ESSAYS ON FRICTIONAL FINANCIAL MARKETS

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ABSTRACT

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This thesis uses theoretical approach to study various types of frictions in financial markets. In the first chapter, "Information Acquisition and Liquidity Traps in OTC Markets," I analyze the interaction between buyers' information acquisition and market liquidity in over-the-counter markets with adverse selection. If a buyer anticipates that future buyers will acquire information about asset quality, she has an incentive to acquire information to avoid buying a lemon that will be hard to sell later. However, when current buyers acquire information, they cream-skim the market, leaving a larger fraction of lemons for sale and giving future buyers an incentive to acquire information. A liquid market can go through a self-fulfilling market freeze when buyers start to acquire information. More importantly, if information acquisition continues for a long enough period of time, the market gets stuck in an information trap with low liquidity: information acquisition worsens the composition of assets remaining on the market, and the bad composition incentivizes information acquisition. This prediction helps explain why the market for non-agency residential mortgage-backed securities experienced a sudden drop in liquidity–as potential buyers realized the need for greater due diligence–but has remained essentially dormant despite a strong recovery in the housing market.

In the second chapter, "Intervention with Screening in Global Games," my coauthor Lin Shen and I propose a novel intervention program to reduce coordination failure. Compared with the conventional government-guarantee type of programs, such as demand deposit insurance, it incurs a lower cost of implementation and suffers less from moral hazard problems. The proposed program effectively screens agents based on their heterogeneous beliefs of the coordination results. In equilibrium, only a small mass of "pivotal agents" self-select to participate in the program. However, the effect is amplified by strategic complementarities, and coordination failure can be significantly reduced. We demonstrate the generality of the proposed program with applications in panic-based bank runs, debt rollover problems, self-fulfilling market freezes, and underinvestment problems in the real economy.

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CHAPTER 1 : Information Acquisition and Liquidity Traps in OTC Markets

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1.1. Introduction

During the 2007–2008 financial crisis, many asset markets suffered from periods of illiquidity—sellers found it increasingly hard to sell assets at acceptable prices. Dry-ups in liquidity are especially prominent among classes of assets that are opaque and traded in over-the-counter (OTC) markets, as in the case with mortgage-backed securities (Gorton, 2009) and collateralized debt obligations (Brunnermeier, 2009). A large literature has sought to explain these events of market freezes through the lens of asymmetric information.[2] The standard narrative is that asset owners are better informed of their assets' quality than potential buyers in these markets. Therefore, when the perceived average quality of assets decreases, markets freeze as a result of the exacerbated adverse selection problem.

One decade after the financial crisis, the US economy is on track for the longest expansion ever, and housing prices are on a path of continued growth.[3] However, the impact of the crisis seems rather persistent. The market for non-agency residential mortgage-backed securities (RMBS), which was at the center of the financial crisis, has yet to come back (Ospina and Uhlig, 2018).[4] At the same time, investors have been conducting more due diligence in inspecting and evaluating securitized products since the crisis. Instead of solely relying on external ratings, investors now develop their own models to provide independent

[1]I am deeply indebted to my committee members: Itay Goldstein, Guillermo Ordonez, Benjamin Lester, Vincent Glode and Harold Cole for their guidance and support. I also thank Mitchell Berlin, Guillermo Calvo, Alessandro Dovis, Ricardo Lagos, Christian Opp, Victor Rios-Rull, Chaojun Wang, Pierre-Olivier Weill, Shengxing Zhang, as well as seminar participants in the Wharton Finance Seminars, the Macro Seminars at Penn and the Federal Reserve Bank of Philadelphia for useful comments. All errors are my own.

[2]See Tirole (2012), Daley and Green (2012), Camargo and Lester (2014), Guerrieri and Shimer (2014) and Chiu and Koeppl (2016), among many other papers.

[3]See All-Transactions House Price Index for the United States, `https://fred.stlouisfed.org/series/USSTHPI`.

[4]Non-agency mortgage-backed securities are issued by private entities, and do not carry an explicit or implicit guarantee by the US government. In contrast, agency MBS are issued and backed by government agencies or government-sponsored enterprises, such as Fannie Mae, Freddie Mac and Ginnie Mae.

assessments of asset quality.[5] These stark differences in market liquidity and the behavior of market participants before and after the crisis, despite similar fundamentals of the market, are hard to reconcile with the standard narrative of adverse selection. Indeed, if the RMBS market freeze was driven by deterioration of the value of the underlying mortgages, the market should have recovered given the current strong economic fundamentals and the bullish housing market.

To explain both the decline in market liquidity and the increase in investors' due diligence, I introduce buyers' information acquisition into a dynamic adverse-selection model with resale considerations. The key result of my model is that an asset market can have multiple steady states, and more importantly, transitions between steady states are asymmetric. Liquid markets are susceptible to a *self-fulfilling market freeze*, in which buyers suddenly start to acquire information and the market quickly transitions from a liquid state to an illiquid one. As illiquid trading and information acquisition continue for an extended period, the market falls into an *information trap* with low liquidity and information acquisition, in which there is no equilibrium path that leads back to the liquid state. Importantly, while some previous papers have studied sudden market freezes in the framework of multiple equilibria, my findings are different in terms of the sharp prediction of whether the market can recover in a self-fulfilling manner after a market freeze.

Before describing these results in greater detail, it makes sense to first lay out the key ingredients of the model. A continuum of investors trades assets of either high or low quality. Gains from trade arise because asset owners are subject to idiosyncratic liquidity shocks that lower the flow payoff from holding assets. Upon receiving a liquidity shock, an asset owner participates in the market as a seller and trades with potential buyers who arrive sequentially. A seller is privately informed of the quality of her own asset, while the

[5] For instance, see *The Economist* in its January 11, 2014, issue: "Before 2008, . . . , investors piled in with no due diligence to speak of. Aware of the reputational risks of messing up again, they now spend more time dissecting three-letter assets than just about anything else in their portfolio." Also, Kaal (2016) finds that since the financial crisis, private funds have hired more analysts to conduct investors' due diligence using textual analysis of the ADV II filings.

buyer can acquire a noisy signal of the asset's quality by incurring a fixed cost. If the asset is traded, the buyer hold the asset and will return to the market as a seller when receiving a liquidity shock in the future. Otherwise, the seller keeps the asset and waits for the arrival of the next buyer. Although this paper is motived by observations in the non-agency RMBS market, the model can be applied to various OTC markets with asymmetric information.

How does buyers' information acquisition interact with market liquidity? If the current composition of assets for sale is good enough to support pooling trading, buyers' information acquisition reduces current market liquidity. Intuitively, if a buyer acquires information and observes a bad signal, she is unwilling to trade at a pooling price because the posterior belief about the asset's quality becomes worse. In addition to the static relationship between buyers' information acquisition and market liquidity, there is also a dynamic strategic complementarity between buyers' current and future incentives to acquire information, and hence a complementarity between current and future market liquidity. On one hand, current buyers' incentive to acquire information depends on future buyers' information acquisition through the *resale consideration*. If a buyer anticipates that future buyers will acquire information about asset quality, she has an incentive to acquire information so as to avoid buying a low-quality asset that will be hard to sell at a later date. In this sense, expected future market liquidity improves current market liquidity. On the other hand, current buyers' information acquisition changes future buyers' incentives to acquire information through the *cream-skimming effect*. When current buyers acquire information, high-quality assets are traded faster than low-quality assets. As low-quality assets accumulate on the market over time, future buyers have more incentive to acquire information. Therefore, current market illiquidity harms future market liquidity.

The dynamic strategic complementarity in buyers' information acquisition gives rise to the possibility of a self-fulfilling market freeze. Suppose the market is in a liquid state, in which buyers do not acquire information and the composition of assets for sale is good. One day, investors suddenly start to worry that in the future buyers will acquire information, lowering market liquidity . As a result, the resale value of low-quality assets drops abruptly and the current buyers start to acquire information. Because of the cream-skimming effect of information acquisition, the composition of assets for sale deteriorates gradually, giving future buyers more incentive to acquire information. This justifies current investors' belief in future low liquidity. A self-fulfilling market freeze takes place when investors coordinate to follow an equilibrium path with information acquisition.

As the self-fulfilling market freeze continues and the composition of assets for sale declines further, it is impossible for the market to return to liquid trading without outside intervention. This dynamic is apparent if we note that buyers' incentives to acquire information depend on both future market liquidity and the current composition of assets for sale. When the composition is bad enough, even if buyers believe the market will be liquid in the future, it is still optimal for them to acquire information today to avoid buying low-quality assets. Their information acquisition in turn keeps the composition of assets for sale at a low level. The market is therefore "trapped" in an illiquid state with information acquisition and longer trading delays.

The key mechanism that generates the asymmetric transitions between states with different liquidity is the slow-moving property of the composition of assets for sale. Buyers' information acquisition worsens the composition of assets for sale through the cream-skimming effect and has a long-lasting negative impact on future market liquidity. The composition will only improve gradually when buyers stop acquiring information. However, even with the most optimistic belief about future market liquidity, buyers will not stop acquiring information unless the composition of assets is good enough. Buyers' information acquisition and the bad composition of assets for sale reinforce each other, preventing the market from recovering without outside intervention to clean the market.

This paper sheds light on the discussion of regulatory reforms to increase transparency in many asset markets. For example, Dodd-Frank Act Section 942 requires issuers of asset-backed securities (ABS) to provide asset-level information according to specified standards.

These measures increase the precision of buyers' idiosyncratic signals when they conduct due diligence. Although these measures can potentially discipline the ABS issuance process, I show that they have the unintended consequence of increasing fragility in the secondary market. When buyers have access to more precise signals, they have a greater incentive to acquire information and provide quotes conditional on the signals. Therefore the cream-skimming effect becomes stronger and the market is more susceptible to an information trap.

This paper also has important implications for the timing of the provision of asset purchase programs aiming to revive the market. During the latest financial crisis, the US Treasury created the Troubled Asset Relief Program (TARP), aimed at restoring a liquid market by purchasing "toxic" assets. I show that the fraction of "toxic" assets on the market is endogenous and depends on investors' information acquisition in the past. As the market gets deeper into a crisis, the asset composition on the market becomes worse and policy makers need to purchase a larger amount of low-quality assets to revive the market.

The paper is organized as follows. I describe the model setup in Section 1.2. Section 1.3 focuses on the equilibrium analysis. The stationary equilibria are studied in Section 1.4. In Section 1.5 I explore the set of non-stationary equilibria that converge to different steady states. Policy implications are studied in Section 1.6. Section 1.7 concludes.

Related Literature

This paper builds on the large literature on adverse selection initiated by the seminal work of Akerlof (1970). Among many other papers, Janssen and Roy (2002); Camargo and Lester (2014); Chari et al. (2014), and Fuchs and Skrzypacz (2015) analyze dynamic-adverse selection models with centralized or decentralized market structures.[6] These models share the common feature that low-quality assets are sold faster than or at the same speed as high-quality assets. None of these papers feature resale considerations or buyers' acquisition of information about assets' quality.

[6] See also Hendel and Lizzeri (1999), Blouin (2003), Hörner and Vieille (2009), Moreno and Wooders (2010).

Taylor (1999), Zhu (2012), Lauermann and Wolinsky (2016), and Kaya and Kim (2018) all considers dynamic adverse-selection models in which each buyer observes a noisy signal about an asset's quality. A new result obtained in this strand of literature is that high-quality assets are traded faster than low-quality assets. This is related to the cream-skimming effect in my model when buyers acquire information. These papers consider a trading environment with a single seller and sequentially arriving buyers, and there is no scope for reselling the asset. In contrast, in my paper, buyers anticipate that they will sell their assets in the same market when they experience liquidity shocks.

In papers that study dynamic adverse-selection models with resale considerations—such as Chiu and Koeppl (2016) and Asriyan et al. (2018)—buyers' valuation of an asset depends on future market liquidity. This gives rise to an intertemporal coordination problem which in turn yields multiple steady states with symmetric self-fulfilling transitions. Another closely related study is by Hellwig and Zhang (2012), who analyze a dynamic adverse-selection model with both resale consideration and endogenous information acquisition. While I allow buyers' signals to be noisy, they focus on the situations in which the signals are precise. Therefore, information acquisition has no cream-skimming effect in their model and transitions between steady states are symmetric. In contrast to all of the above papers, mine has the novel feature of generating multiple steady states with unidirectional transitions.

This paper is also related to work by Daley and Green (2012, 2016), who study the role of a publicly observable "news" process in dynamic-adverse selection models. In my paper, buyers make their own decisions on whether to acquire information and the information is not observable to other market participants.

In terms of modeling search frictions, this paper builds on the theoretical papers on OTC markets. Examples are Duffie et al. (2005, 2007); Vayanos and Weill (2008); and Lagos and Rocheteau (2009). The trading environment is very similar to the investor's life-cycle model in Vayanos and Wang (2007). I contribute to this literature by introducing asymmetric information about asset quality.

There is a large literature that studies information acquisition in financial markets, including Froot et al. (1992); Glode et al. (2012); Fishman and Parker (2015); as well as Bolton et al. (2016).[7] This literature shows that information acquisition can be a strategic complement and excess information acquisition in equilibrium leads to inefficiency. I differ from this line of research by studying information acquisition in a dynamic trading environment. This allows me to characterize transitions between different states of the market, such as episodes of market freezes or recovery. Also, I consider an opaque trading environment in which trading history is not directly observable to other market participants. This differentiates my paper to the literature that features positive spillover effect of information acquisition.[8]

Lastly, this paper contributes to the literature on the role of transparency and information acquisition in financial crises. Gorton and Ordonez (2014) study how a small shock to the collateral value can be amplified into a large financial crisis when it triggers information acquisition. In my model, a market freeze can arise as a self-fulfilling outcome. Also, I study a topic not addressed in their paper: whether a market can recover after a crisis. In terms of policy implications, this paper is related to the recent discussion of optimal disclosure of information by government and regulators, as in Alvarez and Barlevy (2015); Bouvard et al. (2015); Gorton and Ordonez (2017); and Goldstein and Leitner (2018). A closely related study is that of Pagano and Volpin (2012), who also look at the welfare implications of increasing transparency in the securitization process. My work differs from the literature in that I argue that information disclosure does not directly reveal the value of an asset; instead, investors need to conduct due diligence to interpret the disclosed information. The noise in the interpretation of disclosed information reflects the complexity of the underlying assets, such as securitized products. Greater transparency reduces noise, but it can also exacerbate adverse selection in the market through the cream-skimming effect.

[7]See also Barlevy and Veronesi (2000), Veldkamp (2006), Hellwig and Veldkamp (2009), Goldstein and Yang (2015).

[8]See Camargo et al. (2015) for an example.

1.2. The Model

Time is continuous and infinite. There is a continuum of assets with mass 1. The quality of an asset is either high or low, denoted by $j \in \{H, L\}$. The mass of high-quality and low-quality assets is fixed at $\alpha/(1+\alpha)$ and $1/(1+\alpha)$ respectively, so the ratio of high-quality to low-quality assets is α, which is an exogenous parameter that controls the average quality of the assets. Therefore I will refer to α as the *fundamental* of the market.[9]

The trading environment is populated with a continuum of investors. They are risk-neutral and discount time at rate r. Each of them is restricted to holding either 0 units or 1 unit of an asset. Their preference for holding assets can be either *unshocked* or *shocked*, reflecting the fact that some investors experience liquidity shocks and become financially constrained. Whether an investor is shocked is observable or verifiable. When holding an asset of quality $j \in \{H, L\}$, an unshocked investor enjoys a flow payoff designated as rv_j, while a shocked investor enjoys a flow payoff of rc_j. Throughout this paper, I maintain the assumption that $v_H > c_H > v_L \geq c_L > 0$. Thus, the shocked investors enjoy a lower flow payoff from holding both types of assets. Also, $c_H > v_L$, meaning that the common value component dominates the private value component, which is a necessary condition for the existence of the lemons problem.

Following Vayanos and Wang (2007), I consider a life-cycle model of OTC markets. At any time, there is a flow into the economy of unshocked investors without assets, the buyers in the market. They have a one-time opportunity to trade with the shocked asset owners, who are the sellers in the market. After buying an asset, a buyer becomes an unshocked asset owner. Otherwise, if trade is unsuccessful, the buyer exits the market with zero payoff. Since an investor's liquidity shock is observable, there will be no trade between a buyer and an unshocked asset owner.[10] Therefore, unshocked asset owners only passively hold assets

[9] I deviate from the conventional notation of using the fraction of high-quality assets to represent the average quality of the assets. The notation adopted here turns out to be convenient for characterizing investors' beliefs and asset distribution.

[10] This is a direct implication of the No-Trade Theorem in Milgrom and Stokey (1982).

until their preferences change. These investors are labeled as holders. Holders face liquidity shocks that arrive at Poisson rate δ. Upon receiving a liquidity shock, a holder becomes a seller and offers her asset for sale on the market. For simplicity, I assume that the inflow of buyers at any time equals a constant λ times the mass of sellers in the market. These buyers are matched with sellers randomly. Therefore, from a seller's perspective, buyers arrive at a constant Poisson rate λ. Sellers stay in the market until they sell the assets and exit the economy with zero payoff.

The flow of investors in the economy is summarized in Figure 1. Buyers enter the economy from the pool of outsider investors. When a seller sells an asset, she exits the economy and returns to the pool of outside investors. I use the word *market* to represent the two groups of active traders in the economy, the sellers and the buyers. From a buyer's perspective, the severity of the adverse selection problem is determined by the composition of sellers with high-quality and low-quality assets. Notice that sellers are a subset of asset owners who actively participate in the market. Therefore, the composition of assets among sellers can potentially differ from the fundamental of the market, which is the asset composition among all asset owners. In this sense, the level of adverse selection in my model is endogenous and depends on the asset distribution. Later, I use the word *market composition* to represent the composition of high-quality and low-quality assets among sellers.

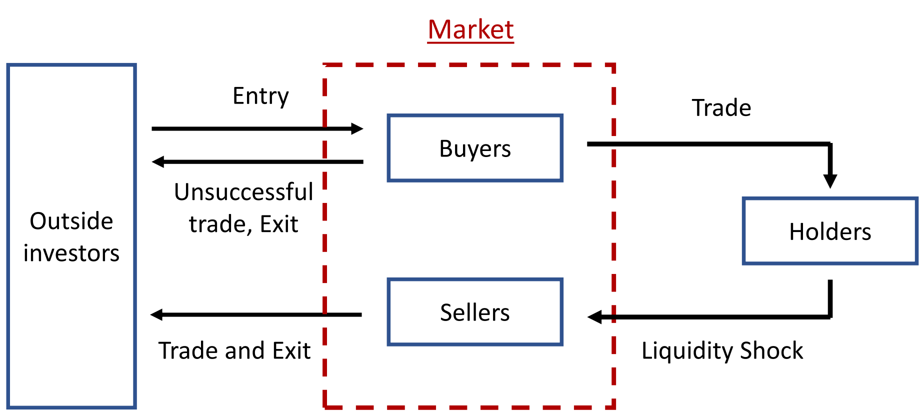

Figure 1: Flow Diagram of the Asset Market

When a buyer meets a seller, the seller is privately informed of the quality of her asset. The buyer does not observe the quality of the seller's asset, nor does she have information regarding the trading history of the seller. Her prior belief is determined by the market composition—i.e., the ratio of high-quality assets and low-quality assets among sellers. In addition, the buyer can pay a fixed cost k to acquire information and obtain a signal $\psi \in \{G, B\}$ of the asset's quality. G represents a good signal and B represents a bad signal. The probability of observing a signal ψ from an asset of quality j is f_j^{ψ}. Signals obtained by different buyers are jointly independent conditional on the quality of the asset. The assumption that a buyer can only observe a noisy signal of the asset's quality captures the opaque nature of the assets. Different buyers may have different evaluations of the same asset. Without loss of generality, I assume $f_H^G > f_L^G$, so a high-quality asset is more likely to generate a good signal than a low-quality asset. This implies that a good signal improves the buyer's posterior belief about the asset's quality. The trading protocol is deliberately simple. The buyer makes a take-it-or-leave-it offer to the seller. The entire transaction takes place instantly, with the seller and buyer separating immediately afterward.

1.3. Equilibrium Analysis

In this section I analyze investors' optimal trading strategies and define the equilibrium of the model. Since investors are infinitesimal, they take the continuation value of leaving a match as given. This allows me to separate the equilibrium analysis into three parts. First, I study a static trading game between a seller and a buyer, taking the continuation values as given. Second, I determine the continuation values of different agents. Lastly, I characterize the evolution of the asset distribution.

1.3.1. The Static Trading Game

The static trading game is played by one seller and one buyer. To define a static trading game, it is sufficient to specify the prior belief of the buyer and the terminal payoffs of both players when they separate. I denote the buyer's prior belief by $\theta(t)$, which equals the probability that the seller carries a high-quality asset divided by the probability that the seller carries a low-quality asset. If θ is small, there is a large fraction of low-quality assets on the market, and the adverse selection problem is severe. In equilibrium, θ must be consistent with the asset distribution among sellers when the buyer meets the seller. If the seller sells the asset or the buyer does not buy the asset, they leave the economy with zero continuation value. If the buyer buys an asset of quality $j \in \{H, L\}$, the continuation value is denoted by $V_j(t)$, which is also the continuation value of a passive holder at time t. If the seller keeps an asset of quality j, the continuation value is denoted by $C_j(t)$. From now on, I omit the time argument of all variables when analyzing the static trading game. A static trading game is therefore defined by the combination of the buyer's prior belief and the continuation values $(\theta; V_H, C_H, V_L, C_L)$. For reasons that will become clear later, we only need to consider the case of $V_H > C_H > V_L, C_L$.

The static game has two stages, the information acquisition stage and the trading stage. We use backward induction to solve the static game. The seller's optimal strategy takes a simple form. A seller with an asset of quality j is going to accept any price higher than the continuation value C_j and reject any offer below C_j. The buyer needs to decide whether to acquire information, and based on her belief about the asset's value after the information acquisition stage, decides upon an optimal offering price. If the buyer acquires information, she will update her belief in a Bayesian way. Her posterior belief about the asset's quality after seeing signal $\psi \in \{G, B\}$ in the form of a high-quality to low-quality ratio is

$$\tilde{\theta}(\theta, \psi) = \frac{f_H^\psi}{f_L^\psi}\theta. \tag{1.1}$$

If the buyer doesn't acquire information, the posterior belief $\tilde{\theta}$ equals the prior belief θ. For the consistency of notation, let $\tilde{\theta}(\theta, N) = \theta$ represent the posterior belief if the buyer has chosen not to acquire information.

The following lemma characterized the optimal offering strategy of the buyer conditional on the posterior belief $\tilde{\theta}(\theta, \psi)$.

Lemma 1.1. *The buyer's strategy is characterized by a threshold belief*

$$\hat{\theta} = \frac{C_H - \min\{C_L, V_L\}}{V_H - C_H}.$$

1. *If* $\tilde{\theta}(\theta, \psi) > \hat{\theta}$*, the buyer makes a pooling offer* C_H*,*

2. *If* $\tilde{\theta}(\theta, \psi) < \hat{\theta}$ *and* $V_L > C_L$*, the buyer makes a separating offer* C_L*,*

3. *If* $\tilde{\theta}(\theta, \psi) < \hat{\theta}$ *and* $V_L < C_L$*, the buyer makes a no-trade offer* $p < C_L$*.*

If the buyer's posterior belief $\tilde{\theta}(\theta, \psi)$ is above the threshold $\hat{\theta}$, the buyer should offer a pooling price C_H to trade with both the high-quality and the low-quality seller. However, if the buyer's posterior belief is not good enough, the optimal price to offer depends on the relationship between V_L and C_L or, alternatively, whether there are gains from trade of a low-quality asset. If $V_L > C_L$, the buyer values a low-quality asset more than the seller does, and the buyer can offer a separating price C_L that will only be accepted by a low-type seller. On the other hand, if $V_L < C_L$, the buyer values a low-quality asset less than the seller does, and it is optimal for the buyer to offer a no-trade price, which is lower than a low-type seller's continuation value, to avoid buying the asset. In the knife-edge case of $\tilde{\theta}(\theta, \psi) = \hat{\theta}$, or $V_L = C_L$, the optimal offering strategy of the buyer can be a mixed strategy.

In the information acquisition stage, the buyer will compare the value of information, which is the increase in the expected payoff after the buyer observes the signal, to the cost of information acquisition. She will only acquire information about the asset when the net gain is positive. The signal is potentially valuable to the buyer because it gives the buyer the option of making offers conditional on the signal. Depending on prior belief, the buyer will either improve the offered price when seeing a good signal, or lower the offered price when seeing a bad signal.

Lemma 1.2. *The value of information is*

$$W(\theta) = \begin{cases} \max\left\{-\frac{\theta}{1+\theta} f_H^B (V_H - C_H) + \frac{1}{1+\theta} f_L^B (C_H - \min\{C_L, V_L\}), 0\right\}, & \text{if } \theta \geq \hat{\theta}, \\ \max\{\frac{\theta}{1+\theta} f_H^G (V_H - C_H) - \frac{1}{1+\theta} f_L^G (C_H - \min\{C_L, V_L\}), 0\}, & \text{if } \theta < \hat{\theta}. \end{cases}$$

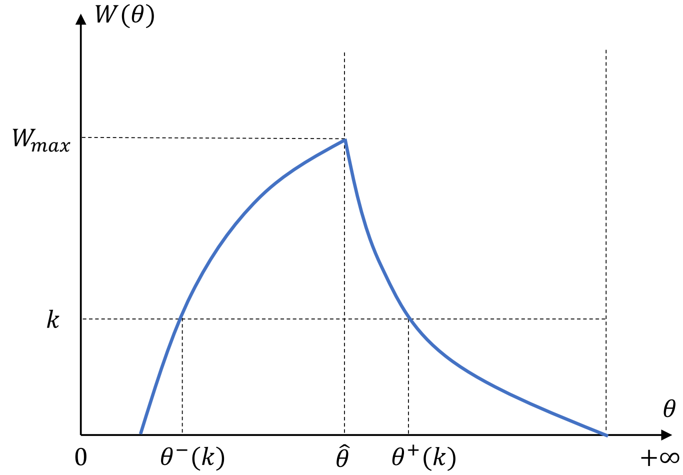

Figure 2: Value of information to the buyer.

Figure 2 depicts the value of information as a function of the prior belief θ. Let W_{max} be the maximum value of information. If the prior belief θ falls at the left or right end of the $[0, 1]$ interval, the value of information is zero. This is because the prior belief is so high (low) that even after observing a bad (good) signal, the posterior is still higher (lower) than the threshold belief. If the prior belief is around the threshold belief $\hat{\theta}$, the value of information first increases from 0, reaches the maximum at $\hat{\theta}$, and then decreases to 0. The buyer will acquire information if and only if the value of information based on the prior belief is greater than the cost of acquiring information. The following lemma summarizes the buyer's optimal strategy in information acquisition.

Lemma 1.3. *If* $k < W_{max}$*, the buyer will acquire information if and only if*

$$\theta^-(k, \min\{C_L, V_L\}) \leq \theta \leq \theta^+(k, \min\{C_L, V_L\}),$$

where the two functions are defined as

$$\theta^-(k,\nu) = \frac{f_L^G(C_H-\nu)+k}{f_H^G(V_H-C_H)-k}, \quad \theta^+(k,\nu) = \frac{f_L^B(C_H-\nu)-k}{f_H^B(V_H-C_H)+k}.$$

Both $\theta^-(k,\nu)$ and $\theta^+(k,\nu)$ are decreasing in ν.

When the value of a low-quality asset ($\min\{C_L, V_L\}$) decreases, the loss of buying a low-quality asset at pooling price C_H is higher. Therefore, the buyer is more inclined to avoid low-quality assets on the right boundary of the information-sensitive region and less willing to rely on the noisy signal on the left boundary. The information-sensitive region $[\theta^-(k), \theta^+(k)]$ moves to the right as both C_L and V_L decrease. As we will show later, C_L and V_L are determined by both the flow payoff from holding the asset and the likelihood that a low-quality asset can be sold at the pooling price in the future. The above comparative statics are important because they are related to the resale consideration that links the current buyers' information acquisition decision to future market liquidity. When the current market composition is relatively good (θ on the right boundary of the information-sensitive region), buyers are more willing to acquire information if their belief about future market liquidity deteriorates.

To conclude the analysis of the static trading game, I summarize the trading probability in the equilibrium of the static trading game (for the non-knife-edge cases) when $k < W_{max}$ in Table 1. When θ falls on the boundary of the information region, the equilibrium is not unique. The buyer will use a mixed strategy of information acquisition. Thus, the set of trading probabilities is the convex combination of the set of trading probabilities of the adjacent regions.

	$\theta < \theta^-(k,\nu)$	$\theta^-(k,\nu) < \theta < \theta^+(k,\nu)$	$\theta > \theta^+(k,\nu)$
$V_L < C_L$	$\rho_H = \rho_L = 0$	$\rho_H = f_H^G, \rho_L = f_L^G$	$\rho_H = \rho_L = 1$
$V_L = C_L$	$\rho_H = 0, \rho_L \in [0,1]$	$\rho_H = f_H^G, \rho_L \in [f_L^G, 1]$	$\rho_H = \rho_L = 1$
$V_L > C_L$	$\rho_H = 0, \rho_L = 1$	$\rho_H = f_H^G, \rho_L = 1$	$\rho_H = \rho_L = 1$

Table 1: Trading probability when $k < W_{max}$

1.3.2. *Continuation Values*

First I introduce some notations that describe the investors' strategy in the full dynamic game, allowing for both pure strategy and mixed strategy. I use $\mu(p, j, t) \in [0, 1]$ to represent the probability of type j seller accepting offer p at time t. The buyer's strategy is more complicated and can be denoted by a couple of functions $\{i(t), \sigma(p, \psi, t)\}$.[11] $i(t) \in [0, 1]$ is the probability that the buyer acquires information at time t. $\sigma(p, \psi, t)$ represents the probability of offering p in a match at time t when seeing signal ψ. If a buyer does not acquire information, $\psi = N$ following the previous notation. Therefore, $\sigma(p, N, t)$ is the buyer's probability of offering p in a match at time t conditional on not acquiring information. In principle, a buyer can draw a price from a mixed distribution. Fortunately, based on the analysis of the static trading game, the buyer will only choose from three relevant offers at any time.[12] Thus it's without loss of generality to assume $\sigma(\cdot, \psi, t)$ is a probability mass function of p.

With the help of the above notations, we can write down $\gamma_j(p, t)$, the probability that a type j seller is offered price p conditional on meeting a buyer at time t.

$$\gamma_j(p, t) = i(t) \sum_{\psi=G,B} f_j^{\psi} \sigma(p, \psi, t) + (1 - i(t))\sigma(p, N, t). \tag{1.2}$$

$\gamma_j(p, t)$ characterizes the market condition faced by a type j seller at time t. If $\gamma_j(p, t)$ has more weights on high prices of p, the market is more liquid for sellers with assets of quality j because it's easier for them to sell the assets at a high price.

The continuation value of sellers with high-quality assets is at least c_H since the sellers can always hold on to their assets. Also, no buyer will offer a price higher than c_H in

[11] Note that the strategy functions are independent of the identity of any given buyer or seller. This means that we will focus on equilibria with symmetric strategies without loss of generality because for any equilibrium with asymmetric strategies, we can find an equilibrium in symmetric strategies with the same path of asset distributions, trading volume, and average prices.

