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Information Asymmetry, Information Precision, and the Cost of Capital

Abstract

This paper examines the relation between information differences across investors (i.e., information asymmetry) and the cost of capital and establishes that with perfect competition information asymmetry makes no difference. Instead, a firm's cost of capital is governed solely by the average precision of investors' information. With imperfect competition, however, information asymmetry affects the cost of capital even after controlling for investors' average precision. In other words, the capital market's degree of competition plays a critical role for the relation between information asymmetry and the cost of capital. This point is important to empirical research in finance and accounting.

Keywords

cost of capital, disclosure, information risk, asset pricing

Disciplines

Accounting

Information Asymmetry, Information Precision, and the Cost of Capital*

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Abstract

The consequences of information differences across investors in capital markets are still much debated. This paper examines the relation between information differences across investors and the cost of capital. Our analysis makes three salient points. First, in models of perfect competition, information differences across investors affect a firm's cost of capital through investors' average information precision, and not information asymmetry *per se*. Second, the average information precision effect is unlikely to diversify away when there exist many firms whose cash flows covary. Finally, in models of imperfect competition information asymmetry affects the willingness to supply liquidity; this, in turn, affects a firm's cost of capital. Thus, the precision of the information and information asymmetry have separate and distinct effects on the cost of capital. Failure to distinguish between them jeopardizes our understanding of the link between information and the cost of capital.

JEL classification: G12, G14, G31, M41

Key Words: Cost of capital, Disclosure, Information risk, Asset pricing

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1 Introduction

Information differences across investors (or groups of investors) have been a long-standing concern to securities regulators and at the core of U.S. disclosure regulation (e.g., Loss, 1983; Loss and Seligman, 2001). For example, the Securities and Exchange Commission (SEC) recently enacted Regulation Fair Disclosure (Reg FD), which intends to equalize information across investors by preventing companies from making disclosures to select groups of investors and analysts. The SEC (2000) argued that selective disclosure allows “those who were privy to the information beforehand...to make a profit or avoid a loss at the expense of those kept in the dark,” and that this practice leads to a loss in investor confidence (see also Levitt, 1998). Particularly, small investors might be unwilling to invest if they fear that insiders gain at their expense; this, in turn, increases firms’ cost of capital to the extent that the risk in the economy has to be borne by fewer investors. Similarly, investors might demand a return premium for investing in the capital markets which exhibit substantial information asymmetry. Critics of Reg FD, however, argued that it could stifle corporate disclosure and in turn increase firms’ cost of capital (AIMR, 2001). For example, SEC Commissioner Unger (2000) voted against the proposed regulation because of concerns that it would “most likely reduce the amount of information available to investors...and the quality of the information that would be produced.”

As this example shows, the consequences of information asymmetry in capital markets and, in particular, its relation to the amount and precision of information available investors and the cost of capital, are still much debated. This paper examines these issues. We show that the precision of the information and information asymmetry have separate and distinct effects on the cost of capital. Thus, it is im-

portant to distinguish between these concepts.

The issue of whether and how information differences across investors affects prices and the cost of capital cannot be addressed in conventional models of asset pricing, such as the Capital Asset Pricing Model (CAPM), because these models generally assume investors have homogeneous beliefs. Prior studies, however, have developed models of capital market equilibria where investors have heterogeneous information. For example, Merton (1987) presents a model in which some investors are less informed than others. In particular, uninformed investors are not aware of the existence of some firms. Merton (1987) argues that information that increases uninformed investors' awareness can lower the cost of capital of these firms.

Recently, O'Hara (2003) and Easley and O'Hara (2004) develop asset pricing models that suggest that "information asymmetry" affects prices and is a determinant of firms' cost of capital. These papers argue that because of information asymmetries, differentially informed traders will choose to hold different portfolios of securities. As a result, they willingly bear "idiosyncratic risk." Less informed traders recognize they are at an information disadvantage and will try to hold assets where their disadvantage is less. This drives down the price of securities with high degrees of asymmetry, thereby increasing the cost of capital for these firms. Easley and O'Hara (2004) also argue that this effect constitutes "information risk"; as such, it is priced in a capital market equilibrium with many assets, i.e., the effect is not diversifiable.

Our paper explores the relation between information differences across investors and the cost of capital by developing a model where investors make wealth allocation decisions amongst securities based on their available information. These allocations, in turn, determine the equilibrium prices of securities and their respective costs of capital. Similar to Admati (1985), our model allows for multiple securities whose cash

flows are correlated. We analyze how the distribution of information across investors affects the cost of capital.

We show that when capital markets are characterized by perfect competition among investors (as most prior models assume), equilibrium prices are a function of two features of the economy's information structure: 1) individual investors' precision-weighted, average assessment of firms' expected end-of-period cash flows; and 2) a discount for the risk investors associate with holding firms' shares that depends on investors' average, assessed precision matrix (i.e., the inverse of the covariance matrix) of the distribution of firms' end-of-period cash flows. In other words, investors' average assessed precision is a key determinant of the expected return on a firm's stock price, and therefore on its cost of capital. The extent to which investors' precision matrices deviate from this average, however, does *not* matter. In particular, information asymmetry across investors does not affect the discount for risk, holding the average assessed precision constant. In other words, investors are not concerned about the precision matrices of other investors, which usually is the primary concern with adverse selection (Akerlof, 1970; Grossman, 1981).

One special setting for our model arises when investors have homogeneous information, in which case our results are equivalent to the Capital Asset Pricing Model (CAPM). Another special setting is one in which the distribution of all firms' cash flows are independent. In this setting, each firm's pricing equation is independent of the information of other firms. This latter setting facilitates comparing our results to the large "noisy rational expectations" literature, which generally examines asset pricing in single-firm settings. Finally, we use our model to examine two special settings where investors have heterogenous information: one where investors are "diversely" informed, as in Kim and Verrecchia (1991), and one where investors are

“asymmetrically informed,” as in Easley and O’Hara (2004).

We show that Easley and O’Hara’s (2004) interpretation of their model and the role of information asymmetry is not correct. In particular, we show that price is discounted because “less informed” investors resolve less uncertainty in the economy about the end-of-period cash flow. In other words, the distribution of information changes the *average* precision of information held by investors, and lowering the precision raises the cost of capital. Importantly, price is not discounted relative to the expected end-of-period cash flow because a “less informed” group of investors faces an adverse selection problem, and thus must price-protect itself. In fact, we show that increasing the degree of information asymmetry between investors can reduce the cost of capital, provided that the average precision increases. Thus, in their model, the dissemination of more information to more investors drives down the cost of capital because it increases the average precision of investors, not because it reduces information asymmetries as they claim.

Next, we extend our model to an economy where firms’ cash flows are correlated. This extension allows us to examine the distinction between diversifiable and non-diversifiable risk. In a large economy, the cost of capital is determined by average assessed *precision-matrix* of the economy and not the average assessed precision for a firm. These results extend the “estimation risk literature” in finance, which assumes all investors have homogeneous information. Extending the homogenous information results in Lambert et al. (2006), we show that higher quality disclosure can lower the assessed covariance between a firm’s cash flows and those of other firms, and thereby lower its cost of capital. We also show that, in a large economy, the effect documented in Easley and O’Hara (2004) is diversifiable.¹

¹ The issue of diversifiability is also discussed in Hughes et al. (2005). But again, they cast the effects in their model as being driven by information asymmetry, when in fact they are precision

Finally, we discuss how information asymmetry affects prices, and hence the cost of capital, in models of imperfect competition (e.g., Kyle, 1985; Diamond and Verrecchia, 1991). In contrast to models of perfect competition, we find asymmetric information can affect to the cost of capital when there is imperfect competition. Broadly stated, in models of imperfect competition information asymmetry affects the willingness of “large” traders to supply liquidity. As liquidity declines, a firm’s extant risk is borne by fewer investors. This, in turn, increases a firm’s cost of capital. As we discuss in greater detail below, this suggests that the information asymmetry effect is separate from (and in addition to) the precision effect discussed above.

As the debate about Regulation FD illustrates, there can be situations where a policy is designed to decrease information asymmetry, but whose unintended consequence is to decrease simultaneously the precision of publicly available information (or vice versa). Thus, understanding the distinction and potential tradeoffs between information-asymmetry effects and precision effects is important. In real capital markets, both effects are likely to be present; this might explain why empirical studies point to an increase (not decrease) in firms’ cost of capital subsequent to Reg FD (e.g., Gomes et al., 2006).

The remainder of this paper is organized as follows. In section 2, we develop the model and derive pricing equations as a function of investors’ information structures. In section 3, we analyze specific information structures commonly used in the literature to derive closed-form solutions to the pricing equations. Section 4 examines the issue of diversifiability by allowing the number of firms and investors to grow large. In section 5, we discuss models of imperfect competition. The last section summarizes the paper and offers suggestions for future research.

effects.

2 Market prices in a multi-firm economy

In this section we introduce a classical, one-period capital market setting in which N investors, indexed by $i = 1, 2, \dots, N$, allocate their wealth over the shares of J firms and a risk-free asset (e.g., bonds). These settings have a long tradition in finance, and include traditional asset pricing models such as the CAPM (see Fama, 1976), as well as “rational expectations” (RE) models (which we discuss in more detail below). Let $\tilde{\mathbf{V}} = \{\tilde{V}_1, \dots, \tilde{V}_j, \dots, \tilde{V}_J\}'$ denote the $J \times 1$ vector of end-of-period cash flows generated by the firms, and $\mathbf{P} = \{P_1, \dots, P_j, \dots, P_J\}'$ the $J \times 1$ vector of beginning-of-period market values, or prices, associated with those firms.² Each investor has information, represented by Φ_i , upon which he forms beliefs about firms’ end-of-period cash flows. We allow for the possibility that Φ_i includes both public and private information, as well as information gleaned from the vector of market (equilibrium) prices, \mathbf{P} . We assume that investors assess the joint distribution of firms’ cash flows to have a multivariate normal distribution based on Φ_i . Specifically, let $\mathbf{E}_i[\tilde{\mathbf{V}}|\Phi_i]$ represent investor i ’s $J \times 1$ vector of firms’ expected values, and \mathbf{Cov}_i investor i ’s $J \times J$ covariance matrix of firms’ end-of-period cash flows, conditional on investor i ’s information, Φ_i . Below, we discuss in greater detail the specific information that underlies investors’ beliefs. For convenience, we represent the precision matrix for investor i ’s beliefs by $\mathbf{\Pi}_i$, where $\mathbf{\Pi}_i$ is the inverse of covariance matrix investor i associates with firms’ end-of-period cash flows: that is, $\mathbf{\Pi}_i = \mathbf{Cov}_i^{-1}$. Finally, let $\mathbf{\Pi}_0 = \sum_{i=1}^N \mathbf{\Pi}_i$ represent the sum of investors’ precision matrices.

