Asymptotic Properties of the Hahn–Hausman Test for Weak-Instruments

Jerry Hausman

James H. Stock

Motohiro Yogo
University of Pennsylvania

Asymptotic Properties of the Hahn–Hausman Test for Weak-Instruments

Abstract

This paper provides weak-instrument asymptotic representations of tests for instrument validity by Hahn and Hausman's (HH) [Hahn, J., Hausman, J., 2002. A new specification test for the validity of instrumental variables. Econometrica 70, 163–189.], and uses these representations to compute asymptotic power against weak or irrelevant instruments. The HH tests were proposed as pretests, and the asymptotic properties of post-test inferences, conditional on the tests failing to reject instrument validity, are also examined.

Disciplines

Econometrics | Finance and Financial Management

Asymptotic Properties of the Hahn-Hausman Test for Weak Instruments

January 2004

Jerry Hausman

Department of Economics, MIT
and the National Bureau of Economic Research

James H. Stock

Department of Economics, Harvard University
and the National Bureau of Economic Research

and

Motohiro Yogo*

Department of Economics, Harvard University

ABSTRACT

This note provides the weak-instrument asymptotic distributions of Hahn and Hausman's (2002) tests for instrument validity. These distributions are used to compute asymptotic rejection rates when instruments are weak and, as a special case, irrelevant. These tests were proposed as pretests, and the asymptotic properties of post-test inferences, conditional on the tests failing to reject instrument validity, are also examined. Monte Carlo simulations show that the weak-instrument asymptotic distributions provide good approximations to the finite sample distributions for samples of size 100.

* We thank Gary Chamberlain and Jin Hahn for helpful discussions, and Jose Tessada for research assistance. This research was supported by NSF grant SBR-0214131.

1. Introduction

Hahn and Hausman (2002; henceforth HH) recently proposed a new test for the validity of inferences based on conventional first-order asymptotics in instrumental variables (IV) regression. Consider the case of a single included endogenous regressor. If the instruments are valid, they reasoned, then standard first-order asymptotics implies that the two stage least squares (TSLS) estimator obtained by regressing one of the endogenous variables, y_1, on the other, y_2, should be close to the reciprocal of the TSLS estimator of the "reverse regression" of y_2 on y_1. Accordingly, HH propose a statistic that is the difference between the forward TSLS estimator and the reciprocal of the reverse TSLS estimator, adjusted for second-order bias and standardized by a second-order expression for the variance of this difference. They also propose a similarly motivated test statistic based on the Nagar (1959) – type bias adjusted TSLS (BTSLS) estimator of Donald and Newey (2001). Hahn and Hausman (2002, 2003a) suggest that a test based on these statistics will reject if one or the other of the conditions for instrument validity fail, that is, if the instruments are weak and/or if they are endogenous.

This note focuses on the first of these possibilities, in which the HH test is used as a test of the null hypothesis that instruments are strong against the alternative that they are weak. Although HH report Monte Carlo results, we are unaware of asymptotic results about the power or consistency of the HH test against weak or irrelevant instruments. Accordingly, Section 2 provides the asymptotic distribution of the HH statistics for the case that sample is large but the instruments are weak or irrelevant. Technically, this entails applying the weak-instrument asymptotics of Staiger and Stock (1997), in which the so-called "concentration parameter," a standard unitless measure of the strength of the instruments and of the quality of the standard large-sample normal approximation (see Rothenberg (1984)), is held constant as the sample size increases. The HH test was proposed as a pretest, and the weak-instrument limiting distribution of the HH statistic is joint with that of k-class estimators obtained in Staiger and Stock (1997); in particular this provides the asymptotic distribution of k-class estimators, conditional on passing the HH pretest (that is, failing to reject the null hypothesis of strong instruments).

Section 3 provides numerical results for asymptotic power functions of the HH test against weak instruments and for the conditional distributions of two *k*-class estimators, the BTSLS estimator and Fuller's (1977) estimator, conditional on passing the HH pretest. Because these results rely on weak-instrument asymptotics, a pertinent question is whether these asymptotic distributions provide good approximations to the finite-sample distribution of the HH statistic and the post-test estimators. Accordingly, Section 4 reports the results of a Monte Carlo study, which finds that the weak-instrument asymptotic distributions provide good approximations to these finite-sample distributions when there are at least 100 (in some cases, fewer) observations.

The scope of this note is limited, and there is room for further work. Although we focus on the case of two endogenous variables, these methods can be applied to the case of multiple endogenous variables. In addition, we examine the power of these tests against weak instruments under the maintained assumption of instrument exogeneity; a complementary exercise would be to examine the power of the HH tests against endogenous instruments.

2. The HH Test Statistics and their Weak-Instrument Asymptotic Distributions

Following HH, consider the IV regression model with a single endogenous regressor:

$$y_1 = y_2\beta + u \tag{1}$$

$$y_2 = Z\Pi + v \tag{2}$$

where y_1 and y_2 are $n\times 1$ vectors of the n observations on the two endogenous variables, Z is a $n\times K$ matrix of observations on the K instrumental variables, β is the unknown scalar coefficient of interest, Π is a $K\times 1$ unknown parameter vector, and u and v are $n\times 1$ vectors of i.i.d. errors with variances σ_u^2 and σ_v^2 and correlation ρ.

