# ESSAYS ON FINANCIAL FRICTIONS, PROPAGATION OF SHOCKS AND MACROECONOMIC VOLATILITY 

Cristina Fuentes-Albero

## A DISSERTATION

in

## Economics

Presented to the Faculties of the University of Pennsylvania
in
Partial Fulfillment of the Requirements for the
Degree of Doctor of Philosophy
2010


Frank Schorfheide, Professor of Economics

> Graduate Group Chairperson

## Dissertation Committee



Francis X. Diebold, Paul F. and Warren S. Miller Professor of Economics Jesús Fernández-Villaverde, Associate Professor, Economics Urban J. Jermann, Safra Professor of International Finance and Capital Markets, Finance

I dedicate this thesis in loving memory of my father and in honor of my mother and sisters because without them I would have never gone this far.

## Acknowledgements

I owe my deepest gratitude to my advisor, Frank Schorfheide, for his guidance, encouragement, and support. I would also like to show my gratitude to Francis X. Diebold, Jesús Fernández-Villaverde, and Urban J. Jermann for insightful comments and discussions. My research has benefited from long conversations with Maxym Kryshko and Leonardo Melosi and the comments of participants at many institutions, but especially at the Board of Governors and the Bank of Spain where I spent some time working on this thesis.

I thank the extraordinary support of Abhinash Bora, Se Kyu Choi, Clement Joubert, Edith Liu, Jon Pogach, and Ximena Saez. Their friendship has been key during all these years. I especially appreciate their words, silences, and gestures over the last year. Thanks for being there when I most needed you.

This thesis would not have been possible without my family. I am indebted to my father, mother, and sisters for their patience, support, encouragement, and unconditional love throughout my life. They are the engine that keeps me going. There are not enough words to express gratitude to my parents for all the effort they have put on me and my sisters. We are who we are just because of them. Last, but not least, I want to have a special thanks to my father for his blind faith on me. He has been my inspiration over time but especially after his death. Because death is not the end, his memory has given me the strength to finish the adventure I started six years ago. I know he is with me and my family celebrating our success as a team.

ABSTRACT<br>ESSAYS ON FINANCIAL FRICTIONS, PROPAGATION OF SHOCKS AND MACROECONOMIC VOLATILITY<br>Cristina Fuentes-Albero<br>Frank Schorfheide, Professor of Economics

I study the evolution of aggregate volatility in the US during the postwar period by assessing the relative role played by financial shocks, technological progress, and changes in the financial system. Balance-sheet variables of firms have been characterized by greater volatility since the early 1970s. This Financial Immoderation has coexisted with the so-called Great Moderation, which refers to the slowdown in volatility of real and nominal variables since the mid 1980s. In the second chapter, I study the moderation in real variables calibrating a real business cycle model with two technology shocks. I consider several statistical specifications for technological progress. A deterministic trend model outperforms in accounting for volatilities, but a stochastic trend model accounts better for the correlation structure of the data. In the third chapter, I account for the divergent patterns in volatility analyzing the role played by financial factors. To do so, I estimate a DSGE model including financial rigidities, allowing for structural breaks in a subset of parameters. I conclude that the Financial Immoderation is driven by larger financial shocks and that the estimated reduction in the size of the financial accelerator in the mid 1980s accounts for $30 \%$ of the decline in the volatilities of investment growth and the nominal interest rate. In the last chapter, I focus on analyzing financial shocks. Using the estimation output, I obtain that the contribution of financial shocks to the variance of investment is increasing over time, reducing the relative importance of the investment-specific technology shock. The estimated reduction in the level of financial rigidities has a significant impact on
the model implied propagation dynamics. Given that the model implies a negative response upon impact of consumption in response to a positive business wealth shock, I empirically characterize the effects of such a financial shock on consumption using sign restrictions. I conclude that documenting the effects on consumption is not a trivial matter since the results vary significantly depending on the variables used to measure business wealth and the cost of external borrowing.

## Contents

Acknowledgements ..... iii
1 Introduction ..... 1
2 Technology Shocks, Statistical Models, and The Great Moderation ..... 6
2.1 Introduction ..... 6
2.2 The Model ..... 9
2.3 Calibration ..... 15
2.3.1 Data set ..... 15
2.3.2 Deterministic trend model ..... 17
2.3.3 Stochastic trend model ..... 20
2.3.4 Comparing statistical models ..... 22
2.4 The Great Moderation ..... 25
2.5 Conclusion ..... 28
3 Financial Frictions, the Financial Immoderation, and the Great Mod- eration ..... 29
3.1 Introduction ..... 29
3.2 Empirical Motivation ..... 34
3.3 The Model ..... 37
3.3.1 Households ..... 38
3.3.2 Retailers ..... 40
3.3.3 Intermediate goods sector ..... 41
3.3.4 Capital producers ..... 43
3.3.5 Entrepreneurs and financial intermediaries ..... 44
3.3.6 Government ..... 54
3.3.7 Competitive equilibrium ..... 55
3.4 Structural Breaks in Parameters ..... 56
3.5 Parameter Estimates ..... 58
3.5.1 Prior distribution of the parameters ..... 60
3.5.2 Posterior estimates of the parameters ..... 63
3.5.3 Model evaluation ..... 67
3.6 Assessing the Drivers of the Financial Immoderation and the Great Moderation ..... 69
3.7 Conclusions ..... 73
4 Financial Shocks: Model Implied versus Empirical Propagation Dy- namics ..... 75
4.1 Introduction ..... 75
4.2 Financial Shocks in a DSGE Model with Financial Frictions: Relative Importance and Propagation Dynamics ..... 78
4.2.1 Variance decomposition ..... 79
4.2.2 Impulse response functions ..... 81
4.3 What are the Effects of Financial Shocks on Consumption? An Em- pirical Assessment ..... 87
4.3.1 Data and statistical model ..... 87
4.3.2 Sign restriction identification: Methodology ..... 89
4.3.3 Results ..... 92
4.4 Conclusion ..... 95
Appendices ..... 97
Appendix A Chapter 2 ..... 97
Appendix A. 1 Tables and figures ..... 97
Appendix A. 3 Balance Growth Path ..... 118
Appendix A. 4 Log-linearization around the steady state ..... 119
Appendix A. 5 Stochastic trend model: closed form solution ..... 121
Appendix A. 6 Extensions ..... 126
Appendix B Chapter 3 ..... 131
Appendix B. 1 Tables ..... 131
Appendix B. 2 Figures ..... 140
Appendix B. 3 Data ..... 141
Appendix B. 4 Methodology ..... 144
Appendix B. 5 Log-linearized equilibrium conditions ..... 149
Appendix C Chapter 4 ..... 155
Appendix C. 1 Tables ..... 155
Appendix C. 2 Figures ..... 158

## List of Tables

1 Calibration Targets ..... 97
2 Deterministic Trend: Calibrated Parameters ..... 98
3 Baseline Stochastic Trend: Calibrated Parameters ..... 98
4 Stochastic Trend with a Moving Average Component: Calibrated Pa- rameters ..... 99
5 The Great Moderation: Empirical Evidence ..... 100
6 Results: Deterministic Trend ..... 101
7 Results: Baseline Stochastic Trend ..... 102
8 Results: Stochastic Trend with Moving Average Component ..... 103
9 Model implied volatility to observed volatility ratio $\left(\sigma_{\text {model }} / \sigma_{\text {data }}\right)$ when $\nu=1$ ..... 104
10 Variance Decomposition for the whole sample under $\nu=1$ ..... 104
11 Model implied autocorrelation to observed coefficient ratio $\left(\rho_{\text {model }} / \rho_{\text {data }}\right)$ when $\nu=1$ ..... 105
12 Correlation coefficients $(\nu=1)$ ..... 105
13 Cross-correlation output with x: Whole sample and $\nu=1$ ..... 106
14 Cross-correlation output with x: 1948:1-1983:4 and $\nu=1$ ..... 107
15 Cross-correlation output with x: 1984:1-2006:4 and $\nu=1$ ..... 108
16 The Great Moderation: Time-invariant coefficients ..... 109
17 The Great Moderation: Time-varying coefficients ..... 110
18 Autocorrelation ( $\nu=1$ ) ..... 111
19 Hansen-Rogerson preferences 1948:1-2006:4 ..... 112
20 VAR approach: Estimated Parameters ..... 113
21 Multivariate Analysis Results ..... 114
22 Multivariate Analysis Results: ratio of standard deviations ..... 115
23 Chow's Breakpoint Test: AR(1) with drift ..... 131
24 Chow's Breakpoint Test: Cyclical component. AR(1) with drift ..... 132
25 Ratio post- to pre- standard deviation: Cyclical component ..... 132
26 Prior ..... 133
27 Marginal Data Densities Comparison ..... 134
28 Posterior estimates ..... 134
29 Posterior estimates ..... 135
30 Model Fit: Standard deviations. Raw variables. ..... 136

31 Model Fit: Standard deviations. Cyclical component using the HP-filter. 137
32 Ratio of standard deviations. Cyclical component using the HP filter. 138
33 Counterfactuals: Percentage of the model-implied change in cyclical standard deviations. 139
34 Variance decomposition at business cycle frequencies ..... 155
35 Variance decomposition at the business cycle frequency ..... 156
36 Sign restrictions. ..... 157
37 Percentage of IRFs delivering a negative response upon impact of con- sumption ..... 157

## List of Figures

A-1 Impulse response functions with respect to a neutral technology shock ..... 116
A-2 Impulse response functions with respect to an investment-specific tech- nology shock ..... 117
B-1 Debt to net worth ratio. Cyclical component. ..... 140
C-1 Impulse Response Functions with respect to a wealth shock. The dot- ted line is the IRF for the 1954-1969 period, the solid line is the IRF for 1970-1983, and the dashed line is the IRF for the post-1984 period ..... 159
C-2 Impulse Response Functions with respect to a shock to the marginal bankruptcy cost. The dotted line is the IRF for the 1954-1969 period, the solid line is the IRF for 1970-1983, and the dashed line is the IRF for the post-1984 period. ..... 160
C-3 Impulse Response Functions with respect to a shock to the marginal bankruptcy cost: 1954-1969 ..... 161
C-4 Impulse Response Functions: A comparison ..... 162
C-5 IRF with respect to a wealth shock. Net worth is measured using FOFA and the external financial premium, using the corporate spread ..... 163
C-6 IRF with respect to a wealth shock. Net worth is measured using industrial Dow Jones and the external financial premium, using the corporate spread ..... 164
C-7 IRF with respect to a wealth shock. Net worth is measured using industrial Dow Jones and the external financial premium, using the prime lending spread ..... 165
C-8 1970-1983: IRF with respect to a wealth shock. Net worth is measured using FOFA and the external financial premium, using the corporate spread ..... 166
C-9 1970-1983: IRF with respect to a wealth shock. Net worth is measured using FOFA and the external financial premium, using the prime lend- ing spread ..... 167

## Chapter 1

## Introduction

In macroeconomics, economic fluctuations are modeled as shocks to the economy. Therefore, studying the shocks driving business cycle fluctuations, as well as their propagation mechanism in the economy, are topics of great interest for researchers. In fact, business cycle literature has been among the most productive areas of economic research over the last decades. The wave of contributions that followed the seminal work by [47] focused on the role of real shocks as drivers of fluctuations at business cycle frequencies. [19] conclude that technology shocks account for more than half of the cyclical variance of output in the postwar period. The empirical success of real business cycle (RBC) models has been questioned by [29] among others. He suggests that the sources of business cycles are non-technology shocks, which is hard to reconcile with a standard RBC model, but consistent with models featuring monopolistic competition and sticky prices. Following [29]'s contribution, there was an expansion of research on the sources of business cycles using a New Keynesian perspective. Many of these contributions focused on characterizing the propagation mechanism of monetary policy shocks.

The debate between defenders of technology and non-technology driven business cycles was heated up by the distinction between neutral and investment-specific technology shocks proposed by [35]. For example, [36] conclude that investment-specific
technology shocks account for $30 \%$ of output volatility. [26] uses a neoclassical growth model to identify the short-run effects of neutral and investment-specific technology shocks. He concludes that the investment-specific technology shock accounts for up to $67 \%$ of the variation in output and $47 \%$ of that in hours. Recently, [43] estimate a New Keynesian model and conclude that the investment-specific technology shock is the main driver of US business cycle fluctuations in the postwar period. They suggest that such a shock is a proxy for financial shocks or developments in the financial sector.

Over the last few years, there has been a growing interest in introducing credit market imperfections in standard macro models to analyze the role played by financial rigidities in the propagation of economic shocks. [8] and [5] consider frameworks in which credit market imperfections arise because there is asymmetric information between borrowers and lenders. This asymmetry translates into external borrowing being more expensive than internal financing. This wedge is the so called external finance premium which is the key relationship in the amplification and propagation mechanism known as the financial accelerator. Once the effects of financial rigidities on the propagation dynamics of technology and monetary shocks was well documented, researchers started to consider shocks originated in the financial sector as potential drivers of the business cycle. For example, [53] construct and study shocks to the efficiency of the financial sector. They conclude that the median contribution of these shocks to the variance of investment and output is $45 \%$.

My dissertation focuses on studying the evolution of aggregate volatility in the US during the postwar period by assessing the relative role played by financial shocks, technological progress, and changes in the financial system. The US economy over the last 55 years has been characterized by two empirical regularities. On the one hand, there has been a slowdown in the magnitude of business cycle fluctuations of
real and nominal variables since the mid 1980s. This empirical regularity was popularized by [57] as the Great Moderation. On the other hand, financial variables have become more volatile since the early 1970s. I refer to this empirical regularity as the Financial Immoderation. In the second chapter, I study the moderation in real variables through the lens of a Neoclassical business cycle model. In the third chapter, I focus on disentangling the role of financial factors in the divergent patterns in volatility using a New Keynesian dynamic stochastic general equilibrium (DSGE) model. In the last chapter, I analyze the propagation of financial shocks in the theoretical economy estimated in chapter 3. Given the lack of guidance on the response of household consumption to a financial shock affecting firms' ability to borrow, I empirically document the effects of a shock to business wealth on consumption using sign restrictions as proposed by [59].

The second chapter, Technology Shocks, Statistical Models, and the Great Moderation, analyzes the cyclical features implied by a simple RBC model with two technology shocks à la [36]. In the spirit of [39], I analyze the performance of the model in accounting for US business cycle features under trend stationary and difference stationary technology processes. Calibrating the model to US data, I conclude that the deterministic trend model outperforms the stochastic trend model in accounting for business cycle volatilities. The trend stationary model, however, underpredicts the correlation of consumption and output at all lead and lags, which is at odds with the data. The difference stationary version of the model overcomes those shortcomings. Therefore, I can conclude that the difference stationary model is more successful in matching the correlation structure of the data. The observed reduction in the volatility of the TFP shock and the price of investment suffices to deliver the magnitude of the Great Moderation in both models.

The aim of the third chapter, Financial Frictions, the Financial Immoderation,
and the Great Moderation, is to account for the immoderation of financial cycles and the moderation of real and nominal cycles analyzing the role played by financial factors. To do so, I use a DSGE model that includes a financial accelerator mechanism à la [5]. Financial rigidities arise form asymmetric information between borrowers and lenders. Costly state verification implies that external borrowing is more expensive than internal financing. The difference is the external finance premium. I enrich the model with financial shocks affecting the two channels of the external finance premium. The balance sheet channel refers to the negative dependence of the premium on the amount of collateralized net worth. The asymmetric information channel establishes that the premium is a positive function of the severity of the agency problem.

I estimate the model economy using Bayesian techniques on a data set containing real, nominal, and financial variables. To account for the breaks in the second moments of the data, I allow for structural breaks in the volatilities of shocks, monetary policy coefficients, and average size of the financial accelerator mechanism. I conclude that the widening of the financial cycle is driven by larger financial shocks and that the estimated reduction in the size of the financial accelerator in the mid 1980s accounts for $30 \%$ of the decline in the volatilities of investment growth and the nominal interest rate.

In the last chapter, I assess the relative importance of financial shocks as drivers of the business cycle in the theoretical economy estimated in chapter 3. I conclude that financial shocks are not only the drivers of balance sheet variables in the business sector, but they also become the main sources of variability in investment during the Great Moderation era, relegating technology shocks to a secondary role. In this chapter, I also document the model implied propagation dynamics of financial shocks and its evolution over time. I obtain that the estimated reduction in the level of financial
rigidity reduces the contemporaneous effects of financial shocks but it enhances their persistence.

The model implied impulse response functions suggest that consumption and investment responses to a positive shock affecting business wealth are of opposite signs. This can be interpreted as being at odds with the common understanding of an expansionary financial shock. I estimate the effects of shocks to business wealth on consumption by imposing sign restrictions on the impulse response functions of investment, business wealth, and the cost of external borrowing. I obtain that expansionary financial shocks affecting net worth in the business sector have an ambiguous effect on household consumption.

## Chapter 2

## Technology Shocks, Statistical Models, and The Great Moderation

### 2.1 Introduction

Technology driven business cycles have been in the core of the Real Business Cycle (RBC) literature from its origins. For example, [54] claims that technology shocks account for more than a half of the US business cycle fluctuations over the postwar period. In [19], technology shocks account for about $75 \%$ of the volatility of output. Such an empirical success has been questioned by [29] and [30] among others. They claim that business cycle features are due mainly to non-technology factors. However, [35] started a new wave of attention on technology-driven business cycles by allowing for not only a neutral technology shock,I but also an investment-specific one. Recent contributions to the empirical macro literature, such as [44], show that investmentspecific technology shocks are the main driver of the US business cycle.

In this paper, I explore the performance of a simple model inspired by [36] under different specifications for the two technology processes. The goal is to determine which statistical model accounts better for the US business cycle features. In particular, I consider three different assumptions regarding the stochastic processes governing technological change. The first statistical model assumes trend stationarity allowing for any persistence level. In the second statistical model, impose difference stationarity by assuming that technological progress is described as a random walk with drift. Finally, I allow for autocorrelated errors in the unit root model. My analysis is in the spirit of [39]. He explores several specifications for the Solow residual and concludes that a trend stationary model accounts better for the US business cycle. [41] revisits [39]'s work by estimating the RBC model using maximum likelihood. He concludes that an increase in the persistence of technological progress improves the performance of the model in accounting for the variance of output and consumption. It deteriorates, however, the success at explaining the volatility of investment and hours worked. I build upon these two papers by incorporating into the analysis the investment-specific technology shock. As in [39], I perform a calibration exercise to assess the ability of the model to describe the US business cycle over the last 50 years.

I conclude, as [39], that trend stationary models account better for the volatility at business cycle frequencies of real variables. Difference stationary environments, however, perform better in capturing the correlation structure of the data. I highlight here that the model implied correlation between consumption and investment under stationary technological progress is at odds with the data. Such correlation, however, has a positive sign when technology shocks follow a random walk process. The statistical model also has a relevant impact on the relative importance of neutral and investment-specific shocks in accounting for the variance of real variables.

In particular, the relative contribution of the investment-specific shock to the cyclical variability of consumption, investment, capital, and hours worked is significantly larger under trend stationarity.

The US economy has been characterized by milder fluctuations over the past two decades. This phenomenon was dated by [45] and [49] and labeled as the Great Moderation by [57]. Thus, it is challenging to analyze the explanatory power of the statistical models of interest when the so called Great Moderation is at hand. I want to determine whether the slowdown in the volatility of the two shocks under analysis suffices to explain a significant part of the Great Moderation. [4] consider a basic RBC model à la [38] with only one technology shock. They conclude that the slowdown in the volatility of productivity shocks can account for about a $50 \%$ decline in business cycle volatility.

My results suggest that 'good luck' in the form of smaller innovations to the technology processes can account for the bulk of the volatility slowdown in my model. I estimate a reduction in the size of technology shocks of about $45 \%$. All specifications are able to generate a slowdown in cyclical volatility of significant magnitude. But the stochastic trend model with autocorrelated errors outperforms the other two statistical models at accounting for the Great Moderation.

The paper proceeds as follows. In section 2.2, I set up my baseline model. In section 2.3, I proceed with my calibration exercises using the three statistical models under analysis. Section 2.4 presents several counterfactuals in order to analyze the Great Moderation in the framework defined by my model economy. Section 2.5 concludes.

### 2.2 The Model

The model is a simplified version of the one proposed by [36]. In particular, I abstract from different capital goods and variable capital utilization. I do preserve the existence of both neutral and investment-specific technology shocks.

I consider three statistical versions of the baseline model in order to assess which one accounts better for the US business cycle features. First, I analyze a deterministic trend version of the model where the stochastic processes are trend stationary. Second, I consider a stochastic trend model where the technology processes follow a random walk with drift. Finally, I allow for some persistence in the innovation of the investment-specific technology in a stochastic trend model. Therefore, in the first case my economy is affected only by temporary shocks. In the second model, all shocks are permanent. In the last model, I am considering both permanent and transitory shocks. In particular, any neutral shock will be permanent, while any investment-specific shock will have both permanent and transitory effects.

Since [51], there has been a large empirical literature about stochastic trends in macro variables. Unit roots and stationary processes differ in their implications at infinite time horizons, but for any given finite sample, there is a representative from either class of models that can account for all the observed features of the data ${ }^{1}$. In addition, the lack of power of univariate classical tests for unit roots ${ }^{2}$ is well known. Therefore, I choose among the three specifications described above using the following criterion: the most preferred statistical model will be the one able to account for a larger proportion of the US business cycle properties.

[^0]In this economy, there is a continuum of households that maximize their expected lifetime utility given by

$$
\begin{equation*}
\mathbb{E}_{0}\left[\sum_{t=0}^{\infty} \beta^{t} U\left(C_{t}, H_{t}\right)\right] \tag{2.1}
\end{equation*}
$$

Both [39] and [41] use Hansen-Rogerson preferences ${ }^{3}$. I divert by using a specification rather conventional in the empirical macro literature.

$$
\begin{equation*}
U\left(C_{t}, H_{t}\right)=\ln C_{t}-B \frac{H_{t}^{1+1 / \nu}}{1+1 / \nu} \tag{2.2}
\end{equation*}
$$

where $C_{t}$ stands for consumption, $H_{t}$ for hours worked, $\nu$ for the short-run (Frisch) labor supply elasticity, and $B$ is a preference weight. It is well known that the log utility in consumption implies a constant long-run labor supply in response to a permanent change in technology. Hence, I do not have to worry about trending hours implied by the model even under the difference-stationary specification ${ }^{4}$.

The representative household supplies labor at the competitive equilibrium wage, $W_{t}$, and rents capital, $K_{t}$, to firms at rental rate, $R_{t}$. The capital stock depreciates at rate $\delta$. Therefore the representative household maximizes (2.1) subject to

$$
\begin{align*}
C_{t}+P_{t}^{k} X_{t} & =W_{t} H_{t}+R_{t} P_{t}^{k} K_{t}  \tag{2.3}\\
(1+\eta) K_{t+1} & =(1-\delta) K_{t}+X_{t} \tag{2.4}
\end{align*}
$$

where $P_{t}^{k}$ is the (relative) price of investment (using the consumption good as a numeraire) and $X_{t}$ stands for quality-adjusted investment. Note that while the budget constraint, equation (2.3), is expressed in consumption units, the capital accumulation equation, (2.4), is expressed in efficiency units. Population in this economy grows at

[^1]rate $(1+\eta)$.

There is also a continuum of firms that rent capital and labor services from households and produce consumption and investment goods. The representative firm solves the following problem:

$$
\begin{array}{ll}
\max & \Pi_{t}=C_{t}+P_{t}^{k} X_{t}-W_{t} H_{t}-R_{t} P_{t}^{k} K_{t} \\
\text { s.t. } & C_{t}+\frac{X_{t}}{V_{t}}=A_{t} K_{t}^{\alpha} H_{t}^{1-\alpha} \tag{2.6}
\end{array}
$$

where $A_{t}$ is the current level of (neutral) technology and $V_{t}$ stands for the current level of the investment-specific technology. Firms produce both consumption and investment goods only if $V_{t}=\frac{1}{P_{t}^{k}}$. A raise in $V_{t}$ implies a fall in the cost of producing a new unit of capital in terms of output, which can also be interpreted as an improvement in the quality of new capital produced with a given amount of resources. Note that investment in consumption units is defined as $I_{t}=P_{t}^{k} X_{t}$. Therefore, (2.6) is identical to the familiar resource constraint.

$$
Y_{t}=C_{t}+I_{t}=A_{t} K_{t}^{\alpha} H_{t}^{1-\alpha}
$$

Let us consider three statistical specifications for the stochastic processes governing the technology levels in this economy. In the deterministic trend model, technology processes are modeled as follows:

$$
\begin{aligned}
A_{t} & =A_{0} e^{\gamma_{a} t+\varepsilon_{a t}} \\
V_{t} & =V_{0} e^{\gamma_{v} t+\varepsilon_{v t}}
\end{aligned}
$$

where $\varepsilon_{a t}$ and $\varepsilon_{a t}$ are autoregressive processes. The explicit structure of the errors is
discussed in section 2.3.

In the stochastic trend version of the model, the processes are given by

$$
\begin{aligned}
A_{t} & =A_{t-1} e^{\gamma_{a}+\varepsilon_{a t}} \\
V_{t} & =V_{t-1} e^{\gamma_{v}+\varepsilon_{v t}}
\end{aligned}
$$

which implies that the log technologies evolve according to a random walk with drift. In the baseline stochastic trend model, the errors are assumed to be white noise. In the stochastic trend model with persistence, the log of investment-specific technology level is assumed to follow a random walk with drift and moving average component.

Under all the specifications, my model economy exhibits long-run growth. Therefore, I transform my economy so that I can work with a detrended version of the original one. In the trend stationary model economy, the following variables are stationary

$$
\frac{Y_{t}}{q^{t}}, \quad \frac{C_{t}}{q^{t}}, \quad \frac{I_{t}}{q^{t}}, \quad \frac{W_{t}}{q^{t}}, \quad \frac{K_{t}}{(q v)^{t}}, \quad H_{t}, \quad R_{t}
$$

where $q=e^{\frac{1}{1-\alpha} \gamma_{a}+\frac{\alpha}{1-\alpha} \gamma_{v}}$ and $v=e^{\gamma_{v}}$.

Let us denote a stationary variable $Z$ by $\tilde{Z}$. Therefore, the stationary equilibrium
conditions for this statistical version of the model are given by:

$$
\begin{align*}
\tilde{Y}_{t} & =\tilde{C}_{t}+\tilde{I}_{t}  \tag{2.7}\\
\tilde{Y}_{t} & =A_{0} e^{\varepsilon_{a t}} \tilde{K}_{t}^{\alpha} H_{t}^{1-\alpha}  \tag{2.8}\\
(1+\eta) q v \tilde{K}_{t+1} & =(1-\delta) \tilde{K}_{t}+V_{0} e^{\varepsilon_{v t}} \tilde{I}_{t}  \tag{2.9}\\
1 & =\beta E_{t}\left[\left(\frac{e^{\varepsilon_{v t}-\varepsilon_{v t+1}}}{q v}\right)\left(\frac{\tilde{C}_{t}}{\tilde{C}_{t+1}}\right)\left(1-\delta+R_{t+1}\right)\right]  \tag{2.10}\\
H_{t} & =\left(\frac{1}{B} \frac{\tilde{W}_{t}}{\tilde{C}_{t}}\right)^{\nu}  \tag{2.11}\\
R_{t} & =\alpha V_{0} e^{\varepsilon_{v t}} \frac{\tilde{Y}_{t}}{\tilde{K}_{t}}  \tag{2.12}\\
\tilde{W}_{t} & =(1-\alpha) \frac{\tilde{Y}_{t}}{H_{t}} \tag{2.13}
\end{align*}
$$

Given the detrended version of my economy, I can solve for the steady state. Let us denote the steady state value of a variable $Z$ by $Z^{*}$.

$$
\begin{align*}
Y^{*} & =C^{*}+I^{*}  \tag{2.14}\\
Y^{*} & =A_{0} K^{* \alpha} H^{*(1-\alpha)}  \tag{2.15}\\
(1+\eta) q v K^{*} & =(1-\delta) K^{*}+V_{0} I^{*}  \tag{2.16}\\
1 & =\beta\left(\frac{1}{q v}\right)\left(1-\delta+R^{*}\right)  \tag{2.17}\\
H^{*} & =\left(\frac{1}{B} \frac{W^{*}}{C^{*}}\right)^{\nu}  \tag{2.18}\\
R^{*} & =\alpha V_{0} \frac{Y^{*}}{K^{*}}  \tag{2.19}\\
W^{*} & =(1-\alpha) \frac{Y^{*}}{H^{*}} \tag{2.20}
\end{align*}
$$

Let us consider now the two difference-stationary models. [6] showed in a model with only one shock that any of the trending variables of these kinds of models can
be decomposed into a permanent component that is a random walk with drift (a stochastic trend) and a stationary stochastic process. In my case I have to take into account that the two stochastic processes have a unit root ${ }^{5}$. Hence, given such a statistical model, I have that the following variables are stationary

$$
\frac{C_{t}}{Q_{t}}, \quad \frac{I_{t}}{Q_{t}}, \quad \frac{Y_{t}}{Q_{t}}, \quad H_{t}, \quad R_{t}, \quad \frac{K_{t+1}}{Q_{t} V_{t}}, \quad \frac{W_{t}}{Q_{t}}
$$

where $Q_{t}=A_{t}^{\frac{1}{1-\alpha}} V_{t}^{\frac{\alpha}{1-\alpha}}$.
The stationary equilibrium conditions are:

$$
\begin{align*}
\tilde{Y}_{t} & =\tilde{C}_{t}+\tilde{I}_{t}  \tag{2.21}\\
\tilde{Y}_{t} & =\left(\frac{1}{q_{t} v_{t}}\right)^{\alpha} \tilde{K}_{t}^{\alpha} H_{t}^{1-\alpha}  \tag{2.22}\\
(1+\eta) \tilde{K}_{t+1} & =(1-\delta)\left(\frac{1}{q_{t} v_{t}}\right) \tilde{K}_{t}+\tilde{I}_{t}  \tag{2.23}\\
1 & =\beta E_{t}\left[\left(\frac{1}{q_{t+1} v_{t+1}}\right)\left(\frac{\tilde{C}_{t}}{\tilde{C}_{t+1}}\right)\left(1-\delta+R_{t+1}\right)\right]  \tag{2.24}\\
H_{t} & =\left(\frac{1}{B} \frac{\tilde{W}_{t}}{\tilde{C}_{t}}\right)^{\nu}  \tag{2.25}\\
R_{t} & =\alpha\left(q_{t} v_{t}\right) \frac{\tilde{Y}_{t}}{\tilde{K}_{t}}  \tag{2.26}\\
\tilde{W}_{t} & =(1-\alpha) \frac{\tilde{Y}_{t}}{H_{t}} \tag{2.27}
\end{align*}
$$

where

$$
\begin{align*}
& q_{t}=\frac{Q_{t}}{Q_{t-1}}=e^{\frac{1}{1-\alpha}\left(\gamma_{a}+\varepsilon_{a t}\right)+\frac{\alpha}{1-\alpha}\left(\gamma_{v}+\varepsilon_{v t}\right)}  \tag{2.28}\\
& v_{t}=\frac{V_{t}}{V_{t-1}}=e^{\gamma_{v}+\varepsilon_{v t}} \tag{2.29}
\end{align*}
$$

[^2]Given that the stationary version of the difference-stationary model satisfies the usual assumptions, I can solve for the steady-state of this transformed economy. Then,

$$
\begin{align*}
Y^{*} & =C^{*}+I^{*}  \tag{2.30}\\
Y^{*} & =\left(\frac{1}{q^{*} v^{*}}\right)\left(K^{*}\right)^{\alpha}\left(H^{*}\right)^{1-\alpha}  \tag{2.31}\\
(1+\eta) K^{*} & =(1-\delta)\left(\frac{1}{q^{*} v^{*}}\right) K^{*}+I^{*}  \tag{2.32}\\
1 & =\beta\left(\frac{1}{q^{*} v^{*}}\right)\left(1-\delta+R^{*}\right)  \tag{2.33}\\
H^{*} & =\left(\frac{1}{B} \frac{W^{*}}{C^{*}}\right)^{\nu}  \tag{2.34}\\
R^{*} & =\alpha q^{*} v^{*} \frac{Y^{*}}{K^{*}}  \tag{2.35}\\
W^{*} & =(1-\alpha) \frac{Y^{*}}{H^{*}} \tag{2.36}
\end{align*}
$$

where $q^{*}=e^{\frac{1}{1-\alpha} \gamma_{a}+\frac{\alpha}{1-\alpha} \gamma_{v}}$ and $v^{*}=e^{\gamma_{v}}$

### 2.3 Calibration

### 2.3.1 Data set

I use the data set constructed by [55]. They use data from NIPA-BEA, FAT-BEA, BLS, and [20] to construct quarterly series of investment-specific technological change and neutral technological change. Basically, they construct a series for the relative price of investment (in terms of the consumption good) that spans from 1948.I to 2006.IV and then proceed with a growth accounting exercise to recover the neutral technological change series. While the investment-specific process is approximated by the inverse of the (relative) price of investment, the neutral technology process is associated with the Solow residual of the economy.

In the literature, there can be found different ways of computing the quarterly Solow residual. [19] claim that as the BEA produces only annual estimates for the capital stock, any quarterly series introduces additional noise in the measure of the Solow residual. Therefore, they propose a 'conservative' approach by omitting capital when computing the neutral technology process. This approach has been widely used in the literature, for a recent example see [4]. [34] establish that another justification for omitting capital could be measurement errors. However, mismeasurement affects the level of the capital stock but not its time series properties. Thus, other approaches construct quarterly capital series by iterating on the law of motion for capital. Note that as [35] point out, I have to be careful when constructing my capital stock series since it must be in efficiency units. In the data base, capital stock series is constructed recursively using the perpetual inventory method

$$
K_{t+1}=(1-\delta) K_{t}+X_{t}
$$

where $X_{t}$ is the total nominal investment deflated by the quality-adjusted price of investment. Therefore, $X_{t}$ stands for investment in efficiency units. $\delta$ is the average depreciation rate of the time-varying physical depreciation rates for total capital available from [20]. The initial capital stock in efficiency units is calibrated using the steady-state investment equation.

I first perform my calibration exercise matching moments of the whole sample, ranging from 1948 to 2006. But, as I state in the introduction to this paper, the US economy has been characterized by milder business cycle fluctuations since the mid 1980s. There is a consensus in the empirical macro literature on dating the Great Moderation as a regularity starting in 1984. Therefore, I also conduct my analysis by dividing the sample in 1984. In this way, I can test whether the empirical success of
my model in delivering the business cycle features characterizing the US economy is homogenous across subsamples. In addition, I can study the ability of the model in delivering the observed slowdown in aggregate volatility.

### 2.3.2 Deterministic trend model

I consider the following statistical specification:

$$
\begin{aligned}
\ln A_{t} & =\ln A_{0}+\gamma_{a} t+\varepsilon_{a t} \\
\ln V_{t} & =\ln V_{0}+\gamma_{v} t+\varepsilon_{v t}
\end{aligned}
$$

which has been estimated using the following econometric strategy:

1. Regress each technological change series on a constant and a linear time trend

$$
\begin{align*}
\ln A_{t} & =\varphi_{a}+\gamma_{a} t+\varepsilon_{a t}  \tag{2.37}\\
\ln V_{t} & =\varphi_{v}+\gamma_{v} t+\varepsilon_{v t} \tag{2.38}
\end{align*}
$$

2. Generate the corresponding residual series $\left\{\hat{\varepsilon}_{a t}\right\}$ and $\left\{\hat{\varepsilon}_{v t}\right\}$.
3. Estimate univariate autoregressive processes for those shocks

$$
\begin{align*}
& \varepsilon_{a t}=\rho_{a} \varepsilon_{a t-1}+\xi_{a t}  \tag{2.39}\\
& \varepsilon_{v t}=\rho_{v 1} \varepsilon_{v t-1}+\rho_{v 2} \varepsilon_{v t-2}+\xi_{v t} \tag{2.40}
\end{align*}
$$

where $\xi_{a} \sim \mathcal{N}\left(0, \sigma_{\xi_{a}}^{2}\right)$ and $\xi_{v} \sim \mathcal{N}\left(0, \sigma_{\xi_{v}}^{2}\right)$. The lag structure for the errors has been chosen following the Akaike Information and the Bayesian Information Criteria.

The estimated parameters are reported in table 2. I observe that in the post-1984 period there has been a $48 \%$ reduction in the volatility of the innovation to the neutral technology and a $40 \%$ reduction in the volatility of the innovation to the investment-specific technology. I analyze in section 2.4 if such a reduction in innovations' volatilities suffices to explain the slowdown in the volatility of the real variables of interest.

In my model the vector of parameters is given by

$$
\left(\alpha, \gamma_{a}, \gamma_{v}, \beta, \delta, B, \nu, \eta, \mu, \varphi_{a}, \varphi_{v}, \rho_{a}, \rho_{v 1}, \rho_{v 2}, \sigma_{\xi_{a}}, \sigma_{\xi_{v}}\right)
$$

where $\mu$ is a scaling parameter chosen so that steady state output is equal to 1 . I can estimate $\left(\alpha, \gamma_{a}, \gamma_{v}, \eta, \varphi_{a}, \varphi_{v}, \rho_{a}, \rho_{v 1}, \rho_{v 2}, \sigma_{\xi_{a}}, \sigma_{\xi_{v}}\right)$ from the data. In order to calibrate the remaining parameters I consider the targets specified in table 1.

Given my specification, I cannot calibrate both $\nu$ and $B$. In fact, my calibrated $B$ will be conditional on the choice for the Frisch elasticity parameter. In the literature I find values for such a parameter in a wide range encompassing values between 0.2 and $\infty$. To keep the analysis simple, I simulate my model considering a small grid for the labor supply elasticity. In particular, $\nu=\{0.5,1,1.5,2\}$. The calibrated parameters are reported in table 2 .

Table 6 in appendix A. 1 reports my results for the grid over the short-run elasticity of labor supply, $\nu$. The ability of my model to account for the US business cycle features is sensitive to the value of the parameter governing the Frisch elasticity of labor supply. Cyclical volatility of all variables but consumption and labor productivity are a positive function of the short-run elasticity of labor. In particular, the volatility of investment in efficiency units, output, and hours worked are significantly closer to the observed variability under $\nu=2$ than with $\nu=0.5$.

