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$\overline{J o s h u a ~ R . ~ K l e i n, ~ P r o f e s s o r ~ o f ~ P h y s i c s ~ \& ~ A s t r o n o m y ~}$

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# ABSTRACT SPECTROSCOPIC ANALYSIS OF STELLAR ROTATIONAL VELOCITY AT THE BOTTOM OF THE MAIN SEQUENCE 

Steven H. Gilhool
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This thesis presents analyses aimed at understanding the rotational properties of stars at the bottom of the main sequence. The evolution of stellar angular momentum is intertwined with magnetic field generation, mass outflows, convective motions, and many other stellar properties and processes. This complex interplay has made a comprehensive understanding of stellar angular momentum evolution elusive. This is particularly true for low-mass stars due to the observational challenges they present. At the very bottom of the main sequence (spectral type $\lesssim \mathrm{M} 4$ ), stars become fully convective. While this 'Transition to Complete Convection' presents mysteries of its own, observing the rotation of stars across this boundary can provide insight into stellar structure and magnetic fields, as well as their role in driving the evolution of stellar angular momentum. I present here a review of our understanding of rotational evolution in stars of roughly solar mass down to the end of the main sequence.

I detail our efforts to determine large numbers of rotational velocity measurements for M dwarfs observed by the Apache Point Galactic Evolution Experiment (APOGEE). We analyzed the 714 M dwarfs as late as spectral type $\sim$ M7, the largest sample of M dwarfs to date. Consistent with the hypothesis that fully-convective M dwarfs spin down more slowly than solar-type stars, we found that the fraction of detectably rotating stars jumped from about $10 \%$ for early to mid M dwarfs, to about $35 \%$ for late M dwarfs. We also found some interesting tension between the rotation fractions from spectroscopic studies of $v \sin i$ like ours, and those expected from rotation periods derived from photometric surveys.

Finally, I describe our novel data-driven technique for rapidly estimating $v \sin i$ in survey data. Rather than directly measuring the broadening of spectral lines, we leveraged the large information content of high-resolution spectral data to empirically estimate $v \sin i$. This computationally efficient technique provides a means of rapidly estimating $v \sin i$ for large numbers of stars in spectroscopic survey data. Indeed, we were able to estimate $v \sin i$ up to $15 \mathrm{~km} \mathrm{~s}^{-1}$ for 27,000 APOGEE spectra, in fractions of a second per spectrum.

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## Chapter 1

## Introduction

The earliest stellar classification systems ordered stars from blue to red, and although it was suspected that color was a function of mass, it was not until 1909 that this was demonstrated conclusively. Since that time, our understanding of the red end of the main sequence, which consists of the faintest and least massive stars, has developed considerably, but is still lacking in many ways. In this work, I present infrared spectroscopic analyses of the rotational velocities of stars at the bottom of the main sequence. I examine stars that range from approximately $1.3 \mathrm{M}_{\odot}$ (spectral type F5) down to $\sim 0.2 \mathrm{M}_{\odot}$ (spectral type $\mathrm{M} 6-8$ ), with a particular interest in shedding light on the mysterious ways that the lowest-mass stars differ from stars like the Sun.

### 1.1 M Dwarfs

The M dwarfs occupy the very bottom end of the main sequence (see Figure 1.1). With the exception of some early L dwarfs, they are the coolest celestial bodies that are able to initiate and sustain hydrogen burning in their cores, thus earning the designation of 'star.' These diminutive stars $\left(M_{\star} \sim 0.1-0.5 M_{\odot}\right)$ are in fact the most numerous stars in the galaxy (Henry et al. 1994), but because they are intrinsically faint, their fundamental properties are not as well measured as


Figure 1.1 The Hertzprung-Russel diagram illustrates the relationship between stellar color and luminosity. This work contains rotational velocity measurements for stars at the bottom half of the main sequence (roughly F5 to M7). The rotational behavior of the least massive stars is of particular interest. Image credit: R. Hollow from CSIRO.
for more massive stars.
The present-day spectral type ordering, O-B-A-F-G-M, emerged from the Harvard system of spectral classification, which was devised by Mrs. W. P. Fleming to complete the Henry Draper Catalog in the early 1900's. Because the observations were made on photographic plates, only stars that were bright at optical wavelengths $\left(m_{\mathrm{V}} \lesssim 9\right)$ were detected. Therefore, the stars first classified as spectral type M were generally giants (Reid \& Hawley, 2005).

As astronomical instrumentation improved, the $M$ dwarf sequence began to emerge, albeit in a somewhat ad hoc manner. The later MKK system for spectral typing (Morgan et al., 1943) included M dwarfs, but only down to spectral type M2. The later MK system (Johnson \& Morgan, 1953) extrapolated the MKK system down to M5, using Barnard's Star as a standard. As observations of later M dwarfs became more commonplace, a number of (mutually incompatible) classification systems and extensions arose to accommodate the M dwarfs. The KHM system (Kirkpatrick et al., 1991), which classifies stars by fitting spectral features, ratios of spectral
features, and spectral slope is most commonly used these days. In fact, the system now includes the L, T, and Y types, which are mostly brown dwarfs (Kirkpatrick et al. 1999, Kirkpatrick 2005).

The distinguishing spectral features of M dwarfs come from the molecules TiO and VO . The TiO bands, which first appear weakly in late K dwarfs, produce strong absorption throughout the $M$ dwarf sequence, and saturate at about spectral type M6. At about M7, VO absorption becomes prominent and TiO begins to weaken as it condenses and settles as dust. Atomic features such as Fe I, Mg I and Ca I are also present in late K through mid M types, but are obscured by the blanketed molecular lines at later types. The atomic lines re-emerge in brown dwarf spectra, as TiO and VO condense and settle below the photosphere (Tinney, 1999). Figure 1.2 shows M dwarf spectra, and their signature atomic and molecular features, at either end of the $M$ dwarf sequence.

### 1.1.1 Observational Challenges

M dwarfs emit most of their radiation at near-infrared wavelengths, reaching a maximum between 0.75 and $1.5 \mu \mathrm{~m}$. The near-infrared is typically defined as the range covered by the JHKL bands (1 to $\sim 4 \mu \mathrm{~m}$ ) Reid \& Hawley, 2005). Within the context of this thesis, I will use 'infrared' and 'near-infrared' interchangeably to refer to that range. Observing large numbers of M dwarfs, to say nothing of the even fainter brown dwarfs, requires instrumentation designed for observing at infrared wavelengths.

Modern optical detectors are charge coupled devices (CCDs). A CCD consists of a doped silicon semi-conductor bonded to an insulating material with an embedded grid of electrodes. A voltage applied to the electrodes defines a grid of pixels. CCDs detect astronomical radiation via the photoelectric effect - incident photons strike the atoms of semi-conductor and dislodge electrons. The free electrons are then confined to the potential well of their given pixel. Once the observation is complete, the voltages are manipulated in order to move the photo-electrons in each pixel to the read-out electronics, and an image is constructed (Reid \& Hawley, 2005). Silicon is
transparent to photons red-ward of $\sim 1.05 \mu \mathrm{~m}$, so different semiconductors are needed to make infrared detectors.

Infrared detectors are typically built out of InSb or HgCdTd . Infrared arrays are importantly not CCDs, although they do operate based on the same photoelectric effect. A key difference between CCDs and infrared arrays is that in the infrared arrays, the charge is not moved to the read-out electronics at the edge of the chip. Rather, each pixel has its own transistor and is read out independently. This eliminates some drawbacks found in CCDs, such as charge transfer inefficiency, but the infrared technology is overall not as mature as CCD technology Reid \& Hawley, 2005). Additionally, infrared detectors and instruments must be cooled to very low temperatures in order to minimize thermal emission.

A further complication of observing $M$ dwarfs is the sky itself, which is bright and highly variable at infrared wavelengths (Reid \& Hawley, 2005). Moreover, at these wavelengths, there is strong absorption of stellar flux due primarily to molecules of water, methane and carbon monoxide in the atmosphere. There are a few relatively clear windows at $1.25 \mu \mathrm{~m}(\mathrm{~J}$ band $), 1.6 \mu \mathrm{~m}(\mathrm{H}$ band), and $2.2 \mu \mathrm{~m}$ (K band). Nevertheless, even these windows are not as clean as at optical wavelengths, as shown in Figure 1.3

### 1.1.2 Astrophysical Importance

M dwarfs are important for a number of reasons. Due to their abundance, they are a major component of the galaxy. The statistics of $M$ dwarfs are necessary to constrain the stellar mass function, which is of prime importance to star formation theories, and to understanding galactic dynamics (Reid \& Hawley, 2005). For example, Tinney (1993) found that the usual assumed form of a simple power-law was a poor representation of the mass function at low mass. The presence of structure in the mass function also shed doubt on fragmentation theories of star formation, which are essentially scale-independent. Furthermore, given ample $M$ dwarf data, the mass function

[^0]

Figure 1.2 Figure 2 from Kirkpatrick et al. (1993) showing the dominant spectral features of M dwarfs at optical and near-infrared wavelengths. Figure 2a shows the spectrum of Gl 411, an M2 dwarf, while Figure 2 b shows that of Gl 752B, an M8 dwarf. The subscript 'FF' denotes Paschen lines introduced by flat-fielding with F and G stars. The ' $\bigoplus$ ' denotes telluric absorption features. Note the shift of the black-body peak toward the infrared with later spectral type. The TiO bands in the red saturate at approximately spectral type M6. Later M dwarfs exhibit more structure at infrared wavelengths.


Figure 1.3 Atmospheric transmission at optical and infrared wavelengths. Reproduced from Figure 1 of Smette et al. (2015).
can be reasonably extrapolated to brown dwarf masses, where much of the mass is fundamentally unobservable. Although Tinney (1999) argued that the hidden brown dwarf mass was insufficient to be the culprit behind dark matter (an intriguing idea), this data is nevertheless important.

M Dwarfs are also promising targets for planet searches since they exhibit larger radial velocity and transit signals than do larger stars (all other things being equal), have close-in habitable zones, and may host many small planets (e.g. Gaidos et al. 2016). Indeed, many current and future exoplanet surveys such as CARMENES Quirrenbach et al. 2014), SPIRou (Thibault et al. 2012), IRD (Tamura et al., 2012), MAROON-X (Seifahrt et al., 2016), MEarth (Irwin et al., 2009), HPF (Mahadevan et al., 2012), MINERVA-Red (Blake et al., 2015), and NIRPS ${ }^{2}$ will target low-mass stars.

On the other hand, the inter-related phenomena of stellar rotation and magnetic activity (discussed in detail in Chapter 2) can degrade radial velocity precision. Stellar rotation results in the Doppler broadening of spectral absorption lines, which reduces the radial velocity information content of the spectrum. Worse still, magnetic activity is connected to flaring, spotting, and other sources of stellar variability, which operate on a wide range of timescales. This so-called 'RV jitter' is a systematic source of error that is very difficult to disentangle from the true radial velocity signal in practice.

Transit surveys searching for rocky habitable-zone planets orbiting M dwarfs are also affected by rotation and activity. The periodic photometric modulation of starspots rotating across the stellar surface can mimic planetary transit signals. In fact, Newton et al. (2016b) found that the typical rotation period of early $M$ dwarfs was on the order of the orbital period of a habitablezone planet, making spurious transit detections likely. Therefore, an understanding of M dwarf rotation is crucial to survey design (e.g. Deshpande et al. 2013) and target selection, and to the disentangling of systematic effects from astrophysical signals.

[^1]
### 1.2 The Transition to Complete Convection

Perhaps the most fascinating feature of the M dwarf sequence is the Transition to Complete Convection (TTCC), also called the M4 Transition. Stellar models suggest that from M0 to about M4 $\left(\sim 0.25 \mathrm{M}_{\odot}\right)$, the radiative core of M dwarfs shrinks from $\sim 0.5 \mathrm{R}_{\star}$ to essentially zero (e.g. Dorman et al. 1989, Chabrier \& Baraffe 1997). Beyond this point, M dwarf interiors are fully convective. Additionally, differential rotation decreases sharply to about 10 times less than solar shear, enabling the star to rotate as a solid body (Donati et al. 2014). This transition is a fundamental divide between the latest dwarfs and much of the rest of the main sequence. Interestingly, some properties of M dwarfs appear unaffected by the TTCC. For example, the radius-luminosity relationship varies smoothly and gradually over the entire sequence. On the other hand, as a function of temperature, the radius drops sharply across the transition. Mid M dwarfs at about 3300 K can have a wide range of radii (from $\sim 0.25 \mathrm{R}_{\odot}$ to $\sim 0.5 \mathrm{R}_{\odot}$ ). Magnetic activity as measured by $\mathrm{H} \alpha$ emission and X-ray emission seems to be unaffected by the transition, but direct observations of large-scale magnetic topology show a clear distinction between earlyand late-M dwarfs. Magnetic dynamo theories also suggest that the kind of dynamo that operates in the Sun (and in stars down to the TTCC), cannot be present in fully convective stars. The long rotation period of the Sun can be explained by magnetic braking, the process by which the stellar magnetic field dissipates angular momentum through interactions with the stellar wind. The extent of the braking depends on the strength and morphology of the magnetic field. Therefore, the difference in magnetic field generation across the TTCC ought to be reflected in the timescale over which the stars spin down. It is hypothesized that fully convective M dwarfs, are less able to efficiently shed angular momentum through magnetic braking, and therefore spin down more slowly over time (Stassun et al. 2011). In the following chapter, I will review our understanding of stellar rotation at the bottom of the main sequence, and sketch out in more detail the interplay between stellar structure, magnetic fields, and rotation.

## Chapter 2

## Stellar Rotation

Measuring stellar rotation is a powerful way to probe many astrophysical phenomena. Due to conservation of angular momentum, stars are born rotating. The initial stellar angular momentum and the rate at which it is transferred to the surrounding medium are closely tied to many astrophysical processes such as the outflow of stellar material; light element abundances (Barnes, 2003); the magnetic dynamo mechanism; and the strength, morphology and lifetime of the stellar magnetic field (Bouvier, 2013). The subsequent evolution of stellar angular momentum is thought to be governed by interactions between a star's magnetic field, winds, and the ambient environment. Stellar rotation is a directly-observable quantity that provides a window into the evolution of stellar structure, magnetic fields, and the interactions between those fields and the relatively cool atmospheres of low-mass stars.

Rotation also connects to fundamental, but not directly observable, quantities such as stellar mass and age. Though the evolution of angular momentum is a complex and not fully-understood process, it is somewhat predictable for mature main sequence stars that are approximately Sunlike $\left(M_{\star} \sim 0.5-1.1 M_{\odot}\right)$. This has lead to the development of 'gyrochronology', a procedure by which ages for individual field stars can be estimated from their color (mass) and rotational velocity (Barnes, 2003)

### 2.1 Observational Techniques

Several methods can be used to measure stellar rotation, with different methods applicable in different areas of parameter space. I will briefly mention two techniques that are beyond the scope of this work. The first is interferometry. Using interferometry, it is possible to resolve bright, nearby stars (e.g. Kervella et al. 2004, Monnier et al. 2007). A rapidly rotating star will contract along the rotation axis and expand at the equator and, due to the differences in surface gravity, will be brighter at the poles than at the equator. While this method is only applicable to a small number of stars, it is powerful; a fully reconstructed surface brightness map can reveal the equatorial velocity, the angle of the rotation axis projected along our line of sight (i.e. inclination) and on the sky (i.e. position angle).

The second technique is seismology, by which rotation (and internal structure) can be inferred from the periodic pulsation of a star. This technique requires long-term, continuous light curves, which are available for thousands of stars thanks to the Kepler mission. This method can measure rotational velocity, differential rotation, and inclination angle.

### 2.1.1 Photometric monitoring

For spotted stars that may exhibit photometric variability related to rotation, photometric rotation periods can be inferred. For example, given the excellent photometric precision of the Kepler satellite, rotation periods for thousands of stars have been measured (e.g., McQuillan et al. 2014). This method is, depending on the time baseline and sampling of the photometry, sensitive to rotation on a wide range of timescales, but is not effective for stars that lack spots. At the same time, spot evolution on timescales that are short compared to the rotation period may lead to ambiguity in the estimated rotation periods.

### 2.1.2 Spectroscopy

The projected rotational velocity, $v \sin i$, can be inferred from the analysis of line broadening in high-resolution spectra. A broadened line profile is equivalent to the convolution of the intrinsic profile and a rotational broadening kernel. In the Fourier domain, this convolution becomes a product, and so the Fourier transform of the broadening kernel can be isolated. The Fourier transform of the rotational broadening kernel has zeros at frequencies inversely proportional to $v \sin i$ (e.g., Dravins et al. 1990, Díaz et al. 2011). Estimating $v \sin i$ from the Fourier transform of line profiles requires isolated lines and high-resolution, high signal-to-noise spectra. Additionally, this approach is best-suited for use with rapid rotators $\left(v \sin i \gtrsim 30 \mathrm{~km} \mathrm{~s}^{-1}\right)$ as the first zero of the Fourier transform of the broadening kernel can be obscured by high-frequency Fourier noise at lower $v \sin i$ Bouvier, 2013).

Alternatively, $v \sin i$ can be measured using the cross-correlation method (e.g., Delfosse et al. 1998, Reiners et al. 2012, Houdebine \& Mullan 2015). Here, a star known to be rotating slowly is used as a template. The template star is broadened at various values of $v \sin i$, and the broadened copies are cross-correlated with the unbroadened template. The widths of the resulting cross-correlation function peaks provide a calibrated measure of $v \sin i$. Observed spectra are then cross-correlated with the template, and their $v \sin i$ is inferred from the width of the crosscorrelation peak. This method requires template stars (known to be rotating slowly) of the same spectral type as the observed stars.

Perhaps most straightforward is the template-fitting technique (e.g., Jenkins et al.|2009, Passegger et al. 2016, Gilhool et al. 2018, Rajpurohit et al. 2018), in which $v \sin i$ is determined through the direct, pixel-by-pixel fitting of theoretical template spectra to observed spectra. Typically, a suite of stellar models is fit to the observed spectrum. The rotational broadening is simulated by the convolution of the templates with a broadening kernel, and the best-fit model yields the $v \sin i$ measurement. This method is limited by its dependence on theoretical spectra, which may
introduce (especially in the case of $M$ dwarfs) biases related to systematic differences between stellar lines and lines in the theoretical templates.