2.1 The HH Test Statistics

Let $\hat{\beta}_{LIML}$ denote the LIML estimator of β, let $\hat{\sigma}^2_{u,LIML}$ denote the estimator of σ^2_u based on the LIML residuals, and let $P_Z = Z(Z'Z)^{-1}Z'$ and $M_Z = I_K - P_Z$, where I_K is the $K\times K$ identity matrix. The HH TSLS-based test statistic is

$$m_1 = \hat{d}_1 / \sqrt{\hat{w}_1}\ , \tag{3}$$

where[1]

$$\hat{d}_1 = \sqrt{n}\left[\frac{y_2'P_Z y_1}{y_2'P_Z y_2} - \frac{y_1'P_Z y_1}{y_2'P_Z y_1} + \frac{n^2\hat{\Xi}}{(y_2'P_Z y_2)(y_2'P_Z y_1)}\right]$$

$$\hat{\Xi} = \frac{K-1}{n^2}\left[\frac{y_2'M_Z y_2}{n-K}\left(y_1'P_Z y_1 - (K-1)\frac{y_1'M_Z y_1}{n-K}\right)\right.$$

$$\left.-\frac{y_2'M_Z y_1}{n-K}\left(2y_2'P_Z y_1 - (K-1)\frac{y_2'M_Z y_1}{n-K}\right) + \frac{y_1'M_Z y_1}{n-K}y_2'P_Z y_2\right]$$

$$\hat{w}_1 = \frac{2(K-1)(n-1)^2\hat{\sigma}^4_{u,LIML}[y_2'P_Z y_2 - (K-1)y_2'M_Z y_2/(n-K)]^2}{(n-K)(y_2'P_Z y_2)^2(y_2'P_Z y_1)^2}.$$

The HH Nagar-based test statistic is

$$m_2 = \hat{d}_2 / \sqrt{\hat{w}_2}\ , \tag{4}$$

where

$$\hat{d}_2 = \sqrt{n}\left[\frac{y_2'P_Z y_1 - (K-2)y_2'M_Z y_1/(n-K+2)}{y_2'P_Z y_2 - (K-2)y_2'M_Z y_2/(n-K+2)}\right.$$

$$\left.-\frac{y_1'P_Z y_1 - (K-2)y_1'M_Z y_1/(n-K+2)}{y_2'P_Z y_1 - (K-2)y_2'M_Z y_1/(n-K+2)}\right]$$

$$\hat{w}_2 = \frac{2(K-1)(n-1)^2 \hat{\sigma}^4_{u,LIML}}{(n-K)\hat{\beta}^2_{LIML}[y_2' P_Z y_2 - (K-1) y_2' M_Z y_2 /(n-K)]^2}.$$

Using second-order asymptotics, HH show that m_1 and m_2 have standard normal null distributions. Experience since Hahn and Hausman (2002) was written suggests that the Nagar form of the test (the m_2 statistic) is to be preferred to the TSLS form (m_1); also, the Nagar form does not entail a bias correction, making it easier to apply.

2.2 Weak Instrument Asymptotic Distribution

Following Staiger and Stock (1997), the weak-instrument asymptotic distributions of m_1 and m_2 are obtained by modeling the coefficient matrix Π as local to zero, specifically, by setting $\Pi = C/\sqrt{n}$, where C is a fixed matrix. Under this nesting, the concentration parameter is

$$\mu^2 = C' Q_{ZZ} C / \sigma_v^2, \tag{5}$$

where $Q_{ZZ} = E(Z'Z/n)$. If $\mu^2 = 0$, then the instruments are irrelevant and β is unidentified.

Define the 2×2 matrices $\bar{\Sigma}$ and B, where $\bar{\Sigma}_{11} = \bar{\Sigma}_{22} = 1$ and $\bar{\Sigma}_{12} = \bar{\Sigma}_{21} = \rho$ and where $B_{11} = \mu^2$ and $B_{12} = B_{21} = B_{22} = 0$. Define Ψ to be a 2×2 random matrix with a noncentral Wishart distribution with K degrees of freedom, covariance matrix $\bar{\Sigma}$, and noncentrality matrix B, and denote the elements of Ψ as

$$\Psi = \begin{bmatrix} \nu_1 & \nu_2 \\ \nu_2 & \nu_3 \end{bmatrix}. \tag{6}$$

It follows from Lemma A1 and Theorem 1 in Staiger and Stock (1997) that the following limits hold jointly:

---

[1] The expression for $\hat{\Xi}$ given here is obtained by substituting $\hat{\alpha}/(1-\hat{\alpha}) = (K-1)/(n-$

$$(y_1'P_Zy_1, y_2'P_Zy_1, y_2'P_Zy_2) \xrightarrow{d} (\sigma_u^2 H_1, \sigma_u\sigma_v H_2, \sigma_v^2 \nu_1), \tag{7}$$

$$(y_1'M_Zy_1/n, y_2'M_Zy_1/n, y_2'M_Zy_2/n) \xrightarrow{p} (\sigma_u^2 J_1, \sigma_u\sigma_v J_2, \sigma_v^2),$$

$$\hat{\beta}_{LIML} \xrightarrow{d} \sigma_u(\bar{\beta} + \Delta_{LIML})/\sigma_v, \text{ and}$$

$$\hat{\sigma}^2_{LIML} \xrightarrow{d} \sigma_u^2 S_{LIML},$$

where $\bar{\beta} = \sigma_v\beta/\sigma_u$, $H_1 = \bar{\beta}^2 \nu_1 + 2\bar{\beta}\nu_2 + \nu_3$, $H_2 = \bar{\beta}\nu_1 + \nu_2$, $J_1 = \bar{\beta}^2 + 2\rho\bar{\beta} + 1$, $J_2 = \bar{\beta} + \rho$, $\Delta_{LIML} = (\nu_2 - \rho\kappa^*_{LIML})/(\nu_1 - \kappa^*_{LIML})$, $S_{LIML} = 1 - 2\rho\Delta_{LIML} + \Delta^2_{LIML}$, and κ^*_{LIML} is the smallest root of $\det(\Psi - \kappa\bar{\Sigma}) = 0$.

