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Joao F. Gomes
University of Pennsylvania

Jeremy Greenwood

Sergio Rebelo

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Disciplines

Finance | Finance and Financial Management

Equilibrium Unemployment[&]

Joao Gomes[a], Jeremy Greenwood[b] and Sergio Rebelo[*,c]

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[a]Department of Finance, The Wharton School, University of Pennsylvania, Philadelphia, PA 19104, USA
[b]Department of Economics, University of Rochester, Rochester, NY 14627, USA
[c]Department of Finance, J.L. Kellogg Graduate School of Management, Northwestern University, Evanston, IL 60208, USA

Abstract
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Key Words: Search; Incomplete Markets; Business Cycles; Unemployment Insurance; Welfare Costs of Business Cycles.

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*Corresponding author. Tel: 847-467-2329; Fax: 847-491-5719; E-mail: s-rebelo@kellogg.nwu.edu

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1 Introduction

What determines the rate of unemployment and its movement over the business cycle? In the U.S. economy the unemployment rate moves countercyclically. So too does the average duration of unemployment, implying that it is easier to find a job in booms than in busts. Furthermore, the flows into and out of unemployment are positively correlated and move countercyclically. Are these key facts about unemployment behavior consistent with a general equilibrium labor search model in which individual job opportunities are affected by both aggregate and idiosyncratic shocks? Such a framework constitutes a natural model of the equilibrium rate of unemployment, and as such, perhaps is the ideal laboratory to examine such questions as the impact of unemployment insurance and the cost of business cycles.

The model in this paper is constructed along the lines of the classic search-theoretic models of unemployment developed by Lucas and Prescott (1974) and Jovanovic (1987).[1] A distinguishing feature of these models is that workers must *choose* whether to work or not at the prevailing wage. In the current paper, employed agents decide whether to keep their current job opportunities or search for better ones. Unemployed agents have to pick between accepting employment or continuing search. This model of job search is embedded into an Aiyagari (1994), Huggett (1997) and Laitner (1992) style model of incomplete markets. An individual's job opportunities are subject to both idiosyncratic and aggregate shocks. Agents cannot completely insure themselves against these shocks, given the lack of an Arrow-Debreu-McKenzie contingent-claims market. The best they can do is to smooth the effects of these shocks by borrowing and lending on an economy-wide capital market using bonds, subject to a borrowing constraint. Given the lack of full insurance, unemployment is meaningful in the sense that the unemployed are generally worse off than the employed.

The model is simulated to see if it can rationalize some key features of the U.S. labor market, both at the micro and macro levels. At the micro level the model has little trouble

[1] During the last decade there has been a resurgence of interest in unemployment models. The literature on search and matching models is reviewed in the Appendix.

matching the average rate and duration of unemployment observed in the U.S. economy. It does reasonably well matching some stylized facts about the effect of job displacement on subsequent labor earning and future job displacements. The model also does well at predicting individual income and consumption dynamics. At the macro level the search-theoretic paradigm presented here is consistent with the cyclical regularities of aggregate consumption, investment and output. It also does a good, but not perfect, job matching the business cycle facts governing the rate and duration of unemployment, and the flows into and out of both employment and unemployment.

Two applications illustrate the utility of the developed framework. The first quantifies the effects of unemployment insurance benefits. Changes in the level of benefits are found to have a large impact on both the unemployment rate and its average duration. The second application analyzes the welfare effects of economic fluctuations. Despite the presence of incomplete markets, business cycles can actually improve welfare in the search-theoretic paradigm developed here.

Here is an itinerary for the rest of the trip. Section 2 develops the model and provides some theoretical results. The model is then parameterized and calibrated in Section 3. The results for the calibrated version of the model are presented in Sections 4 and 5. Findings at the micro level are discussed in Section 4, and then attention turns to those at the macro level in Section 5. The model's implications for the welfare costs of unemployment insurance and business cycles are also addressed in these sections. Some concluding remarks are offered in Section 6, which takes stock of the main findings and discusses possible extensions for the model.

2 The Model

The economy is populated by a continuum of individuals distributed over the unit interval. Each agent seeks to maximize the expected value of lifetime utility:

$$E_0 \sum_{t=0}^{\infty} \beta^t U(\widetilde{c}_t - D(l_t)), \quad 0 < \beta < 1,$$

where $\widetilde{c}$ and l represent consumption and labor effort in the current period. The function $D(l)$ has the property $D(0) = 0$. For future reference it is useful to define consumption net of the disutility of working as

$$c = \widetilde{c} - D(l).$$

When employed, each agent derives income from working and past savings in the form of physical capital. Income can be used for consumption, saving for the future, and to pay taxes. The unemployed live off of past savings and unemployment insurance. An unemployed agent does not pay taxes. Agents can borrow and lend freely in an economy-wide capital market at the real rate of interest, r, subject to a borrowing constraint: the level of assets, a, has to be greater than a minimum level $\overline{a}$ so as to ensure that there is no default. A worker divides his time between work and leisure. In addition, physical capital depreciates at rate δ.

At the beginning of each period, every agent has a job opportunity represented by the production function $O(k, l; \varepsilon, \lambda)$, where l and k are inputs of labor and capital, and ε and λ represent aggregate and idiosyncratic technology shocks, respectively. An agent uses his own labor effort to operate the project. He rents capital from a competitive capital market at a rental rate of $(r + \delta)$. If the agent chooses to take this job opportunity he will earn labor income in the amount

$$\max_k \; [O(k, l; \varepsilon, \lambda) - (r + \delta)k].$$

The aggregate technology shock is drawn from the distribution function $F(\lambda'|\lambda) \equiv \Pr[\lambda_{t+1} \leq \lambda'|\lambda_t = \lambda]$; this is common to all production technologies in the economy. The function F is decreasing in λ (in the sense of first-order stochastic dominance).[2] The idiosyncratic shock for this job opportunity evolves according to the distribution function $G(\varepsilon'|\varepsilon) \equiv \Pr[\varepsilon_{t+1} \leq \varepsilon'|\varepsilon_t = \varepsilon]$, which is decreasing in ε. Take this distribution function as satisfying the Feller property.[3]

Agents are free to accept or reject their employment/production opportunities. Agents who decide to operate this technology are defined as employed. Unemployed agents do not work in the current period and search for a new production opportunity that comes on line next period. To simplify, assume that search is effortless and that a searcher cannot obtain more job offers or better job prospects by increasing the effort devoted to search. In addition, suppose that it is not possible to search on the job.

The timing of events is as follows. At the beginning of each period an agent has a job opportunity described by the pair (ε, λ). Depending on the values of ε and λ he will decide to accept or reject this opportunity. If the agent accepts it he earns labor income in the amount $O(k, l; \varepsilon, \lambda) - (r + \delta)k$. He pays lump-sum taxes in the amount τ. In addition, he receives the amount ra in rental income, where a denotes the units of physical capital accumulated by the agent. Given his capital and labor income, each agent decides how much to consume and save. Denote the value of the agent's idiosyncratic shock for the next period by ε'. If the agent accepts the current job opportunity, then ε' will be drawn from the distribution $G(\varepsilon'|\varepsilon)$.

If the individual instead rejects the job opportunity, he searches for a new production technology. The simplest job sampling rule is to allow a searching agent to sample one new job prospect per period. In line with simplicity, let the agent draw a new technology for operation next period from the distribution function $H(\varepsilon')$. When the agent rejects his job opportunity, he must live solely off his past savings, $(1 + r)a$, and unemployment insurance benefits, μ.

[2] In other words, if $\lambda_1 > \lambda_2$ then $F(\lambda^{\theta}|\lambda_1) \leq F(\lambda^{\theta}|\lambda_2)$, with the inequality holding strictly for some λ^{θ}.

[3] That is, for any continuous and bounded function $X(\cdot, \varepsilon)$ the function $\Xi(\cdot, \varepsilon) = \int X(\cdot, \varepsilon^{\theta})dG(\varepsilon^{\theta}|\varepsilon)d\varepsilon^{\theta}$ is also continuous and bounded.

At the beginning of each period individuals decide whether to work or search. Clearly, the values for the technology shocks, ε and λ, as well as the individual's wealth, a, are relevant for this decision. So too is the economy's distribution of wealth since this determines the rental rate on capital, $r + \delta$. Let $Z(a, \varepsilon)$ represent the cumulative distribution of agents over the state (a, ε). Suppose that this distribution function evolves according to some transition operator T so that $Z' = \mathsf{T}Z$. The equilibrium interest rate will be a function of the aggregate technology shock and the cross-sectional distribution of agents across states, so that $r = R(\lambda; Z)$. The government keeps unemployment insurance benefits fixed at the amount μ, while balancing its budget on a period-by-period basis. Therefore taxes must change with the state of the economy. Thus, let $\tau = T(\lambda; Z)$.

The expected lifetime utility of a worker and a searcher in state $(a, \varepsilon, \lambda; Z)$ are represented by $W(a, \varepsilon, \lambda; Z)$ and $S(a, \lambda; Z)$. Finally, $Y(\varepsilon, \lambda; Z)$ is the income earned by a worker net of the disutility of working so that

$$Y(\varepsilon, \lambda; Z) = \max_{k,l}[O(k, l; \varepsilon, \lambda) - (r + \delta)k - D(l)].$$

The decision rules for k and l are $K(\varepsilon, \lambda; Z)$ and $L(\varepsilon, \lambda; Z)$.

The choice problem for a worker is

$$\begin{aligned} W(a, \varepsilon, \lambda; Z) = {} & \max_{c,a^{\theta}}\{U(c) \\ & + \beta \int \max[W(a', \varepsilon', \lambda'; Z'), S(a', \lambda'; Z')] dG(\varepsilon'|\varepsilon) dF(\lambda'|\lambda) d\varepsilon' d\lambda'\}, \end{aligned} \quad \text{P}(1)$$

subject to

$$\begin{aligned} c + a' &= Y(\varepsilon, \lambda; Z) + [1 + R(\lambda; Z)]\, a - T(\lambda; Z), \quad (1) \\ a' &\geq \overline{a}, \end{aligned}$$

and $Z' = \mathsf{T}Z$. Here $c = \widetilde{c} - D(L(\varepsilon, \lambda; Z))$. The worker's decision rules for c and a' are $C^w(a, \varepsilon, \lambda; Z)$, and $A^w(a, \varepsilon, \lambda; Z)$.

The programming problem for a searcher is

$$S(a,\lambda;Z)=\max_{c,a'}\{U(c)+\ \beta\int\max[W(a',\varepsilon',\lambda';Z'),S(a',\lambda';Z')]dH(\varepsilon')dF(\lambda'|\lambda)d\varepsilon' d\lambda'\},$$

P(2)

subject to

$$\begin{aligned} c+a' &= [1+R(\lambda;Z)]\,a+\mu, \\ a' &\geq \overline{a}, \end{aligned}$$

and $Z'=\mathsf{T}Z$. Since searching requires no effort both l and $D(l)$ are zero for the searcher. The searcher's decision rules for consumption and asset accumulation read $c=C^s(a,\lambda;Z)$ and $a'=A^s(a,\lambda;Z)$. The lemma below establishes some properties on W and S.

Lemma 1 *The functions W and S exist, are continuously increasing in a, and W is continuously increasing in ε.*

Proof. See Appendix. ■

Clearly, an agent will choose to work in the current period if $W(a,\varepsilon,\lambda;Z)\geq S(a,\lambda;Z)$; otherwise he will search. Let $\Omega(a,\varepsilon,\lambda;Z)$ be the decision rule governing whether an individual works or not. This decision rule is specified by

$$\Omega(a,\varepsilon,\lambda;Z)=\begin{cases} 1, \text{ if } W(a,\varepsilon,\lambda;Z)\geq S(a,\lambda;Z), \\ 0, \text{ otherwise.} \end{cases} \tag{2}$$

An agent who finds himself in state $(a,\varepsilon,\lambda;Z)$ will save the amount

$$a'=A(a,\varepsilon,\lambda;Z)\equiv\Omega(a,\varepsilon,\lambda;Z)A^w(a,\varepsilon,\lambda;Z)+(1-\Omega(a,\varepsilon,\lambda;Z))A^s(a,\lambda;Z).$$

Last, the government maintains a balanced budget each period. This requires that

$$\mu\int[1-\Omega(a,\varepsilon,\lambda;Z)]dZ(a,\varepsilon)dad\varepsilon=\tau\int\Omega(a,\varepsilon,\lambda;Z)dZ(a,\varepsilon)dad\varepsilon. \tag{3}$$

The lefthand side gives the amount of benefits paid to unemployed individuals while the righthand side shows taxes paid by workers. Also, in a competitive equilibrium the demand

and supply of capital should always be equal. The market clearing condition for the capital market reads

$$\int K(\varepsilon,\lambda;Z)\Omega(a,\varepsilon,\lambda;Z)dZ(a,\varepsilon)dad\varepsilon = \int adZ(a,\varepsilon)dad\varepsilon. \tag{4}$$

The total demand for capital by working agents is represented by the lefthand side of the above the expression, while the righthand side gives the total supply from all agents.

The model's competitive equilibrium is defined now.

Definition *A competitive equilibrium is a set of decisions rules, A^w, L, K, A^s, Ω, a set of value functions, W, S, pricing and tax functions, R and T, and a law of motion for the aggregate wealth distribution, $Z' = \mathsf{T}Z$, such that:*

1. *The decision rules A^w, L, and K, and value function W, solve problem P(1), given the functions S, R, T and T.*

2. *The decision rule A^s, and value function S, solve problem P(2), given the functions W, R, T and T.*

3. *The work/search decision rule, Ω, is determined by (2), given W and S.*

4. *The government's budget balances and the capital market clears so that (3) and (4) hold.*

5. *The law of motion for the economy-wide distribution of wealth, or $Z' = \mathsf{T}Z$, is described by*

$$\begin{aligned} Z'(a',\varepsilon') &= \int\{I(A(a,\varepsilon,\lambda;Z)-a')[\Omega(a,\varepsilon,\lambda;Z)G(\varepsilon'|\varepsilon) \\ &\quad +(1-\Omega(a,\varepsilon,\lambda;Z))H(\varepsilon')]dZ(a,\varepsilon)dad\varepsilon\}, \end{aligned} \tag{5}$$

   where $I(x)=1$ if $x \leq 0$ and $I(x)=0$ otherwise.

