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unlawful, price, coordination, firms

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Exploring the Boundaries of Unlawful Collusion: Price Coordination when Firms Lack Full Mutual Understanding*

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Abstract

Proving that firms have violated Section 1 of the Sherman Act requires showing that they have a "meeting of minds" regarding coordinated behavior; it is not high prices that are illegal but rather the mutual understanding that produces those high prices. This paper offers a framework for exploring the relationship between mutual understanding and collusive outcomes, and uses it to characterize pricing behavior when it is common knowledge among firms that price increases will at least be matched. An upper bound on price is derived which is less than the equilibrium price. Supracompetitive pricing is sure to emerge even though firms lack mutual understanding as to who will be the price leader.

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1 Introduction

The economic theory of collusion focuses on what outcomes are sustainable and the strategy profiles that sustain them: What prices and market allocations can be supported? What are the most effective strategies for monitoring compliance? What are the most severe punishments that can be imposed in response to evidence of non-compliance? The literature is rich in taking account of the determinants of the set of collusive outcomes including market traits such as product differentiation and demand volatility, firm traits such as capacity, cost, and time preference, and the amount of public and private information available to firms.

In comparison, the primary focus of antitrust law is not on the outcome nor on the strategies that sustain an outcome but rather the means by which a collusive arrangement is achieved. A violation of Section 1 of the Sherman Act requires that firms have an *agreement* to coordinate their behavior. To establish the presence of an agreement, it must be shown that there was "mutual consent" among firms,[1] that firms "had a conscious commitment to a common scheme designed to achieve an unlawful objective,"[2] that firms had a "unity of purpose or a common design and understanding, or a meeting of minds."[3] While economics focuses on outcomes, the law focuses on the mutual understanding among firms ("meeting of minds") that produces those outcomes.[4]

Given that mutual understanding is not something that is directly observed, the judicial approach is to focus on communications among firms and to infer a level of mutual understanding from those communications (while possibly supplementing it with market outcomes in drawing those inferences). From this assessment, the courts seek to determine whether the level of mutual understanding among firm is sufficient to produce (or have the capability to produce) coordinated behavior and thereby to be deemed an unlawful agreement. Express communication among firms involving an exchange of assurances (for example, one firm proposes to raise price and the other firm affirms) is clearly viewed as sufficient to conclude that firms have a "meeting of minds" intended to produce a supra-competitive outcome. The real challenge is evaluating situations in which firms do not engage in such egregious and straightforward means for delivering the requisite mutual understanding. For example, in *United States v. Foley*,[5] there was a dinner among realtors at which one of them announced he was raising his commission rate from six to seven percent. Subsequently, other realtors in attendance did so, and the court convicted them. The raising of commission rates would almost surely had been insufficient by itself for a conviction; it was critical that there was this announcement which provided the basis for

[1] *Esco Corp. v. United States*, 340 F.2d 1000, 1007-08 (9th Cir. 1965).

[2] *Monsanto Co. v. Spray-Rite Serv. Corp.*, 465 U.S. 752 (1984); 753.

[3] *American Tobacco Co. v. United States*, 328 U.S. 781 (1946); 810.

[4] For an examination of the distinction between the economic and legal approaches to collusion, see Kaplow and Shapiro (2007) and Kaplow (2011a, 2011b, 2011c).

[5] 598 F.2d 1323 (4th Cir. 1979), cert. denied, 444 U.S. 1043 (1980).

producing a "conscious commitment to a common scheme." Thus, in that case, there was mutual understanding but surely of a lesser sort than if the realtors had engaged in an unconstrained conversation on the coordination of prices. Moving to a situation with even less direct communication and mutual understanding, Richard Posner has argued - first as a scholar and then as a judge - that firms' pricing behavior could be sufficient to conclude there is an agreement:

> If a firm raises price in the expectation that its competitors will do likewise, and they do, the firm's behavior can be conceptualized as the offer of a unilateral contract that the offerees accept by raising their prices.[6]

At the same time, Judge Posner recognizes that this is not the current judicial view; price movements are not considered to be sufficient to conclude that firms have an unlawful agreement.

The primary legal challenge is where to draw the line regarding when it is appropriate to conclude that firms have mutual understanding of a plan to raise prices, allocate markets, or engage in some other form of coordinated behavior. This challenge can be broken down into addressing two questions: First, how do various forms of communication determine the mutual understanding among firms? Second, how does that mutual understanding determine firm behavior with regards to price and other market outcomes? Addressing these questions - and thereby defining the contours of the law - has largely fallen to lawyers and judges with the conspicuous absence of economists.[7] The primary reason for the irrelevance of economic analysis is that it is predicated upon the assumption of *equilibrium* which presumes *complete* mutual understanding. As a result, economic analysis cannot address the critical question: How much and what type of mutual understanding is necessary to produce collusive outcomes?

It is the objective of the research program described in this paper to use economic analysis to assist in defining the boundaries of a Section 1 violation by putting forth a framework to address the following questions: How much and what type of mutual understanding is *necessary* to produce collusive outcomes? (Falling short of that mutual understanding threshold should not raise antitrust concerns.) How much and what type of mutual understanding is *sufficient* to produce collusive outcomes? (Rising above that mutual understanding threshold should raise antitrust concerns and could invoke per se prohibitions.) When mutual understanding is incomplete but collusive outcomes can still prevail, to what extent does the lack of full mutual understanding limit the extent of collusion? Thus, the focus of the analysis will be on addressing the second question -

[6] *In Re High Fructose Corn Syrup Antitrust Litigation Appeal of A & W Bottling Inc et al*, U.S. Court of Appeals, 295 F3d 652, (7th Cir., 2002). Also see Posner (2001), pp. 94-95.

[7] "On the ultimate issue of whether behavior is the result of a contract, combination, or conspiracy, however, courts routinely prevent economists from offering an opinion, because economics has surprisingly little to say about this issue." Page (2007), p. 424.

how mutual understanding determines behavior - rather than the first question - how communication determines mutual understanding.

The approach begins with a plausible assumption on mutual understanding among firms regarding their strategies where *plausible* means that one can describe forms of communication that could reasonably produce that mutual understanding without involving express communication. This mutual understanding is formally modelled by the assumption that it is common knowledge among firms that their strategies are in a *subset of the strategy space*. As equilibrium is when that subset is a singleton - the strategy profile is common knowledge - this approach can be viewed as a weakening of equilibrium. It is further assumed that each firm acts to maximize the present value of its profit stream and this is common knowledge ("rationality is common knowledge"). The analysis involves deriving the behavioral implications of mutual understanding regarding rationality and some properties of firms' strategies. In particular, is this mutual understanding sufficient to produce collusive outcomes? If it is, is the extent of collusion as great as when there is full mutual understanding (that is, equilibrium)? Or does limited mutual understanding constrain how effectively firms can collude?

While the approach put forth is general, the paper is really about executing it in a specific case. Specifically, it is assumed that it is common knowledge among firms that any price increase will be at least matched by the other firms, and failure to do so results in the competitive outcome. What is not common knowledge is leadership protocol. Which firm will lead by raising price? When will it raise price? What price will it set? Is another firm expected to lead the next round of price hikes? Mutual understanding about price matching but not about price leadership leaves many strategies consistent with firms' common beliefs. The reason for focusing on price leadership and price matching is because it is a well-recognized and documented device for producing supracompetitive outcomes, and I believe the assumptions on common knowledge can be reasonably motivated when firms do not expressly communicate. Both points are elaborated upon and argued to later in the paper.

The first result derives an upper bound on price which is strictly less than that which is achieved under equilibrium. Thus, in this particular setting, less than full mutual understanding limits the extent of collusion. Without further assumptions, no more can be said about the price path; it could involve competitive or supracompetitive prices. Assuming firms Bayesian update in learning about other firms' strategies and placing some additional structure on firms' prior beliefs, a result from the rational learning literature (Kalai and Lehrer, 1993) is used to show that the long-run price is supracompetitive. Thus, sufficient conditions are provided for collusion to prevail when firms' mutual understanding is substantively incomplete. In particular, price leadership emerges even though firms lack common knowledge about who will lead on price and at what levels.

While the contribution of this paper is cast in terms of understanding the relationship between mutual understanding and firm behavior, another perspective is that it offers a theory of tacit collusion. In practice and in the law, there are two types of collusion. *Explicit* *collusion* is when supracompetitive prices are achieved via express communication about an agreement. *Tacit collusion* is when supracompetitive prices are achieved without express communication in which case mutual understanding is likely to be incomplete. Economic theory does not distinguish between these two types because it does not deal with the issue of how firms coordinate when they lack full mutual understanding. Rightfully and frequently, lawyers remind economists of our inadequacy in this regard:

> While properly applied economic science may allow an economist to reach conclusions about "collusion," the term as used by economists may include both tacit and overt collusion among competitors ... and it is unclear whether economists have any special expertise to distinguish between the kinds of "agreement."[8]

The theory of collusion developed here is one that I feel comfortable as espousing to be tacit because it has: 1) a level of mutual understanding among firms that is plausibly achieved without express communication; and 2) a transparent mechanism for coordinating on a collusive outcome.[9], [10]

The model is described in Section 2 - where standard assumptions are made regarding cost, demand, and firm objectives - and in Section 3 - where the assumption of equilibrium is replaced with alternative assumptions on the behavior and beliefs of firms. An upper bound on price is characterized in Section 4. An example is provided in Section 5 to show that, without further structure, either competitive or supracompetitive pricing can arise. Additional structure is provided in Section 6 by way of a firm's prior beliefs and learning about other firms' strategies. There it is shown that the upper bound in Section

[8] Milne and Pace (2003), p. 36.

[9] One should not be misled to believe that the theoretical industrial organization literature is replete with theories of tacit collusion by virtue of use of the expression, as exemplified by the excellent survey "The Economics of Tacit Collusion" (Ivaldi et al, 2003). These theories characterize collusive behavior assuming full mutual understanding of strategies (that is, equilibrium). There is, however, some research that is most naturally considered explicit collusion because it assumes firms expressly communicate within the context of an equilibrium (as opposed to communciate in order to get to an equilibrium). Cheap talk messages about firms' private information on cost are exchanged in Athey and Bagwell (2001, 2008), on demand in Aoyagi (2002), Hanazono and Yang (2007), and Gerlach (2009), and on sales in Harrington and Skrzypacz (2011). There is also a body of work on bidding rings in auctions where participation in the auction is preceded by a mechanism among the ring members that involves the exchange of reported valuations; see, for example, Graham and Marshall (1987) and Krishna (2010).

[10] To my knowledge, the only other theory of tacit collusion is MacLeod (1985), whose approach is very different. To begin, it is based on firms announcing proposed price changes rather than making actual price changes. Axioms specify how firms respond to a price announcement, and these axioms are common knowledge. A firm's price response is allowed to depend on the existing price vector and the announced price change, and it is assumed the firm which announces the price change will implement it. If it is assumed that the price response is continuous with respect to the announcement, invariant to scale changes, and independent of firm identity then the response function must entail matching the announced price change. When firms are symmetric, the theory predicts that the joint profit maximum is achieved. To the contrary, the theory developed here predicts price is always below the price that maximizes joint profit.

4 will eventually be reached with high probability. Section 7 considers a linear example. While the analysis in this paper focuses on price leadership and matching, underlying it is a general approach for exploring the relationship between mutual understanding and collusive outcomes; that approach is described in Section 8. Section 9 offers a few concluding remarks.

2 Assumptions on the Market

Consider a symmetric differentiated products price game with n firms. $\pi(p_i, \mathbf{p}_{-i}) : \Re^n_+ \rightarrow \Re$ is a firm 's profit when it prices at p_i and its rivals price at $\mathbf{p}_{-i} = (p_1, ..., p_{i-1}, p_{i+1}, ..., p_n)$. Assume $\pi(p_i, \mathbf{p}_{-i})$ is bounded, twice continuously differentiable, increasing in a rival's price p_j $(j \neq i)$, and strictly concave in own price p_i. A firm's best reply function then exists:

$$\psi(\mathbf{p}_{-i}) = \arg\max_{p_i} \pi(p_i, \mathbf{p}_{-i}).$$

Further assume

$$\frac{\partial^2 \pi}{\partial p_i \partial p_j} > 0, \ \forall j \neq i$$

from which it follows that $\psi(\mathbf{p}_{-i})$ is increasing in p_j, $j \neq i$. A symmetric Nash equilibrium price, p^N, exists and is assumed to be unique,

$$\psi(p, .., p) \gtreqless p \text{ as } p \lesseqgtr p^N,$$

and let

$$\pi^N \equiv \pi\left(p^N, ..., p^N\right) > 0.$$

Assuming $\pi(p, ..., p)$ is strictly concave in p, there exists a unique joint profit maximum p^M,

$$\sum_{j=1}^{n} \frac{\partial \pi(p, ..., p)}{\partial p_j} \gtreqless 0 \text{ as } p \lesseqgtr p^M,$$

and $p^M > p^N$.

Firms interact in an infinitely repeated price game with perfect monitoring. A collusive price $p' > p^N$ is sustainable with the grim trigger strategy if and only if:[11]

$$\left(\frac{1}{1-\delta}\right) \pi\left(p', ..., p'\right) \geq \max_{p_i < p} \pi\left(p_i, p', ..., p'\right) + \left(\frac{\delta}{1-\delta}\right) \pi^N, \tag{1}$$

where δ is the common discount factor. Define $\widetilde{p}$ as the best price sustainable using the grim trigger strategy:

$$\widetilde{p} \equiv \max\left\{p \in \left[p^N, p^M\right] : \left(\frac{1}{1-\delta}\right) \pi(p, ..., p) \geq \max_{p_i < p} \pi(p_i, p, ..., p) + \left(\frac{\delta}{1-\delta}\right) \pi^N\right\}.$$

[11] The grim trigger strategy has any deviation from the collusive price p' result in a price of p^N forever.

Assume $\widetilde{p} > p^N$ and if $\widetilde{p} \in \left(p^N, p^M\right)$ then

$$\left(\frac{1}{1-\delta}\right)\pi\left(p,...,p\right) \gtreqless \pi\left(\psi\left(p,...,p\right),p,...,p\right)+\left(\frac{\delta}{1-\delta}\right)\pi^N \text{ as } p \lesseqgtr \widetilde{p} \text{ for } p \in \left[p^N, p^M\right]. \quad (2)$$

$\widetilde{p}$ will prove to be a useful benchmark.