Substitution of the expressions in the preceding paragraph into (3) and (4) yields

$$m_1 \xrightarrow{d} \frac{|H_1|}{\sqrt{2(K-1)}S_{LIML}|1-(K-1)/\nu_1|}\left[\frac{H_2}{\nu_1} - \frac{H_1}{H_2} + \frac{\Xi^*}{\nu_1 H_2}\right] \text{ and} \tag{8}$$

$$m_2 \xrightarrow{d} \frac{|\bar{\beta}+\Delta_{LIML}||\nu_1-(K-1)|}{\sqrt{2(K-1)}S_{LIML}}\left[\frac{H_2-(K-2)J_2}{\nu_1-(K-2)} - \frac{H_1-(K-2)J_1}{H_2-(K-2)J_2}\right], \tag{9}$$

where $\Xi^* = (K-1)\{H_1 - (K-1)J_1 - J_2[2H_2 - (K-1)J_2] + J_1\nu_1\}$.

Remarks.

1. Both test statistics m_1 and m_2 have $O_p(1)$ limits. This suggests that neither test will reject with probability one asymptotically, regardless of the value of μ^2, and in particular that neither test is consistent against nonidentification.
2. Like the limiting representation (8) and (9) for the m_1 and m_2 statistics, the limiting representations for k-class estimators and test statistics obtained in Staiger and Stock (1997) follow from (7) and the subsequent (joint) limits. It follows that the limiting representations for k-class estimators and test statistics obtained in Staiger and Stock (1997) are joint with (8) and (9), which in turn

K), as used in HH equation (3.8), into the expression for $\hat{\Xi}$ following HH equation (3.5).

makes it possible to evaluate numerically the distribution of a k-class statistic, conditional on passing the HH pretest (for example, conditional on $|m_1| \leq 1.645$).

3. We followed HH by defining the m_1 and m_2 statistics using LIML to estimate incidental parameters in the second-order expressions, hence the appearance of $\hat{\beta}_{LIML}$ and $\hat{\sigma}^4_{u,LIML}$ in the definitions following (3) and (4). Other IV estimators can be used to estimate these nuisance parameters, however, and in fact numerical work suggests that LIML might not be the best choice because it is prone to outliers when instruments are weak. It follows from the previous remark that weak-instrument limiting representations akin to (8) and (9) can be obtained using the joint limiting representation of a k-class estimator used instead of LIML to calculate the incidental parameters in m_1 and m_2 (we do not provide these limiting expressions here to conserve space). Because LIML and other estimators are not consistent under weak-instrument asymptotics, the weak-instrument asymptotic distribution of m_1 and m_2 in general depends on the choice of estimator used to calculate the incidental parameters.

3. Asymptotic Power and Post-Test Estimator Performance: Numerical Results

This section evaluates the asymptotic properties of the HH tests from two perspectives: as a test for weak instruments, as measured by small values of the concentration parameter; and as a pretest in which the object of interest is subsequent post-test inferences based on an IV estimator. The numerical results were computed by Monte Carlo evaluation of the weak-instrument limits (8) and (9) using 20,000 Monte Carlo draws of the noncentral Wishart random variable Ψ. Following HH (footnote 5), we set $\mathrm{var}(y_{1i}|Z_i) = 1$, $\sigma_v^2 = \mathrm{var}(y_{2i}|Z_i) = 1$, and $\bar{\beta} = -2\rho$ so that $\sigma_u^2 = 1$. With this normalization, the distributions of m_1 and m_2 depend only on K, μ^2, and ρ. Throughout, Fuller's (1977) estimator with c =1 is used to calculate the incidental parameters in m_1

and m_2, that is, with $\hat{\beta}_{Fuller}$ and $\hat{\sigma}^2_{u,Fuller}$ replacing $\hat{\beta}_{LIML}$ and $\hat{\sigma}^2_{u,LIML}$ in the definitions following (3) and (4).[2]

3.1 Asymptotic Power for Small Values of the Concentration Parameter

One definition of weak instruments is that instruments are weak when the concentration parameter is sufficiently small that conventional first-order asymptotics could result in misleading inferences (for further discussion see the survey by Stock, Wright, and Yogo (2002)). Given this definition, the power of the HH test against weak instruments can be assessed by computing the rejection rate as a function of μ^2/K and ρ; power should be high when μ^2/K is small or zero and should equal the size of the test when μ^2/K is large.

Asymptotic rejection rates of the two HH tests, at the 10% significance level, are summarized in Figure 1 as a function of μ^2/K for K = 5 and 30 and for ρ = .9 and .5. As a reference, the figure also plots the bias of the TSLS estimator; under the normalization $\sigma^2_u = \sigma^2_v = 1$ used here, the probability limit of the OLS estimator of β is ρ, which is also the asymptotic bias of the TSLS estimator in the unidentified case $\mu^2/K = 0$. For example, for $K = 5$ and $\rho = .5$ (the second panel), when $\mu^2/K = 2$ the bias of TSLS is .13, so the bias of TSLS, relative to the inconsistency of OLS, is .13/.50 = 26%. Thus, for values of μ^2/K in the range plotted, TSLS bias typically is substantial.

For the cases considered in Figure 1, the asymptotic power of the 10% HH tests against $\mu^2/K < 2$ ranges from 8% to 34%. Generally speaking, the two tests perform similarly. We have considered other values of K, ρ, and $\bar{\beta}$, and the highest rejection rate we found was 34% (we did not conduct an exhaustive search however). For 5% HH tests, the highest rejection rate we found was 27%.

3.2 Asymptotic Performance as a Pretest

---

[2] Fuller's estimator with c =1 has moments for all values of μ^2/K. Hahn, Hausman, and Kuersteiner (2003) provide extensive numerical documentation of the good performance of this estimator, relative to other prominent IV estimators, under weak instruments.

