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Nikolai Roussanov
University of Pennsylvania

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Abstract

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Composition of Wealth, Conditioning Information, and the Cross-Section of Stock Returns *

Nikolai Roussanov

The Wharton School - University of Pennsylvania, and NBER[†]

November 25, 2012

ABSTRACT

Value stocks covary with aggregate consumption more than growth stocks during periods when financial wealth is low relative to consumption. However, the conditional value premium does not exhibit such countercyclical behavior. Consequently, a one-factor conditional consumption-based asset pricing model can be rejected without making any arbitrary assumptions on the dynamics of the price of risk or the conditional moments. Empirical evidence is somewhat more consistent with a consumption-based model augmented with an aggregate wealth growth factor, which can be motivated by recursive preferences or relative wealth concerns.

JEL Classification: G12, G17, C14.

Keywords: consumption-based asset pricing, conditioning information, human capital, stock return predictability, nonparametric regression, value premium, linear factor models, relative wealth concerns.

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[†]Contact: nroussan@wharton.upenn.edu

1 Introduction

The central prediction of the canonical consumption-based asset pricing model (e.g. Breeden (1979)) is that average return on any security is proportional to its risk, measured by the conditional covariance of returns with aggregate consumption growth. This prediction fails dramatically when confronted with the cross-section of unconditional expected returns, and particularly for the equity portfolios of Fama and French (1993). In principle, the consumption-based model could still hold conditionally, if both the price of consumption risk and the covariances of returns with consumption growth vary over time (e.g., Campbell and Cochrane (2000)). In this paper I show that the empirical properties of conditional moments of equity returns and aggregate consumption are inconsistent with the canonical conditional one-factor consumption-based model, without making any assumptions on the time-series behavior of aggregate risk aversion. The observed patterns of expected returns are potentially consistent with a generalization of the conditional consumption-based model that includes the return on the wealth portfolio as an additional priced factor. However, statistical evidence in support of the extended model, which may be hindered by the unobservable nature of aggregate wealth, is somewhat inconclusive.

I identify a key feature of the data that drives the rejection of standard CCAPM: "value" stocks, which have high *unconditional* expected returns, typically do not exhibit a greater increase in *conditional* expected returns than "growth" stocks when their relative exposure to consumption risk rises. This fact is at odds with explanations of the value premium that appeal to a time-varying price of consumption risk (e.g. Lettau and Ludvigson (2001b)) and thus underlies the economic (rather than purely statistical) rejection of the conditional CCAPM.[1] Imposing conditional moment restrictions prescribed by the theory in a flexible way that avoids tight parametric assumptions on the dynamics of conditional moments and

[1]A number of authors have argued that conditioning information substantially improves the empirical performance of consumption-based models by allowing the price of consumption risk to vary over time, in particular Lettau and Ludvigson (2001b), Lustig and Van Nieuwerburgh (2005), Petkova and Zhang (2005), and Santos and Veronesi (2006). However, others have suggested that the superior performance of the conditional models may be an illusion caused by the low statistical power of standard asset pricing tests (e.g. Lewellen and Nagel (2006), Ferson and Siegel (2009), and Nagel and Singleton (2010)).

risk prices reveals a conditional value premium puzzle of essentially the same magnitude as observed unconditionally.

These findings pose a challenge to some of the leading dynamic asset pricing models that rely on time-varying price of consumption risk, driven either by habit-dependent risk aversion (e.g. Campbell and Cochrane (1999)) or by shifts in the distribution of wealth across heterogeneous investors (e.g. Chan and Kogan (2002)). I explore an extension of the standard consumption-beta framework and consider a conditional two-factor model with contemporaneous aggregate consumption growth and aggregate wealth growth (proxied by the stock market return) – CWCAPM. Such a model can be motivated either by recursive preferences (Epstein and Zin (1991), Duffie and Epstein (1992)) or by social status concerns (Bakshi and Chen (1996), Roussanov (2010)). In the former class of models wealth growth is an additional state variable because it captures innovations to the continuation utility that may not be reflected in current consumption, whereas in the latter set of models aggregate wealth enters individual preferences directly.[2] Such a conditional two-factor model substantially reduces the magnitude of pricing errors on the benchmark book-to-market and size portfolios, effectively eliminating the value puzzle. Nevertheless, the evidence in favor of the model is not conclusive as some pricing errors are statistically significant (e.g., large growth stocks actually outperform).

The key innovation in my empirical analysis is testing conditional implications of asset pricing models without specifying a particular parametric structure on the dynamics of returns and factor risk prices.[3] I develop an intuitive econometric procedure based on

[2]Garleanu and Panageas (2009) build a heterogeneous-agents model with recursive preferences in which prices of risk associated with consumption growth and with news about future utility are both functions of the cross-sectional composition of wealth. While their explicit setup features a single source of aggregate uncertainty and thus collapses to a conditional one-factor model, a more general version of such a model can be a seen as an example of a two-factor CWCAPM. Such priced sources of risk that are not fully reflected in contemporaneous consumption are news about long-run growth pioneered by Bansal and Yaron (2004), investment-specific shocks introduced by Papanikolaou (2011), and innovations to uncertainty explored by Bansal, Kiku, Shaliastovich, and Yaron (2012) as well as Campbell, Giglio, Polk, and Turley (2012).

[3]In early contributions to the conditional CAPM/ICAPM literature, Bollerslev, Engle, and Wooldridge (1988) model the dynamics of conditional covariances explicitly using GARCH methodology, Campbell (1987) and Harvey (1989) also model conditional covariances explicitly via linear instrumental variables; Shanken (1990) pursues a similar approach.

nonparametric kernel regression. I estimate the conditional market prices of risk using the information contained in the cross section of asset returns via cross sectional regressions of conditional expected returns on conditional covariances, both estimated nonparametrically for each point in the state space.[4] This approach is robust to misspecification of both the conditional moments and the prices of risk. This is important, since most conditional asset pricing models do not describe explicitly the dependence of covariances or risk prices on the observed conditioning variables, and, as emphasized by Brandt and Chapman (2007), using *ad hoc* specification (e.g., linearity) can lead to spurious rejections. I use Monte Carlo simulation analysis to demonstrate that the pricing error tests based on my estimation methodology have sufficient power to reject a false model, yet also allow for a true conditional model to be detected even when the unconditional tests are likely to reject it (e.g., when the wealth portfolio return is imperfectly observed by the econometrician).

Given the difficulty of measuring the wealth portfolio, I provide additional evidence in support of the CWCAPM that relies on the fact that total wealth returns reflect news about future consumption growth (Bansal and Yaron (2004), Hansen, Heaton, and Li (2008), Hansen, Heaton, Lee, and Roussanov (2007)). This complementary approach involves using long-run rather than contemporaneous consumption growth to test the conditional CCAPM (e.g. as in Parker and Julliard (2005)). I show that covariances of portfolio returns with long-run consumption growth vary less over time than the contemporaneous covariances. Using these covariances in asset pricing tests results in small and insignificant pricing errors, but the advantage over the standard model seems to come primarily from the differences in unconditional rather than conditional covariances across portfolios. This result suggests that the mixed evidence in favor of the CWCAPM may be in part due to the fact that the stock market is a poor proxy for the total wealth portfolio, as originally pointed out by Roll (1977). The latter is especially relevant in the presence of composition effects, whereby the relative

[4] Following Pagan and Schwert (1990) it is common to use nonparametric regression to estimate conditional volatility of stock returns. For other studies that have used nonparametric techniques to identify nonlinearities in stochastic discount factors see, for example, Gallant, Hansen, and Tauchen (1990) and Bansal and Viswanathan (1993); Chen and Fan (1999), Wang (2003), and Chen and Ludvigson (2009) use nonparametric methods to test conditional moment restrictions implied by asset pricing models. The procedure developed here is also related to the conditional method of moments of Brandt (1999).

contributions of financial and human capital to total wealth change over time (Lustig and Van Nieuwerburgh (2008), Lustig, Van Nieuwerburgh, and Verdelhan (2009), Bansal, Kiku, Shaliastovich, and Yaron (2012)).

This paper is structured as follows. Section 2 describes the class of consumption-based conditional asset pricing models that feature composition effects and introduces the new econometric methodology for their estimation and testing. I present the main empirical results in Section 3. In Section 4 I investigate statistical properties of the nonparametric tests using simulation analysis. Section 5 extends the empirical analysis by considering long-run consumption risk and incorporating a larger conditioning information set. Section 6 concludes. Discussion of the underlying economic theory, statistical properties of the estimators, and data description is relegated to the Appendix, as are some of the empirical results confirming the robustness of my main findings.

2 Conditional linear factor models

2.1 Composition of total wealth and conditional CCAPM

A large class of consumption-based asset pricing models implies a relationship between conditional expected returns on risky assets in excess of the risk-free rate and the conditional covariance of excess returns with aggregate consumption growth. This relationship can be written as

$$E\left(R^{ei}_{t+1}|I_t\right) = \gamma_t Cov(R^{ei}_{t+1}, \frac{\Delta C_{t+1}}{C_t}|I_t) \tag{1}$$

where R^{ei}_{t+1} is the excess return and $\frac{\Delta C_{t+1}}{C_t}$ is the growth rate of aggregate consumption. In the classical setting with representative consumer who has power utility γ_t is constant over time and equal to the coefficient of relative risk aversion. More generally, γ_t is a function of variables contained in the information set I_t. This is the case in settings with time-varying risk aversion, such as the habit formation models (Constantinides (1990) and Campbell and Cochrane (1999)) where γ_t depends on the history of past consumption. It is also consistent with heterogeneous investor models in which the price of aggregate consumption risk depends on the evolution of the joint distribution of consumption shares and risk aversion parameters across households (e.g. Grossman and Shiller (1982), Chan and Kogan (2002)).

The possibility that the price of consumption covariance risk γ_t is time varying offers some hope of rationalizing puzzling features of the cross-section of stock returns within the consumption-based asset pricing paradigm, as emphasized by Campbell and Cochrane (2000). Assets that have the same unconditional covariance with consumption growth can earn different average returns if *conditional* covariances differ. Assets that covary more with consumption when the price of consumption risk γ_t is high are riskier, and therefore will have higher expected returns. In particular, Lettau and Ludvigson (2001b) argue that the "value premium" - the tendency of stocks with higher ratios of book to market equity to earn higher returns than do low book to market stocks - can be explained by the fact that "value" stocks comove more with consumption growth during "bad times" when the price of risk is high than do growth stocks, even though the *unconditional* covariances are not very different.

Generic conditional factor models are not testable using discrete-time data without imposing additional restrictions since the econometrician does not necessarily observe the entire conditioning information set (Hansen and Richard (1987)).[5] However one can test specific versions of these models that make predictions regarding specific observable quantities that capture time-variation in risk premia:

$$E\left(R^{ei}_{t+1}|z_t\right) = \gamma_C\left(z_t\right) Cov(R^{ei}_{t+1}, \frac{\Delta C_{t+1}}{C_t}|z_t). \tag{2}$$

where $z_t \in I_t$ are some pre-specified variables that are thought to capture variation in the price of consumption risk so that $\gamma_t = \gamma\left(z_t\right)$.

Here I specify the conditioning information set z_t *a priori* following the recent literature

[5] In continuous time the second moments can be measured arbitrarily precisely (Merton (1980), Andersen, Bollerslev, Diebold, and Labys (2003)) and therefore no conditioning information is required. In discrete-time, similar approach can be used under the assumption that these moments vary sufficiently smoothly over time (Lewellen and Nagel (2006), Ang and Kristensen (2009)) as long as high-frequency data is available. This approach is not applicable to testing models with factors that are not observed at high frequency, such as consumption.

that emphasizes the fluctuations in the composition of aggregate consumption and wealth and restrict it to variables that capture time variation in the shares of financial wealth and human capital in the total aggregate wealth. Economic theory predicts that these variables should be important for capturing time evolution in the conditional covariance between consumption growth and stock returns, as emphasized by Duffee (2005) and Santos and Veronesi (2006). Indeed, if stock market (or, more generally, all non-human) wealth W and a stream of labor income y are the only state variables driving consumption, this covariance can be expressed, for asset i, as

$$Cov_t(R^{ei}, \frac{\Delta C_{t+1}}{C_t}) = \varepsilon_W(z_t)\, Cov_t(R^{ei}_{t+1}, \frac{\Delta W_{t+1}}{W_t}) + \varepsilon_y(z_t)\, Cov_t(R^{ei}_{t+1}, \frac{\Delta y_{t+1}}{y_t}), \tag{3}$$

where $\varepsilon_W(z_t)$ and $\varepsilon_y(z_t)$ are elasticities of consumption with respect to financial wealth and labor income (which are assumed to be the only determinants of consumption). This equality holds exactly in continuous time if W and y follow diffusion processes (see Appendix A) but similar expressions can be derived in discrete time, at least approximately (e.g. Duffee (2005) uses the log-linearized Euler equation framework of Campbell (1996)). It shows that even if conditional covariances of asset returns with the total stock market wealth and with labor income growth are constant, the covariance of returns with consumption growth need not be.[6] For example, if stock returns and labor income growth are uncorrelated, this covariance will be greater when consumption is more sensitive to changes in stock market wealth.[7]

In the case of time-separable preferences with constant relative risk aversion coefficient

[6]The idea that the composition of total wealth might be important for explaining asset returns goes back at least to Roll (1977), who argued that the stock market is a poor proxy for the total wealth portfolio. Fama and Schwert (1977) tested a version of CAPM that includes human capital return as an additional factor. Stambaugh (1982) extended the market portfolio proxy to incorporate non-stock market assets. Ferson, Kandel, and Stambaugh (1987) tested (and rejected) a conditional CAPM in which market betas vary due to the changing composition of the market portfolio, even if the return covariance matrix is constant. More recently, some of the tests of conditional factor models also included proxies for the return to human capital - e.g. Campbell (1996), Jagannathan and Wang (1996), Jagannathan, Kubota, and Takehara (1998), Heaton and Lucas (2000), and Lettau and Ludvigson (2001b). A related, but different, recent strand of literature has focused on the effect of *consumption composition* on asset returns - see Pakos (2004), Piazzesi, Schneider, and Tuzel (2007), and Yogo (2006).

[7]This decomposition relies on deliberately stark assumptions about the joint dynamics of labor income and asset returns. If consumption reflects news about future growth rates (e.g., of labor income) or discount rates, the covariances with these innovations will also enter (3).

γ the conditional moment restriction (2) is equivalent to

$$E\left(R^{ei}_{t+1}|z_t\right) = \lambda_W\left(z_t\right) Cov(R^{ei}_{t+1}, R^{eM}_{t+1}|z_t) + \lambda_y\left(z_t\right) Cov(R^{ei}_{t+1}, \frac{\Delta y_{t+1}}{y_t}|z_t), \tag{4}$$

where R^{eM}_{t+1} is the excess return on the total financial wealth portfolio (i.e. the market return) and the prices of risk are given by $\lambda_W\left(z_t\right) = \gamma\varepsilon_W\left(z_t\right)$ and $\lambda_y\left(z_t\right) = \gamma\varepsilon_y\left(z_t\right)$. This observation that the risk premia associated with assets' covariances with the state variables are equal to the sensitivities of consumption to the state variables scaled by the utility curvature is the central insight of Breeden (1979), which leads to the equivalence between the multi-factor intertemporal CAPM and the single-factor consumption CAPM. In the more general case of recursive preferences (4) may still hold even if (2) does not (although the form of the price of risk functions λ_W and λ_y is more involved - see Appendix *A* for details). In what follows I refer to this model as an *Intertemporal CAPM* (with human capital).

In addition to the canonical consumption CAPM and the human-capital ICAPM above I consider another closely related model, referred to as CWCAPM, in which covariances of returns with both consumption growth and aggregate financial wealth growth (e.g., proxied by the market portfolio as above) contribute to the determination of asset's expected excess return:

$$E\left(R^{ei}_{t+1}|z_t\right) = \lambda_C\left(z_t\right) Cov(R^{ei}_{t+1}, \frac{\Delta C_{t+1}}{C_t}|z_t) + \lambda_W\left(z_t\right) Cov(R^{ei}_{t+1}, R^{eM}_{t+1}|z_t). \tag{5}$$

This specification is motivated by the asset pricing models with recursive utility in which aggregate wealth proxies for the continuation value of future consumption utility (e.g. Epstein and Zin (1989) and Duffie and Epstein (1992)) and models with social status concerns in which aggregate wealth is a state variable as long as it effects investors' relative position (e.g. Bakshi and Chen (1996) and Roussanov (2010)). In the latter case, the ratio of aggregate consumption to aggregate financial wealth is a fundamental state variable that drives time-variation in the two prices of risk $\lambda_C\left(z_t\right)$ and $\lambda_W\left(z_t\right)$ (see Appendix A for a derivation).

Motivated by the role of wealth composition in driving conditional moments of consumption and asset returns I use the following variables in my investigation: the ratio of labor income to consumption introduced by Santos and Veronesi (2006), the cointegrating residual of consumption, financial wealth and labor income developed by Lettau and Ludvigson (2001a), and the ratio of aggregate consumption to financial (stock market) wealth that is similar to the variable used by Duffee (2005).[8] Throughout the remainder of the paper I will adopt the following notation for the four alternative conditioning variables: the cointegrating residual of consumption and wealth is *cay*; by analogy, the labor income to consumption ratio is referred to as *yc*; the consumption to wealth ratio is labeled *ca*.

2.2 Testing conditional restrictions

Linear factor models of empirical asset pricing can be specified as restrictions on first and second moments of (excess) asset returns $\mathbf{R}^e$ and some fundamental factors $\mathbf{f}$ such as

$$E_t\left(\mathbf{R}^e_{t+1}\right) = Cov_t\left(\mathbf{R}^e_{t+1}, \mathbf{f_{t+1}}\right)' \lambda_t, \tag{6}$$

where λ is the vector of risk prices associated with the factors, which generally vary over time. This representation is equivalent to the stochastic discount factor representation and the somewhat more traditional beta representation (see Cochrane (2005) for discussion).

As is well known, the conditional model above does not in general imply the unconditional model

$$E\left(R^{ei}_{t+1}\right) = Cov\left(R^{ei}_{t+1}, \mathbf{f_{t+1}}\right)' \bar{\lambda}.$$

Thus the conditional model cannot be tested directly using standard econometric methods. The usual approach to testing such models (e.g. Cochrane (1996)) amounts to assuming that the conditional covariances and expected returns are (linear) functions of prespecified

[8]This is different from measuring the ratio of consumption to *total* wealth(e.g. as estimated by Lustig, Van Nieuwerburgh, and Verdelhan (2009)), which can vary even in the absence of the composition effect.

conditioning variable(s) and testing the unconditional 'scaled factor' models of the form

$$E\left(R^{ei}_{t+1}\right) = Cov\left(R^{ei}_{t+1}, \mathbf{\tilde{f}_{t+1}}\right)' \tilde{\lambda}, \tag{7}$$

where $\mathbf{\tilde{f}_{t+1}} = \mathbf{f_{t+1}} \otimes [\mathbf{1}, \mathbf{z}_t]$ and $\mathbf{z}$ is the vector of instruments that are assumed to capture *all* of the relevant conditioning information. The focus of this paper is on testing the conditional moment restrictions

$$E\left(R^{ei}_{t+1}|z_t\right) = Cov\left(R^{ei}_{t+1}, \mathbf{f_{t+1}}|z_t\right)' \lambda\left(z_t\right), \tag{8}$$

as well as their unconditional implications

$$E\left(R^{ei}_{t+1}\right) = E\left[Cov\left(R^{ei}_{t+1}, \mathbf{f_{t+1}}|z_t\right)' \lambda\left(z_t\right)\right]. \tag{9}$$

Imposing conditional moment restrictions is equivalent to augmenting the space of test assets with a large number of 'managed' portfolios that use the conditioning variable to determine the portfolio weights (e.g., see Cochrane (1996)). Therefore, doing so yields a much more powerful test of the conditional model than does (7). The challenge in imposing such conditional restrictions is in allowing for a sufficiently general functional form of the conditional moments and prices of risk, given little explicit guidance from economic theory.

2.3 Nonparametric cross-sectional regression

In this section I develop an econometric approach to estimating linear factor models with conditioning information that is robust to misspecification of the functional relationship between factor risk prices and the observed conditioning variables. This class of models can be summarized by the set of N conditional moment restrictions, each corresponding to one of the test assets $i \in \{1, \ldots, N\}$:

$$E\left(R^{ei}_{t+1} - Cov(R^{ei}_{t+1}, \mathbf{f}_{t+1}|z_t)'\lambda\left(z_t\right)|z_t\right) = 0,$$

where R^{ei}_{t+1}denotes excess returns on asset i and $\mathbf{f}_{t+1}$ is the K-vector of factors. The conditioning variable z_t is in general a d-dimensional vector.

For each fixed value z, the estimator of the vector of (conditional) risk prices is then

$$\hat{\lambda}(z) = \arg\min_{\lambda} \left\{ \mathbf{g}(z)' W(z) \mathbf{g}(z) \right\}, \tag{10}$$

where

$$\mathbf{g}(z) = \widehat{E}\left(R^{e}_{t+1}|z\right) - \widehat{Cov}\left(R^{e}_{t+1}, \mathbf{f}_{t+1}|z\right)' \lambda \tag{11}$$

and W is a weighting matrix[9] that can be state-dependent. Letting the vector of conditional mean returns to be denoted $\mathbf{m}(z)$ and the $N \times K$ matrix of conditional covariances between excess returns and factors be $\mathbf{cv}(z)$, the estimator is given by the weighted least-squares regression of conditional mean returns on conditional covariances:

$$\widehat{\lambda}(z) = \left(\widehat{\mathbf{cv}}(z)' W \widehat{\mathbf{cv}}(z)\right)^{-1} \widehat{\mathbf{cv}}(z)' W \widehat{\mathbf{m}}(z), \tag{12}$$

where the hatted variables refer to the estimated quantities, as usual. I use the locally linear estimators of conditional moments in most of my analysis, as they are known to possess somewhat better statistical properties than the simple nonparametric kernel regression approach, in particular lower bias at the boundaries of the state space, although both yields essentially the same results in my setting. See appendix for a detailed description of these estimators.