2.1 Steady-State Results – No Aggregate Uncertainty

The presence of the $\max[W,S]$ operation on the righthand side of P(1) and P(2) greatly complicates the analysis of the model. Still with a few assumptions (satisfied in the computational results) some intuition about the model's economic mechanisms can be developed.

Define $J(a)$ as the value of the shock ε at which an agent is indifferent between working and searching. Formally this job threshold rule is defined by the equation

$$W(a, J(a)) = S(a), \tag{6}$$

where λ and Z have been dropped from the value functions given the focus on a deterministic steady state. Since W is monotonically increasing and continuous in ε, J will be a function and

$$W(a, \varepsilon) \ \mathsf{R} \ S(a) \text{ as } \varepsilon \ \mathsf{R} \ J(a).$$

To further develop intuition (in a heuristic way) make the following assumption:

Assumption *W and S are C^1 functions.*

By the implicit function theorem it then follows that $J(a)$ is a C^1 function too. It transpires that P(1) and P(2) will have the form

$$W(a, \varepsilon) = \max_{a^{\theta} \geq \overline{a}} \{U(Y(\varepsilon) + (1+r)a - \tau - a') + \beta[S(a')G(J(a')|\varepsilon) + \int_{J(a^{\theta})} W(a', \varepsilon')dG(\varepsilon'|\varepsilon)d\varepsilon']\},$$

and

$$S(a) = \max_{a^{\theta} \geq \overline{a}} \{U((1+r)a + \mu - a') + \beta[S(a')H(J(a')) + \int_{J(a^{\theta})} W(a', \varepsilon')dH(\varepsilon')d\varepsilon']\}.$$

Using the envelope theorem it then follows that

$$W_1(a, \varepsilon) = U_1(Y(\varepsilon) + (1+r)a - \tau - a')(1+r) = U_1(C^w(a, \varepsilon))(1+r), \tag{7}$$

and

$$S_1(a) = U_1((1+r)a + \mu - a')(1+r) = U_1(C^s(a))(1+r). \tag{8}$$

These two conditions can be used to gain some useful information about the consumption behavior of workers and searchers.

Lemma 2 *C^w and C^s are strictly increasing in a if and only if W and S are strictly concave in a.*

Proof. Immediate from conditions (7) and (8) together with the fact that U_1 is decreasing. ■
Decreasing marginal utility of wealth associated with strict concavity is a natural property for this type of environment, and it holds in all simulations of the model presented. Unfortunately, this cannot be established as a theoretical property of the model. The problem is that the function $\max[W, S]$ appearing on the righthand side of P(1) and P(2) may not be a concave function of a even if W and S are. Note that the presence of the idiosyncratic shock can be used to smooth out the kinks in the value functions that are due to the $\max[W, S]$ operation and allows strict concavity in W and S to be obtained. The Appendix discusses this in more detail.

The next lemma establishes that workers consume more than searchers, ceteris paribus, and that wealthier agents are choosier about the jobs they accept?

Lemma 3 *$J_1(a)$ R 0 and $C^w(a, J(a))$ R $C^s(a)$ if and only if $S_1(a)$ R $W_1(a, J(a))$.*

Proof. By applying the implicit function theorem to (6) it follows that $J(a)$ is increasing in a if and only if $S_1(a) > W_1(a, J(a))$ since $J_1(a) = [S_1(a) - W_1(a, J(a))]/W_2(a, J(a))$. Equations (7) and (8) imply $C^w(a, J(a))$ R $C^s(a)$ as $S_1(a)$ R $W_1(a, J(a))$. ■

Corollary 4 *$C^w(a, \varepsilon) > C^s(a)$, if $S_1(a) > W_1(a, J(a))$, W_1 is decreasing in ε, and $\varepsilon \geq J(a)$.*

Proof. From (7) it transpires that $C^w(a, \varepsilon)$ is increasing in ε if and only if W_1 is decreasing in ε. Now, when $\varepsilon = J(a)$ an agent is indifferent between working and searching and $C^w(a, \varepsilon) > C^s(a)$ by the previous lemma and the assumption that $S_1(a) > W_1(a, J(a))$. For higher values of ε the agent still prefers to work, and his consumption will be even larger given that W_1 is decreasing in ε. ■

Consider the case where $S_1(a) > W_1(a, J(a))$. At the job threshold an extra unit of wealth will be worth more to a searcher than a worker. Intuitively, this property would seem likely since a searcher must live solely off of his assets. It holds in all of the simulations conducted. Here wealthy agents will be choosier about accepting job opportunities. One interpretation of this result is that richer people are more willing to undertake riskier activities [Danforth (1979)]. Also, in this situation an agent will experience a drop in consumption upon crossing the threshold from work to search. It also seems natural that W_1 should be strictly decreasing in ε. For a worker a higher value for the shock implies higher current income, and a greater likelihood of higher future income, so that an extra unit of savings should be worth less. If this condition holds, a worker's consumption must always exceed a searcher's for the same level of wealth. Again, this property holds for all the experiments conducted.

The upshot of the above analysis is summarized below.[4]

Proposition 5 *$C^w(a,\varepsilon) > C^s(a)$ [when $\varepsilon \geq J(a)$] and $J(a)$ is strictly increasing in a, provided that W and S are strictly concave functions with $S_1(a) > W_1(a, J(a))$, and W_1 is decreasing in ε.*

These properties are portrayed in Figure 1, which plots data obtained from the simulated model.

Agents in the model are unable to insure perfectly against the possibility of becoming unemployed. Upon becoming unemployed they experience a drop in consumption. This clearly may affect an agent's saving behavior. Aiyagari (1994), Huggett (1997) and Laitner (1992) have illustrated how the presence of borrowing constraints in a model with heterogeneous agents leads to over savings in the sense that the equilibrium interest rate lies below the rate of time preference. Their argument would appear to apply here too.[5] Thus, one

[4] If W and S are strictly concave functions then a slight modification of the Benveniste and Scheinkman theorem (to allow for the borrowing constraint) can be used to show that these functions are continuously differentiable in a [see Aiyagari (1994)].

[5] A worker's asset accumulation is determined by the efficiency condition $U_1(C^w(a,\varepsilon)) \geq \beta(1+r)[\int_{J(a')} U_1(C^w(a',\varepsilon'))dG(\varepsilon'|\varepsilon)d\varepsilon' + \int^{J(a')} U_1(C^s(a'))dG(\varepsilon'|\varepsilon)d\varepsilon']$. Likewise, the searcher's asset accumulation is governed by $U_1(C^s(a)) \geq \beta(1+r)[\int_{J(a')} U_1(C^w(a',\varepsilon'))dH(\varepsilon')d\varepsilon' + \int^{J(a')} U_1(C^s(a'))dH(\varepsilon')d\varepsilon']$. These equations hold with equality whenever the borrowing constraint does not bind. Next, integrate both sides of the worker's Euler equation with respect to the stationary distribution Z over the part of the state space applying to him. Perform the analogous operation on the searcher's Euler equation. Sum the resulting equations. Use the definition of a stationary distribution on the righthand side of the resulting expression to get $\int\{\int_{J(a)} U_1(C^w(a,\varepsilon))dZ(a,\varepsilon)d\varepsilon + \int^{J(a)} U_1(C^s(a))dZ(a,\varepsilon)d\varepsilon\}da \geq \beta(1+r)\int\{\int_{J(a')} U_1(C^w(a',\varepsilon'))dZ(a',\varepsilon')d\varepsilon' + \int^{J(a')} U_1(C^s(a'))dZ(a',\varepsilon')d\varepsilon'\}da'$. But this can only hold if $\beta(1+r) \leq 1$ (assuming that the integrals are bounded). If the set of liquidity constrained agents has

cannot assume that in a steady state without aggregate risk $r = 1/\beta - 1$.

3 Calibration

The quantitative properties of the model's competitive equilibrium cannot be established analytically and must be developed via simulation. This task is made difficult by: (i) the form of programming problems P(1) and P(2) that are not readily amenable to linearization or linear-quadratic approximation techniques; and (ii) the necessity to include some measure of the cross-sectional distribution of wealth as a state variable. Computing the competitive equilibrium involves three steps. The first is to impose restrictions on the model's functional forms. The second is to determine as many parameters as possible either by matching properties of the model to U.S. data or by using prior empirical evidence. The last step is to develop a numerical algorithm capable of approximating the competitive equilibrium up to an arbitrarily small error. The first and second part of this procedure are described below. The numerical algorithm employed to simulate the competitive equilibrium is detailed in the Appendix.

The time period chosen for decision making is six weeks. This short time horizon seems appropriate given that the average duration of unemployment is about one quarter. Since most macro-data is only available at the quarterly frequency, the output of the model was aggregated up to this frequency. The functional forms for the production technology, the utility function and the stochastic processes for the shocks are described next.

3.1 Preferences

The momentary utility function is

$$U(\widetilde{c} - D(l)) = \frac{(\widetilde{c} - \frac{l^{1+\theta}}{1+\theta})^{1-\sigma} - 1}{1-\sigma}, \qquad \theta > 0, \sigma > 0. \tag{9}$$

strictly positive measure then the inequality is strict, implying that $(1 + r) < 1/\beta$. Thus, the possibility of over-accumulation continues to hold for this economy despite the presence of search. This argument was developed in Huggett (1997).

Preferences of this sort can be obtained from a more general setup with home production, as Benhabib, Rogerson and Wright (1991) show. In models of labor contracting, the employed typically end up worse off than the nonemployed. Nosel, Rogerson and Wright (1992) illustrate how these preferences can rectify this problem in the well-known Rogerson (1988) /Hansen (1985) indivisible labor model. These preferences are also useful in obtaining a countercyclical trade balance in models of small open economies, something that has proven difficult for the standard form of preferences, as Correia, Neves and Rebelo (1995) demonstrate. Last, Devereux, Gregory and Smith (1992) show how this utility function can rationalize the observed pattern of comovement in income and consumption across countries.

The parameters σ and θ can be interpreted as the coefficient of relative risk aversion and the (inverse) labor supply elasticity. Therefore, σ was set equal to 2, a value within the acceptable range specified by Mehra and Prescott (1985), and θ was set equal to 10, implying a labor supply elasticity of about 0.1, also reasonable according to Ghez and Becker (1975) or MaCurdy (1981) for example. Finally, the intertemporal discount factor β was set to $1/1.06^{1/8}$, a value that is consistent with an equilibrium annual interest rate of (approximately) 6%.

3.2 Technology

The production function available to each individual is assumed to be

$$O(k, l; \varepsilon, \lambda) = \exp(\lambda + \varepsilon) k^{\alpha} l^{1-\alpha},$$

where α denotes the share of capital in production. Following Cooley and Prescott (1995) this share was set equal to 0.36, a value that is also consistent with the large majority of related studies. The depreciation rate δ is set to 0.006 per period (approximately 5% per annum), a value also consistent with the recommendations of Cooley and Prescott (1995).

3.3 Technology Shocks

Aggregate Shocks:

The properties of the aggregate technology shock, λ, are summarized by a three-point Markov chain. This chain is chosen to approximate, using the Tauchen and Hussey (1990) algorithm, an AR(1) process with serial correlation ρ_λ and standard deviation σ_λ. Additionally, $E[\exp(\lambda)] = 1$. The parameters ρ_λ and σ_λ, are restricted by the requirement that the stochastic process for the Solow residual generated by the model,

$$\ln \mathsf{z} = \ln \mathsf{y} - (1-\alpha) \ln \mathsf{l},$$

roughly matches the first-order serial correlation coefficient, ρ_{z}, and the standard deviation, σ_{z}, of the Solow residual, $\ln \mathsf{z}$, as computed by Cooley and Prescott (1995).[6] Specifically, Cooley and Prescott (1995) report that $\sigma_{\mathsf{z}} = 0.0224$ and $\rho_{\mathsf{z}} = 0.950$, while the numbers obtained here are $\sigma_{\mathsf{z}} = 0.0216$ and $\rho_{\mathsf{z}} = 0.905$. Section 5 will discuss the fact that the aggregate technology shock, λ, does not coincide with the logarithm of the Solow residual, $\ln \mathsf{z}$.

Idiosyncratic Shocks:

The worker's shock ε is assumed to evolve according to

$$\varepsilon' = \rho_\varepsilon \varepsilon + \xi,$$

where $\xi \sim N(0, \sigma_\varepsilon^2)$. A searcher draws a value of ε in line with

$$\varepsilon = \nu,$$

where $\nu \sim N(0, \sigma_v^2)$.

Notice that there are only three new parameters introduced relative to a standard real business cycle model (ρ_ε, σ_ε, σ_v). The properties assumed for the idiosyncratic shocks have implications for the average rate and duration of unemployment in the economy. The parameters governing the stochastic processes for the idiosyncratic shocks are chosen to be in accordance with two criteria. First, the model's average rate of unemployment (6.1%)

[6] The lower case bold letters denote aggregate variables.

is close to the average U.S. unemployment rate (5.9%). Second, the average duration of unemployment in the model (11 weeks) is also similar to the average duration in the U.S. (13 weeks). These parameter values will also have implications for the model's predictions about income dynamics at the level of the individual. These predictions will be compared with some evidence from the U.S. data in Section 4.2.

3.4 Government Policy

The final step in the calibration procedure is to specify the details of the unemployment insurance policy. Unemployment compensation is fully described by the following policy for transfers, μ:

$$\mu = \begin{cases} 0, & \text{if employed,} \\ \eta \mathsf{y}^*, & \text{if unemployed,} \end{cases}$$

where y^* is the average (per capita) income in the economy *without* fluctuations and η can be interpreted as the (average) replacement ratio. The value of η adopted was 0.5, which provides a reasonable approximation to the U.S. unemployment insurance system according to Hansen and Imrohoroglu (1992). Given this replacement ratio the requirement that the government balances its budget in every period determines the equilibrium level of taxes in the economy.

