Analyzing Price Limit Policy Under a Theoretical Framework

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Abstract
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ANALYZING PRICE LIMIT POLICY UNDER A THEORETICAL FRAMEWORK

By

Shuangcheng Du

An Undergraduate Thesis submitted in partial fulfillment of the requirements for the

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Faculty Advisor:

Winston (Wei) Dou

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THE WHARTON SCHOOL, UNIVERSITY OF PENNSYLVANIA

May 2018
ABSTRACT

This paper provides a theoretical framework to study the effects of implementing price limit policy on price movements and trading behaviors. Due to the high difficulty of isolating effects of price limits in empirical data, it is useful to develop a model to simulate the theoretically possible effects of enforcing price limits on stock trading. In addition, we offer two ways of expanding the model into 2-segments that could differentiate the effects of having price limits on individual investors from institutional investors. According to our model, hitting price limits can be summarized to eight different short-term scenarios and each results in different effects on the direction of price movement and short-term liquidity in the next trading period. Besides offering a theoretical perspective on price limit effects, our model provides two new ways of approaching price limits in stock market: i) it can be simplified to the identification of the eight types of patterns and ii) it can be transformed to a study of investor composition and their behaviors on risk bearing with the presence of price limits.
INTRODUCTION

Intraday price limit policy has been widely used across regions and markets. It is particularly favored by the emerging markets for stabilization. As of today, it is a notable feature of the Chinese stock market. It was first implemented on December 16th, 1996 at Shanghai and Shenzhen Stock Exchange. The daily price fluctuation of a single stock has been set to be 10 percent of the previous closing price for regular stocks and 5 percent for special treatment (ST) stocks.

The debate over the existence of price limit policy itself has been far from unanimous. On one hand, price limit advocates claim that it effectively manages short-term volatility in stock price fluctuations by offering a “time-out” or “cool-off period”, that it mitigates overreactions of frenzied traders, and that it does not interfere with normal trading activities. On the other hand, critics argue that the price limits cause higher volatility on subsequent trading days, that the policy delays the reaching of market equilibrium level, and that it does interfere with normal trading.

There are two main arguments for the policy to exist in China. First, it keeps the market stabilized by managing short-term volatility during unusual and sudden circumstances such as extreme company news, global financial crisis, systematic economic crush, unforeseeable disasters, and stock manipulation. Second, it protects individual investors by offering them more time to receive and digest information. Whether the government should intervene to stabilize the market and protect individual investors from the institutional investors are beyond the scope of this project. This research only aims to investigate the potential effect of the price limit policy.
While there have been many empirical studies on the effects of the price limit policy in different markets, it is hard to control for all other factors and quantify the impact on stock price and trading behaviors caused by price limit policy. Thus, it is helpful to build a theoretical framework which enables us to isolate the effects of having price limits in trading.

In summary, this research seeks to model the theoretical effects of the intraday price limit. Results from the project are expected to not only add to the debate on price limit policy itself with a theoretical framework, but also to offer some insights to the policy makers and market regulators. Section 2 of the paper presents past literature on the effects of price limits. Section 3 describes our general model for stock trading, price movements, and price limits trigger. Section 4 analyzes both short-term and long-term effects of price limits based on the general model. Section 5 offers two ways of expanding the general model to accommodate two segments of investors. Section 6 acknowledges some limitations of the model and points out potential directions for future studies. We conclude the paper in section 7.

LITERATURE REVIEW

Early research concerning price limit policy first appeared in light of the market crash in October 1987. Greenwald and Stein (1991) stated that panic had contributed to market crash. In response to this, many research was conducted to see if mechanisms such as price limit, trading halt and circuit breaker should be put into place. There was some previous work that touched upon these topics. Lee, Ready, and Seguin (1994)
investigated trading halt and concluded it would increase, rather than decrease, both volume and volatility.

Many papers were written based on the futures market. Ma, Rao, and Sears (1989a), for instance, examined futures market using both daily and intraday level data. They found empirical evidence on the reversal behavior after price limits were hit, disproving the delay hypothesis. This conclusion, however, was challenged by Lehmann (1989) by stating that the study of Ma et al. (1989a) failed to take into the imbalance effect caused by the existence of both patient and impatient traders. In another word, price behaviors in the limit-hitting neighborhood could provide little information. Fama (1989) also challenged the effectiveness of price limits by arguing that the increased volatility found was simply because of the interference to price discovery process.