2.4 Properties of the estimator

Consistency of the price of risk estimates $\widehat{\lambda}(z)$ under the null hypothesis that the asset pricing model holds (i.e. the population moment conditions are satisfied) follows from the

[9]The nonparametric approach used by Wang (2003) can be viewed as a special case of the method considered here. He estimates stochastic discount factor (SDF) loadings under the assumption that the factor mimicking portfolios are priced exactly, and then uses this estimated SDF to test its ability to price a set of portfolio returns. In other words, he uses one set of (conditional) moment conditions for estimation (by setting K conditional moments to zero in sample) and another set of N moment conditions for testing.

consistency of nonparametric conditional moment estimators above. More formal discussion of consistency of the nonparametric price of risk estimators can be found in Appendix *B*. Similar to the standard two-pass method, the usual errors-in-variables problem arising from the fact that the covariances of returns with factors are estimated is also present in the context of conditional estimation considered here. It does not affect the consistency of our estimators as long as the "first-stage" quantities (conditional means and covariances) are estimated consistently, but it does make the market price of risk estimators biased. In addition, the nonparametric regression estimators of conditional moments are also biased. This is the usual cost associated with the flexibility allowed by nonparametric estimation. Of course, a parametric conditional model has the same problem unless economic theory specifies the functional form of the conditional moments and risk prices. Unfortunately, there is no straightforward way to "correct" for these two types of bias since the asymptotic theory for the estimators proposed above is rather involved and its development is beyond the scope of this paper[10]. In practice I use bootstrap methods to conduct statistical inference. Bootstrap allows constructing confidence intervals based on the approximated empirical distribution functions of the estimators under study. I provide the details of the bootstrap approach in Appendix *E*. The main way of controlling both the bias and the variance of the estimators is by choosing the bandwidth h, which essentially specifies how smooth the resulting functional estimates are (usually, too much smoothing increases the bias, whereas too little smoothing increases the variance of the estimators). It is known that the choice of a kernel function does not have a significant effect on the statistical properties of kernel estimators (see Pagan and Ullah (1999)), as long as they satisfy certain simple conditions (see Appendix *B*). I use Epanechnikov kernel, which is known to be optimal (in terms of the trade-off between bias and variance) whenever a single conditioning variable is used (as in my application).

Bandwidth selection is an unresolved issue that plagues much of the nonparametric estimation literature. It is a standard result that the optimal (in the sense that it minimizes the mean integrated square error of the nonparametric regression) smoothing parameter h

[10]Aït-Sahalia (1992) presents a general method for constructing asymptotic distributions of estimators based on nonparametric kernel functionals, which could be applied in the present setup.

is given by

$$h = c\sigma\left(z\right)T^{-\frac{1}{d+4}},$$

where σ is the (vector of) unconditional standard deviation(s) of z, T is the sample size, d is the dimension of z, and c is a constant. Therefore, in practice, one only is given an optimal convergence rate for the bandwidth, since the latter constant is unrestricted. Moreover, when variables in z are highly persistent, which is the case for most of the financial ratios and is true for some of the variables used in this study, larger bandwidths are optimal and convergence rates are slower than in the standard stationary setup (see Bandi (2004)).

There exist a number of techniques for "automatic" choice of the optimal constant c, and therefore of the optimal smoothing parameter. Most of them are based on either leave-one-out cross-validation or bootstrap and concentrate on minimizing the prediction error of the conditional moment estimators. Since in the present context the conditional moment estimators are "first-pass" quantities used in constructing the "second-pass" estimates of the market prices of risk, it is unclear that any of those procedures are equally suitable in the present context. At the same time, given the criterion that the estimators proposed here are based on, it is natural to make the choice of the bandwidth parameter subject to the same criterion. Consider

$$\begin{pmatrix} \hat{\lambda}\left(z\right) \\ \hat{h}(z) \end{pmatrix} = \arg\min_{\lambda,h}\left\{\mathbf{g}\left(z;\lambda,h\right)' W\left(z;h\right)\mathbf{g}\left(z;\lambda,h\right)\right\},$$

where

$$\mathbf{g}\left(z;\lambda,h\right) = \widehat{\mathbf{m}}\left(z;h\right) - \widehat{\mathbf{cv}}\left(z;h\right)'\lambda.$$

Then the first-order conditions still give the estimators $\hat{\lambda}\left(z\right)$ above, but now the bandwidth is chosen automatically. Pending further development of the asymptotic theory for the estimators proposed here there is no claim that this method of choosing the bandwidth is "optimal." I find, however, that the results obtained using this approach do not differ dramatically from those obtained with more standard procedures (for example, minimizing the mean integrated standard error under the bootstrap distribution).

Methodologically, my approach is closely related and complementary to that adopted by Nagel and Singleton (2010). They also impose conditional moment restrictions implied by the asset pricing model. They derive the optimal weights on the test assets and instruments that allow them to maximize the power of asset pricing tests asymptotically. At the same time, their adoption of the GMM framework requires an explicit specification of the prices of risk as functions of the conditioning variables. In contrast, I allow prices of risk to be as flexible as possible by using the fully nonparametric approach. While this approach in general will not be efficient, it minimizes the misspecification bias and gives a (true) model the greatest chance of success by ensuring that it is not rejected due to an incorrectly specified functional form of the risk prices (a non-trivial concern, as shown by Brandt and Chapman (2007)). In addition, in Section 5.2 below I show that my approach allows parsimonious nonparametric modeling of the dependence of the prices of risk on a large number of conditioning variables via a single-index approach similar to that of Aït-Sahalia and Brandt (2001).

3 Empirical results

3.1 Conditional expected returns and conditional covariances

I use excess returns on the six benchmark equity portfolios of Fama and French (1992), which are the intersection of the two portfolios formed on size and three portfolios formed on the ratio of book equity to market equity, to test conditional asset pricing models at quarterly frequency. The time period is fourth quarter of 1952 through the fourth quarter of 2008 (see Appendix D for detailed description of the data). Before evaluating the cross-sectional fit of the asset pricing models I analyze the dynamics of conditional moments of the test returns. All of these quantities are estimated nonparametrically; in order to reduce the bias in the estimates I present the means of the sampling distributions along with the 95% confidence intervals obtained via stationary bootstrap (see Appendix C for details on the bootstrap procedure). Shaded area in the background represents the kernel estimate of the probability density function of the conditioning variable, scaled appropriately.

Figure 1 displays conditional expected excess returns on the six benchmark portfolios as functions of *cay* (solid lines), along with the unconditional average returns (straight dashed lines). Expected returns on all of the portfolios increase throughout most of the range of *cay*, but decline at the high values of the state variable. The strength of the relationship varies across portfolios. For large portfolios, and especially for large growth portfolios, the differences between conditional mean returns in low-*cay* states and the high-*cay* states are a lot more pronounced and more statistically significant than they are for the small portfolios (especially small growth). For the large growth portfolios expected returns vary between being close to zero or slightly negative to over 4% per quarter, around the unconditional mean of about 2%. For the small value portfolio the expected returns vary between 1% and 5%, reverting back to the unconditional mean of 3.5% per quarter in the right tail of the distribution of *cay*. For the small portfolios the variation in expected returns is less detectable statistically than for large portfolios, as the 95% confidence intervals include the unconditional average return throughout most of the range except the lowest values of *cay*.

Figure 2 reports the estimates of conditional covariances of portfolio returns with consumption growth functions of *cay*. The functional relationship between the conditional covariance and the conditioning variable is roughly linear for all portfolios throughout most of range of the state variable, except at the tails of its distribution, where covariances appear concave but poorly estimated due to the relatively small number of extreme observations. All of the covariances are decreasing in *cay* (except in the extreme left tail of the distribution). The decreasing pattern is consistent with the wealth composition effect emphasized by Duffee (2005) if *cay* reflects changes in asset wealth more than changes in the value of human wealth (which is unobservable). The decline appears somewhat steeper for the small and growth portfolios. Since high values of *cay* predict high expected returns, they can be thought of as "bad" states of the world, in which the price of market risk is high. Conversely, low *cay* is associated with low risk premia. Lettau and Ludvigson (2001b) argue that this is the mechanism through which conditional-beta models can explain the high excess returns on value portfolios relative to the growth portfolios.

Are these differences in the direction of conditional covariances as functions of *cay* significant, economically or statistically?[11] I test whether the differences between consumption growth covariances of the value and growth portfolios within the same size grouping are significant, at a given value of the state variable. Figure 3 (lower panels) presents the plots of pairwise differences in conditional covariances between the two large and two small portfolio portfolios along the Value-Growth dimension, along with the 95% confidence bands. Broadly, the differences between the value and growth portfolios described above are marginally significant at the 5% level in the right tail of the distribution of *cay*: when the variable is above 0.02 ("bad states") covariance with aggregate consumption growth is higher for the large value portfolio than for the large growth, and for small value rather than for small growth. Conversely, when *cay* is below -0.02 ("good states"), the covariances are higher for the growth portfolios, although these differences are not significant. Given that in almost 60% of all observations *cay* is in the interval $[-0.01, 0.01]$, *most of the time* there is no statistically detectable difference in conditional covariances between value and growth portfolios.

In order to formally test whether the conditional moments evaluated at high and low values of *cay* are different, I construct bootstrap distributions for the differences between point estimates corresponding to such high and low values. Using these distributions recentered around zero I can test whether the estimated differences between conditional moments of a portfolio excess return evaluated at two different points in the state space are positive (for expected returns) or negative (for conditional covariances). Table I reports the differences between the point estimates of the conditional moments and the bootstrap p-values for these tests. The conditional means and covariances are estimated at values of *cay* equal to -0.019 and 0.02 which correspond approximately to the 10th and 90th percentiles of the empirical distribution of this variable. The differences in expected returns between the high and the low values of *cay* are positive and statistically significant for the basis portfolios, with the

[11]The difference between value and growth portfolios is less pronounced in the covariances with the market return and with the labor income growth (not reported here), which is consistent with the composition effect. The *yc* variable does not appear to capture a substantial cross-sectional variation in the dynamics of conditional covariances, while *ac* works similarly to *cay*. These estimates are omitted here but are available upon request.

one-sided p-values at or below 1 percent. Again, this is consistent with the notion that low values of *cay* represent "good states" and correspond to low risk premia, while high values - "bad states" and high risk premia.

The estimated differences of conditional covariances of basis portfolio returns with aggregate consumption growth are negative, but the p-values are larger. Still, for the Small Growth and the Large Growth portfolios the hypothesis that the difference is non-negative can be rejected as the p-values are below 5 percent. Importantly, however, the conditional covariances of the long-short (Value minus Growth) portfolio excess returns with consumption growth do exhibit the same pattern of time-variation as noted above: value is riskier than growth in "bad times" and vice versa. Indeed, for both large and small stocks the difference between point estimates of the conditional covariances is significantly positive with p-values under 2 percent.

Despite the marginal statistical significance and small economic magnitude of these differences, they have the right sign to be consistent with the value premium. In principle, given a "right" amount of variation in the price of consumption risk it might be possible to reconcile the *unconditional* expected returns predicted by the model with those observed in the data. However, the estimated *conditional* first moments paint a very different picture. The logic of the conditional (C)CAPM implies that value portfolios are riskier because they have higher conditional covariance with the factor (consumption growth) in bad times. It also implies that, as a consequence, conditional expected returns on value portfolios must be especially high in those states of the world, relative to the growth portfolios. This is not the case empirically: as described above, conditional expected returns on value (especially the small value) portfolios are only weakly increasing as a function of *cay*. At the same time, growth portfolios exhibit the strongest predictability, to the extent that the expected returns on large value and small growth are virtually the same in the "bad" states in which *cay* is high, even though they are quite different unconditionally. In particular, the differences of conditional expected returns between value and growth portfolios within each size grouping, plotted in the top two panel of figure 3 are in stark contrast to the corresponding differences in consumption covariances. While differences between covariances increase in "bad states," the differences in conditional expected returns are positive and flat throughout most of the range of *cay* and decrease in the right tail of the distribution, becoming significantly negative. The bootstrap tests reported in Table I indicate that the differences in conditional expected returns on Value minus Growth portfolios between high and low *cay* states are negative, albeit not significantly different from zero at conventional levels, unlike the differences in conditional covariances, which are significantly positive. It appears that utilizing conditioning information poses a challenge for consumption-risk models attempting to explain the value premium, since the dynamics of risk and expected returns appear to have the opposite signs.

3.2 Time-varying price of consumption risk

The nonparametric cross-sectional regression allows me to estimate the price of consumption risk (i.e., risk aversion) as a function of the conditioning variable. Figure 4 depicts the estimated risk price as a function of *cay*. Similarly to the behavior of conditional excess returns, the risk price is increasing as a function of the state variable throughout most of its range, except for the largest values of *cay* where the risk price plummets. The estimate is close to zero (and even slightly negative) for values of *cay* below -0.02, which correspond to "good times" in the Lettau and Ludvigson (2001b) interpretation. It rises to values around 250 and above at the mean of the distribution of *cay* which is equal to zero, becoming statistically reliably different from zero despite the wider confidence band. For values above the mean of *cay* the price of risk rises rapidly, reaching values of 500 and above. Such values for the quantity that is essentially the coefficient of relative risk aversion might appear extremely large, even if they are broadly consistent with the models of time-varying risk aversion such as Campbell and Cochrane (1999). However, after reaching its peak for values of *cay* around 0.02, the risk price starts to decline rapidly as a function of the state variable, plunging below zero for for *cay* above 0.03. While the confidence band is wide for these high levels of the state variable, this nonlinearity in the risk price is statistically significant. The nonlinearity of the estimated price of consumption risk may explain the finding of Nagel and Singleton (2010), who estimate this risk price under a linear specification and report negative estimated risk aversion over much of the state space spanned by *cay*.[12] While the point estimates of the risk aversion obtained using my flexible specification are negative in both tails of the distribution of *cay*, they are not statistically significantly different from zero in those regions, even in the left tail where the confidence bands are fairly narrow.

The fact that the estimated price of risk is not monotonic as a function of *cay*, which appears do be driven by the non-monotonicity of conditional expected returns depicted in Figure 1, may appear surprising. At least in some of the models of time-varying risk premia the effective risk aversion is a monotonic function of the underlying state variable (e.g. the surplus consumption ratio of Campbell and Cochrane (1999)). However, even if such a model were true, the fact that *cay* captures some of the composition effect as well as the time-varying risk aversion, may lead to a non-monotonicity (since the composition effect is, in general not monotonic - see discussion in Santos and Veronesi (2006)). Further, in models with heterogenous agents such as Garleanu and Panageas (2009) the price of aggregate consumption risk is not a monotonic function of the underlying state variable (the consumption share of risk-tolerant investors). If the model of interest did feature a monotonic relationship between a specific conditioning variable and the price of risk, one could in principle impose such a restriction in estimation (e.g. similarly to Aït-Sahalia and Duarte (2003)), potentially improving the efficiency of the estimator as well as increasing the power of the asset pricing tests. In fact, undersmoothing the estimator of the risk price by imposing a tight upper bound on the bandwidth parameter yields an essentially monotonic estimate of the price of risk even using my approach (these results are available upon request).

The relation between the estimated conditional risk-return trade-off and the conditioning variable could be potentially influenced by the small number of observations in both the left and the right tail of the empirical distribution of the *cay*. While there is substantial uncertainty about the estimates near both of the boundaries of the support of this distribu-

[12] Since Nagel and Singleton (2010) use gross returns to pin down the scale of the stochastic discount factor in their estimation, which may effect the price of risk estimates, their results may not be directly comparable to mine, as I only use excess returns.

tion, there is also possibility of a substantial bias. For example Li, Pearson, and Poteshman (2004) show that conditioning on the state variable not crossing the boundary can bias the estimated conditional means in a univariate setting. In the present context, this could mean that the estimates of conditional expected returns estimated as a function of a variable that includes market wealth in the denominator (such as consumption-wealth ratio) may be upwardly biased near the upper bound of the state space, as well as downward biased near the lower bound. Given the evidence in Figure 1 the former bias should not be a concern, since the conditional expected returns appear decreasing rather than increasing near the upper bound of *cay*. The latter bias may be more of a problem, potentially explaining why the estimated price of risk is zero or even negative for low values of *cay*. Employing locally-linear estimators of conditional moments helps mitigate boundary bias in the nonparametric context (e.g., Fan and Gijbels (1996)). It is likely that the actual bias is indeed rather small when *cay* is used as the conditioning variable since, unlike the ratio of consumption to stock market wealth *ca*, the consumption-wealth residual is a composite variable that incorporates information about aggregate labor income as well as non-stock market wealth, and therefore is less directly affected by the condition that the level of stock market wealth stays within the domain spanned by its observed values. Considering state variables that are not directly influenced by the stock market returns may also help reduce the bias. I employ one such variable, the labor income to consumption ratio *yc*, in Section **??** below. As a further extension, I use composites of several conditioning variables in Section 5.2.

3.3 Pricing errors

The ability of the conditional models to explain the cross-section of returns is ultimately judged based on their pricing errors. Table II reports the average pricing error test statistics for the three conditional models, as well as the benchmark unconditional and scaled-factor models. The first model (CCAPM) uses consumption growth as the only factor. The second model (ICAPM) uses market return and labor income growth as the two risk factors. The third model (CWCAPM) uses aggregate consumption and aggregate wealth growth as the two factors. The conditional ICAPM is tested using either *cay* (following Lettau and Ludvigson (2001b)) or *yc* (following Santos and Veronesi (2006)) conditioning variables; for CWCAPM I use *ca* as the conditioning variable, which is most appropriate for this model; I test the CCAPM using all three variables.

The key test statistics of interest are the *unconditional* averages of the *conditional* pricing errors that are being minimized by the conditional method of moments:

$$\alpha = E\left(\mathbf{g}\left(z_t; \lambda, h\right)\right).$$

Thus, conditioning down the expectation (11) implies that the estimated average pricing error, for asset i, is given by

$$\hat{\alpha}_i = \widehat{E}\left[R^{ei}_{t+1} - \widehat{Cov}(R^{ei}_{t+1}, \mathbf{f}_{t+1}|z_t)'\widehat{\lambda}\left(z_t\right)\right], \tag{13}$$

where the conditional moments and prices of risk are estimated using the nonparametric cross-sectional regression approach of Section 2.3.

For the unconditional models and the scaled factor conditional models the corresponding unconditional moments are used. The prices of risk in these latter cases are estimated by cross-sectional regression of expected returns on covariances, which is equivalent to the standard SDF/GMM methodology (e.g. see Cochrane (2005)). For the scaled factor models,

$$\tilde{\mathbf{f}}_{t+1} = \left[\mathbf{f_{t+1}}, \mathbf{f_{t+1}} \otimes z_t\right]$$

is used in place of $\mathbf{f_{t+1}}$, which implicitly assumes that $\widehat{\lambda}\left(z_t\right)$ are linear in z_t but does not place any restrictions on the conditional moments. One could also look at the scaled factor models estimated by imposing the conditional moment restrictions, which would be a special case of the conditional models where the prices of risk are linear functions of the conditioning variables. This is similar to the approach of Nagel and Singleton (2010). Since the restrictions imposed by this estimation method are tighter than those imposed by the nonparametric approach, a model rejected using the latter method would also be rejected using the former.

Instead of testing whether the overall level of pricing errors across the portfolios is zero, I focus on a few salient pricing errors that capture the essential features of the cross-section of stock returns. Namely, I consider the pricing errors of four long-short portfolios: small value minus small growth, small growth minus large growth, small value minus large value, and large value minus large growth. In order to test whether each one of these pricing errors is equal to zero I compute their finite sample distribution by semi-parametric bootstrap. Specifically, I use the estimated values of the covariances and prices of risk (as functions of conditioning variables) to simulate excess returns on the six basis portfolios under the null hypothesis that all of the test portfolios are priced correctly. These are used to obtain p-values for the (two-sided) tests of whether the pricing errors on the four long-short portfolios are different from zero.

The scaled-factor models appear to do a much better job explaining the average returns than the unconditional CCAPM and ICAPM. While for the unconditional consumption CCAPM only the small value minus small growth pricing error is large and statistically significant at 1.6 percent per quarter, the three other pricing errors are also sizable - except for the Large Value - Large Growth spread all of the pricing errors are larger than the average excess returns on the portfolios. The CCAPM scaled with *cay* cuts the Small Value minus Small Growth and Small Growth minus Large Growth pricing errors by a factor of three, and none of the errors are significantly different from zero. The unconditional ICAPM has similar magnitudes of pricing errors and most of them are statistically significant, presumably because the covariances with the market return are estimated much more precisely than covariances of returns with consumption growth. The scaled CCAPM and ICAPM that use the labor-consumption ratio *yc* as the instrument do not perform as well as do their counterparts scaled with *cay*, in that pricing errors are larger and statistically significant, but they still produce smaller pricing errors than the unconditional models.

While there is considerable uncertainty about the estimated conditional pricing errors, the nonparametric tests reveal that the conditional models do not do a nearly as good a job at explaining the cross-section of average returns as the scaled factor models suggest. For example, for the consumption CAPM with either *cay* or *ca* the average pricing errors have essentially the same magnitudes as the unconditional CCAPM pricing errors. The Small Value minus Small Growth pricing error is statistically significant at a 1% or 2% level. For the ICAPM or the CCAPM conditioned on *yc* the rejections are even stronger, as in addition to the Small Value - Small Growth error, they display positive and statistically significant pricing errors on either Large Value minus Large Growth or Small Value minus Large Value. The key pricing errors that represent the value premium puzzle - Small Value minus Small Growth and Large Value minus Large Growth - are of almost the same magnitudes as the average returns on these strategies for most of the conditional (as well as unconditional) models.

The only exception is the CWCAPM. This model has lower pricing errors even unconditionally, with the Small Value minus Small Growth pricing error of 82 basis points per quarter that is not statistically different from zero. It does imply statistically significant pricing errors for the Small Growth minus Large Growth and Large Value minus Large Growth strategies, which are equal to *negative* 94 basis points, and *negative* 35 basis points, respectively. Thus, according to CWCAPM Large Growth stocks actually outperform - the opposite of value puzzle (as well as the size effect). It is not surprising that the scaled version of this model can perform substantially better, since it does not impose the conditional moment restrictions and therefore has more degrees of freedom. Overall, while the CWCAPM can be rejected on purely statistical grounds at least for some of the test assets, it emerges as a clear leader in its ability to explain the value premium.

3.4 Conditional pricing errors

Average pricing errors can understate the extent of mispricing if *conditional* pricing errors are large but volatile. The conditional pricing errors can be assessed by looking directly at their nonparametric estimates. Figure 5 depicts the conditional pricing errors on the selected portfolios for the consumption CAPM as functions of *cay*:

$$\widehat{E}\left(R^{ei}_{t+1}|z_t\right) - \widehat{\lambda_C}\left(z_t\right)\widehat{Cov}(R^{ei}_{t+1}, \frac{\Delta C_{t+1}}{C_t}|z_t). \tag{14}$$

For each of the six portfolios, the solid line gives the estimated conditional mean of the pricing errors with 95% confidence bands around it. The straight dashed line is the pricing error from the unconditional model, while the dash-dotted line gives the pricing error from the scaled factor model (7), both obtained using the standard GMM procedure described in Cochrane (2005). These figures show that most of the conditional pricing errors are significantly larger in absolute value than the corresponding scaled-factor pricing errors, and often bigger (in absolute value) than the unconditional model errors. In the middle of the range of *cay* (which contains the majority of observations) most of the conditional pricing errors coincide with the errors of the unconditional CCAPM.

There is substantial statistical uncertainty about the estimated conditional pricing errors, as evidenced by the wide confidence band around them. Still, some of the conditional errors are statistically significantly different from zero over substantial regions over the state space. In particular, small value and neutral portfolios significantly outperform when *cay* is negative (i.e., in "good" times), while the large neutral portfolio outperforms in "bad" times (when *cay* is positive. The latter pricing error switches from positive to negative but statistically insignificant for negative values of *cay*, suggesting that looking at the average pricing error may be misleading about the model's performance as positive and negative conditional errors cancel out on average.