Finally, we developed a novel, data-driven technique for rapidly estimating $v \sin i$, described in Chapter 4

### 2.2 Rotational Evolution in Low-Mass stars

The evolution of stellar angular momentum is governed by a complex and recursive process. Stellar rotation and the internal dynamics of the star generate the magnetic field, which in turn influences the rotation rate and stellar structure, which in turn re-generate the magnetic field. While our understanding has improved greatly over the past century, the complicated nature of all the processes involved makes it difficult to construct models which can produce the diversity of observed rotation rates, magnetic field properties, etc. as a function of stellar mass, age, and which agree with our detailed observations of the Sun.

### 2.2.1 Average Rotation Rates Across the Main Sequence

The average rotation rate of stars is known to vary across the main sequence, reflecting important differences in stellar structure and magnetic field properties. In the middle of the main sequence, roughly between spectral types mid-F and mid-K, stars are found to be rotating slowly. Sun-like field stars typically rotate at velocities of $v_{\text {rot }} \lesssim 5 \mathrm{~km} \mathrm{~s}^{-1}$. These stars primarily spin down through magnetic braking. The magnetic field, generated at the interface between the star's radiative core and its convective envelope, interacts with the stellar wind and surrounding medium, and causes the star to lose angular momentum at a rapid rate. At either end of the main sequence, however, there is a sharp increase in the average rotation rate (Stauffer \& Hartmann, 1986). Stars more massive than $\sim 1.3 \mathrm{M}_{\odot}(>\mathrm{F} 5)$ lack an outer convective envelope, and therefore do not produce surface magnetic fields of sufficient extent to efficiently brake the star (Schatzman, 1962). The
mean rotation rate for massive stars is on the order of $150 \mathrm{~km} \mathrm{~s}^{-1}$. Meanwhile, at the bottom of the main sequence, late $M$ dwarfs and brown dwarfs are thought to rotate rapidly due to being fully convective. Lacking the radiative core, these stars are thought to produce topologically distinct magnetic fields, which similarly may not be able to efficiently dissipate angular momentum (Stassun et al. 2011, Bouvier 2013, Houdebine et al. 2017).

### 2.2.2 Magnetic Fields

Essentially all young low-mass stars have magnetic fields (Donati 2013). Low-mass stars can sustain long-lived magnetic activity (as traced by $\mathrm{H} \alpha$ emission), with activity lifetimes of nearly 10 Gyr for late M dwarfs. Magnetic fields profoundly affect much of the stellar evolution, influencing the collapse of the molecular cloud, the evolution of the protostellar disk, protostar and protoplanetary system, the stellar structure (i.e. convection zone geometry), internal angular momentum transport (e.g. by coupling the core and convection zone), and late-time rotational evolution through magnetic braking. The magnetic field can modify convective motions, which are partially responsible for re-generating the magnetic field. This interplay manifests in different magnetic field morphology in high- and low- activity regions (Donati, 2013).

The connection between stellar magnetic fields and stellar activity (e.g. flares, spots, prominences) is clear and has been well-observed in the Sun. The solar dynamo is thought to concentrate at the tachocline, the thin interface region between the radiative core and outer convective envelope. The rotational gradient is at a maximum at the tachocline. The differential rotation shears large-scale magnetic topology. The Sun's magnetic field is generated by a so-called $\alpha \Omega$ dynamo. The $\Omega$ effect twists the magnetic field lines of an initially poloidal field into a primarily toroidal field. The $\alpha$ effect in turn twists the toroidal field, generating poloidal loops, but with the opposite polarity of the original field. The Sun displays poloidal surface magnetic fields, and presumably has toroidal fields at the base of the CZ , which dictate the non-stochastic arrangement of sun spots (Donati, 2013). The 'interface' dynamo of the Sun appears to be characteristic of solar- and
late-type stars (Barnes, 2003) ( $\left.\sim 0.3-1.3 \mathrm{M}_{\odot}\right)$. Such magnetic fields appear to efficiently couple the convection zone with the outer environment (Kawaler, 1988), allowing the star to efficiently dissipate angular momentum via the stellar wind (Barnes, 2003). The Sun, at an age of 4.5 Gyr rotates slowly, with a period of approximately 28 days.


Figure 2.1 Figure 1 from Sanchez et al. (2014). Illustration of the $\alpha \Omega$ dynamo. An initially poloidal field (a), is converted to a toroidal field via the $\Omega$ effect (b) and (c). The $\alpha$ effect, shown in (d), (e) and (f), twists the loops of magnetic flux, producing a net poloidal field of opposite polarity in (g). The lower branch, shown in (h), (i) and (j), illustrates the contribution from the Babcock-Leighton effect, whereby buoyant magnetic flux tubes rise to the surface, forming bipolar sunspots. The bipolar regions connect, and are advected by meridional flows, generating a large-scale poloidal field.

### 2.2.3 Magnetic Braking

The basic idea behind magnetic braking is that the ionized stellar wind will remain coupled to the magnetic field out to a radius where the magnetic tension can no longer compensate for the effect of the Coriolis force. The angular momentum of the material that travels beyond this point will be lost to the system, allowing the star to spin down (Kawaler, 1988). This distance where the magnetic energy density equals the kinetic energy density is called the Alfvèn radius,

$$
\begin{equation*}
r_{\mathrm{A}} \approx \frac{B^{2}}{16 \pi \Omega v \rho} \tag{2.2.1}
\end{equation*}
$$

where $B$ is the magnetic field strength, $\Omega$ is the angular velocity, $v$ is the velocity of the poloidal wind, and $\rho$ is its density. Magnetic braking can powerfully slow stellar rotation, spinning down stars like the Sun on scales of about 1 Gyr.

Based on observations of the Sun and of stars in the Pleiades and Hyades clusters, Skumanich (1972) posited a relationship between rotational velocity and stellar age,

$$
\begin{equation*}
v_{e q} \propto \tau^{-1 / 2} \tag{2.2.2}
\end{equation*}
$$

Weber \& Davis (1967) expressed the angular momentum loss rate as,

$$
\begin{equation*}
\frac{d J}{d t}=\frac{2}{3} \Omega \dot{M} r_{\mathrm{A}}^{2}=\frac{J}{\tau_{\mathrm{w}}} \tag{2.2.3}
\end{equation*}
$$

where $\dot{M}$ is the mass-loss rate and $\tau_{\mathrm{W}}$ is the braking timescale. Using some unrealistically simple assumptions, this can be rewritten as,

$$
\begin{equation*}
\frac{d J}{d t} \propto \Omega^{3}=I \frac{d \Omega}{d t} \tag{2.2.4}
\end{equation*}
$$

which asymptotically integrates to the Skumanich law in Equation 2.2.2. Kawaler (1988), expanding on work done by Mestel (1984), developed a parameterized model for the magnetic braking of Sun-like stars, which incorporated a term describing the geometry of the magnetic field. The Kawaler prescription was also tested on models of stars rotating as solid bodies, and models in which the core and convective envelope were decoupled.

Solar-type stars exhibit an angular velocity gradient, in which the core rotates more quickly than the outer layers. The models of Endal \& Sofia (1981), for example, produce this behavior because hydrodynamic transport of angular momentum in the core is much slower than it is in the convective zone. Furthermore, Stauffer et al. (1984) argued that magnetic braking may initially only slow the outer convective envelope. Consequently, angular momentum could be "hidden" in
the core and not reflected in the observed rotation speed, until the core and envelope eventually couple. This seems to fit with the observation that, below F5, more massive stars spin down earlier, but less massive stars (at least down to early $M$ types) eventually reach lower rotational velocities.

Still, the Kawaler models produced braking that was too strong to match observations. This lead to the hypothesis that above some threshold of rotation, the magnetic field saturates, and the star does not spin down as quickly.

### 2.2.4 Magnetic Field Observations

Astronomers use a variety of methods to probe stellar magnetic fields. We can detect magnetic fields using spectroscopy or spectropolarimetry, and observing the effect of the magnetic field on the atomic and molecular energy levels. The magnetic field splits atomic and molecular energy levels (the Zeeman Effect), broadening spectral lines. Direct estimates of magnetic fields in stars other than the Sun were first made by measuring the differential broadening of spectral lines as a function of their magnetic sensitivities (see Robinson et al. 1980). Such observations of Zeeman broadening probe the small-scale magnetic field, which encompasses most of the magnetic energy. More recently, precise spectropolarimetry has allowed for the probing of the medium- and largescale magnetic fields through Zeeman Doppler Imaging (ZDI) (e.g. Donati et al. 1992, Donati et al. 2006). The topology of the medium and large-scale magnetic fields is obtained though tomographic modeling of a time-series of Zeeman signatures. While the medium- and large-scale magnetic fields enclose less energy than the small-scale fields, these observations are crucial for checking the topological predictions of dynamo models (Donati, 2013).

One surprising result that came out of ZDI observations was that strong toroidal fields were discovered at the surface of stars rotating in the saturation regime. Later studies found such fields even in relatively slow rotators. Solar-type stars with periods as long as $\sim 20$ days were found to have surface toroidal fields (Dunstone et al. 2008, Petit et al. 2005, Marsden et al. 2006). This
result was unexpected because the Sun does not exhibit surface toroidal fields, and lead to the idea that dynamo processes may not be confined to the tachocline, but rather distributed throughout the convection zone (see Brandenburg 2005 for sun, Lites et al. 2008).

Other measurements of field strength include X-ray emission, which results from coronal heating, and certain chemical tracers, such as $\mathrm{H} \alpha$ and ionized Ca (Ca II), which trace chromospheric heating.

Many M dwarfs display magnetic activity. The magnetic activity lifetime is a function of stellar mass, with smaller stars staying active for longer. The eventual spin-down of the star is interpreted as the mechanism by which the magnetic activity ceases (Newton et al. 2017). Using a sample of M dwarfs from the MEarth survey, West et al. (2015) found that the fraction of active M dwarfs, and the strength of the activity, decreased with decreasing rotation rates. Late M dwarfs were found to be active out to longer periods, before declining sharply. This, along with additional evidence from MEarth (Irwin et al. 2011, Newton et al. 2016a, Newton et al. 2017), is consistent with the idea that magnetic field strength saturates above some critical rotational velocity. Additionally, Newton et al. (2017) found a mass-dependent threshold in rotation period, beyond which no $\mathrm{H} \alpha$ emission is measured. Interestingly, this threshold increases sharply at masses corresponding to the TTCC. Late K and early M dwarfs were active up to periods of about 10 and 20 days, respectively; mid M dwarfs $\left(\sim 0.3 \mathrm{M}_{\odot}\right)$ were active up to periods of between 20 and 50 days; and late M dwarfs $\left(\lesssim 0.15 \mathrm{M}_{\odot}\right.$ were active up to periods of 80 days or more.

Barnes (2003), found similar rotational behavior in observations of cluster stars. Laying the groundwork for gyrochronology, Barnes (2003) posited that low-mass stars occupy two distinct rotation sequences, starting on the ' C ' (or convective field) sequence, and ending on the ' I ' (or interface sequence). In this picture, the convective fields are weak and disordered, lacking largescale fields, and inefficient at slowing the stars' rotation. As the star evolves, differential rotation increases, eventually turning on the interface field, which couples the core, convective envelope, and stellar wind. This interpretation fit the observed features of the cluster data. At later times
and earlier spectral types, stars were more and more likely to have long periods and exist on a well-defined mass-age-rotation relation. A small number of stars were found with intermediate rotations, suggesting a rapid spin-down from C sequence to the I sequence. While a rapid spindown appears likely, in terms of magnetic topology, this description is in conflict with later ZDI observations.

The overall picture revealed by ZDI observations shows that moderately rotating ( $R_{o} \lesssim 1$ ), earlier low-mass stars $\left(M_{\star}>0.5 M_{\odot}\right)$, have substantial large-scale toroidal surface fields, with non-axisymmetric poloidal components. On the other hand, mid-to-late M dwarfs ( $M<0.5 M_{\odot}$ ) that rotate quickly ( $R_{o} \lesssim 0.1$ ), seem to generate large-scale fields that are mostly poloidal and axisymmetric. Finally, at very low mass $\left(M \lesssim 0.2 M_{\odot}\right)$, and very rapid rotation $\left(R_{o} \sim 0.01\right)$, stars seem capable of generating either strong, axisymmetric poloidal fields, or weak, non-axisymmetric toroidal fields (Donati, 2013). This may point to a bistable dynamo (Morin et al. 2010), such as has been proposed for planetary dynamos (Morin et al. 2011, Gastine et al. 2013).

On either side of the TTCC, M dwarfs with similar rotational velocity show similar X-ray luminosity, suggesting that the global magnetic field strength is unaffected by the structural transition to full convection Donati, 2013). On the other hand, the large-scale topology revealed by ZDI observations show a clear discontinuity across the same boundary. This suggests, contrary to dynamo models and earlier theory, that fully convective dynamos are much more efficient at producing large-scale, axisymmetric poloidal fields (Donati, 2013).

### 2.2.5 Very low mass models

The large-scale magnetic topology of low-mass stars varies widely with mass and rotation rate, as shown by ZDI observations. The field strength of typical stars is approximately equal to the equipartition field strength, but very active stars, such as fully-convective $M$ dwarfs and young low-mass stars show clear deviations from that relation. At later spectral types in the M dwarf sequence, the radiative core shrinks from $\sim 0.5 R_{\star}$ until it becomes vanishingly small at roughly
$0.3 M_{\odot}$. This transition coincides with a strong decrease in differential rotation (roughly 10 times smaller than Solar shear), and with a sharp change in magnetic field strength and morphology, although Donati et al. (2008) found the transition at slightly higher masses $\left(0.4-0.5 M_{\odot}\right)$. The lifetime of magnetic fields in this regime also increases. Many models of fully convective dynamos have been proposed, but much uncertainty remains. Such models included dynamos that generate small-scale fields only (e.g. Durney et al. 1993), non-axisymmetric large scale fields when the convection zone rotates as a solid body (Chabrier \& Küker, 2006), axisymmetric poloidal fields in the presence of significant differential rotation (Dobler et al., 2006), but differential rotation is virtually non-existent for low $\mathrm{R}_{o}$ and fully-convective stars (Browning, 2008). While these models show a variety of plausible dynamo mechanisms, they failed to produce the observed strong dipolar fields in solid-body rotators (Donati, 2013).

### 2.2.6 Schematic of Rotational Evolution

In general, stars spin up as they contract and fall onto the main sequence. Naively, the conservation of angular momentum dictates that all stars would be rapidly rotating at birth. This turns out not to be the case. Observations of clusters show that low-mass stars $\left(\sim 0.5-1.1 \mathrm{M}_{\odot}\right)$ are formed and arrive on the zero-age main sequence (ZAMS) with a wide distribution of rotation speeds, which are on average much lower than expected. The leading theory is that magnetic interactions between the protostar and protostellar disk are responsible for the unexpected slow rotation (see Camenzind 1990, Koenigl 1991 and Edwards et al. 1993). The reasoning behind this theory is based on observations of the classical T Tauri stars (cTTSs). The cTTSs are pre-main sequence stars with accretion disks, and are found to be rotating more slowly than their disk-less counterparts, despite the fact that they are accreting high specific angular momentum material. The accretion disks are often spatially associated with jets that can only be explained by the acceleration of material due to magnetic fields. While this explanation is plausible, models have not yet shown that magnetic fields can realistically slow such stars powerfully enough to counteract the spin-up
due to both contraction and accretion (Donati, 2013).
In any case, the rotation of pre-main sequence low-mass stars is powerfully restrained. After the circumstellar disk is dissipated, these stars do spin up as they continue to contract onto the ZAMS. Once on the main sequence, magnetic braking appears to be the dominant mechanism of angular momentum loss. The large dispersion in rotational velocity is erased on timescales of a few 0.1 Gyr for Sun-like stars, up to timescales of $\sim 1 \mathrm{Gyr}$ for K dwarfs. Observations of stars in open clusters, along with studies of magnetic activity traced by $\mathrm{H} \alpha$ emission, suggest the initial spin-down is rapid, and occurs earlier for more massive stars. After the initial spin-down, these stars exist on a well-defined mass-rotation relation (Bouvier, 2013). The gradual evolution of stars on this relation admits the possibility of "gyrochronology" suggested by Barnes (2003), whereby stellar age can be estimated from the present-day rotational velocity.

Gyrochronology, however, is not currently applicable to very low-mass stars. Low-mass stars arrive on the ZAMS much later than solar-type stars, exhibit a wider distribution of rotation speeds, and a longer spin-down timescale. The observed wide distribution of rotation in mature populations of late $M$ dwarfs may be due to the fact that, at later types, the convective envelope accounts for a greater fraction of the star's moment of inertia. Indeed, past the TTCC, the entire star is convective, and is essentially a solid-body rotator.

Another contributing factor is the saturation of the rotation-activity relation. Activity scales with rotation rate and later spectral type, up to a point where the magnetic field strength saturates (see eg Hall 2008). Even more tightly correlated with activity than rotation rate is the Rossby number (see eg Noyes et al. 1984). The Rossby number is the ratio of the rotation period to the convective turnover time, which indicates the something about differential rotation vs convective eddies. Magnetic activity increases with decreasing Rossby number. The convective turnover time increases at later spectral types, so $R_{\mathrm{o}}$ (at a fixed rotation rate) decreases with later spectral type, suggesting that later types are more active. The increase in magnetic flux with $\frac{1}{R_{\mathrm{o}}}$ is generally attributed to the increase in the fractional area of the stellar surface covered with magnetic fields.

The average magnetic flux at the surface of cool stars generally increases with $\frac{1}{R_{\mathrm{o}}}$ until saturating at $R_{\mathrm{o}} \sim 0.1$. This corresponds to a period of 2 days for a Sun-like star, or $6-10$ days for fully convective M dwarfs (Donati, 2013). Therefore, the latest stars may remain in the saturated regime for much longer than earlier types, spinning down slowly. Then, when the star eventually exits the saturation regime, it spins down rapidly.