We assume that each of the N investors has a negative exponential utility function with a risk tolerance of parameter τ , and chooses his portfolio to maximize the

² Throughout our analysis we denote a random variable by a tilde ($\tilde{}$), and the realization of a random variable and/or a fixed parameter with no tilde. In addition, we put vectors and matrices in bold.

expected utility of his end-of-period wealth. Let $\hat{\mathbf{X}}_i = \{\hat{X}_{i1}, \dots, \hat{X}_{ij}, \dots, \hat{X}_{iJ}\}'$ represent the $J \times 1$ vector of investor i 's endowment of firm shares, and \hat{B}_i his endowment of a risk-free asset (e.g., bonds). Similarly, let $\mathbf{X}_i = \{X_{i1}, \dots, X_{ij}, \dots, X_{iJ}\}'$ represent the $J \times 1$ vector of investor i 's demand for firm shares, and B_i the units of the risk-free asset he chooses to hold. We assume that the risk-free asset has an initial price of \$1, and yields $1 + R_f$ at the end of the period. Finally, for convenience let $\mathbf{X}_0 = \{X_{01}, \dots, X_{0j}, \dots, X_{0J}\}'$ represent the $J \times 1$ vector of total supply of firm shares in the economy, where $X_{0j} = \sum_{i=1}^N X_{ij} = \sum_{i=1}^N \hat{X}_{ij}$.

Given this structure, investor i 's expected utility can be expressed as a certainty equivalent, which has the following special structure:

$$\begin{aligned} & \text{Certainty Equivalent of Investor } i\text{'s Expected Utility of End-of-Period Wealth} \\ = & E[\text{End-of-Period Wealth}|\Phi_i] - \frac{1}{2\tau} \text{Var}[\text{End-of-Period Wealth}|\Phi_i], \end{aligned}$$

where

$$\begin{aligned} E[\text{End-of-Period Wealth}|\Phi_i] &= E_i[B_i(1 + R_f) + \mathbf{X}_i\tilde{\mathbf{V}}], \text{ and} \\ \text{Var}[\text{End-of-Period Wealth}|\Phi_i] &= \text{Var}_i[B_i(1 + R_f) + \mathbf{X}_i\tilde{\mathbf{V}}] \\ &= \mathbf{X}_i\mathbf{Cov}_i\mathbf{X}_i'. \end{aligned}$$

Thus, we can reduce each investor's objective function to choosing \mathbf{X}_i and B_i such that

$$\max_{\mathbf{X}_i, B_i} E_i[B_i(1 + R_f) + \mathbf{X}_i\tilde{\mathbf{V}}|\Phi_i] - \frac{1}{2\tau}\mathbf{X}_i\mathbf{Cov}_i\mathbf{X}_i',$$

subject to

$$\mathbf{X}_i\mathbf{P}' + B_i = \hat{\mathbf{X}}_i\mathbf{P}' + \hat{B}_i.$$

Solving the budget constraint for the risk-free asset implies setting $B_i = \hat{\mathbf{X}}_i\mathbf{P}' + \hat{B}_i -$

$\mathbf{X}_i \mathbf{P}'$; substituting this expression into the objective function above implies

$$\max_{\mathbf{X}_i} \mathbf{X}_i E_i [\tilde{\mathbf{V}}|\Phi_i] + (\hat{\mathbf{X}}_i \mathbf{P}' + \hat{B}_i - \mathbf{X}_i \mathbf{P}') (1 + R_f) - \frac{1}{2\tau} \mathbf{X}_i \mathbf{Cov}_i \mathbf{X}_i'. \quad (1)$$

Computing the first-order condition for investor i 's demand for firms shares, \mathbf{X}_i , yields

$$\mathbf{X}_i = \tau \mathbf{\Pi}_i \left(E_i [\tilde{\mathbf{V}}|\Phi_i] - \mathbf{P} (1 + R_f) \right). \quad (2)$$

To derive an equilibrium set of prices, we sum the demands of all investors as expressed in eqn. (2), and set them equal to the aggregate supply of shares in each firm. This yields the following expression for the vector of equilibrium prices.

Proposition 1. *The equilibrium price depends on investors' information through the following equation*

$$\mathbf{P} = \frac{(\mathbf{\Pi}_0)^{-1} \sum_{i=1}^N \mathbf{\Pi}_i E_i [\tilde{\mathbf{V}}|\Phi_i] - (\frac{1}{N} \mathbf{\Pi}_0)^{-1} \frac{\mathbf{X}_0}{N\tau}}{1 + R_f}. \quad (3)$$

While not a closed-form solution per se, eqn. (3) is useful in illustrating the aggregation properties of price. Specifically, the vector of market prices, \mathbf{P} , is equal to: the vector of investors' precision-weighted, average assessment of firms' expected end-of-period cash flows, $(\mathbf{\Pi}_0)^{-1} \sum_{i=1}^N \mathbf{\Pi}_i E_i [\tilde{\mathbf{V}}|\Phi_i]$; minus a discount that results from the risk the market associates with holding firms' shares, $(\frac{1}{N} \mathbf{\Pi}_0)^{-1} \frac{\mathbf{X}_0}{N\tau}$; where both the average assessment of expected values and risk are discounted back to the beginning of the period at the risk-free rate, $1 + R_f$. Investors' expected values are weighted by their precisions, $\mathbf{\Pi}_i$, because investors' demands are proportionate to their precisions, *ceteris paribus*. The risk associated with holding firms' shares, $(\frac{1}{N} \mathbf{\Pi}_0)^{-1} \frac{\mathbf{X}_0}{N\tau}$, is a function of three elements: 1) the inverse of investors' average precision matrix for the distribution of firms' end-of-period cash flows, $(\frac{1}{N} \mathbf{\Pi}_0)^{-1}$, as an expression of the (average) uncertainty associated with holding firms' shares; 2) investors' average

holding of the total supply of firm shares, $\frac{\mathbf{x}_0}{N}$; and 3) the inverse of investors' tolerance for risk, τ . The precision matrix for investor i is simply the inverse of the covariance matrix he associates with the cash flows of all firms based on his information Φ_i . The result of inverting each investor's covariance matrix, averaging these inverses, and then inverting the average provides a matrix that is analogous to a covariance matrix. Because of the nonlinearities involved in matrix inversion, however, this covariance matrix is not the simple average of investors' individual covariance matrices.

Eqn. (3) does not necessarily represent a closed-form expression for price because when price aggregates information about firms' cash flows, price can also be on the right-hand-side of the equation as an element of Φ_i . That is, price can be a conditioning variable in the expectation and precision terms through investors' demand for firm shares. The notion that investors condition their expectations over price, and thereby glean additional information about firms' cash flows through the price aggregation process, is the central tenet of the so-called RE-literature (e.g., Grossman and Stiglitz, 1980; Hellwig, 1980; Diamond and Verrecchia, 1981; Admati, 1985, etc.) In general, a RE-analysis requires that eqns. (1) and (3) be solved simultaneously, along with the market-clearing condition that investors' demand for firm shares equals the aggregate supply of those shares. For now, and primarily in the interests of motivating the discussion, we abstract from this issue; starting with Section 2 we explicitly study market prices in the context of a RE-analysis.

To start, consider a circumstance in which all investors have homogeneous information, which we represent by Φ_H : that is, $\Phi_i = \Phi_H$ for all i . In addition, let $\mathbf{\Pi}_H$ represent investors' homogeneous precision matrix, and \mathbf{Cov}_H its inverse: that is, $\mathbf{\Pi}_i = \mathbf{\Pi}_H$ and $\mathbf{Cov}_H = \mathbf{\Pi}_H^{-1}$ for all i .

Corollary 1. *When investors have the same information, the pricing equation reduces to*

$$\mathbf{P} = \frac{E[\tilde{\mathbf{V}}|\Phi_H] - \mathbf{\Pi}_H^{-1} \frac{\mathbf{X}_0}{N\tau}}{1 + R_f}, \text{ or } \mathbf{P} = \frac{E[\tilde{\mathbf{V}}|\Phi_H] - \mathbf{Cov}_H \frac{\mathbf{X}_0}{N\tau}}{1 + R_f}. \quad (4)$$

When the vector of the supply of firm shares, \mathbf{X}_0 , is fixed, the expression for prices in eqn. (4) is identical to that found in the Capital Asset Pricing Model (CAPM) (e.g., Sharpe, 1964; Lintner, 1965). In particular, eqn. (4) shows that the price of each firm is equal to its expected end-of-period cash flow, minus a discount for the risk associated with holding that firm's shares. The discount for holding firm j 's shares, say, is $\sum_{k=1}^N Cov[\tilde{V}_j, \tilde{V}_k] \frac{X_{0k}}{N\tau}$, where $\sum_{k=1}^N Cov[\tilde{V}_j, \tilde{V}_k]$ represents firm j 's contribution to the aggregate uncertainty investors associate with the cash flows of all firms, $Var[\sum_{k=1}^N \tilde{V}_k]$.

Note that when all investors have the same information, their beliefs are homogeneous; hence, there is no additional information to glean from price. This implies that Corollary 1 is a closed-form solution for prices and a RE-analysis is unnecessary. This will not be true, however, of our next two results.

Another special case arises when investors have diverse information, but of homogeneous precision (i.e., $\mathbf{\Pi}_i = \mathbf{\Pi}_H$ and $\mathbf{Cov}_i = \mathbf{Cov}_H$ for all i). In this circumstance, eqn. (3) also simplifies.

Corollary 2. *If all investors have the same precision of information, but the information itself is diverse,*

$$\mathbf{P} = \frac{\frac{1}{N} \sum_{i=1}^N E_i[\tilde{\mathbf{V}}|\Phi_i] - \mathbf{\Pi}_H^{-1} \frac{\mathbf{X}_0}{N\tau}}{1 + R_f} = \frac{\frac{1}{N} \sum_{i=1}^N E_i[\tilde{\mathbf{V}}|\Phi_i] - \mathbf{Cov}_H \frac{\mathbf{X}_0}{N\tau}}{1 + R_f}. \quad (5)$$

Because investors' precision matrices are homogeneous, the averaging process involving precision matrices in eqn. (3) disappears. Each price is now a simple average of each investor's expected cash flow (based on his information) less a reduction for

risk that depends on the assessed covariance matrix of cash flows. The assessed covariance matrix is homogeneous across investors. In other words, information affects investors' conditional expected values, but not their conditional covariances. If one were to ignore the possibility of investors conditioning their expectations on price by assuming that there was too much “noise” in prices, then Corollary 2 is analogous to the Lintner's (1969) heterogeneous information version of the CAPM.