The deterministic trend model is able to account for some relevant features of US business cycles irrespective of my choice for $\nu$. In particular, the model accounts for the large fluctuations of investment compared to output and for the small fluctuations of capital and consumption compared to output.

The standard deviation of hours implied by the model is smaller than the standard deviation of labor productivity which is at odds with the data. This is, however, a typical feature of RBC models with utility non-linear in hours. [39]'s deterministic trend model was able to account for the pattern in the data by assuming that labor is indivisible and that agents trade employment lotteries ${ }^{6}$.

The trend stationary model generates too much volatility in consumption in the first subsample for any value of the Frisch elasticity. For $\nu=\{1,1.5,2\}$, the model overestimates capital volatility for the pre-1984 sample.

Finally, this statistical version of my baseline RBC model cannot generate enough correlation between output and consumption. It generates, however, a large correlation between labor productivity and output that is at odds with the data. Moreover, the model cannot account for the change in sign in such a correlation in the second sub-sample.

[^3]
### 2.3.3 Stochastic trend model

## Random walk with drift

Following [46] when addressing the difference stationary specification, I restrict my attention to the following class of parametric forms

$$
\Phi(L)(1-L) \log \left(X_{t}\right)=\gamma_{x}+\Theta(L) \varepsilon_{x t}
$$

where $\Phi(L)$ and $\Theta(L)$ are lag polynomials whose roots are outside the unit circle. The statistical model to be considered in this section is as follows

$$
\begin{aligned}
\ln A_{t} & =\ln A_{t-1}+\gamma_{a}+\varepsilon_{a t} \\
\ln V_{t} & =\ln V_{t-1}+\gamma_{v}+\varepsilon_{v t}
\end{aligned}
$$

which can be rewritten as

$$
\begin{aligned}
\ln A_{t} & =\ln A_{0}+\gamma_{a} t+\sum_{i=0}^{t} \varepsilon_{a t-i} \\
\ln V_{t} & =\ln V_{0}+\gamma_{v} t+\sum_{i=0}^{t} \varepsilon_{v t-i}
\end{aligned}
$$

Note that any shock to the stochastic trend at time $t$ has a permanent effect in the log-level of the technology processes. Therefore, I am abstracting from transitory shocks in this specification which implies that I am just analyzing a lower bound of the effects of technology shocks.

Following [26] and [25], I assume

$$
\begin{equation*}
\binom{\varepsilon_{a t}}{\varepsilon_{v t}} \sim \mathcal{N}\left[\binom{0}{0}, \mathbf{D}\right] \tag{2.41}
\end{equation*}
$$

where $\mathbf{D}$ is a diagonal matrix i.e.

$$
\mathbf{D}=\left(\begin{array}{cc}
\sigma_{a}^{2} & 0 \\
0 & \sigma_{v}^{2}
\end{array}\right)
$$

My estimates are reported in table 3. Under this specification, I estimate a reduction in the volatility of the innovations to the technology shocks of about $48 \%$. In this version of the baseline RBC model, my calibration targets are identical to the ones in the previous subsection. The calibrated parameters are given in table 3.

In table 7 of appendix A.1, I report the results for the different values of the Frisch elasticity. The results for the volatility of output, investment, capital, and hours are also sensitive to the value of such a parameter. This statistical specification accounts for the same qualitative features of the US business cycle as the deterministic trend version.

The difference-stationary model does not overpredict the volatilities of consumption and capital. In fact, this statistical version of the model generates lower volatilities for all the variables than the trend stationary one. In addition, the stochastic trend model is successful in accounting for the correlation of consumption and output. But it shares with the deterministic trend model the remaining unmatched features.

## Random walk with drift and moving average component

Following [11], I allow for a moving average component in the unit root specification for the investment-specific technology process. Thus, (2.41) can be substituted by

$$
\begin{equation*}
\ln V_{t}=\ln V_{t-1}+\gamma_{v}+\rho \varepsilon_{v t-1}+\xi_{t} \tag{2.42}
\end{equation*}
$$

I do not modify my statistical specification for the neutral technology process since there is no empirical evidence for the inclusion of a moving average component in such a representation.

Note that (2.42) allows for both temporary and permanent shocks. In particular, a fraction $1 /(1-\rho)$ of any innovation to the investment-specific shock is permanent. The remainder has a temporary effect.

My estimation results are reported in table 4. I also observe here a reduction in the volatility of the innovations to the technology shocks of about $56 \%$ for the investment-specific technology and $48 \%$ for the neutral one.

The results over the grid for the elasticity of labor supply with respect to real wage are reported in table 8 in appendix A.1. This version of the stochastic trend model shares all the 'virtues' of the baseline stochastic trend model and improves upon some of its shortcomings. For example, the volatility of hours is larger than in the baseline difference-stationary model.

### 2.3.4 Comparing statistical models

From my previous analysis, I can conclude that irrespective of the value for $\nu$, all the statistical models are able to qualitatively reproduce the slowdown in volatility. While the baseline difference-stationary model implies a reduction in the volatility of the
variables at hand of about $52 \%$, the trend-stationary model overpredicts the slowdown for all the variables but output. Even though the baseline stochastic trend model outperforms the other two statistical specifications, it over predicts the slowdown in capital, hours, and labor productivity. The model implies a $48 \%$ reduction while in the data I observe about a $35 \%$ slowdown.

To continue my analysis let us set the Frisch elasticity parameter equal to 1. I have chosen only one value in the grid for expositional purposes. Table 9 reports how much volatility each model is able to account for. I observe that the trend-stationary model outperforms the difference-stationary models for the volatility of all variables but labor productivity. Notice that the stochastic trend model with a moving average component performs relatively better than the baseline stochastic trend model in the first sub-sample under analysis.

In table 10, I report the variance decomposition for the different specifications under analysis. It is remarkable that for the deterministic trend model the investmentspecific shock is the main contributor to the variance of consumption, capital, and hours. Therefore, I conclude that if I were interested in matching volatility levels using a simple level stationary RBC model, I should include not only the usual neutral productivity shock but also an investment specific disturbance. Note that for the stochastic trend versions of my model, the neutral shock accounts for the bulk of the variance for all variables. Therefore, failing to include an investment shock will not worsen the results as much as it would under a deterministic trend environment.

Figures A-1 and A-2 are the impulse response functions for the deterministic trend version and the baseline stochastic trend one. The responses to a neutral innovation only differ in the steady state to which each economy converges. Short run dynamics of consumption, hours, and labor productivity in response to an investment-specific shock are richer in a level stationary environment than in a difference stationary one.

That would help to explain that the deterministic trend model accounts better for macro volatilities.

Let us now analyze the performance of the statistical specifications of my RBC model in terms of accounting for the correlation structure of the data. From table 11, I can conclude that all versions do a similar job replicating the correlation between all the variables of interest and output but consumption. While the stochastic trend versions of the baseline model are able to account fairly well for the correlation between consumption and output, the deterministic trend version falls too short. All the different specifications of the RBC model under analysis perform very poorly in matching the low correlation between output and labor productivity. Moreover, none of them is able to reproduce the change in sign I observe in the post-1984 period.
[39] concluded that the deterministic trend model is the best one accounting for correlations of all the variables with output. Conversely, from my results I conclude that the stochastic trend model outperforms the deterministic trend one.

Given the counterintuitive result obtained for the correlation between consumption and output for the deterministic trend model, I have explored the cross-correlations with output for five lags and leads, and the correlations of other pairs of variables.

Table 15 reports the cross-correlations with output for lags and leads. I conclude that the results for all versions of the model are similar for all variables but consumption. Not only the deterministic trend under predicts the correlation between consumption and output for the current period, but also under predicts for all lags and leads. The stochastic versions of the model, however, account for the relative magnitude and signs at all lags and leads.

Table 12 reports the correlations for different pairs of variables. As expected, none of the versions of the model can capture any of the correlations with labor productivity. For all the other moments not involving consumption, the performance
of all the statistical specifications is fairly uniform. Let us give a closer look to the correlations with consumption. First of all, the deterministic trend model predicts negative correlations between consumption and investment in efficiency units and hours which are at odds with the data. The stochastic specifications account correctly for the sign of the moments of interest. Secondly, I should stress out here that while the stochastic trend model with a moving average component can account for the relative magnitude of the increase in the correlation between consumption and investment, capital, and hours across subsamples, the baseline stochastic trend model fails to do so except for capital.

Given the above, I can conclude that choosing one specification over the others depends upon what I am attempting to explain. If I were interested in matching volatilities I would choose, as [39], the deterministic trend model. However, I would need to include in my RBC model not only a neutral productivity shock, but also an investment-specific one. If I wanted to match correlations ${ }^{7}$, I would choose a stochastic trend model. Finally, if I wanted to match the magnitude of the volatility slowdown in the 1980s, I would also choose a stochastic trend model since the model implied slowdown is the closest to the observed one.

### 2.4 The Great Moderation

So far, I have performed my analysis allowing for changes in all the structural parameters over the two subsamples of interest. In such a way I have shown that any of the statistical versions of my RBC model is able to account for a slowdown in macro volatilities. However, I am more interested in analyzing which percentage of

[^4]the performance of my model due only to 'good luck'.
Thus, to better assess the relative importance of each technology shock in explaining the Great Moderation, I perform some counterfactuals in the spirit of the ones performed by [4]. In particular, I proceed with three experiments in two scenarios. In the first scenario, I calibrate the parameters of the model to match the targets for the whole sample (i.e., I fix them equal to the first column of tables 2, 3, and 4). In the second scenario, I allow for time variation in the coefficients of the laws of motion for the technology processes.

In the first counterfactual I analyze the explicative power of the neutral technology shock. To do so, I set the volatility of the innovation to the investment-specific technology to match its volatility for the entire sample. The standard deviation of the neutral innovation, however, changes across subsamples. The second counterfactual is analogous to the first one but I focus on the investment-specific technology shock. Finally, in the third counterfactual I explore the explicative power of both shocks jointly by letting their standard deviations vary across subsamples.

The results under time invariant coefficients are reported in table 16. For the first experiment, I observe that while the stochastic trend models can reproduce a large fraction of the slowdown observed in the data, the trend-stationary model does only an acceptable job of accounting for the slowdown in output and labor productivity volatilities. My main conclusion from this experiment is that smaller neutral technology innovations suffice to explain a large proportion of the aggregate stability observed in the mid 1980s if the model economy is difference stationary.

From the second counterfactual, I conclude that the role of the investment-specific shock as a single actor is greatly reduced. For example, for the deterministic trend case I have that although the investment-specific shock is $62 \%$ as volatile in the post1984 as in the pre-1984 period, this has a very small effect on the volatility of output,
investment, and labor productivity. However, I observe a reduction of about $22 \%$ in the volatility of consumption, capital, and hours. The relevance of the investment shock to explain the slowdown in real variables in my difference stationary economy is almost negligible.

Using the third experiment, I can quantify the relative importance of the interaction between the two shocks active in my model economy. Here all the models are able to imply volatility slowdowns relatively similar to the ones in the data which implies that the interaction between the two technology shocks is significant.

Let us now perform the same counterfactuals but allowing for time variation not only in the volatilities of the innovation processes, but also in the laws of motion of the technology processes. Results are reported in table 17. The results are qualitatively similar to the ones explained previously. On the one hand, the investment shock in a difference stationary economy is not sufficient to induce a slowdown in macro volatilities of a similar magnitude to the ones observed in the data. The role of such a shock is larger for a level stationary economy. It is remarkable that the role of the investment shock is larger when the law of motion of the technology level is time-varying than when it is assumed to be fixed across subsamples. From the last experiment, I can conclude again that the stochastic trend model accounts better for the magnitude of the slowdown than the deterministic trend one. In this environment, the slowdown implied by the level stationary model is not only larger than the one observed in the data, but also larger than the one implied by the model under timeinvariant laws of motion.

I conclude that while the neutral shock is the main driving force in the slowdown in volatilities generated by my difference-stationary model, allowing for a larger financial flexibility in the form of milder investment-specific shocks substantially improves its ability to reproduce the magnitude of the observed slowdown. Such a financial
flexibility plays an even larger role in a level stationary economy since not only enhances the slowdown due to the neutral shock, but also it is the main driving force in the slowdown of consumption, capital, and hours volatility. Therefore, the Great Moderation in my setting is not due only to 'good luck' but also to the interaction between the two technology shocks.

### 2.5 Conclusion

I find that the choice of the statistical model for the stochastic processes in an RBC model with two technology shocks is not a trivial one. In fact, one model would be preferred to the others depending on the features of the business cycle the researcher wants to match.

I conclude that even though the neutral technology shock is the main driving force in replicating the Great Moderation, having both technology shocks translate into a better accounting for such a macroeconomic phenomenon. Therefore, the cross effects seem to be relevant. However, a bivariate specification of the innovations to the technology processes does not translate into a significative improvement of the performance of the model under analysis(see appendix A.6.2).

I have shown that in a simple RBC model the two technology shocks can explain approximately $70 \%$ of the observed slowdown in volatilities of US macro variables in mid 1980s. The remaining $30 \%$ could be explained as suggested in the literature by a reduction in the standard deviation of other shocks such as preference shocks, by an improved financial environment, or by good policy. Discriminating among those alternatives requires a richer model which is beyond the scope of my analysis.

## Chapter 3

## Financial Frictions, the Financial Immoderation, and the Great Moderation

### 3.1 Introduction

The U.S. economy over the 1954-2006 period has been characterized by two empirical regularities. On the one hand, since the mid 1980s, fluctuations at business cycle frequencies for real and nominal variables are milder. This decline in macroeconomic volatility defines the so-called Great Moderation. On the other hand, financial variables have become more volatile over time. [42] document an increase in the volatility of debt and equity financing in the nonfarm business sector contemporaneous with the slowdown in the amplitude of the real cycle. In this paper, I reconsider the study of the balance-sheet data for the nonfarm business sector along with other financial variables, such as balance-sheet data for households, net private savings, and demand deposits at commercial banks. I document that the widening of the financial cycle
starts in 1970. I label this second empirical regularity the Financial Immoderation.
I account for those divergent patterns in volatility by means of a structural model. I consider a model featuring a standard set of real and nominal frictions as in [56] extended to accommodate financial rigidities as in [5]. I enrich the theoretical environment by including financial shocks affecting the spillovers of credit market imperfections on the economy. This theoretical framework allows us to quantify the relative role played by financial factors, monetary policy, and economic shocks in shaping the evolution of aggregate volatility. To do so, I estimate my model using a data set containing real, nominal, and financial variables. To account for the breaks in the second moments of the data, I allow for structural breaks in the average level of financial rigidity, coefficients in the monetary policy rule, and the size of shocks. As a byproduct of my analysis, I can not only characterize the propagation mechanism of financial shocks in the US economy, but also study its evolution over the last 50 years.

One of the main objectives of this paper is to quantify the relative role played by financial factors in shaping macroeconomic and financial volatilities. However, the workhorse dynamic stochastic general equilibrium (DSGE) model used in the literature abstracts from interactions between credit markets and the rest of the economy. This benchmark macroeconomic model is based on the capital structure irrelevance theorem by [50]; that is, the composition of agents' balance sheets has no effect on their optimal decisions. Nevertheless, episodes such as the Great Depression or the current financial turmoil stand as compelling evidence of the linkage between the developments in the financial and real sectors. Along these lines, recent contributions to the literature have focused on incorporating credit markets in the workhorse DSGE model. For example, [5] and [40] stress the relevance of the balance sheet's condition in determining economic activity. The ability to borrow depends upon borrowers'
wealth, which ultimately affects the demand for capital and the level of economic activity they can engage in.

Following [16], I consider a theoretical framework with real and nominal rigidities as in [56] enriched with frictions in the credit market à la [5]. In this environment, asymmetric information between borrowers and lenders arises because the return to capital depends not only on aggregate but also on idiosyncratic risk. While borrowers freely observe the realization of their idiosyncratic productivity shock, lenders must pay monitoring costs to observe the realized return of a borrower. To minimize monitoring costs, lenders audit borrowers only when they report their inability to pay the loan back under the terms of the contract. In order to be compensated for the risk of default, lenders extend loans at a premium over the risk-free interest rate. The composition of borrowers' balance sheets determines the external finance premium at which the loan is settled. The lower an entrepreneur's net worth (collateral) with respect to her financing needs, the higher the premium required in equilibrium. The external finance premium is at the heart of the mechanics operating in the financial accelerator emphasized by [5]. The financial accelerator hypothesis states that credit market imperfections amplify and propagate economic shocks. For example, in an economic downturn, borrowers' wealth deteriorates because of the decline in asset prices. Such a reduction in the value of collateral translates into a higher premium requested by lenders. Relatively more expensive credit reduces the incentives to engage in investment activities, depressing output production even further. The latter generates an additional drop in asset prices, which feeds the chain again.

In a model à la [5], the external finance premium is driven by two channels: the balance-sheet channel and the information channel. The balance-sheet channel captures the dependence of external financing opportunities on the composition of firms' balance sheets. The information channel implies that the external finance
premium is a positive function of the severity of the agency problem. I enrich the DSGE model by introducing financial shocks affecting those two channels. Exogenous shocks to the balance-sheet channel are introduced in the form of wealth shocks. Shocks to the information channel are modeled as innovations affecting the parameter governing agency costs. In this paper, I study the relative role played by those two shocks in shaping the evolution of aggregate volatility. I also analyze the propagation mechanism of the two financial shocks in the US economy.

I estimate the model economy using Bayesian techniques on a standard data set of real and nominal variables extended to include a series for firms' net worth. I need to take a stand on defining the empirical equivalent to such a model variable. I focus on the data provided by the Flow of Funds Accounts to define net worth as tangible assets minus credit market liabilities for the nonfarm business sector, measured in real per capita terms. As I have stated above, I perform the estimation exercise using the whole data sample, but I allow for structural breaks in the variances of the shocks, the coefficients in the monetary policy rule, and the average size of the financial accelerator. Therefore, I consider three explanations for the Financial Immoderation and the Great Moderation: changes in the size of shocks, changes in the conduct of monetary policy, and changes in the US financial system.

The main empirical findings of the paper are the following. Financial factors play a significant role in shaping financial and macroeconomic volatilities. Financial shocks are the only driver of the variance of financial flows. Therefore, the increase in fluctuations at business cycle frequencies for balance-sheet variables is driven by larger financial shocks hitting the US economy.

I also find that while the average level of financial rigidities do not change in the 1970s, the estimated decrease in the mid 1980s is more than $75 \%$. This decline accounts for more than $30 \%$ of the reduction in the cyclical volatility of investment
and the nominal interest rate. The effect on the remaining variables is, however, negligible.

This paper relates to two strands of the empirical macro literature. The first strand addresses the study of the Great Moderation, that is, the evolution of volatilities at business cycle frequencies during the second half of the last century. The second strand considers the estimation of the financial accelerator model.

Since [45] and [49] dated the start of the Great Moderation, there has been a growing literature on dissecting the possible sources of such a mildness in real business cycle fluctuations. Recent contributions have focused on analyzing the link between financial innovations and aggregate volatility. My paper is along the lines of [42] and [21], who consider credit market frictions only for firms. In particular, I obtain an estimated reduction in the average level of financial rigidities during the Great Moderation similar to the ones provided by those two papers.

The literature on bringing the financial accelerator by [5] to the data through an estimation exercise is less vast than the literature on the Great Moderation. Most of the contributions estimate the theoretical environment using only nominal and real variables and focusing on data from the Volcker-Greenspan era. To the best of my knowledge, besides the study of the Great Depression by [16], the only reference using pre-1980s data is the recent work by [33], whose sample spans 1973 to 2008. They do not address, however, the break in second moments of the data observed in the mid 1980s.

The plan of the paper is as follows. Section 3.2 presents the empirical evidence that motivates the paper. I describe the model in Section 3.3. Section 3.4 discusses the choice of parameters allowed to change over time. I describe the estimation procedure and report the estimation results in Section 3.5. Section 3.6 analyzes the drivers of the divergent patterns in volatility. Section 3.7 concludes.

### 3.2 Empirical Motivation

This section presents the empirical evidence that motivates the paper. It characterizes real, nominal, and financial cycles over the period 1954-2006. I do not consider more recent data for reasons of data accuracy. Revisions of NIPA data within a year of publication and of Flow of Funds Accounts within two or three years of publication are often considerable. In addition, at the end of the sample it is difficult to distinguish trend breaks from cycles.

I set the empirical characterization considering two structural breaks in the data: 1970 and 1984. Let us start by motivating the choice of 1984. Since the contributions by [45] and [49], there has been a consensus in the empirical macro literature about the existence of a break in the second moments characterizing real and nominal cycles around 1984. [57] popularized it as the starting point of the Great Moderation.

The choice of the break in 1970 is based on several observations. First, analyzing the evolution of the cyclical component of balance-sheet variables such as the debt-to-net-worth ratio reported in Figure B-1, I conclude that the cycle becomes wider in the 1970s. Moreover, both inflation and the federal funds rate are more volatile in the 1970s and early 1980s. The high and volatile inflation over the period has been the subject of careful study by researchers such as [15], [52], and [18], among others.

Second, the 1970s are convulsive years in US economic history. There were significant changes not only in the financial system but also in other areas of the economic system. In the financial arena, the 1970s was the decade of the introduction of ATMs, phone transfers for savings balances at commercial banks, NOW (negotiable order of withdraw) accounts, money market certificates with yields tied to US Treasury securities, IRAs (individual retirement accounts), MMMF (market money mutual funds), incorporation of the NYSE, a partial lifting of Regulation Q, the Securities Protection

Act, the Financial Institutions Regulatory and Interest Control Act, the Electronic Fund Transfers Act, the International Banking Act, the Bankruptcy Reform Act, etc. At the same time, the US experienced the collapse of the Bretton Woods currencyexchange mechanism, the appointment of Burns as chairman of the Federal Reserve System after 19 years of Martin, the end of the Vietnam war, the oil crises, the stagflation episode, several government bailouts of the automobile and aviation industries, and the start of the service economy. Therefore, testing for a break at the beginning of the 1970s seems a natural candidate.

Tables 23 and 24 report Chow tests on the average squared residuals of regressing an $\operatorname{AR}(1)$ with drift for the definition of the variables of interest used in the estimation exercise and their cyclical component, respectively. I reject the null of parameter constancy when testing for a break in 1970 for net worth, inflation, and the federal funds rate for both definitions of the variables. I reject the null for all variables except labor share using the two definitions at hand when the break is set in 1984. Finally, I also reject the null for all variables except labor share when considering the two breaks jointly. In particular, the log-likelihood ratio statistic is larger for this scenario than when considering single breaks. Therefore, I can conclude that the data are best represented by a scenario that allows for two breaks in second moments.

I report in Table 25 the ratio of standard deviations of the cyclical component for a set of real, nominal, and financial variables. Although the focus of my paper is on financial variables related to the nonfarm business sector, I analyze here a broader data set, including net worth of households, net private savings, and demand deposits at commercial banks. Following [42], I report in the first column of Table 25 the ratio of cyclical standard deviations when only a break in 1984 is considered. All the variables included in my data set deliver the patterns described by [42]; that is, there is a contemporaneous moderation in the real side of the economy and an
exacerbation in the volatility of financial variables. The magnitude of the changes is also along the lines of the results provided by those authors. The novelty of my analysis is the consideration of two breakpoints. The second and third columns of Table 25 report the relevant statistics to characterize the three subperiods of interest: 1954-1969, 1970-1983, 1984-2006. Therefore, in the remainder of this section I focus my discussion on analyzing the information provided by the last two columns of the table.

Let us start by comparing the standard deviation of the cyclical component in the 1970-1983 sample period with that of the 1954-1969 era. The volatility of real variables is, on average, $50 \%$ greater in the 1970s and early 1980 s than in the pre1970 period. Nominal variables are also more volatile in the 1970-1983 sample period, but the increase in their cyclical volatility is greater than the one observed for real variables. In particular, the standard deviation of the cyclical component of both inflation and nominal interest rates more than doubles in the 1970s and early 1980s with respect to the 1950s and 1960s. Finally, all financial variables are also more volatile over the second sample period. The more dramatic change is the one experienced by demand deposits at commercial banks whose variability triples in the 1970-1983 sample period.

In the last column of Table 25, I compare the standard deviations of the cyclical components for the post-1984 period with that of the 1970-1983 sample period. The volatility of consumption, investment, and output decreases by about $55 \%$. This result is what characterizes the Great Moderation per se. The slowdown in the cyclical variability of hours and labor share is milder. Nominal variables, also in this case, follow the pattern of change of real variables. Financial variables, however, are more volatile in the 1984-2006 sample period. The most significant increases in cyclical variability are the ones for net worth for the nonfarm business sector and net
private savings. Both of them are $45 \%$ more volatile in the Great Moderation era than in the Great Inflation period (1970-1983). Therefore, I can state that the post-1984 period is characterized by an additional increase in the volatility of financial variables at business cycle frequencies.

I can summarize the empirical regularities present in the US aggregate data over the 1954-2006 period as follows. The first subperiod, 1954-1969, is characterized by relatively stable inflation and interest rates. The 1970-1983 sample period constitutes the first stage of the Financial Immoderation. In this period, fluctuations at business cycle frequencies of real, nominal, and financial variables become wider. The last subperiod expands from 1984 to the end of the sample. It is characterized by the coexistence of the second stage of the Financial Immoderation and the Great Moderation.

### 3.3 The Model

My theoretical framework features real and nominal rigidities as in [56] and [12]. However, to assess the role played by financial frictions in the evolution of volatilities in the US economy, I extend the framework including financial rigidities as in [5]. Financial frictions arise because there is asymmetric information between borrowers and lenders. Following Townsend's (1979)'s costly state verification framework, I assume that while borrowers freely observe the realization of their idiosyncratic risk, lenders must pay monitoring costs to observe an individual borrower's realized return.

Since [16] integrated the financial accelerator mechanism of [5] in the workhorse DSGE model, several studies have focused on assessing the empirical relevance of the financial accelerator by comparing the model fit with that of the workhorse DSGE model or on studying the propagation of real and nominal shocks. In this paper, I
focus the analysis on two issues: the role of financial shocks and the model's potential to account for breaks in the second moments of the data. I incorporate in the theoretical framework a shock to firms' wealth and a shock to agency costs. While the former has been previously studied, the inclusion of the latter is a major novelty of this paper.

My model economy is populated by households, financial intermediaries, entrepreneurs, capital producers, intermediate good firms, retailers, and government. Entrepreneurs are the only agents able to transform physical capital into capital services to be used in production. They purchase capital from capital producers and rent it to intermediate goods firms. Capital acquisition can be finance using internal financing and external borrowing. Financial intermediaries capture funds from households in the form of deposits and lend them to entrepreneurs. Intermediate goods firms carry out production by combining capital and labor services. Retailers generate the final good of this economy by combining intermediate goods. The government conducts both fiscal and monetary policy. In order to have non-neutrality of monetary policy, I need to include a nominal rigidity in a monopolistically competitive sector. Assuming entrepreneurs have market power would make it more difficult to solve for the debt contract. Hence, I introduce sticky prices in the intermediate good sector instead.

### 3.3.1 Households

I assume there is a continuum of infinitely lived households whose length is unity. They work, consume, invest savings in a financial intermediary in the form of deposits that pay a risk-free rate of return, purchase nominal government bonds, receive dividends from their ownership of firms, pay lump-sum taxes, and obtain (give) wealth transfers from (to) entrepreneurs. The representative household chooses a plan
for $\left\{C_{t}, D_{t+1}, H_{t}, N B_{t+1}\right\}$ to maximize her expected discounted lifetime utility

$$
\begin{equation*}
\mathbb{E}_{t} \sum_{j=0}^{\infty} \beta^{j} b_{t+j}\left[\ln \left(C_{t+j}-h C_{t+j-1}\right)-\theta_{t+j} \frac{H_{t+j}^{1+1 / \nu}}{1+1 / \nu}\right] \tag{3.1}
\end{equation*}
$$

subject to

$$
\begin{equation*}
C_{t}+D_{t+1}+\frac{N B_{t+1}}{P_{t}} \leq \frac{W_{t}}{P_{t}} H_{t}+R_{t-1} D_{t}+R_{t-1}^{n} \frac{N B_{t}}{P_{t}}+\operatorname{div}_{t}-T_{t}-\operatorname{Trans}_{t}( \tag{3.2}
\end{equation*}
$$

where $C_{t}$ stands for consumption, $h$ for the degree of habit formation, $D_{t+1}$ for today's real deposits in the financial intermediary, $H_{t}$ for hours worked, $\nu$ for the Frisch elasticity of labor, $b_{t}$ for a shock to the stochastic discount factor, $\theta_{t}$ for a labor supply shifter, $P_{t}$ for the price level of the final good, $\frac{W_{t}}{P_{t}}$ for real wage, $R_{t}$ for the risk-free real interest rate paid on deposits, $R_{t}^{n}$ for the risk-free nominal interest rate paid on government bonds, $N B_{t}$ for nominal government bonds, $T_{t}$ for real taxes (subsidies) paid to (received from) the government, $d i v_{t}$ for dividends obtained from ownership of firms, and Trans $_{t}$ for wealth transfers from/to the entrepreneurial sector. The nature of these transfers is described in section 3.3.5.

The shock to the labor supply, $\theta_{t}$, affects the intratemporal tradeoff between leisure and consumption. It is assumed to evolve as

$$
\begin{equation*}
\ln \left(\theta_{t}\right)=\left(1-\rho_{\theta}\right) \ln (\theta)+\rho_{\theta} \ln \left(\theta_{t-1}\right)+\sigma_{\theta} \varepsilon_{\theta, t} \tag{3.3}
\end{equation*}
$$

with $\varepsilon_{\theta, t} \sim \mathcal{N}(0,1)$.

The intertemporal preference shock aims to capture exogenous fluctuations in preferences due to changes in beliefs or in taste. In particular, the stochastic discount factor fluctuates endogenously with consumption and exogenously with the shock $b_{t}$,
which is given by

$$
\begin{equation*}
\ln \left(b_{t}\right)=\rho_{b} \ln \left(b_{t-1}\right)+\sigma_{b} \varepsilon_{b, t} \tag{3.4}
\end{equation*}
$$

where $\varepsilon_{b, t} \sim \mathcal{N}(0,1)$.

Finally, as usual in the literature, I have assumed log-utility in consumption so that the marginal rate of substitution between consumption and leisure is linear in the former, which is necessary to ensure the existence of a balanced growth path.

### 3.3.2 Retailers

The retail sector is populated by infinitely lived and perfectly competitive firms producing final goods, $Y_{t}$, by combining a continuum of intermediate goods, $Y_{t}(s)$. Final goods can be used for consumption and investment. Intermediate goods are transformed into final goods by means of a [24] aggregator.

$$
\begin{equation*}
Y_{t}=\left[\int_{0}^{1}\left(Y_{t}(s)\right)^{\frac{1}{1+\lambda_{t}}}\right]^{1+\lambda_{t}} \tag{3.5}
\end{equation*}
$$

where $\lambda_{t}$ is the markup shock and $\frac{1+\lambda_{t}}{\lambda_{t}}$ measures the elasticity of substitution between differentiated intermediate goods. I assume that the markup evolves as follows

$$
\begin{equation*}
\ln \left(\lambda_{t}\right)=\left(1-\rho_{\lambda}\right) \ln \left(\lambda^{\star}\right)+\rho_{\lambda} \ln \left(\lambda_{t-1}\right)+\sigma_{\lambda} \varepsilon_{\lambda, t} \tag{3.6}
\end{equation*}
$$

where $\varepsilon_{\lambda, t} \sim \mathcal{N}(0,1)$ and $\lambda^{\star}$ stands for the value of the markup at the steady state.
Final goods firms take the prices of intermediate goods as given and choose $Y_{t}(s)$ to minimize costs, given by $\int_{0}^{1} P_{t}(s) Y_{t}(s) d s$ subject to the Dixit-Stiglitz aggregator.

From the first-order condition, I have that the demand function for the $s^{t h}$ intermediate good is given by

$$
\begin{equation*}
Y_{t}(s)=\left[\frac{P_{t}}{P_{t}(s)}\right]^{\frac{1+\lambda_{t}}{\lambda_{t}}} Y_{t} \tag{3.7}
\end{equation*}
$$

Integrating the above and imposing the zero-profit condition, I obtain the following expression for the aggregate price index

$$
\begin{equation*}
P_{t}=\left[\int_{0}^{1} P_{t}(s)^{-1 / \lambda_{t}} d s\right]^{-\lambda_{t}} \tag{3.8}
\end{equation*}
$$

### 3.3.3 Intermediate goods sector

There is a continuum of infinitely lived producers of intermediate goods, indexed by $s \in[0,1]$, operating under monopolistic competition. They produce intermediate inputs, $Y_{t}(s)$, combining labor, $H_{t}$, and capital services, $k_{t}$, using a Cobb-Douglas technology. Labor services are obtained from households and capital services from entrepreneurs.

$$
\begin{equation*}
Y_{t}(s)=\left[Z_{a, t} H_{t}(s)\right]^{1-\alpha} k_{t}(s)^{\alpha} \tag{3.9}
\end{equation*}
$$

where $Z_{a, t}$ stands for the neutral technology shock that evolves as follows

$$
\begin{equation*}
\log \left(Z_{a, t}\right)=\Upsilon_{z}+\log \left(Z_{a, t-1}\right)+\sigma_{Z} \varepsilon_{Z, t} \tag{3.10}
\end{equation*}
$$

$\Upsilon_{z}$ is the average growth rate of the economy and $\epsilon_{Z, t} \sim \mathcal{N}(0,1)$.
Intermediate goods producers solve a two-stage problem. First, they decide on the demand schedule for labor and capital services by minimizing total costs conditional on factor prices. The optimization problem is given by

$$
\begin{equation*}
\min _{H_{t}(s), k_{t}(s)} \frac{W_{t}}{P_{t}} H_{t}(s)+r_{t}^{k} k_{t}(s) \tag{3.11}
\end{equation*}
$$

subject to

$$
Y_{t}(s)=\left[Z_{a, t} H_{t}(s)\right]^{1-\alpha} k_{t}(s)^{\alpha}
$$

where $r_{t}^{k}$ is the rental rate of capital. Therefore, the optimal capital-to-labor ratio is given by

$$
\frac{k_{t}(s)}{H_{t}(s)}=\frac{\alpha}{1-\alpha} \frac{W_{t} / P_{t}}{r_{t}^{k}}
$$

and the real marginal cost by

$$
\chi_{t}(s)=\left(\frac{\alpha}{1-\alpha}\right)^{1-\alpha}\left(\frac{1}{\alpha}\right)^{\alpha} \frac{\left(W_{t} / P_{t}\right)^{1-\alpha}\left(r_{t}^{k}\right)^{\alpha}}{Z_{a, t}}
$$

Given that both the optimal capital-to-labor ratio and the real marginal cost depend only on market prices, common parameters across intermediate producers, and the economy-wide neutral technology shock, I conclude that those two variables are identical for all producers. Hence, I can proceed by assuming a representative agent in the sector.

In the second stage, intermediate goods producers face a pricing problem in a sticky price framework à la Calvo. At any given period, a producer is allowed to reoptimize her price with probability $\left(1-\xi_{p}\right)$. For simplicity, let us assume that those firms that do not re-optimize set their price equal to their last optimized price. When reoptimization is possible, an intermediate firm $s$ will set the price $P_{t}^{\star}(s)$ that maximizes the expected value of the firm

$$
\begin{equation*}
\max _{P_{t}^{\star}(s)} \mathbb{E}_{t} \sum_{j=0}^{\infty}\left[\xi_{p}^{j} \Lambda_{t, t+j}\left(\frac{P_{t}^{\star}(s)}{P_{t+j}}-\chi_{t+j}\right) Y_{t+j}(s)\right] \tag{3.12}
\end{equation*}
$$

subject to

$$
\begin{equation*}
Y_{t}(s)=\left[\frac{P_{t}(s)}{P_{t}}\right]^{-\frac{1+\lambda_{t}}{\lambda_{t}}} Y_{t} \tag{3.13}
\end{equation*}
$$

where $\Lambda_{t, j}$ is the stochastic discount factor between $t$ and $t+j$ for households.
Given that not all retailers are allowed to adjust their prices, the aggregate price index is given by the following weighted average

$$
\begin{equation*}
P_{t}=\left[\xi_{p} P_{t-1}^{1 / \lambda_{t}}+\left(1-\xi_{p}\right)\left(P_{t}^{\star}(s)\right)^{1 / \lambda_{t}}\right]^{\lambda_{t}} \tag{3.14}
\end{equation*}
$$

### 3.3.4 Capital producers

Capital producers are infinitely lived agents operating in a perfectly competitive market. Capital producers produce new physical capital stock, $K_{t+1}$, combining final goods, $I_{t}$, with currently installed capital, $K_{t}$, using a constant returns to scale technology. The new capital is sold to entrepreneurs at price $P_{t}^{k}$. I assume that one unit of time t investment delivers $\zeta_{t}$ units of time $t+1$ physical capital. $\zeta_{t}$ is the investment-specific technology shock along the lines of [36].

$$
\begin{equation*}
\ln \left(\zeta_{t}\right)=\rho_{\zeta, 1} \ln \left(\zeta_{t-1}\right)+\sigma_{\zeta} \varepsilon_{\zeta, t} \quad \varepsilon_{\zeta, t} \sim \mathcal{N}(0,1) \tag{3.15}
\end{equation*}
$$

I assume that capital producers repurchase used capital from entrepreneurs. Since previously installed capital is an input for the production of new physical capital, the marginal rate of transformation between old (conveniently depreciated) and new capital is equal to one. This implies that the price of old and new capital is identical.
[5] assume there are increasing marginal adjustment costs in the production of capital, so that they can obtain time variation in the price of capital. Such a variation contributes to the volatility of entrepreneurial net worth. In my set-up, I can obtain time variation in the price of capital through the investment-specific technology shock. However, I assume adjustment costs to impute some discipline in the volatility of investment. I follow [10] in assuming that capital producers are subject to quadratic
capital adjustment costs specified as $\left[\frac{\xi}{2}\left(\frac{I_{t}}{K_{t}}-\left(\mathfrak{Z}^{*}-1+\delta\right)\right)^{2} K_{t}\right]$, where $\mathfrak{Z}^{*}$ is the growth rate of the economy in the steady state.