[12] We can pick any $p < c_L$ to be the no-trade price.

equilibrium.[13] Therefore

$$C_H(t) = c_H. \tag{1.3}$$

The previous analysis of the static trading game shows that only three types of prices will be offered by a buyer at time t: the pooling price $C_H(t) = c_H$, the separating price $C_L(t)$ or the no-trade price $p < C_L(t)$. Getting an offer at the separating price or the no-trade price will not change the continuation value of the seller. Therefore, to compute the continuation value of a low-quality seller, we consider the hypothetical case where the seller always holds on to the asset unless offered c_H. In fact, $\gamma_j(c_H, t)$ can be viewed as a proxy of endogenous market liquidity for owners of an asset of quality j. This is especially important for investors with low-quality assets because it measures the likelihood of extracting information rent in future meetings. Since the arrival rate of a pooling offer c_H for a low-type seller at time τ is $\lambda\gamma_L(c_H, \tau)$, for a low-quality seller remaining in the market at time t, the distribution function of the arrival time of an offer with pooling price c_H is $1 - e^{-\lambda \int_t^\tau \gamma_L(c_H,u)du}$. A low-quality seller's continuation value is characterized by[14]

$$C_L(t) = \int_t^\infty \left[(1 - e^{-r(\tau-t)})c_L + e^{-r(\tau-t)}c_H\right] d(1 - e^{-\lambda \int_t^\tau \gamma_L(c_H,u)du}). \tag{1.4}$$

The seller enjoys the flow payoff rc_L before a pooling offer arrives, and the value jumps to c_H when the seller accepts the offer. If $\gamma_L(c_H, \tau)$ improves for all future $\tau > t$, the low-type sellers' continuation value $C_L(t)$ increases.

Now let's turn to the continuation value of a holder/buyer. A holder enjoys the flow payoff from an asset and mechanically becomes a seller when hit by a liquidity shock that arrives

[13] Otherwise the price of high-quality asset will be unbounded when t goes to infinity

[14] Equivalently, a low-quality seller's continuation value can be characterized by a differential equation $rC_L(t) = rc_L + \lambda\gamma_L(c_H, t)\,(c_H - C_L(t)) + \frac{\mathrm{d}C_L(t)}{\mathrm{d}t}$.

at Poisson rate δ.[15] The continuation value of a type-j holder at time t is

$$V_j(t) = \int_t^\infty \left[(1 - e^{-r(\tau-t)})v_j + e^{-r(\tau-t)}C_j(\tau)\right] d(1 - e^{-\delta(\tau-t)}). \tag{1.5}$$

To derive the gains from trade at time t, we need to compare the continuation values of sellers and holders. Notice for the high type, $C_H(t) = c_H$,

$$V_H(t) = \frac{rv_H + \delta c_H}{r + \delta}. \tag{1.6}$$

As long as $\delta > 0$, $V_H(t) > C_H(t)$ holds at any time. There are always gains from trade for high-quality assets. However, the same result doesn't necessarily hold for low-quality assets although $v_L \geq c_L$. Taking the difference between (1.5) and (1.4), we have

$$V_L(t) - C_L(t) = \int_t^\infty \left[\underbrace{(1 - e^{-r(\tau-t)})(v_L - c_L)}_{\text{flow payoff}} \ldots \right. \tag{1.7}$$

$$\left. - \underbrace{\int_t^\tau e^{-r(u-t)}\lambda\gamma_L(c_H, u)(c_H - C_L(u))\mathrm{d}u}_{\text{information rent}}\right] d(1 - e^{-\delta(\tau-t)}). \tag{1.8}$$

The first component of the integrand represents the holder's extra benefit from the higher flow payoff. However, the positive gain is offset by the information rent of the low-type seller, represented by the second component of the integrand. Notice $C_L(\tau) \leq \frac{rc_L+\lambda c_H}{r+\lambda} < c_H$. When the low-type seller is likely to be offered a pooling price c_H—i.e., $\gamma_L(c_H, u) > 0$—she can take advantage of the liquid market condition and extract information rent from the buyers. This benefit is not enjoyed by the holder. The buyer/holder has an advantage of holding the asset because of the higher flow payoff. However, she has a disadvantage in reselling the asset because her liquidity shock is observable. The fact that an asset holder seeks to immediately sell her asset on the market reveals that she is holding a low-quality asset. Whether the gain from trade is positive or negative depends on the relative size

[15] The continuation value of a type-j holder can be equivalently characterized by a differential equation $rV_j(t) = rv_j + \delta\left(C_j(t) - V_j(t)\right) + \frac{\mathrm{d}V_j(t)}{\mathrm{d}t}$.

of the two components. As the market condition becomes uniformly more liquid (higher $\gamma_j(c_H, u)$ for all $u > t$), the gains from trade decrease. Here I state the following assumption regarding the information structure of the signal:

Assumption 1.1. $f_L^G > \frac{r+\lambda}{\lambda}\frac{v_L - c_L}{c_H - c_L}$.

Given Assumption 1.1, the gains from trade for low-quality assets could be positive, negative, or zero depending on future market conditions denoted by $\gamma_L(c_H, t)$. A liquid market condition in the future (uniformly higher $\gamma_L(c_H, t)$) increases the low-quality seller's incentive to remain in the market and wait for a pooling offer, therefore lowering the gain from trade. Assumption 1.1 implies that if future buyers always acquire information, the gains from trade of a low-quality asset are negative. This result is formally stated in Lemma 1.4.

Lemma 1.4. *Given Assumption 1.1, $V_L(t) - C_L(t) < 0$ if $\gamma_L(c_H, \tau) \geq f_L^G$ for any $\tau > t$.*

For Assumption 1.1 to hold, the value difference between the high-type and low-type assets can not be too small (v_L is relatively close to c_L instead of c_H). Also, buyers' signals must be inaccurate ($f_L^G > 0$) so that when they acquire information, there is a large enough chance that they will offer a pooling price to a low-quality seller.

1.3.3. The Evolution of Asset Quality

The trading probability of each type of asset at any time can be constructed from the trading strategies. The probability that an asset of quality j is traded in a match at time t is

$$\rho_j(t) = \sum_{\{p:\mu(p,j,t)>0\}} \gamma_j(p, t)\mu(p, j, t). \tag{1.9}$$

The product $\gamma_a(p, t)\mu(p, a, t)$ represents the probability that a type a asset is sold at price p at time t. The summation of the product over p gives us the trading probability.

Let $m_H^S(t)$ and $m_L^S(t)$ represent the masses of high-quality and low-quality assets held by sellers. Since high-quality and low-quality assets are in fixed supply of $\frac{\alpha}{1+\alpha}$ and $\frac{1}{1+\alpha}$ respectively, mass $\frac{\alpha}{1+\alpha} - m_H^S(t)$ of high-quality assets and mass $\frac{1}{1+\alpha} - m_L^S(t)$ of low-quality assets are held by holders. The evolution of asset distribution is fully characterized by the following differential equations:

$$\dot{m}_H^S(t) = \delta\left(\frac{\alpha}{1+\alpha} - m_H^S(t)\right) - \lambda\rho_H(t)m_H^S(t), \tag{1.10}$$

$$\dot{m}_L^S(t) = \delta\left(\frac{1}{1+\alpha} - m_L^S(t)\right) - \lambda\rho_L(t)m_L^S(t). \tag{1.11}$$

In each equation, the right-hand side consists of two terms. The first term represents the inflow of assets brought into the market by holders who just received liquidity shocks. The second term represents the outflow of assets because of trading. Since buyers are assigned to sellers randomly, buyers' prior beliefs about the quality of their counter-parties' assets must be consistent with the market composition of high-quality and low-quality assets. For this reason, we use the same notation $\theta(t)$ to represent both the market composition and the buyers' prior belief

$$\theta(t) = \frac{m_H^S(t)}{m_L^S(t)}. \tag{1.12}$$

Combining (1.10) and (1.11), we can characterize the evolution of the market composition as

$$\frac{\mathsf{d}}{\mathsf{d}t}\ln\theta(t) = \underbrace{\frac{\delta}{m_H^S(t)}\frac{\alpha}{1+\alpha}(1-\theta(t)/\alpha)}_{\text{fundamental reversion}} - \underbrace{\lambda(\rho_H(t)-\rho_L(t))}_{\text{trading probability differential}}. \tag{1.13}$$

The evolution of asset distribution can be equivalently characterized by $m_H^S(t)$ and $\theta(t)$. The change in the quality of assets on the market can be decomposed into two effects. The first effect is the *fundamental reversion*. When $\theta(t) < \alpha$, the composition of assets on the market is worse than the fundamental. Therefore, the inflow of assets because of liquidity shocks improves the quality of assets on the market. On the contrary, the inflow of assets worsens the quality of assets on the market when $\theta(t) > \alpha$. Therefore, the market composition tends to revert to the fundamental. This effect is stronger when the high-quality asset on the market is a smaller fraction of total stock of high-quality asset in the economy. The second term is the *trading-probability differential*. Most previous literature has focused on cases where low-quality assets trade weakly faster than high-quality assets in illiquid markets. In those cases, $\rho_H(t) \leq \rho_L(t)$ so the second effect is always weakly positive. In the analysis of the static trading game, we know that when $\theta(t)$ falls in the information acquisition region and there's negative gain from trade for low-quality assets, $\rho_H(t) > \rho_L(t)$. Therefore, high-quality assets leave the market faster than low-quality assets, so the second effect is negative. The negative trading-probability differential effect generates novel implications for the set of steady states and market transitions in the dynamic equilibrium.

1.3.4. Equilibrium Definition

The equilibrium of the full dynamic game is defined as follows.[16]

Definition 1.1. *Given an initial asset distribution* $\left\{\theta(0), m_H^S(0)\right\}$*, an equilibrium consists of paths of asset distribution* $\left\{\theta(t), m_H^S(t)\right\}$*, buyers' strategies* $\{i(t), \sigma(p, \psi, t)\}$ *and continuation value functions* $V_H(t), V_L(t)$*, sellers' strategies* $\mu(p, a, t)$ *and continuation value functions* $C_H(t), C_L(t)$ *such that*

1. *For any time* t*, given the continuation values* $V_L(t)$*,* $V_H(t)$*,* $C_L(t)$*,* $C_H(t)$ *and the prior belief* $\theta(t)$*, a buyer's strategy* $\{i(t), \sigma(p, \psi, t)\}$ *and a seller's strategy* $\mu(p, a, t)$ *form a sequential equilibrium of the static trading game.*

2. *The sellers' continuation values* $C_H(t)$ *and* $C_L(t)$ *are given by* (1.2), (1.3) *and* (1.4)*. The buyers' continuation values* $V_H(t)$ *and* $V_L(t)$ *are given by* (1.5)*.*

3. *The asset distribution* $\left\{\theta(t), m_H^S(t)\right\}$ *evolves according to* (1.10) *and* (1.13)*.*

[16]This definition makes use of some results in the previous analysis. A complete definition of equilibrium is given in the Appendix.

1.4. Stationary Equilibria

In this section, we characterize the set of stationary equilibria of the dynamic trading game, ignoring the role of the initial asset distribution. A stationary equilibrium is an equilibrium in which the asset distribution and investors' trading strategies remain fixed along the equilibrium path. These stationary equilibria are the steady states of the market in the long run. We mostly focus on the pure-strategy stationary equilibria while leaving most of the analysis of mixed-strategy stationary equilibria in the Appendix. The stationary equilibria can be ranked in terms of the total welfare of the investors.

1.4.1. Construction of Stationary Equilibria

The set of stationary equilibria can be exhausted by guess-and-verify. We start by assuming a trading strategy for all investors and compute the continuation values $\bar{V}_H, \bar{C}_H, \bar{V}_L, \bar{C}_L$. At the same time, we can compute the stationary asset distribution, especially the market composition $\bar{\theta}$, and check if the assumed trading strategies are consistent with the static trading game $(\bar{\theta}; \bar{V}_H, \bar{C}_H, \bar{V}_L, \bar{C}_L)$.

Let $\bar{\rho}_H$ and $\bar{\rho}_L$ be the trading probability of high-quality and low-quality assets in a match. The stationary market composition is

$$\bar{\theta} = \frac{\delta + \lambda \bar{\rho}_L}{\delta + \lambda \bar{\rho}_H} \alpha. \tag{1.14}$$

If high-quality assets are traded with higher probability in the stationary equilibrium (i.e., $\bar{\rho}_H > \bar{\rho}_L$), the stationary market composition is worse than the fundamental α. On the contrary, if low-quality assets are traded faster, the stationary market composition is better than the fundamental.

The analysis of the static trading game shows that along any equilibrium path, the continuation values of high-quality assets are fixed at $\bar{C}_H = c_H$ and $\bar{V}_H = \frac{rv_H + \delta c_H}{r+\delta}$, independent of the market conditions. Let $\bar{\gamma}_L(c_H)$ be the constant probability that a low type is offered the pooling price c_H in any given match in a stationary equilibrium. The low-quality sellers' and buyers' continuation values are

$$\bar{C}_L = \frac{rc_L + \lambda\bar{\gamma}_L(c_H)c_H}{r + \lambda\bar{\gamma}_L(c_H)}, \ \bar{V}_L = \frac{rv_L + \delta\bar{C}_L}{r + \delta}. \tag{1.15}$$

If $\bar{\gamma}_L(c_H)$ is small in a stationary equilibrium, the market features lower liquidity and the value of owning low-quality assets is low.

Depending on the strategy of the buyer, the pure strategy stationary equilibria can be put into three categories. Here we describe the information-insensitive pooling stationary equilibrium and the information-sensitive stationary equilibrium while leaving the analysis of the last case, the information-insensitive separating stationary equilibrium, in the Appendix.

Information-Insensitive Pooling Stationary Equilibrium (S_1)

In the first case, buyers do not acquire information and always offer the pooling price c_H. Therefore, both high-quality and low-quality assets are traded at the same speed, $\bar{\rho}_{H,1} = \bar{\rho}_{L,1} = 1$, and the market composition $\bar{\theta}_1$ is the same as the fundamental α. Since the low-type sellers get a pooling offer in each match, $\gamma_L(c_H) = 1$, the continuation values of the low-type sellers and buyers are

$$\bar{C}_{L,1} = \frac{rc_L + \lambda c_H}{r + \lambda}, \ \bar{V}_{L,1} = \frac{rv_L + \delta\bar{C}_{L,1}}{r + \delta}.$$

Notice Assumption 1.1 implies that $\bar{V}_{L,1} < \bar{C}_{L,1}$, so there are no gains from trade between a buyer and a low type seller. We can check if offering a pooling price without acquiring information is a buyer's optimal trading strategy given the market composition and the continuation values in the stationary equilibrium. To simplify the notation, we use $\theta_1^-(k)$ and $\theta_1^+(k)$ to represent the upper and lower bound of the information region if the continuation values equal to those in the stationary equilibria S_1.

$$\theta_1^-(k) = \theta^-(k, \bar{V}_{L,1}),\ \theta_1^+(k) = \theta^+(k, \bar{V}_{L,1}).$$

Lemma 1.5. *An information-insensitive pooling stationary equilibrium* S_1 *exists when*

$$\alpha \geq \max\left\{\frac{c_H - \bar{V}_{L,1}}{V_H - c_H},\ \theta_1^+(k)\right\}.$$

Lemma 1.5 gives the sufficient and necessary conditions on the fundamental α for the information-insensitive pooling stationary equilibria to exist. It imposes two lower bounds on the fundamental α. If k is large, buyers have no incentive to acquire information for any market composition. In order for buyers to offer a pooling price, $\bar{\theta}_1$ must exceed the threshold for pooling offers. If k is small, $\bar{\theta}_1$ must fall in the information-insensitive pooling region. Notice the threshold $\theta_1^+(k)$ depends on the low type seller's continuation value in the stationary equilibrium.

S_1 is the stationary equilibrium with highest market liquidity subject to search frictions. Both high-type and low-type assets are transferred to the high valuation investors (buyers) whenever a match is formed. Moreover, buyers do not spend resources on inspecting the assets. This resembles the market condition in many liquid OTC markets before the financial crisis. Investors offer similar prices for assets with the same credit ratings without spending resources to acquire private information regarding the quality of the assets. They do it for two reasons. First, lemons only account for a small fraction of the assets for sale, and the composition of assets for sale is unlikely to deteriorate because the fundamental of the market is strong. Second, the expectation that the market will remain liquid in the future keeps investors from worrying about obtaining a lemon because they know that later they will be able to sell it quickly at a high price.

Information-Sensitive Stationary Equilibrium (S_2)

Now let's consider a pure strategy stationary equilibrium with information acquisition (i.e. $\bar{i} = 1$). From the analysis of the static trading game, we know that the pooling price is offered if and only if a good signal is observed. Therefore, the probability of a low-type seller getting a pooling offer is $\bar{\gamma}_L(c_H) = f_L^G$. The continuation values of the low type sellers and buyers in S_2 are

$$\bar{C}_{L,2} = \frac{rc_L + \lambda f_L^G c_H}{r + \lambda f_L^G}, \quad \bar{V}_{L,2} = \frac{rv_L + \delta \bar{C}_{L,2}}{r + \delta}. \tag{1.16}$$

In S_2, low-type sellers expect they will receive the offer c_H with probability f_L^G in a match at any time in the future. Assumption 1.1 implies that $\bar{C}_{L,2} > \bar{V}_{L,2}$, so there's no gain from trade with low-type sellers. Buyers will offer the pooling price c_H after seeing a good signal and offer a no-trade price $p < \bar{C}_{L,2}$ after seeing a bad signal. The probability that an asset is traded in a match is equal to the probability that a good signal is generated by the asset, so $\bar{\rho}_{H,2} = f_H^G$, $\bar{\rho}_{L,2} = f_L^G$. Since the high-quality assets are traded faster, the stationary market composition is worse than the fundamental.

$$\bar{\theta}_2 = \frac{\delta + \lambda f_L^G}{\delta + \lambda f_H^G} \cdot \alpha < \alpha. \tag{1.17}$$

To check whether the assumed trading strategies indeed form a stationary equilibrium, we need to verify that the stationary market composition falls in the information-sensitive region given the continuation values. Let

$$\theta_2^-(k) = \theta^-(k, \bar{V}_{L,2}), \ \theta_2^+(k) = \theta^+(k, \bar{V}_{L,2}).$$

be the lower and upper bounds of the information region when the continuation values are equal to those in S_2, Lemma 1.6 gives the sufficient and necessary conditions for the pure strategy information-sensitive stationary equilibrium to exist.

Lemma 1.6. *Suppose Assumption 1.1 is true. An information-sensitive stationary equilibrium S_2 exists if and only if*

$$\frac{\delta + \lambda f_H^G}{\delta + \lambda f_L^G} \cdot \theta_2^-(k) \le \alpha \le \frac{\delta + \lambda f_H^G}{\delta + \lambda f_L^G} \cdot \theta_2^+(k).$$

Lemma 1.6 puts a lower bound and an upper bound on the fundamental. From the expressions for the information region in Lemma 1.3, we know the information region exists when k is small. Therefore, S_2 doesn't exist when k is above a threshold value. In S_2, the market is less liquid than in the information-insensitive pooling stationary equilibrium S_1. Buyers are cautious about the composition of assets on the market, and they always acquire information. As buyers rely on an inaccurate signal, high-quality sellers sometimes receive bad quotes because their asset is taken to be a lemon. It takes longer for a high-quality seller to find an acceptable price in the market compared with the liquid stationary equilibrium S_1. As for the low-quality sellers, there is still a positive probability that they will receive a pooling offer since the buyers sometimes mistakenly take lemons for good assets. If the signal is noisy enough, as in Assumption 1.1, the expected information rent received by a low-quality seller is higher than the difference in discounted flow payoff between a seller and a buyer. Therefore, low-quality sellers demand a high price that the buyers are not willing to offer unless a good signal is observed. As a result, low-quality sellers stay in the market longer than high-quality sellers. The rent seeking behavior of low-quality sellers has two negative effects on the allocative efficiency in the market. The first effect is direct: low-quality assets are not traded immediately when a buyer arrives, even if the buyer has a higher flow payoff for holding the asset. The second effect is indirect: as low-quality sellers stay longer in the market, the market composition remains below the fundamental and therefore reduces buyers' incentive to offer pooling prices.

Proposition 1.1 shows that the information-insensitive pooling stationary equilibrium S_1 and the information-sensitive stationary equilibrium S_2 coexist when the fundamental α is within an intermediate region.

Proposition 1.1 (Coexistence of S_1 and S_2). *Suppose Assumption 1.1 is true. Let $A_1(k)$ and $A_2(k)$ be*

$$A_1(k) = \max\left\{\theta_1^+(k),\ \frac{\delta+\lambda f_H^G}{\delta+\lambda f_L^G}\cdot\theta_2^-(k)\right\},\quad A_2(k) = \frac{\delta+\lambda f_H^G}{\delta+\lambda f_L^G}\cdot\theta_2^+(k).$$

S_1 and S_2 co-exist if and only if $\alpha \in [A_1(k), A_2(k)]$. When k is small, $A_1(k) < A_2(k)$.

When agents hold the belief that the market will be liquid as in S_1 in the future, the value of a low-quality asset is high for both sellers and buyers. Buyers are willing to offer the pooling price without acquiring information for a wide range of the market composition. Also, as buyers acquire assets without any selection, the market composition remains at the fundamental value. However, when agents believe the market will be partially illiquid as in S_2, the value of a low-quality asset becomes lower. The information-insensitive pooling region shrinks. At the same time, as buyers cream-skim the market, the market composition stays below the fundamental. Both the trading effect and the valuation effect justify the buyers' information acquisition behavior.

1.4.2. Welfare Analysis

The total welfare along an equilibrium path is given by

$$\begin{aligned}\varepsilon =& \frac{\alpha}{1+\alpha}v_H + \frac{1}{1+\alpha}v_L \\ &- \int_0^\infty e^{-rt}\left[rm_H^S(t)(v_H - c_H) + rm_L^S(t)(v_L - c_L) + \lambda(m_H^S(t) + m_L^S(t))i(t)k\right]\mathrm{d}t. \quad (1.18)\end{aligned}$$

The first line of the right-hand side $\frac{\alpha}{1+\alpha}v_H + \frac{1}{1+\alpha}v_L$ represents the welfare in a frictionless benchmark. In the benchmark, assets can be moved from shocked investors to unshocked investors instantaneously. However, due to search frictions and information frictions, some assets are held by shocked investors in equilibrium. The first and the second term in the integrand of (1.18) represents the welfare loss because of market illiquidity. The third term represents the welfare loss from the resources devoted to information acquisition.

From (1.10) and (1.11) we can solve for the stationary asset distribution characterized by the mass of high-quality and low-quality assets held by sellers,

$$\bar{m}_H^S = \frac{\delta\alpha}{(\delta + \lambda\bar{\rho}_H)(1+\alpha)}, \; \bar{m}_L^S = \frac{\delta}{(\delta + \lambda\bar{\rho}_L)(1+\alpha)}. \tag{1.19}$$

Using the trading probability and (1.19) for stationary asset distribution, we can write down the welfare loss $\Delta = \alpha v_H + (1-\alpha)v_L - \varepsilon$ in each stationary equilibrium:

$$\Delta_1 = \frac{\delta\alpha}{\delta+\lambda}(v_H - c_H) + \frac{\delta(1-\alpha)}{\delta+\lambda}(v_L - c_L),$$
$$\Delta_2 = \frac{\delta\alpha}{\delta+\lambda f_H^G}\left(v_H - c_H + \frac{\lambda k}{r}\right) + \frac{\delta(1-\alpha)}{\delta+\lambda f_L^G}\left(v_L - c_L + \frac{\lambda k}{r}\right).$$

The welfare loss in S_1 is lower than that in S_2. In S_2, sellers hold a larger mass of both high-quality and low-quality assets, and buyers are paying extra costs of information acquisition compared to S_1. As we previously pointed out, S_1 is the most efficient stationary equilibrium subject to search frictions.

1.5. Non-Stationary Equilibria

In the previous section we investigated various states of the market in the long run. Now we turn to analyze how investors' trading behavior and market liquidity evolve over time starting from a given initial asset distribution. Particularly, we are interested in the following question. When a liquid steady state and an illiquid steady state co-exist, is it possible for the market to transition from one to the other? In order to answer this question, it is important to study the set of non-stationary equilibria.

To show the existence of a certain equilibrium path from an initial asset distribution to a terminal steady state, we first hypothesize about investors' trading strategies for any $t > 0$. Given the paths of trading probability $\rho_H(t)$ and $\rho_L(t)$ and the initial asset distribution represented by $m_H^S(0)$ and $m_L^S(0)$, the full path of the asset distribution can be analytically solved from (1.10) and (1.11) as follows:

$$m_H^S(t) = e^{-\int_0^t \delta+\lambda\rho_H(s)\mathrm{d}s} m_H^S(0) + \frac{\delta\alpha}{1+\alpha}\int_0^t e^{-(\delta+\lambda\rho_H(u)(t-s))\mathrm{d}u}\mathrm{d}s, \tag{1.20}$$

$$m_L^S(t) = e^{-\int_0^t \delta+\lambda\rho_L(s)\mathrm{d}s} m_L^S(0) + \frac{\delta}{1+\alpha}\int_0^t e^{-(\delta+\lambda\rho_L(u)(t-s))\mathrm{d}u}\mathrm{d}s. \tag{1.21}$$

Next we can compute the paths of continuation values to verify whether the assumed trading strategies form an equilibrium of the static trading game at any $t > 0$.

In the Appendix, I provide sufficient conditions for the market composition $\theta(t)$ to change monotonically along a non-stationary equilibrium path.

1.5.1. Self-fulfilling Market Freeze

Suppose the market has an asset distribution as in the liquid state S_1. Is it possible that all investors suddenly change their beliefs and coordinate to follow an equilibrium path that converges to the illiquid state S_2? This question is answered in Proposition 1.2.

Proposition 1.2 (Self-fulfilling Market Freeze)**.** *If Assumption 1.1 holds, for small k there exists*

$$A_3(k) = \theta_2^+(k) \in (A_1(k), A_2(k)),$$

such that, for any $\alpha \in [A_1(k), A_3(k)]$, starting from an initial asset distribution in the neighborhood of S_1, there is an equilibrium path that converges to S_2.

When $\alpha \in [A_1(k), A_3(k)]$, the model has multiple equilibria starting from the asset distribution of S_1. Proposition 1.2 implies that a liquid market can go through a self-fulfilling market freeze. Starting from the asset distribution in S_1, if all investors believe that future buyers will not acquire information and always offer the pooling price, the current buyers have no incentive to acquire information and they continue to offer the pooling price. The market therefore remains in the liquid steady state of S_1. However, if all investors believe the market liquidity will begin to decline and buyers in the future will begin to acquire information as a way of avoiding low-quality assets, the continuation value of holding low quality assets drops immediately. Thus, for current buyers, the loss incurred by buying a low-quality asset at the pooling price becomes larger, and this gives them more incentive to acquire information. When current buyers acquire information but their independent evaluation of the assets are not accurate enough, high-quality assets are traded faster than low-quality assets, resulting in a cream-skimming effect on the market composition. The market composition deteriorates over time and justifies future buyers' information acquisition. Therefore, the market evolves along a path with information acquisition and converges to the information-sensitive steady state S_2.

Notice that Proposition 1.2 does not imply that the information-insensitive pooling steady state is unstable. In fact, the liquid steady state is locally stable.

Proposition 1.3. *If $\alpha, \theta(0) > \theta_1^+(k)$, there exists an equilibrium path with pooling offers and no information acquisition that converges to S_1.*

The results of Propositions 1.2 and 1.3 can be illustrated graphically. In Figures 3 and 4 I plot the phase diagram of the evolution of asset distributions according to (1.10) and (1.13). The horizontal axis represents the market composition that determines the current investors' trading strategies. The vertical axis represents the mass of sellers with high-quality assets in the market. Although the mass of high-quality sellers does not affect the current investors' trading strategies directly, it shapes the evolution of the asset distribution through the interaction with market composition. Recall that the evolution of the asset distribution depends on the trading probability of different assets, which in turn depends on investors' belief about future market liquidity through resale considerations. Therefore, before we plot a phase diagram, we need to specify investor's continuation values according to their belief about future market liquidity.

Figure 3 shows the phase diagram when all investors believe future buyers will not acquire information but instead will always make pooling offers. Given this belief, the continuation values of owners of low-quality assets are $\bar{V}_{L,1}$ and $\bar{C}_{L,1}$. The corresponding information-sensitive region is given by $[\theta_1^-(k), \theta_1^+(k)]$, represented by the shaded region in the figure. If the fundamental α is above $\theta_1^+(k)$, there exists an information-insensitive pooling steady state, represented by the stationary asset distribution S_1 on the right of the shaded region. If the investors maintain their belief about a liquid market in the future, the market will stay in S_1. Moreover, as Proposition 1.3 shows, starting from any asset distribution to the right of the shaded region, there is a path converging to S_1. Along the path, the asset composition is always above $\theta_1^+(k)$, consistent with the investors' belief that there is no need for information acquisition.

What happens when investors' beliefs shifts? Suppose the market starts out with the asset distribution in S_1, but investors suddenly start to believe that investors in the future will acquire information and the market will become illiquid. The phase diagram changes from Figure 3 to 4. The continuation values of owning low-quality assets drop to $\bar{V}_{L,2}$ and $\bar{C}_{L,2}$, the same as in the information-sensitive steady state. Since the continuation values become lower, the information-sensitive region moves to the right, represented by the shaded region in Figure 4. The asset composition is good enough to support pooling trading in S_1 when investors believe in a liquid market in the future. However, after the shift in the investors' beliefs, S_1 is now in the shaded information-sensitive region, reflecting higher incentives to acquire information when investors anticipate lower liquidity in the future. The market will therefore follow the arrows and move to S_2. The whole path lies within the shaded region, meaning that buyers always acquire information along the path, consistent with investors' belief in low liquidity in the future. The transition from S_1 to S_2 is consistent with an event of a self-fulfilling market freeze, in which trading delays suddenly become longer.

