Review

Corticonic models of brain mechanisms underlying cognition and intelligence

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Abstract

The concern of this review is brain theory or more specifically, in its first part, a model of the cerebral cortex and the way it: (a) interacts with subcortical regions like the thalamus and the hippocampus to provide higher-level-brain functions that underlie cognition and intelligence, (b) handles and represents dynamical sensory patterns imposed by a constantly changing environment, (c) copes with the enormous number of such patterns encountered in a lifetime by means of dynamic memory that offers an immense number of stimulus-specific attractors for input patterns (stimuli) to select from, (d) selects an attractor through a process of "conjugation" of the input pattern with the dynamics of the thalamo–cortical loop, (e) distinguishes between redundant (structured) and non-redundant (random) inputs that are void of information, (f) can do categorical perception when there is access to vast associative memory laid out in the association cortex with the help of the hippocampus, and (g) makes use of "computation" at the edge of chaos and information driven annealing to achieve all this. Other features and implications of the concepts presented for the design of computational algorithms and machines with brain-like intelligence are also discussed. The material and results presented suggest, that a Parametrically Coupled Logistic Map network (PCLMN) is a minimal model of the thalamo–cortical complex and that marrying such a network to a suitable associative memory with re-entry or feedback forms a useful, albeit, abstract model of a cortical module of the brain that could facilitate building a simple artificial brain.

In the second part of the review, the results of numerical simulations and drawn conclusions in the first part are linked to the most directly relevant works and views of other workers. What emerges is a picture of brain dynamics on the mesoscopic and macroscopic scales that gives a glimpse of the nature of the long sought after brain code underlying intelligence and other higher level brain functions.

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1. Introduction

The term *Corticonics*, coined by M. Abeles in 1991 [1] to describe the neural circuits of the cerebral cortex, conjugates cortex in a manner similar to electron and electronics. Here Corticonics, echoing electronics, refers to the art of identifying salient anatomical and physiological attributes of cortical organization to be abstracted and used in devising algorithms and computational models of the cortex, *cognition* and *intelligence* that are more powerful than what is possible today with neural net and connectionist models. Simply put, intelligence is what the brain does in predicting and deciding what to do next to maximize the chances of an organism to survive in its environment; as long as my brain continues to predict and anticipate what will happen next I will survive. Present day neural nets and connectionist models of the cortex have not been very effective in duplicating higher-level brain function, especially the ability of the brain to process and attach meaning to dynamic input patterns. These are spatio-temporal signals furnished by sensory organs responding to the influence of a continually changing, complex, and uncontrolled environment or alternately due to dynamic probing and exploration of the environment by touch, taste, smell or vision, the latter via saccadic eye movement. The reason for this limitation is the simplistic transfer-function description of processing elements used in most present day neural net and connectionist models which, as will be appreciated below, may not adequately represent the way a cortical patch reacts to sensory information relayed to it by the thalamus or to inputs it receives from other parts of the cortex itself. The importance of being able to process dynamic sensory patterns is well recognized by some neuroscientists. For example Miguel Nicolelis wrote in [2] the following passage which is reproduced here verbatim because it happens to express eloquently the need for work and results being reported on or reviewed here:

"The fact that sensory cues in the natural environment typically have intricate time-varying properties and that animals can perceive such sensory stimuli very rapidly suggest that the mammalian central nervous system is capable of processing complex time-dependent signals very efficiently. For example, speech and other vocalization signals have unique spectro-temporal signatures in which the timing and order of basic components (e.g. syllables, phonemes, motifs, etc.) play a critical role in distinguishing the meaning of different sounds. Thus, neural circuits have to decode the temporal structure of these signals in order to allow the subject to perceive and respond adaptively to them". According to Nicolelis, any theory of perception in the central nervous system, i.e., a theory of brain function, must be able to: (a) Accommodate dynamic (spatio-temporal) environmental stimuli, and (b) Form suitable representations for the large number of stimuli an organism is bound to encounter in its lifetime.

The brain's cortex is the seat of all higher-level brain functions such as cognition, thought, language, learning, control of complex motor function and quite likely the more esoteric attributes of awareness and consciousness. Better understanding of cortical dynamics will have profound scientific, technological and economical implications not to mention clinical benefits. This review discusses and demonstrates first the advantages and effectiveness of a novel approach to modeling the cortex which relies on a paradigm different than that used in the conventional neural net and connectionist models. We will do this by adopting a distinctly different view-point than the transfer-function and stimulus-response paradigm mentioned earlier. This approach views the cortex as a high-dimensional nonlinear dynamical system with a default state of continual activity because of extensive feedback and reentrant signaling (see how difficult it is to halt the dynamics by trying not to think of anything or to hold onto a thought). The effect of extrinsic (sensory) input patterns, which are usually dynamic, is to perturb the default pattern of activity in the system's state-space leading to behavioral changes through self-organization, adaptation, and learning. Accordingly the behavior of the cortex is viewed as stemming from "conjugation" of the extrinsic stimulus with the on-going internal dynamics of the system that furnishes the context within which the sensory input gets processed and interpreted. It is clear that this paradigm and that used in conventional recurrent neural networks, where the input or stimulus is normally a static initial condition and the processing elements are quiescent until an input stimulus is received, are different and one may therefore expect it to lead to systems with novel capabilities. This review provides evidence supporting this expectation. For the nonlinear dynamics theorist, the new paradigm advanced here, may conjure a state-space picture in which a dissipative high-dimensional nonlinear dynamical system, such as the brain's cortex, "floats" in its state-space carrying out the representation, encoding, dynamic memory, learning and interpretation of extrinsic sensory patterns rapidly and efficiently in a manner mostly different than in present day recurrent neural net and brain models that rely on seeking minima in the energy landscape. The approach to modeling the cortex given here combines concepts and ideas from neuroscience, neurophysiology, psychology, nonlinear dynamics and information theory to offer a promising new way to represent, process, classify, learn, and recognize (label) spatio-temporal signals (dynamic patterns) of the kind encountered by the brain and by would be man made intelligent systems.

It is worth noting that at first glance, the mixing of dynamical systems which can be chaotic and operating far-fromequilibrium at the edge-of-chaos, and the equilibrium thermodynamics on which information theory is based may seem contradictory. The work reviewed here indicates differently and gives evidence that much in fact can be gained from a judicial mix of the two. This is supported by the work of Egolf [3] who shows that the macroscopic behavior of some far-from-equilibrium systems might be actually understood in terms of equilibrium statistical mechanics.

An important question about brain function, one can ask at this point in relation to the preceding discussion concerns the role of attractors of dynamical systems in cortical cognitive processes. The most obvious role for attractors is to make it possible to operate on or utilize the activity trace left by a stimulus after it has changed or vanished. Because a high-dimensional nonlinear dynamical system like the brain can exhibit in principle all types of attractors in its state space one may also ask: how can one compute with diverse attractors?; is the appearance of a particular attractor associated with the recognition of a particular object or stimulus?; is the brief settling of transitory cortical activity into an attractor state synonymous with recognition?, is the persistent attractor activity while it lasts needed and how does it participate for example in the forming of lasting memory? Answers to these and other relevant questions are attempted and given in this paper.

More specifically, our goal is to present a macroscopic dynamical theory of the cortex in which populations of coupled neurons form basic functional units (netlets, cortical columns, or modules) with emergent functional properties, that are mathematically modeled by *parametrically coupled logistic maps* (PCLMs). The concept of PCLM has the advantage that instead of solving systems of coupled nonlinear differential equations representing the individual neurons in the netlet or column, it reduces the complex fine-grained dynamics to a more tractable coarse-grained model, that of a PCLM, governed by a difference equation. Computationally, this is far more efficient and opens the way for numerical investigation of the spatio-temporal dynamics of large assemblies of PCLMs and in particular to investigating the consequences of modeling the thalamo–cortical complex with networks of interacting PCLMs. Plenty of evidence is available to justify this approach (see Section 4), but the strongest comes ultimately from the results of numerical studies presented in Section 6 that give clear indication of its viability and promise despite the high-level of abstraction of cortical organization it involves. The supporting evidence given, shows that the class of *parametrically*

coupled logistic map networks (PCLMNs) studied, exhibits remarkable *corticomorphic* mechanisms. This includes: the handling of both static or dynamic (spatio-temporal) input patterns, self-organization and autonomous adaptation driven by mutual-information (MI) which is an information theoretic measure of the flow of information between elements of the network, emergence of stimulus (input)—specific attractors in the form of isolated clusters of activity, formation of Hebbian assemblies reminiscent to the hot spots of brain activity routinely pinpointed by functional magnetic resonance imaging (fMRI), dynamic memory, discrimination between redundant (structured) information-bearing and non-redundant stimulus patterns that are void of information, provision of an immense number of coexisting attractors available for input patterns to draw from, computing with diverse attractors, and a role for synchronicity, bifurcation, symmetry-breaking, and chaos in the operation of this new class of networks.

The concept of a recurrent network with dynamic connections and parametrically coupled recursive processing elements, also called bifurcation processing elements or maps, was introduced in 1996 by Farhat and del Moral Hernandez [4] while the addition of nonlinear activity-dependent coupling and introduction of the concept of *corticonic network* and the set of *corticonic equations* discussed below, were made by Farhat [5,6]. Dynamic connectivity has also been used by others (see for example [7]). Parametrically coupled map networks differ from Kaneko's coupled map lattices [8], which use Turing type diffuse coupling in that the coupling between elements in the PCLMN is not direct but indirect through the control parameters or bifurcation parameters of the maps. Here, the control parameter of a map is a function of the extrinsic sensory input (stimulus) and the intrinsic activity of the network, the latter via nonlinear coupling from neighboring elements and includes self-coupling. A variety of parametrically coupled map networks of this type or ones that utilize linear coupling have since been described in the literature [9–18].

2. Dynamic environment and modeling the cortex

Sensory cues in natural environment have intricate characteristic time-varying structures. Animals make sense of dynamic sensory stimuli rapidly and this suggests that the mammalian nervous system is adept at processing complex spatio-temporal signals. For example speech and other vocalization signals have unique spectro-temporal signatures which are ultimately decoded and interpreted by higher-level brain centers: the neural circuitry of the cortex. A similar situation holds for the human visual system, where even when the object is not in motion but is stationary, produces dynamic sensory patterns through saccadic eye movements that scan salient features of the object/scene to produce dynamic sensory data that gets relayed to the cortex for interpretation. Similar observations hold for other sensory modalities: hearing, olfaction, touch and taste. Dynamic sensory patterns are the norm in biological information processing systems and the cortex has evolved to be an exquisite "machine" for the processing and making sense of transient neural activity traces produced by such dynamic sensory signals. Dynamic patterns in the form of complex spatio-temporal signals arise also in many artificial systems. Understanding how the cortex handles (represents and interprets) dynamic stimuli is therefore useful for developing new approaches and algorithms for corticomorphic dynamic information processing. The concern therefore is development of a new class of cortically inspired dynamical networks and algorithms that are naturally suited for handling (accepting, classifying, and recognizing) dynamic input patterns. The class of PCLMNs described, which we can also name *corticonic networks*, incorporate attributes of cortical organization and are shown to possess functional properties that make them potentially useful for the design of a new generation of intelligent machines.

