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Disciplines
Accounting

An Examination of the Efficiency of Resource Allocation Auctions Within Firms[1]

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Abstract

There is growing interest in the use of markets within firms. Proponents have noted that markets are a simple and efficient mechanism for allocating resources in economies in which information is dispersed. In contrast to the use of markets in the broader economy, the efficiency of an *internal market* is determined in large part by the *endogenous* contractual incentives provided to the participating, privately informed agents. In this paper, we study the optimal design of managerial incentives when resources are allocated by an internal auction market, as well as the efficiency of the resulting resource allocations. We show that the internal auction market can achieve first-best resource allocations and decisions, but only at an excessive cost in compensation payments. We identify conditions under which the internal auction market and associated optimal incentive contracts achieve the benchmark second-best outcome; the advantage of the auction is that it is easier to implement than the direct revelation mechanism. When the internal auction mechanism is unable to achieve second-best, we characterize the factors that determine the magnitude of the shortfall. Overall, our results speak to the robust performance of relatively simple market mechanisms and associated incentive systems in resolving resource allocation problems within firms.

1 Introduction

Two key aspects of a firm's management control system are its resource allocation process, and the extent to which that process is supported by the firm's performance evaluation and compensation systems. The importance of these elements is magnified by the fact that most decentralized firms suffer from problems of asymmetric information. At least since Harris et al. (1982), the theoretical literature in management accounting has approached this issue as a problem in mechanism design, and has accordingly dealt with it by solving the associated direct revelation game. The problem with this approach is that such solutions, to the extent they can even be characterized, are complicated and, therefore, expensive to implement. An alternative is to view the firm as an "economy" and to use the power of markets to allocate resources within the firm.

Markets are widely recognized as fostering efficient resource allocations when information about relative values is dispersed across agents in an economy.[1] Furthermore, markets are informationally efficient and relatively simple to implement and maintain in comparison to other, more centralized resource allocation mechanisms (Debreu (1959); Arrow (1964)). Their ability to induce efficient allocations, coupled with their simplicity, has led to calls for the use of markets within firms for allocating resources.[2] In response, a number of organizations have developed and employed internal markets. For example, BP Amoco created an internal market to allocate pollution permits across business units, while the US Navy has utilized an internal market to fill certain jobs.[3] In addition, Ford has employed an experimental auction market for allocating cars to dealers, Intel has developed an internal market for allocating production capacity, and Hewlett-Packard has created markets for allocating computing power and conference rooms.[4]

In proposing the use of internal markets, one must recognize a fundamental difference between internal markets and markets within an economy: preferences are exogenous in an economy and endogenous within an organization. More specifically, preferences in internal markets are determined by the compensation schemes provided to the market participants. Hence, the efficiency of an internal market is a function of the organization's incentive system. In this paper, we study the optimal design of contractual incentives to support the resource allocations implemented via

[1] See Hayek (1945).

[2] See, for example, Halal et al. (1993), Malone (2004), and Stuart (2005).

[3] See House and Victor (2006) and Jaffe (2003), respectively, for further details regarding the BP Amoco and US Navy cases.

[4] For more on these and other examples, see IBM (2006) and Stuart (2005). Taylor (2006) describes an interesting use of an internal market to direct the capital budgeting process.

an internal market, and analyze the efficiency of the resulting allocation of resources.

The model we employ for our analysis consists of an owner (or principal) and two managers (or agents). Each manager is hired to undertake a separate and independent project. The amount of personal cost incurred by each manager to complete his project is uncertain when he is hired and initiates his project. After each manager privately learns his personal cost to complete his project, the principal uses an auction to allocate a single unit of an indivisible resource to one of the managers.[5] The resource reduces a manager's personal cost to complete his project. After the realization of his cost and the allocation of the resource, each manager decides whether or not to complete his project. As a benchmark, we initially consider the first-best setting where the managers' realized costs are contractible. We then study how a second-price sealed-bid auction, coupled with a fixed completion bonus, performs relative to the first-best. The combination of this auction setup and the completion bonus creates a simple mechanism that is relatively easy to maintain and implement.[6]

We find that the auction mechanism induces the first-best resource allocation and completion decisions when each manager's completion bonus equals his project's completion value to the principal. Hence, when the managers' bonuses fully impound the principal's benefit of project completion, the auction yields efficient resource allocation and project completion decisions. While such outcomes may be socially efficient, they do not maximize the value of the principal's expected utility. In particular, inducing efficient resource allocations and project completion decisions requires that the principal make excessively costly bonus payments. As a consequence, the principal chooses a lower bonus, and settles for inefficient allocations and fewer project completions.

In the presence of private information, however, a more appropriate benchmark than first-best is the outcome achieved under a direct revelation mechanism (see Myerson (1981)). As with the second-best direct revelation mechanism, when the principal allocates the resource using the auction mechanism, she chooses the managers' bonuses to trade off the managers' informational rents against the efficiency of the resource allocation and project completion decisions. We show that the

[5] As an example of an indivisible resource, consider a consultant whose time constraints permit him to advise only one of the managers.

[6] The second-price sealed-bid auction is one of the four "standard" types of auctions, the others being Dutch, English, and second-price open-bid. An advantage of the second-price sealed-bid auction is that bidding one's true value for the resource is a dominant strategy. Under a fairly broad set of conditions, all four standard types of auctions yield the same expected revenue to the auctioneer (the Revenue Equivalence Theorem). For a comprehensive discussion of the auction literature, see Klemperer (2004).

optimal trade-off results in the principal designing the managers' bonuses so that the *endogenous* value of the resource to each manager (and hence the amount each bids for the resource) is different from the value to the principal of that manager receiving the resource. Thus, even though the auction always results in the manager with the highest endogenous value for the resource winning it, it may not always result in the manager whom the principal would most want to receive the resource getting it. While our auction mechanism is inherently simpler to implement than the second-best direct revelation mechanism, simplicity comes at a cost. We derive a precise set of conditions under which the auction is able to attain the second-best benchmark. When these conditions are not met, we identify the factors that determine the extent to which the auction mechanism falls short of the second-best direct revelation benchmark.

In the setting discussed so far, each manager's cost to complete his project was assumed to be exogenously determined. In an extension of the model, we address a scenario in which these costs are influenced by investments made by each manager, for example in skill or in cost-reducing activities, where these investments are subject to moral hazard. In this case, the chosen incentives have an additional role, that of influencing the managers' investment in skill. Our main finding is the characterization of settings in which the auction mechanism is robust to the presence of this additional incentive problem.

An internal market is, in essence, a mechanism for establishing prices and allocating resources within an organization. Therefore, our analysis relates to the vast accounting literature on transfer pricing.[7] A primary difference between our paper and the transfer pricing literature is that the latter is concerned with the movement of products across divisions, while we study the competition among divisions for the same resource. As such, our work has more in common with papers that have looked at the allocation of central resources or services, in particular the literature pioneered by Harris et al. (1982) (see Rajan and Reichelstein (2004) for a survey of this area of research).[8]

Our paper is clearly related to the large literature analyzing the efficiency of auctions in an economy. The preponderance of this literature has analyzed the allocative efficiency of different types of auctions when bidders are privately informed as to their valuations for the resource, and those valuations are exogenously specified. In contrast, as noted earlier, when auctions are used

[7]A non-representative sample of recent work in this area includes Christensen and Demski (1998), Baldenius (2000), Arya and Mittendorf (2004), and Baldenius and Reichelstein (2006); for a current survey of transfer pricing practices, see Ernst & Young (2006).

[8]There is also a related theoretical and empirical literature in finance on the use of internal capital markets by firms; see, for example, Lamont (1997), Stein (1997), Scharfstein and Stein (2000), and Gertner et al. (2002).

within the firm to allocate resources, the bidders' valuations of the resource are influenced by their job duties and compensation. Thus, the principal, in choosing the managers' compensation contracts, must consider the effect that the contracts have on their valuations, their bids, and the resulting resource allocation and project completion decisions.

There are two branches of the extant auctions literature that consider endogenous bidder valuations for a resource. The first examines the effect of additional information on a bidder's valuation, the incentives for a bidder to release that information, and the incentives for a bidder to gather additional information (see Milgrom and Weber (1982) and Bergemann and Valimaki (2006) for a survey of the literature). The second examines the implications of changing the form of the payment made by the winning bidder. For example, in the case of an auction for natural resource extraction rights, the winning bidder could pay his bid plus a royalty on the resources extracted in lieu of just paying his bid (see Riley (1988)). Our research is related to this second branch of the auction literature, because we allow the bidder's valuation of the resource to be affected by the design of the auction. Closest to our work is that by Laffont and Tirole (1987), who examine the auctioning of incentive contracts, as in a procurement setting.[9] There are a number of differences between their paper and ours. In Laffont and Tirole (1987), the job itself is auctioned off whereas, in our model, the agents are assigned the job ex ante and a helpful resource is auctioned off. Further, the agents take actions after the auction in their model, while the agents take actions before as well as after the auction in section 4 of our paper. Perhaps the most important difference is that Laffont and Tirole (1987) assume that compensation can vary in the bids and the realized project cost, thus enabling them to separate the auction design from the incentive contract design. In contrast, we restrict attention to auctions with fixed completion bonuses. Our analysis is thus in the spirit of recent work (see, for example, Rogerson (2003) and Chu and Sappington (2005)) that seeks to examine the efficiency of mechanisms that are easy to implement and that impose lower informational requirements than the fully optimal, complex solution to the mechanism design problem.

The following section introduces the model and the first-best benchmark. Section 3 analyzes the auction mechanism and compares the outcome with the second-best direct revelation mechanism. Section 4 introduces moral hazard to the auction mechanism and Section 5 concludes.

[9] See also McAfee and McMillan (1986), McAfee and McMillan (1987) and Riordan and Sappington (1987).

2 The Model

Our model consists of a risk-neutral principal and two risk-neutral managers, 1 and 2. The principal contracts with each manager to work on independent projects. At the time the principal and managers contract, each manager is uncertain as to the personal cost he must incur to complete his project. Once the projects are underway, each manager learns the cost to complete his project. In particular, manager i learns that his cost is $1 - z_i$, where z_i is the realization of a uniformly distributed random variable $\tilde{z}_i$ with (normalized) support [0,1]; $\tilde{z}_1$ and $\tilde{z}_2$ are independent and identically distributed.[10] The principal has available an indivisible resource which, if allocated to a manager, reduces that manager's cost to complete his project. If manager i is allocated the resource, his cost to complete the project is reduced to $(1-\beta)(1-z_i)$ where $\beta \in [0, 1]$ and is publicly known. Additionally, we assume that the managers are constrained in their wealth, and hence the firm cannot "sell" the benefits of the tasks directly to the managers. Moreover, managers are free to leave the firm at any time, and thus must receive their reservation payoff of zero if the principal wants them to stay with the firm and complete their projects. Finally, the firm obtains a benefit $B > 0$ if a project is completed (i.e., if both projects are completed, the firm receives $2B$) and a benefit of 0 if it is not completed. Given our scaling of the costs, and in order to avoid dealing with degenerate settings in our analysis, we specify that $B \leq 2$.

2.1 Benchmark

As a benchmark, consider the first-best case where the project completions and realizations z_1 and z_2 are observable and contractible. Let $\underline{z}$ denote the minimum element of $\{z_1, z_2\}$ and $\overline{z}$ denote the maximum element of $\{z_1, z_2\}$. Table 1 provides the first-best resource allocation decisions, completion decisions, and compensation levels for each possible $\{z_1, z_2\}$ realization.

Insert Table 1 Here

To understand the conditions, note first that it is always optimal to allocate the resource to the project that will be completed and has the largest cost to complete (lowest z) realization. It follows that the resource should be allocated to the manager with realization $\underline{z}$ if both projects are to be completed. However, if only one project is to be completed, it is optimal to complete the project with realization $\overline{z}$ and allocate the resource to that project. Given these two observations,

[10] In Section 4, we allow manager i to improve the distribution of z_i through costly investment.

the first condition in the first column specifies when the completion of both projects dominates the completion of just the most profitable project (i.e., the project associated with the larger z, $\overline{z}$). If this condition is satisfied, completion of both projects also dominates completion of neither project. The first column of the next row contains the condition under which completion of just the most profitable project dominates the completion of both projects and the completion of neither project. The condition in the first column of the final row implies that the completion of neither project dominates the completion of just the most profitable project. If this condition is met, completion of neither project also dominates completion of both projects. The compensation in each case is the amount necessary to satisfy each manager's minimum utility constraint.

3 Analysis of The Auction Mechanism

As pointed out in the Introduction, simple auction mechanisms have proven to be efficient ways to allocate resources when information within economies is dispersed across agents. To assess how well an auction works within the context under consideration, we consider a sealed-bid second-price auction for allocating the resource coupled with a contractual payment that is conditioned on whether or not a manager completes his project. Under this auction mechanism, the manager with the higher bid acquires the resource and has the lower bid, denoted p, deducted from his compensation. We restrict attention to symmetric contracts and let c denote the bonus a manager receives if he completes his project. Given that the managers are unable to ex ante commit to stay with the firm for the entire game, the optimal fixed component of compensation is 0. Hence, we do not incorporate a fixed component of compensation into the managers' contracts.