For the later analysis, consider the "price matching" objective function for a firm:

$$W\left(p_i, \mathbf{p}_{-i}\right) \equiv \pi\left(p_i, \mathbf{p}_{-i}\right) + \left(\frac{\delta}{1-\delta}\right)\pi\left(p_i, ..., p_i\right).$$

Given its rivals price at $\mathbf{p}_{-i}$ in the current period, $W\left(p_i, \mathbf{p}_{-i}\right)$ is firm i's payoff from pricing at p_i *if it believed* that all firms would match that price in all ensuing periods. Consider

$$\frac{\partial W\left(p_i, \mathbf{p}_{-i}\right)}{\partial p_i} = \frac{\partial \pi\left(p_i, \mathbf{p}_{-i}\right)}{\partial p_i} + \left(\frac{\delta}{1-\delta}\right)\sum_{j=1}^{n}\frac{\partial \pi\left(p_i, ..., p_i\right)}{\partial p_j}.$$

If $p_i < p^M$ then the second term is positive; by raising its current price, a firm increases the future profit stream under the assumption that its price increase will be matched by its rivals. If $p_i > \psi\left(p_{-i}\right)$ then the first term is negative. Evaluate $\frac{\partial W\left(p_i, p_{-i}\right)}{\partial p_i}$ when firms price at a common level p:

$$\begin{aligned}\frac{\partial W\left(p, ..., p\right)}{\partial p_i} &= \frac{\partial \pi\left(p, ..., p\right)}{\partial p_i} + \left(\frac{\delta}{1-\delta}\right)\sum_{j=1}^{n}\frac{\partial \pi\left(p, ..., p\right)}{\partial p_j} \\ &= \left(\frac{1}{1-\delta}\right)\left(\frac{\partial \pi\left(p, ..., p\right)}{\partial p_i} + \delta\sum_{j\neq i}^{n}\frac{\partial \pi\left(p, ..., p\right)}{\partial p_j}\right)\end{aligned}$$

Thus, when $p \in \left(p^N, p^M\right)$, raising price lowering current profit, $\frac{\partial \pi(p,...,p)}{\partial p_i} < 0$, and increases future profit, $\sum_{j=1}^{n}\frac{\partial \pi(p,...,p)}{\partial p_j} > 0$. By the preceding assumptions, $W\left(p_i, \mathbf{p}_{-i}\right)$ is strictly concave in p_i since it is the weighted sum of two strictly concave functions. Hence, a unique optimal price exists,

$$\phi\left(\mathbf{p}_{-i}\right) = \max_{p_i} W\left(p_i, \mathbf{p}_{-i}\right). \quad (3)$$

By the preceding assumptions, $\phi\left(\mathbf{p}_{-i}\right)$ is increasing in a rival's price as

$$\frac{\partial \phi\left(\mathbf{p}_{-i}\right)}{\partial p_j} = -\frac{\partial^2 W\left(p_i, \mathbf{p}_{-i}\right)/\partial p_i \partial p_j}{\partial^2 W\left(p_i, \mathbf{p}_{-i}\right)/\partial p_i^2} = -\frac{\frac{\partial^2 \pi(p_i, \mathbf{p}_{-i})}{\partial p_i \partial p_j}}{\frac{\partial^2 \pi(p_i, \mathbf{p}_{-i})}{\partial p_i^2} + \left(\frac{\delta}{1-\delta}\right)\left(\frac{\partial^2 \pi(p,...,p)}{\partial p^2}\right)} > 0.$$

As there is a benefit in terms of future profit from raising price (as long as it does not exceed the joint profit maximum) then the price matching best reply function results in a higher price than the standard best reply function. To show this result, consider

$$\begin{aligned}\frac{\partial W\left(\psi\left(\mathbf{p}_{-i}\right),\mathbf{p}_{-i}\right)}{\partial p_i} &= \frac{\partial \pi\left(\psi\left(\mathbf{p}_{-i}\right),\mathbf{p}_{-i}\right)}{\partial p_i}+\left(\frac{\delta}{1-\delta}\right)\sum_{j=1}^{n}\frac{\partial \pi\left(\psi\left(\mathbf{p}_{-i}\right),...,\psi\left(\mathbf{p}_{-i}\right)\right)}{\partial p_j}\\ &= \left(\frac{\delta}{1-\delta}\right)\sum_{j=1}^{n}\frac{\partial \pi\left(\psi\left(\mathbf{p}_{-i}\right),...,\psi\left(\mathbf{p}_{-i}\right)\right)}{\partial p_j}>0,\end{aligned}$$

which is positive because $\mathbf{p}_{-i}\leq\left(p^M,...,p^M\right)$ implies $\psi\left(\mathbf{p}_{-i}\right)<p^M$.[12] By the strict concavity of W, $\phi\left(\mathbf{p}_{-i}\right)>\psi\left(\mathbf{p}_{-i}\right)$.

ϕ has a fixed point p^* because it is continuous, $\phi\left(p^N,...,p^N\right)>p^N$, and

$$\frac{\partial W\left(p^M,...,p^M\right)}{\partial p_i}=\frac{\partial \pi\left(p^M,...,p^M\right)}{\partial p_i}<0\Rightarrow\phi\left(p^M,...,p^M\right)<p^M.$$

Further assume the fixed point is unique:

$$\phi\left(p,...,p\right)\gtreqqless p \text{ as } p\lesseqqgtr p^*.$$

Thus, if rival firms price at p^*, a firm prefers to price at p^* rather than price differently under the assumption that its price will be matched forever. p^* will prove to be a useful benchmark.

Results are proven when the price set is finite.[13] From hereon, assume the price set is $\Delta_\varepsilon\equiv\{0,\varepsilon,2\varepsilon,...,\}$, where $\varepsilon>0$ and is presumed to be small. For convenience, suppose $p^N,p^*,\widetilde{p}\in\Delta_\varepsilon$.[14] As the finiteness of the price set could generate multiple optima, define the best reply correspondence for the price matching objective function:

$$\overline{\phi}\left(\mathbf{p}_{-i}\right)\equiv\arg\max_{p_i\in\Delta_\varepsilon}\pi\left(p_i,\mathbf{p}_{-i}\right)+\left(\frac{\delta}{1-\delta}\right)\pi\left(p_i,...,p_i\right).$$

The best reply correspondence is assumed to have the following property:[15]

$$\overline{\phi}\left(\mathbf{p}_{-i}\right)\begin{cases}\subseteq\{p'+\varepsilon,...,p^*\} & \text{if } \mathbf{p}_{-i}=\left(p',...,p'\right) \text{ where } p'<p^*-\varepsilon\\ =\{p^*\} & \text{if } \mathbf{p}_{-i}=\left(p_i^*,...,p_i^*\right)\\ \subseteq\{p^*,...,p'-\varepsilon\} & \text{if } \mathbf{p}_{-i}=\left(p',...,p'\right) \text{ where } p'>p^*+\varepsilon\end{cases}\tag{4}$$

[12] Since $\psi\left(p,..,p\right)\gtreqqless p$ as $p\lesseqqgtr p^N$ then $\psi\left(p^M,..,p^M\right)<p^M$. Given that ψ is increasing then $\mathbf{p}_{-i}\leq\left(p^M,...,p^M\right)$ implies $\psi\left(\mathbf{p}_{-i}\right)<\psi\left(p^M,..,p^M\right)<p^M$.

[13] A discussion of the case of an infinite price set is provided at the end of Section 4.

[14] If $\widetilde{p}\in\Delta_\varepsilon$ and $\psi\left(\widetilde{p},...,\widetilde{p}\right)\in\Delta_\varepsilon$ then $\widetilde{p}$ is still the best price sustainable using the grim punishment.

[15] It is shown in Appendix A that a sufficient condition for (4) is $-\frac{\partial^2\pi}{\partial p_i^2}\geq 2\frac{\partial^2\pi}{\partial p_i\partial p_{-i}}$, which holds when demand and cost functions are linear.

Recall that p^* is the unique fixed point for $\phi(\mathbf{p}_{-i})$ and is also a fixed point for $\overline{\phi}(\mathbf{p}_{-i})$. By (4), if all rival firms price at p' then firm i's best reply has it price above p' when $p' < p^* - \varepsilon$. Analogously, if $p' > p^* + \varepsilon$ then firm i's best reply has it price below p'. Note that an implication of (4) is that the set of symmetric fixed points of $\overline{\phi}(\mathbf{p}_{-i})$ is, at most, $\{p^* - \varepsilon, p^*, p^* + \varepsilon\}$.

The case of linear demand and cost functions in Section X satisfies all of the assumptions made in Section 2.

3 Assumptions on Beliefs and Behavior

Let us begin by making the standard assumption that firms are rational, firms believe other firms are rational, and so forth.

Assumption A1: A firm is rational in the sense of choosing a strategy to maximize the present value of its expected profit stream given its beliefs on other firms' strategies, and rationality is common knowledge.

As the focus here is on tacit collusion - in which case firms do not engage in express communication - assuming firms have accurate beliefs as to their rivals' strategies is problematic. The approach taken is to weaken the equilibrium assumption that the strategy profile is common knowledge by instead assuming that only some properties of firms' strategies are common knowledge or, alternatively stated, it is common knowledge that firms' strategies lie in a subset of the strategy space.

Definition: The strategy of firm i has the *price matching plus* (PMP) property if:

$$p_i^t \begin{cases} \in \left\{\max\left\{p_1^{t-1},...,p_n^{t-1}\right\},...,\overline{p}\right\} & \text{if } p_j^\tau \geq \min\left\{\max\left\{p_1^{\tau-1},...,p_n^{\tau-1}\right\},\overline{p}\right\}\ \forall j, \forall \tau \leq t-1 \\ & \text{and } \max\left\{p_1^{t-1},...,p_n^{t-1}\right\} < \overline{p} \\ \\ = \overline{p} & \text{if } p_j^\tau \geq \min\left\{\max\left\{p_1^{\tau-1},...,p_n^{\tau-1}\right\},\overline{p}\right\}\ \forall j, \forall \tau \leq t-1 \\ & \text{and } \max\left\{p_1^{t-1},...,p_n^{t-1}\right\} \geq \overline{p} \\ \\ = p^N & \text{if not } p_j^\tau \geq \min\left\{\max\left\{p_1^{\tau-1},...,p_n^{\tau-1}\right\},\overline{p}\right\} \\ & \forall j, \forall \tau \leq t-1 \end{cases}$$

First note that matching a price increase means setting $p_i^t = \max\left\{p_1^{t-1},...,p_n^{t-1}\right\}$. Thus, as of period t, price increases have always been *at least* matched when $p_j^\tau \geq \max\left\{p_1^{\tau-1},...,p_n^{\tau-1}\right\}\ \forall j, \forall \tau \leq t-1$. Price matching plus behavior is subject to the caveat that firms are only expected to match price as high as $\overline{p}$; thus, firms are not expected to follow an excessively high price increase. Firms have then been complying with this modified price matching when $p_j^\tau \geq \min\left\{\max\left\{p_1^{\tau-1},...,p_n^{\tau-1}\right\},\overline{p}\right\}\ \forall j, \forall \tau \leq t-1$. In that event, the PMP property has a firm price in period t at least as high as $\max\left\{p_1^{t-1},...,p_n^{t-1}\right\}$ with the caveat of not pricing in excess of $\overline{p}$. Finally, if any firm should fail to act in a manner consistent with the previously described behavior then firms revert to pricing at the non-collusive price p^N thereafter. A strategy satisfying the PMP property will be referred to as being PMP-compatible.[16]

Assumption A2: It is common knowledge that a firm's strategy satisfies the price matching plus property.

By A2, there is a "meeting of minds" among firms that: 1) price increases are at least matched as long as past price increases have been at least matched in the past; 2) price increases will be followed only as high as $\overline{p}$; and 3) departure from this price matching behavior results in reversion to non-collusive pricing. I now want to argue how mutual understanding among firms that their strategies have these properties could plausibly be achieved without express communication of the sort associated with explicit collusion. Let us take them in reverse order.

Consider the assumption that it is common knowledge that failure to at least match price increases (up to a maximum price of $\overline{p}$) results in non-collusive pricing thereafter. It is useful to break this assumption into two parts. First, a departure from price matching results in some form of punishment. Second, that punishment is the grim punishment. The second condition is unimportant because results are unchanged if the punishment is at least as severe as the grim punishment, and if it is less severe then results are continuous in the punishment payoff in which case results are robust and the paper's conclusions remain intact. The first condition seems natural in that firms are seeking to collude through the mutual understanding that price increase will be at least matched. Thus, departure from that behavior ought to induce either a breakdown of collusion or a more calculated punishment. The restrictiveness of the condition is really that it is common knowledge as to the low continuation payoff that ensues after a departure from price-matching plus behavior. Though, as stated above, any punishment would work for our analysis, a coordinated punishment - as opposed to simply a breakdown of collusion and a return to competition - would have to be justified in terms of how the punishment is common knowledge without express communication. In light of the focus on motivating the extent of mutual understanding among firms, I feel it is more consistent with the spirit of the enterprise to simply assume a departure results in a breakdown of collusion rather than a coordinated punishment, even though results are not dependent on that specification.

In specifying how firms respond to this incongruity between beliefs and behavior - they expected a price increase to be at least matched and it was not - let me provide a second justification by drawing on Lewis (1969) to argue that the competitive solution is

[16] The potential role of price matching here is to coordinate on a collusive outcome. Price matching has also been explored as a form of punishment; see Lu and Wright (2010) and Garrod (2012). Some papers exploring price leadership as a collusive equilibrium include Rotemberg and Saloner (1990) and Mouraviev and Rey (2011).

salient. Lewis (1969) defines a salient outcome as "one that stands out from the rest by its uniqueness in some conspicuous respect"[17] and that precedence is one source of saliency: "We may tend to repeat the action that succeeded before if we have no strong reason to do otherwise."[18] Cubitt and Sugden (2003) stress the latter qualifier and note that "precedent allows the individual to make inductive inferences in which she has *some* confidence, but which are overridden whenever deductive analysis points clearly in a different direction."[19] With this perspective in mind, the movement from competition to collusion can be seen as a shift from inductive to deductive reasoning. Firms have been competing and, by induction, they would expect to continue to do so. However, through some other coordinating event, firms supplant inductive inferences with deductive reasoning so that a common expectation of competition is replaced with a common expectation of price matching. With this as a backdrop, my claim is that a subsequent departure in behavior from price matching implies a breakdown in the efficacy of deductive reasoning, in response to which firms revert to the original inductive analysis and therefore the competitive solution. Here I am appealing to the view that firms will "tend to pick the salient as a last resort."[20] The saliency of the competitive solution emanates from it being the most recent outcome (prior to the current episode of collusion) that was common knowledge to firms.[21]

The second feature of the PMP property to consider is that a firm does not price in excess of $\overline{p}$, which means that it will follow price increases only as high as $\overline{p}$ and, as a price leader, will not raise price beyond $\overline{p}$. It is surely compelling for a firm to have some upper

[17] Lewis, (1969), p. 35.

[18] Lewis, (1969), p. 37.

[19] Cubitt and Sugden (2003), p. 196. Also see Sugden (2011).

[20] Lewis, (1969), p. 35.

[21] There are two implicit assumptions in the preceding argument that warrant discussion. First, the saliency of the competitive solution relies on it prevailing prior to this episode of tacit collusion. However, that is not essential for the paper's main results. If some other behavior described the pre-collusion setting then that behavior can be assumed instead. What *is* critical is that how firms respond to the departure from price matching is common knowledge and the associated continuation payoff is lower than if firms had abided by the PMP property. A second assumption, which figures prominently in discussions of saliency (such as in Lewis, 1969), is that the current post-collusion situation is sufficiently similar to the pre-collusion situation so that induction on the latter is compelling. It is well-recognized that

> no two interactions are exactly alike. Any two real-world interactions will differ in matters of detail, quite apart from the inescapable fact that "previous" and "current" interactions occur at different points in time. Thus, the idea of "repeating what was done in previous instances of the game" is not well-defined. Precedent has to depend on analogy: to follow precedent in the presence instance is to behave in a way that is *analogous with* behaviour in past instances. ... Inductive inference is possible only because a very small subset of the set of possible patterns is privileged. [Cubitt and Sugden (2003), pp. 196-7.]

The post-collusion scenario most notably differs from the pre-collusion scenario in that the former was preceded by an episode of collusion, while the latter was (probably) not. Though this difference could disrupt the saliency of the pre-collusion outcome when it comes to responding to a departure from the PMP property, it is reasonable for its saliency to remain intact which is the presumption made here.

bound to how high it will price; it would be nonsensical to raise it to where demand is zero or even above the monopoly price. However, it is important to emphasize that this assumption goes further in that this upper bound is presumed common to firms, and is commonly known. It is easy to show that A1-A2 imply that this upper bound on price matching cannot be less than the static Nash equilibrium price and cannot exceed the highest equilibrium price (with the grim punishment). Proofs are in Appendix B.

Lemma 1 *A1-A2 imply* $\overline{p} \in \left\{p^N, ..., \widetilde{p}\right\}$.

The results of Section 4 require no further restriction on $\overline{p}$, while the results of Section 5 are unchanged as long as $\overline{p}$ is not too much less than $\widetilde{p}$.[22] However, it'll make for an easier presentation if it is assumed that the upper bound on price matching is the highest level consistent with it being common knowledge that firms are rational and use PMP-compatible strategies.

Assumption A3: $\overline{p} = \widetilde{p}$.

One could imagine that $\widetilde{p}$ might be a focal point given that it is the *highest* feasible value of $\overline{p}$ and that higher values are weakly more profitable for all firms. S^{PMP} will denote the subset of a firm's strategy space that satisfies the PMP property with $\overline{p} = \widetilde{p}$. Note that A2-A3 can be rephrased as: It is common knowledge that the strategy profile lies in S^{PMP}.[23]

Finally, we arrive at the central element of A2 which is that it is common knowledge that price increases will at least be matched. How could non-express communication produce such mutual understanding? To begin, price leadership and price matching is a common form of tacit collusion; see Markham (1951) and Scherer (1980, Chapter 6) for several examples. Thus, firms might well recognize it as a possible strategy in which case it need not take much in terms of communication to achieve mutual understanding regarding at least price matching. One message that could trigger it is for a firm to publicly announce: "I will not undercut the prices of my rivals." This announcement is not proposing that firms engage in coordinated price increases but only not to reduce price and to follow price increases. There are well-documented cases in which a firm does announce elements of its pricing strategy with an apparent attempt to collude. In the truck rental market,

[22] Specifically, $\overline{p}$ must be at least as high as p^*.