The particular example of corticonic network described here incorporates abstractions of known attributes of cortical organization and function. Despite the wide recognition that the cerebral cortex on the macroscale is a highly ordered structure [19–24], the overwhelming complexity of cortical tissue on the microscale of neurons, synapses, axons, and dynamics of linear and nonlinear membrane patches with their voltage and chemically activated ionic channels, provokes despair of ever describing its circuitry and function in any way that would enable developing a computationally efficient microscopic approach to modeling the cortex and its function. Nevertheless, some early attempts in that direction have been made [25] and despite some notable progress in "reverse-engineering" the cortex on sufficiently detailed level [26], it still appears unlikely that a microscopic theory of the brain/cortex will be developed in the foreseeable future. It can be argued that the large number of neurons per millimeter cube of cortical tissue alone (~10⁵) and the dense connections between them (10³ to 10⁴ connections per neuron leading incidentally to approximately 3 km of axonal fiber lengths per millimeter cube) mean that the computational complexity of a detailed model of even such a small volume of cortex is extensive, let alone an entire cortex or even a cortical patch for which it would be enormous. Against this argument there is the contention that if we continue Moore's Law we expect to

have 10^{10} transistors on a single chip by 2018 which suggests 10^{19} computations per second (CPS) at 1 GHz clock rate exceeding the rough estimate of 10^{17} CPS a brain can do with 10^{11} neurons each firing at maximum rate of 10^{2} spikes per second and having 10^{4} synapses per neuron, and it is perhaps only then one can expect that simulating a whole brain on a digital computer may become feasible.

The challenge in modeling a complex system like the cortex is to decide which features of cortical organization to make salient in the model and which features to ignore while preserving the ability of the model to predict behavior similar to that of the cortex. The test of the model lies ultimately in how well it can reproduce, predict, and synthesize cortical functions. An important question, always to ask about any model, is whether the behavior it exhibits is because of its simplification or if it captures the behavior of the thing being modeled despite the simplification. Modeling the cortex too closely may result in an intractable model which has little chance of uncovering over-arching principles [27].

3. Salient features of cortical organization

Fortunately, there are several salient features and principles of cortical organization that are useful for developing a computationally efficient model and macroscopic theory of the cortex. **One** such organizational principle is the view of an unfolded and flattened cortex as a 2-D array of vertical units (vertically oriented columns, or netlets of neurons) (see for example [19–24]). A **second** organizational principle involves the radical hypothesis that: (a) the basic functional unit for higher-level processing in the cortex is not the single neuron but the cortical column [28,29] or netlet [30,31] consisting of a columnar organization of 10^3-10^4 cortical neurons acting as a functional unit and, (b) a netlet has *emergent* functional behavior that is mathematically captured by a parametrically coupled (driven) logistic map (PCLM), a nonlinear iterative map on the unit interval with rich complex behavior, not present in the constituent neurons. (For definition and properties of the logistic map see for example [32]; for the PCLM see below.)

It is all the more remarkable that the wide range of rich and complex corticomorphic behavior of these corticonic networks comes, as will be seen, from a relatively small set of simple but concise mathematical equations. This situation is not uncommon; there are several instances of simple sets of concise equations capturing the complexity of phenomena in a given field. Ultimate examples are Maxwell's equations in electrodynamics, and the Schrödinger equation in quantum mechanics. More relevant recent examples of simple equations and rules capturing rich complex behavior are cellular automata and the complex valued quadratic map that gives rise to one of the most astonishing objects in mathematics: the Mandelbrot set. Indeed, one general goal of research in the subject matter of this review has been to determine the extent to which the simple set of equations governing the dynamics of PCLMNs or corticonic networks described below have similar predictive and synthesizing power as in these examples and can thus serve as foundation for a new brain theory.

A third organizational principle is that cortical columns (modules, netlets) interact via two types of connections both of which go through the thalamus [33]: local connections, i.e. via close-range connections and longer-range cortico-cortical connections that connect cortical columns in different parts of the cortex or a cortical patch via axonal fibers, also called association fibers, that form the white-matter of the brain as depicted in Fig. 1 in which the thalamus is omitted for clarity. In view of our meager understanding of the way the short- and the long-range connections of the cortex augment one another and participate in giving the cortex its processing power, it is reasonable to make some guesses and picture in forming our model that the short-range intercortical connections between columns make-up in conjunction with the thalamus a network that engages in forming internal representations of sensory input activity relayed to it by the thalamus and the sensory cortices. For detail on thalamic and other inputs to the cortex see Fig. 2 and for example any of references [19–24]. The action of such a network, includes as will be shown by the simulation results to be presented below, is to self-organize, under the influence of inputs it is subjected to in accordance to principles first set-forth by van der Malsburg [34], so as to produce stimulus-specific sparse patterns of activity for all subsequent inputs. Such sparse activity seems necessary for driving the other parts of the cortical columns which are assumed to form another network via the long-range cortico-cortical connections and through interaction with the hippocampus to provide associative learning and memory (engram) formation. The role of short- and long-range connections in associative memory is a topic of persisting interest in cortical modeling (see for example [33] and references therein).

Based on these observations a formal model of the cortex, a *corticonic network*, has been conceived, and studied by numerical simulation. The network includes, as its basic ingredients, specific architecture or connectivity, nonlin-

Cortical modules (columns, netlets) with short-range intercortical (nearest neighbor) connections.



Long-range cortico-cortical connections (Associative fibers).

Fig. 1. Schematized unfolded and flattened cortical patch depicted as array of cortical columns (netlets, modules, neuronal groups) with short-range (here intercortical) and long-range cortico–cortical connections between modules.



Fig. 2. Summary of cortico-thalamo-cortical connectivity.

ear, activity dependent, coupling between processing elements, self-organization through autonomous adaptation of its coupling factors driven by *mutual information*, which is an information theoretic measure of the flow of information between elements of the network, and *symmetry-breaking*. Other ingredients of the model are the parametrically coupled logistic map as the basic processing element in the network that is meant to mathematically model the complex emergent behavior of a cortical column or netlet, and the gradual transfer of control over the dynamics from extrinsic control, i.e. by the input pattern, to intrinsic control, i.e., by internal feedback to generate persistent activity patterns constituting attractors that characterize the input stimulus depending on the state of coupling in the network. Ultimately the persistent activity (attractor) is dissolved or over-ridden by new thalamic input once it is interpreted and not needed anymore. The relevance of attractor dynamics in cortical information processing and cognition is considered in [35] where attention is drawn to the series of neurocognitive experiments carried out by Miyashita and coworkers [36,37] in which they observe selective enhancement of neural activity that persisted for 16 seconds after removal of the stimulus. Before going into the mathematical formulation of the network given in Section 5 we will give a review of the logistic map as emergent property of the neuronal population of a netlet, cortical column or thalamo–cortical module.

4. The parametrically coupled logistic map as emergent property of neuronal populations of the cortico-thalamo-cortical loop

This section makes use of the standard formulation of population dynamics as means for developing the emergent functional properties of a netlet or a cortical column including its projections to and from the thalamus. It begins with the rationale and motivation for the idea of population encoding in the cortex then goes on to explain how the parametrically coupled (driven) logistic map (PCLM), which we use to mathematically model the behavior of a neuronal assembly, is deduced from the well known differential equations of population growth. This approach is taken because it accounts for the roles of inertia and memory in making the transition from the *logistic map* describing the dynamics in the form of a difference equation in discrete-integer time. The transition is justified by a plausible competition for ionic resources and by the refractory period of neurons in the population. The approach enables also explaining time-symmetry breaking and the emergence of the complex dynamics associated with the logistic map as compared to the deterministic and therefore relatively simple solution of the differential equation for population for maximum strength.

4.1. Population encoding

It is well known to neuroscientists, from microelectrode studies, that the response of any single neuron in the cerebral cortex is not repeatable even under carefully controlled experimental conditions: only through averaging of tens of trials are reliable results to be obtained. Sources for the variability in the response are: ambiguity and noise in the spike patterns impinging on the neuron, probabilistic nature of exocytosis (neurotransmitter release in synaptic cleft), and intrinsic fluctuations of membrane potential.

This view inevitably raised the question of how the generally reliable and consistent behavior observed in primates and other organisms can arise from a system (cortex) composed of unreliable components. Mathematical investigations of reliable information processing by systems composed of unreliable components have shown that any prescribed level of fidelity maybe obtained so long as there is sufficient redundancy within these systems [38,39] and that the most efficient systems are those which are comprised of complex components rather than simple ones [39]. This has led a number of investigators [29,31,40–53] to adopt the hypothesis that the basic functional unit in the cortex is a spatially localized aggregate or assembly of neurons defining a cortical column or netlet consisting of a radial organization of interacting neurons and that it is the over-all activity of neurons within such a neuronal mass (netlet or column) that is functionally significant rather than the responses of any single neuron. Such redundancy is not necessarily a simple increase in the number of neurons involved in any given operation but rather an increased complexity of the overall operations served by neurons inter-connected into the radially organized netlets or columns possessing emergent functional properties. In other words, the cortex does not use merely redundancy to overcome this variability but redundancy accompanied by emergence. This gives rise to the idea of "population encoding" of cognitive events in the cortex. It is shown in [41] that localized neuronal activity consistent with the physiological concept of a column or netlet arises as natural consequence of the interplay between topographic afferent projections and self-organizing interactions within the cortex. This view of cortical organization is very similar to that in [21] and [24] in which the cortex is conceived of as a matrix of overlapping radially oriented cylindrical modules, each module being defined by the extent of specific afferent terminal arborization and the lateral spread of dendritic branches of pyramidal neurons with each module possessing emergent complex functional properties that are not seen in, and are considerably more rich than, its constituent neurons as will be discussed below.

4.2. Population growth dynamics in continuous and discrete time

The high density of neurons per millimeter cube of cortical tissue mentioned earlier justifies the term population of neurons. The growth in population size N(t) in the presence of competition for limited resources and overcrowding obeys the differential equation [54]

$$\frac{\mathrm{d}N}{\mathrm{d}t} = F(N) \tag{1}$$

known as the logistic equation, where

$$F(N) = rN\left(1 - \frac{N}{K}\right)$$

r = initial per capita growth rate i.e. $r = \frac{\dot{N}^k}{N}$ when $N \leq K$ and K is called the carrying capacity or maximum population size.

The differential equation (1) is a deterministic equation whose solution,

$$N(t) = \frac{K}{1 + C e^{-rt}}, \quad C = \frac{N_0}{K} - 1$$
(2)

gives the population N(t) given the initial population N(0) and the values of the parameters r and K. The behavior of the population is seen to be completely predictable for all time. In fact knowing the population size at any instant of time enables predicting its size, not only at any future time but also its value at any past time. The solution is entirely deterministic and reversible in time; it is time-symmetric. The solution in (2) is the peculiar property of time reversibility which counters all experience: we know the flow of time is irreversible.