In order to facilitate the characterization of the optimal contract and the resulting allocations, we first assess how the managers behave for a given bonus, c. We assume that the managers play the unique symmetric Nash equilibrium, which has the property that each manager's dominant strategy is to bid his personal valuation for the resource. Agent i's valuation for the resource is given by $v_i(z_i)$:

$$v_i = \begin{cases} 0 & 0 \leq z_i \leq 1 - \frac{c}{1-\beta} \\ c - (1-\beta)(1-z_i) & 1 - \frac{c}{1-\beta} \leq z_i \leq 1 - c \cdot \\ \beta(1-z_i) & 1 - c \leq z_i \leq 1 \end{cases} \tag{1}$$

Notice that the manager's valuation function is non-monotonic in his cost realization, z_i. The reason is that a manager with a realization of $z_i < 1 - \frac{c}{1-\beta}$ will not complete the task, even if he is awarded the resource. On the other hand, a manager with $1 - \frac{c}{1-\beta} \leq z_i < 1 - c$ will complete the project only if he is awarded the resource. Finally, a manager with $z_i \geq 1 - c$ will always complete his project. We refer to these managers as **budget unconstrained** and to managers in the intermediate region ($1 - \frac{c}{1-\beta} \leq z_i < 1 - c$) as **budget constrained**. A budget constrained manager's value for the resource is $c - (1-\beta)(1-z_i)$ because this is the incremental benefit he obtains from the resource and completing the project as opposed to not completing the project. A budget unconstrained manager's value for the resource is $\beta(1 - z_i)$ because this is the incremental benefit he obtains from receiving the resource and completing the project as opposed to completing the project without the resource. Finally, a manager with $z_i < 1 - \frac{c}{1-\beta}$ assigns zero value to the resource because he will not complete his project even if given the resource. Figure 1 graphically illustrates the manager's bidding strategy as a function of his cost realization.

Insert Figure 1 here

The single-peaked valuation function in Figure 1 drives the efficiency losses associated with our internal auction mechanism. In the traditional auction literature, each manager bids based on his *exogenously* endowed value for the resource. Further, his optimal bidding strategy is *monotonic* in his value and, as a result, the resource is always allocated efficiently ex ante because the manager who values the resource the most always bids the most and wins it. In our mechanism, as in the traditional auction, each manager's bid is monotonic in his *value*, which is determined by his *exogenous* cost realization and the *endogenous* completion bonus. Furthermore, the manager who values the resource the most always wins the resource. However, the resource allocation may not be ex ante efficient because efficiency is defined from the principal's perspective and is determined by the agents' cost realizations (z_1, z_2) rather than their endogenous values (v_1, v_2). Hence the resource may not always be allocated efficiently under our auction mechanism.

To illustrate the efficiency loss attributable to the auction mechanism, assume that for realization $\{z_1, z_2\}$ it would be desirable for each manager to complete his project. In that case the resource should be allocated to the manager with the higher cost (i.e. lower z realization). The manager with the higher cost, however, will not necessarily bid a higher value in the auction because he may simply forego completion if he loses the auction. From Figure 1, note that if manager 1 receives a draw $z_1 = x$ and manager 2 receives $z_2 = y'$, and in equilibrium each manager bids his value, manager 2 will be awarded the resource and manager 1 will not complete his project, despite the fact that both managers would have completed their project had manager 1 been awarded the resource.

Of course, the behavior of each manager is determined by his compensation so the potential for the auction to fail to allocate the resource efficiently and induce efficient completion can be mitigated through the principal's choice of contract. We next assess how the first-best completion and allocation outcomes relate to the contract parameter, c. The following table provides the resource allocation and completion strategies for each possible $\{z_1, z_2\}$ under the auction mechanism.

Insert Table 2 here

The most telling observation of the table is that the resource allocation and product completion strategies are "identical" to those in the first-best setting except that the contract bonus parameter, c, replaces the benefit of completion to the firm, B. Hence, the auction mechanism allocates the resource and induces the completion decisions that are efficient for the two managers given their contract parameters; however, this need not be the efficient outcome for the principal. Indeed, the resource allocation and completion decisions replicate those of the first-best if and only if the manager's bonus (i.e., completion benefit), c, equals the firm's completion benefit, B.

3.1 The Optimal Bonus

The analysis above implies that the efficiency of the resource allocation and completion decisions induced with the internal auction mechanism can achieve the first-best if the firm gives the "right" bonus incentive to the managers. Of course, the principal does not care about the ex-ante efficiency of the resource allocations and completion decisions when devising the optimal bonus; she cares only about her own expected profits. Hence, setting $c = B$ to induce efficient resource allocations and completion decisions may not be optimal.

The principal's ex ante payoff is determined by three components: the expected completion benefits, the expected auction revenues (i.e., the expected amount deducted from the winning manager's compensation in payment for the resource), and the expected completion bonus payments. To facilitate the process of solving for the optimal bonus, it is useful to assess each of these components and identify how each varies in the bonus c. Consider first the expected completion benefits, which equals the expected number of projects completed times the completion benefit B. If $c = 1$, any manager type will complete his project, even without help. In fact the same is true for any bonus coefficient c greater than 1. Clearly, as c decreases below 1, an increasing number of managers will not complete their task without help, and for $c < 1 - \beta$, managers with an exceedingly high cost of completing their project will not complete their task even if awarded the resource. From Figure 1, it is clear that decreasing c (when $c < 1$) causes the increasing portion of manager i's valuation, $v_i(z_i)$, to increase which, in turn, decreases the expected number of projects completed. More formally, we have the following lemma.[11]

Lemma 1 *The expected number of projects completed under the auction mechanism with completion bonus c is:*

$$CP = \begin{cases} 2 & c \geq 1 \\ \frac{2\beta-(1-c)^2}{\beta} & 1 > c \geq 1-\beta \\ \frac{(2(1-\beta)-\beta c)c}{(1-\beta)^2} & 0 \leq c < 1-\beta \end{cases}$$

CP is increasing in c.

Consider next the expected amount of compensation that the winning manager will payback the principal in return for the resource. We refer to the payment as the owner's "revenue." From Figure 1 we see that, when c increases, every type of manager z_i has an equal or greater value for the resource, which in turn drives up bids and ultimately the owner's expected revenue. Hence, we have Lemma 2.

Lemma 2 *The principal's expected revenue from allocating the resource under the auction mechanism with completion bonus c is given by:*

$$ER = \begin{cases} \frac{\beta c^3}{3(1-\beta)^2} & 0 \leq c < 1-\beta \\ \frac{\beta^2+(c-1)^3}{3\beta} & 1-\beta \leq c < 1 \\ \frac{\beta}{3} & c \geq 1 \end{cases} \cdot$$

ER is increasing in c.

Insert Figure 2 here

The previous two lemmas indicate that increases in c can increase the principal's welfare in two ways. First, increases in c increase the expected number of projects completed, which increases the expected completion benefits. Second, increases in c increase the revenues obtained from the

[11] Proofs to all results are in the Appendix.

auction. Of course, the principal cannot increase his welfare by raising c indefinitely because c also determines the expected completion bonus the principal must pay each manager. The expected completion bonus equals the expected number of projects completed multiplied by c. It follows that the expected completion bonus increases directly in c and, from Lemma 1, indirectly in c through the effect of c on the expected number of projects completed. Hence, the optimal c must trade off the benefit of a higher c on the first two components, the expected completion benefits and the expected auction revenues, against the cost of a higher c on the third component, the expected bonus payments. A formal analysis of this trade-off yields Proposition 1.

Proposition 1 *The owner's optimal choice of c is given by:*

$$c^* = \begin{cases} \frac{B}{2} & B \leq 2(1-\beta) \\ \frac{1}{4}\left(3 + B - \sqrt{(B-1)^2 + 8\beta}\right) < 1 & B > 2(1-\beta) \end{cases}.$$

The optimal payment c^* is continuous in B and β, however the induced behavior of the managers is vastly different when $B < 2(1-\beta)$ and $B \geq 2(1-\beta)$. When $B < 2(1-\beta)$, it is not worthwhile to the principal for a sufficiently "bad" type manager to complete his project, even if awarded help. In Figure 1, note that types $z_i < 1 - \frac{c}{1-\beta}$ will not complete their tasks even if awarded help, as $(1-z_i)(1-\beta) > c$. On the other hand, when $B \geq 2(1-\beta)$, the optimal bonus c^* is sufficiently large that *any* type of manager z_i will complete his task if assigned the resource.[12]

We turn next to understanding the characteristics of the optimal auction mechanism, as well as the manner in which they respond to changes in the exogenous parameters. We start with a key comparative statics result on the choice of bonus coefficient.[13]

Corollary 1 *When the auction mechanism is employed, the optimal bonus is increasing in the completion benefit and decreasing in the return to the resource:* $\frac{\partial c^*}{\partial B} > 0, \frac{\partial c^*}{\partial \beta} \leq 0.$

Intuitively, the optimal bonus increases in the benefit of completion to the firm because increasing the bonus makes it more likely that the manager will complete the project. In addition, the bonus is decreasing in the return to the help resource, β . The intuition underlying this comparative static is that increases in the productivity of the resource naturally make the value of the resource

[12] We are implicitly assuming that a manager who is indifferent between completing his task and abandoning it, resolves the matter by completing his task.

[13] The proofs to all Corollaries are omitted.

greater for each manager. Hence, it takes a smaller completion bonus to induce the managers to complete the project with help, which allows the firm to retain a larger fraction of the total surplus.

As indicated by Proposition 1, the optimal bonus is always set below that which implements the first-best resource allocation and completion decisions ($c = B$). As a consequence, we have the following result.

Corollary 2 *For any cost realization, $\{z_1, z_2\}$, the number of projects completed in the first-best setting exceeds the number of projects completed when the auction mechanism is employed.*

A direct implication of Corollary 2 is that the auction mechanism leads to poor follow-through on projects relative to the first-best case. With respect to resource allocations, Corollary 2 also implies that, under the auction mechanism and its associated optimal completion bonus, there exist cost realizations where the firm would like to override the auction outcome and allocate the resource to the losing manager. Consider, for example, the case where $B < 2(1-\beta)$, hence $c^* = B/2$, and the cost realization is such that $1 - \overline{z} < c^* < 1 - \underline{z} < c^*/(1-\beta)$ and $\beta(1-\overline{z}) > c^* - (1-\beta)(1-\underline{z})$. In this case, the manager with the low cost realization, $\overline{z}$, wins the auction and pays $c^* - (1-\beta)(1-\underline{z})$. Further, only the low cost manager completes the project. Given that outcome, the firm would prefer to nullify the auction, refund the payment to the low cost manager, and simply give the resource to the high cost manager. Why? Doing so decreases the auction revenues by $c^* - (1-\beta)(1-\underline{z}) = \frac{B}{2} - (1-\beta)(1-\underline{z})$ and, by inducing the high cost manager to also complete the project, increases the net completion benefit by $B - c^* = B/2 > B/2 - (1-\beta)(1-\underline{z})$. In summary, the auction mechanism fails to induce first-best efficient allocations of the resource and the completion decisions.

3.2 The Second-Best Direct Revelation Mechanism

When managers have private information about their operating units, the first-best is an overly stringent benchmark for evaluating the efficiency of the auction mechanism. The more appropriate standard is the outcome attained under the optimal direct revelation mechanism. In this section, we derive the outcomes attained under such an optimal second-best contract, and then compare these outcomes with those attained by the optimal auction mechanism.

In deriving the optimal contract for the second-best case, we continue to assume that each manager must attain a reservation level of expected utility, conditional upon the type realizations, of 0. Let $x_i(z_i, z_{-i}) \geq 0$ denote the payment made to manager i conditional upon i reporting a cost of z_i and the other manager reporting a cost of z_{-i}. Also, let $k_i(z_i, z_{-i})$ be 0 or 1 depending upon whether manager i is required to complete the project conditional upon the two reports, and $p_i(z_i, z_{-i})$ be the probability that manager i receives help conditional upon the two reports. Then, the principal's optimal contracting problem solves:

$$max_{\{x(\cdot),k(\cdot),p(\cdot)\}} \int_0^1 \int_0^1 \sum_{i=1}^{2} \left(Bk_i(z_i, z_{-i}) - x_i(z_i, z_{-i})\right) f_1(z_1) f_2(z_2) dz_1 dz_2$$

subject to:

$$x_i(z_i, z_{-i}) - k_i(z_i, z_{-i})(1 - z_i)(1 - \beta p_i(z_i, z_{-i})) \geq 0 \quad \forall z_i, z_{-i}$$

$$z_i \in \arg\max_s \mathrm{E}_{-i}\left[x_i(s, z_{-i}) - k_i(s, z_{-i})(1 - z_i)(1 - \beta p_i(s, z_{-i}))\right] \quad \forall z_i$$

$$p_i(\cdot, \cdot) \in [0, 1] \quad \forall i$$

$$p_1(z_i, z_{-i}) + p_2(z_i, z_{-i}) \leq 1 \quad \forall z_i, z_{-i}$$

$$x_i(\cdot, \cdot) \geq 0 \quad \forall i$$

The first constraint is the ex-post reservation constraint, the second is the truth-telling constraint, the third and fourth are constraints on the probabilities, and the fifth is the nonnegativity constraint on managerial compensation.