Kim and Rhee (1997) tested the three main hypotheses regarding the effectiveness of price limits, the volatility spillover, the delayed price discovery, and the trading interference hypothesis, by looking at data from Tokyo Stock Exchange. Their evidence suggested that price limits might be ineffective. Similar results were found by Phylaktis, Kavussanos, and Manalis (2000) based on Athens Stock Exchange, showing that price limits had no significant effect on stock volatility. The policy merely slowed down the process of reaching to equilibrium. Yet research based on some markets showed different results. Berkman and Lee (2002) used data from Korean Stock Exchange to study the effect of widened price limits. Evidence showed that widened limits did increase volatility, suggesting that price limits were managing volatility to some extent. Hsieh, Kim, and Yang (2009) examined the magnet effect of price limits based on data from the Taiwan Stock Exchange. They used a logit model to show that the conditional probability
of price movements increased when approaching price limits, which supported the existence of magnet effect. Another study based on the Taiwan Stock Exchange by Chang and Hsieh (2008) showed mixed evidence, supporting only one out of the three hypotheses.

Some research has also been conducted to investigate price limit policy in China. Kim, Liu, and Yang (2013) compared two periods of the Chinese stock market. During 1992 to 1996, China did not have price limits policy. This was changed in 1996. They concluded that price limits in China could help facilitate price discovery and reduce volatility. They claimed that no magnet effects were found. Lu (2016) did a research based on cross-listed stocks in mainland China and Hong Kong. This quasi-natural experiment found that the delay of efficient price discovery, the volatility spillover and the trading interference become statistically insignificant when stocks were actively traded.

Given the controversial debates over the price limits and mixed evidence from previous literature on various datasets, it is worth developing a framework that is helpful to examine the theoretical impacts of price limits in hope of contributing to the ongoing discussions and offering new perspectives to approach future empirical studies.

MODEL DEVELOPMENT

Here we develop a general model to simulate the trading environment and price movement of a single risky asset. More specifically, the model describes the price movement of a stock, which is driven by trading activities of potential investors. The model begins and ends at equilibrium with price being equal to the expected true value of
the stock and with the market being cleared. After establishing the initial model, price limits are added as boundaries for price movement in each period. Ultimately, this model aims to i) describe theoretically how price limits are hit and triggered in trading; ii) offer a way of quantifying the effects from price limits in various perspectives; and iii) explain how each underlying factor contributes to the effects of implementing price limits. A list of notations for the model is shown below:

**Table 1: General model notations**

<table>
<thead>
<tr>
<th>N</th>
<th>Total potential investors for the given risk asset (stock)</th>
</tr>
</thead>
<tbody>
<tr>
<td>B&lt;sub&gt;t&lt;/sub&gt;</td>
<td>Total shares of buy orders at time t</td>
</tr>
<tr>
<td>S&lt;sub&gt;t&lt;/sub&gt;</td>
<td>Total shares of sell orders at time t</td>
</tr>
<tr>
<td>D&lt;sub&gt;t&lt;/sub&gt;</td>
<td>Difference between total shares of buy and sell orders</td>
</tr>
<tr>
<td>p&lt;sub&gt;0&lt;/sub&gt;</td>
<td>Initial (previously formed) equilibrium price</td>
</tr>
<tr>
<td>ṽ&lt;sub&gt;0&lt;/sub&gt; ~ N(v&lt;sub&gt;0&lt;/sub&gt;, σ&lt;sub&gt;0&lt;/sub&gt;&lt;sup&gt;2&lt;/sup&gt;)</td>
<td>Initial true value of the risky asset (stock) at time 0</td>
</tr>
<tr>
<td>ṽ&lt;sub&gt;new&lt;/sub&gt; ~ N(v&lt;sub&gt;new&lt;/sub&gt;, σ&lt;sub&gt;new&lt;/sub&gt;&lt;sup&gt;2&lt;/sup&gt;)</td>
<td>New true value of the risky asset (stock)</td>
</tr>
<tr>
<td>b&lt;sub&gt;t&lt;/sub&gt;</td>
<td>Average risk a buyer takes at time t</td>
</tr>
<tr>
<td>s&lt;sub&gt;t&lt;/sub&gt;</td>
<td>Average risk a seller takes at time t</td>
</tr>
<tr>
<td>β&lt;sub&gt;p&lt;/sub&gt;</td>
<td>Price elasticity of the risky asset</td>
</tr>
<tr>
<td>Φ(p&lt;sub&gt;t&lt;/sub&gt;, v&lt;sub&gt;new&lt;/sub&gt;, σ&lt;sub&gt;new&lt;/sub&gt;)</td>
<td>Cumulative distribution function</td>
</tr>
<tr>
<td>p&lt;sub&gt;t&lt;/sub&gt;'</td>
<td>Price movement with the presence of price limits</td>
</tr>
</tbody>
</table>