This peculiar aspect of the solution in (2) become more so when one considers that real (practical) populations behave in much more complicated ways than that predicted by Eqs. (1) and (2) where in addition to steady states, the behavior of a real population exhibits also periodicities and irregular (chaotic) behavior. This raises a fundamental question: what modification of Eq. (1) would predict such complexity? The answer to this question is related to the fact that real populations have finite reproduction time as opposed to continuous reproduction implicit in Eq. (1) and to issues of inertia and memory and continuous- vs. discrete-time dynamics. To appreciate this, we derive next a discrete time version of the differential equation (1), i.e. a difference equation with finite iteration time that reflects the finite reproduction time in real populations. We begin by expressing Eq. (1) in the equivalent form:

$$\lim_{\Delta t \to 0} \frac{N(t + \Delta t) - N(t)}{\Delta t} = F(N) = rN(t)(1 - N(t))$$
(3)

or

$$N(t + \Delta t) = N(t) + rN(t) \left(1 - \frac{N(t)}{K}\right) \Delta t \dots \quad (\Delta t \to 0)$$
(4)

Relax the requirement $\Delta t \rightarrow 0$ and replace the continuous time variable by discrete time i.e., $t \rightarrow n\Delta t$ and let $\Delta t \rightarrow 1$ to get

$$N(n+1) = N(n) + rN(n) \left(1 - \frac{N(n)}{K}\right)$$

$$= (1+r)N(n) - \frac{r}{K}N^{2}(n)$$
(5)

$$=aN(n) - bN^2(n)$$
(6)

where a = 1 + r and $b = \frac{r}{K}$. Next let,

$$X(n) = -\frac{b}{a}N(n) \tag{7}$$

or,

$$N(n) = \frac{a}{b}X(n) \tag{8}$$

to finally obtain from Eq. (4),

$$X(n+1) = \mu X(n) (1 - X(n))$$
(9)

In which n = 0, 1, 2... is discrete integer time. Since the population N(n) cannot be negative and a and b are positive real, X(n) will range between [0, 1] (see Eq. (7)) and

$$\mu = a = 1 + r \tag{10}$$

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and because $f(X) = \mu X(1 - X)$ has a maximum f(X) = 1 the value of μ cannot exceed 4. Therefore $0 \le \mu \le 4$. Eq. (9) with $0 \le \mu \le 4$ is known as the *logistic map*¹ which is known to exhibit rich array of complex behavior depending on the value of the control (bifurcation or nonlinearity) parameter μ [32]. Such complex behavior is similar to that observed with actual populations having finite reproduction time as for example in a population of bacteria in a petri dish subject to finite resources and crowding or, as will be discussed below, populations of neurons forming a netlet or cortical column where the refractory period of neurons is viewed as the formal analog to the finite reproduction time and neurons competing for ionic resources required for their operation. Eq. (9) expresses the dynamics of the population dynamics more realistically than the continuous time equation the logistic equation. Why? The answer to this interesting question lies intuitively in the finite reproduction time and the role played by inertia and memory in the transition from a differential equation description to a difference equation.

To see this we recall that if one knows the value of N at time t, then Eq. (2) can be used to determine the population size N(t) for any time $t + \Delta t$ in the future ($\Delta t > 0$) or the past ($\Delta t < 0$).

Thus given the population size N(t) and Δt we can obtain its value at $t + \Delta t$ by the Taylor expansion,

$$N(t + \Delta t) = N(t) + \left(\frac{\mathrm{d}N}{\mathrm{d}t}\right)_t \frac{\Delta t}{1!} + \left(\frac{\mathrm{d}^2 N}{\mathrm{d}t^2}\right)_t \frac{\Delta t^2}{2!} + \left(\frac{\mathrm{d}^3 N}{\mathrm{d}t^3}\right)_t \frac{\Delta t^3}{3!} + \cdots$$
(11)

Eq. (11) says that if we know N(t) and want to know $N(t + \Delta t)$ where Δt is finite, one needs to know not only the value of N(t) but also its higher order derivatives at time t which entails storage and memory. The larger is Δt , the larger is the number of higher-order derivatives one needs to know and use and this entails increased memory. This increased memory in Eq. (11) reflects the infinite inertia of the differential equations and their solutions (here Eqs. (1) and (2)) and relates in a subtle way to the time symmetry of Eq. (2) which is shared by the laws of physics except when chaos is present as emphasized below.

Next making use of Eq. (1) we can express Eq. (11) in the form,

$$N(t + \Delta t) = N(t) + F(N)\Delta t + \frac{dF(N)}{dt}\frac{\Delta t^2}{2!} + \frac{d^2F(N)}{dt^2}\frac{\Delta t^3}{3!} + \dots$$
(12)

where, in accordance to Eq. (1) we have set $(\frac{dN}{dt})_t = F(N)$ and $(\frac{d^2N}{dt^2})_t = \frac{dF(N)}{dt}$ and so on. We note next that Eq. (4), which was used to derive the logistic map is exactly Eq. (12) but with all higher-order

We note next that Eq. (4), which was used to derive the logistic map is exactly Eq. (12) but with all higher-order terms in Δt in Eq. (12) ignored. The logistic map (Eq. (9)) describes thus population dynamics under conditions of reduced inertia and memory which permit the appearance of dynamic orbits (sequences), that manifest complexity and unpredictability (chaos) when Eq. (9) is iterated. Such complex behavior emerges despite the fact that the logistic map (Eq. (9)) is entirely deterministic. The complex behavior is seen to arise from the finite reproduction time and restricted memory implicit in the logistic equation where to know X(n + 1), only knowledge of the preceding value X(n) is needed. Complexity, therefore seems to be a consequence of the transition from the continuous-time inherent in the differential equation, the logistic equation, to the discrete-time dynamics of the logistic map.

If brain dynamics were to involve infinite inertia and memory there would be no opportunity for an organism to engage in *protean* (unpredictable behavior) [55,56] which is essential for survival: species with predictable behavior do not survive. A species whose behavior is easily predictable by an adversary would be on the losing if not terminal end of things. It seems therefore crucial that the brain/cortex possess a mechanism that does not preclude the appearance of a certain degree of behavioral unpredictability. Such mechanism might be available in the complex dynamics of neuronal populations and their emergent, the logistic map.

4.3. Time-symmetry breaking

Time is something we know well by intuition but find difficult to explain. We know that the flow of time is inexorable. There is no way to reverse the arrow of time. Yet, the logistic equation of population growth, as we saw, is time reversible. In contrast to the time-reversibility of the logistic equation and its solution, the logistic map is not always

¹ A map, e.g. Eq. (9), is a formula that describes the new state of the system in terms of the past state. A differential equation (e.g. Eq. (1)) is a formula for the instantaneous rate of change of the current state in terms of a function of the current state.

time-reversible. To see this consider Eq. (9) rewritten in the form

$$X^{2}(n) - X(n) + \frac{1}{\mu}X(n+1) = 0$$
(13)

Let X(n) = y and $\frac{1}{\mu}X(n+1) = C$, then

$$y^2 - y + C = 0 (14)$$

a quadratic equation whose solution is,

$$y_{1,2} = \frac{1}{2} \pm \frac{1}{2}\sqrt{1 - 4C} \tag{15}$$

Thus when $\mu = 4$, and $C = \frac{1}{4}X(n+1)$, Eq. (15) becomes,

$$y_{1,2} = \frac{1}{2} \left[1 \pm \sqrt{1 - X(n+1)} \right]$$

Since $X(n + 1) \leq 1$, this result shows that, when $\mu = 4$ the logistic map is chaotic, that is for every value X(n + 1) there are two possible values of *y* and therefore of X(n). This underlies the loss of predictability of chaos and also that we cannot resurrect the past from the present as was the case with the logistic equation; time symmetry is broken. Accordingly, the logistic map (Eq. (9)) is in better agreement with the reality of the irreversibility of time than the logistic equation (1). This property of the logistic map represents a subtle but important role played by chaos: breaking the connection with the non-immediate past, and is responsible for the unique functional capabilities of PCLMN in modeling brain dynamics to be described in Section 6.

4.4. Isomorphism between the dynamics of population growth and the dynamics of a neuronal population or netlet

The preceding results and observations apply to the behavior of neuronal populations in a netlet or thalamo– cortical complex by noting the following possible isomorphism, shown in Table 1, between the vital quantities in the population growth theory and those of neuronal population or netlet dynamics.

Netlet dynamics therefore is described by the logistic map of Eq. (9), X(n) would represent the fraction of neurons active at the *n*-th time step, the control parameter μ furnishes the mechanism by which the behavior of the map can be altered by extrinsic input which would require making μ also function of discrete time. Accordingly, for a netlet, Eq. (9) would be replaced by

$$X(n+1) = \mu(n)X(n)(1 - X(n))$$
(16)

where now $\mu(n)$ is a function of discrete integer time that can be used to incorporate the effects of extrinsic and intrinsic reentrant signals driving the netlet. Because the control parameter $\mu(n)$ is used to account for modulation by extrinsic inputs from other cortical columns and from subcortical (thalamic) nuclei, Eq. (16) is referred to as *Parametrically Coupled Logistic Map* (PCLM) to distinguish it from that of the logistic map, where μ is time invariant. One can also view $\mu(n)$ as modeling the role of the thalamic reticular nucleus (TRN) in the thalamo–cortical complex. PCLMs serve as building block for the *parametrically coupled logistic map network* (PCLMN) described in Section 6.

A large number of authors (see for example [40–54] have studied the collective behavior of a population of cortical neurons tightly coupled to form a functional unit under variety of assumptions and conditions. Several of these authors, [30,31,42,43,46,48,50,53], arrive at description of the dynamics in terms of one-dimensional maps that resemble the logistic map description arrived at here from rather simple considerations of population growth dynamics.

Table 1

Isomorphism between population dynamics and neuronal population or netlet dynamics

Population growth	Neuronal populations or netlet	
Population $N(t)$ Competition for available resources (nutrients, space, etc.)	Fraction of neurons firing at <i>n</i> -th time step, $X(n)$ Competition for ionic species needed for the creation of action potentials and or interplay between inhibitory and excitatory neurons in a cortical column.	
Finite reproduction time	Refractory period	

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The fact that the functional unit in the cortex, maybe represented by PCLM behavior constitutes a high degree of abstraction of cortical function and organization. The major question then is whether such simplification of the description of the processing elements in the cortex can be legitimately used to model a cortical patch and to show that it can exhibit corticomorphic behavior. The simulation results described in Section 6 show the answer is affirmative. The fact that the dynamics of a neuronal population (netlet or cortical column interacting with the thalamus) may be mathematically modeled by a PCLM, that has little resemblance to the behavior of the constituent neurons, is an example of the important role of emergence in cortical modeling. Emergence [27] is a mechanism, whose importance in endowing the cortex not only with complex functional units, but also high-level brain function has, in the most part, not been fully incorporated in the formulation of brain theory. This review stresses the role of emergence in various levels of cortical/brain organization. It is tempting to conjecture that succession of emergences from lower hierarchical-levels of organization to higher-levels may be related to the emergence of self-awareness and consciousness.

It is worth noting that one-dimensional maps are increasingly used to model more complex neuronal dynamics like those of the Hodgkin–Huxley neuron model [57]. Here we propose and give evidence that parametrically coupled maps can be used to model the dynamics of high-dimensional nonlinear dynamical systems like the cortico–thalamo– cortical loop and we introduce the concept of cortex-module that paves the way to creating controlled trajectories in state space that may facilitate perceptual categorization.

