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Capacity Dynamics and Endogenous Asymmetries In Firm Size

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Abstract
Empirical evidence suggests that there are substantial and persistent differences in the sizes of firms in most industries. We propose a dynamic model of capacity accumulation that is consistent with the observed facts. The model highlights the mode of product market competition and the extent of investment reversibility as key determinants of the size distribution of firms in an industry. In particular, if firms compete in prices and the rate of depreciation is large, then the industry moves toward an outcome with one dominant firm and one small firm. Industry dynamics in this case resemble a preemption race. Contrary to the usual intuition, this preemption race becomes more brutal as investment becomes more reversible.

Capacity Dynamics and Endogenous Asymmetries in Firm Size
– Online Appendix –

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1 Extensions and Generalizations

1.1 Cost/Benefit Considerations and Asymmetric Industry Structures

Asymmetries arise and persist provided that one firm has a strategic advantage over the other. The tangible form of this advantage is that one firm can get the other to stop investing. In Section 4 of the paper, we have shown that this is the case under price competition because a firm's profits from product market competition peak in its own capacity. Below we show that cost/benefit considerations can give rise to a strategic advantage and hence asymmetries irrespective of the mode product market competition, but that the dynamics of the industry hinge on the source of the strategic advantage. In particular, the possibility of gaining a strategic advantage based on cost/benefit considerations does not lead to a preemption race.

Benefit of Capacity. Roughly speaking, the benefit of adding a block of capacity is determined by the increase in profits from product market competition that results from an increase in capacity.[1] Recall that demand is given by $Q(P) = a - bP$ or, equivalently, $P(Q) = \frac{a}{b} - \frac{Q}{b}$. An increase in b thus causes an inward rotation of demand around the maximum quantity a. That is, holding a fixed, increasing b corresponds to decreasing consumers' willingness to pay. In fact, profits $\pi(i, j)$ from product market competition are inversely proportional to b, and the marginal benefit of capacity therefore decreases with b.

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[1] The nature of our argument here is more suggestive than formal because it is in fact the value function that determines the marginal benefit of capacity (see equation (4) in the paper). At the same time, however, the value function reflects not only the policy function but also the profits from product market competition.

In other words, if consumers' willingness to pay is low, the marginal benefit of capacity is low.

Table 1 lists the most likely long-run industry structures under quantity and price competition for various values of b.[2] Throughout, we hold the rate of depreciation fixed at $\delta = 0.1$. As can be seen, decreasing b relative to our baseline parameterization of $b = 10$

b	most likely industry structure under quantity competition	most likely industry structure under price competition
0.1	(5,5)	(3,9), (9,3)
1	(5,5)	(3,9), (9,3)
$\boxed{10}$	$\boxed{(4,4)}$	$\boxed{(2,8), (8,2)}$
15	(3,3)	(2,7), (7,2)
20	(3,3)	(0,7), (7,0)
50	(0,4), (4,0)	(0,4), (4,0)
100	(0,0)	(0,0)

Table 1: Most likely industry structures. Quantity and price competition.

changes little. This is in line with our intuition: If consumers' willingness to pay is high, the marginal benefit of capacity is high. Hence, firms have a strong incentive to forestall depreciation by holding extra capacity. Similarly, increasing b from 10 to 20 changes little. Increasing b further to 50, however, produces a noticeable change: Now there is an asymmetry under both modes of product market competition. In fact, one firm accumulates enough capacity to supply the monopoly quantity, the other no capacity at all. With $b = 100$, finally, we have $x(i, j) = 0$ for all i and j. Hence, a symmetric (albeit trivial) industry structure arises. The reason is that consumers' willingness to pay is already so low that adding capacity is simply not worthwhile.

How does an asymmetric industry structure come about under quantity competition? Figure 1 shows the policy function and limiting distribution in case of $b = 20$ and $b = 50$. With $b = 20$, the limiting distribution is unimodal with a mode at state $(3, 3)$. The reason is that a firm cannot gain a strategic advantage over its rival: $i \leq 3$ implies $x(i, j) > 0$ for all j. This is in line with our analysis of capacity-constrained quantity competition in Section 3 of the paper. With $b = 50$, by contrast, a firm has a strategic motive to invest as it can now deter its rival from investing by growing large. In particular, $x(0, j) > 0$ if $j \leq 2$ and $x(0, j) = 0$ if $j \geq 3$. That is, a medium-sized or large firm has a strategic advantage over a small rival because the smaller firm stops to invest if it is sufficiently far behind. One therefore expects the industry to evolve towards an asymmetric structure, and indeed the limiting distribution is bimodal with modes at states $(0, 4)$ and $(4, 0)$.

[2] Decreasing a has very similar effects to increasing b. Here we focus on b because a cannot be meaningfully increased without adjusting the capacity grid. To see this, note that, say, the Cournot quantity $q^D = \frac{a}{3}$ is increasing in a but independent of b.

To better understand what is going on, consider again the single-period profit function under quantity competition as shown in the left panel of Figure 3 and tabulated in Table 5 (up to scale). The key point to note here is that the marginal benefit of capacity (weakly) decreases in the rival's capacity.[3] Consequently, when competing against a medium-sized or large firm, a small firm has a weak incentive to invest and, in fact, may choose not to invest at all. This is the source of the strategic advantage that a medium-sized or large firm enjoys over a small rival. Finally, turning from $b = 50$ to $b = 20$, the marginal benefit of capacity goes up. But if consumers' willingness to pay is high enough, the marginal benefit of capacity more than outweighs the cost irrespective of the rival's capacity. Hence, neither firm is able to gain a strategic advantage over the other, and a symmetric industry structure results.

While the industry structure may be alike, the industry dynamics hinge on the source of the strategic advantage. In particular, the possibility of gaining a strategic advantage based on cost/benefit considerations does not lead to a preemption race. Instead, one of the firms is stuck forever with zero capacity. To see this, consider the marginal distribution of states (i, j) after $T = 5, 15, 25, 50$ periods, starting from state $(0, 0)$ (not shown). Under quantity competition with $b = 50$, the modes of the marginal distribution are states $(0, 1)$ and $(0, 1)$ after $T = 5$ periods; states $(0, 3)$ and $(3, 0)$ after $T = 15$ periods; and states $(0, 4)$ and $(4, 0)$ after $T = 25$ and $T = 50$ periods. In sum, the smaller firm is trapped in its marginal position.

Instead of engaging in a drawn-out preemption race, firms barely put up a fight for market dominance. Suppose the industry starts in state $(0, 0)$ with both firms investing 5.05. If Firm 1 gets the lead, it increases its investment to 6.74 and Firm 2 decreases its investment to 3.01. This most likely moves the industry from state $(1, 0)$ to state $(2, 0)$, where Firm 1 and Firm 2 decrease their investments to 5.41 and 1.01, respectively. But once the industry moves on to state $(3, 0)$, Firm 2 stops investing. In other words, investment is too low to even make sure that a firm leaves the initial zero-capacity state behind. This, of course, contrasts sharply with the heavy investment under capacity-constrained price competition.