HH developed the m statistics to be used as a pretest: if the test fails to reject, then inference should proceed using an estimator that has good second-order properties, such as LIML or BTSLS (HH, p. 179) or Fuller's estimator. Viewed thus, the appropriate way to assess the performance of the statistics is to examine the reliability of post-test inferences based on first-order asymptotics for LIML, BTSLS, or Fuller's estimator, conditional on the HH test failing to reject. Because LIML can produce large outliers, we focus on post-test inferences – point estimates and hypothesis tests – based on BTSLS and Fuller's estimator with $c = 1$.[3]

Figure 2 presents the asymptotic median bias of BTSLS (a) conditional on $|m_1| \leq 1.645$ and (b) conditional on $|m_2| \leq 1.645$ (we consider median instead of mean bias because moments of BTSLS are not guaranteed to exist). As a benchmark, the figure also presents the unconditional asymptotic median bias of BTSLS, that is, the bias that would arise from using BTSLS without any pretest. The conditional asymptotic distribution of the BTSLS estimator was computed by drawing (by Monte Carlo) from the joint weak-instrument asymptotic distribution of m_1, m_2, and the BTSLS estimator. As in Figure 1, the bias is presented as a function of μ^2/K for various values of K and ρ. Figure 3 presents the corresponding asymptotic median bias of Fuller's ($c = 1$) estimator. Comparing the (unconditional) TSLS bias in Figure 1 to the BTSLS and Fuller unconditional median bias in Figures 2 and 3 reveals that the BTSLS and Fuller estimators have substantially less bias than TSLS, at least for $\mu^2/K > 2$. The median bias of the Fuller estimator is close to that of BTSLS, in some cases slightly larger, in others, slightly smaller. There is essentially no difference between the conditional and unconditional BTSLS median bias curves, that is, the median bias of the BTSLS and Fuller estimators is essentially the same unconditionally or conditional on passing the HH pretest.

Figure 4 presents the asymptotic null rejection rate (the asymptotic size) of a nominal 5% Wald test of the hypothesis $\beta = \beta_0$ based on the BTSLS estimator and its

---

[3] An estimator that could be used in the event that the HH statistic rejects is OLS, which can have lower MSE than TSLS if the instruments are invalid (see the discussion in Hahn and Hausman (2003b)), however here we do not examine inference conditional on failing the HH pretest.

standard error (computed using the standard k-class formula), both unconditionally and conditional on passing the HH test. Analogous asymptotic null rejection rates are presented in Figure 5 for the Wald test based on the Fuller (c = 1) estimator. For small values of μ^2/K, the size distortions in both Wald tests can be substantial, especially in the ρ = .9 case, although the size distortions using either the BTSLS or Fuller Wald tests are much less than for the Wald test based on TSLS (Stock and Yogo (2002)). As in Figures 2 and 3, the conditional and unconditional Wald test size curves in Figures 4 and 5 are essentially the same.

4. Monte Carlo Results

The foregoing conclusions were based on the weak-instrument asymptotic distribution of the HH and k-class statistics. Here, we briefly summarize the results of a Monte Carlo experiment that examines whether the weak-instrument asymptotic distributions provide a good approximation to the finite-sample distributions of the HH statistic and to selected k-class statistics, conditional on passing the HH pretest. The finite-sample results were computed using 1000 Monte Carlo draws for the system (1) and (2) with i.i.d. normal errors; the parameter settings are the same as described in the first paragraph of Section 3. The HH pretest is based on comparing the forward and reverse BTSLS estimator (the m_2 statistic), where the incidental parameters are estimated using the Fuller (c = 1) estimator.

The results are summarized in Table 1 (only a subset of the results are reported to save space). First consider the "HH Rejection Rate" column. For a given value of K and μ^2/K, the finite sample rejection rates of the m_2 statistic are close to each other and to the asymptotic limit for all values of n; by n = 100, the finite-sample rates generally are within Monte Carlo error of the asymptotic rejection rates.

The final six columns of Table 1 report the RMSE of three k-class estimators, the Fuller (c = 1) estimator, LIML, and the BTSLS estimator, both unconditionally and conditional on passing the pretest (that is, conditional on $|m_2| \leq 1.645$). First consider the results for the Fuller (c = 1) estimator. The finite-sample RMSE is in most cases close to the asymptotic RMSE for $n = 50$, and in all cases is close for $n = 100$. Consistent with the asymptotic computations in Section 3, the Monte Carlo results confirm that, for $K = 5$, the Fuller ($c = 1$) RMSE is essentially the same unconditionally and conditional on passing the pretest. For $K = 30$ in the nearly unidentified case ($\mu^2/K = .5$), the RMSE of the Fuller estimator is approximately 6% less conditional on passing conditional on passing the pretest, compared to the unconditional RMSE; when $\mu^2/K = 2$, the conditional and unconditional RMSEs of the Fuller estimator are essentially the same. The remaining four columns are more difficult to interpret because moments are not guaranteed to exist for LIML or for BTSLS, so these entries should be viewed just as measures of the spread of the sample of estimates obtained in the Monte Carlo draws. In the cases in which the finite-sample RMSEs are small and are comparable across sample sizes, they effectively converge to the asymptotic limit by $n = 100$.

Comparing RMSEs across estimators reveals that the RMSEs for Fuller ($c = 1$) are always the smallest of the three estimators or are nearly so. In several cases, the RMSEs of LIML and BTSLS are very large, indicative of nonexistent moments. A practical implication, consistent with the extensive simulation results in Hahn, Hausman, and Kuersteiner (2003), is that using LIML and BTSLS in situations with weak instruments can yield very poor estimates because of the presence of outliers, and that inference based on the Fuller estimator is preferable when instruments are weak. The HH pretest appears to be successful at screening severe LIML and BTSLS outliers. Even so, within Monte Carlo error, the RMSE of the unconditional Fuller estimator is never greater than, and typically much less than, the RMSE of LIML and BTSLS, conditional on passing the HH pretest.

References

Donald, S.G. and W.K. Newey (2001): "Choosing the Number of Instruments," *Econometrica* 69, 1161 – 1191.

Fuller, W.A. (1977), "Some Properties of a Modification of the Limited Information Estimator," *Econometrica* 45, 939 – 954.