Finally, it may be the case that the magnetic fields of late $M$ dwarfs are simply unable to produce the powerful magnetic braking of earlier types. The ZDI observations previously discussed show a surprising diversity of magnetic field topology in late $M$ dwarfs. The strong, axisymmetric fields should seemingly be able to produce strong braking, while the weak, disordered fields may not. Because the rapidly rotating late $M$ dwarfs can apparently host fields of either type, it is unclear whether or not this is the case.

### 2.3 Our Contribution

The complementary contributions of modeling and observation, presented above, form a somewhat dissonant picture of stellar angular momentum evolution at the bottom of the main sequence. The underlying physics governing the evolution of stellar angular momentum are incredibly complex, and poorly constrained. As such, we are limited by a lack of observational data. In the following chapters, I will present our contribution to this endeavor. In Chapter 3, I present our $v \sin i$ measurements for a large sample of APOGEE M dwarfs. We added a large number of $v \sin i$ estimates, which lend statistical power to the investigation of M dwarf rotation across the TTCC. In Chapter 4. I present our work in developing a novel data-driven technique for estimating $v \sin i$. As the next generation of surveys promises to deliver massive amounts of data, this efficient technique could be a valuable tool for rapidly for large numbers of stars.

## Chapter 3

## Rotational Velocities of APOGEE

## M Dwarfs

This chapter ${ }^{1}$ details an analysis of the rotation of more than 700 M dwarfs observed as part of the The Apache Point Observatory Galactic Evolution Experiment (APOGEE; Majewski et al. 2017) M Dwarf Survey. This Sloan Digital Sky Survey (SDSS) ancillary science program (Deshpande et al. 2013) was carried out as part of SDSS III (Eisenstein et al. 2011). This data set represents the largest high-resolution spectroscopic survey of M dwarfs to date, which lends unprecedented statistical power to the study of M dwarfs and their fundamental properties. In Section 3.1 we describe the APOGEE data set and the selection criteria used to generate the M star sample. In Section 3.2 we describe our template fitting approach to determining $v \sin i$, as well as the limitations of this technique, and compare our results to those in the literature. In Section 3.3 we compare the distribution of projected rotation velocity to that inferred from photometric rotation periods published in the literature. Finally, in Section 3.4 we examine the possible implications of our results and summarize our findings.

[^2]
### 3.1 Data Selection

We analyzed APOGEE spectra from SDSS data release 13 (SDSS Collaboration et al., 2016), observed using the SDSS main 2.5 m telescope (Gunn et al. 2006). The APOGEE spectrograph is a multiplexed, cryogenic, high-resolution $(R \approx 22,000)$ fiber-fed instrument. It covers the H-band $(\lambda=1.514 \mu \mathrm{~m}-1.696 \mu \mathrm{~m})$ across three near infrared detectors; blue $(\lambda=1.52 \mu \mathrm{~m}-1.58 \mu \mathrm{~m})$, green $(\lambda=1.59 \mu \mathrm{~m}-1.64 \mu \mathrm{~m})$, and red $(\lambda=1.65 \mu \mathrm{~m}-1.69 \mu \mathrm{~m})$ Wilson et al. 2010; Skrutskie \& Wilson 2015). The APOGEE data pipeline produces a range of spectral products that correct for the effects of atmospheric emission and absorption and also combine spectra obtained at different epochs into single, high signal-to-noise stellar spectra in the rest frame of the star (Nidever et al. 2015). In this work, we analyzed apStar spectra, which are weighted combinations of multiple spectra of each star gathered over different epochs. For all APOGEE targets that were observed more than once, the apStar files contain two coadded spectra - one is generated using a pixel-based weighting scheme, and the other is generated using a global weighting scheme. In the pixel-based weighting scheme, the $i^{\text {th }}$ pixel in the coadded spectrum is the signal-to-noise-per-pixel weighted combination of the $i^{\text {th }}$ pixels of the individual spectra. In the global weighting scheme, the coadded spectrum is the total signal-to-noise-per-spectrum weighted combination of the individual spectra. In most cases, there is little difference between the two. All of the stars in our sample were observed at least twice, so we used the apStar spectra with the pixel-based weighting scheme and found no measurable difference in our results when instead using the coadded spectra with the global weighting scheme.

We also utilized data from the APOGEE Stellar Parameters and Chemical Abundance Pipeline (ASPCAP) (García Pérez et al., 2016). Each APOGEE target has a corresponding aspcapStar file, which contains a pseudo-continuum normalized spectrum, along with parameters output from the ASPCAP pipeline. The ASPCAP pipeline produces $T_{\text {eff }},[\mathrm{M} / \mathrm{H}], \log g$, and $v \sin i$ measurements, in addition to estimates of up to 15 chemical abundances for most APOGEE stars. The ASPCAP
spectral libraries cover a wide range of temperatures and chemical compositions for both dwarf and giant stars. Of interest to our work are the GK dwarf grid ( $3500 \mathrm{~K} \leq T_{\text {eff }} \leq 6000 \mathrm{~K}$ ) and the M dwarf grid ( $2500 \mathrm{~K} \leq T_{\text {eff }} \leq 4000 \mathrm{~K}$ ), which was added in Data Release 13. The GK dwarf grid uses ATLAS9 models, while the M dwarf grid uses MARCS models (Mészáros et al., 2012). In order to preserve continuity in the derived parameters of the GK grid, the ATLAS9 models are used in the overlapping temperature range ( $3500 \mathrm{~K} \leq T_{\text {eff }} \leq 4000 \mathrm{~K}$ ). This, however, produces a discontinuity in the ASPCAP parameters for the M dwarf sequence $\left(2500 \mathrm{~K} \lesssim T_{\text {eff }} \lesssim 4000 \mathrm{~K}\right)$. Applying a consistent suite of models, targeted specifically at measuring M dwarf parameters, was a primary motivation for this work.

In this analysis of M dwarf rotation, we made use of the ASPCAP $T_{\text {eff }}$ measurements. We tested the ASPCAP $T_{\text {eff }}$ measurements against $T_{\text {eff }}$ estimates based on data from the NASAInfrared Telescope Facility (IRTF) SpeX Spectrograph Terrien et al., 2015). These estimates are based on K-band $\mathrm{H}_{2} \mathrm{O}$-index relations from Mann et al. (2013) for $T_{\text {eff }}>3300 \mathrm{~K}$, and V-K colortemperature relations for $T_{\text {eff }}<3300 \mathrm{~K}$. Generally, the temperature data are consistent, although there appears to be a constant offset between the data sets below $T_{\text {eff }}<3300 \mathrm{~K}$, where the V-K color-temperature relation is used instead of the IRTF spectra. As a secondary check, we compared our rough $T_{\text {eff }}$ results from template-fitting (described in Section 3.2 to the ASPCAP $T_{\text {eff }}$ Even though we use a completely different suite of theoretical stellar models, and our technique is not designed to measure $T_{\text {eff }}$ precisely, the temperature data is broadly consistent at the $\sim 100 \mathrm{~K}$ level (see Figure 3.1). In the absence of a de-facto standard for measuring the temperatures of cool M dwarfs, we adopt the ASPCAP $T_{\text {eff }}$ values.


Figure 3.1 Left: Comparison of ASPCAP-pipeline $T_{\text {eff }}$ to values based on IRTF SpeX spectra, using $\mathrm{H}_{2} \mathrm{O}$-index relations for stars with $T_{\text {eff }}>3300 \mathrm{~K}$ (Mann et al., 2013), and a V-K colortemperature relation for $T_{\text {eff }}<3300 \mathrm{~K}$. There appears to be a constant offset between the ASPCAP $T_{\text {eff }}$ and the V-K color-derived $T_{\text {eff }}$. The RMS of the residuals between the ASPCAP $T_{\text {eff }}$ and $T_{\text {eff, IRTF }}>3300 \mathrm{~K}$ is 70 K . The RMS of the residuals between the ASPCAP $T_{\text {eff }}$ and $T_{\text {eff, IRTF }}<$ 3300 K is 73 K . Right: Comparison of $T_{\text {eff }}$ estimates from the ASPCAP pipeline and the best-fit $T_{\text {eff }}$ from template-fitting. The points and error bars represent the mean and standard deviation of the estimated $T_{\text {eff }}$ in each ASPCAP $T_{\text {eff }}$ bin. Our template-fitting procedure is not optimized to measure temperature, nor do we interpolate between the grid points, which are in increments of 100 K . Nevertheless, the data agree at the $\sim 100 \mathrm{~K}$ level.

We chose our sample starting with the 1350 M Dwarfs observed as a part of the APOGEE M Dwarf Survey, demarcated by bit 19 in the APOGEE_TARGET1 flag. We then made cuts, requiring that:

1. $2600 \mathrm{~K} \leq T_{\text {eff }} \leq 4000 \mathrm{~K}$ (excluded 114 stars)
2. $\mathrm{SNR} \geq 50$ (excluded 14 stars)
3. The star did not fail in the ASPCAP pipeline. The ASPCAP pipeline logs various warnings and flags in the APOGEE_ASPCAPFLAGS field of the ASPCAP headers. We cut all spectra with the 'STAR_BAD' flag, which is triggered by most failure modes (excluded 508 stars).

The final sample consists of 714 stars. The number of stars, range of magnitudes, and SNR per $T_{\text {eff }}$ bin are shown in Table 1

Table 1. Description of APOGEE M Dwarf Sample

| $T_{\text {eff }}$ <br> $(\mathrm{K})$ | N | H <br> $(\mathrm{mag})$ | J <br> $(\mathrm{mag})$ | K <br> $(\mathrm{mag})$ | $\mathrm{SNR}^{\mathrm{a}}$ |
| :---: | :--- | :---: | :---: | :---: | ---: |
| $2600-2700$ | 5 | $10.4-11.8$ | $11.1-12.4$ | $10.0-11.4$ | $108-217$ |
| $2700-2800$ | 9 | $7.19-12.5$ | $7.79-13.1$ | $6.85-12.1$ | $65.0-567$ |
| $2800-2900$ | 7 | $10.5-12.4$ | $11.1-13.0$ | $10.2-12.1$ | $58.4-264$ |
| $2900-3000$ | 12 | $9.49-12.5$ | $10.0-13.1$ | $9.17-12.2$ | $50.9-589$ |
| $3000-3100$ | 35 | $9.16-13.2$ | $9.75-13.8$ | $8.89-13.0$ | $51.1-493$ |
| $3100-3200$ | 58 | $8.35-12.3$ | $8.87-12.9$ | $8.05-12.0$ | $51.5-773$ |
| $3200-3300$ | 106 | $8.05-12.3$ | $8.59-12.9$ | $7.77-12.0$ | $50.5-1470$ |
| $3300-3400$ | 139 | $7.03-12.4$ | $7.58-12.9$ | $6.81-12.2$ | $64.1-985$ |
| $3400-3500$ | 105 | $7.56-12.0$ | $8.12-12.6$ | $7.32-11.8$ | $50.3-1390$ |
| $3500-3600$ | 72 | $7.12-11.9$ | $7.70-12.5$ | $6.89-11.7$ | $54.8-1740$ |
| $3600-3700$ | 70 | $7.03-12.0$ | $7.57-12.5$ | $6.77-11.7$ | $50.9-1100$ |
| $3700-3800$ | 43 | $7.32-11.8$ | $7.89-12.4$ | $7.09-11.6$ | $58.0-1150$ |
| $3800-3900$ | 41 | $7.26-11.6$ | $7.85-12.3$ | $7.04-11.5$ | $57.5-1170$ |
| $3900-4000$ | 14 | $7.37-11.8$ | $7.96-12.5$ | $7.18-11.7$ | $146-1190$ |
|  |  |  |  |  |  |
| $2609-3999$ | 714 | $7.03-13.2$ | $7.57-13.8$ | $6.77-13.0$ | $50.3-1740$ |

${ }^{\text {a }}$ According to the APOGEE documentation, due to unquantified systematic errors, the maximum SNR is likely limited to $\sim 200$. The SNR quoted here is based on the statistical flux error estimates, which do not necessarily reflect the SNR limit.

### 3.2 Method

### 3.2.1 Overview of $v \sin i$ measurement techniques

There are two primary methods by which $v \sin i$ is measured: the cross-correlation technique, and the template-fitting technique (see Section 2.1.2). We tested the cross-correlation technique, but our sample does not include known slow rotators at the lowest temperatures. We experimented with using theoretical templates instead of empirical onces, but our results were dominated by systematic disagreement between spectral features in the templates and those in the observations.

Instead, we used the template-fitting technique, the results of which, we denote by 'VFIT.' We make a direct, pixel-to-pixel comparison between a high signal-to-noise spectrum and a library of theoretical templates spanning a wide range of stellar parameters. The template is convolved with a rotational broadening kernel, and the kernel that produces the best fit to the data constitutes the measured value of $v \sin i$. This technique is limited by the resolution of the data ( $\mathrm{R} \approx 13 \mathrm{~km} \mathrm{~s}^{-1}$ for APOGEE) and also by systematic differences between spectral features found in the star and those in the spectral library.

### 3.2.2 Overview of template-fitting technique

Our approach to measuring $v \sin i$ was to forward-model the APOGEE apStar spectra using a library of theoretical templates, broadened by a theoretical broadening kernel to account for rotation. We used BT-Settl model spectra, calculated using the PHOENIX code Allard et al. 2012 Baraffe et al. 2015), for a grid of model parameters shown in Table 2. These spectr22 were calculated at a wavelength grid spacing of $0.02 \AA$, and have an output grid spacing of $0.2 \AA$. The templates with $[\mathrm{M} / \mathrm{H}] \geq 0$ are not alpha-enhanced, templates with $[\mathrm{M} / \mathrm{H}]=-0.5$ have $[\alpha / \mathrm{H}]=+0.2$ and templates with $[\mathrm{M} / \mathrm{H}]<-0.5$ have $[\alpha / \mathrm{H}]=+0.4$.

We continuum normalized the apStar and PHOENIX template spectra with our own code.

[^3]Table 2. PHOENIX template grid parameters

| Quantity | Range | Increment |
| :--- | :---: | :---: |
|  |  |  |
| $\log g$ | $4.5-5.5$ dex | 0.5 dex |
| $T_{\text {eff }}$ | $2600-4000 \mathrm{~K}$ | 100 K |
| $[\mathrm{M} / \mathrm{H}]$ | -2.5 to +0.5 dex | varies |
| $v \sin i$ | $1-100 \mathrm{~km} \mathrm{~s}^{-1}$ | varies |

We fit a robust $5^{\text {th }}$-degree polynomial to the spectrum, rejecting points that deviate significantly from the estimated continuum, and then re-fit. The process goes through 10 iterations to arrive at our final estimate of the continuum. It is worth noting that the continuum level of M dwarfs is often difficult to discern, due to line blanketing. We compared our continuum-normalized apStar spectra against the continuum-normalized spectra produced by the ASPCAP pipeline and found no significant differences.

We used only the blue portion of the APOGEE spectra. This span of approximately $620 \AA$ $(15163.52 \leq \lambda \leq 15783.38 \AA)$, was chosen because it contains stellar spectral lines that tend to agree well with the theoretical spectra, and contains ample spectral information while being small enough to keep computation times reasonable.

The fitting process proceeds iteratively. In the first step, we fit each APOGEE spectrum to the full grid of spectral templates in order to determine the best-fit template. The template spectra are broadened by seven different values of $v \sin i$, ranging from $2-70 \mathrm{~km} \mathrm{~s}^{-1}$ in order to ensure that fast rotators are not misidentified. Next, we divide the APOGEE spectrum into 100pixel segments and re-fit using the best-fit template only using a more finely spaced $v \sin i$ grid. Segmenting the spectrum gives us a means of excluding sections of the spectrum which are poorly modeled by the templates, and/or are contaminated by bad pixels, bright sky lines, or telluric lines, while also yielding an estimate of our single-measurement uncertainty. Finally, we used a Gaussian Mixture Model to estimate the overall best fit $v \sin i$, properly accounting for outlier
segments and our $v \sin i$ detection limit.

### 3.2.3 Detection Limit

We estimated the $v \sin i$ detection floor based on simulations with synthetic spectra, and comparisons with literature $v \sin i$. To simulate the deteciton limit, we added Gaussian noise to our templates, convolved them with a fiducial APOGEE LSF, and degraded them to the APOGEE resolution. We artificially broadened the synthetic spectra, and then attempted to recover the $v \sin i$ using our fitting process. Given the perfect match between the stellar template and simulated spectra in this test, we were able to recover $v \sin i$ down to $v \sin i \sim 5 \mathrm{~km} \mathrm{~s}^{-1}$. However, we expect that systematic differences between the stellar templates and the actual features in the stellar spectra will degrade our ability to detect small $v \sin i$.

Deshpande et al. (2013) estimated the detection floor to be $v \sin i>4 \mathrm{~km} \mathrm{~s}^{-1}$, which is the minimum velocity at which the broadening kernel is resolved at APOGEE resolution and sampling. We adopted a more conservative detection limit here. Comparisons with a small number of available literature values of $v \sin i$ suggested that we can recover $v \sin i \sim 5 \mathrm{~km} \mathrm{~s}^{-1}$ in most cases, but setting the detection limit at $v \sin i>5 \mathrm{~km} \mathrm{~s}^{-1}$ would also yield a number of false detections. Furthermore, early results using the $v \sin i>5 \mathrm{~km} \mathrm{~s}^{-1}$ threshold showed an unexpected rising $v \sin i$ floor with decreasing temperature. At $T_{\text {eff }}<3000 \mathrm{~K}$, nearly all $v \sin i$ measurements were greater than $5 \mathrm{~km} \mathrm{~s}^{-1}$, which is statistically unlikely, given the projection effect alone. It is possible that the suite of PHOENIX models that we are using does not capture a systematic change in line width with effective temperature. For example, the model suite may not span a large enough range of surface gravity at low temperatures, resulting in artificially inflated $v \sin i$ at low effective temperature if there is a physical correlation between $\log g$ and $T_{\text {eff }}$. However, no such systematic effect has been found by other authors using PHOENIX templates Maldonado et al. 2017, Houdebine \& Mullan 2015). Tests with template grids spanning a wide range of $\log g(4.5 \leq \log g \leq 5.5)$ indicate that systematic bias of $v \sin i$ with $\log g$ is small, and does not
have a large impact on our detection threshold.
Since there are no late M dwarfs in our sample that are independently known to be nonrotating, we are unable to decisively test the detection limit as a function of effective temperature. Ultimately, we set a conservative detection limit of $v \sin i>8 \mathrm{~km} \mathrm{~s}^{-1}$, which eliminates any inconsistencies with the literature, and admits the possibility that we cannot reliably measure $v \sin i \sim 5 \mathrm{~km} \mathrm{~s}^{-1}$ for all $T_{\text {eff }}$.