Cost of Capacity. We now turn from the benefit to the cost of capacity (as embodied in the transition probabilities). Recall that we choose the effectiveness of investment α given the rate of depreciation δ such that the probability of success equals $\bar{\theta}$ at an investment of $\bar{x}$, i.e., $\alpha = \frac{\bar{\theta}}{(1-\delta-\bar{\theta})\bar{x}}$. Given our baseline parameterization of $\bar{x} = 20$ and $\bar{\theta} = 0.5$, the cost of a 50-percent-chance of adding a block of capacity is therefore 20. To put these numbers into perspective, recall that the monopoly quantity corresponds to 4 blocks of capacity and that the monopoly profit is 40. Clearly, a higher cost of adding a block of capacity goes hand-in-hand with a lower effectiveness of investment because $\bar{x}$ and α are inversely

[3]With the exception of state $(2, 2)$ to be precise.

proportional. We focus on $\bar{x}$ in what follows because it is easier to interpret than α.

Table 2 lists the most likely long-run industry structures under quantity and price competition for various values of $\bar{x}$. Strikingly, the entries in Table 2 are identical to the ones

$\bar{x}$	most likely industry structure under quantity competition	most likely industry structure under price competition
0.2	(5,5)	(3,9), (9,3)
2	(5,5)	(3,9), (9,3)
$\boxed{20}$	$\boxed{(4,4)}$	$\boxed{(2,8), (8,2)}$
30	(3,3)	(2,7), (7,2)
40	(3,3)	(0,7), (7,0)
100	(0,4), (4,0)	(0,4), (4,0)
200	(0,0)	(0,0)

Table 2: Most likely industry structures. Quantity and price competition.

in Table 1. Of course, this does not imply that, say, doubling $\bar{x}$ is equivalent to doubling b. In fact, the investment policy with $b = 10$ and $\bar{x} = 40$ is different from the one with $b = 20$ and $\bar{x} = 20$. However, these differences are small, indicating that the same mechanism is at work in both cases. In particular, with $\bar{x} = 100$, there is again an asymmetry under quantity as well as price competition. This time, however, it arises not because the marginal benefit of capacity is low but because the cost is high. Finally, with $\bar{x} = 200$, the marginal cost of capacity is already so high that adding capacity is simply not worthwhile and we again have $x(i, j) = 0$ for all i and j.

Pakes & McGuire's (1994) Quality Ladder Model. Cost/benefit considerations are the driving force in Pakes & McGuire's (1994) quality ladder model. In their model, the product market is characterized by price competition with differentiated products. Firms invest to increase the quality of their product. There is an outside good. The industry is hit by a shock once the quality of the outside good goes up. Given the functional form of demand, this translates into a (potential) decrease in the quality of all inside goods.

There are three similarities between Pakes & McGuire's (1994) quality ladder model and our capacity accumulation model with capacity-constrained *quantity* competition. First, depending on the parameterization, both models may lead to a symmetric or an asymmetric industry structure. Second, just like capacity-constrained quantity competition, cost/benefit considerations give rise to a strategic advantage in Pakes & McGuire's (1994) quality ladder model. Third, again just like capacity-constrained quantity competition, the possibility of gaining a strategic advantage does not lead to a preemption race in Pakes & McGuire's (1994) quality ladder model. Instead, one of the firms is stuck forever with a good of the lowest possible quality.

Turning to our model of capacity accumulation with *price* competition, recall that a firm's profits from product market competition peak in its own capacity. Consequently, it is often better for the small firm to be considerably smaller than the large firm rather than to be slightly smaller. Cost/benefit considerations therefore play a minor role in our model. In fact, because more capacity leads to less profits, in our model the small firm would be better off staying put rather than trying to grow even if capacity were costless. For the same reason it is also in the self-interest of a firm to withdraw capacity from the industry once it has lost the preemption race, which is precisely why firms engage in the preemption race in the first place. In this sense, both the strategic advantage that renders asymmetries persistent and the preemption race that is fought to determine the identity of the industry leader are intimately connected to the nature of capacity-constrained price competition. This differs markedly from Pakes & McGuire's (1994) quality ladder model.

1.2 Product Differentiation

Up to this point, we have focused on product market competition with homogeneous products. Turning to the opposite extreme of independent goods, it should be clear that asymmetric industry structures can no longer arise: After all, with independent goods, both firms are monopolists, and each firm therefore accumulates enough capacity to supply the monopoly quantity.

The question then is: What happens in intermediate cases? To answer this question, we incorporate product differentiation into our model of price competition. This allows us to study how the degree of product differentiation affects the structure of an industry.

Unfortunately, there is no "off-the-shelf" model of capacity-constrained price competition with differentiated products. Below we therefore derive the single-period profit functions from first principles. In doing so, we replace the "hard" capacity constraints of Kreps & Scheinkman (1983), Deneckere & Kovenock (1996), and Allen, Deneckere, Faith & Kovenock (2000) with "soft" capacity constraints. Hard capacity constraints imply that the cost of producing beyond capacity is infinite. With soft capacity constraints, on the other hand, a firm can produce any quantity, albeit at an exploding cost. This is in line with the notion that capacity choice is really a choice of scale that determines the cost structures of firm and thus sets the conditions for price competition (Tirole 1988, p. 218). This in turn suggests that the idiosyncratic shocks to firms' capacities that play such an important role in shaping industries are really tantamount to cost shocks.

Soft capacity constraints offer an additional advantage: They allow us to assume that firms have a "common carrier requirement". That is, a firm is obliged to satisfy all of its demand, it cannot turn away customers. This avoids specifying a rationing scheme and gives rise to a Nash equilibrium in pure strategies in the product market game (see Maggi (1996b) among others).

Demand. The utility-maximization problem of the representative consumer is given by

$$\max_{q_0 \geq 0, q_1 \geq 0, q_2 \geq 0} q_0 + \alpha q_1 + \alpha q_2 - \frac{\beta}{2} q_1^2 - \frac{\beta}{2} q_2^2 - \gamma \beta q_1 q_2$$

subject to the budget constraint $q_0 + p_1 q_1 + p_2 q_2 = y$, where q_0 is the numéraire good and y the consumer's income. $\gamma \in [0, 1)$ measures the degree of product differentiation, ranging from zero for independent goods to unity for homogeneous goods (perfect substitutes). Solving the utility-maximization problem yields the linear demand system

$$\begin{aligned} q_1(p_1, p_2) &= \frac{1}{1-\gamma^2} \left(a(1-\gamma) - b p_1 + \gamma b p_2 \right), \\ q_2(p_1, p_2) &= \frac{1}{1-\gamma^2} \left(a(1-\gamma) - b p_2 + \gamma b p_1 \right), \end{aligned}$$

where $a = \frac{\alpha}{\beta}$ and $b = \frac{1}{\beta}$. This specification was originally proposed by Bowley (1924) and has subsequently been used by Spence (1976) and Dixit (1979) to model differentiated product oligopolies. Note that there are no income effects because utility is quasilinear.