Hahn, J. and J. Hausman (2002): "A New Specification Test for the Validity of Instrumental Variables," *Econometrica* 70, 163 – 189.

Hahn, J. and J. Hausman (2003a): "Weak Instruments: Diagnosis and Cures in Empirical Economics," *American Economic Review* 93, No. 2 (May), 118 – 125.

Hahn, J. and J. Hausman (2003b), "Instrumental Variable Estimation with Valid and Invalid Instruments," manuscript, MIT.

Hahn, J., J. Hausman, and G. Kuersteiner (2003): "Estimation with Weak Instruments: Accuracy of Higher Order Bias and MSE Approximations," manuscript, MIT.

Nagar, A.L. (1959): "The Bias and Moment Matrix of the General *k*-Class Estimators of the Parameters in Simultaneous Equations," *Econometrica* 27, 575 – 595.

Rothenberg, T.J. (1984): "Approximating the Distributions of Econometric Estimators and Test Statistics," Ch. 15 in *Handbook of Econometrics*, *Vol. II*, ed. by Z. Griliches and M.D. Intriligator. Amsterdam: North Holland, 881 – 935.

Staiger, D. and J.H. Stock (1997): "Instrumental Variables Regression with Weak Instruments," *Econometrica* 65, 557 – 586.

Stock, J.H. and M. Yogo (2002), "Testing for Weak Instruments in Linear IV Regression," forthcoming, *Identification and Inference in Econometric Models: Essays in Honor of Thomas J. Rothenberg,* D.W.K. Andrews and J.H. Stock (eds.), Cambridge University Press.

Stock, J.H., J.H. Wright, and M. Yogo (2002): "A Survey of Weak Instruments and Weak Identification in Generalized Method of Moments," *Journal of Business and Economic Statistics* 20, 518 – 529.

Table 1

Monte Carlo Comparison of Finite Sample and Weak-Instrument Asymptotic Distributions:

m_2 HH Test Rejection Rates and Estimator RMSE's with and without HH Pretest

					HH Test Rejection Rate	RMSE for Fuller (c = 1)		RMSE for LIML		RMSE for BTSLS	
K	μ^2/K	ρ	n	R^2		Uncond'l	Cond'l	Uncond'l	Cond'l	Uncond'l	Cond'l
5	0.5	0.5	50	0.0476	0.128	0.579	0.578	24.027	5.395	196.478	1.310
5	0.5	0.5	100	0.0244	0.135	0.585	0.570	91.599	13.203	5.893	1.512
5	0.5	0.5	500	0.0050	0.139	0.593	0.585	9.663	7.702	6.666	1.516
5	0.5	0.5	∞		0.135	0.596	0.592	79.091	62.704	28.725	1.878
5	2.0	0.5	50	0.1667	0.098	0.351	0.339	2.535	0.630	1.175	0.671
5	2.0	0.5	100	0.0909	0.078	0.365	0.348	1.523	0.695	0.915	0.657
5	2.0	0.5	500	0.0196	0.085	0.354	0.339	4.365	4.084	2.815	0.658
5	2.0	0.5	∞		0.089	0.361	0.349	5.940	2.537	18.439	0.726
5	0.5	0.9	50	0.0476	0.221	0.542	0.547	6.205	4.565	5.374	1.742
5	0.5	0.9	100	0.0244	0.205	0.573	0.577	8.438	4.612	14.620	0.929
5	0.5	0.9	500	0.0050	0.240	0.550	0.548	18.152	1.771	9.527	0.935
5	0.5	0.9	∞		0.223	0.574	0.580	503.857	82.400	41.288	1.134
5	2.0	0.9	50	0.1667	0.099	0.284	0.283	1.217	0.711	8.973	0.682
5	2.0	0.9	100	0.0909	0.088	0.265	0.258	2.039	1.018	9.826	0.598
5	2.0	0.9	500	0.0196	0.101	0.267	0.266	1.298	0.710	9.506	0.677
5	2.0	0.9	∞		0.097	0.271	0.261	6.400	0.587	21.134	0.774

Table 1, ctd.

K	μ^2/K	ρ	n	R^2	HH Test Rejection Rate	RMSE for Fuller (c = 1)		RMSE for LIML		RMSE for BTSLS	
						Uncond'l	Cond'l	Uncond'l	Uncond'l	Cond'l	Uncond'l
30	0.5	0.5	100	0.1304	0.065	0.532	0.498	3.538	2.652	2.515	1.554
30	0.5	0.5	200	0.0698	0.074	0.496	0.468	3.041	2.181	2.871	1.597
30	0.5	0.5	500	0.0291	0.087	0.510	0.475	1.943	0.989	3.152	2.523
30	0.5	0.5	∞		0.079	0.481	0.451	20.159	20.554	7.739	1.710
30	2.0	0.5	100	0.3750	0.069	0.174	0.171	0.187	0.181	0.189	0.188
30	2.0	0.5	200	0.2308	0.088	0.157	0.153	0.162	0.158	0.177	0.176
30	2.0	0.5	500	0.1071	0.116	0.158	0.152	0.165	0.157	0.179	0.178
30	2.0	0.5	∞		0.085	0.156	0.154	0.162	0.160	0.177	0.175
30	0.5	0.9	100	0.1304	0.095	0.320	0.304	12.896	0.974	33.008	1.201
30	0.5	0.9	200	0.0698	0.094	0.280	0.264	15.730	3.009	5.156	1.401
30	0.5	0.9	500	0.0291	0.126	0.296	0.285	0.968	0.886	20.550	1.377
30	0.5	0.9	∞		0.107	0.285	0.265	1.429	0.514	26.292	1.552
30	2.0	0.9	100	0.3750	0.079	0.138	0.139	0.147	0.147	0.221	0.199
30	2.0	0.9	200	0.2308	0.087	0.138	0.137	0.146	0.145	0.205	0.189
30	2.0	0.9	500	0.1071	0.126	0.137	0.135	0.145	0.143	0.216	0.186
30	2.0	0.9	∞		0.092	0.139	0.139	0.147	0.148	0.206	0.193

Notes: The "HH Rejection Rate" is the fraction of times that the m_2-based HH test, calculated using the Fuller (c = 1) estimator for the incidental parameters, rejects at the 10% significance level (that is, $|m_2| > 1.645$). The final six columns report the RMSE of the indicated estimator, either unconditionally (without a pretest) or conditional on passing the HH pretest (that is, if $|m_2| \leq 1.645$). The finite-sample results were computed by Monte Carlo using 1000 draws, using the design described in the text; the results for $n = \infty$ were computed using 20,000 draws from the weak-instrument asymptotic distribution.

