatically using standard image processing software. However, the evolution of most of these shape descriptors under curvature-driven abrasion is unknown. Nevertheless, we are aware of two 2-D-shape descriptors, which are known to change monotonically under curvature-driven abrasion—and hence may be used to test the geometric theory. The first one is the isoperimetric ratio defined as IR

Figure 3. Shape descriptors. (a) Axis dimensions $L > I > S$, as they are measured for pebbles in the field. (b) Isoperimetric ratio IR measured from 2-D projections of pebbles. (c) Convexity $Conv_{2D}$ measured from 2-D projections of pebbles.

(isoperimetric ratio) = $4\pi A/P^2$, where A is the area enclosed by the evolving 2-D curve and P is the perimeter of the curve (Figure 3). IR is often referred to as circularity [Blott and Pye, 2008] or roundness [Cox, 1927] in the literature. For a perfect circle $IR = 1$, and for any other curve $IR < 1$. It was proven by Gage [1983] that IR increases monotonically under curvature-driven abrasion. Although the shape evolution of a 2-D curve differs from the shape evolution of the 2-D projection of a 3-D particle, we expect similar behavior for the projections.

The second shape descriptor is the entropy defined by the curvature distribution along the perimeter of the 2-D curve, which we will refer to as the curvature entropy. Curvature entropy was originally defined for smooth, convex curves by Chow [1991], who showed that if the perimeter of the curve is normalized to unity ($P = 1$) then the curvature entropy increases under curvature-driven abrasion. However, a pebble's surface is naturally nonsmooth [Domokos et al., 2012] and, moreover, a photo taken of the 2-D projection of a pebble has a finite resolution, so in our approximation pebble contours are represented by convex polygons. By suitable interpolation we replace the original polygon by a polygon with equal sides. In this case, instead of using the curvature entropy as described in Chow [1991], we apply its discrete analogue, the so-called

Shannon information entropy [Shannon, 1948], $H_S = -\sum_{i=1}^m \frac{\alpha_i}{2\pi} \log\left(\frac{\alpha_i}{2\pi}\right)$. Here α_i is the external angle at the i th vertex of the m -sided convex hull of the pixel contour resulting from the image processing (Figure 4). Based on the results of Chow [1991], it can be shown that for fixed values of m , the Shannon curvature entropy H_S also increases under curvature-driven abrasion. The physical interpretation is as follows: as the 2-D contour of a pebble evolves toward a circle under abrasion, the curvature along the pebble's perimeter

Figure 4. Image processing and curvature entropy. (a) Raw image of pebble taken in the field. (b) Image prepared for bulk-shape data analysis by converting to binary image. (c) Interpolated polygon with equal sides. (d) External angles used to calculate curvature entropy.

Table 1. Expected Evolution of Various 3-D and 2-D Shape Descriptors Under Curvature-Dependent Abrasion

Shape Descriptor	Measurement Method (This Study)	Expected Behavior	
		Phase I	Phase II
Axis ratios (S/L and I/L)	Hand measure	Constant	Increase $\rightarrow S/L = I/L = 1$
Convexity (Conv_{3-D})	Image technique (Conv_{2-D})	Increase $\rightarrow \text{Conv}_{3-D} = 1$	Constant ($\text{Conv}_{3-D} = 1$)
Equilibrium points (n_S and n_U)	Hand measure	Decrease $\rightarrow n_S = n_U = 2$	Constants ($n_S = n_U = 2$)
Isoperimetric ratio (IR)	Image technique	Increase $\rightarrow \text{IR} = 1$	
Curvature entropy (H_S)	Image technique	Increase	

becomes more uniform (and curvature entropy increases); for a perfect circle, curvature is equal at all points on the curve (and entropy is maximized).

The last shape descriptor applied in our study is the number of static equilibrium points, which was recently proposed to classify pebble shape [Domokos *et al.*, 2010]. Equilibrium points are points on an object's surface where the object may rest stationary on a horizontal surface. Stable and unstable equilibrium points correspond to local minima and maxima of the object's radius from its center of gravity. One advantage of measuring equilibrium points is that they are integers that may be objectively counted in the field by simple balancing (as long as grains can be manually lifted). Figure 2 illustrates that the numbers of stable (n_S) and unstable (n_U) equilibria are expected to decrease during Phase I abrasion, as corners round and the initially angular shape with many equilibrium points approaches an ellipse-like shape with only two stable and two unstable equilibrium points. In Phase II, this decreasing trend stops and the number of equilibrium points is expected to remain $n_S = n_U = 2$.