**Trading Behavior**
At given time t, the amount of total buy and sell orders is determined by three factors. First, it is affected by total number of potential investors that are interested in this asset. For instance, total number of potential investors is likely to be less for a tobacco company. Second, it is influenced by the percentage of the potential investors whose perceived true values of stock are higher or lower than the current market price. In another word, it is calculated by using the cumulative distribution function with the current price and the normally distributed true value perception. See figure 1 below. Lastly, it is affected the average amount of risk that each buyer and seller choose to take. This factor can also be understood as a function of the investor’s amount of available capital plus the degree of risk aversion.

\[
\text{Buy (Sell) orders} = \#\text{potential investors} \cdot \text{Prob of Buy (Sell)} \cdot \text{Risk Factor}
\]

\[
B_t = N \cdot [1 - \Phi(p_{t-1}, v_{\text{new}}, \sigma_{\text{new}})] \cdot b_t
\]

\[
S_t = N \cdot \Phi(p_{t-1}, v_{\text{new}}, \sigma_{\text{new}}) \cdot s_t
\]

Figure 1

**Price Movement**

Many previous research have touched upon potential reasons for price impacts and price elasticity. Shleifer (1986), for instance, tested a downward sloping demand curve for
stock. Scholes (1972) presented the three hypotheses: the price-pressure hypothesis, substitution hypothesis, and the information hypothesis. These three hypotheses were frequently mentioned and tested empirically in later studies such as Mikkelson and Partch (1985), Harris and Gurel (1986), Chan and Lakonishok (1993) etc. In addition, Loderer, Cooney, and Van Drunen (1991) tested the price elasticity of demand for common stocks. The underlying assumption of our model draws insight from these three hypotheses.

Price movement in our model is strictly subject to the changes in the demand of the stock and the elasticity of the demand curve. That is, change in demand of a stock is represented as the differences between total buy and sell orders in the current market. In another word, the imbalance of total buy and sell orders can be seen as a shortage of liquidity and the difference between buy and sell intentions will put pressure on the price movement by the measure of this stock’s price elasticity. Our assumption can also be interpreted as a slow process of adjusting to new information regarding the true value of the stock.

Our model starts with a risky asset (stock) at equilibrium where market is cleared with the same amount of buy and sell orders. In addition, initial price is equal to the true value of the asset.

\[
\begin{align*}
D_0 &= B_0 - S_0 = 0 \\
p_0 &= v_0
\end{align*}
\]

There is a shock to the expected true value of the stock. True value of the stock is shifted to \( \bar{v}_{\text{new}} \sim N(v_{\text{new}}, \sigma_{\text{new}}^2) \), throwing the model off from equilibrium with an unbalanced and uncleared market being formed. Note that we adopt a normal distribution for the expected true value of assets similar to Kyle (1985).

\[
\begin{align*}
D &\neq 0 \\
p &\neq v_{\text{new}}
\end{align*}
\]
Given the amount of unbalanced orders on the market, price will move towards the new expected true value of stock and the amount of movement is determined by the stock’s price elasticity. While it is assumed that the price elasticity is unique to individual stocks, we take it as a relatively stable parameter for each stock and will remain constant in our model. The model is rebalanced when the new equilibrium is reached at \( t=T \).

\[
p_t = p_{t-1} + \beta_p \cdot D_t
\]

until

\[
\begin{cases}
D_T = B_T - S_T = 0 \\
p_T = v_{\text{new}}
\end{cases}
\]

**Price Limits**

Including price limits in our model is simply adding upper and lower bounds for the price movements in each period. Like what China has been using for its stock market, we set the price limits to be \( \pm 10\% \) for each period.

\[
0.9p_{t-1} \leq p_t \leq 1.1p_{t-1}
\]