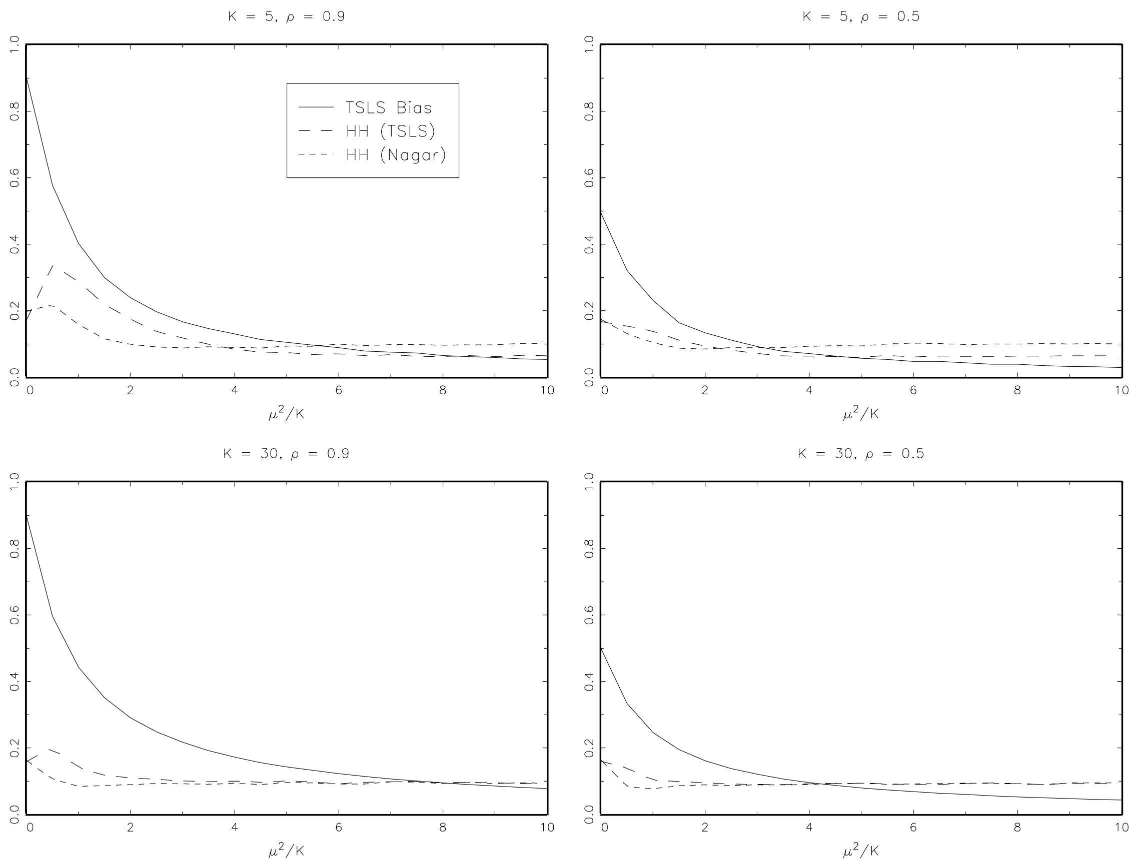

Figure 1. Asymptotic power of 10% HH tests using m_1 ("TSLS") and m_2 ("Nagar") against weak instruments, and the asymptotic bias of TSLS (solid line), as a function of the concentration parameter divided by the number of instruments (μ^2/K)