To summarize, we expect all of the shape descriptors to change monotonically with distance downstream in a river under curvature-driven abrasion. Further evidence for curvature-driven abrasion from river data would be the delineation of the two phases of shape evolution described above (see Table 1). An individual particle traveling downstream is expected to undergo a sharp phase transition in its shape evolution. We expect, however, that the statistical behavior of an ensemble of particles in a natural river—where sediment input is spatially distributed—will exhibit a more subtle transition, because any point in a river contains a mixture of pebbles that are at different points in their respective trajectories.

2.3. The Box Equations for Modeling Phase II Abrasion

While equation (3) can capture the shape evolution in both Phases I and II, this equation and especially its 3-D equivalent [Bloore, 1977] are difficult to analyze both analytically and numerically. It also assumes an invariable environment, i.e., that there is only one abraded particle and that impacting particles are all identical and unchanging in shape and size. Another drawback is that it only treats collisional abrasion, although frictional abrasion (rolling and sliding) is likely to be important in Phase II of curvature-driven abrasion. Thus, while equation (3) and its 3-D equivalent offer an adequate tool to understand the abrasion of a single particle and also offer a good approximation to the abrasion in Phase I, they are, in their original form, inappropriate to numerically simulate the shape evolution of large particle populations in the second phase where the shape evolution is dominated by collective (particles abrade each other) and frictional abrasion.

A suitable solution for these problems is the use of the box equations recently published by Domokos and Gibbons [2012]. The box equations were derived from the 3-D equivalent of equation (3) by assuming that pebble shape is always a triaxial ellipsoid. Thus, the box equations are limited in that shape evolution may only be tracked in Phase II, where the assumption of ellipsoidal pebbles is valid. However, the main advantage of the box equations is that they are based on the concept of mutual abrasion, and therefore, they offer a model for the collective evolution of a large number of pebbles through binary collisions. Additionally, frictional abrasion can be easily included as an additive term. The original concept of the box equations was developed further in Domokos and Gibbons [2013], incorporating an independent physical model for volume diminution. The general form of the box equations is

$$\dot{\mathbf{y}} = \mathbf{C}_y^c \cdot \mathbf{F}^c(\mathbf{y}, \mathbf{z}) + \mathbf{C}_y^f \cdot \mathbf{F}^f(\mathbf{y}) \quad (4)$$

$$\dot{\mathbf{z}} = \mathbf{C}_z^c \cdot \mathbf{F}^c(\mathbf{z}, \mathbf{y}) + \mathbf{C}_z^f \cdot \mathbf{F}^f(\mathbf{z}) \quad (5)$$

Figure 5. Conceptual illustration of the effects of frictional and collisional abrasion in equations (4) and (5) on the plane S/L - I/L . Sliding drives particles toward infinitely flat shapes ($S/L = 0$), rolling results in an infinitely thin needle-like shape ($S/L = I/L = 0$), while collisions between like size particles produce spheres ($S/L = I/L = 1$). Figure adapted from Domokos and Gibbons [2012].

(cf. Domokos and Gibbons [2013] where the exact formulation of functions $\mathbf{F}^c$ and $\mathbf{F}^f$ can be found). In this system of equations, $\mathbf{y}$ and $\mathbf{z}$ are two interacting particles where $\mathbf{y}$ represents the abrading environment for $\mathbf{z}$ and vice versa and $\dot{(\cdot)}$ denotes differentiation with respect to time. Both $\mathbf{y}$ and $\mathbf{z}$ are three-component vectors with components S_y/L_y , I_y/L_y , and $L_y/2$ and S_z/L_z , I_z/L_z , and $L_z/2$, respectively, so the box equations aim to track the evolution of the axis ratios and the size (the semimajor axis length) of the pebbles. The first additive term on the right-hand sides of equations (4) and (5), with superscripts c , describes collisional abrasion, i.e., the result of many binary collisions between $\mathbf{y}$ and $\mathbf{z}$, accordingly the arguments include both $\mathbf{y}$ and $\mathbf{z}$. The second additive term, with superscripts f , describes frictional abrasion, i.e., the rolling or sliding of a pebble on a substrate. The arguments here include only the particle in question, i.e., either $\mathbf{y}$ or $\mathbf{z}$. The separate effects of frictional and collisional abrasion in equations (4) and (5) are

illustrated in Figure 5. Coefficients C_y^c , C_z^c and C_y^f , C_z^f represent the intensity of collisional and frictional abrasion, respectively. These coefficients may depend on the size of the particle since it is well known that the mode of transport (sliding, rolling, saltation, and suspension), and thus, the intensity of frictional versus collisional abrasion depends on the size of the particle [Abbott and Francis, 1977; Drake et al., 1988]. Below, the box equations are developed into a numerical model that when applied to field data allows us to quantify the contribution of abrasion to downstream fining in a natural river.