Letting $P = \frac{p_1 + p_2}{2}$ be the average price, aggregate demand $q_1(p_1, p_2) + q_2(p_1, p_2)$ can be written as $Q(P) = \frac{2}{1+\gamma} (a - bP)$. Our earlier demand specification for homogeneous goods (equation (1) in the paper) is thus the limit of the above demand system for differentiated goods. For this reason, we retain $a = 40$ and $b = 10$ from the case of homogenous goods, but vary the degree of product differentiation by choosing $\gamma \in \{0.9, 0.7\}$. The top left and bottom left panels of Figure 2 show the resulting profits from product market competition.

Soft Capacity Constraints. Given that a firm holds $\overline{q} > 0$ units of capacity, we assume that the total cost of producing q units of output is

$$c(q, \overline{q}) = \frac{1}{1+\eta} \left(\frac{q}{\overline{q}} \right)^{\eta} q,$$

where $\eta > 0$ measures the severity of the capacity constraint. The larger is η, the closer we are to hard capacity constraints: marginal cost, $\left(\frac{q}{\overline{q}} \right)^{\eta}$, remains small as long as $q < \overline{q}$ (because $\left(\frac{q}{\overline{q}} \right)^{\eta} \to 0$ as $\eta \to \infty$), but becomes large once $q > \overline{q}$ (because $\left(\frac{q}{\overline{q}} \right)^{\eta} \to \infty$ as $\eta \to \infty$).

Since total cost is not defined at zero capacity, we set $M = 9$ with $\overline{q}_1 = 5$, $\overline{q}_2 = 10$ up to $\overline{q}_9 = 45$ in what follows. We set $\eta = 10$ to approximate hard capacity constraints. If $\overline{q} = \overline{q}_3 = 15$, for example, then marginal cost is essentially zero as long as output is below 12.5 units (about 83 percent of capacity). Marginal cost then rises gradually in a neighborhood of capacity, and finally increases sharply once output is in excess of 16 units (about 107 percent of capacity). Finally, we hold the rate of depreciation fixed at $\delta = 0.1$.

Price Competition. Suppose that firms' capacities are given by $(\overline{q}_i, \overline{q}_j)$ and that they compete in the product market by setting prices (p_1, p_2). Because firms produce to satisfy demand, the profit-maximization problem for, say, Firm 1 is given by

$$\max_{p_1 \geq 0} p_1 q_1(p_1, p_2) - c(q_1(p_1, p_2), \overline{q}_i).$$

Firm 1's profit-maximization problem and the corresponding problem for Firm 2 give rise to a system of FOCs. The Nash equilibrium can be computed easily by numerically solving this system.

Results. Figure 2 shows the profits from product market competition, policy functions, and limiting distributions for $\gamma \in \{0.9, 0.7\}$. If $\gamma = 0.9$, the policy function is reminiscent of the one we obtain under price competition, while it resembles the one that arises under quantity competition if $\gamma = 0.7$. In other words, how much a firm invests depends critically on its rival's capacity if the degree of product differentiation is low (i.e., $\gamma = 0.9$, see top middle panel), whereas a firm's investment is fairly insensitive to its rival's capacity if it is high (i.e., $\gamma = 0.7$, see bottom middle panel). In particular, industry dynamics resemble a preemption race if the degree of product differentiation is low, and the industry evolves towards an asymmetric structure (top right panel). In contrast, the industry evolves towards a symmetric structure if the degree of product differentiation is high (bottom right panel).

Table 3 lists the most likely long-run industry structures along with the corresponding cross-price elasticities for various values of γ. As can be seen, the cross-price elasticities are declining rapidly with the degree of product differentiation, whereas the size differences are declining slowly as we move away from homogeneous goods. In sum, asymmetric industry structures arise and persist as long as products are not too differentiated.

γ	most likely industry structure	cross-price elasticity of small firm	cross-price elasticity of large firm
0.95	(1,5), (5,1)	20.56	4.62
0.9	(1,5), (5,1)	10.54	2.61
0.8	(3,4), (4,3)	1.62	1.37
0.7	(4,4)	0.87	0.87
0.5	(4,4)	0.57	0.57

Table 3: Most likely industry structures and corresponding cross-price elasticities: Price competition with differentiated products.

1.3 Number of Firms

In contrast to the Kreps & Scheinkman (1983) model of capacity-constrained price competition with homogenous products, our model of capacity-constrained price competition with differentiated products is easily extended from $N = 2$ to $N > 2$ firms. Table 4 lists the five most likely size distributions along with their respective probabilities. The left panel pertains to $N = 2$ firms, the middle panel to $N = 3$ firms, and the right panel to $N = 4$ firms. Throughout, we hold the degree of product differentiation fixed at $\gamma = 0.9$ and the rate of depreciation at $\delta = 0.1$. Note that our notion of a size distribution differs somewhat from the one of an industry structure or state. For example, the industry structures (i, j) and (j, i) both give rise to the size distribution $\{i, j\}$. In general, a size distribution encompasses all permutations of an industry structure and is thus invariant to a relabelling of firms.

$N = 2$		$N = 3$		$N = 4$	
distribution	prob.	distribution	prob.	distribution	prob.
$\{1, 5\}$	0.32	$\{1, 1, 5\}$	0.33	$\{1, 1, 1, 4\}$	0.48
$\{2, 5\}$	0.24	$\{1, 2, 4\}$	0.20	$\{1, 1, 1, 3\}$	0.16
$\{1, 4\}$	0.15	$\{1, 1, 4\}$	0.14	$\{1, 1, 2, 4\}$	0.13
$\{2, 4\}$	0.11	$\{1, 2, 5\}$	0.12	$\{1, 1, 2, 3\}$	0.09
$\{3, 5\}$	0.06	$\{1, 2, 3\}$	0.05	$\{1, 1, 3, 4\}$	0.04

Table 4: Most likely size distributions along with their respective probabilities: Price competition with differentiated products.

As can be seen from Table 4, with $N > 2$ firms, price competition still leads to firms of unequal size. In general, the most likely industry structure consists of one medium-sized and $N - 1$ small firms. Moreover, industry dynamics continue to resemble a preemption race. Once a firm falls behind, it stops to invest, whereas its rivals continue to invest. In short, firms drop out of the race one-by-one, thus propelling the last remaining firm into a position of dominance.

1.4 Other Extensions and Robustness Checks

Below we briefly discuss two other extensions of the model, namely entry and exit and demand uncertainty. We then argue that our results are robust to the chosen capacity grid. To facilitate comparisons with earlier results, we return to the case of homogenous products.

Entry and Exit. Entry and exit can be incorporated in straightforward ways into our model. Indeed, the situation considered in this paper can be interpreted as one in which, although entry and exit are feasible, entry barriers are insurmountable (infinite setup costs) and investments in capacity are completely sunk (zero salvage value).