The representative capital producer chooses the level of investment that maximizes her profits, which are given by ${ }^{1}$

$$
\begin{equation*}
P_{t}^{k} \zeta_{t} I_{t}-P_{t} I_{t}-P_{t} \frac{\xi}{2}\left(\frac{I_{t}}{K_{t}}-\left(\mathfrak{Z}^{*}-1+\delta\right)\right)^{2} K_{t} \tag{3.16}
\end{equation*}
$$

Let $Q_{t}=\frac{P_{t}^{k}}{P_{t}}$ be the relative price of capital,

$$
\begin{equation*}
Q_{t}=\frac{1}{\zeta_{t}}\left[1+\xi\left(\frac{I_{t}}{K_{t}}-\left(\mathfrak{Z}^{*}-1+\delta\right)\right)\right] \tag{3.17}
\end{equation*}
$$

which is the standard Tobin's q equation. In the absence of capital adjustment costs, the relative price for capital, $Q_{t}$, is equal to the inverse of the investment-specific shock. The quantity and price of capital are determined in the market for capital. The supply of capital is given by equation (3.17). The demand curve will be determined by the entrepreneurial sector (equation 3.20).

The aggregate capital stock of the economy evolves according to

$$
\begin{equation*}
K_{t}=(1-\delta) K_{t}+\zeta_{t} I_{t} \tag{3.18}
\end{equation*}
$$

### 3.3.5 Entrepreneurs and financial intermediaries

Entrepreneurs are finitely lived risk-neutral agents who borrow funds captured by financial intermediaries from households. Borrowing and lending occur in equilibrium because entrepreneurs and households are two different types of agents. As I

[^5]have stated above, financial rigidities arise because there is asymmetric information between borrowers and lenders. While entrepreneurs can freely observe the realization of their idiosyncratic risk, financial intermediaries must pay an auditing cost to observe it. To minimize monitoring costs, lenders will audit borrowers only when they report their inability to pay the loan back under the terms of the contract. I assume that the auditing technology is such that, when monitoring occurs, the lender perfectly observes the borrower's realized return. Monitoring or bankruptcy costs are associated with accounting and legal fees, asset liquidation, and interruption of business.

Since financial intermediaries may incur these costs in the event of default by a borrower, loans are made at a premium over the risk-free interest rate. Such an external finance premium captures the efficiency of financial intermediation. The external finance premium is affected by two channels: the balance-sheet channel and the information channel. The balance-sheet channel implies that as the share of capital investment funded through external financing increases, the probability of default also rises. Lenders request compensation for the higher exposure to risk with a higher premium. The information channel is linked to the elasticity of the external finance premium with respect to the entrepreneurial leverage ratio. This channel states that the larger the rents generated by asymmetric information, the more sensitive the premium is to the leverage ratio. Therefore, the external finance premium is an increasing function of the level of financial rigidity, which is measured by the agency cost. I enrich the model by introducing financial shocks affecting both the balance-sheet and the information channels of the external finance premium.

In a costly state verification set-up, entrepreneurs try to avoid the financial constraint by accumulating wealth. However, the assumption of a finite lifetime implies
that financial intermediation is necessary; that is, entrepreneurs cannot be fully selffinanced. In addition, the deceased fraction, $\gamma$, of the population of borrowers transfers wealth to the pool of active entrepreneurs. This transfer of resources guarantees that any active entrepreneur has nonzero wealth so she can gain access to external financing.

## Individual entrepreneur's problem

Entrepreneurs own the capital stock, $K_{t}$, of the economy. At the beginning of the period, an entrepreneur is hit by an idiosyncratic shock, $\omega_{t}^{j}$, that affects the productivity of her capital holdings. This idiosyncratic shock is at the center of the informational asymmetry, since it is only freely observed by the entrepreneur. For tractability purposes, I assume $\omega_{t}^{j}$, for all $j$, is i.i.d lognormal with c.d.f. $F(\omega)$, parameters $\mu_{\omega}$ and $\sigma_{\omega}$, such that $\mathbb{E}\left[\omega^{j}\right]=1$. After observing the realization of the idiosyncratic shock, entrepreneurs choose the capital utilization rate, $u_{t}^{j}$, that solves the following optimization problem

$$
\begin{equation*}
\max _{u_{t}^{j}}\left[u_{t}^{j} r_{t}^{k, j}-a\left(u_{t}^{j}\right)\right] \omega_{t}^{j} K_{t}^{j} \tag{3.19}
\end{equation*}
$$

where, around the steady state, $a(\cdot)=0, a^{\prime}(\cdot)>0, a^{\prime \prime}(\cdot)>0$ and $u^{\star}=1$. Therefore, capital services, $k_{t}^{j}$, rented to intermediate goods producers are given by $k_{t}^{j}=u_{t}^{j} \omega_{t}^{j} K_{t}^{j}$.

The capital demand for entrepreneur $j$ is given by her expected gross returns on holding one unit of capital from $t$ to $t+1$

$$
\begin{equation*}
\mathbb{E}_{t}\left[R_{t+1}^{k, j}\right]=\mathbb{E}_{t}\left[\frac{r_{t+1}^{k, j}+\omega_{t+1}^{j}(1-\delta) Q_{t+1}}{Q_{t}}\right] \tag{3.20}
\end{equation*}
$$

where $\omega_{t+1}^{j}(1-\delta) Q_{t+1}$ is the return to selling the undepreciated capital stock back to
capital producers.
As I pointed out before, I can write the equilibrium conditions for intermediate goods producers in terms of aggregate variables. Therefore, I have

$$
r_{t}^{k, j}=\omega_{t}^{j} \frac{\alpha \chi_{t}(s) Y_{t}(s)}{k_{t}(s)}=\omega_{t}^{j} \frac{\alpha \chi_{t} Y_{t}}{k_{t}}=\omega_{t}^{j} r_{t}^{k}
$$

and, hence,

$$
\begin{equation*}
\mathbb{E}_{t}\left[R_{t+1}^{k, j}\right]=\mathbb{E}_{t}\left[\omega_{t+1}^{j} R_{t+1}^{k}\right] \tag{3.21}
\end{equation*}
$$

where $R_{t+1}^{k}$ is the aggregate gross return on capital.

## Debt contract

Conditional on survival, an entrepreneur $j$ purchases physical capital, $K_{t+1}^{j}$, at relative price $Q_{t}$. An entrepreneur can finance the purchasing of new physical capital investing her own net worth, $N_{t+1}^{j}$, and using external financing, $B_{t+1}^{j}$, to leverage her project. Therefore, she can finance her investment in capital goods as follows:

$$
\begin{equation*}
Q_{t} K_{t+1}^{j}=B_{t+1}^{j}+N_{t+1}^{j} \tag{3.22}
\end{equation*}
$$

Given that the entrepreneur is risk neutral, she offers a debt contract that ensures the lender a return free of aggregate risk. The lender can diversify idiosyncratic risks by holding a perfectly diversified portfolio. A debt contract is characterized by a triplet consisting of the amount of the loan, $B_{t+1}^{j}$, the contractual rate, $Z_{t+1}^{j}$, and a schedule of state-contingent threshold values of the idiosyncratic shock, $\bar{\omega}_{n, t+1}^{j}$, where $n$ refers to the state of nature. For values of the idiosyncratic productivity shock above the threshold, the entrepreneur is able to repay the lender at the contractual rate. For values below the threshold, the borrower defaults, and the lender steps in
and seizes the firm's assets. A fraction of the realized entrepreneurial revenue is lost in the process of liquidating the firm. In this case, the financial intermediary obtains

$$
\begin{equation*}
\left(1-\mu_{t+1}\right) \omega_{n, t+1}^{j} R_{n, t+1}^{k} Q_{t} K_{t+1}^{j} \tag{3.23}
\end{equation*}
$$

where $\omega_{n, t+1}^{j} R_{n, t+1}^{k}$ stands for the ex post gross return on capital for a given entrepreneur $j$ (see equation 3.21) and $\mu_{t+1}$ for the marginal bankruptcy cost. In the literature, the marginal bankruptcy cost is assumed to be a constant parameter. I assume, however, that it is a drifting parameter so that exogenous changes in the level of financial rigidities affect the business cycle properties of the model. In section 3.3.5, I describe in detail the relevance of this assumption and the stochastic specification chosen.

For a given state n, the threshold value for the idiosyncratic productivity shock is defined as

$$
\begin{equation*}
\bar{\omega}_{t+1}^{j} R_{t+1}^{k} Q_{t} K_{t+1}^{j}=Z_{t+1}^{j} B_{t+1}^{j} \tag{3.24}
\end{equation*}
$$

where $Z_{t+1}^{j}$ is the contractual rate whose dynamics, ceteris paribus, are governed by those of $\bar{\omega}_{t+1}^{j}$. Hence, I set up the debt contract only in terms of the idiosyncratic productivity threshold. From this equation, I can determine the payoffs for the borrower and lender as a function of the realized idiosyncratic risk. If $\omega_{t+1}^{j} \geq \bar{\omega}_{t+1}^{j}$, then the entrepreneur can satisfy the terms of the contract. She pays the lender $Z_{t+1}^{j} B_{t+1}^{j}$ and keeps $\left(\omega_{t+1}^{j} R_{t+1}^{k} Q_{t} K_{t+1}-Z_{t+1}^{j} B_{t+1}^{j}\right)$. If $\omega_{t+1}^{j}<\bar{\omega}_{t+1}^{j}$, the entrepreneur declares bankruptcy; that is, she defaults on her loans. In this case, the financial intermediary liquidates the firm, obtaining $\left(1-\mu_{t+1}\right) \omega_{t+1}^{j} R_{t+1}^{k} Q_{t} K_{t+1}$ and leaving the lender with zero wealth.

The terms of the debt contract are chosen to maximize expected entrepreneurial
profits conditional on the return of the lender, for each possible state of nature, being equal to the real riskless rate. That is, the participation constraint is given by the zero profit condition for the financial intermediary.

$$
\begin{equation*}
\max _{\left\{\bar{\omega}_{n, t+1}^{j}, K_{t+1}^{j}\right\}} \sum_{n} \Xi_{n}\left[\int_{\bar{\omega}_{n, t+1}^{j}}^{\infty} \omega d F(\omega)-\left[1-F\left(\bar{\omega}_{n, t+1}^{j}\right)\right] \bar{\omega}_{n, t+1}^{j}\right] R_{n, t+1}^{k} Q_{t} K_{t+1}^{j} \tag{3.25}
\end{equation*}
$$

st

$$
\begin{gather*}
\left(\left[1-F\left(\bar{\omega}_{n, t+1}^{j}\right)\right] \bar{\omega}_{n, t+1}^{j}+\left(1-\mu_{t+1}\right) \int_{0}^{\bar{\omega}_{n, t+1}^{j}} \omega d F(\omega)\right) R_{n, t+1}^{k} Q_{t} K_{t+1}^{j} \\
=R_{t}\left(Q_{t} K_{t+1}^{j}-N_{t+1}^{j}\right) \tag{3.26}
\end{gather*}
$$

where $\Xi_{n}$ stands for the probability of reaching state $n, F\left(\bar{\omega}_{n, t+1}^{j}\right)$ is the default probability, $R_{t}\left(Q_{t} K_{t+1}^{j}-N_{t+1}^{j}\right)$ is the cost of funds, $\left(1-\mu_{t+1}\right) \int_{0}^{\bar{\omega}_{n, t+1}^{j}} \omega R_{n, t+1}^{k} Q_{t} K_{t+1}^{j} d F(\omega)$ is the payoff if the entrepreneur defaults on the loan, and $\left[1-F\left(\bar{\omega}_{n, t+1}^{j}\right)\right] \bar{\omega}_{n, t+1}^{j} R_{n, t+1}^{k}$ $Q_{t} K_{t+1}^{j}$, which is equal to $\left[1-F\left(\bar{\omega}_{n, t+1}^{j}\right)\right] Z_{t+t}^{j} B_{t+1}^{j}$, stands for the revenue if the loan pays. Therefore, the left-hand side in equation (3.26) is the expected gross return on a loan for the financial intermediary.

Let $\varrho_{t+1}^{j}=\frac{B_{t+1}^{j}}{N_{t+1}^{j}}$ be the debt-to-wealth ratio, $\Gamma\left(\bar{\omega}_{t+1}^{j}\right)=\int_{0}^{\bar{\omega}_{t+1}^{j}} \omega f(\omega) d \omega+\bar{\omega}_{t} \int_{\bar{\omega}_{t+1}^{j}}^{\infty}$ $f(\omega) d \omega$, the expected share of gross entrepreneurial earnings going to the lender, $1-\Gamma\left(\bar{\omega}_{t+1}^{j}\right)$, the share of gross entrepreneurial earnings retained by borrowers, and $\mu_{t+1} G\left(\bar{\omega}_{t+1}^{j}\right)=\mu_{t+1} \int_{0}^{\bar{\omega}_{t+1}^{j}} \omega f(\omega) d \omega$, the expected monitoring costs. Then I can rewrite the standard debt contract problem as

$$
\begin{array}{ll}
\max _{\left\{\bar{\omega}_{n, t+1}^{j}, \varrho_{t+1}^{j}\right\}} & \sum_{n} \Xi_{n}\left\{\left[1-\Gamma\left(\bar{\omega}_{n, t+1}^{j}\right)\right] \frac{R_{n, t+1}^{k}}{R_{t}}\left(1+\varrho_{t+1}^{j}\right)\right. \\
& \left.+\Psi\left(\bar{\omega}_{n, t+1}^{j}\right)\left[\frac{R_{n, t+1}^{k}}{R_{t}}\left[\Gamma\left(\bar{\omega}_{n, t+1}^{j}\right)-\mu_{t+1} G\left(\bar{\omega}_{n, t+1}^{j}\right)\right]\left(1+\varrho_{t+1}^{j}\right)-\varrho_{t+1}^{j}\right]\right\}
\end{array}
$$

where $\Psi\left(\bar{\omega}_{n, t+1}^{j}\right)$ is the Lagrange multiplier linked to the participation constraint. From the first-order condition with respect to the debt-to-wealth ratio

$$
0=\mathbb{E}_{t}\left[\left(1-\Gamma\left(\bar{\omega}_{t+1}^{j}\right)\right) \frac{R_{t+1}^{k}}{R_{t}}+\psi\left(\bar{\omega}_{t+1}^{j}\right)\left(\left[\Gamma\left(\bar{\omega}_{t+1}^{j}\right)-\mu_{t+1} G\left(\bar{\omega}_{t+1}^{j}\right)\right] \frac{R_{t+1}^{k}}{R_{t}}-1\right)\right],
$$

I can conclude that the schedule of threshold values for the idiosyncratic productivity shock depends upon aggregate variables so that it is common for all entrepreneurs. I can proceed, hence eliminating the superscript in $\omega_{t+1}$. From the participation constraint for the financial intermediary, it directly follows that the debt-to-wealth ratio, $\varrho_{t+1}^{j}$, is identical for all $j$. Therefore, I perform the remainder of the analysis dropping all superscripts.

I derive the supply for loans from the zero profit condition for the financial intermediary

$$
\begin{equation*}
\frac{R_{t+1}^{k}}{R_{t}}\left[\Gamma\left(\bar{\omega}_{t+1}\right)-\mu_{t+1} G\left(\bar{\omega}_{t+1}\right)\right]=\left(\frac{Q_{t} K_{t+1}-N_{t+1}}{Q_{t} K_{t+1}}\right) \tag{3.27}
\end{equation*}
$$

The above states that the external finance premium, $\mathbb{E}_{t}\left[\frac{R_{t+1}^{k}}{R_{t}}\right]$, is an increasing function of the debt-to-assets ratio and of the severity of the agency problem between borrowers and lenders. Equation (3.27) provides one of the foundations of the financial accelerator mechanism: a linkage between the entrepreneur's financial position and the cost of external funds, which ultimately affects the demand for capital.

The other main component of the financial accelerator is the evolution of entrepreneurial net worth. Note that the return on capital and, hence, the demand for capital by entrepreneurs depends on the dynamics of net worth. Let $V_{t}$ be entrepreneurial equity and $W_{t}^{e}$ be the wealth transfers made by exiting firms to the pool of active firms. Then, aggregate entrepreneurial net worth (average net worth across
entrepreneurs) is given by the following differential equation

$$
\begin{aligned}
N_{t+1} & =x_{t} \gamma V_{t}+W_{t}^{e} \\
& =x_{t} \gamma\left[R_{t}^{k} Q_{t-1} K_{t}-R_{t-1} B_{t}-\mu_{t} R_{t}^{k} Q_{t-1} K_{t} \int_{0}^{\bar{\omega}_{t}} \omega f(\omega) d \omega\right]+W_{t}^{e} \\
& =x_{t} \gamma\left[R_{t}^{k} Q_{t-1} K_{t}-R_{t-1} B_{t}-\mu_{t} G\left(\bar{\omega}_{t}\right) R_{t}^{k} Q_{t-1} K_{t}\right]+W_{t}^{e}
\end{aligned}
$$

where $x_{t}$ is a wealth shock, $\left[R_{t}^{k} Q_{t-1} K_{t}^{j}-R_{t-1} B_{t}\right]$ is the gross return on capital net of repayment of loans in the nondefault case, and $\mu_{t} G\left(\bar{\omega}_{t}\right) R_{t}^{k} Q_{t-1} K_{t}$ is the gross return lost in case of bankruptcy. Therefore, equity stakes for entrepreneurs that survive to period $t$ are given by the aggregate return on capital net of repayment of loans.

Wealth shocks can be interpreted as shocks to the stock market that generate asset price movements that cannot be accounted for by fundamentals. [16] suggest that shocks to entrepreneurial wealth capture the so-called irrational exuberance. I can also consider wealth shocks as a reduced form for changes in fiscal policy that have redistributive effects between firms and households. Exogenously driven changes in the valuation of entrepreneurial equity need to be financed by another sector of my model economy. I assume that the household sector receives (provides) wealth transfers from (to) the entrepreneurial sector, which are defined as

$$
\begin{equation*}
\operatorname{Trans}_{t}=N_{t+1}-\gamma V_{t}-W_{t}^{e}=\gamma V_{t}\left(x_{t}-1\right) \tag{3.28}
\end{equation*}
$$

where $\gamma V_{t}+W_{t}^{e}$ is the value that entrepreneurial equity would have taken if there were no wealth shocks.

## Financial shocks

In a model with informational asymmetries, financing capital acquisitions with internally generated funds is preferred to external borrowing since it is less costly. The difference between external and internal financing is the so-called external finance premium. In my environment, this premium is defined as the expected discounted return to capital

$$
\begin{equation*}
\mathbb{E}_{t}\left[\frac{R_{t+1}^{k}}{R_{t}}\right]=\mathbb{E}_{t}\left[\left(\frac{1}{\Gamma\left(\bar{\omega}_{t+1}\right)-\mu_{t+1} G\left(\bar{\omega}_{t+1}\right)}\right)\left(\frac{Q_{t} K_{t+1}-N_{t+1}}{Q_{t} K_{t+1}}\right)\right] \tag{3.29}
\end{equation*}
$$

The external finance premium is determined by two channels: the balance-sheet channel, through the debt-to-assets ratio

$$
\frac{Q_{t} K_{t+1}-N_{t+1}}{Q_{t} K_{t+1}}
$$

and the information channel, through the elasticity of the external finance premium with respect to the leverage ratio, which is given by

$$
\frac{1}{\Gamma\left(\bar{\omega}_{t+1}\right)-\mu_{t+1} G\left(\bar{\omega}_{t+1}\right)}
$$

The external finance premium is the key relationship of the financial accelerator, since it determines the efficiency of the contractual relationship between borrowers and lenders. I enrich the theoretical framework by assuming that this essential mechanism is affected exogenously by two financial shocks: a wealth shock and a shock to the marginal bankruptcy cost.

The balance-sheet channel states the negative dependence of the premium on the amount of collateralized net worth, $N_{t+1}$. The higher the stake of a borrower in the
project, the lower the premium over the risk-free rate required by the intermediary. I introduce shocks to this channel through an entrepreneurial equity shifter. These types of wealth shocks were first introduced by [32]. Recently, they have been explored by [17], [53], and [33].

Recently, [23] has explored shocks to the elasticity of the risk premium with respect to the entrepreneurial leverage ratio. He solves the model discarding the contribution of the dynamics of the idiosyncratic productivity threshold to the dynamics of the remaining variables. ${ }^{2}$ Hence, those shocks can refer to shocks to the standard deviation of the entrepreneurial distribution, to agency costs paid by financial intermediaries to monitor entrepreneurs, and/or to the entrepreneurial default threshold. He cannot, however, discriminate among the sources of the shock. [17] solve the model completely so that they can introduce a specific type of shock affecting the efficiency of the lending activity. In particular, they propose riskiness shocks affecting the standard deviation of the entrepreneurial distribution. A positive shock to the volatility of the idiosyncratic productivity shock widens the distribution so that financial intermediaries find it more difficult to distinguish the quality of entrepreneurs.

I introduce exogenous disturbances affecting the elasticity of the premium with respect to the leverage ratio by assuming the marginal bankruptcy cost is time-variant. The information channel, therefore, establishes that the external finance premium is a positive function of the severity of the agency problem measured by the marginal bankruptcy cost, $\mu_{t}$. An increase in the level of financial rigidity implies an enlargement of the informational asymmetry rents which translates into a higher premium on external funds. To the best of my knowledge, only [48] have explored time variation

[^6]along this margin. They estimate a partial equilibrium version of the BGG model using a panel of 900 US nonfinancial firms over the period 1997:1 to 2003:3. They find evidence of significant time variation in the marginal bankruptcy cost. In particular, they conclude that time variation in the parameter of interest is the main driver of the swings in the model-implied external finance premium. I assume that the shock to entrepreneurial wealth follows the following process
\[

$$
\begin{equation*}
\ln \left(x_{t}\right)=\rho_{x} \ln \left(x_{t-1}\right)+\sigma_{x} \varepsilon_{x, t}, \quad \varepsilon_{x, t} \sim \mathcal{N}(0,1) \tag{3.30}
\end{equation*}
$$

\]

and the shock to the marginal bankruptcy cost

$$
\begin{equation*}
\ln \left(\mu_{t}\right)=\left(1-\rho_{\mu}\right) \ln \left(\mu^{\star}\right)+\rho_{\mu} \ln \left(\mu_{t-1}\right)+\sigma_{\mu} \varepsilon_{\mu, t}, \quad \varepsilon_{\mu, t} \sim \mathcal{N}(0,1) \tag{3.31}
\end{equation*}
$$

The unconditional mean of the process governing the agency problem between borrowers and lenders, $\mu^{\star}$, determines the average level of financial rigidity in the model economy. This parameter governs, then, the size of the financial accelerator. In particular, $\mu^{\star}$ stands for the steady-state level of the marginal bankruptcy cost.

### 3.3.6 Government

Government spending is financed by government nominal bonds sold to households and by lump-sum taxes.

$$
\begin{equation*}
N B_{t+1}+P_{t} T_{t}=P_{t} G_{t}+R_{t-1}^{n} N B_{t} \tag{3.32}
\end{equation*}
$$

where the process for public spending $G_{t}$ is given by $G_{t}=g Y_{t}$, with the government spending-to-output ratio, $g$, being constant. The monetary authority follows a Taylortype interest rate rule. I assume the authority adjusts the short-term nominal interest rate responding to deviations of inflation and output growth from the target, i.e., their steady-state values.

$$
\begin{equation*}
\left(\frac{R_{t}^{n}}{R^{n \star}}\right)=\left(\frac{R_{t-1}^{n}}{R^{n \star}}\right)^{\rho_{R}}\left(\frac{\pi_{t}}{\pi^{\star}}\right)^{\left(1-\rho_{R}\right) \psi_{\pi}}\left(\frac{\Delta Y_{t}}{\Upsilon_{z}}\right)^{\left(1-\rho_{R}\right) \psi_{y}} e^{\sigma_{R} \varepsilon_{R, t}} \tag{3.33}
\end{equation*}
$$

with $\rho_{R}>0,\left(1-\rho_{R}\right) \psi_{\pi}>0,\left(1-\rho_{R}\right) \psi_{y}>0$, and $\varepsilon_{R, t} \sim \mathcal{N}(0,1)$. If $\psi_{\pi}>1$, then monetary policy is consistent with stabilizing inflation. If $\psi_{y}>0$, then monetary policy is consistent with stabilizing output growth.

### 3.3.7 Competitive equilibrium

Definition 3.1. A competitive equilibrium is defined by a sequence of prices

$$
\left\{P_{t}, P_{t}(s), W_{t}, R_{t}, R_{t}^{k}, R_{t}^{n}, Q_{t}, Z_{t+1}\right\}_{t=0}^{\infty}
$$

decisions rules for

$$
\left\{C_{t}, N B_{t}, D_{t+1}, H_{t}, I_{t}, Y_{t}(s), Y_{t}, u_{t}, K_{t+1}\right\}_{t=0}^{\infty}
$$

and laws of motion for $\left\{N_{t+1}, K_{t+1}\right\}_{t=0}^{\infty}$ such that all of the above optimality conditions are satisfied, the monetary authority follows its policy rule, and all markets clear.

Let us state here the final goods market clearing condition (total resources constraint)

$$
\begin{equation*}
Y_{t}=C_{t}+I_{t}+G_{t}+a\left(u_{t}\right) K_{t}+\mu_{t} G\left(\bar{\omega}_{t}\right) R_{t}^{k} Q_{t-1} K_{t} \tag{3.34}
\end{equation*}
$$

and the credit market clearing condition

$$
\begin{equation*}
D_{t+1}=B_{t+1}=Q_{t} K_{t+1}-N_{t+1} \tag{3.35}
\end{equation*}
$$

### 3.4 Structural Breaks in Parameters

Traditional approaches to the Great Moderation have focused on two explanations for the slowdown in real and nominal volatilities: smaller shocks hitting the US economy and tougher reaction to inflation by the monetary authority. Those two hypotheses are, however, insufficient to account for the empirical evidence since the mid 1980s. On the one hand, smaller shocks cannot account for more volatile financial cycles. On the other hand, it is hard to reconcile that better monetary policy translates into more stable real and nominal cycles and a destabilization of the financial cycle. Among other researchers, [42] highlight the potential relevance of changes in the US financial system to account for the contemporaneous divergence of volatility patterns.

In this paper, I test the relative role that changes in the size of the shocks hitting the economy, changes in monetary policy, and changes in the financial system played in the two stages of the Financial Immoderation and the Great Moderation. To do so, I allow for structural breaks in three sets of parameters intimately linked to each of these potential explanations: variance of the innovations, monetary policy coefficients, and the average level of financial rigidity. I use, however, a relatively naïve approach in treating structural breaks. I assume economic agents do not face an inference problem to learn endogenously about the regimes. When forming rational expectations about the dynamic economy, they take regime changes as completely exogenous events and assume that the current regime will last forever. Thus, once a structural break in parameters happens, agents learn about it immediately and conveniently readjust
their choices. This simplifying assumption facilitates the estimation when, as in my case, breaks in the steady state of the economy are allowed.

In this section, I first discuss how breaks in parameters affect the system matrices of the state space representation of the solution to the linear rational expectations (LRE) model. The system of log-linearized equilibrium conditions can be represented as

$$
\begin{equation*}
\Gamma_{0}(\varrho) \widetilde{s}_{t}=\Gamma_{1}(\varrho) \widetilde{s}_{t-1}+\Psi(\varrho) \varepsilon_{t}+\Pi(\varrho) \eta_{t} \tag{3.36}
\end{equation*}
$$

where $\widetilde{s}_{t}$ is a vector of model variables expressed in deviations from steady state, $\varepsilon_{t}$ is a vector of exogenous shocks, $\eta_{t}$ is a vector of rational expectations errors with elements $\eta_{t}^{x}=\widetilde{x}_{t}-\mathbb{E}_{t-1}\left[\widetilde{x}_{t}\right]$, and $\varrho$ is the vector of structural parameters. The solution to the LRE model can be cast in state space form as

$$
\begin{array}{rll}
\text { Transition equations: } & s_{t}=[I-\Phi(\varrho)] \bar{s}+\Phi(\varrho) s_{t-1}+\Phi_{\varepsilon}(\varrho) \varepsilon_{t} \\
\text { Measurement equations: } & y_{t}=B(\varrho) s_{t} \tag{3.38}
\end{array}
$$

where $s_{t}=\widetilde{s}_{t}+\ln (\bar{s})$ and $\bar{s}$ is the state vector evaluated in the steady state. Breaks in any parameter affect $\varrho$. However, while changes in monetary policy affect $\Phi(\varrho)$ and variations in the size of exogenous shocks shift $\Phi_{\varepsilon}(\varrho)$, structural breaks in the average level of financial rigidities have an impact on $\Phi(\varrho)$ and $\bar{s}$. That is, changes in $\mu^{\star}$ not only affect the coefficient matrices but also the steady state of the economy. This poses a challenge in the estimation exercise, since I need to conveniently adapt the filter used to evaluate the likelihood of the data.

In the remainder of the section, I discuss how structural breaks in the parameters of my choice help the model to account for the empirical evidence. For example, in my theoretical framework, an increase (decrease) in the size of a disturbance generates a
nonnegative (nonpositive) change in the volatility of all model variables. Therefore, an enlargement in the variability of the shocks hitting the economy could account for the empirical evidence of the 1970s and early 1980s since the volatility of all variables of interest moved in the same direction.

Recent US economic history highlights the relevance of monetary policy to the level and stability of inflation. That is, changes in the degree of response to objectives by the monetary authority will have a larger impact in shaping nominal cycles. In particular, I should expect a loosening of the monetary authority's reaction to deviations of inflation from the target during the Burns-Miller era and a tightening in the Volcker-Greenspan era.

Changes in the average level of bankruptcy costs imply changes in the level of financial frictions due to asymmetric information. An increase (decrease) in the average marginal bankruptcy cost enhances (weakens) the transmission of exogenous shocks to entrepreneurial wealth and costs of capital. Consequently, the responses of investment and output to shocks are more active (muted), since the sensitivity of borrowing costs to leverage increases (decreases). Given that the 1960s, 1970s, and early 1980s were years of profound changes in the US financial system, I should expect a decrease in the unconditional average of the level of financial rigidities in the model economy.

### 3.5 Parameter Estimates

I estimate the model with standard Bayesian estimation techniques using eight macroeconomic quarterly US time series as observable variables: the growth rate of real per capita net worth in the nonfarm business sector, the growth rate of real per capita gross value added (GVA) by the nonfarm business sector, the growth rate of real per
capita consumption defined as nondurable consumption and services, the growth rate of real per capita investment defined as gross private investment, log hours worked, the $\log$ of labor share, the $\log$ difference of the GVA deflator, and the federal funds rate. A complete description of the data set is given in Appendix 4.4. The model is estimated over the full sample period from 1954.4 to 2006.4.

All the series enumerated above except net worth in the nonfarm business sector are standard in the data sets used in the empirical macro literature. I discuss in further detail the inclusion of such financial variable in my set of observable variables. My theoretical framework describes the evolution of three financial series: entrepreneurial wealth, debt, and the external finance premium. Therefore, the estimation exercise could aim to match the behavior of all of those. Net worth for a firm is generally defined as total assets minus total liabilities. However, in order to be consistent with the model, I define net worth as tangible assets minus credit market liabilities. First, the model is a model of tangible assets purchased by firms so that it has nothing to say about financial assets held by entrepreneurs. Second, external financing in the model relates only to that obtained in credit markets. Hence, I do not consider trade and taxes payable nor miscellaneous liabilities provided in the Flow of Funds Accounts. An alternative measure for entrepreneurial wealth used by [17] is stock market data. This measure contains information only for publicly traded firms, which are a smaller set of firms than the one linked to the aggregate macroeconomic variables of my data set. In addition, in my model there is no role for equity finance.

Following the reasoning provided in the previous paragraph, my definition of debt is given by credit market liabilities in the nonfarm business sector. This information is contained in the series for entrepreneurial wealth. Therefore, if I am to consider only one financial variable in my empirical analysis, it seems reasonable to include net worth, since its informational content includes that of the dynamics of debt.

The external finance premium is essentially an unobservable variable. Hence, any empirical counterpart to be used in the estimation exercise is a proxy for the model concept of interest. [5] suggest considering the prime lending rate and the 6 -month Treasury bill rate in defining the external finance premium. [16] define the external finance premium as the premium on Baa bonds. Recently, [33] have used individual security-level data to construct a corporate credit spread index. They use such a credit spread as a proxy for the fluctuations in the unobservable external finance premium. I refrain from using a proxy for the external finance premium for several reasons. First, constructing a measure for such a variable using individual firm data is beyond the scope of this paper. Second, I focus on the analysis of the nonfarm business sector, which includes both corporate and noncorporate US firms. Therefore, using corporate credit or bond spreads and real variables for the nonfarm business sector introduces a discrepancy between financial and macroeconomic variables that would make it harder to evaluate the goodness of fit of my analysis. Third, in my theoretical framework, external financing is modeled using a simple debt contract. However, as long as it is the only form of external financing, the external finance premium is interpreted in the literature (see De Graeve, 2008) as pertaining to all forms of external finance. Therefore, there is no choice of approximation to this model variable that is free of controversy.

### 3.5.1 Prior distribution of the parameters

In this section, I discuss the prior information on the parameters used in the estimation exercise (see Table 26). First, I provide a thorough description of my prior choice for the parameters linked to the financial accelerator. Then, I discuss the priors on the remaining parameters. My prior choice for these parameters is fairly standard in
the literature. I use identical priors across subsamples for those parameters subject to structural breaks. I let the data speak about the size of the structural break without imposing any additional a priori information.

As is standard in the literature, I use degenerate priors on the default probability, $F(\bar{\omega})$, and the survival probability, $\gamma$. [3] report historical default rates for US bonds over the period 1971-2005 and deliver an average equal to $3 \%$. This is the value for the annual default rate widely used in the literature on the financial accelerator to pin down the quarterly default probability. I obtain the survival probability, $\gamma$, from the steady state of the economy given that I set the debt-to-wealth ratio to its historical average. The value for $\gamma$ is $98.54 \%$, which implies that firms live, on average, 17 years. This tenure is close to the median tenure reported by [48] from a panel of 900 nonfinancial firms.

Conversely, I use an informative prior for the unconditional average of financial rigidity, $\mu^{\star}$. Such a parameter captures the steady state value of the marginal bankruptcy cost. Therefore, it must lie inside the unit interval. A beta distribution guarantees that the parameter of interest belongs to the $0-1$ interval. In order to determine the location parameter of the beta prior distribution, I consider micro evidence on bankruptcy costs. [2], using data from 26 firms, concludes that bankruptcy costs are about $20 \%$ of the firm's value prior to bankruptcy and in the range 11-17\% of firm's value up to three years prior to bankruptcy. [1] analyze 201 firms that completed Chapter 11 bankruptcies during the period 1982-1993 to determine that the mean liquidation costs are $36.5 \%$. Using those two results, [8] conclude that the interval empirically relevant for the marginal bankruptcy cost parameter is [0.20, 0.37]. [48] estimate a partial equilibrium version of the model by [5] using panel data over the period 1997 to 2003. As a byproduct of their estimation, they obtain the model implied time series for the marginal bankruptcy cost. Their estimates lie in the range
of $7 \%$ to $45 \%$. Therefore, I assume the beta distribution for the unconditional average level of financial rigidity is centered at 0.28 . I choose the diffusion parameter to be equal to 0.05 so that the $95 \%$ credible set, $[0.13,0.41]$, encompasses most of the values provided in the literature.

My priors on the autoregressive coefficients of the stochastic exogenous processes are beta distributions with mean 0.6 and standard deviation 0.1 . The priors on the innovations' standard deviations are quite diffuse. In particular, I assume inverse gamma distributions centered at 0.01 with 4 degrees of freedom. The covariance matrix of the innovations is diagonal.

Following [56], I assume Gaussian priors on the monetary policy coefficients. I center the prior for the response of the monetary authority to deviations of inflation from the target at 1.50. I consider a diffuse enough prior by setting the standard deviation equal to 0.35 . The coefficient governing the response to deviations of output growth from the target is assumed to be Normal, around a mean 0.5 with standard deviation 0.1. The persistence of the monetary policy rule is assumed to follow a beta distribution, with mean 0.6 and standard deviation 0.2.

I assume Gaussian priors for $\Upsilon_{z}$ and $\log \left(H^{\star}\right)$ centered at zero and at its empirical historical average respectively and with standard error of 0.01 . I use a diffuse gamma distribution for the net annualized inflation rate in the steady state with mean 3 and standard deviation 1. I assume that the capital share in the Cobb-Douglas production function, $\alpha$, is described by a normal distribution with mean 0.3 and standard deviation 0.05 . The price markup at the steady state follows a beta distribution centered at 0.15 and standard deviation of 0.02 . I choose a beta distribution for the Calvo parameter with a location parameter equal to 0.75 and dispersion of 0.1 . The capital adjustment cost parameter is assumed to follow a gamma distribution with location and diffusion parameters equal to 2 and 1 , respectively. The gamma prior for the

Frisch elasticity is centered at the balance growth path of $\nu=2$, but I consider a disperse prior by setting its standard deviation to 1 . The habit parameter is assumed to have a beta distribution with mean 0.6 and standard deviation 0.1 . The elasticity of capital adjustment costs follows a gamma centered at 0.5 and with standard deviation 0.3.

Finally, three more parameters are fixed in the estimation procedure: government spending share, depreciation rate, and discount rate. The exogenous government spending to GVA ratio is set to the historical average $g^{\star}=0.20$. The depreciation rate, $\delta$, is set to 0.025 so that the annual depreciation rate is $10 \%$. The value for households' discount rate, $\beta$, is chosen so that, in the steady state, the nominal riskfree interest rate matches the historical quarterly gross federal funds rate. Therefore,

$$
\beta=\frac{\left(1+\pi^{\star} / 400\right) \exp \left\{\Upsilon_{z}\right\}}{R^{n \star}}
$$

where $\pi^{\star}$ and $\Upsilon_{z}$ are set equal to their observed average.