3. Field Setting and Measuring Methods

3.1. Field Setting

We seek a demonstration that curvature-driven abrasion—predicted by the idealized geometric model of a single abrader colliding with an infinite plane—occurs in the downstream evolution of pebbles undergoing collision due to bed load transport in a natural river. In particular, we predict (1) a monotonic downstream increase in IR , H , convexity, and the aspect ratios S/L and I/L ; (2) a monotonic downstream decrease in the number of equilibrium points; and (3) the appearance of the two phases of abrasion described above. In addition, we aim to quantify the contributions of abrasion and size-selective sorting to downstream diminution of pebble mass and diameter in a river, by employing the box equations. We expect, in general, that the dominant process governing pebble evolution in rivers changes from abrasion in the energetic headwaters to size-selective sorting in the depositional alluvial plain [Shaw and Kellerhals, 1982; Dawson, 1988; Paola and Seal, 1995; Gasparini et al., 1999; Jerolmack and Brzinski, 2010]. Accordingly, an ideal field setting would be a wadeable river that may be traversed from source to sink, with a point source of sediment input at its headwaters. The river should be of very steep slope in the upper portions, with a bedrock channel bottom and no floodplain, to facilitate abrasion and suppress size-selective sorting. The lower reaches of the stream should be characterized by a low-gradient alluvial channel with well-developed floodplains, to allow the effect of size-selective transport to manifest through deposition.

Our river of study is the Rio Mameyes and its steep tributary Bisley 3, located in the Luquillo Critical Zone Observatory in northeastern Puerto Rico (Figure 6). Sediments in the channel are composed almost exclusively of volcanoclastic lithology, which are fine-grained sedimentary rocks [Seiders and Pease, 1971;

Figure 6. Map of field site, located within the Luquillo Critical Zone Observatory in northeastern Puerto Rico. The red line outlines the Rio Mameyes watershed, and the blue line denotes the channel. Circles mark sampling sites. Yellow circles represent detailed sampling sites where equilibrium points and axis dimensions were measured in addition to images of pebbles (grain populations A and B). Red circles represent sampling sites where only images of pebbles were taken (grain population C).

Briggs and Aguilar-Cortés, 1980]. The section of river under study is ~10 km long, and its profile exhibits a concave shape, but with a clear break in slope at the junction of Bisley 3 with the Mameyes (Figure 7a). This tributary was selected because of its continuous accessibility from the headwaters to the gravel-sand transition in the mainstem. Bisley 3 contains a ~10 m high knickpoint in its upper reaches. The knick point appears to be a significant source of sediment to the channel downstream, as piles of rocks up to ~1 m in diameter may be seen just below it. Beyond 100 m downstream of the knickpoint, rocks within the stream exhibit no visible weathering rinds and are angular and irregular in shape (Figure 7c). We performed Schmidt hammer tests on ~10 particles larger than 1 m (the minimum size required for reliable measurements) at each pebble count site to assess material strength. With the exception of the weathered boulders in the vicinity of the knickpoint, sampled particles had consistent strength values (mean of 85 MPa) with little variability and no downstream trend

(Figure 7b). Results suggest that bed load sediments should have approximately uniform susceptibility to abrasion downstream. The lack of weathering rinds also indicates that abrasion is rapid compared to in-stream weathering. We take the knickpoint as the beginning of bed load transport in the river, and the limiting source location for sediment in the stream; it is thus the origin of our profile ($x = 0$ km). Unfortunately, it is not the only source of sediment; landslides are prevalent along the steep valley walls of Bisley 3 and are capable of delivering very coarse and angular particles to the stream. Thus, sediment input is spatially distributed rather than from a point source. Numerical models and field studies have shown that spatial variations in sediment supply can produce either downstream fining [Pizzuto, 1995; Sklar *et al.*, 2006] or coarsening [Attal and Lavé, 2006] grain size trends. The potential effects of spatially varying sediment supply could obscure expected patterns from abrasion and must be carefully considered when interpreting observed trends and model results.

Along the ~2 km distance from the knickpoint to its junction with the Mameyes, Bisley 3 exhibits sporadic bedrock exposure, slopes generally greater than 0.1, and no floodplain; it is a partially alluviated bedrock river [Howard, 1980; Whipple, 2004]. We expect abrasion to be dominant in this tributary with little to no size-selective transport, due to the general preference for deposition in fully alluviated reaches and the lack of sediment storage in bedrock reaches [Hodge *et al.*, 2011]. At the junction of the Bisley 3 tributary with the mainstem Mameyes, the Mameyes is an alluviated bedrock channel confined in a valley. It transitions at $x = 5$ km to a fully alluvial stream with a well-developed floodplain on its exit from the mountains (Figure 7a). River rocks in the Mameyes are rounded and nearly ellipsoidal in shape (Figure 7d). Our study region ends at the upstream boundary of the gravel-sand transition on the Mameyes—i.e., we only examine the gravel portion of the river where bed load predominates. We expect size-selective transport to dominate over abrasion in the lower alluvial portion of the Mameyes.