Finally, we consider diverse information in conjunction with diverse precision. A point of emphasis in the next section is that the vast majority of research on investors with diverse information, heterogeneous precision, and “rational expectations” has been couched in the context of a single-, and not multi-, firm setting.³ As we show below, single-firm results will hold in a multi-firm economy under the additional assumption that all cash flows are independently distributed across firms. Therefore, to facilitate comparisons with prior work, we examine briefly the special case where each investor assesses firms' cash flows to be distributed independently. When investors assess that cash flows are independent, their demand functions in eqn. (2) reduce to

$$X_{ij} = \tau \pi_i [\tilde{V}_j] \left(E_i [\tilde{V}_j | \Phi_i] - (1 + R_f) P_j \right), \quad (6)$$

where $\pi_i [\tilde{V}_j]$ denotes the precision (i.e., the reciprocal of the variance) investor i associates with the end-of-period cash flow of firm j based on his information, Φ_i . Note that, *ceteris paribus*, investors who assess the riskiness of cash flows to be lower (higher) will purchase more (fewer) shares in the firm. Similarly, investors with higher (lower) risk tolerances will purchase more (fewer) shares. Let $\pi_0 [\tilde{V}_j] = \sum_{i=1}^N \pi_i [\tilde{V}_j]$ represent the sum of the precisions investors associate with the cash flow of firm j . Extending the expression for prices in eqn. (3) to a circumstance where cash flows are independent yields the following result.

³ An exception is Admati (1985).

Corollary 3. *When investors assess firms' cash flows to be distributed independently, the price for firm j , P_j , will be equal to*

$$P_j = \frac{\frac{1}{\pi_0[\tilde{V}_j]} \sum_{i=1}^N \pi_i [\tilde{V}_j] E_i [\tilde{V}_j] - \frac{1}{N} \frac{X_{0j}}{\pi_0[\tilde{V}_j] N\tau}}{1 + R_f}, \quad j = 1, 2, \dots, J. \quad (7)$$

As in eqn. (3), price in eqn. (7) is composed of two terms. The first term on the right-hand-side of eqn. (7) is a precision-weighted, average assessment of investors' expected end-of-period cash flow for firm j . Investors' expected values are weighted by their precisions because their demands are proportionate to their precisions, *ceteris paribus*. The second term on the right-hand-side of eqn. (7) represents a discount for risk; this is based on investors' *average* precision of the distribution of the end-of-period cash flow for firm j (i.e., $\frac{1}{N} \pi_0 [\tilde{V}_j]$). In particular, note that *other* information characteristics are irrelevant to a firm's equilibrium price and cost of capital. For example, it does not matter whether there exists a high degree of information asymmetry among investors, as is the case with a large number of relatively better informed investors (i.e., investors with high precision) and a large number of relatively uninformed investors (i.e., investors with low precision), or a low degree of information asymmetry, as is the case where all investors have approximately the same amount of information. As long as the *average* precision is the same in the two economies, prices will also be the same.

As alluded to above, with the exception of Corollary 1 (which assumes that investors have homogeneous beliefs), none of our results are expressed in closed-form in that they do not explicitly incorporate the additional information investors' glean by conditioning their expectations over price. To solve the pricing system when investors condition their expectations over price, we need to place more structure on the notion of information. This is done in the next section.

3 Market prices with heterogeneous information

In this section we extend our analysis to heterogeneous information across investors in conjunction with investors conditioning their expectations over price. As discussed previously, price is as an aggregator of diverse information across investors. Rational investors are cognizant of the aggregation process, and thus use price as an additional conditioning variable in forming their expectations about future cash flows; this is what is meant by investors having “rational expectations.” To solve for price in closed-form, we assume that the structure of information in the economy is as follows. First, we assume that investors have a common prior over the distribution of the vector of end-of-period cash flows, $\tilde{\mathbf{V}}$. Let $\mathbf{m} = \{m_1, \dots, m_j, \dots, m_J\}'$ represent the $J \times 1$ vector of common prior beliefs about the expected value of $\tilde{\mathbf{V}}$, and Ψ the $J \times J$ common prior precision matrix of $\tilde{\mathbf{V}}$. Note that in this formulation we allow for the cash flows to be correlated across firms.

Each firm has associated with it a public announcement. Let $\tilde{\mathbf{y}} = \{\tilde{y}_1, \dots, \tilde{y}_j, \dots, \tilde{y}_J\}'$ represent the $J \times 1$ vector of announcements. We assume that announcements have a normal distribution, where η represents the $J \times J$ precision matrix of $\tilde{\mathbf{y}}$.⁴ We assume that announcements are unbiased, $E[\tilde{\mathbf{y}} | \tilde{\mathbf{V}} = \mathbf{V}] = \mathbf{V}$, but impose no structure on η . For example, neither the variances across announcements, nor the covariances across pairs of announcements, need be identical.

Each investor has private information about each firm. Let $\tilde{\mathbf{z}}_i = \{\tilde{z}_{i1}, \dots, \tilde{z}_{ij}, \dots, \tilde{z}_{iJ}\}'$ represent the $J \times 1$ vector of information available to investor i . As with public announcements, we assume that private information is unbiased and has a normal distribution. Let \mathbf{S}_i represent investor i 's $J \times J$ precision matrix for $\tilde{\mathbf{z}}_i$ conditional

⁴ It is straightforward to extend the model to allow for a vector of announcements about each firm, or for each investor to observe a vector of private information about each firm.

on $\tilde{\mathbf{V}} = \mathbf{V}$. As is standard practice in the literature, we assume that $\tilde{\mathbf{y}}$ and $\tilde{\mathbf{z}}_i$ are independent conditional on $\tilde{\mathbf{V}} = \mathbf{V}$.

We impose no structure on \mathbf{S}_i : for any investor the variance in private information can be different across firms, and \mathbf{S}_i can differ across investors. In particular, this implies that investors need not receive private information of equal quality. For example, some investors might not be endowed with any private information, in which case \mathbf{S}_i is a $J \times J$ matrix of 0's. In fact, it is precisely this feature - how the quality of information differs across investors - that we are interested in examining. As discussed above, information (both public and private) is useful in investors' demands for *all* firms. That is, public information (say, accounting earnings) for firm j will, in general, will be useful for both revising the assessment of expected end-of-period cash flow for firm j and for other firms whose cash flows covary with firm j .

In this section we specialize our analysis to consider two information structures that have been analyzed in the single-firm literature. First we consider a structure analogous to that in Kim and Verrecchia (1991) (KV). In KV, the distribution of the private information, $\tilde{\mathbf{z}}_i$, conditional on $\tilde{\mathbf{V}} = \mathbf{V}$, is independent across investors. That is, each investor observes a vector of information whose error terms can be correlated across firms, but uncorrelated across investors. Below we extend KV to a multi-firm setting where the price for one firm can be informative about the future cash flows for all firms.

Second we consider an information structure analogous to that in Easley and O'Hara (2004) (EOH). In EOH, there are two groups of investors, "less informed" investors and "more informed" investors. Members of the "less informed" group observe only the vector of public announcements, $\tilde{\mathbf{y}}$. In addition to the public announcements, members of the "more informed" group also observe a vector of private information,

$\tilde{\mathbf{z}}_i$. All members of the “more informed” group, however, receive exactly the same private information. Thus, the error terms in the signals of the “more informed” group are identical. Consequently, prices are only incrementally informative for investors in the “less informed” group. Eventually we also extend EOH to a multi-firm setting.

As is standard in the RE-literature, we prevent prices from being fully revealing by assuming that the aggregate supply of firm shares is uncertain. Specifically, we assume that the aggregate supply vector, $\tilde{\mathbf{X}}_0 = \{\tilde{X}_{01}, \dots, \tilde{X}_{0j}, \dots, \tilde{X}_{0J}\}'$, has a normal distribution. This implies that prices, as information sources, incorporate noise in the form of an aggregate supply shock. As is also the convention in the RE-literature, we perform our analysis in the context of the distribution of the supply shock *per-capita*: that is, let $\tilde{\mathbf{x}}_0 = \left\{ \tilde{x}_{01} = \frac{\tilde{X}_{01}}{N}, \dots, \tilde{x}_{0j} = \frac{\tilde{X}_{0j}}{N}, \dots, \tilde{x}_{0N} = \frac{\tilde{X}_{0J}}{N} \right\}'$ represent the $J \times 1$ vector of per-capita supply shocks. Note that the vector of per-capita supply shocks, $\tilde{\mathbf{x}}_0$, also has a normal distribution; let $\bar{\mathbf{x}}_0 = \{\bar{x}_{01}, \dots, \bar{x}_{0j}, \dots, \bar{x}_{0N}\}'$ represent the $J \times 1$ vector of mean values of $\tilde{\mathbf{x}}_0$, and \mathbf{W} the $J \times J$ precision matrix of $\tilde{\mathbf{x}}_0$. We assume that the distribution of $\tilde{\mathbf{x}}_0$ is independent of all other variables.

3.1 Diversely informed investors

The goal of this subsection is to extend the KV analysis to a multi-firm setting with correlated cash flows; in the next subsection we extend Easley and O’Hara (2004) in a similar fashion. To start, as described above, each investor receives a vector of private information, $\tilde{\mathbf{z}}_i$, where each vector has associated with it a conditional precision matrix \mathbf{S}_i . A solution to a RE-equilibrium typically starts by first requiring that investors conjecture that the vector of prices has the following linear functional form:

$$\tilde{\mathbf{P}} = \mathbf{a} + \mathbf{b}\tilde{\mathbf{V}} + \mathbf{c}\tilde{\mathbf{y}} - \mathbf{d}\tilde{\mathbf{x}}_0 + \mathbf{e}\bar{\mathbf{x}}_0. \quad (8)$$

That is, investors conjecture that the price vector is a linear function of true cash flow, $\tilde{\mathbf{V}}$, public announcements, $\tilde{\mathbf{y}}$, and supply shocks, $\tilde{\mathbf{x}}_0$ and $\bar{\mathbf{x}}_0$, where \mathbf{a} is a $J \times 1$ vector of intercepts, and \mathbf{b} , \mathbf{c} , \mathbf{d} , and \mathbf{e} are $J \times J$ matrices of coefficients. Note that in this formulation the price of each firm is allowed to be a function of the information signals of all other firms in the marketplace. For this same reason, the price of one firm also conveys information about other firms. In particular, we describe the additional information investors glean by conditioning their expectations over price as the “orthogonal” information in price. We represent the orthogonal information in price with a variable $\tilde{\mathbf{u}}$, where $\tilde{\mathbf{u}}$ is defined by

$$\tilde{\mathbf{u}} = \mathbf{b}^{-1} \left(\tilde{\mathbf{P}} - \mathbf{a} - \mathbf{c}\tilde{\mathbf{y}} + (\mathbf{d} - \mathbf{e})\bar{\mathbf{x}}_0 \right) = \tilde{\mathbf{V}} - \mathbf{b}^{-1}\mathbf{d}(\tilde{\mathbf{x}}_0 - \bar{\mathbf{x}}_0). \quad (9)$$

Note that $\mathbf{b}^{-1}\mathbf{d}$ is a $J \times J$ matrix. Conditional on $\tilde{\mathbf{V}} = \mathbf{V}$, the covariance matrix for orthogonal information is

$$\mathbf{b}^{-1}\mathbf{d} \cdot \mathbf{W}^{-1} \cdot (\mathbf{b}^{-1}\mathbf{d})'.$$

Finally, let $\boldsymbol{\theta}$ represent the precision matrix for orthogonal information conditional on $\tilde{\mathbf{V}} = \mathbf{V}$: that is,

$$\boldsymbol{\theta} = \left(\mathbf{b}^{-1}\mathbf{d} \cdot \mathbf{W}^{-1} \cdot (\mathbf{b}^{-1}\mathbf{d})' \right)^{-1}.$$

Note that even though investors have diverse private information, the precision of the orthogonal information, i.e., the incremental information investors glean from price, is the same across investors.