5. Mathematical formulation—the corticonic equations and modeling the cortico–thalamo–cortical interactions

A formal description of the corticonic network or PCLMN model of the cortico-thalamo-cortical system is presented next. It is worth remarking first that a model is a convenient way of thinking about the system. A measure of the validity of any model, is how well it predicts and synthesizes behavior associated with or observed in the system being modeled. The results presented in Section 6 will show that this measure is well satisfied by our model. One dimensional topology is adopted to simplify plotting the time evolution of the state vector $\bar{X}(n)$ of the network but the given formulation is easily extendable to two-dimensional or higher topology. The network employs nonlinear activity dependent coupling functions between processing elements. The nonlinear coupling enables the network to self-organize via competing forces of order and disorder as demonstrated below. In addition, control over the dynamics of the network transitions gradually from initially entirely extrinsic control by the applied stimulus (input) to eventually intrinsic control via self- and nearest-neighbor coupling as will be described. The network has also dual dynamics characterized by: fast dynamics controlling the evolution of the state vectors $\bar{X}(n)$, i.e. the states of the processing elements (PEs) which are the PCLMs, and slower dynamics controlling the evolution of the nonlinear coupling strengths between PEs.

As discussed in Section 4, a PCLM, is the common logistic map [30] but with control parameter modulated in time. Accordingly the orbit $X_i(n)$ of the *i*-th PCLM or PE is expressed by Eq. (16) reproduced here for convenience,

$$X_i(n+1) = \mu_i(n)X_i(n)(1 - X_i(n)) \quad i = 0, 1...N - 1, n = 0, 1, 2...$$
(16)

where $X_i(n)$ is confined to the interval [0, 1] and $\mu_i(n)$ to the interval [0, 4]. Typically a small amount of noise is added to r.h.s. of Eq. (16') to prevent iterations from getting trapped at $X_i = 0$ or 1. The initial state $X_i(0)$ in Eq. (16') is selected randomly in [0, 1] with uniform probability, while the parameter $\mu_i(n)$ is taken to be function of the extrinsic stimulus (input) $X_i^s(n)$ and the intrinsic input (feedback) from all elements in the network connecting to *i*-th element, including self-connection, in accordance to,

$$\mu_{i}(n) = u_{i}^{s}(n) e^{-\alpha n} + \frac{1 - e^{-\alpha n}}{N_{i}} \sum_{j \in |N_{i}|} g_{ij} (X_{j}(n))$$
(17)

where

$$u_{i}^{s}(n) = 4 \left(X_{i}^{s}(n) \right)^{C_{i}^{s}}$$
(18)

$$g_{ij}(X_j(n)) = 4(X_j(n))^{C_{ij}}$$
⁽¹⁹⁾

are nonlinear activity dependent *coupling functions*, the first of which represents the coupling of the *i*-th input $X_i^s(n)$ into the network, and the second the coupling of the *j*-th element to the *i*-th. C_i^s and C_{ij} are positive real *coupling*



Fig. 3. The nonlinear activity dependent coupling function $g_{ij}(X_j) = 4(Xj(n))^{C_{ij}}$ plotted for several values of C_{ij} ranging between zero and infinity.

factors that control the form of the activity dependent coupling functions and furnish, as will be seen below, the means for incorporating autonomous adaptation in the network. $|N_i|$ is the set of elements connecting to the *i*-th element. A plot of $g_{ij}(X_j)$ is shown in Fig. 3 for different values of C_{ij} . In anticipation of the discussion of adaptation, note that the coupling function is never eliminated even for large values of C_{ij} because for $X_j = 1$, $g_{ij} = 4$ independent of the value of C_{ij} .

The first term on the r.h.s. of Eq. (17) represents the effect of the sensory (external) input $X_i^s(n)$ and is assumed to decay exponentially in time with rate constant α ; the second term represents self-feedback and the effect of internal feedback from other elements which is seen to grow exponentially in time with rate constant α . In this fashion, as the effect of the sensory input diminishes getting weaker in time, that of the self- and internal feedback grows stronger affecting thereby the transfer of control over the dynamics of PCLMN from extrinsic to intrinsic control. This handingover of control over the dynamics from extrinsic to intrinsic, introduced by Farhat [59], is crucial and is meant to avoid having the network be permanently forced by the input, giving thereby an opportunity for the interactions between its elements to have their say in determining the final state (convergent state or attractor) of the network. This does not mean that the final state bears no relation to $\bar{X}^{s}(n)$; to the contrary, the final state, is stimulus-specific. The first term in Eq. (17) is an exponentially decaying forcing function that acts in effect as a "soft" initial condition that guides the network towards certain region of its state-space where, all along, the exponentially increasing intrinsic dynamics, represented by the second term in Eq. (17), takes over completely leading the network to an attractor, a persistent stimulus-specific state characteristic of the particular $\bar{X}^{s}(n)$. In this manner we ensure that the PCLMN operation is due to the trace or after-glow of the environmental stimulus. The simulation results presented below show that the final state (attractor) reached by PCLMN is dependent on $\bar{X}^{s}(n)$, on its coupling factors matrices (\bar{C}^{s} and \bar{C}), on α and is independent of the initial state $\bar{X}(0)$ as should be the case.

The argument for the handing-over control over the dynamics as expressed by the pair of Eqs. (16) and (17), is in concert with the biophysical observation made by Freeman and coworkers [58] pertaining to the gradual disappearance of the trace of a sensory stimulus applied to the olfactory bulb of rabbit as it is followed deeper in the cortex where it was found to eventually vanish in a sea of intrinsically dominated activity. Similar behavior was apparently observed by Freeman's group for other sensory modalities. The argument is reinforced by another view, held by brain scientists (see for example [60]), namely that an animal or organism surviving in a complex uncontrolled environment is constantly being buffeted by energy and forces created by the environment that act on its sensory surfaces where

they get converted into sensory data from which information and meaning are extracted by the cortex. To do this effectively, the cortex must be left unhampered by disruptive influence of sensory data that occur while it is making sense of (interpreting) the afterglow left by sensory data it has just received. This suggests that the cortex receives the next gradually fading burst or transient influence of sensory data relayed to it in a reentrant manner by other parts of the brain e.g. the thalamus and the sensory cortices (see Fig. 2) only after the present input is dealt with which may be synonymous with reaching an attractor state. Once reached, the attractor state would unravel to free the network to handle the next input. This is consistent with unity of consciousness: one can be conscious or aware of no more than one thing at a time. It is worth noting that this process of handing-over control of dynamics from extrinsic to intrinsic influences agrees also with the picture of a cortex continually at work (dynamic cortex) whose state-space behavior is intermittently redirected by bursts of thalamo–cortical sensory-encoding signals. This picture is also consistent with the functional state-space hypotheses of Koukkou and Lehmann [61]. We expect understanding and modeling this intriguing view of biological information processing, and appreciating how it distinctly differs from the standard input–output approach to the processing of information, will have increasing influence on the design philosophy of future brain-like artificial intelligent systems.

Next we express Eqs. (17)–(19) in the equivalent form,

$$\mu_i(n) = 4 \left(X_i^s(n) \right)^{C_i^s} e^{-\alpha n} + \frac{1 - e^{-\alpha n}}{N_i} \sum_{j \in |N_i|} 4 \left(X_j(n) \right)^{C_{ij}}$$
(20)

and note because both $X_i^s(n)$ and $X_j(n)$ range in [0, 1], the value of $\mu_i(n)$ will range in [0, 4] as required by Eq. (1). Also the properties of the logistic map (see for example [32]) suggest that when $\mu_i(n)$ assumes small values confined to [0, 1] the orbit of the *i*-th map or PE in the network will tend to zero, $X_i(n) = 0$, while high values of $\mu_i(n)$, close to 4, cause the orbit to be chaotic. In between, a whole range of periodic (period-m)² activities can occur. Bifurcations (rapid switching) between all these modalities can also take place as the value of $\mu_i(n)$ passes through critical values. Thus as the value of $\mu_i(n)$ is modulated by the combined influences of the extrinsic stimulus and the intrinsic activity, the orbit $X_i(n)$ can change in a complicated way that covers ordered (fixed-point and periodic) and disordered activity including intermittency and the two extremes of activity ($X_i(n) = 0$) and chaos. Note the parametric modulation $\mu_i(n)$ of the *i*-th PCLM, occurs at the same discrete-integer-time rate as the state variable $\bar{X}(n)$ and the stimulus $\bar{X}^s(n)$. This is not unusual and in fact parallels a similar mechanism in the neural realm. The hypothesis that transmission efficiencies of synapses in the central nervous system (CNS) undergo modifications on a short-time scale has been investigated by von der Malsburg and Bienenstock [62] who point out that such short-term synaptic plasticity is an almost necessary condition for the appearance of persistent activity states that are stable for about 1 [sec], the time scale of psychological processes.

In what follows all C_i^s in Eq. (20) will be set uniformly and arbitrarily to 0.5 which would achieve a certain degree of equalization where values of $X_i^s(n)$ falling in the lower ranges of their [0, 1] interval would be enhanced more relative to those falling in the upper ranges (see Fig. 3). The initial values $C_{ij}(0)$ are set randomly in a suitably selected range [A, B] with uniform probability. The exact values of A and B are not critical but are selected as shown below, to cause the PCLMN to burst into irregular activity that is at the *edge of chaos*: a mix of order and chaos. The coupling factor C_{ij} in Eqs. (19) and (20) determining the value of the coupling function $g_{ij}(X_j)$ from the *j*-th PE to the *i*-th PE is adapted from its initial value $C_{ij}(0)$ according to the following algorithm driven by mutual information (MI):

$$C_{ij}(m+1) = C_{ij}(m) \left(1 + \Delta C_{ij}(m)\right)$$
(21)

$$\Delta C_{ij}(m) = \delta \tanh \beta I_{ij} \tag{22}$$

where *m* is the index of the *m*-th adaptation, with adaptation occurring every T_{tr} iterations as discussed further below. δ and β are positive real constants that control the rate of punctuated adaptation, and its sensitivity to changes in the mutual information I_{ij} through β . In Eq. (22),

$$I_{ij} = H_i + H_j - H_{ij} \tag{23}$$

² Cyclic activity consisting of a repeating sequence of m distinct values of $X_i(n)$.

is the normalized mutual information between the orbits of the *j*-th and *i*-th maps (PEs); H_i and H_j are the normalized entropies of *i*-th and *j*-th orbits respectively, and H_{ij} is the normalized cross-entropy of the joint variable (X_i, X_j) determined by Eqs. (24) and (25). (For definition of entropies and mutual information see for example [58].)

$$H_{i} = -\frac{1}{\log N_{div}} \sum_{k=1}^{N_{div}} p_{k}^{(i)} \log p_{k}^{(i)}$$
(24)

$$H_{ij} = -\frac{1}{2\log N_{div}} \sum_{k=1}^{N_{div}} \sum_{\ell=1}^{N_{div}} p_{k\ell}^{(ij)} \log p_{k\ell}^{(ij)}$$
(25)

To estimate the probability $p_k^{(i)}$, the entire range [0, 1] of the state variable $X_i(n)$ is divided into N_{div} equal intervals. The probability $p_k^{(i)}$ is then estimated based on the relative frequency of the state variable visiting the *k*-th interval during the past T_{tr} time steps or iterations where T_{tr} is sufficiently large, typically 100 in the simulations below. The probability $p_k^{(j)}$ is estimated in a similar manner and so is the probability $p_{k\ell}^{(ij)}$ except for the natural extension to the 2-dimensional range for the joint variable (X_i, X_j) . The determination of entropies and the process of MI driven adaptation are commenced after a brief interval of n_{tr} iterations following the application of a stimulus to exclude transients. The mutual information function I_{ij} is an information theoretic measure of the degree with which the orbit $X_i(n)$ of the *i*-th PE (the PCLMs) is influenced by the orbit $X_j(n)$ of the *j*-th PE. As such it is Hebbian in nature. (Reasons why MI is a better measure of similarity between dynamic sequences than correlation functions are discussed in [63].)