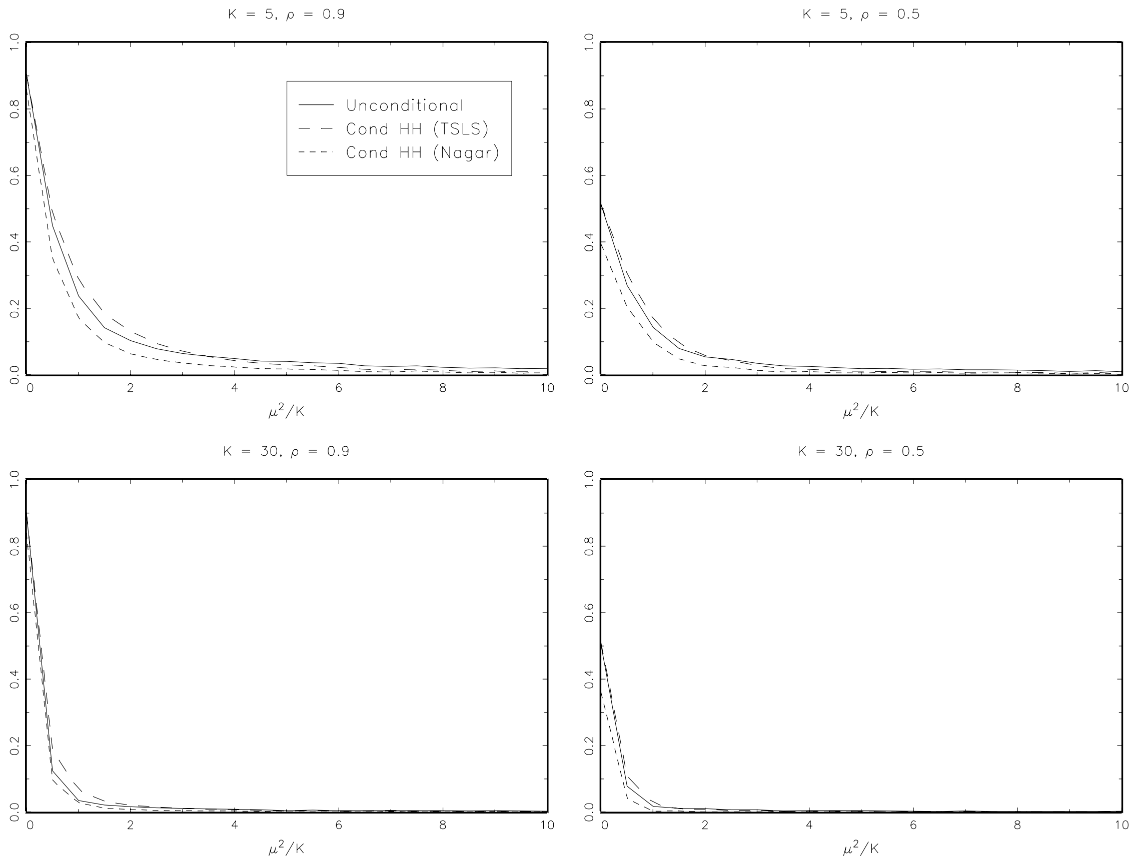

Figure 2. Asymptotic median bias of BTSLS, conditional on acceptance of a 10% HH pretest

"Cond HH (TSLS)" is conditional on $|m_1| \leq 1.645$

"Cond HH (Nagar)" is conditional on $|m_2| \leq 1.645$

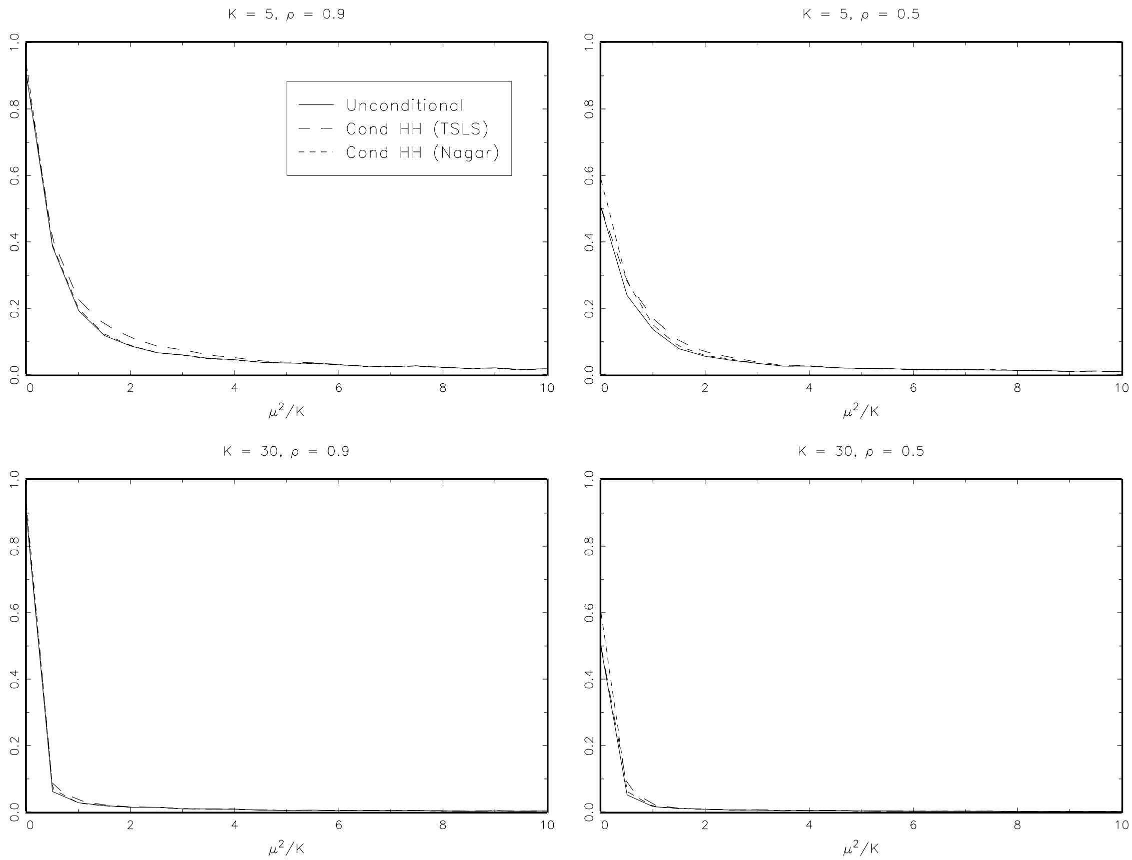

Figure 3. Asymptotic median bias of the Fuller estimator, conditional on acceptance of a 10% HH pretest