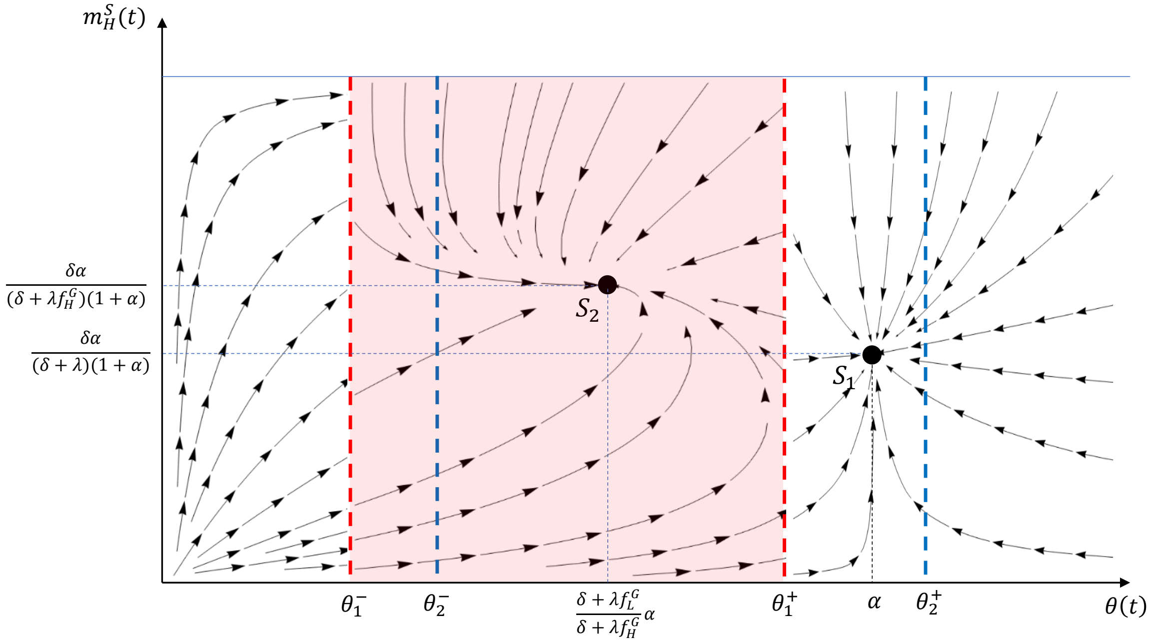

Figure 3: Phase Diagram ($V_L(t) = \bar{V}_{L,1},\ C_L(t) = \bar{C}_{L,1}$).

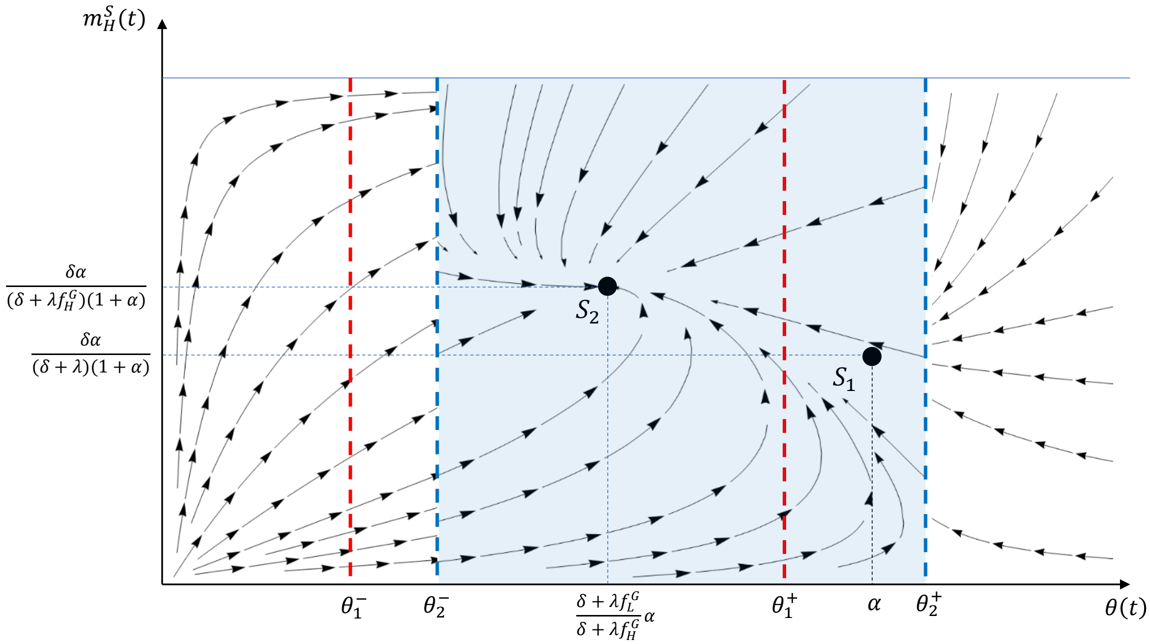

Figure 4: Phase Diagram ($V_L(t) = \bar{V}_{L,2},\ C_L(t) = \bar{C}_{L,2}$).

1.5.2. Information Trap

If the market's initial asset distribution is in the illiquid state S_2, is there a non-stationary equilibrium path that converges to liquid trading? The answer depends on the relationship between the market composition in S_2 and the information-sensitive region $[\theta_1^-(k), \theta_1^+(k)]$ in S_1. This can be illustrated in the same set of phase diagrams. In Figure 3 and Figure 4, the information acquisition regions in S_1 and S_2 overlap and the illiquid state S_2 falls in the overlapping region. Starting from the initial asset distribution in S_2, if all investors hold the belief that future buyers will acquire information, S_2 is in the shaded information-sensitive region in Figure 4, consistent with the investors' belief. Now suppose all investors believe that in the future, buyers will not acquire information and will always offer the pooling price. This optimistic belief in future market liquidity improves the continuation values, changing the phase diagram to Figure 3 and shifting the information-sensitive region to $[\theta_1^-(k), \theta_1^+(k)]$. However, since S_2 is also in the shaded information-sensitive region in Figure 3, current buyers will still acquire information and cream-skim the market. Their trading behavior keeps the asset distribution at S_2 and prevents the market from recovering to S_1. To summarize, if the market composition in S_2 satisfies $\bar{\theta}_2 < \theta_1^+(k)$, there is no equilibrium path that converges to the liquid state S_1.

Now let's consider the opposite case if $\bar{\theta}_2 \geq \theta_1^+(k)$. Starting from the initial asset distribution in S_2, when investors believe the market will be liquid in the future, the optimal strategy for a buyer is to stop acquiring information and to instead offer the pooling price. As a result, the market composition will gradually improves and converges to $\bar{\theta}_1$, the market composition in S_1. Along this path, buyers do not acquire information. Therefore, if $\bar{\theta}_2 \geq \theta_1^+(k)$, there exists a non-stationary equilibrium path that transitions from S_2 to S_1.

Assumption 1.2. $\frac{f_H^G f_L^B}{f_L^G f_H^B} > \frac{c_H - \bar{V}_{L,2}}{c_H - \bar{V}_{L,1}}$.

Assumption 1.2 is equivalent to the condition $\theta_2^-(0) < \theta_1^+(0)$. If Assumption 1.2 is true, $\theta_2^-(k) < \theta_1^+(k)$ holds for small k so that the two information acquisition regions overlap.

The intuitive interpretation of Assumption 1.2 is that it requires the signal to be relatively accurate so given any set of continuation values, information acquisition is optimal for a wide range of market composition. Otherwise, if the information available to be buyers is very noisy, information acquisition is irrelevant most of the time.[17]

I call the overlapping part of the two information-sensitive regions $[\theta_2^-(k), \theta_1^+(k)]$ the *information trap* whenever it exists. The information trap is different from the information sensitive regions we just discussed. At any time t, the information sensitive region depends on the continuation values of owning low-quality assets $V_L(t), C_L(t)$. However, by definition, the information trap is time and strategy invariant so it is independent of investors' beliefs and the continuation values. When the market composition is within the information trap, whether or not investors believe that future buyers will acquire information or not, the optimal strategy is to acquire information today, and the cream-skimming effect will be in play. Intuitively speaking, the market composition will be trapped in the region and dragged into the "sink," which is the information-sensitive state S_2.[18]

Proposition 1.4 formally conveys the condition in which there is no non-stationary equilibrium path that transitions from S_2 to S_1.

Proposition 1.4 (Information Trap)**.** *If Assumption 1.1 and 1.2 hold, for small k there exists*

$$A_4(k) = \frac{\delta + \lambda f_H^G}{\delta + \lambda f_L^G} \cdot \theta_1^+(k) \in (A_1(k), A_2(k)), \tag{1.22}$$

such that, for any $\alpha \in [A_1(k), A_4(k)]$, if the initial asset distribution is in the neighborhood of S_2, there is no equilibrium path converging to pooling trading.

[17] Assumption 1.2 is not in conflict with Assumption 1.1. Assumption 1.1 requires that f_L^G is not too small so the buyer can make a mistake in the inspection and take a low-quality asset as a "good" one. However, it does not put any restrictions on the signals observed from a high-quality asset. When f_H^G gets closer to 1, the left-hand side of Assumption 1.2 goes to ∞.

[18] In Appendix 1.12, I consider whether there exists a non-stationary equilibrium path that converges to the liquid state S_1, starting from an arbitrary initial market composition $\theta(0)$ in the information trap. I provide the sufficient and necessary conditions such that the equilibrium path exists.

Propositions 1.2 and 1.4 jointly imply that, for $\alpha \in [A_1(k), \min\{A_3(k), A_4(k)\}]$, the liquid steady state S_1 and the illiquid steady state S_2 coexist. More importantly, the transitions between the two steady states are asymmetric. Suppose the market is in the liquid state S_1 where buyers are not paying any attention to the idiosyncratic features of the assets. They simply buy assets at the pooling price from any seller they meet in the market. The market composition remains at a high level. A self-fulfilling market freeze starts from a market-wide panic about a decline in future market liquidity. Investors worry that if they hold low-quality assets in the portfolio, in the future, it will be hard for them to sell these assets at good prices. Because of this concern, buyers start to collect information and carefully evaluate the assets they see on the market. They are only willing to offer a good price for an asset if the aspects of the asset satisfy their own criteria. However, because buyers' evaluations of assets are not perfect, sellers who receive a bad quote will stay in the market with the hope that they will receive a high quote from the next buyer. The trading speeds of both types of assets drop immediately, and the value of low-quality assets to the current owners decline. As the market goes further down the illiquid path, the market composition deteriorates gradually as low-quality assets accumulate in the market. At some point, the market composition becomes bad enough that it falls into the information trap. Even if buyers have optimistic beliefs about future market liquidity, since the current market composition is bad, they keep acquiring information to avoid buying low-quality assets at high prices. The low liquidity and the bad market composition reinforce each other through buyers' information acquisition, and the market can not recover to the liquid state.

1.6. Policy Implications

In this section we explore two policy implications of the model.

1.6.1. Issuance Transparency

Transparency in the issuance process of ABS was low before the latest financial crisis. The low issuance transparency has been criticized for generating moral hazard problems in the securitization process and adverse selection problems in the secondary market, which played important roles in the creation and propagation of the financial crisis. After the financial crisis, regulators moved toward a more transparent issuance process. For example, Dodd-Frank Act Section 942 requires issuers of asset-backed securities (ABS) to provide asset-level information according to specified standards.[19] In the context of my model, these regulatory changes could lower the cost of information acquisition and increase the precision of buyers' signals.

Definition 1.2. *A signal ψ' is (weakly) more precise than a signal ψ if and only if $f_H^{G'} \geq f_H^G$ and $f_L^{G'} \leq f_L^G$.*

We use two simple criteria to evaluate the effect of increasing transparency on the liquidity of the secondary market. First, we look at $\theta_1^+(k)$, since the liquid steady state $S1$ exists if and only if $\alpha > \theta_1^+(k)$. Second, we consider $A_4(k)$. When $\alpha > A_4(k)$, there is no steady state in the information trap.

Proposition 1.5. *If both ψ' and ψ satisfy Assumption 1.1 and 1.2, and ψ' is more precise than ψ, both $\theta_1^+(k)$ and $A_4(k)$ increase when switching from the signal structure ψ to ψ', and when k decreases.*

Proposition 1.5 implies that increasing transparency in the issuance process can harm market liquidity, judging by our simple criteria. An intuitive explanation of this result is that when issuers provide more information regarding the pool of assets backing the ABS, future investors can better evaluate the assets' quality upon conducting due diligence. This gives buyers more incentive to acquire information, and when they do so, the cream-skimming effect is stronger. It is worth mentioning that I only consider the impact of increasing transparency on the liquidity of the secondary market and ignore the impact on disciplining the issuance process. A complete evaluation of these types of polices should take effects on both the primary and the secondary markets into consideration.

[19] See https://www.sec.gov/spotlight/dodd-frank-section.shtml#942.

1.6.2. Asset Purchase Programs

When a market freezes because of the adverse selection problem, a natural solution is to clean the market by removing low-quality assets from the market. Many theoretical papers have studied the design of asset purchase programs in the presence of severe adverse selection, including Philippon and Skreta (2012), Tirole (2012), Camargo and Lester (2014) and Chiu and Koeppl (2016). During the latest financial crisis, the US Treasury created the Public-Private Investment Program (PPIP) to purchase "toxic" assets, aiming at restoring liquidity in the markets for legacy Commercial MBS and non-agency RMBS.

Asset purchase programs can help the target market restore liquid trading through two channels. First, it removes lemons from the market, so the fundamentals of the market improves. Second, if the government purchases assets at a higher price than the market would offer, or selling assets to the government is easier than locating a buyer in the private sector, the asset purchase program effectively increases the value of lemons. As a result, the lemon's problem is mitigated and buyers in the market are more willing to offer pooling prices.

In my model, when the market goes through a self-fulfilling market freeze from S_1 to S_2, the market composition deteriorates gradually and the mass of "toxic" assets on the market increases over time. In the proof of Proposition 1.2, I show that $\theta(t)$ decreases and $m_H^s(t)$ increases over time along the path of market freeze. There exists a time $\hat{t}$ such that $\theta(\hat{t}) = \theta_1^+(k)$. If the government intervenes before $\hat{t}$, the market composition is above the information trap. There still exists an equilibrium path that converges to liquid trading. Therefore, market liquidity can be boosted by a plan that guarantees a floor-price for all assets. The government does not need to actually purchase assets from the market since the market will immediately return to liquid trading as buyers all stop acquiring information. However, after $\hat{t}$, the market enters the information trap and there is no self-fulfilling equilibrium path that returns to S_1. The government needs to purchase a positive amount of assets to revive the market.

Chiu and Koeppl (2016) study the *announcement effect* of asset purchase programs. Specifically, when the government announces that it will purchase a given amount of lemons at a given price later at a given time, it is possible that the market will restore to liquid trading even before the government actually purchases these assets. Thus the government may be justified in delaying the purchase to lower the intervention cost. However, a direct implication of Proposition 1.4 is that in an illiquid steady state within the information trap, there is no announcement effect for any asset purchase program with purchasing price $p \in [\bar{C}_{L,2}, \bar{C}_{L,1}]$.

1.7. Conclusions

In this paper, I present a model for studying the interaction between buyers' information acquisition and market liquidity in over-the-counter markets with adverse-selection problems. Buyers can acquire information to avoid buying low-quality assets, and their incentive for doing so is strong if they expect that the market will be illiquid when they resell their assets. When buyers' signals are inaccurate, information acquisition has a cream-skimming effect on the composition of assets for sale and harms future market liquidity. The interaction of resale consideration and the cream-skimming effect gives rise to multiple steady states and asymmetric transitions between steady states. Specifically, the market can transition from a liquid state without information acquisition to an illiquid state with information acquisition, but it can not transition back. This uni-directional transition between different steady states is a novel feature of my model that, to the best of my knowledge, is not present in the models used in previous papers on dynamic adverse selection. This result helps explain the continued low liquidity in the non-agency residential mortgage-backed-security market in spite of the recovery of the US economy and the housing markets.

1.8. Appendix 1: Alternative Definition of Equilibrium

Here I provide a formal but less intuitive equilibrium definition which is equivalent to the definition provided in Section 1.3.

Definition 1.3. *A equilibrium consists of paths of asset distribution* $\left\{\theta(t), m_H^S(t), m_L^S(t)\right\}$, *buyers' policy functions* $\{i(t), \sigma(p, \psi, t)\}$ *and value functions* $\{V_H(t), V_L(t)\}$, *seller's policy function* $\mu(p, j, t)$ *and value functions* $\{C_H(t), C_L(t)\}$, *which satisfy the following conditions:*

1. *Seller's optimality condition: For any* $j \in \{H, L\}$,

$$\mu(p, j, t) = \begin{cases} 1, & \text{if } p > C_j(t), \\ [0, 1], & \text{if } p = C_j(t), \\ 0, & \text{if } p < C_j(t). \end{cases} \tag{1.23}$$

2. *Buyer's optimality conditions:*

   (a) *For* $\psi \in \{G, B\}$, $\sigma(p, \psi, t) > 0$ *only if* p *solves*

$$J(\psi, t) = \max_p \frac{\theta(t)}{\theta(t)+1} f_H^\psi \mu(p, H, t) \left[V_H(t) - p\right] + \frac{1}{\theta(t)+1} f_L^\psi \mu(p, L, t) \left[V_L(t) - p\right];$$

   (b) $\sigma(p, N, t) > 0$ *only if* p *solves*

$$J(N, t) = \max_p \frac{\theta(t)}{\theta(t)+1} \mu(p, H, t) \left[V_H(t) - p\right] + \frac{1}{\theta(t)+1} \mu(p, L, t) \left[V_L(t) - p\right];$$

   (c) *The value of information* $W(t)$ *is*

$$W(t) = \max\left\{J(G, t) + J(B, t) - J(N, t), 0\right\},$$

   and $i(t)$ *satisfies*

$$i(t) = \begin{cases} 1, & \text{if } W(t) > k, \\ [0, 1], & \text{if } W(t) = k, \\ 0, & \text{if } W(t) < k. \end{cases} \tag{1.24}$$

3. *The continuation values of sellers* $C_j(t)$ *are given by* (1.2),(1.3) *and* (1.4)*. The continuation values of buyers/holders* $V_j(t)$ *are given by* (1.5)*.*

4. *The asset distribution, characterized by* $m_H^S(t)$*,* $m_L^S(t)$ *and* $\theta(t)$ *evolves according to* (1.11)-(1.13)*.*

1.9. Appendix 2: Other Stationary Equilibria

1.9.1. Pure-Strategy Stationary Equilibria

Information-Insensitive Separating Stationary Equilibrium (S_3)

When the stationary market composition falls in the information-insensitive region with separating offers, the market is in an information-insensitive separating stationary equilibrium. This is the third and the last type of stationary equilibrium with pure strategies. In S_3, buyers do not acquire information and only offers the separating price. Therefore, the low-quality assets are traded with probability 1 in each match and the high-quality assets are never traded. $\bar{\rho}_{H,3} = 0,\ \bar{\rho}_{L,3} = 1$. The stationary equilibria market composition is better than the fundamental.

$$\bar{\theta}_3 = \frac{\delta + \lambda}{\delta} \cdot \alpha > \alpha. \tag{1.25}$$

Since the pooling price is never offered in equilibrium, the continuation values of low-quality asset owners are

$$\bar{C}_{L,3} = c_L,\ \ \bar{V}_{L,3} = \frac{rv_L + \delta c_L}{r + \delta}.$$

It's easy to verify that $\bar{V}_{L,3} > \bar{C}_{L,3}$ so there are gains from trade for low-quality assets. Similarly, let

$$\theta_3^-(k) = \theta^-(k, \bar{C}_{L,3}),\ \ \theta_3^+(k) = \theta^+(k, \bar{C}_{L,3})$$

be the lower and upper bounds of the information-sensitive region when the continuation values are equal to those in S_3

Lemma 1.7. *An information-insensitive separating stationary equilibrium S_3 exists if and only if*

$$\alpha \leq \frac{\delta}{\delta+\lambda} \min\left\{\frac{c_H - c_L}{V_H - c_H}, \theta_3^-(k)\right\}.$$

In S_3, all high-quality assets and a fraction of low-quality assets are on the market. Yet, the fundamental of the market is so bad that the amount of lemons on the market is large enough to prevent any pooling offers or information acquisition from buyers. The continuation values of low-quality asset owners are the lowest in all possible equilibria.

1.9.2. Mixed-Strategy Equilibria

Here we provide two useful results that restrict the set of possible mixed strategies in equilibrium.

Lemma 1.8. *In any equilibrium, if $i(t) > 0$, $\sigma(c_H, G, t) = 1$ and $\sigma(c_H, B, t) = 0$.*

Lemma 1.8 applies to all equilibrium path. It implies a buyer will offer the pooling price c_H if and only if a good signal is observed. The proof is intuitive. Based on the analysis of the static trading game, it is clear that given any set of continuation values, buyers only choose between two price. Without loss of generality, assume the buyer offers price p_1 after seeing a good signal and mix between p_1 and p_2 after seeing a bad signal. Since the buyer uses mixed strategy after seeing a bad signal, then the expected payoff from offering the two prices based on the posterior belief of seeing a bad signal must be the same. Therefore, the expected payoff doesn't change if the buyer offer p_1 with probability 1 after seeing a bad signal. This makes the buyer's offer independent of the signal. Thus, the buyer can simply offer p_1 without information acquisition and save the fixed cost. The above reasoning shows the sub-optimality of using mixed strategy after acquiring information. We can us Lemma 1.8 to simplify (1.2), in any equilibrium,

$$\gamma_L(t) = i(t) f_L^G + \left(1 - \bar{i}(t)\right) \sigma(c_H, N, t). \tag{1.26}$$

Do sellers randomize in equilibrium? Obviously, sellers of low-quality assets always accept the pooling price c_H. Also, sellers of high-quality assets always accept the pooling price c_H in any equilibrium. If sellers of high-quality assets accept price c_H with a probability less than 1, a buyer can raise the offer by a tiny amount and increase the surplus by $V_H - C_H$ with a strictly positive probability. Following the same logic, if sellers of low-quality assets randomize when offered a separating price, $\bar{C}_L$ must be equal to $\bar{V}_L$. In stationary equilibria, this implies that $\bar{\gamma}_L = \frac{r}{\lambda}(v_L - c_L)/(c_H - v_L)$. By Assumption 1.1, $\bar{\gamma}_L < f_L^G$. Using (1.26), we immediately have the following lemma.

Lemma 1.9. *If Assumption 1.1 holds, in any stationary equilibria with sellers of low-quality assets using mixed strategies, we have* $\bar{i} < 1$ *and* $\bar{\sigma}(c_H, N) < f_L^G$ *.*

If buyers randomize between a separating offer and a no-trade offer, the gains from trade of low-quality assets must be zero, $V_L(t) = C_L(t)$. We say two equilibria are equivalent when sellers and buyers of both high-quality and low-quality assets have the same trading probability and continuation values at any give time. Any equilibrium with buyer mixing between a separating offer and a no-trade offer is equivalent to an equilibrium with buyers only offering the separating price and sellers rejecting the offer with a positive probability. This equivalence allows us to focus on mixed-strategy equilibria in which buyers only choose between the separating offer and the pooling offer.

Mixed-Strategy Stationary Equilibrium without Information Acquisition

Any mixed strategy stationary equilibrium without information acquisition must have buyers using mixed strategies. It is sufficient to consider buyers mixing between the pooling price c_H and the separating price $\bar{C}_L$. Notice in any equilibrium without information acquisition, the probability of buyer offering c_H is equal to γ_L. When buyers do not acquire information, whether they offer the separating price or the no-trade price depends on the relationship between $\bar{V}_L$ and $\bar{C}_L$. Since in a stationary equilibrium, $\bar{V}_L$ is a weighted average of v_L and $\bar{C}_L$, it's equivalent to compare $\bar{C}_L$ and v_L. There are three cases:

1. (S_4) $\bar{C}_L > v_L$. This is the case when buyers offer c_H with probability $\bar{\gamma}_{L,4}$ and the no trade price with probability $1 - \bar{\gamma}_{L,4}$. In each match, either type of asset is traded with probability $\bar{\gamma}_{L,4}$. (1.15) implies that $\bar{\gamma}_{L,4} > \frac{r}{\lambda}(v_L - c_L)/(c_H - v_L)$. This stationary equilibria exists when the following conditions are satisfied:

$$\frac{c_H - \bar{V}_{L,1}}{V_H - c_H} < \alpha < \frac{c_H - v_L}{V_H - c_H}, \tag{1.27}$$

$$k \geq (f_L^B - f_H^B)(V_H - c_H)\frac{\alpha}{1+\alpha}. \tag{1.28}$$

The market liquidity $\bar{\gamma}_{L,4}$ is determined by $\alpha = \frac{c_H - \bar{V}_{L,4}}{V_H - c_H}$ and (1.15).

2. (S_5) $\bar{C}_L < v_L$. In this stationary equilibrium buyers offer c_H with probability $\bar{\gamma}_{L,5}$ and the separating price $\bar{C}_{L,5}$ with probability $1 - \bar{\gamma}_{L,5}$. Low-quality sellers accept the separating offer for sure. In each match, a high-quality asset is traded with probability $\bar{\gamma}_{L,5}$ and a low-quality asset is always traded. If this stationary equilibrium exists, (α, k) must satisfy the following conditions given a market liquidity $\bar{\gamma}_{L,5} \in (0, \frac{r}{\lambda}(v_L - c_L)/(c_H - v_L))$.

$$\bar{C}_{L,5} = \frac{rc_L + \lambda\bar{\gamma}_{L,5}c_H}{r + \lambda\bar{\gamma}_{L,5}},$$

$$\frac{c_H - \bar{C}_{L,5}}{V_H - c_H} = \frac{\delta + \lambda}{\delta + \lambda\bar{\gamma}_{L,5}} \cdot \alpha,$$

$$k \geq (f_L^B - f_H^B)(V_H - c_H) \cdot \frac{c_H - \bar{C}_{L,5}}{V_H - \bar{C}_{L,5}}.$$

3. (S_6) $\bar{C}_L = v_L$. In this stationary equilibria, buyers offer c_H with probability $\bar{\gamma}_{L,6} = \frac{r}{\lambda}(v_L - c_L)/(c_H - v_L)$ and the separating price $\bar{c}_{L,6}$ with probability $1 - \bar{\gamma}_{L,6}$. Low-quality

sellers accept the separating offer with probability $\bar{\mu}(v_L, L) \in (0, 1)$. For the stationary equilibria to exist, (α, k) must satisfy the following conditions

$$\frac{\delta + \lambda \bar{\gamma}_{L,6}}{\delta + \lambda} \cdot \frac{c_H - v_L}{V_H - c_H} < \alpha < \frac{c_H - v_L}{V_H - c_H},$$
$$k \geq (f_L^b - f_H^b)(V_H - c_H) \cdot \frac{c_H - v_L}{V_H - v_L}.$$

where $\bar{\mu}(v_L, L)$ is the solution to

$$\frac{\delta + \lambda \bar{\gamma}_{L,6}}{\delta + \lambda \left[\bar{\gamma}_{L,6} + \bar{\mu}(v_L, L)(1 - \bar{\gamma}_{L,6})\right]} \cdot \frac{c_H - v_L}{V_H - c_H} = \alpha. \tag{1.29}$$

Mixed-strategy equilibrium with partial information acquisition

Now let's turn to the mixed-strategy stationary equilibria with $\bar{i} \in (0, 1)$. In any equilibrium, buyers always offer c_H after observing a good signal.

1. (S_7) First let's consider stationary equilibria with $\bar{\theta}$ located on the right boundary of the information-sensitive region. Since $\bar{\theta} > \hat{\theta}$, when buyers do not acquire information, they offer the pooling price. Therefore $\bar{\gamma}_{L,7} = \bar{i}_7 f_L^G + 1 - \bar{i}_7$. Notice $\bar{\gamma}_{L,7} > f_L^G$. Assumption 1.1 implies that $\bar{C}_{L,7} > v_L$, so there's no gain from trade for low-quality assets. Low-quality assets will not be traded if a bad signal is observed. High-quality and low-quality assets are traded with probability $\bar{\rho}_{H,7} = \bar{i}_7 f_H^G + 1 - \bar{i}_7$, while low-quality assets are traded with probability $\bar{\rho}_{L,7} = \bar{i}_7 f_L^G + 1 - \bar{i}_7$. The stationary equilibria market composition $\bar{\theta}_7$ is given by (1.14). S_7 exists if and only if the following conditions are satisfied:

$$\theta^+(k, \bar{V}_{L,7}) \geq \frac{c_H - \bar{V}_{L,7}}{V_H - c_H}, \tag{1.30}$$
$$\alpha = \frac{\delta + \lambda \bar{\rho}_{H,7}}{\delta + \lambda \bar{\rho}_{L,7}} \cdot \theta^+(k, \bar{V}_{L,7}) \tag{1.31}$$

2. (S_8) The next group of stationary equilibria we investigate has $\bar{\theta}$ located on the left

boundary of the information-sensitive region. Since $\bar{\theta} < \hat{\theta}$, buyers never offer the pooling price without information acquisition. Therefore $\bar{\gamma}_{L,8} = \bar{i}_8 f_L^G$. High-quality assets are traded with probability $\bar{\rho}_{H,8} = \bar{i}_8 f_H^G$. The probability that a low type asset is traded depends on whether there's gain from trade. Given different $\bar{i}_8$, there are three cases:

- If $\bar{i}_8 > \frac{r}{\lambda f_L^G}(v_L - c_L)/(c_H - v_L)$, the gain from trade of low-quality assets is negative. Low-quality assets are traded with probability $\bar{\rho}_{L,8} = \bar{\gamma}_{L,8}$.
- If $\bar{i}_8 < \frac{r}{\lambda f_L^G}(v_L - c_L)/(c_H - v_L)$, the gain from trade of low-quality assets is positive. Low-quality assets are traded with probability $\bar{\rho}_{l,8} = 1$.
- If $\bar{i}_8 = \frac{r}{\lambda f_L^G}(v_L - c_L)/(c_H - v_L)$, the gain from trade of low-quality assets is zero. Sellers of low-quality assets can use mixed strategies when offered the separating price. Low-quality assets are traded with probability $\bar{\rho}_{l,8} \in [\bar{\gamma}_{L,8}, 1]$.

The continuation values of the owners of low-quality assets are given by (1.15). The stationary equilibria market composition $\bar{\theta}_8$ is given by (1.14). Let $\bar{\nu}_8 = \min\left\{\bar{V}_{L,8}, \bar{C}_{L,8}\right\}$. S_8 with a given $\bar{i}_8 \in (0,1)$ exists if and only the following conditions are satisfied:

$$\theta^{-}(k, \nu_8) \geq \frac{c_H - \bar{\nu}_8}{V_H - c_H}, \tag{1.32}$$

$$\alpha = \frac{\delta + \lambda \bar{\rho}_{H,8}}{\delta + \lambda \bar{\rho}_{L,8}} \cdot \theta^{-}(k, \bar{\nu}_8). \tag{1.33}$$

3. (S_9) The last group of stationary equilibria features buyer's partial information acquisition and mixed offering strategy when information is not acquired. Buyers acquire information with probability $\bar{i}_9$. In case the buyers do not acquire information, they offer the pooling price with probability $\bar{\sigma}(c_H, N)$. Therefore, $\bar{\gamma}_{L,9} = \bar{i}_9 f_L^G + \bar{\sigma}(c_H, N)$. High-quality assets are traded with probability $\bar{\rho}_{h,9} = \bar{i}_9 f_L^G + \bar{\sigma}(c_H, N)$. The probability that low type assets are traded depends on the gain from trade of low-quality assets. There are three cases depending on $\bar{\gamma}_{L,9}$:

- If $\bar{i}_9 > \frac{r}{\lambda}(v_L - c_L)/(c_H - v_L)$, the gain from trade of low-quality assets is negative. Low-quality assets are traded with probability $\bar{\rho}_{L,9} = \bar{\gamma}_{L,9}$.

- If $\bar{i}_9 < \frac{r}{\lambda}(v_L - c_L)/(c_H - v_L)$, the gain from trade of low-quality assets is positive. Low-quality assets are traded with probability $\bar{\rho}_{L,9} = 1$.

- If $\bar{i}_9 = \frac{r}{\lambda}(v_L - c_L)/(c_H - v_L)$, the gain from trade of low-quality assets is zero. Sellers of low-quality assets can use mixed strategies when offered the separating price. Low-quality assets are traded with probability $\bar{\rho}_{l,9} \in [\bar{\gamma}_{L,9}, 1]$.