### 3.5.2 Posterior estimates of the parameters

The estimation procedure is as follows. First, I obtain the posterior mode by maximizing the posterior distribution, which combines the prior distribution of the structural parameters with the likelihood of the data. By assuming $\varepsilon_{t} \sim_{i i d} \mathcal{N}\left(\mathbf{0}, \Sigma_{\varepsilon}\right)$ in equation 3.37, I can use the Kalman filter to evaluate the likelihood function. I modify the Kalman filter to accommodate for changes in the system matrices. A full description of the modification used in the estimation exercise is given in Appendix 4.4. Second, I use the random walk Metropolis-Hastings algorithm to obtain draws from the posterior distribution. In particular, I run 3 chains of 500,000 draws using a burn-in period of $20 \%$ of the draws.

Tables 28 and 29 report the posterior median and the $95 \%$ credible intervals obtained by the Metropolis-Hastings algorithm. Let us first analyze Table 28, which contains those parameters not allowed to change over time. Some of the estimates are fairly standard, such as the inflation rate in the steady state, log hours in the steady state, the average growth rate, the adjustment cost parameter, the elasticity of capital utilization costs, the markup in the steady state, the backward looking parameter of the monetary policy rule, and the autoregressive coefficients. The first three parameters of the previous enumeration are close to their historical averages. The remaining ones are close enough to the widely accepted values in the literature so I do not discuss them further. I just highlight here that while the shock to the information channel of the external finance premium is highly persistent, $\rho_{\mu}=0.97$, the wealth shock is much less persistent, $\rho_{x}=0.52$.

The posterior median estimate for the Frisch elasticity, $\nu=1.03$, is inside the bounds found in the literature but slightly higher than the values obtained in DSGE models with sticky wages. Since I have a flexible labor market, I need a large enough Frisch elasticity to match the dynamics of hours worked. The median estimate for the capital share, $\alpha=0.28$, lies in between the values recently obtained in the DSGE literature (Smets and Wouters, 2007 and Justiniano, Primiceri, and Tambalotti, 2009), and the standard values used in the RBC literature.

The estimated degree of habit formation, $h=0.34$, is lower than the traditional 0.60 advocated in the literature. The estimated Calvo parameter, $\widehat{\xi}_{p}=0.40$, implies that the nominal friction is not too relevant, since firms re-optimize their prices every six and a half months approximately. since the financial accelerator mechanism not only amplifies but also propagates the shocks hitting the economy, I do not need a high degree of habit formation or of nominal rigidity to match the persistence of the data.

Table 29 reports the estimates for those parameters allowed to change in 1970 and 1984. The first group of parameters is formed by the average level of financial rigidity, $\mu^{\star}$, the size of the shock to the marginal bankruptcy cost, $\sigma_{\mu}$, and the size of the neutral technology shock, $\sigma_{z}$. These three parameters are characterized by presenting only one structural break in 1984. The neutral technology shock post1984 is $50 \%$ smaller than in the previous decades. The size of the structural break estimated in 1984 implies a significant change in the nature of the process governing agency costs.

On the one side, the estimated reduction in the size of the unconditional mean of the process is above $75 \%$. This result is along the lines of [42], who obtain that after the mid 1980s, the model economy is in a virtually frictionless environment, and [21] who estimates an $80 \%$ reduction in monitoring costs in a model based on [8]. The reduction in the average level of financial rigidities accounts not only for the decrease in bankruptcy costs linked to the Bankruptcy Reform Act of 1978 (see White, 1983) but also for other changes in the US financial system. The decades under analysis are characterized by the IT revolution, waves of regulation and deregulation, development of new products, and improvements in the assessment of risk. All these factors define the level of financial rigidity in terms of the model economy. Therefore, the Great Moderation period is characterized by easier access to credit, which accounted for a reduction in $\mu^{\star}$. On the other side, the size of the shock post-1984 is four times larger than in the pre-1984 period. Therefore, the unconditional average of the process governing the level of financial rigidity is smaller but the variability of the disturbance to the process is larger. I can reconcile these two results by noting that a reduction in $\mu^{\star}$ increases the average recovery rate for financial intermediaries. Hence, intermediaries are willing to enlarge their exposure to risk, which is captured by the increase in $\sigma_{\mu}$.

The second set of parameters in Table 29 contains only the size of the shock to
the balance-sheet channel of the external finance premium. The size of the wealth shock is an increasing step function. Larger balance-sheet shocks affecting the model economy reflect the increasing sensitivity of the system to asset price movements. This result does not come as a surprise, since the US data have been characterized by several price "bubbles" over the last few decades: the dramatic rise in US stock prices during the late 1990s or the housing bubble during the early 2000s, for example. One possible interpretation of wealth shocks is that they stand for asset price changes not driven by fundamentals.

The remaining standard deviations of innovations increase in the 1970s and decrease in the last sample period. The investment-specific and the intertemporal preference shock are smaller, in the post-1984 than in the pre-1970 sample period. The post-1984 values of the monetary policy shock, the intratemporal preference shock, and the markup shock are, however, in the same neighborhood as those taken in the pre-1970 sample period. That is, the 1970s and early 1980s were an "exception," in the sense of [7], for these parameters.

Finally, I describe the results for the monetary policy reaction function parameters. The mean of the long-run reaction to deviations of inflation from the target is larger than the standard values in the literature. As pointed out elsewhere in the literature, the monetary authority chooses a looser reaction to inflation in the 1970s. Post-1984, however, there is a tightening in the response to inflation. As long as the reaction to inflation post-1984 is similar to the one pre-1970, I can say, in simplistic terms, that it seems Volcker overcame Burns-Miller's will in terms of inflation by reusing Martin's recipes. The monetary authority responds strongly to changes in the growth rate of output (changes in the output gap) over the whole sample period. The authority started to respond more tightly in the 1970s and kept that level in the post-1984 era.

### 3.5.3 Model evaluation

In this section, I evaluate the model fit using two approaches. First, I provide a relative measure of fit by performing Bayesian model comparison. Second, I assess the absolute fit of the model using a posterior predictive check.

In Bayesian econometrics, model comparison is performed using the marginal likelihood of the data or marginal data density. This statistic is defined as the weighted average of the likelihood where the weights are given by the prior

$$
p\left(Y \mid \mathcal{M}_{i}\right)=\int p\left(Y \mid \varrho, \mathcal{M}_{i}\right) p\left(\varrho \mid \mathcal{M}_{i}\right) d \varrho
$$

where $\mathcal{M}_{i}$ stands for model i. Table 27 reports the differences of log-marginal data densities with respect to a model without breaks in parameters. I conclude that the model with breaks in the three set of parameters not only outperforms with respect to the model with no breaks but also to any partial model in which only one set of parameters is allowed to be subject to structural breaks. Therefore, I conclude that the data at hand are best represented by the theoretical framework that allows for structural breaks in the size of shocks, the average level of financial rigidity, and monetary policy coefficients.

I study the absolute model fit of the data using the posterior distribution. In particular, I compare model-implied statistics with those as in the data. I generate samples of the same length as the data (after a burn-in period of 100 observations) from the model economy using 1000 posterior draws. Table 30 reports the median of the model-implied moments and the $90 \%$ credible intervals for raw data and Table 31 that for the cyclical component.

The model overpredicts the volatility of net worth growth, consumption growth,
and inflation across subsamples. It overpredicts the volatility of all variables except the nominal interest rate and hours for the post-1984 period. Let us analyze the performance of the model in accounting for relative standard deviations with respect to the standard deviation of output growth. The model matches the relative standard deviations of net worth growth, investment growth, consumption growth, and inflation over this period fairly well. Moreover, the model is able to generate relative standard deviations of the magnitude of the observed ones in all sample periods of interest. Let us analyze, for example, the relative standard deviation of the net worth growth rate. Pre-1984 this variable is less volatile than output; post-1984 it is almost twice as volatile. The model is able to capture such a change in relative volatilities. Moreover, the model displays an increase in the relative volatility of net worth growth in the 1970-1983 sample period as observed in the data. Finally, the model is able to deliver the main characteristics of the generalized immoderation in the 1970s and the subsequent moderation in the mid 1980s in real and nominal variables and the additional immoderation on the financial side of the economy. The model is not able to capture, however, the enlargement of the volatility of the raw series for hours in the post-1984 period and the slowdown in the volatility of labor share in the 1970s.

In the literature characterizing the business cycle, model fit is performed using the moments of the cyclical component of the variables. Therefore, I compute the cyclical component of the observable variables in log-levels and the model-implied series using the Hodrick-Prescott filter.

The model overpredicts the volatility of net worth, labor share, and inflation and it underpredicts that of hours for all periods. It also fails to deliver the large increase in the cyclical volatility of output and consumption in the 1970s. The main failure is, however, that the model delivers the result that the standard deviations for net worth in the pre-1984 subsamples are larger than these of output. But my environment is
successful in many other dimensions. It captures the fact that hours are less volatile than output pre-1984 but more volatile afterwards. In accounting for the ratio of standard deviations across subperiods, the model is even more successful than in the case for the raw series. In particular, the model delivers changes in volatilities in the same direction as in the data for all variables. Moreover, the magnitude of the increases in volatility in the 1970-1983 sample period and the decreases in the last sample period are closer to the observed ones.

I can conclude that the model proposed in this paper fits the data fairly well. It delivers moments in consonance with the data both for the raw and filtered series. Therefore, my model is a good candidate for analyzing the US business cycle.

### 3.6 Assessing the Drivers of the Financial Immoderation and the Great Moderation

In this section, I analyze the contribution of each of the potential candidates, size of the shocks, monetary policy stance, and severity of financial rigidities, to the model-implied changes in business cycle properties. To do so, I perform two sets of counterfactual exercises: one for the first stage of the Financial Immoderation and another for the second stage and the Great Moderation.

Counterfactuals 1-4 refer to the first stage of the Financial Immoderation. I perform simulations using the following procedure:

1. Simulate the model economy for 200 periods (after a burn-in of 100 observations) using the parameter vector characterizing the 1954-1969 sample period.
2. Simulate the model economy for 200 periods (after a burn-in of 100 observations) using the parameter vector characterizing the 1970-1983 sample period.
3. Compute the ratio of standard deviations.
4. Simulate the model economy for 200 periods (after a burn-in of 100 observations) using the parameter vector of the counterfactual.
5. Compute the ratio of standard deviations with respect to those obtained in step 1.
6. Compute the percentage of the ratio obtained in step 3 attributable to the counterfactual.
7. Repeat the above 10,000 times.
8. Compute $90 \%$ credible intervals.

In Table 32, I report the observed and model-implied ratios of standard deviations of the cyclical component. The first three columns focus on the comparison between the 1954-1969 and 1970-1983 sample periods. The last three columns consider the ratio of standard deviations of the post-1984 period with respect to the 1970s and early 1980s computed following the procedure described above. Table 33 delivers the percentage of the total increase or decrease in standard deviation generated by the model that can be accounted for by the corresponding counterfactual.

In Counterfactual 1, I analyze the role played by the estimated changes in 1970 in the response of the monetary authority to deviations of inflation and output growth from the target. In particular, I simulate the model economy as described above, using a parameter vector with the same entries as the one characterizing the 1954-1969 sample period but with the monetary policy coefficients of the 1970-1983 parameter vector. The contemporaneous loosening in the response to inflation and the tightening in the response to output observed in the 1970s and early 1980s account for the
following percentages of the model-implied increase in cyclical volatility: $46 \%$ for inflation, $32 \%$ for the nominal interest rate, $15 \%$ for labor share, $3 \%$ for hours worked, and $5 \%$ for net worth.

In Counterfactual 2, I study the relative significance of the estimated $3 \%$ increase in the level of financial rigidity. Such an increase in agency costs accounts for an average of $5 \%$ of the model-implied increase in the volatility of the cyclical component of net worth, output, investment, consumption, hours, and the nominal interest rate.

I analyze the role played by the financial shocks in the immoderation of the 1970s and early 1980s in Counterfactual 3. The change in the size of financial shocks accounts completely for the increase in the cyclical volatility of net worth. It also accounts for the following percentages of the widening of business cycle fluctuations: $8 \%$ for output, $18 \%$ for investment, $6 \%$ for consumption, $6 \%$ for hours, $2 \%$ for labor share, and $5 \%$ for the nominal interest rate.

Counterfactual 4 assesses the relative importance of changes in the remaining shocks of the economy. The estimated changes in the size of the shocks account for $9 \%$ of the increase in the cyclical volatility of net worth, $100 \%$ of that in investment variability, $88 \%$ of investment, $94 \%$ of consumption, $94 \%$ of hours, $83 \%$ of labor share, $40 \%$ of inflation, and $53 \%$ of the nominal interest rate.

I conclude that the change in behavior of the monetary authority explains a large fraction of the increase in the variability of nominal variables observed in the 1970s and early 1980s. The immoderation observed in real and financial variables is driven by larger shocks hitting the US economy. In particular, the increase in the size of the wealth shock suffices to deliver the increase in the cyclical volatility of net worth.

In Counterfactuals 5-8, I study the drivers of the empirical evidence of the post1984 sample period, which is characterized by a contemporaneous enlargement of the financial cycle and a smoothing of real and nominal cycles. I proceed as described
above but the baseline parameter vector is the one linked to the 1970-1983 period and the parameter vector used in step 2 of the procedure is the one for the 1984-2006 sample period.

In Counterfactual 5, I study the relative contribution of the tightening of monetary policy in response to inflation to the Great Moderation and the widening of the financial cycle. Stricter monetary policy accounts for $41 \%$ of the model-implied reduction in the cyclical volatility of inflation, $22 \%$ of the decrease in the variability of the nominal interest rate, $15 \%$ of the slowdown in the volatility of labor share, and $3 \%$ of the increase in the standard deviation of the cyclical component of net worth. It has, however, a negligible effect on the variability at business cycle frequencies of investment, consumption, and hours.

I analyze the role played by the reduction in the unconditional average level of financial rigidity in Counterfactual 6. A model with a smoother financial sector accounts for $34 \%$ of the model-implied slowdown in investment and nominal interest rate volatility. It also accounts for $9 \%$ of the decrease in the cyclical volatility of inflation. The effect on the remaining variables is almost negligible.

I study the effect of the estimated increase in the size of financial shocks in the mid 1980s in Counterfactual 7. It has a negligible effect on the volatility of the labor share and inflation. However, it generates an increase in the magnitude of the cyclical variation for the remaining variables. The most remarkable changes are the $70 \%$ increase in the volatility of investment and the $93 \%$ increase in that of net worth, which stands for $150 \%$ of the total model-implied immoderation.

Counterfactual 8 analyzes the effect of the decrease in the size of all the remaining shocks in the model economy. I obtain the result that smaller real and nominal shocks overpredict the slowdown in output and the volatility of hours worked. These changes in the size of shocks account for the fraction of the reduction in the amplitude of
the nominal cycle not accounted for by the tightening of monetary policy and the relaxation of financial rigidity.

From the counterfactual exercises, I conclude that the behavior of the monetary authority has a significant impact on shaping the nominal cycle. Changes in the financial system are relevant for the variability of investment and nominal interest rates. The remaining swings in the amplitude of fluctuations at business cycle frequencies are driven by changes in the size of shocks hitting the economy.

### 3.7 Conclusions

I have studied two empirical regularities characterizing the US aggregate data over the last 55 years. The Great Moderation is related to the significant slowdown in the amplitude of the real and nominal cycles since the mid 1980s. The Financial Immoderation refers to the enlargement of the cyclical volatility of financial variables present since 1970. In this paper, I have made inference on the size of the structural breaks in parameters needed to account for the evolution of the second moments of the data in a model featuring nominal, real, and financial frictions. In particular, I have focused on breaks in the size of shocks, monetary policy coefficients, and the average size of the financial accelerator to disentangle the role played by changes in luck, in the conduct of monetary policy, and in the financial system respectively.

I conclude that while changes in the conduct of monetary policy account for a relevant proportion of the changes in the volatility of nominal variables, its effect on the variability of the remaining variables is small. Financial factors are relevant in shaping the business cycle properties of financial variables, investment, and the nominal interest rate. The estimated reduction in the size of the financial accelerator allows the model to account for $30 \%$ of the slowdown in the volatility of investment
and the nominal interest rate. In the next chapter of this thesis, I illustrate the growing relative significance of financial shocks in accounting for the variability of these variables in detriment of technology shocks.

My study reaffirms the growing convention in the literature on integrating credit market imperfections in otherwise standard macroeconomic models. I have documented the importance of including financial shocks in the analysis. Moreover, I highlight the relevance of taking into account structural breaks in the data, since my conclusions, in terms of assessing the main drivers of the cycle or characterizing the propagation dynamics of shocks, may differ significantly.

## Chapter 4

## Financial Shocks: Model Implied versus Empirical Propagation Dynamics

### 4.1 Introduction

Standard dynamic stochastic general equilibrium (DSGE) models, such as [14] and [56], abstract from interactions between credit markets and the rest of the economy. Those models are based on the capital structure irrelevance theorem by [50]; that is, the composition of agents' balance sheets has no effect on their optimal decisions. Nevertheless, episodes such as the Great Depression or the 2007-2009 financial turmoil stand as compelling evidence of the linkage between the developments in the financial and real sectors. Along these lines, recent contributions to the literature have focused on incorporating credit markets in the workhorse DSGE model. For example, [5] and [40] stress the relevance of the balance sheet's condition in determining economic activity. The ability to borrow depends upon borrowers' wealth, which ultimately
affects the demand for capital and the level of economic activity they can engage in.
Subsequent contributions to the literature on macroeconomic models with financial frictions have explored the transmission of shocks originated in the financial sector. In the model estimated in chapter 3, I introduce two financial shocks: a shock to business wealth and a shock to the marginal bankruptcy cost. In this chapter, I study the relative importance of these shocks in explaining the variance of the variables of interest. I also use the estimation output to characterize the model implied responses to financial shocks.

I conclude that the relevance of financial shocks in accounting for investment volatility increases over time. While before the Great Moderation, technology shocks (neutral and investment-specific) are the main drivers of investment variance, explaining more than $40 \%$, after the mid 1980s, they account for only $24 \%$. Financial shocks, however, explain $42 \%$ of investment variance at business cycle frequencies in the Great Moderation era. If I abstract from financial shocks, their relative contribution to investment variability is absorbed by technology shocks. Therefore, I can conclude that failing to include financial shocks results in an overstatement of the relative role played by technology shocks.

The estimated reduction in the average level of financial rigidities in the mid 1980s has important implications for the propagation mechanism of financial shocks. On the one hand, a smaller financial accelerator induces more muted responses to financial innovations; that is, the amplification mechanism linked to imperfections in the credit market gets reduced. On the other hand, a reduction in financial frictions enhances the persistence of the responses to financial shocks in the US economy.

The investment-based channel of models of the financial accelerator such as [5] and [8] suggests that the response of investment to an expansionary (contractionary) financial shock is positive (negative). But there is no consensus among empirical
macro researchers on the response of consumption to financial shocks affecting firms' ability to borrow. Using the model described in chapter 3, I obtain that expansionary financial shocks generate different responses upon impact depending on the nature of the financial shock. For example, a positive wealth shock delivers a negative response upon impact for consumption, but an expansionary shock to the marginal bankruptcy cost implies a positive response.

The negative response upon impact of consumption to a positive shock to the value of firms could be considered as a striking implication of the model. However, the responses of consumption and investment to a shock affecting firms' wealth are also of opposite signs in [17] and [53]. To the best of my knowledge, the only paper able to deliver a positive response upon impact of consumption after a positive wealth shock affecting the business sector is [33]. It is hard to assess which model does actually account better for the evidence since there has not been a careful empirical documentation of the effects of financial shocks on real variables.

In this chapter, I directly address the empirical assessment of the effects on consumption of a financial shock affecting business wealth. To do so, I estimate a vector autoregressive model on consumption, investment, business wealth, and financial spreads. I identify the financial shock using sign restrictions as proposed by [59]. I assume that a shock to business wealth generates responses of identical sign in business wealth and investment and of opposite sign in credit spreads. I remain silent on the response of consumption to the financial shock since the objective of the paper is to assess such a response.

From my empirical analysis, I conclude that the response of consumption to a financial shock affecting business wealth is not only ambiguous, but also it may dramatically vary depending on the series used to measure business wealth and the external finance premium. After a positive shock to business wealth, only $6 \%$ of the
responses satisfying the identification restrictions imply a negative response upon impact of consumption when business wealth is measured using data provided by the Flow of Funds Accounts and the external finance premium is proxied by the corporate spread. However, $55 \%$ of the responses imply a negative response of consumption when business wealth is measured by the industrial Dow Jones index and I use the prime lending spread ${ }^{1}$ as the proxy for the cost of external financing for firms. Moreover, the percentage of contractionary responses of consumption to an expansionary business wealth shock varies significantly across subsamples. However, I can state that financial shocks can generate responses of opposite signs for investment and consumption since there is no scenario under which we obtain zero negative responses upon impact for consumption.

The plan of the paper is as follows. In section 4.2, I discuss the model implied impulse response functions and the variance decomposition for the variables used in the estimation exercise. Section 4.3 reports the empirical assessment of the effects of financial shocks on consumption. I conclude in section 4.4.

### 4.2 Financial Shocks in a DSGE Model with Financial Frictions: Relative Importance and Propagation Dynamics

In this section, I focus on the study of the two financial shocks introduced in the model economy of chapter 3 . To do so, I analyze the variance decomposition and the impulse response functions.

[^7]
### 4.2.1 Variance decomposition

Table 34 indicates the variance decomposition at business cycle frequencies for output, investment, consumption, net worth, hours worked, labor share, inflation, and the nominal interest rate. I compute the spectral density of the observable variables implied by the DSGE model and use an inverse difference filter to obtain the spectrum for the level of output, investment, consumption, and net worth (see Appendix 4.4). I define business cycle fluctuations as those corresponding to cycles between 6 and 32 quarters.

The main driver of output variance is the neutral technology shock. The relative significance of this shock decreases over time from $67 \%$ to $39 \%$. The markup shock and the intratemporal shock become more relevant over time. In particular, their contribution to the variance of output doubles from the 1954-1969 sample period to the Great Moderation era. Since the markup shock determines the variance of the labor share and the labor supply shifter is the main driver of the variance in hours, I can conclude that the dynamics of output have shifted from being determined by capital to being determined by labor services. The variance of consumption is mainly shaped by the neutral technology shock pre-1984 but driven by a richer set of shocks afterwards, since the largest contributor accounts only for $31 \%$ of the total variance. Nominal interest rate variance is driven by the investment-specific shock, the shock to the stochastic discount factor, and the wealth shock, in decreasing order of relevance. The relative contribution of the two financial shocks increases post-1984 from accounting for $12 \%$ of the cyclical variance of the interest rate to $20 \%$. Such an increase in the proportion of the variance of the federal funds rate accounted for by financial factors is taken away from the contribution of the intertemporal preference shock. The variance of inflation is explained to a large extent by the monetary
policy shock. This result is at odds with the standard results in the literature in which the relative contribution of the monetary policy shock is small. One of the usual main contributors to the variance of inflation is the markup shock. But, in my environment, the markup shock completely drives the variance of labor share. Therefore, the model faces a tradeoff when using the realization of the markup shock to match up the dynamics of either the labor share or the inflation rate. In my set-up, the model solves the issue by drawing the dynamics of the labor share through the markup shock and the dynamics of the inflation rate using the monetary shock, which is irrelevant for any other variable.

The cyclical variance of net worth is driven by the wealth shock. Even though the size of the shock to the marginal bankruptcy cost increases dramatically post-1984, the relative contribution of this shock to the variance of net worth decreases over time.

The most remarkable entry in Table 34 is the variance decomposition of investment. First, the contribution of the investment-specific technology shock is smaller than in the literature. This result is not just an artifact linked to the assumption of adjustment costs in capital instead of in investment. I use the specification for the adjustment costs proposed by [10]. However, they note that the variance of investment is explained to a large extent by the investment-specific technology shock. In my case, the lesser relevance of the investment-specific shock is due to the presence of financial shocks. An environment identical to the current one but without financial shocks would deliver a relative contribution of the investment shock above $50 \%$. In the first subsample, the contributions of the two technology shocks and the two financial shocks are $42 \%$ and $35 \%$ respectively. In the 1970s and early 1980s, those contributions are $49 \%$ and $26 \%$,respectively. Post-1984, financial shocks are the main driver of investment variance, accounting for $42 \%$. Technology shocks account only
for $24 \%$ of the variance of investment.
From the above, I conclude that the relative contribution of technology shocks, in particular, of the investment-specific technology shock, is overstated in the literature. Once financial shocks are ain play, the contribution of technology shocks is significantly smaller, since they no longer account for financial factors in a reduced-form style.

### 4.2.2 Impulse response functions

The propagation of real and nominal shocks in the context of a model of the financial accelerator has already been studied in the literature. Therefore, in this section, I focus only on the study of the propagation dynamics of financial shocks. For both the wealth shock and the innovation to the marginal bankruptcy cost, I plot the responses in the first 40 quarters in terms of percentage deviations with respect to the steady state. Each plot contains three impulse response functions (IRFs). The dotted line is the IRF computed using the parameter vector characterizing the 1954-1969 sample period. The solid line is the IRF for the 1970s and early 1980s. The dashed line is the IRF of the post-1984 period.

## Wealth shock

Figure C-1 reports the impulse response functions following a wealth shock that, upon impact, induces an increase in entrepreneurial net worth equal to a $1 \%$ deviation from its steady-state value. The size of the shock is constant across subsamples so I can better analyze changes in the propagation mechanism of this financial shock. The response upon impact for net worth is identical because of all the defining elements of entrepreneurial wealth are predetermined when the wealth shock is realized. The
main messages from the figure are: (i) the impulse response functions in the 19541969 and 1970-1983 sample periods are almost identical for net worth, consumption, investment, output, hours worked, and the external finance premium; (ii) responses upon impact for all variables, except net worth, are smaller in the 1984-2006 sample period; and (iii) the responses become more persistent post-1984.

Let us first analyze the impulse response functions for the pre-1984 sample periods. The response of net worth is very persistent, which is the source of the large contribution of the wealth shock to the low frequency fluctuations of entrepreneurial wealth. A positive wealth shock that increases the value of collateral reduces the probability of default so that financial intermediaries are willing to lend at a lower premium. Therefore, the response of the external finance premium upon impact is negative. This immediate improvement in credit markets has a significant amplification effect on investment so that the response of investment upon impact more than doubles the initial response of net worth. The initial response of output is positive but smaller than the boost in investment because consumption decreases upon impact and the total resources constraint needs to be satisfied. The negative response of consumption upon impact is linked to the general equilibrium effects of my model. A nonfundamental increase in entrepreneurial wealth is financed through a reduction in household wealth. The reduction in total disposable income is not large enough to generate a decrease in consumption of the same magnitude as the increase in entrepreneurial wealth. This is due to the fact that other sources of household wealth, such as labor income, react positively to the wealth shock, since hours worked increase upon impact. The positive response of inflation and the nominal interest rate suggests that the wealth shock displays the features of a standard demand shock: quantities and prices move in the same direction, leading to a tightening of monetary policy.

The responses of labor share, inflation, and nominal interest rates for the 19541969 and 1970-1983 periods are not as similar as the IRFs for the other variables. The IRF of labor share undertakes very small values, the largest one being $0.008 \%$. The differences between the responses in the 1950s-1960s and the 1970s-early 1980s are driven by small differences in the IRFs of output and hours worked across subsamples. The larger response of inflation upon impact in 1970s-early 1980s is due to a less active response to deviations of inflation from the target by the monetary authority. That is, inflation is left to vary more ad libitum. I can explain the larger response upon impact of interest rates in the 1970-1983 period by noting that the monetary authority also responds to deviations of output growth from its steady-state value and that such a response is tighter over this period than in the 1950s and 1960s.

In the Great Moderation era, the response of net worth to the same wealth shock peaks at a higher value and a quarter later. From the second quarter onward, the response function post-1984 always lies above those for the pre-1984 sample periods. This can be easily reconcilable from the definition of aggregate net worth. Lower average agency costs alleviate the deadweight loss associated with bankruptcy, $\mu_{t} G\left(\bar{\omega}_{t}\right) R_{t}^{k} Q_{t-1} K_{t}$, which implies that for the same initial increase in wealth, the effects are more long-lasting, since more resources are accumulated from period to period. Higher persistence induced by the lower dependence on the financial accelerator mechanism translates into more persistent responses for all variables except labor share. Therefore, the persistence implied by the financial accelerator is a negative function of the size of financial rigidity. The responses for all variables except net worth are also characterized by a significantly smaller response upon impact. This is driven by the smaller size of the financial accelerator mechanism. Lower levels of credit market imperfections reduce the elasticity of the external finance premium with respect to the leverage ratio. Therefore, the amplification effect linked to the
improvement in credit market conditions is more muted.

## Shock to the marginal bankruptcy cost

Figure C-2 reports the impulse response functions to shocks to the marginal bankruptcy cost. I focus on a negative shock that generates a reduction upon impact in the external financial premium of $0.06 \%$ in the pre-1984 sample period. Such a shock also generates a response upon impact of similar magnitude to the one generated by the wealth shock in the post-1984 sample period. I define the innovation under analysis so as to make the comparison with the impulse response functions to a wealth shock reported in the previous section easier.

As in the previous section, I first discuss the IRFs for the pre-1984 period. A negative shock to agency costs creates an incentive for entrepreneurs to select contractual terms with a larger debt-to-net worth ratio, since the deadweight loss linked to bankruptcy is smaller. There are two opposing effects operating as a result of higher debt-to-net worth ratios. On the one hand, both the default probability and the default productivity threshold increase, offsetting the effect of lower bankruptcy costs in determining entrepreneurial net worth. I label this effect the default effect. On the other hand, there is a mass effect that stays for the increase in capital investment linked to a larger set of resources available. Larger amounts of capital holdings imply a larger equity value through an increase in total capital returns. Given that the response upon impact of entrepreneurial net worth is negative, the default effect dominates the mass effect. After a few quarters, however, the mass effect becomes the dominant one, since the response of the debt-to-net worth ratio is decreasing given that the IRF for the external finance premium is increasing. The response of net worth is increasing until quarter 38 and very persistent. Figure C-3 reports the IRFs for the 1954-1969 sample period considering 200 quarters. I observe that net
worth, consumption, and output need more than 150 quarters, i.e., 38 years, to get back to their steady-state values. I can conclude that the effects of a shock to the marginal bankruptcy cost have an "almost permanent" flavor. I use quotation marks because given that the model economy is stationary, a transitory shock does not imply a permanent effect per se, but it can have a very long-lasting one.

The response of investment upon impact is above $3.5 \%$, which is larger than the response I obtained to a wealth shock. This result is driven by the mass effect explained above. Irrespective of the relative dominance of this effect in terms of shaping the response of entrepreneurial wealth, the increase in the pool of resources available for purchasing capital enhances investment activity in the economy. In particular, note that after a wealth shock, the debt-to-net worth ratio does not increase, which explains the difference in the response of investment after the two financial shocks.

Consumption responds to the expansionary shock positively. Hours worked, however, decrease upon impact. The enlargement in investment raises future productivity and, hence, future wages. Households perform intertemporal substitution by decreasing current hours and increasing them in the future when wages are higher. As a consequence of the reduction of hours upon impact, output responds negatively to an expansionary shock to agency costs. However, as with net worth, after a few periods output's response is positive, large, and long-lasting. The positive slope of the response of output over the first 40 quarters or so is driven by the positive slope of the responses of both consumption and hours worked.

Given the expansionary effect on investment of the shock at hand, inflation increases upon impact. As before, the difference in the initial response in the 1950s1960s versus the 1970s-early 1980s is driven by monetary policy. In the latter subperiod, inflation floats more significantly without meeting a strong enough response from the monetary authority.

The response of the nominal interest rate is negative pre-1984 and positive afterwards. The federal funds rate, in my model, responds to both deviations of inflation and output growth from their respective steady-state values. The initial negative response of output in the pre-1984 subperiods translates into negative output growth upon impact of the shocks. The monetary authority responds by decreasing the interest rate. The positive response of inflation requires an increase in the federal funds rate. However, the reduction in output growth dominates the increase in inflation, forcing the monetary authority to use an expansionary monetary policy. However, post-1984, the response of output upon impact is negligible. Therefore, the rise in inflation, even though it is $50 \%$ smaller than in the 1970s and the early 1980s, dominates monetary policy-making. The risk-free nominal interest rate, then, increases upon impact.

Given the significant decline in the size of the financial accelerator, the post-1984 impulse response functions are all characterized by smaller responses for all variables.

## Comparison of impulse response functions

In this section, I compare the impulse response functions to the two expansionary financial shocks. Let us focus on the responses for the 1954-1969 sample period. The same rationale follows for the other sample periods of interest. Both financial shocks generate the sample response upon impact for the external finance premium. The responses upon impact of net worth, output, consumption, and hours worked, reported in Figure C-4, are very different. Therefore, I can learn about the source of the responses.

While net worth responds positively upon impact to a wealth shock, its initial response to a shock to the marginal bankruptcy cost is negative. In the latter, the response upon impact is determined by the dominance of the default effect over the
mass effect. After a wealth shock, the default probability decreases, the default productivity threshold decreases, and the recovery rate remains unchanged. Therefore, irrespective of the response of the debt-to-net worth ratio, the response of net worth is always positive upon impact.

The response upon impact of consumption to an expansionary financial shock is a function of the nature of the financial shock. A change in entrepreneurial wealth driven by an equity valuation shifter, such as my wealth shock, modifies consumption in the reverse direction, since exogenously driven variations in entrepreneurs' net worth are financed by the household sector. However, consumption responds to an expansionary financial shock as to any expansionary shock hitting the economy whenever the financial shock affects the marginal bankruptcy cost.

Hours worked respond to a positive technology shock in a framework with sticky prices when the financial shock decreases the marginal monitoring cost. The sign of the response to a wealth shock is identical to the one implied by an expansionary technology shock in a standard RBC model. Consequently, the signs of the responses of output to expansionary wealth shocks are determined by those of the responses of output.

### 4.3 What are the Effects of Financial Shocks on Consumption? An Empirical Assessment

### 4.3.1 Data and statistical model

There is a lot of diversity in the literature on financial frictions about the financial series used to perform estimation exercises. [42] and [28] use data from the Flow of Funds accounts provided by the Board of Governors of the Federal Reserve System.
[17] use several financial variables in their estimation exercise: as a measure of real net worth for entrepreneurs, the value of the Dow Jones industrial average scaled by the GDP deflator; bank reserves; external finance premium defined as the difference between Baa and Aaa yields on corporate bonds; and the spread between the 10-year government rate and the federal funds rate. [33] use a proxy for the external finance premium based on a sample of credit spreads constructed from bond data provided by Lehman/Warga and Merril Lynch. [5], however, set the risk spread at the steady state considering the historical average of the difference between the prime loan rate and the six-month T-bill.

In this paper, I estimate four-variate Bayesian vector autoregressive (VAR) models including consumption growth, investment growth, business wealth growth, and a proxy for the external finance premium. While consumption is defined as durable consumption plus services provided by NIPA, investment is constructed using gross private investment. Both variables are corrected by the size of the nonfarm business sector in the economy. I consider two different measures of business wealth: (i) net worth defined as tangible assets minus credit market liabilities using the Flow of Funds Accounts provided by the Board of Governors and (ii) the real counterpart of the Industrial Dow Jones index. I perform the analysis considering the two most common proxies in the literature for the premium on external borrowing faced by the business sector: the corporate bond spread and a measure of the prime lending spread. The corporate bond spread is defined as the Baa rate over the Aaa rate. I define the prime lending rate as the difference between the prime loan rate and the 3-month T-bill. In order to make my purely empirical analysis comparable with the results obtained under the theoretical framework estimated in Chapter 3, I use data from 1954:IV to 2006:IV.

Let me assume that the $m \times 1$ vector $y_{t}$ is described by the following reduced form
representation of a vector autoregressive (VAR) model

$$
\begin{equation*}
y_{t}=\Phi_{1} y_{t-1}+\ldots+\Phi_{p} y_{t-p}+u_{t} \tag{4.1}
\end{equation*}
$$

where $u_{t}$ are the one-step ahead forecast errors with variance-covariance matrix $\Sigma$ so that $u_{t} \sim_{i i d} \mathcal{N}(0, \Sigma)$. I can add a constant and a time trend to the above representation. I assume that the prediction errors, $u_{t}$, are a function of $m$ fundamental innovations, $v_{t}$, which are assumed to be mutually independent with unit variance, $\mathbb{E}_{t}\left[v_{t} v_{t}^{\prime}\right]=I_{m}$. I need to find an identifying matrix, $A$, that defines a one-to-one mapping from the vector of structural shocks, $v_{t}$, to the reduced form residuals, $u_{t}$. Therefore, the matrix $A$ is such that $u_{t}=A v_{t}$ and $\Sigma=\mathbb{E}\left[u_{t} u_{t}^{\prime}\right]=A \mathbb{E}\left[v_{t} v_{t}^{\prime}\right] A^{\prime}=A A^{\prime}$. Because of the orthogonality assumption for the structural innovations and the symmetry of $\Sigma$, I need to impose at least $\frac{m(m-1)}{2}$ restrictions on $A$ in order to achieve identification.

### 4.3.2 Sign restriction identification: Methodology

[59] proposes an agnostic identification procedure based on sign restrictions on the impulse response functions implied by the VAR model. Before describing the identification strategy, let us obtain the representation for the impulse response function. Let us rewrite the VAR defined in equation 4.1 in companion form

$$
\begin{equation*}
Y_{t}=\Phi Y_{t-1}+u_{t} \tag{4.2}
\end{equation*}
$$

The impulse responses to fundamental shocks are given by $\frac{\partial Y_{t+j}}{\partial v_{t}}=\Phi^{j} A$. I aim to identify the columns of $A$ associated to financial shocks. The matrix $A$ is unique up to an orthonormal transformation, that is, for any $\Omega \Omega^{\prime}=I_{m}$ I have $\Sigma=A \Omega \Omega^{\prime} A$.

As [27] describe, there are two approaches to pin the matrix $\Omega$ down: Givens and Householder transformations.