[23] One might ask why it isn't reasonable to suppose it is common knowledge that firms use some typical punishment strategy and price immediately at $\widetilde{p}$. Let us consider firm i using a grim trigger strategy: i) price at p^N in periods $1, ..., t_i - 1$; ii) price at $\widetilde{p}$ in period t_i; ii) price at $\widetilde{p}$ in period $t\,(> t_i)$ if all firms priced at $\widetilde{p}$ in periods $t = t_i, ..., t-1$; and iv) price at p^N in period $t\,(> t_i)$ otherwise. To begin, it is a strong assumption for it to be common knowledge that firms use a strategy of that form but even that is not enough to ensure collusive pricing. Collusive prices are charged in the long-run if and only if $t_1 = \cdots = t_n$ so that all firms begin collusive pricing in the same period. There is nothing to make period 1 a focal point as that is just where the analysis begins. It is then an open question whether reasonable assumptions on mutual understanding can be made to result in firms pricing at $\widetilde{p}$.

U-Haul CEO Joe Schoen - during an earnings conference call with analysts - commented that it was "very much trying to function as a price leader" and was seeking to discourage competitor Budget from undercutting price: "As a strategy I believe the Budget Truck Rental Company is trying to take U-Haul's price in every single corridor and drop it 1 or 2 or 3 or 4 ... percent so that they can just price off of us but down. Does that make sense?"[24] The Federal Trade Commission argued under Section 5 ("invitation to collude") that U-Haul was trying to create the understanding among firms that U-Haul is the price leader and that Budget should follow U-Haul's price increases. That is actually more mutual understanding than is assumed here which is only common knowledge about price matching, but not about price leadership. In conclusion, it would seem at least plausible that non-express communication - such as through a public announcement - could produce mutual understanding among firms that price increases are not to be undercut.

4 An Upper Bound on the Collusive Price

The first step in the analysis is to show that A1-A3 are compatible so that if a firm believes its rivals use PMP-compatible strategies then it is optimal for that firm to use a PMP-compatible strategy. This is Lemma 2. Theorem 3 presents the main result of this section which is to provide an upper bound on price. Theorem 4 then shows that this upper bound is strictly below a benchmark equilibrium price.

Recall that S^{PMP} is the subset of a firm's strategy space that satisfies A2-A3. Lemma 2 shows that if firm i's beliefs over other firms' strategies have support in S^{PMP} then firm i's best reply lies in S^{PMP}; that is, the set S^{PMP} is closed under the best reply operator.

Lemma 2 *Assume* $\max\left\{p_1^0,...,p_n^0\right\} \geq p^N$*. If firm i's beliefs over other firms' strategies have support in S^{PMP} then, for all histories, firm i's best reply lies in S^{PMP}.*

Theorem 3 shows that if it is common knowledge that firms are rational and that firms use strategies satisfying the PMP property then price is bounded above by (approximately) p^*.[25]

Theorem 3 *Assume A1-A3.* $\left\{\left(p_1^t,...,p_n^t\right)\right\}_{t=1}^{\infty}$ *is weakly increasing over time and there exists finite T such that $p_1^t = \cdots = p_n^t = \widehat{p}\ \forall t \geq T$ where $\widehat{p} \leq p^* + \varepsilon$.*

In explaining the basis for Theorem 3, first note that while A2 leaves unspecified whether some firm will initiate a price increase, it is fully consistent with A1-A3 for a firm to be a price leader. For example, if a firm believed other firms would not raise price

[24] *Matter of U-Haul Int'l Inc. and AMERCO* (FTC File No. 081-0157, July 10, 2010).

[25] Theorems 3 and 5 require that the initial price vector is not too high: $\left(p_1^0,...,p_n^0\right) \in \left\{p^N,...,p^*+\varepsilon\right\}^n$. The most natural assumption is $\left(p_1^0,...,p_n^0\right) = \left(p^N,...,p^N\right)$ so that firms are initially competing.

then it would be rational for it to increase price (as long as the current price is not too high). The issue is how far would it go in raising price. If the firm expected that its price increase would only be met and never exceeded by a rival (for example, rivals are believed to only match price increases) then it would not want to raise price above (approximately) p^*.[26] Recall that p^* is the price at which a firm, if it were to raise price from p^* to any higher level (call it p'), it would lose more in current profit (because of lower demand from pricing above the level p^* set by its rivals) than it would gain in future profit (from all firms pricing at p'). Thus, a firm that believed its rivals would never initiate price increases would not raise price beyond p^*. However, a firm might be willing to lead a price increase above p^* *if* it believed it would induce a rival to further increase price; for example, if the firm believed that firms would take turns leading price increases. The essence of the proof of Theorem 3 is showing that cannot happen.

By A2-A3, a firm will never price above $\widetilde{p}$. If $\widetilde{p} > p^*$ then it furthermore means that a firm would never raise price *to* $\widetilde{p}$ since such a price increase would only induce its rivals to match that price; it would not induce them to further raise price. Thus, if a firm is rational and believes the other firms use PMP-compatible strategies then it will not raise price to $\widetilde{p}$. This puts an upper bound on price of $\widetilde{p} - \varepsilon$. We next build on that result to argue that $\widetilde{p} - 2\varepsilon$ is an upper bound on price. Given it is common knowledge that firms are rational and firms use PMP-compatible strategies, firm i then believes firm $j\,(\neq i)$ is rational and also that firm j believes firm h uses a PMP-compatible strategy (for all $h \neq j$); hence, firm i knows that firm j will not raise price to $\widetilde{p}$. This means that firm i knows that if it raises price to $\widetilde{p} - \varepsilon$ that this price increase will only be matched and not exceeded, which then makes a price increase to $\widetilde{p} - \varepsilon$ unprofitable (as long as $\widetilde{p} - \varepsilon > p^*$). Given that all firms are not willing to raise price to $\widetilde{p} - \varepsilon$ then $\widetilde{p} - 2\varepsilon$ is an upper bound on price. The proof is completed by induction - with each step using another layer of common knowledge - to end up with the conclusion that a firm would never raise price to a level exceeding p^*. Hence, price is bounded above by p^*.

In deriving this upper bound, the punishment for deviation from (at least) matching price is reversion to a stage game Nash equilibrium. With the full mutual understanding associated with equilibrium, such a punishment could sustain a price as high as $\widetilde{p}$. With the more limited mutual understanding that the strategy profile lies in S^{PMP}, price can only rise as high as p^* which the next result shows is strictly less than $\widetilde{p}$.

Theorem 4 $p^* \in \left(p^N, \widetilde{p}\right)$.

Recall that p^* is the price at which the reduction in current profit from a marginal increase in a firm's price is exactly equal in magnitude to the rise in the present value of the future profit stream when that higher price is matched by all firms for the infinite future. Equivalently, p^* is the price at which the increase in current profit from a marginal

[26] In discussing results, I will generally refer to the upper bound as p^* rather than $p^* + \varepsilon$ since ε is presumed to be small.

decrease in price to $p^* - \varepsilon$ is exactly equal in magnitude to the *fall* in the present value of the future profit stream *when the firm's rivals lower price to* $p^* - \varepsilon$ (when ε is small). In comparison, $\widetilde{p}$ is the price for a firm at which the increase in current profit from a marginal decrease in price is exactly equal in magnitude to the fall in the present value of the future profit stream *when the firm's rivals lower price to* p^N.[27] Given that the punishment is more severe in the latter case, it follows that the maximal sustainable price is higher: $\widetilde{p} > p^*$.

With common knowledge that the strategy profile lies in S^{PMP}, the steady-state price is bounded above by p^* even though higher prices are sustainable. In other words, if firms started at a price of $\widetilde{p}$ then such a price would persist under A1-A3. But if firms start with prices below p^*, such as at the non-collusive price p^N, then prices will not go beyond p^*, even though higher prices are sustainable. The obstacle is that it is not in the interests of any firm to lead a price increase beyond p^*. Thus, with only mutual understanding that the strategy profile lies in S^{PMP}, what is constraining how high price can go is the trade-off a firm faces when it acts as a price leader: It foregoes current demand and profit in exchange for higher future profit from its rivals having raised their prices to match its price increase. This is to be contrasted with equilibrium where what limits how high price goes is whether the price is stable or whether instead a firm finds it profitable to undercut it. In a sense, coordination comes for free with equilibrium. Limited mutual understanding makes the coordination on price (through price leadership) the constraining factor, not the stability of the price which is eventually coordinated upon. Finally, note that Theorems 3 and 4 are robust to the form of the punishment. p^* is independent of the punishment and, given another punishment, $\widetilde{p}$ would just be the highest sustainable price with that punishment. In particular, if the punishment is at least as severe as the grim punishment then Theorems 3 and 4 are unchanged.[28]

In concluding, let me discuss the role of the finiteness of the price set. p^* is the highest price to which a firm will raise price if it can only anticipate that other firms will match its price. Thus, a firm is willing to take the lead and price above p^* only if, by doing so, it induces a rival to enact further price increases. Since no firm will price above $\widetilde{p}$ then raising price to $\widetilde{p}$ cannot induce rivals to lead future price increases. Thus, a firm will not raise price to a level beyond $\widetilde{p} - \varepsilon$, which means $\widetilde{p} - \varepsilon$ is an upper bound on price. This argument works iteratively to ultimately conclude that p^* is (approximately) an upper bound on price. The finiteness of price is critical in this proof strategy for it allows $\widetilde{p} - \varepsilon$ to be well-defined. However, even with an infinite price set, it is still the case that a necessary condition for a firm to lead and raise price above p^* is that it will induce a rival to enact

[27] That is, $\widetilde{p}$ is the highest price for which a firm incentive compatibility constraint, (1), holds. For this discussion, suppose $\widetilde{p} < p^M$.

[28] The property that price falls short of the equilibrium price is similar to a finding in Lockwood and Thomas (2002). They consider a multi-player infinite horizon setting in which actions are irreversible; that is, the minimum element of player i's period t action set is the action the player selected in period $t-1$. The equilibrium path is uniformly bounded above by the first-best for basically the same reasons that price converges to a value less than $\widetilde{p}$. I want to thank Thomas Mariotti for pointing out this connection.

further price increases. As that must always be true then, if the limit price exceeds p^* when there is an infinite price set, price cannot converge in finite time. But since it is still the case that $\widetilde{p}$ is an upper bound on price, the price increases must then get arbitrarily small; eventually, each successive price increase will bring forth a smaller future price increase by a rival. I am not arguing that this argument will prevent Theorem 3 from extending to the infinite price set but rather that it is the only argument that could possibly do so. Therefore, either Theorem 3 extends to when the price set is infinite or, if it does not, it implies a not very credible price path with never-ending price increases that eventually become arbitrarily small. The oddity of such a price path is an artifact of assuming an infinite set of prices when, in fact, the set of prices is finite.

5 Example: Price Can be Competitive or Supracompetitive

By Theorem 3, if it is common knowledge that firms are rational and that their strategies satisfy the PMP property then price is bounded above by p^*. But is p^* the least upper bound? And is there a lower bound on price exceeding p^N? The purpose of the current section is to show, by way of example, that it is consistent with A1-A3 for price to converge to p^* and also fail to rise above p^N.Thus, a tighter result than Theorem 3 will require additional assumptions, which is taken up in the next section.

For the duopoly case, suppose the price set is composed of just three elements, $\left\{p^N, p', p^M\right\}$, and $p' \equiv \left(p^M + p^N\right)/2$. Assume δ is sufficiently close to one which has the implication: $p^* = \widetilde{p} = p^M$.[29] Consider the following pair of functions which map from the lagged maximum price to current price:

$$\begin{aligned} S^L\left(p^N\right) &= p', S^L\left(p'\right) = p^M, S^L\left(p^M\right) = p^M \\ S^F\left(p^N\right) &= p^N, S^F\left(p'\right) = p', S^F\left(p^M\right) = p^M \end{aligned} \tag{5}$$

S^L (where L denotes "leader") has a firm raise price to p' when the lagged maximum price is p^N, to p^M when the lagged maximum price is p', and price at p^M when the lagged maximum price is p^M. S^F (where F denotes "follower") has a firm's price equal the lagged maximum price. When S^L (or S^F) is referred to as a strategy, it is meant that the specification in (5) applies when both firms have priced at least as high as the previous period's maximum price in all past periods, and otherwise a firm prices at p^N. Thus, these strategies satisfy the PMP property. It is shown in Appendix C that $\left(S^L, S^F\right)$ is a subgame perfect equilibrium when $\delta \simeq 1$ and

$$\pi\left(p', p^N\right) + \pi\left(p^M, p'\right) > \pi\left(p^M, p^N\right) + \pi\left(p^M, p^M\right). \tag{6}$$

[29] In this example, as opposed to elsewhere in the paper, p^* is defined for when the feasible price set is $\left\{p^N, p', p^M\right\}$ rather than $\Re_+$. This, however, is a good approximation when $\delta \simeq 1$ as then $p^* \simeq p^M$ when the price set is $\Re_+$. Of course, if $\delta \simeq 1$ then $\widetilde{p} = p^M$ whether the price set is $\left\{p^N, p', p^M\right\}$ or $\Re_+$.

It is also shown that (6) holds for the case of linear demand and cost when products are sufficiently differentiated and/or cost is sufficiently low.

If $\left(S^L, S^F\right)$ is a subgame perfect equilibrium then it immediately follows that both S^L and S^F are rationalizable strategies (that is, consistent with A1). Thus, a price path of $\left(\left(p', p^N\right), \left(p^M, p'\right) \left(p^M, p^M\right), ...\right)$, with a steady-state price of p^M, is consistent with A1-A3. It is achieved by having firm 1 use S^L based upon the belief that firm 2 use S^F, and firm 2 uses S^F based upon the belief that firm 1 uses S^L; and these beliefs are consistent with A1. However, it is also the case that a price path of $\left(\left(p^N, p^N\right), ...\right)$ is consistent with A1-A3. It occurs when each firm uses S^F based upon the belief that the other firm uses S^L, and these beliefs are also consistent with A1. Thus, A1-A3 could produce supracompetitive prices or competitive prices.

6 Learning and the Steady State Price

Thus far, assumptions have been made on a firm's beliefs regarding price matching - specifically, other firms will at least match price up to a maximum level of $\widetilde{p}$ - and regarding what happens when behavior is contrary to such price matching - firms revert to competitive prices. Common knowledge of those properties along with rationality is sufficient to place an upper bound on price of (approximately) p^*. What is not yet clear is whether price is assured of reaching p^* or whether supracompetitive pricing would emerge at all or instead price is mired at the competitive level. Though firms have mutual understanding about price matching, they lack common knowledge about price leadership and some firm must take the the initiative if prices are to rise above the competitive level. In this section, sufficient conditions are provided for price leadership to emerge.

In some markets, a particular firm may be the salient leader by virtue of its size or access to information (what is referred to as barometric price leadership; see, for example, Cooper, 1997). However, there is no such presumption here. Furthermore, firms prefer to be price followers than price leaders because it is costly to initiate a price hike as a firm will lose demand prior to its price being matched.[30] Hence, each firm would prefer another firm to take the lead in raising price. Lacking common knowledge as to who will lead, this incentive to wait and follow could indeed prevent supracompetitive prices from ever emerging. It is possible to show, by way of example, that firms always pricing at

[30] Wang (2009) provides indirect evidence of the costliness of price leadership. In a retail gasoline market in Perth, Australia, Shell was the price leader over 85% of the time until a new law increased the cost of price leadership, after which the three large firms - BP, Caltex, and Shell - much more evenly shared the role of price leader. The law specified that every gasoline station was to notify the government by 2pm of its next day's retail prices, and to post prices on its price board at the start of the next day *for a duration of at least 24 hours*. Hence, a firm which led in price could not expect its rivals to match its price until the subsequent day. The difference between price being matched in an hour and in a day is actually quite significant given the high elasticity of firm demand in the retail gasoline market. For the Quebec City gasoline market, Clark and Houde (2011, p. 20) find that "a station that posts a price more than 2 cents above the minimum price in the city loses between 35% and 50% of its daily volume."

p^N is consistent with A1-A3 and firms pricing at p^* in finite time (and thereafter) is also consistent with A1-A3. The latter price path arises if there is a firm that believes all other firms will never lead on price in which case it is optimal for that firm to take the initiative and raise price (and those beliefs are consistent with A1-A3). The former price path arises if each firm believes some other firm will lead on price, in which case it is optimal for it to wait in anticipation and simply match the price increase; but if all firms believe that then none will lead and price remains at the competitive level. Thus, a tighter result than Theorem 3 will require additional assumptions.