The continuation values of the owners of low-quality assets are given by (1.15). The stationary equilibria market composition $\bar{\theta}_9$ is given by (1.14). Let $\bar{\nu}_9 = \min\left\{\bar{V}_{L,9}, \bar{C}_{L,9}\right\}$. S_9 with given $\bar{i}_9 \in (0,1)$ and $\bar{\sigma}(c_H, N)$ exists if and only if the following conditions are satisfied:

$$k = (f_L^B - f_H^B)(V_H - c_H) \cdot \frac{c_H - \bar{\nu}_9}{V_H - \bar{\nu}_9}, \tag{1.34}$$

$$\alpha = \frac{\delta + \lambda \bar{\rho}_{H,9}}{\delta + \lambda \bar{\rho}_{L,9}} \cdot \frac{c_H - \bar{\nu}_9}{V_H - c_H}. \tag{1.35}$$

1.10. Appendix 3: Monotonicity of Paths of Market Composition

Define $\bar{\rho}_{H0}$ and $\bar{\rho}_{L0}$ as

$$\bar{\rho}_{H0} = \frac{\delta}{\lambda}\left(\frac{\alpha}{m_H^S(0)(1+\alpha)} - 1\right), \; \bar{\rho}_{L0} = \frac{\delta}{\lambda}\left(\frac{1}{m_L^S(0)(1+\alpha)} - 1\right). \tag{1.36}$$

Compared with (1.19), if the initial asset distribution is an stationary distribution, $\bar{\rho}_{H0}$ and $\bar{\rho}_{L0}$ are the corresponding trading probability of high-quality and low-quality assets. A higher $\bar{\rho}_{H0}$ ($\bar{\rho}_{L0}$) is related to a smaller initial mass of high-quality(low-quality) assets in the market. Note that $\bar{\rho}_{H0} > \bar{\rho}_{L0}$ if and only if $\theta(0) < \alpha$, while $\bar{\rho}_{H0} < \bar{\rho}_{L0}$ if and only if $\theta(0) > \alpha$. In the follow lemma, we give two scenarios in which the market composition $\theta(t)$ converges monotonically to a new steady state along an equilibrium path.

Lemma 1.10. *Assume $\rho_H(t) = \bar{\rho}_H$ and $\rho_L(t) = \bar{\rho}_L$,*

1. *$\theta(t)$ is decreasing (increasing) in $t \in (0, +\infty)$ if $\bar{\rho}_{L0} \geq \bar{\rho}_{H0} \geq \bar{\rho}_H \geq \bar{\rho}_L$ ($\bar{\rho}_{H0} \geq \bar{\rho}_{L0} \geq \bar{\rho}_L \geq \bar{\rho}_H$);*

2. *if $\bar{\rho}_H = \bar{\rho}_L$, $\theta(t)$ is decreasing (increasing) in $t \in (0, +\infty)$ if and only if $\bar{\rho}_{H0} \leq \bar{\rho}_{L0}$ ($\bar{\rho}_{H0} \geq \bar{\rho}_{L0}$).*

1.11. Appendix 4: Alternative Assumptions on Buyers' Entry and Exit

In the model, I make a simplifying assumption with respect to buyers' entry and exit. Namely, the inflow of buyers is proportional to the mass of sellers at any given time, and buyers exit the market immediately if no trade happens within matches. This assumption helps me highlight the effect of buyers' trading strategy on market liquidity without considering the changes in the meeting rate. Here I analyze the robustness of the main results in a model with more conventional assumptions on buyers' entry and exit.

Let's consider a market with a fixed inflow of buyers denoted by ϵ. After unsuccessful trade, buyers do not exit the market. Instead, they stay on the market and are matched randomly with sellers. Denote the mass of buyers at time t by $m^B(t)$. The matching function takes a multiplicative form of $\hat{\lambda} m^B(t) \left[m_H^S(t) + m_L^S(t)\right]$. Therefore, each seller meets a buyer at Poisson rate $\hat{\lambda} m^B(t)$, and each buyer meets a seller at Poisson rate $\hat{\lambda} \left[m_H^S(t) + m_L^S(t)\right]$. Since the matching process is random, the prior belief of a seller—the probability of meeting a high-quality seller to the probability of meeting a low-quality seller—is still $\theta(t)$. Compared to the model described in Section 1.2, the market liquidity is affected by both the endogenous meeting rate and buyers' trading strategy. In addition, buyers now take into consideration the option value of waiting to buy assets later. Both factors complicate the analysis of the model, especially the analytical characterization of the non-stationary equilibria.

To characterize the equilibrium in the revised model, we need to introduce more notations. Let $\hat{J}(t)$ be the ex ante expected value of a matched buyer and $J(t)$ be the continuation value of an unmatched buyer at time t. They are linked through the following expression.

$$J(t) = \int_t^{+\infty} e^{-r(\tau-t)} \hat{J}(\tau) \mathrm{d} \left(1 - e^{-\int_t^\tau \lambda m^S(t) \mathrm{d}u}\right).$$

The continuation values $C_H(t)$, $V_H(t)$ and $V_L(t)$ still satisfy (1.3), (1.5) and (1.6), while $C_L(t)$ is different because the matching function is different.

$$C_L(t) = \int_t^{\infty} \left[(1 - e^{-r(\tau-t)})c_L + e^{-r(\tau-t)}c_H\right] d(1 - e^{-\lambda \int_t^\tau m^B(u)\gamma_L(c_H,u)du}).$$

For the static trading game, the previous analyses still apply if we replace the continuation values with $\hat{C}_H(t) = C_H(t)$, $\hat{C}_L(t) = C_L(t)$, $\hat{V}_H(t) = V_H(t) - J(t)$ and $\hat{V}_L(t) = V_L(t) - J(t)$. Let $\nu(t) = \min\left\{\hat{V}_L(t), \hat{C}_L(t)\right\}$, the expected value of being matching at time t is

$$\hat{J}(t) - J(t) = \begin{cases} \frac{1}{1+\theta(t)}\left(\hat{V}_L(t) - \nu(t)\right), & \theta(t) < \hat{\theta}^-(k, \nu(t)), \\ \frac{1}{1+\theta(t)}\left[\hat{V}_L(t) - f_L^G \hat{C}_H(t) - f_L^B \nu(t)\right] \dots & \\ +\frac{\theta(t)}{1+\theta(t)} f_H^G \left(\hat{V}_H(t) - \hat{C}_H(t)\right) - k, & \theta^-(k, \nu(t)) \leq \theta(t) < \theta^+(k, \nu(t)), \\ \frac{1}{1+\theta(t)}\left(\hat{V}_L(t) - \hat{C}_H(t)\right) + \frac{\theta(t)}{1+\theta(t)}\left(\hat{V}_H(t) - \hat{C}_H(t)\right), & \theta(t) \geq \theta^+(k, \nu(t)). \end{cases}$$

Although the characterization is more complicated, the main result still holds—given certain parametric restrictions, there exists two steady states, a liquid one without information acquisition and an illiquid one with information acquisition. Moreover, given the initial condition in the illiquid steady state, there is no equilibrium that converges to the liquid steady state. Here I provide the intuition without giving the details of the analysis. First, since the static trading game can be represented with a set of modified continuation values, the equilibrium of the static trading game does not change qualitatively. Specifically, the information-sensitive region lies to the left of the information-insensitive pooling region. Second, when buyers acquire information, high-quality assets are still traded faster than low-quality assets. Therefore, the cream-skimming effect of information acquisition is still present in the revised model. Third, although the rate at which sellers meet buyers is higher in an illiquid market, it does not offset the low liquidity caused by buyers' information acquisition. To summarize, the above three components that drive the main results are all present in the revised model.

1.12. Appendix 5: Non-Stationary Equilibria from the Information Trap

The following proposition shows that when the current market composition falls in the overlapping part of the two information-sensitive region $[\theta_2^-(k), \theta_1^+(k)]$, it is hard for the market to recover to the liquid state S_1, even if an information-insensitive pooling stationary equilibria exists for the same set of parameters and fundamental α.

Proposition 1.6. *If $\theta_2^-(k) \leq \theta(0) < \theta_1^+(k)$, there exists an equilibrium path that converges to pooling trading if and only if the dynamics of the asset distribution characterized by (1.10) and (1.11) with $\rho_H(t) \equiv f_H^G$ and $\rho_L(t) \equiv f_L^G$ satisfy $\theta(t) = \theta_1^+(k)$ for some $t \geq 0$.*

1.13. Appendix 6: Proofs

Proof of Lemma 1.1-1.3 (Solutions to the Static Trading Game). $\mathbf{V_L < C_L}$**, no gains from trade for low-quality assets.** The buyer has lower continuation value of the low-quality asset than the seller. Therefore, no trade will take place at any price lower than C_H. The buyer will compare the expected payoff from offering the lowest pooling price and withdrawing from trading (or offering a price lower than V_L). The buyer finds it optimal to offer the pooling price C_H if and only if

$$\tilde{\theta} V_H + V_L - (1 + \tilde{\theta}) C_H \geq 0.$$

It can be written as

$$\tilde{\theta} \geq \hat{\theta} = \frac{C_H - V_L}{V_H - C_H}. \tag{1.37}$$

where $\hat{\theta}$ is the threshold belief.

If the prior belief $\theta \geq \hat{\theta}$, the optimal strategy of a buyer without information is to offer the lowest pooling offer C_H and get the expected revenue $\frac{\theta}{1+\theta}V_H + \frac{1}{1+\theta}V_L - C_H$. However, when observing the signal, the buyer can make offers conditional on the signal. Specifically, if $\theta \geq \hat{\theta}$ and $\tilde{\theta}(\theta, B) \leq \hat{\theta}$, the buyer will offer pooling price C_H when observing G and withdraw from trade if observing B. The expected revenue is $\frac{\theta}{1+\theta}f_H^G(V_H - C_H) + \frac{1}{1+\theta}f_L^G(V_L - C_H)$. If $\tilde{\theta}(\theta, B) > \hat{\theta}$, the buyer is willing to offer the pooling price C_H no matter what the signal is. The expected revenue is $\frac{\theta}{1+\theta}V_H + \frac{1}{1+\theta}V_L - C_H$, the same as if there's no information. Therefore, the value of information for the buyer can be written in the form of an option value

$$W(\theta) = \max\left\{-\frac{\theta}{1+\theta}f_H^B(V_H - C_H) + \frac{1}{1+\theta}f_L^B(C_H - V_L), 0\right\}.$$

The intuition is as following. For prior belief $\theta \geq \hat{\theta}$, the signal allow the buyer to avoid loss $C_H - V_L$ from buying a low-quality asset with probability $\frac{1}{1+\theta}f_L^B$. However the signal can be "false negative" with probability $\frac{\theta}{1+\theta}f_H^B$ and by making conditional offers the buyer loses the trade surplus $V_H - C_H$ from buying a high-quality asset.

On the other hand, if $\theta < \hat{\theta}$, there will be no trade for both types if there's no information. Therefore, using the same reasoning as above, we find the value of information for the buyer is

$$W(\theta) = \max\left\{\frac{\theta}{1+\theta}f_H^G(V_H - C_H) - \frac{1}{1+\theta}f_L^G(C_H - V_L), 0\right\}.$$

After observing the signal, the buyer has the option to make conditional offers. Doing so, the buyer gains the surplus of trading with the high type with probability $\frac{\theta}{1+\theta}f_H^G$, but incurs a loss of trading with the low type with probability $\frac{1}{1+\theta}f_L^G$. The buyer will make conditional offers only if the net gain is positive.

$\mathbf{V_L \geq C_L}$, non-negative gains from trade for low-quality assets. There's a non-negative gain if the buyer offers a low price to only buy low-quality assets. Therefore, the buyer compares the expected gain from offering a pooling price with only buying low-quality assets. The buyer find it optimal to offer pooling price if and only if

$$\frac{\tilde{\theta}}{1+\tilde{\theta}}V_H + \frac{1}{1+\tilde{\theta}}V_L - C_H \geq \frac{1}{1+\tilde{\theta}}(V_L - C_L),$$

which translates into

$$\tilde{\theta} \geq \hat{\theta} = \frac{C_H - C_L}{V_H - C_H}.$$

If $\theta \geq \hat{\theta}$, the buyer will offer pooling price C_H without information. By making conditional offers, the buyer can reduce the price paid for a low-quality asset from C_H to C_L with probability $\frac{\theta}{1+\theta}f_L^B$, but with probability $\frac{\theta}{1+\theta}f_H^B$ she will lose the revenue $V_H - C_H$ from buying a high-quality asset. The value of information to the buyer is

$$W(\theta) = \max\left\{-\frac{\theta}{1+\theta}f_H^B(V_H - C_H) + \frac{1}{1+\theta}f_L^B(C_H - C_L), 0\right\}.$$

If $\theta < \hat{\theta}$, the buyer will only trade with the low type at price C_L without information. By making conditional offers, the buyer can get revenue of $V_H - C_H$ with probability $\frac{\theta}{1+\theta}f_H^G$ from buying a high-quality asset, while loss $C_H - C_L$ with probability $\frac{1}{1+\theta}f_L^G$ buying a low-quality asset at the pooling price. The value of information to the buyer is therefore

$$W(\theta) = \max\left\{\frac{\theta}{1+\theta}f_H^G(V_H - C_H) - \frac{1}{1+\theta}f_L^G(C_H - C_L), 0\right\}.$$

Let $\nu = \min\{V_L, C_L\}$, the value of information can be written in a synthetic form,

$$W(\theta) = \begin{cases} \max\left\{-\frac{\theta}{1+\theta}f_H^B(V_H - C_H) + \frac{1}{1+\theta}f_L^B(C_H - \nu), 0\right\}, & \text{if } \theta \geq \hat{\theta}, \\ \max\{\frac{\theta}{1+\theta}f_H^G(V_H - C_H) - \frac{1}{1+\theta}f_L^G(C_H - \nu), 0\}, & \text{if } \theta < \hat{\theta}. \end{cases}$$

Notice $W(\theta)$ remains at zero for θ close to 0, then increases to its maximum value $W_{max}(\nu) =$

$(f_L^B - f_H^B)(v_H - c_H) \cdot \frac{C_H - \nu}{V_H - \nu}$ at $\theta = \hat{\theta} = \frac{C_H - \nu}{V_H - C_H}$, and decreases to zero at a finite value of θ. For $k < W_{max}(\nu)$, the boundaries of the information-sensitive region can be solved by equating $W(\theta)$ and k,

$$\theta^-(k,\nu) = \frac{f_L^G(C_H - \nu) + k}{f_H^G(V_H - C_H) - k}, \quad \theta^+(k,\nu) = \frac{f_L^B(C_H - \nu) - k}{f_H^B(V_H - C_H) + k}.$$

□

Proof of Lemma 1.4. First note that

$$C_L(t) \leq \frac{rc_L + \lambda c_H}{r + \lambda}.$$

If $\gamma_L(c_H, \tau) \geq f_L^G$ for any $\tau > t$,

$$\begin{aligned}
&(1 - e^{-r(\tau - t)})(v_L - c_L) - \int_t^\tau e^{-r(u-t)} \lambda \gamma_L(c_H, u)(c_H - C_L(u)) \mathrm{d}u, \\
\leq &(1 - e^{-r(\tau - t)})(v_L - c_L) - \int_t^\tau e^{-r(u-t)} \lambda f_L^G \left(c_H - \frac{rc_L + \lambda c_H}{r + \lambda} \right) \mathrm{d}u, \\
= &(1 - e^{-r(\tau - t)})(v_L - c_L) - \lambda f_L^G \frac{r(c_H - c_L)}{r + \lambda} \int_t^\tau e^{-r(u-t)} \mathrm{d}u, \\
= &(1 - e^{-r(\tau - t)})(v_L - c_L) \left(v_L - c_L - f_L^G \frac{\lambda}{r + \lambda}(c_H - c_L) \right).
\end{aligned}$$

If Assumption 1.1 holds, the above expression is negative for any $\tau > t$. Therefore $V_L(t) - C_L(t) < 0$. □

Proof of Prosposition 1.1. Since $f_L^G < f_H^G$ and $f_L^B > f_H^B$, the interval defined in Lemma 1.6 has positive measure for small k. Also, when k is small, the condition for the existence of

S_1 becomes

$$\alpha \geq \frac{f_L^B(c_H - \bar{V}_{L,1}) - k}{f_H^B(V_H - c_H) + k}.$$

Lemma 1.5 and 1.6 jointly imply that S_1 and S_2 coexist if and only if $\alpha \in [A_1(k), A_2(k)]$. To show the interval has positive measure for small k, it's sufficient to show that

$$\frac{f_L^B(c_H - \bar{V}_{L,1})}{f_H^B(V_H - c_H)} < \frac{\delta + \lambda f_H^G}{\delta + \lambda f_L^G} \cdot \frac{f_L^B(c_H - \bar{V}_{L,2})}{f_H^B(V_H - c_H)}.$$

In fact, the above inequality always holds since $\bar{V}_{L,1} > \bar{V}_{L,2}$ and $f_L^G > f_L^G$. □

Proof of Lemma 1.10. When $\rho_H(t)$ and $\rho_L(t)$ are constants, they can be further simplified as

$$m_H^S(t) = \frac{\delta\alpha}{\delta + \lambda\rho_H} + \left(m_H^S(0) - \frac{\delta\alpha}{\delta + \lambda\rho_H}\right) e^{-(\delta+\lambda\rho_H)t}, \tag{1.38}$$

$$m_L^S(t) = \frac{\delta(1-\alpha)}{\delta + \lambda\rho_L} + \left(m_L^S(0) - \frac{\delta(1-\alpha)}{\delta + \lambda\rho_L}\right) e^{-(\delta+\lambda\rho_L)t} \tag{1.39}$$

Plugging in (1.38) and (1.39), we can show that the sign of $\frac{\mathrm{d}\theta(t)}{\mathrm{d}t}$ is the same as the sign of

$$\frac{(\delta + \lambda\bar{\rho}_{H0}) - (\delta + \lambda\bar{\rho}_H)}{1 + (\delta + \lambda\bar{\rho}_{H0})\frac{e^{(\delta+\lambda\bar{\rho}_H)t} - 1}{\delta + \lambda\bar{\rho}_H}} - \frac{(\delta + \lambda\bar{\rho}_{L0}) - (\delta + \lambda\bar{\rho}_L)}{1 + (\delta + \lambda\bar{\rho}_{L0})\frac{e^{(\delta+\lambda\bar{\rho}_L)t} - 1}{\delta + \lambda\bar{\rho}_L}}. \tag{1.40}$$

Note that for any $t > 0$ the function $\frac{x-y}{1+x\frac{e^{yt}-1}{y}}$ is strictly increasing in x and strictly decreasing in y for any $y \leq x$. Thus, if $\bar{\rho}_{L0} \geq \bar{\rho}_{H0} \geq \bar{\rho}_H \geq \bar{\rho}_L$ ($\bar{\rho}_{H0} \geq \bar{\rho}_{L0} \geq \bar{\rho}_L \geq \bar{\rho}_H$), (1.40) is non-positive (non-negative), which implies $\theta(t)$ is decreasing (increasing) in t. Similarly, if $\bar{\rho}_H = \bar{\rho}_L$, the sign of (1.40) is the same as the sign of $\bar{\rho}_{H0} - \bar{\rho}_{L0}$. Therefore, $\theta(t)$ is decreasing (increasing) in t if and only if $\bar{\rho}_{H0} \leq \bar{\rho}_{L0}$ ($\bar{\rho}_{H0} \geq \bar{\rho}_{L0}$). □

Proof of Proposition 1.2. Notice

$$A_2(k) = \frac{\delta + \lambda f_H^G}{\delta + \lambda f_L^G} \cdot \theta^+(k, \bar{V}_{L,2}) = \frac{\delta + \lambda f_H^G}{\delta + \lambda f_L^G} \cdot A_3(k) > A_3(k). \tag{1.41}$$

$A_1(k)$ is the maximum of two values. By Lemma 1.3 we know $\theta^+(k, \bar{V}_{L,2}) > \theta^+(k, \bar{V}_{L,1})$. To show that $\theta^+(k, \bar{V}_{L,2}) > A_1(k)$ for small enough k, it is sufficient to show that

$$\frac{f_L^B(c_H - \bar{V}_{L,2})}{f_H^B(V_H - c_H)} > \frac{\delta + \lambda f_H^G}{\delta + \lambda f_L^G} \cdot \frac{f_L^G(c_H - \bar{V}_{L,2})}{f_H^G(V_H - c_H)}. \tag{1.42}$$

It follows directly from $f_L^B > f_H^B$ and $f_H^G > f_L^G$.

Given any $\alpha \in (A_1(k), A_3(k))$, the no information pooling stationary equilibria features $\bar{\theta}_1 = \alpha > \theta^+(k, \bar{V}_{L,1})$. Suppose the market starts from an initial asset distribution with $\theta(0)$ in the neighbourhood of α. Let's consider two paths. On the first path buyers always offer the pooling price c_H without acquiring information. Therefore, $\rho_H(t) = \rho_L(t) = 1$. Lemma 1.10 implies that $\theta(t)$ converges monotonically to α. Since the continuation values are the same as in the no information pooling stationary equilibria, it is easy to verify that $\theta(t)$ falls in the pooling no information region for any $t > 0$. The first path is indeed an equilibrium path converging to S_1.

For the second path, assume buyers always acquire information and offer the pooling price c_H only if a good signal is observed. Thus, the continuation values are the same as in the information stationary equilibria. Moreover, $\rho_H(t) = f_H^G$ and $\rho_L(t) = f_L^G$ for any $t > 0$. Lemma 1.10 implies that starting from the initial distribution close to S_1, $\theta(t)$ decreases monotonically to $\bar{\theta}_2$. Notice by assumption

$$\theta(0) = \alpha < A_3(k) = \theta^+(k, \bar{V}_{L,2}),$$

$$\bar{\theta}(+\infty) = \frac{\delta + \lambda f_L^G}{\delta + \lambda f_H^G} \cdot \alpha \geq \frac{\delta + \lambda f_L^G}{\delta + \lambda f_H^G} \cdot A_1(k) \geq \bar{\theta}^-(k, \bar{V}_{L,2}).$$

The whole path of $\theta(t)$ lies within the information sensitive region. Since $\bar{\theta}_2$ is the only sink in the information region, when starting from an initial distribution close to that of S_1, the path of $\theta(t)$ also stays in the information sensitive region. Therefore, the second path is an equilibrium path converging to S_2. $\square$

Proof of Proposition 1.3. Assume buyers do not acquire information and always offer the pooling price c_H for any $t > 0$. Therefore, both high-quality and low-quality assets are traded with probability 1. Also, the continuation values of owners of low-quality assets are fixed at $V_L(t) = \bar{V}_{L,1}$ and $C_L(t) = \bar{C}_{L,1}$. To show the assumed path is indeed an equilibrium path, we only need to verify that the whole path of market composition falls in the pooling information-insensitive region. In fact, Lemma 1.10 implies that the market composition $\theta(t)$ increases monotonically from $\theta(0)$ to α. Given that $\alpha, \theta(0) > \theta_1^+(k)$, we know $\theta(t) > \theta_1^+(k)$ for any $t > 0$. The assumed path is an equilibrium path that converges to S_1. $\square$

Proof of Proposition 1.6. First, we prove a lemma that characterizes any equilibrium path that converges to pooling trading.

Lemma 1.11. *If* $\frac{\delta+\lambda f_L^G}{\delta+\lambda f_H^G}\alpha \leq \theta^+(k, \bar{V}_{L,1})$, *along any equilibrium path that converges to pooling trading,* $\theta(t)$ *must be weakly increasing whenever* $\theta(t) < \theta^+(k, \bar{V}_{L,1})$.

Proof of Lemma 1.11. This can be proved by contradiction. Suppose there exist t_1 such that $\dot{\theta}(t_1) < 0$ and $\theta(t_1) < \theta^+(k, \bar{V}_{L,1})$. By continuity of $\theta(t)$, there exists $t_3 > t_2 \geq t_1$ such that $\dot{\theta}(t_2) < 0$, $\theta(t) < \theta^+(k, \bar{V}_{L,1})$ for any $t_2 \leq t < t_3$ and $\theta(t) \geq \theta^+(k, \bar{V}_{L,1})$ for any $t \geq t_3$. Namely, t_3 is the last time that $\theta(t)$ enters the region $\theta \geq \theta^+(k, \bar{V}_{L,1})$ from the left. $\theta(t)$ decreases at t_2 and stays below $\theta^+(k, \bar{V}_{L,1})$ for $t_2 < t < t_3$.

Since $\theta(t) > \theta^+(k, \bar{V}_{L,1})$ for any $t > t_3$, using backward induction, we can show that $C_L(t_3) > V_L(t_3) = \bar{V}_{L,1}$. For t slightly less than t_3, $\theta^-(k, \bar{V}_{L,1}) < \theta(t) < \theta^+(k, \bar{V}_{L,1})$, therefore, buyers acquire information and only offers the pooling price when signal G is observed. So $\rho_H(t) = f_H^G$ and $\rho_L(t) = f_L^G$. Since $\theta(t)$ crosses $\theta^+(k, \bar{V}_{L,1})$ from the left, for t slightly less than t_3, we have

$$\frac{\mathrm{d}}{\mathrm{d}t} \ln \theta(t) = \frac{\delta\alpha}{m_H^S(t)(1+\alpha)} (1 - \theta(t)/\alpha) - \lambda(f_H^G - f_L^G) > 0, \tag{1.43}$$

Taking the limit of t to t_3, it yields

$$\frac{\delta\alpha}{m_H^S(t_3)(1+\alpha)} \left(1 - \theta^+(k, \bar{V}_{L,1})/\alpha\right) - \lambda(f_H^G - f_L^G) \geq 0. \tag{1.44}$$

Evaluating the derivative of $\theta(t)$ at $t = t_2$, we have

$$\frac{\delta\alpha}{m_H^S(t_2)(1+\alpha)} (1 - \theta(t_2)/\alpha) - \lambda(\rho_H(t_2) - \rho_L(t_2)) < 0. \tag{1.45}$$

By construction, $\theta(t_2) < \theta^+(k, \bar{V}_{L,1}) < \alpha$. Also notice $\rho_H(t_2) - \rho_L(t_2) < f_H^G - f_L^G$. Comparing (1.44) and (1.45), we have

$$m_H^S(t_2) > m_H^S(t_3). \tag{1.46}$$

On the other hand, since $\theta^+(k, \bar{V}_{L,1}) \geq \frac{\delta + \lambda f_L^G}{\delta + f_H^G} \alpha$, from (1.44) we know

$$m_H^S(t_3) \leq \frac{\delta\alpha}{1+\alpha} \frac{1 - \theta^+(k, \bar{V}_{L,1})/\alpha}{\lambda(f_H^G - f_L^G)} \leq \frac{\delta}{\delta + \lambda f_H^G} \frac{\alpha}{1+\alpha}.$$

Rewrite (1.10),

$$\frac{\mathrm{d}}{\mathrm{d}t}\left(m_H^S(t) - \frac{\delta}{\delta + \lambda f_H^G}\frac{\alpha}{1+\alpha}\right) = -(\delta + \lambda f_H^G)\left(m_H^S(t) - \frac{\delta}{\delta + \lambda f_H^G}\frac{\alpha}{1+\alpha}\right) - \lambda\left(\rho_H(t) - f_H^G\right) m_H^S(t). \tag{1.47}$$

Since $\theta(t) < \theta^+(k, \bar{V}_{L,1})$ for $t_2 \le t < t_3$, from Table 1 we know $\rho_H(t) \le f_H^G$. Therefore

$$\frac{\mathrm{d}}{\mathrm{d}t}\left(m_H^S(t) - \frac{\delta}{\delta + \lambda f_H^G}\frac{\alpha}{1+\alpha}\right) \ge -(\delta + \lambda f_H^G)\left(m_H^S(t) - \frac{\delta}{\delta + \lambda f_H^G}\frac{\alpha}{1+\alpha}\right), \tag{1.48}$$

or equivalently

$$\frac{\mathrm{d}}{\mathrm{d}(-t)}\left(\frac{\delta}{\delta + \lambda f_H^G}\frac{\alpha}{1+\alpha} - m_H^S(t)\right) \ge (\delta + \lambda f_H^G)\left(\frac{\delta}{\delta + \lambda f_H^G}\frac{\alpha}{1+\alpha} - m_H^S(t)\right), \tag{1.49}$$

Given $m_H^S(t_3) \le \frac{\delta}{\delta+\lambda f_H^G}\frac{\alpha}{1+\alpha}$, (1.49) implies that $m_H^S(t_2) \le m_H^S(t_3)$. This is in contradiction with (1.46). Therefore, $\theta(t)$ must be weakly increasing when $\theta(t) < \theta^+(k, \bar{V}_{L,1})$ along any equilibrium path that converges to pooling trading. $\square$

Now we can move on to prove the necessity of the given condition. Notice, if $\frac{\delta+\lambda f_L^G}{\delta+\lambda f_H^G}\alpha > \theta(k, \bar{V}_{L,1})$, the path with constant $\rho_H(t) = f_H^G$ and $\rho_L(t) = f_L^G$ converges to $\frac{\delta+\lambda f_L^G}{\delta+\lambda f_H^G}\alpha > \theta(k, \bar{V}_{L,1})$ in the end. On the other hand, if $\frac{\delta+\lambda f_L^G}{\delta+\lambda f_H^G}\alpha \le \theta(k, \bar{V}_{L,1})$, Lemma 1.11 indicates that any path that starts from $\theta(0) < \theta(k, \bar{V}_{L,1})$ and converges to pooling trading only crosses $\theta^+(k, \bar{V}_{L,1})$ once. Again, let t_3 be the earliest time such that $\theta(t_3) = \theta^+(k, \bar{V}_{L,1})$. For any $0 \le t < t_3$, we must have $\theta^-(k, \bar{V}_{L,2}) \le \theta(0) \le \theta(t) < \theta^+(k, \bar{V}_{L,1})$. Using backward induction, it can be easily shown that $\bar{V}_{L,1} < V_L(t) < \bar{V}_{L,2}$ for any $0 \le t < t_3$. Therefore, from the monotonicity of $\theta^-(k, \cdot)$ and $\theta^+(k, \cdot)$ we know that $\theta^-(k, V_L(t)) < \theta^-(k, V_{L,2}) < \theta(t) < \theta^+(k, \bar{V}_{L,1}) < \theta^+(k, V_L(t))$ for any $0 \le t < t_3$. Also, Assumption 1.1 implies that $V_L(t) < C_L(t)$ for any $t \ge 0$. Referring to Table 1, we know buyers acquire information with probability 1, $\rho_H(t) = f_H^G$, $\rho_L(t) = f_L^G$ for any $0 \le t < t_3$. This shows that if we fix $\rho_H(t) = f_H^G$ and $\rho_L(t) = f_L^G$ for any $t \ge 0$, we must have $\theta(t_3) = \theta^+(k, \bar{V}_{L,1})$.