In the context of the information structure we consider in this subsection, investor i 's expected value of firms' cash flows becomes

$$E_i \left[\tilde{\mathbf{V}} \right] = \boldsymbol{\Pi}_i^{-1} (\boldsymbol{\Psi}\mathbf{m} + \boldsymbol{\eta}\mathbf{y} + \mathbf{S}_i\mathbf{z}_i + \boldsymbol{\theta}\mathbf{u}), \quad (10)$$

where $\boldsymbol{\Pi}_i = \boldsymbol{\Psi} + \boldsymbol{\eta} + \mathbf{S}_i + \boldsymbol{\theta}$ is the *total* precision matrix of investor i . The total precision matrix is comprised of an investor's prior, $\boldsymbol{\Psi}$, the public announcements,

$\boldsymbol{\eta}$, his private information, \mathbf{S}_i , and orthogonal information, $\boldsymbol{\theta}$. The key insight here is that when investor i conditions his expectations on his priors, public information, private information, and price, the total precision of his information is simply the sum of the precision matrices of his priors, the public announcements, his private information, and the precision matrix of the orthogonalized price vector.

Let $Avg[\cdot] = \frac{1}{N} \sum_{i=1}^N [\cdot]$ represent an averaging function. Then the average precision of information across investors, which eqn. (3) indicates is the key variable in determining the discount for risk in pricing, becomes

$$Avg[\boldsymbol{\Pi}_i] = \boldsymbol{\Psi} + \boldsymbol{\eta} + Avg[\mathbf{S}_i] + \boldsymbol{\theta}. \quad (11)$$

The only remaining task is to solve for $\boldsymbol{\theta}$. To do this, we must solve for the parameters \mathbf{b} and \mathbf{d} in the pricing eqn. (8). Substituting the expected cash flows and precisions into the pricing eqn. (3) and summing across all investors gives us an equation for price. We can then equate the coefficients from this equation with those from the pricing eqn. (8) to solve for the parameters \mathbf{b} and \mathbf{d} .

Proposition 2. *The precision of the orthogonal information in price is*

$$\boldsymbol{\theta} = \tau^2(Avg[\mathbf{S}_i]) \cdot \mathbf{W} \cdot (Avg[\mathbf{S}_i])',$$

and the average total precision of information across investors is

$$Avg[\boldsymbol{\Pi}_i] = \boldsymbol{\Psi} + \boldsymbol{\eta} + Avg[\mathbf{S}_i] + \tau^2(Avg[\mathbf{S}_i]) \cdot \mathbf{W} \cdot (Avg[\mathbf{S}_i])'.$$

The contribution of Proposition 2 toward the goal of this paper is that now we can solve the discount in price relative to the expected end-of-period cash flow in closed-form. Specifically, Proposition 1 demonstrates that the discount in price is governed by the *average* precision of information across investors. By determining

the average precision in closed-form, we can represent the discount in the vector of prices in a RE-setting as

$$\begin{aligned} \left(\frac{1}{N}\mathbf{\Pi}_0\right)^{-1}\frac{\mathbf{X}_0}{N\tau} &= (\text{Avg}[\mathbf{\Pi}_i])^{-1}\frac{\mathbf{X}_0}{N\tau} \\ &= (\mathbf{\Psi} + \boldsymbol{\eta} + \text{Avg}[\mathbf{S}_i] + \tau^2(\text{Avg}[\mathbf{S}_i]) \cdot \mathbf{W} \cdot (\text{Avg}[\mathbf{S}_i])')^{-1}\frac{\mathbf{X}_0}{N\tau}. \end{aligned}$$

Once again, it is important to emphasize that the discount is governed by the *average* precision, and not the *asymmetry* in the precision of information across investors.

For the special case where firms' cash flows are distributed independently, the market prices each firm independently. Specifically, in this circumstance the $\mathbf{\Psi}$, $\boldsymbol{\eta}$, \mathbf{S}_i , and $\boldsymbol{\theta}$ matrices are all diagonal, and the average precision of information across investors for each firm j is

$$\Psi_j + \eta_j + \text{Avg}[s_{ij}] + \tau^2(\text{Avg}[s_{ij}])^2 w_j, \quad (12)$$

where Ψ_j , η_j , s_{ij} , and w_j represent the j -th diagonal term of the (diagonal) matrices $\mathbf{\Psi}$, $\boldsymbol{\eta}$, \mathbf{S}_i , and \mathbf{W} , respectively. The result for firms with independent cash flows is tantamount to the result in KV for an economy with a single firm. Note that the average precision of information across investors increases with the precision of the prior, Ψ_j , the precision of the public announcement, η_j , and/or the average precision of private information, $\text{Avg}[s_{ij}]$. This implies that firm j 's cost of capital decreases with an increase in the precision of any of these terms.

3.2 Asymmetrically informed investors

Next, we examine an information structure analogous to that studied in Easley and O'Hara (2004), (EOH). A salient claim in the EOH analysis is that information asymmetries are "priced"; our goal is to re-examine that claim in the context of our

analysis. To facilitate this comparison, we begin with the special case where all firms' cash flows are distributed independently. When all firms' cash flows are distributed independently, for all intent and purpose each firm can be analyzed independent of every other firm. Thus, in this subsection we suppress the firm- j subscript, and couch our analysis in the context of a “generic” firm: this eases the notation burden.

As in EOH, in this subsection we assume there are two groups of investors. The first group is comprised of N_1 “less informed” investors, who receive Q_1 public information signals, \tilde{y}_h , $h = 1, \dots, Q_1$. The second group is comprised of N_2 “more informed” investors, who receive not only the Q_1 public signals, but also Q_2 private signals, \tilde{z}_k , $k = 1, \dots, Q_2$. All investors in the “more informed” group observe the same signals. As stated before, this differs from KV, where private information is distributed independently across investors. Also as in EOH, in this subsection we assume that all signals (both public and private) have the same precision, s (i.e., here $\eta \equiv s$). We assume that each of the signals \tilde{y}_h and \tilde{z}_k is conditionally independent of every other signal. Also as before, each investor has a common prior on the distribution of the end-of-period cash flow, \tilde{V} ; specifically, investors believe that \tilde{V} has a normal distribution with mean m and precision Ψ .

The “more informed” group observes all public and private information available in the economy. Hence, the “more informed” group cannot learn any additional information from price. In effect, for the “more informed” group, price is a redundant source of information. Let $\tilde{Y} = \sum_{h=1}^{Q_1} \tilde{y}_h$ and $\tilde{Z} = \sum_{k=1}^{Q_2} \tilde{z}_k$ represent summary statistics for the vector of public and private signals, respectively. Conditional upon the realization of public and private information (i.e., conditional on $\tilde{Y} = Y$ and $\tilde{Z} = Z$), the conditional expectation of end-of-period cash flow of “more informed” investors

is

$$E \left[\tilde{V} | y_1, \dots, y_{Q_1}, z_1, \dots, z_{Q_2} \right] = \frac{\Psi m + sY + sZ}{\Psi + (Q_1 + Q_2) s},$$

where the total precision of their information is $\Psi + (Q_1 + Q_2) s$. Note that the total precision is increasing in the prior precision, Ψ , the precision per signal, s , and the total number of signals, $Q_1 + Q_2$.

Members of the “less informed” group base their investment decisions on the Q_1 public signals and price. As before, to ensure that price is not fully revealing, we assume the shock to the aggregate supply creates noise. Specifically, we assume that the aggregate per-capita supply for the firm, \tilde{x}_0 , has a normal distribution with mean \bar{x}_0 and precision w , and is independent of each y_h and z_k .

As in the previous subsection, a solution to a RE-equilibrium typically starts by requiring first that investors conjecture that “less informed” investors conjecture that price has the form

$$\tilde{P} = a + b\tilde{Z} + c\tilde{Y} + d\tilde{x}_0 - e\bar{x}_0. \quad (13)$$

Here we represent the orthogonal information in price with a variable \tilde{u} , where \tilde{u} is defined by

$$\tilde{u} = \frac{\tilde{P} - a - c\tilde{Y} + \bar{x}_0(d - e)}{bQ_2} = \frac{\tilde{Z}}{Q_2} - \frac{d}{b} \frac{\tilde{x}_0 - \bar{x}_0}{Q_2}.$$

As before, the orthogonal information in price represents the incremental information “less informed” investors glean by conditioning their expectations on price. Note that the presence of an aggregate supply shock makes orthogonal information in price a noisy measure of the private information available to “more informed” investors, \tilde{Z} .

We denote the precision of \tilde{u} by θ . Unlike the diverse information case considered above where the precision of orthogonal information was defined conditional on the realization of firms’ cash flows, here θ represents the total precision of \tilde{u} .

As a consequence of conditioning their expectations on price, the total precision of investors in the “less informed” group is $\Psi + Q_1 s + \theta$. As above, Ψ represents investors’ common prior precision. While “less informed” investors have no private information, they learn the private information of “more informed” investors (with noise) when they condition their expectations on price. Here, θ represents the precision of the additional information “less informed” investors glean from price.

EOH use a slightly different (but equivalent) parameterization in their model that facilitates some of the comparative statics analysis they do. In particular, they define Q as the total number of information signals observed by “more informed” investors, where $Q \equiv Q_1 + Q_2$, and α as the percentage of all signals that are private (i.e., $\alpha = \frac{Q_2}{Q_1 + Q_2}$). The fraction of investors in the informed group is μ , where $\mu = \frac{N_2}{N_1 + N_2}$.