As just described, competitive pricing can occur - in spite of price matching being common knowledge - because firms' beliefs are inconsistent; each believes someone else will lead on price and each takes the role of follower with no price leader emerging. However, if firms could learn over time about rivals' strategies then perhaps a firm would learn that other firms are not likely to lead which would could induce it to raise price. As it seems reasonable to allow firms to update their beliefs over other firms' strategies as pricing behavior is observed, this section explores the implications of such learning. In doing so, I will draw on a seminal result of Kalai and Lehrer (1993) regarding the learning of strategies in an infinitely repeated game.

With technical details being available in Kalai and Lehrer (1993), let me convey in the simplest terms what assumptions are needed to use their result. Assume each firm has prior beliefs on the true strategy profile.[31] From these beliefs are generated beliefs on infinite price paths, Λ^∞ where $\Lambda \equiv \{0, \varepsilon, ..., P\}^n$ and P is some upper bound on the price set so that the price set is finite. A4 is referred to as the "Grain of Truth" assumption.

Assumption A4: Each firm's prior beliefs on infinite price paths assigns positive probability to the true price path.

Equilibrium requires that a firm's beliefs put probability one on the true strategy profile and thus a firm accurately predicts the true path of play. A4 only requires that positive probability be attached to the true path of play. Though vastly weaker than equilibrium, A4 is not a weak assumption given there are an infinite number of possible paths.[32] It is next assumed that firms are Bayesian learners.

Assumption A5: In response to the realized price path, a firm updates its beliefs as to the future price path using Bayes Rule.

What transpires is that a firm starts with prior beliefs about the strategy profile and the price path. As of period t, it will have observed the first $t-1$ periods of the true

[31] Kalai and Lehrer (1993) allow for strategies in which players randomize. I will focus on when the true strategy profile is a pure strategy profile. However, given that their analysis models a player's beliefs on other players' strategies as a point belief, it is important that beliefs are over mixed (behavior) strategies.

[32] It is also worth noting that A4 is weaker than assuming positive probability is assigned to the true strategy profile since there may be many strategy profiles that produce the same realized price path.

underlying price path and then update its beliefs as to the strategies other firms are using and what it implies about the future price path. The main result of Kalai and Lehrer (1993) is that if players are Bayesian learners and their prior beliefs assign positive probability to the true path then players will eventually have beliefs over the future path of play that are "close" to the true future path of play. Thus, players will come to almost learn the true path and, as they are rational, play will approximate a Nash equilibrium. Of course, there are many Nash equilibria in almost any infinitely repeated game so there are no further predictions from this result about behavior.[33]

What will allow us to derive more precise predictions than in Kalai and Lehrer (1993) is the additional structure of A1-A3.[34] A1-A3 imply that firms' prior beliefs over the true strategy profile have support which is the set of rationalizable strategies based on the initial strategy set S^{PMP}. By Lemma 1, we know that the set of rationalizable strategies (drawn from the entire strategy space) on S^{PMP} is a subset of S^{PMP}. By Bernheim (1984), the set of rationalizable strategies based on S^{PMP} is non-empty; let us denote it $Rat\left(S^{PMP}\right)$. Under the assumptions of Kalai and Lehrer (1993), firms would be learning over the entire strategy space, while here firms are learning over the more restricted space of $Rat\left(S^{PMP}\right)$.

A1-A4 put structure on firms' beliefs at the start of play, while A5 describes how firms modify their beliefs over the course of play. Under those assumptions, Theorem 5 shows that firms are very likely to eventually price close to the supracompetitive price p^*.

Theorem 5 *Assume A1-A5. For all $\eta > 0$, there exists T such that, with probability of at least $1 - \eta$, $p_1^t = \cdots = p_n^t = \widehat{p}$ $\forall t \geq T$ where $\widehat{p} \in \{p^* - \varepsilon, p^*, p^* + \varepsilon\}$.*

Learning implies price leadership will eventually emerge even though only price matching - and not price leadership - is common knowledge. What A4-A5 avoid is firms endlessly miscoordinating whereby each firm does not raise price on the anticipation that some other firm will raise price. As long as positive probability is given to the true price path, Bayesian updating results in some firm eventually raising price and that will continue to occur until price reaches p^*.

7 Linear Example: Explicit vs. Tacit Collusion

In this section, we use the preceding theory along with equilibrium theory to draw comparisons between tacit and explicit collusion. The theory of tacit collusion is that which has been characterized in this paper. Firms have mutual understanding of price matching

[33] Note that collusive prices need not prevail even if firms use collusive strategies and eventually learn the future price path. Consider the grim trigger strategies in footnote 24. Unless $t_1 = \cdots = t_n$, the long-run price is the static Nash equilibrium price p^N. Though firms are using collusive strategies and eventually learn the future price path, they are in the punishment phase by the time that path is learned.

[34] They also assume that a player knows only its own payoffs, while the more standard assumption is made here that the game is common knowledge.

which results in a steady-state price of p^*. In specifying explicit collusion, I will assume that firms, if they could expressly communicate, would agree to simultaneously raise price to the best equilibrium price of $\widetilde{p}$. The steady-state price differential between explicit and tacit collusion will then be measured by $\widetilde{p} - p^*$. This measure does implicitly assume that the same punishment is deployed with explicit collusion as with tacit collusion which is likely not to be the case since presumably more punishments are available to firms if they can coordinate through express communication.[35] It is then best to think of $\widetilde{p} - p^*$ as isolating the effect of the method of coordination - price leadership and matching versus express communication - while controlling for the mechanism that sustains the collusive outcome.

Assuming linear demand and cost functions, a firm's profit function is

$$\pi\left(p_i, \mathbf{p}_{-i}\right) = \left(a - bp_i + d\left(\frac{1}{n-1}\right)\sum_{j\neq i} p_j\right)\left(p_i - c\right), \text{where } a > bc > 0, b > d > 0.$$

The non-collusive stage game Nash equilibrium price and the joint profit-maximizing price are, respectively,

$$p^N = \frac{a+bc}{2b-d},\ p^M = \frac{a+(b-d)\,c}{2\,(b-d)}.$$

The price matching best reply function is

$$\phi\left(\mathbf{p}_{-i}\right) = \frac{a+(b-\delta d)\,c}{2\,(b-\delta d)} + \left(\frac{(1-\delta)\,d}{2\,(b-\delta d)}\right)\left(\frac{1}{n-1}\right)\sum_{j\neq i} p_j,$$

from which we can derive p^*:

$$p^* = \phi\left(p^*, ..., p^*\right) \Rightarrow p^* = \frac{a+(b-\delta d)\,c}{2\,(b-\delta d)} + \left(\frac{(1-\delta)\,d}{2\,(b-\delta d)}\right)p^* \Rightarrow$$

$$p^* = \frac{a+(b-\delta d)\,c}{2b-(1+\delta)\,d}$$

p^* is an increasing convex function of the discount factor:

$$\frac{\partial p^*}{\partial \delta} = \frac{d\,(a-(b-d)\,c)}{(2b-(1+\delta)\,d)^2} > 0,\ \frac{\partial^2 p^*}{\partial \delta^2} = \frac{d^2\,(a-(b-d)\,c)}{(2b-(1+\delta)\,d)^3} > 0.$$

It is straightforward to derive price under explicit collusion by solving (2):

$$\widetilde{p} = \min\left\{\frac{4ab^2+ad^2+4b^3c+bcd^2-4b^2cd-ad^2\delta-4abd+4abd\delta+3bcd^2\delta-4b^2cd\delta}{6bd^2-12b^2d+d^3\delta+8b^3-d^3-2bd^2\delta}, \frac{a+(b-d)\,c}{2\,(b-d)}\right\}.$$

[35] Though there is also the argument that express communication allows for re-negotiation which can weaken punishments; see McCutcheon (1997).

If $\widetilde{p} < p^M$ then $\widetilde{p}$ is also an increasing convex function of the discount factor:

$$\frac{\partial \widetilde{p}}{\partial \delta} = \frac{4bd\,(a - bc + cd)\,(2b - d)}{\left(4b\,(b-d) + d^2\,(1-\delta)\right)^2} > 0,\ \frac{\partial^2 \widetilde{p}}{\partial \delta^2} = \frac{8bd^3\,(a - (b-d)\,c)\,(2b-d)}{\left(4b\,(b-d) + d^2\,(1-\delta)\right)^3} > 0.$$

Define $\delta^* \in (0,1)$ by: if $\delta < (>)\,\delta^*$ then $\widetilde{p}\,(\delta) < (=)\,p^M$.

The next result shows that $\widetilde{p} - p^*$ is increasing in δ when δ is low - so that a higher discount factor exacerbates the cost from coordinating through price leadership - but is decreasing in δ when δ is high.

Theorem 6 *Assume linear demand and cost functions. Then* $\frac{\partial(\widetilde{p} - p^*)}{\partial \delta} > (<)\,0$ *as* $\delta < (>)\,\delta^*$.

Illustrating this result for $a = 1, b = 1, d = .9, c = 0$, Figures 1 and 2 compare price under explicit collusion and tacit collusion, and how this comparison varies with the discount factor. To begin, the forces determining the steady-state price depends on the method of coordination. With tacit collusion through price leadership and matching, a firm that leads on price trades off lower current profit - as its demand falls by raising its price - and higher future profit - as rivals subsequently match that price. With a current loss and a future gain, a firm is more willing to engage in price leadership when its discount factor is higher; hence p^* is increasing in δ. It is then the profitability of leading that determines the steady-state price under tacit collusion. By comparison, explicit collusion allows firms to simultaneously raise price so there is no price leader and thus no current loss incurred; what constrains the collusive price is sustainability and, by the usual argument, $\widetilde{p}$ is increasing in δ (when $\widetilde{p} < p^M$).[36] In sum, the steady-state price under explicit collusion is determined by the profitability of not undercutting that price, while the profitability of leading a price increase is what drives the steady-state price under tacit collusion.

When the discount factor is low, price under explicit collusion is near the competitive price because only prices close to the competitive price are sustainable. Price under tacit collusion is also near the competitive price because only for small price increases above the competitive price is the current loss exceeded by the future gain, and that is because the first-order current loss is zero when all firms price at p^N. Hence, when the discount factor is low, the type of coordination mechanism makes little difference. When the discount factor is high, the collusive price is near the joint profit maximum under either explicit or tacit collusion. Given firms' long-run view, high prices are sustainable and firms are strongly inclined to lead price increases. It is when the discount factor is moderate that the coordination mechanism makes the biggest difference. Firms are able to sustain high prices but no firm is willing to act as a price leader to get price to that level. For the numerical example in Figure 1 with $\delta = .7$, the competitive price is .91 and explicit collusion results

[36] Of course, sustainability is also an issue with tacit collusion. However, since $\widetilde{p} > p^*$ then incentive compatibility constraints are not binding under tacit collusion (at least when firms are not too asymmetric).

in a price of 4.47 which is close to the joint profit maximum of 5.00; however, tacit collusion with price leadership results in a price of only 2.13. It is when firms are moderately patient that the means of coordination has the most significant impact on the steady-state price.

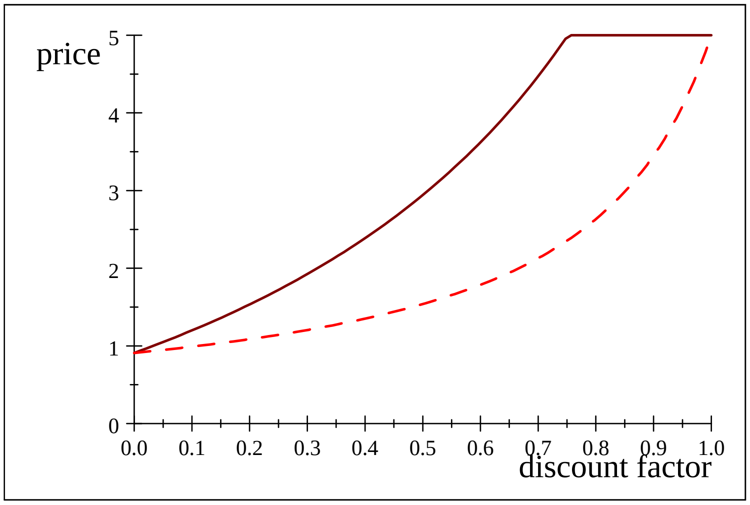

Figure 1: Price under explicit collusion (solid line) and tacit collusion (dashed line)

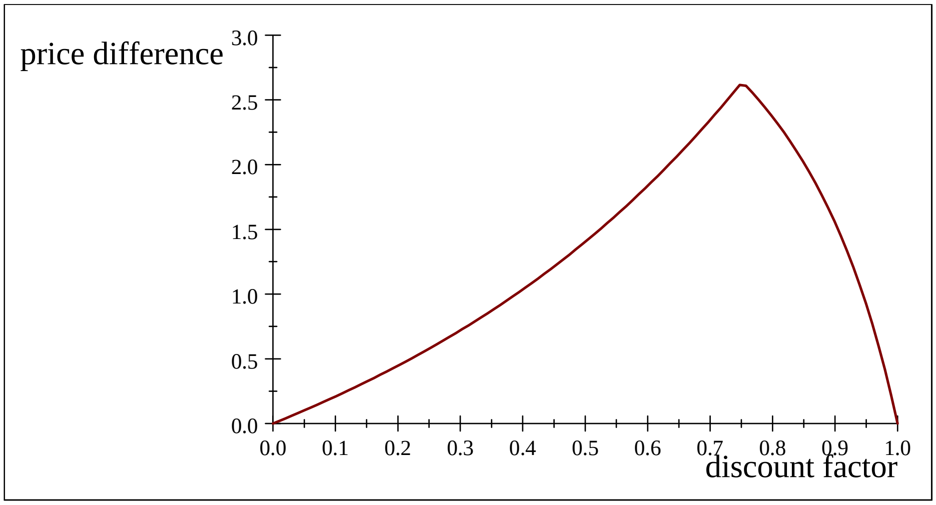

Figure 2: Price difference between explicit and tacit collusion, $\widetilde{p} - p^*$

This insight may also have implications for when cartel formation (that is, explicit collusion) is most likely. When the discount factor is sufficiently low, cartel formation is unlikely because the rise in price is small (whether firms, in the absence of cartel formation, would compete or tacitly collude). When the discount factor is sufficiently high, cartel formation is not likely either if the alternative is tacit collusion because tacit collusion does nearly as well.[37] It is when the discount factor is moderate that cartel formation is most attractive because it results in a much higher price than if firms either competed or tacitly colluded. While the attractiveness of tacit collusion (compared to competition) is always greater when the discount factor is higher, that is not the case with the attractiveness of explicit collusion (compared to tacit collusion). What are conditions promoting collusion can then depend on whether collusion is explicit or tacit.

8 Description of General Framework

The focus of this paper has been to derive the implications of firms having mutual understanding that price increases will at least be matched. That objective was originally cast within the broader objective of understanding the relationship between mutual understanding among firms and market outcomes, which was motivated by the desire to identify the types of "meeting of minds" that produce collusive outcomes. While the analysis has focused on price matching, the approach is quite general and it is this approach that is described in this section.

As is well known, equilibrium is an assumption on a firm's behavior (a firm selects a strategy to maximizes its expected payoff given its beliefs as to other firms' strategies) and on a firm's beliefs (a firm's beliefs as to another firm's strategy coincides with that firm's actual strategy; that is, beliefs are correct). In considering weaker assumptions, one could loosen either or both assumptions. The approach here is to focus on loosening the assumption on correct beliefs, while maintaining the assumption of optimal behavior given beliefs. The framework is constructed on two assumptions. First, it is common knowledge that firms are rational. Second, it is common knowledge that firms' strategies lie in a subset of the strategy space, which I will denote $\widehat{S}$. There are two criteria that $\widehat{S}$ should satisfy. First, $\widehat{S}$ should be closed under the best reply operator. If it is not then common knowledge that firms' strategies lie in $\widehat{S}$ is incompatible with rationality's being common knowledge. Second, $\widehat{S}$ should be motivated as to how firms might have common knowledge regarding $\widehat{S}$ without having engaged in the express communication associated with explicit collusion. The method and content of communication should be described that could lead to mutual understanding that strategies lie in $\widehat{S}$. Candidate methods are firm announcements, recommendations of "best practices" by a trade association, past practices in this market or closely related markets, and focal points (such as may be provided by government regulations[38]). Available evidence and common sense are used to make a plausible assumption about what is commonly understood among firms about

[37] This results is at best suggestive because it comes with at least two serious caveats. First, if more severe punishments can be coordinated upon under explicit collusion then price will be higher. Second, the comparison focuses on steady-state profit and ignores how the transition path might differ between tacit and explicit collusion.