### 3.2.4 The Fitting Process in Detail

The spectral fitting is performed using the IDL package mpfit, which performs a least-squares optimization Markwardt, 2009). Each fit has three free parameters:

1. A constant line depth scale factor, $k$, to account for overall systematic differences between the observed depths of the stellar lines and those in the spectral library. The relative intensity of the template at a given wavelength, $I(\lambda)$, is determined by the optical depth, $\tau$ :

$$
I(\lambda)=e^{-\tau(\lambda)}
$$

which we scale such that

$$
I(\lambda)=e^{-k \tau(\lambda)}
$$

This changes the depth of all spectral lines, while maintaining their relative opacities. Although scaling the line depths may undermine the inferred metallicity, the inclusion of this parameter significantly improved the overall quality of the fits. We tested the code without the scaling parameter and found the comparison to literature $v \sin i$ to be much worse. For the three stars with higher-resolution literature $v \sin i$ which are detectably rotating, the RMS of the residuals was $\sim 0.4 \mathrm{~km} \mathrm{~s}^{-1}$ when we used the scaling parameter, and $\sim 7 \mathrm{~km} \mathrm{~s}^{-1}$ without the scaling parameter. Furthermore, our non-detections are fully consistent with
the literature when using the scaling parameter, but several become inconsistent without the use of the scaling parameter. The Pearson, Spearman and Kendall correlation coefficients all suggest that there is no correlation between the scale parameter and $v \sin i$.
2. A constant multiplicative offset to the continuum level
3. A linear term for the multiplicative offset to the continuum level

The adjusted template is then convolved with a rotational broadening kernel whose width is determined by $v \sin i$ :

$$
G(x)= \begin{cases}\frac{2(1-\epsilon)\left(1-x^{2}\right)^{1 / 2}+\frac{\pi \epsilon}{2}\left(1-x^{2}\right)}{\pi\left(1-\frac{\epsilon}{3}\right)} & |x|<1 \\ 0 & |x|>1\end{cases}
$$

where $x=\frac{\Delta \lambda}{\lambda_{o}} \cdot \frac{c}{v \sin i}$ (see Gray 1992). The limb darkening parameter, $\epsilon$, also affects the shape of the rotational broadening kernel. We calculated $\epsilon$ using the $j k t l d$ code (Southworth, 2015) with a linear limb darkening law Claret, 2000). The $j k t l d$ grids are calculated at solar metallicity and only go up to $\log g=5.0$ so we use the same values of $\epsilon$ for the $\log g=5.0$ and $\log g=5.5$ templates. Otherwise, we construct the broadening kernel using the value of $\epsilon$ corresponding to the $T_{\text {eff }}$, and $\log g$ of the current template. These are shown in Figure 3.2 . While the Claret (2000) calculations give us $0.35 \lesssim \epsilon \lesssim 0.59$, other authors have used a fixed value of $\epsilon=0.6$ (see eg. Mohanty \& Basri 2003, Tinney \& Reid 1998). Díaz et al. (2011) caution that incorrect values of $\epsilon$ can lead to error of up to $15 \%$ in $v \sin i$. We tested our procedure using fixed values of $\epsilon=[0.4,0.6,0.8]$ and found only about a $5 \%$ difference in $v \sin i$ over that range, with lower values of $\epsilon$ tending to produce higher values of $v \sin i$. Therefore, we adopted the Claret values of $\epsilon$ and conclude that our results are relatively insensitive to $\epsilon$.


Figure 3.2 Limb darkening parameter values for our range of stellar parameters, calculated with $j k t l d$ code (Southworth, 2015), using a linear limb darkening law from Claret (2000). Values range from $\epsilon=0.35$ to 0.59 , with a mean value of $\epsilon=0.44$.

Next, we convolve the broadened template spectrum with the APOGEE Line Spread Function (LSF) in order to simulate the instrumental broadening. We use the LSF determined by the APOGEE pipeline, which is described as a set of 26 coefficients. The coefficients control the construction of the LSF from a series of Gauss-Hermite polynomials. We used the IDL code, $l s f_{-} g h$, from the SDSS idlutils packag\& ${ }^{3}$ The LSF is wavelength dependent, but only changes slightly over our spectral range (see Figure 3.3). We use the LSF corresponding to the central wavelength of the spectral range.


Figure 3.3 Example of an APOGEE LSF for a single observation. The LSFs corresponding to the central wavelength of each chip (blue, green, and red) are shown. The LSF changes slightly across the detector.

Finally, we calculate $\chi^{2}$. We use the APOGEE_PIXMASK vector (HDU 3 from the apStar

[^4]files) to mask bad pixels. We consider any pixel flagged with bits $0,1,2,3,4,5,6,12$, or 14 to be a bad pixel. We do not exclude pixels based on bit 13 , which corresponds to the flag for pixels near significant telluric features. We found no systematic biases in the analyses with and without these pixels. All flagged pixels were masked during the $\chi^{2}$ minimization process used to determine the best-fit $v \sin i$. We also mask the five pixels at either end of the spectral region to minimize edge effects resulting from convolutions in the forward modeling process.

In the first pass of the $v \sin i$ fitting process, the above fitting process is performed 1,890 times (three values of $\log g, 15$ values of $T_{\text {eff }}$, six values of $[\mathrm{M} / \mathrm{H}]$, and seven values of $v \sin i$ ) per APOGEE star. The result is a data cube with $\chi^{2}$ as a function of $\log g, T_{\text {eff }},[\mathrm{M} / \mathrm{H}]$, and $v \sin i$. At each set of $\log g, T_{\text {eff }}$, and $[\mathrm{M} / \mathrm{H}]$, we perform a cubic spline interpolation to find the minimum of $\chi^{2}$ as a function of $v \sin i$. For each star, we then take the set of $\log g, T_{\text {eff }}$, and $[\mathrm{Fe} / \mathrm{H}]$ corresponding to the global minimum of $\chi^{2}$ to be the best-fit template parameters.

In the second pass of the $v \sin i$ fitting process, $\log g, T_{\text {eff }}$, and $[\mathrm{Fe} / \mathrm{H}]$ are fixed at the previously determined best-fit values. We then re-fit each apStar spectrum by dividing the spectrum up into 28 100-pixel chunks and stepping through a fine grid of $1<v \sin i<100 \mathrm{~km} \mathrm{~s}^{-1}$ in $1 \mathrm{~km} \mathrm{~s}^{-1}$ steps. The measured value of $v \sin i$ for the segment is that which minimizes the value of $\chi^{2}$. The other details of the fitting process (pixel masking, limb darkening, LSF) are the same as in the first pass.

We use a Gaussian Mixture Model (McLachlan \& Peel, 2000) to determine the final $v \sin i$ based on the $v \sin i$ estimates from each of the individual spectral chunks. The model is the weighted combination of two Gaussians; one representing the 'true' distribution from which the measurements are drawn, and one representing some unknown mechanism responsible for producing outliers. The model parameters are then: $\boldsymbol{\theta}=\left[\alpha, \mu_{\text {true }}, \sigma_{\text {true }}, \mu_{\text {out }}, \sigma_{\text {out }}\right]$, where $\alpha$ is the probability that a data point is drawn from the true distribution, and $\mu$ and $\sigma$ are the Gaussian means and widths for the true distribution and the outlier distribution, respectively. The probability of measuring $v \sin i_{i}$, given $\boldsymbol{\theta}$ is:

$$
\begin{align*}
& p\left(v \sin i_{i} \mid \boldsymbol{\theta}\right)=\frac{\alpha}{\sigma_{\text {true }} \sqrt{2 \pi}} \exp \left[-\frac{1}{2}\left(\frac{v \sin i_{i}-\mu_{\text {true }}}{\sigma_{\text {true }}}\right)^{2}\right] \\
&+\frac{1-\alpha}{\sigma_{\text {out }} \sqrt{2 \pi}} \exp \left[-\frac{1}{2}\left(\frac{v \sin i_{i}-\mu_{\text {out }}}{\sigma_{\text {out }}}\right)^{2}\right] \tag{3.2.1}
\end{align*}
$$

In principal, we could determine the model parameters by maximizing the likelihood, $L$, of the data:

$$
\begin{equation*}
L=\prod_{i}^{N} p\left(v \sin i_{i} \mid \boldsymbol{\theta}\right) \tag{3.2.2}
\end{equation*}
$$

The presence of non-detections in our data, however, necessitates a modified approach. The numerical values of $v \sin i$ output by our code for non-detections are not reliable, so we cannot include them in the usual likelihood calculation. On the other hand, the number of non-detections places a constraint on the model, so it is necessary to incorporate them into the analysis. We used a modified form of the likelihood equation developed in the statistical field of survival analysis, which incorporates left-censored data (Feigelson \& Nelson, 1985). The likelihood equation then becomes:

$$
\begin{equation*}
L\left(v \sin i_{i} \mid \boldsymbol{\theta}\right)=\prod_{i}^{N}\left[p\left(v \sin i_{i} \mid \boldsymbol{\theta}\right)\right]^{\delta_{i}}\left[P\left(c_{i} \mid \boldsymbol{\theta}\right)\right]^{\left(1-\delta_{i}\right)} \tag{3.2.3}
\end{equation*}
$$

In this formulation, $\delta=1$ for detections and $\delta=0$ for non-detections. $P\left(c_{i} \mid \boldsymbol{\theta}\right)$ is the cumulative distribution function, evaluated at the upper-limit for detection $\left(c_{i}=8 \mathrm{~km} \mathrm{~s}^{-1}\right)$. As is common practice, we actually maximize the log likelihood, which simplifies the computation by turning the product into a summation. We used the IDL code, amoeba, which performs a downhill-simplex optimization Nelder \& Mead, 1965). The final $v \sin i$ measurement is the expectation value of the Gaussian defined by $\mu_{\text {true }}$ and $\sigma_{\text {true }}$.

### 3.2.5 Results

Figure 3.4 shows two example APOGEE spectra with their best-fit models superimposed in red. The upper spectrum is considered a non-detection $\left(v \sin i<8 \mathrm{~km} \mathrm{~s}^{-1}\right)$, while the lower spectrum is that of a rapid rotator $\left(v \sin i=22.6 \mathrm{~km} \mathrm{~s}^{-1}\right)$. The rotational line broadening in the rapid rotator is clearly discernible to the eye. These stars have similar $T_{\text {eff }}$ and $[\mathrm{Fe} / \mathrm{H}]$. In Figure 3.5 we examine the relationship between $v \sin i$ and $T_{\text {eff }}$ in our sample. Overall, our $v \sin i$ results show a lower frequency of rapid rotators for early M dwarfs and a higher frequency of rapid rotators for late M dwarfs. This is consistent with other spectroscopic studies of $M$ dwarf rotation in the literature (e.g. Reiners 2007).

There are 67 APOGEE M dwarfs with previously published $v \sin i$, but only 16 that we could use for comparison. Of the 67 published values, 9 were from previous work on APOGEE (Deshpande et al. 2013), 31 were for stars with $T_{\text {eff }}>4000 \mathrm{~K}$, and 11 were for stars that failed in the ASPCAP pipeline. As shown in Figure 3.6 and Table 3, the data are broadly consistent, aside from '2MASS J02085359+4926565,' which is from a lower resolution survey ( $R \approx 19000$; Gizis et al. 2002). The remaining measurements all come from surveys with $R \geq 31,000$. We found that our measured $v \sin i$ values agreed with this small number of available literature values to within approximately $3 \mathrm{~km} \mathrm{~s}^{-1}$ (or $0.4 \mathrm{~km} \mathrm{~s}^{-1}$ if we only consider literature values derived from spectra with higher resolution than the APOGEE spectra).

We estimated our $v \sin i$ uncertainty for each star as the quadrature sum of three sources of error. Based on the results of the Gaussian Mixture Model fits to the chunk-based $v \sin i$ estimates, we scale $\sigma_{\text {true }}$ by the square root of the number of non-outlier chunks $(\sqrt{\text { floor }(\alpha \cdot 28)})$ as an estimate of the single measurement uncertainty in our fitting process. We include the $0.4 \mathrm{~km} \mathrm{~s}^{-1}$ uncertainty in the absolute scale of $v \sin i$ based on the comparison to literature values, and also a $1 \mathrm{~km} \mathrm{~s}^{-1}$ uncertainty resulting from systematic template mismatch due to the coarse sampling in $[\mathrm{M} / \mathrm{H}]$ and $T_{\text {eff }}$ in our template grid. This was based on tests where fits for $v \sin i$ were forced with
Table 3. Comparison to $v \sin i$ Measurements from the Literature

| 2MASS ID | Name | $\begin{gathered} \text { RA } \\ (\text { deg. J2000) } \end{gathered}$ | $\begin{gathered} \text { DEC } \\ \text { (deg. J2000) } \end{gathered}$ | $\begin{aligned} & T_{\text {eff }} \\ & (\mathrm{K}) \end{aligned}$ | Spectral Type ${ }^{\text {a }}$ | $\begin{gathered} v \sin i_{\text {VFIT }} \\ \left(\mathrm{km} \mathrm{~s}^{-1}\right) \end{gathered}$ | $\begin{aligned} & v \sin i_{\mathrm{LIT}} \\ & \left(\mathrm{~km} \mathrm{~s}^{-1}\right) \end{aligned}$ | Resolution $(\mathrm{R} / 1000)$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| J02085359+4926565 | GJ 3136 | 32.223315 | 49.449055 | 3340 | M 4.0 V | $22.9 \pm 1.5$ | $30.0 \pm 5.0^{1}$ | 19 |
| J03212176+7958022 | GJ 133 | 50.340691 | 79.967285 | 3586 | M 2.0 Ve | < 8.0 | $<1.0^{2}$ | 75/115 |
| J04584599+5056378 | GJ 1074 | 74.691634 | 50.943859 | 3807 | M 1.0 Ve | $<8.0$ | $<4.0^{3}$ | 40/48 |
| J05470907-0512106 | LHS 1785 | 86.787800 | -5.2029690 | 3149 | M 4.5 V | < 8.0 | $4.50 \pm 0.60^{4}$ | 37 |
| J06421118+0334527 | G 108-21 | 100.54659 | 3.5813060 | 3437 | M3V | < 8.0 | $0.900^{2}$ | 75/115 |
| J09005033+0514293 | Ross 687 | 135.20971 | 5.2414950 | 3392 | M3.0Ve | < 8.0 | $<3.0^{5}$ | 40 |
| J09422327+5559015 | GJ 363 | 145.59698 | 55.983776 | 3390 | M3V | < 8.0 | $<3.0^{5}$ | 40 |
| J $10355725+2853316$ | UCAC4 595-047332 | 158.98856 | 28.892134 | 3442 | M3.0V | $<8.0$ | $4.00 \pm 2.0^{6}$ | 50 |
| J13085124-0131075 |  | 197.21351 | -1.5187690 | 3498 | M3.0V | < 8.0 | $<2.0{ }^{9}$ | 65/68 |
| J13455527+2723131 | LHS 2795 | 206.48032 | 27.386990 | 3318 | ... | < 8.0 | $6.40 \pm 1.0^{2}$ | 75/115 |
| J13564148+4342587 | LP 220-13 | 209.17285 | 43.716324 | 2626 | M8V | $13.5 \pm 1.6$ | $14.0 \pm 2.0^{7}$ | 31/32 |
| J 14333985+0920094 | HD 127871B | 218.41606 | 9.3359630 | 3384 | M3.5V | <8.0 | $5.30 \pm 1.0^{2}$ | 75/115 |
| J $14573227+3123446$ | Ross 53 | 224.38448 | 31.395733 | 3884 | K5V | < 8.0 | $2.63 \pm 1.0^{2}$ | 75/115 |
| J 16404891+3618596 | Ross 812 | 250.20383 | 36.316566 | 3662 | M2V | < 8.0 | $<4.0^{3}$ | 40/48 |
| J $19454969+3223132$ | LP 337-3 | 296.45707 | 32.387005 | 3623 | M1.5Ve | < 8.0 | $<4.0^{3}$ | 40/48 |
| J 19510930+4628598 | GJ 1243 | 297.78877 | 46.483295 | 3205 | M4.0V | $22.5 \pm 1.6$ | $22.0 \pm 3.0^{3}$ | 40/48 |
| J 19535443+4424541 | G 208-44 | 298.47680 | 44.415043 | 2749 | M 5.5 Ve | $22.6 \pm 1.5$ | $22.5{ }^{8}$ | 31 |

an incorrect template, off in $T_{\text {eff }},[\mathrm{M} / \mathrm{H}]$, and/or $\log g$ by up to two grid points. As shown in Figure 3.7 the impact on $v \sin i$ is small, approximately $1 \mathrm{~km} \mathrm{~s}^{-1}$. We estimate a total $v \sin i$ measurement uncertainty between 1 and $3 \mathrm{~km} \mathrm{~s}^{-1}$ for the majority of our targets.

We note that our typical reduced- $\chi^{2}$ is much greater than 1 , due to systematic disagreement between template and observation, and the generally underestimated errors supplied by the APOGEE pipeline. Additionally, different M dwarf templates have been shown to produce different results. Either of these effects can potentially bias our results. However, we designed our fitting procedure to mitigate the former, and we chose the BT-Settl models to be the most appropriate for M dwarfs in regards to the latter. We tested the sensitivity of $v \sin i$ to the model parameters by creating a synthetic spectrum and performing the fit using templates with parameters which were off by $\pm 1$ grid point in $[\mathrm{Fe} / \mathrm{H}]$ and $\log g$, and off by up to $\pm 300 \mathrm{~K}$. The resulting $v \sin i$ error was approximately $1 \mathrm{~km} \mathrm{~s}^{-1}$.