An orthonormal matrix $\Omega$ for an $m$-variate VAR can be obtained as the product of $\frac{m(m-1)}{2}$ Givens matrices so that there are $\frac{m(m-1)}{4}$ combinations of rotations of different elements. Rotations are always bivariate so in an $m$-dimensional system rows $m$ and $n$ are rotated by

$$
\Omega_{m, n}=\left(\begin{array}{cccccc}
1 & 0 & 0 & \ldots & 0 & 0  \tag{4.3}\\
0 & 1 & 0 & \ldots & 0 & 0 \\
\ldots & \ldots & \ldots & \ldots & \ldots & \ldots \\
0 & 0 & \cos \varphi & \ldots & -\sin \varphi & 0 \\
\ldots & \ldots & \ldots & 1 & \ldots & \ldots \\
0 & 0 & \sin \varphi & \ldots & \cos \varphi & 0 \\
\ldots & \ldots & \ldots & \ldots & \ldots & \ldots \\
0 & 0 & 0 & 0 & 0 & 1
\end{array}\right)
$$

For expositional purpose, let us consider $m=3$, then the orthonormality restrictions can be imposed using the following

$$
\Omega=\left(\begin{array}{ccc}
\cos \varphi_{1} & \sin \varphi_{1} & 0  \tag{4.4}\\
-\sin \varphi_{1} & \cos \varphi_{1} & 0 \\
0 & 0 & 1
\end{array}\right)\left(\begin{array}{ccc}
\cos \varphi_{2} & 0 & \sin \varphi_{2} \\
0 & 1 & 0 \\
-\sin \varphi_{2} & 0 & \cos \varphi_{2}
\end{array}\right)\left(\begin{array}{ccc}
1 & 0 & 0 \\
0 & \cos \varphi_{3} & \sin \varphi_{3} \\
0 & -\sin \varphi_{3} & \cos \varphi_{3}
\end{array}\right)
$$

with $\left\{\varphi_{1}, \varphi_{2}, \varphi_{3}\right\} \in[0,2 \pi]$. Note that the matrix $\Omega$ does not span the space of all orthonormal matrices but only the subspace of orthonormal matrices with a unit determinant.

Householder transformation are based on delivering an orthonormal matrix $\Omega$
by generating draws from a standard Gaussian distribution and computing the QR decomposition. [59] shows that for an arbitrary decomposition $\tilde{A}$ of the matrix $\Sigma$, any structural impulse vector $a$ from the identifying matrix $A$ can be represented as $\tilde{A} \alpha$, where $\alpha$ is a vector belonging to the hypersphere of unit radius in $R^{m 2}$. The natural candidates for the arbitrary decomposition $\tilde{A}$ of the identifying matrix $A$ are the QR decomposition (Householder transformation) and the Cholesky decomposition of the matrix $\Sigma$.

I use a Bayesian approach to estimate the VAR model. Under this framework, sign restrictions are equivalent to assign a zero probability to reduced-form parameters that deliver impulse response functions that do not satisfy the identification restrictions. I use a Minnesota prior for the reduced-form VAR parameters and an independent uniform prior on $\alpha$. Therefore, given Gaussian errors in (4.1), I can conclude that the posterior distribution for $\Phi$ and $\Sigma$ is of the Inverted Wishart-Normal family. I obtain posterior draws from the posterior for the VAR coefficients and draws from the space of possible impulse vectors. I make inference using the joint draws that satisfy the identifying sign restrictions for the impulse response functions. The posterior sampler can be described as follows:

1. Obtain draws for $\Phi, \Sigma$ using a Gibbs sampler. Take $n_{1}$ draws. Compute $\widetilde{A}=$ $\operatorname{chol}(\Sigma)$.
2. For each draw,
(a) generate a draw for $\alpha$ from the $m$-dimensional unit sphere by drawing $\widetilde{\alpha}$ from an $m$-dimensional standard normal distribution and normalizing its length to unity.

$$
\begin{equation*}
\alpha=\frac{\widetilde{\alpha}}{\|\widetilde{\alpha}\|} \tag{4.5}
\end{equation*}
$$

[^8](b) construct the impulse vector $a=\widetilde{A} \alpha$.
(c) compute the impulse response functions.
(d) if the sign restrictions for the $k=\{0,1, .$.$\} horizons are satisfied, store the$ impulse response functions. Otherwise, discard.
3. Repeat steps $2 \mathrm{a}-2 \mathrm{~d} n_{2}$ times.

In my case, I perform the estimation exercise for all the variables but the proxy for the external finance premium in growth rates. But, I impose the sign restrictions on the variables in levels, that is, I impose the constraints on the accumulated impulse response functions. Not only my theoretical model but also any economic model would suggest that a shock reinforcing the value of firms has two effects. On the one hand, it reduces the cost of borrowing since the value of the collateral is larger. On the other hand, it enhances investment spending. Therefore, I can identify an expansionary shock to business wealth by imposing a positive response during two quarters for net worth and investment, and a negative response during two quarters for the credit spread. The choice of the number of periods during which the sign restrictions are imposed is not trivial. It would be of great interest to perform robustness checks using different values for $k$. Note that I have left unspecified the response of consumption since my object of interest is to assess empirically how consumption responds to a shock to the value of firms. The sign restrictions used to identify the financial shock under analysis are given in Table 36.

### 4.3.3 Results

In the appendix, I report the median and the $95 \%$ credible interval set for the points on the impulse response functions satisfying the sign restrictions. I perform the
analysis not only using the whole sample, but also dividing it up using the breakpoints identified in Chapter 3. Table 37 delivers the percentage of responses satisfying the identification restrictions that imply a negative response upon impact of consumption. From the table, the main conclusion is that there is ambiguity about the sign of the response of consumption to a financial shock affecting business wealth.

Let us first consider the data set in which net worth is measured using data from the Flow of Funds accounts (FOFA) and the proxy for the external finance premium is the corporate bond spread. In this case, $10 \%$ of the responses satisfying the identification restrictions deliver a negative response upon impact of consumption when considering the whole sample under analysis. By subsamples, I obtain that while in 1970-1983 only $2 \%$ of the responses are negative upon impact, both in 1954-1969 and 1984-2006 more than $30 \%$ of the responses satisfying the sign restrictions generate responses of opposite signs for consumption and investment. Figure C-5 reports the impulse response functions for the whole sample and figure C-8 the ones for the subperiod 1970-1983. I can highlight several differences when comparing those two sets of impulse response functions: (i) the $95 \%$ bands for consumption in the 19701983 subsample are in the positive real line, (ii) investment peaks at a higher value and stabilizes at a larger value in the 1970-1983 subsample due to the fact that, as both investment and consumption responses are positive, the enhancement of output after a financial shock is larger which boosts investment even further, and (iii) while it takes 10 quarters to cross the zero point for the corporate spread response for the whole sample, it only takes 6 quarters in the 1970-1983 subperiod.

When net worth is measured using FOFA but the cost of external funding is measured with the prime lending spread, I obtain that $57 \%$ of the responses satisfying the restrictions deliver a negative response upon impact for consumption during the 1970-1983 sample. This result sharply differs from the $2 \%$ obtained when the external
finance premium was measured using the corporate bond spread. Comparing the impulse response function for consumption in figure C-9 with that using the corporate bond spread (figure C-8), I observe that while in the former, the median response is always negative; in the latter, the $95 \%$ credible bands are positive. Therefore, I can conclude that assessing the effects of financial shocks on consumption is not a trivial matter since it significantly depends upon the choice of observable variables in the empirical analysis.

If I measure business wealth with the firm's value in the stock market, that is, with the real counterpart of the industrial Dow Jones index and the cost of external borrowing using the corporate bond spread, I obtain that the percentages of responses upon impact of opposite sign for consumption and investment across subsamples range from 30 to $49 \%$. Figure C-6 reports the impulse response functions for the whole sample under this framework.

Finally, when I use the Dow Jones as proxy for business wealth and the prime lending rate as proxy for the cost of borrowing, I obtain that $55 \%$ of the responses satisfying the identification restrictions imply a negative response upon impact of consumption when using the whole sample. In the 1970-1983 subperiod, that percentage is equal to $65 \%$. During 1954-1969, however, only $10 \%$ of the responses deliver a response upon impact of consumption of the opposite sign to the response of investment. I deliver the impulse response functions for the whole sample in figure C-7. It is remarkable that the median response for consumption is below zero for almost 5 quarters.

### 4.4 Conclusion

I have analyzed the model implied dynamics in response to the financial shocks introduced in the third chapter of this thesis. I conclude that financial factors are relevant in shaping the business cycle properties of financial variables, investment, and the nominal interest rate. Financial shocks are not only the only drivers of the variance of net worth, but also the main drivers of investment variance in the Great Moderation era. The relative contribution of technology shocks to the variance of real variables decreases significantly over time. The estimated reduction in the size of the financial accelerator changes the propagation mechanism of financial shocks to the economy. The responses upon impact are smaller for both financial shocks and the responses to a wealth shock are more persistent.

I obtain that the model implied response upon impact of consumption to an expansionary financial shock affecting business wealth is negative. Although this result is widespread in the literature on the financial accelerator, it seems to be at odds with the popular understanding of an expansionary shock.

The best way of assessing the goodness of fit of my model in this dimension is by comparing my results with the empirical responses. To the best of my knowledge, there is no empirical characterization of the effects of a shock to business wealth on household consumption in the literature. I do perform such an analysis in this chapter using sign restrictions and several measures of business wealth and the external finance premium.

I conclude that assessing the effects of financial shocks on consumption is not a trivial issue since my results depend significantly upon the choice of observable variables to proxy both firms' value and the cost of borrowing. In any case, I can state that the response of consumption to a financial shock of the nature I am interested in
is ambiguous since I always obtain a non-zero percentage of responses of consumption and investment of opposite signs.

It would be of great interest to address the empirical characterization of the response of consumption to a shock to business wealth using the measures of the external finance premium constructed by [33] using micro data.

## Appendices

## Appendix A Chapter 2

Appendix A. 1 Tables and figures

Table 1: Calibration Targets

|  | 1948:1-2006:4 | 1948:1-1983:4 | $1984: 1-2006: 4$ |
| :--- | :---: | :---: | :---: |
| $H^{*}$ | 0.31 | 0.31 | 0.31 |
| $Y^{*}$ | 1 | 1 | 1 |
| $\left(\frac{K / V}{Y}\right)^{*}$ | 10.288 | 10.502 | 9.953 |
| $\left(\frac{X}{K}\right)^{*}$ | 0.0277 | 0.0276 | 0.0279 |
| $\left(\frac{I}{Y}\right)^{*}$ | 0.28 | 0.29 | 0.28 |

Table 2: Deterministic Trend: Calibrated Parameters

|  | $1948: 1-2006: 4$ | $1948: 1-1983: 4$ | $1984: 1-2006: 4$ |
| :--- | :---: | :---: | :---: |
| $\alpha$ | 0.36 | 0.36 | 0.36 |
| $\gamma_{a}$ | 0.000131 | 0.001413 | -0.000824 |
| $\gamma_{v}$ | 0.006760 | 0.005311 | 0.009438 |
| $\beta$ | 0.995 | 0.992 | 0.998 |
| $\delta$ | 0.0135 | 0.0131 | 0.0121 |
| $\eta$ | 0.0034 | 0.0037 | 0.0030 |
| $\mu$ | 0.0086 | 0.0091 | 0.0074 |
| $\varphi_{a}$ | 4.67 | 4.59 | 4.83 |
| $\varphi_{v}$ | -0.16 | -0.07 | -0.65 |
| $\rho_{a}$ | 0.98 | 0.97 | 0.96 |
| $\rho_{v 1}$ | 1.77 | 1.71 | 1.73 |
| $\rho_{v 2}$ | -0.77 | -0.76 | -0.73 |
| $\sigma_{\xi_{a}}$ | 0.0073 | 0.0086 | 0.0045 |
| $\sigma_{\xi_{v}}$ | 0.0033 | 0.0037 | 0.0023 |
| $\nu$ | $\{0.5,1,1.5,2\}$ | $\{0.5,1,1.5,2\}$ | $\{0.5,1,1.5,2\}$ |
| $B$ | $\{30.02,9.31,6.30,5.18\}$ | $\{30.21,9.36,6.34,5.21\}$ | $\{29.73,9.22,6.24,5.13\}$ |
|  |  |  |  |

Table 3: Baseline Stochastic Trend: Calibrated Parameters

|  | $1948: 1-2006: 4$ | $1948: 1-1983: 4$ | $1984: 1-2006: 4$ |
| :--- | :---: | :---: | :---: |
| $\alpha$ | 0.36 | 0.36 | 0.36 |
| $\gamma_{a}$ | 0.000619 | 0.001489 | -0.000734 |
| $\gamma_{v}$ | 0.00643 | 0.00493 | 0.008762 |
| $\beta$ | 0.9894 | 0.9897 | 0.9889 |
| $\delta$ | 0.01348 | 0.01401 | 0.01267 |
| $\eta$ | 0.0034 | 0.0037 | 0.0030 |
| $\mu$ | 0.92443 | 0.91670 | 0.93694 |
| $\sigma_{a}$ | 0.0074 | 0.0087 | 0.0045 |
| $\sigma_{v}$ | 0.0054 | 0.0060 | 0.0030 |
| $\nu$ | $\{0.5,1,1.5,2\}$ | $\{0.5,1,1.5,2\}$ | $\{0.5,1,1.5,2\}$ |
| $B$ | $\{30.02,9.31,6.30,5.18\}$ | $\{30.21,9.36,6.34,5.21\}$ | $\{29.73,9.22,6.24,5.13\}$ |

Table 4: Stochastic Trend with a Moving Average Component: Calibrated Parameters

|  | $1948: 1-2006: 4$ | $1948: 1-1983: 4$ | $1984: 1-2006: 4$ |
| :--- | :---: | :---: | :---: |
| $\alpha$ | 0.36 | 0.36 | 0.36 |
| $\gamma_{a}$ | 0.000619 | 0.001489 | -0.000734 |
| $\gamma_{v}$ | 0.006404 | 0.004911 | 0.008739 |
| $\beta$ | 0.9894 | 0.9897 | 0.9889 |
| $\delta$ | 0.01352 | 0.01404 | 0.01271 |
| $\eta$ | 0.0034 | 0.0037 | 0.0030 |
| $\mu$ | 0.92439 | 0.91667 | 0.93692 |
| $\rho$ | 0.618159 | 0.6359218 | 0.5071769 |
| $\sigma_{a}$ | 0.0074 | 0.0087 | 0.0045 |
| $\sigma_{v}$ | 0.0025 | 0.0028 | 0.0012 |
| $\nu$ | $\{0.5,1,1.5,2\}$ | $\{0.5,1,1.5,2\}$ | $\{0.5,1,1.5,2\}$ |
| $B$ | $\{30.02,9.31,6.30,5.18\}$ | $\{30.21,9.36,6.34,5.21\}$ | $\{29.73,9.22,6.24,5.13\}$ |

Notes: I use Ríos-Rull et al. (2007) Data Set. I have HP-filtered the $\log$ of real variables. Standard deviations are in percentage terms.
Table 6: Results: Deterministic Trend

Table 7: Results: Baseline Stochastic Trend

|  | 1948:1-2006:4 |  |  |  | 1948:1-1983:4 |  |  |  | 1984:1-2006:4 |  |  |  | $\sigma_{\text {post }} / \sigma_{\text {pre }}$ |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | $\% \sigma_{\text {data }}$ | $\rho_{z, y}^{\text {data }}$ | $\% \sigma_{\text {model }}$ | $\rho_{z, y}^{\text {model }}$ | $\% \sigma_{\text {data }}$ | $\rho_{z, y}^{\text {data }}$ | $\% \sigma_{\text {model }}$ | $\rho_{z, y}^{\text {model }}$ | $\sigma_{d a t a}$ | $\rho_{z, y}^{\text {data }}$ | $\% \sigma_{\text {model }}$ | $\rho_{z, y}^{\text {model }}$ | Data | Model |
|  |  |  |  |  | $\nu=0.5$ |  |  |  |  |  |  |  |  |  |
| C | 0.92 | 0.78 | 0.68 | 0.89 | 1.07 | 0.78 | 0.82 | 0.90 | 0.62 | 0.81 | 0.41 | 0.90 | 0.58 | 0.50 |
| X | 5.77 | 0.89 | 2.67 | 0.84 | 6.85 | 0.89 | 3.01 | 0.84 | 3.47 | 0.92 | 1.57 | 0.85 | 0.53 | 0.52 |
| y | 1.73 | 1 | 1.06 | 1 | 2.07 | 1 | 1.26 | 1 | 0.99 | 1 | 0.63 | 1 | 0.48 | 0.50 |
| k | 0.59 | 0.36 | 0.26 | 0.33 | 0.68 | 0.39 | 0.30 | 0.32 | 0.44 | 0.27 | 0.16 | 0.31 | 0.65 | 0.53 |
| h | 1.88 | 0.87 | 0.18 | 0.83 | 2.11 | 0.87 | 0.21 | 0.83 | 1.46 | 0.90 | 0.11 | 0.83 | 0.69 | 0.52 |
| $y / h$ | 0.94 | 0.11 | 0.91 | 0.99 | 1.06 | 0.23 | 1.09 | 0.99 | 0.72 | -0.46 | 0.55 | 0.99 | 0.68 | 0.50 |
|  |  |  |  |  | $\nu=1$ |  |  |  |  |  |  |  |  |  |
| C | 0.92 | 0.78 | 0.69 | 0.87 | 1.07 | 0.78 | 0.80 | 0.89 | 0.62 | 0.81 | 0.41 | 0.90 | 0.58 | 0.51 |
| X | 5.77 | 0.89 | 2.91 | 0.85 | 6.85 | 0.89 | 3.15 | 0.86 | 3.47 | 0.92 | 1.73 | 0.87 | 0.53 | 0.54 |
| y | 1.73 | 1 | 1.11 | 1 | 2.07 | 1 | 1.27 | 1 | 0.99 | 1 | 0.68 | 1 | 0.48 | 0.54 |
| k | 0.59 | 0.36 | 0.30 | 0.33 | 0.68 | 0.39 | 0.31 | 0.32 | 0.44 | 0.27 | 0.18 | 0.33 | 0.65 | 0.58 |
| h | 1.88 | 0.87 | 0.30 | 0.83 | 2.11 | 0.87 | 0.33 | 0.84 | 1.46 | 0.90 | 0.18 | 0.86 | 0.69 | 0.55 |
| $y / h$ | 0.94 | 0.11 | 0.87 | 0.98 | 1.06 | 0.23 | 1.00 | 0.98 | 0.72 | -0.46 | 0.53 | 0.98 | 0.68 | 0.53 |
|  |  |  |  |  | $\nu=1.5$ |  |  |  |  |  |  |  |  |  |
| C | 0.92 | 0.78 | 0.70 | 0.89 | 1.07 | 0.78 | 0.83 | 0.89 | 0.62 | 0.81 | 0.42 | 0.89 | 0.58 | 0.51 |
| X | 5.77 | 0.89 | 2.93 | 0.87 | 6.85 | 0.89 | 3.45 | 0.88 | 3.47 | 0.92 | 1.85 | 0.88 | 0.53 | 0.54 |
| y | 1.73 | 1 | 1.15 | 1 | 2.07 | 1 | 1.37 | 1 | 0.99 | 1 | 0.71 | 1 | 0.48 | 0.52 |
| k | 0.59 | 0.36 | 0.29 | 0.31 | 0.68 | 0.39 | 0.35 | 0.33 | 0.44 | 0.27 | 0.19 | 0.34 | 0.65 | 0.54 |
| h | 1.88 | 0.87 | 0.37 | 0.85 | 2.11 | 0.87 | 0.44 | 0.86 | 1.46 | 0.90 | 0.23 | 0.86 | 0.69 | 0.52 |
| $y / h$ | 0.94 | 0.11 | 0.85 | 0.97 | 1.06 | 0.23 | 1.02 | 0.98 | 0.72 | -0.46 | 0.52 | 0.98 | 0.68 | 0.51 |
|  |  |  |  |  | $\nu=2$ |  |  |  |  |  |  |  |  |  |
| C | 0.92 | 0.78 | 0.73 | 0.88 | 1.07 | 0.78 | 0.85 | 0.90 | 0.62 | 0.81 | 0.43 | 0.89 | 0.58 | 0.51 |
| X | 5.77 | 0.89 | 3.16 | 0.88 | 6.85 | 0.89 | 3.57 | 0.89 | 3.47 | 0.92 | 1.89 | 0.88 | 0.53 | 0.53 |
| y | 1.73 | 1 | 1.22 | 1 | 2.07 | 1 | 1.42 | 1 | 0.99 | 1 | 0.73 | 1 | 0.48 | 0.51 |
| k | 0.59 | 0.36 | 0.31 | 0.33 | 0.68 | 0.39 | 0.35 | 0.33 | 0.44 | 0.27 | 0.19 | 0.32 | 0.65 | 0.54 |
| h | 1.88 | 0.87 | 0.45 | 0.86 | 2.11 | 0.87 | 0.51 | 0.87 | 1.46 | 0.90 | 0.26 | 0.87 | 0.69 | 0.51 |
| $y / h$ | 0.94 | 0.11 | 0.86 | 0.96 | 1.06 | 0.23 | 0.97 | 1.01 | 0.72 | -0.46 | 0.52 | 0.97 | 0.68 | 0.51 |

Table 8: Results: Stochastic Trend with Moving Average Component

|  | 1948:1-2006:4 |  |  |  | 1948:1-1983:4 |  |  |  | 1984:1-2006:4 |  |  |  | $\sigma_{\text {post }} / \sigma_{\text {pre }}$ |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | $\% \sigma_{\text {data }}$ | $\rho_{z, y}^{\text {data }}$ | $\% \sigma_{\text {model }}$ | $\rho_{z, y}^{\text {model }}$ | $\% \sigma_{\text {data }}$ | $\rho_{z, y}^{\text {data }}$ | $\% \sigma_{\text {model }}$ | $\rho_{z, y}^{\text {model }}$ | $\% \sigma_{\text {data }}$ | $\rho_{z, y}^{\text {data }}$ | $\% \sigma_{\text {model }}$ | $\rho_{z, y}^{\text {model }}$ | Data | Model |
|  |  |  |  |  | $\nu=0.5$ |  |  |  |  |  |  |  |  |  |
| c | 0.92 | 0.78 | 0.69 | 0.87 | 1.07 | 0.78 | 0.82 | 0.86 | 0.62 | 0.81 | 0.40 | 0.95 | 0.58 | 0.49 |
| x | 5.77 | 0.89 | 2.60 | 0.85 | 6.85 | 0.89 | 3.12 | 0.85 | 3.47 | 0.92 | 1.50 | 0.93 | 0.53 | 0.48 |
| y | 1.73 | 1 | 1.06 | 1 | 2.07 | 1 | 1.26 | 1 | 0.99 | 1 | 0.66 | 1 | 0.48 | 0.52 |
| k | 0.59 | 0.36 | 0.27 | 0.31 | 0.68 | 0.39 | 0.34 | 0.31 | 0.44 | 0.27 | 0.16 | 0.32 | 0.65 | 0.47 |
| h | 1.88 | 0.87 | 0.19 | 0.80 | 2.11 | 0.87 | 0.23 | 0.80 | 1.46 | 0.90 | 0.10 | 0.90 | 0.69 | 0.43 |
| $y / h$ | 0.94 | 0.11 | 0.92 | 0.99 | 1.06 | 0.23 | 1.08 | 0.99 | 0.72 | -0.46 | 0.57 | 1 | 0.68 | 0.53 |
|  |  |  |  |  | $\nu=1$ |  |  |  |  |  |  |  |  |  |
| c | 0.92 | 0.78 | 0.70 | 0.85 | 1.07 | 0.78 | 0.84 | 0.85 | 0.62 | 0.81 | 0.40 | 0.94 | 0.58 | 0.48 |
| x | 5.77 | 0.89 | 2.77 | 0.86 | 6.85 | 0.89 | 3.31 | 0.87 | 3.47 | 0.92 | 1.54 | 0.93 | 0.53 | 0.47 |
| y | 1.73 | 1 | 1.10 | 1 | 2.07 | 1 | 1.33 | 1 | 0.99 | 1 | 0.66 | 1 | 0.48 | 0.50 |
| k | 0.59 | 0.36 | 0.29 | 0.33 | 0.68 | 0.39 | 0.35 | 0.34 | 0.44 | 0.27 | 0.16 | 0.33 | 0.65 | 0.46 |
| h | 1.88 | 0.87 | 0.31 | 0.81 | 2.11 | 0.87 | 0.38 | 0.82 | 1.46 | 0.90 | 0.16 | 0.90 | 0.69 | 0.42 |
| $y / h$ | 0.94 | 0.11 | 0.87 | 0.98 | 1.06 | 0.23 | 1.05 | 0.98 | 0.72 | -0.46 | 0.52 | 0.99 | 0.68 | 0.50 |
|  |  |  |  |  | $\nu=1.5$ |  |  |  |  |  |  |  |  |  |
| c | 0.92 | 0.78 | 0.72 | 0.85 | 1.07 | 0.78 | 0.87 | 0.84 | 0.62 | 0.81 | 0.42 | 0.94 | 0.58 | 0.48 |
| x | 5.77 | 0.89 | 3.06 | 0.88 | 6.85 | 0.89 | 3.55 | 0.87 | 3.47 | 0.92 | 1.64 | 0.93 | 0.53 | 0.46 |
| y | 1.73 | 1 | 1.18 | 1 | 2.07 | 1 | 1.40 | 1 | 0.99 | 1 | 0.71 | 1 | 0.48 | 0.51 |
| k | 0.59 | 0.36 | 0.32 | 0.32 | 0.68 | 0.39 | 0.38 | 0.33 | 0.44 | 0.27 | 0.17 | 0.32 | 0.65 | 0.45 |
| h | 1.88 | 0.87 | 0.42 | 0.83 | 2.11 | 0.87 | 0.49 | 0.81 | 1.46 | 0.90 | 0.21 | 0.91 | 0.69 | 0.43 |
| $y / h$ | 0.94 | 0.11 | 0.87 | 0.96 | 1.06 | 0.23 | 1.04 | 0.96 | 0.72 | -0.46 | 0.53 | 0.99 | 0.68 | 0.51 |
|  |  |  |  |  | $\nu=2$ |  |  |  |  |  |  |  |  |  |
| c | 0.92 | 0.78 | 0.72 | 0.84 | 1.07 | 0.78 | 0.87 | 0.84 | 0.62 | 0.81 | 0.42 | 0.93 | 0.58 | 0.48 |
| x | 5.77 | 0.89 | 3.14 | 0.88 | 6.85 | 0.89 | 3.59 | 0.88 | 3.47 | 0.92 | 1.74 | 0.94 | 0.53 | 0.48 |
| y | 1.73 | 1 | 1.20 | 1 | 2.07 | 1 | 1.42 | 1 | 0.99 | 1 | 0.73 | 1 | 0.48 | 0.51 |
| k | 0.59 | 0.36 | 0.33 | 0.33 | 0.68 | 0.39 | 0.37 | 0.35 | 0.44 | 0.27 | 0.18 | 0.34 | 0.65 | 0.49 |
| h | 1.88 | 0.87 | 0.48 | 0.84 | 2.11 | 0.87 | 0.57 | 0.83 | 1.46 | 0.90 | 0.25 | 0.91 | 0.69 | 0.44 |
| $y / h$ | 0.94 | 0.11 | 0.85 | 0.95 | 1.06 | 0.23 | 1.01 | 0.95 | 0.72 | -0.46 | 0.52 | 0.98 | 0.68 | 0.51 |

Table 9: Model implied volatility to observed volatility ratio ( $\sigma_{\text {model }} / \sigma_{\text {data }}$ ) when $\nu=1$

Table 10: Variance Decomposition for the whole sample under $\nu=1$ |  | DT |  | ST |  | ST-MA |  |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: |
|  | A | V | A | V | A | V |
| c | 26 | 74 | 86 | 14 | 84 | 16 |
| x | 54 | 46 | 65 | 35 | 67 | 33 |
| y | 91 | 9 | 99 | 1 | 96 | 4 |
| k | 25 | 75 | 67 | 33 | 63 | 37 |
| h | 35 | 65 | 68 | 32 | 60 | 40 |
| $\mathrm{y} / \mathrm{h}$ | 91 | 9 | 99 | 1 | 97 | 3 |

Table 11: Model implied autocorrelation to observed coefficient ratio ( $\rho_{\text {model }} / \rho_{\text {data }}$ ) when $\nu=1$

[^9]Table 13: Cross-correlation output with x : Whole sample and $\nu=1$

|  | $\mathrm{x}(-5)$ | $\mathrm{x}(-4)$ | $\mathrm{x}(-3)$ | $\mathrm{x}(-2)$ | $\mathrm{x}(-1)$ | $\mathrm{x}(0)$ | $\mathrm{x}(1)$ | $\mathrm{x}(2)$ | $\mathrm{x}(3)$ | $\mathrm{x}(4)$ | $\mathrm{x}(5)$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Data |  |  |  |  |  |  |  |  |  |  |  |
| Y | -0.14 | 0.05 | 0.31 | 0.60 | 0.84 | 1 | 0.84 | 0.60 | 0.31 | 0.05 | -0.14 |
| V | -0.30 | -0.22 | -0.12 | -0.01 | 0.09 | 0.17 | 0.25 | 0.29 | 0.30 | 0.30 | 0.29 |
| A | 0.19 | 0.31 | 0.46 | 0.60 | 0.65 | 0.65 | 0.32 | 0.003 | -0.27 | -0.43 | -0.49 |
| C | -0.04 | 0.16 | 0.37 | 0.59 | 0.75 | 0.78 | 0.66 | 0.47 | 0.27 | 0.09 | -0.06 |
| X | -0.04 | 0.13 | 0.36 | 0.60 | 0.79 | 0.89 | 0.71 | 0.46 | 0.17 | -0.07 | -0.22 |
| I | 0.02 | 0.18 | 0.41 | 0.64 | 0.82 | 0.91 | 0.71 | 0.43 | 0.12 | -0.13 | -0.29 |
| K | -0.51 | -0.44 | -0.31 | -0.12 | 0.11 | 0.36 | 0.55 | 0.66 | 0.67 | 0.61 | 0.51 |
| H | -0.26 | -0.09 | 0.14 | 0.42 | 0.69 | 0.87 | 0.87 | 0.75 | 0.54 | 0.30 | 0.10 |
| LBP | 0.26 | 0.28 | 0.29 | 0.28 | 0.19 | 0.11 | -0.19 | -0.39 | -0.51 | -0.51 | -0.45 |
| Deterministic Trend |  |  |  |  |  |  |  |  |  |  |  |
| Y | 0.01 | 0.15 | 0.32 | 0.52 | 0.75 | 1 | 0.75 | 0.52 | 0.32 | 0.15 | 0.01 |
| C | -0.22 | -0.18 | -0.12 | -0.02 | 0.10 | 0.26 | 0.24 | 0.22 | 0.21 | 0.20 | 0.18 |
| X | 0.11 | 0.24 | 0.38 | 0.54 | 0.71 | 0.89 | 0.65 | 0.42 | 0.23 | 0.06 | -0.07 |
| K | -0.28 | -0.21 | -0.11 | 0.01 | 0.16 | 0.54 | 0.46 | 0.53 | 0.55 | 0.55 | 0.51 |
| H | 0.16 | 0.26 | 0.38 | 0.51 | 0.64 | 0.67 | 0.55 | 0.35 | 0.17 | 0.02 | -0.11 |
| LBP | -0.11 | 0.02 | 0.18 | 0.37 | 0.60 | 0.86 | 0.67 | 0.50 | 0.35 | 0.21 | 0.10 |
| Stochastic Trend |  |  |  |  |  |  |  |  |  |  |  |
| Y | 0.03 | 0.15 | 0.30 | 0.50 | 0.73 | 1 | 0.73 | 0.50 | 0.30 | 0.15 | 0.03 |
| C | -0.06 | 0.04 | 0.19 | 0.37 | 0.60 | 0.87 | 0.68 | 0.51 | 0.37 | 0.25 | 0.15 |
| X | 0.09 | 0.19 | 0.32 | 0.47 | 0.65 | 0.85 | 0.59 | 0.37 | 0.19 | 0.05 | -0.06 |
| K | -0.36 | -0.30 | -0.21 | -0.08 | 0.10 | 0.33 | 0.48 | 0.57 | 0.60 | 0.60 | 0.57 |
| H | 0.13 | 0.23 | 0.34 | 0.49 | 0.65 | 0.83 | 0.55 | 0.32 | 0.13 | -0.01 | -0.12 |
| LBP | -0.01 | 0.11 | 0.27 | 0.46 | 0.70 | 0.98 | 0.73 | 0.52 | 0.34 | 0.20 | 0.08 |
| Stochastic Trend-MA |  |  |  |  |  |  |  |  |  |  |  |
| Y | -0.00 | 0.13 | 0.30 | 0.50 | 0.72 | 1 | 0.72 | 0.50 | 0.30 | 0.13 | -0.00 |
| C | -0.09 | 0.03 | 0.18 | 0.36 | 0.58 | 0.85 | 0.67 | 0.52 | 0.38 | 0.24 | 0.13 |
| X | 0.06 | 0.18 | 0.33 | 0.48 | 0.66 | 0.86 | 0.57 | 0.35 | 0.16 | 0.00 | -0.12 |
| K | -0.37 | -0.31 | -0.21 | -0.08 | 0.10 | 0.33 | 0.48 | 0.56 | 0.58 | 0.57 | 0.52 |
| H | 0.08 | 0.19 | 0.32 | 0.47 | 0.62 | 0.81 | 0.52 | 0.29 | 0.10 | -0.05 | -0.16 |
| LBP | -0.04 | 0.09 | 0.26 | 0.46 | 0.69 | 0.98 | 0.73 | 0.52 | 0.34 | 0.18 | 0.05 |