3.2.1 Second-Best Resource Allocation and Completion Decisions

Below, we characterize the optimal second-best solution. As before, we let $\underline{z}$ denote the minimum element of the realization $\{z_1, z_2\}$ and $\overline{z}$ denote the maximum element of the realization $\{z_1, z_2\}$.

Proposition 2 *Under any second-best contract the resource is allocated to the manager with the highest cost that is required to complete his project:* $p_i(z_i, z_{-i}) = 1$ *if and only if* $z_i = \underline{z}$ *and* $k_i(z_i, z_{-i}) = 1$ *or* $z_i = \overline{z}$ *and* $k_{-i}(z_i, z_{-i}) = 0$*. Furthermore, manager* i *is asked to complete his project if and only if* $B \geq 2(1 - z_i)(1 - \beta p_i(z_i, z_{-i}))$*.*

We can employ the conditions in Proposition 2 to summarize the second-best resource allocation and completion decisions in a manner that facilitates comparison with those of the first-best and those attained under the auction mechanism.

Insert Table 3 here

The resource allocation and completion table in the second-best setting is the same as that in the first-best setting except for the first column, where the B expressions in Table 1 have been replaced by $B/2$ expressions in Table 3. Table 3 halves the benefit relative to the first-best setting, because in the second-best setting, the principal must pay the managers informational rents which effectively doubles their individual cost of completing each project. In particular, when the principal decides which projects should go forward, he faces the *virtual cost* which, because of the uniform probability assumption, is double each manager's true cost of completing his project. Because the managers' virtual costs exceed their costs in the first-best setting, the principal optimally asks that fewer projects be completed. This observation that the firm foregoes completed projects to induce revelation in a less costly manner is consistent with the general adverse selection literature.

What is of particular interest here is a comparison of the resource allocation and completion decisions under our auction mechanism with those under the second-best contract. Proposition 1, coupled with Tables 2 and 3, yield Corollary 3.

Corollary 3 *If $B \leq 2(1-\beta)$, the resource allocation and completion decisions in the second-best are replicated with the optimal auction mechanism. If $B > 2(1-\beta)$, the number of projects completed for any cost realization, $\{z_1, z_2\}$, in the second-best setting exceeds the number of projects completed when the auction mechanism is employed.*

Corollary 3 implies that the auction mechanism replicates the resource allocations and completion decisions attained under a second-best contract in cases where the completion benefit is small or the returns to the resource are small. This observation may appear surprising in light of our analysis of the auction mechanism, which suggested inherent resource allocation inefficiencies arising from the bidding behavior in the auction. The corollary demonstrates that even the second-best mechanism incorporates such inefficiencies in order to mitigate the informational rents earned by the managers. However, in other cases, the auction mechanism fails to replicate the second-best resource allocation and completion decisions. Instead, fewer projects are completed with the market mechanism than would be under a second-best mechanism.

The fact that the auction mechanism replicates the second-best resource allocation and project completion decisions when $B \leq 2(1-\beta)$ suggests that the auction mechanism may attain the second-best benchmark in such cases. The following proposition demonstrates that this is indeed true.

Proposition 3 *The auction mechanism attains the benchmark second-best level of performance if* *and only if* $B \leq 2(1-\beta)$ *.*

Note that the second-best regime allocates the resource, determines which projects are completed, and compensates the employees as a function of the two announcements made.[14] The auction mechanism, in contrast, does not have the same degrees of freedom to establish state contingent pay-offs. Instead, the auction mechanism uses the two announcements (i.e., the bids) to determine who should be allocated the resource and how much the winner should pay for the resource. A fixed bonus c is then employed to motivate the managers in their bidding and project completion decisions. Surprisingly, even with the disadvantage of fewer degrees of freedom, the auction mechanism manages to attain the second-best benchmark when the principal's benefit B and the helpfulness of the resource β are limited (i.e., $B \leq 2(1-\beta)$).

In contrast, for larger values of B and β (i.e., $B > 2(1-\beta)$) the auction mechanism does not attain the second-best benchmark. For large values of B, the principal decides not to pay $B/2$ as a completion bonus, because the bonus becomes exceedingly large. Instead, the principal opts for a smaller bonus of $c^* < B/2$. The lowered bonus, in turn, implies fewer projects will be completed with the auction mechanism relative to the second-best mechanism.

Because the auction mechanism fails to achieve the second-best when $B > 2(1-\beta)$, it is important to assess the determinants of the opportunity loss from using the auction. The next result provides a complete characterization of the impact of the exogenous factors in the model, B and β, on the difference in overall value from employing the auction relative to the full-blown second-best mechanism.

Proposition 4 *If* $B > 2(1-\beta)$*, the value of the principal's objective under the second-best less the value under the auction mechanism is an increasing function of* B *and* β*.*

Under the second-best, an increase in B or β causes the principal to induce completion for more state realizations due to the increased direct return to completion, captured by B, or the decreased cost of completion, captured by β. In order for the auction mechanism to replicate the second-best resource allocation and completion decisions, the completion bonus paid to the managers must be given by $c = B/2$. For large values of B, however, paying the managers $B/2$ to replicate the second-best resource allocation and completion decisions is too costly because the

[14] We should also note that the degrees of contracting freedom are sufficiently large, because of the managers' risk-neutrality, that there are a continuum of optimal second-best contracts. This implies that revelation can be efficiently induced by any compensation scheme that has the appropriate expected information rents.

expected payments received in the auction plus the expected returns from completion are insufficient to cover the greater expected compensation. Instead, the principal optimally pays the managers $c = \frac{1}{4}\left(3 + B - \sqrt{(B-1)^2 + 8\beta}\right) < B/2$. The extent of this deviation from the second-best is increasing in B and β, and, as a consequence, the auction mechanism falls farther from the second-best when B or β increase.

As an example, Figure 3 plots the efficiency of the auction mechanism relative to the second-best mechanism. The graphs only plot the relative efficiency of the auction mechanism for $B > 2(1-\beta)$, because the auction mechanism attains the second-best benchmark when $B \leq 2(1-\beta)$.

Insert Figure 3 here

4 Moral Hazard in Cost Reduction

To this point we have considered a scenario in which the managers are exogenously endowed with their cost-to-complete realizations. This is arguably restrictive since managers often have the ability to influence the cost of their projects via ex-ante activities such as research, planning or improved search. Moreover, such investments are typically either privately undertaken or are otherwise extremely costly to verify and use in a formal contract. In this section, we examine the robustness of the auction mechanism to the presence of such investments, as well as the impact of the moral hazard considerations they introduce.

To model ex-ante investments, we specify that each manager can initially expend effort that affects the distribution of his type z_i. In particular, we assume that manager i can take action e_i, where $e_i \in \{0, 1\}$, and that action affects the probability distribution over the cost to complete his project. Without loss of generality, suppose the cost to taking action $e_i = 0$ is zero and the manager incurs disutility ν when he selects $e_i = 1$. As before, we will assume that manger i realizes a cost of $1 - z_i$ to finish his project. However we now assume the distribution of z_i is related to the manager's choice of effort e_i in the following way:

$$f(z_i, e_i) = \begin{cases} 1 & e_i = 1 \\ 2 - 2z_i & e_i = 0 \end{cases}.$$

Thus, z_i is uniformly distributed over $[0, 1]$ if $e_i = 1$. If $e_i = 0$, the support for z_i is still $[0, 1]$, but the marginal density is increasing in z_i.[15]

[15]This is the same formulation used to derive the probabilities of completion from Lemma 1.

When the manager picks $e_i = 0$, we shall say that he has taken no effort, and when he chooses $e_i = 1$ we shall say that he has exerted effort. We continue to assume that each manager privately observes his own cost to complete. Moreover, each manager's action e_i is unobserved by the principal and the other manager, and is subject to moral hazard. Finally, we continue to assume the managers are free to leave the firm at any time, and hence, each manager must always receive a level of utility at least as large as that offered by their outside option, which is 0.

The second-best benchmark contract changes in this setting because the principal's preference over e_1 and e_2 must be considered. We will assume that the principal prefers $e_1 = e_2 = 1$, which implicitly imposes an upper limit on ν, each manager's cost of effort. Thus, the only difference between this section and the preceding one is that the principal must also now motivate the managers to expend effort $e_i = 1$ to reduce their anticipated completion costs.

We begin by examining manager i's preferences over e_i in the auction mechanism. Managers face two countervailing incentives with respect to their choice of e_i. By exerting effort, manager i increases his probability of getting a large z_i. A large z_i, in turn, increases the probability that the manager completes his project and secures the completion bonus c. On the other hand. by exerting no effort, manager i increases the probability that he will receive the rents associated with obtaining the help resource. Thus, each manager must balance the advantage of exerting effort – increasing the probability of completing the project and receiving the bonus c – versus the disadvantage of exerting effort – completing the project without the cost-reducing resource. The following lemma characterizes the incentives facing for each manager to exert effort as a function of the other manager's choice of effort.

Lemma 3 *Agent i's incentive to exert effort is decreasing in manager $-i$'s effort.*

The proof to Lemma 3 shows that manager i's preference for shirking increases as the other manager works. To understand why, recall that the resource is awarded to the manager with the highest cost to completion, conditional on the manager choosing to complete his project. As such, if manager $-i$ works, the probability of manager i being awarded the resource increases if he decides to shirk. The only countervailing force to induce the manager i to work is the opportunity to earn rents upon completing a project, i.e., the probability of completing a project increases if manager i works.

To simplify the notation, let $N(\beta, c, \nu) = s(1,1) - s(0,1) - \nu$, the marginal surplus or loss a manager obtains by exerting effort assuming that the other manager is exerting effort as well.

If $N(\beta, c, \nu)$ is positive, both managers exerting effort is a Nash equilibrium. For sufficiently large values of ν, however, the principal will be unable to motivate both managers to exert effort regardless of c. This follows from the fact that the manager will opt to incur more expected costs ex post (i.e., a higher expected z_i) in lieu of incurring higher costs ex ante (i.e., incurring ν). As shown in the following proposition, when ν is sufficiently small, the principal will be able to induce $\{e_1, e_2\} = \{1, 1\}$ as a Nash equilibrium for a sufficiently large completion bonus, c.

Proposition 5 *For sufficiently small ν, the principal can induce $e_1 = e_2 = 1$ to be a Nash equilibrium by setting $c \geq \hat{c}(\beta, \nu)$. Moreover:*

$$\frac{\partial}{\partial \beta}\hat{c}(\beta, \nu) \geq 0; \quad \frac{\partial}{\partial \nu}\hat{c}(\beta, \nu) \geq 0.$$

When the completion bonus exceeds the threshold $\hat{c}(\beta, \nu)$, each manager prefers effort to no effort. Although exerting no effort increases the probability that the manager will win the resource, it also increases the probability that the manager does not finish his project and forfeits the bonus.

The analysis to this point raises the question of whether the no-moral-hazard optimal completion bonus, c^*, can ever suffice to motivate effort. The following example demonstrates the possibility of c^* being sufficiently large to satisfy the moral hazard problem.

Example: Suppose the Principal earns $B = 3/2$ dollars for the completion of either project. Moreover, suppose the resource would cut either manager's cost in half, that is $\beta = 1/2$. The optimal bonus without moral hazard is given by $\frac{1}{8}(9 - \sqrt{17})$. A manager's ex-ante expected surplus from exerting effort is given by $\frac{1}{192}(123 - 19\sqrt{17})$. On the other hand, the manager's expected surplus if he exerts no effort is $\frac{1}{768}(295 - 47\sqrt{17})$. Clearly, for any cost of effort $\nu < \frac{1}{192}(123 - 19\sqrt{17}) - \frac{1}{768}(295 - 47\sqrt{17}) \approx 0.10082$, the manager will find it worthwhile to work hard in order to improve his chances of obtaining a low cost (large z). On the other hand, for $\nu > 0.1$, the principal must increase the size of the bonus in order to motivate the manager.