The conditional pricing errors are also informative in the case of CWCAPM. Figure 6 presents conditional pricing errors for this specification as functions of *ca*:

$$\widehat{E}\left(R^{ei}_{t+1}|z_t\right) - \widehat{\lambda_C}\left(z_t\right)\widehat{Cov}(R^{ei}_{t+1}, \frac{\Delta C_{t+1}}{C_t}|z_t) - \widehat{\lambda_W}\left(z_t\right)\widehat{Cov}(R^{ei}_{t+1}, \frac{\Delta W_{t+1}}{W_t}|z_t).$$

It is evident that for the three large-capitalization portfolios (bottom three panels) the hypothesis that the pricing errors are zero cannot be rejected. While there is some variation in the pricing errors as a function of the state variable, the bootstrap distributions of error estimates are centered near zero for most of the range. However, for the small-capitalization portfolios (top three panels) this is not the case. The conditional pricing errors are typically almost as large than the unconditional pricing errors, and their 95-percent confidence bands do not include zero over ranges of ca that contain a nontrivial fraction of the data. In particular, for the small growth portfolio, the pricing errors are significantly negative in the region of high ca, with the exception of the extreme right tail which has very few observations and wide error bounds. The small value portfolio has significantly positive errors in the same (but smaller) range, implying that the value premium in small stocks is not explained as well by the conditional model as the value effect in large stocks. This evidence suggests that tests of conditional models based on unconditional pricing errors (or averages of conditional error) may have low power as they ignore some of the information contained in the conditional moment restrictions.

4 Statistical properties of the nonparametric tests

The empirical results above rely on the ability of the nonparametric tests to distinguish between the conditional and the unconditional moments of asset returns and state variables, especially since these results indicate that conditional models do not seem to substantially outperform unconditional ones. Further, it is important to determine whether the apparent superior performance of a two-factor model with consumption and aggregate wealth factors (CWCAPM) relative to the one-factor consumption-based model is due to the tests' lack of statistical power.

In order to investigate the statistical properties of the nonparametric tests and confirm the reliability of my empirical results above I conduct a simulation exercise. As a null hypothesis for the Monte Carlo study I use a simplified version of the social status model described in Appendix A, combined with exogenously parameterized dynamics of conditional covariances of portfolio returns that are broadly consistent with the estimates in Section 4 above. This model has a conditional two-factor structure of returns with rich yet tractable risk price dynamics and thus provides a convenient laboratory for analyzing statistical properties of conditional asset pricing tests. I generate artificial data by simulating the model, and then apply the nonparametric estimation and testing methodology developed in Section 3 to the simulated data in order to evaluate both the size and the power of the tests.

4.1 Simulation setup

I specify conditional expected excess returns as linear functions of conditional covariances with aggregate consumption and wealth growth, scaled by the corresponding risk prices:

$$E\left(R^{ei}_{t+1}|z_t\right) = \lambda_C\left(z_t\right) Cov(R^{ei}_{t+1}, \frac{\Delta \bar{C}_{t+1}}{\bar{C}_t}|z_t) + \lambda_W\left(z_t\right) Cov(R^{ei}_{t+1}, \frac{\Delta \bar{W}_{t+1}}{\bar{W}_t}|z_t),$$

where

$$\lambda_C = \frac{\gamma}{1+\eta\left(\frac{\bar{C}_t}{\bar{W}_t}\right)^{\gamma}}, \lambda_W = \frac{\gamma\eta\left(\frac{\bar{C}_t}{\bar{W}_t}\right)^{\gamma}}{1+\eta\left(\frac{\bar{C}_t}{\bar{W}_t}\right)^{\gamma}}, \tag{15}$$

which is a special case of (A-17) with identical households (so that $s^i_t = 1$ for $\forall i, t$). I assume that the aggregate consumption-wealth ratio is exponentially affine in the conditioning variable z_t, which is meant to mimic the behavior of the cointegrating residual *cay* in the data:

$$\frac{\bar{C}_t}{\bar{W}_t} = \exp\left(\zeta_0 + \zeta_1 z_t\right),$$

where z_t itself follows an AR(1) process:

$$z_{t+1} = \varphi_z z_t + \sigma_z \omega^z_{t+1}.$$

I assume that the logarithm of the aggregate consumption process follows a random walk:

$$\log \bar{C}_{t+1} = \log \bar{C}_t + \mu_c + \sigma_c \omega^c_{t+1},$$

while the stationarity of the consumption-wealth ratio implies that aggregate consumption and aggregate wealth are cointegrated (in logs):

$$\log \bar{W}_t = \log \bar{C}_t - \left(\zeta_0 + \zeta_1 z_t\right).$$

The parameters are chosen to match the moments of aggregate consumption as well as the ratio of financial wealth to aggregate consumption. These parameters are summarized in Table III (panel A).

I impose the following exogenous structure on the conditional covariances of returns with state variables:

$$\begin{aligned}
Cov(R^{ei}_{t+1}, \frac{\Delta \bar{C}_{t+1}}{\bar{C}_t}|z_t) &= \rho^i_c(z_t)\,\sigma_c\sigma_i, \\
Cov(R^{ei}_{t+1}, \frac{\Delta \bar{W}_{t+1}}{\bar{W}_t}|z_t) &\approx Cov(R^{ei}_{t+1}, \frac{\Delta \bar{C}_{t+1}}{\bar{C}_t}|z_t) - Cov(R^{ei}_{t+1}, \frac{\bar{C}_{t+1}}{\bar{W}_{t+1}}|z_t) \\
&\approx \rho^i_c(z_t)\,\sigma_c\sigma_i - \zeta_1\rho^i_z(z_t)\,\sigma_z\sigma_i,
\end{aligned}$$

where σ_i is the standard deviation of portfolio i excess return.

Thus, the only source of time-variation in conditional covariances is in the time-varying correlations for returns with the two factors - the consumption growth innovations and innovations in the consumption-wealth residual. I exogenously specify simple functional forms for the dynamics of these conditional covariances:

$$\rho^i_c(z_t) = \frac{l_0}{1 + e^{l^i_1 z_t}}$$

and

$$\rho^i_z(z_t) = k^i_0 + k^i_1 z_t.$$

For parsimony, I parameterize only two pairs of these conditional correlations, representing the extreme Value and Growth portfolios, so that the conditional correlations for the six test portfolio returns are determined in the following manner:

$$\begin{aligned}
\rho^1_f(z_t) &= \rho^4_f(z_t) \doteq \rho^G_f(z_t), \\
\rho^3_f(z_t) &= \rho^6_f(z_t) \doteq \rho^V_f(z_t), \\
\rho^2_f(z_t) &= \rho^4_f(z_t) \doteq \frac{1}{2}\left(\rho^G_f(z_t) + \rho^V_f(z_t)\right),
\end{aligned}$$

for $f = \{c, z\}$. Using empirical estimates for the conditional standard deviations implies reasonable magnitudes for resulting conditional covariances of portfolio returns with consumption growth.The nonlinear structure of conditional correlations allows me to capture the "composition effect" (i.e. covariances of returns with consumption growth decline as consumption-wealth ratio increases) as well as the fact the key observation of Lettau and Ludvigson (2001b) that Value portfolio returns covary more highly with consumption growth than Growth portfolios in "bad times," when z_t is high, but not on average.

I then parameterize the price of risk functions (15) and the covariances of returns with the cointegrating residual z_t (which in turn feed into covariances with wealth growth) so as to match the level of expected excess returns in the data, as well as to ensure that the conditional expected returns are broadly increasing as functions of z_t (albeit not necessarily monotonically).All of the parameters are summarized in Table III Panels A and B (all moments are quarterly).

Finally, I simulate realized portfolio excess returns by adding innovations to the expected returns (which are constructed as functions of the state variable z_t):

$$R^{ei}_{t+1} = E\left(R^{ei}_{t+1}|z_t\right) + \sigma_i \left[\rho^i_c\left(z_t\right)\omega^c_{t+1} + \rho^i_z\left(z_t\right)\omega^z_{t+1} + \sqrt{1 - \rho^i_c\left(z_t\right)^2 - \rho^i_z\left(z_t\right)^2}\omega^i_{t+1}\right],$$

where ω^i_{t+1} is the idiosyncratic (i.e., unpriced) component of portfolio i excess return. All shocks ω^i_{t+1} as well as ω^c_{t+1} and ω^z_{t+1} are independently and identically distributed, drawn from the standard normal distribution. The statistics of these simulated excess portfolio returns are summarized in Panel C of Table III, together with those of the key state variables: aggregate consumption growth, aggregate wealth growth, and the consumption-wealth ratio.

4.2 Monte Carlo analysis: pricing errors

I use the stylized model of expected portfolio excess returns described above to evaluate the size and power properties of the nonparametric conditional asset pricing tests by means of Monte Carlo simulation. Specifically, I simulate $N_{MC} = 10000$ draws from the model and compute the resulting distribution of pricing error test statistics. For each Monte Carlo draw I simulate the model for $T_{long} = 1000$ periods, and use the last $T_{short} = 221$ observations, corresponding to the length of the empirical sample, for testing. Table IV reports the results. I estimate and test the following models using the simulated data: the one-factor consumption CAPM (CCAPM), the two-factor consumption/wealth CAPM (CWCAPM), which is the true model, and the two-factor model where the total stock market returns (obtained as equal weighted portfolio of the six simulated tests assets) is used as a proxy for the unobserved wealth portfolio return.

For each simulated series, I estimate average pricing errors by applying the conditional method of moments estimation developed in Section 3 to the six simulated portfolio returns and evaluate the average pricing errors on the four long-short corner portfolios, thus replicating the empirical analysis in Section 4. I use bootstrap with 100 replications to test whether each of the portfolio pricing errors is significantly greater (or smaller) than zero. I report the mean average pricing error across the Monte Carlo simulations for each model, as well as the fraction of Monte Carlo draws in which the bootstrap tests reject the hypothesis that the average pricing error is equal to zero (reported in the parentheses). The table also provides the mean average excess returns on each of the test portfolios combined with the standard deviation of these average excess returns across the Monte Carlo draws, as a measure of sampling uncertainty about the mean (in brackets). There is considerable uncertainty about the magnitude of the size premium: the differences between Small and Large portfolio returns are on average around 80 basis points per quarter but are well within two standard errors from zero. At the same time, the value premium is very clearly pronounced: the differences between Value and Growth portfolios are on average around 90 basis points per quarter but clearly statistically significant as standard errors are less than 40 basis points for these portfolios. Therefore, these test assets provide a suitable setting for evaluating the power of different asset pricing tests to distinguish between the models aimed at explaining the value premium.

I consider three models. First, I test the the one-factor consumption-based model (CCAPM) - while this model is false by construction in the simulated data, I am interested in evaluating the power of the tests to reject it. Second, I test the two factor model with consumption growth as the first factor and the excess return on the market portfolio (defined as an equal weighted average of the six basis portfolios) as the second factor (CW-CAPM(M)). While this model does not hold exactly in the simulation, it is meant to proxy for the true model (CWCAPM) when the wealth portfolio is unobserved. Finally, I test the true model with the consumption growth and wealth growth factors (CWCAPM). The latter allows me to evaluate the size of the tests (i.e., the probability of rejecting the true model).

I first test the unconditional versions of the models (reported in the upper panel of Table IV)). The pricing error tests reject the unconditional CCAPM very strongly: average pricing errors on the Value minus Growth portfolios are 1.44% and 1.05% for Small and Large pairs, respectively, and in each case the pricing error is statistically significant in approximately 90% of the Monte Carlo draws. For the CWCAPM(M) model these pricing errors are smaller but still substantial (0.98 and 0.73 percent, respectively) and each statistically significant in 60% of the Monte Carlo draws. In contrast, the bootstrap tests rarely reject the unconditional version of the true (conditional) model CWCAPM: the largest pricing error, for the Small Value minus Small Growth portfolio, is only 25 percent, and each of the Value minus Growth pricing errors is statistically significant in at most 18 percent of cases (interestingly, Small minus Large pricing errors, while essentially zero on average, are more often statistically significant, about 22% of the time for either the Value or the Growth portfolios). In sum, the unconditional bootstrap pricing error tests have substantial power to reject the wrong model, and small probability of rejecting the unconditional approximation of the true conditional model.

The middle panel of the table presents the results using unconditional tests of the conditional models using the scaled factor approach. All three models produce average pricing errors that are essentially zero, with each of the portfolios producing a statistically significant pricing error in about 15% of Monte Carlo draws for the one-factor model (CCAPM) and between 7% and 10% of the time for both of the two-factor models. Consequently, the scaled factor tests do not have much power to distinguish between the models, and in particular to reject the false model, as it is rejected only slightly more often than the true model.

Finally, the bottom panel reports tests using the nonparametric conditional method of moments to test the conditional implications of the three models. The conditional CCAPM has estimated pricing errors of almost the same magnitudes on average as the unconditional CCAPM: the Value minus Growth strategies are mispriced by 1.07% and 0.80% in Small and Large pairs, and statistically different from zero in 74 and 76 percent of the Monte Carlo draws, respectively. For the conditional CWCAPM(M) the pricing errors on the same test assets are only half as large, and statistically significant in only about 10 to 13 percent of the draws. Thus, from the perspective of the conditional asset pricing tests, the model that uses the market portfolio as a proxy for the wealth portfolio return is a reasonably good approximation to the true model.

For the true model CWCAPM itself the conditional pricing errors are 0.14% for the Small Value minus Small Growth and 0.11% for the Large Value minus Large Growth strategy. Each of the average pricing error is statistically significantly different from zero in at most 6 percent of the Monte Carlo draws. Therefore, the conditional test has the correct size: the true model is rarely rejected based on these pricing error tests. For the Small minus Large portfolios the pricing errors are small for all three models, and rarely significant, so the power of the nonparametric test clearly comes from its ability to distinguish between the different dynamics of the conditional means and covariances of Value portfolios vis-a-vis Growth portfolios, rather than simply differences in the amount of uncertainty in returns (by construction, conditional moments of the simulated Small and Large portfolio returns within the same book-market grouping only differ in volatility).

Overall, the evidence from the Monte Carlo simulations indicates that imposing conditional moment restrictions in estimating the conditional asset pricing models substantially improves the ability of cross-sectional asset pricing tests to distinguish between the model, even if the tests are based on the unconditional pricing errors. The nonparametric tests have sufficient power to reject a false conditional model as well as the ability to identify the correct model. This is in contrast to the scaled-factor tests that do not impose conditional moment restrictions by only testing the unconditional implications of the conditional models, effectively allowing the model too many degrees of freedom. Interestingly, the standard unconditional model tests have a reasonably good ability to distinguish between the false model and the true model, but only when the true model is correctly specified. In the case of the unobserved wealth portfolio the unconditional approximation of the model that uses the stock market return as a proxy is rejected much more often than is the conditional model, since the latter uses additional information about the conditional covariance structure of observed returns to identify the risk prices. This finding helps to interpret one of the empirical findings in Section 4.3 (the conditional CWCAPM is not rejected while the unconditional version is rejected): the lack of rejection is not due to the low power of the nonparametric test, but rather due to its superior ability to detect the conditional risk-return trade-off.

4.3 Monte Carlo analysis: conditional moments

Are the pricing error tests used to evaluate the conditional models based on reasonably accurate estimates of the conditional moments of excess returns? This question can also be answered within the same Monte Carlo simulation setup. I simulate the model 10000 times and estimate the conditional expected excess returns on the six benchmark portfolios, as well as the conditional covariances of returns with simulated consumption growth, using short samples of $T_{short} = 221$ observations in each draw, following the same approach as employed in Section 3.1. I report mean estimate across the Monte Carlo draws for each moment as a function of the conditioning variable, alongside the true value of the conditional moment, as well as the 95% confidence intervals for the estimates. Figure 7 presents estimates of conditional expected excess returns on the six portfolios while Figure 8 presents estimates of conditional covariances of returns with consumption growth. Clearly, the estimated conditional moments are quite close to the true moments, although there is considerable uncertainty about them. For the conditional mean excess returns, the 95% confidence bands include zero in both the right and the left tail of the distribution of the conditioning variable, as there are too few observations to estimate the conditional moments reliably. Still, the average conditional mean estimate virtually coincides with the true conditional mean function throughout the range of the state variable. The same is essentially true for the estimates of conditional covariances, although the latter appear slightly noisier near the boundaries of the state space, with potentially a very slight slight bias toward zero in the upper tail of the distribution of the conditioning variable. Overall, I conclude that the nonparametric conditional moment estimators implemented as part of the conditional method of moments have sufficiently reliable to permit robust interpretation of the estimation results.

5 Extensions

In this section I employ measures of long-run consumption risk to further corroborate the evidence in support of the CWCAPM. I then show that extending the conditioning information set does not substantially improve the performance of the canonical conditional CCAPM.

5.1 Long-run consumption growth risk

The fact that the stock market may be a poor proxy for the total wealth portfolio could be harming the empirical performance of the CWCAPM, as verified by the simulation evidence above. The opposite concern is that the apparently superior performance of the CWCAPM may be too good to be true. As a conditional two-factor model it may have enough degrees of freedom to generate a spurious fit for the cross-section of *conditional* expected excess returns due to the factor structure that is present in the returns on the benchmark portfolios. Such concerns are raised by Daniel and Titman (2005) and Lewellen, Nagel, and Shanken (2010) in the case of testing unconditional multi-factor models, and could potentially apply if the factor structure carries over to the conditional covariances of portfolios returns.

I address both of these concerns simultaneously by relying on the key insight of Bansal and Yaron (2004) that the total wealth return reflects news about future, as well as contemporaneous, consumption growth. Indeed, Parker (2003) shows that the variation in consumption risk as measured by the conditional covariances of stock returns with long-run consumption growth over time is much better aligned with time-variation in expected stock returns than is the case when contemporaneous consumption growth is used.[13] Parker and Julliard (2005) and Hansen, Heaton, and Li (2008) show that exposures to news about future aggregate consumption help explain the differences in *unconditional* expected returns between value and growth stocks (see also the discussion in Hansen, Heaton, Lee, and Roussanov (2007)). Here I extend the analysis by using a single conditional covariance with long-run consumption growth to capture the effect of both consumption risk and total wealth risk on the *conditional* expected returns simultaneously.

I estimate conditional covariances with long-run consumption growth as

$$Cov\left(R_{t+1}, \frac{C_{t+1+S}}{C_t}|z_t\right), \tag{16}$$

for S equal to either 11 or 19 quarters. I then estimate the one-factor conditional CCAPM as before, using this covariance as the measure of consumption risk.

Table V displays the tests statistics for the differences in conditional moments between the high and low *cay* states. Consistent with the findings of Parker (2003) the composition effect is not present in the covariances of returns with long-run consumption growth, in contrast to the contemporaneous covariances: for most of the basis portfolios, the difference in covariances between "bad" and "good" state are positive, and none are significantly different from zero. The fact that we cannot reject that the differences are zero may be due to the lack of statistical power in estimating time-variation in long-run covariances rather than to the lack of comovement between long-run consumption risk and conditional expected returns.

However, the cross-sectional patterns of time-varying long-run consumption covariances are similar to those identified over the short run, albeit weaker. The differences in conditional covariances on long-short Value minus Growth portfolios are still positive, and statistically significant only for the Small portfolios. As before, the differences in average returns on Value minus Growth strategies between "bad" and "good" states have the opposite (i.e. negative) sign from the differences in covariances but are not statistically different from zero.

[13]A measure of consumption risk based on longer horizons can be rationalized by appealing to models of slow adjustment of consumption in response to wealth returns (e.g. Gabaix and Laibson (2002)).

Table VI presents the corresponding pricing error tests for the unconditional, scaled and conditional versions of the long-run CCAPM with *cay* as the conditioning variable. When consumption growth is computed over 3-year interval (i.e., $S = 11$) the unconditional CCAPM performs rather well, producing small pricing errors, none of which are statistically different from zero. This is consistent with findings of Parker and Julliard (2005) who argue that CCAPM with long-run consumption growth is able to explain the cross-section of stock returns. Interestingly, for the 5-year horizon ($S = 19$) the CCAPM does not perform as well - pricing errors are larger in magnitude, and, in particular, Small minus Large Value pricing error is large (80 basis points per quarter) and statistically significant at a 4% level. It is not surprising that in both cases the scaled version of the model performs better, displaying small and insignificant pricing error. Imposing the conditional moment restrictions nonparametrically reduces the advantage of the conditional models over the unconditional ones as is the case in all of the situations analyzed above. Still, for both $S = 11$ and $S = 19$ none of the pricing errors are statistically different from zero, and most are smaller than the unconditional ones. Overall, using measures of long-run consumption risk results in a substantial improvement in explaining the value premium relative to the canonical CCAPM, both unconditionally and conditionally.

5.2 Expanding the conditioning set: single-index approach

One concern is that the above results are due to a particular set of conditioning variables used and other variables that are potentially important for capturing the joint dynamics of asset returns and consumption, as well as for the price of consumption risk, are omitted. The reason for focusing on composition of wealth variables is that models linking these variables to risk prices have been established in the literature as described above, and using these variables is sufficient for testing the predictions of these models. However, these models may be misspecified, so that it is possible that a slightly more general version of the CCAPM still holds. In principle, it is impossible to fully allay such concerns, as the Hansen-Richard critique still applies. In practice, however, it is unlikely to be a serious problem, since the evidence documented above puts fairly stringent requirements on the joint dynamics of asset returns and consumption growth that would have to be satisfied by the "true" conditional model. Consider the conditional asset pricing relation (1) implied by the consumption CAPM where the information set I_t is partitioned into the subset spanned by a given conditioning variable z_t and the orthogonal component $\tilde{I}_t$:

$$E\left(R^{ei}_{t+1}|z_t, \tilde{I}_t\right) = \gamma_t Cov(R^{ei}_{t+1}, \frac{\Delta C_{t+1}}{C_t}|z_t, \tilde{I}_t). \tag{17}$$

Conditioning this down to the information set spanned by z_t yields

$$\begin{aligned} E\left(R^{ei}_{t+1}|z_t\right) &= E\left(\gamma_t|z_t\right) Cov\left[R^{ei}_{t+1}, \frac{\Delta C_{t+1}}{C_t}\middle| z_t\right] && (18) \\ &+ Cov\left[\gamma_t, Cov_t(R^{ei}_{t+1}, \frac{\Delta C_{t+1}}{C_t})\middle| z_t\right] && (19) \\ &- E\left(\gamma_t|z_t\right) Cov\left[E_t R^{ei}_{t+1}, E_t \frac{\Delta C_{t+1}}{C_t}\middle| z_t\right], && (20) \end{aligned}$$

so that the tests that use one of composition of wealth variables as z_t effectively ignore the last two terms of this moment condition. Whether this omission can lead to a spurious rejection of the model depends on the signs (and magnitudes) of these two terms. The empirical results above indicate that the left hand side of this relation (the conditional expected return) for Value minus Growth portfolios is greater than the first term on the left hand side in "good times" (i.e. when conditional expected returns on all portfolios are low), and smaller in "bad times" (when conditional expected returns are high). The last term is plausibly of second order as even if consumption growth is predictable, the amount time-variation in the conditional covariance between expected consumption growth and expected excess returns is likely to be small. Focusing on the second term, the observed pattern of conditional pricing errors implies that the covariance between the consumption risk of Value minus Growth portfolios with the time-varying price of consumption risk γ_t must be higher in "good" times than in "bad" times, as measured by the composition of wealth variables. While it cannot be ruled out *a priori*, such a pattern of joint dynamics of returns and consumption risk appears highly implausible.