Now we want to show the given condition is also sufficient. This is done by guess-and-verify. Let t_3 be the first positive value that satisfies $\theta(t) = \theta(k, \bar{V}_{L,1})$ in the hypothetical path with $\rho_H(t) \equiv f_L^G$ and $\rho_L(t) \equiv f_L^G$. Let $i(t) = 1$ and $\sigma(c_H, G, t) = 1$ for any $t < t_3$ and $i = 0$, $\sigma(c_H, N, t) = 1$ for any $t \ge t_3$. It is easy to construct an equilibrium path that's consistent with the above offering strategy. $\square$

Proof of Proposition 1.4. Since $\bar{V}_{L,1} > \bar{V}_{L,2}$, by Lemma 1.3, $\theta^+(k, \bar{V}_{L,1}) < \theta^+(k, \bar{V}_{L,2})$, therefore $A_4(k) < A_2(k)$. Also, Assumption 1.2 implies that $\theta^-(k, \bar{V}_{L,2}) < \theta^+(k, \bar{V}_{L,1})$. It immediately follows that $A_1(k) < A_4(k)$ for small $k > 0$. By Proposition 1.1, we know when k is small, for any $\alpha \in (A_1(k), A_4(k))$, S_1 and S_2 coexist. Moreover, the market composition in the information stationary equilibria S_2 satisfies

$$\theta^-(k, \bar{V}_{L,2}) < \bar{\theta}_2 < \theta^+(k, \bar{V}_{L,1}).$$

Therefore, the asset distribution in S_2 falls in the information trap. By Proposition 1.6, when the asset distribution is in the neighbourhood of S_2, there's no equilibrium path that converges to S_1. $\square$

Proof of Proposition 1.5. It is a direct implication of the monotonicity of $\theta_1^+(k)$ and $A_4(k)$. $\square$

CHAPTER 2 : Intervention with Screening in Global Games

Lin Shen Junyuan Zou[1 2]

2.1. Introduction

In many economic environments, strategic complementarities among agents can give rise to coordination failure.[3] To reduce the welfare loss from coordination failure, policy makers may intervene by providing incentives for agents to play the socially desirable equilibrium. For instance, during the recent financial crisis, governments around the world provided explicit and implicit guarantees on debt obligations of financial institutions to prevent "runs" on the financial systems. While these policies proved to be effective in restoring financial stability, some drawbacks also emerged. First, implementing guarantee programs at such large scale exposes the policy maker to large costs, which jeopardized sovereign debt sustainability and led to the sovereign debt crisis in many European countries (Acharya et al., 2014; Farhi and Tirole, 2016). Second, the policies were criticized for their vulnerability to moral hazard problems (Kareken and Wallace, 1978; Keeley, 1990; Cooper and Ross, 2002).[4]

Given that such large-scale interventions are costly, a natural question is whether it is possible to reduce the size of intervention programs without compromising the effectiveness. To answer this question in a general context, we consider a coordination game with incomplete information as in standard global games (Morris and Shin, 2003). When agents receive pri-

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[3]Examples of coordination failures include but not limited to bank run (Diamond and Dybvig, 1983), currency attack (Obstfeld, 1996), macroeconomic coordination failure (Cooper and John, 1988) and technological development (Bresnahan and Trajtenberg, 1995).

[4]Allen et al. (2017) endogenizes the effect of government guarantees on banks' excessive risk taking and shows that guarantees are overall welfare improving even with moral hazard problems.

vate signals, they form interim beliefs regarding the expected payoffs from taking different actions. We propose a group of programs with voluntary participation that screen agents based on their interim beliefs. Compared with the conventional government-guarantee type of programs, it has two main advantages. First, in equilibrium, only a small group of marginal investors self-select into the program, which reduces the implementation costs. Also, our proposed programs have the advantage that moral hazard problems are limited to the small group of participating agents.

This paper provides novel insights for the design of intervention policies to reduce coordination failure in various economic contexts. Some existing literature (Sakovics and Steiner, 2012; Choi, 2014) has studied policies that target ex-ante important agents based on their payoff functions. We contribute to this literature by highlighting the role of agents' interim beliefs of the economic fundamental and other agents' actions. If an ex-ante important agent is very optimistic about the coordination result, there's no need to provide extra incentives for her to take the socially desirable action. It is more cost-efficient if the resources are allocated to agents who have medium beliefs and are at the margin of taking the socially desirable action.

In our benchmark model, we explore a canonical binary-action coordination game under the global games framework. Global games are useful for linking coordination outcome to the underlying fundamental and determining the unique equilibrium. More importantly, they highlight the strategic interactions of agents with heterogeneous private information. In the model, a continuum of agents is each endowed with an investment opportunity. Their investments feature strategic complementarities. Specifically, the investments are successful if and only if the mass of agents investing exceeds a threshold which decreases in the fundamental of the economy. In addition, each agent receives a noisy private signal of the fundamental and makes inferences about the other agents' investment decisions. The game has a unique equilibrium where all agents follow the same threshold strategy. In terms of welfare, there exists a region of weak fundamentals in which agents do not invest, however, the investments would have been successful if all agents were to invest. Therefore, social welfare will be improved if the policy maker can lower the investment threshold and reduce the coordination failure region. The setup of the model is fairly general such that it can be applied to various economic contexts. In section 6, we discuss three coordination problems and make policy recommendations based on the proposed intervention policy.

Next, we allow the policy maker to offer a subsidy-tax program with voluntary participation to all agents who invest. If an investor accepts the offer, she receives a direct subsidy. In return, she is required to pay tax when the investment is successful. We classified the intervention programs into three categories based on the subsidy-to-tax ratio. If a program is too austere, i.e. has a low subsidy-to-tax ratio, no agents will participate. We call this type of programs the *zero-participation programs*. If a program is too generous such that all investors participate, we call it a *full-participation program*. Many existing intervention policies, including government guarantees and direct subsidies, benefit all agents uniformly and therefore fall into this category. We show that full-participation programs can effectively reduce coordination failure, however, are costly to implement. To reduce the costs of implementation, we propose *partial-participation programs* with medium subsidy-to-tax ratios. A partial participation program is equivalent to a costly insurance policy and screens agents based on their interim belief of success. The most optimistic investors who believe in a high probability of paying the tax do not take the offer. At the same time, the most pessimistic agents who believe in a high probability of coordination failure do not find it worthwhile to invest solely to take advantage of the offer. Only agents with intermediate beliefs will participate in the program since it provides protection against coordination failure and investment loss. We show that with a partial-participation program there is a unique Bayesian Nash equilibrium, in which all agents follow the same threshold strategy with two thresholds. An agent will invest and reject the offer if she receives a high signal; she will invest and accept the offer if her signal is medium and between the two thresholds; she will not invest if she receives a low signal. When the information friction goes to zero, the two thresholds converge, and the expected mass of agents who accept the offer goes to zero, which implies zero expected cost of implementation for the policy maker. Furthermore, with proper choice of subsidy and tax, coordination failures can be eliminated, and the first-best investment threshold can be achieved.

To understand intuitively how partial-participation programs can improve coordination results at a minimal cost, let us start with the original threshold equilibrium without intervention programs. For agents receiving signals right below the investment threshold, without any intervention policy, they will not invest in fear of coordination failure and investment loss. The partial-participation programs provide protection against investment loss and give them extra incentive to make the investment. Therefore, with the partial-participation programs, all agents rationally expect the mass of agents who invest to increase and the strategic complementarities strengthen all agents' incentive to invest. Hence, agents receiving even lower signals would be willing to accept the offer and invest, which is also expected by all agents in the economy and gives them more incentive to invest. Repeating the thought process, the extra incentive to invest provided by the partial-participation programs is amplified by higher-order beliefs, and the investment threshold can be reduced significantly in equilibrium. Given that all agents are more optimistic and less worried about coordination failure, the downside protection of the partial-participation programs becomes less appealing, and the mass of investors who accept the offer in equilibrium is actually small.

We then compare government guarantee programs with partial-participation programs in the presence of moral hazard problems. Government guarantee programs are a special case of full-participation programs and have been widely used to reduce coordination failure. We extend the benchmark model by assuming that after investment, an investor can earn private benefit by shirking, which will reduce the success probability of her own investment. Both types of intervention programs reduce investors' "skin in the game" hence induce shirking at the expense of the policy maker and social welfare. For example, in the context of credit freeze when banks abstain from lending, government guarantees reduce banks' incentive to screen and monitor borrowers. Moral hazard problem critically limits the scale of the government-guarantee type of programs. Specifically, if a government guarantees a large amount of investment losses, all investors, including the most optimistic ones, would participate and shirk. In contrast, for partial-participation programs, the moral hazard problem is limited to the program participants. For the optimistic agents, rejecting the offer and exerting effort gives higher payoff than participating and shirking. Hence, the social welfare loss only incurs for medium-belief agents, the mass of whom goes to zero in the limit of vanishing information frictions. As a result, in the limit, there exist partial-participation programs that can restore the first best, yet no government-guarantee type of programs can restore the first best.

Besides the benchmark model, we also show that the results could be generalized to allow unobservable ex-ante heterogeneity in agents' payoff and information structure. Regarding ex-ante agent heterogeneity, a closely related paper is Sakovics and Steiner (2012). The difference is that they only allow the policy maker to provide direct subsidies conditional on agents' observable heterogeneities. Under their setup, the most cost-efficient subsidies should target the important agents with specific ex-ante characteristics. However, their policy space falls into the category of full-participation programs in our model, and the policy maker can save costs and limit moral hazard problems by switching to a partial-participation program. In other words, we show that subsidization should target the interim rather than ex-ante "pivotal" types. Moreover, since the "pivotal" agents self-select to participate in partial-participation programs, the policy maker does not need to observe agents' ex-ante characteristics. We also show that the binary payoff structure in the baseline model can be generalized to a continuous monotonic payoff function.

Our paper is related to two lines of literature. First, our model is built on the literature of global games which was pioneered by Carlsson and Van Damme (1993). Researchers have applied the global games techniques to analyze coordination failures in different contexts, to name a few, bank runs (Rochet and Vives, 2004; Goldstein and Pauzner, 2005), currency attack (Morris and Shin, 1998), credit freeze (Bebchuk and Goldstein, 2011), debt rollovers (Morris and Shin, 2004; He and Xiong, 2012), and political revolutions (Edmond, 2013). We take a general approach and propose intervention programs that can be applied to reduce coordination failure in different contexts. Morris and Shin (2003) reviews the most commonly applied setup and applications of global games. Our main model in section 2 is a special case with binary payoffs. In section 5, we discuss a generalized payoff structure as in Morris and Shin (2003). In both cases, we show that there exists costless intervention to reduce the coordination threshold and eliminate coordination failures in the limit of zero information friction.

Second, our mechanism shares similar ideas found in the literature that explores policies targeting a specific group of agents to reduce coordination failures. For example, within the contracting literature, Segal (2003) and Bernstein and Winter (2012) show that the optimal policy is to *divide and conquer*, i.e. subsidize a subset of players so that they invest even if no one else invests, then the surplus of players in the no-subsidy set can be fully extracted. Sakovics and Steiner (2012) and Choi (2014) analyzed a coordination game with ex-ante heterogeneous agents and showed that different types should be subsidized in a certain order. These papers all demonstrate that subsidizing a subset of agents to ensure their participation can efficiently encourage the participation of the rest of the agents and reduce coordination failure. Our proposed intervention program is different in terms of implementation. The policy maker offers the same option to all agents, and a subset of agents self-select to participate in the program. In the generalization of unobservable ex-ante heterogeneity, we show that our proposed intervention program is more cost-efficient and does not require information about agents' heterogeneity. Cong et al. (2017) and Basak and Zhou (2017) analyze intervention policies under dynamic settings. In both papers, the policy maker target a subset of agents in each period. The coordination result of the current period serves as a public signal of the fundamental of the economy. They emphasize the effect of the public signal on agents' beliefs and behaviors in the subsequent period(s). Another closely related paper is Morris and Shadmehr (2017), which analyzes the reward schemes for a revolutionary leader to elicit effort from citizens. The optimal reward scheme also screens citizens for their optimism. However, they consider bounded reward schemes imposed on a continuous and unbounded effort choice set, while we focus on subsidy-tax programs that agents can voluntarily choose to participate in. More importantly, while they assume zero cost for implementing any reward scheme, we target minimizing the cost of intervention.

The rest of the paper is organized as follows. In section 2, we present a benchmark model of a binary-action investment game and introduce intervention policies that can reduce coordination failures. Section 3 and 4 compare the proposed program with government-guarantee type of programs in terms of implementation cost and robustness to moral hazard problems. Two extensions of the benchmark model are discussed in section 5. Section 6 presents several applications of the benchmark model and discusses policy recommendations in each context. Finally, section 7 concludes.

2.2. The Benchmark Model

In this section, we analyze a binary-action investment game in which each agent's investment outcome depends on the aggregate investment in the economy. In such an environment, inefficient coordination failure can arise in which agents abstain from investment because of their self-fulfilling expectation that other agents will not invest. Then we introduce intervention policies and show how they can encourage investment and reduce coordination failure.

2.2.1. Setups

There is a unit mass of ex-ante identical infinitesimal agents, indexed by $i \in [0, 1]$. These agents are endowed with the same investment opportunity, and they simultaneously make investment decisions $a_i \in \{0, 1\}$. $a_i = 1$ if agent i invests, and $a_i = 0$ if agent i does not invest. Not investing results in zero payoffs, while investing incurs a fixed cost $c > 0$ and generates a profit of $b > c$ if agent i's project is successful and 0 if it fails. We assume all agents' investment payoffs are perfectly correlated. The investments would be successful when the fundamentals of the economy are strong enough or a sufficient number of agents invest. Specifically, the payoff from an investment project is

$$\pi(\theta, I) = \begin{cases} b - c, & \text{if } I \geq 1 - \theta, \\ -c, & \text{if } I < 1 - \theta. \end{cases}$$

where $I = \int_0^1 a_i \mathrm{d}i$ represents the fraction of investors or the aggregate investment level, and θ stands for the fundamentals of the economy. Note that agents' investment decisions feature strategic complementarities, because each project is more likely to succeed when more agents choose to invest. When the fundamentals are higher, it requires less aggregate investment to make the projects successful. Without information friction, when $\theta \in [0, 1)$, all agents investing ($I = 1$) and all agents not investing ($I = 0$) are both Nash equilibria. However, all agents investing is strictly more efficient than the other equilibrium. Therefore, the first-best outcome is that all agents coordinate to invest when $\theta \geq 0$ and not to invest when $\theta < 0$.

We follow the standard global games setup and assume the following information structure. The fundamental θ is drawn from a uniform distribution with support $[\underline{\theta}, \bar{\theta}]$ and it is not directly observable to the agents when they make investment decisions.[5] Instead, each agent receives a noisy signal about the fundamental $x_i = \theta + \sigma\varepsilon_i$, where ε_i is identically and independently distributed with a continuous and strictly increasing c.d.f. $F(\varepsilon)$, the support of which is $[-\frac{1}{2}, \frac{1}{2}]$. Furthermore, we assume that $\underline{\theta} < -\sigma$ and $\bar{\theta} > 1 + \sigma$. Under this assumption, there exist two dominance regions of signals, $[-\underline{\theta} - \frac{1}{2}\sigma, \underline{x})$ and $(\bar{x}, \bar{\theta} + \frac{1}{2}\sigma]$, with $\underline{x}$ and $\bar{x}$ defined as

$$\Pr(\theta \geq 1 | x = \bar{x}) = \frac{c}{b},$$
$$\Pr(\theta \geq 0 | x = \underline{x}) = \frac{c}{b}.$$

[5] We assume a uniform prior to obtain an analytical solution to the coordination game. This is without loss of generality since it can be viewed as a limiting case as the size of the information friction goes to zero.

Intuitively, with the lowest aggregate investment level $I = 0$, an agent is indifferent between the two actions when she receives signal $\bar{x}$. Therefore, her dominant strategy when signal $x > \bar{x}$ is to invest. Similarly, with the highest aggregate investment level $I = 1$, an agent is indifferent between the two actions if she observes signal $\underline{x}$. Hence, when $x < \underline{x}$, not investing is the dominant strategy.

2.2.2. Equilibrium without Intervention

In this subsection, we analyze the equilibrium without intervention and identify the inefficiencies due to coordination failure. Proposition 2.1 characterizes the equilibrium.

Proposition 2.1. *Without intervention, there is a unique equilibrium in which all agents follow the same strategy*

$$a_i(x_i) = \begin{cases} 1, & \text{if } x_i \geq \xi_0^*, \\ 0, & \text{if } x_i < \xi_0^*. \end{cases}$$

where $\xi_0^* = \frac{c}{b} + \sigma F^{-1}\left(\frac{c}{b}\right)$.

Since there is a continuum of agents, given the realization of fundamentals θ, we can apply the law of large numbers to calculate the aggregate investment I and predict the coordination outcomes. In equilibrium, all agents follow the same threshold strategy. Therefore, the coordination outcome also has a threshold above which the investment projects are successful. Let $\theta^*(\xi)$ denote the fundamental threshold when all agents follow the threshold strategy ξ, then it is defined by

$$F\left(\frac{\theta^*(\xi) - \xi}{\sigma}\right) = 1 - \theta^*(\xi).$$

In words, at the fundamental threshold, the fraction of investors I equals the cutoff $1 - \theta$.

Then the fundamental threshold in equilibrium is given by

$$\theta^*(\xi_0^*) = \frac{c}{b}.$$

The fundamental realizations can be divided into three regions as shown below. In the

Figure 5: Coordination Results

middle region $\theta \in \left[0, \frac{c}{b}\right)$, if all agents coordinate to invest, the investment projects would have been successful. However, the agents have self-fulfilling beliefs that other agents do not invest. As a result, they rationally choose not to invest. Since a unit of successful investment generates a positive surplus of $b-c$, in the middle region, coordination failure leads to social welfare loss of $b-c$. Hence, the first-best scenario has a fundamental threshold θ^* equal to zero. And in the next section, we will show how our proposed intervention program can lower this cutoff and reduce inefficiencies caused by coordination failure.

2.2.3. Intervention Program

Having characterized the equilibrium in the game without intervention, we now describe the subsidy-tax intervention program that the policy maker can use to boost investment and reduce coordination failure. The intervention program consists of two parts, a direct subsidy $s \in [0, c]$ and a contingent tax $t \in [0, b)$. Specifically, if an investor decides to accept the offer, she receives an upfront subsidy s regardless of the investment outcome and pays a lump-sum tax t only if the investment succeeds.[6] The program is only available to the investors and they voluntarily decide whether to participate in the program. Note that there is an implicit assumption that the actions taken by the agents are observable to the policy maker and can be contracted on. We make this assumption because, as shown in Bond

[6]Since in the benchmark model there's only two possible payoffs from investing, we only need to specify a contingent lump-sum tax. In section 2.5.2, we analyze a more general setup where there's a continuum of investment outcomes and we allow tax to be proportional to the investment revenue.

and Pande (2007), if the policy maker cannot observe individual actions, its ability to use subsidy-tax schemes as a coordination device is greatly limited. This assumption imposes certain limitations on the application of our proposed intervention mechanism. For example, in the context of currency attack, it is hard to trace agents' action and tax conditional on agents' investment behavior. Therefore, the intervention program discussed in this paper cannot be applied to solving currency deflation caused by coordination failure (Morris and Shin, 1998). Despite this limitation, there is a wide range of real-world applications. In section 6, we discuss three representative examples.

Mathematically, if an investor accepts the offer, her payoff is modified to

$$\tilde{\pi}(\theta, l) = \begin{cases} b - t - (c - s), & \text{if } l \geq 1 - \theta, \\ -(c - s), & \text{if } l < 1 - \theta. \end{cases}$$

The upfront subsidy s reduces the cost of investment and encourages agents to invest. The contingent tax t directly helps the policy maker recover the cost of providing subsidies. More importantly, it will become clear later that the contingent tax t indirectly saves cost by deterring participation of optimistic agents. The timeline of the coordination game with the intervention program is modified as follows. At the beginning of the game, the policy maker announces the intervention program (s, t). Note that since the subsidy s and tax t are both state independent, the announcement of the intervention program does not convey any information possessed by the policy maker. Angeletos et al. (2003) demonstrates that the informational role of state contingent policy can lead to multiple equilibria in global games. Therefore, the intervention programs analyzed in this paper are free from the signaling concern of state contingent policies and do not require the policy maker to have superior information about the fundamentals of the economy. Then the fundamental θ is realized, and each agent receives a noisy signal of the fundamental. After observing the signal, agents simultaneously make their decisions on whether to invest and if so, whether to participate in the intervention program. As soon as the decisions are made, active investors pay the cost c, and the policy maker transfers the subsidy s to all investors participating in the intervention program. Then the aggregate investment I and the investment returns are realized. Finally, the policy maker collects tax t from the investors participating in the intervention program if the investments are successful. The timeline is summarized in Figure 6 below.

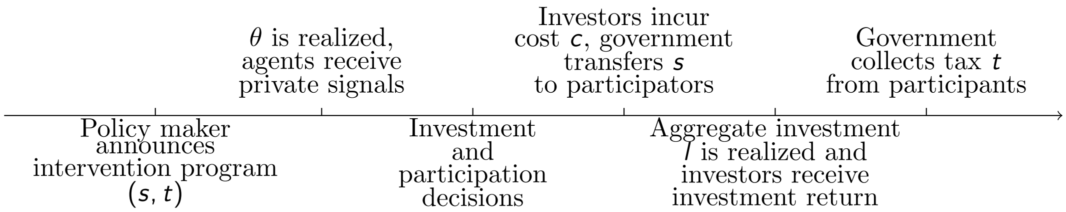


Figure 6: Timeline of the Investment Game

Although the intervention program is specified as a subsidy-tax program, it can be interpreted as other forms of intervention with transfers between the policy maker and the investors, contingent on the coordination result. For example, a government-guarantee type program that promises to cover the loss of failed investment up to $s^g \leq c$ is equivalent to a subsidy-tax program with $s = t = s^g$. To see this, under both programs, the net transfer from the government to any participating investor is 0 in the case of successful investments and s^g in the case of failed investments. Similarly, an asset purchase program in which the policy maker buys $\frac{t}{b}$ fraction of the project with price s is equivalent to a subsidy-tax program (s, t).

2.2.4. Equilibrium with Intervention

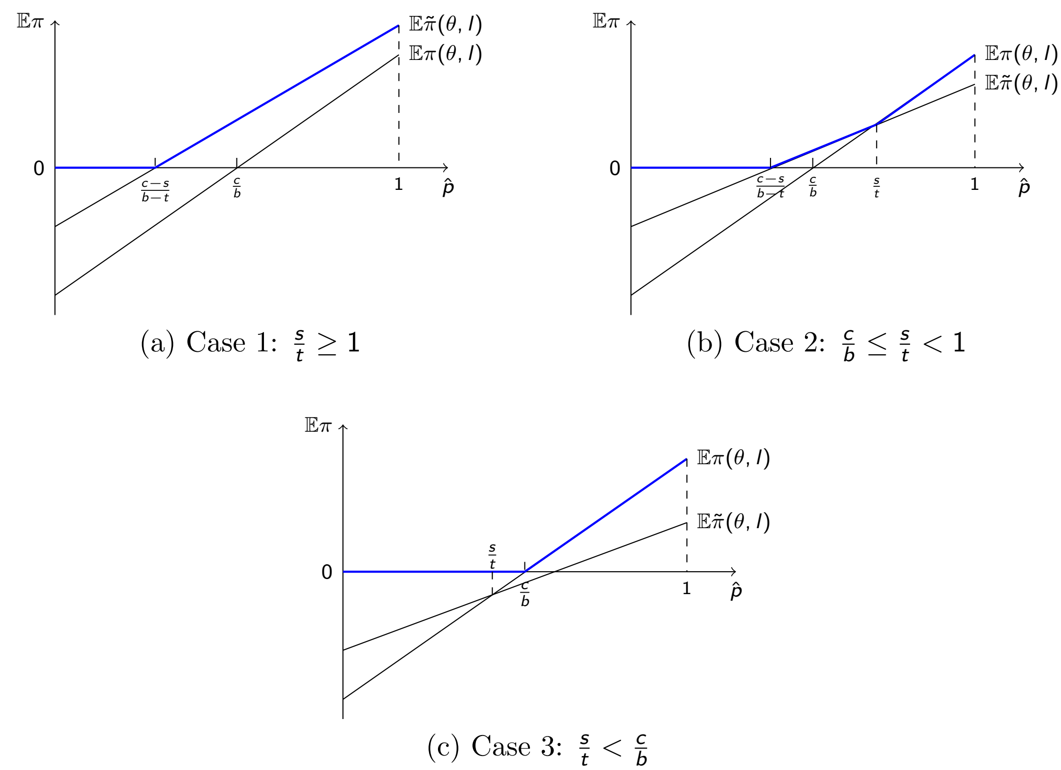
We now analyze the equilibrium with intervention and demonstrate how the intervention program works to reduce coordination failure. With the intervention program, an agent has three choices: $\{a = 1, Reject\}$, $\{a = 1, Accept\}$, and $\{a = 0\}$. Note that although agents make two decisions, whether to invest and conditional on investing, whether to accept the offer, only their investment decisions affect the coordination results. Therefore, an agent only cares about the investment decisions of the others but not their participation in the intervention program. As a result, to analyze the best response and equilibrium strategies, it is sufficient to condition on other agents' investment strategies. Let $\hat{p}_i = \Pr[I \geq 1 - \theta | x_i]$

denote the interim belief of success of agent i given her private signal x_i and other agents' investment strategies $a_{-i}(x)$. The expected payoffs from $\{a = 1, Reject\}$ and $\{a = 1, Accept\}$ are

$$\mathbb{E}[\pi(\theta, l)|x_i] = \hat{p}_i b - c, \tag{2.1}$$

$$\mathbb{E}[\tilde{\pi}(\theta, l)|x_i] = \hat{p}_i(b - t) - (c - s) \tag{2.2}$$

respectively. And the expected payoff from $\{a = 0\}$ is zero. Figure 7 depicts the expected payoff as a function of the interim belief $\hat{p}$. It can be divided into three cases according to the subsidy-tax ratio $\frac{s}{t}$. In the first case when $\frac{s}{t} \geq 1$, accepting the offer dominates rejecting



(a) Case 1: $\frac{s}{t} \geq 1$

(b) Case 2: $\frac{c}{b} \leq \frac{s}{t} < 1$

(c) Case 3: $\frac{s}{t} < \frac{c}{b}$

Figure 7: Expected Payoffs and Interim Beliefs

the offer. This is because investors always receive a higher subsidy s than their tax payment required by the intervention program. We call this type of programs the full-participation programs. Without intervention, the belief threshold for investment is the cost-benefit ratio

$\frac{c}{b}$. With a full-participation program, the threshold is lowered to $\frac{c-s}{b-t}$. In the third case with $\frac{s}{t} < \frac{c}{b}$, rejecting dominates accepting the offer. We call this type of programs the zero-participation programs. Thus, the threshold belief under the zero-participation program is the same as the original cost-benefit ratio $\frac{c}{b}$. The second case is the most interesting. When $\frac{c}{b} \leq \frac{s}{t} < 1$ (figure 7.b), an agent would only accept the offer and invest when she has an intermediate belief $\hat{p} \in [\frac{c-s}{b-t}, \frac{s}{t}]$. We call this type of programs the partial-participation programs. Notice in both case 1 and case 2, the provision of the intervention program lowers the threshold belief to $\frac{c-s}{b-t}$. The difference is that, in case 2, the most optimistic agents do not participate in the intervention program, which is cost saving especially when the information friction is small. We will analyze the cost of the programs in detail in section 3.

Next we sketch the analyses of equilibrium with intervention. It will become clear later that iterated deletion of dominated strategies allows us to focus on cutoff investment strategies. We say an agent follows a cutoff investment strategy with threshold k, if her investment strategy is

$$a_i(x; k) = \begin{cases} 1, & \text{if } x \geq k, \\ 0, & \text{if } x < k. \end{cases} \tag{2.3}$$

Let $p(x; k)$ denote the interim belief of success when an agent receives private signal x and all other agents follow a cutoff investment strategy k,

$$p(x; k) = Pr(\theta > \theta^*(k)|x) = F\left(\frac{x - \theta^*(k)}{\sigma}\right), \tag{2.4}$$

where $\theta^*(k)$ is the fundamental threshold for successful investment and satisfies $F\left(\frac{k-\theta^*(k)}{\sigma}\right) = \theta^*(k)$. An agent's interim belief of success $p(x; k)$ increases in x and decreases in k, because a high private signal x indicates a high realization of fundamentals θ, and a low investment threshold k implies a high aggregate investment I. Both imply a high probability of success.

In all three cases depicted in figure 2.3, the optimal investment strategy is that an agent invests if and only if her belief $p(x, k)$ exceeds a threshold. Since $p(x, k)$ is monotonic in both x and k, an agent's best response to other agents' cutoff strategy k is also a cutoff investment strategy based on her own signal. The two dominance regions form two extreme cutoff investment strategies. Starting there, by iterated deletion of dominated strategies, we are able to prove the uniqueness of the equilibrium with intervention. The details of the analyses can be found in the proof of proposition 2.2 below. The following proposition characterizes the equilibrium with a subsidy-tax intervention program (s, t).[7]

Proposition 2.2. *When the policy maker offers a subsidy-tax intervention program $(s, t) \gg 0$, the game has a unique equilibrium. There are three different cases,*

1. *When $\frac{s}{t} \geq 1$, the equilibrium is for any agent i,*

$$a_i = 1, \textit{Accept}, \quad \textit{if} \quad x_i \geq \xi^*(s, t),$$
$$a_i = 0, \quad \textit{if} \quad x_i < \xi^*(s, t).$$

where

$$\xi^*(s, t) = \frac{c - s}{b - t} + \sigma F^{-1}\left(\frac{c - s}{b - t}\right),$$

2. *When $\frac{c}{b} \leq \frac{s}{t} < 1$, the equilibrium is for any agent i,*

$$a_i = 1, \textit{Reject}, \quad \textit{if} \quad x_i \geq \eta^*(s, t),$$
$$a_i = 1, \textit{Accept}, \quad \textit{if} \quad \xi^*(s, t) \leq x_i < \eta^*(s, t),$$
$$a_i = 0, \quad \textit{if} \quad x_i < \xi^*(s, t),$$

[7] Frankel et al. (2003) prove existence, uniqueness and monotonicity in multi-action global games. However, our setup does not satisfy the continuity assumption. Therefore, we provide our own proof in the Appendix.

where

$$\xi^*(s,t) = \frac{c-s}{b-t} + \sigma F^{-1}\left(\frac{c-s}{b-t}\right),$$
$$\eta^*(s,t) = \frac{c-s}{b-t} + \sigma F^{-1}\left(\frac{s}{t}\right).$$

3. *When* $\frac{s}{t} < \frac{c}{b}$, *the equilibrium is for any agent* i,

$$a_i = 1, \textit{Reject}, \quad \textit{if} \quad x_i \geq \xi^*(s,t),$$
$$a_i = 0, \quad \textit{if} \quad x_i < \xi^*(s,t),$$

where

$$\xi^*(s,t) = \frac{c}{b} + \sigma F^{-1}\left(\frac{c}{b}\right).$$

The ratio of the upfront subsidy s and the ex-post tax t can be interpreted as the generosity of the program. If the offer is generous (case 1), all investors find it profitable to accept the offer and the equilibrium investment cutoff depends on the modified cost $c' = c - s$ and benefit $b' = b - t$. If the offer is austere (case 3), all investors will not be interested in the offer. Therefore the equilibrium investment cutoff is the same as the original cutoff without the intervention program. The most interesting case is case 2, in which the generosity of the offer is medium. Investors with high private signals have strong beliefs in the success of the project, so they will reject the subsidy offer since they believe in a high probability of paying a net tax in the future. However, even without subsidies, these optimistic agents would invest anyway. Agents with low private signals have strong beliefs in the failure of the project, so even with the subsidy s, they still suffer a loss of $c - s$ from investing. Therefore, these agents would not invest regardless of the intervention program. In contrast, investors receiving signals around the threshold do not have strong beliefs about the coordination results. Without the intervention program, some of these agents would not invest. The intervention program provides insurance against losses in case of failed investment and gives these agents extra incentive to invest. With the extra incentive, these agents' decisions are effectively altered and the aggregate action I therefore increases. The increase in I, in turn, strengthens all agents' incentive to invest. Agents with even lower signals would participate in the program and change their decisions to invest. Through iterations of higher-order beliefs, the action cutoff is significantly lowered. Moreover, agents with signals around the old cutoff are significantly more optimistic, and therefore the intervention program is no longer appealing to them. In equilibrium, the mass of investors accepting the offer is rather small. We call these investors the "pivotal" investors, since the equilibrium investment cutoff is determined by their modified cost and benefit.