As the example shows, the no-moral-hazard contract may offer sufficient rents to induce the managers to engage in costly effort to lower their expected completion costs. To get a better handle as to when effort can be induced at no incremental cost, we exploit some properties of $N(\beta, c, \nu)$. In the proof to Proposition 5, $N(\beta, c, \nu)$ is shown to be minimized at $c = \frac{1-\beta}{1+2\beta}$. Hence, if effort is implemented as a Nash equilibrium when $c^* > \frac{1-\beta}{1+2\beta}$, as is always the case when $B \geq 2(1 - \beta)$, then any upward shock to the principal's project completion benefit B will result in a new bonus, c^*, which will again implement effort by both managers in a Nash equilibrium. Similarly, if $B < 2(1-\beta)$ and $c^* < \frac{1-\beta}{1+2\beta}$, if c^* motivates managers to exert effort, then the managers will continue to exert effort following any revision of the bonus, c^*, following a downward shock to the principal's completion benefit B.

Characterizing the optimal second-best solution to this problem would be very complicated. If the second-best mechanism without moral hazard does not induce the managers to exert effort, then the principal must increase the rents paid to the managers with high type realizations, as those realizations are more likely to be the product of the manager having exerted effort. Further, the principal would have to determine whether or not to alter the quantity of projects completed.[16] Although previous work has considered moral hazard alongside adverse selection, in these models the moral hazard does not alter the managers' private information. Without a second-best benchmark, we are unable to assess the efficiency loss associated with the use of our auction mechanism. Instead, for the remainder of this section, we characterize the optimal auction mechanism when the managers' efforts influence their type realizations.

Proposition 5 established that for a suitably large completion bonus c, the principal can make $\{e_1, e_2\} = \{1, 1\}$ a Nash equilibrium, yet the principal may wish to make both managers exerting effort the *unique* Nash equilibrium. There exists a literature on implementation of a unique Nash equilibrium, albeit the literature relies on increasing the message space.[17] The following proposition, however, demonstrates that whenever the auction mechanism supports a Nash equilibrium in which both managers exert effort, that equilibrium is unique.

Proposition 6 *If both managers exerting effort is a Nash equilibrium ($c \geq \hat{c}$), then both managers exerting effort is the unique Nash equilibrium.*

The proposition shows that if both managers exert effort in a Nash equilibrium, then that equilibrium must be the unique Nash equilibrium. To illustrate, consider our earlier example where $B = 3/2$ and $\beta = 1/2$. In order for effort to be a unique Nash equilibrium choice for both managers, from the proposition, the principal must ensure that exerting effort is a dominant strategy. We have already shown that exerting effort is a best response to the other manager exerting effort. Hence, we assess whether exerting effort is also a best response to the other manager not exerting effort.

[16] For a simple example where moral hazard followed by adverse selection calls for a first-best production schedule, see Laffont and Martimort (2002) p. 296-298.

[17] See, for example, Ma (1988) and Moore and Repullo (1990).

A manager who exerts effort while his peer does not expects to earn a surplus of $\frac{1}{768}(359-47\sqrt{17})$, whereas if the manager also does not exert effort, he only expects to earn $\frac{1}{1280}(43\sqrt{17}-35)$. The difference, which is approximately 0.10396, is positive which, consistent with the result in Proposition 6, implies that exerting effort is a dominant strategy.

5 Conclusion

In this paper we address the issue of allocating resources within a firm in which two managers are hired to implement two independent projects. Each manager's cost to complete his project is a random variable that he privately observes after being hired. Subsequently, the firm must allocate a resource, which reduces either manager's cost of completing his project, to one of the managers. The standard approach to this type of problem in the economics and accounting literature since the work of Harris et al. (1982) and Myerson (1981) has been to formulate the problem as a revelation game and employ a direct revelation mechanism. However, direct mechanisms are generally complex and, as a result, are costly to implement. Increasingly, firms are turning to the internal use of markets, which have a demonstrated capacity for efficient allocation of resources within an economy. Our analysis characterizes the optimal specification of a simple internal auction and examines its effectiveness relative to that of a second-best mechanism.

We highlight that a major difference between an auction in an economy and within a firm is that the valuations of the participants are (typically) exogenously given in the former case, whereas they are endogenously determined through contractual incentives established by the principal in the latter case. We show that this difference between implementing an auction in an economy and a firm results in different optimal bidding strategies: in economies, the bidding strategies are monotonic in the underlying types of the participants whereas within firms, the bidding strategies may be non-monotonic. This difference implies that, within a firm, while the manager with the highest endogenous value for the resource always wins the resource, he is not always the manager whom the principal wishes to receive the resource. Not surprisingly, we find that this source of inefficiency may cause the auction mechanism to fall short of the first-best. However, we provide necessary and sufficient conditions for our auction mechanism to attain second-best efficiency. In cases where the auction falls short of that benchmark, we identify the primary factors that determine the loss in efficiency from implementing an auction. Finally, we extend the primary model to a setting where each manager affects the distribution of his cost realization through the expenditure of private effort. In this setting, we identify circumstances under which the additional moral hazard problem is settled at no incremental cost to the principal. Overall, our results speak to the dramatic performance of relatively simple market mechanisms and associated incentive systems in resolving resource allocation problems within firms.

From a technical standpoint, the results in our paper can be extended in at least two directions. One extension would solve for the optimal second-best revelation mechanism when the managers also face a moral hazard problem, and compare the results to those generated by our market mechanism. We conjecture that the market mechanism continues to come close to the second-best outcome. A second extension would generalize the moral hazard problem of Section 4 to a continuous task with a generic distributional outcome. Again, we expect our results to be robust to such a generalized setting, although it would be interesting to characterize precisely how the moral hazard environment affects the performance of the market mechanism relative to the second-best benchmark.

6 Appendix

Proof of Lemma 1

Let $P2$ denote the probability of completing both projects, and similarly $P1$ and $P0$ denote the probability of completing one project and no projects, respectively. Under the auction mechanism, we will show that:

$$P2 = \begin{cases} 1 & c \geq 1 \\ \frac{\beta-(1-c)^2}{\beta} & 1 > c \geq 1-\beta \\ \frac{c^2}{1-\beta} & 0 \leq c < 1-\beta \end{cases}$$

$$P1 = \begin{cases} 0 & c \geq 1 \\ \frac{(1-c)^2}{\beta} & 1 > c \geq 1-\beta \\ \frac{(2+\beta(-2+c)-2c)c}{(1-\beta)^2} & 0 \leq c < 1-\beta \end{cases}$$

$$P0 = \begin{cases} 0 & c \geq 1 \\ 0 & 1 > c \geq 1-\beta \\ \left(1-\frac{c}{1-\beta}\right)^2 & 0 \leq c < 1-\beta \end{cases}$$

The expression for CP and its monotonicity follow directly from the expressions above. We present the analysis for a more general distribution function so that the results derived in this section can be reused in Section 4, where managers can affect their cost realizations by undertaking cost-reducing activities. We assume that the density of manager i's cost realization z_i is given by $f(z_i, e_i) = (2-2z_i)(1-e_i)+e_i$, where e_i denotes manager i's choice of effort. We define $F(z_i, e_i)$ to be the corresponding cumulative distribution function. Until Section 4, $e_i \equiv 1$, so the density is uniform over the interval $[0, 1]$.

If $c \geq 1$, then both managers will always find it profitable to complete their projects, because their payoff for completion c always exceeds their cost of completion.

Next, suppose $1-\beta \leq c < 1$. Now a manager will always complete his task if he receives the resource (by construction), and hence, assuming the principal always offers the resource to one of the managers, the probability of no tasks being completed is zero. On the other hand, the probability that both managers complete their projects is given by the probability that both managers are budget unconstrained ($z_i > 1-c$ for $i = 1, 2$) plus the probability that only one manager is budget constrained, yet values the resource more than the unconstrained manager. Formally, the probability of both tasks being completed, which we denote by $P2(\cdot)$, is given by:

$$(1 - F(1-c, e_1))(1 - F(1-c, e_2)) + \int_0^{1-c} [(1 - F(Z(s), e_1))f(s, e_2) + (1 - F(Z(s), e_1))f(s, e_1)]\, ds. \quad (2)$$

Here, $Z(s)$ is the type of budget unconstrained manager that values the resource as much as a budget constrained manager with type s.[18] In particular, $Z(s) = 1 - \frac{c-(1-\beta)(1-s)}{\beta}$. Letting $P2(e_1, e_2, \beta, c)$ denote the probability that both tasks are completed given that manager 1 puts forth effort e_1 and manager 2 works e_2, we can solve (2) to obtain:

$$P2(e_1, e_2, \beta, c) = \frac{-1}{3\beta^2}\begin{pmatrix} 6\beta - 3\beta^2 - 3 + 8c - 8\beta c - 6c^2 + c^4 + 2\beta c^4 \\ -(e_1 + e_2)(1-c)^2(c^2 + c - 2 + 2\beta(1 + c + c^2)) \\ +e_1 e_2(c^2(1+2\beta) + \beta - 1) \end{pmatrix} \quad (3)$$

Setting $e_1 = e_2 = 1$ in (3), the probability of both tasks being completed when $1 - \beta \le c < 1$ reduces to $P2(1, 1, \beta, c) = \frac{\beta-(1-c)^2}{\beta}$.

Finally, if $c < 1-\beta$, then we are no longer assured that a manager who receives help will complete his task (see Figure 1). In particular, if $z_i < 1 - \frac{c}{1-\beta}$, then manager i will not complete his task, regardless of whether the resource is allocated to him or not. In this instance, the probability of both tasks being completed is given by:

$$(1 - F(1-c, e_1))(1 - F(1-c, e_2)) + \int_{1-\frac{c}{1-\beta}}^{1-c} [(1 - F(Z(s), e_1))f(s, e_2) + (1 - F(Z(s), e_1))f(s, e_1)]\, ds. \quad (4)$$

Note that the integral in (4) is identical to that of (2), except that the bounds of integration are changed to reflect the fact that not all managers will complete their tasks in the presence of the resource. Solving for (4), we obtain:

$$P2(e_1, e_2, \beta, c) = \frac{c^2}{3(1-\beta)^2}\begin{pmatrix} (2\beta - 3)(e_2(c-1) - c)c \\ +e_1\left((2\beta - 3)(c-1)c + e_2(3(c-1)^2 + \beta(4c - 2c^2 - 3))\right) \end{pmatrix} \quad (5)$$

Again, assuming $e_1 = e_2 = 1$ in (5), when $c < 1-\beta$, we can write $P2(1, 1, \beta, c) = \frac{c^2}{1-\beta}$. Rather than solve directly for $P1$, we instead solve for $P0$, and obtain $P1 = 1 - P0 - P2$. Neither manager will complete his project if both have a realization $z \in \left[0, 1 - \frac{c}{1-\beta}\right]$, hence we can write:

[18] In Figure 1, $Z(x) = y$.

$$P0 = F\left[1-\frac{c}{1-\beta}, e_1\right] F\left[1-\frac{c}{1-\beta}, e_2\right] = \frac{(\beta+c-1)^2(c(e_1-1)+\beta-1)(c(e_2-1)+\beta-1)}{(1-\beta)^4}.$$

When $e_1 = e_2 = 1$, $P0$ reduces to $\left(1-\frac{c}{1-\beta}\right)^2$.$\blacksquare$

Proof of Lemma 2

In a second price auction, the principal's expected revenue is given by the expected second-order statistic. To facilitate the proof, we make a change of variables from the managers' types, z_i, to their valuations, $v_i(z_i)$. The distribution of the managers' values when $c < 1-\beta$ is different from the distribution when $c \geq 1-\beta$; in the former case, because managers with $z_i \in [0, 1-\frac{c}{1-\beta}]$ place no value on the resource, a mass exists at the zero valuation.

We begin with the case where $1 > c \geq 1-\beta$, which corresponds to the dotted line in figure 2. Let $g(v, e_i)$ denote the density of manager i's value for the resource after taking action e_i. Then, from the valuations found in (1), we can write:

$$g(v; e_i) = \begin{cases} \frac{1}{\beta} f\left(1-\frac{v}{\beta}, e_i\right) & v < c+\beta-1 \\ \frac{1}{\beta} f\left(1-\frac{v}{\beta}, e_i\right) + \frac{1}{1-\beta} f\left(1+\frac{v-c}{1-\beta}, e_i\right) & c+\beta-1 < v < c\beta \end{cases} \tag{6}$$

The split in the distribution of values reflects the fact that when $c \geq 1-\beta$, the budget constrained and budget unconstrained managers share a limited set of values. Note that only budget unconstrained managers will have values less than or equal to $c+\beta-1$, whereas both types of managers can have valuations between $c+\beta-1$ and $c\beta$.

