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Examination of the Stoney Equation and a Practical Determination of the Error in Residual Film Stress Measurements

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Disciplines
Nanoscience and Nanotechnology

Comments
The purpose of this document is to show how common practical errors in substrate thickness and film thickness impact the resulting residual stress measurements using an analysis of the Stoney Equation.

Goal:

To demonstrate a mathematical analysis of how the errors in substrate thickness, film thickness, and stress and material distribution manifest in uncertainties in film stress as calculated by the Stoney equation.

Derivation:

The Stoney Equation can be written as:

$$\sigma_s = \frac{E_s h_s^2}{6h_f(1-\nu_s)R}$$

Where:
σ_s=Stress on the substrate
h_f=Film thickness
h_s=Substrate thickness
R=Radius of curvature
E_s=Young's Modulus
ν_s=Poisson ratio

Realizing that a large amount of these values are relatively constant within the use of one type of deposition metal, some of the terms can be grouped into a singular term:

$$\beta = \frac{E_s}{6(1-\nu_s)} = const.$$
$$\Rightarrow$$
$$\sigma_s = \beta \frac{h_s^2}{h_f R}$$

Where, in this equation, the only terms that contribute to the error associated with the film stress are the film thickness and the substrate thickness.

Knowing that for general cases where T=T(X,Y,Z) has the form:

$$T = \frac{X * Y}{Z}$$

The error is given by

$$\delta T = |T| * \sqrt{\left(\frac{\delta X}{X}\right)^2 + \left(\frac{\delta Y}{Y}\right)^2 + \left(\frac{\delta Z}{Z}\right)^2}$$

And that for an equation of the form

$$F = X^n$$

The error is given by

$$\delta F = |n| * \left(\frac{\delta X}{X}\right) * |F|$$

And that for an equation of the form

$$D = \alpha X$$

The error is given by

$$\delta D = |D| * \frac{\delta X}{|X|}$$

It is found that the error propagation for the Stoney equation can be written as:

$$\delta \sigma_s = |\sigma_s| * \sqrt{\left(\frac{\delta h_f}{h_f}\right)^2 + \left(\frac{\delta R}{R}\right)^2 + 4\left(\frac{\delta h_s}{h_s}\right)^2}$$

It should then follow that whatever error calculated in regards to the stress on the substrate should fall within the error bars governed by this equation, with most weight being given to the error relating to the substrate thickness.

The rrror in film stress is equal to the film stress multiplied by the compounding errors that come from the film thickness, substrate thickness, and radius of curvature.

Considering the fact that the substrate thickness, Poisson ratio, and Young's modulus are known values, the error in the film stress can be further simplified to an equation that only has values relating to the film as its variables. Towards this end, the following known values for silicon are used (obtained via https://www.memsnet.org/material/siliconsibulk/):

ν_s=.278 (Poisson's Ratio)
E_s=163 GPa (Young's Modulus)
h_s=525 µm (Substrate Thickness)
δh_s=25 µm (Upper limit for error in Substrate Thickness)
ß=37.63 GPa (Combination of Constant values [described above])
Using these known values, the error in the film stress can be rewritten as follows:

$$\delta\sigma_s = |\sigma_s| * \sqrt{\left(\frac{\delta h_f}{h_f}\right)^2 + \left(\frac{\delta R}{R}\right)^2 + \frac{4}{441}}$$

A Numerical Example

From the data set of copper deposited at 400W with 7 mTorr gas, let the following values be assumed:
h_f=324 nm=3.24x10^{-7} m (Film Thickness)
δh_f=38 nm=.38x10^{-7} m (Error in Film Thickness)
R=55.45 m (Radius of Curvature)
δR=5.76 m (Error in Radius of Curvature)
σ_s=461 MPa=4.61x10^8 Pa (Stress on the Substrate)
From which we obtain the following:

$$\delta\sigma_s = |4.61x10^8| * \sqrt{\left(\frac{.38x10^{-7}}{3.24x10^{-7}}\right)^2 + \left(\frac{55.45}{5.76}\right)^2 + \frac{4}{441}} = 7.54x10^7$$

Which shows that the associated error in the film stress should fall within 75.4 MPa about this stress point. This implies that for each film stress point, there is an associated error in said film stress, and these points should be within each other's error range.