Table 1 summarizes the model's baseline calibration.

4 Micro-Level Findings

4.1 Unemployment and Employment

What are the job prospects of an agent who becomes unemployed? To address this question, turn off the aggregate technology shock ($\sigma_\lambda = 0$) and focus on the resulting stationary distribution for the model. Figure 2 shows the cumulative hazard rates associated with exiting from unemployment. Sixty five percent of agents exit the pool of unemployed one period after entry with another 22% leaving after two periods. The model, therefore, is consistent with the observation by Clark and Summers (1979) that a large fraction of unemployment spells (79% in 1969, 60% in 1974 and 55% in 1975) end within one month. Hall (1995) argues that the Clark-Summers exit rates exaggerate the extent to which it is easy to find employment because they include workers who take temporary jobs. Those temporary jobs are, however, also present in the model. There are workers who accept jobs with a short expected duration because their productivity is just slightly above their reservation productivity.

Table 2 compares Ruhm's (1991) estimates of the effects of job displacement on the subsequent level of earnings and future spells of unemployment with the model's implications for the same variables.[7] This table shows that in the U.S. economy a displaced worker tends to experience an additional 8.35 weeks of unemployment in the displacement year, 4.32 weeks in the following year, etc. The model is consistent with the post-displacement behavior of unemployment in the U.S. data. The effects of entering the unemployment state on the wage are slightly more severe in the model than in the data.

Dynarski and Sheffrin (1987) report that consumption drops when an individual becomes unemployed. Gruber (1997) estimates the drop in food consumption to be 6.8% under the current unemployment insurance system and 22% in its absence. These findings are usually interpreted as evidence of the lack of full insurance against the possibility of unemployment.[8] The model is consistent with this property of consumption behavior: in the model agents reduce their annual consumption by 6% upon becoming unemployed.

A key implication of the model is that an agent's threshold job productivity is increasing in his financial wealth, a. Richer agents are more selective about the jobs that they are willing to take. In a recent study Rendón (1996) has found empirical evidence for this effect

[7] One caveat with this comparison is that the Ruhm (1991) data excludes quits, including only workers displaced as a result of plant closings and layoffs. Ideally the model's implications should be compared with the effects of total job separations, not just displacements.

[8] This consumption drop is consistent with complete markets, however, if the marginal utility of consumption is declining in leisure as in (9). This is not a property of standard utility functions such as a Cobb-Douglas in consumption and leisure.

in a sample of white male high school graduates who did not attend college.[9]

Finally, the model is consistent with the findings by Abraham and Farber (1987) and Altonji and Shakotko (1987) that labor income increases modestly with tenure. While there are other explanations for the effect of tenure on wages, such as on-the-job learning, it is worth noting that a search model can also generate this effect. This results from the persistent character of the worker's idiosyncratic shock. Workers who find a high-productivity job are less likely to see their productivity fall below the job threshold in the near future. In other words, workers in high-paying jobs are likely to remain there longer than workers in low-paying ones. The slope of the regression of (ln) wages on tenure (measured in years) generated by the model is 0.2%. This estimate compares well with the Abraham and Farber (1987) one that labor earnings rise for every year on the job by 0.3% for blue collar nonunion workers and 0.6% for managerial and professional nonunion ones.[10]

4.2 Income Dynamics

What are the model's predictions for income dynamics? Income dynamics for an individual are influenced by the idiosyncratic shock parameters ρ_ε, σ_ε, and σ_v. The reasonableness of the choices made for these parameters can be gauged by comparing the model's predictions for income dynamics with estimates from the U.S. data. Heaton and Lucas (1996) estimate a process for individual income using annual data from the PSID for the period 1969-84. They used an equation of the form[11]

$$\ln(y_{it}/y_{it-1}) = \upsilon_0 + \upsilon_1 \ln(y_{it-1}/y_{it-2}) + \upsilon_2 \ln(\mathsf{y}_t/\mathsf{y}_{t-1}) + \mu_{it}.$$

[9] Rendón's (1996) findings may be contaminated by unobserved human capital elements – for example wealthier individuals may have more highly educated parents who devoted more time to their offspring's education.

[10] See equation 2 in Tables 4a and 4b.

[11] Here y_{it} denotes individual's i income (which includes unemployment insurance) and y_t is aggregate income.

This equation can also be estimated using simulated data from the model.[12] To do this, the model's data must be aggregated up to the annual level. Like the Heaton and Lucas (1996) study, the measure of income used includes unemployment insurance and taxes. The results of this exercise are reported in Table 3. The model does fairly well in accounting for most of the persistence and volatility in individual income.

Hubbard, Skinner and Zeldes (1995) also estimate the process governing individual income. They specify an AR(1) equation of the form $\ln(y_{it}) = \nu_1 \ln(y_{it-1}) + \mu_{it}$. This equation can also be estimated using time-aggregated simulated data from the model. The resulting estimate is $\nu_1 = 0.50$ and $\sigma_{\mu_i} = 0.19$. This compares with Hubbard, Skinner and Zeldes's (1995) estimate of $\nu_1 = 0.95$ and $\sigma_{\mu_{it}} = 0.14$ for high school graduates (Table 2).[13] Given the high degree of persistence, the Hubbard, Skinner and Zeldes (1995) estimate implies greater long-run variability in labor income than does the Heaton and Lucas (1996) one. Given that the Heaton and Lucas (1996) and Hubbard, Skinner and Zeldes (1995) estimates differ, it is no surprise that the model can only match one of them.

The model's ability to match the income and wealth statistics is very similar to that of Aiyagari's (1994) model.[14] Here the coefficient of variation in income across individuals is 0.21, a little below the value of 0.28 ($= 0.241/\sqrt{1 - 0.527^2}$) that obtains from the Heaton and Lucas (1996) estimate in Table 3. As one might expect, however, the model can not generate nearly enough skewness in the distribution of wealth. The Gini coefficient for wealth is only 0.38, significantly smaller than the value of 0.81 that Kessler and Wolff (1991, Table 3) observe in the U.S. economy. Since the analysis abstracts from such important issues as lifecycle savings, human capital formation and family dissolution, this is no surprise.

[12] The aggregate shock is reintroduced here.

[13] Unfortunately for the purposes here, they run this regression for different education classes. Their median estimate is focused on here.

[14] The aggregate shock is now shut down again.

4.3 Comparative Statics Results

Comparative statics experiments can provide some intuition regarding the economic mechanisms at work in the model. The qualitative results of these experiments are summarized in Table 4. The discussion focuses on the parameters that exert the strongest influence on the rate of unemployment — the replacement ratio, η, and the parameters that determine the distribution of the idiosyncratic shocks, σ_ν, ρ_ε, and σ_ε. The different rows report the new values of the unemployment rate that result from changing each individual parameter to the value indicated.

4.3.1 Idiosyncratic shocks

If the variance of the worker's shock, σ_ε, is reduced it becomes less likely that a worker will experience bad luck with his job. This reduces the number of agents engaging in job search, lowering the rate of unemployment. Reducing σ_ε by 20% results in a large decline in the unemployment rate: a drop from 6.1% to 4.9%. Note that without idiosyncratic shocks there would be no unemployment in the model. Nobody would expect that they could improve their lot by quitting their current job and searching for a better one because all jobs would be the same.

Reducing the persistence of the worker's shock, ρ_ε, lowers the rate of unemployment. When shocks are less persistent a worker who receives a bad shock is less likely to remain in a low-productivity state. This raises the opportunity cost of search. When ρ_ε declines, a worker who receives a high ε is less likely to remain highly productive for a longer period of time. This lowers the rewards to searching. Both of these effects conspire to make search less attractive, lowering the unemployment rate. A small (5%) change in the persistence parameter has a large impact on the equilibrium rate of unemployment.

A decline in the standard deviation of the searcher's idiosyncratic shock, σ_ν, reduces the rate of unemployment because the value of search has now declined. The probability of getting a good job has decreased. Note that while the odds of getting a bad job offer have also been reduced, the agent does not have to accept bad realizations of the idiosyncratic shock. Thus, a decline in the variance of job seeker's shock lowers the option value of search.

The message from the above experiments is that idiosyncratic shocks are important for determining the equilibrium rate of unemployment. Thus, improving the calibration of the idiosyncratic shock process is likely to be an important step for future research.

4.3.2 Application 1: The Role of Unemployment Insurance

An interesting application of this model is to study of the economic consequences of different unemployment insurance (UI) schemes. The benchmark replacement ratio is 50%, a value consistent with the evidence for the U.S. In contrast, many European countries adopt replacement ratios around 70% [Martin (1996)]. The model does not distinguish between unemployment due to quits and layoffs. On the one hand, all job separations are voluntary and could be labelled as quits. On the other hand, all separations are due to lost productivity and, on this account, could be viewed as layoffs.[15] In reality only worker who get laid off are eligible for unemployment benefits. Therefore, for this application it might have been better to calibrate the model to the average rate and duration of unemployment due to layoffs. With this caveat in mind, Table 4 illustrates that a change in the replacement rate can have a dramatic effect on the rate of unemployment. With this change alone the model can account for most of the difference between the average unemployment rate in the U.S. (6.1% in the benchmark calibration) and that of a typical European economy (13.9%). The average duration of unemployment also rises from 11 weeks in the benchmark economy to over 14 weeks in the high unemployment insurance model. About 20% of unemployment spells now last at least 6 months. Finally, increasing the replacement rate leads to a decline in welfare equal to 4.4% of consumption – net of the cost of supplying labor effort.[16]

[15] Suppose productivity fell close to zero. Is it reasonable to expect someone to work for next to nothing?

[16] This welfare loss was computed using the same procedure described in Section 5.2 to evaluate the welfare costs of cyclical fluctuations.

5 Macro-Level Findings

5.1 Business Cycle Facts

Solving the model with aggregate shocks requires keeping track of the evolution of the wealth distribution, Z. In order to make their decisions, individuals have to forecast the future values of the real interest rate. These values are influenced by the wealth distribution, Z. The algorithm employed to compute the equilibrium of this economy is described in the Appendix. It approximates the distribution Z by the mean level of wealth in the economy. By (4) the mean level of wealth in the economy equals the aggregate capital stock, k.

The behavior of unemployment in the model depends critically on the response of the threshold rule to an aggregate productivity shock. This rule now takes the form $\varepsilon = J(a, \lambda, Z) = J(a, \lambda, \mathsf{k})$. It is difficult to predict theoretically the response of this threshold rule to a change in λ because there are two contradictory effects at play. When λ rises, the opportunity cost of search increases, which should lead to less search. Yet, since λ is serially correlated, an increase in λ raises the conditional dispersion of future productivity $\exp(\varepsilon' + \lambda')$. As the comparative statics experiments of Section 4.3 have shown, this increased dispersion raises the option value of search, which should lead to more search. Figure 3 depicts the threshold rules for the three possible values of λ and for a value of k equal to the mean value obtained in the simulations. This figure shows that the opportunity cost effect dominates: when λ increases the threshold line shifts downward, leading to less search.

In the model recessions have both a "cleansing" and a "sullying" effect. These effects can be gleaned from Figure 3. The cleansing effect of recessions results from the following. In expansions, jobs with low idiosyncratic productivity are not abandoned because agents want to take advantage of the temporarily high aggregate productivity. For any given level of assets the threshold value for the idiosyncratic shock rises, other things equal. It is in recessions that these low-productivity jobs are eliminated as workers search for better opportunities. The sullying effect of recessions results from wealth dropping during recessions, thus increasing agents' willingness to accept low-productivity jobs.[17] As Figure 3 shows the threshold value for the idiosyncratic shock falls with assets, *ceteris paribus*.[18] Which effect dominates on net? The mean value of the idiosyncratic shock *across workers* moves countercyclically in model. Its correlation with output is -0.82. Therefore, the cleansing effect dominates.

The cleansing effect has implications for the measurement of Solow residuals. Recall that λ does not coincide with the logarithm of the Solow residual, measured as the logarithm of aggregate output minus the product of labor's share of income and total hours worked. The source of the discrepancy is the cyclical movement in the threshold rule depicted in Figure 3. In an expansion many low-ε production opportunities are retained, only to later be discarded in a recession. Therefore the volatility of the measured Solow residual underestimates the volatility of the true productivity shock.[19] Hence, on this account, aggregate productivity shocks may be larger than they appear to be.

Figures 5, 6, and 7 depict the average response of the system across all instances in the 1,500 agent, 10,000 period simulation in which the aggregate productivity shock transited from its mean value to its high value. The diagrams portray the response of the system to a 1% increase in the shock. Figures 8, 9, and 10 report the same information for transitions from the mean value of the shock to its lowest value. These figures are analogous to impulse response functions. They show clearly that the model is capable of retaining the successful features of real business cycle models, in terms of the behavior of consumption, output and investment, while, at the same time, being consistent with some key regularities of

[17] Barlevy (2000) stresses an alternative mechanism (on-the-job search) that also generates a sullying effect of recessions.

[18] Additionally, in recessions the aggregate capital stock falls. This lowers the threshold value for the idiosyncratic shock as well. Figure 4 depicts the threshold rules for two values of k, the mean plus or minus four standard deviations. An increase in the aggregate capital stock lowers the real interest rate. This increases the probability that an individual will work for two reasons: (i) the rental price of capital drops, raising the value of labor income associated with a given idiosyncratic productivity level; and (ii) it reduces ra, thus lowering end-of-period wealth. The figure shows, however, that these effects are quantitatively small.

[19] Recall that values of ρ_λ and σ_λ^2 were chosen so that the Solow residual, as conventionally measured using simulated data generated by the model, exhibits roughly the same serial correlation and variance reported by Cooley and Prescott (1995) for their estimate of the Solow residual for the U.S. economy.

unemployment behavior.