We can calculate the cumulative distribution functions from (6) as:

$$G(v; e_i) = \begin{cases} \int_0^{c-(1-\beta)} \left(\frac{1}{\beta} f\left(1-\frac{s}{\beta}, e_i\right)\right) ds & v < c+\beta-1 \\ \int_{c-(1-\beta)}^{v} \left(\frac{1}{\beta} f\left(1-\frac{s}{\beta}, e_i\right) + \frac{1}{1-\beta} f\left(1+\frac{s-c}{1-\beta}, e_i\right)\right) ds & c+\beta-1 \leq v \leq c\beta \end{cases}$$

Solving, we have:

$$G(v; e_i) = \begin{cases} \frac{v(e_i(\beta-v)+v)}{\beta^2} & v < c+\beta-1 \\ \frac{1}{(1-\beta)^2\beta^2} \begin{pmatrix} \beta^4 + \beta^3(e_i c-2) - (e_i-1)v^2 + \beta v(e_i - 2v(1-e_i)) \\ +\beta^2(1-c^2+2cv+e_i(c^2-c-2cv-v)) \end{pmatrix} & c+\beta-1 \leq v \leq c\beta \end{cases}$$

Because the expected revenue is given by the expected second order statistic of the managers' values, we can use standard formulas involving the cumulative distribution function to obtain:

$$ER(e_1, e_2) = \int_0^{c\beta} s\left(g(s, e_1)(1 - G(s, e_2)) + g(s, e_2)(1 - G(s, e_1)\right) ds$$

Solving, we find that:

$$ER(e_1, e_2, c) = \frac{1}{60\beta^2}\begin{pmatrix} 4(8\beta^3 + (1-c)^4(4+c) + 2\beta(c-1)^3(6+3c+c^2)) \\ +(e_1+e_2)(-7\beta^3 - (1-c)^4(11+4c) - 2\beta(c-1)^3(9+7c+4c^2)) \\ +2e_1e_2(\beta^3 + (1-c)^4(3+2c) + 2\beta(c-1)^3(2+c+2c^2) \end{pmatrix}$$

If $e_1 = e_2 = 1$, then $ER(1, 1, c) = \frac{\beta^2+(c-1)^3}{3\beta}$ when $1 - \beta < c \leq 1$. When $c > 1$, the managers' valuations follow the thin line in Figure 2 where both managers always finish their projects (as in the case of $c = 1$). Hence, when $c > 1$, the bonus parameter, c, does not play any role in either the probability of finishing or the value either manager attaches to winning the resource (beyond the case where $c = 1$).

Finally, when $c < 1 - \beta$, both types of managers can have valuations between 0 and $c\beta$, as the boldface line indicates in Figure 2. However, there exists a mass of types with value 0. We can thus write the density of values as:

$$g(v; e_i) = \frac{1}{\beta} f\left(1 - \frac{v}{\beta}, e_i\right) + \frac{1}{1-\beta} f\left(1 + \frac{v-c}{1-\beta}, e_i\right) \quad 0 < v \leq c\beta \tag{7}$$

$$P(v = 0; e_i) = \int_{c-(1-\beta)}^{0} \frac{1}{1-\beta} f\left(1 + \frac{s-c}{1-\beta}, e_i\right) ds \tag{8}$$

From (8), we can write the cumulative distribution function as:

$$G(v; e_i) = \int_0^v \left(\frac{1}{1-\beta} f\left(\frac{s-c}{1-\beta}, e_i\right) + \frac{1}{\beta} f\left(1 - \frac{s}{\beta}, e_i\right)\right) ds + \int_{c-(1-\beta)}^{0} \frac{1}{1-\beta} f\left(1 + \frac{s-c}{1-\beta}, e_i\right) ds.$$

We can now solve for the expected revenue as follows:

$$\begin{aligned} ER(e_1, e_2) &= \int_0^{c\beta} s\left(g[s, e_1](1 - G(s, e_2)) + g[s, e_2](1 - G(s, e_1))\right) ds \\ &= \frac{1}{60(1-\beta)^4}\begin{pmatrix} \beta c^3(10e_1e_2(1-\beta)^2 - 5c(\beta-1)(2e_2 + e_1(3 - 5e_2 + 2\beta(e_2-1)))) \\ +2(e_1-1)(e_2-1)(8+\beta(2\beta-7)) \end{pmatrix}. \end{aligned}$$

Note that the expected revenue calculation above omits the calculation at the mass $v = 0$ because whenever at least one manager values the resource at zero, the other manager will obtain the resource free of charge. In the present setting where $e_1 = e_2 = 1$, we have $ER(e_1, e_2) = \frac{\beta c^3}{3(1-\beta)^2}$ when $c < 1 - \beta$.■

Proof of Proposition 1

Let π^{cb} denote the principal's total expected profits. Recall that for values of c greater than $1 - \beta$, any type of manager who receives the resource will complete his task; when c is less than $1-\beta$, a manager may not wish to complete his task even if he is awarded the resource. If $c = 1-\beta$, the worst type of manager is indifferent between completing his task and abandoning it. We will show that the two different representation of π^{cb} yield the same value when the bonus coefficient c is set at $1 - \beta$.

We begin by considering the case where the principal sets $c > 1 - \beta$. Then we can write:

$$\pi^{cb} = ER[1,1] + (B - c)(1 + P2(1,1)) = \frac{\beta^2 + 6\beta(B - c) - (1 + 3B - 4c)(1 - c)^2}{3\beta}. \tag{9}$$

Note that π^{cb} is a cubic in c, and the coefficient of c^3 is positive. The two solutions to $\frac{\partial}{\partial c}\pi^{cb} = 0$ are given by $\frac{1}{4}(3 + B - \sqrt{(B-1)^2 + 8\beta})$ and $\frac{1}{4}(3 + B + \sqrt{(B-1)^2 + 8\beta})$. As such, the principal's profits are increasing for $c \leq \frac{1}{4}(3 + B - \sqrt{(B-1)^2 + 8\beta})$ and $c \geq \frac{1}{4}(3 + B + \sqrt{(B-1)^2 + 8\beta})$. However, note that $\frac{1}{4}(3 + B + \sqrt{(B-1)^2 + 8\beta}) \geq 1$, and hence cannot be a valid solution as we have already shown that $c \leq 1$. Thus, when we restrict $c > 1 - \beta$, the solution to the principal's problem will always be $\frac{1}{4}(3+B-\sqrt{(B-1)^2 + 8\beta})$, as long as this expression is greater than $1-\beta$, or equivalently, provided $B > 2(1-\beta)$. Alternatively, if $B \leq 2(1-\beta)$, i.e., $\frac{1}{4}(3+B-\sqrt{(B-1)^2 + 8\beta}) \leq 1 - \beta$, then the optimal solution sets the bonus $c = 1 - \beta$.

We now drop the earlier assumption that the principal requires any recipient of the resource to complete his project, and hence restrict the bonus coefficient c to be less than $1 - \beta$. The principal now maximizes:

$$\begin{aligned} \pi^{cb} &= ER[1,1] + (B - c)(2 \cdot P2(1,1) + P1(1,1)) \\ &= \frac{c}{3(1-\beta)^2} \cdot \left[2c(\beta(3 + 2c) - 3) - 3B(\beta(2 + c) - 2)\right]. \end{aligned} \tag{10}$$

The cubic has two extreme points, $c = \frac{1-\beta}{\beta}$ and $c = \frac{B}{2}$. Examining the second order conditions, $c = \frac{B}{2}$ is a maximum whenever $B < \frac{2}{\beta} - 2$ whereas $c = \frac{1-\beta}{\beta}$ is a maximum whenever $B > \frac{2}{\beta} - 2$.

However, $\frac{1-\beta}{\beta} > 1-\beta$ and hence is never a feasible solution. In order for $B/2$ to belong to the interval $[0, 1-\beta)$, we must have $B < 2(1-\beta)$, which in turn implies that $B < \frac{2}{\beta} - 2$. If $B \geq 2(1-\beta)$, the best the principal can do is to set c equal to $1-\beta$, since her objective function is increasing over the entire range $c \in [0, 1-\beta)$. Whereas, if $B < 2(1-\beta)$, then $B < \frac{2}{\beta} - 2$, and hence the optimal solution calls for setting $c = B/2$.

Now, because the two expressions for the principal's profits (9) and (10) share the same value at $c = 1-\beta$, when $B < 2(1-\beta)$, revealed preference shows that the optimal solution calls for setting $c = B/2$. When $B > 2(1-\beta)$, the same argument demonstrates that $c = \frac{1}{4}\left(3 + B - \sqrt{(B-1)^2 + 8\beta}\right)$ is optimal. Finally, if $B = 2(1-\beta)$, the optimal solution calls for setting $c = \frac{1}{4}\left(3 + B - \sqrt{(B-1)^2 + 8\beta}\right) = B/2$.■

Proof of Proposition 2

We first prove two lemmas which are useful in the proof of the proposition.

Lemma 4 *Any incentive compatible set $(k(z), p(z), x(z))$ must satisfy the following:*

$$E_{-i}[k_i(z_i, z_{-i})(1-\beta p_i(z_i, z_{-i}))] \quad \textit{is increasing in } z_i.$$

Proof: Truth-telling mandates that:

$$\begin{aligned}
\mathrm{E}_{-i}\left[x_i(z_i, z_{-i}) - k_i(z_i, z_{-i})(1-z_i)(1-\beta p_i(z_i, z_{-i}))\right] &\geq \mathrm{E}_{-i}\left[x_i(\tilde{z}_i, z_{-i}) - k_i(\tilde{z}_i, z_{-i})(1-z_i)(1-\beta p_i(\tilde{z}_i, z_{-i}))\right] \\
\mathrm{E}_{-i}\left[x_i(\tilde{z}_i, z_{-i}) - k_i(\tilde{z}_i, z_{-i})(1-\tilde{z}_i)(1-\beta p_i(\tilde{z}_i, z_{-i}))\right] &\geq \mathrm{E}_{-i}\left[x_i(z_i, z_{-i}) - k_i(z_i, z_{-i})(1-\tilde{z}_i)(1-\beta p_i(z_i, z_{-i}))\right]
\end{aligned}$$

Summing the pair yields:

$$\begin{aligned}
&\mathrm{E}_{-i}\left[(1-\tilde{z}_i)\left(k_i(z_i, z_{-i})(1-\beta p_i(z_i, z_{-i})) - k_i(\tilde{z}_i, z_{-i})(1-\beta p_i(\tilde{z}_i, z_{-i}))\right)\right] \geq \\
&\quad \mathrm{E}_{-i}\left[(1-z_i)\left(k_i(z_i, z_{-i})(1-\beta p_i(z_i, z_{-i})) - k_i(\tilde{z}_i, z_{-i})(1-\beta p_i(\tilde{z}_i, z_{-i}))\right)\right]
\end{aligned}$$

If $\tilde{z}_i > z_i$ then it must be the case that $\mathrm{E}_{-i}\left[k_i(z_i, z_{-i})(1-\beta p_i(z_i, z_{-i})) - k_i(\tilde{z}_i, z_{-i})(1-\beta p_i(\tilde{z}_i, z_{-i}))\right]$ is non-positive, i.e.

$$\mathrm{E}_{-i}\left[k_i(z_i, z_{-i})(1-\beta p_i(z_i, z_{-i}))\right] \leq \mathrm{E}_{-i}\left[k_i(\tilde{z}_i, z_{-i})(1-\beta p_i(\tilde{z}_i, z_{-i}))\right].$$

■

Lemma 5 *The expected payment to agent i can be expressed as the sum of his expected cost of completion and an informational rent term. That is:*

$$E_{-i}\left[x_i(z_i)\right] = E_{-i}\left[k_i(z_i,z_{-i})(1-z_i)(1-\beta p_i(z_i,z_{-i})) + \int_0^{z_i} k_i(s,z_{-i})(1-\beta p_i(s,z_{-i}))ds\right].$$

Proof: Let $\pi_i(z_i) = \pi_i(z_i,z_i) = \mathrm{E}_{-i}[\pi_i(z_i,z_i,z_{-i})]$, where $\pi_i(a,b,d)$ is manager i's payoff for announcing b when his true type is a and the other manager announces d, and $\pi_i(a,b)$ is the expected value of $\pi_i(a,b,d)$ over d. The Envelope Theorem yields:

$$\pi_i'(z_i) \quad = \quad \frac{\partial}{\partial s}\pi_i(s,z_i)\bigg|_{s=z_i} + \frac{\partial}{\partial s}\pi_i(z_i,s)\bigg|_{s=z_i}$$

The second term on the r.h.s. is zero if local IC holds, so applying the Fundamental Theorem of Calculus to the FOC yields:

$$\pi_i(z_i) - \pi_i(0) = \mathrm{E}_{-i}\left[\int_0^{z_i} k_i(s,z_{-i})(1-\beta p_i(s,z_{-i}))ds\right].$$

Abusing notation and letting z denote the vector of realizations $\{z_i,z_{-i}\}$, by the definition of π_i, the expected payment above can be expressed as:

$$\mathrm{E}_{-i}\left[x_i(z)\right] = \mathrm{E}_{-i}\left[k_i(z_i,z_{-i})(1-z_i)(1-\beta p_i(z_i,z_{-i})) + \int_0^{z_i} k_i(s,z_{-i})(1-\beta p_i(s,z_{-i}))ds\right] + \pi_i(0).$$