We also investigated the relationship between the signal-to-noise of our spectra and the estimated $v \sin i$. Since cooler stars will tend to be fainter, and therefore have lower signal-to-noise spectra, a bias in our analysis could artificially inflate the trend we see of increasing rotation with decreasing $T_{\text {eff }}$. We simulated this effect and found no such bias. As shown in Figure 3.8, on average we recover the known $v \sin i$ in simulated spectra with a fidelity of better than $1 \mathrm{~km} \mathrm{~s}^{-1}$ even for the faintest stars in our sample. The full results of the fitting process are reported in Table 4.


Figure 3.4 Sample of the template fitting process. An identical portion of the blue chip is shown for two APOGEE spectra, with the best-fit PHOENIX model in red. These stars are similar in $T_{\text {eff }}$ and $[\mathrm{M} / \mathrm{H}]$, but the upper panel shows the spectrum of a slow rotator (non-detection), whereas the bottom panel shows a rapid rotator $\left(v \sin i=22.6 \mathrm{~km} \mathrm{~s}^{-1}\right)$. The broadening and blending of lines due to rotation is evident.


Figure $3.5 v \sin i$ results from our template fitting approach. The red downward facing triangles represent non-detections ( $v \sin i<8 \mathrm{~km} \mathrm{~s}^{-1}$ ) and are plotted at the calculated value for visualization purposes. The blue circles are measured values of $v \sin i$. Effective temperatures are those from Data Release 13 of the ASPCAP pipeline. The typical error for $T_{\text {eff }}$ is shown in the legend. Uncertainties in $v \sin i$ range from $1-3 \mathrm{~km} \mathrm{~s}^{-1}$.


Figure 3.6 Comparison of our $v \sin i$ measurement with literature values, color-coded by reference. See Table 3 for full citations. The arrows denote upper limits for non-detections. The fastest literature rotator, '2MASS J02085359+4926565,' (unfilled square with dashed error bars) is from a lower resolution survey ( $R \approx 19000$; Gizis et al. 2002 ). Our results are entirely consistent with the literature.


Figure 3.7 Test of the effect of misidentifying the best-fit PHOENIX template. We fit the set of stars with previously published $v \sin i$ using a mini-grid of templates adjacent to the best-fit template. The results for '2MASS J19510930+4628598' are shown here. The dashed line shows our reported value of $v \sin i$ and each point is a $v \sin i$ measurement, corresponding to a point in the mini-grid. In each panel, $v \sin i$ is plotted against one grid parameter, while the other two grid parameters are encoded in the shape and color of the plot symbol (denoted in the legend). The RMS of the residuals is $0.53 \mathrm{~km} \mathrm{~s}^{-1}$, and the maximum deviation is $\Delta v \sin i=1.2 \mathrm{~km} \mathrm{~s}^{-1}$. We assume a $1 \mathrm{~km} \mathrm{~s}^{-1}$ systematic error from possible template mismatches.


Figure 3.8 Deviation in $v \sin i$ as a function of SNR. We added Gaussian noise to the spectra of several rotators and measured the $v \sin i$ of the degraded spectra. A robust linear fit yields an average deviation of about $0.75 \mathrm{~km} \mathrm{~s}^{-1}$ at a SNR of 50 , which is the minimum allowed in our sample. We also performed this procedure on non-detections. A small minority of trials ( $2 \%$ at $50<\mathrm{SNR}<70$ ) yielded $v \sin i$ above the detection limit. There are only 37 stars in our sample with SNR in this range, so it is unlikely that any are false positives due to flux error alone.
Table 4. $\quad v \sin i$ Results

| 2MASS ID | $\begin{gathered} \mathrm{RA} \\ \text { (degrees J2000) } \end{gathered}$ | $\begin{gathered} \text { DEC } \\ \text { (degrees J2000) } \end{gathered}$ | $\begin{gathered} T_{\text {eff, }}, \text { ASPCAP } \\ (\mathrm{K}) \end{gathered}$ | $\begin{aligned} & \mathrm{T}_{\text {eff, VFIT }} \\ & (\mathrm{K}) \end{aligned}$ | $\underset{(\mathrm{dex})}{[\mathrm{M} / \mathrm{H}]_{\mathrm{VFIT}}}$ | $\underset{(\mathrm{dex})}{\log g_{\mathrm{VFIT}}}$ | $\begin{gathered} v \sin i_{\mathrm{VFIT}} \\ \left(\mathrm{~km} \mathrm{~s}^{-1}\right) \end{gathered}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| J00004701+1624101 | 0.19587600 | 16.402811 | 3725 | 3800 | +0.5 | 5.0 | < 8.0 |
| J00034394+8606422 | 0.93308800 | 86.111732 | 2893 | 2800 | -0.0 | 5.0 | $13.2 \pm 1.5$ |
| J00255540+5749320 | 6.4808560 | 57.825562 | 3400 | 3300 | +0.5 | 5.0 | < 8.0 |
| J00255888+5559296 | 6.4953360 | 55.991581 | 3589 | 3500 | -0.0 | 4.5 | $11.5 \pm 1.5$ |
| J00262872+6747026 | 6.6196670 | 67.784073 | 3756 | 3800 | -0.0 | 4.5 | < 8.0 |
| J00270673+4941531 | 6.7780790 | 49.698093 | 3297 | 3300 | -0.0 | 5.0 | < 8.0 |
| J00285391+5022330 | 7.2246660 | 50.375839 | 3236 | 3200 | +0.5 | 5.0 | $14.2 \pm 1.5$ |
| J00301250+5028392 | 7.5520980 | 50.477570 | 3209 | 3200 | -0.0 | 5.0 | < 8.0 |
| J02081218+1508424 | 32.050754 | 15.145118 | 3174 | 3200 | +0.5 | 5.5 | < 8.0 |
| J02081366+4949023 | 32.056958 | 49.817318 | 2824 | 2600 | -0.0 | 5.0 | < 8.0 |
| J02085359+4926565 | 32.223315 | 49.449055 | 3340 | 3300 | +0.5 | 5.0 | $22.9 \pm 1.5$ |
| J02122001+1249287 | 33.083411 | 12.824662 | 3459 | 3700 | +0.5 | 5.5 | < 8.0 |
| J02144781+5334438 | 33.699241 | 53.578857 | 3333 | 3400 | -0.5 | 5.0 | < 8.0 |
| J05320969+2754534 | 83.040394 | 27.914845 | 2865 | 2900 | -0.0 | 5.5 | $8.36 \pm 1.7$ |
| J05325989+2608271 | 83.249549 | 26.140879 | 3342 | 3400 | +0.5 | 5.5 | $47.7 \pm 2.3$ |

### 3.3 Comparison with Rotation Periods from Photometry

In this analysis, we used spectroscopic data to measure the projected rotational velocity, but it is also possible to infer rotation periods (without the projection effect) from periodic photometric variations due to star spots rotating across the stellar surface (eg. Irwin et al. 2011, McQuillan et al. 2014. Newton et al. 2016a). If the stellar radius is known, the rotation period, $P$, can be converted to an equatorial rotational velocity: $v_{r o t}=\frac{2 \pi \mathrm{R}_{\star}}{P}$. Newton et al. 2016a), for example, uses the mass-radius relation from Boyajian et al. (2012) to estimate the stellar radius and convert period measurements from the MEarth survey to rotational velocities. Spectroscopic and photometric methods are sensitive to different regimes of rotation velocity. While our $v \sin i$ detection limit means that we are typically not sensitive to rotation periods longer than $\sim 1-2$ days, photometric surveys are sensitive to periods shorter than tens of days. However, comparisons can be made between the two techniques. For example, photometric studies suggest a larger proportion of slowly rotating late-type stars that may not be present in our data. Newton et al. 2016a) compared their rotation period measurements from the MEarth survey to $v \sin i$ measurements from Delfosse et al. (1998), Mohanty \& Basri (2003) and Browning et al. (2010). Both Delfosse et al. and Mohanty \& Basri found that approximately $50 \%$ of mid M dwarfs were detectably rotating, while Browning et al. reported a rotation fraction of $30 \%$. Assuming a stellar radius of $0.2 R_{\odot}$, the $v \sin i$ detection limit of those surveys $\left(\sim 3 \mathrm{~km} \mathrm{~s}^{-1}\right)$, corresponds to rotation periods of $P \lesssim 3.3$ days. Newton et al. (2016a), however, finds just $18 \pm 2 \%$ of mid M dwarfs in their sample would be detected as rotating in the aforementioned $v \sin i$ studies.

We sought to make a quantitative comparison between our spectroscopic projected rotation velocities and the rotation velocities derived from photometric periods in the literature. In Figure 3.9. we directly compared our $v \sin i$ to the $v_{r o t}$ from Newton et al. 2016a). Since $v \sin i$ is the minimum possible rotational velocity, the shaded region $\left(v \sin i>v_{r o t}\right)$ is non-physical. Even though we are comparing $v \sin i$ to $v_{r o t}$, we do expect most data points to fall near the one-to-one


Figure 3.9 Comparison of our $v \sin i$ measurement versus equatorial velocities calculated from photometric periods in Newton et al. (2016a). Since $v \sin i$ is the minimum possible equatorial velocity, the gray shaded region is non-physical. The downward triangles are non-detections in our analysis, and are therefore not inconsistent with the MEarth data.
correspondence line, because the distribution of $\sin i$ peaks sharply at $\sin i=1$ (assuming the spin axes are randomly oriented). Specifically, half of the stars should have $\sin i \geq 0.86$, and that is in fact the case for the 10 detections in the sample. A Kolmogorov-Smirnov test yields a $77 \%$ confidence that the data are consistent with randomly distributed spin axes. Therefore, the direct comparison of $v \sin i$ and $v_{r o t}$ for stars with both measurements appears consistent, given the uncertainties.

We also explored the relationship between rotation and effective temperature using our APOGEE data and literature sources. We calculated the fraction of rotators as a function of effective tem-
perature defined as having $v \sin i>8 \mathrm{~km} \mathrm{~s}^{-1}$. In Figure 3.10, we show the fraction of rotators in temperature bins. The error bars bracket $90 \%$ confidence intervals assuming binomial statistics Gehrels, 1986). The red points are from earlier $v \sin i$ analyses (see Reiners et al. 2012 and references therein). The literature $v \sin i$ are binned by spectral type, and mapped onto the temperature scale based upon relations between stellar temperature and spectral type, derived from

Table 5 of Pecaut \& Mamajek (2013).


Figure 3.10 Fraction of rapid rotators as a function of effective temperature. The black points (circles) are based on our $v \sin i$ measurements, with $90 \%$ confidence intervals from binomial statistics. The red points (squares) are based on $v \sin i$ from literature sources (Reiners et al. 2012 and references therein) also with $90 \%$ confidence intervals from binomial statistics. These are binned by spectral type, and mapped onto our $T_{\text {eff }}$ scale. The green region with the dotted outline is the $90 \%$ confidence interval of simulated rotation fractions based on rotation periods measured from Kepler photometry (McQuillan et al. 2014). The blue region with the solid outline is the $90 \%$ confidence interval of simulated rotation fractions based on rotation periods measured from MEarth photometry (Newton et al., 2016a).

The shaded regions in Figure 3.10 are estimated rotation fractions based upon two sets of photometric period data. We simulated the expected $v \sin i$ distributions and the resulting rotation fractions using a Monte-Carlo analysis. The green region is based on rotation periods from Kepler photometry (McQuillan et al. 2014), and the blue region is based on rotation periods from MEarth photometry (Newton et al. 2016a). The procedure was performed as follows:

1. We bin the data in bins of 100 K for $2600 \leq T_{\text {eff }} \leq 4000 \mathrm{~K}$.

Kepler: We used the $T_{\text {eff }}$ data from Table 3 of McQuillan et al. (2014).
MEarth: No $T_{\text {eff }}$ estimates were provided, so we used the stellar mass estimates from Newton et al. (2016a) and inferred a temperature from the stellar mass using a relation based on Pecaut \& Mamajek (2013).
2. We generate a realistic set of rotational velocities, $v_{r o t}$, for each temperature bin.

Kepler: We used the quoted rotation periods, $P$, and errors, $\sigma_{P}$. We had to infer a radius for each star from its temperature. We estimated the stellar radii using an empirical relation from Table 1 of Mann et al. (2015). This relation is quoted as having an uncertainty of $13.4 \%$, which we applied to the radius estimates.

MEarth: We used the $v_{r o t}$ from Newton et al. (2016a), which were calculated from their period and radius measurements, with a $13 \%$ uncertainty, based on an assumed $10 \%$ uncertainty in both quantities. The measurements are tagged with a quality flag: (A) and (B) for confident detections, $(\mathrm{U})$ for possible detections, and ( N ) for non-detections. We use only the stars with (A) and (B) flags.

## 3. We generate a corresponding set of $\sin i$ values.

Assuming a random distribution of spin axis orientations, the probability density function of $\sin i \in[0,1]$ is:

$$
p(\sin i) \propto \frac{\sin i}{\sqrt{1-\sin ^{2} i}}
$$

This is the same as a uniform distribution of $\cos i$. We generated random inclinations by generating a uniform distribution of $\cos i$ and transforming using $\sin i=\sqrt{1-\cos ^{2} i}$.
4. We combine (2) and (3) to $\operatorname{simulate} v \sin i=v_{r o t} \cdot \sin i$, and calculate the rotation fraction.

Since our simulated populations of $v \sin i$ were generated from the stars with detected periods, we had to account for the number of stars observed which had no detectable period.

Kepler: McQuillan et al. (2014) noted that the periodic fraction of stars with $T_{\text {eff }}<$ 4000 K is $83 \%$, so we scaled the total number of stars in each bin accordingly. They report completeness of $\sim 95 \%$, which we take to be $100 \%$ for our purposes.

MEarth: We added the stars with (U) and (N) flags in each bin as non-detections. We also scaled up the number of detections to account for the difference in sensitivity between Kepler and MEarth. Figure 3.11 shows the cumulative distribution of period detections as a function of amplitude for Kepler (in black) and MEarth (in red). Approximately $40 \%$ of the Kepler detections fall below the minimum sensitivity of MEarth, so we scaled up the number of detections to account for the expected missing detections.

In Figure 3.10 we show the results of a Monte Carlo simulation to determine the expected rotation fraction as a function of $T_{\text {eff }}$ based on the Kepler and MEarth photometric periods. Although the spectroscopic and photometric results appear consistent for the sample of stars with both measurements (see Figure 3.9), we found that the distribution of rotations as a function of temperature exhibited some disagreement for stars later than $\sim$ M3. The spectroscopicallyderived $v \sin i$ estimates seem to suggest a higher fraction of late, fully-convective $M$ dwarfs are rapidly rotating ( $v \sin i>8 \mathrm{~km} \mathrm{~s}^{-1}$ ) as compared to the photometrically-derived rotation periods. It is interesting to note that $v \sin i$ studies, both ours and those in the literature, tend to show a sharp change in behavior at the M4 transition, while the photometric studies show rotation fractions that are fairly constant along the $M$ dwarf sequence.


Figure 3.11 Comparison of Kepler and MEarth sensitivity to periodic photometric variations. Approximately $40 \%$ of the periods detected in McQuillan et al. (2014) have amplitudes that are too small to be detected and classified as (A) or (B) rotators in Newton et al. (2016a).

### 3.4 Discussion and Conclusions

We present results from an analysis of high-resolution spectroscopic observations of 714 M dwarfs obtained by the SDSS APOGEE survey. We derive estimates of projected rotation velocity, $v \sin i$, by fitting a large suite of rotationally broadened theoretical templates to these observations. We analyze these spectra in chunks and use a Gaussian Mixture Model approach to estimate our measurement uncertainty based on individual fits to 28 chunks of each spectrum. For each of our targets, we estimate an overall uncertainty of 1 to $3 \mathrm{~km} \mathrm{~s}^{-1}$ and find good agreement between our measurements and a small number of previously published values.

Through a Monte Carlo simulation, we attempt to make a direct comparison between the overall distribution of our projected rotational velocities and the photometric periods published in the literature. While we do find broad agreement in that rotation fraction increases with lower stellar temperature in all data sets, for $T_{\text {eff }}<3200 \mathrm{~K}$ we see a rotation fraction that is a factor of $\sim 2$ higher than the rotation distribution that would be inferred from the MEarth photometry (see Figure 3.10). There are a number of factors which could explain this tension. One important factor concerning the distribution of rotational velocities is the stellar age of the population. We have not made any explicit selection cuts based on age here, though it is possible that the reduced proper motion criteria used in Deshpande et al. (2013) to originally select our sample could induce an age bias. The APOGEE M Dwarf sample was assembled using two different catalogs with two different proper motion cuts. Part of the APOGEE sample was selected with $\mu>150 \mathrm{mas} / \mathrm{yr}$, and part was selected with $\mu>40 \mathrm{mas} / \mathrm{yr}$. Similarly, Newton et al. (2016a) noted that MEarth proper motion cuts of $\mu>150$ mas/yr likely excluded some kinematically cold stars, which would tend to be younger and more rapidly rotating. We applied the kinematic age estimation method presented in Newton et al. (2016a) to our APOGEE targets and found a slightly larger proportion of likely young stars than in the MEarth sample.

There are at least two factors which could contribute to the high rotation fractions for late M dwarfs seen in our $v \sin i$ analyses. The first is the possibility that the detection threshold is higher than assumed. If, for example, our detection threshold were $v \sin i=12 \mathrm{~km} \mathrm{~s}^{-1}$, then the fraction of rotators at $T_{\text {eff }}<3200 K$ would shift down in Figure 3.10 and would be almost entirely consistent with the inferred rotational velocities from MEarth photometry. Another systematic error which may bias our results is binary contamination. We have no particular mechanism for filtering out binaries.