Figures 6 and 9 show that unemployment is clearly countercyclical. This is what the threshold rules in Figure 3 had suggested: workers are willing to accept a low-ε job when aggregate productivity is high. The flow into unemployment declines in an expansion as workers become less willing to quit their jobs. Average duration declines in the first period as searchers become more inclined to accept low-ε offers. Further declines take place up until four periods, reflecting the fact that most of the agents who become unemployed in period one accept jobs in periods two, three and four. The flow out of unemployment increases slightly initially, as agents become employed to take advantage of the high aggregate productivity. This flow subsequently declines as the number of unemployed workers is sharply reduced.

Table 5 shows that the volatility and comovement of consumption, output and investment are similar to those in the U.S. data. In both Table 5 and Table 6, discussed below, the model and U.S. data series are detrended using the Hodrick-Prescott filter with a smoothing parameter of 1,600. The degree of amplification generated by the model can be measured in two different ways. The first measure is the ratio of the (HP-detrended) standard deviation of output relative to the standard deviation of the Solow residual. This statistic is 0.59 for Hansen's (1985) standard divisible labor model and 0.51 for the baseline search model. The second measure is the ratio of the (HP-detrended) standard deviation of output relative to the standard deviation of true technology shocks. This statistic is 0.59 for the standard model and 0.25 for the search model. According to the first measure the model provides the same amplification as standard real business cycle (RBC) models, while according to the second measure, the model provides much less amplification than RBC models. The first-order serial correlation of (HP filtered) output in the model is 0.53, which is typical for RBC models.

Table 6 confirms that the model reproduces the comovement patterns of labor market variables that characterize U.S. data. Average hours and employment are procyclical, while the unemployment rate, and the duration of unemployment are countercyclical. Employment is a little less volatile in the model than it is in the data. A version of Okun's law for the model economy implies that a fall in GDP by 2.1% corresponds to a 1% rise in unemployment. This relation is thought to be 2 for 1 in the U.S. data.

One salient feature of labor market data is the countercyclical character of flows into and out of unemployment. This feature has been documented for the U.S. [Davis, Haltiwanger and Schuh (1996), Merz (1996)] and for several European countries [Burda and Wyplosz (1994)]. The model matches the negative correlation between unemployment flows and output and correctly predicts the relative magnitudes of these correlations. The model generates too much volatility in these series, however, and implies a similar volatility for inflows and outflows. Blanchard and Diamond (1990) stress that in the U.S. flows into unemployment are more volatile than flows out of it. Merz (1996) has, however, recently disputed these findings, arguing that the difference between the volatility in these two flows is not statistically significant.

The model does a slightly better job in matching the volatility of flows into and out of employment. It fails to replicate the procyclical nature of flows into employment (or the negative correlation between employment inflows and outflows). There may be a good reason for this failure. In the real world, many individuals may choose not to participate in the labor force. Flows between employment and nonparticipation may be as large as flows between employment and unemployment. The model does not permit nonparticipation. Hence, flows into (out of) unemployment must equal flows out of (into) employment. Allowing for a home state, along the lines Andolfatto and Gomme (1996), may improve the model's ability to match labor market flows.

5.2 Application 2: The Welfare Cost of Business Cycle Fluctuations

5.2.1 The Lucas (1987) Calculation

In a classic work, Lucas (1987) calculates the potential welfare benefits of business cycle stabilization. To set the stage for the current analysis, it is fruitful to go through his calculation for the model economy developed here. Imagine a representative agent living in world where *aggregate* consumption, $\widetilde{\mathsf{c}}$, follows some stationary stochastic process. Denote the long-run mean and variance of this process by $\overline{\widetilde{\mathsf{c}}}$ and $\sigma^2_{\widetilde{\mathsf{c}}}$. How much would the agent be willing to pay to eliminate all business cycle risk?

To answer this question, denote expected utility in a world with business cycles by[20]

$$E[\sum_{t=0}^{\infty}\beta^t U(\widetilde{\mathsf{c}}_t)].$$

By taking a second-order Taylor expansion of the momentary utility function (and dropping the remainder term), expected lifetime utility can be expressed as

$$E[\sum_{t=0}^{\infty}\beta^t U(\widetilde{\mathsf{c}}_t)] \simeq \frac{1}{1-\beta}\{U(\overline{\widetilde{\mathsf{c}}}) + U_{11}(\overline{\widetilde{\mathsf{c}}})\sigma^2_{\widetilde{\mathsf{c}}}/2\}.$$

Thus, the per-period benefit (expressed in units of consumption as a fraction of average consumption) from eliminating the variability in consumption is

$$\frac{1}{2}\frac{U_{11}(\overline{\widetilde{\mathsf{c}}})\sigma^2_{\widetilde{\mathsf{c}}}}{\overline{\widetilde{\mathsf{c}}}} \times \frac{1}{U_1(\overline{\widetilde{\mathsf{c}}})} = \frac{1}{2}\frac{\overline{\widetilde{\mathsf{c}}}U_{11}(\overline{\widetilde{\mathsf{c}}})}{U_1(\overline{\widetilde{\mathsf{c}}})}\frac{\sigma^2_{\widetilde{\mathsf{c}}}}{\overline{\widetilde{\mathsf{c}}}^2} = \frac{1}{2}\sigma(\frac{\sigma_{\widetilde{\mathsf{c}}}}{\overline{\widetilde{\mathsf{c}}}})^2, \text{ [cf. Lucas (1987, eq. 8)]} \tag{10}$$

where σ is the coefficient of relative risk aversion.[21]

In the model economy the coefficient of variation for aggregate consumption, $\sigma_{\widetilde{\mathsf{c}}}/\overline{\widetilde{\mathsf{c}}}$, is 0.0113 (per quarter). The potential welfare benefits from eliminating the variability in aggregate consumption would therefore amount to 0.013% of aggregate consumption, given that $\sigma = 2$. This is extremely close to the number derived by Lucas (1987).[22] This calculation does not factor in that leisure also fluctuates over the business cycle, a point recognized by Lucas (1987, p. 28). Effective aggregate consumption fluctuates in the model by 0.08% (its coefficient of variation×100).[23] Thus, the welfare benefits don't change appreciably when leisure is taken into account.

[20] To be clear, note that utility is defined as a function of aggregate consumption, $\widetilde{\mathsf{c}}$, not aggregate consumption net of aggregate labor effort, l, or $\mathsf{c} = \widetilde{\mathsf{c}} - D(\mathsf{l})$.

[21] In Lucas (1987) this formula gives the exact benefits of reducing variability in aggregate consumption, given the assumed stochastic process. Aiyagari (1994) uses it as an approximation.

[22] Lucas (1987) estimates the coefficient of variation to be 0.013, a number similar to that obtained in the model economy. Lucas does not do the welfare calculation for $\sigma = 2$, just for $\sigma = 1$ and $\sigma = 5$. Lucas also uses equation (10). Therefore, plugging his number into this formula gives a welfare cost of 0.017%. The costs he reports for $\sigma = 1$ and $\sigma = 5$ are 0.008% and 0.042%.

[23] Effective aggregate consumption is defined by $\mathsf{c} = \widetilde{\mathsf{c}} - D(\mathsf{l})$, where l is aggregate labor effort.

5.2.2 General Equilibrium Results

"It is remarkable about how much one can say about the importance of macroeconomics questions on the basis of preferences alone," Lucas (1987, p. 20) has noted. So, how do general equilibrium considerations refine this answer? To address this question, let

$$E_b[\sum_{t=0}^{\infty} \beta^t U(c_t)],$$

represent the expected utility for an agent living in the economy with business cycles shocks.[24]
Similarly, let

$$E_n[\sum_{t=0}^{\infty} \beta^t U(c_t)],$$

denote the expected utility for an agent living in the economy with no business cycles shocks; i.e., one where $\sigma_\lambda = 0$ and $E[\exp(\lambda)] = 1$. So how much would an agent have to be compensated to move from the economy without business cycle shocks to the one with them. This compensating variation, ϖ, is defined (in proportional terms) by the equation

$$E_b[\sum_{t=0}^{\infty} \beta^t U(c_t)] = E_n[\sum_{t=0}^{\infty} \beta^t U(\varpi c_t)].$$

Given the form of the momentary utility, ϖ can be expressed simply by

$$\varpi = \{\frac{E_b[\sum_{t=0}^{\infty} \beta^t c_t^{1-\sigma}]}{E_n[\sum_{t=0}^{\infty} \beta^t c_t^{1-\sigma}]}\}^{1/(1-\sigma)}.$$

For the model economy $\varpi - 1 = 0.0056$. Thus, an agent would be willing to *pay* 0.56% of his consumption in order to move to the economy *with* aggregate fluctuations! Consequently, the *actual* benefits from eliminating aggregate fluctuations are much lower (they are actually a loss here) than the *potential* benefits reported by Lucas (1987). The Lucas calculations were intended as an *upper bound* on the welfare benefits of eliminating aggregate fluctuations, a fact often forgotten.[25]

[24] Now, c is defined as *individual* consumption net of the disutility of working so that $c = \widetilde{c} - D(l)$.

[25] To quote Lucas (1987, p. 27): "I want to propose taking these numbers seriously as giving the order-of-magnitude of the potential marginal social product of additional advances in business cycle theory — or more accurately, as a loose upper bound, since there is no reason to think that eliminating *all* consumption variability is either a feasible or desirable objective of policy."

Although the exact computation is cumbersome the basic intuition for this results is fairly simple and can be understood by looking at the first panel in Figure 1. The agent desires to work when high values for ε or λ are drawn and search when draws are low. Hence, an agent's expected lifetime utility is given by the outer envelope of the S and W curves. Since this envelope is convex the agent likes risk.[26] In response to a mean-preserving spread in risk the agent becomes choosier about the job he accepts. He now has better odds of drawing a good job prospect. This is true for drawing bad jobs as well, but the agent has the option of rejecting them. An example of this basic mechanism for a simplified environment is provided in the Appendix.

6 Conclusions

A search model of equilibrium unemployment was developed here. Job opportunities are subject to both aggregate and idiosyncratic productivity shocks. Each period a worker decides whether to stay with his current job or quit and search for a better one. Likewise, a unemployed person chooses whether to accept his current job offer or to continue searching for a better prospect. Financial markets are incomplete so individuals must self insure against the possibility of unemployment by building up their savings.

The framework is successful in accounting for some key labor market regularities, both at the macro and micro levels. The model is calibrated to match the average rate and duration of U.S. unemployment. It can be judged on how well it matches other features of the data. Consider the micro-level findings first. In the U.S. most spells of unemployment last less than a month. The model is consistent with these rapid exit rates from unemployment. Additionally, the model also successfully mimics the impact that a spell of unemployment has both on subsequent wages and future spells of unemployment. The pattern of individual-level income dynamics generated by the model fits the U.S. data well. In the U.S. data

[26] The W curve is in fact slightly convex because the reduced-form production function is convex in the shock, as seen in equation (19). Even if W were concave, the effect of the kink imparted from the $\max[W, S]$ operation works to put a nonconcave zone in an agent's expected lifetime utility.

consumption drops significantly when an individual enters unemployment. This same drop in consumption, accompanied by lower welfare, takes place in the model when agents become unemployed. Finally, the model predicts that changes in the unemployment insurance replacement ratio have a significant impact on both the average rate and duration of unemployment. At the macro level, the model does quite well in duplicating the standard set of business cycle facts summarizing fluctuations in output, consumption, investment and hours worked. It also replicates the countercyclical nature of the unemployment rate and its duration, as well as the movements in the flows into and out of unemployment. Last, the model can be used to gauge the welfare cost of business cycles. In search-theoretic models aggregate fluctuations may actually improve welfare, notwithstanding the absence of complete markets.

Naturally, the model can be improved. In the U.S. flows between employment and nonparticipation may be as large as flows between employment and unemployment. A home state, representing withdrawal from the labor market, could be added. A home state may also help the model replicate the procyclical nature of flows into employment, and the negative correlation between employment inflows and outflows. There are also some important features of the labor market that the model is not equipped to address. The model abstracts from vacancies; the number of new job openings always equals the number of unemployed workers. This prevents the model from confronting regularities such as the negative relation between vacancies and unemployment (the Beveridge curve). One way to proceed here may be to build a bilateral search model where entrepreneurs search for workers and workers search for entrepreneurs. With a home state the number of entrepreneurs and workers in the labor market would be variable. A vacancy occurs whenever an entrepreneur is searching for a worker. For a job to be filled *both* the parties must agree. Here, a job is like a marriage between the entrepreneur and the worker. A separation is really a divorce between the entrepreneur and worker. Additionally, such a framework will permit a distinction between quits and layoffs. A quit occurs when the worker severs the relationship with the entrepreneur, while a layoff happens when the entrepreneur terminates the relationship.

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A Appendix

A.1 Literature Review

One widely used class of unemployment models builds upon the influential *matching* paradigm of Mortensen and Pissarides (1994). Examples that incorporate this paradigm into real business cycle models include Andolfatto (1996) and Merz (1995). The Andolfatto (1996) model does well at mimicking the Beveridge curve, the statistical relationship between unemployment and vacancies. Den Haan, Ramey and Watson (2000) endogenize the job destruction rate in this framework by modelling the employment relationship. They argue that this improves the model's propagation mechanism: there is an amplified and more persistent response of macroaggregates to shocks. The Mortensen and Pissarides framework has recently been evaluated by Cole and Rogerson (1999). Alvarez and Veracierto (1998) cross a version of the Mortensen and Pissarides (1994) framework with the well-known Hopenhayn and Rogerson (1993) industry dynamics model. Here an unemployed worker can improve the odds of getting a job by expending some labor effort. They also allow for incomplete markets.