EOH show that price can be expressed as:⁵

$$E [\tilde{V} - \tilde{P}|\text{Info}] = \frac{1}{\tau} \frac{\bar{x}_0}{\Psi + (1 - \alpha) Q s + \mu \alpha Q s + (1 - \mu) \theta}. \quad (14)$$

In particular, the denominator of this expression indicates how the distribution of information across investors affects the cost of capital. EOH discuss their results in terms of the *asymmetry* of the information between the two groups of investors. Note, however, that the denominator of their expression can be re-written as

$$\begin{aligned} & \mu (\Psi + Q s) + (1 - \mu) (\Psi + (1 - \alpha) Q s + \theta) \\ &= \mu (\Psi + Q s) + (1 - \mu) (\Psi + Q_1 s + \theta). \end{aligned} \quad (15)$$

The expression on the right-hand-side of eqn. (15) is simply the *average* precision of information across investors. This renders their pricing equation equivalent to our eqn. (7). The weights are the percentage of investors in each group: μ for the

⁵ EOH assume the risk free rate is zero.

informed group and $1 - \mu$ for the less informed group. This equation clearly shows that it is the *average* precision of information that determines the magnitude of the discount relative to the expected cash flow.

EOH interpret their analysis and comparative statics results as being driven by information asymmetries. Each of their results, however, can also be explained using the average precision metric. For example, EOH show that increasing the parameter α - the fraction of signals that are private - makes the cost of capital go up. They claim that this occurs because the information asymmetry between the groups increases. Note, however, that EOH hold constant the total number of information signals “more informed” investors observe, Q . Thus, while increasing α does result in an increase in information asymmetry, it also *reduces* the average precision of information by reducing the amount of information the “less informed” receive without having any effect on the amount of information the “more informed” receive (because Q is fixed). In other words, “more informed” investors receive the same number of signals regardless of the magnitude of α . Thus, the average precision goes down, which implies that the cost of capital goes up.

EOH also show that increasing the parameter μ - the proportion of investors who get private information - makes the cost of capital go down. Again, while increasing μ does reduce information asymmetry across investors, by the same token it also increases the average precision of information in the market. Consistent with an increase in average precision, our average precision metric predicts that the cost of capital go down.

But it is important to emphasize that the cost of capital is lower *because* of the existence of investors with private information, not in spite of it. To see this, note that if no one in the economy received any private information (i.e., $\mu = 0$), the

denominator in the cost of capital equation reduces to $\Psi + (1 - \alpha) Qs = \Psi + Q_1 s$.⁶ This is strictly smaller than the denominator in the expression where there is a group that receives private information in addition to the Q_1 public signals. EOH's results hint at this when they show that if $\alpha = 1$, implying that "less informed" investors learn nothing (other than what they can learn through price), increasing the number of signals the informed group receives, Q , makes the cost of capital *go down*. But this also makes the degree of information asymmetry *go up*, which should increase the cost of capital in the context of the EOH information-risk story. Instead, what is really happening is that the *average* precision of information across investors is going up: the more informed group receives more information, while the less informed group receives less. Therefore, the cost of capital goes down.

As further evidence, consider the situation where the number of public signals is non-zero, and the number of signals that are private increases. This increases information asymmetry, but it also increases the weighted-average precision of information.⁷ Does the cost of capital *go down* (because the average precision of investors goes up), or does it *go up* (because the degree of asymmetry goes up)?

Given the way EOH parameterize their model, this comparison is difficult to do because it requires examining the effect of simultaneously increasing Q and decreasing α . Our simple re-parameterization makes the analysis more straightforward. In our formulation, we can hold constant the number of public signals by fixing Q_1 , yet increase the number of private signals by increasing Q_2 . Recall that the average

⁶ Note that price is no longer informative in this case, so that $\theta = 0$.

⁷ Increasing the number of private signals will increase the precision of information for *both* groups. The more informed group benefits directly, while the less informed group benefits from price being a more informative signal. It is easy to show, however, that the precision of the informed group increases by more than the precision of the less informed group. Therefore, the difference in the precisions between the two groups increases, which means the degree of information asymmetry increases.

precision of the information in the economy weights the precisions of the two groups according to their relative sizes. From above, the precision of the more informed group is $\Psi + Qs$ and the precision of the less informed group is $\Psi + Q_1s + \theta$. Solving for the precision of the information obtained from price in a RE-context yields the following result.

Proposition 3. *The average precision of investors' information is*

$$\mu (\Psi + Qs) + (1 - \mu) (\Psi + Q_1s + \theta) ,$$

where the precision of orthogonal information that arises from “less informed” investors conditioning their expectations on price is

$$\theta = \left(\frac{1}{Q_2s} + \left(\frac{1}{\frac{N_2}{N}\tau s} \right)^2 \frac{1}{Q_2^2w} \right)^{-1} . \quad (16)$$

Eqn. (16) is analogous to eqn. (A3) in EOH.

Now consider what happens to the cost of capital if we hold Q_1 fixed, but increase Q_2 . The precision of the “more informed” group obviously increases. Moreover, the precision of the information “less informed” investors glean from price *also increases*. Thus, the precision of the information available to each group increases. This implies that the average precision of investors goes up, and the cost of capital goes down.

To summarize, contrary to the suggestion in EOH, price is not discounted because “less informed” investors face an adverse selection problem and price protect themselves. Instead, price is discounted because “less informed” investors simply have more uncertainty about the end-of-period cash flow, which reduces the average precision of information across investors in the economy. The implication is that the cost of capital does not decline as a result of reducing information asymmetries. Instead, the cost of capital declines as more persons in the economy receive more

information. In other words, a greater transmission of the “more informed” investors’ private information to the “less informed” group through price does lower the cost of capital, but this is simply because the average precision of investors’ information increases - it is not because information asymmetry has been reduced. In the same way, giving “more informed” investors even more private information also lowers the cost of capital. In this case, information asymmetry may actually increase. The average precision of information for investors goes up, however, so the cost of capital goes down. In fact, the precision of the information available to “less informed” investors actually improves here, because price contains more information.⁸

It is straightforward to extend the analysis of asymmetric information across investors to an economy in which the cash flows are cross-sectionally correlated. For convenience, assume that the same N_1 investors receive only the public signals about each firm, whereas that the remaining N_2 investors also receive private signals about each firm. Assume that there exist Q_1 public signals about each firm and Q_2 private signals. The only complicating feature is that now each public signal and each private signal conveys information about the future cash flows of all firms. Assume all signals (both public and private) have the same precision matrix, \mathbf{S} .

As before, “more informed” investors cannot learn anything from price. “Less informed” investors base their investment decisions on the Q_1 public signals and price. Again, the price of one firm can be potentially informative about the cash

⁸ As further evidence, suppose we hold constant the average precision across investors of their “basic” information (e.g., their information not including any inferences they glean from prices), but increase the information asymmetry of the “basic” information of the two groups. That is, suppose we hold constant $\mu(Q_1 + Q_2) + (1 - \mu)Q_1$, but simultaneously increase Q_2 and decrease Q_1 . Even though information asymmetries increase, such a change will *decrease* the cost of capital. The reason is that the average precision of the “total” information (i.e., the information investors possess including their inferences from prices) held by the two groups will increase. In particular, the decrease in the precision of the “less informed” investors that results from lowering Q_1 is partially offset by their ability to infer from price the information that results from an increase in Q_2 .

flows of all firms. Solving for the precision of the information gleaned from price in a RE-context yields the following result.

Proposition 4. *The average precision of investors' information is*

$$\mu(\Psi + Q\mathbf{S}) + (1 - \mu)(\Psi + Q_1\mathbf{S} + \boldsymbol{\theta}), \quad (17)$$

where the precision of the incremental information conveyed by price is

$$\boldsymbol{\theta} = \left[\frac{1}{Q_2}\mathbf{S}^{-1} - \left(\frac{1}{\tau} \frac{N}{N_2} \frac{1}{Q_2} \right)^2 \mathbf{S}^{-1} \cdot \mathbf{W} \cdot (\mathbf{S}^{-1})' \right]^{-1}.$$

The first term in the average precision equation, eqn. (17), is the precision of more informed investors, weighted by their relative population in the economy (μ), and the second term is the precision of less informed investors, weighted by their relative population ($1 - \mu$). Note again that an increase in Q_2 increases the average precision of information, which lowers the cost of capital.

The significance of Proposition 4 is that it demonstrates how the notion of average precision extends to an EOH setting with multiple firms whose cash flows may covary. Average precision governs the discount in price associated with the market holding firms' shares. Thus, as the expression for average precision manifest in eqn. (17) goes up or down, cost of capital correspondingly decreases or increases.

4 Disclosure and diversification

Section 3 establishes that the discount the market associates with holding firms' shares relies on investors' average precision for the distribution of firms' end-of-period cash flows. An interesting ancillary question, then, is whether an individual firm can affect its discount through investors' average precision by disclosing additional information about its cash flow. To address this question, recall that in Proposition 1 the discount

was characterized by $(\frac{1}{N}\mathbf{\Pi}_0)^{-1}\frac{\mathbf{X}_0}{N\tau}$, where $(\frac{1}{N}\mathbf{\Pi}_0)^{-1}$ represents the inverse of investors' average precision matrix for the distribution of firms' end-of-period cash flows, $\frac{\mathbf{X}_0}{N}$ represents investors' per-capita holding of firm shares, and τ represents investors' tolerance for risk. It is straightforward to show that when there are only a finite number of investors in the economy (i.e., N is finite), additional disclosure by firm j will reduce its discount by increasing the average precision in investors' information, $\mathbf{\Pi}_0$. Nonetheless, it could be argued that the finite- N case is uninteresting in the context of the CAPM in that it fails to capture the effect of "diversification." Thus, in this section we study the effect of disclosure by firm j on its discount in the presence of diversification.

Diversification is achieved typically by appealing to the notion of a "large economy." So as to avoid cases as uninteresting as the finite- N case, however, some consideration should be given to how one defines a "large economy." For example, if by "large economy" one means that the number of investors in the economy gets large (i.e., N gets large), while all other features of the economy remain finite, then diversification alone eliminates any discount because $\frac{\mathbf{X}_0}{N}$ converges to 0. But just as the finite- N case is an uninteresting setting to study diversification, this setting is equally uninteresting in that the economy only becomes large in the number of investors, not in the number of firms (or the total supply of firm shares). Thus, we define a "large economy" as one in which *both* the number of investors, N , and the number of firms, J , become large. In other words, we characterize a "large economy" as one in which the number of firms becomes large in relation to the number of investors.