Table 14: Cross-correlation output with x: 1948:1-1983:4 and $\nu=1$

|  | $\mathrm{x}(-5)$ | $\mathrm{x}(-4)$ | $\mathrm{x}(-3)$ | $\mathrm{x}(-2)$ | $\mathrm{x}(-1)$ | $\mathrm{x}(0)$ | $\mathrm{x}(1)$ | $\mathrm{x}(2)$ | $\mathrm{x}(3)$ | $\mathrm{x}(4)$ | $\mathrm{x}(5)$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Data |  |  |  |  |  |  |  |  |  |  |  |
| Y | -0.18 | 0.00 | 0.28 | 0.58 | 0.84 | 1 | 0.84 | 0.58 | 0.28 | 0.00 | -0.18 |
| V | -0.30 | -0.22 | -0.12 | -0.01 | 0.84 | 0.18 | 0.25 | 0.29 | 0.31 | 0.31 | 0.29 |
| A | 0.20 | 0.33 | 0.50 | 0.65 | 0.84 | 0.71 | 0.36 | 0.03 | -0.25 | -0.44 | -0.50 |
| C | -0.07 | 0.14 | 0.36 | 0.58 | 0.84 | 0.78 | 0.64 | 0.43 | 0.21 | 0.02 | -0.14 |
| X | -0.09 | 0.08 | 0.32 | 0.57 | 0.84 | 0.89 | 0.70 | 0.43 | 0.13 | -0.11 | -0.26 |
| I | -0.03 | 0.14 | 0.37 | 0.61 | 0.81 | 0.91 | 0.70 | 0.40 | 0.07 | -0.18 | -0.33 |
| K | -0.53 | -0.47 | -0.33 | -0.12 | 0.12 | 0.39 | 0.58 | 0.68 | 0.68 | 0.59 | 0.46 |
| H | -0.36 | -0.20 | 0.05 | 0.36 | 0.66 | 0.87 | 0.88 | 0.75 | 0.52 | 0.27 | 0.06 |
| LBP | 0.34 | 0.39 | 0.43 | 0.42 | 0.33 | 0.23 | -0.11 | -0.35 | -0.50 | -0.54 | -0.49 |
| Deterministic Trend |  |  |  |  |  |  |  |  |  |  |  |
| Y | 0.02 | 0.16 | 0.33 | 0.53 | 0.75 | 1 | 0.75 | 0.53 | 0.33 | 0.16 | 0.02 |
| C | -0.26 | -0.26 | -0.24 | -0.19 | -0.10 | 0.04 | 0.05 | 0.08 | 0.13 | 0.17 | 0.21 |
| X | 0.15 | 0.28 | 0.41 | 0.56 | 0.72 | 0.86 | 0.64 | 0.42 | 0.23 | 0.06 | -0.09 |
| K | -0.35 | -0.27 | -0.17 | -0.03 | 0.14 | 0.33 | 0.47 | 0.55 | 0.59 | 0.59 | 0.55 |
| H | 0.19 | 0.30 | 0.42 | 0.54 | 0.66 | 0.76 | 0.56 | 0.36 | 0.18 | 0.02 | -0.12 |
| LBP | -0.15 | -0.04 | 0.10 | 0.28 | 0.51 | 0.78 | 0.60 | 0.45 | 0.33 | 0.23 | 0.15 |
| Stochastic Trend |  |  |  |  |  |  |  |  |  |  |  |
| Y | -0.00 | 0.13 | 0.28 | 0.48 | 0.71 | 1 | 0.71 | 0.48 | 0.28 | 0.13 | -0.00 |
| C | -0.09 | 0.03 | 0.18 | 0.37 | 0.61 | 0.89 | 0.69 | 0.51 | 0.35 | 0.22 | 0.11 |
| X | 0.07 | 0.17 | 0.30 | 0.45 | 0.64 | 0.87 | 0.58 | 0.35 | 0.17 | 0.03 | -0.08 |
| K | -0.38 | -0.32 | -0.23 | -0.10 | 0.08 | 0.32 | 0.47 | 0.55 | 0.59 | 0.58 | 0.54 |
| H | 0.11 | 0.21 | 0.32 | 0.47 | 0.64 | 0.84 | 0.54 | 0.31 | 0.12 | -0.02 | -0.13 |
| LBP | -0.04 | 0.09 | 0.25 | 0.45 | 0.69 | 0.98 | 0.72 | 0.50 | 0.32 | 0.17 | 0.04 |
| Stochastic Trend-MA |  |  |  |  |  |  |  |  |  |  |  |
| Y | 0.01 | 0.14 | 0.31 | 0.50 | 0.73 | 1 | 0.73 | 0.50 | 0.31 | 0.14 | 0.01 |
| C | -0.10 | 0.02 | 0.17 | 0.37 | 0.59 | 0.85 | 0.68 | 0.52 | 0.37 | 0.24 | 0.14 |
| X | 0.11 | 0.21 | 0.35 | 0.50 | 0.68 | 0.87 | 0.60 | 0.37 | 0.18 | 0.03 | -0.09 |
| K | -0.36 | -0.29 | -0.20 | -0.06 | 0.12 | 0.34 | 0.49 | 0.57 | 0.60 | 0.59 | 0.55 |
| H | 0.13 | 0.23 | 0.35 | 0.48 | 0.64 | 0.82 | 0.55 | 0.32 | 0.13 | -0.02 | -0.13 |
| LBP | -0.03 | 0.10 | 0.27 | 0.47 | 0.70 | 0.98 | 0.74 | 0.53 | 0.34 | 0.19 | 0.06 |



|  | $\mathrm{x}(-5)$ | $\mathrm{x}(-4)$ | $\mathrm{x}(-3)$ | $\mathrm{x}(-2)$ | $\mathrm{x}(-1)$ | $\mathrm{x}(0)$ | $\mathrm{x}(1)$ | $\mathrm{x}(2)$ | $\mathrm{x}(3)$ | $\mathrm{x}(4)$ | $\mathrm{x}(5)$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Data |  |  |  |  |  |  |  |  |  |  |  |
| Y | 0.27 | 0.45 | 0.61 | 0.78 | 0.89 | 1 | 0.88 | 0.78 | 0.61 | 0.45 | 0.27 |
| V | -0.23 | -0.15 | -0.08 | -0.01 | 0.07 | 0.12 | 0.20 | 0.24 | -0.33 | 0.25 | 0.25 |
| A | 0.20 | 0.17 | 0.13 | 0.18 | 0.15 | 0.21 | -0.06 | -0.18 | -0.33 | -0.36 | -0.42 |
| C | 0.10 | 0.27 | 0.44 | 0.61 | 0.72 | 0.82 | 0.78 | 0.73 | 0.62 | 0.47 | 0.32 |
| X | 0.39 | 0.53 | 0.66 | 0.79 | 0.88 | 0.92 | 0.82 | 0.69 | 0.52 | 0.35 | 0.16 |
| I | 0.43 | 0.56 | 0.68 | 0.80 | 0.88 | 0.92 | 0.80 | 0.67 | 0.49 | 0.32 | 0.13 |
| K | -0.38 | -0.27 | -0.17 | -0.04 | 0.08 | 0.27 | 0.39 | 0.51 | 0.60 | 0.65 | 0.66 |
| H | 0.25 | 0.44 | 0.61 | 0.74 | 0.84 | 0.90 | 0.89 | 0.83 | 0.71 | 0.55 | 0.39 |
| LBP | -0.14 | -0.27 | -0.40 | -0.44 | -0.50 | -0.46 | -0.61 | -0.62 | -0.61 | -0.51 | -0.43 |
| Deterministic Trend |  |  |  |  |  |  |  |  |  |  |  |
| Y | 0.03 | 0.16 | 0.31 | 0.50 | 0.73 | 1 | 0.73 | 0.50 | 0.31 | 0.16 | 0.03 |
| C | -0.12 | -0.09 | -0.04 | 0.02 | 0.11 | 0.20 | 0.22 | 0.22 | 0.22 | 0.20 | 0.16 |
| X | 0.07 | 0.19 | 0.33 | 0.50 | 0.70 | 0.94 | 0.66 | 0.43 | 0.24 | 0.09 | -0.03 |
| K | -0.36 | -0.29 | -0.20 | -0.07 | 0.10 | 0.31 | 0.45 | 0.53 | 0.56 | 0.56 | 0.53 |
| H | 0.09 | 0.20 | 0.32 | 0.47 | 0.64 | 0.85 | 0.58 | 0.36 | 0.18 | 0.04 | -0.06 |
| LBP | -0.03 | 0.09 | 0.23 | 0.41 | 0.64 | 0.89 | 0.69 | 0.50 | 0.35 | 0.21 | 0.10 |
| Stochastic Trend |  |  |  |  |  |  |  |  |  |  |  |
| Y | 0.00 | 0.13 | 0.30 | 0.49 | 0.73 | 1 | 0.73 | 0.49 | 0.30 | 0.13 | 0.00 |
| C | -0.09 | 0.03 | 0.19 | 0.38 | 0.61 | 0.90 | 0.70 | 0.53 | 0.38 | 0.24 | 0.12 |
| X | 0.07 | 0.18 | 0.32 | 0.47 | 0.66 | 0.87 | 0.60 | 0.37 | 0.19 | 0.04 | -0.08 |
| K | -0.39 | -0.33 | -0.23 | -0.09 | 0.09 | 0.33 | 0.48 | 0.57 | 0.60 | 0.60 | 0.56 |
| H | 0.11 | 0.22 | 0.35 | 0.49 | 0.67 | 0.86 | 0.56 | 0.32 | 0.13 | -0.02 | -0.14 |
| LBP | -0.03 | 0.10 | 0.26 | 0.46 | 0.70 | 0.98 | 0.73 | 0.52 | 0.34 | 0.18 | 0.05 |
| Stochastic Trend-MA |  |  |  |  |  |  |  |  |  |  |  |
| Y | -0.01 | 0.12 | 0.28 | 0.48 | 0.72 | 1 | 0.72 | 0.48 | 0.28 | 0.12 | -0.01 |
| C | -0.08 | 0.04 | 0.20 | 0.39 | 0.64 | 0.94 | 0.72 | 0.53 | 0.36 | 0.22 | 0.10 |
| X | 0.05 | 0.16 | 0.31 | 0.49 | 0.69 | 0.93 | 0.63 | 0.39 | 0.19 | 0.03 | -0.08 |
| K | -0.42 | -0.36 | -0.26 | -0.12 | 0.07 | 0.33 | 0.49 | 0.59 | 0.62 | 0.61 | 0.57 |
| H | 0.09 | 0.20 | 0.33 | 0.50 | 0.69 | 0.90 | 0.59 | 0.33 | 0.13 | -0.03 | -0.14 |
| LBP | -0.04 | 0.09 | 0.25 | 0.45 | 0.70 | 0.99 | 0.73 | 0.50 | 0.31 | 0.16 | 0.04 |

Table 16: The Great Moderation: Time-invariant coefficients

|  | DT |  |  | ST |  |  | ST-MA |  |  | $\begin{gathered} \text { Data } \\ \sigma_{\text {post }} / \sigma_{\text {pre }} \end{gathered}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | $\% \sigma_{\text {pre }}$ | $\% \sigma_{\text {post }}$ | $\sigma_{\text {post }} / \sigma_{\text {pre }}$ | $\% \sigma_{\text {pre }}$ | $\% \sigma_{p o s t}$ | $\sigma_{\text {post }} / \sigma_{\text {pre }}$ | $\% \sigma_{\text {pre }}$ | $\% \sigma_{p o s t}$ | $\sigma_{\text {post }} / \sigma_{\text {pre }}$ |  |
| Experiment 1 |  |  |  |  |  |  |  |  |  |  |
| c | 0.90 | 0.79 | 0.88 | 0.79 | 0.47 | 0.59 | 0.82 | 0.47 | 0.57 | 0.58 |
| x | 4.11 | 3.01 | 0.73 | 3.13 | 2.19 | 0.70 | 3.21 | 2.11 | 0.66 | 0.53 |
| y | 1.48 | 0.84 | 0.57 | 1.28 | 0.70 | 0.54 | 1.34 | 0.68 | 0.51 | 0.48 |
| k | 0.57 | 0.49 | 0.86 | 0.31 | 0.22 | 0.71 | 0.34 | 0.23 | 0.68 | 0.65 |
| h | 0.71 | 0.59 | 0.83 | 0.33 | 0.23 | 0.69 | 0.35 | 0.25 | 0.71 | 0.69 |
| $y / h$ | 1 | 0.56 | 0.56 | 1.01 | 0.55 | 0.54 | 1.05 | 0.53 | 0.50 | 0.68 |
| Experiment 2 |  |  |  |  |  |  |  |  |  |  |
| c | 0.93 | 0.69 | 0.74 | 0.70 | 0.65 | 0.93 | 0.71 | 0.67 | 0.94 | 0.58 |
| x | 3.78 | 3.25 | 0.86 | 2.93 | 2.49 | 0.83 | 2.90 | 2.46 | 0.86 | 0.53 |
| y | 1.28 | 1.23 | 0.96 | 1.10 | 1.09 | 0.99 | 1.11 | 1.12 | 1.01 | 0.48 |
| k | 0.55 | 0.43 | 0.78 | 0.29 | 0.24 | 0.83 | 0.31 | 0.25 | 0.81 | 0.65 |
| h | 0.69 | 0.54 | 0.78 | 0.31 | 0.26 | 0.84 | 0.33 | 0.26 | 0.79 | 0.69 |
| $y / h$ | 0.88 | 0.84 | 0.95 | 0.87 | 0.86 | 0.98 | 0.87 | 0.88 | 1.01 | 0.68 |
| Experiment 3 |  |  |  |  |  |  |  |  |  |  |
| c | 0.98 | 0.57 | 0.58 | 0.80 | 0.43 | 0.53 | 0.84 | 0.42 | 0.50 | 0.58 |
| x | 4.20 | 2.37 | 0.56 | 3.23 | 1.71 | 0.53 | 3.35 | 1.65 | 0.49 | 0.53 |
| y | 1.47 | 0.78 | 0.53 | 1.29 | 0.69 | 0.54 | 1.34 | 0.68 | 0.50 | 0.48 |
| k | 0.61 | 0.36 | 0.59 | 0.32 | 0.17 | 0.53 | 0.36 | 0.17 | 0.47 | 0.65 |
| h | 0.75 | 0.44 | 0.59 | 0.34 | 0.18 | 0.53 | 0.37 | 0.18 | 0.49 | 0.69 |
| $y / h$ | 0.99 | 0.52 | 0.53 | 1.02 | 0.55 | 0.54 | 1.06 | 0.54 | 0.51 | 0.68 |

Table 17: The Great Moderation: Time-varying coefficients

|  | DT |  |  | ST |  |  | ST-MA |  |  | $\begin{gathered} \text { Data } \\ \sigma_{\text {post }} / \sigma_{\text {pre }} \end{gathered}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | $\% \sigma_{\text {pre }}$ | $\% \sigma_{\text {post }}$ | $\sigma_{\text {post }} / \sigma_{\text {pre }}$ | $\% \sigma_{\text {pre }}$ | $\% \sigma_{\text {post }}$ | $\sigma_{\text {post }} / \sigma_{\text {pre }}$ | $\% \sigma_{\text {pre }}$ | $\% \sigma_{\text {post }}$ | $\sigma_{\text {post }} / \sigma_{\text {pre }}$ |  |
| Experiment 1 |  |  |  |  |  |  |  |  |  |  |
| C | 1.23 | 0.64 | 0.52 | 0.79 | 0.49 | 0.62 | 0.80 | 0.44 | 0.55 | 0.58 |
| x | 5.19 | 2.71 | 0.52 | 3.20 | 2.12 | 0.66 | 3.26 | 1.84 | 0.56 | 0.53 |
| y | 1.55 | 0.83 | 0.54 | 1.30 | 0.70 | 0.54 | 1.30 | 0.67 | 0.52 | 0.48 |
| k | 0.65 | 0.40 | 0.62 | 0.30 | 0.22 | 0.73 | 0.33 | 0.21 | 0.64 | 0.65 |
| h | 0.98 | 0.51 | 0.52 | 0.33 | 0.22 | 0.67 | 0.36 | 0.20 | 0.56 | 0.69 |
| $y / h$ | 1.01 | 0.53 | 0.52 | 1.02 | 0.56 | 0.55 | 1.02 | 0.53 | 0.52 | 0.68 |
| Experiment 2 |  |  |  |  |  |  |  |  |  |  |
| c | 1.29 | 0.55 | 0.43 | 0.71 | 0.66 | 0.93 | 0.73 | 0.65 | 0.89 | 0.58 |
| x | 4.94 | 3.24 | 0.66 | 2.89 | 2.49 | 0.86 | 3.02 | 2.34 | 0.77 | 0.53 |
| y | 1.33 | 1.25 | 0.94 | 1.11 | 1.11 | 1 | 1.12 | 1.10 | 0.98 | 0.48 |
| k | 0.64 | 0.39 | 0.61 | 0.28 | 0.26 | 0.93 | 0.31 | 0.24 | 0.77 | 0.65 |
| h | 0.97 | 0.53 | 0.55 | 0.30 | 0.26 | 0.87 | 0.34 | 0.25 | 0.74 | 0.69 |
| $y / h$ | 0.88 | 0.80 | 0.91 | 0.88 | 0.88 | 1 | 0.89 | 0.87 | 0.98 | 0.68 |
| Experiment 3 |  |  |  |  |  |  |  |  |  |  |
| c | 1.30 | 0.46 | 0.35 | 0.79 | 0.44 | 0.56 | 0.83 | 0.41 | 0.49 | 0.58 |
| x | 5.30 | 2.26 | 0.43 | 3.27 | 1.72 | 0.39 | 3.32 | 1.56 | 0.47 | 0.53 |
| y | 1.58 | 0.79 | 0.50 | 1.28 | 0.71 | 0.56 | 1.31 | 0.68 | 0.52 | 0.48 |
| k | 0.65 | 0.31 | 0.48 | 0.30 | 0.18 | 0.60 | 0.34 | 0.17 | 0.50 | 0.65 |
| h | 1.00 | 0.40 | 0.40 | 0.34 | 0.18 | 0.53 | 0.37 | 0.17 | 0.46 | 0.69 |
| $y / h$ | 1.04 | 0.51 | 0.49 | 1.01 | 0.57 | 0.56 | 1.03 | 0.54 | 0.52 | 0.68 |

Table 18: Autocorrelation $(\nu=1)$

|  | Data |  |  |  | DT |  |  | ST |  |  | ST-MA |  |  |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Whole | Pre | Post | Whole | Pre | Post | Whole | Pre | Post | Whole | Pre | Post |  |
| y | 0.84 | 0.84 | 0.88 | 0.76 | 0.76 | 0.73 | 0.73 | 0.73 | 0.72 | 0.71 | 0.73 | 0.72 |  |
| c | 0.81 | 0.81 | 0.83 | 0.86 | 0.92 | 0.72 | 0.74 | 0.75 | 0.74 | 0.73 | 0.74 | 0.74 |  |
| x | 0.82 | 0.81 | 0.90 | 0.81 | 0.85 | 0.72 | 0.72 | 0.72 | 0.70 | 0.75 | 0.76 | 0.74 |  |
| k | 0.93 | 0.93 | 0.93 | 0.97 | 0.97 | 0.97 | 0.96 | 0.96 | 0.96 | 0.96 | 0.97 | 0.96 |  |
| h | 0.90 | 0.89 | 0.94 | 0.83 | 0.88 | 0.71 | 0.71 | 0.72 | 0.70 | 0.70 | 0.71 | 0.72 |  |
| lbp | 0.71 | 0.71 | 0.74 | 0.76 | 0.78 | 0.74 | 0.73 | 0.74 | 0.73 | 0.72 | 0.73 | 0.72 |  |

Table 19: Hansen-Rogerson preferences 1948:1-2006:4

|  | Data |  | Both shocks |  |  | Only N-shock |  |  | Only I-shock |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | $\% \sigma_{z}$ | $\rho_{z y}$ | $\% \sigma_{z}$ | $\rho_{z y}$ | $\sigma_{\text {model }} / \sigma_{\text {data }}$ | $\% \sigma_{z}$ | $\rho_{z y}$ | $\sigma_{\text {model }} / \sigma_{\text {data }}$ | $\% \sigma_{z}$ | $\rho_{z y}$ | $\sigma_{\text {model }} / \sigma_{\text {data }}$ |
| DTM |  |  |  |  |  |  |  |  |  |  |  |
| c | 0.92 | 0.78 | 0.89 | 0.08 | 0.97 | 0.55 | 0.94 | 0.60 | 0.70 | -0.84 | 0.76 |
| i | 5.44 | 0.91 | 6.99 | 0.93 | 1.28 | 4.01 | 0.99 | 0.73 | 5.72 | 0.97 | 1.05 |
| y | 1.73 | 1 | 2.07 | 1 | 1.20 | 1.68 | 1 | 0.97 | 1.21 | 1 | 0.70 |
| k | 0.59 | 0.36 | 0.81 | 0.35 | 1.37 | 0.39 | 0.33 | 0.66 | 0.71 | 0.44 | 1.20 |
| h | 1.88 | 0.87 | 2.19 | 0.91 | 1.16 | 1.18 | 0.99 | 0.63 | 1.84 | 0.98 | 0.98 |
| $y / h$ | 0.94 | 0.11 | 0.89 | 0.08 | 0.95 | 0.55 | 0.94 | 0.59 | 0.70 | -0.84 | 0.74 |
| STM |  |  |  |  |  |  |  |  |  |  |  |
| c | 0.92 | 0.78 | 0.75 | 0.87 | 0.82 | 0.72 | 0.98 | 0.78 | 0.21 | -0.81 | 0.23 |
| i | 5.44 | 0.91 | 3.80 | 0.91 | 0.70 | 3.08 | 0.99 | 0.57 | 2.23 | 0.99 | 0.41 |
| y | 1.73 | 1 | 1.41 | 1 | 0.82 | 1.38 | 1 | 0.81 | 0.32 | 1 | 0.18 |
| k | 0.59 | 0.36 | 0.37 | 0.33 | 0.63 | 0.30 | 0.35 | 0.51 | 0.22 | 0.38 | 0.37 |
| h | 1.88 | 0.87 | 0.85 | 0.90 | 0.45 | 0.69 | 0.98 | 0.37 | 0.50 | 0.97 | 0.27 |
| $y / h$ | 0.94 | 0.11 | 0.75 | 0.87 | 0.80 | 0.72 | 0.98 | 0.77 | 0.21 | -0.81 | 0.22 |

Table 20: VAR approach: Estimated Parameters

Table 21: Multivariate Analysis Results

|  | 1948:1-2006:4 |  |  |  | 1948:1-1983:4 |  |  |  | 1984:1-2006:4 |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | $\% \sigma_{\text {data }}$ | $\rho_{\text {data }}$ | $\% \sigma_{\text {model }}$ | $\rho_{\text {model }}$ | $\% \sigma_{\text {data }}$ | $\rho_{\text {data }}$ | $\% \sigma_{\text {model }}$ | $\rho_{\text {model }}$ | $\% \sigma_{\text {data }}$ | $\rho_{\text {data }}$ | $\% \sigma_{\text {model }}$ | $\rho_{\text {model }}$ |
| DTM |  |  |  |  |  |  |  |  |  |  |  |  |
| c | 0.92 | 0.78 | 0.97 | 0.33 | 1.07 | 0.78 | 1.37 | 0.24 | 0.62 | 0.81 | 0.51 | -0.23 |
| i | 5.44 | 0.91 | 4.26 | 0.71 | 6.40 | 0.91 | 5.99 | 0.69 | 3.42 | 0.92 | 2.87 | 0.88 |
| y | 1.73 | 1 | 1.26 | 1 | 2.07 | 1 | 1.49 | 1 | 0.99 | 1 | 0.88 | 1 |
| k | 0.59 | 0.36 | 0.51 | 0.34 | 0.68 | 0.39 | 0.63 | 0.15 | 0.44 | 0.27 | 0.42 | 0.51 |
| h | 1.88 | 0.87 | 0.66 | 0.71 | 2.11 | 0.87 | 0.89 | 0.65 | 1.46 | 0.90 | 0.56 | 0.89 |
| $y / h$ | 0.94 | 0.11 | 0.92 | 0.86 | 1.06 | 0.23 | 1.13 | 0.80 | 0.72 | -0.46 | 0.46 | 0.84 |
| STM |  |  |  |  |  |  |  |  |  |  |  |  |
| c | 0.92 | 0.78 | 0.68 | 0.95 | 1.07 | 0.78 | 0.78 | 0.96 | 0.62 | 0.81 | 0.41 | 0.95 |
| i | 5.44 | 0.91 | 2.17 | 0.91 | 6.40 | 0.91 | 2.47 | 0.91 | 3.42 | 0.92 | 1.34 | 0.90 |
| y | 1.73 | 1 | 1.05 | 1 | 2.07 | 1 | 1.22 | 1 | 0.99 | 1 | 0.63 | 1 |
| k | 0.59 | 0.36 | 0.22 | 0.32 | 0.68 | 0.39 | 0.25 | 0.31 | 0.44 | 0.27 | 0.13 | 0.32 |
| h | 1.88 | 0.87 | 0.23 | 0.89 | 2.11 | 0.87 | 0.26 | 0.90 | 1.46 | 0.90 | 0.14 | 0.88 |
| $y / h$ | 0.94 | 0.11 | 0.86 | 0.99 | 1.06 | 0.23 | 0.99 | 0.99 | 0.72 | -0.46 | 0.51 | 0.99 |

Table 22: Multivariate Analysis Results: ratio of standard deviations

|  | $\sigma_{\text {post }} / \sigma_{\text {pre }}$ <br> Data |  |
| :--- | :--- | :--- |
| Model |  |  |$|$| DTM |  |  |
| :--- | :--- | :--- |
| c | 0.58 | 0.37 |
| i | 0.53 | 0.48 |
| y | 0.48 | 0.59 |
| k | 0.65 | 0.67 |
| h | 0.69 | 0.63 |
| $\mathrm{y} / \mathrm{h}$ | 0.68 | 0.41 |
| STM |  |  |
| c | 0.58 | 0.53 |
| i | 0.53 | 0.54 |
| y | 0.48 | 0.52 |
| k | 0.65 | 0.52 |
| h | 0.69 | 0.54 |
| $\mathrm{y} / \mathrm{h}$ | 0.68 | 0.51 |

Figure A-1: Impulse response functions with respect to a neutral technology shock
IRF for $1 S D$ in NEUTRAL innovation






$$
\begin{aligned}
& -D T \\
& ----. S T \\
& - \\
& \hline
\end{aligned}
$$

Figure A-2: Impulse response functions with respect to an investment-specific technology shock

IRF for 1SD in INVESTMENT SPECIFIC innovation







| $-\quad----S T$ |
| :--- |
| - |

## Appendix A. 3 Balance Growth Path

From the feasibility constraint I can conclude that output, consumption, and investment grow at the same rate

$$
\begin{aligned}
Y_{t} & =C_{t}+I_{t} \\
\frac{Y_{t}}{Y_{t-1}} & =\frac{C_{t}}{C_{t-1}} \frac{C_{t-1}}{Y_{t-1}}+\frac{I_{t}}{I_{t-1}} \frac{I_{t-1}}{Y_{t-1}} \\
g_{Y} & =g_{C} \frac{C_{t-1}}{Y_{t-1}}+g_{I} \frac{I_{t-1}}{Y_{t-1}}
\end{aligned}
$$

Therefore, $g_{Y}$ is constant if and only if $g_{Y}=g_{C}=g_{I}$. Let us consider the investment equation

$$
\begin{aligned}
(1+\eta) K_{t+1} & =(1-\delta) K_{t}+V_{t} I_{t} \\
(1+\eta) \frac{K_{t+1}}{K_{t}} & =(1-\delta)+\frac{V_{t} I_{t}}{K_{t}} \\
(1+\eta) g_{K} & =(1-\delta)+\frac{V_{t} I_{t}}{K_{t}}
\end{aligned}
$$

$g_{K}$ is constant if and only if $(V \times I)$ grows at the same rate as $K$ which requires that $g_{K}=g_{I} g_{V}$.

Let us analyze the production function

$$
\begin{aligned}
Y_{t} & =A_{t} K_{t}^{\alpha} H_{t}^{1-\alpha} \\
g_{Y} & =g_{A} g_{K}^{\alpha} g_{H}^{1-\alpha}
\end{aligned}
$$

Given that hours are stationary, that is, $g_{H}=1$, I have that $g_{Y}=g_{A} g_{K}^{\alpha}$, where
$g_{K}=g_{Y} g_{V}$. Therefore,

$$
g_{Y}=g_{A}^{\frac{1}{1-\alpha}} g_{V}^{\frac{\alpha}{1-\alpha}}
$$

In the deterministic trend model, $g_{Y}=e^{\frac{1}{1-\alpha} \gamma_{a}+\frac{\alpha}{1-\alpha} \gamma_{v}}$ and $g_{V}=e^{\gamma_{v}}$, which implies that $g_{K}=e^{\frac{1}{1-\alpha}\left(\gamma_{a}+\gamma_{v}\right)}$. In the stochastic trend model, $g_{Y}=A_{t}^{\frac{1}{1-\alpha}} V_{t}^{\frac{\alpha}{1-\alpha}}$ and $g_{K}=$ $A_{t}^{\frac{1}{1-\alpha}} V_{t}^{\frac{1}{1-\alpha}}$.

## Appendix A. 4 Log-linearization around the steady state

Let us define $\hat{x}_{t}=\ln \left(\tilde{X}_{t} / X^{*}\right)$, then the $\log$-linearized system of equations for the deterministic trend model is given by:

$$
\begin{aligned}
\hat{y}_{t} & =\frac{C^{*}}{Y^{*}} \hat{c}_{t}+\frac{I^{*}}{Y^{*}} \hat{i}_{t} \\
\hat{y}_{t} & =\alpha \hat{k}_{t}+(1-\alpha) \hat{h}_{t}+\varepsilon_{a t} \\
q v \hat{k}_{t+1} & =(1-\delta) \hat{k}_{t}+V_{0} \frac{I^{*}}{K^{*}}\left[e^{\varepsilon_{v t}}\left(1+\hat{i}_{t}\right)-1\right] \\
0 & =E_{t}\left[\hat{c}_{t}-\hat{c}_{t-1}+\varepsilon_{v t}-\varepsilon_{v t+1}+\left(\frac{R^{*}}{1-\delta+R^{*}}\right) \hat{r}_{t+1}\right] \\
\hat{h}_{t} & =\nu\left(\hat{w}_{t}-\hat{c}_{t}\right) \\
\hat{r}_{t} & =\hat{y}_{t}-\hat{k}_{t}+\varepsilon_{v t} \\
\hat{w}_{t} & =\hat{y}_{t}-\hat{h}_{t}
\end{aligned}
$$

The log-linearized system of equation for the stochastic trend model is given by:

$$
\begin{aligned}
& \hat{y}_{t}=\hat{c}_{t} \frac{C^{*}}{Y^{*}}+\hat{i}_{t} \frac{I^{*}}{Y^{*}} \\
& \hat{y}_{t}=-\alpha\left(\hat{q}_{t}+\hat{v}_{t}\right)+\alpha \hat{k}_{t}+(1-\alpha) \hat{h}_{t}
\end{aligned}
$$

$$
\begin{aligned}
\hat{k}_{t+1} & =(1-\delta)\left(\frac{1}{q^{*} v^{*}}\right)\left[\hat{k}_{t}-\left(\hat{q}_{t}+\hat{v}_{t}\right)\right]+\hat{i}_{t} \frac{I^{*}}{K^{*}} \\
0 & \left.=E_{t}\left[\hat{c}_{t}-\hat{c}_{t-1}-\left(\hat{q}_{t+1}+\hat{v}_{t+1}\right)+\hat{r}_{t+1}^{k}\right]\right] \\
\hat{r_{t}^{k}} & =\left(\frac{R^{*}}{R^{k *}}\right) \hat{r}_{t} \\
\hat{h}_{t} & =\nu\left(\hat{w}_{t}-\hat{c}_{t}\right) \\
\hat{r}_{t} & =\hat{y}_{t}-\hat{k}_{t}+\hat{q}_{t}+\hat{v}_{t} \\
\hat{w}_{t} & =\hat{y}_{t}-\hat{h}_{t} \\
\hat{q}_{t} & =\frac{1}{1-\alpha} \varepsilon_{a t}+\frac{\alpha}{1-\alpha} \varepsilon_{v t} \\
\hat{v}_{t} & =\varepsilon_{v t}
\end{aligned}
$$

The above is a system of 11 equations and 11 unknowns: $\left\{\hat{c}_{t}, \hat{i}_{t}, \hat{y}_{t}, \hat{k}_{t}, \hat{h}_{t}, \hat{b}_{t}, \hat{r}_{t}, \hat{r}_{t}^{k}, \hat{q}_{t}, \hat{v}_{t},\right\}$.
To proceed with estimation I need to also consider the following conditions:

$$
\begin{aligned}
q^{*} & =e^{\frac{1}{1-\alpha} \gamma_{a}+\frac{\alpha}{1-\alpha} \gamma_{v}} \\
v^{*} & =e^{\gamma_{v}} \\
R^{*} & =\frac{q^{*} v^{*}}{\beta}-(1-\delta) \\
R^{k *} & =(1-\delta)+R^{*}=\frac{q^{*} v^{*}}{\beta} \\
\frac{I^{*}}{K^{*}} & =1-(1-\delta) \frac{1}{q^{*} v^{*}} \\
\frac{K^{*}}{Y^{*}} & =\frac{1}{\alpha q^{*} v^{*}} R^{*} \\
\frac{C^{*}}{Y^{*}} & =\left(\frac{1-\delta}{q^{*} v^{*}}-1\right) \frac{K^{*}}{Y^{*}}+1 \\
\frac{I^{*}}{Y^{*}} & =1-\frac{C^{*}}{Y^{*}} \\
H^{*} & =\left(\frac{1}{B^{*}}\right)^{\frac{\nu}{1+\nu}}\left[(1-\alpha) \frac{Y^{*}}{C^{*}}\right]^{\frac{\nu}{1+\nu}}
\end{aligned}
$$

## Appendix A. 5 Stochastic trend model: closed form solution

In the economy under analysis both welfare theorems hold, therefore I can solve the planner's problem which is given by

$$
\begin{aligned}
\max _{C_{t}, H_{t}} U & =\mathbb{E}\left[\sum_{t=0}^{\infty} \beta^{t}\left(\ln C_{t}-B \frac{H_{t}^{1+1 / \nu}}{1+1 / \nu}\right)\right] \\
\text { s.t. } & \\
C_{t}+I_{t} & =A_{t} K_{t}^{\alpha} H_{t}^{1-\alpha} \\
K_{t+1} & =(1-\delta) K_{t}+V_{t} I_{t} \\
A_{t} & =A_{t-1} e^{\gamma_{a}+\epsilon_{a t}} \\
V_{t} & =V_{t-1} e^{\gamma_{v}+\epsilon_{v t}} \\
A_{0}, V_{0}, K_{0} & \text { given }
\end{aligned}
$$

I will proceed first by combining the resource constraint and the law of motion for capital. Thus, my equilibrium conditions are given by the Euler equation, the labor supply, and the new resources constraint. Secondly, as my economy is non-stationary I need to transform it to be able to solve my model. All variables but hours and capital grow at rate $Q_{t}=A_{t}^{\frac{1}{1-\alpha}} V_{t}^{\frac{\alpha}{1-\alpha}}$. Capital grows at rate $Q_{t} V_{t}=A_{t}^{\frac{1}{1-\alpha}} V_{t}^{\frac{1}{1-\alpha}}$ and hours are stationary. Therefore, the equilibrium conditions for the transformed model
economy are given by $^{3}$ :

$$
\begin{gathered}
1=\beta \mathbb{E}_{t}\left(\frac { \tilde { C } _ { t } } { \tilde { C } _ { t + 1 } } \left((1-\delta) e^{-\frac{1}{1-\alpha}\left(\gamma_{a}+\gamma_{v}+\epsilon_{a t+1}+\epsilon_{v t+1}\right)}\right.\right. \\
\left.\left.+\alpha e^{-\frac{\alpha}{1-\alpha}\left(\gamma_{a}+\gamma_{v}+\epsilon_{a t+1}+\epsilon_{v t+1}\right)} \tilde{K}_{t+1}^{\alpha-1} H_{t+1}^{1-\alpha}\right)\right) \\
B H_{t}^{1 / \nu}=(1-\alpha) \frac{\tilde{Y}_{t} / H_{t}}{\tilde{C}_{t}} \\
\tilde{C}_{t}+\tilde{K}_{t+1}=(1-\delta) e^{-\frac{1}{1-\alpha}\left(\gamma_{a}+\gamma_{v}+\epsilon_{a t}+\epsilon_{v t}\right)} \tilde{K}_{t}+e^{-\frac{\alpha}{1-\alpha}\left(\gamma_{a}+\gamma_{v}+\epsilon_{a t}+\epsilon_{v t}\right)} \tilde{K}_{t}^{\alpha} H_{t}^{1-\alpha}
\end{gathered}
$$

Let us assume there is full depreciation (i.e. $\delta=1$ ). Thus, the above reduces to:

$$
\begin{align*}
1 & =\beta \mathbb{E}_{t}\left[\frac{\tilde{C}_{t}}{\tilde{C}_{t+1}}\left(\alpha e^{-\frac{\alpha}{1-\alpha}\left(\gamma_{a}+\gamma_{v}+\epsilon_{a t+1}+\epsilon_{v t+1}\right)} \tilde{K}_{t+1}^{\alpha-1} H_{t+1}^{1-\alpha}\right)\right]  \tag{6}\\
B H_{t}^{1 / \nu} & =(1-\alpha) \frac{\tilde{Y}_{t} / H_{t}}{\tilde{C}_{t}}  \tag{7}\\
\tilde{C}_{t}+\tilde{K}_{t+1} & =e^{-\frac{\alpha}{1-\alpha}\left(\gamma_{a}+\gamma_{v}+\epsilon_{a t}+\epsilon_{v t}\right)} \tilde{K}_{t}^{\alpha} H_{t}^{1-\alpha} \tag{8}
\end{align*}
$$

## Appendix A.5.1 Baseline system: An exact solution

My guess for policy function for capital will be

$$
\begin{equation*}
\tilde{K}_{t+1}=\alpha \beta e^{-\frac{\alpha}{1-\alpha}\left(\gamma_{a}+\gamma_{v}+\epsilon_{a t}+\epsilon_{v t}\right)} \tilde{K}_{t}^{\alpha} H_{t}^{1-\alpha} \tag{9}
\end{equation*}
$$

[^10]Then, from (8) I have that the policy function for consumption is given by:

$$
\begin{equation*}
\tilde{C}_{t}=e^{-\frac{\alpha}{1-\alpha}\left(\gamma_{a}+\gamma_{v}+\epsilon_{a t}+\epsilon_{v t}\right)} \tilde{K}_{t}^{\alpha} H_{t}^{1-\alpha}(1-\alpha \beta) \tag{10}
\end{equation*}
$$

Let us plug (9) and (10) in (6)

$$
\begin{aligned}
1 & =\beta \mathbb{E}_{t}\left[e^{-\frac{\alpha}{1-\alpha}\left(\epsilon_{a t}+\epsilon_{v t}-\epsilon_{a t+1}-\epsilon_{v t+1}\right)} \frac{\tilde{K}_{t}^{\alpha} H_{t}^{1-\alpha}}{\tilde{K}_{t+1}^{\alpha} H_{t+1}^{1-\alpha}} \alpha e^{-\frac{\alpha}{1-\alpha}\left(\gamma_{a}+\gamma_{v}+\epsilon_{a t+1}+\epsilon_{v t+1}\right)} \tilde{K}_{t+1}^{\alpha-1} H_{t+1}^{1-\alpha}\right] \\
& =\alpha \beta \mathbb{E}_{t}\left[\frac{1}{\tilde{K}_{t+1}} e^{-\frac{\alpha}{1-\alpha}\left(\gamma_{a}+\gamma_{v}+\epsilon_{a t}+\epsilon_{v t}\right)} \tilde{K}_{t}^{\alpha} H_{t}^{1-\alpha}\right]
\end{aligned}
$$

which implies

$$
\tilde{K}_{t+1}=\alpha \beta e^{-\frac{\alpha}{1-\alpha}\left(\gamma_{a}+\gamma_{v}+\epsilon_{a t}+\epsilon_{v t}\right)} \tilde{K}_{t}^{\alpha} H_{t}^{1-\alpha}
$$

since $\tilde{K}_{t+1}$ is a choice variable at time $t$ not an unknown variable dated at time $t+1$. Therefore, as my guess satisfies the equilibrium conditions, I can ensure the policy function for capital is of the form given by (9). Consequently, the guess for the consumption policy rule is also part of the solution to my model. Note that I constructed such a guess by using (9) and the resources constraint. Note that both policy rules depend on model parameters, current realizations of shocks (I assume current shocks are observed before current decisions are taken), capital at time $t$ which is a predetermined variable (chosen at time $t-1$ ), and current labor decision. Therefore, to completely characterize the policy rules of interest it remains to provide the labor supply policy function. To do so let us consider (7) and plug (10) so that:

$$
\begin{aligned}
B H_{t}^{1 / \nu} & =\frac{\tilde{W}_{t}}{\tilde{C}_{t}} \\
& =\frac{(1-\alpha) \tilde{Y}_{t} / H_{t}}{(1-\alpha \beta) \tilde{Y}_{t}}
\end{aligned}
$$

Thus,

$$
\begin{equation*}
H_{t}=\left(\frac{1-\alpha}{B(1-\alpha \beta)}\right)^{\frac{\nu}{1-\nu}} \tag{11}
\end{equation*}
$$

which is a constant.
Therefore, by substituting (11) in (9) and (10) I have my policy rules as functions of model parameters and current state variables.