The model in this paper belongs to a another class of models which features *search* with incomplete markets. Wright (1986) is an example of an early general equilibrium search model. Other examples of prior work along these lines includes Andolfatto and Gomme (1996), Hansen and Imrohoroglu (1992), Ljundqvist and Sargent (1998), and Zhang (1995). These papers study the effects of unemployment insurance. Linear utility is used by Ljundqvist and Sargent (1998), hence the completeness of markets is not an issue. Andolfatto and Gomme (1996) model the institutional detail of the Canadian unemployment insurance system. An novel feature of their analysis is that they allow for nonparticipation in the labor force. They do not allow for personal asset holdings. The Hansen and Imrohoroglu (1992) paper abstracts from physical capital. The features omitted from these papers are not essential to the analyses; a line must be drawn somewhere in any abstraction. None of this work incorporates aggregate uncertainty, a significant complexity that is necessary for undertaking business cycle analysis.

A.2 Computation

The algorithm used to compute the model's competitive equilibrium approximates the wealth distribution, Z, by a limited set of statistics, such as a set of points characterizing a frequency distribution or a set of moments – see Den Haan (1997) or Krusell and Smith (1998). A law of motion is also specified for the statistics characterizing Z. In line with the findings of Krusell and Smith (1998), it will be assumed that approximating the wealth distribution Z by its means is adequate for the analysis. Denote the mean level of the capital stock by k so that $\mathsf{k} = \int a dZ(a, \varepsilon) da d\varepsilon$. In order to solve the model parametric forms must be specified for the law of motion for the aggregate capital stock, the equilibrium interest rate, and the level of taxes. Assume that the aggregate capital stock has a law of motion of the following form:

$$\mathsf{k}' = \kappa_0 + \kappa_1 \mathsf{k} + \kappa_2 \lambda = \mathsf{K}(\mathsf{k},\lambda), \tag{11}$$

and that the equilibrium interest rate and tax functions can be written as

$$\ln r = \iota_0 + \iota_1 \ln \mathsf{k} + \iota_2 \lambda = \ln R(\mathsf{k},\lambda), \tag{12}$$

and

$$\ln \tau = \vartheta_0 + \vartheta_1 \ln \mathsf{k} + \vartheta_2 \lambda = \ln T(\mathsf{k},\lambda). \tag{13}$$

A.2.1 Computing the Model's General Equilibrium

The algorithm for computing the solution to the model with aggregate shocks proceeds as follows.

1. *Initialization.* Generate $n(m + 1)$ normally distributed random variables. Here n represents the number of periods in the simulation and m is the number of agents. Initialize each agent i's asset holdings at some level, say $a_{i,0}$. This could be done in accordance with the stationary distribution obtained from the deterministic version of

the model. Next, an initial guess for the laws of motion for the aggregate capital stock, interest rate and lump-sum taxes is made:

$$\mathsf{k}' = \kappa_0^0 + \kappa_1^0 \mathsf{k} + \kappa_2^0 \lambda,$$

$$\ln r = \iota_0^0 + \iota_1^0 \ln \mathsf{k} + \iota_2^0 \lambda,$$

and

$$\ln \tau = \vartheta_0^0 + \vartheta_1^0 \ln \mathsf{k} + \vartheta_2^0 \lambda.$$

A good guess for the ι's and ϑ's comes from (17) and (18) below. A good guess for κ_0^0 and κ_1^1 may come from the law of motion for the standard neoclassical growth model.

2. *Computing the Sample Path (Iteration $j+1$)* . The first step is to solve the dynamic programming problems for workers and searchers, taking as given the law of motions for the aggregate capital stock, the equilibrium interest rate and lump-sum taxes:

$$\mathsf{k}_{t+1} = \kappa_0^j + \kappa_1^j \mathsf{k}_t + \kappa_2^j \lambda_t, \tag{14}$$

$$\ln r_t = \iota_0^j + \iota_1^j \ln \mathsf{k}_t + \iota_2^j \lambda_t, \tag{15}$$

and

$$\ln \tau_t = \vartheta_0^j + \vartheta_1^j \ln \mathsf{k}_t + \vartheta_2^j \lambda_t. \tag{16}$$

This gives a solution for the value functions W^{j+1} and S^{j+1}. The procedure for obtaining these solutions is discussed in detail in the section below. Now, suppose that agent i's state in period t is characterized by $(a_{i,t}, \varepsilon_{i,t}, \lambda_t, \mathsf{k}_t)$. To compute his state for $t+1$:

(a) Check whether $W^{j+1}(a_{i,t}, \varepsilon_{i,t}, \lambda_t, \mathsf{k}_t) > S^{j+1}(a_{i,t}, \lambda_t, \mathsf{k}_t)$ to determine whether agent i will work or not in the current period.

(b) Compute the agent's asset holding for period $t+1$, or $a_{i,t+1}$. If the agent is a worker compute his asset holdings for period $t+1$, or $a_{i,t+1}$, using the decision rule $a_{i,t+1} = A^{w,j+1}(a_{i,t}, \varepsilon_{i,t}, \lambda_t, \mathsf{k}_t)$. Alternatively, one could solve the worker's decision problem at the point $(a_t, \varepsilon_{i,t}, \lambda_t, \mathsf{k}_t)$. Note that worker i will hire capital in the amount $k_{i,t} = K(\varepsilon_{i,t}, \lambda_t, r_t)$.

(c) If agent i is a searcher compute his asset holdings using decision rule $a_{i,t+1} = A^{s,j+1}(a_{i,t}, \lambda_t, \mathsf{k}_t)$. Again, one could instead solve the searcher's decision problem at the point $(a_{i,t}, \lambda_t, \mathsf{k}_t)$. A searcher hires no capital so that $k_{i,t} = 0$.

(d) The aggregate supply of capital stock can be computed by calculating $\sum_{i=0}^{m} a_{i,t} = \mathsf{k}_t$. The demand for capital is calculated by computing $\sum_{i=0}^{m} k_{i,t}$. If i is a worker $k_{i,t} = Const\exp(\varepsilon_{i,t} + \lambda_t)^{(1+\theta)/[\theta(1-\alpha)]} \times (1/r_t)^{(\alpha+\theta)/[\theta(1-\alpha)]}$, otherwise $k_{i,t} = 0$. Therefore, the equilibrium interest rate can be computed from the formula

$$r_t = [\frac{Const}{\mathsf{k}_t} \sum_{i \in \mathcal{W}_t} \exp(\varepsilon_{i,t} + \lambda_t)^{(1+\theta)/[\theta(1-\alpha)]}]^{\theta(1-\alpha)/(\alpha+\theta)}, \tag{17}$$

where $\mathcal{W}_t$ is the set of worker's indices.

(e) Similarly, the budget balancing lump-sum tax is given by

$$\tau_t = \mu \frac{m - \#\mathcal{W}_t}{\#\mathcal{W}_t}, \tag{18}$$

where $\#$ denotes the number of elements in a set.

3. *Updating the Aggregate Law of Motion.* By collecting the time series $\{\mathsf{k}_t\}_{t=0}^{n}$ and $\{\lambda_t, r_t, \tau_t\}_{t=0}^{n}$ revised aggregate laws of motion can be computed by running the following regressions[27]

$$\mathsf{k}_{t+1} = \kappa_0^{j+1} + \kappa_1^{j+1}\mathsf{k}_t + \kappa_2^{j+1}\lambda_t = \mathsf{K}^{j+1}(\mathsf{k},\lambda),$$

[27] The R^2 of these regressions is in practice very close to one, after the first few iterations. Theoretically the lagged value of the aggregate unemployment rate should have been included as a state variable as well. This would have increased *significantly* the computational burden while increasing the explanatory power of these regressions only slightly. While this is comforting, it is hard to say whether including the lagged unemployment rate as a state variable matters without doing the full analysis. At this point in time, the general applicability of the numerical method used here is an open question.

$$\ln r_t = \iota_0^{j+1} + \iota_1^{j+1} \ln \mathsf{k}_t + \iota_2^{j+1} \lambda_t = \ln R^{j+1}(\mathsf{k},\lambda),$$

$$\ln \tau_t = \vartheta_0^{j+1} + \vartheta_1^{j+1} \ln \mathsf{k}_t + \vartheta_2^{j+1} \lambda_t = \ln T^{j+1}(\mathsf{k},\lambda).$$

4. Step 2 should be repeated using the revised laws of motion until $\text{dist}([\kappa^{j+1}, \iota^{j+1}, \vartheta^{j+1}], [\kappa^j, \iota^j, \vartheta^j]) <$ tol.

A.2.2 Computing the Value Functions

In Step 1 of the algorithm the value functions W^{j+1} and S^{j+1} needed to be computed. A loop is nested within the main algorithm to do this. Suppose one had a guess for the functions (11), (12) and (13) as given by (14), (15) and (16). Given this guess the dynamic programming problems P(1) and P(2) can be solved. In particular the worker's problem would have the general form

$$\begin{aligned} W^{j+1}(a, \varepsilon, \lambda, \mathsf{k}) \quad = \quad & \max_{c,a^0,k,l} \{U(c) + \\ & \beta \int \max[W^{j+1}(a', \varepsilon', \lambda', \mathsf{K}^j(\mathsf{k}, \lambda)), S^{j+1}(a', \lambda', \mathsf{K}^j(\mathsf{k},\lambda))] \\ & \times dG(\varepsilon'|\varepsilon) dF_1(\lambda'|\lambda) d\varepsilon' d\lambda'\}, \end{aligned}$$

subject to

$$c + a' = Y(\varepsilon, \lambda; R^j(\mathsf{k}, \lambda)) + [1 + R^j(\mathsf{k}, \lambda)]a - T^j(\mathsf{k}, \lambda),$$

where the functions R^j, T^j and K^j are defined by (15), (16) and (14). The searcher's problem would appear as

$$\begin{aligned} S^{j+1}(a, \lambda, \mathsf{k}) \quad = \quad & \max_{c,a^0,k,l} \{U(c) + \\ & \beta \int \max[W^{j+1}(a', \varepsilon', \lambda', \mathsf{K}^j(\mathsf{k}, \lambda)), S^{j+1}(a', \lambda', \mathsf{K}^j(\mathsf{k},\lambda))] dH(\varepsilon') dF(d\lambda'|\lambda) d\varepsilon' d\lambda'\}, \end{aligned}$$

subject to

$$c + a' = [1 + R^j(\mathsf{k}, \lambda)]a + \mu.$$

The functions W and S need not be concave, because the operators defined by P(1) and P(2) do not map concave functions into strictly concave ones. To see why, consider two strictly concave functions $X(a)$ and $Y(a)$. The function $Z(a) = \max[X(a), Y(a)]$, however, may not be concave. For example, let $X(a) = -a^{1-\sigma_x}/(1-\sigma_x)$ and $Y(a) = -a^{1-\sigma_y}/(1-\sigma_y)$, with $\sigma_x = 2$ and $\sigma_y = 1.5$. Figure 11 plots the two functions. Observe that their outer envelope, or $Z(a)$, is not concave due to the depression at the point where X and Z intersect.

Concavity is a highly desirable property both for theoretical and computational reasons. For instance, theoretically C^w and C^s are increasing in a if and only if W and S are strictly concave, a fact demonstrated in Section 2.1. Computationally, when W and S are strictly concave, solving the first-order conditions to P(1) and P(2) is enough to find the decision rules. Furthermore, when W and S are well-behaved strictly concave functions they can be approximated well by low-order polynomials. So, how can the functions W and S be made strictly concave? The trick employed here is to use the idiosyncratic shock ε to render the functions W and S strictly concave. To see how this works, add a continuously distributed random variable ε to the function X in the above example. Specifically, define Y by $X(a, \varepsilon) = -(a+\varepsilon)^{1-\sigma_x}/(1-\sigma_x)$. Let ε be distributed normally with $E[\varepsilon] = 0$. Figure 12 plots $Z(a) = E[\max[X(a, \varepsilon), Y(a)]]$ when the standard deviation for ε varies over 0.05, 0.10, and 0.15 (or so that the standard deviation of ε is approximately 5, 10 and 15% of the value for a at the kink). Increases in the variance of ε smooth out the depression in $Z(a)$ and make it more concave.

In order to solve these problems the functions W^{j+1} and S^{j+1} are approximated by low-order polynomials, specifically quadratics. First, a grid was specified over the model's state space for the continuous variables a, ε, and k — recall that λ only has three values contained in the some set $\mathcal{L}$. Denote these sets of grid points by $\mathcal{A}, \mathcal{E}$, and $\mathcal{K}$. Second, an initial guess is made for the second-degree polynomials used to approximate W^{j+1} and S^{j+1}. Denote this guess by $W^{j+1,0}$ and $S^{j+1,0}$. A good initial guess may be the solution for the value functions obtained on the previous iteration of the main algorithm, or W^j and S^j. Third, given a guess for W^{j+1} and S^{j+1} at the i-th iteration of this inner loop, or $W^{j+1,i}$ and $S^{j+1,i}$, problems

P(1) and P(2) are solved at each point in the set $\mathcal{A} \times \mathcal{E} \times \mathcal{L} \times \mathcal{K}$ by using this guess on the righthand side of P(1) and P(2). This results in lefthand values for W^{j+1} and S^{j+1} at each of these points. Fourth, two new second-degree polynomials are then fitted to these points via least squares.[28] The new functions are represented by $W^{j+1,i+1}$ and $S^{j+1,i+1}$. Fifth, the procedure is repeated until convergence is obtained.

Remark The deterministic version of the model can easily be computed by just solving this inner loop for computing W and S for a given value of r, which is adjusted iteratively until the demand and supply of capital are equated. Here W and S are just functions of a and ε so there is no need to find the laws of motion (11), (12) and (13).

A.3 Properties of W and S

A few properties about W and S are established here.[29]

Lemma 1 *The functions W and S exist, are continuously increasing in a, and W is continuously increasing in ε.*

Proof. Consider the mapping defined by equations P(1) and P(2): $(W^{j+1}, S^{j+1}) = \mathsf{M}(W^j, S^j)$. By applying the Theorem of the Maximum it is straightforward to see that the operator M maps W^j's and S^j's that are continuous in a and ε into W^{j+1}'s and S^{j+1}'s that are also continuous in a and ε. By Blackwell's sufficient conditions the operator M defines a contraction mapping in the space of continuous functions with the uniform norm. Hence, W and S exist and are continuous functions (in a and ε).