Note that this fixes manager i's expected payment conditional on observing z_i. If we define $x_i(z)$ as the term inside the brackets above, then $x_i(z) \geq 0$ for all values of z such that the manager is at worst indifferent to finishing if he is asked to (i.e., not just in expectation). The fact that $\pi_i(0) = 0$ is optimal can be seen by assuming the contrary, say $\pi_i(0) > 0$, then noting that we can reduce each manager's compensation by $\pi_i(0)$ and the schedule will continue to satisfy both IC and IR. ■

We can now prove the proposition. Integration by parts allows us to rewrite

$$\mathrm{E}_{-i}\left[\int_0^1 \left(k_i(t,z_{-i})(1-t)(1-\beta p_i(t,z_{-i})) + \int_0^t k_i(s,z_{-i})(1-\beta p_i(s,z_{-i}))ds\right) f_i(t)dt\right]$$

as

$$\mathrm{E}_{-i}\left[\int_0^1\left(k_i(t,z_{-i})(1-t)(1-\beta p_i(t,z_{-i}))+\left(\frac{1-F(t)}{f(t)}\right)k_i(t,z_{-i})(1-\beta p_i(t,z_{-i}))\right)f_i(t)dt\right]$$

Therefore using Lemma 5 we can re-write the Principal's problem as:

$$\max\int_0^1\int_0^1\left(\sum_{i=1}^2 B(k_i(z_i,z_{-i}))-\left[\begin{array}{c}[k_i(z_i,z_{-i})(1-z_1)(1-\beta p_i(z_i,z_{-i}))\\ +\left(\frac{1-F(z_i)}{f(z_i)}\right)k_i(z_i,z_{-i})(1-\beta p_i(z_i,z_{-i}))\end{array}\right]\right)f_i(z_i)f_{-i}(z_{-i}) \quad (11)$$

Because we are using the uniform density over $[0,1]$ we can replace $\frac{1-F(z_i)}{f(z_i)}$ with $1-z_i$ allowing us to rewrite (11) as:

$$\max\int_0^1\int_0^1\left(\sum_{i=1}^2 B(k_i(z_i,z_{-i}))-2\left[k_i(z_i,z_{-i})(1-z_i)(1-\beta p_i(z_i,z_{-i}))\right]\right)f_i(z_i)f_{-i}(z_{-i}) \quad (12)$$

From (12), it is obvious that the principal will set $p_i(z_i,z_{-i})=1$ to the manager with $k(z_i,z_{-i})=1$ and the smallest value of z_i, otherwise $p_i(z_i,z_{-i})=0$. The principal will set $k_i(z_i,z_{-i})=1$ whenever doing so maximizes the integrand in (12). In other words, when $B/2\geq\beta(1-\overline{z})+(1-\beta)(1-\underline{z})$, then $k_1(z_1,z_2)=k_2(z_2,z_1)=1$, whereas when $(1-\overline{z})\beta+(1-\underline{z})(1-\beta)>B/2\geq(1-\beta)(1-\overline{z})$, the manager with the higher realization of z is asked to complete the project. Finally, in all other cases, the principal prefers for both managers not to complete their projects. Hence, the monotonicity requirements from Lemma 4 are satisfied.■

Proof of Proposition 3

We begin by calculating the principal's expected surplus in the second-best setting. Recall that from (12), the principal's expected payment to each manager is given by two times the manager's realized cost. The probability of *both* managers finishing their tasks times the principal's payoff when two tasks are finished is given by the sum of (13) and (14), below:

$$2\int_{1-\frac{B}{2}}^1\int_s^1(2B-2((1-t)+(1-\beta)(1-s)))\,dtds; \quad (13)$$

$$2\int_{1-\frac{B}{2(1-\beta)}}^{1-\frac{B}{2}}\int_{1-\frac{\frac{B}{2}-(1-\beta)(1-s)}{\beta}}^1(2B-2((1-t)+(1-\beta)(1-s)))\,dtds. \quad (14)$$

In the two expressions above, t denotes the larger of the two z_i draws, and s the smaller of the two. The integrals in (13) and (14) are pre-multiplied by 2 because either manager 1 or manager

2 could receive the smaller cost realization z. When both managers are asked to complete their projects, the manager with the greatest cost $(1-s)$ is always awarded the resource. The integral in (13) refers to the case where $(1-s)$ is sufficiently small that, regardless of the other manager's cost realization, $B/2 \geq \beta(1-t)+(1-\beta)(1-s)$; i.e., from Table 3, the two managers will always be asked to complete their tasks. On the other hand, the integral in (14) considers larger realizations of $(1-s)$, and thus, in order to again satisfy $B/2 \geq \beta(1-t)+(1-\beta)(1-s)$ requires that the larger of the two realizations, t, be restricted. The lower bound of integration, $1-\frac{B/2-(1-\beta)(1-s)}{\beta}$ for t is the edge where $B/2 = \beta(1-t)+(1-\beta)(1-s)$, i.e., the cost boundary where both managers are asked to complete their tasks. The sum of (13) and (14) is given by $\frac{(3+\beta)B^3}{12(1-\beta)}$.

On the other hand, the probability of a single manager completing his task times the payoff to the principal under the second-best is given by the sum of the following expressions, (15) and (16):

$$2\int_{1-\frac{B}{2(1-\beta)}}^{1-\frac{B}{2}} \int_{s}^{1-\frac{\frac{B}{2}-(1-\beta)(1-s)}{\beta}} \left(B-2((1-t)(1-\beta))\right) dtds; \tag{15}$$

$$2\int_{0}^{1-\frac{B}{2(1-\beta)}} \int_{1-\frac{B}{2(1-\beta)}}^{1} \left(B-2((1-t)(1-\beta))\right) dtds. \tag{16}$$

Again, t denotes the larger of the two draws. Recall that only one manager is asked to complete his project whenever the second entry in the first column of Table 3 holds. The second integral in (15) is the complement to the second integral in (14), i.e., the bounds ensure that the entry in the first row and column of Table 3 is violated, and instead, the second row of the first column holds. The first integral in (16) is the complement to the first integral in (14) in that it allows the smaller of the two cost realizations to belong to an interval which would never satisfy the first row and column of Table 3. The second integral in (16) simply limits the larger of the two cost realizations such that the principal is assured that at least one task is accomplished, i.e. it limits the larger of the two realization to satisfy $(1-t)\beta+(1-s)(1-\beta) > B/2 \geq (1-\beta)(1-t)$. The sum of (15) and (16) is given by $\frac{B^2(6+\beta(B-6)-3B+\beta^2 B)}{12(1-\beta)^2}$. Summing the expected payoffs times their respective probabilities yields the principal's total expected payoff which is given by:

$$\frac{B^2(6-\beta(6+B))}{12(1-\beta)^2}. \tag{17}$$

We now turn to the principal's expected payoff using the market mechanism. Substituting the optimal completion bonus $c = B/2$ into the principal's objective function (10) yields (17).■

Proof of Proposition 4

We begin with the principal's expected payoff from the second-best regime. If both managers complete their tasks, the principal's expected payoff is given by:[19]

$$\begin{aligned}
&2\int_{1-\frac{B}{2}}^{1}\int_{s}^{1}\left(2B-2((1-t)+(1-\beta)(1-s))\right)dtds\\
+2\int_{0}^{1-\frac{B}{2}}&\int_{1-\frac{\frac{B}{2}-(1-\beta)(1-s)}{\beta}}^{1}\left(2B-2((1-t)+(1-\beta)(1-s))\right)dtds\\
&=\frac{16\beta^3+(1-4\beta)(B-2)^3-\beta^2(40-36B+B^3)}{12\beta^2} \qquad (18)
\end{aligned}$$

Similarly, if only a single manager completes his task, then the principal's expected payoff in the second-best setting is given by:

$$\begin{aligned}
2\int_{0}^{1-\frac{B}{2}}&\int_{s}^{1-\frac{\frac{B}{2}-(1-\beta)(1-s)}{\beta}}\left(B-2((1-t)(1-\beta))\right)dtds\\
&=\frac{(B-2)^2(2+3\beta(B-2)-B+\beta^2(4+B))}{12\beta^2}. \qquad (19)
\end{aligned}$$

Summing (18) and (19), we obtain the principal's expected payoff which is given by:

$$\frac{16\beta^2-(B-2)^3+24\beta(B-1)}{12\beta}. \qquad (20)$$

Similarly, the principal's payoff using the auction mechanism is derived by inserting the optimal completion bonus, $c^*=\frac{1}{4}\left(3+B-\sqrt{(B-1)^2+8\beta}\right)$, into the principal's objective function in (9) to yield:

$$\frac{1}{24\beta}\left(8\beta^2+\left(1+\sqrt{(B-1)^2+8\beta}-B\right)(B-1)^2+4\beta\left(9B-9+2\sqrt{(B-1)^2+8\beta}\right)\right). \qquad (21)$$

The difference between the principal's payoff in the second-best setting, (20), and the auction mechanism, (21), is given by:

$$D(B,\beta)=-\frac{1}{24\beta}\left(\begin{array}{c}8\beta^2-2(16\beta^2-(B-2)^3+24\beta(B-1))\\+\left(1+\sqrt{(B-1)^2+8\beta}-B\right)(B-1)^2+4\beta\left(2\sqrt{(B-1)^2+8\beta}+9(B-1)\right)\end{array}\right).$$

[19] Note that the integrands in this proof mirror those in the proof of Proposition 3; however, the bounds of integration differ because now $B>2(1-\beta)$.

Ignoring the scalar constant $\frac{1}{24}$, the above expression can be simplified as follows:

$$D(B,\beta)=\frac{1}{\beta}\left(24\beta^2-2(B-2)^3+(B-1)^3+12\beta(B-1)-\left((B-1)^2+8\beta\right)^{\frac{3}{2}}\right). \tag{22}$$

To show that $D(B,\beta)$ is increasing in B for $B\geq 2(1-\beta)$, note that:

$$\frac{\partial D}{\partial B}=\frac{3}{\beta}\left(-2(B-2)^2+(B-1)^2+4\beta-(B-1)\sqrt{(B-1)^2+8\beta}\right). \tag{23}$$

Label the expression in brackets in (23) as $j(\beta)$, and note that $j\left(1-\frac{B}{2}\right)=0$ as:

$$\begin{aligned} j\left(\frac{2-B}{2}\right) &= -2(B-2)^2+(B-1)^2+2(2-B)-(B-1)\sqrt{(B-1)^2+4(2-B)} \\ &= 2(2-B)(B-1)+(B-1)^2-(B-1)(3-B) \\ &= (B-1)(4-2B+B-1-3+B)=0. \end{aligned}$$

We then differentiate $j(\beta)$ to obtain:

$$\frac{\partial j}{\partial \beta}=\frac{4}{\sqrt{(B-1)^2+8\beta}}\left(\sqrt{(B-1)^2+8\beta}-(B-1)\right). \tag{24}$$

For $B\geq 2(1-\beta)$, or equivalently, $\beta\geq 1-\frac{B}{2}$, note that $\sqrt{(B-1)^2+8\beta}\geq\sqrt{(B-1)^2+8-4B}=(3-B)>1$. Since $(B-1)<1$, the expression in (24) is strictly positive, i.e., $j(\beta)$ is monotone increasing in β for $\beta\geq 1-\frac{B}{2}$. As $j\left(1-\frac{B}{2}\right)=0$, we have thus demonstrated that (23) >0, as was to be shown.