While expanding the conditioning set using a range of known predictive variables is simple in principle, it raises difficult econometric challenges. In particular, conditional method of moments is subject to the "curse of dimensionality" that comes with nonparametric estimation of conditional moments (e.g. as discussed by Brandt (1999)). The same problem is faced by Nagel and Singleton (2010), who consider one conditioning variable at a time. I address this issue by employing a semiparametric approach similar to that used by Aït-Sahalia and Brandt (2001). This method allows the information contained in a $K \times 1$ vector of instruments Z_t to be condensed into a single conditioning variable z_t via a linear function

$$z_t = \theta' Z_t,$$

where θ is a $K \times 1$ vector of index weights. This single index can be used in place of the single conditioning variable in the nonparametric estimation and asset pricing tests, thus potentially capturing more information about the dynamics of consumption risk, asset returns, and risk prices while maintaining the flexibility of nonparametric estimation. The construction of a single index summarizing information about conditional moments of returns is in the spirit of Ludvigson and Ng (2007) who propose a method for condensing information in a large number of variables into a small number of instruments via factor analysis. In order to impose additional discipline on the estimation, I impose the restriction that the index weights are the same for all of the conditional moments across the tests assets. This allows me to estimate the dependence of the factor risk prices on the single index directly. Finally, I use a constant bandwidth parameter in estimating the conditional moments, which is estimated jointly with the vector of index weights (e.g., Hardle, Hall, and Ichimura (1993)):

$$\begin{pmatrix} \hat{\theta} \\ \hat{h} \end{pmatrix} = \arg\min_{\theta,h} \left\{ \mathbf{G}_T\left(\theta,h\right)' W\left(\theta,h\right) \mathbf{G}_T\left(\theta,h\right) \right\},$$

where

$$
\begin{aligned}
\mathbf{G}_T(\theta, h) &= \frac{1}{T}\sum_{t=1}^{T}\mathbf{g}(z_t; h), \\
z_t &= \theta' Z_t \\
\mathbf{g}(z; h) &= \widehat{\mathbf{m}}(z; h) - \widehat{\mathbf{cv}}(z; h)' \hat{\lambda}(z; h), \\
\widehat{\lambda}(z) &= \left(\widehat{\mathbf{cv}}(z; h)' \widehat{\mathbf{cv}}(z; h)\right)^{-1} \widehat{\mathbf{cv}}(z; h)' \widehat{\mathbf{m}}(z; h),
\end{aligned}
$$

and $\widehat{\mathbf{m}}$ and $\widehat{\mathbf{cv}}$ are the nonparametric estimators of conditional means and covariances of returns and factors as described above. I use the identity matrix in place of $W(\theta, h)$ for simplicity, although any matrix that converges in probability to some positive-definite matrix can be used, including the optimal GMM weighting matrix of Hansen (1982) or the weighting matrix used in the continuously-updated updated GMM estimator of Hansen, Heaton, and Yaron (1996).

I use this framework to test whether conditioning variables other than the composition of wealth variables used above help capture time-variation in the price of consumption risk. As such I add the following variables to the conditioning set: IP, the 12-month growth rate of the monthly U.S. industrial production index; the term spread $term$ (the difference between the 10-year and the 3-month U.S. treasury bond yields); the default spread def (the difference between the BAA and the AAA rated bond yields); and the dividend yield dp (the ratio of the quarterly dividends on the value-weighted market portfolio to the total market capitalization of the portfolio at the end of the quarter). I also consider the first 8 principal components extracted from the panel of 131 macroeconomic time series in Ludvigson and Ng (2009).

I conduct inference in the following way. First, I compute the probability that each of the elements of $\hat{\theta}$ that is estimated to be positive is negative under the bootstrap distribution, and vise versa for the negative ones (I refer to these as non-parametric p-values). Second, I compute the semiparametric p-values for these one-sided tests under the null hypothesis that the index weights are equal to zero for all variables except for one of the composition of wealth variables (e.g., *cay*). In order to do this I estimate the model using the single given variable, resample the residuals as described in the appendix, and re-estimate the model using the single-index approach described above. The resulting bootstrap distribution provides the probabilities of generating obtained point estimates when the true coefficients are zero.

The results of single-index estimation and tests are presented in Table VII. I consider four different specifications of the single-index model. The first one includes *cay* as well as IP, $term$, and def. While the signs of the point estimates are consistent with the notion of counter-cyclical price of consumption risk (the yield spreads as well as *cay* come in positively, where as IP has a negative coefficient), only *cay* has a statistically significant weight in the index (the probability that it has a 'wrong' sign under the bootstrap distribution is 1%, while it is above 15% for all the other variables). Imposing the null hypothesis that only *cay* enters the index yields very large p-values for the other three variables. Adding the dividend-price ratio to the index (the second specification) reduces the magnitude of the *cay* coefficient as well as that for the term spread, and even flips the sign for def. Still, only the *cay* coefficient is the only variable that is strongly statistically significant (for the dividend yield, the bootstrapped coefficient is negative 10% of the time, so its significance is marginal). Further, under the null that only *cay* enters the index, the *dp* can have coefficient at least as large as estimated with a probability of 44%.

In the third specification I add the labor-consumption ratio *yc* to all of the above variables. Like *cay*, this variable's weight in the index is very precisely estimated, with essentially zero a zero probability that the coefficient is negative (rather than positive). It is not clear, however, whether *cay* and *yc* contain independent information about the price of consumption risk. In fact, under the null that only *cay* enters the index the coefficient for *yc* is not statistically significant, and vice versa: under the null that only *yc* matters, *cay* coefficient could be as large as observed over 31% of the time (all of the other variables are not significant in either case). As it appears that the two composition of wealth variables essentially drive each other out in these tests, it is likely that the common information that is contained in both of them is sufficient to capture the time-variation in the price of consumption risk. As none of the other variables have a statistically detectable contribution to the index, this evidence validates the focus on the composition of wealth variables in conditional asset pricing tests.

Finally, the fourth specification presented in Panel B of the Table considers the index comprised of *cay* and the eight principal components of the macroeconomic panel. Unlike the variables in Panel A, the weights of all of the principal components in the single index are estimated very precisely, since they are orthogonal to each other by construction. Under the null hypothesis that *cay* is the only variable driving the price of consumption risk only two of the eight principal principal components have estimated coefficients that are statistically significantly different from zero: $PC2$ and $PC6$.

Does a specification in which these two macroeconomic variables enter the single index along with *cay* improve the performance of the conditional asset pricing model? To answer this question I use the composite index consisting only of three variables (*cay*, $PC2$ and $PC6$) to estimate the conditional price of consumption risk together with the conditional moments and test the asset pricing model as before. Panel C of the same Table presents the coefficients of the single index together with their p-values under the null that *cay* is the only variable entering the index, showing that under this specification neither of the principal components is significant. Adding these two additional degrees of freedom makes it harder to reject the asset pricing model using the average pricing error tests: the pricing errors on the four long-short portfolios presented in Panel E are all insignificantly different from zero, even though their economic magnitudes are large, especially for the key Small Value minus Small Growth portfolio, which has pricing error greater than its average return (1.83% percent vs. 1.63%). Nevertheless, examining the pricing errors on the six basis portfolios themselves, displayed in Panel D, reveals statistically detectable mispricing. In particular, the Small Value and Small Intermediate portfolios both have large and statistically significant pricing errors. Therefore, even with the flexibility attained by adding macroeconomic factors to the conditioning set describing the dynamics of the price of consumption risk the model is not able to explain the value and size anomalies.

6 Conclusion

This paper investigates the empirical performance of conditional asset pricing models in which conditioning information captures the changing composition of total wealth, and as such is a source of time-variation in expected returns and covariances. The main finding is that the time-series behavior of consumption risk associated with the trading strategies that capture the "value premium" in the cross-section of stock returns are is at odds with the dynamics of conditional expected returns on these strategies. The evidence I present is consistent with the argument of Lettau and Ludvigson (2001b) that value portfolio returns covary with aggregate consumption growth more during "bad times", when risk premia are high, than during "good times," while the opposite is true for growth portfolios. At the same time, the conditional expected returns on value portfolios do not increase by more than those of growth portfolios in "bad states," as predicted by the conditional CCAPM. This central conclusion is largely robust to the alternatives ways of measuring time-varying consumption risk of equity portfolios.

The evidence presented here suggests that greater covariation of returns with the measure of consumption growth might not be sufficient to explain the value premium by itself. This finding mirrors the theoretical arguments of Lettau and Wachter (2007) and Santos and Veronesi (2010) that models with time-varying risk aversion driven by habit formation cannot explain the value premium if growth stocks are viewed as long duration assets and therefore more sensitive to variation in discount rates (see also Lynch and Randall (2010)). The fact that the conditional covariances and conditional expected returns on value portfolios do not move in the same direction as functions of conditioning information suggests that another risk factor might be required whose dynamics would play an offsetting role.

The conditional models that are not rejected on the basis of average pricing errors are the CCAPM with long-horizon consumption growth and the two-factor CWCAPM in which consumption growth and aggregate wealth growth are two separately priced sources of risk, which suggests that the cross-section of average returns reflects long-run consumption risk that is partly captured in the return on the market portfolio (e.g. Bansal and Yaron (2004), Hansen, Heaton, and Li (2008)). However, the relative success of these models appears to be driven primarily by their unconditional, rather than conditional, properties.

Better measurement of consumption risk could be part of the solution to the remaining puzzle, e.g. by allowing infrequent adjustment of consumption to wealth shocks, as advocated by Jagannathan and Wang (2007), and by measuring long-run (rather than contemporaneous) consumption risk of households that participate in the stock market as in Malloy, Moskowitz, and Vissing-Jørgensen (2005). Applying the methodology developed here to testing the conditional implications of the asset pricing models considered in these recent studies should yield further insights into the role of consumption risk in explaining the cross-section of stock returns, but is fraught with difficulties as estimation of conditional covariance may not be feasible given the data available to researchers at present.

References

Aït-Sahalia, Yacine, 1992, The delta and bootstrap methods for nonlinear functionals of nonparametric kernel estimators based on dependent multivariate data, Working Paper.

——— , and Michael Brandt, 2001, Variable selection for portfolio choice, *Journal of Finance* 56, 1297–1351.

Aït-Sahalia, Yacine, and Jefferson Duarte, 2003, Nonparametric option pricing under shape restrictions, *Journal of Econometrics* 116, 9–47.

Andersen, Torben G., Tim Bollerslev, Francis X. Diebold, and Paul Labys, 2003, Modeling and forecasting realized volatility, *Econometrica* 71, 579–625.

Ang, Andrew, and Dennis Kristensen, 2009, Testing conditional factor models, NBER Working Paper.

Bakshi, Gurdip S., and Zhiwu Chen, 1996, The spirit of capitalism and stock market prices, *American Economic Review* 86, 133–157.

Bandi, Federico, 2004, On persistence and nonparametric estimation (with an application to stock return predictability), University of Chicago Working Paper.

Bansal, Ravi, Dana Kiku, Ivan Shaliastovich, and Amir Yaron, 2012, Volatility, the macroeconomy and asset prices, Working Paper 18104 National Bureau of Economic Research.

Bansal, Ravi, and S. Viswanathan, 1993, No arbitrage and arbitrage pricing: A new approach, *Journal of Finance* 48, 1231–1262.

Bansal, Ravi, and Amir Yaron, 2004, Risks for the long run: A potential resolution of asset pricing puzzles, *Journal of Finance* 59, 1481 – 1509.

Bollerslev, Tim, Robert F. Engle, and Jeffrey M. Wooldridge, 1988, A capital asset pricing model with time varying covariances, *Journal of Political Economy* 96, 116–131.

Brandt, Michael W., 1999, Estimating portfolio and consumption choice: a conditional Euler equation approach, *Journal of Finance* 54, 1609–1645.

———, and David Chapman, 2007, Linear approximations and tests of conditional pricing models, NBER Working Paper No. W12513.

Breeden, Douglas T., 1979, An intertemporal asset pricing model with stochastic consumption and investment opportunities, *Journal of Financial Economics* 7, 265–96.

Campbell, John Y., 1987, Stock returns and the term structure, *Journal of Financial Economics* 18, 373–399.

———, 1996, Understanding risk and return, *Journal of Political Economy* 104, 298–345.

———, and John H. Cochrane, 1999, By force of habit: A consumption based explanation of aggregate stock market behavior, *Journal of Political Economy* 107, 205–251.

———, 2000, Explaining the poor performance of consumption-based asset pricing models, *Journal of Finance* 55, 2863–2878.

Campbell, John Y., Stefano Giglio, Christopher Polk, and Robert Turley, 2012, An intertemporal CAPM with stochastic volatility, Working Paper 18411 National Bureau of Economic Research.

Chan, Yeung Lewis, and Leonid Kogan, 2002, Catching up with the joneses: Heterogeneous preferences and the dynamics of asset prices, *Journal of Political Economy* 110, 1255–1285.

Chen, Hui, and Scott Joslin, 2010, Generalized transform analysis of affine processes: with an application to a model of time-varying labor income risk, working paper, MIT Sloan.

Chen, Xiaohong, and Yanqin Fan, 1999, Consistent hypothesis testing in semiparametric and nonparametric models for econometric time series, *Journal of Econometrics* 91, 373–401.

Chen, Xiaohong, and Sydney C. Ludvigson, 2009, Land of addicts? An empirical investigation of habit-based asset pricing models, *Journal of Applied Econometrics* 24, 1057–1093.

Cochrane, John H., 1996, A cross-sectional test of an investment-based asset pricing model, *Journal of Political Economy* 104, 572–621.

———, 2005, *Asset Pricing* (Princeton University Press: Princeton, NJ).

———, Francis A. Longstaff, and Pedro Santa-Clara, 2008, Two Trees, *Review of Financial Studies* 21, 347–385.

Constantinides, George M., 1990, Habit formation: A resolution of the equity premium puzzle, *Journal of Political Economy* 98, 519–543.

Cuoco, Domenico, 1997, Optimal consumption and equilibrium prices with protfolio constraints and stochastic income, *Journal of Economic Theory* 72, 33–73.

Daniel, Kent D., and Sheridan Titman, 2005, Testing factor-model explanations of market anomalies, working paper.

Duffee, Gregory, 2005, Time-variation in the covariance between stock returns and consumption growth, *Journal of Finance* 60, 1673–1712.

Duffie, Darrell, and Larry G. Epstein, 1992, Asset pricing with stochastic differential utility, *Review of Financial Studies* 5, 411–436.

Epstein, Larry, and Stanley Zin, 1989, Substitution, risk aversion, and the temporal behavior of consumption growth and asset returns I: A theoretical framework, *Econometrica* 57, 937–969.

———, 1991, Substitution, risk aversion, and the temporal behavior of consumption growth and asset returns II: An empirical analysis, *Journal of Political Economy* 99, 263–286.

Fama, Eugene F., and Kenneth R. French, 1992, The cross-section of expected stock returns, *Journal of Finance* 47, 427–465.

——— , 1993, Common risk factors in the returns on stocks and bonds, *Journal of Financial Economics* 33, 3–56.

Fama, Eugene F., and James MacBeth, 1973, Risk, return and equilibrium: Empirical tests, *Journal of Political Economy* 81, 607–636.

Fama, Eugene F., and G. William Schwert, 1977, Human capital and capital market equilibrium, *Journal of Financial Economics* 4, 115–146.

Fan, Jianqing, and Irene Gijbels, 1996, *Local Polynomial Modelling and Its Applications* (Chapman and Hall: London).

Ferson, Wayne E., and Campbell R. Harvey, 1999, Conditioning variables and the cross section of stock returns, *Journal of Finance* 54, 1325 – 1360.

Ferson, Wayne E., Shmuel Kandel, and Robert F. Stambaugh, 1987, Tests of asset pricing with time-varying expected risk premiums and market betas, *Journal of Finance* 42, 201–220.

Ferson, Wayne E., and Andrew F. Siegel, 2009, Testing Portfolio Efficiency with Conditioning Information, *Review of Financial Studies* 22, 2535–2558.

Gabaix, X., and D. Laibson, 2002, The 6D bias and the equity-premium puzzle, in *NBER Macroeconomics Annual 2001, Volume 16* . pp. 257–330 (MIT Press).

Gallant, A. Ronald, Lars P. Hansen, and George Tauchen, 1990, Using conditional moments of asset returns to infer the volatility of intertemporal marginal rates of substitution, *Journal of Econometrics* 45, 141–180.

Garleanu, Nicolae, and Stavros Panageas, 2009, Young, old, conservarive and bold. the implications of finite lives and heterogeneity for asset pricing, working paper.

Ghysels, Eric, 1998, On stable factor structures in the pricing of risk: Do time-varying betas help or hurt?, *Journal of Finance* 53, 549–573.

Gourieroux, Christian, Alain Monfort, and Carlos Tenreiro, 2000, Kernel M-estimators and functional residual plots, in Jaya Krishnakumar, and Elvezio Ronchetti, ed.: *Panel Data Econometrics: Future Directions* . chap. 12 (Elsevier Science B.V.: Amsterdam).

Grossman, Sanford J., Angelo Melino, and Robert J. Shiller, 1987, Estimating the continuous-time consumption-based asset-pricing model, *Journal of Business and Economic Statistics* 5, 315–327.

Grossman, Sanford J., and Robert J. Shiller, 1982, Consumption correlatedness and risk measurement in economies with non-traded assets and heterogeneous information, *Journal of Financial Economics* 10, 195–210.

Hansen, Lars, and Scott F. Richard, 1987, The role of conditioning information in deducing testable restrictions implied by dynamic asset pricing models, *Econometrica* 55, 587–613.

Hansen, Lars P., 1982, Large sample properties of generalized method of moments estimators, *Econometrica* 50, 1029–1054.

Hansen, Lars Peter, John Heaton, Junghoon Lee, and Nikolai Roussanov, 2007, Intertemporal substitution and risk aversion, , vol. 6, Part 1 of *Handbook of Econometrics* . pp. 3967 – 4056 (Elsevier).

Hansen, Lars P., John Heaton, and Amir Yaron, 1996, Finite-sample properties of some alternative GMM estimators, *Journal of Business and Economic Statistics* 14, 262–280.

Hansen, Lars Peter, John C. Heaton, and Nan Li, 2008, Consumption strikes back? measuring long-run risk, *Journal of Political Economy* 116, 260–302.

Hansen, Lars P., and Ravi Jagannathan, 1997, Assessing specification errors in stochastic discount factor models, *Journal of Finance* 52, 557–590.

Hardle, Wolfgang, Peter Hall, and Hidehiko Ichimura, 1993, Optimal smoothing in single-index models, *The Annals of Statistics* 21, pp. 157–178.

Härdle, Wolfgang Karl, 1992, *Applied Nonparametric Regression* (Cambridge University Press).

Harvey, Campbell, 1989, Time-varying conditional covariances in tests of asset pricing models, *Journal of Financial Economics* 24, 289–317.

He, Hua, and Henri Pagés, 1993, Labor income, borrowing constraints, and equilibrium asset prices, *Economic Theory* 3, 663–696.

Heaton, John C., and Deborah J. Lucas, 2000, Portfolio choice and asset prices: The importance of entrepreneurial risk, *Journal of Finance* 55, 1163–1198.

Horowitz, Joel L., 2001, The bootstrap, in James J. Heckman, and Edward Leamer, ed.: *Handbook of Econometrics* . , vol. 5 (Elsevier Science B.V.: Amsterdam).

Jagannathan, Ravi, Keiichi Kubota, and Hitoshi Takehara, 1998, Relationship between labor-income risk and average return: Empirical evidence from the Japanese stock market, *Journal of Business* 71, 319–348.

Jagannathan, Ravi, and Yong Wang, 2007, Lazy investors, discretionary consumption, and the cross-section of stock returns, *Journal of Finance* 62, 1623–1661.

Jagannathan, Ravi, and Zhenyu Wang, 1996, The conditional CAPM and the cross-section of expected returns, *Journal of Finance* 51, 3–53.

Koo, Hyeng Keun, 1998, Consumption and portfolio selection with labor income: a continuous-time approach, *Mathematical Finance* 8, 49–65.

Kreps, David, and Evan Porteus, 1978, Temporal resolution of uncertainty and dynamic choice theory, *Econometrica* 46, 185–200.

Lettau, Martin, and Sydney Ludvigson, 2001a, Consumption, aggregate wealth and expected stock returns, *Journal of Finance.*

——— , 2001b, Resurrecting the (C)CAPM: a cross-sectional test when risk premia are time-varying, *Journal of Political Economy* 109, 1238–1287.

Lettau, Martin, and Jessica A. Wachter, 2007, Why Is Long-Horizon Equity Less Risky? A Duration-Based Explanation of the Value Premium, *Journal of Finance* 62, 55–92.

Lewellen, Jonathan, and Stefan Nagel, 2006, The conditional CAPM does not explain asset-pricing anomalies, *Journal of Financial Economics* 82, 289–314.

——— , and Jay Shanken, 2010, A skeptical appraisal of asset pricing tests, *Journal of Financial Economics* 96, 175–194.

Li, Minqiang, Neil D. Pearson, and Allen M. Poteshman, 2004, Conditional estimation of diffusion processes, *Journal of Financial Economics* 74, 31 – 66.

Ludvigson, Sydney C., and Serena Ng, 2007, The empirical risk-return relation: A factor analysis approach, *Journal of Financial Economics* 83, 171–222.

——— , 2009, Macro Factors in Bond Risk Premia, *Review of Financial Studies* 22, 5027–5067.

Lustig, Hanno, and Stijn Van Nieuwerburgh, 2005, Housing collateral, consumption insurance, and risk premia: An empirical perspective, *Journal of Finance* 60, 1167–1219.

——— , 2008, The Returns on Human Capital: Good News on Wall Street is Bad News on Main Street, *Review of Financial Studies* 21, 2097–2137.

——— , and Adrien Verdelhan, 2009, The wealth-consumption ratio, working paper.

Lynch, Anthony W., and Oliver Randall, 2010, Why Surplus Consumption in the Habit Model May Be Less Persistent than You Think, NBER Working Paper.

Malloy, Christopher, Tobias Moskowitz, and Annette Vissing-Jørgensen, 2005, Long-run stockholder consumption risk and asset returns, *Journal of Finance* 64, 2427–2479.

Martin, Ian, 2009, The lucas orchard, working paper, Stanford University.

Mayers, David, 1972, Nonmarketable assets and capital market equilibrium under uncertainty, in Michael Jensen, ed.: *Studies in the theory of capital markets* . pp. 223–248 (Praeger: New York).

Merton, Robert C., 1973, An intertemporal capital asset pricing model, *Econometrica* 41, 867–887.

——— , 1980, On estimating the expected return on the market: an exploratory investigation, *Journal of Financial Economics* 8, 323–361.