The drainage area of the Mameyes watershed is 44 km², with a mean annual rainfall of >4500 mm/yr at the headwaters and 1500 mm/yr at the mouth [Garcia-Martino *et al.*, 1996]. Orographic effects and hurricanes produce intense rainfall events and frequent bed load transport [Scatena *et al.*, 2004; Heartsill-Scalley *et al.*,

Figure 7. Photographs of the field site. (a) River elevation profile with important transitions labeled. (b) Plot of uniaxial compressive strength (UCS) measured by Schmidt hammer (converted from hammer rebound to UCS using relation in Kahraman [2001]) versus distance from headwaters. (c) Headwater tributary Bisley 3 is steep and contains large angular pebbles. (d) Lower alluvial mainstem Rio Mameyes has shallow slope and contains smaller rounded rocks.

2007; Pike *et al.*, 2010]. A recent study of tagged cobbles in Bisley 3 and the Mameyes showed that pebbles up to 0.3 m in diameter are mobilized approximately 20 times per year and that some traveled up to 1.2 km over a 2 year study period [Phillips *et al.*, 2013]. Based on the description above, we expect that rapid Phase I abrasion occurs in the steep and energetic Bisley 3 tributary and then transitions to Phase II in the Mameyes. Because mass loss from abrasion reduces the collision energy of a pebble (equation (2)) [Jerolmack and Brzinski, 2010], we expect a downstream gradient of decreasing abrasion rate along the study profile, while fining due to size-selective deposition should begin in the Mameyes on its exit from the mountains. A key assumption in our approach is that abrasion occurs by chipping and planing, i.e., that fragmentation due to crushing is not significant. Rock fragmentation would partially reset particle shape evolution by creating sharp edges. If this process were dominant, none of the observed downstream trends in particle shape would be consistent with the geometric theory. It is likely that some degree of fragmentation

occurs, which would slow the observed rate of downstream rounding of grains, but the trends we present below indicate that it cannot be dominant. Indeed, visual inspection of bed sediments showed very few fresh fracture faces at each site, indicating that fragmentation was not significant in this river. However, no attempt was made to quantify the occurrence of fragmentation as the required judgment was deemed too subjective.

3.2. Measuring Methods

We selected nine sites along Bisley 3 and eight sites along the Rio Mameyes for detailed study (Figure 6). At each site, we performed measurements on two grain populations. For grain population A, we collected 100–150 grains randomly, from the size range 20–200 mm (in terms of axis length L). The lower limit in size was based on our desire to sample only particles transported primarily in bed load; the upper limit represents the maximum reasonable size of a rock that could be lifted. We measured the three axis lengths ($L > I > S$) of each pebble and counted the number of stable (n_S) and unstable (n_U) equilibrium points by hand [Domokos *et al.*, 2010]. These grains were also placed on a rigid board, with axis S perpendicular to the board, and photographed from above to obtain images of the maximum 2-D projection of the grain. Axis ratios S/L and I/L were computed from the measured axis lengths, while the images were used to compute the isoperimetric ratio (IR), the curvature entropy (H_S) and a 2-D version of convexity defined as $\text{Conv}_{2-D} = A_P/A_H$, where A_P is the area of the grain's projection and A_H is the area of the convex hull of the projection (Figure 3c). This convexity index is sometimes referred to as solidity [Rashband, 1997]. Measurements at each site were averaged in stratified grain size ranges—20–64 mm, 65–128 mm, and larger than 128 mm—to compensate for noise while allowing some assessment of relations between size and shape that may arise, for example, by differing modes or frequency of transport. For grain population B, Wolman pebble counts [Wolman, 1954] were performed by randomly selecting 100 particles from the surface of the bed and measuring L , I , and S for each of them. There was no size restriction for measurement B. The manual measurements provide rich data but are time intensive. To complement and extend the spatial range of these data, we selected an additional 58 sites along the Rio Mameyes and Bisley 3 where only image-based data were collected by taking photographs of 40 randomly selected grains; we denote these measurements as grain population C. Shape parameters estimated from all grains at each site were averaged together to produce a single value per site.