Next, we show that $D(B,\beta)$ is increasing in β for $\beta\geq 1-\frac{B}{2}$. From (22), the partial derivative $\frac{\partial D(B,\beta)}{\partial\beta}$ is non-negative if and only if any of the following equivalent inequalities hold:

$$\begin{pmatrix} \beta\left(48\beta+12(B-1)-\frac{24}{2}\left(8\beta+(B-1)^2\right)^{\frac{1}{2}}\right) \\ -24\beta^2+12\beta(B-1)-2(B-2)^3+(B-1)^3-\left(8\beta+(B-1)^2\right)^{\frac{3}{2}} \end{pmatrix} \geq 0 \tag{25}$$

$$24\beta^2+2(B-2)^3-(B-1)^3-\left(4\beta-(B-1)^2\right)\sqrt{(B-1)^2+8\beta} \geq 0. \tag{26}$$

Let $g(\beta)$ denote the expression on the left hand side of (26). Note that the function $g(\beta)$ is zero when $\beta=1-\frac{B}{2}$, since:

$$\begin{aligned} g\left(\frac{2-B}{2}\right) &= 24\frac{(B-2)^2}{4}+2(B-2)^3-(B-1)^3-(3-B)\left(2(2-B)-(B-1)^2\right) \\ &= 2(B-2)\left(3(B-2)+B^2-4B+4+3-B\right)+(B-1)^2(3-B-B+1) \\ &= 2(B-2)(B^2-2B+1)+2(B-1)^2(2-B)=0. \end{aligned}$$

Moreover, the derivative of $g(\beta)$ is given by:

$$\begin{aligned}
\frac{\partial g(\beta)}{\partial \beta} &= 48\beta - \left(4\beta - (B-1)^2\right) 4 \left((B-1)^2 + 8\beta\right)^{-\frac{1}{2}} - 4\sqrt{(B-1)^2 + 8\beta} \\
&= 48\beta - \frac{4}{\sqrt{8\beta + (B-1)^2}} \left(4\beta - (B-1)^2 + (B-1)^2 + 8\beta\right) \\
&= \frac{48\beta}{\sqrt{8\beta + (B-1)^2}} \left(\sqrt{8\beta + (B-1)^2} - 1\right). \qquad (27)
\end{aligned}$$

As before, $\beta \geq 1 - \frac{B}{2}$ implies $\sqrt{8\beta + (B-1)^2} \geq 3 - B > 1$, so the above expression (27) is positive. Because $g(\beta)$ is zero at $\beta = 1 - B/2$, and increasing for all $\beta \geq 1 - \frac{B}{2}$, we have established that $\frac{\partial D(B,\beta)}{\partial \beta} \geq 0$ for all $\beta \geq 1 - \frac{B}{2}$.■

Proof of Lemma 3

Let $s(e_i, e_{-i})$ denote manager i's, ex-ante expected surplus from taking action e_i when the other manager picks effort e_{-i}. We first show that agent i's expected surplus is given by:

$$s(e_i, e_{-i}) = -\frac{1}{60\beta^2} \begin{pmatrix} 4((1-c)^4(4+c) - 2\beta^3 - 5\beta^2(3c-2) + 2\beta(c-1)^3(6+3c+c^2)) \\ -e_{-i}(7\beta^3 + (1-c)^4(11+4c) + 2\beta(c-1)^3(9+7c+4c^2)) \\ -e_i(10\beta^2 - 3\beta^3 + (1-c)^4(11+4c) + 2\beta(c-1)^3(9+7c+c^2)) \\ +2e_ie_{-i}(\beta^3 + (1-c)^4(3+2c) + 2\beta(c-1)^3(2+c+2c^2)) \end{pmatrix} \qquad (28)$$

To derive (28) we proceed in two steps. We first find the manager's expected payoff c minus the cost of effort, and in the second step, we subtract the manager's expected payment.

To simplify notation, as before, let $Z[s] = 1 - \frac{c-(1-\beta)(1-s)}{\beta}$ denote the type of budget- unconstrained manager with the same value for the resource as a budget-constrained manager with cost $(1-s)$. Let $ZZ[t] = 1 + \frac{\beta(1-t)-c}{1-\beta}$ denote the type of budget constrained manager with the same valuation for the resource as a budget unconstrained manager with cost $(1-t)$. The following table shows manager i's payoff for all possible realizations of z_i and z_{-i}:

Payoff	z_i Condition	z_{-i} Condition	Relative z_i
$c-(1-z_i)$	$z_i \geq 1-c$	$z_{-i} \geq 1-c$	$z_i > z_{-i}$
$c-(1-z_i)$	$z_i \geq 1-c$	$z_{-i} < 1-c$	$z_i > Z[z_{-i}]$
$c-(1-\beta)(1-z_i)$	$z_i \geq 1-c$	$z_{-i} \geq 1-c$	$z_i < z_{-i}$
$c-(1-\beta)(1-z_i)$	$z_i < 1-c$	$z_{-i} < 1-c$	$z_i \geq z_{-i}$
$c-(1-\beta)(1-z_i)$	$z_i < 1-c$	$z_{-i} \geq 1-c$	$Z[z_i] < z_{-i}$
$c-(1-\beta)(1-z_i)$	$1-\frac{c-1+\beta}{\beta} \geq z_i \geq 1-c$	$z_{-i} < 1-c$	$Z[z_{-i}] > z_i$
0	$1-c > z_i$	$1-c > z_{-i}$	$z_i < z_{-i}$
0	$1-c > z_i$	$1-c \leq z_{-i} \leq 1-\frac{c-1+\beta}{\beta}$	$z_i < ZZ[z_{-i}]$

To obtain each manager's expected surplus, we first sum the expected values in the table above and use the probabilities previously computed in the proof of Lemma 1 to obtain:

$$\frac{1}{60\beta^2}\begin{pmatrix} 2(12\beta^3-(1-c)^4(4+c)+10\beta^2(3c-2)-2\beta(c-1)^3(6+3c+c^2)) \\ +2e_{-i}(3\beta^3+(1-c)^4(9+c)+2\beta(c-1)^3(6+3c+c^2)) \\ -e_i(9\beta^3-10\beta^2-(1-c)^4(2c-7)-2\beta(1-c)^4(3+2c)) \\ -e_ie_{-i}(\beta^3+(1-c)^4(3+2c)+2\beta(c-1)^3(2+c+2c^2)) \end{pmatrix}. \tag{29}$$

Using the cumulative distribution of values, $G(\cdot)$, from Lemma 2, we can calculate the expected payment as:

$$\int_0^{c\beta} sg(s,e_{-i})(1-G(s,e_i))ds$$

and can subtract it from (29) to obtain the manager's expected surplus $s(e_i,e_{-i})$ in (28).

Now, we can characterize the cross partial:

$$\frac{\partial^2 s(e_i,e_{-i})}{\partial e_i \partial e_{-i}} = \frac{-\left(\beta^3+(1-c)^4(3+2c)+2\beta(c-1)^3(2+c+2c^2)\right)}{30\beta^2}. \tag{30}$$

It remains to be shown that (30) is negative. To this end, note that we can disregard the denominator of (30). The numerator of (30) is concave in β hence we can solve for the maximizing value of β via the first order approach to obtain $\hat{\beta}=(1-c)\sqrt{\frac{2}{3}(2+c+2c^2)(1-c)}$. Clearly, when $\hat{\beta}>1$, $\beta=1$ maximizes the numerator of (30) over the feasible region $\beta\in[0,1]$. Moreover, $\hat{\beta}>1$ whenever $c<.11$, and in this region, substituting $\beta=1$ into the numerator of (30) yields $c^3(-6c^2+15c-10)$, which is clearly non-positive on the entire real line. Now, when $\hat{\beta}\leq 1$, plugging $\hat{\beta}$ into the numerator of (30) and simplifying yields:

$$-(1-c)^4\left(3+2c-\frac{4}{3}(2+c+2c^2)\sqrt{\frac{2}{3}(1-c)(2+c+2c^2)}\right). \tag{31}$$

It remains to be shown that the expression in (31) is non-positive, which is equivalent to the following series of inequalities:

$$\begin{aligned}
3+2c &\geq \frac{4}{3}(2+c+2c^2)\sqrt{\frac{2}{3}(1-c)(2+c+2c^2)} \\
9(3+2c)^2 &\geq 16(2+c+2c^2)^2\frac{2}{3}(2+c+2c^2) \\
27(9+4c^2+12c)^2 &\geq 32(2+c+2c^2)^3 \\
(2c-1)^2(64c^5+96c^4+224c^3+160c^2+144c-13) &\geq 0.
\end{aligned} \tag{32}$$

The first expression on the left-hand side of (32) is always positive, and hence can be ignored. The second expression on the left-hand side of (32) has a single sign-change among its coefficients and hence, by Descartes' Rule of Signs, has a single positive real root. Let $f(c) = 64c^5+96c^4+224c^3+160c^2+144c-13$, and note that $f(0) < 0 < f(.1)$; hence, the unique root of the polynomial $f(c)$ belongs to the interval $(0, .1)$, which lies in the range where $\hat{\beta} > 1$. As the coefficient on the highest order of the polynomial, c^5 is positive, $f(c)$ is strictly positive for all $c > 0.1$, thus concluding the proof.■

Proof of Proposition 5

We can write $N(\beta, c, \nu)$ as:

$$N(\beta,c,\nu) = \frac{1}{12\beta^2}\left(2\beta^2-\beta^3+(1-c)^4+2\beta(c-1)^3(1+c)\right)-\nu \tag{33}$$

The equation $\frac{\partial}{\partial c}N(\beta,c,\nu)=0$ has two solutions in c: $c=1$ with multiplicity 2, and $c=\frac{1-\beta}{1+2\beta}$. However, $\frac{\partial^2}{\partial c^2}N(\beta,c,\nu)\Big|_{c=1} = 0$ and $\frac{\partial^2}{\partial c^2}N(\beta,c,\nu)\Big|_{c=\frac{1-\beta}{1+2\beta}} = \frac{3}{1+2\beta} > 0$, therefore, the function $N(\beta,c,\nu)$ is minimized at $c=\frac{1-\beta}{1+2\beta} < 1-\beta$. In fact, $N(\beta,c,\nu)$ is increasing in the range $c \in [1-\beta, 1]$. This follows because:

$$\frac{\partial}{\partial c}N(\beta,c,\nu)\Big|_{c=1-\beta} = \frac{2}{3}\beta(1-\beta) \geq 0,$$

and we know that $\frac{\partial}{\partial c}N(\beta,c,\nu)$ does not cross 0 before $c=1$. By the intermediate value theorem, $N(\beta,c,\nu)$ is therefore increasing over $c \in \left[\frac{1-\beta}{1+2\beta}, 1\right]$. Since $\frac{\partial}{\partial \nu}N(\cdot) = -1$, any increase in ν will increase $\hat{c}$, the solution to $N(\beta,\hat{c},\nu)=0$, and decrease the set of values of c that generate a Nash equilibrium. Similarly, we have $\frac{\partial}{\partial\beta}N(\beta,c,\nu) = -\frac{1}{12\beta^3}(\beta^3 + 2(1-c)^4 + 2\beta(c-1)^3(1+c))$, which is always non-positive; hence, $N(\beta,c,\nu)$ is decreasing in β and thus, $\hat{c}$ must be increasing in β.■

Proof of Proposition 6

Examine the difference $N2(\beta,c,\nu) = s(1,0) - s(0,0)$. Label the difference $N2(\beta,v,\nu)$. We have:

$$N2(\beta,c,\nu) = \frac{1}{60\beta^2}\left(10\beta^2 - 3\beta^3 + (1-c)^4(11+4c) + 2\beta(c-1)^3(9+7c+4c^2)\right) - \nu.$$

To show that if $e_1 = e_2 = 1$ is a Nash equilibrium, then it is the unique Nash, it is sufficient to show that $N(\beta,c,\nu) \geq 0 \Rightarrow N2(\beta,c,\nu) \geq 0$. To demonstrate this, we will show that $N2(\beta,c,\nu) \geq N(\beta,c,\nu)$ by proving that their difference is always non-negative. To this end, we have:

$$ND(\beta,c,\nu) = N2(\beta,c,\nu) - N(\beta,c,\nu) = \frac{\beta^3 + (1-c)^4(3+2c) - 2\beta(1-c)^3(2c^2+2+c)}{30\beta^2}.$$

We first show that both $ND(\beta,0,\nu)$ and $ND(\beta,1,\nu)$ are non-negative. We have:

$$ND(\beta,0,\nu) = \frac{3 - 4\beta + \beta^3}{30\beta^2}. \tag{34}$$

The sign of (34) is determined by its numerator, which decreases on the range $\beta \in [0, 2/\sqrt{3}]$, equals 0 when $\beta = 1$, and is strictly positive for $\beta \in [0,1)$. On the other hand, we can write:

$$ND(\beta,1,\nu) = \frac{\beta}{30}.$$

Clearly, $ND(\beta,1,\nu) \geq 0$. Now, it remains to be shown that $ND[\beta,c,\nu] \geq 0$ for $c \in (0,1)$. To this end, note that $\frac{\partial}{\partial c}ND = 0$ has three solutions: $c = 1$, $c = -\frac{\sqrt{1-\beta}}{\sqrt{1+2\beta}}$ and $c = \frac{\sqrt{1-\beta}}{\sqrt{1+2\beta}}$, and only the last root is possible in the range $(0,1)$. Moreover, we have $\frac{\partial^2}{\partial c^2}ND(\beta,c,\nu)\Big|_{c=\frac{\sqrt{1-\beta}}{\sqrt{1+2\beta}}} \geq 0$, hence $c = \frac{\sqrt{1-\beta}}{\sqrt{1+2\beta}}$ is a local minimum. If $ND\left(\beta, \frac{\sqrt{1-\beta}}{\sqrt{1+2\beta}}, \nu\right) \geq 0$, then the proof is complete. To this end, we have:

$$\begin{aligned} \text{sign}\left(ND\left(\beta, \frac{\sqrt{1-\beta}}{\sqrt{1+2\beta}}, \nu\right)\right) &= \text{sign}\left((\beta-1)\frac{\sqrt{1-\beta}(8+12\beta) + \sqrt{1+2\beta}(2\beta^3+3\beta^2-8)}{30\beta^2(1+2\beta)^{3/2}}\right) \\ &= -\text{sign}\left(\sqrt{1-\beta}(8+12\beta) + \sqrt{1+2\beta}(2\beta^3+3\beta^2-8)\right) \end{aligned} \tag{35}$$

The expression inside the sign function in (35) has two zeroes: $\beta = 0, 1/2$. Moreover, for $\beta = 1/4$ and $\beta = 3/4$, the expression is positive, hence (35) is non-negative on the entire interval $\beta \in [0,1]$.■

References

Kenneth Arrow. The role of securities in the optimal allocation of risk-bearing. *Review of Economic Studies*, 31(2):91–96, April 1964.