**MODEL ANALYSIS**

**General Model Behavior**

We now move to model analysis and draw some insights. First, it is worth noting that the vanilla model, the general model without price limits, is not a closed form model. In another word, it does not guarantee to reach and stabilize at \( v_{\text{new}} \). This is mainly due to the unpredictable risk factors at any given period. See figure 2 below for an example, where \( p_0 = 15, v_{\text{new}} = 20 \). Furthermore, due to the nature of the risk factor, we set risk factors to be nonstationary in our model to focus on the study of price limits.
Another interesting observation from the simulation is that having price limits can be helpful in certain situations where new equilibriums are not reached due to a combination of all factors. While in some of these situations price movement will not converge both with and without price limits, there are times when price limits can make the price converge to the new equilibrium. See figure 3 below for an example. Due to the assumption that buying and selling risk factors remain constant through our interested period, a stable and dynamic price bounce is formed instead of a single new equilibrium. Adding the price limits in the model, however, will lead to a stabilized equilibrium. Note that the new equilibrium formed with price limits is not equal to the expected true value of the asset $v_{\text{new}}$. 

Figure 2

[Graph of Unreaching of Equilibrium with and without price limit]
Under our model, price limits are hit and triggered when

\[
\frac{\beta_p}{p_{t-1}} \cdot |N\{[1 - \Phi(p_{t-1}, v_{new}, \sigma_{new})] \cdot b_t - \Phi(p_{t-1}, v_{new}, \sigma_{new}) \cdot s_t)| > 0.1
\]

We can draw some quick insights from this equation. First, all else equal, a bigger pool of potential investors of a given stock leads to higher probability of hitting price limits. Second, all else equal, a higher price elasticity of stock leads to higher probability of hitting price limits. Third, all else equal, stocks with smaller closing price from the previous period are more likely to hit the price limits. Fourth, there exists asymmetry between effects from the risk factors of buyers and sellers.

**Short-term Behaviors**

In this section, we analyze the potential short-term effects that price limits can cause under the theoretical framework of our model. Here we assume that unit time is equal to a single trading day. When price limits are hit, there are only eight categories of scenarios based on the relative positions of current price with price limits \(p'_t\), current price without
price limits \( p_t \), and the expected true value of the asset \( v_{\text{new}} \). After presenting the eight types of situations, we focus on three potential short-term effects that can be caused by hitting price limits: i) “distance” to the new equilibrium price; ii) change in size and direction of price movement; and iii) market liquidity in the next period.

First, we look at eight possible scenarios after price limits are hit, which are illustrated below as figure 4 to 11. Notice that the four scenarios on the left-hand side, i.e. figure 4, 6, 8, and 10, all result in the upper limits being hit and the current prices with price limits \( p_t' \) are lower than \( p_t \). And the four scenarios on the right-hand side, i.e. figure 5, 7, 9, and 11, are the opposite where lower limits are hit and current prices with price limits are higher than \( p_t \). Also note that it is entirely possible for price to move farther away from the new equilibrium. This can happen when the “minorities” of likely investors, either buyers or sellers, are somehow more certain with their beliefs of the true value or are willing to take more risks and therefore heavier positions.
The effect of price limits on “distance” to the new equilibrium is most straightforward. In situations like figure 4 and 5, $p'_t$ is farther away from $v_{new}$ than $p_t$. That is,

$$|v_{new} - p'_t| > |v_{new} - p_t|$$

In situations like figure 8, 9, 10, and 11, $p'_t$ is closer to $v_{new}$ than $p_t$. That is,

$$|v_{new} - p'_t| < |v_{new} - p_t|$$

And in situations like figure 6 and 7, it is undetermined. Thus, if given the frequency of appearances for each type of scenarios, we can have a sense of how often price limits cause a short-term delay and acceleration of price moving into the new equilibrium.

Next, we will look at the potential effects that price limits can bring in terms of changes in sizes and directions of price movements. Theoretically, the size and direction of price movement in the next trading period depend on the difference between next period’s total buy and sell orders. More specifically, we look at

$$D_{t+1} = B_{t+1} - S_{t+1} = N[[1 - \Phi(p_t, v_{new}, \sigma_{new})] \cdot b_{t+1} - \Phi(p_t, v_{new}, \sigma_{new}) \cdot s_{t+1}]$$

and
\[ D_{t+1}' = B_{t+1}' - S_{t+1}' = N\{[1 - \Phi(p_t', v_{new}, \sigma_{new})] \cdot b_{t+1}' - \Phi(p_t', v_{new}, \sigma_{new}) \cdot s_{t+1}'\} \]

Thus, there will be a change in direction of next period’s price movement if

\[ \frac{D_{t+1}}{D_{t+1}'} < 0 \]

And the impact on the size of price movements will be

\[ \beta_p \cdot (D_{t+1}' - D_{t+1}) \]

Thus, it is obvious that the short-term effects on size and direction of price movement depend on both the investors’ risk-taking behaviors and the relative positions of prices and new equilibrium in the eight scenarios. If we assume that the settings of risk factors allow the price to move towards the new equilibrium in the next period, then the effects on direction of price movements will only rely on which of the eight situations we are in.