Nagel, Stefan, and Kenneth Singleton, 2010, Estimation and evaluation of conditional asset pricing models, *Journal of Finance, forthcoming.*

Pagan, Adrian, and Aman Ullah, 1999, *Nonparametric Econometrics* (Cambridge University Press: New York).

Pagan, Adrian R., and G. William Schwert, 1990, Alternative models for conditional stock volatility, *Journal of Econometrics* 45, 267–290.

Pakos, Michal, 2004, Asset pricing with durable goods and non-homothetic preferences, Working paper, University of Chicago.

Palacios, Miguel, 2010, Human capital as an asset class implications from a general equilibrium model, working paper, Vanderbilt University.

Papanikolaou, Dimitris, 2011, Investment shocks and asset prices, *Journal of Political Economy* 119, 639 – 685.

Parker, Jonathan A., 2003, Consumption risk and expected stock returns, *American Economic Review* 93, 376–382.

——— , and Christian Julliard, 2005, Consumption risk and the cross section of expected returns, *Journal of Political Economy* 113, 87–131.

Petkova, Ralitsa, and Lu Zhang, 2005, Is value riskier than growth?, *Journal of Financial Economics* 78, 187–202.

Piazzesi, Monika, Martin Schneider, and Selale Tuzel, 2007, Housing, consumption and asset pricing, *Journal of Financial Economics* 83, 531–569.

Politis, D., and J. Romano, 1994, The stationary bootstrap,, *Journal of the American Statistical Association* 89, 1303–1313.

Roll, Richard W., 1977, A critique of the asset pricing theory's tests, *Journal of Financial Economics* 4, 129–176.

Roussanov, Nikolai, 2010, Diversification and its discontents: Idiosyncratic and entrepreneurial risk in the quest for social status, *Journal of Finance* 65, 1755–1788.

Santos, Tano, and Pietro Veronesi, 2006, Labor income and predictable stock returns, *Review of Financial Studies* 19, 1–44.

———, 2010, Habit formation, the cross section of stock returns and the cash-flow risk puzzle, *Journal of Financial Economics* 98, 385–413.

Shanken, Jay, 1990, Intertemporal asset pricing : An empirical investigation, *Journal of Econometrics* 45, 99–120.

Stambaugh, Robert F., 1982, On the exclusion of assets from tests of the two-parameter model: A sensitivity analysis, *Journal of Financial Economics* 10, 237–268.

Wang, Kevin Q., 2003, Asset pricing with conditioning information: a new test, *Journal of Finance* 58, 161–196.

Weil, Philippe, 1990, Non-expected utility in macroeconomics, *Quarterly Journal of Economics* 105, 29–42.

Yogo, Motohiro, 2006, A consumption-based explanation of expected stock returns, *Journal of Finance* 61, 539580.

Table I: **Differences in conditional moments of portfolio returns**

Bootstrap tests of differences in conditional moments of returns for the benchmark portfolios, using $z = cay$ as the conditioning variable, where $z^L = -0.019$ and $z^H = 0.02$ correspond to the 10th and 90th percentiles of the distribution of cay, respectively. The test statistics are differences in point estimates of conditional moments evaluated at these two states for each test portfolio. The p-values for the one-sided tests reported in the parentheses are computed using the bootstrap distributions of the corresponding test statistics centered at zero. Data is for the time period IV.1952 - IV.2008.

	$E(R\|z^H) - E(R\|z^L)$	$100 \times (cov(R, \Delta c\|z^H) - cov(R, \Delta c\|z^L))$
Small Growth	4.06	-1.13
	(0.01)	(0.04)
Small Value	3.63	-0.41
	(0.01)	(0.23)
Large Growth	5.21	-1.12
	(0.00)	(0.01)
Large Value	3.62	-0.33
	(0.01)	(0.24)
Small Value minus Growth	-0.43	0.72
	(0.41)	(0.01)
Large Value minus Growth	-1.59	0.79
	(0.09)	(0.02)

Table II: **Average pricing errors: quarterly data,** *cay*

Unconditional pricing errors for the conditional model are given by

$$\alpha_i = \widehat{E}\left[R^{ei}_{t+1} - \widehat{Cov}(R^{ei}_{t+1}, \mathbf{f}_{t+1}|z_t)'\widehat{\lambda}\,(z_t)\right],$$

where i is one of the four long-short portfolio returns that are combinations of the original 6 portfolios used to estimate the model: Small Value minus Small Growth (SV-SG), Small Growth minus Large Growth (SG-LG), Small Value minus Large Value (SV-LV) and Large Value minus Large Growth (LV-LG).
P-values for the test that individual pricing errors are equal to zero given in the parentheses are computed using (semi)parametric stationary bootstrap with 10000 replications.

Model	SV-SG	SG-LG	SV-LV	LV-LG
unconditional CCAPM	1.75	-0.70	0.54	0.51
	(0.01)	(0.13)	(0.19)	(0.24)
unconditional (I)CAPM	2.05	-0.11	0.93	1.01
	(0.00)	(0.64)	(0.00)	(0.01)
unconditional CWCAPM	0.83	-0.73	0.61	-0.51
	(0.37)	(0.14)	(0.09)	(0.00)
CCAPM scaled with *cay*	0.52	-0.22	0.74	-0.44
	(0.26)	(0.35)	(0.07)	(0.02)
(I)CAPM scaled with *cay*	0.25	-0.34	0.29	-0.37
	(0.54)	(0.16)	(0.44)	(0.08)
CCAPM scaled with *yc*	1.09	-0.83	0.48	-0.21
	(0.02)	(0.02)	(0.10)	(0.28)
(I)CAPM scaled with *yc*	0.71	-0.64	0.63	-0.56
	(0.01)	(0.00)	(0.01)	(0.00)
CWCAPM scaled with *ca*	0.15	-0.15	0.10	-0.10
	(0.55)	(0.41)	(0.65)	(0.53)
conditional CCAPM with *cay*	1.41	-0.58	0.66	0.17
	(0.01)	(0.10)	(0.06)	(0.55)
conditional ICAPM with *cay*	1.57	-0.32	0.47	0.78
	(0.00)	(0.18)	(0.13)	(0.03)
conditional CCAPM with *yc*	1.48	-0.54	0.71	0.24
	(0.00)	(0.11)	(0.05)	(0.50)
conditional ICAPM with *yc*	1.84	-1.09	0.52	0.22
	(0.00)	(0.00)	(0.05)	(0.49)
conditional CCAPM with *ca*	1.44	-0.94	0.36	0.15
	(0.02)	(0.02)	(0.27)	(0.57)
conditional CWCAPM with *ca*	0.82	-0.94	0.23	-0.35
	(0.19)	(0.01)	(0.19)	(0.01)
average returns	1.59	0.13	0.88	0.84

Table III: Simulated model - CWCAPM

Panel A lists the parameters of the conditional two-factor asset pricing model with consumption risk and aggregate wealth risk (CWCAPM) used in the Monte Carlo simulation exercise.

Panel B lists the parameters governing the conditional correlations of simulated returns with the pricing factors.

Panel C displays summary statistics of the simulated data.

Panel A. Preference parameter and aggregate dynamics

γ	η	ζ_0	ζ_1
20	25×10^{15}	-1.66	8.80

Panel B. Conditional correlation parameters

	l_0	l_1	k_0	k_1
Value	0.30	2.00	-0.75	5.00
Growth	0.30	100.00	-0.40	-2.00

Panel C. Summary statistics of simulated data

	ΔC	ΔW	$\frac{C}{W}$	Portfolio excess returns					
				SG	SI	SV	LG	LI	LV
Mean (%)	0.50	0.68	18.91	2.79	2.87	3.65	1.85	2.05	2.82
Std.Dev. (%)	0.46	6.11	2.42	11.49	8.83	9.23	7.59	6.33	7.22
Autocorrelation	0.00	-0.06	0.89	-0.00	-0.00	-0.00	-0.01	-0.01	-0.01

Table IV: **Average pricing errors: Monte Carlo simulations**

A version of the conditional two-factor model CWCAPM (the true model) is simulated $N_{MC} = 10000$ times. The artificial data in each replication constitutes a sample of $T = 221$ observations from the simulated series.

The following models are estimated using either conditional or unconditional moments: CCAPM is the one-factor consumption-based model; the two-factor model CWCAPM(M) uses the simulated stock market portfolio return as a proxy for the wealth portfolio return, where as the model CWCAPM uses the true wealth return.

Unconditional average pricing errors are estimated for the four long-short portfolio returns that are combinations of the 6 basis portfolios used to estimate the model: Small Value minus Small Growth (SV-SG), Small Growth minus Large Growth (SG-LG), Small Value minus Large Value (SV-LV) and Large Value minus Large Growth (LV-LG). Values in the parentheses indicate the fraction of Monte Carlo draws for which the p-values for the test that individual pricing errors are equal to zero are less than 5%. The p-values are computed using (semi)parametric stationary bootstrap with 100 replications.

The means of portfolio excess returns and standard errors for the means (in brackets) are average and standard deviations of estimates of means across the Monte Carlo draws, respectively.

Model	SV-SG	SG-LG	SV-LV	LV-LG
unconditional CCAPM	1.44	-0.20	0.19	1.06
	(0.89)	(0.16)	(0.15)	(0.90)
unconditional CWCAPM(M)	0.98	-0.20	0.05	0.73
	(0.60)	(0.46)	(0.49)	(0.60)
unconditional CWCAPM	0.25	-0.02	0.04	0.19
	(0.17)	(0.22)	(0.23)	(0.18)
CCAPM scaled with z	0.03	0.00	0.01	0.02
	(0.16)	(0.15)	(0.15)	(0.16)
CWCAPM(M) scaled with z	0.03	-0.03	0.01	-0.00
	(0.09)	(0.09)	(0.10)	(0.09)
CWCAPM scaled with z	0.01	-0.00	-0.00	0.01
	(0.07)	(0.08)	(0.09)	(0.07)
conditional CCAPM with z	1.07	-0.09	0.17	0.80
	(0.74)	(0.14)	(0.14)	(0.76)
conditional CWCAPM(M) with z	0.41	-0.26	-0.17	0.32
	(0.10)	(0.07)	(0.08)	(0.13)
conditional CWCAPM with z	0.14	0.02	0.05	0.11
	(0.05)	(0.06)	(0.08)	(0.06)
average returns	0.87	0.86	0.77	0.97
	[0.38]	[0.67]	[0.58]	[0.25]

Table V: **Differences in conditional moments of portfolio returns - long-run consumption risk**

Bootstrap tests of differences in conditional covariances of returns on the benchmark portfolios with long-run aggregate consumption growth and differences in conditional mean excess returns, estimated jointly using $z = cay$ as the conditioning variable, where $z^L = -0.019$ and $z^H = 0.02$ correspond to the 10th and 90th percentiles of the distribution of cay (in the entire sample IV.1952 - IV.2008), respectively. Consumption growth is calculated over $S+1$ quarters.

Panel A: $S = 11$

	$E(R\|z^H) - E(R\|z^L)$	$100 \times (cov(R, \Delta c\|z^H) - cov(R, \Delta c\|z^L))$
Small Growth	4.28	-1.31
	(0.01)	(0.34)
Small Value	3.26	4.24
	(0.01)	(0.91)
Large Growth	5.29	4.00
	(0.00)	(0.96)
Large Value	3.48	6.11
	(0.01)	(0.99)
Small Value minus Growth	-1.03	5.55
	(0.26)	(0.01)
Large Value minus Growth	-1.80	2.10
	(0.07)	(0.11)

Panel B: $S = 19$

	$E(R\|z^H) - E(R\|z^L)$	$100 \times (cov(R, \Delta c\|z^H) - cov(R, \Delta c\|z^L))$
Small Growth	5.08	-4.10
	(0.01)	(0.23)
Small Value	3.85	1.83
	(0.01)	(0.64)
Large Growth	5.89	1.90
	(0.00)	(0.71)
Large Value	4.32	3.67
	(0.00)	(0.83)
Small Value minus Growth	-1.23	5.93
	(0.23)	(0.01)
Large Value minus Growth	-1.57	1.77
	(0.10)	(0.23)

Table VI: **Average pricing errors: long-run consumption risk**

CCAPM estimated using quarterly aggregate data, with consumption risk measured by covariances with long-run consumption growth over $S + 1$ quarters.
P-values for the test that individual pricing errors are equal to zero given in the parentheses are computed using (semi)parametric stationary bootstrap with 10000 replications.

Model	SV-SG	SG-LG	SV-LV	LV-LG
unconditional CCAPM, $S = 11$	0.40	0.33	0.47	0.25
	(0.53)	(0.17)	(0.22)	(0.45)
unconditional CCAPM, $S = 19$	0.72	0.10	0.80	0.02
	(0.43)	(0.28)	(0.04)	(0.70)
CCAPM scaled with *cay*, $S = 11$	0.21	0.15	0.49	-0.13
	(0.18)	(0.19)	(0.06)	(0.24)
CCAPM scaled with *cay*, $S = 19$	0.18	0.14	0.39	-0.07
	(0.29)	(0.18)	(0.12)	(0.37)
conditional CCAPM with *cay*, $S = 11$	-0.10	0.49	0.69	-0.30
	(0.25)	(0.17)	(0.08)	(0.17)
conditional CCAPM with *cay*, $S = 19$	0.40	0.10	0.60	-0.10
	(0.62)	(0.31)	(0.11)	(0.23)
average returns	1.63	0.15	0.97	0.81

Table VII: **Multiple conditioning variables - single index**

Single-index semiparametric estimation of the time-varying price of consumption growth risk. The estimated coefficients contained in the vector θ are weights of individual variables in the single index used in the nonparametric estimation of the conditional prices of risk. The bootstrap p-values in the parentheses are computed using two methods: fully nonparametrically, by estimating the frequency of bootstrap samples producing the estimate of an opposite sign from the point estimate (p_{nonp}), and semi-parametrically under the null hypothesis that only one variable z enters the index.

The variables are the consumption-wealth residual cay, together with a set of macroeconomic and financial variables, such as the 12-month growth rate of industrial production IP ; the term spread $term$; the default spread def; the dividend yield dp; and the labor income to consumption ratio yc (Panel A), and with the first 8 principal components extracted from 131 macroeconomic series in Ludvigson and Ng (2009) (Panel B).

Panel A: standard predictors

Variables:	cay	IP	$term$	def	dp	yc
$\hat{\theta}$	1.22	-0.50	0.43	1.85	1.13	1.51
p_{nonp}	(0.01)	(0.08)	(0.07)	(0.20)	(0.16)	(0.03)
p_{null}, $z = cay$		(0.84)	(0.70)	(0.23)	(0.42)	(0.19)

Panel B: factor-based predictors

Variables:	cay	$PC1$	$PC2$	$PC3$	$PC4$	$PC5$	$PC6$	$PC7$	$PC8$
$\hat{\theta}$	1.33	-1.12	1.35	0.90	0.93	0.72	1.42	0.65	0.94
p_{nonp}	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)
p_{null}, $z = cay$		(0.15)	(0.01)	(0.84)	(0.72)	(0.93)	(0.01)	(0.96)	(0.81)

Panel C: coefficients on individual predictors

Variables:	cay	$PC2$	$PC6$
	1.04	1.62	0.30
p_{null}, $z = cay$		(0.10)	(0.95)

Panel D: pricing errors on benchmark portfolios

Portfolio	SG	SI	SV	LG	LI	LV
	-0.60	0.87	1.23	0.28	0.70	0.73
	(0.82)	(0.05)	(0.04)	(0.10)	(0.06)	(0.07)
average returns	1.60	2.79	3.27	1.30	1.59	2.09

Panel E: pricing errors on long-short strategies

Portfolio	SV-SG	SG-LG	SV-LV	LV-LG
	1.83	-0.88	0.50	0.45
	(0.25)	(0.34)	(0.21)	(0.66)
average returns	1.63	0.15	0.97	0.81

Figure 1: **Conditional expected excess returns using** *cay*
Each panel depicts the conditional expected excess returns on a portfolio over the range of the conditioning variable, *cay*. The top row contains Small stock portfolios, the leftmost column - Growth stock portfolios. The bold solid line is the mean of the sampling distribution of the nonparametric estimate, the dash-dotted lines are 95% confidence bounds, all obtained via stationary bootstrap. Rescaled kernel density of the conditioning variable is shaded in the background.

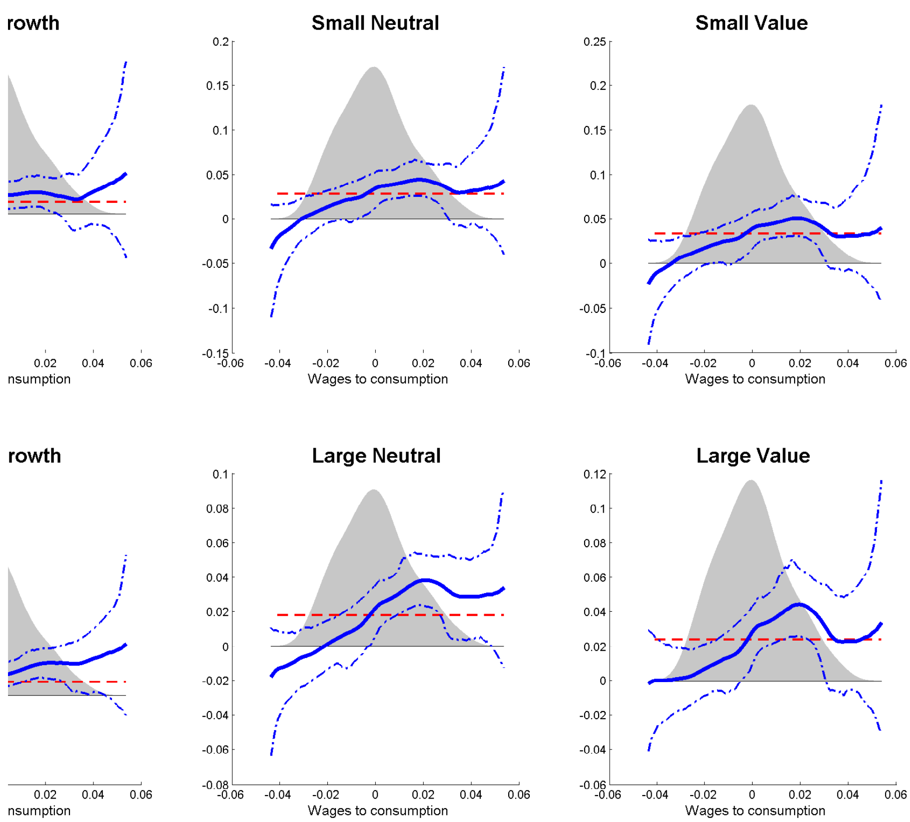


Figure 2: **Conditional covariances of portfolio returns with consumption growth using** *cay*
Each panel depicts the conditional covariance of a portfolio excess return with the with aggregate consumption growth over the range of the conditioning variable, *cay*. The solid line is the mean of the sampling distribution of the nonparametric estimate, the dash-dotted lines are 95% confidence bounds, all obtained via stationary bootstrap. Rescaled kernel density of the conditioning variable is shaded in the background.

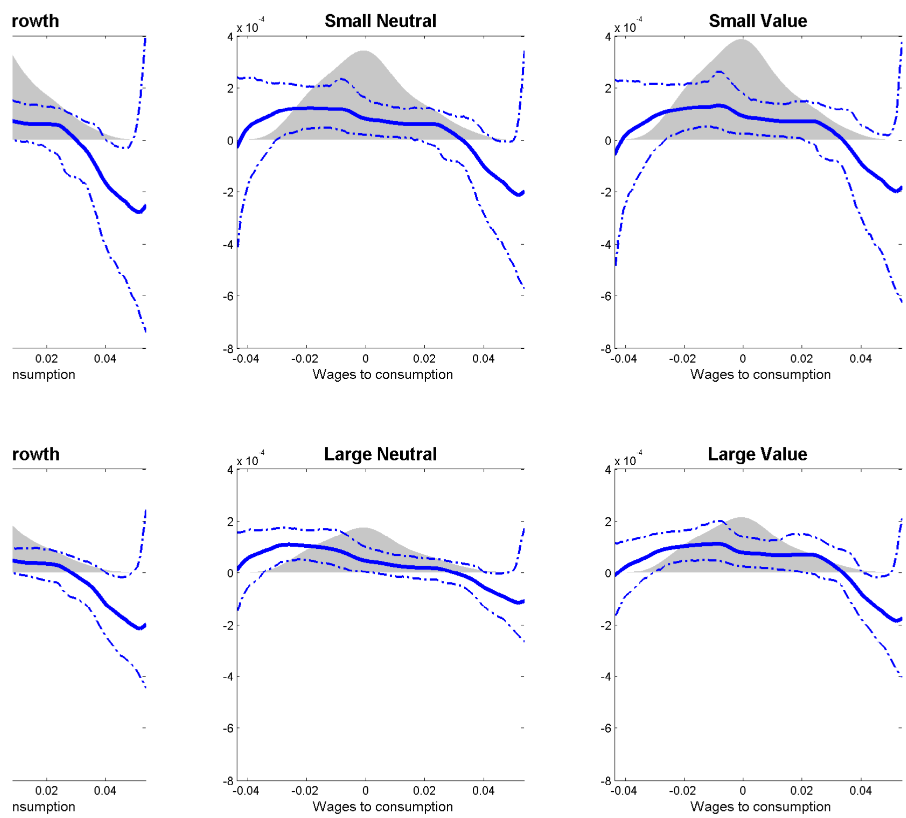


Figure 3: **Difference in conditional expected returns and conditional covariances of portfolio returns with consumption growth using** *cay*

Each panel depicts differences in either the conditional expected returns or the conditional covariance of a portfolio excess return with the aggregate consumption growth over the range of the conditioning variable, *cay* for the two long short portfolios:

SV - SG (small value minus small growth)

LV - LG (large value minus large growth)

The solid line is the mean of the sampling distribution of the nonparametric estimate, the dash-dotted lines are 95% confidence bounds, all obtained via stationary bootstrap. Rescaled kernel density of the conditioning variable is shaded in the background.