In case 1 and 2, the fundamental cutoff above which the investment projects are successful is

$$\theta^*(\xi^*(s,t)) = \frac{c-s}{b-t}. \tag{2.5}$$

Note that the new fundamental cutoff is lower than that without government intervention. Therefore, the provision of the intervention program successfully reduces the inefficient coordination failure region. If the government picks $s = c$ and $t \in [s, b)$, the fundamental cutoff can be reduced to 0, eliminating the whole region of inefficient coordination failure as demonstrated in Figure 8.

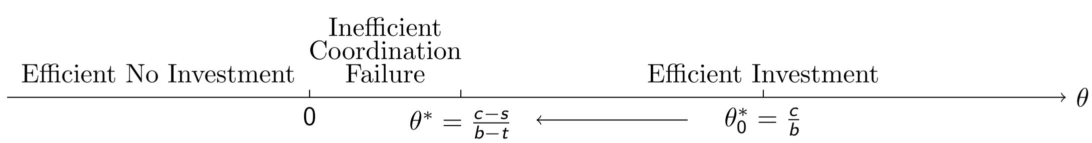


Figure 8: Coordination Results after Intervention

2.3. Cost of Implementation

In this section, we compare the implementation cost of partial-participation and full-participation intervention programs in two cases, one with negligible information frictions and one with non-negligible information frictions. We then discuss the intuitions why partial-participation is more cost-efficient than full-participation programs.

2.3.1. Cost of the Intervention Programs

We compare the expected cost of the partial-participation and full-participation programs conditional on the same target fundamental threshold θ^* of successful investment. To allow for the possibility that the policy maker values tax and subsidy differently, the value of tax for the policy maker is normalized to 1 and the cost of providing subsidy is assumed to be τ. The ex-post cost of providing the intervention program to an individual investor is

$$\hat{c}(\theta, s, t) = \begin{cases} \tau s - t, & \text{if } l \geq 1 - \theta, \\ \tau s, & \text{if } l < 1 - \theta. \end{cases} \tag{2.6}$$

When $\hat{c}(\theta, s, t)$ is negative, the policy maker profits from providing this intervention program.

For the rest of the analyses, we focus on $\tau \geq 1$ for two reasons. First, we believe it is a realistic characterization. If subsidy is provided before tax collection, $\tau > 1$ reflects the funding cost of the policy maker due to the opportunity cost of other welfare-improving programs. Alternatively, if the program is government guarantee, $\tau > 1$ reflects the cost of commitment, such as setting aside funds specifically for the program. Moreover, any administrative cost incurred by providing subsidy or collecting tax can raise τ. Secondly, if $\tau < 1$, given negligible information frictions, the policy maker can easily restore the first best and profit at the same time by offering $t = s = c$.[8] The coordination problem then becomes trivial. Therefore, for the rest of the paper, we assume $\tau \geq 1$.

Let $C(\theta, s, t; \sigma)$ denote the ex-post total cost of providing a subsidy-tax intervention program (s, t) given the realized fundamental θ and the information friction σ. For full-participation programs, i.e., $\frac{s}{t} \geq 1$, all investors participate in the intervention program. The ex-post

[8]To be precise, the policy maker should set $t = s = c - \varepsilon$ with a very small ε to avoid over-investment when $\theta < 0$ and keep the left dominance region.

cost of implementation is

$$C(\theta, s, t; \sigma) = \begin{cases} (\tau s - t)\left[1 - F\left(\frac{\xi^*(s,t)-\theta}{\sigma}\right)\right], & \text{if } \theta \geq \frac{c-s}{b-t}, \\ \tau s\left[1 - F\left(\frac{\xi^*(s,t)-\theta}{\sigma}\right)\right], & \text{if } \theta < \frac{c-s}{b-t}. \end{cases} \tag{2.7}$$

For partial-participation programs $\frac{c}{b} \leq \frac{s}{t} < 1$, only pivotal investors participate in the intervention program. In this case,

$$C(\theta, s, t; \sigma) = \begin{cases} (\tau s - t)\left[F\left(\frac{\eta^*(s,t)-\theta}{\sigma}\right) - F\left(\frac{\xi^*(s,t)-\theta}{\sigma}\right)\right], & \theta \geq \frac{c-s}{b-t}, \\ \tau s\left[F\left(\frac{\eta^*(s,t)-\theta}{\sigma}\right) - F\left(\frac{\xi^*(s,t)-\theta}{\sigma}\right)\right], & \theta < \frac{c-s}{b-t}. \end{cases} \tag{2.8}$$

If $\frac{s}{t} < \frac{c}{b}$, no agents will find it profitable to opt in to the intervention program, therefore $C(\theta, s, t; \sigma) = 0$.

Proposition 2.3 below compares the ex-post and ex-ante expected cost of partial-participation programs and full-participation programs, which restore first best in the limit of vanishing information frictions.

Proposition 2.3. *With strictly costly subsidy $\tau > 1$, when the information friction σ goes to 0, there exists a continuum of full-participation programs (s, t) and a continuum of partial-participation programs (s', t') achieving the first-best outcome, where $s = s' = c$ and $t \leq c < t' \leq b$.*

For any such (s, t) and (s', t'), given θ, the full-participation program (s, t) is ex-post more costly than the partial-participation program (s', t'). Specifically,

$$\begin{aligned} \lim_{\sigma \to 0} C(\theta, s, t; \sigma) = \tau s - t > \lim_{\sigma \to 0} C(\theta, s', t'; \sigma) &= 0, && \textit{if } \theta > 0; \\ \lim_{\sigma \to 0} C(\theta, s, t; \sigma) = \tau s - t > \lim_{\sigma \to 0} C(\theta, s', t'; \sigma) &= \frac{s'}{t'}(\tau s' - t'), && \textit{if } \theta = 0; \\ \lim_{\sigma \to 0} C(\theta, s, t; \sigma) = \lim_{\sigma \to 0} C(\theta, s', t'; \sigma) &= 0, && \textit{if } \theta < 0. \end{aligned}$$

Moreover, the full-participation program (s, t) is ex-ante strictly more costly than the partial-

participation program (s', t'). *Specifically,* $\lim_{\sigma \to 0} \mathbb{E}_\theta C(\theta, s, t; \sigma) > \lim_{\sigma \to 0} \mathbb{E}_\theta C(\theta, s', t'; \sigma) = 0$.

The proof is in the Appendices. When the information friction is small, although both full-participation programs and partial-participation programs can effectively reduce coordination failures and restore the first-best outcome, the partial-participation programs are ex-post weakly less costly than the full-participation programs in all states. Intuitively, compared with full-participation programs, partial-participation programs have less participants since the optimistic investors are deterred from participating. This subsequently reduces the cost of implementation. If the policy maker evaluates the ex-ante expected cost of the programs, in the limit of negligible information frictions, the partial-participation programs incur zero cost and strictly dominate the full-participation programs.

Now we extend the analysis to the case of non-negligible information frictions $\sigma > 0$. To facilitate the comparison of the cost of different programs given the same fundamental θ^*, we introduce an alternative parameterization of the intervention programs. Specifically, an intervention program (s, t) can be equivalently parameterized by (θ^*, λ) as follows,

$$s = \frac{c - \theta^* b}{1 - \theta^*} + \theta^* \lambda,$$
$$t = \frac{c - \theta^* b}{1 - \theta^*} + \lambda.$$

$\theta^* = \frac{c-s}{b-t}$ is the target fundamental threshold, and $\lambda = \frac{t-s}{1-\theta^*}$ is proportional to the net tax charged by the program when the project succeeds. λ can also be interpreted as the scale of the program because given the same target θ^*, both tax and subsidy are strictly increasing in λ. Intuitively, when λ increases, the intervention program charges a higher net tax and becomes less attractive. To achieve the same target, the government needs to increase the direct subsidy s (and the tax t at the same time) to provide more downside protection to the investors. When $\lambda \in [-\frac{c-\theta^* b}{1-\theta^*}, 0]$, the subsidy-to-tax ratio $\frac{s}{t} \geq 1$, and the program is a full-participation program. When $\lambda \in (0, \frac{b-c}{1-\theta^*})$, $\frac{c}{b} < \frac{s}{t} < 1$, and the program is a partial-participation program. Note that to achieve the same target θ^*, the partial-participation programs are larger in scale λ than the full-participation programs. The reason is that partial-participation programs are less generous, i.e. have lower subsidy-to-tax ratio than full-participation programs, the magnitude of partial-participation programs need to be larger to provide more downside protection as compensation.

Suppose the policy maker is considering switching from a full-participation program (θ^*, λ) to a partial-participation program (θ^*, λ'). The change in the expected cost of implementation comes from both the extensive and the intensive margin. On the extensive margin, the most optimistic investors will no longer enter the program. Hence, this effect always reduces the expected cost of intervention. However, on the intensive margin, the cost of providing the program to an individual investor could increase or decrease. Formally, the difference in unit cost is

$$\hat{c}(\theta, s', t') - \hat{c}(\theta, s, t) = \begin{cases} (\tau\theta^* - 1)(\lambda' - \lambda), & \text{if } \theta \geq \theta^*, \\ \tau\theta^*(\lambda' - \lambda), & \text{if } \theta < \theta^*. \end{cases}$$

With vanishing information frictions, the effect on the intensive margin is negligible because the mass of participants in partial-participation programs goes to zero except for the knife-edge case of $\theta = \theta^*$. Therefore, switching to any partial-participation program will always reduce the cost of implementation. This is no longer true with non-negligible information frictions. In proposition 2.4, we provide two sufficient conditions such that switching to a partial-participation program reduces the expected cost of implementation.

Proposition 2.4. *For any $\sigma > 0$, if $1 \leq \tau < G(\theta^*, 1)$ or $\theta^*(1 + \sigma) < 1$, there exists a partial-participation program (θ^*, λ) which achieves θ^* at lower expected cost than any full-participation program targeting θ^*, where $G(\alpha, \beta)$ is defined for any $0 \leq \alpha \leq \beta \leq 1$ as*

$$G(\alpha, \beta) = \frac{\int_{F^{-1}(\alpha)}^{F^{-1}(\beta)} F(x)\mathrm{d}x}{\alpha(F^{-1}(\beta) - F^{-1}(\alpha))}.$$

The proof involves technical details and is included in the Appendix. Here we provide some intuitions. Since partial-participation programs provide more subsidy and charges more tax to each participants, the effect on the intensive margin depends on the ratio of expected mass of taxpayers to the expected mass of subsidy receivers. This ratio is equal to $G(\theta^*, 1)$ in a partial-participation program (θ^*, λ) when λ approaches 0. If $\tau < G(\theta^*, 1)$, the ratio is large enough such that the increase in expected tax revenue is greater than the increase in expected subsidy provision. Hence, the effect on the intensive margin also works in favor of the partial-participation programs, and switching to a partial-participation program with small λ reduces the expected cost. Notice for any given $\theta^* < 1$, $G(\theta^*, 1) > 1$. Therefore, the special case of $\tau = 1$ always satisfies the first condition. The second condition governs the relative importance of the two margins. If θ^* and σ are jointly small, the participation threshold η^* for partial-participation programs is also small, therefore the mass of participants is significantly reduced. In particular, if the second condition holds, the effect on the extensive margin dominates that on the intensive margin, making the proposed partial-participation program less costly than any full-participation programs. In summary, Proposition 2.4 gives three circumstances in which the most cost-efficient subsidy-tax program is a partial-participation program: ambitious target (small θ^*), small information frictions (small σ), or small cost of subsidy τ. Note that as a special case, if the policy maker targets at the first-best $\theta^* = 0$, there always exists a partial-participation program that dominates all full-participation programs.

We use a numerical example to demonstrates how switching to a partial-participation program from a full-participation program can reduce the expected cost of the intervention. Suppose the prior on θ is uniformly distributed on $[\underline{\theta}, \bar{\theta}] = [-0.2, 1.2]$. The private noise ε follows a uniform distribution over $[-\frac{1}{2}, \frac{1}{2}]$ and $\sigma = 0.2$. c and b are set to 1 and 1.25, so the benchmark success threshold is $\frac{c}{b} = 0.8$. The policy maker has a cost parameter $\tau = 1.05$ and targets a success threshold $\theta^* = 0.2$. The least costly full-participation program to achieve the equilibrium threshold is $s = t = 0.9375$. The ex-post cost as a function of the realized fundamental is represented by the solid blue line in Figure 9. The cost is

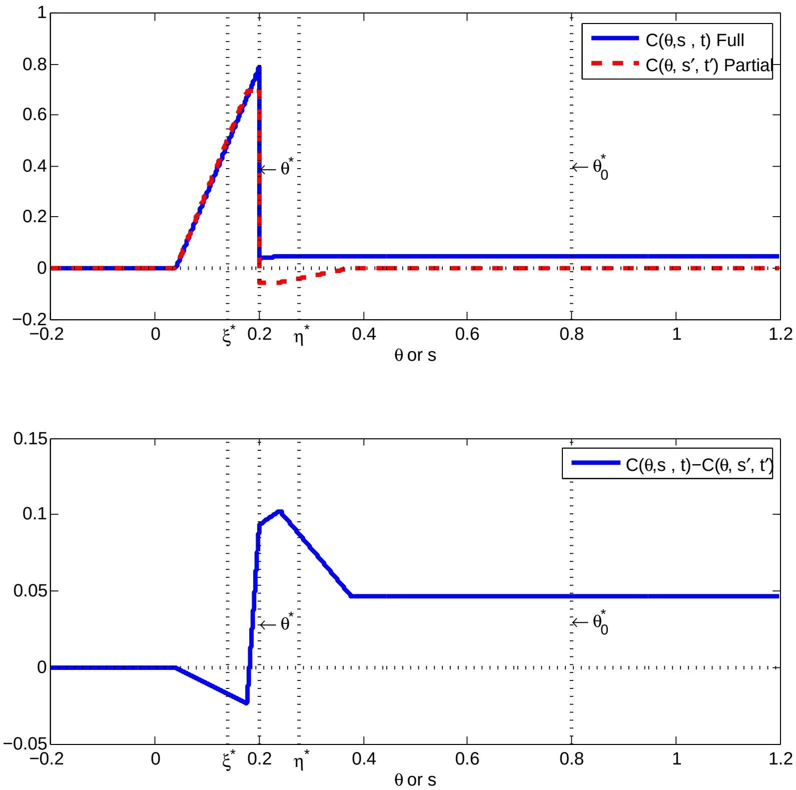


Figure 9: Cost Functions

positive for all $\theta > \theta^*$ because all investing agents sign up for the program and there's a positive cost $\tau s - t$ of providing this program to each agent. When θ falls below θ^*, the cost surges because the investment projects fail and the policy maker can't recover t. Now the policy maker switches to a partial-participation program. There's a continuum of partial-participation programs that targets the same threshold θ^*. We take $(s', t') = (0.97, 1.1)$ for an example. The red dashed line in the top panel of Figure 9 represents the ex-post cost function of program (s', t'). It has a similar shape as the cost function of the full-participation program. However, it converges to 0 when θ is large enough so that all agents receive signals higher than η^* and no agents participate in the intervention program. The difference between the two cost functions is plotted in the bottom panel. Compared to the full-participation program, the partial-participation program incurs lower cost when $\theta > \theta^*$ because of the higher tax charge and the lower participation rate. When $\theta < \theta^*$, since the partial-participation program provides higher subsidy, it incurs higher cost than the full-participation program. On average, the partial-participation program incurs lower expected cost.

2.3.2. Discussions

From previous analyses, we show that partial-participation intervention programs can improve the coordination results to the first-best outcome in the investment game, yet has zero cost when the information friction vanishes. This result seems striking at first glance. The most important reason why the partial-participation intervention program works effectively at a minimal cost is that it targets precisely the marginal agents who are on the investment threshold and can be incentivized to invest relatively easily. These agents are also the "pivotal" investors whose investment decisions are crucial in the determination of the investment threshold. The figure below demonstrates how through higher-order beliefs, our proposal effectively reduces coordination failure.

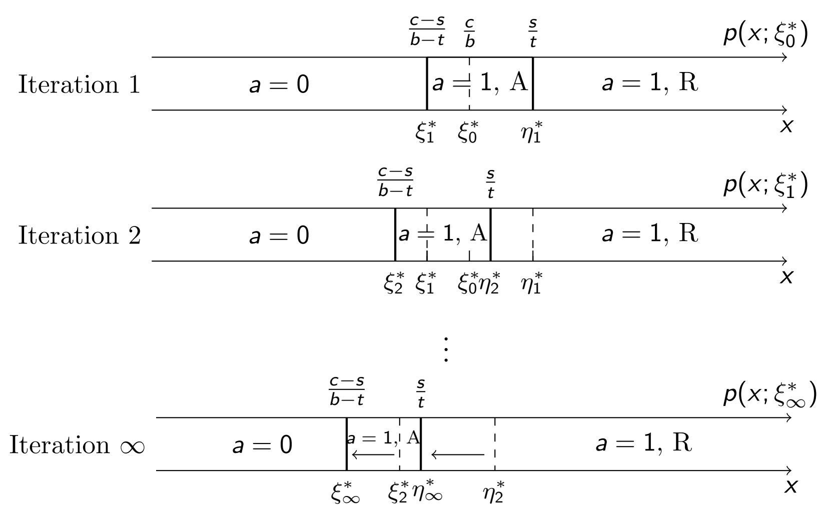


Figure 10: Role of Higher-Order Beliefs

In each iteration, the lower axis denotes the signal received by an agent, and the upper axis denotes the corresponding belief. Start from the cutoff strategy ξ_0^*, which is the original cutoff without intervention. The partial intervention program incentivizes agents to lower the investment threshold to ξ_1^*. Since all agents understand that more agents are willing to invest, given the same private signals, they all believe in a higher aggregate action I and a higher probability of successful investment $p(x; \xi_1^*)$. Therefore, they are willing to lower their investment threshold further to ξ_2^*. Similarly, with the additional mass of agents receiving signals between ξ_1^* and ξ_2^* investing, all agents are more optimistic about the success of the investment and therefore further lower their investment threshold to ξ_3. At the same time, as the agents become more optimistic about their investments, the intervention program becomes less attractive, which implies a decreasing sequence of participation thresholds η_n^*. With an infinite number of iterations, both the investment threshold and the participation threshold are significantly lowered. As the information friction decreases, investors become more certain about the coordination results, so the mass of "pivotal" investors shrinks to zero. However, as long as there exist a few pivotal investors, the intervention program will have a significant effect on the investment threshold due to higher-order beliefs.

Our partial-participation programs share similar spirit to the targeted intervention programs. Sakovics and Steiner (2012) analyze coordination games with heterogeneous agents and argue that the optimal subsidy schedule is to target a certain type of agent. In section 5, we examine an extension with heterogeneous agents and show that there exist partial-participation programs that incur zero cost to restore first-best outcome in the limit of negligible information frictions. Similar to the main model, in equilibrium, only a small mass of "pivotal" agents self-select to accept the policy maker's offer. The only difference is that different agent types have different thresholds, and the "pivotal" agents are the ones receiving signals around their own thresholds. The result conveys one message contrasting Sakovics and Steiner (2012) that policy makers should target interim rather than ex-ante important types. Also, one common problem with targeted intervention programs is that information acquisition to identify the targeted type(s) can be costly. The policy maker needs to correctly identify each agent's type to implement the targeted intervention programs. In contrast, our proposed intervention programs incentivize the "pivotal" agents to self-reveal their types, therefore the implementation only requires information on the payoff structure of different types. As a result, our proposed program is superior to the targeted intervention programs in terms of reducing the costs of collecting information.

2.4. Interventions in the Presence of Moral Hazard

In this section, we address the concern of moral hazard problem of government guarantees and demonstrate our proposal's robustness to moral hazard problems. For example, in the context of self-fulfilling credit freeze (Bebchuk and Goldstein, 2011), banks may abstain from lending in fear that the other banks will withdraw lending, which results in a coordination failure of credit crunch. If the government provides guarantees on bank losses, the banks may have the incentive to shirk in screening and monitoring the borrowers, since the losses caused by shirking is guaranteed by the government. In order to do incorporate moral hazard problems in the model, we modify the game into two stages. The first stage is the same as the benchmark model with an intervention program, except that the payoffs are not realized until the second stage. If the realized fundamental $\theta < 1 - l$, we say the aggregate state is *Bad*. In this case, the investment project fails and the game ends immediately. If the realized fundamental $\theta \geq 1 - l$, we say the aggregate state is *Good*. In this case, the game enters the second stage, in which investors make their effort choices. If an investor exerts effort, the investor pays a cost of effort c^e, and her project succeeds with probability 1.[9] On the other hand, if an investor shirks, her own project succeeds with probability $1 - \gamma$. As in the benchmark model, the project generates b in case of success and 0 in case of failure. And for the participants in the intervention program, they are required to pay tax t if their investments are successful. We make the following assumption on the parameters.

Assumption 2.1. *The investment opportunity has the following properties,*

a) shirking is inefficient, $c^e < \gamma b$*;*

[9]The results hold as long as the success probability when exerting effort is between $1 - \gamma$ and 1, which prevents the policy maker from inferring effort choice based on ex-post investment outcome. Otherwise, the moral hazard problem can potentially be solved by imposing ex-post punishment when the policy maker observes failed investment.

b) the investment projects are ex-ante efficient, $b > c + c^e$.

Given the assumptions above, the first-best scenario is that all agents invest and exert effort if the fundamental $\theta \geq 0$, and all agents do not invest otherwise.

The equilibrium with moral hazard problem can be solved backward. In the second stage, an investor would exert effort if and only if

$$b - t - c^e \geq (1 - \gamma)(b - t). \tag{2.9}$$

This condition can be interpreted as a constraint on the size of the tax t,

$$t \leq b - \frac{c^e}{\gamma}. \tag{2.10}$$

When the tax is above the threshold, participating investors has too little "skin in the game" to exert effort, resulting in inefficient outcomes. Intuitively, with a higher cost of effort c^e or lower losses caused by shirking γ, the incentive problem is more severe, imposing a tighter constraint on the size of tax t.

Next, we will analyze the equilibrium under different programs and examine whether a full-participation program like government guarantee or a partial-participation program can achieve first best when there is moral hazard problem in the private investment project. In the context of our model, we interpret the government guarantee program as a subsidy-tax program (s, t) with $s = t$, which is the full-participation programs with least cost. Since participating in the government guarantee program weakly dominates investing alone, every investor will take advantage of this program.

Government Guarantee. The moral hazard problem in the second stage imposes an upper limit on the scale of the government guarantee program if the policy maker wants to enforce effort.

The expected payoff from investing with the government guarantee program is

$$\mathbb{E}[\tilde{\pi}(\theta, l)|x_i] = \begin{cases} \hat{p}_i(b - t - c^e) - (c - s), & \text{if } t \leq b - \frac{c^e}{\gamma}, \\ \hat{p}_i(1 - \gamma)(b - t) - (c - s), & \text{if } t > b - \frac{c^e}{\gamma}. \end{cases} \tag{2.11}$$

From the analysis of the benchmark model, we know that in the unique Bayesian Nash equilibrium, the fundamental threshold above which the aggregate state is good is equal to the belief of the marginal investor. Given a program with $t \leq b - c^e/\gamma$ that prevents shirking, the fundamental threshold in equilibrium is

$$\theta^* = \frac{c - s}{b - t - c^e}. \tag{2.12}$$

Given a program with $t > b - c^e/\gamma$ that tolerates shirking, the fundamental threshold in equilibrium is

$$\theta^* = \frac{c - s}{(1 - \gamma)(b - t)}. \tag{2.13}$$

In both cases, reducing the fundamental threshold to the first best $\theta^* = 0$ requires the subsidy s to be as close to c as possible. However, by the nature of the intervention program, this also requires the contingent tax $t = s$ to be as close to c as possible. The scale of the intervention program is constrained by the incentive constraint as shown in (2.10), and whether the constraint is binding depends on the severity of the moral hazard problem.

Assumption 2.2. *The moral hazard problem is severe,* $\frac{c^e}{\gamma} > b - c$.

Given Assumption 2.2 above, the maximum program size t that prevents shirking in the second period is strictly less than c, the cost of the investment project. Therefore, the government guarantee program cannot achieve efficient fundamental threshold in the first stage and prevent shirking in the second period at the same time. The result is summarized in Proposition 2.5 below. When Assumption 2.2 does not hold, the government guarantee program with $t = c$ achieves the first-best outcome.

Proposition 2.5. *Given Assumption 2.1 and 2.2, no government guarantee program can restore the first-best outcome when $\sigma \to 0$.*

Partial-participation Programs. Now let us consider a subsidy-tax program with $\frac{s}{t} \in [\frac{c}{b}, 1)$. Given that the tax is higher than the subsidy, whether to participate in the program depends on investors' idiosyncratic beliefs of the probability that the aggregate state is good. As in the benchmark model, the program is the most attractive to agents with intermediate beliefs. What complicates the analyses is that agents will take into account their effort decisions in the second period when they compare the cost and benefit of participating in the program. When the moral hazard problem in the second period is not severe, i.e., Assumption 2.2 does not hold, the policy maker can choose $s = c$ and $t \in [c, b)$ to implement the first-best outcome, which is the same as government guarantee programs. In the following analyses, we focus on the case when the moral hazard problem is severe, i.e., Assumption 2.2 holds, and full-participation government guarantee programs cannot achieve the first best.

Given that Assumption 2.2 holds and $t > c$, the optimistic agents will reject the intervention offer and exert effort, the agents with medium beliefs will accept the intervention offer and shirk. Intuitively, the intervention offer reduces participant's investment risk as well as "skin in the game". The most optimistic agents who strongly believe in the success of investment do not want to share the profits with the policy maker. Therefore, they will reject the offer and fully endogenize the payoff from investment which incentivizes them to make the first-best effort choice. In contrast, the agents with medium beliefs are willing to invest only if the policy maker bears part of the investment risk. However, the intervention program also reduces their "skin in the game" because they need to share the investment profits with the policy maker but bare the full cost of effort. As a result, these agents will participate in the intervention program and shirk.

Formally, given the optimal effort choices in the second stage, the expected payoffs from $\{a = 1, Reject\}$ and $\{a = 1, Accept\}$ for an agent who receives signal x_i and forms belief $\tilde{p}_i$ are

$$\mathbb{E}[\pi(\theta, l)|x_i] = \tilde{p}_i(b - c^e) - c, \tag{2.14}$$

$$\mathbb{E}[\tilde{\pi}(\theta, l)|x_i] = \tilde{p}_i(1 - \gamma)(b - t) - (c - s). \tag{2.15}$$

The expected payoffs are linear and increasing in the belief $\tilde{p}_i$, and the slopes are different. The difference in the slopes of $\mathbb{E}\pi(\theta, l)$ and $\mathbb{E}\tilde{\pi}(\theta, l)$,

$$(b - c^e) - (1 - \gamma)(b - t) = \gamma b + t(1 - \gamma) - c^e > 0 \tag{2.16}$$

is strictly positive given Assumption *2.1a*. Investing alone is the optimal choice if and only if the belief $\tilde{p}_i$ exceeds the critical participation belief

$$p_2^*(s, t) \equiv \frac{s}{\gamma b + t(1 - \gamma) - c^e}. \tag{2.17}$$

Not investing is the optimal action choice if and only if the belief of the agent is worse than the critical investment belief

$$p_1^*(s, t) \equiv \frac{c - s}{(1 - \gamma)(b - t)}. \tag{2.18}$$

The optimal action choice if the belief of success probability is between $p_1^*(s, t)$ and $p_2^*(s, t)$ is to invest and accept the offer.

Similar to those in the benchmark model, the critical beliefs determine the equilibrium thresholds of investment and participation regarding the private signal x. Investment efficiency in the first stage requires the critical investment belief $p_1^*(s, t)$ to be as close to 0 as possible, which implies that the policy maker should choose subsidy $s = c$. On the other hand, if t can be selected properly such that the critical participation belief $p_2^*(s, t) < 1$, the investors who are very optimistic about the aggregate state would choose to invest and reject the offer. The exclusion of optimistic investors from the program improves efficiency in the second stage game and reduces the policy maker's cost from inefficient failures due to shirking. As the information friction goes to zero, the mass of "pivotal" investors who participate in the program goes to zero. The following proposition summarizes the result.

Proposition 2.6. *Given Assumption 2.1 and 2.2, the equilibrium outcome given a subsidy-tax program* (s, t) *with* $s = c$ *and* $\frac{c+c^e-\gamma b}{1-\gamma} < t < b$ *converges to the first best when* $\sigma \to 0$*. The ex-ante cost of providing such program also converges to 0 when* $\sigma \to 0$*.*

The above proposition demonstrates the advantage of the partial-participation programs compared with full-participation programs like government guarantee when the moral hazard problem is relatively severe. In the benchmark model, both types of programs can achieve the first-best outcome at zero cost with diminishing information friction if $\tau = 1$. They are different in terms of the program size: full-participation programs invite all investors, while partial-participation programs only target the "pivotal" investors. Absent other frictions, the size of a program does not alter the efficiency or the cost of implementing the program. However, the moral hazard problem causes welfare losses in proportion to the size of a program. When using a government guarantee program, the policy maker faces a trade-off between the first-stage investment efficiency and the second-stage effort efficiency. A program with high subsidy over tax ratio ($\frac{s}{t}$) encourages investment in the first stage but deters effort input in the second stage. This trade-off limits the role of the government guarantee program in improving social efficiency. On the contrary, despite the moral hazard problem, a partial-participation program still achieves the first-best outcome at zero cost. The advantage of partial-participation programs in dealing with moral hazard is that they only involve a small mass of investors. Although these participating investors shirk in the second stage, it will have a limited impact on the social welfare since the mass of these participating investors goes to zero as the information friction vanishes. In general, the partial-participation program proposed in this paper is superior to the full-participation programs such as government guarantee in the presence of any size-related inefficiency.

2.5. Extensions

2.5.1. Unobservable Ex-ante Heterogeneity

In this part, we study whether the existence of ex-ante heterogeneity in agents' payoff structure and information structure changes our results. The assumptions on the heterogeneity resemble those in Sakovics and Steiner (2012). Our analyses differ from their paper in two dimensions. First, they studied the optimal intervention when the policy maker can only provide a lump-sum subsidy, while we consider subsidy-tax programs. Second, they assume the types of agents are observable, while we allow for hidden types.