One possible systematic effect arising from the photometric analyses is aliasing. It is possible that some of the reported periods are harmonics of the true rotation period. In Newton et al. (2016a), for example, the authors addressed this by listing alternate rotation periods for stars
which disagreed with the $v \sin i$ from the literature. Also, being a ground-based survey, MEarth is sensitive to aliasing at 1-day periods. Similarly, the photometric periods may not be strictly indicative of rotation periods, rather a combination of rotation and any periodic behavior intrinsic to spots themselves. This effect is difficult for us to quantify as it is unknown what the characteristic spot evolution timescales are for cool stars. Finally, we may be underestimating the rotation fraction by counting all stars without periods as non-detections. It is very likely, for example, that some of the MEarth stars with the (U) flag are rotating. In fact, the authors note that even the $(\mathrm{N})$ flag does not mean that the star is not rotating; it simply means that they were unable to detect a periodic modulation. We also note that even though our sample represents the largest set of high-resolution spectra of $M$ dwarfs, there are fewer than 10 stars in each of the three coolest temperature bins.

One interesting feature of our analysis is the comparatively small fraction of non-detections in our sample below $T_{\text {eff }} \sim 3000 \mathrm{~K}$. Approximately $65 \%$ of the these coolest dwarfs in our sample are non-detections of rotation given our conservative $8 \mathrm{~km} \mathrm{~s}^{-1}$ detection limit. This is consistent with the trend toward faster rotation at later spectral types observed in earlier $v \sin i$ analyses. It has been suggested based on photometric data that the rotation periods of low-mass stars may be bi-modal (e.g. Irwin et al. 2011|McQuillan et al. 2014, Newton et al. 2016a). For example, Figure 15 of Newton et al. (2016a) shows a substantial population of stars with masses less than $0.3 \mathrm{M}_{\odot}$ and rotation periods longer than 10 days. These would all be non-detections in our analysis, even if we were to assume a less conservative $v \sin i$ detection limit.

Overall, our $v \sin i$ results show a low frequency of rapid rotators for early M dwarfs and a high frequency of rapid rotators for late $M$ dwarfs, with a sharp transition which roughly coincides with the M4 transition to fully-convective stellar interiors (see Figure 3.5). This is consistent with other spectroscopic studies of $M$ dwarf rotation in the literature (e.g. Reiners 2007). As shown in Figure 3.9, we find good agreement for individual targets between our $v \sin i$ estimates and published photometric periods. Since $\sin i$ can not be larger than 1, a given equatorial rotation
velocity derived from a photometric period and stellar radius sets a physical limit on $v \sin i$. Within a sample of 19 stars with photometric periods for which we measured $v \sin i$, we found no examples non-physical rotational velocities. Through a Monte Carlo simulation, we make a direct comparison between the overall distribution of our projected rotational velocities and the photometric periods published in the literature. While we do find broad agreement in that rotation fraction increases with lower stellar temperature in all data sets, for $T_{\text {eff }}<3200 \mathrm{~K}$ we see a rotation fraction that is a factor of $\sim 2$ higher than the rotation distribution that would be inferred from the MEarth photometry (see Figure 3.10). This rotation fraction depends both on the detection limit in our analysis, which we conservatively set at $8 \mathrm{~km} \mathrm{~s}^{-1}$ as well as the fraction of stars with a given rotation rate that exhibit detectable photometric variability. While we attempt to quantify this effect by comparing the distribution of photometric amplitudes found in the MEarth survey to those in the Kepler survey (which has substantially better photometric precision), it is possible that a significant population of rotating stars without photometric variability remains. While the fraction of non-detections at $T_{\text {eff }}<3000 K$ depends sensitively on our assumed $v \sin i$ detection limit, the fraction of rotators in our sample appears consistent with the bi-modal photometric period distribution seen in the MEarth and Kepler studies.

## Chapter 4

## Data-Driven Method for

## Measuring Rotational Velocity

This chapter ${ }^{1}$ describes a novel, data-driven approach to measuring $v \sin i$. Leveraging the large information content of high-resolution stellar spectra, we use regression techniques to estimate $v \sin i$ empirically. Our approach is a modification of The Cannon (Ness et al. 2015), a technique that trains a generative model of the stellar flux as a function of wavelength using high-fidelity reference data, and then produces estimates of stellar parameters and abundances for other stars directly from their spectra. Such a technique requires that the data set includes objects with high-fidelity measurements of $v \sin i$ and other stellar parameters, spanning the parameter space of the full data set. We analyzed SDSS APOGEE spectra, constructing a model informed by highfidelity stellar parameter estimates derived from high-resolution California Kepler Survey spectra of the same stars. The data are described in Section 4.1.

The major change that we introduce to The Cannon framework is that, rather than modeling the flux as a function of wavelength, we model the first derivative of the flux, because we expect

[^5]the slopes of spectral lines to depend strongly on $v \sin i$. We describe the technique in detail in Section 4.2

While this approach to $v \sin i$ estimation has its limitations, it is computationally efficient and provides a means of rapidly estimating $v \sin i$ for large numbers of stars in spectroscopic survey data. We estimate $v \sin i$ up to $15 \mathrm{~km} \mathrm{~s}^{-1}$ for 27, 000 APOGEE spectra, in fractions of a second per spectrum. Our estimates agree with the APOGEE pipeline $v \sin i$ estimates to within $1.2 \mathrm{~km} \mathrm{~s}^{-1}$. Our results are discussed in Section 4.3. Finally, we discuss the advantages and limitations of our approach, outline some possible improvements, and summarize our findings in Section 4.4.

### 4.1 Data Used in This Analysis

### 4.1.1 Spectral Data from APOGEE

We measure $v \sin i$ using near-infrared spectra from Data Release 14 of the Sloan Digital Sky Survey (SDSS; Abolfathi et al. 2017). The SDSS Apache Point Observatory Galactic Evolution Experiment (APOGEE) has produced near infrared spectra and associated data products, including estimates of stellar parameters and chemical abundances, for over 250,000 giant and dwarf stars. The APOGEE instrument (Wilson et al. 2010, Skrutskie \& Wilson 2015) is a cryogenic, multiplexed near-infrared spectrograph with resolving power of $R \sim 22,500$. Its three H-band detectors span a wavelength range of $\lambda=1.514 \mu \mathrm{~m}-1.696 \mu \mathrm{~m}$. The three chips are referred to respectively as 'blue' $(\lambda=1.52 \mu \mathrm{~m}-1.58 \mu \mathrm{~m})$, 'green' $(\lambda=1.59 \mu \mathrm{~m}-1.64 \mu \mathrm{~m})$, and 'red' $(\lambda=1.65 \mu \mathrm{~m}-1.69 \mu \mathrm{~m})$.

The main APOGEE pipeline reduces observations, removes telluric absorption and sky emission lines, and combines individual visit spectra from multiple epochs into single, high signal-tonoise (typically $>100$ ) coadded spectra in the rest frame of the star. These apStar files contain the coadded and individual visit spectra for each star, along with important header information (Nidever et al., 2015). Additionally, APOGEE spectra are processed by the APOGEE Stellar

Parameter and Chemical Abundance Pipeline (ASPCAP; García Pérez et al. 2016), which produces psuedo-continuum normalized spectra, stellar parameter estimates and estimates of up to 15 individual chemical elements. We applied the techniques described here to the apStar spectra. We used the ASPCAP parameter estimates to select the survey sample (see Section 4.1.4) from the full APOGEE catalog, and for later testing our results. We restricted our sample to stars that were observed by the SDSS main 2.5 m telescope (Gunn et al. 2006), and that have spectra with average signal-to-noise $>100$ per pixel.

### 4.1.2 Reference Data from the California Kepler Survey

The data-driven technique described here depends on having reference objects with high-fidelity parameter measurements, preferably derived from spectra of higher resolution than that of APOGEE. For this purpose, we used stellar parameter estimates from the California Kepler Survey (CKS), a project designed to precisely measure the properties of Kepler planets' host stars (mainly F-G-K dwarfs). The CKS data set consists of 1305 high-resolution $(R=60,000)$ optical $(3,640 \AA \leq \lambda \leq$ $7,990 \AA$ ) spectra from the Keck HIRES instrument (see Petigura et al. 2017 and Johnson et al. 2017). While we performed early tests using the CKS spectra, the analysis presented here makes use of the CKS parameter estimates only.

The CKS team measured stellar parameters using two different spectroscopic pipelines - SpecMatch Petigura, 2015 and SME@XSEDE Valenti \& Piskunov, 1996). In most cases, the CKS determinations of $T_{\text {eff }},[\mathrm{Fe} / \mathrm{H}]$, and $\log g$ are the arithmetic means of the values output by the two pipelines. The internal precision of CKS $T_{\text {eff }}$ and $[\mathrm{Fe} / \mathrm{H}]$ measurements are quoted at $\sigma_{\mathrm{Teff}}=60 \mathrm{~K}$ and $\sigma_{[\mathrm{Fe} / \mathrm{H}]}=0.04$ dex, respectively. Adding systematic uncertainties based upon comparison with other measurement techniques, the quoted total uncertainties are $\sigma_{\text {Teff }}=117 \mathrm{~K}$ and $\sigma_{[\mathrm{Fe} / \mathrm{H}]}=0.07$ dex. The total uncertainty in $\log g$ is $\sigma_{\log g}=0.1$ dex (Petigura et al., 2017). The CKS $v \sin i$ values were determined solely by the SpecMatch pipeline. Based on a comparison between SpecMatch $v \sin i$ and Rossiter-McLaughlin measurements (Albrecht et al., 2012) for a
subset of the CKS data, the uncertainty in $v \sin i$ was determined to be $\sigma_{v \sin i}=1 \mathrm{~km} \mathrm{~s}^{-1}$ for $v \sin i \geq 1 \mathrm{~km} \mathrm{~s}^{-1}$. Stars with SpecMatch $v \sin i<1 \mathrm{~km} \mathrm{~s}^{-1}$ are to be considered non-detections with upper-limits of $v \sin i<2 \mathrm{~km} \mathrm{~s}^{-1}$ Petigura, 2015). Out of the 1305 CKS targets, 362 were also observed by the APOGEE survey. The stars observed by both surveys formed the basis for our training sample.

### 4.1.3 Training Sample

Our training sample is a subset of the 362 stars with both APOGEE spectra and CKS parameter estimates. We made quality cuts on the APOGEE spectra by requiring average signal-to-noise $\geq 100$ per pixel, and removing spectra with the ASPCAP_FLAG set to STAR_BAD, which indicates a critical failure in the ASPCAP pipeline. Using the CKS data, we further restricted the sample to stars that are likely main sequence F-G-K dwarfs by requiring $\log g_{\mathrm{CKS}} \geq 3.9$. Finally, we restricted the range of $v \sin i$, in order to have a sufficient density of training data spanning the entire range. Only seven of the APOGEE/CKS stars have $v \sin i>14 \mathrm{~km} \mathrm{~s}^{-1}$, so we removed them. At the other extreme, 34 stars had $v \sin i_{\mathrm{CKS}}<1 \mathrm{~km} \mathrm{~s}^{-1}$. We dealt with these non-detections by creating two copies of the training sample. In the first, we used only the stars with detectably measured $v \sin i\left(1 \mathrm{~km} \mathrm{~s}^{-1} \leq v \sin i_{\mathrm{CKS}} \leq 13.8 \mathrm{~km} \mathrm{~s}^{-1}\right)$. This sample contained 236 stars. In the second, we incorporated the non-detections as synthetic fast rotators. That is, we treated each of the non-detections as a non-rotating star, and broadened their spectra with uniform random values of $v \sin i$ between 10 and $15 \mathrm{~km} \mathrm{~s}^{-1}$, in order to supplement the number of training stars with $v \sin i \geq 10 \mathrm{~km} \mathrm{~s}^{-1}$ where the data was relatively sparse. This training sample contained 270 spectra. We determined the complexity of the model by performing cross-validation tests on both of these training samples. The model selection is described in more detail in Section 4.2.3. Because the inclusion of artificial rotators significantly improved our results at $v \sin i \geq 10 \mathrm{~km} \mathrm{~s}^{-1}$, we adopted the latter set of 270 APOGEE/CKS stars as our final training sample.

As measured by CKS, the training sample spans $4675 \mathrm{~K} \leq T_{\text {eff }} \leq 6508 \mathrm{~K},-0.46 \leq[\mathrm{Fe} / \mathrm{H}] \leq$
0.38 , and $3.9 \leq \log g \leq 4.6$. The parameter space spanned by the CKS reference labels is described in Figure 4.1. The data from the final training set are plotted in various projections, with histograms showing the distributions of the training data for each label individually. Even with the addition of synthetic rotators, the training data are mostly slow rotators. Though the ASPCAP measurements of the training sample stars are not used in this analysis, we note that they are in close agreement with the CKS measurements. The ASPCAP values range from $4660 \mathrm{~K} \leq T_{\text {eff }} \leq 6448 \mathrm{~K}$, and $-0.55 \leq[\mathrm{Fe} / \mathrm{H}] \leq 0.47$. None of the stars in the training sample have $\log g$ measurements reported by ASPCAP. The ASPCAP headers do, however, contain uncalibrated $\log g$ estimates in the FPARAM vector. The $\log g$ are not reported because ASPCAP does not have a $\log g$ calibration for dwarf stars. The APOGEE website ${ }^{2}$ notes that ASPCAP $\log g$ estimates are typically underestimated for dwarfs, but the FPARAM values happen to be in excellent agreement with the CKS $\log g$. We did, however, make use of the ASPCAP measurements (including the uncalibrated $\log g$ ) in selecting an appropriate survey sample out of the full APOGEE data set.

[^6]

Figure 4.1 Illustration of the CKS training data parameter space. The scatter plots show the training labels in several projections. The histograms show the distribution of training data for each label.

### 4.1.4 Survey Sample

The survey sample is the set of APOGEE stars similar to those in the training sample, but without CKS measurements. We selected APOGEE stars that were fit with the appropriate grids for F-GK dwarfs (ASPCAP_CLASS $=$ ' GKd ' or ' Fd '), again with signal-to-noise $>100$, and in the same range of $T_{\text {eff }},[\mathrm{Fe} / \mathrm{H}], \log g$, and $v \sin i$ as spanned by the training set. The exact ranges of the ASPCAP measurements of survey sample stars are $4660 \mathrm{~K} \leq T_{\text {eff }} \leq 6439 \mathrm{~K},-0.48 \leq[\mathrm{Fe} / \mathrm{H}] \leq 0.37$, and $3.9 \leq \log g \leq 4.6$. The survey sample consists of precisely 27,000 stars.

The aim of this method is to propagate the reference $v \sin i$ measurements from the training sample to the stars in the survey sample. We now turn to describing, in detail, how we train the model, and how we use it to estimate $v \sin i$.

### 4.2 Empirical Data-driven Technique

As the basis of our data-driven approach, we adopted the statistical framework of The Cannon (Ness et al. 2015, Casey et al. 2016). The Cannon estimates stellar parameters by training a generative model, which describes the flux at each rest wavelength as a probability distribution in terms of the reference stellar parameters, hereafter referred to as stellar labels. The model can then be used to estimate labels for unlabeled survey data, by determining the labels that most nearly generate the observed spectrum.

A key feature of The Cannon is that the model is trained at each wavelength independently. This rests on the assumption that stars with the same labels will have similar-looking spectra, and that the flux at each pixel will generally change smoothly as a function of the stellar labels. Ness et al. (2015) showed that this assumption was warranted in the case of the three most important fundamental parameters, $T_{\text {eff }},[\mathrm{Fe} / \mathrm{H}]$, and $\log g$, which the Cannon recovers at higher precision than the reference ASPCAP data.

Rotation, however, broadens and blends spectral lines, redistributing flux along the spectral
dimension. Given two stars with identical $T_{\text {eff }},[\mathrm{Fe} / \mathrm{H}]$, and $\log g$, but very different rotation rates, the spectra will not look similar in terms of flux. As the Cannon sample was primarily composed of giant stars, rotational broadening was negligible in that analysis. In fact, the few stars that were flagged as rapidly rotating by the ASPCAP pipeline were excluded from the analysis. Our generative model instead describes the slope of the spectrum at a given wavelength, rather than the flux.

While differentiating the spectrum does not add information to the problem, and indeed, complicates the noise model by introducing correlation between pixels, we chose to work in fluxslope space for a few reasons. The first is precisely because of the introduction of correlation between pixels. Ness et al. (2015) showed that it was reasonable to ignore the (actual) correlation between pixels when inferring $T_{\text {eff }},[\mathrm{Fe} / \mathrm{H}]$, and $\log g$, but because rotation increases the correlation of the flux between pixels, we felt that the assumption of independence between pixels becomes more problematic as $v \sin i$ increases. Instead, we make the assumption that we can treat the fluxslope at each pixel as independent. The second is that it is possible that there is less correlation between the labels ( $T_{\text {eff }}$ and $[\mathrm{Fe} / \mathrm{H}]$, for example) when working in flux-slope space. Finally, we expect the slope to be less affected by errors in continuum normalization. Ness et al. (2015) and Casey et al. (2016) took great care to properly continuum-normalize the spectra used in the Cannon, arguing that standard continuum normalization techniques are signal-to-noise dependent, and small errors in continuum normalization can lead to large errors in label estimates. The fluxslope, however, should be less sensitive to such errors, and completely insensitive to constant offsets in the continuum.

In order to perform the differentiation, we used a Savitsky-Golay (S-G) filter. We chose the S-G filter because it can efficiently smooth and differentiate the data simultaneously. Another desirable feature of the S-G filter is that, unlike most other smoothing filters, it can preserve the higher order moments of spectral lines. We used the S-G filter to calculate the slope and slope error at each pixel based on a window which could be arbitrarily wide. We ran the analysis
using a 3-pixel window and a second degree S-G polynomial. The 3-pixel window corresponds to $\Delta v \sin i \sim 7 \mathrm{~km} \mathrm{~s}^{-1}$. This minimum $\mathrm{S}-\mathrm{G}$ window width is the most physically sensible choice given the range of $v \sin i$ spanned by the data. Repeating the analysis with larger windows (5-, 7- and 9-pixels) yielded inferior results when comparing our $v \sin i$ estimates to those generated by ASPCAP. The calculation of slope and slope error using the S-G filter is described in detail in Appendix A .