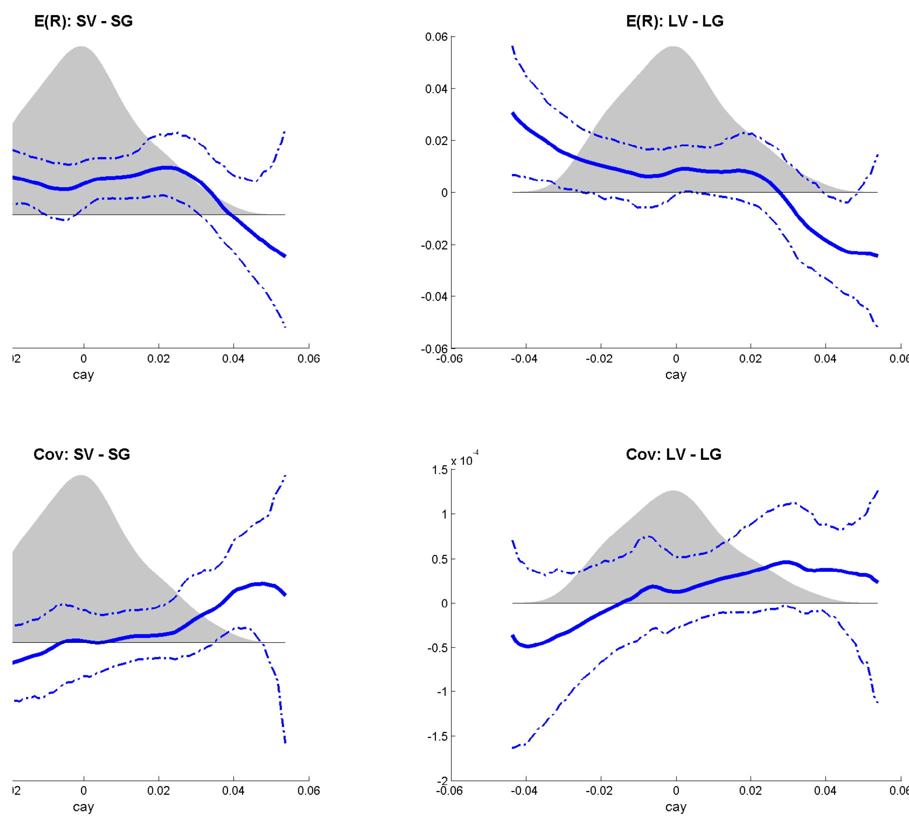


Figure 4: **Conditional price of consumption risk using** *cay*
The figure depicts the estimated price of consumption covariance risk (risk aversion) implied by the cross-section of stock returns, as a function of the consumption-wealth residual *cay*. The solid line is the mean of the sampling distribution of the nonparametric estimate, the dash-dotted lines are 95% confidence bounds, all obtained via stationary bootstrap. In addition, the pricing errors corresponding to the unconditional version of the model, as well as the scaled-factor conditional version are shown in the bottom set of panels (dashed and dotted straight lines, respectively).

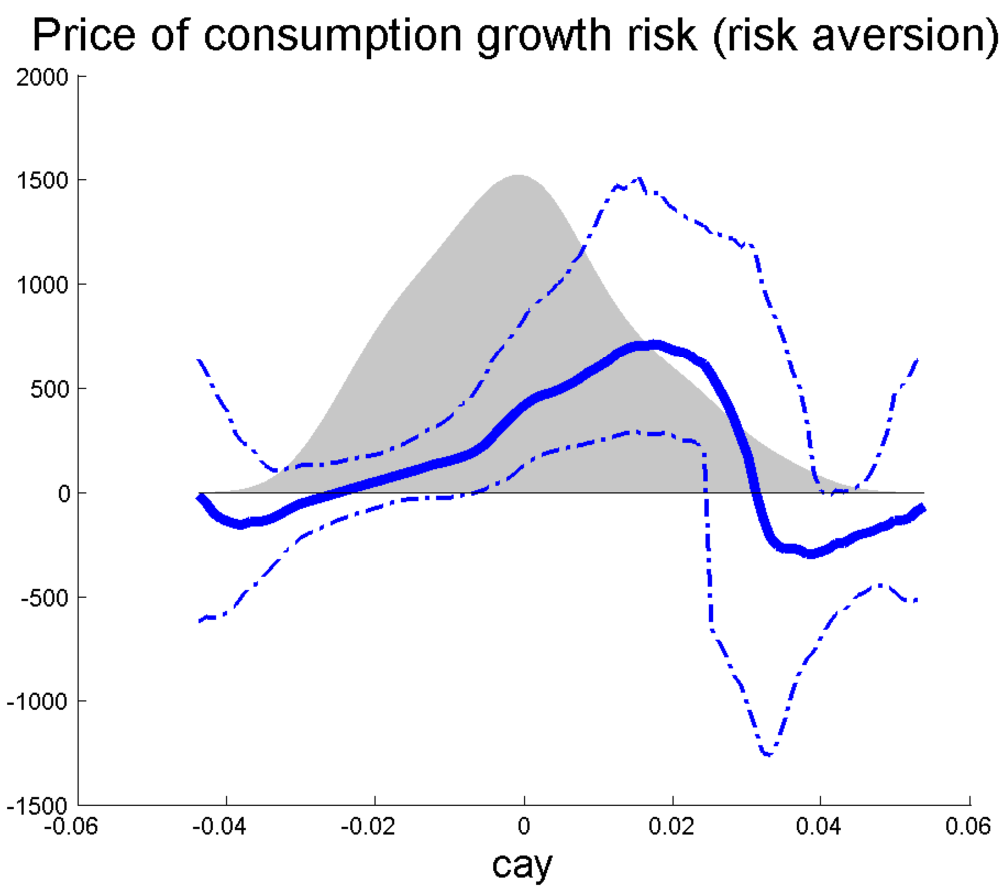


Figure 5: **Conditional pricing errors for CCAPM using** *cay*

Each panel depicts the conditional pricing error for the portfolio. The bold solid line is the mean of the sampling distribution of the nonparametric estimate, the dash-dotted lines are 95% confidence bounds, all obtained via stationary bootstrap. In addition, the pricing errors corresponding to the unconditional version of the model, as well as the scaled-factor conditional version are shown in the bottom set of panels (dashed and dotted straight lines, respectively). Rescaled kernel density of the conditioning variable is shaded in the background.

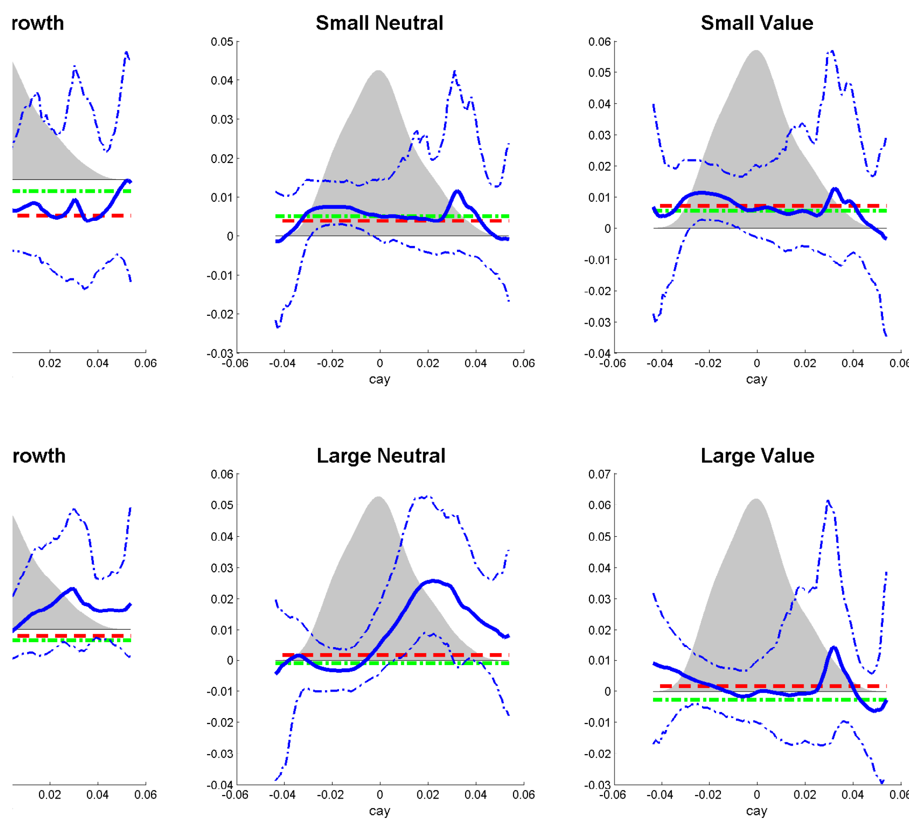


Figure 6: **Conditional pricing errors for CWCAPM using** *ca*
Each panel depicts the conditional pricing error for the portfolio. The bold solid line is the mean of the sampling distribution of the nonparametric estimate, the dash-dotted lines are 95% confidence bounds, all obtained via stationary bootstrap. In addition, the pricing errors corresponding to the unconditional version of the model, as well as the scaled-factor conditional version are shown in the bottom set of panels (dashed and dotted straight lines, respectively).

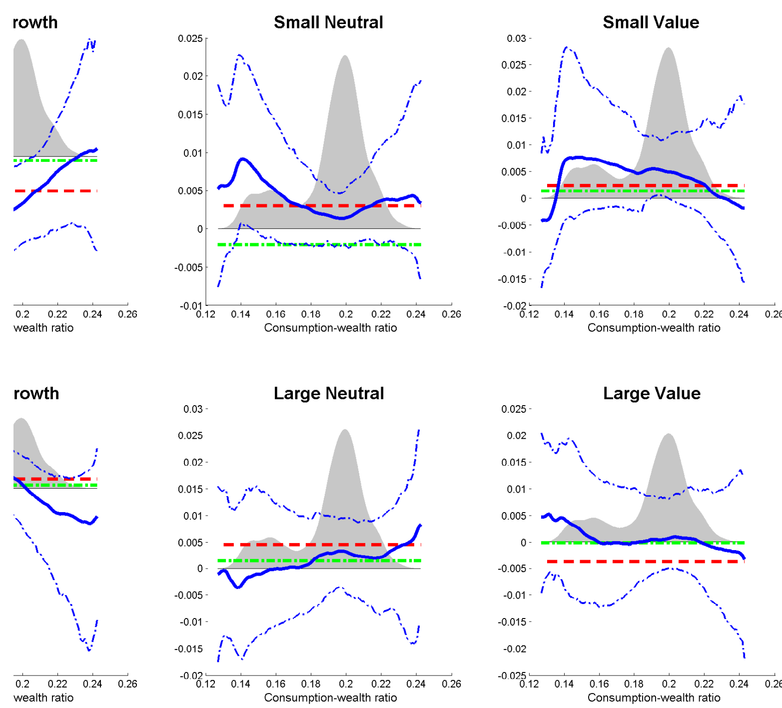


Figure 7: **Conditional means of portfolio excess returns - Monte Carlo estimates** The two-factor conditional model CWCAPM is simulated 10000 times; the conditional expected excess returns on the six benchmark portfolios are estimated using short samples of $T_{short} = 221$ observations for each draw. The solid line is the average estimate across the Monte Carlo draws, and the dash-dotted lines are the 95% confidence intervals. The solid red line is the true conditional moment function.

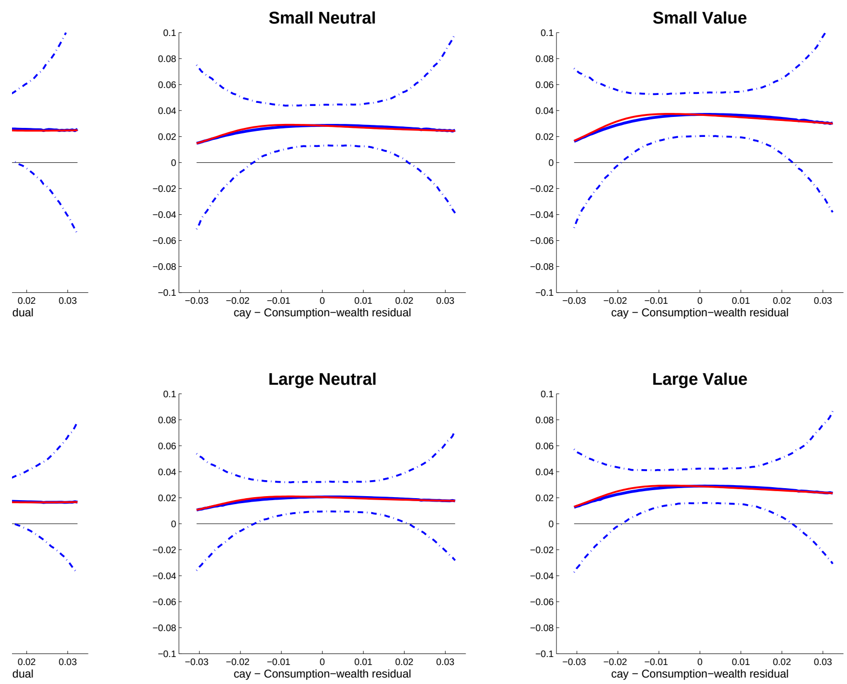
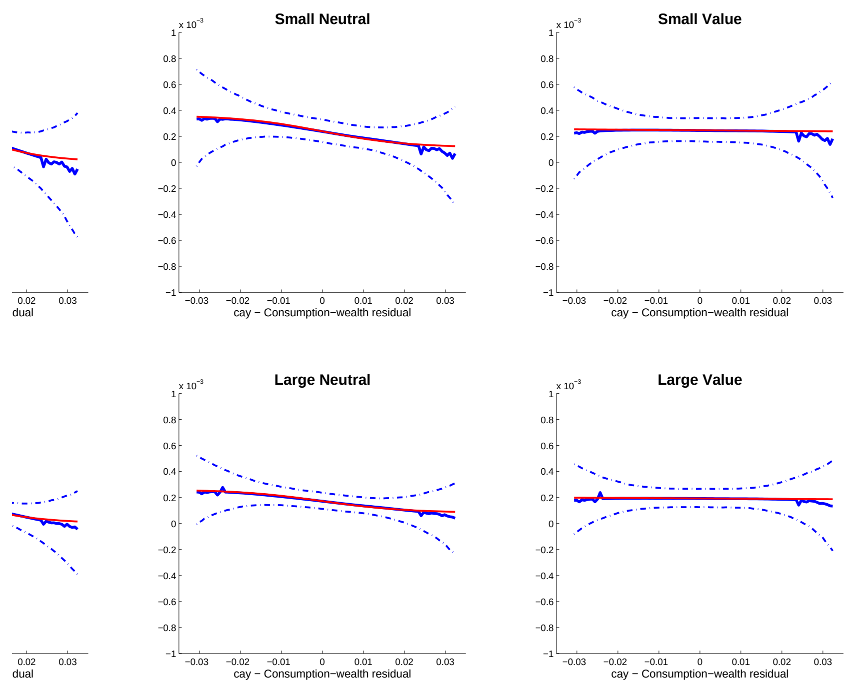


Figure 8: **Conditional covariances of portfolio returns with aggregate consumption growth - Monte Carlo**
The two-factor conditional model CWCAPM is simulated 10000 times; the conditional covariances of excess returns on the six benchmark portfolios with the simulated consumption growth are estimated using short samples of $T_{short} = 221$ observations for each draw. The solid line is the average estimate across the Monte Carlo draws, and the dash-dotted lines are the 95% confidence intervals. The solid red line is the true conditional moment function.



Appendix to

Composition of Wealth, Conditioning Information, and the Cross-Section of Stock Returns

A Intertemporal CAPM with the Composition Effect

Here I present a stylized model that illustrates the potential role for wealth composition in supplying conditioning information for asset pricing tests. In order to highlight the basic intuition and allow straightforward interpretation of parameters that can be estimated, I first restrict my attention to economies populated by representative consumer(s) who derive income from financial assets and human capital (in the form of a single stream of labor income). The consumer may or may not be restricted from borrowing against her human wealth. The derivation follows standard ICAPM arguments as in Merton (1973) and Breeden (1979), slightly generalized using the methodology developed by Duffie and Epstein (1992). I then consider a stylized economy with heterogeneous investors who have relative wealth concerns as in Roussanov (2010).

Since the primary focus of this paper is empirical, I do not prove that the model presented here possesses an equilibrium solution. Provided that an equilibrium exists, I characterize the testable restrictions it places on the cross-section of asset returns as well as on aggregate consumption. [14]

The technologies available to the investor consist of a vector of K risky stocks $\mathbf{S} = [S^1, \ldots, S^K]'$, a riskless bond B, and a stream of aggregate labor income y, with the dynamics

[14] In endowment economy settings Santos and Veronesi (2006) and Cochrane, Longstaff, and Santa-Clara (2008) are able to characterize the equilibrium quantities more explicitly by making specific assumptions that restrict the dynamics of asset returns; Martin (2009) and Chen and Joslin (2010) provide explicit solutions of similar exchange economies with more general underlying dynamics. Palacios (2010) solves a model of a production economy with Epstein-Zin preferences, which is most closely related to the environment considered here. The issues of equilibrium existence in the more general environments featuring labor income have been addressed rigorously by Cuoco (1997) and He and Pagés (1993).

given by

$$\begin{aligned}\frac{d\mathbf{S}_t}{\mathbf{S}_t} &= \mu_t dt + \sigma_t dZ_t, \\ \frac{dB_t}{B_t} &= r_t dt, \\ dy_t &= m y_t dt + \sigma_y y_t dZ_t,\end{aligned}$$

where Z_t is an M-dimensional Brownian motion with $E(dZ_t, dZ_t') = I$, σ is a $N \times M$ matrix, σ_y is a $1 \times M$ vector, $N \leq M$ (i.e. markets are not necessarily complete).

The dynamic budget constraint gives the law of motion for financial wealth:

$$dW_t = [(r_t + \alpha'(\mu_t - r_t \mathbf{1}))W_t + y_t - c_t]dt + \alpha' W_t \sigma_t dZ_t,$$

where α is the vector of wealth shares invested in each risky asset.

In order to simplify exposition and focus on the composition effect as the sole driver of time-variation in conditional moments, assume that there are only two state variables affecting the conditional moments of returns and entering the consumer's dynamic optimization problem. In particular, assume that m and σ_y are constant, while μ_t, σ_t and r_t are adapted to the filtration generated by $[W, y]$ (in what follows I suppress the time subscripts). That is, conditional expected returns and the conditional covariance matrix of returns can potentially depend only on the total value of the market portfolio and aggregate labor income y.

In the most general case that is relevant to the empirical discussion in this paper, the representative agent's preferences are represented by stochastic differential utility (see Duffie and Epstein (1992) for details). These preferences are given by a tuple (f^*, A), referred to as "aggregator", where f^* is a "felicity" function of the current consumption and of the continuation utility (thus responsible for intertemporal substitution) and A is the "variance multiplier" of the utility process (reflecting risk aversion). It turns out that for any such aggregator there exists a *normalized* aggregator $(f, 0)$, which represents the same preferences (i.e. the two are ordinally equivalent). This simplifies calculation significantly. In particular, using a normalized aggregator, the Bellman equations is given by the following equation:

$$0 = \max_{c,\alpha} DV + (rW + y)\, V_w + myV_y + \frac{1}{2}\sigma_y \sigma_y' y^2 V_{yy}, \tag{A-1}$$

where

$$DV = (\alpha'(\mu - r\mathbf{1}) - c)WV_w + \frac{1}{2}\alpha'\sigma\sigma'\alpha W^2 V_{WW} + \alpha'\sigma\sigma_y' W y V_{Wy} + f(c, V(W, y))$$

In general, the standard first order conditions characterize the optimal consumption and investment policies.

- Consumption:

$$V_W(W, y) = f_c(c, V) \tag{A-2}$$

- Portfolio weights:

$$\alpha = -\frac{V_W}{WV_{WW}} (\sigma\sigma')^{-1} (\mu - r\mathbf{1}) - (\sigma\sigma')^{-1} \sigma\sigma_y' \frac{yV_{Wy}}{WV_{WW}}. \tag{A-3}$$

From the latter we can again obtain the restriction of conditional expected returns:

$$\mu_i - r = -\frac{V_{WW}}{V_W} W Cov(R_i, R_M) - y\frac{V_{Wy}}{V_W} Cov(R_i, \frac{dy}{y}).$$

But now differentiating the envelope condition yields

$$\begin{aligned} V_{WW} &= f_{cc}C_W + f_{cV}V_W \quad \text{and} \\ V_{Wy} &= f_{cc}C_y + f_{cV}V_y \end{aligned}$$

Then the conditional moment restrictions on asset returns[15] can be rewritten as

$$\mu_i - r = -W\left(\frac{f_{cc}}{f_c}C_W + f_{cV}\right)Cov(R_i, R_M) - y\left(\frac{f_{cc}}{f_c}C_y + \frac{f_{cV}V_y}{f_c}\right)Cov(R_i, \frac{dy}{y}), \quad \text{(A-4)}$$

where $C_W = \frac{\partial C^*(W,y)}{\partial W}$ and $C_y = \frac{\partial C^*(W,y)}{\partial y}$ for the optimal consumption policy $C^*(W, y)$. Alternatively, using the CES properties of the aggregator, we can write

$$\mu_i - r = -\left(\frac{f_{cc}C}{f_c}\varepsilon_W + Wf_{cV}\right)Cov(R_i, R_M) - \left(\frac{f_{cc}C}{f_c}\varepsilon_y + \frac{yf_{cV}V_y}{f_c}\right)Cov(R_i, \frac{dy}{y}), \quad \text{(A-5)}$$

where $\varepsilon_W = \frac{WC_W}{C}$ and $\varepsilon_y = \frac{yC_y}{C}$ are elasticities of consumption with respect to financial wealth and labor income. I use the CES specification of Kreps and Porteus (1978) for the SDU aggregator

$$f^* = \frac{\delta}{1-\gamma}\frac{c^{1-\gamma} - V^{1-\gamma}}{V^{-\gamma}}, \quad A = \frac{\alpha - 1}{V},$$

which has a normalized aggregator with

$$f = \frac{\delta}{1-\gamma}\frac{c^{1-\gamma} - (\alpha V)^{\frac{1-\gamma}{\alpha}}}{(\alpha V)^{\frac{1-\gamma}{\alpha}-1}},$$

where $-\frac{f_{cc}C}{f_c} = \gamma$ is the reciprocal of the constant elasticity of intertemporal substitution and α is the risk aversion parameter. These preferences are the continuous-time limit of the recursive utility introduced by Epstein and Zin (1989) and Weil (1990). If $\alpha = 1 - \gamma$ these preferences collapse to the standard additive isoelastic utility with curvature γ.

In this case we have

$$\mu_i - r = \gamma\varepsilon_W Cov(R_i, R_M) + \gamma\varepsilon_y Cov(R_i, \frac{dy}{y}), \quad \text{(A-6)}$$

which, as a consequence of the Itô's lemma is equivalent to the consumption CAPM restric-

[15] Notice that in general this relation cannot be reduced to the familiar two-factor representation involving the market return and the consumption growth due to the presence of labor income.

tion

$$\mu_i - r = \gamma Cov(R_i, \frac{dC}{C}) \tag{A-7}$$

In the case with no labor income $\varepsilon_y = 0$, and by homotheticity of the value function, $\varepsilon_W = 1$, so we obtain the usual (conditional) CAPM:

$$\mu_i - r = \gamma Cov(R_i, R_M).$$

In the more general case, using the CES aggregator we can see that the market prices of risk associated with the representation (A-5) are given by

$$\lambda_W = \gamma \varepsilon_W - W f_{cV} \quad \text{and} \quad \lambda_y = \gamma \varepsilon_y - \frac{y f_{cV} V_y}{f_c},$$

where the second additive component of each of the risk prices does not allow a simple interpretation[16]. As we can see, the basic two-factor ICAPM relation (8) obtains:

$$\mu_i - r = \lambda_W Cov(R_i, R_M) + \lambda_y Cov(R_i, \frac{dy}{y}), \tag{A-8}$$

However, the market prices of risk λ corresponding to the two conditional covariances become additive functions of the consumption elasticities ε_W and ε_y, so that (4) does not hold. The analogous expression would be instead a three-factor model:

$$\mu_i - r = \gamma Cov(R_i, \frac{dC}{C}) - W f_{cV} Cov(R_i, R_M) - \frac{y f_{cV} V_y}{f_c} Cov(R_i, \frac{dy}{y}). \tag{A-9}$$

Although functional form of the second additive component of a risk price is difficult to characterize, it is apparent what properties it needs to posses in order for the market prices of risk to match their empirical counterparts. In particular, these functions need to move in the opposite direction from consumption elasticities (as functions of conditioning variables).