Of interest are conditions under which the state vector X(n) would converge, under MI driven adaptation or annealing, to an ordered attractor state, e.g. into a fixed-point attractor, specific to the particular stimulus vector $\bar{X}^{s}(n)$. In the PCLMN configuration considered here, every element is connected to itself with coupling strength specified by coupling factor $C_{ii}(0)$ selected randomly in the range $[a, b], b > a \ge 0$, with uniform probability $P_r = 1$. Every element is also connected to its nearest neighbors with finite probability $P_r < 1$ with initial coupling strength specified by coupling factor $C_{ij}(0)$ randomly set in the range $[A, B], B > A \ge 0$. The nearest-neighbor coupling is meant to be an abstraction of the close-range intercortical connections of the cortex. In the following we select $P_r < 1$ which means that not every PE is connected to its nearest neighbors and constitutes another mechanism for symmetry-breaking in the network besides the temporal symmetry breaking discussed earlier. The symmetry breaking here was found to enable the PCLMN to possess the desired property of converting every input or stimulus pattern $\bar{X}^{s}(n)$ to a stimulus specific fixed-point attractor state \bar{X}^* , a persistent activity of the network in which the active PEs have fixed analog values (fixed-point orbits). The symmetry-breaking here refers to departure from the symmetrical condition when every element is connected to its nearest neighbors ($P_r = 1$). It is meant to represent the plausible situation when certain percentage of cortical columns have weak or no local connections to their neighbors. The simulation results presented below were carried out for nearest-neighbor connections made with probability $P_r = 0.5$ i.e. only about 50% of the PEs have nearest neighbor connections. Similar results were obtained with $P_r = 0.75$, but $P_r = 1$ as stated earlier, usually prevented the network from showing any interesting behavior. This appears to be in agreement with the point of view that anatomical variability in the cortex is unavoidable from a developmental point of view [64] and would apply to the local connectivity of cortical modules resulting in a situation where a certain percentage might end up with weak or no local connections. A systematic investigation of the effect of setting the nearest-neighbor connections with different values of P_r on the behavior of the network is however needed to determine whether there is an optimal value for P_r for which the network performance is superior and whether this can be related in any way to any available estimates of local (intercortical) connectivity in cortical tissue, [65] or to the concept of small-world network [84,106,116].

6. Numerical simulation results and their implications

For the case of self-connection and nearest-neighbor connections, Eq. (20) can be expressed in the more explicit form.

$$\mu_{i}(n) = 4 \left(X_{i}^{s}(n) \right)^{C_{i}^{s}} e^{-\alpha n} + \frac{4}{3} \left(1 - e^{-\alpha n} \right) \left\{ \left(X_{i-1}(n) \right)^{C_{i,i-1}} + \left(X_{ii}(n) \right)^{C_{ii}} + \left(X_{i+1}(n) \right)^{C_{i,i+1}} \right\}$$

$$i = 0, 1, 2, \dots, N - 1$$
(26)

Parameters of the PCLMN used in the simulation results presented and discussed below were:

$$\alpha = 0.1, \quad C_i^s = 0.5, \quad C_{ii}(0) \in [0, 0.5] \text{ with probability } P_r = 1$$

 $C_{ii}(0) \in [0, 0.5] \text{ with probability } P_r = 0.5$
(27)

The above ranges of the parameters $C_{ii}(0)$ and $C_{ij}(0)$ were deliberately chosen because they produced initial irregular "chaotic"³ activity which helped the network self-organize under MI driven adaptation into a form (coupling strengths matrix) that produced stimulus specific attractors. The way this occurs is documented in Fig. 4. This gives an example of the behavior of a PCLMN of size N = 100 under the specified parameters (Eq. (27)) showing the time evolution of the state vector $\bar{X}(n)$ and the changes occurring in the coupling factors matrix as result of the network adapting (self-organizing) for two different input (stimulus) vectors $\bar{X}^s(n)$. A large library of stimulus vectors $\bar{X}^s(n)$ (about 100) was available for evaluating the performance of PCLMN. The library was produced by an array of 100 uncoupled logistic maps whose control or nonlinearity parameters vector $\bar{\mu}$ was set by a computer and the posttransient orbits vector produced by the array was taken as $\bar{X}^s(n)$ and stored in memory for different $\bar{\mu}$ vectors. Typically each stimulus vector $\bar{X}^s(n)$ stored in memory consisted of few hundred iterations. This produced a large variety of spatio-temporal activity patterns with varying degrees of similarity and dissimilarity between them to be used in studying the behavior of the PCLMN. The parameters used in the MI driven adaptation process described by Eqs. (20)–(25) were $\beta = 0.1$, $\delta = 3$, $T_{tr} = 100$, $n_{tr} = 100$ and $N_{div} = 100$.

Examples of numerical simulations of the PCLMN are given in Figs. 4(a) to 4(c). Fig. 4(a) shows the time evolution of the state vector $X(n) = \{X_i(n), i = 1, 2, ..., N = 100\}$ (top two central panels in which discrete time n is the horizontal axis and the vertical axis is the element index *i* of the network for one stimulus). The narrow leftmost panel mus at the top left is a plot of the values of the control vector $\bar{\mu}^s = \{\mu_i^s, i = 1, 2, ..., N\}$ of a stimulus generating layer consisting of N = 100 individual (uncoupled) logistic maps producing a dynamic (spatio-temporal) input vector pattern $\bar{X}^{s}(n) = \{X_{i}^{s}(n), i = 1, 2, ..., N = 100\}$ a snippet of which designated XS is shown immediately to the right of the mus panel for a small number of time steps (n = 90 to 99). The right-most narrow panel gives the color code scale of the orbits $0 \le X_i(n) \le 1$ of the individual elements of the network, while SZ is the spatial power spectrum of $e^{j2xX_i(n)}$ where $X_i(n)$ is regarded as normalized phase variable. It is worth noting that the actual extrinsic input to the network, the first term in Eq. (26), exerts influence for only the first ten or so iterations of $\bar{X}^{s}(n)$ because of the exponential decay constant $\alpha = 0.1$. The next two panels from the top show the time evolution of $X_i(n)$ for the i = 0 element. The bottom square pattern is the mutual information matrix computed at the 499-th iteration using the preceding 100 iteration. The top left entry in Fig. 4(b) shows the time evolution state variable vector X(n) of the network following the first MI (mutual information) driven adaptation of the initial coupling factors matrix $\bar{C}(0)$ executed at the 400 iteration together with the MI matrix computed from $\bar{X}(n)$, $400 \le n \le 499$ which is shown at the top right. The bottom panels in Fig. 4(b) show the convergent state vector of the network \bar{X}^* (left panel) reached after four MI driven adaptations executed at n = 400, 500, 600, and 700 using always the preceding 100 iterations to compute the MI. The corresponding MI matrix of the convergent network is given at the bottom right panel showing its elements I_{ij} have converged to zero. Because I_{ij} is a measure of the flow of information between the *i*-th and *j*-th elements, convergence of the network is synonymous with a convergent \overline{C} matrix that eliminates the flow of information between the elements (PCLMs) of the network. We call this process *MI driven self-organization*. In the convergent state all elements converge to fixed analog values ranging between zero to a maximum value determined by an extended upper limit of the fixed-point regime of the bifurcation diagram of the logistic map as discussed in Section 6.2. This section also demonstrates the above MI driven annealing process of the PCLMN guarantees convergence to a fixed point attractor. It is important to point out that the evolution of the PCLMN described above is not dependent on initial state, $\bar{X}(0)$ of the PCLMN. This is indeed in agreement with what one would expect in biological systems where the ability of an organism to recognize an object or event should not depend on the state of its cortex at the instant an environmental event, or the sensation induced by it, arrives. Attention is also drawn to the way the initial irregular "chaotic" activity, which is actually a mix of ordered and chaotic orbits that place the network at the edge of chaos, is altered by the paced or punctuated adaptation process into a static (fixed point) attractor specific to the applied stimulus given in the XS pane display. Also note the zero activity orbits (dark horizontal lines) clearly visible in the initial stages of the network's evolution (top two panels of Fig. 4(a)). These lines are result

 $^{^{3}}$ From here on the term "chaotic" is used to designate edge of chaos activity.



Fig. 4. (a) Time evolution of the state vector $\bar{X}(n)$ (top), $X_i(n)$, i = 0 (middle) and mutual information (bottom) before the first MI driven adaptation of the network. (b) (Top) State vector $\bar{X}(n)$ and MI matrix following first adaptation. (Bottom) Fixed-point attractor and MI matrix showing the flow of information in convergent network is zero. (c). Another instance of input-specific attractor and its MI matrix. Note different input pattern $\bar{X}^s(n)$.



of the symmetry-breaking in the network connectivity which is found to be instrumental in enabling the network to converge to stimulus specific fixed point attractors an example of which was given in the bottom left panel of Fig. 4(b). The symmetry-breaking lines occur for all isolated PEs in the network i.e. elements that have no nearest neighbor connections and whose extrinsic stimulus $X_i^s(n)$ happen to be zero. The number of the symmetry-breaking lines is determined therefore by both the nature of the input stimulus and the number of elements without nearest neighbor connections which is determined in turn by the value of P_r used in setting the initial coupling factors $C_{ij}(0)$. The joint requirement of isolation and zero extrinsic input suggest that the role of these lines is to carry out some sort of initial coarse discrimination between applied stimuli that gets subsequently refined by MI driven adaptation. Without symmetry-breaking the stimulus-specificity of the convergent states (the attractors) is significantly diminished. The rightmost displays in all color plates of Fig. 4 is the color code for $\overline{X}^s(n), \overline{X}(n)$ and $\overline{SZ}(n)$. The $\overline{SZ}(n)$ display is a snapshot of the power spectrum of $\exp(-j2\pi X_i(n))$ for the noted number of time-steps. The similarity of this display to the familiar code product labels, when the network converges to an attractor, is worth noting because it identifies attractors by means of a color-bar code reader as an alternate scheme for labeling of attractors with an associative memory as discussed in Section 7. Finally in this section examples of the initial coupling factors matrix $\overline{C}(0)$ and the



Fig. 5. Initial coupling factors matrix $\bar{C}(0)$ (left) and convergent (adapted) matrix \bar{C} for which the flow of information in the network is zero.

convergent coupling factor matrix \bar{C} for which the network converged to fixed point attractor state \bar{X}^* , for which the flow of information between elements is zero are shown in Fig. 5.