[38] For example, Knittel and Stango (2003) argue that a regulatory cap on credit card interest rates served as a focal point for tacit collusion.

their strategies.[39]

Having made these assumptions, the analysis involves deriving their implications for firm behavior and market outcomes. If it is common knowledge that firms' strategies are in some subset of the strategy space, what are the implications of firms' rationality being common knowledge? Some basic questions to address are: 1) Is there a lower bound on price exceeding the static Nash equilibrium (competitive) price? If so then this level of mutual understanding is sufficient to induce supracompetitive prices. 2) Is there an upper bound on price that is below that which can be achieved through equilibrium? If so then less than full mutual understanding is constraining firms. Finally, more precise results on the long-run price may be had by deriving the implications of firm learning and utilizing the result of Kalai and Lehrer (1993). Assuming it is common knowledge that each firm is rational and each firm's strategy is in $\widehat{S}$ constrains a firm's prior beliefs over other firms' strategies to be in the set of rationalizable strategies based on a game in which the strategy set is $\widehat{S}$. Assuming a firm Bayesian updates its beliefs over other firms' strategies and its prior beliefs assign positive probability to the true path could deliver precise results about the long-run market outcome.

9 Concluding Remarks

In his classic examination of imperfect competition, Chamberlain (1948) originally argued that collusion would naturally emerge because each firm would recognize the incentive to maintain a collusive price, rather than undercut its rivals' prices and bring forth retaliation. We now know that it is a non-trivial matter for firms to coordinate on a collusive solution because there are so many collusive equilibria. These equilibria differ in terms of the mechanism that sustains collusion as well as the particular outcome that is sustained. Modern oligopoly theory has generally ignored the question of how a collusive arrangement is achieved and instead focused on what can be sustained; that is, the properties of equilibrium outcomes. While the mutual understanding implicit in equilibrium could be acquired through express communication, this leaves unaddressed non-explicit forms of collusion, which are accepted by economists and the courts to occur in practice and are well-documented by experimental evidence.[40] This lack of theoretical attention regarding collusive behavior with less than full mutual understanding has limited the role of economic analysis in defining the contours of what is legal and illegal according to antitrust law.

[39] More easily motivated are ordinal properties of firms' strategies. For example, A2 specified that firms would at least match price increases without specifying the sequence of prices. I want to thank David Miller for making this observation.

[40] Some recent work showing the emergence of tacit collusion in an experimental setting includes Fonseca and Normann (2011) - who investigate when express means of coordination are especially valuable relative to tacit means - and Rojas (2012) - who shows that tacit collusion in the lab can be quite sophisticated in that the degree of collusion varies with the current state of demand. For general references on tacit collusion in experiments, see Huck, Normann, and Oechssler (2004) and Engel (2007).

This paper has proposed a framework within which to explore the relationship between mutual understanding among firms and the outcomes that are produced. When it is common knowledge that price increases will be at least matched (up to some maximum level) and failure to do so results in a return to competition, there is an upper bound on price that is strictly below what equilibrium sustains. If firms' prior beliefs recognize the future price path as a possibility then Bayesian learning about other firms' strategies result in supracompetitive pricing emerging almost for sure. Thus, mutual understanding that price increases will be matched - but not about who will enact those price increases - is sufficient to produce collusion. In terms of defining the boundaries of unlawful collusion, this theory provides a foundation for making unlawful a firm announcing that it will not undercut other firms' prices. Though such an announcement does not directly communicate an intent or plan to coordinate on future prices, if it produces mutual understanding among firms that price increases will be at least matched then firms will eventually coordinate on raising price. As exemplified here, economic analysis can indeed shed light on what forms of limited mutual understanding are capable of producing collusive outcomes and, in this manner, contribute to the determination of what should be prohibited under Section 1 of the Sherman Act.

10 Appendix A

For when the price set is Δ_ε and $p^* \in \Delta_\varepsilon$, let us derive sufficient conditions for the property in (4) to hold, which is reproduced here:

$$\overline{\phi}(\mathbf{p}_{-i}) \begin{cases} \subseteq \{p' + \varepsilon, ..., p_i^*\} & \text{if } \mathbf{p}_{-i} \leq (p', ..., p') \text{ where } p' < p^* - \varepsilon \\ = \{p^*\} & \text{if } \mathbf{p}_{-i} = (p^*, ..., p^*) \\ \subseteq \{p^*, ..., p' - \varepsilon\} & \text{if } \mathbf{p}_{-i} \geq (p', ..., p') \text{ where } p' > p^* + \varepsilon \end{cases}$$

To show that this holds for $p' < p^* - \varepsilon$, it is sufficient to establish that a lower bound on $\overline{\phi}(p^* - \eta\varepsilon, ..., p^* - \eta\varepsilon)$ is $p^* - \eta\varepsilon + \varepsilon$ when $\eta \in \{2, 3, ...\}$. If the unconstrained optimum is at least $p^* - \eta\varepsilon + \varepsilon$ then that is indeed the case.

Define $\widehat{\phi}(p) \equiv \phi(p, ..., p)$ as the best reply function when all other firms price at p, and $\widehat{\phi} : \Re_+ \to \Re_+$. We want to show: if $\eta \in \{2, 3, ...\}$ then $\widehat{\phi}(p^* - \eta\varepsilon) \geq p^* - (\eta - 1)\varepsilon$. It will be shown that a sufficient condition for this result is $\widehat{\phi}'(p) \leq 1/2$. Note that:

$$\widehat{\phi}'(p) = -\frac{\frac{\partial^2 \pi}{\partial p_i \partial p_{-i}}}{\frac{\partial^2 \pi}{\partial p_i^2} + \left(\frac{\delta_i}{1-\delta_i}\right)\left(\frac{d^2\pi}{dp^2}\right)} \leq -\frac{\frac{\partial^2 \pi}{\partial p_i \partial p_{-i}}}{\frac{\partial^2 \pi}{\partial p_i^2}},$$

so $\widehat{\phi}'(p) \leq 1/2$ holds when

$$-\frac{\partial^2 \pi}{\partial p_i^2} \geq 2\frac{\partial^2 \pi}{\partial p_i \partial p_{-i}}.$$

Using the functional forms in Section 8, $\widehat{\phi}'(p) < 1/2$ holds for the case of linear demand and cost:

$$\widehat{\phi}'(p) = \frac{(1-\delta)d}{2(b - \delta d)} < \frac{d}{2b} < \frac{1}{2}, \ \forall \delta \in (0,1).$$

First note:

$$\widehat{\phi}(p^*) = p^*$$

and

$$\widehat{\phi}(p^*) = \widehat{\phi}(p^* - \varepsilon) + \int_{p^*-\varepsilon}^{p^*} \widehat{\phi}'(p)\, dp.$$

Given $\widehat{\phi}' \leq 1/2$, it follows from the previous equality:

$$\begin{aligned} \widehat{\phi}(p^*) &\leq \widehat{\phi}(p^* - \varepsilon) + \frac{\varepsilon}{2} \\ \widehat{\phi}(p^* - \varepsilon) &\geq \widehat{\phi}(p^*) - \frac{\varepsilon}{2} \Rightarrow \widehat{\phi}(p^* - \varepsilon) \geq p^* - \frac{\varepsilon}{2} \end{aligned}$$

Next consider:

$$\begin{aligned} \widehat{\phi}(p^* - \varepsilon) &= \widehat{\phi}(p^* - 2\varepsilon) + \int_{p^*-2\varepsilon}^{p^*-\varepsilon} \widehat{\phi}'(p)\, dp \\ \widehat{\phi}(p^* - \varepsilon) &\leq \widehat{\phi}(p^* - 2\varepsilon) + \frac{\varepsilon}{2} \Rightarrow \widehat{\phi}(p^* - 2\varepsilon) \geq \widehat{\phi}(p^* - \varepsilon) - \frac{\varepsilon}{2} \end{aligned}$$

Using $\widehat{\phi}(p^* - \varepsilon) \geq p^* - \frac{\varepsilon}{2}$, the previous inequality implies:

$$\widehat{\phi}(p^* - 2\varepsilon) \geq p^* - \frac{\varepsilon}{2} - \frac{\varepsilon}{2} \Rightarrow \widehat{\phi}(p^* - 2\varepsilon) \geq p^* - \varepsilon$$

which is the desired result for the case of $\eta = 2$. The proof is completed by induction.

Suppose for $\eta \geq 2$, it is true that:

$$\widehat{\phi}(p^* - \eta\varepsilon) \geq p^* - (\eta - 1)\varepsilon.$$

Consider:

$$\begin{aligned}
\widehat{\phi}(p^* - \eta\varepsilon) &= \widehat{\phi}(p^* - (\eta + 1)\varepsilon) + \int_{p^*-(\eta+1)\varepsilon}^{p^*-\eta\varepsilon} \widehat{\phi}'(p)\, dp \\
\widehat{\phi}(p^* - \eta\varepsilon) &\leq \widehat{\phi}(p^* - (\eta + 1)\varepsilon) + \frac{\varepsilon}{2} \\
\widehat{\phi}(p^* - (\eta + 1)\varepsilon) &\geq \widehat{\phi}(p^* - \eta\varepsilon) - \frac{\varepsilon}{2}
\end{aligned}$$

Using $\widehat{\phi}(p^* - \eta\varepsilon) \geq p^* - (\eta - 1)\varepsilon$ in the preceding inequality,

$$\begin{aligned}
\widehat{\phi}(p^* - (\eta + 1)\varepsilon) &\geq p^* - (\eta - 1)\varepsilon - \frac{\varepsilon}{2} \\
\widehat{\phi}(p^* - (\eta + 1)\varepsilon) &\geq p^* - \eta\varepsilon + \frac{\varepsilon}{2} > p^* - \eta\varepsilon,
\end{aligned}$$

which proves the result. The proof when $p' > p^* + \varepsilon$ is analogous.

11 Appendix B

Proof of Lemma 1. Suppose not so that $\overline{p} \notin \{p^N, ..., \widetilde{p}\}$. If $\overline{p} < p^N$ and $\max\{p_1^{t-1}, ..., p_n^{t-1}\} = \overline{p}$ then, by A2, firms price at $\overline{p}$ for the infinite future. However, a firm could do better in current and future profit by pricing at $\psi(\overline{p}, ..., \overline{p})\,(> \overline{p})$,

$$\pi(\psi(\overline{p}, ..., \overline{p}), \overline{p}, ..., \overline{p}) + \left(\frac{\delta}{1-\delta}\right)\pi^N > \left(\frac{1}{1-\delta}\right)\pi(\overline{p}, ..., \overline{p}),$$

because $\pi^N > \pi(\overline{p}, ..., \overline{p})$. This violates A1. Next suppose $\overline{p} > \widetilde{p}\,(> p^N)$ and $\max\{p_1^{t-1}, ..., p_n^{t-1}\} = \overline{p}$. A2 implies that firms price at $\overline{p}$ for the infinite future. Again, a firm could do better by pricing at $\psi(\overline{p}, ..., \overline{p})\,(< \overline{p})$:

$$\pi(\psi(\overline{p}, ..., \overline{p}), \overline{p}, ..., \overline{p}) + \left(\frac{\delta}{1-\delta}\right)\pi^N > \left(\frac{1}{1-\delta}\right)\pi(\overline{p}, ..., \overline{p}),$$

which is true by the definition of $\widetilde{p}$. Again, A1 is violated. ■

A useful property of other firms using PMP-compatible strategies is that a lower bound on a rational firm's period t continuation payoff is the payoff associated with all firms pricing at $\min\left\{\max\left\{p_1^{t-1},...,p_n^{t-1}\right\},\widetilde{p}\right\}$ in all periods (Lemma 7). Intuitively, if the rivals to firm i are using PMP-compatible strategies then they will price at least as high as $\min\left\{\max\left\{p_1^{t-1},...,p_n^{t-1}\right\},\widetilde{p}\right\}$ in all ensuing periods, as long as firm i does not violate the PMP property and induce a shift to p^N. Hence, firm i can at least earn the profit from all firms (including i) pricing at $\min\left\{\max\left\{p_1^{t-1},...,p_n^{t-1}\right\},\widetilde{p}\right\}$.

Lemma 7 *Let V^t denote a firm's continuation payoff for period t. If the other firms' strategies are PMP-compatible and*

$$p_j^\tau \geq \min\left\{\max\left\{p_1^{\tau-1},...,p_n^{\tau-1}\right\},\widetilde{p}\right\}\ \ \forall j, \forall\tau \leq t-1.$$

Then, for a rational firm,

$$V^t \geq \frac{\pi\left(\min\left\{\max\left\{p_1^{t-1},...,p_n^{t-1}\right\},\widetilde{p}\right\},...,\min\left\{\max\left\{p_1^{t-1},...,p_n^{t-1}\right\},\widetilde{p}\right\}\right)}{1-\delta}.$$

Proof of Lemma 7. Wlog, the analysis will be conducted from the perspective of period 1 (and suppose 0 was a period of collusion). Consider firm i pricing at $\min\left\{\max\left\{p_1^{t-1},...,p_n^{t-1}\right\},\widetilde{p}\right\}$ in the current period and then, in all ensuing periods, matching the maximum price of the other firms' in the previous period:

$$p_i^1 = \min\left\{\max\left\{p_1^{t-1},...,p_n^{t-1}\right\},\widetilde{p}\right\};\ p_i^t = \max\left\{\mathbf{p}_{-i}^{t-1}\right\}\ \text{ for } t=2,...$$

where

$$\max\left\{\mathbf{p}_{-i}^{t-1}\right\} \equiv \max\left\{p_1^{t-1},...,p_{i-1}^{t-1},p_{i+1}^{t-1},...,p_n^{t-1}\right\}.$$

Given this strategy for firm i and that the other firms' strategies are PMP-compatible, there will never be a violation of the PMP property. Hence, firm i's payoff is

$$\pi\left(p_i^1,\mathbf{p}_{-i}^1\right)+\sum_{t=2}^{\infty}\delta^{t-1}\pi\left(\max\left\{\mathbf{p}_{-i}^{t-1}\right\},\mathbf{p}_{-i}^t\right).$$

Since $p_i^1 \leq \max\left\{\mathbf{p}_{-i}^{t-1}\right\}$ (as all firms' strategies satisfy the PMP property) and $\max\left\{\mathbf{p}_{-i}^{t-1}\right\} \leq p_j^t\ \forall j\neq i,\ \forall t\geq 2$, it follows from firm i's profit being increasing in the other firms' prices that

$$\begin{aligned} & \pi\left(p_i^1,\mathbf{p}_{-i}^1\right)+\sum_{t=2}^{\infty}\delta^{t-1}\pi\left(\max\left\{\mathbf{p}_{-i}^{t-1}\right\},\mathbf{p}_{-i}\right) \\ \geq\ & \pi\left(p_i^1,...,p_i^1\right)+\sum_{t=2}^{\infty}\delta^{t-1}\pi\left(\max\left\{\mathbf{p}_{-i}^{t-1}\right\},...,\max\left\{\mathbf{p}_{-i}^{t-1}\right\}\right). \end{aligned} \tag{7}$$

Next note that $p_i^1 \le \max\left\{\mathbf{p}_{-i}^{t-1}\right\} \le \widetilde{p} \le p^M$ which implies

$$\pi\left(\max\left\{\mathbf{p}_{-i}^{t-1}\right\}, ..., \max\left\{\mathbf{p}_{-i}^{t-1}\right\}\right) \ge \pi\left(p_i^1, ..., p_i^1\right).$$

Using this fact on the RHS of (7),

$$\begin{aligned} & \pi\left(p_i^1, ..., p_i^1\right) + \sum_{t=2}^{\infty} \delta^{t-1} \pi\left(\max\left\{\mathbf{p}_{-i}^{t-1}\right\}, ..., \max\left\{\mathbf{p}_{-i}^{t-1}\right\}\right) \qquad (8) \\ \ge \quad & \pi\left(p_i^1, ..., p_i^1\right) + \sum_{t=2}^{\infty} \delta^{t-1} \pi\left(p_i^1, ..., p_i^1\right) = \frac{\pi\left(p_i^1, ..., p_i^1\right)}{1-\delta_i}. \end{aligned}$$

(7) and (8) imply

$$\pi\left(p_i^1, \mathbf{p}_{-i}^1\right) + \sum_{t=2}^{\infty} \delta^{t-1} \pi\left(\max\left\{\mathbf{p}_{-i}^{t-1}\right\}, \mathbf{p}_{-i}^t\right) \ge \frac{\pi\left(p_i^1, ..., p_i^1\right)}{1-\delta_i},$$

from which we conclude $V^t \ge \pi\left(p_i^1, ..., p_i^1\right) / (1-\delta)$. ■

Proof of Lemma 2. Suppose the period t history is such that

$$p_j^\tau < \min\left\{\max\left\{p_1^{\tau-1}, ..., p_n^{\tau-1}\right\}, \widetilde{p}\right\} \text{ for some } j \text{ and some } \tau \le t-1.$$

A PMP-compatible strategy has a firm price at p^N in the current and all future periods. Thus, if firm i's beliefs over other firms' strategies has support in S^{PMP} then pricing at p^N is clearly optimal. Hence, a PMP-compatible strategy is uniquely optimal for firm i for those histories.