4. Results

4.1. Field Data and Two-Phase Abrasion

We first examine the downstream trend in axis dimensions measured from pebble counts. Throughout the length of Bisley 3 there is no discernible trend for any of the axis dimensions (Figure 8c). Beyond this distance in the mainstem Mameyes, axis dimensions decline with distance downstream (Figure 8c). The data permit—but do not confirm—an exponential fit to this downstream trend (equation (1)). The axis ratios S/L and I/L fluctuate but show no trend over the first ~1.5 km studied and then begin to slowly increase (Figures 8a/8b). These two patterns are compatible with the constant and increasing trends expected between Phases I and II, respectively, of the geometric abrasion theory but are not conclusive (Table 1).

Convexity (Conv_{2-D}) shows a more robust and smooth pattern with distance downstream (Figure 8h); it first rapidly increases over a distance of 1.5 km and then appears to saturate at a value of approximately 0.98 indicating almost completely convex shapes. Values for IR increase rapidly over the same distance as convexity and then continue to increase but at a lower rate over the remaining distance downstream (Figure 8g). Similarly, the trend for entropy tracks convexity and IR (Figure 8i). Finally, the number of equilibrium points declines rapidly over the same distance as other rapid shape changes and then fluctuates widely in the lower 8.5 km of the river. All of the observed shape parameter trends are in agreement with qualitative predictions of curvature-driven abrasion (Table 1). The exact location of the transition from Phases I to II cannot be identified, as the variability inherent in this natural system precludes a sharp transition. Shape data indicate, however, that the first 1 km of Bisley 3 is within Phase I and that the transition to Phase II has occurred by $x = 2$ km where this tributary joins the mainstem Mameyes. It is interesting to note that despite the large change in channel slope and discharge at this junction, particle size and shape exhibit no discontinuity.

Figure 8. Field results—Plots of site-averaged shape descriptors as a function of distance downstream of the origin. Dashed line denotes the location of the tributary junction between Bisley 3 and the Rio Mameyes. Data in Figures 8a, 8b, 8e, and 8f are averaged at a site by the following size classes: 20 to 64 mm (open circles), 65 to 128 mm (black squares), and greater than 128 mm (gray triangles). (a) Average axis ratio S/L by different size class for grain population A; (b) average axis ratio I/L by different size class for grain population A; (c) measured grain size for all three axis dimensions, S (gray triangles), I (black squares), and L (open circles) for grain population B; (d) estimated total pebble volume for grain population B; (e) average stable equilibrium points by different size class for grain population A; (f) average unstable equilibrium points by different size class for grain population A; (g) average 2-D convexity for grain population A and C; (h) average isoperimetric ratio for grain population A and C; and (i) average curvature entropy of 2-D image contours for grain population A and C.

Average values for equilibrium points indicate that pebbles in Phase II are not ellipsoids ($n_s = 3.1$ and $n_u = 2.9$), which may be a consequence of natural heterogeneity or effects such as friction that are not accounted for in equation (3) [Szabó *et al.*, 2013]. Nonetheless, they are almost entirely convex and smooth, indicating that describing pebbles in Phase II as triaxial ellipsoids—a prerequisite for applying the box equations—is a reasonable approximation. Although pebble volume was not measured, it may be estimated for pebbles in Phase II from measured axis dimensions (grain population B) by using the assumption of triaxial ellipsoidal shape: $V = \frac{\pi}{6} SIL$. Results show that pebble volume decreases downstream in Phase II (Figure 8d), in a manner consistent with an exponential form, i.e., equation (2). However, this volume decline may combine effects from both abrasion and size-selective transport.

4.2. Numerical Model

Here we develop and implement a simple numerical model based on equations (4) and (5) presented above. The box equations are capable of modeling the collective evolution of a large population of particles through binary collisions, assuming Phase II abrasion. Additional terms may be added to account for deposition and frictional effects; by adjusting the magnitude of these terms in the model in order to match field observations, we may assess the relative contribution of different processes to downstream changes in pebble size and shape. Downstream pebble evolution is modeled below for the portion of the Mameyes

over which we infer that Phase II abrasion is operative. Therefore, the model is only run to simulate grain evolution down the mainstem Mameyes, not Bisley 3. The box equations require as input an initial distribution of grain sizes and axis ratios. The pebbles from Bisley 3 do not exhibit any systematic trend in either parameter; moreover, data demonstrate that pebbles at the mouth of Bisley 3 have achieved Phase II abrasion, a prerequisite for modeling shape evolution with the box equations. In order to obtain robust statistical distributions of initial particle sizes and aspect ratios, we use all data collected along Bisley 3 and at the upstream most site in the Rio Mameyes as the model input conditions.