In situations like figure 6 and 7, for instance, where \( p_t' \) and \( p_t \) are on different side of \( v_{new} \), price will move towards different directions. These can be seen as effective “cool-off” periods for overreacting investors.

Lastly, we can draw some insights regarding the impact of price limits on short-term liquidity. Here we define short-term liquidity as the number of shares traded each period.

This can be written as \( \min(B_{t+1}, S_{t+1}) \) versus \( \min(B_{t+1}', S_{t+1}') \). If we assume that hitting price limits does not alter the risk factor of buyers and sellers and the risk factors remain stable from period to period, then the impact on short-term liquidity can be transformed to the first effect, i.e. the “distance” to new equilibrium price. That is, if current price with price limits is farther away from the new equilibrium price and the assumptions we have just made for the risk factors hold, then there will be a shortage of short-term liquidity in the next trading period.
Long-term Behaviors

Here we show some results of how price limits can affect price movement and trading in the long-term. Long-term effects are harder to gauge since it is a cumulation of different short-term effects. We first lay out the theoretical impact below. Then we display several long-run simulations.

According to our model, differences between total buy and sell orders at any given time ($D_t$) in the market is the primary driver of price movement. Thus, it is essential to show how the price limit policy can influence $D$ in the long-run. First, we assume the risk factors $b_t$ and $s_t$ is unaffected by hitting the price limits. Then the cumulative difference caused by price limits will be

$$Total \ Diff \ in \ D = N \cdot \sum_{t=0}^{T} (b_{t+1} + s_{t+1}) \cdot [\Phi(p_t, v_{new}, \sigma_{new}) - \Phi(p_t', v_{new}, \sigma_{new})]$$

Now we allow the risk factors to change in the event of price limits being triggered. That is, people will act differently when price limits are hit due to signal effects. Then the total cumulative difference in $D$ will become

$$Total \ Diff \ in \ D'_t - D_t = N \cdot \sum_{t=0}^{T} b_{t+1}'[1 - \Phi(p'_t)] - s_{t+1}' \Phi(p_t') - b_{t+1}[1 - \Phi(p_t)] + s_{t+1} \Phi(p_t)$$

In addition to knowing the cumulative differences in $D$, we can also calculate the total shares traded during the entire course of equilibrium reaching. As we specified above, for each time period $t$, we say that the number of traded shares is equal to $\min(B_t, S_t)$. Thus, the total amount of long-term liquidity $L$ during the path to equilibrium can be described as:

$$Total \ Diff \ in \ D'_t - D_t = N \cdot \sum_{t=0}^{T} b_{t+1}'[1 - \Phi(p'_t)] - s_{t+1}' \Phi(p_t') - b_{t+1}[1 - \Phi(p_t)] + s_{t+1} \Phi(p_t)$$
\[
L = \sum_{t=0}^{T} \min(B_t, S_t)
\]
\[
L' = \sum_{t=0}^{T} \min(B'_t, S'_t)
\]

And the cumulative effects of price limits on long-term liquidity can thus be written as

\[L' - L\]

Again, the long-run effects can be interpreted as cumulations of short-term effects that we have analyzed above. Since the theoretical results are not very intuitive, we ran some simulations with assumptions on variables and parameters to show some of the potential impacts.

Before getting into the long-term effect of price limits, one thing that is not so obvious from the model itself and the price limit hitting equation above is the role of standard deviation. Standard deviation can be interpreted as an indicator of heterogeneity in people’s perception of the true value. For instance, a large standard deviation indicates that within the group of investors the perception of true asset value is widely distributed. It is certain that the relationship between standard deviation and the time of reaching equilibrium is monotonic. A simulation with all else equal and variation in the standard deviation is shown below. Notice from the graph that when \(\sigma_{\text{new}} = 2\), it takes longer for price to reach the new equilibrium with more fluctuation.
Next, in the cases where a new equilibrium can be reached, we found that having a price limit could lead to different long-term effects. That is, the reaching of equilibrium can be delayed, accelerated, or unaffected. Each case can be found in the figures below.
In summary, while price limits have been triggered and hit in all three scenarios above and caused different short-term effects, having price limits can have quite different impacts over price movements and the reaching of equilibrium in the long-run.