Let

$$U_w(a, a', \varepsilon, \cdot) = U(Y(\varepsilon) + (1+r)a - \tau - a'),$$

[28] The R^2 of these regressions is in practice very close to one after the first few iterations.

[29] The key reference on these properties is Stokey and Lucas with Prescott (1989).

$$U_s(a, a', \cdot) = U(\mu + (1+r)a - a'),$$

$$Q_w(a', \varepsilon, \cdot) = \int \max[W^j(a', \varepsilon', \cdot), S^j(a', \cdot)] dG(\varepsilon'|\varepsilon) dF(\lambda'|\lambda) d\varepsilon' d\lambda',$$

and

$$Q_s(a', \cdot) = \int \max[W^j(a', \varepsilon', \cdot), S^j(a', \cdot)] dH(\varepsilon') dF(\lambda'|\lambda) d\varepsilon' d\lambda'.$$

Now, it needs to be shown that the operator M maps W's and S's that are nondecreasing in a into ones that are increasing in a. To see this, consider two levels of asset holdings $a_1 < a_2$. So the question is: If W^j and S^j are nondecreasing in a then will W^{j+1} and S^{j+1} be increasing in a? The answer is yes since

$$\begin{aligned} W^{j+1}(a_1, \varepsilon, \cdot) &= \max_{a^0 \geq \overline{a}} \{U_w(a_1, a', \varepsilon, \cdot) + \beta Q_w(a', \varepsilon, \cdot)\} \\ &< \max_{a^0 \geq \overline{a}} \{U_w(a_2, a', \varepsilon, \cdot) + \beta Q_w(a', \varepsilon, \cdot)\} \equiv W^{j+1}(a_2, \varepsilon, \cdot). \end{aligned}$$

A similar argument can be used to establish that $S^{j+1}(a_1, \cdot) < S^{j+1}(a_2, \cdot)$.

To show that W^{j+1} is increasing in ε consider two levels of idiosyncratic shock $\varepsilon_1 < \varepsilon_2$ It is easy to see that if W^j is nondecreasing in ε then W^{j+1} is increasing in ε as

$$\begin{aligned} W^{j+1}(a, \varepsilon_1, \cdot) &= \max_{a^0 \geq \overline{a}} \{U_w(a, a', \varepsilon_1, \cdot) + \beta Q_w(a', \varepsilon_1, \cdot)\} \\ &< \max_{a^0 \geq \overline{a}} \{U_w(a, a', \varepsilon_2, \cdot) + \beta Q_w(a', \varepsilon_2, \cdot)\} \equiv W^{j+1}(a, \varepsilon_2, \cdot), \end{aligned}$$

where the second line follows from the facts that (i) $Y(\varepsilon_1) < Y(\varepsilon_2)$ and (ii) the distribution function $G(\varepsilon'|\varepsilon_2)$ stochastically dominates the one $G(\varepsilon'|\varepsilon_1)$ so that

$$\int \max[W^j(a', \varepsilon', \cdot), S^j(a', \cdot)] dG(\varepsilon'|\varepsilon_1) dF(\lambda'|\lambda) d\varepsilon' d\lambda' \leq$$
$$\int \max[W^j(a, \varepsilon', \cdot), S^j(a', \cdot)] dG(\varepsilon'|\varepsilon_2) dF(\lambda'|\lambda) d\varepsilon' d\lambda'.$$

■

A.4 An Example where Welfare can Increase with Risk

Let $U(\widetilde{c} - D(l)) = \widetilde{c} - l^{1+\theta}/(1+\theta)$, $O(k, l; \varepsilon) = \exp(\varepsilon)k^{\alpha}l^{1-\alpha}$, and $\delta = \mu = 0$. Ignore the borrowing constraint. In a stationary equilibrium this will imply that $1 + r = 1/\beta$. Next, define the return to working by

$$\begin{aligned} Y(\varepsilon) &= \max_{l,k}[\exp(\varepsilon)k^{\alpha}l^{1-\alpha} - l^{1+\theta}/(1+\theta) - rk] \\ &= [(1-\alpha)^{(1+\theta)/\theta}\theta/(1+\theta)][\alpha\beta/(1-\beta)]^{\alpha(1+\theta)/[\theta(1-\alpha)]} \\ &\times \exp\{(1+\theta)/[\theta(1-\alpha)]\varepsilon\}. \end{aligned} \tag{19}$$

Note that Y is strictly positive, increasing and *convex* in ε. Let a searcher draw his ε from the cumulative distribution function $H : [-\infty, \overline{\varepsilon}] \to [0, 1]$. For a worker, let ε' evolve according to

$$\varepsilon' = \begin{cases} \varepsilon, & \text{with probability } p \text{ (job continues)}, \\ -\infty, & \text{with probability } 1-p, \text{ (job ends)}. \end{cases}$$

This setup is as close as one can get to the model in text while retaining a tractable solution.

The Bellman equation for a searcher is

$$\widetilde{S}(a) = \max_{a^{\theta}}\{(1+r)a - a' + \beta E\{\max[\widetilde{W}(a', \varepsilon'), \widetilde{S}(a')]\}\}, \tag{20}$$

while the one for a worker is

$$\widetilde{W}(a, \varepsilon) = \max_{a^{\theta}}\{Y(\varepsilon) + (1+r)a - a' + \beta E\{\max[\widetilde{W}(a', \varepsilon'), \widetilde{S}(a')]\}\}. \tag{21}$$

Given the linear form of the utility function these programming problems can be significantly simplified.

Lemma 6 *The value functions (20) and (21) have the forms*

$$\widetilde{S}(a) = S + (1+r)a, \tag{22}$$

and

$$\widetilde{W}(a, \varepsilon) = W(\varepsilon) + (1+r)a. \tag{23}$$

Proof. It is readily verifiable that when a guess of this form is inserted into the righthand sides of (20) and (21) a solution of this form is obtained on the lefthand sides of these equations. ■

Note from (20) and (21) that S and W will satisfy

$$S = \beta E\{\max[W(\varepsilon'), S]\}, \tag{24}$$

and

$$W(\varepsilon) = Y(\varepsilon) + \beta E\{\max[W(\varepsilon'), S]\}. \tag{25}$$

Clearly then an agent will choose to work or to search depending on whether $W(\varepsilon)$ is greater than or less than S.[30] Define the threshold shock J by the equation

$$W(J) = S. \tag{26}$$

Solutions for S and W can now be obtained. Using (24), (25) and (26) it is easy to see that

$$S = \beta \int_J W(\varepsilon')dH(\varepsilon') + \beta H(J)S, \tag{27}$$

and

$$W(\varepsilon) = Y(\varepsilon) + \beta p W(\varepsilon) + \beta(1-p)S. \tag{28}$$

Note from (28) that

$$S = \frac{Y(J)}{1-\beta}. \tag{29}$$

and

$$W(\varepsilon) = \frac{Y(\varepsilon) + \beta(1-p)S}{1-\beta p}. \tag{30}$$

[30] Or equivalently, depending on whether $\widetilde{W}(a,\varepsilon)$ is greater than or less $\widetilde{S}(a)$, by (22) and (23).

The solution will be complete if a condition characterizing $Y(J)$ or J can be found. To this end, use (27) and (30) to get

$$S = \beta \int_J [\frac{Y(\varepsilon') + \beta(1-p)S}{1-\beta p}] dH(\varepsilon') + \beta H(J)S,$$

or

$$S = \beta \int_J \frac{Y(\varepsilon')}{1-\beta p} dH(\varepsilon') + \frac{\beta^2(1-p)S[1-H(J)]}{1-\beta p} + \beta H(J)S,$$

which implies

$$S = \beta \int_J \frac{Y(\varepsilon')}{1-\beta p} dH(\varepsilon') + \frac{\beta^2(1-p)S}{1-\beta p} + \frac{\beta H(J)(1-\beta)S}{1-\beta p}.$$

Equation (29) then allows this to be rewritten as

$$[\frac{1-\beta p-\beta^2(1-p)}{1-\beta} - \beta H(J)]Y(J) = \beta \int_J Y(\varepsilon') dH(\varepsilon'),$$

so that

$$[1+\beta(1-p) - \beta H(J)]Y(J) = \beta \int_J Y(\varepsilon') dH(\varepsilon'). \tag{31}$$

Lemma 7 *There exits a unique J solving (31) — or equivalently (26).*

Proof. Integrating the righthand side of (31) by parts generates

$$\begin{aligned} \beta \int_J Y(\varepsilon') dH(\varepsilon') &= \beta Y(\varepsilon')H(\varepsilon')|_J^{\overline{\varepsilon}} - \beta \int_J^{\overline{\varepsilon}} Y_1(\varepsilon')H(x)dx \\ &= \beta Y(\overline{\varepsilon}) - \beta Y(J)H(J) - \beta \int_J^{\overline{\varepsilon}} Y_1(\varepsilon')H(x)dx. \end{aligned}$$

Therefore (31) can be rewritten as

$$[1+\beta(1-p)]Y(J) = \beta Y(\overline{\varepsilon}) - \beta \int_J^{\overline{\varepsilon}} Y_1(\varepsilon')H(\varepsilon')d\varepsilon'.$$

The lefthand side of this equation is increasing in J and has slope $[1+\beta(1-p)]Y_1(J)$. It starts at 0 (when $J = -\infty$) and rises to $[1+\beta(1-p)]Y(\overline{\varepsilon}) = \beta Y(\overline{\varepsilon}) + (1-\beta p)Y(\overline{\varepsilon})$. The righthand side is also increasing in J but has a lower slope $\beta Y_1(J)H(J)$. It begins at the higher intercept $\beta \int Y(\varepsilon)dH(\varepsilon)$ and increases to the lower point $\beta Y(\overline{\varepsilon})$.[31] ■

So the question is how will an increase in risk affect the agent? To answer the question the concept of an increase in risk needs to operationalized.

Assumption *Let the cumulative distribution function $\mathcal{H}$ be smaller than H in terms of second-degree stochastic dominance. Also, assume that $\mathcal{H}$ and H have the same means. (Note the variance of ε is higher with the distribution function $\mathcal{H}$ than with H).*

The effect of an increase in risk on the reservation wage and an agent's welfare can now be established.

Lemma 8 *The reservation wage, J, is higher with $\mathcal{H}$ than with H (i.e., the threshold wage rises with mean-preserving spread in H).*

Proof. It is sufficient to show that for a given value of J the righthand side of (31) is higher with $\mathcal{H}$ than with H. Formally, it is required that

$$\beta Y(\overline{\varepsilon}) - \beta \int_J Y_1(\varepsilon')\mathcal{H}(\varepsilon')d\varepsilon' \geq \beta Y(\overline{\varepsilon}) - \beta \int_J Y_1(\varepsilon')H(\varepsilon')d\varepsilon',$$

or

$$\beta \int_J Y_1(\varepsilon')[H(\varepsilon') - \mathcal{H}(\varepsilon')]d\varepsilon' \geq 0.$$

Integration by parts then yields

$$\begin{aligned}\int_J^{\overline{\varepsilon}} Y_1(\varepsilon')[H(\varepsilon') - \mathcal{H}(\varepsilon')]d\varepsilon' \quad &= \quad Y_1(\varepsilon') \int_{-\infty}^{\varepsilon^\theta} [H(x) - \mathcal{H}(x)]dx \,\Big|_J^{\overline{\varepsilon}} \\ &\quad - \int_J^{\overline{\varepsilon}} Y_{11}(\varepsilon') \int_{-\infty}^{\varepsilon^\theta} [H(x) - \mathcal{H}(x)]dxd\varepsilon'.\end{aligned}$$

[31] Clearly for search to be optimal, at least in some states of nature, it must be the case that $Y(\overline{\varepsilon}) > 0$. It follows that $[1 + \beta(1-p)]Y(\overline{\varepsilon}) = \beta Y(\overline{\varepsilon}) + (1 - \beta p)Y(\overline{\varepsilon}) > \beta Y(\overline{\varepsilon})$.

Now, the fact that H and $\mathcal{H}$ have the same means implies that $\int_{-\infty}^{\overline{\varepsilon}}[H(x)-\mathcal{H}(x)]dx=0$. Hence, the above equation can be simplified to

$$\begin{aligned}\int_J^{\overline{\varepsilon}} Y_1(\varepsilon')[H(\varepsilon')-\mathcal{H}(\varepsilon')]d\varepsilon' &= -Y_1(J)\int_{-\infty}^{J}[H(x)-\mathcal{H}(x)]dx \\ &\quad -\int_J^{\overline{\varepsilon}} Y_{11}(\varepsilon')\int_{-\infty}^{\varepsilon^{\theta}}[H(x)-\mathcal{H}(x)]dxd\varepsilon' \geq 0.\end{aligned}$$

The direction of the inequality follows from the facts that Y is an increasing convex function and that $\int_{-\infty}^{z}[H(x)-\mathcal{H}(x)]dx \leq 0$ for all z by the definition of second-degree stochastic dominance. ■

Proposition 9 *The individual is better off with $\mathcal{H}$ than H (i.e., welfare rises with a mean-preserving spread in H).*

Proof. From (29) it is apparent that a searcher must be better off since S is increasing in J. Then (30) implies that the same is true for a worker, since W rises with S.[32] ■

[32] This implies that $\widetilde{S}$ and $\widetilde{W}$ are also increasing in J [by (22) and (23)].