6.1. Dynamic memory

To the author's knowledge, the PCLMN is the first realization of the concept of dynamic memory. The PCLMN is a dynamic memory in the sense that it is an algorithm, the corticonic equations in Section 5, designed to hold an immense number of coexisting attractors (patterns) any one of which can be selected and brought to the fore by a process of conjugation of the incoming signal (stimulus pattern) with the network dynamics set forth in the algorithm. The selected attractor pattern is stimulus-specific in the sense that it is one that agrees or achieves maximum consensus with the incoming signal in accordance with pre-established criterion embedded in the network parameters and dynamics, namely by the way the PCLMN gets self-organized by MI-driven adaptation of its initial connection strengths matrix.

The number of patterns produced in this manner is immense. To see this consider a PCLMN of N elements whose analog values are discernible (measured) over L levels. It will have $N_A = L^N$ coexisting attractor patterns. For N =100 and L = 256 (8 bit resolution) this amounts to an immense or astronomical number of $N_A = 2.5610^{200}$ stimulusspecific attractor patterns and all evidence we obtained so far from numerical simulations and analytical considerations [66] point to all attractors being accessible to external inputs. Thus for all practical considerations, the PCLMN is able to produce an *internal representation*, a fixed-point attractor, for every imaginable input it receives. In comparison, pre-storing such a large number of patterns in a static memory to be addressed by a sensory pattern is impractical. A dynamic memory has therefore decided advantages over a static memory. The labeling of internal representations by means of a suitable associative memory or a lookup table or a bar-code reader of the convergent \overline{SZ}^* pattern for example, can be used now to attach meaning (name or label) to the representations/attractors forming thereby what can be called a *cortical module* able to carry out perceptual categorization as will be expanded upon in Section 7.

6.2. Redundancy detection

This is the ability to distinguish structured (redundant) input (stimulus) patterns that carry information and produce therefore distinct attractors, from non-redundant (structureless or uniformly random) input patterns that do not convey meaningful information. A redundant pattern or picture contains correlated detail (pixels) while a non-redundant one is void of any correlation [66]. The detection of structure and the ability to distinguish between redundant and non-redundant input patterns is a recognized property of the brain that was considered by Barlow [66]. According to Barlow,

"Our nervous system handles regularity and pattern in a special way. It may be important to realize that, in informational terms, it is redundancy that is singled out for special treatment, and I think this principle gives unity to an astonishing range of nervous functions reaching up to quite complex mental acts."

"Regularity and pattern in sensory stimuli are certainly important. Recognizing that this is informational redundancy suggests that it may be detected by distributed, local processes rather than global operations involving much of the sensory input. Much of the influence of the environment on the brain may operate in this way rather than through trial and error learning or the even slower and more drastic method of genetic selection. Redundancy can be exploited to transmit or display information more compactly and it is suggested that it may also be a criterion for preservation or rejection of information."

The PCLMN is a redundancy detector where all uniformly random (white noise) input patterns are characterized by the same attractor. This is illustrated in the plots in the top two rows of Fig. 6. The two noise-like stimulus patterns shown there in the left column are distinct, as can be verified by scrutinizing and comparing their detail, and yet their attractors, shown in the right column, are identical. The bottom two rows in Fig. 6 show two distinct chaotic input patterns (left column) and their attractors (right column) which are seen to be different. This shows that chaos and white noise are different which is in agreement with the widely accepted view. Obviously this capability of the PCLMN is functionally useful because it provides a mechanism that prevents overloading a cognitive processor, like the brain, with useless "information".



Fig. 6. Example of the ability of the PCLMN to discern redundancy or structure in input patterns $\bar{X}^{s}(n)$ (left column) and their convergent stimulus-specific attractors (right column). Top two rows are two distinct random (noise like) patterns that lack any structure and their attractors that are seen to be identical. In contrast (bottom two rows) distinct chaotic patterns give rise to distinct attractors as shown in the bottom two rows.



Fig. 7. Example of rapid convergence to stimulus-specific fixed-point attractor in a cumulatively adapted PCLMN via a brief cascade of state determined bifurcations clearly seen in the leftmost part of the figure that search the state-space of the PCLMN and converge autonomously to a stimulus-specific attractor.

6.3. Cumulative adaptation

The PCLMN has another mode of operation that speeds-up the convergence to a fixed-point attractor. This mode involves *cumulative adaptation* of the network by several distinct input patterns that leads to MI driven cumulatively adapted network capable of converging directly without further MI driven adaptation to an input-specific attractor in about 50 iterations as illustrated in Fig. 7 instead of the 800 or 900 iterations needed to converge in the previous mode of operation described earlier in Fig. 4. Note the direct and rapid convergence to an attractor in Fig. 7 takes place through a cascade of state-determined bifurcations seen to the left in the figure, that search the state-space of the network for a stimulus-specific attractor.

In the cumulative adaptation process the final convergent coupling factors \overline{C} matrix, arrived at as result of applying a first input to the un-adapted network with $\overline{C}(0)$, is used as the initial coupling factors matrix of the network when a second distinct input is applied to the network and the new resulting convergent \overline{C} matrix is used as the initial \overline{C} for a third distinct applied input and the process is repeated. For the network parameters given earlier, normally a total of three or four such cumulative adaptations with distinct inputs are sufficient to make the values in the coupling factors matrix stabilize to a \overline{C} matrix whose components do not change with further application of distinct inputs. Such cumulatively self-organized network retains the same remarkable ability of the PCLMN discussed in the preceding Section 6.2 to produce stimulus-specific attractors and to distinguish between redundant (structured) and non-redundant (unstructured) inputs except the process is now much faster.

The self-organization under MI driven adaptation and the final stabilized coupling strengths matrix depend on the nature of the first few occurring (applied) stimuli and their order of occurrence. This does not however pose a problem because what matters is that self-organization produces a network with consistent behavior; that is, once self-organization is complete, a specific input would choose a specific attractor. What counts therefore in self-organization here is consistency and not the requirement for example that two identical PCLMNs that were exposed to and self-organized by different sets of stimuli give later the same set of attractors when subjected to an identical set of stimuli. Their attractors can be different but perceptual categorization of these attractors, or more precisely the objects (stimuli) that spawned them, as would be acquired through behavior and learning must be the same. How this may be realized by combining the PCLMN with an associative memory is discussed below in Section 7. It is important to note that convergence of the PCLMN to an attractor did not involve an energy function and search for local energy minima as is the case in conventional attractor type neural networks. This distinction together with the earlier observation that the PCLMN is a dynamic memory are responsible for the immense number of attractors furnished by the PCLMN in contrast to conventional networks and connectionist models.



Fig. 8. Components of the convergent vectors $\bar{\mu}_i^*$ and \bar{X}_i^* , the large dark points always fall in the fixed point regime of the bifurcation diagram of the logistic map (left). Moreover, because of coupling and collective dynamics the fixed point regime reaches into the period 2 and period 4 regime (right).

6.4. Evidence of guaranteed convergence to stimulus-specific attractors

As can be seen in the examples in Figs. 4, 6 and 7 applying of input (stimulus) patterns to the PCLMN cause its convergent \bar{X}^* and $\bar{\mu}^*$ vectors to have components (large dark points) with values distributed over the fixed-point regime (see [32]) of the bifurcation diagram characterizing the logistic map. This is illustrated in Fig. 8 where the components of the convergent vectors $\bar{\mu}^*$ and \bar{X}^* belonging to several applied stimulus patterns are seen to fall in the range $0 \le \mu_i^* \le 3.5$ and $0 \le \bar{X}_i^* \le 0.76$. This takes place via the MI driven annealing process demonstrated earlier in Fig. 4 or via a faster process in the cumulatively adapted PCLMN via the initial cascade of state-driven bifurcations as seen in Fig. 7. In the $0 < \mu_i^* < 3.5$ range, the fixed-point attractor of the *i*-th map is either $X_i^* = 0$ for $0 \le \mu_i^* \le 1$ or $X_i^* = 1 - \frac{1}{\mu_i^*}$ for $1 < \mu_i^* < 3.5$ with merging of the period-2 and period-4 branches in the bifurcation diagram of the isolated logistic map occurring in the $3.0 < \mu < 3.5$ range (shown in right part of Fig. 8). The merging of branches constitutes yet another manifestation of symmetry breaking which suggests an important role for it in brain dynamics. Once the values of μ_i , as the network evolves, enter the fixed-point regime of the logistic map, the operation of the PCLMN begins to resemble that of the brain-state-in-a-box (BSB) [67] as far as convergence is concerned, especially because both networks have strongly diagonal-dominant coupling (connection) matrices (see Fig. 5 for the PCLMN). One expects then that the convergence proof of the BSB, based on the strongly diagonal-dominant property, would be applicable to the adapted PCLMN when its $\bar{\mu}$ vector has components that have entered the fixed-point regime of the bifurcation diagram of the logistic map. In fact, the convergence proof of the BSB may (as elaborated upon in [68] and [69]) be strictly more applicable to a network such as the PCLMN possessing analog states than to a network with binary states such as the BSB.

As seen in Fig. 8, all \bar{X}^* , $\bar{\mu}^*$ vector pairs obtained in several runs of the network have components X_i^* , μ_i^* , i = 1, 2, ..., N, falling in the fixed point regime of the bifurcation diagram of the isolated logistic map except now this regime is seen to extend into the period-2 and part of the period-4 solutions of the logistic map. In other words the fixed point regime $X_i^* = 1 - \frac{1}{\mu_i^*}$ of the isolated logistic map extends now to approximately $\mu_i^* = 3.5$ as a result of the parametric coupling. It is noting in this respect that the standard analysis of the period-doubling bifurcations of the logistic map (see for example [70] means the above behavior of the PCLMN's convergence to an extended fixed-point trace suppresses the period-m solutions (branches) (see Fig. 17.4 in [70]) and stabilizes the period 1 (fixed-point) solution of the period-m regime.

7. Cortex-module and perceptual categorizations

This section presents ideas, some speculative, as to how the PCLMN may be used to form a cortical-module capable of perceptional categorizations which is the ability to recognize an item or object, for example chair, despite all its normally occurring manifestations, that is despite the variations that it could normally have in shape, color, texture, lighting, location, etc. Achieving such higher-level perception is not an easy task because it requires deciphering and



Fig. 9. Cortical-module formed by a PCLMN in tandem with a hetero-associative memory. Because of the enormous number of input-specific attractors that can be furnished by the PCLMN, every input pattern, here feature-vector $\bar{X}^f(n)$, like that shown in the top left color pane, is guaranteed to produce a fixed-point attractor \bar{X}^* as output following a brief cascade of state-determined bifurcations lasting few tens of network iterations (top right color pane). The role of the hetero-associative memory is to tag the attractors of the PCLMN, produced by individual inputs, with labels that identify the input. With feedback (dotted line), and systematic pairing of the attractor-label associations stored in the associative network (memory), it becomes possible to form a predetermined sequence of associations (patterns) in response to an input. This is equivalent to forming a desired prescribed trajectory in state-space of the network in response.

emulating several aspects of neural and cortical information processing. How the PCLMN may facilitate this task is outlined next.