On the other hand, some preliminary work indicates that our conclusions remain intact with less extreme assumptions. To see this, consider the case of price competition and positive depreciation. If exit is incorporated (positive salvage value), a firm that falls behind its rival first stops investing and then exits the market. This adds value to becoming the dominant firm, and thus tends to make the preemption race more brutal. On the other hand, the dominant firm is not free of all competitive pressures once the small firm has exited. In fact, as long as entry is possible (small setup costs), the incumbent firm continues to sustain high levels of capacity in order to ward off potential entrants. Hence, as in the model without entry and exit, the large firm has to perpetually reinforce its dominant position.

Demand Uncertainty. The papers by Maggi (1996a), Gabszewicz & Poddar (1997), and Reynolds & Wilson (2000) emphasize the role demand uncertainty plays in shaping the structure of an industry. Demand uncertainty is easily incorporated in our model by introducing demand states $d \in \{1, \ldots, D\}$ and specifying a demand function of the form $Q(P) = a_d - b_d P$. By choosing $a_d \geq 0$ and $b_d \geq 0$ appropriately, we are able to consider a range of different forms of demand uncertainty, including isoelastic rotations of demand around a constant choke-off price. The transition between demand states is governed by a Markov process.

The first scenario we explored was demand growth. Since firms anticipate the ultimate state of demand, their investment behavior in earlier demand states is similar to their investment behavior in the ultimate demand state. Moreover, all demand states but the ultimate are transitory. This in turn implies that the ergodic distribution is identical to the one that results from an industry without demand uncertainty. In sum, demand growth has no permanent impact on the structure of the industry.

We then turned to demand cycles. Compared to an industry without demand uncertainty, firms invest more in the upswing of a demand cycle and less in the downswing. Moreover, firms investment behavior is remarkably similar across demand states, again because firms anticipate the upcoming changes in demand. While the states in a demand cycle are recurrent, the ergodic distribution is again similar across demand states. Compared to an industry without demand uncertainty, the ergodic distribution is somewhat more spread out due to the additional variability added by the demand uncertainty. In sum, if firms compete in prices and investment is reversible, the industry ends up with one dominant firm and one small firm in the presence of demand cycles. Hence, although demand cycles have a lasting impact on the industry, they are relatively unimportant in comparison to the forces unleashed by the competitive interactions among firms.

Capacity Grid. We have repeated our computations with a finer capacity grid. In particular, we set $M = 19$ with $\overline{q}_0 = 0$, $\overline{q}_1 = 2.5$, $\overline{q}_2 = 5$ up to $\overline{q}_{19} = 45$ (instead of $M = 10$ with $\overline{q}_0 = 0$, $\overline{q}_1 = 5$, $\overline{q}_2 = 10$ up to $\overline{q}_9 = 45$). To keep the cost of adding a unit of capacity comparable between the two parameterizations, we have increased α, the effectiveness of investment, by 50%.

In case of quantity competition, the most likely long-run industry structures (in units of capacity) are now $(12.5, 12.5)$, $(12.5, 12.5)$, $(15, 15)$, and $(15, 15)$ for $\delta = 0, 0.01, 0.1, 0.3$. In particular, firms now fail to make maximal profits from product market competition even with $\delta = 0$. In other words, our earlier finding that the combined profits are maximal at the steady-state (Section 3 of the paper) is an artifact of the capacity levels we use. Moreover, the most likely long-run industry structures with $\delta = 0$ and $\delta = 0.01$ now coincide.

Turning to the case of price competition, the most likely long-run industry structures are now $(12.5, 12.5)$, $(12.5, 12.5)$, $(12.5, 20)$ and $(20, 12.5)$, and $(12.5, 25)$ and $(25, 12.5)$ for $\delta = 0, 0.01, 0.1, 0.3$. Again, the most likely long-run industry structures with $\delta = 0$ and $\delta = 0.01$ coincide. This is particularly striking in case of $\delta = 0.01$, where our original parameterization yielded $(10, 25)$ and $(25, 10)$ as the modes of the ergodic distribution. Note, however, that while the most likely industry structure is symmetric, the ergodic distribution also puts considerable mass on asymmetric structures: $(12.5, 15)$ and $(15, 12.5)$ each have a probability of 0.18 and $(10, 15)$ and $(15, 10)$ each have a probability of 0.03, whereas $(12.5, 12.5)$ has a probability of 0.55. Finally, the gap between firms remains substantial in case of $\delta = 0.1$ and $\delta = 0.3$.

In sum, small changes are magnified by the lumpiness of capacity. On the other hand, the basic shape of the policy functions remains unchanged with a finer capacity grid. Consequently, we continue to obtain preemption races and asymmetric industry structures under capacity-constrained price competition and symmetric industry structures under quantity competition.

2 Omitted Tables and Figures

- Tables 5 and 6 and Figure 3: Profits $\pi(i, j)$ from product market competition.
- Figures 4-11: Transition distribution after $T = 5, 15, 25, 50$ periods.

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$\pi(i,j)$		$j=0$ $\bar{q}_j=0$	$j=1$ $\bar{q}_j=5$	$j=2$ $\bar{q}_j=10$	$j=3$ $\bar{q}_j=15$	$j=4$ $\bar{q}_j=20$	$j=5$ $\bar{q}_j=25$	$j=6$ $\bar{q}_j=30$	$j=7$ $\bar{q}_j=35$	$j=8$ $\bar{q}_j=40$	$j=9$ $\bar{q}_j=45$
$i=0$	$\bar{q}_i=0$	$\boxed{0}$	0	0	0	0	0	0	0	0	0
$i=1$	$\bar{q}_i=5$	17.50	$\boxed{15.00}$	12.50	10.00	8.75	8.75	8.75	8.75	8.75	8.75
$i=2$	$\bar{q}_i=10$	30.00	25.00	$\boxed{20.00}$	15.00	15.00	15.00	15.00	15.00	15.00	15.00
$i=3$	$\bar{q}_i=15$	37.50	30.00	22.50	$\boxed{17.78}$	17.78	17.78	17.78	17.78	17.78	17.78
$i=4$	$\bar{q}_i=20$	40.00	30.63	22.50	17.78	$\boxed{17.78}$	17.78	17.78	17.78	17.78	17.78
$i=5$	$\bar{q}_i=25$	40.00	30.63	22.50	17.78	17.78	$\boxed{17.78}$	17.78	17.78	17.78	17.78
$i=6$	$\bar{q}_i=30$	40.00	30.63	22.50	17.78	17.78	17.78	$\boxed{17.78}$	17.78	17.78	17.78
$i=7$	$\bar{q}_i=35$	40.00	30.63	22.50	17.78	17.78	17.78	17.78	$\boxed{17.78}$	17.78	17.78
$i=8$	$\bar{q}_i=40$	40.00	30.63	22.50	17.78	17.78	17.78	17.78	17.78	$\boxed{17.78}$	17.78
$i=9$	$\bar{q}_i=45$	40.00	30.63	22.50	17.78	17.78	17.78	17.78	17.78	17.78	$\boxed{17.78}$

Table 5: Profits $\pi(i,j)$ from product market competition: Quantity competition.