Using this definition of a "large economy," we begin our study of diversification with the special case in which the cash flows for all firms are distributed independently. Recall from Corollary 3 that when cash flows are independent, investors' covariance

and precision matrices are diagonal, and the discount for the risk for firm j reduces to

$$\frac{1}{\frac{1}{N}\pi_0 [\tilde{V}_j]} \frac{X_{0j}}{N\tau} = \frac{1}{\frac{1}{N}\sum_{i=1}^N \pi_i [\tilde{V}_j]} \frac{X_{0j}}{N\tau}.$$

Here, irrespective of the number of investors in the economy, the average precision of information, $\frac{1}{N}\sum_{i=1}^N \pi_i [\tilde{V}_j]$, remains finite. As N increases, however, the per-capita supply of firm j 's shares, $\frac{X_{0j}}{N}$, steadily diminishes. This is consistent with the concept of “diversifiable risk.” When the per-capita supply approaches 0 as N gets large (i.e., as $\frac{X_{0j}}{N} \rightarrow 0$), the discount for the risk for firm j approaches 0. Thus, here the average precision of investors’ information is irrelevant; as such, any disclosure by firm j intended to affect the average precision is also irrelevant.

Now consider the case where all investors have the same precision of information, but the information itself is diverse. Recall from Corollary 2 that when all investors have the same precision of information, the discount for the risk in the vector of firm share prices, \mathbf{P} , reduces to

$$\mathbf{Cov}_H \frac{\mathbf{X}_0}{N\tau}.$$

This expression implies that the discount to the price of firm j 's shares is

$$\frac{1}{N\tau} \sum_{k=1}^J Cov_{j,k} X_{0k} = \frac{1}{N\tau} \left(Cov_{j,j} X_{0j} + \sum_{k \neq j} Cov_{j,k} X_{0k} \right), \quad (18)$$

where $Cov_{j,k}$ represents the j -th, k -th element of the matrix \mathbf{Cov}_H . The right-hand-side of eqn. (18) is analogous to the discount for risk in the traditional CAPM, except in our analysis the covariances are multiplied by the supply of firms’ shares; in the CAPM, the aggregate supply of firms’ shares (in percentage terms) is 1. As discussed above, if the off-diagonal elements of the covariance matrix \mathbf{Cov}_H are zero, this reduces to

$$\frac{1}{N\tau} Cov_{j,j} X_{0j},$$

which approaches 0 as N gets large. If the off-diagonal elements are *non-zero*, however, $\sum_{k \neq j}^J Cov_{j,k} X_{0k}$ will grow as the number of firms in the economy, J , grows; this implies that

$$\frac{1}{N\tau} \sum_{k \neq j}^J Cov_{j,k} X_{0k}$$

will *not* approach 0. In short, for the discount in the price for firm j 's shares to remain positive in a “large economy” (as we have defined this concept), firms’ cash flows must covary.

When firms’ cash flows covary, there exists a role for disclosure by firm j . Specifically, let $\tilde{\delta}_j$ represent the error in the disclosure firm j provides about its cash flow; we assume that $\tilde{\delta}_j$ is independent of all other variables.

Proposition 5. *When all investors have the same precision of information, but the information itself is diverse, firm j 's discount, $\frac{1}{N\tau} \sum_{k=1}^J Cov_{j,k} X_{0k}$, moves closer to 0 as the precision in its disclosure about its cash flow increases.*

Proposition 5 demonstrates that disclosure by a firm attenuates the discount for the risk the market associates with holding shares of that firm. It also extends results in the “estimation risk” literature that concern information structures based on the historical time-series of return variables from a stationary process (Brown, 1979; Barry and Brown, 1984 and 1985; etc.). Finally, Proposition 5 extends results in Lambert et al. (2006) that are based on homogeneous beliefs to the case of diverse information and homogeneous precisions.

To illustrate Proposition 5, recall that $\tilde{\delta}_j$ represents the error in the disclosure firm j provides about its cash flows. Let $Var[\tilde{V}_j]$ and $Var[\tilde{\delta}_j]$ represent the variances of firm j 's cash flow, and the error in disclosure about that cash flow, respectively. Then disclosure by firm j reduces $Cov_{j,k}$, the j -th, k -th element of the matrix \mathbf{Cov}_H ,

from $Cov_{j,k}$ to

$$\frac{Var [\tilde{\delta}_j]}{Var [\tilde{V}_j] + Var [\tilde{\delta}_j]} Cov_{j,k}.$$

This, in turn, implies that $\frac{1}{N\tau} \sum_{k=1}^J Cov_{j,k} X_{0k}$ reduces to

$$\frac{Var [\tilde{\delta}_j]}{Var [\tilde{V}_j] + Var [\tilde{\delta}_j]} \frac{1}{N\tau} \sum_{k=1}^J Cov_{j,k} X_{0k}. \quad (19)$$

Thus, as the precision in the error in firm j 's disclosure about its cash flow increases (i.e., as $Var [\tilde{\delta}_j]$ decreases), the expression in eqn. (19) gets closer to 0. In short, disclosure by a firm attenuates the discount for the risk the market associates with holding shares of that firm. Proposition 5 extends easily to the situation in which investors' precision matrices for are not homogeneous.

5 Imperfect competition

To this point, we have shown that in a CAPM or RE-setting, the discount to prices that results from the risk the market associates with holding firms' shares is a function of the average precision of information across investors, not information asymmetry *per se*. Information asymmetry does not manifest in cost of capital in conventional characterizations of the CAPM or RE models because these settings are based on perfect competition. In a model of perfect competition, each investor determines his demand for firm shares based on a conjecture that his demand cannot affect price. In particular, more informed investors do not strategically reduce their demand for fear of revealing their information to others, thereby adversely impacting the price at which they trade. Moreover, no trading takes place until an equilibrium price is set. Less informed investors rationally use price as a conditioning variable in setting their expectations and assessing risk when they submit their demand order.

Under perfect competition, eqn. (2) shows that each investor's demand for shares of firm j is decreasing in his assessed degree of uncertainty (increasing in his assessed precision) about j 's future cash flow. While less informed investors will demand fewer shares when they perceive uncertainty to be high, more informed investors will demand more shares. Because demand is linear in each investor's precision, the relevant metric when investors' demands are aggregated to clear the market is the average assessed precision. Moreover, the assessed uncertainty about firm j 's cash flow is not greater when the investor perceives that other investors possess more precise information. In fact, an investor's assessed degree of uncertainty decreases when other investors acquire more information, because this information becomes (partially) transmitted through price when investors condition their expectations over price in determining their demand. In other words, the intuition that information asymmetry drives up the cost of capital in a model of perfect competition because less informed investors reduce their demand for firm shares is flawed.

This being the case, how does information asymmetry manifest in cost of capital? We argue that the effect of information asymmetry may arise in the context of imperfect competition. Imperfect competition results when investors' demand orders are sufficiently "large" in relation to the market as a whole such that a conjecture that their demand cannot affect price is unsustainable. Thus, investors must take into account the effect of their actions on the market price at which their trades are executed.

While there exist a variety of characterizations of imperfect competition in the literature (e.g., Copeland and Galai, 1983; Kyle, 1985; Glosten and Milgrom, 1985; Admati and Pfleiderer, 1988; etc.), the characterization of this phenomenon offered in Diamond and Verrecchia (1991) is perhaps closest to our modeling in the prior

sections. In Diamond and Verrecchia (DV), there are two classes of investors: two large, risk neutral traders who are (potentially) informed; and N less informed “market makers” who are risk averse with risk tolerance τ . In each period, the risk neutral traders submit their demands, and then the market-makers set the price at which they execute those demands based on publicly available information and inferences about firm value based on the order flow. The risk-averse market makers also provide the risk-bearing capacity for the market by absorbing whatever shares are not demanded by the risk neutral traders. For all intent and purpose, the risk-averse market makers serve the role of generic investors in a model of perfect competition: they are risk-averse, perfectly competitive, and are informed indirectly by order flow, in the same fashion that uninformed investors are informed indirectly in a RE-model by conditioning their expectations over price. In other words, the risk-averse market makers serve the same role as the less well-informed investors in Grossman and Stiglitz (1980), Easley and O’Hara (2004), etc.

In a model of perfect competition, risk neutral traders with private information will take infinite positions in a firm’s stock. In a world of imperfect competition, however, such large positions will influence the beliefs of other investors about future cash flows (because the risk neutral traders may be informed); this, in turn, affects the price at which trade takes place. In particular, in DV the risk neutral traders conjecture rightly that price will be a linear, increasing function of their demand for shares.

DV construct a three-date model in which the firm’s uncertain cash flow is realized at date three. At the beginning of the model, the large, risk neutral traders do not know whether they will become informed or receive liquidity shocks (i.e., they do not know their type). At date 2, one large risk-neutral trader receives private information

about the firm's cash flow and the other receives a liquidity shock that requires him to sell a random number of his existing shares. Trade occurs both at date 1, before any public information is disseminated or the risk-neutral traders learn their type, and at date 2, after information and type is revealed.

If we fold the pricing back to the beginning of the model - before the risk neutral investors know their type - DV's Proposition 1 shows that the equilibrium price is lower than the ex-ante expected cash flow. Thus, despite the existence of risk neutral traders in the economy, pricing is not done on a risk neutral basis. That is, the cost of capital exceeds the risk-free rate. Moreover, increasing the precision of the public information, which reduces information asymmetry between traders and market makers, causes price to increase (which decreases the cost of capital).

These results follow from the risk neutral traders' unwillingness to take a large position in the stock at date 1 because they anticipate this may lead to problems at date 2. In particular, they are concerned that if they incur a liquidity shock at date 2, they will have to sell their holdings at a low price. The low price at date 2 will occur because market makers observing a large order to sell shares will be unable to distinguish whether it is a result of an uninformed trader's need for liquidity or from an informed trader's decision to sell based on the receipt of bad news. The market makers' lowering of price in response to an order to sell shares is the classic price-protection response to an adverse selection problem. The greater the asymmetry of information between (potentially) better informed traders and less well informed market makers, the greater will be the reaction of price to the order flow, including the reaction to a liquidity trader's sale of shares at date 2. Because the risk neutral traders are unwilling to take a large position at date 1, the risk-averse market makers must absorb more of the shares at that time. As a consequence, the stock price at

date 1 is accordingly lower.

If there were no information asymmetries, the risk neutral investors will hold all the firm's shares at all dates, and pricing will be done on a risk neutral basis. There will be no price reaction to a sell order at date 2 because it will be known to be liquidity-driven. The greater the degree of information asymmetry at date 2 between informed, risk neutral traders and risk-averse market makers, however, the more price (mistakenly) reacts to a liquidity-based need to sell (buy) shares. Anticipating this, the risk neutral traders purchase fewer shares at date 1, which means that more shares are held by the risk-averse market makers. The compensation market-makers require for bearing a greater share of risk drives the date 1 price downward. Therefore, more information asymmetry increases the cost of capital.