There are N groups of infinitesimal agents indexed by g, each group with mass m^g. There are three folds of heterogeneity. First, the agents differ in their profitability. They pay the same investment cost c yet earn different revenue b^g from successful investment. Assume there is no inefficient project, so $b^g > c$ for all g. Second, the agents impose different levels of externalities for the coordination results. Specifically, the aggregate action $I = \sum_{g=1}^N \int_0^{m^g} w^g a_i^g di$. Same as in the benchmark model, the condition that investment is successful is $I \geq 1 - \theta$. The weights are normalized such that $\sum_{g=1}^N w^g m^g = 1$. Lastly, each agent receives a private signal $x_i^g = \theta + \sigma \varepsilon_i^g$, where ε_i^g is independent across agents and follows a group-specific distribution with c.d.f. $F^g(\varepsilon)$, the support of which is $\left[-\frac{1}{2}, \frac{1}{2}\right]$. We assume an agent's group is not observable to the policy maker. However, the policy maker knows the composition of agents.

The equilibrium without intervention is summarized by the following proposition.

Proposition 2.7. *Without intervention, there is a unique equilibrium in which an agent in group g invests if and only if her private signal is greater or equal to ξ_0^g, which is given by*

$$\xi_0^g = \sum_{g=1}^N m^g w^g \frac{c}{b^g} + \sigma F_g^{-1}\left(\frac{c}{b^g}\right). \tag{2.19}$$

From the above proposition, we can calculate the fundamental threshold θ^* above which the investments are successful. The expression for the fundamental threshold is given by

$$\theta^* = \sum_{g=1}^{N} m^g w^g \frac{c}{b^g}, \tag{2.20}$$

which is a weighted average of the cost-benefit ratio of different types of agents. Let $b_{min} = \min\{b^g\}_{g=1}^{N}$. The following proposition shows our previous results still hold when there is unobservable heterogeneity among agents.

Proposition 2.8. *Given a subsidy-tax program with $s < c$ and $s < t < b_{min}$, there exists a unique equilibrium in which a type j agent follows the strategy below,*

$$\begin{aligned} a = 1, \textit{Reject}, &\quad \textit{if} \;\; x \geq \eta_g^*(s,t), \\ a = 1, \textit{Accept}, &\quad \textit{if} \;\; \xi_g^*(s,t) \leq x < \eta_g^*(s,t), \\ a = 0, &\quad \textit{if} \;\; x < \xi_g^*(s,t), \end{aligned}$$

where

$$\begin{aligned} \xi_g^*(s,t) &= \sum_{g=1}^{N} m^g w^g \frac{c-s}{b^g - t} + \sigma F_g^{-1}\left(\frac{c-s}{b_g - t}\right), \\ \eta_g^*(s,t) &= \sum_{g=1}^{N} m^g w^g \frac{c-s}{b^g - t} + \sigma F_g^{-1}\left(\frac{s}{t}\right). \end{aligned}$$

When $s = c$ and $c < t < b_{min}$, the equilibrium outcome converges to the first-best outcome and the expected cost of the program converges to 0 when $\sigma \to 0$.

If agents also differ in the cost of investment, i.e., c^g can be different across groups, we need to relax the assumption that type are unobservable to the government. Instead, we assume the government can observe c_i for each individual agent. If $c_i = c^{g1} = c^{g2}$, the government does not need to know whether agent i is from $g1$ or $g2$. Under this setup, it is equivalent to solve the problem with $\tilde{c^g} = 1$ and $\tilde{b^g} = \frac{b^g}{c^g}$, then scale up agent i's offer by c_i^g.

The intuition for how our proposed intervention program works in the case with ex-ante heterogeneous agents is essentially the same as in the benchmark model. The intervention program incentivizes "pivotal" agents who originally choose not to invest to change their decisions. All agents knowing that there is an increase in the aggregate action I all believe in a higher probability of success. Amplified by higher-order beliefs, the intervention program can efficiently restore the first-best coordination results. Note that the notion of "pivotal" agents refers to the interim type of agents. Since different groups earn different profitabilities from successful investments, they require a different success probability to agree to invest. Our intervention program identifies and targets agents with beliefs right below the cutoffs of their own group. In Sakovics and Steiner (2012), they only look at direct subsidy programs and argue that an efficient program should target the ex-ante "pivotal" group, the group with low b^g and high w^g in our setup. Our results above demonstrate that by allowing an additional intervention tool, the contingent tax t, we are able to reduce coordination failure at a much lower cost. Moreover, the implementation of our proposed program does not require information on an agent's group, therefore our proposed program could save the potential cost of information acquisition.

2.5.2. General Payoff Structure

In this section, we follow the setups of the symmetric binary-action global games in Morris and Shin (2003) and allow for general monotonic payoff functions.

As in the benchmark model in section 2, an agent's payoff from not investing ($a_i = 0$) is normalized to zero. An agent's payoff from investing ($a_i = 1$) is modified to be a continuous function $\pi(x, I)$, which weakly increases in both the private signal x and the aggregate action $I = \int_0^1 a_i di$.[10] The fundamental θ follows a uniform distribution on $[\underline{\theta}, \bar{\theta}]$. The private signal

[10] We assume the payoff is a function of the private signal instead of the fundamental for simplicity of demonstration. Our results still hold under the alternative setup. See Morris and Shin (2003) for the discussion of the two setups.

received by agent i is $x_i = \theta + \sigma\varepsilon_i$, where ε_i are i.i.d. and has a density function $f(\varepsilon)$ and a distribution function $F(\varepsilon)$ with support $[-\frac{1}{2}, \frac{1}{2}]$.

For simplicity, we only consider the family of linear intervention programs. In general, we could allow transfer as a non-linear function of the agents' payoff. The intervention program (s, t) consists of two parts, a direct subsidy $s \geq 0$ and a proportional tax $t \in [0, 1]$. If an agent accepts the offer, she receives the direct upfront subsidy s and pays the proportional tax after the realization of the investment outcome. Her payoff from accepting the offer is[11]

$$\tilde{\pi}(x, l) = (1 - t)\pi(x, l) + s. \tag{2.21}$$

Agents who receive low private signals believe in low realization of the fundamental θ and low aggregate action l, so they are pessimistic about their payoffs from investments. Therefore, they expect to pay low tax and are more willing to accept the offer than optimistic agents. Recall the partial-participation programs in the benchmark model. These programs do not appeal to the optimistic agents who do not need extra incentive to invest, which efficiently saves resources and reduces the cost of the program. The proportional tax t captures this feature and helps to target agents receiving medium signals.

We adopt the standard assumptions on the payoff function in the literature.

Assumption 2.3. *The payoff function $\pi(x, l)$ satisfy the following properties:*

1. *(Monotonicity) The payoff function $\pi(x, l)$ is weakly increasing in both arguments.*

2. *(Strict Laplacian State Monotonicity) $\int_0^1 \pi(x, l)\mathrm{d}l$ is strictly increasing in x.*

[11] One might notice that when $\pi(x, l) < 0$, investors end up paying a negative "tax". In fact, let $\underline{\pi} = \pi(\underline{\theta} - \frac{1}{2}\sigma, 0)$ be the lower bound of the payoff. The intervention program can be implemented by providing a positive subsidy $s - t\underline{\pi}$ and imposing a proportional tax t on the positive tax base $\pi(x, l) - \underline{\pi}$.

3. *(Limit Dominance) There exists* $\theta_0, \theta_1 \in (\underline{\theta} + \frac{1}{2}\sigma, \bar{\theta} - \frac{1}{2}\sigma)$ *such that*

$$\pi(x, 1) < 0, \text{ for all } x < \theta_0, \tag{2.22}$$

$$\pi(x, 0) > 0, \text{ for all } x > \theta_1, \tag{2.23}$$

4. *(Continuity)* $\int_0^1 g(l)\pi(x, l)\mathrm{d}l$ *is continuous in* x *for any density function* g.

The first assumption states the strategic complementarities among the investment choices of different agents. The individual payoff of investing increases when more agents invest. Also, a higher fundamental increases everyone's incentive to invest, given the same aggregate investment. Note that the payoff function need not be strictly increasing or continuous. For example, the payoff function in our benchmark model in Section 2 is a step function. The role of the second assumption is to make sure the equilibrium is unique when it exists, with or without the intervention program. The third assumption ensures the existence of two dominance regions so that we can adopt the iterated deletion of dominated strategies from both sides. The last assumption regulates integration of the payoff function so the equilibrium always exists.

The equilibrium without intervention is characterized in the proposition below. The "natural outcome" serves as a benchmark to analyze the effect of intervention programs.

Proposition 2.9. *Without intervention* $(s = t = 0)$*, when the information friction* σ *is small enough, there is a unique equilibrium in which each agent invests if and only if her private signal* $x \geq \xi_0^*$ *given by*

$$\int_0^1 \pi(\xi_0^*, l)dl = 0.$$

Compare the coordination results characterized in the above proposition with the first-best outcome. In the first-best scenario, if all agents investing can generate positive surplus, the social optimal outcome is for all agents to invest. In other words, the first-best scenario is that all agents follow the same cutoff strategy θ_0, the upper bound for the left dominance region. By Assumption 2.3, unless $\pi(\theta_0, l) = 0$ for any $l \in [0, 1]$, the natural coordination outcome $\xi^* > \theta_0$. Therefore if the realized fundamental $\theta \in (\xi^*, \theta_0)$, there would be a coordination failure. And the goal of intervention is to reduce the coordination threshold from ξ^* to as close to θ_0 as possible.

Next we analyze the equilibrium with an intervention program (s, t). We focus on the partial-participation programs and demonstrate its zero cost of implementation in the limiting case. Proposition 2.10 summarizes the conditions for such partial-participation programs.

Definition 2.1. *A intervention program (s, t) is a partial-participation program with target ξ^* if and only if it satisfy the following three conditions,*

1. *(Intervention Target)* $\int_0^1 \pi(\xi^*, l) dl = -\frac{s}{1-t}$.

2. *(Optimism Exclusion)* $\pi(\xi^*, 1) > \frac{s}{t}$,

3. *(Left Dominance Region)* $\pi(\underline{\theta}, 1) < -\frac{s}{1-t}$,

Denote a coordination game with information friction σ and intervention program (s, t) by $G(\sigma; s, t)$, we can prove the following proposition.

Proposition 2.10. *Given a partial-participation program (s, t) with target ξ^*, the following two properties must be satisfied in any Bayesian Nash equilibrium of the coordination game $G(\sigma; s, t)$,*

1. *Agents invests if and only if their private signal $x > \xi^*$;*

2. *There exists a threshold $\eta^*(\sigma)$ such that investing agents strictly prefer not to participate in the intervention program if and only if their private signal $x > \eta^*(\sigma)$.*

When $\sigma \to 0$, $\eta^*(\sigma)$ converge to ξ^*.

The above proposition provides conditions under which there exist partial-participation programs to reduce the investment threshold to ξ^*. Same as in the benchmark regime-change model, in the limit, ex ante expected mass of participants goes to zero, which implies zero cost of implementation. The question remaining is whether there exist such programs to costlessly restore the first-best scenario, i.e. $\xi^* = \theta_0$. The proposition below answers this question.

Proposition 2.11. *If $\int_0^1 \pi(\theta_0, l)dl \geq \pi(\underline{\theta}, 1)$, for any $\xi^* \in (\theta_0, \xi_0^*)$, there exists a partial-participation program with target ξ^*.*

With the intervention program, all agents become more optimistic about their investment payoff. Therefore, the left dominance region, where agents prefer not to invest even if $l = 1$, shrinks. The condition specified in Proposition 2.11 guarantees that the left dominance region still exists with the intervention program. If the condition is violated, there might be multiple equilibria when targeting ξ^* close to θ_0. However, if we follow the equilibrium refinements proposed in Goldstein and Pauzner (2005), we can select the equilibrium described in Proposition 2.10 even without the left dominance region. Therefore, following the refinements, there always exists a partial-participation program that restores the first-best scenario. Moreover, the left dominance region may disappear because we limit our attention to programs with linear transfers. Linear transfer schedules generally gives a lot of subsidies in case of low fundamental. If the policy maker lowers subsidies in the case of very low fundamental realizations (when $\theta < \theta_0$) or adds convexity to the tax schedule properly, the left dominance region as well as the uniqueness of the equilibrium can be recovered. Either way, there always exists an intervention program that can restore the first-best scenario.

2.6. Selected Applications

The partial-participation programs can be applied to various contexts with coordination problems. In this section, we discuss three representative applications.

2.6.1. Debt Rollover

It has been widely recognized in the literature that panic-based debt run can leads to inefficient firm default. Specifically, consider a firm with many small debt-holders. The firm is more likely to survive if more debt-holders roll over their debts. Therefore, debt-holders' rollover decisions features strategic complementarities. When the fundamental of the firm is weak, debt-holders might stop rolling over their debts because they worry the others would also stop, which can leads to self-fulfilling debt run. Our analyses suggest tranching can be a cost-efficient way to reduce such coordination failure. Instead of one standard debt contract, the firm can issue two types of debts with different seniorities. The senior debt promises lower return yet provides higher payment than the junior debt when the firm defaults. Without tranching, debt-holders who have medium beliefs and coordination concerns would not roll over their standard debts. With the safer option of senior debt, they are willing to lend to the firm which eases the liquidity concern of the firm and boosts all debt-holders' beliefs in the firm's survival. This effect can be amplified by higher order beliefs. In equilibrium, only the pivotal debt-holders choose the senior option. However, the availability of the safer senior debt improves all debt-holders' belief in that the firm can raise enough funds to survive.

Bank run is another similar application. To implement the partial-participation programs, the government can offer optional but costly deposit insurance. In fact, Alipay, the largest online payment platform in China, offers all users an option to purchase insurance against losses on their associated financial accounts. The insurance is costly if their accounts are safe yet provides protection when the platform fails. Therefore, it would work in a similar way as the senior debt option to reduce coordination failure. It is less costly than the mandatory deposit insurance because it screens for the "pivotal depositors" and leaves out the optimistic depositors who would not run even without insurance protection.

2.6.2. Market Freeze

During the 2008 financial crisis, many financial institutions and investors significantly reduced their leverage. This process pushed down the market prices of Commercial Mortgage-Backed Securities (CMBS) and Residual Mortgage-Backed Securities (RMBS). The markets for RMBS and CMBS froze, and prices were well below their fundamentals. Among others, coordination failure can prevent the market from thawing. If only a few investors participate in the market for Mortgage-Backed Securities (MBS), the liquidity in the market is not enough to drive the prices back to the fundamental and the participating investors suffer losses on their investments. However, if a significant amount of liquidity is injected in the market, the prices are more likely to be driven back to reflect the fundamental and investors who bought at a discount can profit from the investment.

In March of 2009, the US Treasury announced the Legacy Securities Public-Private Investment Program (PPIP). Under the program, private equity was matched by government equity and debt to form Public-Private Investment Funds (PPIFs) and purchase highly rated legacy MBS from financial institutions. Private investors in the PPIFs effectively receive investment subsidies from the government and are levered up for their investment. They earn higher investment return in good times and are protected by limited liabilities in bad times. Hence, PPIP is uniformly beneficial to all qualifying private investors and can be interpreted as full-participation programs in our model. PPIP is not efficient in resource allocation in the sense that part of the government funding is provided to the optimistic investors who would have invested in MBS market without PPIP. According to our analyses, the government can reduce the cost of rejuvenating the market by offering a partial-participation program instead. Mapping into the context of PPIP, the government could offer to inject equity into PPIFs in proportion to debt holdings by private investors. This option of debt investment reduces the losses from freezing the MBS market. As a return, the government shares the profit of investment if the market for MBS is successfully rejuvenated. This offer incentivizes the pivotal investors to invest in the MBS market. Since all investors are aware of the offer, they know that the aggregate investment will increase and hence also have more incentive to invest.

2.6.3. Shopping Mall Investment

We analyze a real investment problem in this section. Pashigian and Gould (1998) documents the strategic complementarities among department stores in the same shopping mall. Specifically, department stores with reputations can bring in mall traffic and increase the sales of less-known stores. As discussed in Sakovics and Steiner (2012), the difference in reputation maps into w_g, the importance in coordination outcome of different groups in section 5.1.

Consider a newly opened shopping mall inviting different brands to open new stores. Since all stores benefit from customers' visit to the shopping mall, all stores' investment return increases in the occupancy ratio of the shopping mall. Therefore, coordination failure could lead to low occupancy ratio and failure of the shopping mall. In order to boost investment, according to our analyses, the shopping mall manager could offer an equity injection option. Specifically, if a brand accepts the equity injection offer and opens a new store in the shopping mall, the shopping mall manager pays part of the investment cost and receives proportional profit made by the store as a return. This offer is not appealing to the optimistic brands because they do not want to share the profits with the shopping mall. For brands that are around investment threshold, the equity injection offer reduces their investment risk and increases their expected payoff from the investment. Amplified by higher-order beliefs, all brands significantly lower their investment threshold. Moreover, in equilibrium, only the "pivotal" brands accept the offer. Therefore the resources to finance the intervention program are efficiently allocated.

It is reasonable to assume different brands have different profit functions. We have shown in section 5.2 that the interim critical agents who are around their own investment thresholds self-select to accept our offer. The result that the equity injection offer effectively reduces coordination failure and incurs low financing cost for the shopping mall owner still holds.

2.7. Conclusions

In this paper, we analyze a canonical coordination game under global games framework and propose a novel intervention program for a policy maker to reduce coordination failures. The intervention program screens for the marginal agents who receive medium signals, which reduces the cost of implementation for the program. At the same time, correctly incentivizing the marginal agents have a significant impact on all agents due to strategic complementarities and the amplification through higher order beliefs. In the limit of zero noise in agents' private signals, our proposed program eliminates all coordination failures at zero cost since the expected mass of marginal investors goes to zero. Compared with conventional government guarantee type of programs, our proposed program not only incurs lower cost of implementation but also is shown to be more robust to moral hazard problems.

We demonstrate with three examples that our proposed program has a wide range of applications in improving coordination failures. As a concluding remark, we would like to point out some limitations of the proposed program. First, the program requires the policy maker to observe and condition the provision of the program on agents' action choices, which might not be feasible. For example, in the context of panic-based currency attack, it is hard to trace the identities of the currency holders and give them an optional offer. Second, the effectiveness of the proposed program relies on agents' rationality. If agents possess bounded rationality, the amplification effect through higher order beliefs will be limited.

Proof of Proposition 2.1. It can be proved by iterated deletion of dominated strategies. Let $p(x; k)$ denote the interim belief of success when an agent receives private signal x and all other agents follow a cutoff investment strategy k as defined in (2.4). First, we want to show that strategies survive n rounds of iterated deletion of dominated strategies if and only if

$$a(x) = 0, \text{ if } x < \underline{\xi}_n, \tag{2.24}$$

$$\text{and } a(x) = 1, \text{ if } x \geq \bar{\xi}_n. \tag{2.25}$$

where $\left\{(\underline{\xi}_n, \bar{\xi}_n)\right\}_{n=0}^{\infty}$ satisfies

$$-\infty = \underline{\xi}_0 < \underline{\xi}_1 \leq \cdots \leq \underline{\xi}_n \leq \cdots \leq \bar{\xi}_n \leq \cdots \leq \bar{\xi}_1 < \bar{\xi}_0 = +\infty. \tag{2.26}$$

This result can be proved by induction. Let $\underline{\xi}_0 = -\infty$ and $\bar{\xi}_0 = +\infty$, so the first round of deletion starts with the full set of strategies. Suppose round $n \in \mathbb{N}$ of deletion has been completed. In round $n+1$, the best scenario for an agent to invest is that all other agents follow a cutoff strategy with threshold $\underline{\xi}_n$. Therefore, for any x such that $p(x; \underline{\xi}_n) < \frac{c}{b}$, $a(x) = 1$ is strictly worse than $a(x) = 0$. Similarly, the best scenario for an agent to choose $a_i = 1$ is that all other agents follow a cutoff strategy with threshold $\underline{\xi}_n$. As a result, for x such that $p(x; \bar{\xi}_n) > \frac{c}{b}$, any strategy profile with $a(x) = 1$ is strictly better than $a(x) = 0$.

Given $p(x; k)$ is non-decreasing in x, the strategy profiles that survives deletion of dominated strategies can be summarized in the form of (2.24)(2.25), with $(\underline{\xi}_{n+1}, \bar{\xi}_{n+1})$ defined inductively as

$$\underline{\xi}_{n+1} = \inf\left\{x : p(x; \underline{\xi}_n) \geq \frac{c}{b}\right\} \tag{2.27}$$

and

$$\bar{\xi}_{n+1} = \sup \left\{ x : p(x; \bar{\xi}_n) \leq \frac{c}{b} \right\} \tag{2.28}$$

The monotonicity of $p(x;k)$ guarantees that $\underline{\xi}_{n+1} \leq \bar{\xi}_{n+1}$ given $\underline{\xi}_n \leq \bar{\xi}_n$. Note the dominance region assumption implies that $\underline{\xi}_1 > -\infty$ and $\bar{\xi}_1 < +\infty$ when σ is small enough. Therefore, $\left\{ (\underline{\xi}_n, \bar{\xi}_n) \right\}_{n=0}^{\infty}$ is a well-defined sequence of real couple which satisfies (2.26).

Now we've proved that $\{\underline{\xi}_n\}_{n=1}^{\infty}$ and $\{\bar{\xi}_n\}_{n=1}^{\infty}$ are both monotonic and bounded sequences. Thus, they converges to two finite numbers $\underline{\xi}$ and $\bar{\xi}$ respectively when $n \to \infty$. And the two limits satisfy

$$\underline{\xi} \leq \bar{\xi}. \tag{2.29}$$

The definition (2.27)(2.28) implies that $p(\underline{\xi}; \underline{\xi}) \geq \frac{c}{b}$ and $p(\bar{\xi}; \bar{\xi}) \leq \frac{c}{b}$. Note that

$$p(\xi; \xi) = F\left(\frac{\xi - \theta^*(\xi)}{\sigma} \right) = \theta^*(\xi), \tag{2.30}$$

is strictly increasing in ξ. Therefore $\underline{\xi} = \bar{\xi}$ must be the unique solution to $\theta^*(\xi) = \frac{c}{b}$, which is

$$\xi_0^* = \frac{c}{b} + \sigma F^{-1}\left(\frac{c}{b} \right). \tag{2.31}$$

Since there's only one strategy that survives the iterated deletion of dominated strategies, the equilibrium of the game is unique and the associated equilibrium strategy is the cutoff investment strategy with threshold ξ_0^*. □

Lemma 2.1. *Suppose the optimal strategy of an agent as a function of her interim belief* *of success* $\hat{p}_i$ *can be characterized as*

$$a_i = 1, \textit{Reject}, \quad \textit{if} \quad \hat{p}_i > p_2^*,$$
$$a_i = 1, \textit{Accept}, \quad \textit{if} \quad p_1^* < \hat{p}_i \leq p_2^*,$$
$$a_i = 0, \quad \textit{if} \quad \hat{p}_i \leq p_1^*,$$

where p_1^* *and* p_2^* *are two threshold beliefs that satisfy* $0 \leq p_1^* < p_2^* \leq 1$. *There is a unique Bayesian Nash equilibrium and the equilibrium strategy of any agent is*

$$a_i = 1, \textit{Reject}, \quad \textit{if} \quad x_i \geq \eta^*,$$
$$a_i = 1, \textit{Accept}, \quad \textit{if} \quad \xi^* \leq x_i < \eta^*,$$
$$a_i = 0, \quad \textit{if} \quad x_i < \xi^*,$$

where $\xi^* = p_1^* + \sigma F^{-1}(p_1^*)$ *and* $\eta^* = p_1^* + \sigma F^{-1}(p_2^*)$

Proof of Lemma 2.1. We want to find a sequence $\left\{(\underline{\xi}_n, \bar{\xi}_n)\right\}_{n=0}^{\infty}$ such that strategies survives n rounds of iterated deletion of dominated strategies only if

$$a(x) = 0, \ \text{if } x < \underline{\xi}_n, \tag{2.32}$$
$$\text{and } a(x) = 1, \ \text{if } x \geq \bar{\xi}_n. \tag{2.33}$$

The reason that we can only iterate on the investment cutoff without keeping track of the participation decisions is that an agent's investment decision is independent of other agents' participation decisions. The recursive expression for $\left\{(\underline{\xi}_n, \bar{\xi}_n)\right\}_{n=0}^{\infty}$ is

$$\underline{\xi}_{n+1} = \inf\{x : p(x; \underline{\xi}_n) \geq p_1^*\}, \tag{2.34}$$
$$\bar{\xi}_{n+1} = \sup\{x : p(x; \bar{\xi}_n) \leq p_1^*\}. \tag{2.35}$$

Applying the same techniques in the proof of Proposition 2.1, it becomes clear that the limit of the two cutoff sequences converges to

$$\xi^*(s,t) = p_1^* + \sigma F^{-1}\left(p_1^*\right), \tag{2.36}$$

which is the investment cutoff in the unique Bayesian Nash equilibrium of the global game. The associated participation cutoff η is the solution to

$$p(\eta;\xi^*(s,t)) = p_2^*. \tag{2.37}$$

Solving the above equation yields

$$\eta^*(s,t) = p_1^* + \sigma F^{-1}\left(p_2^*\right). \tag{2.38}$$

□

Proof of Proposition 2.2. In case 1, *invest-and-reject* is dominated by *invest-and-accept*. Therefore, we can rewrite the investment payoff by letting $b' = b - t$ and $c' = c - s$ and directly apply Proposition 2.1. Similarly, *invest-and-accept* is jointly dominated by *invest-and-accept* and *not-invest* in case 3. Since the intervention program is never going to be accepted, the equilibrium is the same as that described in Proposition 2.1. Case 2 is a direct implication of Lemma 2.1. □

Proof of Proposition 2.3. As specified in equation 5, with program (s,t), the fundamental cutoff is $\frac{c-s}{b-t}$. Therefore, the programs targeting at the first-best fundamental cutoff 0 should satisfy $s = c$. Hence, the subsidy to tax ratio of a program targeting at the first best is $\frac{s}{t} = \frac{c}{t}$. If the ratio is greater than 1, the program is a full-participation program. Otherwise, it is a partial-participation program.

As a result, if (s,t) satisfies the following two conditions, it is a full-participation program targeting the first best.

1. $0 \leq t \leq c$,

2. $s = c$.

If (s', t') satisfies the following two conditions, it is a partial-participation program targeting the first best.

1. $c < t' \leq b$,

2. $s' = c$.

Lastly, we calculate the limit of the cost functions as specified in equation 7 and 8. For any $\theta > 0$,

$$\lim_{\sigma \to 0} C(\theta, s, t) = \lim_{\sigma \to 0} (\tau s - t) \left[1 - F\left(\frac{0 - \theta}{\sigma} + F^{-1}(0) \right) \right] = (\tau s - t)\left[1 - F(-\infty)\right] = \tau s - t$$

$$\begin{aligned} \lim_{\sigma \to 0} C(\theta, s', t') &= \lim_{\sigma \to 0} (\tau s' - t') \left[F\left(\frac{0 - \theta}{\sigma} + F^{-1}\left(\frac{s'}{t'}\right) \right) - F\left(\frac{0 - \theta}{\sigma} + F^{-1}(0) \right) \right] \\ &= (\tau s' - t')\left[F(-\infty) - F(-\infty)\right] = 0 \end{aligned}$$

If $\theta = 0$,

$$\lim_{\sigma \to 0} C(\theta, s, t) = \lim_{\sigma \to 0} (\tau s - t) \left[1 - F\left(F^{-1}(0)\right) \right] = \tau s - t$$

$$\lim_{\sigma \to 0} C(\theta, s', t') = \lim_{\sigma \to 0} (\tau s' - t') \left[F\left(F^{-1}\left(\frac{s'}{t'}\right) \right) - F\left(F^{-1}(0)\right) \right] = \frac{s'}{t'}(\tau s' - t')$$

The cost of a partial-participation program is strictly less than that of a full-participation program.

$$\frac{s'}{t'}(\tau s' - t') = \tau c \frac{c}{t'} - c < \tau c - c \leq \tau s - t$$

For any $\theta < 0$,

$$\lim_{\sigma\to 0} C(\theta, s, t) = \lim_{\sigma\to 0} \tau s \left[1 - F\left(\frac{0-\theta}{\sigma} + F^{-1}(0)\right)\right] = \tau s\left[1 - F(\infty)\right] = 0$$

$$\begin{aligned}\lim_{\sigma\to 0} C(\theta, s', t') = & \lim_{\sigma\to 0} \tau s'\left[F\left(\frac{0-\theta}{\sigma} + F^{-1}\left(\frac{s'}{t'}\right)\right) - F\left(\frac{0-\theta}{\sigma} + F^{-1}(0)\right)\right] \\ & = \tau s'\left[F(\infty) - F(\infty)\right] = 0\end{aligned}$$

$\square$

Proof of Proposition 2.4. We compare the expected cost of a full-participation program with (s, t) a partial-participation program (s', t') with small enough $\lambda > 0$.

The expected cost of the full-participation program is

$$\mathbb{E}_\theta[C(\theta, s, t)] = \frac{\tau s}{\bar{\theta} - \underline{\theta}} \int_{\underline{\theta}}^{\bar{\theta}} \left[1 - F\left(\frac{\xi^* - \theta}{\sigma}\right)\right] \mathrm{d}\theta - \frac{t}{\bar{\theta} - \underline{\theta}} \int_{\theta^*}^{\bar{\theta}} \left[1 - F\left(\frac{\xi^* - \theta}{\sigma}\right)\right] \mathrm{d}\theta,$$

and that of the partial-participation program (s', t'),

$$\mathbb{E}_\theta[C(\theta, s', t')] = \frac{\tau s'}{\bar{\theta} - \underline{\theta}} \int_{\underline{\theta}}^{\bar{\theta}} \left[F\left(\frac{\eta^* - \theta}{\sigma}\right) - F\left(\frac{\xi^* - \theta}{\sigma}\right)\right] \mathrm{d}\theta - \frac{t'}{\bar{\theta} - \underline{\theta}} \int_{\theta^*}^{\bar{\theta}} \left[F\left(\frac{\eta^* - \theta}{\sigma}\right) - F\left(\frac{\xi^* - \theta}{\sigma}\right)\right] \mathrm{d}\theta,$$

where ξ^* and η^* are the investment threshold and participation threshold defined as in Proposition 2.2, $\xi^* = \theta^* + \sigma F^{-1}(\theta^*)$, $\eta^*(s', t') = \theta^* + \sigma F^{-1}\left(\frac{s'}{t'}\right)$. To suppress notations, we omit the dependence of η^* on (s', t'). The difference between the cost of full-participation program (s, t) and that of partial-participation program (s', t') can be decomposed into two parts, $\mathbb{E}_\theta[C(\theta, s, t)] - \mathbb{E}_\theta[C(\theta, s', t')] = \Delta_1 + \Delta_2$, where

$$\Delta_1 = \frac{\tau s - t}{\bar{\theta} - \underline{\theta}} \int_{\theta^*}^{\bar{\theta}} \left[1 - F\left(\frac{\eta^* - \theta}{\sigma}\right)\right] \mathrm{d}\theta + \frac{\tau s}{\bar{\theta} - \underline{\theta}} \int_{\underline{\theta}}^{\theta^*} \left[1 - F\left(\frac{\eta^* - \theta}{\sigma}\right)\right] \mathrm{d}\theta,$$

$$\Delta_2 = -\frac{\tau\theta^*(t' - t)}{\bar{\theta} - \underline{\theta}} \int_{\underline{\theta}}^{\bar{\theta}} \left[F\left(\frac{\eta^* - \theta}{\sigma}\right) - F\left(\frac{\xi^* - \theta}{\sigma}\right)\right] \mathrm{d}\theta + \frac{t' - t}{\bar{\theta} - \underline{\theta}} \int_{\theta^*}^{\bar{\theta}} \left[F\left(\frac{\eta^* - \theta}{\sigma}\right) - F\left(\frac{\xi^* - \theta}{\sigma}\right)\right] \mathrm{d}\theta.$$

Δ_1 and Δ_2 are the cost difference on the extensive margin and intensive margin respectively.