$\pi(i,j)$		$j=0$ $\bar{q}_j=0$	$j=1$ $\bar{q}_j=5$	$j=2$ $\bar{q}_j=10$	$j=3$ $\bar{q}_j=15$	$j=4$ $\bar{q}_j=20$	$j=5$ $\bar{q}_j=25$	$j=6$ $\bar{q}_j=30$	$j=7$ $\bar{q}_j=35$	$j=8$ $\bar{q}_j=40$	$j=9$ $\bar{q}_j=45$
$i=0$	$\bar{q}_i=0$	$\boxed{0}$	0	0	0	0	0	0	0	0	0
$i=1$	$\bar{q}_i=5$	17.50	$\boxed{15.00}$	12.50	10.00	7.66	6.13	5.10	4.38	5.16	5.16
$i=2$	$\bar{q}_i=10$	30.00	25.00	$\boxed{20.00}$	15.00	11.25	9.00	7.50	6.43	6.77	6.77
$i=3$	$\bar{q}_i=15$	37.50	30.00	22.50	$\boxed{15.63}$	11.72	9.38	7.81	6.70	6.58	6.58
$i=4$	$\bar{q}_i=20$	40.00	30.63	22.50	15.63	$\boxed{10.00}$	8.00	6.67	5.71	5.36	5.36
$i=5$	$\bar{q}_i=25$	40.00	30.63	22.50	15.63	10.00	$\boxed{5.63}$	4.69	4.02	3.65	3.65
$i=6$	$\bar{q}_i=30$	40.00	30.63	22.50	15.63	10.00	5.63	$\boxed{2.50}$	2.14	1.91	1.91
$i=7$	$\bar{q}_i=35$	40.00	30.63	22.50	15.63	10.00	5.63	2.50	$\boxed{0.63}$	0.55	0.55
$i=8$	$\bar{q}_i=40$	40.00	30.63	22.50	15.63	10.00	5.63	2.50	0.63	$\boxed{0}$	0
$i=9$	$\bar{q}_i=45$	40.00	30.63	22.50	15.63	10.00	5.63	2.50	0.63	0	$\boxed{0}$

Table 6: Profits $\pi(i,j)$ from product market competition: Price competition.

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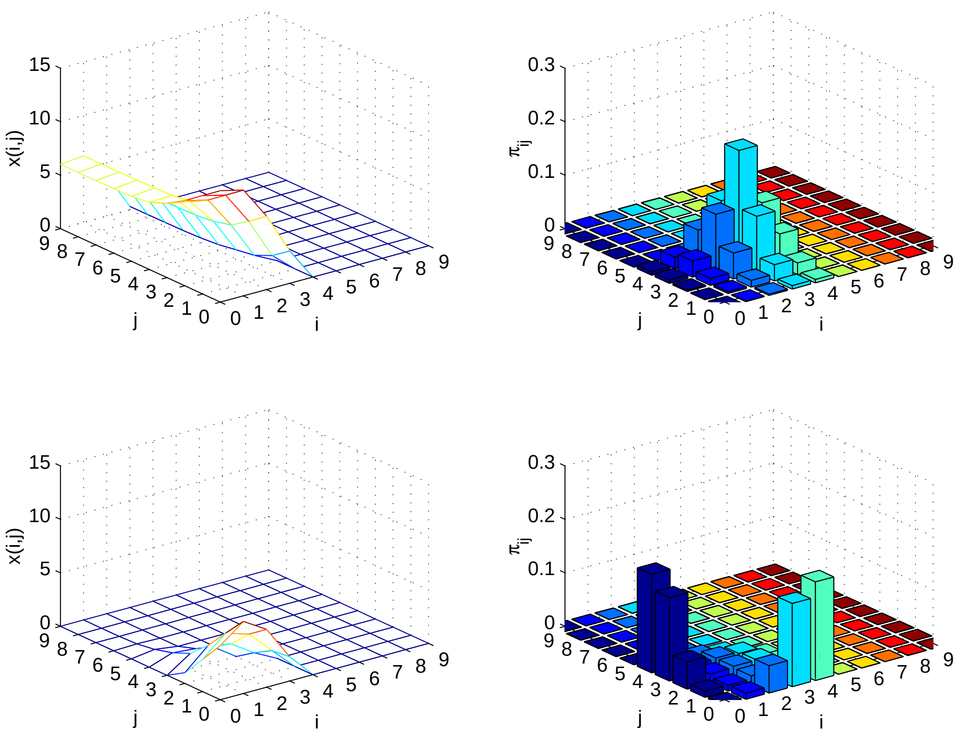

Figure 1: Policy function $x(i, j)$ and limiting distribution. Quantity competition with $b = 20$ (top panels) and $b = 50$ (bottom panels).

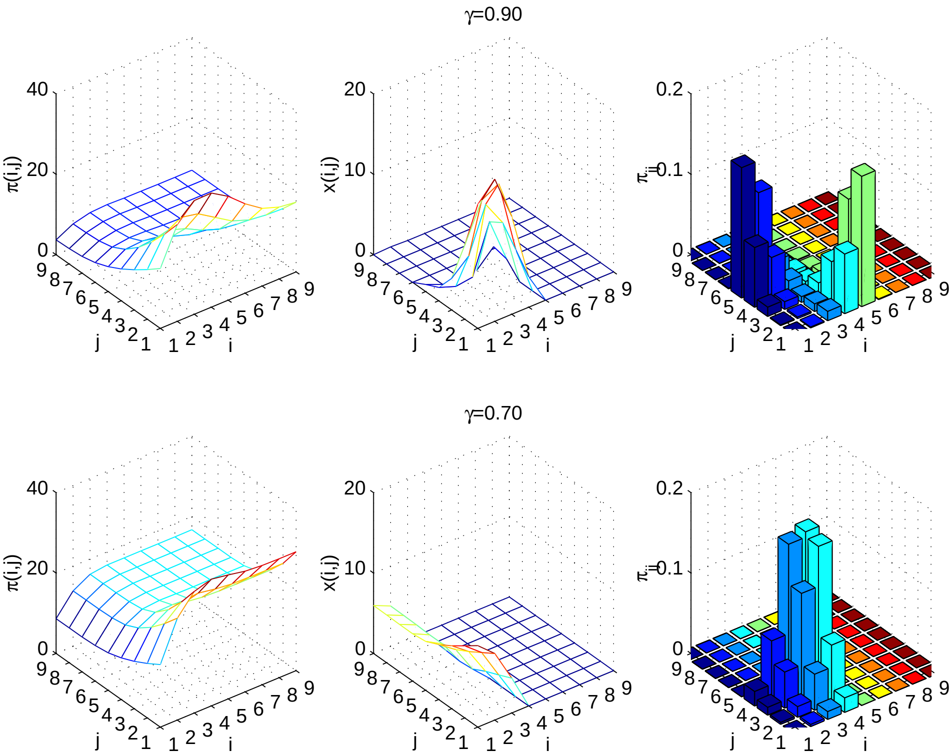

Figure 2: Profits $\pi(i,j)$ from product market competition with differentiated products, policy function $x(i,j)$, and limiting distribution. Price competition with $\gamma = 0.9$ (top panels) and $\gamma = 0.7$ (bottom panels).