Thus, the salient difference between DV's model of imperfect competition versus perfect competition models in the RE-literature is that in the former the number of shares held *in toto* by risk-averse market makers (equivalently, generic investors in models of perfect competition) is endogenous, whereas in the latter it is exogenous. In particular, the number of shares held by the risk-averse market makers is a function of the information structure in the economy, and more specifically, by the degree of information asymmetry.

Interestingly, the results in DV are consistent with the intuition offered in Easley and O'Hara that asymmetric information will cause investors to withdraw from the market, thereby lowering the price. It is important to note, however, that it is not the smaller, uninformed investors who withdraw. Instead, it is the larger investors who withdraw from the market, because they are the ones who bear liquidity costs when exogenous circumstances force them to liquidate their large positions.

6 Conclusion

This paper analyzes how the distribution of information across investors affects firms' cost of capital. We find that when investors have different information, prices are a function of two features of the economy's information structure: 1) individual investors' precision-weighted, average assessment of firms' expected end-of-period cash flows; and 2) a discount for the risk investors associate with holding firms' shares that depends on the investors' average, assessed precision matrix (i.e., the inverse of the covariance matrix) of the distribution of firms' end-of-period cash flows. In other words, investors' average, assessed precision is a key determinant of the expected return on a firm's stock price, and therefore on its cost of capital. The extent to which investors' precision matrices deviate from this average, however, does not matter. In particular, the asymmetry of information across investors does not affect the discount for risk, holding the average assessed precision constant.

Our results imply that Easley and O'Hara's (2004) interpretation of their "information asymmetry" determinant of the cost of capital model is not correct. In particular, we show that in their model price is not discounted relative to the expected end-of-period cash flow because a "less informed" group of investors faces an adverse selection problem, and thus must price-protect itself. Instead, price is discounted because "less informed" investors simply manifest more uncertainty about the end-of-period cash flow. This lowers the *average* precision of information held by investors, which raises the cost of capital. In fact, we show that increasing the degree of information asymmetry between investors can reduce the cost of capital, not increase it. The dissemination of more information to more investors drives down the cost of capital, not the reduction in information asymmetries *per se*.

Interestingly, many of the same factors that are thought to reduce information asymmetries (e.g., improved accounting and disclosure policies, increased following by analysts, etc.) will also increase investors' average, assessed precision of information. Therefore, it may be difficult to distinguish empirically between the two constructs. It is important to emphasize, however, that an information asymmetry story is inconsistent with a model based on perfect competition.

When firms' cash flows are correlated, the average assessed precision of a firm's cash flow is diversifiable; that is, it becomes an increasingly smaller portion of a firm's risk as the economy grows large. What is relevant is not the average assessed precision for a firm, but rather the average assessed *precision-matrix* for the economy. That is, as in the CAPM, what is important is the assessed covariance between firms. We show that improved information about a firm's cash flow can lower the assessed covariance between its cash flows and those of other firms, and thereby lower its cost of capital.

While asymmetric information across investors plays no role in models that depend on perfect competition, in models of imperfect competition asymmetric information affects the willingness of "large" traders to supply liquidity. In models of imperfect competition information asymmetry affects the willingness of "large" traders to supply liquidity. As liquidity declines, a firm's extant risk is borne by fewer investors. This, in turn, increases a firm's cost of capital rises. In other words, a salient implication of our analysis is that information asymmetry is a separate and distinct effect from information precision effect, and both contribute to a firm's cost of capital in unique ways.

APPENDIX

Proof of Proposition 1.

From eqn. (2), investor i 's demand for shares of firms (in vector format) is

$$\mathbf{X}_i = \tau \mathbf{\Pi}_i \left(E_i [\tilde{\mathbf{V}}] - \mathbf{P} (1 + R_f) \right). \quad (\text{A1})$$

Summing the demand function across investors yields

$$\mathbf{X}_0 = \sum_{i=1}^N \mathbf{X}_i = \tau \sum_{i=1}^N \mathbf{\Pi}_i \left(E_i [\tilde{\mathbf{V}}] - \mathbf{P} (1 + R_f) \right);$$

recall that \mathbf{X}_0 is the aggregate supply vector. Solving for \mathbf{P} , the vector of firm share prices, yields

$$\mathbf{P} = \frac{(\mathbf{\Pi}_0)^{-1} \sum_{i=1}^N \mathbf{\Pi}_i E_i [\tilde{\mathbf{V}}] - \left(\frac{1}{N} \mathbf{\Pi}_0\right)^{-1} \frac{\mathbf{X}_0}{N\tau}}{1 + R_f}.$$

Q.E.D.

Proof of Proposition 2.

First, we re-express \mathbf{P} as characterized in Proposition 1 as follows:

$$\mathbf{P} = \left(\frac{1}{N} \mathbf{\Pi}_0\right)^{-1} \frac{\frac{1}{N} \sum_{i=1}^N \mathbf{\Pi}_i E_i [\tilde{\mathbf{V}}] - \frac{\mathbf{X}_0}{N\tau}}{1 + R_f}.$$

Next, substitute into this expression for \mathbf{P} the characterization of $E_i [\tilde{\mathbf{V}}]$ offered in eqn. (10):

$$\begin{aligned} \mathbf{P} &= \left(\frac{1}{N} \mathbf{\Pi}_0\right)^{-1} \frac{\frac{1}{N} \sum_{i=1}^N \mathbf{\Pi}_i \left(\mathbf{\Pi}_i^{-1} (\mathbf{\Psi} \mathbf{m} + \boldsymbol{\eta} \mathbf{y} + \mathbf{S}_i \mathbf{z}_i + \boldsymbol{\theta} \mathbf{u}) \right) - \frac{\mathbf{X}_0}{N\tau}}{1 + R_f} \\ &= \left(\frac{1}{N} \mathbf{\Pi}_0\right)^{-1} \frac{\mathbf{\Psi} \mathbf{m} + \boldsymbol{\eta} \mathbf{y} + \frac{1}{N} \sum_{i=1}^N \mathbf{S}_i \mathbf{z}_i + \boldsymbol{\theta} \mathbf{u} - \frac{\mathbf{X}_0}{N\tau}}{1 + R_f}. \end{aligned}$$

Thus, when investors condition their expectations over \mathbf{P} , they interpret \mathbf{P} a random variable characterized as

$$\tilde{\mathbf{P}} = \left(\frac{1}{N} \mathbf{\Pi}_0\right)^{-1} \frac{\mathbf{\Psi} \mathbf{m} + \boldsymbol{\eta} \tilde{\mathbf{y}} + \frac{1}{N} \sum_{i=1}^N \mathbf{S}_i \tilde{\mathbf{z}}_i + \boldsymbol{\theta} \tilde{\mathbf{u}} - \frac{1}{\tau} \tilde{\mathbf{x}}_0}{1 + R_f}, \quad (\text{A2})$$

where for convenience we now express the aggregate supply on a per-capita basis: that is, $\tilde{\mathbf{x}}_0 = \frac{\bar{\mathbf{x}}_0}{N}$. The law of large numbers implies that $\frac{1}{N} \sum_{i=1}^N \mathbf{S}_i \tilde{\mathbf{z}}_i$ converges to $\frac{1}{N} \sum_{i=1}^N \mathbf{S}_i \tilde{\mathbf{V}}$ because the idiosyncratic elements in $\tilde{\mathbf{z}}_i$ average out. Thus, we substitute this into eqn. (A2), along with the expression for $\tilde{\mathbf{u}}$ suggested in eqn. (9):

$$\begin{aligned} \tilde{\mathbf{P}} &= \left(\frac{1}{N} \mathbf{\Pi}_0\right)^{-1} \frac{\mathbf{\Psi} \mathbf{m} + \boldsymbol{\eta} \tilde{\mathbf{y}} + \frac{1}{N} \sum_{i=1}^N \mathbf{S}_i \tilde{\mathbf{V}} + \boldsymbol{\theta} \left(\tilde{\mathbf{V}} - \mathbf{b}^{-1} \mathbf{d} (\tilde{\mathbf{x}}_0 - \bar{\mathbf{x}}_0)\right) - \frac{1}{\tau} \tilde{\mathbf{x}}_0}{1 + R_f} \\ &= \left(\frac{1}{N} \mathbf{\Pi}_0\right)^{-1} \frac{\mathbf{\Psi} \mathbf{m} + \boldsymbol{\eta} \tilde{\mathbf{y}} + \left(\boldsymbol{\theta} + \frac{1}{N} \sum_{i=1}^N \mathbf{S}_i\right) \tilde{\mathbf{V}} - \left(\boldsymbol{\theta} \mathbf{b}^{-1} \mathbf{d} + \frac{\mathbf{I}}{\tau}\right) \tilde{\mathbf{x}}_0 + \boldsymbol{\theta} \mathbf{b}^{-1} \mathbf{d} \bar{\mathbf{x}}_0}{1 + R_f}. \end{aligned}$$

Now recall that investors conjecture that $\tilde{\mathbf{P}}$ is of the form $\tilde{\mathbf{P}} = \mathbf{a} + \mathbf{b} \tilde{\mathbf{V}} + \mathbf{c} \tilde{\mathbf{y}} - \mathbf{d} \tilde{\mathbf{x}}_0 + \mathbf{e} \bar{\mathbf{x}}_0$.

For this conjecture to be self-fulfilling, it must be the case that

$$\begin{aligned} \mathbf{b} &= \frac{\left(\frac{1}{N} \mathbf{\Pi}_0\right)^{-1}}{1 + R_f} \left(\boldsymbol{\theta} + \frac{1}{N} \sum_{i=1}^N \mathbf{S}_i\right), \text{ and} \\ \mathbf{d} &= \frac{\left(\frac{1}{N} \mathbf{\Pi}_0\right)^{-1}}{1 + R_f} \left(\boldsymbol{\theta} \mathbf{b}^{-1} \mathbf{d} + \frac{\mathbf{I}}{\tau}\right). \end{aligned}$$

Thus,

$$\begin{aligned} \mathbf{b}^{-1} \mathbf{d} &= \left(\boldsymbol{\theta} + \frac{1}{N} \sum_{i=1}^N \mathbf{S}_i\right)^{-1} \left(\frac{1}{N} \mathbf{\Pi}_0\right) \left(\frac{1}{N} \mathbf{\Pi}_0\right)^{-1} \left(\boldsymbol{\theta} \mathbf{b}^{-1} \mathbf{d} + \frac{\mathbf{I}}{\tau}\right) \\ &= \left(\boldsymbol{\theta} + \frac{1}{N} \sum_{i=1}^N \mathbf{S}_i\right)^{-1} \left(\boldsymbol{\theta} \mathbf{b}^{-1} \mathbf{d} + \frac{\mathbf{I}}{\tau}\right). \end{aligned}$$

Hence, we can solve to get

$$\mathbf{b}^{-1} \mathbf{d} = \frac{\left(\frac{1}{N} \sum_{i=1}^N \mathbf{S}_i\right)^{-1}}{\tau}.$$

Substituting this expression for $\mathbf{b}^{-1} \mathbf{d}$ back into the expression for the precision of the orthogonal information in price, $\boldsymbol{\theta}$, yields

$$\boldsymbol{\theta} = \tau^2 \left[\frac{1}{N} \sum_{i=1}^N \mathbf{S}_i \right] \cdot \mathbf{W} \cdot \left[\frac{1}{N} \sum_{i=1}^N \mathbf{S}_i \right]'$$

Q.E.D.