Notice $\mathbb{E}[C(\theta, s, t)]$ is linear in s and t. Therefore, the expected cost of any full-participation program lies between the cost of the guarantee program $\lambda_1 = 0$ with $(s, t) = (\frac{c-\theta^* b}{1-\theta^*}, \frac{c-\theta^* b}{1-\theta^*})$, and the pure subsidy program $\lambda_2 = -\frac{c-\theta^* b}{1-\theta^*}$, with $(s, t) = (c - \theta^* b, 0)$. In the remaining part of the proof, we show that if either of the two conditions is satisfied, the proposed partial-participation program $(s', t') = (\frac{c-\theta^* b}{1-\theta^*} + \theta^* \lambda, \frac{c-\theta^* b}{1-\theta^*} + \lambda)$ with small positive λ has lower cost than both the guarantee program and the pure subsidy program.

Consider the pure subsidy program $(s, t) = (c - \theta^* b, 0)$. Plugging (s, t) into the expression of Δ_1, we have

$$
\begin{aligned}
\Delta_1 =& \frac{\tau(c - \theta^* b)}{\bar{\theta} - \underline{\theta}} \int_{\underline{\theta}}^{\bar{\theta}} \left[1 - F\left(\frac{\eta^* - \theta}{\sigma}\right)\right] \mathrm{d}\theta, \\
=& \frac{\tau(c - \theta^* b)}{\bar{\theta} - \underline{\theta}} \int_{\underline{\theta}}^{\bar{\theta}} \int_{\eta^*}^{\bar{\theta} + \frac{1}{2}\sigma} \frac{1}{\sigma} f\left(\frac{x - \theta}{\sigma}\right) \mathrm{d}x \mathrm{d}\theta, \\
=& \frac{\tau(c - \theta^* b)}{\bar{\theta} - \underline{\theta}} \int_{\eta^*}^{\bar{\theta} + \frac{1}{2}\sigma} \int_{\underline{\theta}}^{\bar{\theta}} \frac{1}{\sigma} f\left(\frac{x - \theta}{\sigma}\right) \mathrm{d}\theta \mathrm{d}x, \\
=& \frac{\tau(c - \theta^* b)}{\bar{\theta} - \underline{\theta}} \int_{\eta^*}^{\bar{\theta} + \frac{1}{2}\sigma} \left[1 - F\left(\frac{x - \bar{\theta}}{\sigma}\right)\right] \mathrm{d}x, \\
=& \frac{\tau(c - \theta^* b)}{\bar{\theta} - \underline{\theta}} \left[\bar{\theta} + \frac{1}{2}\sigma - \eta^* - \sigma \int_{-\frac{1}{2}}^{\frac{1}{2}} F(y) \mathrm{d}y\right] > \frac{\tau(c - \theta^* b)}{\bar{\theta} - \underline{\theta}} (1 - \theta^*),
\end{aligned}
$$

which is strictly positive.

For Δ_2, notice

$$
\begin{aligned}
\int_{\alpha}^{\beta} \left[F\left(\frac{\eta^* - \theta}{\sigma}\right) - F\left(\frac{\xi^* - \theta}{\sigma}\right)\right] \mathrm{d}\theta &= \int_{\alpha}^{\beta} \int_{\xi^*}^{\eta^*} \frac{1}{\sigma} f\left(\frac{x - \theta}{\sigma}\right) \mathrm{d}x \mathrm{d}\theta, \\
&= \int_{\xi^*}^{\eta^*} \int_{\alpha}^{\beta} \frac{1}{\sigma} f\left(\frac{x - \theta}{\sigma}\right) \mathrm{d}\theta \mathrm{d}x, \\
&= \int_{\xi^*}^{\eta^*} \left[F\left(\frac{x - \alpha}{\sigma}\right) - F\left(\frac{x - \beta}{\sigma}\right)\right] \mathrm{d}x,
\end{aligned}
$$

therefore

$$\begin{aligned}
\Delta_2 &= \left(\frac{c-\theta^* b}{1-\theta^*}+\varepsilon\right)\frac{1}{\bar{\theta}-\underline{\theta}}\left[\int_{\xi^*}^{\eta^*} F\left(\frac{x-\theta^*}{\sigma}\right)\mathrm{d}x - \tau\theta^*(\eta^*-\xi^*)\right], \\
&= \left(\frac{c-\theta^* b}{1-\theta^*}+\varepsilon\right)\frac{\theta^*(\eta^*-\xi^*)}{\bar{\theta}-\underline{\theta}}\left[\int_{F^{-1}(\theta^*)}^{F^{-1}\left(\frac{s'}{t'}\right)} \frac{F(y)}{\theta^*(F^{-1}\left(\frac{s'}{t'}\right)-F^{-1}(\theta^*))}\mathrm{d}y - \tau\right], \\
&= \left(\frac{c-\theta^* b}{1-\theta^*}+\varepsilon\right)\frac{\theta^*(\eta^*-\xi^*)}{\bar{\theta}-\underline{\theta}}\left[G\left(\theta^*,\frac{s'}{t'}\right)-\tau\right].
\end{aligned}$$

Taking λ to 0, we have

$$\lim_{\lambda\to 0^+}\Delta_2 = \left(\frac{c-\theta^* b}{1-\theta^*}\right)\frac{\theta^*\sigma(\frac{1}{2}-F^{-1}(\theta^*))}{\bar{\theta}-\underline{\theta}}[G(\theta^*,1)-\tau] = \frac{c-\theta^* b}{\bar{\theta}-\underline{\theta}}\theta^*\sigma[G(\theta^*,1)-\tau].$$

If the first condition holds, $\tau < G(\theta^*,1)$, $\lim_{\varepsilon\to 0^+}\Delta_2 > 0$, $\Delta_1+\Delta_2$ is strictly positive for small enough λ. Also, if the second condition holds, $\theta^*+\sigma<1$,

$$\lim_{\lambda\to 0^+}\Delta_1+\Delta_2 > \frac{\tau(c-\theta^* b)}{\bar{\theta}-\underline{\theta}}(1-\theta^*-\theta^*\sigma) > 0.$$

Now let's turn to the guarantee program with $s=t=\frac{c-\theta^* b}{1-\theta^*}$. For Δ_1, since $\eta^*=\theta^*+\sigma F^{-1}(\frac{s'}{t'}) < \theta^*+\frac{1}{2}\sigma$, we have

$$\Delta_1 > \frac{(\tau-1)s}{\bar{\theta}-\underline{\theta}}\int_{\theta^*}^{\bar{\theta}}\left[1-F\left(\frac{\theta^*+\frac{1}{2}\sigma-\theta}{\sigma}\right)\right]\mathrm{d}\theta + \frac{\tau s}{\bar{\theta}-\underline{\theta}}\int_{\underline{\theta}}^{\theta^*}\left[1-F\left(\frac{\theta^*+\frac{1}{2}\sigma-\theta}{\sigma}\right)\right]\mathrm{d}\theta \geq 0.$$

The last inequality is strict when $\tau>1$. For Δ_2, we have

$$\Delta_2 = \lambda\frac{\sigma\theta^*(F^{-1}\left(\frac{s'}{t'}\right)-F^{-1}(\theta^*))}{\bar{\theta}-\underline{\theta}}\left[G\left(\theta^*,\frac{s'}{t'}\right)-\tau\right].$$

If $\tau>1$, $\lim_{\lambda\to 0^+}\Delta_1>0$, $\lim_{\lambda\to 0^+}\Delta_2=0$. Thus, $\mathbb{E}_\theta[C(\theta,s,t)]-\mathbb{E}_\theta[C(\theta,s',t')]=\Delta_1+\Delta_2>0$ for small enough λ.

If $\tau=1$, since $\frac{s'}{t'}>\frac{c}{b}>\theta^*$, $G\left(\theta^*,\frac{s'}{t'}\right)>1=\tau$, $\Delta_2>0$ for any positive λ. Combining with

$\Delta_1 \geq 0$, we have $\mathbb{E}_\theta[C(\theta, s, t)] - \mathbb{E}_\theta[C(\theta, s', t')] = \Delta_1 + \Delta_2 > 0$ for any positve λ.

To sum up, in either case, when λ being positve and small enough, the partial particiaption program $(s', t') = (\frac{c-\theta^* b}{1-\theta^*} + \theta^* \lambda, \frac{c-\theta^* b}{1-\theta^*} + \lambda)$ has lower expected cost than any full-participation program targeting θ^*. □

Proof of Proposition 2.6. If we can choose (s, t) properly such that $0 < p_1^*(s, t) < p_2^*(s, t) < 1$, Lemma 2.1 implies in the unique Bayesian Nash equilibrium, agents follow a threshold strategy

$$
\begin{aligned}
a_i = 1, \text{Reject}, &\quad \text{if} \quad x_i \geq \eta^*(s, t), \\
a_i = 1, \text{Accept}, &\quad \text{if} \quad \xi^*(s, t) \leq x_i < \eta^*(s, t), \\
a_i = 0, &\quad \text{if} \quad x_i < \xi^*(s, t),
\end{aligned}
$$

where

$$
\begin{aligned}
\xi^*(s, t) &= p_1^*(s, t) + \sigma F^{-1}(p_1^*(s, t)), \\
\eta^*(s, t) &= p_1^*(s, t) + \sigma F^{-1}(p_2^*(s, t)).
\end{aligned}
$$

Moreover, $\xi^*(s, t)$ and $\eta^*(s, t)$ both converges to $p_1^*(s, t)$ when $\sigma \to 0$. Thus, for any continuous belief of the fundamental held by the government, the ex-ante cost of the program converges to 0 when $\sigma \to 0$.

Now we want to show that it is possible to choose (s, t) such that $0 < p_1^*(s, t) < p_2^*(s, t) < 1$ and $p_1^*(s, t)$ can be arbitrarily close to 0. Let $s = c - \varepsilon$ and $\frac{c+c^e-\gamma b}{1-\gamma} < t < b$. The choice of t is feasible since Assumption 2.1b implies $\frac{c+c^e-\gamma b}{1-\gamma} < b$. Note $\frac{c+c^e-\gamma b}{1-\gamma} < t$ implies

$$
\begin{aligned}
p_2^*(s, t) &= \frac{s}{\gamma b + t(1-\gamma) - c^e} < \frac{c - \varepsilon}{c}, \\
p_1^*(s, t) &= \frac{c - s}{(1-\gamma)(b-t)} = \frac{\varepsilon}{(1-\gamma)(b-t)}.
\end{aligned}
$$

Therefore, for any fixed t, when $\varepsilon \to 0$, $p_1^*(s,t)$ converges to 0 and $p_2^*(s,t)$ converges to a positive number which is strictly less than 1. $\square$

Proof of Proposition 2.7. The proof is similar to the proof of Lemma 2.1. We want to find a sequence $\left\{(\underline{\xi}_n^g, \bar{\xi}_n^g)_{g=1}^N\right\}_{n=0}^{\infty}$ such that the strategies of group g agents survive n rounds of iterated deletion of dominated strategies only if

$$a^g(x) = 0, \text{ if } x < \underline{\xi}_n^g, \tag{2.39}$$

$$\text{and } a^g(x) = 1, \text{ if } x \geq \bar{\xi}_n^g. \tag{2.40}$$

To simplify notations, let $\underline{\xi}_n = (\underline{\xi}_n^g)_{g=1}^N$ and $\bar{\xi}_n = (\bar{\xi}_n^g)_{g=1}^N$ be the vectors of threshold signals. The recursive expression for $\left\{(\underline{\xi}_n^g, \bar{\xi}_n^g)_{g=1}^N\right\}_{n=0}^{\infty}$ is

$$\underline{\xi}_{n+1}^g = \inf_x \{x : p^g(x; \underline{\xi}_n) \geq \frac{c}{b^g}\}, \tag{2.41}$$

$$\bar{\xi}_{n+1}^g = \sup_x \{x : p^g(x; \bar{\xi}_n) \leq \frac{c}{b^g}\}. \tag{2.42}$$

We can prove by induction that

$$-\infty = \underline{\xi}_0 < \underline{\xi}_1 \leq \cdots \leq \underline{\xi}_n \leq \cdots \leq \bar{\xi}_n \leq \cdots \leq \bar{\xi}_1 < \bar{\xi}_0 = +\infty. \tag{2.43}$$

Since any bounded monotonic sequence has a finite limit, take n to ∞, we have

$$\bar{\xi} \geq \underline{\xi}. \tag{2.44}$$

Now we want to show $\bar{\xi} = \underline{\xi}$. It can be proved by contradiction. Suppose $\bar{\xi} > \underline{\xi}$. Let h be the group such that $\bar{\xi}^h - \underline{\xi}^h = \max_g \left\{\bar{\xi}^g - \underline{\xi}^g\right\} > 0$. Note that $\theta^*(\bar{\xi})$ is the solution to

$$\sum_{g=1}^N w^g m^g F^g \left(\frac{\bar{\xi}^g - \theta}{\sigma}\right) = \theta. \tag{2.45}$$

Therefore, $\theta^*(\bar{\xi}) - (\bar{\xi}^h - \underline{\xi}^h)$ is the solution to

$$\sum_{g=1}^{N} w^g m^g F^g \left(\frac{\bar{\xi}^g - (\bar{\xi}^h - \underline{\xi}^h) - \theta}{\sigma} \right) - \theta - (\bar{\xi}^h - \underline{\xi}^h) = 0. \tag{2.46}$$

Also notice $\theta^*(\underline{\xi})$ is the solution to

$$\sum_{g=1}^{N} w^g m^g F^g \left(\frac{\underline{\xi}^g - \theta}{\sigma} \right) - \theta = 0. \tag{2.47}$$

Let's compare (2.46) and (2.47). Since $\underline{\xi}^g > \bar{\xi}^g - (\bar{\xi}^h - \underline{\xi}^h)$ and $\bar{\xi}^h - \underline{\xi}^h > 0$, the left hand side of (2.47) is strictly larger than the left hand side of (2.46) for any given θ. Given the left hand side of (2.47) is strictly decreasing in θ, we must have $\theta^*(\bar{\xi}) - (\bar{\xi}^h - \underline{\xi}^h) < \theta^*(\underline{\xi})$. Therefore,

$$\begin{aligned}
p^h(\bar{\xi}^h; \bar{\xi}) =& Pr^h[\theta > \theta^*(\bar{\xi}) | \bar{\xi}^h], \\
=& F^h \left(\frac{\bar{\xi}^h - \theta^*(\bar{\xi})}{\sigma} \right), \\
=& F^h \left(\frac{\underline{\xi}^h - [\theta^*(\bar{\xi}) - (\bar{\xi}^h - \underline{\xi}^h)]}{\sigma} \right), \\
>& F^h \left(\frac{\underline{\xi}^h - \theta^*(\theta^*(\underline{\xi}))}{\sigma} \right), \\
=& p^h(\underline{\xi}^h; \underline{\xi}).
\end{aligned}$$

However. (2.41) and (2.42) implies $p^h(\bar{\xi}^h; \bar{\xi}) = p^h(\underline{\xi}^h; \underline{\xi}) = \frac{c}{b^h}$. Contradiction. This implies $\bar{\xi} = \underline{\xi} = \xi_0$.

To solve for ξ_0, note ξ_0 and θ_0 are the solutions to

$$\sum_{g=1}^{N} w^g m^g F^g \left(\frac{\xi^g - \theta}{\sigma} \right) = \theta, \tag{2.48}$$

$$F^g \left(\frac{\xi^g - \theta}{\sigma} \right) = \frac{c}{b^g}, \quad \text{for any } g = 1, \ldots, N. \tag{2.49}$$

Plugging (2.49) into (2.48) we have

$$\theta_0 = \sum_{g=1}^{N} m^g w^g \frac{c}{b^g}, \tag{2.50}$$

$$\xi_0^g = \sum_{g=1}^{N} m^g w^g \frac{c}{b^g} + \sigma F_g^{-1}\left(\frac{c}{b^g}\right), \quad \text{for any } g = 1, \dots, N. \tag{2.51}$$

□

Proof of Proposition 2.8. The optimal response of an agent in group g is

$$\begin{aligned} a_i = 1, \text{Reject}, \ \ \text{if} \ \ \hat{p}_i \geq \frac{s}{t}, \\ a_i = 1, \text{Accept}, \ \ \text{if} \ \ \frac{c-s}{b^g - t} \leq \hat{p}_i < \frac{s}{t}, \\ a_i = 0, \ \ \text{if} \ \ \hat{p}_i < \frac{c-s}{b^g - t}; \end{aligned}$$

We can apply the same method in the proof of Proposition 2.7 and show that in any equilibrium, agents of group g invest if and only if their private signal is greater or equal to

$$\xi_g^*(s,t) = \sum_{g=1}^{N} m^g w^g \frac{c-s}{b^g - t} + \sigma F_g^{-1}\left(\frac{c-s}{b_g - t}\right). \tag{2.52}$$

Given the investment thresholds, we know the fundamental threshold above which there will be successful investment is

$$\theta^*(s,t) = \sum_{g=1}^{N} m^g w^g \frac{c-s}{b^g - t}. \tag{2.53}$$

Therefore, the signal $\eta^*(s,t)$ that makes an agent from group g indifferent between accepting and rejecting the intervention program is

$$\eta^*(s,t) = \sum_{g=1}^{N} m^g w^g \frac{c-s}{b^g - t} + \sigma F_g^{-1}\left(\frac{s}{t}\right). \tag{2.54}$$

$\square$

Proof of Proposition 2.9. Consider an agent who receives private signal x and knows that all other agents invest if and only if observing private signal k. The expected payoff from investing is

$$U(k,x) = \int_{\underline{\theta}}^{\bar{\theta}} \frac{1}{\sigma} f\left(\frac{x-\theta}{\sigma}\right) \pi\left(\theta, 1 - F\left(\frac{k-\theta}{\sigma}\right)\right) d\theta$$

Note that $U(k,x)$ weakly decreases in k and weakly increases in x. Intuitively, an agent has higher expected payoff if everyone else is more willing to invest or the agent receives a high signal indicating a high fundamental θ. Also note that $U(-\infty, x) < 0$ for $x < \theta_0$ and $U(+\infty, x) > 0$ for $x > \theta_1$.

Next we prove the uniqueness of equilibrium by iterated deletion of dominated strategies. The strategy profile of an agent is the action as a function of the private signal received. We denote it by $a(x) : \mathbb{R} \to \{0,1\}$. We will prove that strategy survives n rounds of iterated deletion of dominated strategies if and only if

$$a(x) = 0, \text{ if } x < \underline{\xi}_n, \tag{2.55}$$

$$\text{and } a(x) = 1, \text{ if } x \geq \bar{\xi}_n. \tag{2.56}$$

where $\left\{(\underline{\xi}_n, \bar{\xi}_n)\right\}_{n=0}^{\infty}$ satisfies

$$-\infty = \underline{\xi}_0 < \underline{\xi}_1 \leq \cdots \leq \underline{\xi}_n \leq \cdots \leq \bar{\xi}_n \leq \cdots \leq \bar{\xi}_1 < \bar{\xi}_0 = +\infty. \tag{2.57}$$

This result can be proved by induction. Let the starting node be $\underline{\xi}_0 = -\infty$ and $\bar{\xi}_0 = +\infty$, meaning that there is no restrictions on agents' strategy. Suppose round $n \in \mathbb{N}$ of deletion has been completed. In round $n+1$, the most optimistic belief for an agent is that all other agents follow a cutoff strategy with threshold $\underline{\xi}_n$. Therefore, for any x such that

$U(\underline{\xi}_n, x) < 0$, $a(x) = 1$ is strictly dominated by $a(x) = 0$. Similarly, the most pessimistic belief for an agent is that all other agents follow a cutoff strategy with threshold $\bar{\xi}_n$. As a result, for x such that $U(\bar{\xi}_n, x) > 0$, any strategy profile with $a(x) = 0$ is strictly dominated by $a(x) = 1$.

Given $U(k, x)$ is non-decreasing in x, the strategy profiles that survives deletion of dominated strategies must satisfy the restrictions in (2.55) and (2.56), with $(\underline{\xi}_{n+1}, \bar{\xi}_{n+1})$ defined inductively as

$$\underline{\xi}_{n+1} = \inf\{x : U(\underline{\xi}_n, x) \geq 0\} \tag{2.58}$$

and

$$\bar{\xi}_{n+1} = \sup\{x : U(\bar{\xi}_n, x) \leq 0\} \tag{2.59}$$

The monotonicity of $U(k, x)$ guarantees that $\underline{\xi}_{n+1} \leq \bar{\xi}_{n+1}$. Note that the dominance region assumption implies that $\underline{\xi}_1 > -\infty$ and $\bar{\xi}_1 < +\infty$. Therefore, $\left\{(\underline{\xi}_n, \bar{\xi}_n)\right\}_{n=0}^{\infty}$ is a well-defined sequence of real couples which satisfies (2.57).

Now we've proved that $\{\underline{\xi}_n\}_{n=1}^{\infty}$ and $\{\bar{\xi}_n\}_{n=1}^{\infty}$ are both monotonic and bounded sequences. Thus, they converges to two finite numbers $\underline{\xi}$ and $\bar{\xi}$ respectively when $n \to \infty$. The definition (2.58) and (2.59) imply that $U(\underline{\xi}, \underline{\xi}) \geq 0$ and $U(\bar{\xi}, \bar{\xi}) \leq 0$. Notice for $y \in [\theta_0, \theta_1]$,

$$U(y, y) = \int_{\underline{\theta}}^{\bar{\theta}} \frac{1}{\sigma} f\left(\frac{y - \theta}{\sigma}\right) \pi\left(y, 1 - F\left(\frac{y - \theta}{\sigma}\right)\right) d\theta = \int_0^1 \pi\,(y, l)\, dl,$$

strictly increases in y and $\underline{\xi} \leq \bar{\xi}$, it must be the case that $U(\underline{\xi}, \underline{\xi}) = U(\bar{\xi}, \bar{\xi}) = 0$.

Since $U(y, y)$ is continuous in y, $U(\underline{\theta}, \underline{\theta}) \leq 0$, $U(\bar{\theta}, \bar{\theta}) \geq 0$, there is a unique solution to $U(y, y) = \int_0^1 \pi\,(y, l)\, dl = 0$. Denote the solution by ξ_0^*, and we have $\underline{\xi} = \bar{\xi} = \xi_0^*$. Therefore, the only strategy that survives the iterated deletion of dominated strategies is the cutoff investment strategy with cutoff ξ_0^*. $\square$

Proof of Proposition 2.10. Consider an agent who receives private signal x and knows that all other agents invest if and only if their signal is above k. The expected payoff from investing and rejecting the intervention offer is

$$U^R(k,x) = \int_{\underline{\theta}}^{\bar{\theta}} \frac{1}{\sigma} f\left(\frac{x-\theta}{\sigma}\right) \pi\left(\theta, 1 - F\left(\frac{k-\theta}{\sigma}\right)\right) d\theta.$$

The expected payoff from investing and accepting the offer is

$$U^A(k,x) = (1-t)U^R(k,x) + s$$

Therefore, the maximum expected payoff from investing is

$$U(k,x) = \max\{U^R(k,x), U^A(k,x)\}$$

We prove a lemma that will be useful later.

Lemma 2.2. *Given that all other agents invest if and only if their signal is above k, there exist two functions $k_1^*(k)$ and $k_2^*(k)$ such that an agent strictly prefers not investing if her private signal $x < k_1^*(k)$ and strictly prefers investing if $x > k_2^*(k)$. $k_1^*(k)$ and $k_2^*(k)$ are given by*

$$k_1^*(k) = \inf\left\{k^* : U^R(k,k^*) \geq -\frac{s}{1-t}\right\},$$

$$k_2^*(k) = \sup\left\{k^* : U^R(k,k^*) \leq -\frac{s}{1-t}\right\},$$

Both $k_1^*(k)$ and $k_2^*(k)$ are weakly increasing in k.

Proof of Lemma 2.2. The Left Dominance Region assumption in Definition 2.1 and Limit Dominance in Assumption 2.3 make sure that the two function $k_1^*(k)$ and $k_2^*(k)$ are well defined. By continuity of $U^R(k,x)$ in x, we have

$$U^R(k,k_1^*(k)) = U^R(k,k_2^*(k)) = -\frac{s}{1-t},$$

For any $x < k_1^*(k)$, $U^R(k,x) < -\frac{s}{1-t}$, $U^A(k,x) = (1-t)U^R(k,x) + s < 0$. Therefore, $U(k,x) = \max\{U^R(k,x), U^A(k,x)\} < 0$, the agent will not invest if observing $x < k_1^*(k)$. On the other hand, for any $x > k_2^*(k)$, $U^R(k,x) > -\frac{s}{1-t}$, $U^A(k,x) = (1-t)U^R(k,x) + s > 0$. Therefore, $U(k,x) = \max\{U^R(k,x), U^A(k,x)\} > 0$, the agent will invest after observing signal $x > k_2^*(k)$.

Since $U^R(k,x)$ is weakly decreasing in k, we can easily show that both $k_1^*(k)$ and $k_2^*(k)$ are weakly increasing in k. □

With Lemma 2.2, we can prove the uniqueness of equilibrium by iterated deletion of dominated strategies. Denote the investment strategy by $a(x)$. We want to show a strategy survives n rounds of iterated deletion of dominated strategies if and only if

$$a(x) = \begin{cases} 0, & \text{if } x < \underline{\xi}_n, \\ 1, & \text{if } x > \bar{\xi}_n, \end{cases}$$

where $\underline{\xi}_0 = -\infty$, $\bar{\xi}_0 = \infty$. $\underline{\xi}_n$ and $\bar{\xi}_n$ are defined inductively by $\underline{\xi}_{n+1} = k_1^*(\underline{\xi}_n)$, $\bar{\xi}_{n+1} = k_2^*(\bar{\xi}_n)$.

Since $k^*(\xi)$ increases in ξ, $\underline{\xi}_n$ and $\bar{\xi}_n$ are increasing and decreasing sequences, respectively. As $n \to \infty$, $\underline{\xi}_n \to \underline{\xi}$ and $\bar{\xi}_n \to \bar{\xi}$. Therefore, $\underline{\xi} = k_1^*(\underline{\xi})$ and $\bar{\xi} = k_2^*(\bar{\xi})$. $\underline{\xi}$ and $\bar{\xi}$ must both be the solution to

$$U^R(\xi,\xi) = -\frac{s}{1-t}.$$

Let $l = 1 - F\left(\frac{\xi-\theta}{\sigma}\right)$, the equation can be written as

$$\int_0^1 \pi(\xi, l)\, dl = -\frac{s}{1-t} \tag{2.60}$$

By Strict Laplacian State Monotonicity in Assumption 2.3, the left hand side is continuous and strictly increasing in ξ. Also, $\int_0^1 \pi(\underline{\theta}, l)\, dl < -\frac{s}{1-t}$, $\int_0^1 \pi(\bar{\theta}, l)\, dl > 0 \geq -\frac{s}{1-t}$, there is a unique solution to the equation above, $\underline{\xi} = \bar{\xi} = \xi^*$. Notice ξ^* is independent of σ. Then by iterated deletion of dominated strategies, it is the unique investment cutoff in equilibrium.

Given the investment cutoff, we can solve for the private signal x such that $U^A(\xi^*, x) = U^R(\xi^*, x)$, or equivalently $U^R(\xi^*, \eta^*(\sigma)) = \frac{s}{t}$. Let $\eta^*(\sigma)$ be the maximum value that satisfies

$$U^R(\xi^*, \eta^*) = \int_{\underline{\theta}}^{\bar{\theta}} \frac{1}{\sigma} f\left(\frac{\eta^* - \theta}{\sigma}\right) \pi\left(\eta^*, 1 - F\left(\frac{\xi^* - \theta}{\sigma}\right)\right) d\theta = \frac{s}{t}. \tag{2.61}$$

For any signal $x > \eta^*(\sigma)$, an agent strictly prefers investing and not participating in the intervention program. Notice when $x > \xi^* + \sigma$, $U^R(\xi^*, x) = \pi(x, 1) > \frac{s}{t}$, therefore, $\eta^*(\sigma)$ is well-defined.

Since $U^R(k, x)$ increases in x, and $U^R(\xi^*, \xi^*) = -\frac{s}{1-t} \leq \frac{s}{t} = U^R(\xi^*, \eta^*(\sigma))$, therefore, $\eta^*(\sigma) \geq \xi^*$. It immediately follows that $\lim_{\sigma \to 0} \eta^*(\sigma) = \eta \geq \xi^*$. Next, we prove $\eta = \xi^*$ by contradiction. Suppose $\eta > \xi^*$, take $\sigma \to 0$ in the left hand side of (2.61), we have

$$\lim_{\sigma \to 0} \int_{\underline{\theta}}^{\bar{\theta}} \frac{1}{\sigma} f\left(\frac{\eta^*(\sigma) - \theta}{\sigma}\right) \pi\left(\eta^*(\sigma), 1 - F\left(\frac{\xi^* - \theta}{\sigma}\right)\right) d\theta = \pi(\eta, 1) \geq \pi(\xi^*, 1) > \frac{s}{t}$$

Contradiction to (2.61). Therefore, $\lim\limits_{\sigma \to 0} \eta^*(\sigma) = \eta = \xi^*$. □

Proof of Proposition 2.11. According to Definition 2.1, a partial-participation program with target ξ^* should satisfy the following conditions

1. $\pi(\underline{\theta}, 1) < -\frac{s}{1-t}$
2. $\pi(\xi^*, 1) > \frac{s}{t}$
3. $\int_0^1 \pi(\xi^*, l) dl = -\frac{s}{1-t}$

As long as the government offers (s, t) given by

$$\left(-\frac{\pi(\xi^*, 1)}{\int_0^1 \pi(\xi^*, l) dl} + 1\right)^{-1} < t < 1, \tag{2.62}$$

and

$$s = -(1-t)\int_0^1 \pi(\xi^*, l)dl, \tag{2.63}$$

the three conditions listed above are satisfied. First, by assumption, $\pi(\underline{\theta}, 1) \leq \int_0^1 \pi(\theta_0, l)dl < \int_0^1 \pi(\xi^*, l)dl = -\frac{s}{1-t}$. Second, (2.62) can be written as $\pi(\xi^*, 1) > -\frac{1-t}{t}\int_0^1 \pi(\xi^*, l)dl = \frac{s}{t}$. Finally, the third condition directly follows equation (2.63). $\square$

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