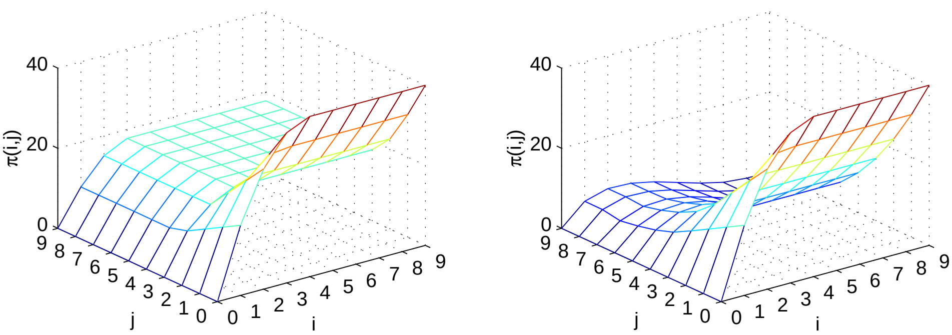

Figure 3: Profits $\pi(i, j)$ from product market competition: Quantity competition (left panel) and price competition (right panel).

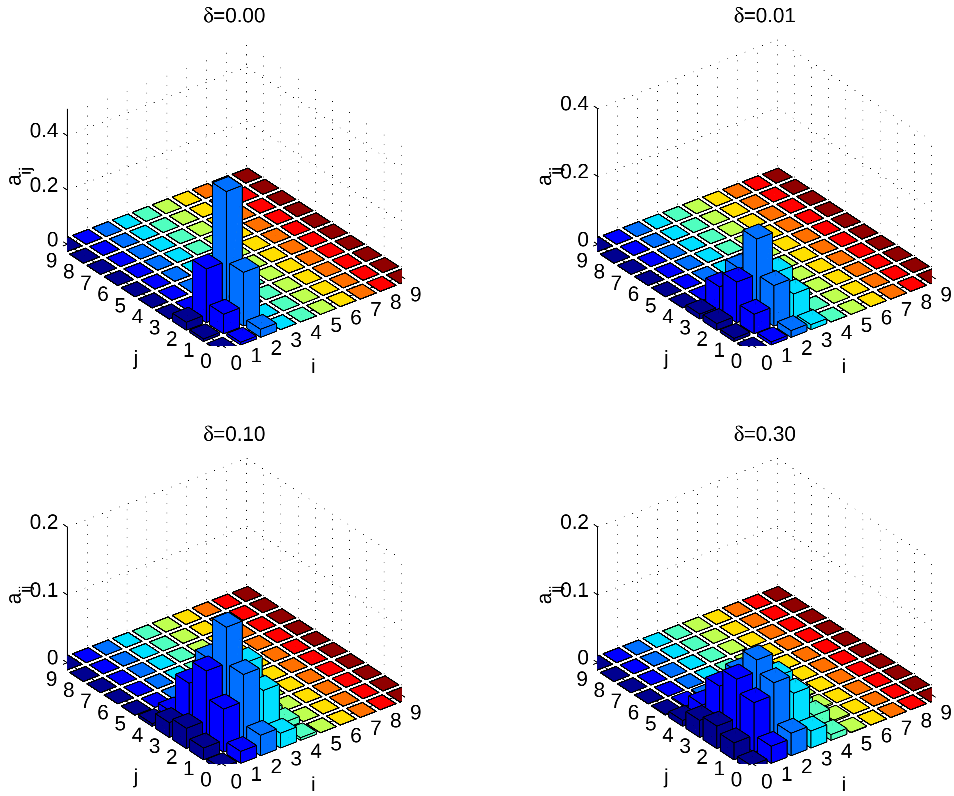

Figure 4: Transient distribution after $T = 5$ periods with initial state $i_0 = j_0 = 1$. Quantity competition with $\delta = 0$ (top left panel), $\delta = 0.01$ (top right panel), $\delta = 0.1$ (bottom left panel), and $\delta = 0.3$ (bottom right panel).

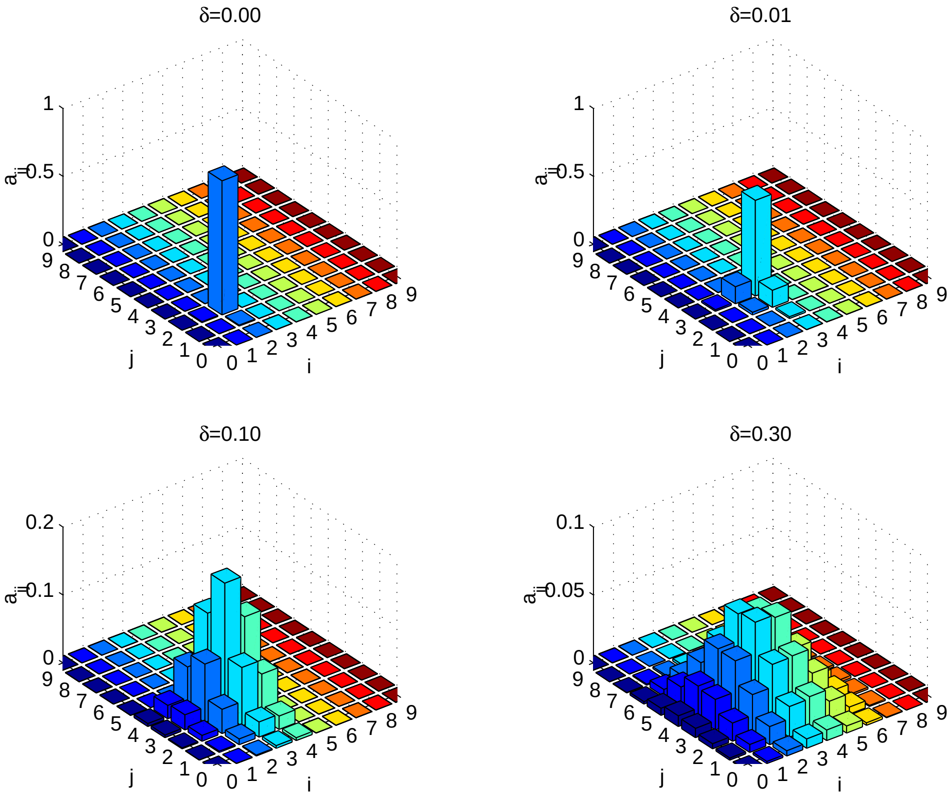

Figure 5: Transient distribution after $T = 15$ periods with initial state $i_0 = j_0 = 1$. Quantity competition with $\delta = 0$ (top left panel), $\delta = 0.01$ (top right panel), $\delta = 0.1$ (bottom left panel), and $\delta = 0.3$ (bottom right panel).