The framework for our approach is as follows. While Ness et al. (2015) formulate a model that describes the flux at each wavelength (Equation 1, in their paper):

$$
\begin{equation*}
f_{n \lambda}=g\left(\boldsymbol{\ell}_{n} \mid \boldsymbol{\theta}_{\lambda}\right)+\text { noise } \tag{4.2.1}
\end{equation*}
$$

we write an analogous model that instead describes the slope of the spectral flux at each wavelength:

$$
\begin{equation*}
f_{n \lambda}^{\prime}=g\left(\boldsymbol{\ell}_{n} \mid \boldsymbol{\theta}_{\lambda}\right)+\text { noise } \tag{4.2.2}
\end{equation*}
$$

We can write a linear form without much loss of generality, similar to their Equation 2:

$$
\begin{equation*}
f_{n \lambda}^{\prime}=\boldsymbol{\theta}_{\lambda}^{\boldsymbol{T}} \cdot \ell_{n}+\text { noise } \tag{4.2.3}
\end{equation*}
$$

where $\boldsymbol{\ell}_{n}$ is the $k$-element label vector belonging to star $n$, and $\boldsymbol{\theta}_{\lambda}^{T}$ is the $k$-element vector of coefficients. The noise term is:

$$
\begin{equation*}
\text { noise }=\xi\left(\sigma_{n \lambda}^{\prime 2}+s_{\lambda}^{2}\right) \tag{4.2.4}
\end{equation*}
$$

where $\sigma_{n \lambda}^{\prime 2}$ is the uncertainty of the slope for star $n$ at wavelength $\lambda, s_{\lambda}^{2}$ is the intrinsic scatter of the model at $\lambda$, and $\xi$ is a Gaussian random number. We proceed in two steps. In Section 4.2.1. we describe how we train the model at each wavelength by optimizing for the coefficients, $\boldsymbol{\theta}_{\lambda}$, and the scatter term, $s_{\lambda}^{2}$. Then, in Section 4.2.2. we describe how we use the trained model to determine $v \sin i$ from the flux-slope data alone.

### 4.2.1 Training Step

Once we have determined a suitable form for our linear model (see Section 4.2.3), we train the model parameters $\boldsymbol{\theta}_{\lambda}$ using the reference data from the California Kepler Survey. Like Ness et al. (2015), we build successively more complicated models in order to determine the necessary level of complexity. The simplest model is one which is linear in the labels:

$$
\begin{equation*}
\ell_{n} \equiv\left[1, \frac{\ell_{n 1}-\overline{\ell_{1}}}{2 \sigma_{\ell 1}}, \frac{\ell_{n 2}-\overline{\ell_{2}}}{2 \sigma_{\ell 2}}, \cdots, \frac{\ell_{n k}-\overline{\ell_{k}}}{2 \sigma_{\ell k}}\right] \tag{4.2.5}
\end{equation*}
$$

Here, the " 1 " is the intercept term, and subtracting the mean of each label 'centers' the data. The division by 2 standard deviations 'standardizes' the data, so that labels with larger dynamic ranges do not dominate the regression. We denote standardized, centered labels with the hat notation, $\hat{\ell}_{n k} \equiv \frac{\ell_{n k}-\overline{\ell_{k}}}{2 \sigma_{\ell k}}$. While the model is linear, the label vector need not be linear in the labels. In fact, it can be composed of any set of arbitrary functions of the labels. The model will nevertheless generate a flux-slope from a linear combination of these functions, weighted by the coefficients, $\boldsymbol{\theta}_{\lambda}$. On the other hand, the problem will become non-linear in the estimation step, as the elements of the label vector will no longer be linearly independent in label space.

We know the $f_{n \lambda}^{\prime}$ and $\sigma_{n \lambda}^{\prime}$ from the apStar spectra, and the $\ell_{n}$ from the CKS data, so we are solving for $\boldsymbol{\theta}_{\lambda}^{T}$ and $s_{\lambda}^{2}$. The log likelihood of the slope data in spectrum $n$ at pixel $\lambda$ is:

$$
\begin{equation*}
\ln p\left(f_{n \lambda}^{\prime} \mid \boldsymbol{\theta}_{\lambda}^{T}, \boldsymbol{\ell}_{n}, s_{\lambda}^{2}\right)=-\frac{1}{2} \frac{\left[f_{n \lambda}^{\prime}-\boldsymbol{\theta}_{\lambda}^{T} \cdot \boldsymbol{\ell}_{n}\right]^{2}}{s_{\lambda}^{2}+\sigma_{n \lambda}^{\prime}{ }^{2}}-\frac{1}{2} \ln \left(s_{\lambda}^{2}+\sigma_{n \lambda}^{\prime}{ }^{2}\right) \tag{4.2.6}
\end{equation*}
$$

At fixed $s_{\lambda}^{2}$, the maximum-likelihood $\boldsymbol{\theta}_{\lambda}^{T}$ can be solved for through straightforward linear algebra operations. For simplicity of implementation, we optimize for $s_{\lambda}^{2}$, and the resulting $\boldsymbol{\theta}_{\lambda}^{T}$, using the IDL AMOEBA code, which performs a downhill simplex optimization (Nelder \& Mead, 1965).

In practice, at each pixel $\lambda$, AMOEBA iteratively solves the matrix equation,

$$
\begin{equation*}
\boldsymbol{M} \boldsymbol{\theta}_{\lambda}=\boldsymbol{Y}_{\lambda} \tag{4.2.7}
\end{equation*}
$$

where $\boldsymbol{M}$ is the design matrix whose rows are the label vectors of each star, $\boldsymbol{\theta}_{\lambda}$ is the set of regression coefficients at pixel $\lambda$, and $\boldsymbol{Y}_{\lambda}$ is the data vector, composed from each star's spectral slope at pixel $\lambda$. More explicitly,

$$
\left[\begin{array}{cccc}
1 & \hat{\ell}_{n=1, k=1} & \cdots & \hat{\ell}_{n=1, k=k}  \tag{4.2.8}\\
1 & \hat{\ell}_{2,1} & \cdots & \hat{\ell}_{2, k} \\
& \vdots & \ddots & \vdots \\
1 & \hat{\ell}_{N, 1} & \cdots & \hat{\ell}_{N, k}
\end{array}\right]\left[\begin{array}{c}
\theta_{0} \\
\theta_{1} \\
\vdots \\
\theta_{k}
\end{array}\right]_{\lambda}=\left[\begin{array}{c}
f_{n=1}^{\prime} \\
f_{2}^{\prime} \\
\vdots \\
f_{N}^{\prime}
\end{array}\right]_{\lambda}
$$

With the assumptions of Gaussian likelihoods and no covariance between pixels (as in Eq. 4.2.6), the solution to this equation is:

$$
\begin{equation*}
\boldsymbol{\theta}_{\lambda}=\left[\boldsymbol{M}^{T} \boldsymbol{C}_{\lambda}^{-1} \boldsymbol{M}\right]^{-1} \boldsymbol{M}^{T} \boldsymbol{C}_{\lambda}^{-1} \boldsymbol{Y}_{\lambda} \tag{4.2.9}
\end{equation*}
$$

where $\boldsymbol{C}$ is the covariance matrix with diagonal entries:

$$
\begin{equation*}
\operatorname{diag}\left(\boldsymbol{C}_{\lambda}\right)=\left[\sigma_{1}^{\prime 2}+s^{2}, \sigma_{2}^{\prime 2}+s^{2}, \cdots, \sigma_{N}^{\prime 2}+s^{2}\right]_{\lambda} \tag{4.2.10}
\end{equation*}
$$

Each AMOEBA iteration takes a trial value of $s_{\lambda}^{2}$, which determines the covariance matrix $\boldsymbol{C}$. The coefficient vector $\boldsymbol{\theta}_{\lambda}$ is then given by Equation 4.2.9. The optimization converges on the value of $s_{\lambda}^{2}$ (and the resulting $\boldsymbol{\theta}_{\lambda}$ ) that maximizes the likelihood of the data, given the model. The scatter term represents the accuracy of the model at each wavelength. Wavelengths at which the data have large scatter about the model will be down-weighted by the additional error term.

The above optimization is carried out at each pixel, except those in the gaps between the three APOGEE chips. Pixels with little usable data (ie. those masked by the APOGEE bitmasks in most spectra), pixels where the AMOEBA optimization does not converge, and pixels where AMOEBA converges to $s_{\lambda}=0$ are flagged and masked. We set their coefficient vectors to $\mathbf{0}$, which enforces that the model is insensitive to the input labels at those wavelengths. Furthermore, we set their scatter terms to a high value ( $10^{3}$ times the scatter value of the worst good pixel). In this way, these bad pixels are automatically incorporated into the generative model, but do not contribute any information.

At the end of the training step, a coefficient vector and a scatter term has been derived for each pixel. This constitutes the generative model. In the estimation step, we determine the labels that generate the best-fit model to the observed spectra, given the trained model.

### 4.2.2 Estimation Step

In the estimation step, we seek to determine the label vector $\ell_{n}$ for stars without reference labels. Again, we maximize the log likelihood described in Eq. 4.2.6, but rather than summing the likelihood over the training spectra at each pixel, we now sum the likelihood over all pixels for each unlabeled spectrum. That is, in contrast to the matrix representation in Eqns. 4.2.7 and
4.2.8, we have:

$$
\begin{equation*}
\Theta \ell_{n}=\boldsymbol{Y}_{n} \tag{4.2.11}
\end{equation*}
$$

where $\boldsymbol{\Theta}$ is the design matrix with each row $i$ composed of the derived $\boldsymbol{\theta}_{\lambda=i}^{T}$. The matrices are of the form:

$$
\left[\begin{array}{llll}
\theta_{0, \lambda=1} & \theta_{1, \lambda=1} & \cdots & \theta_{k, \lambda=1}  \tag{4.2.12}\\
\theta_{0,2} & \theta_{1,2} & \cdots & \theta_{k, 2} \\
\vdots & \vdots & \ddots & \vdots \\
\theta_{0, \mathrm{Npix}} & \theta_{1, \mathrm{Npix}} & \cdots & \theta_{k, \mathrm{Npix}}
\end{array}\right]\left[\begin{array}{c}
1 \\
\hat{\ell}_{1} \\
\vdots \\
\hat{\ell}_{k}
\end{array}\right]_{n}=\left[\begin{array}{c}
f_{\lambda=1}^{\prime} \\
f_{2}^{\prime} \\
\vdots \\
f_{\mathrm{Npix}}^{\prime}
\end{array}\right]_{n}
$$

In the simplest case of a linear-in-labels model, the labels can again be determined by ordinary linear algebra. For more complicated models, such as the quadratic-in-labels model, this is not the case. As a general solution, we again use AMOEBA to optimize the labels. Each iteration of AMOEBA takes a trial set of stellar label values and constructs the $\boldsymbol{\ell}_{n}$, including non-linear terms, if necessary. The model is generated from the matrix multiplication of the label vector and the coefficient matrix from the training step. We calculate the likelihood of the data, given the model, and converge upon the set of labels that maximize the likelihood. We adopt the $v \sin i$ from the best-fit label vector as our $v \sin i$ estimate.

### 4.2.3 Model Selection Using Cross-Validation

Ness et al. (2015) found that for their implementation of The Cannon, the simplest sufficient model was quadratic in three labels $\left(T_{\text {eff }},[\mathrm{Fe} / \mathrm{H}]\right.$, and $\left.\log g\right)$, which was a label vector of the form:

$$
\begin{align*}
& {\left[1, \hat{\ell}_{n 1}^{2}, \hat{\ell}_{n 2}^{2}, \hat{\ell}_{n 3}^{2},\right.} \\
\ell_{n}=\quad & \hat{\ell}_{n 1} \hat{\ell}_{n 2}, \hat{\ell}_{n 2} \hat{\ell}_{n 3}, \hat{\ell}_{n 3} \hat{\ell}_{n 1},  \tag{4.2.13}\\
& \left.\hat{\ell}_{n 1}, \hat{\ell}_{n 2}, \hat{\ell}_{n 3}\right]
\end{align*}
$$

We tested the model's ability to recover $v \sin i$ using a 10 -fold cross-validation Ivezić et al. 2014) on the two training samples described in Section 4.1. We tested many combinations of those 3 fundamental parameters and $v \sin i$, along with a number of functions of the labels. Ultimately, we achieved the best results with a label vector that is quadratic in the three labels intrinsic to the star (identical to the Cannon's label vector in Equation 4.2.13) and independently quadratic in the extrinsic label $v \sin i$. In other words, denoting $v \sin i$ as label four, our label vector is:

$$
\begin{equation*}
\ell_{n} \equiv\left[\ell_{\mathrm{Eq}}\right. \tag{4.2.14}
\end{equation*}
$$

### 4.3 Results

### 4.3.1 Training Sample Cross-Validation Results

The $v \sin i$ results of our cross-validation on the training sample are shown in Figures 4.2 and 4.3. The error bars in both figures are the formal uncertainties derived from the parameter covariance matrix in the test step, and range from $0.1 \mathrm{~km} \mathrm{~s}^{-1}$ to $1 \mathrm{~km} \mathrm{~s}^{-1}$. These should be added in quadrature to the CKS $v \sin i$ uncertainty of $1 \mathrm{~km} \mathrm{~s}^{-1}$. Figure 4.2 shows the results for the subset of the training sample without artificially broadened non-detections. Because the data is sparse at $v \sin i \gtrsim 9 \mathrm{~km} \mathrm{~s}^{-1}$, the high $v \sin i$ results are biased. Figure 4.3 shows the results for the full training set, including the artificially broadened spectra. Adding the synthetic fast rotators (red points) dramatically improved the model's ability to recover large $v \sin i$.


Figure 4.2 Results of 10 -fold cross-validation on the subset of the training sample that contains only the stars which were measured to be detectably rotating by CKS. The model is quadratic in the three fundamental labels, and independently quadratic in $v \sin i$. Due to the sparsity of high $v \sin i$ training data, the model is noticeably biased above $v \sin i \sim 9 \mathrm{~km} \mathrm{~s}^{-1}$.


Figure 4.3 Results of 10 -fold cross-validation on the full training sample, including artificially broadened spectra of non-rotators. The broadened spectra are plotted in red. The addition of data with $v \sin i$ between $10 \mathrm{~km} \mathrm{~s}^{-1}$ and $15 \mathrm{~km} \mathrm{~s}^{-1}$ improves the model's performance at high $v \sin i$.

We show two examples of our data in comparison with the generated models in Figure 4.5. The spectral slope is displayed in the upper panel in black, arbitrarily scaled to range from -1 to 1 . The model generated from the trained parameters (with these spectra held out from the training set), and from which we infer $v \sin i$, is overplotted in red. The bottom panel shows the residuals, color-coded by the scatter term associated with each wavelength. Masked pixels are highlighted by vertical gray bands. The models generally fit the data well. Areas where the residuals are large have large scatter terms, which automatically down-weight those wavelengths in the likelihood calculation. One star, KOI 3052, appeared to be an obvious outlier. Details of what we observed and how we treated this star are described in Appendix B.

The first derivatives of flux-slope model with respect to the labels + are shown in Figure 4.4 A portion of the model generated from the median of the label values is shown in the upper panel. The lower panel displays the linear coefficients of the model for each of the stellar labels.


Figure 4.4 A portion of the model and its first-order derivatives with respect to the labels. The upper panel is the flux-slope model generated from the median labels. The lower panel shows the linear coefficients for each of the labels, color-coded by label.


Figure 4.5 A portion of the spectral slope data, along with the generated model, for two of our training spectra. The spectral slope is arbitrarily scaled to range from -1 to 1 , and plotted in black in the upper panels. The model is generated by the trained parameters with the spectra in question left out from the training set. It is overplotted in red, with masked pixels highlighted by the vertical gray bands. The residuals are shown in the bottom panel, and color-coded by the scatter term associated with each wavelength.

The overall results of the 10 -fold cross-validation of the training sample are summarized in Figure 4.6. The residuals are plotted in various projections of label space. Other than in the $[\mathrm{M} / \mathrm{H}]-T_{\text {eff }}$ space, the residuals appear largely uncorrelated between labels. The distribution of residuals in each individual label are illustrated in the histograms. The distributions are apparently non-Gaussian, but still symmetric and unimodal. They exhibit only a small amount of bias, and have uncertainties similar to those of the CKS reference data.

Figure 4.7 summarizes the precision and accuracy of $v \sin i$ estimated by our approach, as a function of $v \sin i$. The data are binned by CKS $v \sin i$ in $1 \mathrm{~km} \mathrm{~s}^{-1}$ increments, and the mean values of the model $v \sin i$ are plotted versus the mean CKS $v \sin i$. The horizontal error bars represent the uncertainty in the mean of the CKS $v \sin i$, and the vertical error bars represent the scatter in the model $v \sin i$ (standard deviation of the residuals). The data from the detections-only subsample are plotted in black, and the data from the full training sample are plotted in red. The scatter in the recovered $v \sin i$ is approximately $1 \mathrm{~km} \mathrm{~s}^{-1}$ down to $v \sin i \sim 2 \mathrm{~km} \mathrm{~s}^{-1}$.


Figure 4.6 Residuals from the 10 -fold cross validation of the training data, plotted in various projections of the labels. The scatter plots show that the residuals are typically not correlated between labels. There does, however, appear to be some correlation in the $[\mathrm{Fe} / \mathrm{H}]-T_{\text {eff }}$ plane. The contours denote 1- $\sigma$ confidence levels. The residuals in each label are shown in the histograms. Overall, the residuals are only slightly biased, have symmetrical distributions, and uncertainty comparable to that of the input labels.


Figure 4.7 Results from cross-validation of the training sample, binned in units of $1 \mathrm{~km} \mathrm{~s}^{-1}$. The binned data from the training sample subset are plotted in black, and the data from the full training sample, including the artificially broadened spectra of non-rotators, are plotted in red. The horizontal error bars show the uncertainty in the CKS mean $v \sin i$. The vertical error bars show the standard deviation of the residuals in each bin.