Table 1: Baseline Parameterization		
Parameter	Benchmark Value	Description
Preferences		
β	$1/1.06^{1/8}$	Time Preference
σ	2	Risk Aversion
θ	10	Inverse of Labor Supply Elasticity
Technology		
α	0.36	Share of Capital in Production
δ	0.006	Depreciation Rate
Policy		
η	0.5	Replacement Ratio (UI)
Shocks		
Aggregate		
ρ_λ	0.98	Persistence, Aggregate Shock
σ_λ	0.009	Std. Dev. Innovation, Agg. Shock
Workers		
ρ_ε	0.9	Persistence, Worker's Shock
σ_ε	0.052	Std. Dev. Innovation, Worker's Shock
Searchers		
σ_ν	0.085	Std. Dev. Innovation, Searcher's Shock

Table 2: Effects of Job Displacement				
Years After Displacement	Post-Displacement Unemployment (weeks)		Post-Displacement Change in Wages (%)	
	Data[1]	*Model*	*Data*[1]	*Model*
0	8.35	9.89	-10.6	-19.6
1	4.32	3.44	-17.5	-21.2
2	2.08	1.08	-16.2	-22.1
3	1.45	0.28	-14.9	-21.0
4	1.27	0.15	-14.7	-16.7

[1]Ruhm (1991, Table 1)

Table 3: Income Distribution		
Parameter	Data[1]	Model
υ_0	-3.564	-3.459
υ_1	0.527	0.501
υ_2	0.081	0.067
Std. Dev. (u_{it})	0.241	0.186

[1] Heaton and Lucas (1996), Table A2

Table 4: Comparative Statics Exercises		
Benchmark Value	New Value	Unemployment Rate
Worker		
$\sigma_\varepsilon = 0.025$	$\sigma_\varepsilon = 0.052 \times 0.80 = 0.042$	4.9
	$\sigma_\varepsilon = 0.052 \times 1.2 = 0.062$	7.2
$\rho_\varepsilon = 0.9$	$\rho_\varepsilon = 0.9 \times 0.95 = 0.855$	5.5
	$\rho_\varepsilon = 0.9 \times 1.05 = 0.945$	6.3
Searcher		
$\sigma_\nu = 0.085$	$\sigma_\nu = 0.085 \times 0.80 = 0.68$	5.7
	$\sigma_\nu = 0.085 \times 1.2 = 0.102$	6.8
Government		
$\eta = 0.5$	$\eta = 0.7$	13.9

Table 5: Standard Business Cycle Facts		
Variable	Rel. Std. Dev. (%)	Corr. with Output
U.S. Quarterly Data — 1954:1 - 1991:2[1]		
Output	1.72	1.00
Consumption	0.74	0.83
Investment	2.97	0.79
Hours	0.98	0.92
Labor Productivity	0.42	0.34
Model		
Output	1.10	1.00
Consumption	0.69	0.97
Investment	3.00	0.95
Hours	0.61	0.91
Labor Productivity	0.52	0.87

Source: Cooley and Prescott (1995, Table 1.1)

[1] All standard deviations (except output) are reported relative to output.

Table 6: Labor Market Facts

Variable	Rel. Std. Dev. (%)	Corr. with Output
U.S. Quarterly Data		
Employment[1]	0.82	0.89
Average Weekly Hours[1]	0.28	0.62
Unemployment[2]	7.68	-0.87
Duration[2]	6.87	-0.37
Unemployment — Flow In	3.11[3]	-0.78[4]
Unemployment — Flow Out	2.50[3]	-0.51[4]
Employment – Flow In	3.84[5]	0.18[6]
Employment – Flow Out	8.42[5]	-0.65[6]
Corr(U. Flow In, U. Flow Out) = 0.64[4]		
Corr(E. Flow In, E. Flow Out) = -0.32[6]		
Model		
Employment	0.62	0.80
Average Weekly Hours	0.26	0.24
Unemployment	7.27	-0.80
Duration	3.60	-0.14
Unemployment — Flow In	7.95	-0.69
Unemployment — Flow Out	6.87	-0.43
Employment — Flow In	6.87	-0.43
Employment — Flow Out	7.95	-0.69
Corr(U. Flow In, U. Flow Out) = 0.09		
Corr(E. Flow In, E. Flow Out) = 0.09		

[1] Cooley and Prescott (1995), Table 1.1; period 1954:1-1991:2.

[2]Computed for period 1954:1-1991:2.

[3] Merz (1996), Table 1; period 1959:1-1988:2.

[4] Merz (1996), Table 2; period 1959:1-1981:4.

[5] Merz (1996), Table 3; period 1959:1-1981:4.

[6] Merz (1996), Table 4; period 1959:1-1981:4.

FIGURE 1: Determination of Consumption.

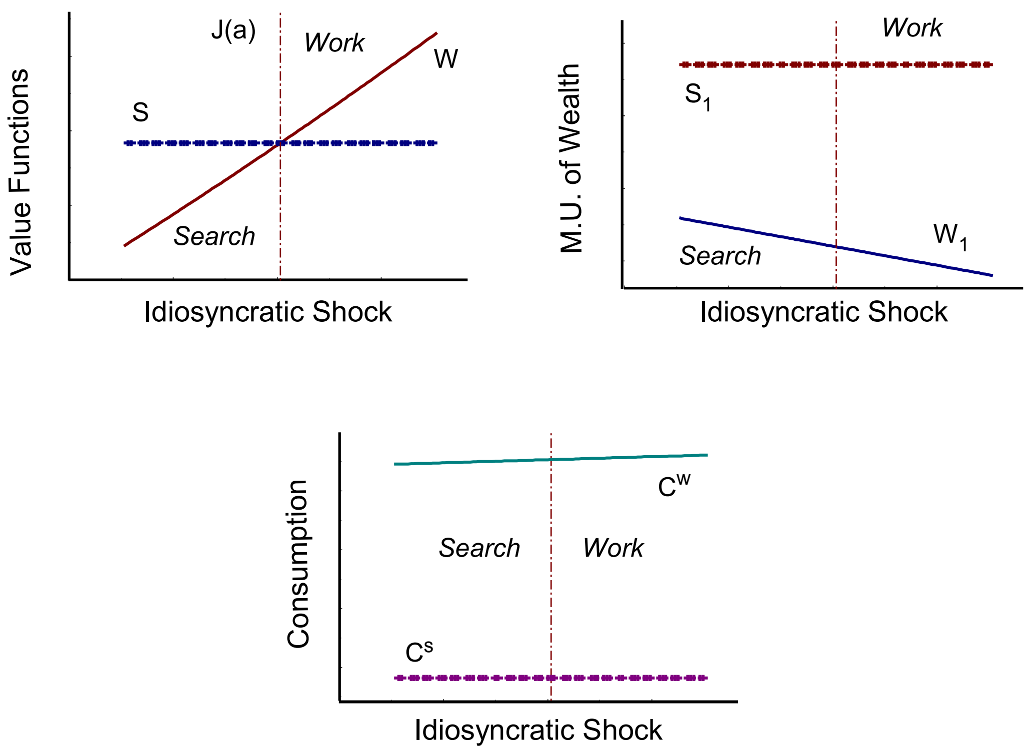

FIGURE 2: Cumulative Hazard Rate.

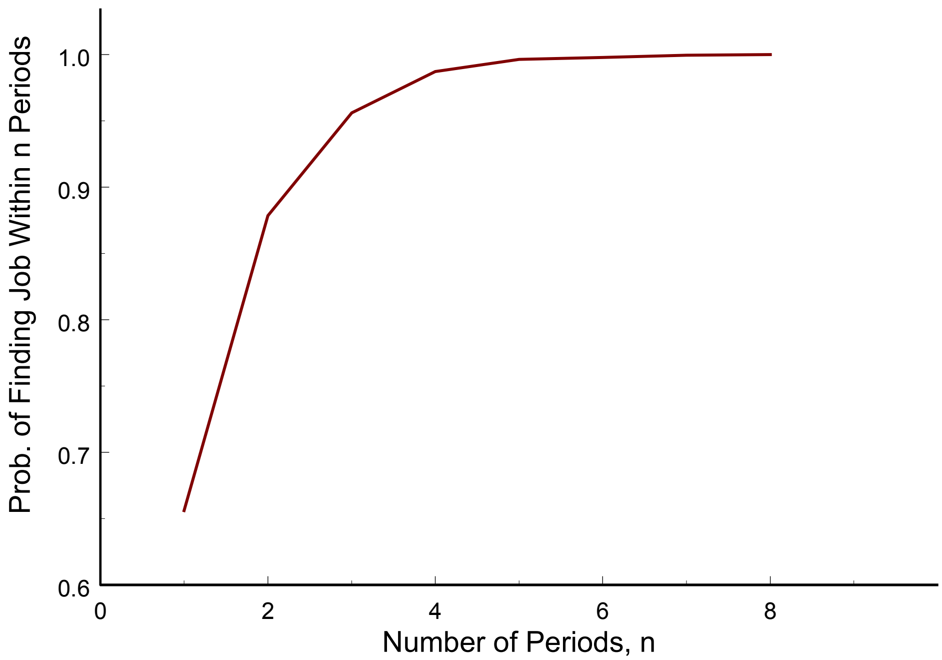

FIGURE 3: Threshold Rules – Aggregate Shock.

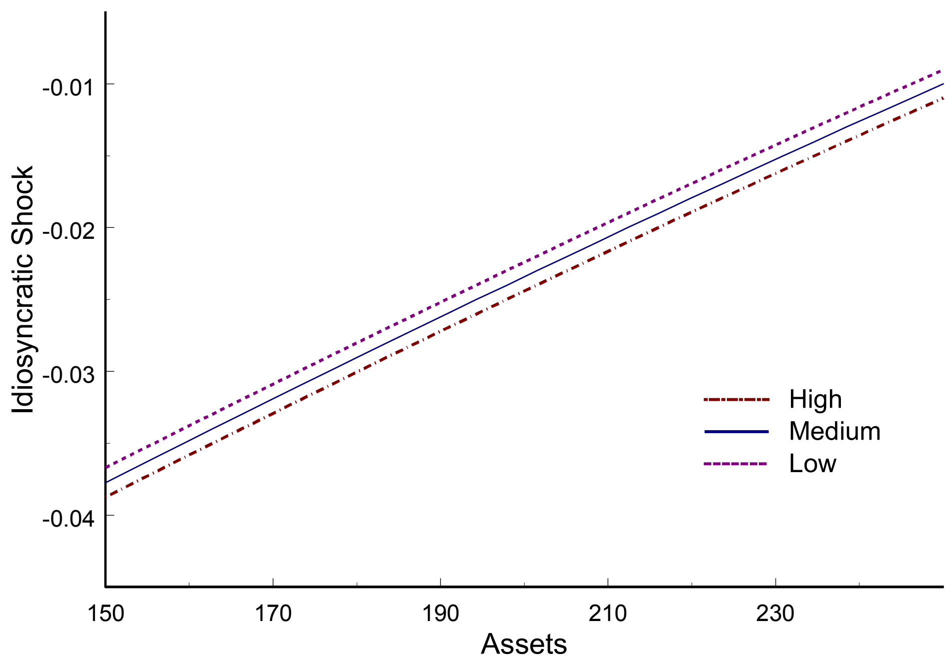

FIGURE 4: Threshold Rules – Aggregate Capital Stock.

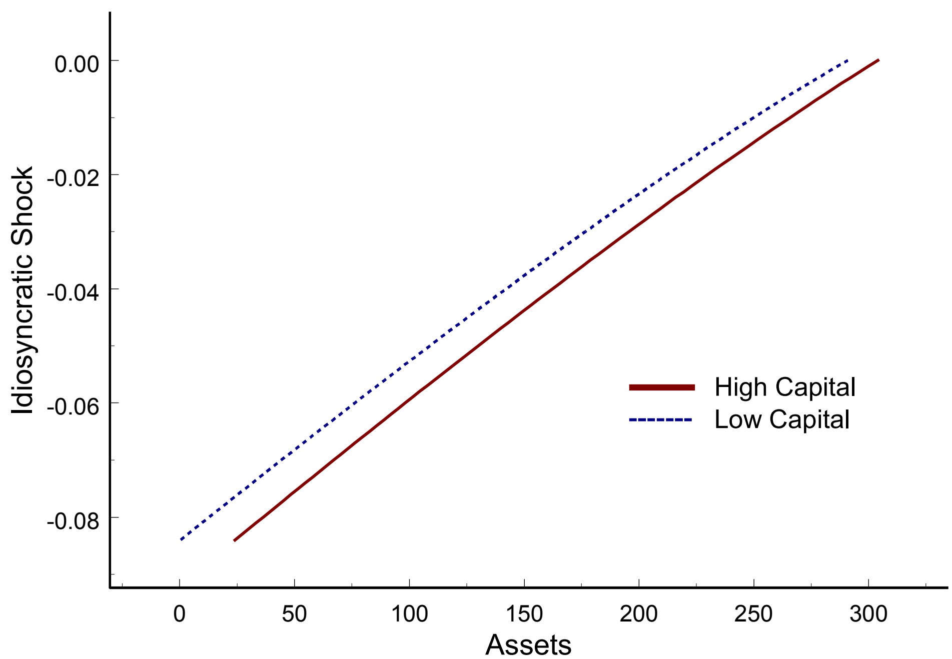

FIGURE 5: Impulse Response, Positive Shock – Consumption, Investment, and Output.

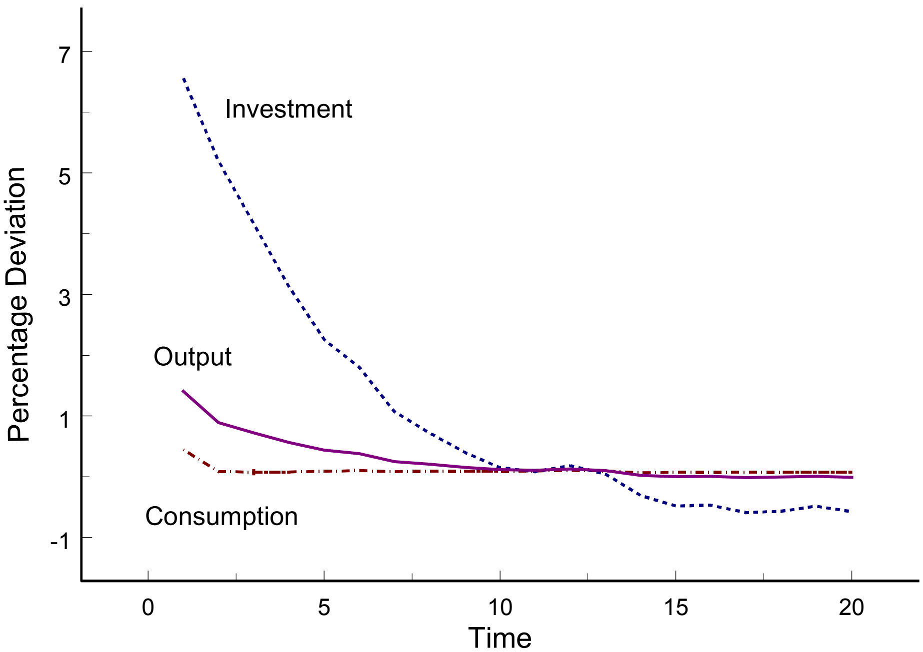

FIGURE 6: Impulse Response, Positive Shock – Employment, Unemployment, and Average Hours Worked.