[16]Campbell (1996) uses the discrete-time recursive utility to derive a similar intertemporal asset pricing model that includes the market return and the labor income growth as factors. In his framework, however, risk prices depend only on risk aversion, and do not include either the intertemporal elasticity or the intratemporal consumption elasticities

In addition, in order to be consistent with high values of the reciprocal of EIS, these functions need to be of the same order of magnitude of consumption elasticities, but of the opposite sign.

Most of the discussion in what follows, as well as the main body of the paper, refers to the state-separable case with CRRA utility.

Via a discrete-time approximation the equilibrium relationship in (A-8) implies a conditional linear factor model

$$E_t\left(R^{ei}_{t+1}\right) \approx \gamma\varepsilon_W\left(z_t\right)Cov_t(R^{ei}_{t+1},R^{M}_{t+1}) + \gamma\varepsilon_y\left(z_t\right)Cov_t(R^{ei}_{t+1},\frac{\Delta y_{t+1}}{y_t}), \tag{A-10}$$

or in the more familiar reduced form notation

$$E_t\left(R^{ei}_{t+1}\right) \approx Cov_t\left(R^{ei}_{t+1},\mathbf{f_{t+1}}\right)'\lambda\left(z_t\right), \tag{A-11}$$

Note that this representation does not require the knowledge of the present value of human wealth (which is endogenous to the model), which is useful for empirical work, since the latter is not observable to an econometrician.

The moments of asset returns (means and covariances) can vary over time as functions of the state variables W and y. In general, the consumption elasticities also vary over time, which leads to time-varying prices of risk associated with the two factors/state variables, $\lambda_W = \gamma\varepsilon_W$ and $\lambda_y = \gamma\varepsilon_y$. Consequently, (A-5) or, equivalently, (A-8) are *conditional* moment restriction. In principle, these moment restrictions can be tested without the use of conditioning information, as long as the relevant data is observed at a high frequency (Andersen, Bollerslev, Diebold, and Labys (2003)). However, in practice this is not feasible, since, unlike financial asset returns, neither labor income nor consumption are observed frequently by the econometrician.

Therefore, in order to proceed with empirical analysis we need to specify the set of variables that must be included in the conditioning information set. In what follows we assume that the functions μ_t, σ_t and r_t are homogeneous of degree zero in the state variables

W and y. That is, these (equilibrium) quantities are functions only of $x = \frac{W}{y}$, the ratio of financial wealth to labor income[17]. This assumption also implicitly requires the ratio x to be stationary in order for the agent's optimization problem to have a solution. This condition is economically intuitive and is a convenient starting point for empirical implementation.

Under the above assumption the value function given by (A-12) is homogeneous (of degree $1-\gamma$) in the two state variables W and y. Then one can show (along the lines of Koo (1998), Appendix A) that the optimal consumption function is of the form

$$C = Q(x)(W + yP(x)),$$

for some functions $Q(x)$, $P(x)$. This highlights the difference between the static CAPM that includes human capital as a component of the total wealth portfolio and the intertemporal CAPM in which the composition of total wealth changes over time. In the former case market prices of risk are fixed, sice the consumption function is constant, whereas in the latter case the marginal propensity to consume out of total wealth (essentially controlled by $Q(x)$) as well as the present value of total wealth (i.e. $W + yP(x)$) endogenously depend on the composition of total wealth and on the intertemporal hedging demands that arise from its variation over time.

Unfortunately the closed-form solution for the consumption function is not available within this framework even if the dynamics of asset returns were restricted further (and even attempting to solve for it numerically would be a daunting task). However, we can express the elasticities of consumption (and therefore the market prices of risk) in terms of observable variables that reflect time-variation in the composition of total wealth. In particular, this will enable us to estimate these quantities from the data and therefore to test the model's restrictions on consumption and asset returns jointly.

From homogeneity of the consumption function it follows that the consumption elastici-

[17]Santos and Veronesi (2006) in effect make a similar assumption by treating conditional betas as functions only of the shares of labor income in consumption instead of the entire cross sectional distribution of shares

ties can be expressed as functions of the wealth to income ratio x:

$$
\begin{aligned}
C_W &= Q'(x)(x+P(x)) + Q(x)(1+P'(x)), \\
C_y &= Q'(x)\left(xP(x) - x^2\right) - Q(x)(P(x) + xP'(x)), \\
\varepsilon_W &= \frac{WC_W}{C} = \frac{xC_W}{Q(x)(x+P(x))} \equiv \varphi(x), \\
\varepsilon_y = \frac{yC_y}{C} &= \frac{C_y}{Q(x)(x+P(x))} \equiv \psi(x), \\
1 &= \varepsilon_W + \varepsilon_y
\end{aligned}
$$

It can be easily seen that this representation also implies that the market prices of risk are functions of the ratio of labor income to consumption derived by Santos and Veronesi (2006)[18]. Let $\varsigma(x) = \frac{C}{y} = Q(x)(x+P(x))$; then $C_W = \varsigma'(x) \Rightarrow \varepsilon_W = \frac{\varsigma'(x)}{\varsigma(x)}x$, and $\varepsilon_y = 1 - \frac{\varsigma'(x)}{\varsigma(x)}x$. As long as the consumption function is monotonic, there is a one-to-one mapping between the financial wealth to labor income ratio and the labor income to consumption ratio.

The cointegrating residual of consumption, financial wealth, and labor income, introduced by Lettau and Ludvigson (2001a), is interpreted by these authors as a proxy for (the logarithm of) the ratio of consumption to total wealth. In my notation, the latter quantity is represented by

$$
\frac{C(W,y)}{W + yP(x)} = Q(x).
$$

Therefore, this variable can also be viewed as conveying the same information about the composition of total wealth as the wealth to income ratio. Clearly, the same can be said for the wealth to income ratio $\frac{W}{C}$, which is similar to the stock market wealth to consumption ratio of Duffee (2005), since in the present model the entire financial wealth is represented by the total stock market.

[18] One can also easily verify that if the financial wealth to labor income ratio x is fixed (i.e. there is no variation in the composition of wealth), the consumption elasticities are constant and equal to the shares of human and nonhuman wealth in the total wealth portfolio. Thus, there is no time-variation in market prices of risk and the Intertemporal CAPM reduces to the standard two-factor CAPM with human capital, as in Mayers (1972), which must hold unconditionally.

Thus, in light of the fact that any two of the three variables W, y, and C provide a sufficient statistic for the conditioning information implied by the model (and therefore for the market prices of risk and the conditional moments of returns) any one of the three variables introduced above could be used in empirical tests of the equilibrium condition (A-8).

Heterogeneity and social status concerns

Departing from the assumption of representative investor, I now assume that there are N households, each household j has its own labor/proprietary income process given by

$$dy_t^j = my_t^j dt + \sigma_y y_t^j dZ_t^j,$$

which is driven by the Brownian vector $dZ_t^j = \left[\begin{array}{cc} dZ_t & d\tilde{Z}_t^j \end{array} \right]'$, whose components dZ_t and $d\tilde{Z}_t^j$ are independent so that the latter captures the idiosyncratic part of household's wealth and consumption growth (in general, markets are incomplete).

Preferences exhibit social status externalities of a type introduced in Roussanov (2010). Households solve

$$V_t \left(W_t^j, y_t^j, \bar{W}_t \right) = \max \int_t^\infty e^{-\rho(s-t)} U(C_s^j, \bar{W}_s) ds$$

with the period utility function:

$$U(C_t^j, \bar{W}_t) = \frac{\left(C_t^j\right)^{1-\gamma}}{1-\gamma} + \eta \bar{W}_t^{1-\gamma} \left(\frac{C_t^j}{\bar{W}_t} \right),$$

where individual households view the aggregate wealth process

$$d\bar{W}_t = \mu_t^w \bar{W}_t dt + \sigma_t^{\bar{W}} \bar{W}_t dZ_t$$

as exogenous.

This problem can be represented by the Bellman equation

$$0 = \max_{c,\alpha} DV + (rW + y)\, V_w + myV_y + \mu_t^w \bar{W} V_{\bar{W}} \tag{A-12}$$

$$+\frac{1}{2}\sigma_t^{\bar{W}}\sigma_t^{\bar{W}\prime} y^2 V_{\bar{W}\bar{W}} + \sigma_t^{\bar{W}}\sigma_y' y^2 V_{yy} + \frac{1}{2}\sigma_y\sigma_y' y^2 V_{yy} - \rho V, \tag{A-13}$$

where

$$DV = (\alpha'(\mu - r\mathbf{1}) - c)WV_w + \frac{1}{2}\alpha'\sigma\sigma'\alpha W^2 V_{WW} + \alpha'\sigma\sigma_y' WyV_{Wy} + \alpha'\sigma\sigma^{\bar{W}\prime} W\bar{W}V_{W\bar{W}} + U(c, \bar{W}_t)$$

As before, standard first order conditions characterize the optimal consumption and portfolio allocations.

- Consumption:

$$V_W(W, y) = U_c(c, \bar{W}_t) \tag{A-14}$$

- Portfolio weights:

$$\alpha = -\frac{V_W}{WV_{WW}} \left(\sigma\sigma'\right)^{-1} (\mu - r\mathbf{1}) - \left(\sigma\sigma'\right)^{-1} \sigma\sigma_y' \frac{yV_{Wy}}{WV_{WW}} - \left(\sigma\sigma'\right)^{-1} \sigma\sigma^{\bar{W}\prime} \frac{\bar{W}V_{W\bar{W}}}{WV_{WW}}. \tag{A-15}$$

The restriction on conditional expected returns is now (for individual investor j):

$$\mu_i - r = -\left(W\frac{V_{WW}}{V_W} + \bar{W}\frac{V_{W\bar{W}}}{V_W}\right) Cov(dR_i, dR_M) - y\frac{V_{Wy}}{V_W} Cov(dR_i, \frac{dy^j}{y^j}).$$

Now differentiating the envelope condition yields

$$\begin{aligned} V_{WW} &= U_{cc}C_W, \\ V_{Wy} &= U_{cc}C_y \text{ , and} \\ V_{W\bar{W}} &= U_{cc}C_{\bar{W}} + U_{c\bar{W}} \end{aligned}$$

Then the conditional moment restrictions on asset returns can be rewritten as

$$\mu_i - r = -\left(W_t^j \frac{U_{cc}}{U_c} C_W + \bar{W}_t \frac{U_{c\bar{W}}}{U_c}\right) Cov(R_i, R_M) - y_t^j \frac{U_{cc}}{U_c} C_y Cov(R_i, \frac{dy^j}{y^j}),$$

or, alternatively,

$$\mu_i - r = -C_t^j \frac{U_{cc}}{U_c} Cov(R_j, \frac{dC_t^j}{C_t^j}) - \bar{W}_t \frac{U_{c\bar{W}}}{U_c} Cov(R_j, R_M), \tag{A-16}$$

where $-C_t^j \frac{U_{cc}}{U_c} = \gamma \frac{\left(C_t^j\right)^{-\gamma}}{\left(C_t^j\right)^{-\gamma} + \eta \bar{W}_t^{-\gamma}}$ and $-\bar{W}_t \frac{U_{c\bar{W}}}{U_c} = \gamma \frac{\eta \bar{W}_t^{-\gamma}}{\left(C_t^j\right)^{-\gamma} + \eta \bar{W}_t^{-\gamma}}$.

Let $s_t^j = \frac{C_t^j}{\bar{C}_t}$ be the ratio of individual to per-capita consumption. Then, by following the arguments of Grossman and Shiller (1982), averaging this expression across households (and assuming that all households participate in the equity market) obtains

$$\mu_j - r = \lambda_C Cov(R_j, \frac{d\bar{C}_t}{\bar{C}_t}) + \lambda_W Cov(R_j, R_M), \tag{A-17}$$

where the risk prices are $\lambda_C = \gamma E_t \left(\frac{s_t^{-\gamma}}{s_t^{-\gamma} + \eta \left(\frac{\bar{C}_t}{\bar{W}_t}\right)^{\gamma}} \right)$, and $\lambda_W = \gamma E_t \left(\frac{\eta \left(\frac{\bar{C}_t}{\bar{W}_t}\right)^{\gamma}}{s_t^{-\gamma} + \eta \left(\frac{\bar{C}_t}{\bar{W}_t}\right)^{\gamma}} \right)$.

Thus, the prices of aggregate consumption risk and aggregate wealth risk both vary over time as functions of the ratio of aggregate consumption to financial wealth $\frac{\bar{C}_t}{\bar{W}_t}$, as well as, potentially, the cross-sectional distribution of consumption.

B Consistency of nonparametric price of risk estimators

In order to establish the uniform consistency of the estimators of market prices of risk $\widehat{\lambda}(z)$ it is enough to show the uniform weak convergence of the objective function,

$$Q_T(z; \lambda) = \mathbf{g}_T(z)' W \mathbf{g}_T(z),$$

to its population analogue,

$$Q_\infty(z;\lambda) = \mathbf{g}_\infty(z)' W \mathbf{g}_\infty(z),$$

where

$$g^i_\infty(z) = E\left(R^{ei}_{t+1} - Cov(R^{ei}_{t+1}, \mathbf{f}_{t+1}|z)'\lambda(z)\,|z\right) = 0.$$

This is true since the population objective reaches its minimum (since W is assumed to be positive semidefinite) at the true value of the functional parameter $\tilde{\lambda}(z)$:

$$Q_\infty\left(z;\tilde{\lambda}\right) = 0 \qquad \text{for all} \quad z \in Z$$

and identification is ensured as long as the number of moment conditions N (i.e. the number of test assets) is at least as large as the number of functional parameters K (i.e. the number of factors): $\tilde{\lambda}(z)$ is unique for each $z \in Z$ (here Z denotes the domain of conditioning variable(s), $Z \subset \mathbb{R}^d$). The aim is therefore to show that

$$\sup_{z\in Z}\sup_{\lambda\in\Lambda} \|Q_T(z;\lambda) - Q_\infty(z;\lambda)\| \xrightarrow{p} 0 \quad \text{as} \quad T \to \infty, \tag{B-1}$$

which would imply that

$$\sup_{z\in Z}\left\|\widehat{\lambda}(z) - \tilde{\lambda}(z)\right\| \xrightarrow{p} 0 \quad \text{as} \quad T \to \infty.$$

To simplify exposition, I consider only the special case that factors have conditional mean equal to zero. Then the conditional moment restrictions can be written as

$$g^i_\infty(z) = E\left(R^{ei}_{t+1} - (R^{ei}_{t+1}\mathbf{f}_{t+1})'\lambda(z)\,|z\right) = 0.$$

The sample analogues of these moment conditions are

$$g_T^i(z) = \frac{1}{\sum_{t=1}^{T-1} K\left(\frac{z-z_t}{h}\right)} \sum_{t=1}^{T-1} \left[R_{t+1}^{ei} - \left(R_{t+1}^{ei} \times \mathbf{f}_{t+1}\right)' \lambda\right] K\left(\frac{z-z_t}{h}\right) .$$

They can be alternatively represented as

$$g_T^i(z) = \frac{L_T^i}{\hat{f}_T(z)},$$

where

$$L_T^i(z;\lambda) = \frac{1}{Th^d} \sum_{t=1}^{T-1} \Psi\left(R_{t+1}^e, \mathbf{f}_{t+1}; \lambda\right) K\left(\frac{z-z_t}{h}\right)$$

with

$$\Psi\left(R_{t+1}^e, \mathbf{f}_{t+1}; \lambda\right) = R_{t+1}^e - \left(R_{t+1}^e \times \mathbf{f}_{t+1}\right)' \lambda,$$

and $\hat{f}_T(z)$ is the kernel estimator of the marginal density $f(z)$ of z:

$$\hat{f}_T(z) = \frac{1}{Th^d} \sum_{t=1}^{T-1} K\left(\frac{z-z_t}{h}\right).$$

Now we can appeal to the standard results for kernel M-estimators and kernel density estimators to establish the uniform convergence of these quantities to their population counterparts $L_\infty^i(z;\lambda) = f(z) E\left[\Psi(R, \mathbf{f}; \lambda) | z\right]$ and $f(z)$, respectively. Following Brandt (1999) one can use the result by Gourieroux, Monfort, and Tenreiro (2000) who show that, under a set of conditions described below,

$$\sup_{z \in Z} \sup_{\lambda \in \Lambda} \left\| L_T^i(z;\lambda) - L_\infty^i(z;\lambda) \right\| \overset{a.s.}{\to} 0 \quad \text{as} \quad T \to \infty.$$

Uniform consistency of kernel density estimators is a standard result (e.g. Pagan and Ullah (1999), Theorem 2.8). Combining the two and applying the continuous mapping theorem yields B-1. The following conditions are required in order establish the above results:

1. The kernel function $K(.)$ is Lipschitz continuous, has bounded support and

$$\int_{\mathbb{R}^d} K(u)\, du = 1$$

2. The sets Z and Λ are compact

3. The bandwidth $h \to 0$ as $T \to \infty$ and there exists such $\beta \in (0,1)$ that $\frac{T^{(1-\beta)/2} h^d}{\log T} \to \infty$ as $T \to \infty$

4. $\left(R^e_{t+1}, \mathbf{f}_{t+1}, z_t\right)$ form a strictly stationary process with the geometric mixing property:

$$\sup_{A \in \mathfrak{F}_0, B \in \mathfrak{F}_k} \left[P(A \cap B) - P(A)P(B)\right] < \alpha \rho^k, \forall k \in \mathbb{N}^*,$$

where $\alpha \geq 0$, $0 \leq \rho < 1, \mathfrak{F}_0 = \sigma\left(R^e_{\tau+1}, \mathbf{f}_{\tau+1}, z_\tau, \tau \leq 0\right), \mathfrak{F}_k = \sigma\left(R^e_{\tau+1}, \mathbf{f}_{\tau+1}, z_\tau, \tau \geq k\right)$.

5. The distribution of z_t exists, is continuous, and has uniformly continuous strictly positive pdf and absolutely integrable characteristic function.

6. $\Psi(R, \mathbf{f}; \lambda)$ is (Lipschitz) continuous on Λ for all $R, \mathbf{f}$ and measurable in $R, \mathbf{f}$ for all λ; $\exists \delta > 0$: $E\left[\sup_{\lambda \in \Lambda} |\Psi\left(R^e_{t+1}, \mathbf{f}_{t+1}; \lambda\right)|^{\frac{2}{\beta}+\delta}\right] < \infty$, where β from condition (3) on the bandwidth.

7. $L^i_\infty(z; \lambda)$ are uniformly equicontinuous for all i:

$$\forall \varepsilon > 0, \exists \delta > 0 : \sup_{z \in Z} \sup_{\|u-s\| < \delta} \sup_{\lambda \in \Lambda} \left|L^i_\infty(u; \lambda) - L^i_\infty(s; \lambda)\right| < \varepsilon$$

Remark A-1 *In place of the fixed matrix W the objective function can be specified using some positive definite matrix $W_T(z)$, which uniformly consistently estimates some $W_\infty(z)$ used in the population objective. A relevant example is a conditional version of the weighting matrix based on the Hansen and Jagannathan (1997) measure of pricing errors, $E(R^e R^{e\prime}|z)^{-1}$, which is replaced by its nonparametric estimate $\widehat{E(R^e R^{e\prime}|z)}^{-1}$ in a finite sample.*

C Bootstrap

Since stationarity of the conditioning variable (z) is a maintained assumption throughout the empirical investigation in this paper, I use stationary bootstrap in order to construct confidence intervals for nonparametric and semiparametric estimates. The bootstrap procedure allows one to approximate the entire sampling distribution of the estimators using their empirical distribution (EDF).

For a sample of length T, the stationary bootstrap procedure introduced by Politis and Romano (1994) amounts to constructing R resampled sets of T observations, which consist of overlapping blocks of observations from the original set. Each observation includes the vector of realized portfolio returns and the realized consumption growth at time $t+1$ as well as the vector of conditioning information known at time t. The block lengths are sampled randomly from a geometric distribution. This ensures that the resulting time-series remain stationary.

In order to minimize the bias in the distribution of nonparametric estimators I undersmooth the estimates (i.e. use low values of the bandwidth parameter h). See Horowitz (2001) for an extensive discussion on the use of bootstrap procedures in various settings, including nonparametric estimation and dependent data.

I use fully non-parametric bootstrap to construct point-wise confidence bands for the functional estimates of conditional expected returns and conditional covariances, as well as for the tests of differences in conditional moments across points in the state space (e.g., Härdle (1992)).

For pricing error tests I use a semi-parametric bootstrap procedure. I use bootstrap to simulate the return and covariance realizations under the null hypothesis that the average conditional pricing error is equal to zero for each portfolio. Specifically, I recenter the residuals

$$u_{t+1}^{i} = R_{t+1}^{ei} - \widehat{Cov}(R_{t+1}^{ei}, \frac{C_{t+1}}{C_t}|z_t)\widehat{\lambda}\,(z_t)$$

around zero, resample them jointly with z_t and consumption growth realization using the stationary block-bootstrap method described above, and calculate excess returns corresponding to each bootstrapped observation that corresponds to period τ as

$$\tilde{R}^{ei}_{\tau+1} = \widehat{Cov}(R^{ei}_{\tau+1}, \frac{C_{\tau+1}}{C_{\tau}}|z_\tau)\widehat{\lambda}\left(z_\tau\right) + u^i_{\tau+1}.$$

I then re-estimate the model on each of the bootstrapped samples in order to construct the distribution of average pricing errors for each portfolio.

D Data

The equilibrium pricing relations (2), (4) and (5) hold exactly in continuous time. Both consumption and labor income data are time-averaged, which might potentially bias the estimates. There is no simple solution to this problem (e.g., see Grossman, Melino, and Shiller (1987)), since high-frequency macroeconomic data is either unavailable or of poor quality. In all of the tests I use quarterly data for consumption and labor income data (results are very similar if monthly data are used instead). A number of authors, such as Campbell (1996) have formulated their models explicitly in discrete time in order to circumvent this issue. Doing so, however, requires *ad hoc* assumptions on the dynamics of human capital and asset returns.[19] One of the purposes of the nonparametric estimation methodology employed here is precisely to avoid making such auxiliary assumptions.

The proxy for the portfolio of traded assets that I use in empirical tests is the value-weighted portfolio of NYSE, NASDAQ and Amex stocks. The universe of traded assets used in cross-sectional tests consists of the 6 portfolios of NYSE, NASDAQ and Amex stocks sorted annually on size and book to market equity, which are used by Fama and French (1993) to construct their benchmark factor returns SMB and HML. Monthly returns are compounded to obtain quarterly returns. Excess returns are constructed using the one-month and three-month Treasury bill rates in place of the riskless rate at monthly and quarterly frequency,

[19]For example, Lustig and Van Nieuwerburgh (2008) argue that rates of return on human capital have a complicated relationship with financial asset returns that makes proxying for the human wealth return with either labor income growth or stock market return inappropriate. See also the discussion in Hansen, Heaton, Lee, and Roussanov (2007) and Bansal, Kiku, Shaliastovich, and Yaron (2012).

respectively.