4.2.1. Abrasion

In the numerical simulation of the equations (4) and (5), following Domokos and Gibbons [2012] we consider n particles and in each iterative step we choose the two particles $\mathbf{y}$ and $\mathbf{z}$ randomly from the population and run the discretization of equations (4) and (5) for a short time period:

$$\mathbf{y}^{i+1} = \mathbf{y}^i + \Delta t \left[C_y^c \cdot \mathbf{F}^c(\mathbf{y}, \mathbf{z}) + C_y^f \cdot \mathbf{F}^f(\mathbf{y}) \right] \quad (6)$$

$$\mathbf{z}^{i+1} = \mathbf{z}^i + \Delta t \left[C_z^c \cdot \mathbf{F}^c(\mathbf{z}, \mathbf{y}) + C_z^f \cdot \mathbf{F}^f(\mathbf{z}) \right] \quad (7)$$

Following the argument laid out in section 2.1, we assume that the pebble travel distance and model time are linearly related [also see Szabó *et al.*, 2013]. This assumption presumes a constant transport velocity as a first-order approximation, recognizing that the actual virtual velocity of particles may vary downstream [Hassan *et al.*, 1992; Ferguson *et al.*, 1996]. We begin the simulation with an initial pebble population obtained from the field measurements and apply equations (6) and (7) iteratively for randomly chosen pebble pairs; the model result generates a time evolution for the axis dimensions of each pebble that is equivalent to a downstream evolution (Figure 5).

The mode of sediment transport depends on the size of the particle; small pebbles are mainly saltating, while larger particles experience rolling and sliding [Abbott and Francis, 1977; Drake *et al.*, 1988]. To take this effect into account, we assume that the intensity of frictional abrasion grows linearly with the size of the particle, that is $C_y^f(L_y) = c_1 L_y$ and $C_z^f(L_z) = c_1 L_z$, where c_1 is a constant and L is measured in millimeters. For the coefficient of collisional abrasion, we assume constants $C_y^c = C_z^c = 1$. Since C_y^c and C_z^c do not depend on the size of the particles, our assumption allows that even large boulders can collide with each other sometimes. While this is probably not physically realistic, due to its rarity it has little effect on the results. Also, by assuming a constant value for C_y^c and C_z^c we can allow the physically relevant situation of a large particle (cobble and boulder) impacted by a mobile pebble.

4.2.2. Selective Deposition

We use the numerical box model to analyze the role of abrasion and selective transport simultaneously in the Rio Mameyes. Although several physical models of selective transport have been proposed in the literature [Fedele and Paola, 2007; Ferguson *et al.*, 1996; Paola and Seal, 1995], and these models could, in principle, be integrated into the box equations, this is beyond the scope of this paper. Instead, we couple the box equations with a simplistic, phenomenological selective deposition rule. Each pebble has an expected value for the final distance traveled to deposition, X . Tracer measurements from the Rio Mameyes showed that normalized step lengths are exponentially distributed in the river [Phillips *et al.*, 2013]; thus, we assumed that the final distance X is a random variable with exponential distribution, where the parameter of the distribution is $1/E[X]$. We assumed that the expected value of X depends on the maximal size L_y of the particle: $E[X] = c_2 e^{-c_3 L_y}$, where c_2 and c_3 are constants. We implemented this simple deposition rule into the numerical box model equations (6) and (7) in the following way: in each iterative step, both for particles $\mathbf{y}$ and $\mathbf{z}$, we randomly draw a value for the final travel distance X using the above exponential distribution. Then, if the actual distance from the source is larger than X , the particle is deposited out of the flow, i.e., we remove it from the particle population.

4.2.3. Abrasion and Selective Deposition

The three parameters (c_1 , c_2 , and c_3) of the numerical model were fitted to obtain the best agreement with the measured field data for the Rio Mameyes. The numerical simulation began with approximately 3000 particles in the system, whose size ranged from $L = 20$ mm to 4 m. The time step was $= 1/1000$, and the total number of iterative steps was 500,000. We found that the optimal value for c_1 (the coefficient in the assumed linear size dependence of friction intensity) is around 0.005. For the selective deposition law we used $c_2 = 10^8$ and $c_3 = 0.006$. This produces an expected travel distance $E[X] = 3.8$ mm for the upper limit

Figure 9. Plots of field data (black triangles) overlaid with results from full box model simulations, showing trends for downstream distance for Phase II abrasion (only mainstem Mameyes). (a) Average long axis for pebbles under 200 mm in length of L . (b) Axis ratio S/L for pebbles under 200 mm in length of L . (c) Axis ratio I/L for pebbles under 200 mm in length of L . (d–f) Same as Figures 9a–9c but for the entire size range of pebbles.