## Appendix A.5.2 Log-linearized system: An approximate solution

Let us consider the following log-linearized system (under the assumption of full depreciation).

$$
\begin{align*}
\hat{y}_{t} & =\frac{C^{*}}{Y^{*}} \hat{c}_{t}+\frac{I^{*}}{Y^{*}} \hat{i}_{t}  \tag{12}\\
\hat{y} & =-\frac{\alpha}{1-\alpha}\left(\epsilon_{a t}+\epsilon_{v t}\right)+\alpha \hat{k}_{t}+(1-\alpha) \hat{h}_{t}  \tag{13}\\
\hat{k}_{t+1} & =\frac{I^{*}}{K^{*}} \hat{i}_{t}=\frac{I^{*} / Y^{*}}{K^{*} / Y^{*}} \hat{i}_{t}  \tag{14}\\
0 & =\mathbb{E}\left[\hat{c}_{t}-\hat{c}_{t+1}-\frac{1}{1-\alpha}\left(\epsilon_{a t+1}+\epsilon_{v t+1}\right)+r_{t+1}\right]  \tag{15}\\
\hat{h}_{t} & =\nu\left(\hat{w}_{t}-\hat{c}_{t}\right)  \tag{16}\\
\hat{r}_{t} & =\hat{y}_{t}-\hat{k}_{t}+\frac{1}{1-\alpha}\left(\epsilon_{a t}+\epsilon_{v t}\right)  \tag{17}\\
\hat{w}_{t} & =\hat{y}_{t}-\hat{h}_{t} \tag{18}
\end{align*}
$$

Plugging (14) into (12) I obtain

$$
\begin{equation*}
\hat{y}_{t}=\frac{C^{*}}{Y^{*}} \hat{c}_{t}+\frac{K^{*}}{Y^{*}} \hat{k}_{t+1} \tag{19}
\end{equation*}
$$

Substitute (17) in (15)

$$
\begin{equation*}
0=\mathbb{E}\left[\hat{c}_{t}-\hat{c}_{t+1} \hat{y}_{t+1}-\hat{k}_{t+1}\right] \tag{20}
\end{equation*}
$$

My guesses for policy rules are given by:

$$
\begin{align*}
\hat{k}_{t+1} & =-\frac{\alpha}{1-\alpha}\left(\epsilon_{a t}+\epsilon_{v t}\right)+\alpha \hat{k}_{t}+(1-\alpha) \hat{h}_{t}  \tag{21}\\
\hat{c}_{t} & =\frac{1}{C^{*} / Y^{*}}\left(1-\frac{K^{*}}{Y^{*}}\right) \hat{k}_{t+1} \tag{22}
\end{align*}
$$

Plugging (21) and (22) into (15)

$$
\begin{aligned}
& 0=\mathbb{E}\left(\frac { 1 } { C ^ { * } / Y ^ { * } } ( 1 - \frac { K ^ { * } } { Y ^ { * } } ) \left(-\frac{\alpha}{1-\alpha}\left(\epsilon_{a t}+\epsilon_{v t}-\epsilon_{a t+1}-\epsilon_{v t+1}\right)+\alpha\left(\hat{k}_{t}-\hat{k}_{t+1}\right)\right.\right. \\
& \left.+(1-\alpha)\left(\hat{h}_{t}-\hat{h}_{t+1}\right)\right)-\frac{\alpha}{1-\alpha}\left(\epsilon_{a t+1}+\epsilon_{v t+1}\right)+\alpha \hat{k}_{t+1}+(1-\alpha) \hat{h}_{t+1}+ \\
& \left.\frac{\alpha}{1-\alpha}\left(\epsilon_{a t}+\epsilon_{v t}-\alpha \hat{k}_{t}-(1-\alpha) \hat{h}_{t}\right)\right) \\
& 0=\mathbb{E}\left[\left(1-\frac{K^{*}}{Y^{*}}\right) \hat{y}_{t}-\frac{C^{*}}{Y^{*}} \hat{y}_{t}-\left(1-\frac{K^{*}}{Y^{*}}\right) \hat{y}_{t+1}+\frac{C^{*}}{Y^{*}} \hat{y}_{t+1}\right]
\end{aligned}
$$

where

$$
\begin{equation*}
\left(1-\frac{K^{*}}{Y^{*}}\right) \hat{y}_{t}-\frac{C^{*}}{Y^{*}} \hat{y}_{t}-\left(1-\frac{K^{*}}{Y^{*}}\right) \hat{y}_{t+1}+\frac{C^{*}}{Y^{*}} \hat{y}_{t+1}=0 \tag{23}
\end{equation*}
$$

since

$$
\begin{equation*}
1=\frac{C^{*}}{Y^{*}}+\frac{K^{*}}{Y^{*}} \tag{24}
\end{equation*}
$$

Therefore, I can conclude that the policy functions for capital and consumption are given by (21) and (22) respectively. I need to provide also a policy rule for hours. Intuitively, the policy rule should be equal to zero. Remember that in the exact solution hours were constant over time. Therefore, the deviation from steady state
should be zero. I show below that given (22), $\hat{h}_{t}=0$ for all $t$.

$$
\begin{aligned}
\hat{h}_{t} & =\nu \hat{w}_{t}-\nu \hat{c}_{t}=\nu\left(\hat{y}_{t}-\hat{h}_{t}\right)-\nu \hat{c}_{t} \\
\hat{h}_{t} & =\frac{\nu}{1-\nu}\left(\hat{y}_{t}-\hat{c}_{t}\right) \\
& =\frac{1}{C^{*} / Y^{*}}\left[\frac{C^{*}}{Y^{*}}+\frac{K^{*}}{Y^{*}}-1\right] \hat{k}_{t+1} \\
& =0
\end{aligned}
$$

## Appendix A. 6 Extensions

## Appendix A.6.1 Hansen-Rogerson Preferences

So far my analysis have only considered the intensive margin of the labor input. Here I will assume another specification for household's preferences so that I will analyze the extensive margin of the labor input. To do so let us assume the following:

1. Labor is indivisible.
2. Agents can trade employment lotteries.
3. Households have a constant relative risk-aversion utility function with a coefficient of risk-aversion equal to 1 .

Therefore, (2.2) will be substituted by

$$
\begin{equation*}
U\left(C_{t}, H_{t}\right)=\ln C_{t}-B H_{t} \tag{25}
\end{equation*}
$$

which implies that the short-run Frisch elasticity of labor supply is infinite.
It is obvious that the (detrended) equilibrium conditions under all the statistical models are identical but the one associated with the labor supply. In particular,
(2.11) and (2.25) will be substituted by

$$
\begin{equation*}
\frac{\tilde{W}_{t}}{\tilde{C}_{t}}=B \tag{26}
\end{equation*}
$$

I need to recalibrate only the parameter linked to the weight of hours in the utility function i.e. B. In fact, I have that

$$
\begin{equation*}
B=(1-\alpha) \frac{Y^{*}}{C^{*}} \frac{1}{H^{*}} \tag{27}
\end{equation*}
$$

I simulate my model only for the deterministic trend case and the baseline stochastic trend one. I perform my analysis only for the whole sample.

First, I allow for the presence of both technology shocks. Then, I perform two counterfactuals in order to asses the relative importance of each technology shock in accounting for the business cycles features observed in the US data. On the one hand, I shut down the investment specific shock and investigate the volatilities implied by my model. On the other hand, I shut down the neutral shock and perform the same analysis.

From table 19 I conclude that the volatilities of all the variables at hand are larger than in the divisible labor economy. Moreover, the Hansen-Rogerson economy overstates the volatilities of investment, output, capital, and hours when both shocks are at hand. It is remarkable that, as in [39], the volatility of hours is larger than the volatility of labor productivity.

The model performs better, in terms of accounting for volatilities, when there is only an investment-specific shock than when there is only a neutral one. In fact, a model with Hansen-Rogerson preferences and only an I-shock is able to replicate almost perfectly the standard deviation of hours.

From table 19, I conclude that a stochastic trend model is not able to generate enough volatility in this scenario either. Under this specification, the neutral shock is the one able to account for the bulk of the volatility for all the variables at hand.

## Appendix A.6.2 Multivariate analysis

I have performed univariate analysis of the error structure associated to the different specifications for the technology processes.I am interested here in exploring a multivariate error structure in order to analyze the interaction between both innovations.

## Deterministic Trend Model

I will consider the following specification

$$
\begin{align*}
\ln A_{t} & =\varphi_{a}+\gamma_{a} t+\varepsilon_{a t}  \tag{28}\\
\ln V_{t} & =\varphi_{v}+\gamma_{v} t+\varepsilon_{v t} \tag{29}
\end{align*}
$$

and I assume

$$
\begin{equation*}
\binom{\varepsilon_{a t}}{\varepsilon_{v t}}=\Gamma_{1}\binom{\varepsilon_{a t-1}}{\varepsilon_{v t-1}}+\Gamma_{2}\binom{\varepsilon_{a t-2}}{\varepsilon_{v t-2}}+\binom{\xi_{a t}}{\xi_{v t}} \tag{30}
\end{equation*}
$$

where

$$
\begin{equation*}
\binom{\xi_{a t}}{\xi_{v t}} \sim \mathcal{N}\left(\mathbf{0}, \Sigma_{\xi}\right) \tag{31}
\end{equation*}
$$

I will restrict my attention to the performance of the model under a unit Frisch elasticity. My estimates are reported in the following table. All the vector autoregressive processes estimated satisfy the stability condition i.e. there is no root that lies outside the unit circle.

The results obtained from the stochastic simulation of my model economy are summarized in the table 21. Let us compare these results with those in table 7. The direction of change for the volatilities of the different variables of interest is not unilateral. For example, while the volatility of consumption is larger in the multivariate setting, the volatility of capital is lower. The performance of the deterministic trend model, however, improves in accounting for the volatility slowdown of investment, capital, and hours.

## Stochastic Trend Model

Let us consider the following

$$
\begin{align*}
\ln A_{t} & =\ln A_{t-1}+\gamma_{a} t+\varepsilon_{a t}  \tag{32}\\
\ln V_{t} & =\ln V_{t-1}+\gamma_{v} t+\varepsilon_{v t} \tag{33}
\end{align*}
$$

and I assume

$$
\begin{equation*}
\binom{\varepsilon_{a t}}{\varepsilon_{v t}}=\Gamma_{1}\binom{\varepsilon_{a t-1}}{\varepsilon_{v t-1}}+\binom{\xi_{a t}}{\xi_{v t}} \tag{34}
\end{equation*}
$$

where

$$
\begin{equation*}
\binom{\xi_{a t}}{\xi_{v t}} \sim \mathcal{N}\left(\mathbf{0}, \Sigma_{\xi}\right) \tag{35}
\end{equation*}
$$

The results from the estimation of the above specification are reported in table 20. The moments implied by this specification are in table 21. Comparing these results with those reported in table 7 I conclude that the multivariate specification implies even lower volatilities for all the variables at hand for all the periods. The performance in terms of replicating the magnitude of the Great Moderation, however, does not change significantly.

From this analysis, I conclude that there is no a significative gain from using a multivariate specification for the innovations.

## Appendix B Chapter 3

## Appendix B. 1 Tables

Table 23: Chow's Breakpoint Test: AR(1) with drift

|  | $\mathbf{1 9 7 0}$ |  | $\mathbf{1 9 8 4}$ |  | $\mathbf{1 9 7 0 - 1 9 8 4}$ |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | F | LR | F | LR | F | LR |
| $\Delta$ Net worth | $6.51^{* *}$ | $6.47^{* *}$ | $10.18^{* * *}$ | $10.03^{* * *}$ | $5.52^{* * *}$ | $10.90^{* * *}$ |
| $\Delta$ Output | $3.29^{*}$ | $3.30^{*}$ | $24.20^{* * *}$ | $23.11^{* * *}$ | $12.69^{* * *}$ | $24.28^{* * *}$ |
| $\Delta$ Investment | $2.87^{*}$ | $2.87^{*}$ | $15.32^{* * *}$ | $14.92^{* * *}$ | $7.80^{* * *}$ | $15.25^{* * *}$ |
| $\Delta$ Consumption | 1.47 | 1.48 | $10.75^{* * *}$ | $10.58^{* * *}$ | $5.65^{* * *}$ | $11.16^{* * *}$ |
| Hours | 0.11 | 0.11 | $7.59^{* * *}$ | $7.53^{* * *}$ | $4.94^{* * *}$ | $9.80^{* * *}$ |
| Labor share | 0.60 | 0.61 | 0.18 | 0.18 | 0.30 | 0.61 |
| Inflation | $2.92^{*}$ | $2.93^{*}$ | $10.99^{* * *}$ | $10.81^{* * *}$ | $16.66^{* * *}$ | $31.33^{* * *}$ |
| Federal Funds Rate | $3.54^{*}$ | $3.55^{*}$ | $6.24^{* * *}$ | $6.21^{* * *}$ | $12.10^{* * *}$ | $23.21^{* * *}$ |

Notes: F refers to the F-statistic, which is distributed as $F(k, T-2 k)$ where $k$ is the number of parameters to be tested and $T$ the total number of observations. The critical values are 2.73 at $10 \%$ significance level, 3.89 at $5 \%$, and 6.76 at $1 \%$ when I test for one break. For two breaks, the critical values are 2.33 at $10 \%, 3.04$ at $5 \%$, and 4.71 at $1 \%$. LR refers to the log-likelihood ratio statistic, which is distributed as $\chi^{2}$ with $(m-1) k$ degrees of freedom, where $m$ is the number of subsamples. The critical values when there is only one break are 2.71 at $10 \%$ significance level, 3.84 at $5 \%$, and 6.64 at $1 \%$. For two breaks, the critical values are 4.61 at $10 \%, 5.99$ at $5 \%$, and 9.21 at $1 \%$. If the statistic is above the critical value, the null hypothesis of no structural change can be rejected. The symbol * indicates I can reject the null of parameter constancy at $10 \%,{ }^{* *}$, at $5 \%$, and ${ }^{* * *}$, at $1 \%$.

Table 24: Chow's Breakpoint Test: Cyclical component. AR(1) with drift

|  | $\mathbf{1 9 7 0}$ |  | $\mathbf{1 9 8 4}$ |  | $\mathbf{1 9 7 0 - 1 9 8 4}$ |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | F | LR | F | LR | F | LR |
| Net worth | $4.01^{* *}$ | $4.01^{* *}$ | $6.03^{* *}$ | $6.00^{* *}$ | $3.29^{* *}$ | $6.58^{*}$ |
| Output | 2.66 | 2.67 | $20.45^{* * *}$ | $19.69^{* * *}$ | $10.79^{* * *}$ | $20.82^{* * *}$ |
| Investment | 1.92 | 1.93 | $13.54^{* * *}$ | $13.24^{* * *}$ | $7.08^{* * *}$ | $13.89^{* * *}$ |
| Consumption | 0.86 | 0.86 | $14.50^{* * *}$ | $14.14^{* * *}$ | $8.36^{* * *}$ | $16.31^{* * *}$ |
| Hours | 0.54 | 0.54 | $11.23^{* * *}$ | $11.04^{* * *}$ | $6.62^{* * *}$ | $13.02^{* * *}$ |
| Labor share | 0.82 | 0.83 | 0.06 | 0.06 | 0.46 | 0.94 |
| Inflation | $3.68^{*}$ | $3.69^{*}$ | $11.50^{* * *}$ | $11.30^{* * *}$ | $18.77^{* * *}$ | $34.98^{* * *}$ |
| Federal Funds Rate | $3.88^{* *}$ | $3.88^{* *}$ | $7.47^{* * *}$ | $7.41^{* * *}$ | $14.17^{* * *}$ | $29.93^{* * *}$ |

Table 25: Ratio post- to pre- standard deviation: Cyclical component

|  | $\frac{1984-2006}{1954-1983}$ | $\frac{1970-1983}{1954-1969}$ | $\frac{1984-2006}{1970-1983}$ |
| :---: | :---: | :---: | :---: |
| Output | 0.49 | 1.55 | 0.41 |
| Investment | 0.60 | 1.47 | 0.51 |
| Consumption | 0.54 | 1.74 | 0.44 |
| Hours | 0.74 | 1.47 | 0.63 |
| Labor share | 0.88 | 1.23 | 0.80 |
| Inflation | 0.46 | 2.58 | 0.34 |
| Federal funds rate | 0.65 | 2.81 | 0.47 |
| Net worth business | 1.67 | 1.32 | 1.47 |
| Debt business | 1.41 | 1.71 | 1.13 |
| Net worth households | 1.23 | 1.94 | 1.04 |
| Net private savings | 1.55 | 1.16 | 1.44 |
| Demand deposits | 1.57 | 3.09 | 1.14 |

Notes: The cyclical component is extracted using the Hodrick-Prescott filter for the quarterly frequency $(\lambda=1600)$.
Table 26: Prior

| Name | Description | Density | Median | St. dev. | 95\% CI |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $\beta$ | Discount rate | Fixed | 0.9988 | - | - |
| $\delta$ | Depreciation rate | Fixed | 0.025 | - | - |
| $\left(\frac{G}{Y}\right)^{\star}$ | Public spending share | Fixed | 0.20 | - | - |
| $\gamma$ | Survival probability | Fixed | 0.9854 | - | - |
| $[F(\bar{\omega})]^{\star}$ | Default probability | Fixed | 0.0075 | - | - |
| $\alpha$ | Capital share | Beta | 0.3 | 0.05 | $[0.20,0.40]$ |
| $\lambda^{\star}$ | Markup in the steady state | Beta | 0.15 | 0.02 | $[0.11,0.19]$ |
| $h$ | Degree of habit formation | Beta | 0.6 | 0.1 | $[0.40,0.79]$ |
| $\pi_{n}^{\star}$ | Inflation in steady state | Gamma | 2.79 | 1 | $[0.55,5.98]$ |
| $\ln \left(H^{\star}\right)$ | Log hours at the steady state | $\mathcal{N}$ | 0.03 | 0.01 | $[0.01,0.04]$ |
| $\xi_{p}$ | Calvo parameter | Beta | 0.75 | 0.05 | $[0.65,0.85]$ |
| $\xi$ | Adjustment cost | Gamma | 1.83 | 1 | $[0.33,3.85]$ |
| $\nu$ | Frisch elasticity | Gamma | 1.84 | 1 | $[0.35,3.89]$ |
| $\left(\mu^{\star}\right)_{l}$ | Level of financial friction | Beta | 0.28 | 0.05 | $[0.14,0.38]$ |
| $a^{" \prime}$ | Elasticity of capital utilization costs | Gamma | 0.44 | 0.3 | $[0.04,1.09]$ |
| $\rho_{r}$ | Degree of backward looking in MP | Beta | 0.63 | 0.2 | $[0.25,0.97]$ |
| $\left(\psi_{\pi}\right)_{l}$ | MP reaction to inflation | $\mathcal{N}$ | 1.54 | 0.35 | $[1.00,2.09]$ |
| $\left(\psi_{y}\right)_{l}$ | MP reaction to output growth | $\mathcal{N}$ | 0.46 | 0.1 | $[0.30,0.70]$ |
| $\rho_{i}$ | Persistence of shocks | Beta | 0.60 | 0.1 | $[0.41,0.79]$ |
| $\Upsilon_{z}$ | Drift in the neutral technology process | $\mathcal{N}$ | 0.03 | 0.01 | $[-0.015,0.020]$ |
| $\left(\sigma_{i}\right)_{j}$ | Std i shock, subsample j | $\mathcal{I G}$ | 0.01 | $d . f .4$ | $[0.005,0.025]$ |

Notes: The prior median and the credible intervals have been obtained using the draws that imply determinacy of the DSGE model from a set of 100,000 draws from the prior distribution with a $25 \%$ burn-in. $j=1,2,3$ and $l=1: 2,3$.

Table 27: Marginal Data Densities Comparison

| Model | MDD(model)-MDD(no breaks) |
| :---: | :---: |
| Breaks in 1970 and 1984 |  |
| Only $\mu^{\star}$ | -4 |
| Only $\psi_{i}$ | 37 |
| Only $\sigma_{j}$ | 34 |
| $\mu^{\star}, \psi_{i}, \sigma_{j}$ | 52 |

Notes: The marginal data density or marginal likelihood of the data is defined as the expectation taken over the likelihood with respect to the prior distribution of the parameters: $p\left(Y \mid \mathcal{M}_{i}\right)=\int p\left(Y \mid \varrho, M_{i}\right) p\left(\varrho \mid M_{i}\right) d \varrho$. MDD above refers to the natural $\log$ of the marginal data density.

Table 28: Posterior estimates

| Name |  |  | Name |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  | Median | 95\%C.I. |  | Median | 95\%C.I. |
| $\alpha$ | 0.28 | $[0.27,0.30]$ | $\rho_{r}$ | 0.44 | $[0.26,0.58]$ |
| $\lambda^{\star}$ | 0.25 | $[0.22,0.28]$ | $\rho_{\zeta}$ | 0.94 | $[0.92,0.97]$ |
| $h$ | 0.34 | $[0.27,0.41]$ | $\rho_{\mu}$ | 0.97 | $[0.95,0.98]$ |
| $\nu$ | 1.03 | $[0.49,1.79]$ | $\rho_{\lambda}$ | 0.88 | $[0.84,0.92]$ |
| $a^{\prime \prime}$ | 0.82 | $[0.31,1.55]$ | $\rho_{\theta}$ | 0.98 | $[0.97,0.99]$ |
| $\pi^{\star}$ | 2.72 | $[2.34,3.12]$ | $\rho_{b}$ | 0.90 | $[0.87,0.93]$ |
| $\ln \left(H^{\star}\right)$ | 0.03 | $[0.01,0.05]$ | $\rho_{x}$ | 0.52 | $[0.44,0.60]$ |
| $\xi_{p}$ | 0.40 | $[0.33,0.46]$ | $100 \Upsilon_{z}$ | 0.54 | $[0.42,0.66]$ |
| $\xi$ | 4.55 | $[2.59,7.01]$ |  |  |  |

Table 29: Posterior estimates

| Name | Pre 1970 |  | 1970-1983 |  | Post 1984 |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Median | $95 \%$ C.I. | Median | $95 \%$ C.I. | Mean | $95 \%$ C.I. |
| $\mu^{\star}$ | 0.24 | $[0.19,0.28]$ | 0.24 | $[0.19,0.30]$ | 0.05 | $[0.03,0.06]$ |
| $\sigma_{\mu}$ | 1.67 | $[1.23,2.18]$ | 1.62 | $[1.19,2.14]$ | 7.18 | $[4.77,10.7]$ |
| $100 \sigma_{Z}$ | 1.52 | $[1.27,1.82]$ | 1.56 | $[1.29,1.88]$ | 0.82 | $[0.71,0.95]$ |
| $100 \sigma_{x}$ | 0.75 | $[0.63,0.89]$ | 0.91 | $[0.75,1.08]$ | 1.52 | $[1.30,1.79]$ |
| $100 \sigma_{\zeta}$ | 1.11 | $[0.75,1.69]$ | 1.64 | $[1.15,2.32]$ | 0.90 | $[0.72,1.11]$ |
| $100 \sigma_{b}$ | 1.70 | $[1.32,2.19]$ | 2.38 | $[1.77,3.10]$ | 1.25 | $[0.98,1.57]$ |
| $100 \sigma_{R}$ | 0.52 | $[0.39,0.70]$ | 0.85 | $[0.62,1.14]$ | 0.47 | $[0.35,0.63]$ |
| $100 \sigma_{\theta}$ | 1.48 | $[1.06,2.11]$ | 1.82 | $[1.28,2.65]$ | 1.37 | $[1.06,1.81]$ |
| $100 \sigma_{\lambda}$ | 3.58 | $[2.86,4.42]$ | 4.66 | $[3.71,5.79]$ | 3.77 | $[3.18,4.47]$ |
| $\psi_{\pi}$ | 2.65 | $[2.24,3.06]$ | 1.84 | $[1.58,2.14]$ | 2.43 | $[2.03,2.90]$ |
| $\psi_{y}$ | 0.31 | $[0.18,0.45]$ | 0.48 | $[0.32,0.64]$ | 0.47 | $[0.32,0.63]$ |

Table 30: Model Fit: Standard deviations. Raw variables.

| Series | $1954-1969$ |  |  | $1970-1983$ |  |  | $1984-2006$ |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Data | Model |  | Data | Model |  | Data | Model |  |
|  | Median |  |  | $90 \%$ | Median |  |  | $90 \%$ |  |
| Median | $90 \%$ |  |  |  |  |  |  |  |  |
| $\Delta$ Net worth | 0.67 | 0.96 | $[0.86,1.06]$ | 0.95 | 1.15 | $[1.02,1.26]$ | 1.35 | 1.79 | $[1.59,1.96]$ |
| $\Delta$ Output | 1.46 | 1.24 | $[1.15,1.33]$ | 1.60 | 1.39 | $[1.27,1.50]$ | 0.69 | 0.91 | $[0.84,0.98]$ |
| $\Delta$ Investment | 5.73 | 4.89 | $[4.50,5.28]$ | 6.20 | 6.26 | $[5.75,6.77]$ | 3.14 | 4.04 | $[3.73,4.34]$ |
| $\Delta$ Consumption | 0.80 | 1.04 | $[0.95,1.12]$ | 0.92 | 1.19 | $[1.08,1.29]$ | 0.51 | 0.72 | $[0.65,0.77]$ |
| Hours | 2.60 | 3.18 | $[2.12,4.41]$ | 2.80 | 3.99 | $[2.74,5.47]$ | 3.48 | 3.04 | $[2.00,4.16]$ |
| Labor share | 2.44 | 1.49 | $[1.17,1.80]$ | 1.75 | 1.98 | $[1.63,2.41]$ | 1.49 | 1.53 | $[1.20,1.84]$ |
| Inflation | 0.37 | 0.40 | $[0.36,0.44]$ | 0.76 | 0.92 | $[0.83,1.02]$ | 0.27 | 0.37 | $[0.34,0.40]$ |
| Nominal interest rate | 0.42 | 0.51 | $[0.39,0.63]$ | 0.91 | 0.88 | $[0.66,1.08]$ | 0.58 | 0.34 | $[0.25,0.43]$ |

Notes: For each parameter draw, I generate 1000 samples with the same length as the data after discarding 100 initial observations.
Table 31: Model Fit: Standard deviations. Cyclical component using the HP-filter.

| Series | $1954-1969$ |  |  | $1970-1983$ |  |  | $1984-2006$ |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Data | Model |  | Data | Model |  | Data | Model |  |
|  | Median |  |  | $90 \%$ |  | Median | $90 \%$ |  | Median |
| Net worth | 1.25 | 1.73 | $[1.42,2.07]$ | 1.65 | 2.10 | $[1.70,2.51]$ | 2.42 | 3.34 | $[2.70,3.97]$ |
| Output | 1.96 | 1.69 | $[1.42,1.95]$ | 3.04 | 1.88 | $[1.55,2.17]$ | 1.24 | 1.21 | $[1.02,1.41]$ |
| Investment | 7.10 | 6.22 | $[5.15,7.04]$ | 10.4 | 8.16 | $[6.87,9.30]$ | 5.29 | 5.29 | $[4.41,6.14]$ |
| Consumption | 1.11 | 1.62 | $[1.32,1.88]$ | 1.93 | 1.85 | $[1.51,2.16]$ | 0.84 | 1.12 | $[0.94,1.33]$ |
| Hours | 1.58 | 1.34 | $[1.13,1.56]$ | 2.32 | 1.74 | $[1.47,1.99]$ | 1.46 | 1.31 | $[1.12,1.15]$ |
| Labor share | 0.77 | 0.95 | $[0.82,1.08]$ | 0.95 | 1.32 | $[1.16,1.50]$ | 0.76 | 0.96 | $[0.84,1.10]$ |
| Inflation | 0.24 | 0.36 | $[0.33,0.39]$ | 0.62 | 0.81 | $[0.75,0.88]$ | 0.21 | 0.34 | $[0.31,0.37]$ |
| Nominal interest rate | 0.21 | 0.30 | $[0.25,0.34]$ | 0.59 | 0.50 | $[0.43,0.57]$ | 0.28 | 0.18 | $[0.15,0.20]$ |

Table 33: Counterfactuals: Percentage of the model-implied change in cyclical standard deviations.

|  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Counterfactual | Net worth | Output | Investment | Consumption | Hours | Labor Share | $\pi$ | $R$ |
| Counterfactual 1 | 5 | 0 | 0 | 0 | 3 | 15 | 46 | 32 |
| Counterfactual 2 | 5 | 8 | 6 | 6 | 3 | 0 | 0 | 3 |
| Counterfactual 3 | 100 | 8 | 18 | 6 | 6 | 2 | 0 | 5 |
| Counterfactual 4 | 9 | 100 | 88 | 94 | 94 | 83 | 40 | 53 |
| Counterfactual 5 | 3 | 0 | 0 | 0 | 0 | 15 | 41 | 22 |
| Counterfactual 6 | - | - | 34 | 0 | - | 0 | 9 | 34 |
| Counterfactual 7 | 150 | - | - | - | - | 0 | 0 | - |
| Counterfactual 8 | 0 | 105 | 83 | 95 | 113 | 96 | 74 | 59 |

Notes: I include a dash (-) when the direction of the counterfactual implied change is at odds with the data.

## Appendix B. 2 Figures

Figure B-1: Debt to net worth ratio. Cyclical component.


## Appendix B. 3 Data

I use US data from NIPA-BEA,CPS-BLS, the FRED database, and the Flow of Funds Accounts from the Federal Reserve Board for the period 1954.4-2006.4.

## Appendix B.3.1 Data used in estimation

- Growth rate of real per capita gross value added by the nonfarm business sector. Data on nominal gross value added are available in NIPA Table 1.3.5. I have deflated such a series using the the implicit price index from table 1.3.4. I divide the new series by the Civilian Noninstitutional +16 (BLS ID LNU00000000) series to obtain per capita variables. The data provided by the BEA are annualized so I divide by 4 to obtain quarterly values for the measures of interest.
- Growth rate of real per capita investment. Investment is defined as gross private domestic investment from NIPA Table 1.1.5. I deflate the nominal variables using the GDP deflator provided by NIPA Table 1.1.4. I weight the resulting series using the relative significance of the nonfarm business sector in total GDP. Finally, I do the same correction described above to render the investment series in per capita quarterly terms.
- Growth rate of real per capita consumption. Consumption is defined as the sum of personal consumption expenditures of nondurables and services from NIPA Table 1.1.5. I deflate the nominal variables using the GDP deflator provided by NIPA Table 1.1.4. I weight the resulting series using the relative significance of the non-farm business sector in total GDP. Finally, I do the same correction described above to have the series in per capita quarterly terms.
- Growth rate of net worth. I define net worth as the real per capita weighted average of net worth for the corporate and noncorporate nonfarm business sector. To ensure the measure of net worth from the data is close enough to the series the model can actually account for, I define net worth as tangible assets minus credit market instruments at market value. On the one hand, I use tangible assets only as a measure for assets because, in my model, collateral is related only to physical capital and inventories; that is, there is no role for financial capital. On the other hand, I evaluate net worth at current (market) prices, since such a variable in my theoretical framework stands for the value of the collateral perceived by lenders. Credit market liabilities from the Flow of Funds Accounts (the weighted sum of series FL104104005.Q from Table B. 102 and series FL114102005.Q from Table B.103) stand for entrepreneurial debt. Tangible assets are given by the weighted sum of series FL102010005.Q from Table B. 102 and series FL112010005.Q from Table B. 103.
- Hours worked is defined, following [56], as the log level of the BLS series PRS85006023 divided by 100 and multiplied by the ratio of civilian population over 16 (CE16OV) to a population index. The population index is equal to the ratio of population at the corresponding quarter divided by the population in the third quarter of 1992. This transformation is necessary, since the series on hours is an index with $1992=100$.
- Labor share is defined as the ratio of total compensation of employees (NIPA Table 2.1) corrected by the size of the non-farm business sector to the gross value added by the nonfarm business sector.
- Inflation is defined as the log difference of the price index for gross value added by the nonfarm business sector (NIPA Table 1.3.4).
- The Federal funds rate is taken from the Federal Reserve Economic Data (FRED).


## Appendix B.3.2 Data used in the empirical evidence section

In addition to the series described above, I also consider the following ones

- Net private savings: Data on nominal net private savings are available in the NIPA Table 5.1. I have deflated such a series using the implicit price index from Table 1.3.4. I divide the new series by the Civilian Noninstitutional +16 (BLS ID LNU00000000) series to obtain per capita variables. The data provided by the BEA are annualized, so I divide by 4 to obtain quarterly values for the measures of interest. I weight the resulting series using the relative significance of the nonfarm business sector in total GDP.
- Debt in the nonfarm business sector: I define debt as the real per capita weighted average of credit market liabilities for the corporate and noncorporate nonfarm business sector. Debt is defined as the weighted sum of series FL104104005.Q from Table B. 102 and series FL114102005.Q from Table B.103.
- Net worth of households (and nonprofit organizations): It is given by the real per capita transformation of the series FL152090005 from Table B. 100 from the Flow of Funds Accounts.
- Demand deposits: It stands for real per capita demand deposits at commercial banks provided by the series DEMDEPSL in the FRED database. Data are available from 1959.


## Appendix B. 4 Methodology

## Appendix B.4.1 MCMC Algorithm

1. Posterior Maximization: The aim of this step is to obtain the parameter vector to initialize my posterior simulator. To obtain the posterior mode, $\widetilde{\varrho}$, I iterate over the following steps:
(a) Fix a vector of structural parameters $\varrho^{\prime}$.
(b) Solve the DSGE model conditional on $\varrho^{\prime}$ and compute the system matrices.

I restrict myself to the determinacy region of the parameter space.
(c) Use the Kalman filter to compute the likelihood of the parameter vector $\varrho^{\prime}, p\left(Y^{T} \mid \varrho^{\prime}\right)$.
(d) Combine the likelihood function with the prior distribution.
2. Compute the numerical Hessian at the posterior mode. Let $\widetilde{\Sigma}$ be the inverse of such a numerical hessian.
3. Draw the initial parameter vector, $\varrho^{(0)}$, from $\mathcal{N}\left(\widetilde{\varrho}^{(0)}, c_{0}^{2} \widetilde{\Sigma}\right)$ where $c_{0}$ is a scaling parameter. Otherwise, directly specify a starting value for the posterior simulator.
4. Posterior Simulator: for $s=1, \ldots, n_{\text {sim }}$, draw $\vartheta$ from the proposal distribution $\mathcal{N}\left(\varrho^{(s-1)}, c^{2} \widetilde{\Sigma}\right)$, where $c$ is a scaling parameter ${ }^{4}$. The jump from $\varrho^{(s-1)}$ is accepted with probability

$$
\min \left\{1, r\left(\varrho^{(s-1), \vartheta \mid Y}\right)\right\}
$$

[^11]and rejected otherwise. Note that
\[

$$
\begin{equation*}
r\left(\varrho^{(s-1), \vartheta \mid Y}\right)=\frac{\mathcal{L}(\vartheta \mid Y) p(\vartheta)}{\mathcal{L}\left(\varrho^{(s-1)} \mid Y\right) p\left(\varrho^{(s-1)}\right)} \tag{36}
\end{equation*}
$$

\]

5. Approximate the expected value of a function $h(\varrho)$ by

$$
\frac{1}{n_{s i m}} \sum_{s=1}^{n_{s i m}} h\left(\varrho^{(s)}\right)
$$

## Appendix B.4.2 Kalman Filter

Let us cast the log-linearized dynamic system in state-space form:

- Transition equation:

$$
\begin{aligned}
s_{t} & =(I-T) \ln (\bar{s})+T s_{t-1}+R \epsilon_{t} \\
& =J+T s_{t-1}+R \epsilon_{t}
\end{aligned}
$$

where $T=\Phi(\varrho), R=\Phi_{\varepsilon}, \epsilon \sim(\mathbf{0}, Q), s_{t}$ stand for the vector of DSGE state variables in log levels, and $\ln (\bar{s})$ is the vector of steady-state log-values of these state variables. Let $\mathfrak{s}=\operatorname{rows}\left(s_{t}\right)$.

- Measurement equation:

$$
y_{t}=Z s_{t}
$$

where $Z=B(\varrho)$ and I have imposed the assumption of zero measurement error in the system.

Linearity and Gaussian errors allow us to use the Kalman filter to evaluate the likelihood function. I give an overview here of such a filter; for a complete description, please see Chapter 13 in [37].

1. Initialization: The filter is initialized with the unconditional distribution of the state vector.

- Initial mean:

$$
\hat{s}_{0 \mid 0}=\ln (\bar{s})
$$

- Initial variance: $P_{0 \mid 0}$ is given by the solution to the following discrete Lyapunov equation

$$
P_{0 \mid 0}=T P_{0 \mid 0} T^{\prime}+R Q R^{\prime}
$$

## 2. Forecasting step

$$
\begin{aligned}
\widehat{s}_{t \mid t-1} & =J+T \widehat{s}_{t-1 \mid t-1} \\
P_{t \mid t-1} & =T P_{t-1 \mid t-1} T^{\prime}+R Q R^{\prime} \\
\widehat{y}_{t \mid t-1} & =Z \widehat{s}_{t \mid t-1} \\
F_{t \mid t-1} & =Z P_{t \mid t-1} Z^{\prime}
\end{aligned}
$$

## 3. Evaluation of the log-likelihood

## 4. Updating step

$$
\begin{aligned}
\widehat{s}_{t \mid t} & =\widehat{s}_{t \mid t-1}+P_{t \mid t-1} Z^{\prime} F_{t \mid t-1}^{-1}\left(y_{t}^{o b s}-\widehat{y}_{t \mid t-1}\right) \\
P_{t \mid t} & =P_{t-1 \mid t-1}-P_{t-1 \mid t-1} Z^{\prime} F_{t \mid t-1}^{-1} Z^{\prime} P_{t-1 \mid t-1}
\end{aligned}
$$

So far, I have assumed that the system matrices were all constant. The Kalman filter, however, is also suitable for state-space models in which those matrices vary over time. The filter needs simply to be modified so that the appropriate matrix is used at each $t$. Given that the state-space system under analysis is a reduced-form representation of a structural model, I should be careful when extending the filter to allow for breaks in the system matrices. Note that if I allow for structural breaks in the size of the shocks and/or the monetary policy coefficients, the system matrices vary but there is no effect on the definition of the steady state of the economy. However, if there is a break in a parameter defining the steady state of my model economy, the econometrician needs to make sure she is using the same information set as the economic agents.

Let us assume there is a shift in the steady state of the economy so that I go from $\bar{s}_{1}$ to $\bar{s}_{2}$. This implies a shift in the entries of $T$ and, hence, $J$. I need to introduce the following modification in the forecasting step

- If $t<t^{\star}$,

$$
\begin{aligned}
\widehat{s}_{t \mid t-1} & =J_{1}+T_{1} \widehat{s}_{t-1 \mid t-1} \\
P_{t \mid t-1} & =T_{1} P_{t-1 \mid t-1} T_{1}^{\prime}+R Q_{1} R^{\prime}
\end{aligned}
$$

- If $t=t^{\star}$,

$$
\begin{aligned}
\widehat{s}_{t \mid t-1} & =J_{2}+T_{2}\left(\ln \left(\bar{s}_{2}\right)-\ln \left(\bar{s}_{1}\right)\right)+T_{2} \widehat{s}_{t-1 \mid t-1} \\
P_{t \mid t-1} & =T_{2} P_{t-1 \mid t-1} T_{2}^{\prime}+R Q_{2} R^{\prime}
\end{aligned}
$$

- If $t>t^{\star}$,

$$
\begin{aligned}
\widehat{s}_{t \mid t-1} & =J_{2}+T_{2} \widehat{s}_{t-1 \mid t-1} \\
P_{t \mid t-1} & =T_{2} P_{t-1 \mid t-1} T_{2}^{\prime}+R Q_{2} R^{\prime}
\end{aligned}
$$

## Appendix B.4.3 Variance decomposition

My data set contains the following series

$$
\left\{\Delta Y, \Delta I, \Delta C, \Delta N, \log (H), \log (L S), \log \left(1+\frac{\pi}{400}\right), \log \left(1+\frac{R^{n}}{400}\right)\right\}
$$

I am interested, however, in the second moments and dynamic properties of

$$
\left.\log (Y), \log (I), \log (C), \log (N), \log (H), \log (L S), \log \left(1+\frac{\pi}{400}\right), \log \left(1+\frac{R^{n}}{400}\right)\right\}
$$

Therefore, I use an inverse difference filter for the first four components on the spectrum implied by the DSGE model. The spectral density is obtained using the state-space representation of the DSGE model and 500 bins for frequencies in the range of periodicities of interest. In particular, I compute the variance decomposition at business cycle frequencies, that is, I focus on those periodic components with cycles between 6 and 32 quarters.