2-SEGMENT MODEL

Model Expansion

Building on the general model described above, we can incorporate segmentation of investors to further differentiate the effects of price limits on individual versus
institutional investors. Here we offer two different methods of model expansion that will rely on slightly different assumptions.

**Expansion 1**

We assume that the normal distribution of the precepted true values are jointly formed by both individual and institutional investors. In addition, we assume that individual and institutional investors have different “risk factors”, which means different size of available capital size and degree of risk aversion. While it is reasonable to assume that institutional investors as a group should always have more available capital for any stock they are interested in, the degree of risk aversion is much harder to assume in advance. That is, instead of representing the average risk buyers (sellers) choose to take with $b_t, s_t$, it will be calculated as weighted average of individual and institutional buyers (sellers).

*Table 2: 2-Segment Model-1 Notations*

<table>
<thead>
<tr>
<th>Notations</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\gamma$</td>
<td>Ratio of individual investors over institutional investors</td>
</tr>
<tr>
<td>$\pi_t^b$</td>
<td>Percentage of buyer being individual investors at time t</td>
</tr>
<tr>
<td>$\pi_t^s$</td>
<td>Percentage of seller being individual investors at time t</td>
</tr>
<tr>
<td>$b_t^{ind}$</td>
<td>Average risk an individual buyer takes</td>
</tr>
<tr>
<td>$s_t^{ind}$</td>
<td>Average risk an individual seller takes</td>
</tr>
<tr>
<td>$b_t^{ins}$</td>
<td>Average risk an institutional buyer takes</td>
</tr>
<tr>
<td>$s_t^{ins}$</td>
<td>Average risk an institutional seller takes</td>
</tr>
</tbody>
</table>

The overall risk factor for buyers and sellers at time $t$ is calculated as weighted average of individual and institutional investors’ risk factors at time $t$. See table 3 below.
Furthermore, it is important to bear in mind that boundaries exist for some of these parameters.

Table 3

<table>
<thead>
<tr>
<th></th>
<th>Buyer</th>
<th>Seller</th>
</tr>
</thead>
<tbody>
<tr>
<td>Individual (%)</td>
<td>$\pi^b_t$</td>
<td>$\pi^s_t$</td>
</tr>
<tr>
<td>Institutional (%)</td>
<td>$1 - \pi^b_t$</td>
<td>$1 - \pi^s_t$</td>
</tr>
<tr>
<td>Weighted Average</td>
<td>$\pi^b_t \cdot b^{ind}_t + (1 - \pi^b_t) \cdot b^{ins}_t$</td>
<td>$\pi^s_t \cdot b^{ind}_t + (1 - \pi^s_t) \cdot b^{ins}_t$</td>
</tr>
</tbody>
</table>

$$\gamma = \frac{N \cdot [1 - \Phi(p_{t-1}, v_{new}, \sigma_{new})] \cdot \pi^b_t + N \cdot \Phi(p_{t-1}, v_{new}, \sigma_{new}) \cdot \pi^s_t}{N \cdot [1 - \Phi(p_{t-1}, v_{new}, \sigma_{new})] \cdot (1 - \pi^b_t) + N \cdot \Phi(p_{t-1}, v_{new}, \sigma_{new}) \cdot (1 - \pi^s_t)}$$

$$\begin{cases} 
0 \leq \pi^b_t \leq 1 \\
0 \leq \pi^s_t \leq 1 \\
0 \leq \gamma \leq 1
\end{cases}$$

Thus, $D_t = B_t - S_t$, where

$$\begin{cases} 
B_t = N \cdot [1 - \Phi(p_{t-1}, v_{new}, \sigma_{new})] \cdot [\pi^b_t \cdot b^{ind}_t + (1 - \pi^b_t) \cdot b^{ins}_t] \\
S_t = N \cdot \Phi(p_{t-1}, v_{new}, \sigma_{new}) \cdot [\pi^s_t \cdot b^{ind}_t + (1 - \pi^s_t) \cdot b^{ins}_t]
\end{cases}$$