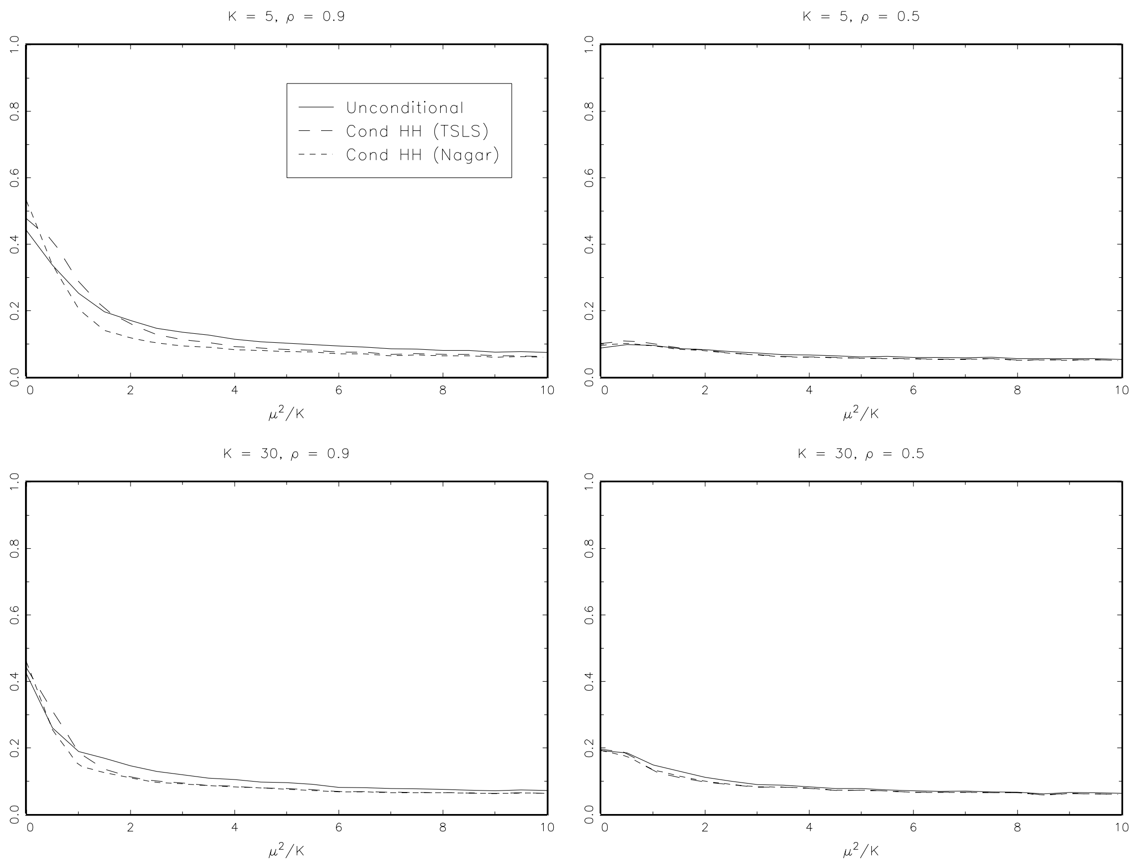

Figure 4. Asymptotic size of the nominal 5% BTSLS-based Wald test of $\beta = \beta_0$, conditional on acceptance of 10% HH pretest

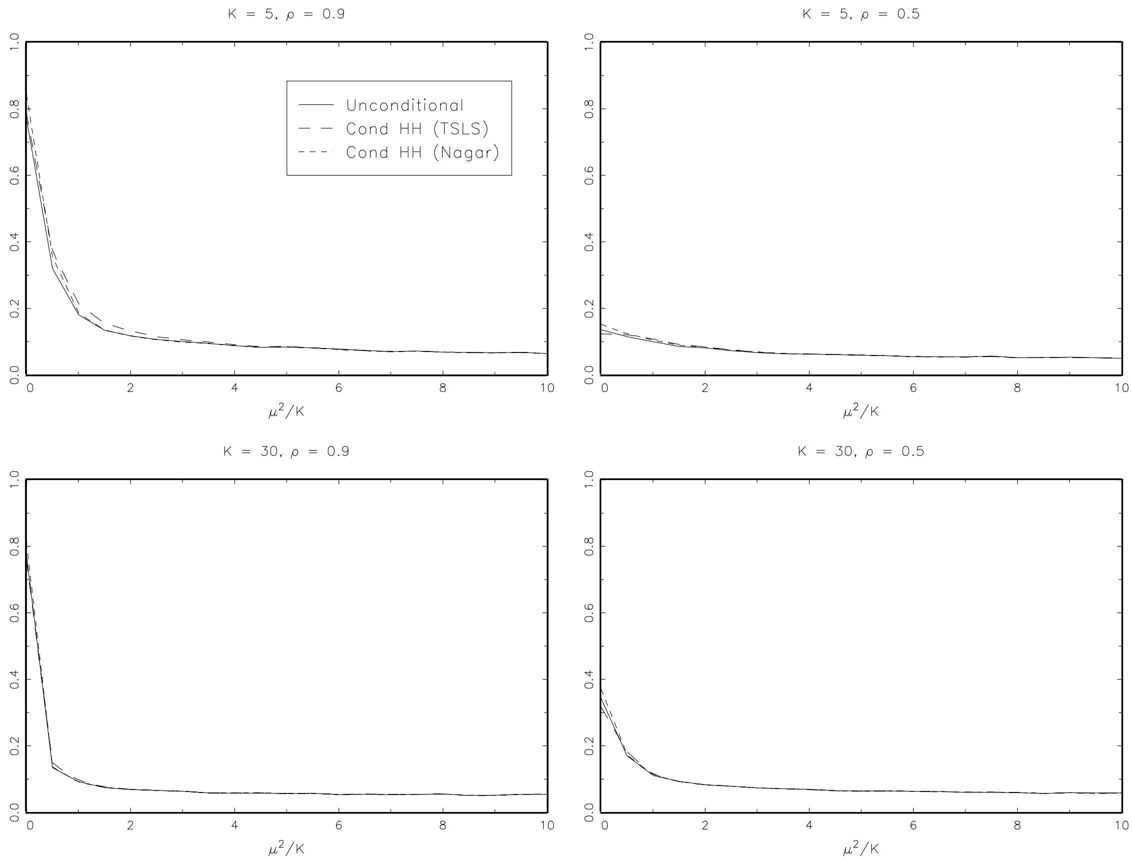

Figure 5. Asymptotic size of the nominal 5% Fuller estimator-based Wald test of $\beta = \beta_0$, conditional on acceptance of 10% HH pretest