For the remainder of the proof, suppose $p_j^\tau \ge \min\left\{\max\left\{p_1^{\tau-1}, ..., p_n^{\tau-1}\right\}, \widetilde{p}\right\}$ $\forall j, \forall \tau \le t-1$. To prove this lemma, we'll show that, for any strategy for firm i that does not satisfy the PMP property, there is a PMP-compatible strategy that yields a strictly higher payoff. Thus, regardless of firm i's beliefs over the other firms' strategies (as long as they have support in S^{PMP}), its expected payoff is strictly higher with some PMP-compatible strategy than with any PMP-incompatible strategy.

Given $p_j^\tau \ge \min\left\{\max\left\{p_1^{\tau-1}, ..., p_n^{\tau-1}\right\}, \widetilde{p}\right\}$ $\forall j, \forall \tau \le t-1$, firm i's strategy can violate the PMP property either by pricing above $\widetilde{p}$ or below $\max\left\{p_1^{t-1}, ..., p_n^{t-1}\right\}$. Let us begin by considering a PMP-incompatible strategy that has firm i price at $p' > \widetilde{p}$. When its rivals price at $\mathbf{p}_{-i}^t$, a PMP-compatible strategy that has firm i price at $\widetilde{p}$ is more profitable than pricing at p' iff:

$$\pi\left(\widetilde{p}, \mathbf{p}_{-i}^t\right) + \left(\frac{\delta}{1-\delta}\right) \pi\left(\widetilde{p}, ..., \widetilde{p}\right) > \pi\left(p', \mathbf{p}_{-i}^t\right) + \left(\frac{\delta}{1-\delta}\right) \pi\left(\widetilde{p}, ..., \widetilde{p}\right), \qquad (9)$$

where recall that the other firms will only follow price as high as $\widetilde{p}$. (9) holds iff

$$\pi\left(\widetilde{p}, \mathbf{p}_{-i}^t\right) > \pi\left(p', \mathbf{p}_{-i}^t\right). \qquad (10)$$

Given that $\mathbf{p}_{-i}^t \leq (\widetilde{p}, ..., \widetilde{p})$ and $p^N < \widetilde{p}$ then $\psi\left(\mathbf{p}_{-i}^t\right) \leq \psi(\widetilde{p}, ..., \widetilde{p}) < \widetilde{p}$. By the strict concavity of π in own price and that $\psi\left(\mathbf{p}_{-i}^t\right) < \widetilde{p} < p'$, (10) is true.

Next consider a PMP-incompatible strategy that has firm i price at $p'' < \max\left\{p_1^{t-1}, ..., p_n^{t-1}\right\}$. Let us show that a PMP-compatible strategy that has firm i price at $\max\left\{p_1^{t-1}, ..., p_n^{t-1}\right\}$ is more profitable than pricing at p'' for any $\mathbf{p}_{-i}^t \in \left[\max\left\{p_1^{t-1}, ..., p_n^{t-1}\right\}, \widetilde{p}\right]^{n-1}$.[41] A sufficient condition for the preceding claim to be true is:

$$\begin{aligned} & \pi\left(\max\left\{p_1^{t-1}, ..., p_n^{t-1}\right\}, \mathbf{p}_{-i}^t\right) \\ & +\left(\frac{\delta}{1-\delta}\right)\pi\left(\max\left\{\mathbf{p}_{-i}^t\right\}, ..., \max\left\{\mathbf{p}_{-i}^t\right\}\right) \\ > \quad & \pi\left(p'', \mathbf{p}_{-i}^t\right) + \left(\frac{\delta}{1-\delta}\right)\pi^N, \end{aligned} \tag{11}$$

where the LHS of (11) is a lower bound on the payoff from pricing at $\max\left\{p_1^{t-1}, ..., p_n^{t-1}\right\}$ and the RHS is the payoff from pricing at p''. In examining the LHS, note that $p_i^t = \max\left\{p_1^{t-1}, ..., p_n^{t-1}\right\}$ and that the other firms' strategies are PMP-compatible imply

$$\max\left\{p_1^t, ..., p_n^t\right\} = \max\left\{\mathbf{p}_{-i}^t\right\}.$$

Using Lemma 7,

$$\left(\frac{\delta}{1-\delta}\right)\pi\left(\max\left\{\mathbf{p}_{-i}^t\right\}, ..., \max\left\{\mathbf{p}_{-i}^t\right\}\right)$$

is a lower bound on the future payoff, which gives us the LHS of (11). When

$$\mathbf{p}_{-i}^t = \left(\max\left\{p_1^{t-1}, ..., p_n^{t-1}\right\}, ..., \max\left\{p_1^{t-1}, ..., p_n^{t-1}\right\}\right), \tag{12}$$

(11) is

$$\begin{aligned} & \pi\left(\max\left\{p_1^{t-1}, ..., p_n^{t-1}\right\}, ..., \max\left\{p_1^{t-1}, ..., p_n^{t-1}\right\}\right) \\ & +\left(\frac{\delta}{1-\delta}\right)\pi\left(\max\left\{p_1^{t-1}, ..., p_n^{t-1}\right\}, ..., \max\left\{p_1^{t-1}, ..., p_n^{t-1}\right\}\right) \\ > \quad & \pi\left(p'', \max\left\{p_1^{t-1}, ..., p_n^{t-1}\right\}, ..., \max\left\{p_1^{t-1}, ..., p_n^{t-1}\right\}\right) + \left(\frac{\delta}{1-\delta}\right)\pi^N. \end{aligned} \tag{13}$$

Since $\max\left\{p_1^{t-1}, ..., p_n^{t-1}\right\} \leq \widetilde{p}$ then (13) is true for all $p'' < \max\left\{p_1^{t-1}, ..., p_n^{t-1}\right\}$ as it is the equilibrium condition for a grim trigger strategy with collusive price $\max\left\{p_1^{t-1}, ..., p_n^{t-1}\right\}$.[42] Thus, (11) holds for (12).

[41] Actually, it is shown to be only weakly as profitable when $\mathbf{p}_{-i}^t = (\widetilde{p}, ..., \widetilde{p})$.

[42] Recall that $\widetilde{p}$ is the highest price consistent with the grim trigger strategy being an equilibrium. Note that (13) holds with equality when $\max\left\{p_1^{t-1}, ..., p_n^{t-1}\right\} = \widetilde{p}$ and $p'' = \psi(\widetilde{p}, ..., \widetilde{p})$ and otherwise is a strict inequality.

To complete the proof, it will be shown that the LHS of (11) is increasing in $\mathbf{p}_{-i}^{t}$ at a faster rate than the RHS in which case (11) holds for all

$$\mathbf{p}_{-i}^{t} \geq \left(\max\left\{p_1^{t-1}, ..., p_n^{t-1}\right\}, ..., \max\left\{p_1^{t-1}, ..., p_n^{t-1}\right\}\right).$$

The derivative with respect to p_j^t, $j \neq i$, of the LHS of (11) is

$$\frac{\partial \pi\left(\max\left\{p_1^{t-1}, ..., p_n^{t-1}\right\}, \mathbf{p}_{-i}^{t}\right)}{\partial p_j} + \left(\frac{\delta}{1-\delta}\right) \frac{\partial \max\left\{\mathbf{p}_{-i}^{t}\right\}}{\partial p_j} \sum_{k=1}^{n} \frac{\partial \pi\left(\max\left\{\mathbf{p}_{-i}^{t}\right\}, ..., \max\left\{\mathbf{p}_{-i}^{t}\right\}\right)}{\partial p_k}, \tag{14}$$

and of the RHS of (11) is

$$\frac{\partial \pi\left(p'', \mathbf{p}_{-i}^{t}\right)}{\partial p_j}. \tag{15}$$

(14) exceeds (15) because the second term in (14) is non-negative, given that $\mathbf{p}_{-i}^{t} \leq \left(p^M, ..., p^M\right)$, and the first term of (14) exceeds (15) because $\max\left\{p_1^{t-1}, ..., p_n^{t-1}\right\} > p''$ and $\frac{\partial^2 \pi}{\partial p_i \partial p_j} > 0$, $j \neq i$. ■

Proof of Theorem 3. Given firms' strategies are PMP-compatible then it immediately follows that each firm's price is weakly increasing. Given a finite price set and the boundedness and monotonicity of prices, prices converge in finite time. Hence, there exists $\widehat{p} \in \left\{p^N, ..., \widetilde{p}\right\}$ and finite T such that $p_1^t = \cdots = p_n^t = \widehat{p}$ for all $t \geq T$.

The remainder of the proof entails proving $\widehat{p} \leq p^* + \varepsilon$. If $p^* + \varepsilon \geq \widetilde{p}$ then, given that PMP-compatible strategies do not have firms pricing above $\widetilde{p}$, it is immediate that $\widehat{p} \leq p^* + \varepsilon$. Thus, suppose $p^* + \varepsilon < \widetilde{p}$. Before going any further, an overview of the proof is provided. First it is shown that if a firm is rational and believes the other firms use PMP-compatible strategies then a firm will not price at $\widetilde{p}$. The reason is that firm i would find it optimal to price above $p^* + \varepsilon$ only if it induced at least one of its rivals to enact further price increases (and not just match the firm's price). However, if a firm believes its rivals will not price above $\widetilde{p}$ (which follows from believing its rivals use PMP-compatible strategies) then it is not optimal for a firm to raise price to $\widetilde{p}$ because it can only expect its rivals to match a price of $\widetilde{p}$, not exceed it. This argument works as well to show that each of the other firms will not raise price to $\widetilde{p}$. Hence, there is an upper bound on price of $\widetilde{p} - \varepsilon$. The proof is completed by induction using the common knowledge in A1-A3. If a firm believes its rivals will not price above p' then it can be shown that a firm will find it optimal not to price above $p' - \varepsilon$. This argument works only when $p' \geq p^* + 2\varepsilon$ which implies that an upper bound on price is $p^* + \varepsilon$, which is the desired result.

Define $\overline{\phi}^U(\mathbf{p}_{-i})$ to be the maximal element of $\overline{\phi}(\mathbf{p}_{-i})$ and let us show that $\overline{\phi}^U(\mathbf{p}_{-i})$ is non-decreasing in $\mathbf{p}_{-i}$. By the definition of $\overline{\phi}^U(\mathbf{p}_{-i})$, we know that:

$$W\left(\overline{\phi}^U\left(\mathbf{p}'_{-i}\right), \mathbf{p}'_{-i}\right) - W\left(p_i, \mathbf{p}'_{-i}\right) > 0,\ \forall p_i \in A \equiv \left\{p \in \Delta_\varepsilon : p > \overline{\phi}^U\left(\mathbf{p}'_{-i}\right)\right\}.$$

Since $\frac{\partial^2 W(p_i,\mathbf{p}_{-i})}{\partial p_i \partial p_j} = \frac{\partial^2 \pi(p_i,\mathbf{p}_{-i})}{\partial p_i \partial p_j} > 0$ then $p_i > \overline{\phi}^U\left(\mathbf{p}'_{-i}\right)$ and $\mathbf{p}''_{-i} \leq \mathbf{p}'_{-i}$ imply

$$W\left(p_i, \mathbf{p}'_{-i}\right) - W\left(p_i, \mathbf{p}''_{-i}\right) \geq W\left(\overline{\phi}^U\left(\mathbf{p}'_{-i}\right), \mathbf{p}'_{-i}\right) - W\left(\overline{\phi}^U\left(\mathbf{p}'_{-i}\right), \mathbf{p}''_{-i}\right), \ \forall p_i \in A,$$

and, re-arranging, we have

$$W\left(\phi^U\left(\mathbf{p}'_{-i}\right), \mathbf{p}''_{-i}\right) - W\left(p_i, \mathbf{p}''_{-i}\right) \geq W\left(\overline{\phi}^U\left(\mathbf{p}'_{-i}\right), \mathbf{p}'_{-i}\right) - W\left(p_i, \mathbf{p}'_{-i}\right), \ \forall p_i \in A.$$

Therefore,

$$W\left(\overline{\phi}^U\left(\mathbf{p}'_{-i}\right), \mathbf{p}''_{-i}\right) - W\left(p_i, \mathbf{p}''_{-i}\right) > 0, \ \forall p_i \in A,$$

which, along with $\mathbf{p}''_{-i} \leq \mathbf{p}'_{-i}$ and the strict concavity of W in own price, imply $\overline{\phi}^U\left(\mathbf{p}''_{-i}\right) \leq \overline{\phi}^U\left(\mathbf{p}'_{-i}\right)$. Hence, $\overline{\phi}^U\left(\mathbf{p}_{-i}\right)$ is non-decreasing.

By (4), $p' > p^* + \varepsilon$ implies $\overline{\phi}^U\left(p', ..., p'\right) \leq p' - \varepsilon$. Given $\overline{\phi}^U\left(\mathbf{p}_{-i}\right)$ is non-decreasing in $\mathbf{p}_{-i}$, it follows:

$$\text{if } \mathbf{p}_{-i} \leq \left(p', ..., p'\right) \text{ and } p' > p^* + \varepsilon \text{ then } \overline{\phi}^U\left(\mathbf{p}_{-i}\right) \leq p' - \varepsilon. \tag{16}$$

From the strict concavity of W in own price, we have:

$$\text{if } p'' > p' \geq \overline{\phi}^U\left(\mathbf{p}_{-i}\right) \text{ then} \tag{17}$$

$$\pi\left(p', \mathbf{p}_{-i}\right) + \left(\frac{\delta}{1-\delta}\right)\pi\left(p', ..., p'\right) > \pi\left(p'', \mathbf{p}_{-i}\right) + \left(\frac{\delta}{1-\delta}\right)\pi\left(p'', ..., p''\right).$$

This property will be used in the ensuing proof.

Let us show that if firm i believes the other firms' strategies are PMP-compatible then a price of $\widetilde{p} - \varepsilon$ is strictly preferred to $\widetilde{p}$. Firm i's beliefs on $\mathbf{p}^t_{-i}$ have support $\left[\max\left\{p_1^{t-1}, ..., p_n^{t-1}\right\}, \widetilde{p}\right]^{n-1}$. For any $\mathbf{p}^t_{-i} \in \left[\max\left\{p_1^{t-1}, ..., p_n^{t-1}\right\}, \widetilde{p}\right]^{n-1}$, Lemma 7 implies that a lower bound on its payoff from $p_i^t = \widetilde{p} - \varepsilon$ is

$$\pi\left(\widetilde{p} - \varepsilon, \mathbf{p}^t_{-i}\right) + \left(\frac{\delta}{1-\delta}\right)\pi\left(\widetilde{p} - \varepsilon, ..., \widetilde{p} - \varepsilon\right). \tag{18}$$

For any $\mathbf{p}^t_{-i} \in \left[\max\left\{p_1^{t-1}, ..., p_n^{t-1}\right\}, \widetilde{p}\right]^{n-1}$, it follows from all firms using PMP-compatible strategies that firm i's payoff from $p_i^t = \widetilde{p}$ is

$$\pi\left(\widetilde{p}, \mathbf{p}^t_{-i}\right) + \left(\frac{\delta}{1-\delta}\right)\pi\left(\widetilde{p}, ..., \widetilde{p}\right). \tag{19}$$

Given that $\widetilde{p} > p^* + \varepsilon$ then $\overline{\phi}^U\left(\mathbf{p}^t_{-i}\right) \leq \widetilde{p} - \varepsilon$ for all $\mathbf{p}^t_{-i} \leq \left(\widetilde{p}, ..., \widetilde{p}\right)$ by (16). It then follows from (17) that (18) strictly exceeds (19). Therefore, for any beliefs of firm i with support $\left[\max\left\{p_1^{t-1},...,p_n^{t-1}\right\},\widetilde{p}\right]^{n-1}$, a price of $\widetilde{p}-\varepsilon$ is strictly preferred to $\widetilde{p}$. It follows that if a firm is rational and believes the other firms use PMP-compatible strategies then its optimal price does not exceed $\widetilde{p}-\varepsilon$.