(a 4 m grain), practically meaning that such a large boulder does not move. For the lower size limit (a 20 mm grain) we have $E[X] = 89,000$ km, i.e., such a small pebble will never be deposited in the model. Figure 9 shows that the site-averages of the measured field data and the corresponding model results match well using the above parameters. Figures 9a–9c show the averages from grain population A, where

Figure 10. Plots of field data (black triangles) overlaid with results from box model simulations in mainstem Mameyes with limiting conditions. Solid line is for model run with no selective deposition, dotted line for no abrasion at all, and dashed line for no frictional abrasion. (a–f) The same as for Figure 9.

corresponding model results were computed only from the particles which fell into the size range of measured field data, i.e., 20–200 mm. Figures 9d–9f shows the shape and size evolution of the whole size range (grain population B).

To better understand the role of competing physical processes in the numerical model, Figure 10 shows the main limiting cases. Model results with no selective deposition (solid line) are not in agreement with the data for the entire size range, indicating that abrasion significantly underpredicts the degree of downstream fining. However, the predicted size and shape evolution in the 20–200 mm size range is reasonable, suggesting that size-selective deposition is ineffective in this restricted size range. For the second limiting case of no abrasion (dotted line), we see that selective deposition alone cannot reproduce the observed increase in the axis ratios of particles in the 20–200 mm size range. However, the results for the whole particle population are reasonably good. We conclude that the strong downstream fining observed in the whole size range is essentially due to selective deposition; however, the role of abrasion is significant for particles in the size range of 20–200 mm. The third limiting case includes both selective deposition and collisional abrasion but neglects frictional abrasion (dashed line). Here we see that the predicted evolution of the axis ratios in the whole size range is incorrect; simulated axis ratios increase, indicating movement toward the sphere. This is because frictional and collisional abrasion work against each other (Figure 5); while particles get flatter and thinner under frictional abrasion (axis ratios decrease), particles colliding with similar size particles converge toward the sphere (axis ratios increase) [Domokos and Gibbons, 2012]. Thus, the constant axis ratios measured for the whole grain population in the field indicate that collisional abrasion of pebbles is “balanced” by frictional abrasion of larger particles such as boulders.

5. Discussion

Data strongly indicate that curvature-driven abrasion occurs in the downstream evolution of pebbles along the Mameyes-Bisley 3 river system. Although downstream trends of individual shape parameters are scattered, the collection of independent parameters all behave as predicted from the geometric theory (Table 1); in particular, the monotonic trends for all shape parameters and the appearance of two phases. Phase I abrasion occurs in the upper ~1 km of the energetic and steep Bisley 3 stream, where pebble shapes evolve rapidly toward smooth ellipsoids, but axis dimensions remain approximately constant. This result is consistent with anecdotal reports that rapid rounding occurs “in the first few kilometers” of a river [Krumbein, 1941; Kuenen, 1956; Adams, 1979; Parker, 1991]. Phase II abrasion plays out in the Mameyes River, where axis ratios slowly increase while all other shape parameters remain approximately constant. At the tributary junction between Bisley 3 and the mainstem Mameyes, shape descriptors all show a smooth transition; this suggests that the transition to Phase II abrasion occurs within Bisley 3 before the junction and that downstream trends are not an artifact of merging these two different rivers into one profile. That predictions from an idealized geometric theory—of a single particle colliding with an infinite plain—are supported by field data from a highly heterogeneous system of mutually colliding pebbles under bed load transport, provides compelling evidence that curvature-driven abrasion should be a general phenomenon. A major difference in shape data from the field as compared to the idealized drum experiments of Domokos *et al.* [2014], however, is that pebbles in the Mameyes never become completely ellipsoidal (Figures 8e and 8f). It appears that collision-induced abrasion drives initially blocky pebbles toward ellipsoids; but, as pebbles move into lower gradient reaches of the river, frictional abrasion from sliding and rolling prevents pebbles from further evolution along this trajectory.

If sorting is minimal in Bisley 3, as expected, we can observe the isolated effects of abrasion in the upper portion of this steep, bedrock channel. In the alluvial portion of the Mameyes stream, however, sorting exerts a strong influence on downstream trends of particle size. Grain diameter data show significant decreases in pebble size, consistent with observations in other alluvial rivers [Adams, 1978; Lewin and Brewer, 2002]. Grain shape data, in particular the axis ratios, show that abrasion is also occurring in these lower alluvial reaches. One central question is “how much of a pebble’s mass is lost due to abrasion?”; this requires separating and removing the effects of sorting. If pebble volume were known along the entire stream profile, one could simply fit the “Generalized Sternberg’s law” (equation (2)) to Phase I data—where we assume that no sorting occurs—to produce a model for mass loss due to abrasion over the entire river length. However, it was not feasible to measure volume for all pebbles in Bisley 3 and the Mameyes (and is likely not feasible for many rivers) due to their large size. Separating the effects of abrasion and sorting from data alone