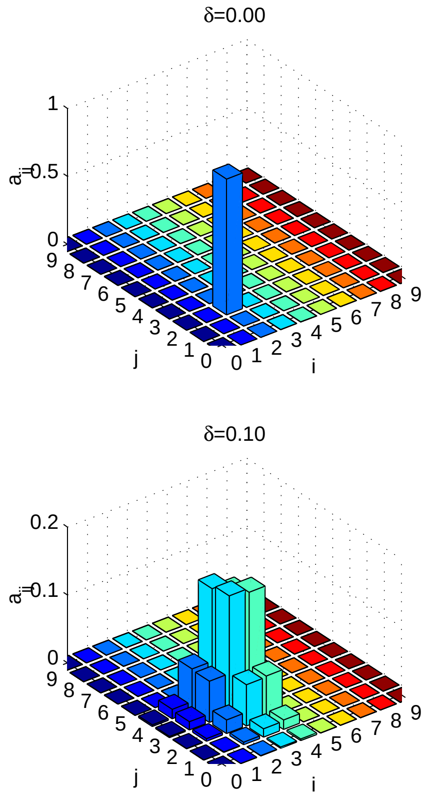

Figure 6: Transient distribution after $T = 25$ periods with initial state $i_0 = j_0 = 1$. Quantity competition with $\delta = 0$ (top left panel), $\delta = 0.01$ (top right panel), $\delta = 0.1$ (bottom left panel), and $\delta = 0.3$ (bottom right panel).

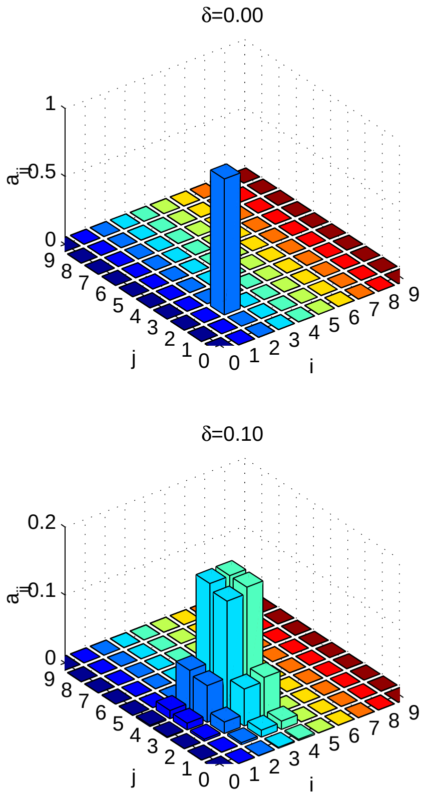

Figure 7: Transient distribution after $T = 50$ periods with initial state $i_0 = j_0 = 1$. Quantity competition with $\delta = 0$ (top left panel), $\delta = 0.01$ (top right panel), $\delta = 0.1$ (bottom left panel), and $\delta = 0.3$ (bottom right panel).

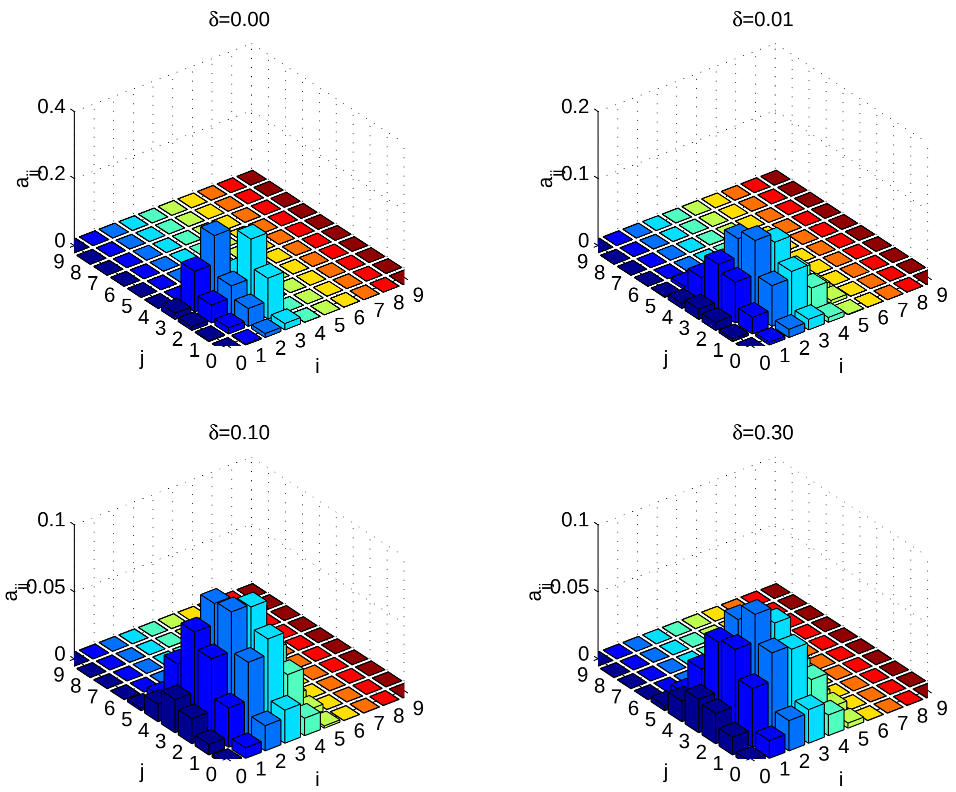

Figure 8: Transient distribution after $T = 5$ periods with initial state $i_0 = j_0 = 1$. Price competition with $\delta = 0$ (top left panel), $\delta = 0.01$ (top right panel), $\delta = 0.1$ (bottom left panel), and $\delta = 0.3$ (bottom right panel).