Real world events or object manifestations, whether the object is in motion or is stationary and subjected to saccadic eye movement, produce characteristic sequences of patterns (dynamic images or signatures) where the time order of their occurrence is important for cognition because it conveys information the brain is very adept at using. How the brain succeeds to categorize large number of patterns as belonging to the same object is a longstanding problem in cognition called *perceptual categorization*. A PCLMN coupled to a hetero-associative memory, as depicted in Fig. 9, forms a *cortical module*, a canonical higher-level processing unit that provides basis for answering this question. To see this, consider the PCLMN alone and recall that the PCLMN accepts also static input patterns and furnishes an enormous number of stimulus-specific attractors. This means it is possible to use its stimulus-specific fixed-point attractor as a next input to obtain a next attractor and then iterate the process. This feedback process is frequently called *reentry* because an entire pattern is returned to the input. The outcome would be a sequence of stimulus-specific attractors that forms a discrete-point trajectory in attractor space of the module. Such trajectory depends only on the initial stimulus pattern; it would be repeatable but cannot be predetermined, that is engineered by design; it would meander aimlessly in state space. To produce a desired prescribed trajectory one needs the cortical module of Fig. 9.

The feedback or reentry represented by the dotted line in the arrangement in Fig. 9 of the PCLMN in tandem with hetero-associative memory can lead to novel powerful capabilities an example of which is ability to characterize an input to the PCLMN not by a stimulus-specific fixed-point attractor and the pre-stored label it extracts from the hetero-associative memory label but by any desired structured self-terminating or repeating sequence or string of attractor-label pairs (associations) stored in the hetero-associative memory that are cycled or looped through by reentry. This would be important because: (a) it is widely agreed that such sequential and cyclic activity in the cortex underlies control of complex motor function as for example in speech and execution of coordinated motion; (b) it would be possible to organize the attractor-label pairs in the cortex-module in such a way that a label of a PCLMN

attractor, when it is fed back to the input of the PCLMN, produces a desired next (target) label from the memory and the new label when fed back in turn as new input would produce a next desired label and so on. This makes it possible to synthesize any desired discrete-time trajectory in the state space of the network that depends on the attractor-label pairs stored in the hetero-associative memory. Call this trajectory "primary or back-bone trajectory" and the final terminal label "primary label". It would be then possible, to trigger or initiate such a back-bone trajectory from any one of its components. But more interesting would be the possibility to trigger it by other strings of attractor-label pairs, belonging to other inputs that are designed, with the help of their own sets of attractor label pairs stored in the associative memory, to terminate on one of the labels in the main back-bone trajectory. In this manner, the primary label of the back-bone trajectory can be reached from trajectory strings, belonging to other inputs, that hone onto the back-bone trajectory, and happen to belong to different manifestation of the same object even if these manifestations and the inputs (feature vectors) they produce are quite dissimilar. This makes it possible to form an intricate web of trajectories terminating into the back-bone trajectory that define a large filamentary basin of attraction, possibly fractal, for the primary label in the state space of the module. The primary label is reached from any point of the back-bone of the branch trajectories connecting to the back-bone one. This can give rise to "perceptual categorization" where all manifestations of an object, chair for example, end up being classified by one concept/label: chair, the terminal attractor of the back-bone trajectory, while allowing for, a yet to be determined, possible means for differentiating between distinct chairs.

The back-bone trajectory and the web of trajectories terminating on it form an intricate engineered filamentary basin of attraction of the final label (primary label) on the main backbone trajectory. This would provide an ability to sculpt the basin of attraction in state space. Any motion triggered in the dendrite-like attractor in state space would correspond to fleeting activation and deactivation of Hebbian assemblies [42] in brain tissue. Because points in state space of the cortical-module correspond to regions of cortical activity, they can be viewed as fleetingly activated glimmering chains of Hebbian assemblies. One can imagine how the ability to carry out such operations, which can be rightly referred to as "phase-space engineering", could lead to novel and more complex operations that permit imitating higher-level brain function in future *cognitive machines*. For example the ability to produce such sequences of labels or patterns may be compared to the formation of trains of thought in the brain and the ability to form any desired state-space trajectory as crucial operation for complex motor control or motion synthesis. This is something that current neural networks and connectionist models have not been able to do because of the limited number of attractors they furnish as compared to the PCLMN. The number and lengths of controlled trajectories in the state-space of such cortical module (the domain of the dendritic-like attractor) is limited by the (capacity) of the hetero-associative memory which can be made arbitrarily large through the use of a bank of hetero-associative memories with the PCLMN output (the fixed-point attractor pattern), the vector \bar{X}^* or a 2-D binary encoded version of it that is optically broadcast in parallel to all members of the bank. This is feasible with 3-D holographic hetero-associative memories formed in photorefractive materials [71,72] that make use of present day state-of-the-art spatial-light modulators to impose the attractor information onto the coherent laser beams needed to form and read-out such memories. Holographic memories of this kind can make it possible to carry out fast parallel search of the vast associative memory bank for a next associated label to be reapplied at the input. In addition, variants of this scheme can be devised to allow distributed storage and recall to achieve robustness similar to that of the association cortex and hippocampal system that is the gateway of long term memory.

The cortical-module concept described here introduces a new paradigm in higher level information processing, involving novel state-space engineering capabilities, that appear to be more viable model of brain function than what has been reported or proposed in the neural networks literature so far.

8. Summary

The rich behavior one observes with the PCLMN has no close parallels in neural network and connectionist models, or in coupled map lattices and cellular automata. We believe that the use of PCLMNs to model thalamo–cortical networks and higher-level brain functions provides a unique opportunity and tool for further development of Corticonics and the eventual realization of intelligent brain-like systems capable of operating in complex uncontrolled environments both natural and artificial.

In support of these statements, and at the risk of repetition, we draw attention to the following observations and results:

• According to Nicholas Humphrey [73]

"Most stimuli in the real world are a matter of fact relatively brief, not least because our bodies are continually moving on and our sense organs exploring different parts of the environment. The result, presumably, is that what constitutes the conscious present is largely the immediate sensory *afterglow* of stimuli that have just passed by—the dying away of activity in reverberatory sensory loops. And it would follow that the temporal depth and subjective richness of this conscious present is bound to be determined by just how long this activity survives".

This statement suggests the brain/cortex is required to produce after activity from every transient sensory input feature-vector relayed to it from the thalamus and the sensory cortices. The PCLMN model of the thalamo-cortical loop presented in this review furnishes a mechanism for realizing such behavior. Humphrey states also that the effect of psychedelic drugs is by enhancing sensations through the lengthening of the after activity which is something that can be modeled and investigated with the PCLMN.

- The clustering of activity seen in the stimulus-specific fixed-point attractors is consistent with the "cell assemblies" hypothesis put forth by Donald Hebb in 1947 [42]. Hebb's basic idea was that there are interconnected, self-reinforcing subsets of neurons that formed representations of information in the nervous system. Single cells might belong to more than one assembly or clique depending on context. Multiple cell assemblies could be active at the same time corresponding to complex perception and thought. The applicability of this statement to the behavior of the PCLMN is striking when one replaces the term "subsets of neurons" by subsets of PCLMs (recall that a PCLM represents cortical column reciprocally interacting with the thalamus). The discussion in Section 7 makes therefore an intriguing correspondence between the cortical-module concept and its activity representation in state-space on the one hand and the idea of dynamic Hebbian assemblies in the cortex.
- An interesting aspect of the isolated clustering of activity in the convergent PCLMN is its reminiscence to *functional magnetic resonance imaging* (fMRI) and *positron emission tomography* (PET) images of brain activity in individuals subjected to a sensory stimulus or engaged in solving or executing an assigned cognitive or motor task.
- An important handing-over of control over dynamics from extrinsic to intrinsic control, together with the fact that $\mu_i(n)$, i = 1, 2, ..., N, the control parameters of the processing elements (PEs) in the network, are state-dependent, lead to *ultra-stability* of the PCLMN. Ultra-stability means that network parameters can always be found for which every applied stimulus leads always to a stimulus-specific static (fixed-point) or dynamic⁴ attractor in the state space of the network depending on the mode of operation, that is, either via information driven annealing or rapid cascade of state-determined bifurcations (rapid switching of behavior at critical points determined by the changing state of the network).
- Cognitive theories usually hypothesize a set of internal representations subjected to computational processes that extract from them meaning in order to produce meaningful (intelligent) behavior. The PCLMN produces such representations in the form of stimulus-specific fixed-point attractors and the cortex-module is an example of how meaning and intelligent control (behavior) can be produced from such representations.
- The brain has evolved to make sense of the world by looking for patterns and extracting meaning from them. Through this, the brain gets to know the world, a process known as cognition. More specifically, our brains look for redundancy (patterns, structure) in the world. We classify all non-redundant (structureless or random patterns) as belonging to the same meaningless set. The adapted PCLMN exhibits exactly such behavior when it forms distinct internal representations (stimulus-specific attractors) for redundant input patterns but forms the same attractor for all meaningless (non-redundant) patterns. The existence of the "meaningless attractor" would conceivably furnish a signal for increased vigilance or scrutiny.
- The challenging issues of separating figure from background and the differentiation and recognition of multiple objects in a scene are not addressed in the review. They may however be investigated by applying a PCLMN approach to the LGN-visual cortex complex.

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⁴ Although not described in this review, parameter-space points for the PCLMN do exist for which one observes dynamic attractors that are a mix of static (fixed-point) and dynamic (period-m or chaotic) orbits.

- The PCLMN is also a tool for demonstrating how order emerges from chaos and how complexity is increased through a process of annealing driven by information flow in the network. Thus reaching an attractor, where the flow of information in the PCLMN is zero, is consistent with the idea of an energy efficient brain.
- The PCLMN models emergent properties of the thalamo-cortical complex and every component (PCLM) involves a large number of neurons, something of the order of 10^4-10^5 neurons. If so, it can be argued that the processing power of a PCLMN, of say N = 100 elements equals that of 10^6-10^7 neurons. This, together with the fact that the PCLMN has local (self and nearest neighbor) connectivity is facilitating hardware implementation of corticonic networks [74–76].

In addition to the attributes listed above, the PCLMN, as dynamic memory, is scalable both in size and dimension. Points in parameter-space can always be found for which stimulus-specific-attractors are obtained regardless of topology or size or the nature of the dynamic stimulus (input) patterns which makes it useful in data fusion.

The above capabilities of the PCLMN model bring to mind the words of Richard Feyman. "We do not need to study the neurologic minutiae of living things to produce useful technologies, but even wrong theories may help in designing machines" [77]. They also provide convincing evidence that computer simulations of well informed models can reveal phenomena whose existence is difficult to prove by analytical means.

9. Relevant other work

Results of the numerical simulations discussed above and the conclusions drawn from them mutually illuminate several other studies. Here, discussed are the most directly relevant.