therefore is not possible. Here the box equations with deposition can be applied to interpret field data, in regions where Phase II abrasion is operative. We examine the predicted trend in volume diminution (in m^3) for the box equations with no deposition included; the result, considering the full particle population, is $V = (0.012)e^{-0.053x}$. Because volume diminution by abrasion should primarily be a function of pebble volume (equation (2)) [Domokos *et al.*, 2014], we use this expression to extrapolate upstream to $x = 0$ km. The model result is that 38% of a pebble's mass, on average, is lost over the 10 km distance from the headwaters to the gravel-sand transition.

That pebbles could lose approximately 40% of their mass along a relatively short (~10 km) distance implies that pebble mobility changes significantly due to abrasion. Parameters that assess mobility, such as threshold Shields stress and sediment transport equations, may produce misleading results when applied to steeper rivers where abrasion is significant, because they assume that particles are spherical and may be represented by a single diameter. In addition, the inferred pebble mass loss implies that significant quantities of sand and silt are produced in situ. This generation of fine sediment may be a significant part of the sediment budget, but it has never been quantified. Future field studies should aim to determine if and what fraction of fine sediment in a river is the product of abrasion. Whether there are geophysical and geochemical signatures of abrasion that may be used to separate its products from other fine sediment sources is unknown to these authors.

It is encouraging that 2-D shape parameters measured from images—in particular the isoperimetric ratio, convexity, and entropy—are in agreement with the more laborious, manual 3-D measurements. Results suggest that the curvature-driven abrasion may be identified from images alone, which should encourage researchers to test the generality of these findings in other rivers, and also aeolian environments where abrasion and sorting have been observed [e.g., Jerolmack *et al.*, 2011]. While these data serve to demonstrate the existence of collisional abrasion, they are not sufficient to quantify pebble mass loss. A practical guide for this problem, based on our findings here, is as follows: (1) Use 2-D image data to identify Phases I and II of abrasion; (2) measure all three axis lengths of pebbles contained within the regime of Phase II abrasion to determine pebble volume (or, measure the masses of all pebbles if they are small enough to be lifted; if so, then no further work is needed); (3) fit the box model with deposition to the downstream pattern of axis ratios in Phase II; and (4) use model results to identify the rate of volume diminution that is due to abrasion alone.

A final note of caution is warranted in the interpretation of our observations and modeling results. While it is beyond the scope of this work to explicitly model the effect of spatially varying sediment input on particle size and shape trends, we must acknowledge that sediment input in our study river (and indeed, in most other rivers) is spatially variable. The effects of spatial variability have been explored in models [Pizzuto, 1995; Attal and Lavé, 2006; Sklar *et al.*, 2006; Chatanantavet *et al.*, 2010], which have demonstrated that the combination of lithologic changes and tributary inputs may cause downstream trends in grain size that are independent of either abrasion or size-selective sorting. It is possible that spatially varying sediment inputs, and spatial trends in input shape, could conspire to produce the observed downstream patterns of size and shape in the Mameyes watershed. This is unlikely, however, and strength measurements (Figure 6b) indicate at least that the observed trends are not related to variation in material properties. The most likely influence of spatially varying sediment input would be to obscure the trends of curvature-driven abrasion, rather than to introduce new trends. The primary contribution of the work presented here is the demonstration of the significance of curvature-driven abrasion in a natural stream, which we believe to be qualitatively robust. The quantitative results and modeling efforts illustrate the potential magnitudes of abrasion versus sorting; however, spatially varying sediment input likely exerts an influence on the reported numerical values of each, and smooths the transition of abrasion between Phases I and II.

6. Conclusion

To summarize, this field investigation has demonstrated curvature-driven abrasion in a natural setting using a set of shape descriptors determined from simple hand measurements and image analysis techniques. Phase I abrasion takes place over the first kilometer in the steep headwater channel. This work provides a way to determine the relative importance of abrasion versus selective transport for a given river system; the results of the box model simulations give evidence that abrasion and selective deposition are both important to reproduce observed size and shape patterns in the Mameyes watershed. Although the relative importance

of abrasion versus sorting can vary due to sediment supply, lithology, and transport conditions, application of the geometric theory suggests that abrasion controls the downstream reduction in pebble mass while sorting determines the downstream trend in diameter. Incorporating explicit measures of pebble shape into future studies should allow researchers to assess the contribution of abrasion in other river systems. To truly test the generality of the curvature-driven abrasion model, future studies should replicate and expand on this analysis, in other river systems and also in aeolian dune fields.

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