### 4.3.2 Survey Sample Results

Once the model determination was complete, we carried out the full analysis. We trained the model parameters $\left(\boldsymbol{\Theta}, s^{2}\right)$ using the full training sample, and then empirically determined $v \sin i$ for the 27,000 stars in the survey sample. The $v \sin i$ results are shown in Figure 4.8, plotted against the $v \sin i$ estimated by ASPCAP. Although there are clear outliers, and there is structure in the residuals, the model $v \sin i$ are generally consistent with the ASPCAP $v \sin i$. The RMS of the residuals is $1.2 \mathrm{~km} \mathrm{~s}^{-1}$. In Figure 4.9 , we show that the reduced $\chi^{2}$ exhibits no systematic dependence on $v \sin i$. The right panel of Figure 4.9 shows the overall distribution of reduced $\chi^{2}$ for the full survey sample. The dashed histogram shows the reduced $\chi^{2}$ distribution for the training sample, obtained from the cross-validation step. In short, our method can reliably estimate $v \sin i$ in the parameter space of the training sample at an uncertainty only slightly greater than that of the CKS reference $v \sin i$.The reduced $\chi^{2}$ of the models suggest that the flux-slope based generative model is good description of the APOGEE observations.


Figure 4.8 Results of the full analysis on 27, 000 APOGEE survey spectra, using the full training sample of 270 APOGEE/CKS stars. The APOGEE spectra and the CKS labels of the training stars are used to train the model. The trained model is then used to estimate $v \sin i$ for the 27,000 stars in the survey sample. The model $v \sin i$ estimates are plotted versus the ASPCAP $v \sin i$. The RMS of the residuals is $1.2 \mathrm{~km} \mathrm{~s}^{-1}$


Figure 4.9 The reduced $\chi^{2}$ of the generated models. Left: The reduced $\chi^{2}$ of the models from the APOGEE survey sample of 27,000 stars, as a function of $v \sin i$. The $\chi^{2}$ distribution does not display any $v \sin i$-dependence. Right: The overall reduced $\chi^{2}$ distribution of both the training data models (generated in the 10 -fold cross validation), and the full survey sample models. The dashed line indicates the training data distribution, and the gray-filled histogram indicates the survey sample distribution.

### 4.4 Discussion and Conclusion

This data-driven approach to $v \sin i$ estimation is a powerful tool for the study of stellar angular momentum. Especially in this era of large scale spectroscopic surveys, our technique provides a (largely) model-independent, computationally inexpensive means of measuring $v \sin i$ for large numbers of stars at once. Using a UNIX workstation with four 3.4 GHz CPUs, estimating $v \sin i$ for the 27,000 stars in the test sample took only 3.3 hours ( $\sim 0.44$ seconds per spectrum), and roughly half of that time was spent on calculating the slope and slope error with our custom S-G filter, which could be optimized to perform faster. This technique is orders of magnitude faster than fitting suites of theoretical templates, for example. Potentially, this method could also be used to cross-calibrate data sets from different surveys, which may have conflicting measurements due to the use of different techniques or stellar models.

We have already discussed some of the limitations of the framework we have chosen, such as the independent treatment of pixels which are, in reality, correlated. Our use of the spectral line slope, rather than spectral flux, is a shortcut that allowed us to capture some of the relationship between the flux at each pixel within this framework. The slope at each pixel incorporates flux information from a 3-pixel window, which is roughly the width of a $\sim 7 \mathrm{~km} \mathrm{~s}^{-1}$ broadening kernel at the APOGEE resolution and sampling. The model described here may not be sufficient at very high $v \sin i$, where the broadening kernel is much larger than 3-pixels.

Another possible limitation may be the use of polynomials as a basis for our model. The broadening and blending of spectral lines results in a complicated relationship between $v \sin i$ and line slope, which can vary widely between pixels. Although we found no functions of the labels that performed better than the quadratic model that we adopted, there is periodic structure in the residuals that is likely due to the polynomial form of the model. Furthermore, it is not clear that a quadratic model would work well for data extending to higher $v \sin i$. One possible improvement would be to use a Gaussian Process regression, which would obviate the need to
specify a functional form for the model. The trade-off is that Gaussian Process regression is more computationally expensive (naively $\mathcal{O}\left(N^{3}\right)$ ).

Another important feature to consider is that the training step does not incorporate label errors. This could aversely affect the accuracy of the model, particularly at low $v \sin i$, (e.g., $v \sin i_{\mathrm{CKS}}=1 \pm 1 \mathrm{~km} \mathrm{~s}^{-1}$ ), where the label errors are a non-negligible fraction of the reference value. A more sophisticated approach could treat the labels as probability distributions and propagate those uncertainties into posterior distributions through the use of Markov Chain Monte Carlo techniques. Again, such improvements would come at the cost of decreased speed.

Finally, there are some limitations inherent to the data-driven aspect of this approach, regardless of the model complexity. One is the necessity of high-fidelity training data spanning label space. The amount of training data needed increases exponentially with the number of labels. Furthermore, the local density of training data within the spanned parameter space can limit the model's accuracy. In the case of $v \sin i$, we have shown that artificially-broadened non-detections can be used to supplement the bona fide rotators at $v \sin i$ where training data is sparse.

In this work, we demonstrated a novel data-driven method for estimating $v \sin i$ from stellar spectra. We used the first derivative of APOGEE spectra, along with high-fidelity parameter estimates from the California Kepler Survey, to train a generative model from which we estimated $v \sin i$ for 27,000 APOGEE F-G-K dwarfs. In the range $0 \leq v \sin i \leq 15 \mathrm{~km} \mathrm{~s}^{-1}$, the model produced $v \sin i$ estimates that agreed with ASPCAP measurements to within $1.2 \mathrm{~km} \mathrm{~s}^{-1}$, in a fraction of the time required by standard $v \sin i$ measurement techniques.

## Chapter 5

## Conclusion

I presented results from our analysis of high-resolution spectroscopic observations of 714 M dwarfs obtained by the SDSS APOGEE survey. We derived estimates of projected rotation velocity, $v \sin i$, by fitting a large suite of rotationally broadened theoretical templates to these observations. We analyzed these spectra in chunks and use a Gaussian Mixture Model approach to estimate our measurement uncertainty based on individual fits to 28 chunks of each spectrum. For each of our targets, we estimated an overall uncertainty of 1 to $3 \mathrm{~km} \mathrm{~s}^{-1}$ and find good agreement between our measurements and a small number of previously published values.

One interesting feature of our analysis is the comparatively comparatively high fraction of rotators in our sample below $T_{\text {eff }} \sim 3000 \mathrm{~K}$. Approximately $35 \%$ of the these coolest dwarfs in our sample have detections of rotation given our conservative $8 \mathrm{~km} \mathrm{~s}^{-1}$ detection limit. This is consistent with the trend toward faster rotation at later spectral types observed in earlier $v \sin i$ analyses.

Through a Monte Carlo simulation, we attempted to make a direct comparison between the overall distribution of our projected rotational velocities and the photometric periods published in the literature. While we do find broad agreement in that rotation fraction increases with lower stellar temperature in all data sets, for $T_{\text {eff }}<3200 \mathrm{~K}$ we see a rotation fraction that is a factor of
$\sim 2$ higher than the rotation distribution that would be inferred from the MEarth photometry.
In examining this discrepancy, we consider the possibility that the detection threshold is higher than assumed. If, for example, our detection threshold were $v \sin i=12 \mathrm{~km} \mathrm{~s}^{-1}$, then the fraction of rotators at $T_{\text {eff }}<3200 K$ would be almost entirely consistent with the inferred rotational velocities from MEarth photometry. However, our data-driven analysis of F-G-K dwarfs showed APOGEE resolution is sufficient for estimating $v \sin i$ down to about $2 \mathrm{~km} \mathrm{~s}^{-1}$. We are confident that a threshold of $8 \mathrm{~km} \mathrm{~s}^{-1}$ is adequate. The difficulty in estimating low $v \sin i$ for low-mass stars may be intrinsic to the stars due to pressure broadening and line blending, although we found no evidence to support this. Systematic errors introduced by the theoretical template spectra are likely a contributing factor in this difficulty.

I also detailed our development of a novel data-driven approach to estimating $v \sin i$. We developed it using F-G-K dwarfs, but it could also be applied to M dwarfs, given adequate training data. This would minimize systematic errors due to imperfect theoretical templates. Especially with upcoming surveys like LSST producing massive amounts of data, our technique's computationally efficiency makes it a timely and valuable addition to the set of $v \sin i$ measurement techniques. Using a UNIX workstation with four 3.4 GHz CPUs, estimating $v \sin i$ for the 27,000 stars in the test sample took only 3.3 hours ( $\sim 0.44$ seconds per spectrum), and that could likely be cut in half with further optimization of the code. This is one to two orders of magnitude faster than our template-fitting technique.

There are some promising avenues to consider for future developments of this work. First of all, the recent (April, 2018) release of GAIA astrometry data opens up the possibility for constraining the ages of the APOGEE M dwarfs. Because the sample is not coeval, we were unable to determine whether or not the high fraction of late $M$ dwarf rotators was due to their slow spindown or simply a selection bias. Similarly, our best estimates of age based on kinematics suggested that our sample was similar to the MEarth sample, but perhaps a bit younger. A significantly younger APOGEE sample may explain the observed disagreement in rotation fraction. Although
kinematic age estimates are fairly uncertain, they would nevertheless provide valuable information in determining the rotational evolution of low-mass stars, and all of the other processes involved.

I would also be very interested in modifying our data-driven $v \sin i$ technique to include Gaussian Process regression. This extremely flexible regression method seems like the perfect fit for modeling the slope of spectral lines, which, especially at late types with strong line blending, is a very complicated function of $v \sin i$. Also, incorporating this into more general frameworks like The Cannon, or Starfish (Czekala et al., 2015) could make for a powerful tool for spectroscopic inference.

## Appendix

## Appendix A

## Calculating Slope with the

## Savitsky-Golay Filter

The S-G filter effectively performs a least-squares polynomial fit of order $m$ to the data in a window of width $w$, centered at each pixel. The $0^{\text {th }}$ order coefficient gives the value of the smoothed data at the central pixel, and successive coefficients yield the derivatives of the smoothed data. This is a computationally efficient operation because the fit can be performed using linear algebra. The IDL library function SAVGOL returns the desired S-G kernel, but does not incorporate slope error estimates. We wrote our own S-G filter function, which smooths and differentiates the spectrum, and simultaneously returns slope error estimates derived from the input APOGEE flux errors. The function performs the following calculation.

We describe the polynomial fit about a given pixel $\lambda_{0}$, as

$$
\begin{equation*}
Y(z)=a_{0}+a_{1} z+\cdots+a_{m} z^{m} \tag{A.0.1}
\end{equation*}
$$

where $z$ is a coordinate describing the distance from the central point $\lambda_{0}, z=\frac{\lambda-\lambda_{0}}{\Delta \lambda}$. The coeffi-
cients are found by solving

$$
\begin{equation*}
\mathbf{J} \boldsymbol{a}=\boldsymbol{y} \tag{A.0.2}
\end{equation*}
$$

where $\mathbf{J}$ is a matrix whose columns are $\left[\mathbf{1}, \boldsymbol{z}, \boldsymbol{z}^{2}, \ldots, \boldsymbol{z}^{m}\right]$, and $\boldsymbol{y}$ is the data in the window of width $w$, centered at $\lambda_{0}$.

The solution, given perfect data, is:

$$
\begin{equation*}
\boldsymbol{a}=\left(\mathbf{J}^{T} \mathbf{J}\right)^{-1} \mathbf{J}^{T} \boldsymbol{y} \tag{A.0.3}
\end{equation*}
$$

We incorporate the flux errors through the covariance matrix in the following way:

$$
\begin{equation*}
\boldsymbol{a}=\left(\mathbf{J}^{T} \mathbf{C}^{-1} \mathbf{J}\right)^{-1} \mathbf{J}^{T} \mathbf{C}^{-1} \boldsymbol{y} \equiv \mathbf{K} \boldsymbol{y} \tag{A.0.4}
\end{equation*}
$$

The result, $\mathbf{K}$, is an $m$ degree by $w$ pixel matrix. The $j^{\text {th }}$ row of $\mathbf{K}$ is a Savitsky-Golay convolution kernel corresponding to order $j$. The smoothed (and optionally differentiated) spectrum is given by the convolution of the kernel of desired order and the data, scaled by a normalization term for orders $m \geq 1$. That is,

$$
\begin{equation*}
\frac{\mathrm{d}^{m} f}{\mathrm{~d} \lambda^{m}}=\frac{m!}{\Delta \lambda^{m}} \mathbf{K}_{m} * I(\lambda) \tag{A.0.5}
\end{equation*}
$$

where $m$ is the desired order of differentiation and $\Delta \lambda$ is the wavelength spacing of the spectrum. Specifically, our first derivative is given by:

$$
\begin{equation*}
\frac{\mathrm{d} f}{\mathrm{~d} \lambda}=\frac{1}{\Delta \lambda} \mathbf{K}_{1} * I(\lambda) \tag{A.0.6}
\end{equation*}
$$

The uncertainties of the filtered data of order $m$ at wavelength $\lambda$ are the diagonal elements of the coefficient covariance matrix:

$$
\begin{equation*}
\Sigma_{\lambda}=\left(\mathbf{J}^{T} \mathbf{C}^{-1} \mathbf{J}\right)^{-1} \tag{A.0.7}
\end{equation*}
$$

Therefore, the variance of the slope at pixel $\lambda, \sigma_{\lambda}^{\prime 2}$, is element $[1,1]$ of $\Sigma_{\lambda}$. We compared the results of our custom S-G filter with those output by the IDL SAVGOL function, and found them to be in agreement. We tested our error estimates against those computed by two other methods. The first method was the straight-forward propagation of error, assuming a simple calculation of the slope $f_{\lambda}^{\prime}=\frac{f_{\lambda, i+1}-f_{\lambda, i-1}}{2 \Delta \lambda}$. The second method was the sampling of the flux errors through a Monte Carlo approach. For each spectrum, we generated $1 \times 10^{3}$ realizations with random fluctuations dictated by the $A P O G E E$ flux error vectors. For each realization, we computed the slope at each pixel using the S-G filter, and took the standard deviation of the slopes as the slope error. All three methods showed broad agreement.

We use the $A P O G E E$ pixel masks (HDU 3 of the apStar files) to identify bad pixels in the apStar spectra, which are effectively masked out in the flux slope estimates. Like Casey et al. (2016), we use all mask bits other than 9,10 , and 11, which correspond to persistence effects. We note that for the purpose of this step, we set the error in masked pixels to $\sigma_{\lambda \text { masked }}=1$, and cap the error in unmasked pixels at the same value. We also cap the flux in masked pixels at 1. Although we are artificially reducing flux errors, we argue that all pixels in a continuumnormalized spectrum should, astrophysically speaking, have flux between 0 and 1 (modulo noise). Capping the flux error at 1 allows us to compute slopes that are still extremely uncertain while avoiding floating point overflows.

## Appendix B

## KOI 3052

One star in our training sample appeared as an obvious outlier. For KOI 3052, the CKS pipeline measured $v \sin i_{\mathrm{CKS}}=3.4 \mathrm{~km} \mathrm{~s}^{-1}$, and we consistently measured $v \sin i_{\text {MODEL }} \sim 10 \mathrm{~km} \mathrm{~s}^{-1}$. While the CKS estimates are made from higher-resolution spectra, the ASPCAP estimate supported our finding with $v \sin i_{\text {ASPCAP }}=9.7 \mathrm{~km} \mathrm{~s}^{-1}$. Upon further visual inspection, the spectra suggest that both estimates are correct. That is, the CKS spectrum appears to be relatively unbroadened, while the APOGEE spectrum appears to be noticeably broadened. The period of KOI 3052 has been measured in several analyses of Kepler photometric data to be between $P=27.75$ days and $P=30.17$ days McQuillan et al. 2013, Reinhold et al. 2013, Walkowicz \& Basri 2013, Mazeh et al. 2015). Using the CKS estimate of $R_{*}=0.83 \mathrm{R}_{\odot}$, the periods suggest an equatorial velocity of $v_{\mathrm{eq}} \sim 1.4 \mathrm{~km} \mathrm{~s}^{-1}$. Because of the projection effect, it must be the case that $v \sin i \leq v_{\mathrm{eq}}$. Although unlikely, it is possible that the measured rotation period is a harmonic of the 'true' rotation period, as a period of $P \sim 14$ days would be within one sigma of the CKS $v \sin i$. Of course, the CKS $v \sin i$ may be overestimated, as it is close to the resolution-limited detection floor of the CKS spectra. The much broader APOGEE spectrum may be the result of an unresolved binary companion. Because the analysis presented here relies upon the agreement between the labels and the data, and because the ASPCAP $v \sin i$ appears consistent with the observed near-
infrared spectrum for KOI 3052, we used the ASPCAP $v \sin i$ value as the reference label for this star only.

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[^0]:    ${ }^{1}$ http://www.astro.caltech.edu/~george/ay1/lec_pdf/Ay1_Lec03.pdf

[^1]:    ${ }^{2}$ http://www.eso.org/public/teles-instr/lasilla/36/nirps/

[^2]:    ${ }^{1}$ This chapter is a slightly modified form of the paper "The Rotation of M Dwarfs Observed by the Apache Point Galactic Evolution Experiment" (Gilhool et al. 2018), published in the Astronomical Journal. The work presented herein was performed by Steven Gilhool under the guidance of Cullen Blake.

[^3]:    2 https://phoenix.ens-lyon.fr/Grids/BT-Settl/AGSS2009/SPECTRA/

[^4]:    $\sqrt[3]{\text { http://www.sdss.org/dr13/software/idlutils/ }}$

[^5]:    ${ }^{1}$ This chapter is a modified version of the paper "A Data-Driven Technique for Measuring Stellar Rotation" by Steven Gilhool and Cullen Blake. The paper was submitted to the AAS Journals in March, 2018. The work described herein was performed by Steven Gilhool under the guidance of Cullen Blake.

[^6]:    2 http://www.sdss.org/dr14/irspec/parameters/