In the introduction of a recent volume [78], Perlovsky and Kozma make the following remarks in mutual agreement with certain functional properties and traits of the PCLMN model and venture into the domains of consciousness and chaos:

"A ubiquitous property of neurodynamics of consciousness is evolution from vague, fuzzy, and unconscious states to more concrete, and conscious. These neurodynamical processes are the essence of perception, cognition, and behavioral decision-making. More specific and conscious states correspond to recognized patterns and executed decisions. Neurodynamics proceeds from less knowledge to more knowledge and from less consciousness to more consciousness. Records of brain activity using EEG arrays and single-neuron evoked potential measurements indicate that brain states exhibit dynamic features of chaotic attractors [79,80]. Vague and less conscious states are characterized by high-dimensional chaotic attractors. More concrete and conscious states are characterized by lower-dimensional "less chaotic" attractors.

Transitions from high-dimensional chaotic states to lower-dimensional and "less chaotic" states form a sequence of increasingly structured dynamics. Ultimately the dynamics leads to conditions facilitating conscious decision making and deliberate action by the individual [80,81]. Following the action, a new situation is generated with modified internal state and goals and with changed environmental conditions. In this novel situation the evolution of the neurodynamics starts again from high dimensional to lower-dimensional "less chaotic" states, which is called the intentional action-perception cycle.

In a cognitive cycle, neurodynamics evolves from less conscious to more conscious states, from vague and uncertain to more concrete knowledge, which is described at an abstract level of dynamic logic [82]. According to dynamic logic, brain states reflect the surrounding world and they are characterized by representations-models and by measures of similarity between the models and input signals. Vague, less conscious states are described by uncertain models with low similarity values. More conscious states correspond to concrete perceptions, cognitions, and decisions; they are described by concrete models and high similarity values. These more conscious models are better adapted-matched to input signals. Adaptation of models is driven by maximization of similarity. This drive is a mathematical representation of a fundamental instinct for more knowledge and more consciousness [83].

To summarize, chaotic neurodynamics and dynamic logic are equivalent descriptions of the dynamics of brain states. Dynamic logic-based modeling field theory provides the modeling framework, which evolves through the action-perception cycle. Dynamic logic can provide a cognitively-motivated model-based approach to describe the emergence of models of increasing clarity as the cognitive cycle progresses. Initial states are vague, uncertain, and less conscious. They are described by highly chaotic neurodynamics. They evolve into more concrete, certain,

conscious states, described by less chaotic neurodynamics. Transitions from more chaotic states to less chaotic ones correspond to processes of perception, cognition, and decision making".

A second body of work pioneering the role of chaos in olfaction, EEG and brain theory is that of Freeman and colleagues who have completed extensive brain modeling studies in recent decades and years leading to a brain field theory summarized in [84], which is relevant to the topic of this review. Below, a few relevant excerpts from this article are given: (The numbering of the references cited in these excerpts are changed to fit in the list of references of this review).

• Networks vs populations

"Attempts to model brain function with model neurons and neural networks have not fared well. The limitation of networks stems from its failure to deal with the hierarchical organization of brains: microscopic, mesoscopic and macroscopic. At the microscopic cellular level the key module is the neuron. Network theory suffices to model neural chains in sensorimotor systems to and from cortex in vertebrate brain stem and spinal cord and in ganglionic chains of invertebrate 'simpler systems'. At the macroscopic brain level the key module is the local area of cortex or nucleus in the cerebral hemisphere that is labeled by imaging differential blood flow and other signs of metabolic enhancement. The intervening mesoscopic level of integration of neural activity into cortical areas, where each neuron interacts with many thousands of other neurons in tissue called neuropil, is not well accessed by either sets of methods. Neuropil cannot be reduced to discrete networks of equivalent neurons; its three types—laminar (cortex), nuclear (basal ganglia), and reticular (periventricular grey matter in the brain stem and spinal cord)—have differing architectures and state variables. The aim of this report is to introduce a field theory as an alternative approach that requires differing definitions, premises, rules of evidence, and classic experiments to address the distinctive properties of neural populations".

"In classical unit experiments a brief stimulus is typically confined to a single sense modality. The resulting unit activity is tracked through relays that process the stimulus information, and a network is constructed to simulate the processing. In classical behavioral experiments the subjects deal not with isolated stimuli but with environmental *situations*, which are signaled by swarms of sensory receptor action potentials that propagate through the spinal cord and brain, where they modify pre-existing unit activity. The impacts of sensory-evoked swarms destabilize cortical neuropil, causing it to jump through successive states [bifurcations, author's addition]. The goal of field theory is to model these states and state transitions as large-scale spatial patterns of neural activity that quickly lead to motor activity governing the engagement of subjects with their situations. The cortical states constitute 'wave packets' that resemble frames in a motion picture [85–89]. The rules of evidence in field theory require that brain states be defined as spatial patterns formed by phase transitions; that state variables include carrier waveforms and gating frequencies; and that EEG parameters be validated as meaningful by demonstrating correlations among EEG patterns, situations, subjects' actions, human subjects' descriptions of multi-sensory perceptions (Gestalts) and their meanings [86]".

• On EEG

"Dendritic current regulates action potentials. That same current in cortex is the main source of the electroencephalogram (EEG) as it flows across the fixed extracellular impedance of the neuropil. Due to summation of extracellular electric potentials of dendrites that are oriented in palisades in cortical neuropil, the EEG amplitude expresses the local mean field of unit activity in the neighborhoods of interacting pyramidal cells sustaining local field potentials. This feature endows the EEG with its value for estimating the magnitudes of spatiotemporal state variables in neuropil both at the mesoscopic network level and at the macroscopic level of the hemisphere but not at the microscopic level. In order to search in the EEG for neural correlates in the pulse activity of single neurons, the assumption is required that there is synchronization of the oscillations in dendritic current of the majority of neurons that contribute to the EEG. Their cooperation is established by the excitatory synapses they give to each other, by which they sustain background activity by mutual excitation [90]. The steady activity from this positive feedback loop is made oscillatory by the negative feedback from inhibitory interneurons. Now the positive feedback is not just disynaptic but also serial through three, four, and indefinitely more synapses, so that the feedback path resembles a one dimensional diffusion process, and the return of output from the pulse of each neuron to itself is a gamma distribution of order one half, that is, randomized [91]. The inference follows that the EEG manifests time multiplexing of neurons in cortical hypercolumns that share the load of signal transmission by random rotation of the duty cycle, thereby minimizing the disturbances in extracellular media of the metabolites

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that accrue with each action potential. The EEG is a correlate of the multiplexed, space-averaged signal output of the local neighborhood, which is very different from the time-averaged output of spike trains from single neurons, because the ergodic hypothesis doesn't hold for activity of cortical neurons in behaving subjects".

"Scalp and intracranial EEGs have value for both unit and field studies in relation to behaviors, but with different preprocessing. Analysis of units treats cortex and its affiliated components in the basal ganglia and cerebellum as a network of mesoscopic modules [92]. For purposes of analysis the EEG is interpreted as containing a mixture of time-dependent signals like voices at a cocktail party, so the signals are localized with high-pass spatial filters (Laplacian operators for EEG, synthetic aperture magnetometry for MEG) to focus modular signals, and PCA or ICA is applied to represent the sources of the "voices" by equivalent dipoles and the time-varying signal of each source as a component of the EEG. Field theory sees EEG as containing a cortical signal that is widely distributed in the neuropil. This component of the coordinated activity is observed in EEG through low-pass spatial filters to emphasize the spatial commonality. Its spatial domain is described by its boundary condition and the textured pattern that is imposed on its oscillatory carrier wave. That pattern is displayed as a vector with its tip inscribing an itinerant chaotic trajectory [93] in brain state space. Its sequential spatial patterns of amplitude modulation (AM) of the shared aperiodic carrier wave vary in sizes and textures that correlate with cognitive situational behaviors (Ohl, Scheich and Freeman, [94]; Ohl et al., [95]; Freeman, [86,96]) of the kind documented by Köhler's field theory [97], Tolman's [98] cognitive map, and Gibson's [99] ecological psychology".

On Paleocortex vs. neocortex

"Paleocortex (olfactory system) in mammals is supplemented with a far more complex form of neuropil: 6-layered neocortex. Despite its enormous size in humans, neocortex retains its anatomical unity and its dynamics for extremely rapid, repetitive phase transitions with formation of spatial patterns. The specialized neural networks embedded in neocortex operate as input/output interfaces for seeing, hearing, touching and intentional action [92]. Understanding the integration of their roles in perception is a major aim of field theory".

On K-sets

"The proposed basis for a field theory is a hierarchy of nonlinear ordinary differential equations, having noninteractive populations (KO) near equilibrium at its base [91,100]. By synaptic transmission KO sets create interactive populations of excitatory (KIe) or inhibitory (KIi) neurons with nonequilibrium point attractors. KI populations of olfactory excitatory or inhibitory neurons comprise the KII set having also a limit cycle attractor used to model olfactory bulb or prepyriform cortex. The KIII set incorporates 3 KII sets to model chaotic attractors governing the olfactory EEG [101], and to replicate olfactory capacity for classification of incomplete patterns embedded in complex backgrounds with facilitation by noise [102,103]. The KIV model of the primordial amphibian paleocortex comprising its limbic system is currently under development as an interactive network of 3 KIII sets being used by autonomous vehicles for navigation [104,105]. The new goal is to construct a KV set that models the unique neurodynamics of neocortex found only in mammals.

It is important to distinguish the neuropil as an embedding medium for modules from the nets with precisely organized connections that define the modules within areas of cortex and to and from the thalamus, basal ganglia and pons [92]. Anatomical data already make clear that neocortical neuropil should be seen not only as a network of hypercolumns but also as a unified structure comprising an entire cerebral hemisphere, Its global connectivity cannot be modeled as a random graph with uniform probability of connection lengths. Short connections approximating nearest neighbor predominate; the significant proportion of long connections leads to small-world effects [106] by which local activity is seeded to distant sites, so that no neuron in the neuropil is more than a few synapses from any other. Several types of network have been proposed [107–110] to model the long-distance synchronization that is readily observed in scalp EEG [111–113]. Quantitative anatomical data have for years [114] shown exponential connectivity distributions among cortical neurons; recent data [115] show fractal and power-law distribution of cortical connectivity of scale-free networks [116]. A goal of field theory is to suggest how to use these concepts to model neocortex as a global organ that stabilizes itself by mechanisms that are sketched in [88,89]".

10. Outlook

Crucial to dealing with brain complexity is the development of a unifying theory or model that abstracts and simplifies and yet has predictive and synthesizing power. Parametrically coupled networks of bifurcation processing

elements (here logistic or quadratic maps), are capable of imitating a good number of functional behaviors attributed to the cortex and the way it interacts with subcortical areas like the thalamus and the hippocampus to produce higher-level brain function. The Corticonic equations and the PCLMN simulations described here give rise to several mechanisms associated with higher-level brain function such as: handling both static and dynamic sensory input patterns, provision of an immense pool of attractors for inputs to choose form, dynamic memory, clustering capability, formation of Hebbian assemblies, information driven annealing into stimulus-specific attractors that does not involve search for local minima of an energy function, immense information carrying capacity, ability to distinguish between redundant and non-redundant input pattern, and ability to form a cortical module that facilitates categorical perception. These attributes are in general agreement with or have been anticipated in other relevant work. They furnish basis for the design of future machines with brain-like intelligence and for development of a cognitive technology including rudimentary artificial brain.

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