Given the common knowledge from Assumptions A1-A3, it is also the case that firm i believes firm $j\,(\neq i)$ is rational and that firm j believes firm h (for all $h\neq j$) uses a PMP-compatible strategy. Hence, applying the preceding argument to firm j, firm i believes firm j will not price above $\widetilde{p}-\varepsilon$. Firm i's beliefs on $\mathbf{p}_{-i}^t$ then have support $\left[\max\left\{p_1^{t-1},...,p_n^{t-1}\right\},\widetilde{p}-\varepsilon\right]^{n-1}$. If $\widetilde{p}-\varepsilon>p^*+\varepsilon$ then, by (16), $\overline{\phi}^U\left(\mathbf{p}_{-i}^t\right)\leq\widetilde{p}-2\varepsilon$ for all $\mathbf{p}_{-i}^t\leq(\widetilde{p}-\varepsilon,...,\widetilde{p}-\varepsilon)$.[43] By the same logic as above, a lower bound on firm i's payoff from $p_i^t=\widetilde{p}-2\varepsilon$ is

$$\pi\left(\widetilde{p}-2\varepsilon,\mathbf{p}_{-i}^t\right)+\left(\frac{\delta}{1-\delta}\right)\pi\left(\widetilde{p}-2\varepsilon,...,\widetilde{p}-2\varepsilon\right), \tag{20}$$

while its payoff from $p_i^t=\widetilde{p}-\varepsilon$ is

$$\pi\left(\widetilde{p}-\varepsilon,\mathbf{p}_{-i}^t\right)+\left(\frac{\delta}{1-\delta}\right)\pi\left(\widetilde{p}-\varepsilon,...,\widetilde{p}-\varepsilon\right). \tag{21}$$

With (21), we used the fact that firms will not price above $\widetilde{p}-\varepsilon$, which was derived in the first step. Again using (17), it is concluded that (20) strictly exceeds (21). Therefore, for any beliefs of firm i over $\mathbf{p}_{-i}^t$ with support $\left\{\max\left\{p_1^{t-1},...,p_n^{t-1}\right\},...,\widetilde{p}-\varepsilon^{n-1}\right\}$, a price of $\widetilde{p}-2\varepsilon$ is strictly preferred to $\widetilde{p}-\varepsilon$. It follows that if a firm is rational and a firm believes other firms' strategies are PMP-compatible, believes the other firms are rational, and believes each of the other firms believes its rivals use PMP-compatible strategies then a firm's optimal price does not exceed $\widetilde{p}-2\varepsilon$. Hence, all firms will not price above $\widetilde{p}-2\varepsilon$.

The proof is completed by induction. Suppose we have shown that firm i believes that the other firms will not price above p' so firm i's beliefs on $\mathbf{p}_{-i}^t$ have support $\left\{\max\left\{p_1^{t-1},...,p_n^{t-1}\right\},...,p\right\}^{n-1}$. (That we can get to the point that firms have those beliefs relies on rationality and that firms use PMP-compatible strategies are both common knowledge.) If $p'>p^*+\varepsilon$ then $\overline{\phi}^U\left(\mathbf{p}_{-i}^t\right)\leq p'-\varepsilon$ for all $\mathbf{p}_{-i}^t\leq(p',...,p')$. A lower bound on firm i's payoff from $p_i^t=p'-\varepsilon$ is

$$\pi\left(p'-\varepsilon,\mathbf{p}_{-i}^t\right)+\left(\frac{\delta}{1-\delta}\right)\pi\left(p'-\varepsilon,...,p'-\varepsilon\right), \tag{22}$$

while its payoff from $p_i^t=p'$ is

$$\pi\left(p',\mathbf{p}_{-i}^t\right)+\left(\frac{\delta}{1-\delta}\right)\pi\left(p',...,p'\right), \tag{23}$$

since all firms have an upper bound of p' on their prices. Using (17), it is concluded that (22) strictly exceeds (23). Therefore, for any beliefs of firm i over $\mathbf{p}_{-i}^t$ with support

[43] If instead $\widetilde{p}-\varepsilon\leq p^*+\varepsilon$ then, given that it has already been shown $\widetilde{p}-\varepsilon$ is an upper bound on the limit price, it follows that $p^*+\varepsilon$ is an upper bound and we're done.

$\left[\max\left\{p_1^{t-1},...,p_n^{t-1}\right\},p'\right]^{n-1}$, a price of $p'-\varepsilon$ is strictly preferred to p'. It follows that firms' prices are bounded above by $p'-\varepsilon$. The preceding argument is correct as long as $p'>p^*+\varepsilon$; therefore, price is bounded above by $p^*+\varepsilon$. ■

Proof of Theorem 4. p^* is defined by

$$\frac{\partial W\left(p^*,...,p^*\right)}{\partial p_i}=\frac{\partial \pi\left(p^*,...,p^*\right)}{\partial p_i}+\left(\frac{\delta}{1-\delta}\right)\sum_{j=1}^{n}\frac{\partial \pi\left(p^*,...,p^*\right)}{\partial p_j}=0$$

or

$$\frac{\partial \pi\left(p^*,...,p^*\right)}{\partial p_i}+\delta\sum_{j\neq i}^{n}\frac{\partial \pi\left(p^*,...,p^*\right)}{\partial p_j}=0.$$

For all $p\geq p^M$,

$$\frac{\partial \pi\left(p,...,p\right)}{\partial p_i}<0 \text{ and } \sum_{j=1}^{n}\frac{\partial \pi\left(p,...,p\right)}{\partial p_j}\leq 0,$$

which implies $p^*<p^M$ by the strict concavity of W in own price. To show $p^*>p^N$, note that $\phi\left(p,...,p\right)>\psi\left(p,...,p\right)$ and $\psi\left(p,...,p\right)\geq p\ \forall p\leq p^N$ implies $\phi\left(p,...,p\right)>p\ \forall p\leq p^N$. Since $\phi\left(p,...,p\right)\gtreqqless p$ as $p\lesseqqgtr p^*$ then $p^*>p^N$. We have then shown $p^*\in\left(p^N,p^M\right)$.

If $\widetilde{p}=p^M$ then $p^*\in\left(p^N,\widetilde{p}\right)$ and we are done. From hereon, suppose $\widetilde{p}<p^M$ in which case the incentive compatibility constraint (ICC) binds:

$$\frac{\pi\left(\widetilde{p},...,\widetilde{p}\right)}{1-\delta}=\pi\left(\psi\left(\widetilde{p},...,\widetilde{p}\right),\widetilde{p},...,\widetilde{p}\right)+\left(\frac{\delta}{1-\delta}\right)\pi\left(p^N,...,p^N\right)\ . \tag{24}$$

As $p\in\left(p^N,p^*\right]$ implies $\psi\left(p,...,p\right)<p\leq\phi\left(p,...,p\right)$ then, by the strict concavity of W,

$$W\left(p,...,p\right)>W\left(\psi\left(p\right),p,...,p\right),$$

which is equivalently expressed as

$$\frac{\pi\left(p,...,p\right)}{1-\delta}>\pi\left(\psi\left(p\right),p,...,p\right)+\left(\frac{\delta}{1-\delta}\right)\pi\left(\psi\left(p\right),...,\psi\left(p\right)\right). \tag{25}$$

$p>p^N$ implies $\psi\left(p,...,p\right)\in\left(p^N,p\right)$. Next note $\psi\left(p,...,p\right)<p\leq p^*<p^M$ implies $\psi\left(p,...,p\right)<p^M$. It then follows from $\psi\left(p,...,p\right)\in\left(p^N,p^M\right)$ that $\pi\left(\psi\left(p\right),...,\psi\left(p\right)\right)>\pi\left(p^N,...,p^N\right)$. Using this property in (25), we have

$$\frac{\pi\left(p,...,p\right)}{1-\delta}>\pi\left(\psi\left(p\right),p,...,p\right)+\left(\frac{\delta}{1-\delta}\right)\pi\left(p^N,...,p^N\right),\ \forall p\in\left(p^N,p^*\right]. \tag{26}$$

Therefore, $p\in\left(p^N,p^*\right]$ is sustainable with the grim trigger strategy. Given (24) - where the ICC binds for $p=\widetilde{p}$ - and evaluating (26) at $p=p^*$ - so the ICC does not bind - it follows from (2) that $\widetilde{p}>p^*$. ■

Proof of Theorem 5. To draw on the result of Kalai and Lehrer (1993), I need to introduce some notation and definitions. If $g \in Rat\left(S^{PMP}\right)^n$ is a strategy profile and $Q \subset \Lambda$ is a collection of infinite price paths for the game then define $\mu_g(Q)$ to be the probability measure on Q induced by g. We know that a strategy profile that is an element of $\left(S^{PMP}\right)^n$ - and thus of $Rat\left(S^{PMP}\right)^n$ - converges in finite time. Suppose, contrary to the statement of the Theorem 5, the true (pure) strategy profile, denoted f, has price converge to $p' < p^* - \varepsilon$. It is then the case that there exists T' such that $\mu_f\left(Q_{\widehat{p}}\right) = 1$ for all infinite price paths $Q_{\widehat{p}}$ with the property: $p_1^t = \cdots = p_n^t = p'$ $\forall t \geq T'$. By Theorem 1 of Kalai and Lehrer (1993), $\exists T(\eta)$ such that firm i's beliefs on the infinite price paths, denoted μ^i, have the property that μ_f is η-*close* to μ^i for price paths starting at t, $\forall t \geq T(\eta)$.[44] Given that $\mu_f\left(Q_{\widehat{p}}\right) = 1$ $\forall t \geq \max\{T', T(\eta)\}$ then μ^i must assign probability of at least $1-\eta$ to the future price path having the property: $p_1^t = \cdots = p_n^t = p'$ $\forall t \geq \max\{T', T(\eta)\}$.[45]

Given these beliefs, let us evaluate optimal play for firm i. For $t \geq \max\{T', T(\eta)\}$, firm i's expected payoff from acting according to its strategy and pricing at p' has an upper bound of

$$(1-\eta)\left(\frac{\pi(p', ..., p')}{1-\delta}\right) + \eta\left[\pi\left(p', p^m ..., p^m\right) + \left(\frac{\delta}{1-\delta}\right)\pi(p^m, ..., p^m)\right]. \qquad (27)$$

Probability $1-\eta$ is assigned to the "true" future price path in which price is always p', and the remaining probability is assigned to all rivals raising price to p^m in the current period (in order to give us an upper bound on the payoff). Now consider firm i deviating from its strategy of pricing at p' by pricing instead at $\phi(p', ..., p')$. The resulting payoff has a lower bound of

$$(1-\eta)\left[\pi\left(\phi\left(p', ..., p'\right), p', ..., p'\right) + \left(\frac{\delta}{1-\delta}\right)\pi\left(\phi\left(p', ..., p'\right), ..., \phi\left(p', ..., p'\right)\right)\right]. \qquad (28)$$

Probability $1-\eta$ is assigned to other firms pricing at p' in the current period and, as a worst case scenario, only matching firm i's price of $\phi(p', ..., p')$ thereafter (which they must do in order to be consistent with A2). A zero payoff is assigned to the remaining probability η to give us a lower bound. We want to show that pricing at p' is not optimal - and thus inconsistent with A1 - which is the case if (28) exceeds (27):

$$\begin{aligned} & (1-\eta)\left[\pi\left(\phi\left(p', ..., p'\right), p', ..., p'\right) + \left(\frac{\delta}{1-\delta}\right)\pi\left(\phi\left(p', ..., p'\right), ..., \phi\left(p', ..., p'\right)\right)\right] \\ > \ & (1-\eta)\left(\frac{\pi(p', ..., p')}{1-\delta}\right) + \eta\left[\pi\left(p', p^m ..., p^m\right) + \left(\frac{\delta}{1-\delta}\right)\pi(p^m, ..., p^m)\right] \end{aligned}$$

[44] Definition 1 (Kalai and Lehrer, 1993): Let $\eta > 0$ and let μ and $\widetilde{\mu}$ be two probability measures defined on the same space. μ is said to be η-*close* to $\widetilde{\mu}$ if there is a measurable set Q satisfying: (i) $\mu(Q)$ and $\widetilde{\mu}(Q)$ are greater than $1-\eta$; and (ii) for every measurable set $X \subseteq Q$: $(1-\eta)\widetilde{\mu}(X) \leq \mu(X) \leq (1+\eta)\widetilde{\mu}(X)$.

[45] I am only using property (i) in Definition 1 of Kalai and Lehrer (1993); see footnote 46.

or

$$
\begin{aligned}
& (1-\eta)\left[\pi\left(\phi\left(p^{\prime},...,p^{\prime}\right),p^{\prime},...,p^{\prime}\right)+\left(\frac{\delta}{1-\delta}\right)\pi\left(\phi\left(p^{\prime},...,p^{\prime}\right),...,\phi\left(p^{\prime},...,p^{\prime}\right)\right)-\frac{\pi\left(p^{\prime},...,p^{\prime}\right)}{1-\delta}\right] \\
> \; & \eta\left[\pi\left(p^{\prime},p^{m}...,p^{m}\right)+\left(\frac{\delta}{1-\delta}\right)\pi\left(p^{m},...,p^{m}\right)\right].
\end{aligned}
\tag{29}
$$

Given that $p^{\prime}<p^{*}-\varepsilon$ then the first bracketed term in (29) is strictly positive. Hence, for η sufficiently small, (29) holds. Thus, if t is sufficiently great, firm i's beliefs are such that pricing at $p^{\prime}$ in period t is non-optimal. Therefore, I conclude that price cannot converge to a value below $p^{*}-\varepsilon$. Given that Theorem 3 showed that an upper bound on convergence is $p^{*}+\varepsilon$, it is concluded that prices converges to some value in $\{p^{*}-\varepsilon,p^{*},p^{*}+\varepsilon\}$. ■

Proof of Theorem 6. Given that $\frac{\partial p^{*}}{\partial\delta}>0$ and $\frac{\partial\widetilde{p}}{\partial\delta}=0$ for $\delta>\delta^{*}$ then: if $\delta>\delta^{*}$ then $\frac{\partial(\widetilde{p}-p^{*})}{\partial\delta}<0$. The remainder of the proof focuses on showing: if $\delta<\delta^{*}$ then $\frac{\partial(\widetilde{p}-p^{*})}{\partial\delta}>0$.