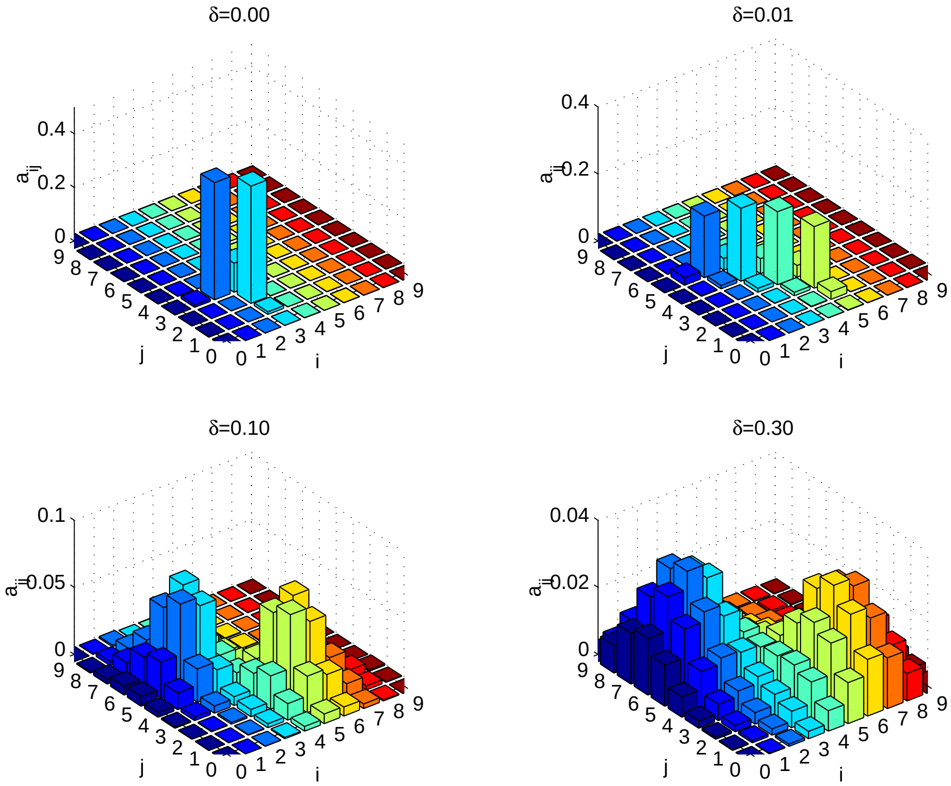

Figure 9: Transient distribution after $T = 15$ periods with initial state $i_0 = j_0 = 1$. Price competition with $\delta = 0$ (top left panel), $\delta = 0.01$ (top right panel), $\delta = 0.1$ (bottom left panel), and $\delta = 0.3$ (bottom right panel).

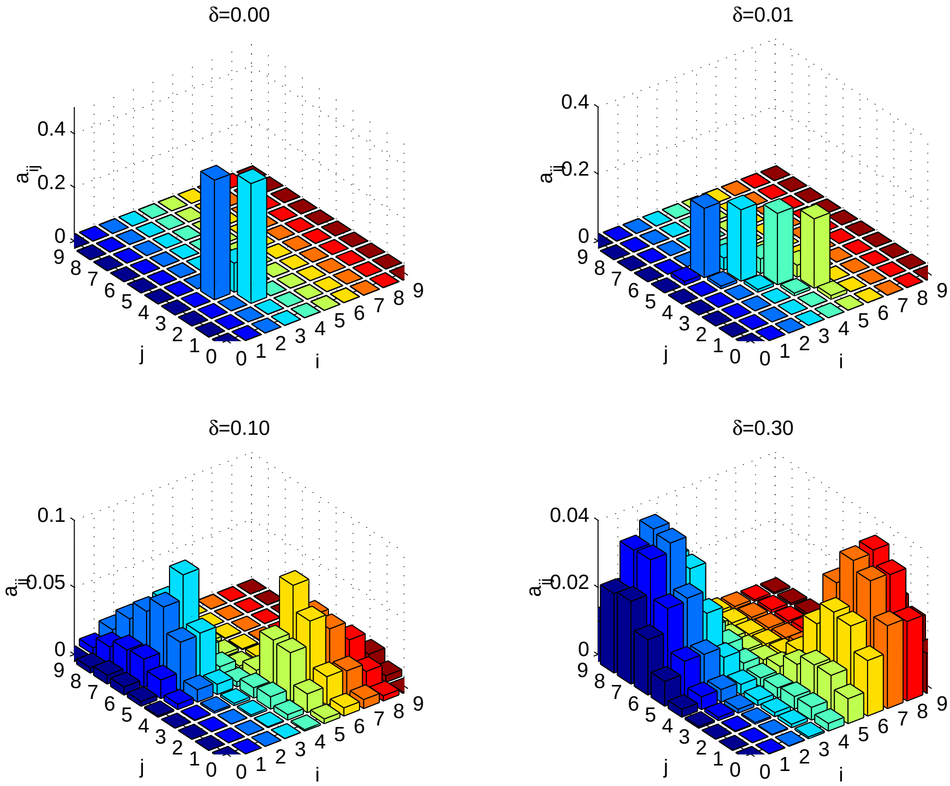

Figure 10: Transient distribution after $T = 25$ periods with initial state $i_0 = j_0 = 1$. Price competition with $\delta = 0$ (top left panel), $\delta = 0.01$ (top right panel), $\delta = 0.1$ (bottom left panel), and $\delta = 0.3$ (bottom right panel).

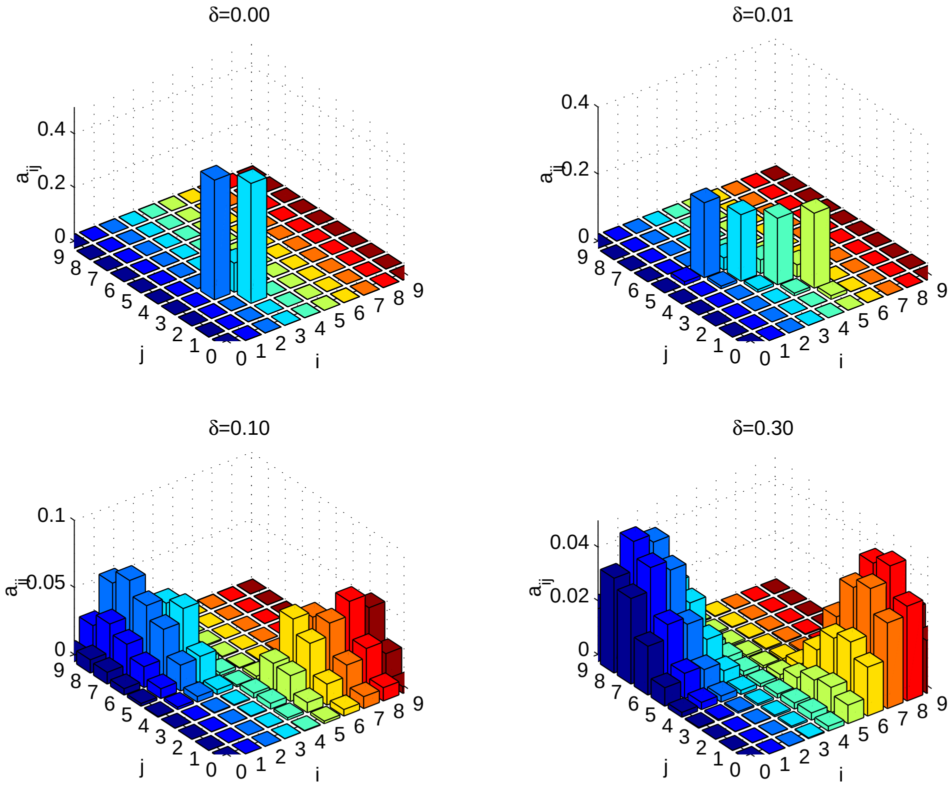

Figure 11: Transient distribution after $T = 50$ periods with initial state $i_0 = j_0 = 1$. Price competition with $\delta = 0$ (top left panel), $\delta = 0.01$ (top right panel), $\delta = 0.1$ (bottom left panel), and $\delta = 0.3$ (bottom right panel).