## Inverse difference filter

Let $X_{t}$ be univariate data in log-levels and $Y_{t}=(1-L) X_{t}$. Note that

$$
\begin{aligned}
X_{t} & =(1-L)^{-1} Y_{t} \\
& =\sum_{h=0}^{\infty} L^{h} Y_{t-h} \\
& =\sum_{h=0}^{\infty} \exp (-i \omega j h) Y_{t-h}
\end{aligned}
$$

Then, the spectral density of $X_{t}$ is given by

$$
s_{X}(\omega)=\left|\sum_{h=0}^{\infty} \exp (-i \omega j h)\right|^{2} s_{Y}(\omega)
$$

which can be approximated by

$$
s_{X}(\omega)=\left|\frac{1}{1-\exp (-i \omega j h)}\right|^{2} s_{Y}(\omega)
$$

at any frequency by 0 .

## Appendix B. 5 Log-linearized equilibrium conditions

Let $\widetilde{Y}_{t}=\frac{Y_{t}}{Z_{a, t}}$ for $C, I, K, G, W / P, M_{t+1} / P_{t}, N B_{t+1} / P_{t}, D_{t+1}$, div, $T, N_{t+1}$. Let $\widehat{\varsigma}=$ $\log \left(\frac{\varsigma}{\varsigma^{\star}}\right)$ where $\varsigma^{\star}$ is the steady state value of the variable $\varsigma$.

1. Trend variable

$$
\widehat{\mathfrak{Z}}_{t}=\varepsilon_{a, t}
$$

2. Household's FOC with respect to $N B_{t+1}$

$$
\widehat{\widetilde{\Lambda}}_{t}=\widehat{R}_{t}^{n}+\mathbb{E}_{t}\left[\widehat{\widetilde{\Lambda}}_{t+1}-\widehat{\pi}_{t+1}+\widehat{\mathfrak{Z}}_{t+1}\right]
$$

where $\Lambda_{t}$ is the Lagrange multiplier linked to the budget constraint.
3. Household's FOC with respect to $H_{t}$

$$
\widehat{b}_{t}+\widehat{\theta}+\frac{1}{\nu} H_{t}=\widehat{\widetilde{W}}_{t}+\widehat{\widetilde{\Lambda}}_{t}
$$

4. Household's FOC with respect to $D_{t+1}$

$$
\widehat{\widetilde{\Lambda}}_{t}=\widehat{R}_{t}+\mathbb{E}_{t}\left[\widehat{\widetilde{\Lambda}}_{t+1}-\widehat{\mathfrak{Z}}_{t+1}\right]
$$

5. Household's FOC with respect to $C_{t}$

$$
\begin{aligned}
\widehat{\widetilde{\Lambda}}_{t}= & \frac{\mathfrak{Z}^{\star}-\beta h \rho_{b}}{\mathfrak{Z}^{\star}-\beta h} \widehat{b}_{t}-\frac{\mathfrak{Z}^{\star} h}{\left(\mathfrak{Z}^{\star}-h\right)\left(\mathfrak{Z}^{\star}-\beta h\right)} \widehat{\mathfrak{Z}}_{t}-\frac{\left(\mathfrak{Z}^{\star}\right)^{2}+\beta h^{2}}{\left(\mathfrak{Z}^{\star}-h\right)\left(\mathfrak{Z}^{\star}-\beta h\right)} \widehat{\widetilde{C}}_{t} \\
& +\frac{\beta \mathfrak{Z}^{\star} h}{\left(\mathfrak{Z}^{\star}-h\right)\left(\mathfrak{Z}^{\star}-\beta h\right)} \widehat{\widetilde{C}}_{t-1}+\frac{\left.\beta \mathfrak{Z}^{\star}-h\right)\left(\mathfrak{Z}^{\star}-\beta h\right)}{\mathbb{E}_{t}} \widehat{\widetilde{C}}_{t+1}
\end{aligned}
$$

6. Price of capital (from capital producers)

$$
\widehat{Q}_{t}=\xi \frac{\widetilde{I}^{\star} \mathfrak{Z}^{\star}}{\widetilde{K}^{\star}}\left(\widehat{\widetilde{I}}_{t}+\widehat{\mathfrak{Z}}_{t}-\widehat{\widetilde{K}}_{t}\right)-\widehat{\zeta}_{t}
$$

7. Capital accumulation

$$
\widehat{\widetilde{K}}_{t+1}=\frac{1-\delta}{\mathfrak{Z}^{\star}}\left(\widehat{\widetilde{K}}_{t}-\widehat{\mathfrak{Z}}_{t}\right)+\frac{\widetilde{I}^{\star}}{\widetilde{K}^{\star}}\left(\widehat{\zeta}_{t}+\widehat{\widetilde{I}}_{t}\right)
$$

8. New Keynesian Phillips curve

$$
\widehat{\pi}=\kappa \widehat{\chi}_{t}+\beta \mathbb{E}_{t} \widehat{\pi}_{t+1}+\kappa \frac{\lambda}{1+\lambda} \widehat{\lambda}_{t}
$$

where $\kappa=\frac{\left(1-\xi_{p}\right)\left(1-\xi_{p} \beta\right)}{\xi_{p}}$ and $\chi_{t}$ is the real marginal cost.
9. Government constraint

$$
\widehat{\widetilde{G}}_{t}=\hat{\widetilde{Y}}_{t}
$$

10. Taylor rule

$$
\widehat{R}_{t}^{n}=\rho_{R} \widehat{R}_{t-1}^{n}+\left(1-\rho_{R}\right) \rho_{\pi} \widehat{\pi}_{t}+\left(1-\rho_{R}\right) \rho_{Y}\left(\widehat{\tilde{Y}}_{t}-\widehat{\tilde{Y}}_{t-1}+\widehat{\mathfrak{Z}}_{t}\right)+\varepsilon_{R, t}
$$

11. Definition of effective capital

$$
\widehat{\widetilde{k}}_{t}=\widehat{u}_{t}+\widehat{\widetilde{K}}_{t}-\widehat{\mathfrak{Z}}_{t}
$$

12. Optimal capital utilization

$$
\widehat{r}_{t}^{k}=\frac{\left(a^{\prime \prime}\right)^{\star}}{\left(r^{k}\right)^{\star}} \widehat{u}_{t}
$$

13. Production technology

$$
\widehat{\tilde{Y}}_{t}=\alpha \widehat{\widetilde{k}}_{t}+(1-\alpha) \widehat{H}_{t}
$$

14. Optimal capital-to-labor ratio for intermediate goods producers

$$
\widehat{\widetilde{k}}_{t}-\widehat{H}_{t}=\widehat{\widetilde{W}}_{t}-\widehat{r}_{t}^{k}
$$

15. Real marginal cost

$$
\widehat{\chi}_{t}=(1-\alpha) \widehat{\widetilde{W}}_{t}+\alpha \widehat{r}_{t}^{k}
$$

16. Expected gross return on capital

$$
\mathbb{E}_{t}\left[\widehat{R}_{t+1}^{k}\right]=\frac{\left(r^{k}\right)^{\star}}{\left(R^{k}\right)^{\star}} \mathbb{E}_{t}\left[\widehat{r}_{t+1}^{k}\right]-\widehat{Q}_{t}+\frac{1-\delta}{\left(R^{k}\right)^{\star}} \mathbb{E}_{t}\left[\widehat{Q}_{t+1}\right]
$$

17. Supply for loans

$$
\begin{aligned}
& {\left[\frac{\widetilde{K}^{\star}-\widetilde{N}^{\star}}{\widetilde{N}^{\star}}\right]\left(\widehat{R}_{t+1}^{k}-R_{t}\right)+\frac{R^{k \star}}{R^{\star}} \frac{\widetilde{K}^{\star}}{\widetilde{N}^{\star}}\left[\Gamma_{\omega}\left(\bar{\omega}^{s s}\right)-\mu G_{\omega}\left(\bar{\omega}^{s s}\right)\right] \bar{\omega}^{s s} \widehat{\omega}_{t+1}} \\
& -\frac{R^{k \star}}{R^{\star}} \frac{\widetilde{K}^{\star}}{\widetilde{N}^{\star}} \mu G\left(\bar{\omega}^{s s}\right) \widehat{\mu}_{t+1}=\widehat{Q}_{t}+\widehat{\widetilde{K}}_{t+1}-\widehat{\widetilde{N}}_{t+1}
\end{aligned}
$$

18. Net worth

$$
\begin{aligned}
\frac{\widetilde{N}^{\star}}{\gamma} \widehat{\widetilde{N}}_{t+1}= & \left(\left[1-\mu^{\star} G\left(\bar{\omega}^{\star}\right)\right] R^{k \star}-R^{\star}\right) \frac{\widetilde{K}^{\star}}{\mathfrak{Z}^{\star}}\left(\widehat{Q}_{t-1}+\widehat{\widetilde{K}}_{t}-\widehat{\mathfrak{Z}}_{t}\right) \\
& +\left(1-\mu^{\star} G\left(\bar{\omega}^{\star}\right)\right) R^{k \star} \frac{\widetilde{K}^{\star}}{\mathfrak{Z}^{\star}} \widehat{R}_{t}^{k}-\left(\widehat{N}^{\star}-\widehat{K}^{\star}\right) \frac{R^{\star}}{\mathfrak{Z}^{\star}} \widehat{R}_{t-1} \\
& -\mu^{\star} G\left(\bar{\omega}^{\star}\right) R^{k \star} \frac{\widetilde{K}^{\star}}{\mathfrak{Z}^{\star}} \widehat{\mu}_{t}-\mu^{\star} G_{\omega}\left(\bar{\omega}^{\star}\right) R^{k \star} \frac{\widetilde{K}^{\star}}{\mathfrak{Z}^{\star}} \bar{\omega}^{\star} \widehat{\bar{\omega}}_{t}+\frac{R^{\star}}{\widetilde{\mathcal{J}}^{\star}} \widehat{\widetilde{N}}_{t}
\end{aligned}
$$

19. First-order condition with respect to the debt-to-wealth ratio

$$
\begin{aligned}
& \mu^{\star}\left[\frac{\mu^{\star} \Psi_{\mu}\left(\bar{\omega}^{\star}, \mu^{\star}\right) G\left(\bar{\omega}^{\star}\right)+\Psi\left(\bar{\omega}^{\star}, \mu^{\star}\right) G\left(\bar{\omega}^{\star}\right)}{1-\Gamma\left(\bar{\omega}^{\star}\right)+\Psi\left(\bar{\omega}^{\star}, \mu^{\star}\right)\left(\Gamma\left(\bar{\omega}^{\star}\right)-\mu^{\star} G\left(\bar{\omega}^{\star}\right)\right)}\right] \mathbb{E}_{t} \mu_{t+1} \\
& =\mathbb{E}_{t}\left[\widehat{R}_{t+1}^{k}-R_{t}\right] \\
& -\left[\frac{R^{k \star}}{R^{\star}}\left[\frac{R^{\star}}{R^{k \star}} \frac{\Psi_{\omega}\left(\bar{\omega}^{\star}, \mu\right)}{\Psi\left(\bar{\omega}^{\star}, \mu\right)}-\frac{\Psi_{\omega}\left(\bar{\omega}^{\star}, \mu\right)}{\Psi\left(\bar{\omega}^{\star}, \mu\right)}\left(\Gamma\left(\bar{\omega}^{\star}\right)-\mu^{\star} G\left(\bar{\omega}^{\star}\right)\right)\right]\right] \bar{\omega}^{\star} \mathbb{E}_{t} \widehat{\bar{\omega}}_{t+1}
\end{aligned}
$$

20. Market clearing conditions
(a) Credit market

$$
\widetilde{D}^{\star} \widehat{\widetilde{D}}_{t+1}=\widetilde{Q}^{\star} \widetilde{K}^{\star}\left(\widehat{Q}_{t}+\widehat{\widetilde{K}}_{t+1}\right)-\widetilde{N}^{\star} \widehat{\tilde{N}}_{t+1}
$$

Debt

$$
\widehat{\widetilde{B}}_{t+1}=\widehat{\widetilde{D}}_{t+1}
$$

(b) Total Resources

$$
\begin{aligned}
\widehat{\tilde{Y}}_{t}= & \frac{\widetilde{C}^{\star}}{\widetilde{Y}^{\star}} \widehat{\widetilde{C}}_{t}+\frac{\widetilde{I}^{\star}}{\widetilde{Y}^{\star}} \widehat{\widetilde{I}}_{t}+\frac{\widetilde{G}^{\star}}{\widetilde{Y}^{\star}} \widehat{\widetilde{G}}_{t} \\
& +\frac{\mu^{\star} G\left(\bar{\omega}^{\star}\right) R^{k \star} Q^{\star} \widetilde{K}^{\star}}{\widetilde{Y}^{\star} \mathfrak{Z}^{\star}}\left[\widehat{R}_{t}^{k}+\widehat{Q}_{t-1}+\widehat{\widetilde{K}}_{t}-\widehat{\mathfrak{Z}}_{t}+\frac{G_{\omega}\left(\bar{\omega}^{\star}\right)}{G\left(\bar{\omega}^{\star}\right)} \bar{\omega}^{\star} \widehat{\omega}_{t}\right] \\
& +\mu^{\star} R^{k \star} \frac{G\left(\bar{\omega}^{\star}\right) \widetilde{K}^{\star}}{\widetilde{Y}^{\star} \widehat{\mathfrak{Z}}^{\star}}+r^{k \star} \frac{\widetilde{K}^{\star}}{\widetilde{Y}^{\star}} \widehat{u}_{t}
\end{aligned}
$$

21. Some definitions

$$
\begin{aligned}
F(\bar{\omega}) & =\int_{0}^{\bar{\omega}} \frac{1}{\omega \sigma_{\omega} \sqrt{2 \pi}} e^{-\frac{\ln (\omega)+0.5 \sigma_{\omega}^{2}}{2 \sigma_{\omega}^{2}}} d \omega \\
F_{\omega}(\bar{\omega}) & =\frac{1}{\omega \sigma_{\omega} \sqrt{2 \pi}} e^{-\frac{\ln (\omega)+0.55 \sigma_{\omega}^{2}}{2 \sigma_{\omega}^{2}}}
\end{aligned}
$$

$$
\begin{aligned}
& F_{\omega \omega}(\bar{\omega})=-\frac{1}{\omega} F_{\omega}(\bar{\omega})\left[1+\frac{\ln (\omega)+0.5 \sigma_{\omega}^{2}}{\sigma_{\omega}^{2}}\right] \\
& G(\bar{\omega})=\int_{0}^{\bar{\omega}} \omega f(\omega) d \omega=1-\Phi\left(\frac{0.5 \sigma_{\omega}^{2}-\ln (\bar{\omega})}{\sigma_{\omega}}\right) \\
& G_{\omega}(\bar{\omega})=\bar{\omega} F_{\omega}(\bar{\omega}) \\
& \Gamma(\bar{\omega})=\int_{0}^{\bar{\omega}} \omega f(\omega) d \omega+\bar{\omega} \int_{\bar{\omega}}^{\infty} f(\omega) d \omega=\bar{\omega}(1-F(\bar{\omega}))+G(\bar{\omega}) \\
& \Gamma_{\omega}(\bar{\omega})=1-F(\bar{\omega}) \\
& \Psi(\bar{\omega}, \mu)=\frac{\Gamma_{\omega}(\bar{\omega})}{\Gamma_{\omega}(\bar{\omega})-\mu G_{\omega}(\bar{\omega})} \\
& \Psi_{\mu}(\bar{\omega}, \mu)=\frac{\left.G_{\omega}(\bar{\omega}) \Psi(\bar{\omega}, \mu)\right)}{\Gamma_{\omega}(\bar{\omega})-\mu G_{\omega}(\bar{\omega})} \\
& \Psi_{\omega}(\bar{\omega}, \mu)=\frac{1}{\left(1-F(\bar{\omega})-\mu \bar{\omega} F_{\omega}(\bar{\omega})\right)^{2}}\left(-F_{\omega}(\bar{\omega})\left[1-F(\bar{\omega})-\mu \bar{\omega} F_{\omega}(\bar{\omega})\right]\right. \\
&\left.-[1-F(\bar{\omega})]\left[-F_{\omega}(\bar{\omega})-\mu F_{\omega}(\bar{\omega})-\mu \bar{\omega} F_{\omega \omega}(\bar{\omega})\right]\right)
\end{aligned}
$$

## Appendix C Chapter 4

## Appendix C. 1 Tables

Table 34: Variance decomposition at business cycle frequencies

| Output |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | BC | Wealth | Neutral | I-shock | Markup | Intra | Inter | MP |
| Pre-1970 | 1 | 2 | 67 | 4 | 6 | 17 | 2 | 0 |
| 1970-1984 | 0 | 3 | 56 | 8 | 8 | 21 | 3 | 1 |
| Post-1984 | 0 | 4 | 39 | 6 | 14 | 31 | 5 | 0 |
| Investment |  |  |  |  |  |  |  |  |
|  | BC | Wealth | Neutral | I-shock | Markup | Intra | Inter | MP |
| Pre-1970 | 22 | 13 | 7 | 35 | 4 | 2 | 17 | 0 |
| 1970-1984 | 7 | 19 | 4 | 45 | 4 | 2 | 18 | 1 |
| Post-1984 | 26 | 16 | 4 | 20 | 6 | 5 | 22 | 0 |
| Consumption |  |  |  |  |  |  |  |  |
|  | BC | Wealth | Neutral | I-shock | Markup | Intra | Inter | MP |
| Pre-1970 | 1 | 6 | 60 | 11 | 1 | 14 | 7 | 0 |
| 1970-1984 | 1 | 7 | 47 | 18 | 1 | 15 | 10 | 0 |
| Post-1984 | 1 | 12 | 31 | 15 | 2 | 20 | 19 | 0 |
| Net Worth |  |  |  |  |  |  |  |  |
|  | BC | Wealth | Neutral | I-shock | Markup | Intra | Inter | MP |
| Pre-1970 | 6 | 92 | 1 | 0 | 0 | 0 | 1 | 0 |
| 1970-1984 | 4 | 93 | 1 | 0 | 0 | 0 | 1 | 1 |
| Post-1984 | 1 | 99 | 0 | 0 | 0 | 0 | 0 | 0 |

Notes: It corresponds to periodic components of cycles between 6 and 32 quarters. BC refers to shocks to the marginal bankruptcy cost and MP to monetary policy shocks

Table 35: Variance decomposition at the business cycle frequency

| Hours |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | BC | Wealth | Neutral | I-shock | Markup | Intra | Inter | MP |
| Pre-1970 | 1 | 8 | 1 | 14 | 18 | 52 | 6 | 1 |
| 1970-1984 | 1 | 7 | 1 | 19 | 18 | 47 | 6 | 2 |
| Post-1984 | 0 | 7 | 0 | 9 | 23 | 51 | 9 | 1 |
| Labor share |  |  |  |  |  |  |  |  |
|  | BC | Wealth | Neutral | I-shock | Markup | Intra | Inter | MP |
| Pre-1970 | 0 | 0 | 1 | 0 | 95 | 0 | 0 | 4 |
| 1970-1984 | 0 | 0 | 2 | 0 | 86 | 1 | 0 | 11 |
| Post-1984 | 0 | 0 | 1 | 0 | 95 | 0 | 0 | 3 |
| Inflation |  |  |  |  |  |  |  |  |
|  | BC | Wealth | Neutral | I-shock | Markup | Intra | Inter | MP |
| Pre-1970 | 0 | 3 | 15 | 8 | 1 | 4 | 11 | 59 |
| 1970-1984 | 0 | 2 | 11 | 8 | 1 | 5 | 14 | 58 |
| Post-1984 | 1 | 1 | 10 | 3 | 3 | 8 | 6 | 68 |
| Nominal rate |  |  |  |  |  |  |  |  |
|  | BC | Wealth | Neutral | I-shock | Markup | Intra | Inter | MP |
| Pre-1970 | 0 | 15 | 3 | 41 | 0 | 1 | 39 | 1 |
| 1970-1984 | 0 | 12 | 1 | 44 | 0 | 0 | 41 | 1 |
| Post-1984 | 6 | 14 | 1 | 48 | 0 | 0 | 29 | 1 |

Notes: It corresponds to periodic components of cycles between 6 and 32 quarters. BC refers to shocks to the marginal bankruptcy cost and MP to monetary policy shocks

Table 36: Sign restrictions.

| $\mathbf{k}$ | Consumption | Investment | Net worth | Spread |
| :---: | :---: | :---: | :---: | :---: |
| 0 | $?$ | + | + | - |
| 1 | $?$ | + | + | - |

Table 37: Percentage of IRFs delivering a negative response upon impact of consumption

| Net worth | Spread | Whole | $1954-1969$ | $1970-1983$ | $1984-2006$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| FOFA | corporate | 6 | 35 | 2 | 31 |
| FOFA | prime | 33 | 18 | 57 | 10 |
| Dow Jones | corporate | 30 | 45 | 30 | 49 |
| Dow Jones | prime | 55 | 10 | 65 | 22 |

## Appendix C. 2 Figures

Figure C-1: Impulse Response Functions with respect to a wealth shock. The dotted line is the IRF for the 1954-1969








Figure C-2: Impulse Response Functions with respect to a shock to the marginal bankruptcy cost. The dotted line is the IRF for the 1954-1969 period, the solid line is the IRF for 1970-1983, and the dashed line is the IRF for the post-1984




Figure C-3: Impulse Response Functions with respect to a shock to the marginal bankruptcy cost: 1954-1969




CONSUMPTION
Figure C-4: Impulse Response Functions: A comparison







Figure C-5: IRF with respect to a wealth shock. Net worth is measured using FOFA and the external financial premium,
using the corporate spread




Figure C-6: IRF with respect to a wealth shock. Net worth is measured using industrial Dow Jones and the external
financial premium, using the corporate spread




Figure C-7: IRF with respect to a wealth shock. Net worth is measured using industrial Dow Jones and the external


financial premium, using the prime lending spread



Figure C-8: 1970-1983: IRF with respect to a wealth shock. Net worth is measured using FOFA and the external





Figure C-9: 1970-1983: IRF with respect to a wealth shock. Net worth is measured using FOFA and the external
financial premium, using the prime lending spread
Figure C-9: 1970-1983: IRF with respect to a wealth shock.
financial premium, using the prime lending spread





## Bibliography

[1] M.J. Alderson and B.L. Betker. Liquidation costs and capital structure. Journal of Financial Economics, 39:45-69, 1995.
[2] E. I. Altman. A further investigation of the bankruptcy cost question. Journal of Finance, 39(4):1067-89, 1984.
[3] E.I. Altman and B. Pasternack. Defults and returns in the high yield bond market: The year 2005 in review and market outlook. Journal of Applied Research in Accounting and Finance, 1(1):3-30, 2006.
[4] Andrés Arias, Gary D. Hansen, and Lee E. Ohanian. Why have business cycle fluctuations become less volatile? Economic Theory, 32:43-58, 2007.
[5] B. S. Bernanke, M. Gertler, and S. Gilchrist. The financial accelerator in a quantitative business cycle framework. In J. Taylor and M. Woodford, editors, The Handbook of Macroeconomics, pages 1341-1393. Elsevier Science B.V., Amsterdam, The Netherlands, 1999.
[6] S. Beveridge and C. Nelson. A new approach to decomposition of economic time series into permanent and transitory components with particular attention to measurement of the 'business cycle'. Journal of Monetary Economics, 7:151174, 1981.
[7] Olivier Blanchard and John Simon. The long and large decline in u.s. output volatility. Brookings Papers on Economic Activity, 1:135-164, 2001.
[8] C. Carlstrom and T. Fuerst. Agency costs, net worth, and business fluctuations: A computable general equilibrium analysis. American Economic Review, 87(5):893-910, 1997.
[9] Yongsung Chang, Taeyoung Doh, and Frank Schorfheide. Non-stationary hours in a dsge model. Journal of Money, Credit, and Banking, Forthcoming, 2007.
[10] I. Christensen and A. Dib. The financial accelerator in an estimated new keynesian model. Review of Economic Dynamics, 11:155-178, 2008.
[11] L. J. Christiano. Why does inventory investment fluctuate so much? Journal of Monetary Economics, 21:247-280, 1988.
[12] L. J. Christiano, M. Eichenbaum, and C. Evans. Nominal rigidities and the dynamic effects of a shock to monetary policy. Journal of Political Economy, 113:1-45, 2005.
[13] Lawrence J. Christiano, Martin Eichenbaum, and Robert Vigfusson. What happens after a technology shock? NBER Working Paper 9819, 2003.
[14] L.J. Christiano, M. Eichenbaum, and C.L. Evans. The effects of monetary policy shocks: Evidence from the flow of funds. The Review of Economics and Statistics, 78(1):16-34, 1996.
[15] L.J. Christiano and C. Gust. The great inflation of the 1970s. Manuscript, 1999.
[16] L.J. Christiano, R. Motto, and M. Rostagno. The great depression and the friedman-schwartz hypothesis. Journal of Money, Credit, and Banking, 35(6 (Part 2)):1119-1197, 2003.
[17] L.J. Christiano, R. Motto, and M. Rostagno. Financial factors in economic fluctuations. mimeo, 2009.
[18] F. Collard and H. Dellas. The great inflation of the 1970s. Journal of Money, Credit and Banking, 39:713-731, 2007.
[19] Thomas F. Cooley and Edward C. Prescott. Economic growth and business cycle. In Thomas F. Cooley, editor, Frontiers of Business Cycle Research, chapter 1. Princeton University Press, Princeton, 1995.
[20] Jason G. Cummins and Giovanni L. Violante. Investment-specific technical change in the united states (1947-2000): Measurement and macroeconomic consequences. Review of Economic Dynamics, 5:243-284, 2002.
[21] B. deBlas. Can financial frictions help explain the performance of the u.s. fed? B.E. Journal of Macroeconomics, 9(1 (Contributions)):Art. 27, 2009.
[22] F. DeGraeve. The external finance preimum and the macroeconomy: Us postwwii evidence. Journal of Economic Dynamics and Control, 32(11):3415-3440, 2008.
[23] A. Dib. Banks, credit market frictions, and business cycles. mimeo, 2009.
[24] A. Dixit and J. Stiglitz. Monopolistic competition and optimum product diversity. American Economic Review, 67:297-308, 1977.
[25] Jesús Fernández-Villaverde and Juan F. Rubio-Ramírez. Estimating macroeconomic models: A likelihood approach. Mimeo, Duke University, 2006.
[26] Jonas D.M. Fisher. The dynamic effects of neutral and investment-specific technology shocks. Journal of Political Economy, 114(3):413-451, 2006.
[27] R. Fry and A. Pagan. Sign restrictions in structural vector autoregressions: A critical review. Mimeo, 2009.
[28] C. Fuentes-Albero. Financial frictions, the financial immoderation, and the great moderation. Mimeo, 2009.
[29] Jordi Galí. Technology, employment, and the business cycle: Do technology shocks explain aggregate fluctuations? The American Economic Review, 89(1):249-271, 1999.
[30] Jordi Galí and Pau Rabanal. Technology shocks and aggregate fluctuations: How well does the real business cycle model fit postwar us data? IMF Working Paper WP/04/234, 2004.
[31] A. Gelman, J. B. Carlin, H. S. Stern, and D. B. Rubin. Bayesian Data Analysis. Chapman and Hall/CRC, 2004.
[32] S. Gilchrist and J.V. Leahy. Monetary policy and asset prices. Journal of Monetary Economics, 49:75-97, 2002.
[33] S. Gilchrist, Alberto Ortiz, and Egon Zakrajs̆ek. Credit risk and the macroeconomy: Evidence from an estimated dsge model. mimeo, 2009.
[34] Paul Gomme and Peter Rupert. Theory, measurement, and calibration of macroeconomic models. Journal of Monetary Economics, 54:460-497, 2007.
[35] Jeremy Greenwood, Zvi Hercowitz, and Per Krusell. Long-run implications of investment-specific technological change. The American Economic Review, 83:342-362, 1997.
[36] Jeremy Greenwood, Zvi Hercowitz, and Per Krusell. The role of investmentspecific technological change in the business cycle. European Economic Review, 44:91-115, 2000.
[37] James D. Hamilton. Time Series Analysis. Princeton University Press, 1994.
[38] Gary D. Hansen. Indivisible labor and the business cycle. Journal of Monetary Economics, 16:309-327, 1985.
[39] Gary D. Hansen. Technical progress and aggregate fluctuations. Journal of Economic Dynamics and Control, 21:1005-1023, 1997.
[40] M. Iacoviello. House prices, borrowing constraints, and monetary policy in business cycles. American Economic Review, 95:739-764, 2005.
[41] P. N. Ireland. Technology shocks and the business cycle: An empirical investigation. Journal of Economic Dynamics E Control, 25:703-719, 2001.
[42] Urban Jermann and Vicenzo Quadrini. Financial innovations and macroeconomic volatility. NBER Working Paper 12308, 2008.
[43] A. Justiniano, G. Primiceri, and A. Tambalotti. Investment shocks and the relative price of investment. CEPR discussion paper No. 7597, 2010.
[44] A. Justiniano, G.E. Primiceri, and A. Tambalotti. Investment shocks and business cycles. Journal of Monetary Economics, 57(2):132-145, 2010.
[45] Chang-Jin Kim and Charles R. Nelson. Has the u.s. economy become more stable? a bayesian approach based on a markov-switching model of business cycle. Review of Economics and Statistics, 81(4):608-616, 1999.
[46] Robert G. King, Charles I. Plosser, and Sergio T. Rebelo. Production, growth, and business cycles ii. new directions. Journal of Monetary Economics, 21:309341, 1988.
[47] F. E. Kydland and E. C. Prescott. Time to build and aggregate fluctuations. Econometrica, 50:1345-1370, 1982.
[48] A.T. Levin, F.M. Natalucci, and E. Zakrajšek. The magnitude and cyclical behvior of financial market frictions. Finance and Economics Discussion Series. Division of Research \& Statistics and Monetary Affairs, Federal Reserve System, Board of Governors, 2004-70, 2004.
[49] Margaret M. McConnell and Gabriel Pérez-Quirós. Output fluctuations in the united states: What has changed since the early 1980's? American Economic Review, 90(5):1464-1476, 2000.
[50] F. Modigliani and M.H. Miller. The cost of capital, corporate finance, and the theory of investment. American Economic Review, 38:261-297, 1958.
[51] C. Nelson and C. Plosser. Trends and random walks in macroeconomic time series: Some evidence and implications. Journal of Monetary Economics, 10:139162, 1982.
[52] E. Nelson. The great inflation of the seventies: What really happened? Advances in Macroeconomics, 5:Article 3, available at http://www.bepress.com/bejm/advances/vol5/iss1/art3, 2005.
[53] Charles Nolan and Christpoh Thoenissen. Financial shocks and the us business cycle. Journal of Monetary Economics, 56(4):596-604, 2009.
[54] Edward C. Prescott. Response to a skeptic. Quarterly Review. Minneapolis Federal Reserve Bank, 10:28-33, 1986.
[55] José Víctor Ríos-Rull, Frank Schorfheide, Cristina Fuentes-Albero, Maxym Kryshko, and Raül Santaeulàlia-Llopis. Calibration, estimation, and the effects of technology shocks. Manuscript. University of Pennsylvania, 2007.
[56] F. Smets and R. Wouters. Shocks and frictions in us business cycles: A bayesian dsge approach. American Economic Review, 97:586-606, 2007.
[57] James H. Stock and Mark W. Watson. Has the business cycle changed and why? NBER Macroeconomics Annual, NBER Working Paper 9127, 2002.
[58] R.M. Townsend. Optimal contracts and competitive markets with costly state verification. Journal of Economic Theory, 21:265-293, 1979.
[59] H. Uhlig. What are the effects of monetary policy on output? results from an agnostic identification procedure. Journal of Monetary Economics, 52:381-419, 2005.
[60] M. J. White. Bankruptcy costs and the new bankruptcy code. Journal of Finance, XXXVIII(2):477-488, 1983.


[^0]:    ${ }^{1}$ For a more detailed discussion on nonstationary time series see [37]
    ${ }^{2}$ I have performed ADF (Augmented Dickey-Fuller) tests on all of the variables of interest. I were not able to reject the null of unit root for all the variables but (log) hours and (log) labor productivity.

[^1]:    ${ }^{3}$ I report in appendix 4.4 the analysis under Hansen-Rogerson preferences.
    ${ }^{4}$ See [9] for an interesting treatment of the stationarity of hours issue and [13] for an analysis of the implications of different labor input measures in a SVAR framework.

[^2]:    ${ }^{5}$ For detrending issues there is no difference between having just a random walk with drift or a random walk with drift plus a moving average component.

[^3]:    ${ }^{6}$ The results under those assumptions for my model are reported in appendix A.6.1. I conclude that if the stochastic processes are trend stationary, a model à la Hansen overstates the volatilities of investment, output, capital, and hours. In such a setting, a model economy with only an investmentspecific technology shock is able to replicate the volatility of hours. I also conclude that under a difference stationary framework my model economy is still not able to generate enough volatility for all the variables at hand.

[^4]:    ${ }^{7}$ Let us use the term correlation in a broad sense ie it refers not only to the correlation with output, but also to the cross-correlations considering lags and leads, and the correlation for any other pair of variables

[^5]:    ${ }^{1}$ Note that one unit of $t+1$ capital is produced by the following technology $(1-\delta) K_{t}+\zeta I_{t}$. Old capital is bought at price $P_{t}^{k}$. Therefore, the cost term cancels out the revenue term.

[^6]:    ${ }^{2}$ [5] perform simulation exercises under a parameterization that implied a negligible contribution of the dynamics of the cutoff. However, most of the contributions to the financial accelerator literature have adopted this result as a feature of the model. Therefore, they proceed by setting those dynamics to zero.

[^7]:    ${ }^{1}$ The prime lending spread is defined as the difference between the prime lending rate and the three-month T-bill.

[^8]:    ${ }^{2}$ Essentially, $\alpha$ is a column of the $\Omega$ matrix.

[^9]:    

    |  | Data |  |  |  | DT |  |  |  | ST |  |  | ST-MA |  |  |
    | :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
    |  | Whole | Pre | Post | Whole | Pre | Post | Whole | Pre | Post | Whole | Pre | Post |  |  |
    | c_x | 0.61 | 0.58 | 0.75 | -0.21 | -0.46 | -0.18 | 0.53 | 0.57 | 0.57 | 0.50 | 0.51 | 0.74 |  |  |
    | c_k | 0.25 | 0.21 | 0.50 | 0.15 | 0.15 | 0.06 | 0.34 | 0.37 | 0.41 | 0.31 | 0.31 | 0.44 |  |  |
    | c_h | 0.70 | 0.69 | 0.75 | -0.41 | -0.61 | -0.37 | 0.50 | 0.54 | 0.54 | 0.40 | 0.40 | 0.67 |  |  |
    | c_lbp | 0.03 | 0.14 | -0.40 | 0.69 | 0.68 | 0.61 | 0.96 | 0.96 | 0.96 | 0.94 | 0.94 | 0.97 |  |  |
    | x_k | 0.21 | 0.24 | 0.09 | 0.28 | 0.24 | 0.31 | 0.24 | 0.23 | 0.24 | 0.24 | 0.23 | 0.24 |  |  |
    | x_h | 0.81 | 0.81 | 0.81 | 0.98 | 0.98 | 0.98 | 0.99 | 1 | 1 | 0.98 | 0.97 | 0.99 |  |  |
    | x_lbp | 0.03 | 0.12 | -0.39 | 0.56 | 0.34 | 0.68 | 0.75 | 0.77 | 0.77 | 0.75 | 0.76 | 0.87 |  |  |
    | k_h | 0.54 | 0.59 | 0.35 | 0.22 | 0.18 | 0.29 | 0.32 | 0.34 | 0.37 | 0.20 | 0.20 | 0.17 |  |  |
    | k_lbp | -0.41 | -0.42 | -0.35 | 0.33 | 0.35 | 0.30 | 0.32 | 0.34 | 0.37 | 0.32 | 0.31 | 0.38 |  |  |
    | h_lbp | -0.40 | -0.28 | -0.80 | 0.38 | 0.16 | 0.51 | 0.73 | 0.75 | 0.75 | 0.67 | 0.68 | 0.83 |  |  |

[^10]:    ${ }^{3}$ Maybe it is more intuitive to write the Euler equation as

    $$
    1=\beta \mathbb{E}_{t}\left[e^{-\frac{1}{1-\alpha}\left(\gamma_{a}+\epsilon_{a t+1}+\alpha\left(\gamma_{v}+\epsilon_{v t+1}\right)\right)} \frac{\tilde{C}_{t}}{\tilde{C}_{t+1}}\left((1-\delta) e^{-\gamma_{v}-\epsilon_{v t+1}}+\alpha e^{\gamma_{a}+\epsilon_{a t+1}} \tilde{K}_{t+1}^{\alpha-1} H_{t+1}^{1-\alpha}\right)\right]
    $$

[^11]:    ${ }^{4}$ The scale factor is set to obtain efficient algorithms. [31] argue that the scale coefficient should be set $c \approx 2.4 \sqrt{d}$, where $d$ is the number of parameters to be estimated. However, I will fine tune the scale factor to obtain a rejection rate of about $25 \%$