If $\delta<\delta^{*}$ then

$$
\begin{aligned}
\widetilde{p}-p^{*} &= \frac{4ab^{2}+ad^{2}+4b^{3}c+bcd^{2}-4b^{2}cd-ad^{2}\delta-4abd+4abd\delta+3bcd^{2}\delta-4b^{2}cd\delta}{6bd^{2}-12b^{2}d+d^{3}\delta+8b^{3}-d^{3}-2bd^{2}\delta} \\
&\quad -\frac{a+(b-\delta d)\,c}{2b-(1+\delta)\,d}
\end{aligned}
$$

$$
\frac{\partial\left(\widetilde{p}-p^{*}\right)}{\partial\delta}=\left[\frac{d\left(a-bc+dc\right)}{\left(8b^{3}-4b^{2}d\delta-12b^{2}d+2bd^{2}\delta+6bd^{2}+d^{3}\delta^{2}-d^{3}\right)^{2}}\right]\times\Psi\left(\delta\right), \tag{30}
$$

where

$$
\begin{aligned}
\Psi\left(\delta\right) \;\equiv\; & 16b^{4}-32b^{3}d\delta-16b^{3}d+8b^{2}d^{2}\delta^{2}+40b^{2}d^{2}\delta- \\
& 4bd^{3}\delta^{2}-16bd^{3}\delta+4bd^{3}-d^{4}\delta^{2}+2d^{4}\delta+d^{4}.
\end{aligned}
$$

As the term in [ ] in (30) is positive then

$$
sign\left\{\frac{\partial\left(\widetilde{p}-p^{*}\right)}{\partial\delta}\right\}=sign\left\{\Psi\left(\delta\right)\right\}
$$

In evaluating the sign of $\Psi\left(\delta\right)$, first note it is positive at the extreme values of δ:

$$
\Psi\left(0\right)=16b^{4}-16b^{3}d+4bd^{3}-d^{4}=16b^{3}\left(b-d\right)+d^{3}\left(4b-d\right)>0
$$

$$
\begin{aligned}
\Psi\left(1\right) &= 16b^{4}-48b^{3}d+48b^{2}d^{2}-16bd^{3}=16b\left(b^{3}-3b^{2}d+3bd^{2}-d^{3}\right) \\
&= 16b\left(b^{2}\left(b-d\right)-2bd\left(b-d\right)+d^{2}\left(b-d\right)\right)=16b\left(b-d\right)\left(b^{2}-2bd+d^{2}\right) \\
&= 16b\left(b-d\right)\left(b-d\right)^{2}>0,
\end{aligned}
$$

which follow from $b > d > 0$. Given $\Psi(0), \Psi(1) > 0$, if $\Psi(\delta)$ is weakly monotonic then $\Psi(\delta) > 0 \forall \delta \in [0,1]$ and thus $\Psi(\delta) > 0 \forall \delta \in [0, \delta^*)$. Let us show $\Psi'(\delta) < 0$. Consider:

$$\Psi'(\delta) = 40b^2d^2 - 32b^3d - 2d^4\delta - 16bd^3 + 2d^4 + 16b^2d^2\delta - 8bd^3\delta.$$

Since

$$\Psi''(\delta) = -2d^4 + 16b^2d^2 - 8bd^3 = 2d^2\left(8b^2 - 4bd - d^2\right) > 0,$$

$\Psi'(1) < 0$ is a sufficient condition to establish that $\Psi'(\delta) < 0 \forall \delta \in [0,1]$. Given that

$$\begin{aligned} \Psi'(1) &= 56b^2d^2 - 32b^3d - 24bd^3 = 8bd\left(7bd - 4b^2 - 6d^2\right) \\ &= -8bd(b-d)(4b-3d) < 0, \end{aligned}$$

we are done. ■

12 Appendix C

In deriving sufficient conditions for $\left(S^L, S^F\right)$ to be a subgame perfect equilibrium, let us first consider S^L have ρ denote the lagged maximum price. If $\rho = p^N$ then $S^L\left(p^N\right) = p'$ which is optimal iff p' is at least as profitable as p^N,

$$\begin{aligned} & \pi\left(p', p^N\right) + \delta\pi\left(p^M, p'\right) + \left(\frac{\delta^2}{1-\delta}\right)\pi\left(p^M, p^M\right) \\ \geq\ & \pi\left(p^N, p^N\right) + \delta\pi\left(p', p^N\right) + \delta^2\pi\left(p^M, p'\right) + \left(\frac{\delta^3}{1-\delta}\right)\pi\left(p^M, p^M\right) \end{aligned} \tag{31}$$

and at least as profitable as p^M,

$$\begin{aligned} & \pi\left(p', p^N\right) + \delta\pi\left(p^M, p'\right) + \left(\frac{\delta^2}{1-\delta}\right)\pi\left(p^M, p^M\right) \\ \geq\ & \pi\left(p^M, p^N\right) + \left(\frac{\delta}{1-\delta}\right)\pi\left(p^M, p^M\right). \end{aligned} \tag{32}$$

(31) and (32) can be simplified to:

$$\pi\left(p', p^N\right) + \delta\pi\left(p^M, p'\right) + \delta^2\pi\left(p^M, p^M\right) \geq \pi\left(p^N, p^N\right) + \delta\pi\left(p', p^N\right) + \delta^2\pi\left(p^M, p'\right) \tag{33}$$

$$\pi\left(p', p^N\right) + \delta\pi\left(p^M, p'\right) \geq \pi\left(p^M, p^N\right) + \delta\pi\left(p^M, p^M\right) \tag{34}$$

If $\delta \simeq 1$ then (33) is true, and (34) is true when:

$$\pi\left(p', p^N\right) + \pi\left(p^M, p'\right) > \pi\left(p^M, p^N\right) + \pi\left(p^M, p^M\right) \tag{35}$$

Now suppose $\rho = p'$. $S^L(p') = p^M$ is optimal iff p^M is at least as profitable as p^N,

$$\pi\left(p^M, p'\right) + \left(\frac{\delta}{1-\delta}\right)\pi\left(p^M, p^M\right) \geq \pi\left(p^N, p'\right) + \left(\frac{\delta}{1-\delta}\right)\pi\left(p^N, p^N\right), \tag{36}$$

and at least as profitable as p',

$$\begin{aligned}\pi\left(p^M, p'\right) + \left(\frac{\delta}{1-\delta}\right)\pi\left(p^M, p^M\right) \quad &\geq \quad \pi\left(p', p'\right) + \delta\pi\left(p^M, p'\right) \\ &\quad + \left(\frac{\delta^2}{1-\delta}\right)\pi\left(p^M, p^M\right).\end{aligned} \tag{37}$$

If $\delta \simeq 1$ then (36) and (37) hold. Finally, if $\rho = p^M$ then $S^L\left(p^M\right) = p^M$ is optimal iff:

$$\left(\frac{1}{1-\delta}\right)\pi\left(p^M, p^M\right) \geq \max\left\{\pi\left(p^N, p^M\right), \pi\left(p', p^M\right)\right\} + \left(\frac{\delta}{1-\delta}\right)\pi\left(p^N, p^N\right), \tag{38}$$

which holds if $\delta \simeq 1$. In sum, S^L is subgame perfect if $\delta \simeq 1$ and (35) holds.

Next, let us turn to S^F. If $\rho = p^N$ then $S^F\left(p^N\right) = p^N$ is optimal iff p^N is at least as profitable as p',

$$\pi\left(p^N, p'\right) + \delta\pi\left(p', p^M\right) + \left(\frac{\delta^2}{1-\delta}\right)\pi\left(p^M, p^M\right) \geq \pi\left(p', p'\right) + \delta\pi\left(p', p^M\right) + \left(\frac{\delta^2}{1-\delta}\right)\pi\left(p^M, p^M\right), \tag{39}$$

and is at least as profitable as p^M,

$$\pi\left(p^N, p'\right) + \delta\pi\left(p', p^M\right) + \left(\frac{\delta^2}{1-\delta}\right)\pi\left(p^M, p^M\right) \geq \pi\left(p^M, p'\right) + \left(\frac{\delta}{1-\delta}\right)\pi\left(p^M, p^M\right). \tag{40}$$

Both conditions hold for all δ.[46] If $\rho = p'$ then $S^F(p') = p'$ is optimal iff p' is at least as profitable as p^N,

$$\pi\left(p', p^M\right) + \left(\frac{\delta}{1-\delta}\right)\pi\left(p^M, p^M\right) \geq \pi\left(p^N, p'\right) + \left(\frac{\delta}{1-\delta}\right)\pi\left(p^N, p^N\right), \tag{41}$$

and is at least as profitable as p^M,

$$\pi\left(p', p^M\right) + \left(\frac{\delta}{1-\delta}\right)\pi\left(p^M, p^M\right) \geq \left(\frac{1}{1-\delta}\right)\pi\left(p^M, p^M\right). \tag{42}$$

(41) holds for $\delta \simeq 1$, and (42) holds for all δ. Finally, if $\rho = p^M$ then $S^F\left(p^M\right) = p^M$ is optimal iff (38) is true. In sum, S^F is subgame perfect if $\delta \simeq 1$.

[46] Note that $\pi\left(p^N, p'\right) > \pi\left(p', p'\right)$ for if that was not the case then p' would be a static Nash equilibrium and thereby violation the assumption that p^N is the unique Nash equilibrium. Similarly, it must be true that $\pi\left(p', p^M\right) > \pi\left(p^M, p^M\right)$.

To evaluate when (35) holds, consider:

$$\begin{aligned}
\pi\left(p', p^N\right) + \pi\left(p^M, p'\right) &> \pi\left(p^M, p^N\right) + \pi\left(p^M, p^M\right) \Leftrightarrow \\
\pi\left(\frac{p^M + p^N}{2}, p^N\right) - \pi\left(p^M, p^N\right) &> \pi\left(p^M, p^M\right) - \pi\left(p^M, \frac{p^M + p^N}{2}\right) \Leftrightarrow \\
-\int_{\frac{p^M+p^N}{2}}^{p^M} \left(\frac{\partial \pi\left(p, p^N\right)}{\partial p_1}\right) dp_1 > \int_{\frac{p^M+p^N}{2}}^{p^M} & \left(\frac{\partial \pi\left(p^M, p\right)}{\partial p_2}\right) dp_2. \qquad (43)
\end{aligned}$$

Assuming linear demand and constant marginal cost,

$$\pi\left(p_i, \mathbf{p}_{-i}\right) = \left(a - bp_i + d\left(\frac{1}{n-1}\right)\sum_{j\neq i} p_j\right)\left(p_i - c\right), \text{where } a > bc > 0, b > d > 0,$$

(43) is

$$\begin{aligned}
-\int_{\frac{p^M+p^N}{2}}^{p^M} \left(a + bc - 2bp_1 + dp^N\right) dp_1 &> \int_{\frac{p^M+p^N}{2}}^{p^M} d\left(p^M - c\right) dp_2 \Leftrightarrow \\
-\left(a + bc + dp^N\right)\left(\frac{p^M - p^N}{2}\right) + b\left[\left(p^M\right)^2 - \left(\frac{p^M + p^N}{2}\right)^2\right] &> d\left(p^M - c\right)\left(\frac{p^M - p^N}{2}\right)
\end{aligned}$$

which, after some manipulations, is equivalent to

$$3bp^M + bp^N > 2a + 2bc + 2dp^N + 2dp^M - 2dc. \qquad (44)$$

Substituting

$$p^N = \frac{a + bc}{2b - d},\ p^M = \frac{a + (b - d)\,c}{2\,(b - d)}$$

and again performing some manipulations, (44) is equivalent to

$$[a + (b - d)\,c]\left[(6b - 4d)\,(b - d) + d^2\right] + 2\,(b - 2d)\,(b - d)\,dc > 0. \qquad (45)$$

The first term is positive because $b > d$, while the second term is non-negative when $b \geq 2d$. Hence, if products are sufficiently differentiated then (45) is true. When instead $b < 2d$ then (45) holds when $c \simeq 0$. Hence, if cost is sufficiently small then (45) is true.

References

[1] Aoyagi, Masaki, "Collusion in Dynamic Bertrand Oligopoly with Correlated Private Signals and Communication." *Journal of Economic Theory*, 102 (2002), 229-248.

[2] Athey, Susan and Kyle Bagwell, "Optimal Collusion with Private Information." *RAND Journal of Economics*, 32 (2001), 428-465.

[3] Athey, Susan and Kyle Bagwell, "Collusion with Persistent Cost Shocks." *Econometrica*, 76 (2008), 493-540.

[4] Chamberlain, Edward, *The Theory of Monopolistic Competition*, Cambridge: Harvard University Press, 1938.

[5] Clark, Robert and Jean-François Houde, "Collusion with Asymmetric Retailers: Evidence from a Gasoline Price-Fixing Case," HEC Montréal, February 2011.

[6] Cooper, David J., "Barometric Price Leadership," *International Journal of Industrial Organization*, 15 (1997), 301-325.

[7] Cubitt, Robin P. and Robert Sugden, "Common Knowledge, Salience and Convention: A Reconstruction of David Lewis' Game Theory," *Economics and Philosophy*, 19 (2003), 175-210.

[8] Engel, Christoph, "How Much Collusion? A Meta-Analysis of Oligopoly Experiments," *Journal of Competition Law and Economics*, 3 (2007), 491-549.

[9] Fonseca, Miguel A. and Hans-Theo Normann, "Explicit vs. Tacit Collusion - The Impact of Communication in Oligopoly Experiments," Duesseldorf Institute for Competition Economics, January 2011 (*European Economic Review*, forthcoming)..

[10] Garrod, Luke, "Collusive Price Rigidity under Price-Matching Punishments," Loughborough University, September 2011.

[11] Gerlach, Heiko, "Stochastic Market Sharing, Partial Communication and Collusion," *International Journal of Industrial Organization*, 27 (2009), 656-666.

[12] Graham, Daniel A. and Robert C. Marshall, "Collusive Behavior at Single-Object Second-Price and English Auctions," *Journal of Political Economy*, 95 (1987), 1217-1239.

[13] Hanazono, Makato and Huanxing Yang, "Collusion, Fluctuating Demand, and Price Rigidity," *International Economic Review*, 48 (2007), 483-515.

[14] Harrington, Joseph E. Jr., "Posted Pricing as a Plus Factor," *Journal of Competition Law and Economics*, 7 (2011), 1-35.

[15] Harrington, Joseph E. Jr. and Andrzej Skrzypacz, "Private Monitoring and Communication in Cartels: Explaining Recent Collusive Practices," *American Economic Review*, 101 (2011), 2425-2449.

[16] Huck, Steffen, Hans-Theo Normann, and Jörg Oechssler, "Two are Few and Four are Many: Number Effects in Experimental Oligopolies," *Journal of Economic Behavior and Organization*, 53 (2004), 435-446.

[17] Ivaldi, Marc, Bruno Jullien, Patrick Rey, Paul Seabright, and Jean Tirole, "The Economics of Tacit Collusion," IDEI Toulouse, Final Report for DG Competition, European Commission, March 2003.

[18] Kalai, Ehud and Ehud Lehrer, "Rational Learning Leads to Nash Equilibrium," *Econometrica*, 561 (1993), 1019-1045.

[19] Kaplow, Louis and Carl Shapiro, "Antitrust," in *Handbook of Law and Economics, Volume 2*, A. Mitchell Polinsky and Steven Shavell, eds., Amsterdam: Elsevier B.V., 2007.

[20] Kaplow, Louis, "On the Meaning of Horizontal Agreements in Competition Law," *California Law Review*, 99 (2011a), 683-818.

[21] Kaplow, Louis, "An Economic Approach to Price Fixing," *Antitrust Law Journal,* 77 (2011b), 343-449.

[22] Kaplow, Louis, "Direct Versus Commnications-Based Prohibitions on Price Fixing," John M. Olin Center for Law, Economics, and Business, Harvard Law School, Discussion Paper. 703, July 2011c (forthcoming, *Journal of Legal Analysis*).

[23] Krishna, Vijay, *Auction Theory*, Second Edition, Amsterdam: Academic Press, 2010.

[24] Lewis, David, *Convention: A Philosophical Study*, Cambridge: Harvard University Press, 1969.

[25] Lu, Yuanzhu and Julian Wright, "Tacit Collusion with Price-Matching Punishments," *International Journal of Industrial Organization*, 28 (2010), 298-306.

[26] MacLeod, W. Bentley, "A Theory of Conscious Parallelism," *European Economic Review*, 27 (1985), 25-44.

[27] Markham, Jesse W., "The Nature and Significance of Price Leadership," *American Economic Review*, 41 (1951), 891-905.

[28] McCutcheon, Barbara, "Do Meetings in Smoke-Filled Rooms Facilitate Collusion?," *Journal of Political Economy*, 105 (1997), 330-350.

[29] Milne, Robert A. and Jack E. Pace, III, "The Scope of Expert Testimony on the Subject of Conspiracy in a Sherman Act Case," *Antitrust*, 17 (2003), 36-43

[30] Mouraviev, Igor and Patrick Rey, "Collusion and Leadership," *International Journal of Industrial Organization*, 29 (2011), 705–717.

[31] Page, William H., "Communication and Concerted Action," *Loyola University Chicago Law Journal*, 38 (2007), 405-460.

[32] Posner, Richard A., *Antitrust Law*, Second Edition, Chicago: University of Chicago Press, 2001.

[33] Rojas, Christian, "The Role of Information and Monitoring on Collusion," *RAND Journal of Economics*, 43 (2012), 78-109.

[34] Rotemberg, Julio J. and Garth Saloner, "Price Leadership," *Journal of Industrial Economics,* 39 (1990), 93-111.

[35] Scherer, F. M., *Industrial Market Structure and Economic Performance*, 2nd Edition, Boston: Houghton Mifflin, 1980.

[36] Wang, Zhongmin, "(Mixed) Strategy in Oligopoly Pricing: Evidence from Gasoline Pricing Cycles Before and Under a Timing Regulation," *Journal of Political Economy*, 117 (2009), 987-1030.