Expansion 2

Instead of doing the segmentation of individual versus institutional investors on the same normal distribution, we can argue that individual and institutional investors have their own distributions for the expected true value of the stock, which include both different expected true value and the standard deviation of the consensus distributions. Similar to the first expansion above, we assume different risk factors for the two groups of investors.
Table 4: 2-Segment Model-2 Notations

<table>
<thead>
<tr>
<th>Notation</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \gamma )</td>
<td>Ratio of individual investors over institutional investors</td>
</tr>
<tr>
<td>( \tilde{v}_{new}^{ind} )</td>
<td>New true value of the risky asset (stock) for individual investors</td>
</tr>
<tr>
<td>( \tilde{v}_{new}^{ins} )</td>
<td>New true value of the risky asset (stock) for institutional investors</td>
</tr>
<tr>
<td>( b_t^{ind} )</td>
<td>Average risk an individual buyer takes</td>
</tr>
<tr>
<td>( s_t^{ind} )</td>
<td>Average risk an individual seller takes</td>
</tr>
<tr>
<td>( b_t^{ins} )</td>
<td>Average risk an institutional buyer takes</td>
</tr>
<tr>
<td>( s_t^{ins} )</td>
<td>Average risk an institutional seller takes</td>
</tr>
</tbody>
</table>

\[ D_t = B_t - S_t, \text{ where} \]
\[ B_t = b_t^{ind} + \frac{\gamma}{\gamma + 1} \cdot N \cdot [1 - \Phi(p_{t-1}, \tilde{v}_{new}^{ind}, \sigma_{new}^{ind})] \cdot b_t^{ind} + \frac{1}{\gamma + 1} \cdot N \cdot [1 - \Phi(p_{t-1}, \tilde{v}_{new}^{ins}, \sigma_{new}^{ins})] \cdot b_t^{ins} \]
and
\[ S_t = s_t^{ind} + \frac{\gamma}{\gamma + 1} \cdot N \cdot [1 - \Phi(p_{t-1}, \tilde{v}_{new}^{ind}, \sigma_{new}^{ind})] \cdot s_t^{ind} + \frac{1}{\gamma + 1} \cdot N \cdot [1 - \Phi(p_{t-1}, \tilde{v}_{new}^{ins}, \sigma_{new}^{ins})] \cdot b_t^{ins} \]

The new equilibrium is reached when price is stabilized at \( p_T \).

Model Discussion

While the 2-segment models can only offer us a theoretical expansion and it is hard to gauge the impact from price limits on institutional versus individual investors without running complex simulations, we can still draw some limited insights from it. First, it serves as a reminder that merely differentiating individual and individual investors based on the speed of identifying information is a limited view. We must also consider their risk factors, that is, their ability to take risk and allocate capital at any given time. Second,
price limits can cause different effects can only effectively protect individual investors if they are positioned in the “correct” directions. However, it seems that even we can simulate different scenarios for individual versus institutional investors, it is hard to justify the implementation of price limit policy with providing protections for individual investors.

LIMITATIONS AND FUTURE STUDIES

Despite the inferences and insights that this theoretical framework can provide us, we also acknowledge some short-comings of the analysis. First, this model is built upon various assumptions that could be challenged in the real world. For instance, it is very likely that the new equilibrium price, i.e., $v_{\text{new}}$, is constantly changing before the equilibrium is reached. Similarly, the standard deviation of expected true value of the asset could also change instantaneously as time progresses.

Another limitation of this paper exists with the fact that the model was not applied to empirical data. Future studies can rely on the model to test several things, including the frequency of appearances for the eight different scenarios. It would also be helpful if future studies can use this framework to assess how price limits serve as signal to affect risk bearing activities of both individual and institutional investors.

CONCLUSION

This paper offers a theoretical model for stock trading that can incorporate price limits. This model allows us to examine the effect of price limit policy in stock market by both looking at some theoretical results and running simulations under the framework. The
general model identifies eight scenarios of hitting price limit, each of which will be likely to result in different consequences on the following trading period and thus turns the problem of price limit study into the identification of the frequency of these eight patterns in real world data. We also offer two possible model expansions to include the perspective of individual versus institutional investors. In conclusion, under our model, the existence of price limits cannot be theoretically justified due to a lack of consensual and beneficial impact.
REFERENCE