Anil Arya and Brian Mittendorf. Linking markets to get a leg up on rivals. *Working paper, Yale University School of Management*, 2004.

Tim Baldenius. Intrafirm trade, bargaining power, and specific investments. *Review of Accounting Studies*, 5(1):27–56, March 2000.

Tim Baldenius and Stefan Reichelstein. External and internal pricing in multidivisional firms. *Journal of Accounting Research*, 44(1):1–28, March 2006.

Dirk Bergemann and Juuso Valimaki. Information in mechanism design. In Richard Blundell, Whitney Newey, and Torsten Persson, editors, *Proceedings of the 9th World Congress of the Econometric Society*, Cambridge, 2006. Cambridge University Press.

John Christensen and Joel Demski. Profit allocation under ancillary trade. *Journal of Accounting Research*, 36(1):71–89, Spring 1998.

Leon Chu and David Sappington. Simple cost-sharing contracts. *Working Paper*, May 2005.

Gerard Debreu. *Theory of Value.* John Wiley & Sons, Inc., New York, NY, 1959.

Ernst & Young. 2005-2006 global transfer pricing surveys. *www.ey.com/transferpricingsurvey*, 2006.

Robert Gertner, Eric A. Powers, and David S. Scharfstein. Learning about internal capital markets from corporate spin-offs. *Journal of Finance*, 57(6):2479–2506, December 2002.

William Halal, Ali Geranmayeh, and John Pourdehnad. *Internal Markets: Bringing the Power of Free Enterprise Inside Your Organization.* Wiley, New York, 1993.

Milton Harris, Charles Kriebel, and Artur Raviv. Asymmetric information, incentives and intrafirm resource allocation. *Management Science*, 28(6):604–620, June 1982.

Freidrich Hayek. The use of knowledge in society. *The American Economic Review*, 35(4):519–530, September 1945.

Joshua House and David Victor. BP's emissions trading system. *Energy Policy*, 34(15):2100–2112, October 2006.

IBM. Putting market forces to work inside your business. `http://www-306.ibm.com/e-business/ondemand/us/growth/markets.shtml`, 2006.

Greg Jaffe. Navy turns auctioneer, lets sailors bid for unpopular posts. *The Wall Street Journal*, page B1, August 11, 2003.

Paul Klemperer. *Auctions: Theory and Practice*. Princeton University Press, Princeton, NJ, 2004.

Jean-Jacques Laffont and David Martimort. *The Theory of Incentives*. Princeton University Press, Princeton, NJ, 2002.

Jean-Jacques Laffont and Jean Tirole. Auctioning incentive contracts. *Journal of Political Economy*, 95(5):921–937, October 1987.

Owen Lamont. Cash flow and investment: Evidence from internal capital markets. *Journal of Finance*, 52(1):83–109, March 1997.

Ching-to Ma. Unique implementation of incentive contracts with many agents. *The Review of Economic Studies*, 55(4):555–571, October 1988.

Thomas Malone. *The Future of Work: How the New Order of Business Will Shape Your Organization, Your Management Style, and Your Life*. Harvard Business School Press, Boston, 2004.

Preston McAfee and John McMillan. Bidding for contracts: A principal-agent analysis. *The RAND Journal of Economics*, 17(3):326–338, Autumn 1986.

Preston McAfee and John McMillan. Competition for agency contracts. *The RAND Journal of Economics*, 18(2):296–307, Summer 1987.

Paul Milgrom and Robert Weber. A theory of auctions and competitive bidding. *Econometrica*, 50(5):1089–1122, 1982.

John Moore and Rafael Repullo. Nash implementation: A full characterization. *Econometrica*, 58 (5):1083–1099, September 1990.

Roger Myerson. Optimal auction design. *Mathematics of Operations Research*, 6(1):58–73, 1981.

Madhav Rajan and Stefan Reichelstein. A perspective on "Asymmetric Information, Incentives and Intrafirm Resource Allocation". *Management Science*, 50(12):1615–1623, December 2004.

John G. Riley. Ex post information in auctions. *The Review of Economic Studies*, 55(3):409–429, July 1988.

Michael H. Riordan and David E. M. Sappington. Awarding monopoly franchises. *The American Economic Review*, 77(3):375–387, June 1987.

William Rogerson. Simple menus of contracts in cost-based procurement and regulation. *American Economic Review*, 93(3):919–926, 2003.

David S. Scharfstein and Jeremy C. Stein. The dark side of internal capital markets: Divisional rent-seeking and inefficient investment. *Journal of Finance*, 55(6):2537–2564, December 2000.

Jeremy C. Stein. Internal capital markets and the competition for corporate resources. *Journal of Finance*, 52(1):111–133, March 1997.

Alix Stuart. Market magic. *CFO Magazine*, page 1, November 2005.

William C. Taylor. Here's an idea: Let everyone have ideas. *The New York Times*, 26(Business): 3, March 2006.

Table 1

Characterization of the First Best						
	Manager with realization $\underline{z}$			Manager with realization $\overline{z}$		
	Allocated Resource?	Complete?	Compensation	Allocated Resource?	Complete?	Compensation
$B \geq \beta(1-\overline{z}) + (1-\beta)(1-\underline{z})$	Yes	Yes	$(1-\beta)(1-\underline{z})$	No	Yes	$1-\overline{z}$
$(1-\overline{z})\beta + (1-\underline{z})(1-\beta)$ $> B \geq (1-\beta)(1-\overline{z})$	No	No	0	Yes	Yes	$(1-\overline{z})(1-\beta)$
$(1-\overline{z})(1-\beta) > B$	Indiff	No	0	Indiff	No	0

Table 2

Characterization of the Auction Mechanism				
	Manager with realization $\underline{z}$		Manager with realization $\overline{z}$	
	Allocated Resource?	Complete?	Allocated Resource?	Complete?
$c \geq \beta(1-\overline{z}) + (1-\beta)(1-\underline{z})$	Yes	Yes	No	Yes
$(1-\overline{z})\beta + (1-\underline{z})(1-\beta) > c \geq (1-\beta)(1-\overline{z})$	No	No	Yes	Yes
$(1-\overline{z})(1-\beta) > c$	Indiff	No	Indiff	No

Table 3

Characterization of the Second Best				
	Manager with realization $\underline{z}$		Manager with realization $\overline{z}$	
	Allocated Resource?	Complete?	Allocated Resource?	Complete?
$B/2 \geq \beta(1-\overline{z}) + (1-\beta)(1-\underline{z})$	Yes	Yes	No	Yes
$(1-\overline{z})\beta + (1-\underline{z})(1-\beta)$ $> B/2 \geq (1-\beta)(1-\overline{z})$	No	No	Yes	Yes
$(1-\overline{z})(1-\beta) > B/2$	Indiff	No	Indiff	No

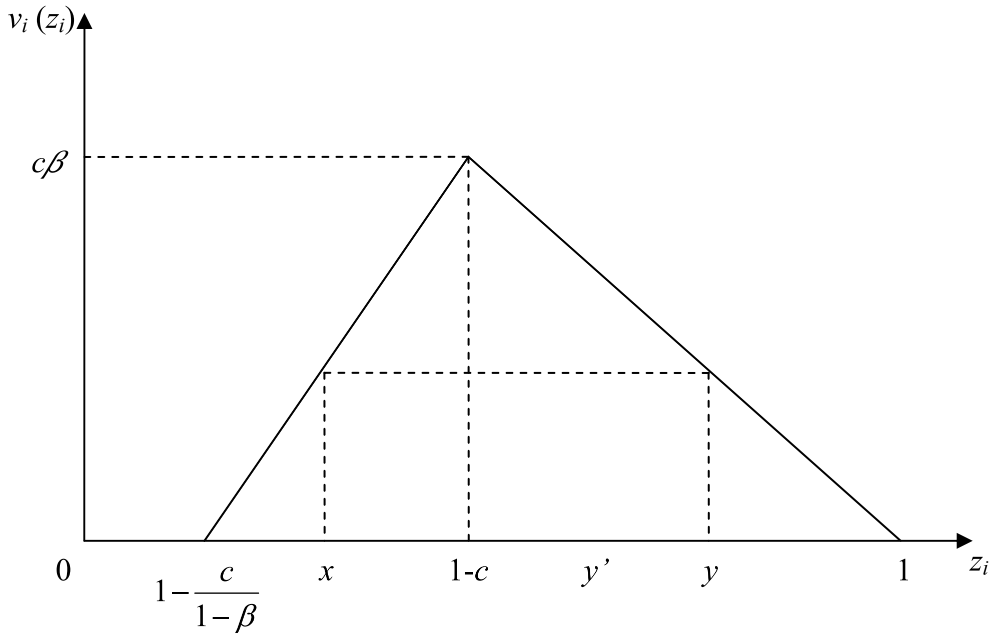

Figure 1: A manager with draw $z_i < 1 - \frac{c}{1-\beta}$ will never complete his project, even with help, and hence values the resource at zero dollars. For managers with $1 - \frac{c}{1-\beta} < z_i < 1 - c$, valuations are increasing in z_i because they only complete the project with help; consequently, the larger the draw, the more money they keep after successful completion of the project. Managers with a draw $z_i > 1-c$ have a value for the resource which is decreasing in z_i, because the better their draw (the larger z_i), the less the resource reduces their costs of completion.

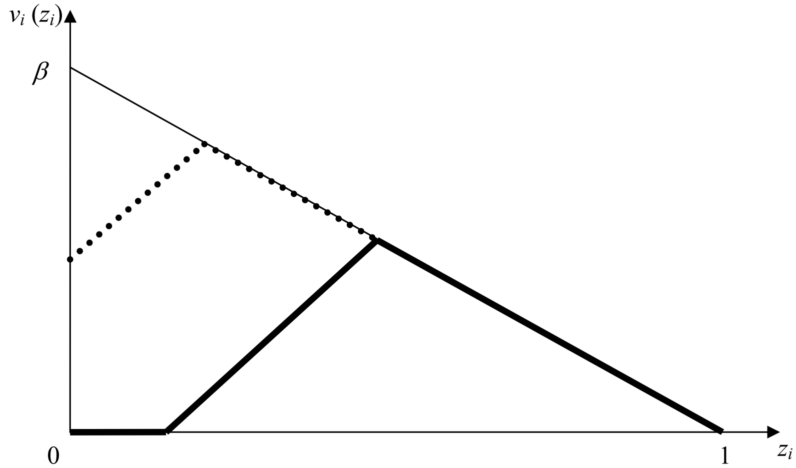

Figure 2: For bonuses c such that $c < 1 - \beta$, the bold face curves depict a manager's valuation for the resource as a function of his type. Note that the managers with sufficiently small z_i (where the valuation is flat) will not complete their project even if they receive the resource whereas managers with an increasing valuation will only complete their project if they receive the resource. For $1 - \beta < c < 1$, a manager's value for the resource is given by the dotted curve, and again, managers with an increasing valuation will only complete their task if they receive help. Finally, managers' valuations for $c \geq 1$ is given by the thin curve and in this case all managers are willing to complete their project, regardless of whether they are awarded the resource or not.

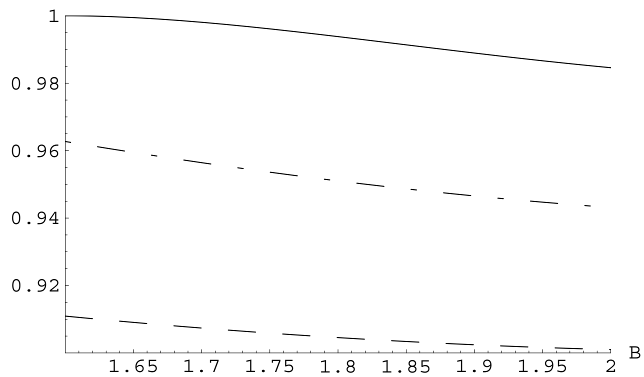

Figure 3: The solid, dashed-dotted and dashed curves plot the efficiency of the auction mechanism relative to the second-best mechanism for $\beta = 0.2, 0.5$ and 0.8, respectively, for values of $B \in [2(1-\beta), 2]$. Clearly, for larger values of β and B, the auction mechanism becomes less efficient at replicating the second-best outcome.