Proof of Proposition 3.

The average precision of investors' information is $\mu(\Psi + Qs) + (1 - \mu)(\Psi + Q_1s + \theta)$. It remains to solve for θ , the precision of the incremental information provided by price, \tilde{u} . To expedite the proof, first we appeal to the (general) expression for $\tilde{\mathbf{P}}$ as given in eqn. (A2)

$$\tilde{\mathbf{P}} = \frac{1}{\frac{1}{N}\mathbf{\Pi}_0} \frac{\Psi\mathbf{m} + \eta\tilde{\mathbf{y}} + \frac{1}{N}\sum_{i=1}^N \mathbf{S}_i\tilde{\mathbf{z}}_i + \theta\tilde{\mathbf{u}} - \frac{1}{\tau}\tilde{\mathbf{x}}_0}{1 + R_f}. \quad (\text{A2})$$

In the EOH setting, and in particular when firms' cash flows are independent, this expression for $\tilde{\mathbf{P}}$ implies that the price of firm j , P_j , can be written as

$$\tilde{P}_j = \frac{(N_1 + N_2)\Psi_j m_j + (N_1 + N_2)s_j\tilde{Y}_j + N_1\theta_j\tilde{u} + N_2s_j\tilde{Z}_j - \frac{1}{\tau}\tilde{x}_j}{(1 + R_f)(N_1(\Psi_j + Q_1s_j + \theta_j) + N_2(\Psi_j + (Q_1 + Q_2)s_j))}, \quad (\text{A3})$$

where, consistent with EOH: 1) s_j describes the precision of both public and private information (i.e., here $\eta_j \equiv s_j$); 2) only "more informed" investors have private information, so $\sum_{i=1}^N \mathbf{S}_i\tilde{\mathbf{z}}_i$ in eqn. (A2) reduces to $N_2s_j\tilde{Z}_j$ in this setting; and 3) only "less informed" investors condition their expectations on price, so $\theta\tilde{\mathbf{u}}$ in eqn. (A2) reduces to $N_1\theta_j\tilde{u}$ in this setting. Henceforth, to ease the notational burden, we suppress the subscript j in eqn. (A3). Recall that \tilde{u} is defined as

$$\tilde{u} = \frac{\tilde{P} - a - c\tilde{Y} + \bar{x}(d - e)}{bQ_2} = \frac{\tilde{Z}}{Q_2} - \frac{d\tilde{x} - \bar{x}}{bQ_2}.$$

Substituting this into eqn. (A3) yields

$$\begin{aligned} \tilde{P} &= \frac{(N_1 + N_2)\Psi m + (N_1 + N_2)s\tilde{Y} + N_1\theta\left(\frac{\tilde{Z}}{Q_2} - \frac{d\tilde{x} - \bar{x}}{bQ_2}\right) + N_2s\tilde{Z} - \frac{1}{\tau}\tilde{x}}{(1 + R_f)(N_1(\Psi + Q_1s + \theta) + N_2(\Psi + (Q_1 + Q_2)s))} \\ &= \frac{(N_1 + N_2)\Psi m + (N_1 + N_2)s\tilde{Y} + \left(\frac{N_1\theta}{Q_2} + N_2s\right)\tilde{Z} - \left(\frac{N_1\theta d}{Q_2 b} + \frac{N_1 + N_2}{\tau}\right)\tilde{x} + N_1\theta\frac{d}{b}\frac{\bar{x}}{Q_2}}{(1 + R_f)(N_1(\Psi + Q_1s + \theta) + N_2(\Psi + (Q_1 + Q_2)s))}. \end{aligned}$$

But recall once again that price is also equal to $P = a + b\tilde{Z} + c\tilde{Y} + dx - e\bar{x}$. Equating coefficients yields

$$\frac{b}{d} = \frac{\frac{N_1\theta}{Q_2} + N_2s}{\frac{N_1\theta d}{Q_2 b} + \frac{N_1 + N_2}{\tau}},$$

or

$$\frac{b}{d} = \frac{N_2}{N} \tau s,$$

where $N \equiv N_1 + N_2$. This implies that

$$\begin{aligned} \text{Var} [\tilde{u}] &= \frac{1}{Q_2 s} + \left(\frac{d}{b}\right)^2 \frac{1}{Q_2^2 w} \\ &= \frac{1}{Q_2 s} + \left(\frac{1}{\frac{N_2}{N} \tau s}\right)^2 \frac{1}{Q_2^2 w}, \end{aligned}$$

where $\theta = \text{Var} [\tilde{u}]^{-1}$. Q.E.D.

Proof of Proposition 4.

The proof of Proposition 4 parallels that of Proposition 3, except here we assume that cash flows are cross-sectionally correlated. Cross-sectional correlation requires that the proof to Proposition 4 be couched in terms of matrices, as opposed to scalars; nonetheless, the underlying logic is identical to that of the proof to Proposition 3. Thus, in the interests of economy we leave this proof for the motivated reader.

Proof of Proposition 5.

Let $\mathbf{\Pi}$ represent investors' homogeneous, total precision matrix. From the proof to Proposition 2, we know that investors' precision is additive in the precision of public information about firms' cash flows (additive in $\boldsymbol{\eta}$). Without loss of generality, assume that firm 1 discloses additional information about its cash flow with error $\tilde{\delta}$, and $\tilde{\delta}$ is independent of all other variables. Let $\mathbf{\Pi}^*$ represent investors' homogeneous, total precision matrix that results from firm 1's additional disclosure, and \mathbf{Cov}^* its inverse: that is, $\mathbf{Cov}^* = \mathbf{\Pi}^{*-1}$. With firm 1's additional disclosure, investors' precision matrix

about firms' cash flows becomes

$$\mathbf{\Pi}^* = \mathbf{\Pi} + \begin{pmatrix} Var[\tilde{\delta}_j] & 0 & \dots & 0 \\ 0 & 0 & \dots & 0 \\ \cdot & \cdot & \cdot & \cdot \\ 0 & 0 & \dots & 0 \end{pmatrix}^{-1} = \begin{pmatrix} \Pi_{1,1} + Var[\tilde{\delta}_j]^{-1} & \Pi_{1,2} & \dots & \Pi_{1,J} \\ \Pi_{2,1} & \Pi_{2,2} & \dots & \Pi_{2,J} \\ \cdot & \cdot & \cdot & \cdot \\ \Pi_{J,1} & \Pi_{J,2} & \dots & \Pi_{J,J} \end{pmatrix},$$

where $\Pi_{j,k}$ represents the j -th, k -th element of the matrix $\mathbf{\Pi}$, and

$$\mathbf{Cov}^* = \begin{pmatrix} \Pi_{1,1} + Var[\tilde{\delta}_j]^{-1} & \Pi_{1,2} & \dots & \Pi_{1,J} \\ \Pi_{2,1} & \Pi_{2,2} & \dots & \Pi_{2,J} \\ \cdot & \cdot & \cdot & \cdot \\ \Pi_{J,1} & \Pi_{J,2} & \dots & \Pi_{J,J} \end{pmatrix}^{-1}.$$

Let $Cov_{j,k}^*$ and $\Pi_{j,k}^*$ represent the j -th, k -th element of the matrices \mathbf{Cov}^* and $\mathbf{\Pi}^*$, respectively. Then

$$\begin{aligned} Cov_{1,j}^* &= \frac{\text{Cofactor}_{j,1} \text{ of } \mathbf{\Pi}^* \text{ matrix}}{\text{Determinant of } \mathbf{\Pi}^* \text{ matrix}} \\ &= \frac{\text{Cofactor}_{j,1} \text{ of } \mathbf{\Pi}^* \text{ matrix}}{\sum_{k=1}^J \Pi_{1,k}^* \cdot \text{Cofactor}_{1,k} \text{ of } \mathbf{\Pi}^* \text{ matrix}} \\ &= \frac{\text{Cofactor}_{j,1} \text{ of } \mathbf{\Pi} \text{ matrix}}{\Pi_{1,1}^* \cdot \text{Cofactor}_{1,1} \text{ of } \mathbf{\Pi}^* \text{ matrix} + \sum_{k=2}^J \Pi_{1,k}^* \cdot \text{Cofactor}_{1,k} \text{ of } \mathbf{\Pi}^* \text{ matrix}} \\ &= \frac{\text{Cofactor}_{j,1} \text{ of } \mathbf{\Pi} \text{ matrix}}{Var[\tilde{\delta}_j]^{-1} \cdot \text{Cofactor}_{1,1} \text{ of } \mathbf{\Pi} \text{ matrix} + \sum_{k=1}^J \Pi_{1,k} \cdot \text{Cofactor}_{1,k} \text{ of } \mathbf{\Pi} \text{ matrix}} \\ &= \frac{\text{Cofactor}_{j,1} \text{ of } \mathbf{\Pi} \text{ matrix}}{\sum_{k=1}^J \Pi_{1,k} \cdot \text{Cofactor}_{1,k} \text{ of } \mathbf{\Pi} \text{ matrix}} \\ &= \frac{Var[\tilde{\delta}_j]^{-1} \cdot \text{Cofactor}_{1,1} \text{ of } \mathbf{\Pi} \text{ matrix}}{\sum_{k=1}^J \Pi_{1,k} \cdot \text{Cofactor}_{1,k} \text{ of } \mathbf{\Pi} \text{ matrix}} + 1 \\ &= \frac{Var[\tilde{\delta}_j]}{Cov_{1,1} + Var[\tilde{\delta}_j]} Cov_{1,j} \\ &= \frac{Var[\tilde{\delta}_j]}{Var[\tilde{V}_1] + Var[\tilde{\delta}_j]} Cov_{1,j}. \end{aligned}$$

Q.E.D.

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