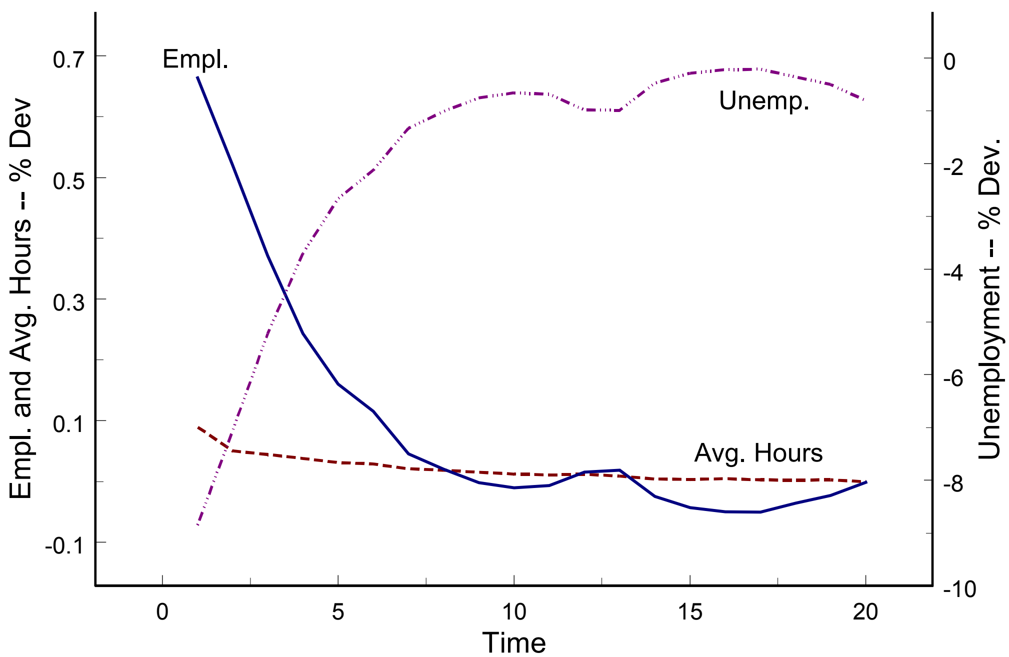

FIGURE 7: Impulse Response, Positive Shock – Duration of Employment, Flow into Unemployment and Flow out of Employment.

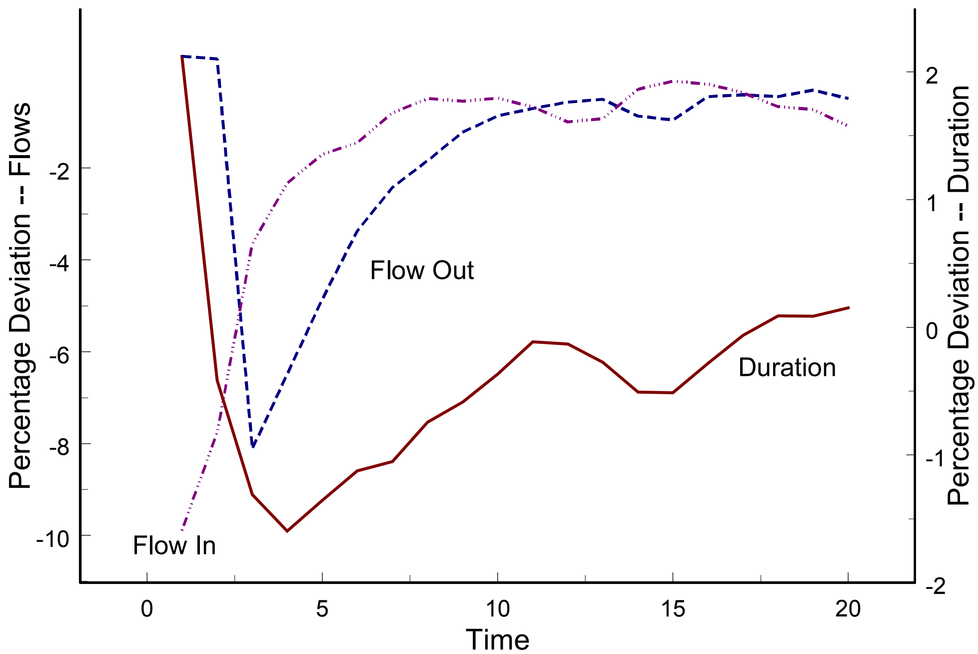

FIGURE 8: Impulse Response, Negative Shock – Consumption, Investment, and Output.

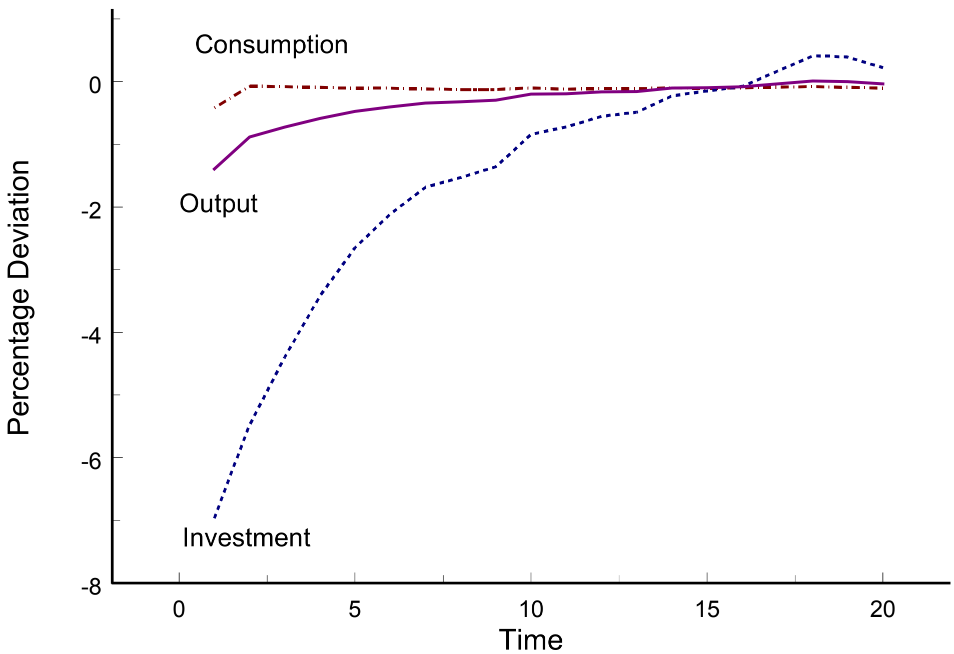

FIGURE 9: Impulse Response, Negative Shock – Employment, Unemployment, and Average Hours Worked.

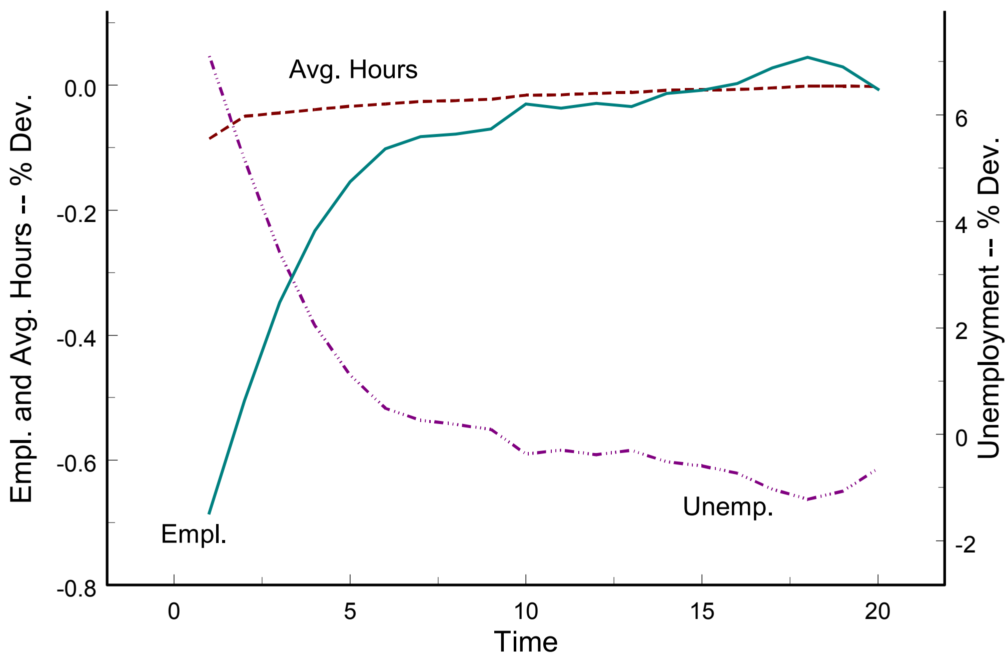

FIGURE 10: Impulse Response, Negative Shock – Duration of Employment, Flow into Unemployment and Flow out of Employment.

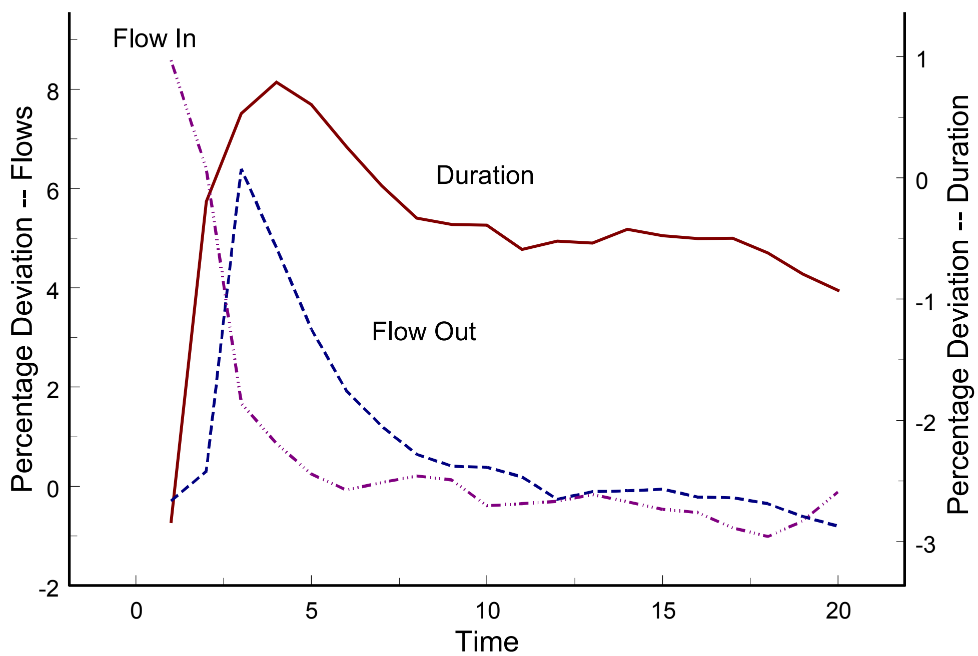

FIGURE 11: X(a) and Y(a).

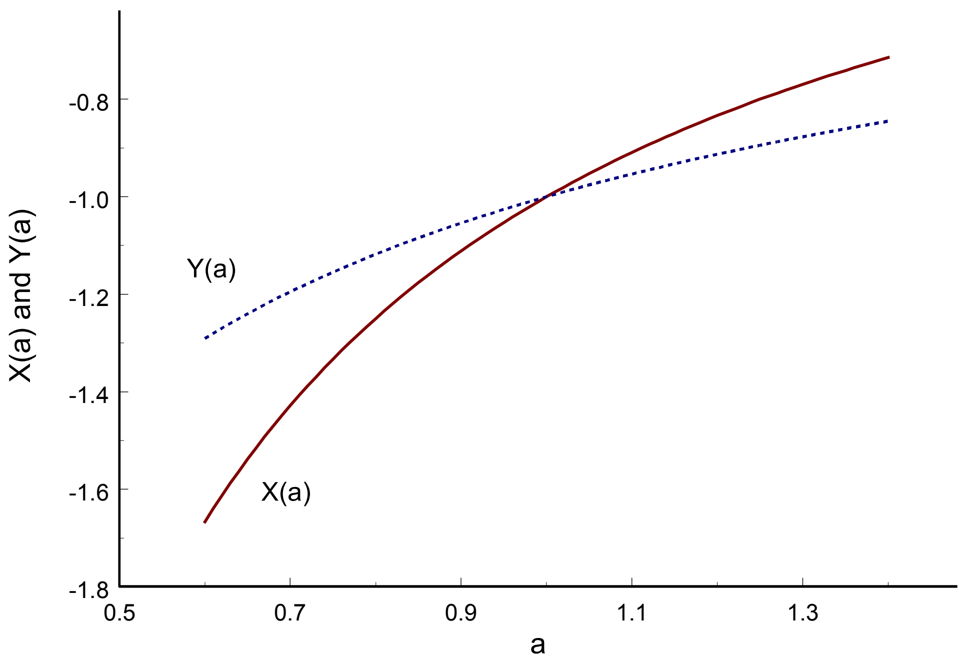

FIGURE 12: Smoothing Effect of Uncertainty.