In order to maintain consistency with previous studies and, in particular, to facilitate the comparison with Lettau and Ludvigson (2001b) and Santos and Veronesi (2006), I use the consumption, financial wealth, and labor income series constructed by Lettau and Ludvigson (2001a) (obtained from Sydney Ludvigson's website). I also use their *cay* variable. The financial wealth variable a is used for constructing the consumption-wealth ratio ca. Consumption series is NIPA nondurable consumption (excluding shoes and clothing at quarterly frequency, following Lettau and Ludvigson (2001a)) and services. I use total stock market capitalization (i.e. NYSE, NASDAQ and Amex, obtained from CRSP) as a proxy for total financial wealth in constructing the ca variable, following Duffee (2005). Quarterly stock market wealth, labor income, and consumption are all deflated with the price deflator of nondurables and services. All data is ranging from the fourth quarter of 1952 to the fourth quarter of 2008. Labor income and consumption data are from the *U.S. National Income and Product Accounts.*

E Comparison with parametric approaches

Could the conclusions reached above be obtained using more standard econometric approaches? Assume that the conditional means of consumption growth and excess returns, as well as their conditional covariance - $E_t\left(\frac{\Delta C_{t+1}}{C_t}\right)$, $E_t R^{ei}_{t+1}$, and $Cov_t(R^{ei}_{t+1}, \frac{\Delta C_{t+1}}{C_t})$ - are all linear in the vector of conditioning variables z_t (which includes the constant). Then we can estimate (e.g. as in Duffee (2005)) the following system:

$$\begin{aligned} \frac{\Delta C_{t+1}}{C_t} &= \kappa' z_t + u^c_{t+1}, \\ R^{ei}_{t+1} &= \mu_i' z_t + u^i_{t+1}, \\ \widetilde{Cov}^{\,i}_{t+1} &= \delta_i' z_t + u^{ci}_{t+1} \end{aligned}$$

where $\widetilde{Cov}^{\,i}_{t+1} = \left(\frac{\Delta C_{t+1}}{C_t} - E_t \frac{\Delta C_{t+1}}{C_t}\right)\left(R^{ei}_{t+1} - E_t R^{ei}_{t+1}\right) = u^c_{t+1} u^i_{t+1}$ is the 'ex-post' covariance of consumption growth and excess returns on asset i, so that the ex ante conditional covariance is given by its projection on the vector of conditioning variables:

$$Cov_t(R^{ei}_{t+1}, \frac{\Delta C_{t+1}}{C_t}) = E_t \widetilde{Cov}^{i}_{t+1} = \delta_i' z_t. \tag{E-1}$$

Table VIII shows the coefficients from the regressions of returns and the ex-post consumption covariances on z_t for several choices of the conditioning variable. The assets used are three portfolios formed from the 6 benchmark portfolios sorted on market capitalization on book/market equity ratios used by Fama and French (1992). The growth portfolio is the equal-weighted average of the small and large growth portfolios, the value and neutral portfolios are, similarly, equal-weighted averages across value and neutral portfolios, respectively.

If high values of z_t are associated with "bad times" and, consequently, a high price of consumption risk, the assets whose covariances with consumption growth are increasing in z_t are riskier. If the CCAPM holds, their expected excess returns should also increase in z_t. Duffee (2005) finds that an increase in the ratio of stock market wealth to consumption is associated with a rise in the covariance of the aggregate stock market return and consumption growth. However, it is also associated with *low* expected stock returns. The top panel illustrates that the same is true for each of the book/market-sorted portfolios. In fact, their does not appear to be much difference in the sensitivities of either conditional expected returns or conditional covariances to this variable, despite the fact that it appears to be a useful scaling variable as shown in section **??**.

The two middle panels of table VIII display the sensitivities of first and second moments of returns to *cay*. It does appear that *cay* plays a similar role at quarterly frequency to the role played by *ac* at monthly frequency: rising *cay* not only predicts higher expected returns, but also lower covariances of consumption with returns, presumably due to the declining share of financial assets in total wealth. The expected return sensitivities exhibit the pattern familiar from section 3.1: value returns are not quite as predictable as growth returns (in terms of the slope coefficient). There is virtually no difference in covariances if the entire sample is used for the estimation. However, using a shorter subsample ending in the second quarter of 2003, which is closer to the sample used by Lettau and Ludvigson (2001b), I find that the covariance of value returns with consumption growth actually increases when *cay* goes up, while growth returns' covariance declines. This is consistent with the argument of Lettau and Ludvigson (2001b) that value is riskier in "bad times," but inconsistent with the fact that value's expected returns are not more but less sensitive than growth's expected returns. Further, the coefficients for the conditional covariances are not statistically significantly different from zero, as their standard errors are very large. This might be in part due to the fact that the linear model is misspecified. Finally, using the labor-to-consumption ratio as the predictive variable (bottom panel) leads to similar conclusions: covariances and expected returns appear to move in the opposite directions for all portfolios, and while there is some heterogeneity across covariance sensitivities, there is much less difference in expected return sensitivities.

In principle, one could go further and impose conditional moment restrictions on the asset returns jointly. This entails making parametric assumptions on the functional form of risk prices. For example, one could follow Duffee (2005) and assume that $\gamma_t = \gamma_0 + \gamma_1 x_t$. Then the model could be estimated using the instrumental variables GMM approach of Campbell (1987) and Harvey (1989). However, such a model would be misspecified by construction, since expected returns, covariances, and prices of risk cannot be all linear. Thus even if the true conditional model holds, it could produce non-trivial pricing errors. Brandt and Chapman (2007) emphasize that the nonlinearity need not be large to produce a spurious rejection. Alternatively, one could avoid imposing parametric structure on the prices of risk and only make assumptions about the dynamics of conditional second moments, as done, for example, by Ferson and Harvey (1999), among others. I discuss this approach in Appendix E and show that, indeed, one can reject the conditional CCAPM using *cay*. Still, the conditional restrictions imposed using this method rely crucially on the linear specification of conditional betas. Therefore, if the linear model for conditional betas is misspecified, it is possible that the conditional tests will reject even the true conditional model. Ghysels (1998) argues that this problem is potentially quite severe, to the extent that the conditional beta models can perform even worse empirically than the unconditional models. Given the substantial difference in the estimated sensitivities of consumption covariances to the conditioning variable between the samples the concern over misspecification should make it hard to argue in favor of using the parametric approaches for imposing conditional moment restrictions.

F Testing conditional factor models using beta representation

Consider the setup of Lettau and Ludvigson (2001b), who specify a conditional consumption CAPM with a single conditioning variable, *cay* - the cointegrating residual of consumption, financial wealth and labor income, so that $\tilde{\mathbf{f}}_{\mathbf{t+1}} = \left[\frac{\Delta C_{t+1}}{C_t}, \frac{\Delta C_{t+1}}{C_t} \times cay_t\right]$ in (7) above. Their tests concentrate on the beta representation

$$E(R^{ei}_{t+1}) = \eta_0 + \eta_1 \beta^i_{cay_t} + \lambda_0 \beta^i_{\Delta C_{t+1}} + \lambda_1 \beta^i_{\Delta C_{t+1} \times cay_t}, \tag{F-1}$$

which is equivalent to (7) except that they allow a non-zero (and time-varying) cross-sectional intercept $(\eta_0 + \eta_1 cay_t)$, which implies that the conditional zero-beta rate is not necessarily equal to the risk-free interest rate. The estimate and test this specification using the standard cross-sectional regression methodology of Fama and MacBeth (1973), first estimating the betas (loadings) of returns on the scaled factors $\left[cay_t, \frac{\Delta C_{t+1}}{C_t}, \frac{\Delta C_{t+1}}{C_t} \times cay_t\right]$ by time-series regression and then regressing the cross-section of returns on the cross-section of betas to obtain the risk premium estimates λ (and η).

An alternative approach would be to test the conditional implications of the consumption CAPM using *cay* as the conditioning variable. The conditional beta representation is given[20]

[20] Lettau and Ludvigson (2001b) start with the stochastic discount factor model $E_t[M_{t+1} R^i_{t+1}] = 1$, where $M_{t+1} = a_t + b_t \frac{\Delta C_{t+1}}{C_t}$. Taking the unconditional expectation and assuming the SDF coefficients are linear functions of the conditioning variable yields

$$E[(a_0 + a_1 cay_t + (b_0 + b_1 cay_t)\frac{\Delta C_{t+1}}{C_t})R^i_{t+1}] = 1$$

and standard manipulations produce the expected return-beta representation (F-1). Alternatively, working with the conditional expectation directly, the conditional expected returns are given by

$$E_t(R^i_{t+1}) = \frac{1}{a_t} - \frac{b_t}{a_t} E_t[\frac{\Delta C_{t+1}}{C_t} R^i_{t+1}],$$

which leads to the beta representation for excess returns (F-2).

by

$$E_t(R^{ei}_{t+1}) = \eta_t + \lambda_t \beta^i_t, \tag{F-2}$$

where η_t, λ_t, and β^i_t are all functions of *cay*. Conditioning down obtains

$$E(R^{ei}_{t+1}) = E\left(\eta_t + \lambda_t \beta^i_t\right).$$

Assuming, as Lettau and Ludvigson (2001b) do, that conditional betas (and risk premia) are linear, i.e. $\beta^i_t = \beta^i_0 + \beta^i_1 cay_t$, these pricing implications can also be tested using the Fama-Macbeth methodology (e.g. Ferson and Harvey (1999)). Specifically, the parameters β^i_0 and β^i_1 can be estimated as factor loadings in the time series regressions

$$R^{ei}_{t+1} = \alpha_0 + \alpha_1 cay_t + \beta^i_0 \frac{\Delta C_{t+1}}{C_t} + \beta^i_1 \frac{\Delta C_{t+1}}{C_t} cay_t$$

Then the fitted conditional betas $\hat{\beta}^i_t = \hat{\beta}^i_0 + \hat{\beta}^i_1 cay_t$ can be used in the cross-sectional regressions (at each date t) to estimate η_t and λ_t. The latter can be used to obtain either the unconditional averages of the risk premium and the zero-beta rate, or can be projected on the conditioning information set. Average of the conditional pricing errors for each asset are then given straightforwardly as

$$u^i = E(R^{ei}_{t+1}) - E\left(\hat{\eta}_t + \hat{\lambda}_t \hat{\beta}^i_t\right).$$

Both of these are valid approaches to testing a conditional factor model. However, the latter approach has more power, since it imposes additional restrictions on the dynamics of conditional betas and expected returns. A simple way to illustrate the dramatic differences between the two approaches is to compare the average pricing errors. Figure 9 plots the average returns on the 25 portfolios formed on size and book-to-market (see Appendix for data description) against the average returns predicted by four empirical models: the unconditional consumption CAPM, the unconditional scaled-factor specification of conditional

CCAPM in (F-1), the three-factor model of Fama and French (1993), and the conditional specification of conditional CCAPM in (F-2). The unconditional consumption CAPM (top left panel) is well-known to have virtually no explanatory power for the average returns of the Fama-French portfolios. In contrast, the scaled CCAPM of Lettau and Ludvigson (2001b) does a relatively good job at lining up the predicted mean returns against the actual ones (top right panel), reducing the square root of the average (squared) pricing errors (alphas) by a third compared to the unconditional CCAPM (from 0.6% to 0.4% for quarterly returns). This performance is comparable to the well-known ability of the Fama-French portfolio-based model to explain the cross-section of value and size-sorted portfolios (bottom left panel). However, imposing the conditional restrictions (F-2) eliminates virtually all of the advantage of the conditional model over the unconditional one. The conditional model generates very little dispersion in the predicted average returns (bottom right panel), thus failing to explain any of the variation in the observed mean portfolio returns.

G Consumption of stockholders

The fact that not all households participate in the equity market suggests an alternative interpretation of the composition effect, i.e. the tendency of the conditional covariances of stock returns with aggregate consumption growth to decline as a the contribution of financial wealth to consumption decreases. Since equity, which represents a large fraction of total financial wealth, is concentrated in the hands of stockholders, their consumption is likely to be disproportionately effected by stock market fluctuations, relative to the consumption of non-stockholders. Thus, a decrease in the value of equity would reduce the stockholders' relative share of aggregate consumption, and therefore reduce the sensitivity of aggregate consumption to the fluctuations in stock market wealth. Indeed, consistent with this interpretation, Malloy, Moskowitz, and Vissing-Jørgensen (2005) use household-level data from the Consumer Expenditure Survey (CEX) to show that the consumption-wealth residual *cay* is highly negatively correlated with the time-varying share of stockholders' consumption in the aggregate consumption.

The direct implication of this interpretation of the composition effect is that the canonical asset pricing relation 2 is misspecified as long as the measure of aggregate consumption includes all households rather than just those that are marginal in the asset market of interests (i.e., stockholders in the case where stock returns are the test assets). In order to verify whether my conclusions are robust to this type of misspecification I use the data from Malloy, Moskowitz, and Vissing-Jørgensen (2005) to test the conditional CCAPM. Their measure of quarterly stockholder consumption growth is available at a monthly frequency (i.e., for overlapping quarterly growth rates), but for a shorter time period (03.1983 - 11.2004) than the aggregate data used elsewhere in the paper. As a benchmark comparison, I also use the monthly series of quarterly aggregate consumption growth based on the NIPA data constructed by Malloy, Moskowitz, and Vissing-Jørgensen (2005) for the same time period. I construct the monthly analog of the *cay* variable as a cointegrating residual of monthly series for aggregate consumption, stock market wealth, and labor income; the resulting series has very similar properties to the *cay* variable of Lettau and Ludvigson (2001b).

As before, I estimate conditional expected returns and conditional covariances of returns with consumption growth jointly, by selecting kernel bandwidth so as to minimize the conditional pricing errors for the cross-section of portfolio returns. The evidence in table IX shows that if differences between "good" and "bad" states in conditional covariances of returns and consumption growth are measured the same way as above, the composition effect is statistically detectable for stockholder consumption, at least for the large growth portfolio, while the differences are not statistically significant for the NIPA aggregate consumption growth measure over the same sample period (however, in both cases statistical significance is somewhat sensitive to the choice of "high" and "low" states. Moreover, the magnitudes of differences in covariances between high and low states are greater for stockholder consumption than for aggregate consumption, which is likely due to the fact that levels of covariances are proportionally higher for latter than for the former. For the Value minus Growth portfolio returns, in both cases the difference is positive and statistically significant for the small portfolios, consistent with the conditional CCAPM of the value effect, but not for the large portfolios. As before, however, the differences in expected returns on these portfolios are negative, albeit not statistically significantly.

In terms of the average pricing errors, the consumption CCAPM, both unconditional and conditional, that uses stockholder consumption does appear to perform somewhat better than the model with aggregate consumption estimated over the same sample period. Table X displays the average pricing errors for the two sets of models, using either *cay* or the stock market wealth-consumption ratio *ac*. While all of the versions of the CCAPM that uses NIPA aggregate consumption growth have large and highly statistically significant pricing errors on the Small Value minus Small Growth and Small Growth minus Large Growth portfolios, for the stockholder consumption CAPM these pricing errors are smaller (although still substantial) and not statistically different from zero, with the exception of the conditional CCAPM using *ac* where it is significant. However, for the stockholder consumption CAPM the Small Value minus Large Value portfolio has a large (2 % per quarter) and statistically significant pricing error, either unconditionally or when *cay* is used as the conditioning variable. Moreover, the lack of statistical significance might be in part attributed to the short sample, which makes estimated pricing errors highly imprecise, especially in the nonparametric setting. Overall, there is evidence that using stockholder consumption to measure risk in asset returns improves the performance of a canonical consumption-based asset pricing model, but does not fully explain the cross section of equity returns. This conclusion is consistent with the evidence documented above that high average return portfolios (e.g. small value) do not seem to have higher conditional expected returns than low average return portfolios at times their risk measured by conditional covariance with consumption growth is higher.

Table VIII: **Sensitivity of conditional moments to conditioning variables**
Regression slope coefficients of portfolio excess returns and their ex-post covariances with consumption growth on the lagged conditioning variable. Standard errors are given in the parentheses.

***ac* - monthly data**

	$E(R^i)$	R^2	Cov^i	R^2
Growth	-0.77	0.01	0.51	0.00
	(0.45)		(2.32)	
Neutral	-0.57	0.01	0.68	0.00
	(0.34)		(1.47)	
Value	-0.64	0.01	0.64	0.00
	(0.34)		(1.44)	

***cay* - quarterly data**

	$E(R^i)$	R^2	Cov^i	R^2
Growth	1.35	0.03	-4.47	0.01
	(0.42)		(3.74)	
Neutral	1.11	0.03	-4.29	0.02
	(0.35)		(2.99)	
Value	1.03	0.03	-4.60	0.02
	(0.38)		(3.31)	

***cay* - quarterly data up to 2003**

	$E(R^i)$	R^2	Cov^i	R^2
Growth	2.35	0.07	-1.29	0.00
	(0.57)		(9.54)	
Neutral	1.87	0.07	1.22	0.00
	(0.47)		(8.12)	
Value	1.79	0.05	2.46	0.00
	(0.50)		(8.32)	

***yc* - quarterly data**

	$E(R^i)$	R^2	Cov^i	R^2
Growth	-0.25	0.01	0.11	0.00
	(0.19)		(19.97)	
Neutral	-0.15	0.00	0.44	0.00
	(0.17)		(17.61)	
Value	-0.18	0.00	0.70	0.00
	(0.21)		(20.72)	

Table IX: **Differences in conditional moments of portfolio returns - stockholders** Bootstrap tests of differences in conditional covariances of returns on the benchmark portfolios with stockholder consumption growth and differences in conditional mean excess returns, estimated jointly using $z = cay$ as the conditioning variable, where $z^L = -0.0174$ and $z^H = 0.02$ correspond to the 10th and 90th percentiles of the distribution of cay (in the entire sample IV.1952 - IV.2008), respectively. The test statistics are differences in point estimates of conditional moments evaluated at these two states for each test portfolio. The p-values for the one-sided tests reported in the parentheses are computed using the bootstrap distributions of the corresponding test statistics centered at zero. Conditional means and covariances are estimated jointly using monthly observations of quarterly consumption growth measures based on, alternatively, the NIPA aggregate data, or the stockholder consumption data from the CEX, both for the period 03.1983 - 11.2004 (see Malloy, Moskowitz, and Vissing-Jørgensen (2005) for detailed description).

Panel A: NIPA

	$E(R\|z^H) - E(R\|z^L)$	$100 \times (cov(R, \Delta c\|z^H) - cov(R, \Delta c\|z^L))$
Small Growth	1.75	-1.82
	(0.25)	(0.06)
Small Value	0.76	-0.12
	(0.37)	(0.45)
Large Growth	2.64	-1.13
	(0.06)	(0.09)
Large Value	1.14	-0.35
	(0.25)	(0.30)
Small Value minus Growth	-0.99	1.69
	(0.33)	(0.04)
Large Value minus Growth	-1.50	0.79
	(0.17)	(0.08)

Panel B: CEX stockholders

	$E(R\|z^H) - E(R\|z^L)$	$100 \times (cov(R, \Delta c\|z^H) - cov(R, \Delta c\|z^L))$
Small Growth	2.14	-9.73
	(0.16)	(0.06)
Small Value	0.33	-3.92
	(0.41)	(0.21)
Large Growth	2.83	-7.93
	(0.03)	(0.05)
Large Value	0.88	-5.35
	(0.25)	(0.07)
Small Value minus Growth	-1.81	5.82
	(0.16)	(0.05)
Large Value minus Growth	-1.95	2.58
	(0.07)	(0.16)

Table X: **Average pricing errors: stockholder consumption**

CCAPM estimated using monthly observations of quarterly consumption growth measures based on, alternatively, the NIPA aggregate data, or the stockholder consumption data from the CEX, both for the period 03.1983 - 11.2004 (see Malloy, Moskowitz, and Vissing-Jørgensen (2005) for detailed description).

P-values for the test that individual pricing errors are equal to zero given in the parentheses are computed using (semi)parametric stationary bootstrap with 10000 replications.

Model	SV-SG	SG-LG	SV-LV	LV-LG
unconditional CCAPM (NIPA)	3.43	-3.16	-0.13	0.40
	(0.00)	(0.00)	(0.35)	(0.31)
unconditional CCAPM (stockholders)	1.84	1.11	2.26	0.69
	(0.10)	(0.16)	(0.01)	(0.22)
CCAPM (NIPA) scaled with *cay*	3.08	-3.20	-0.30	0.18
	(0.00)	(0.00)	(0.17)	(0.52)
CCAPM (stockholders) scaled with *cay*	1.23	-1.15	1.41	-1.33
	(0.15)	(0.05)	(0.06)	(0.00)
CCAPM (NIPA) scaled with *ac*	-0.33	-1.53	-0.83	-1.03
	(0.03)	(0.27)	(0.10)	(0.09)
CCAPM (stockholders) scaled with *ac*	0.62	-0.39	-0.27	0.51
	(0.62)	(0.45)	(0.07)	(0.23)
conditional CCAPM (NIPA) with *cay*	3.45	-3.15	-0.12	0.42
	(0.00)	(0.00)	(0.44)	(0.36)
conditional CCAPM (stockholders) with *cay*	1.88	1.01	2.15	0.74
	(0.13)	(0.24)	(0.05)	(0.28)
conditional CCAPM (NIPA) with *ac*	3.29	-2.96	-0.10	0.43
	(0.00)	(0.00)	(0.46)	(0.21)
conditional CCAPM (stockholders) with *ac*	2.19	0.41	1.74	0.86
	(0.05)	(0.60)	(0.21)	(0.14)
average returns	2.28	-0.79	1.16	0.34

Figure 9: **Fama-MacBeth regressions**

Each panel plots the average excess returns on the 25 portfolios sorted on size (S, 1 = low, 5 = high) and book-to-market (B, 1 = low, 5 = high)), against the average returns predicted by one of the four models:
unconditional consumption CAPM, $E(R^{ei}_{t+1}) = \eta + \lambda\beta^i_{\Delta C_{t+1}}$;
Fama-French three-factor model, $E(R^{ei}_{t+1}) = \eta + \lambda_M\beta^i_{RMRF} + \lambda_S\beta^i_{SMB} + \lambda_H\beta^i_{HML}$;
unconditional version of the conditional consumption CAPM scaled with cay,

$$E(R^{ei}_{t+1}) = \eta_0 + \eta_1\, cay_t + \lambda_0\beta^i_{\Delta C_{t+1}} + \lambda_1\beta^i_{\Delta C_{t+1}\times cay_t};$$

conditional consumption CAPM using cay as the conditioning variable:

$$E(R^{ei}_{t+1}) = E\left(\eta_t + \lambda_t\beta^i_t\right), \text{ where } \beta^i_t = b^i_0 + b^i_1\, cay_t.$$