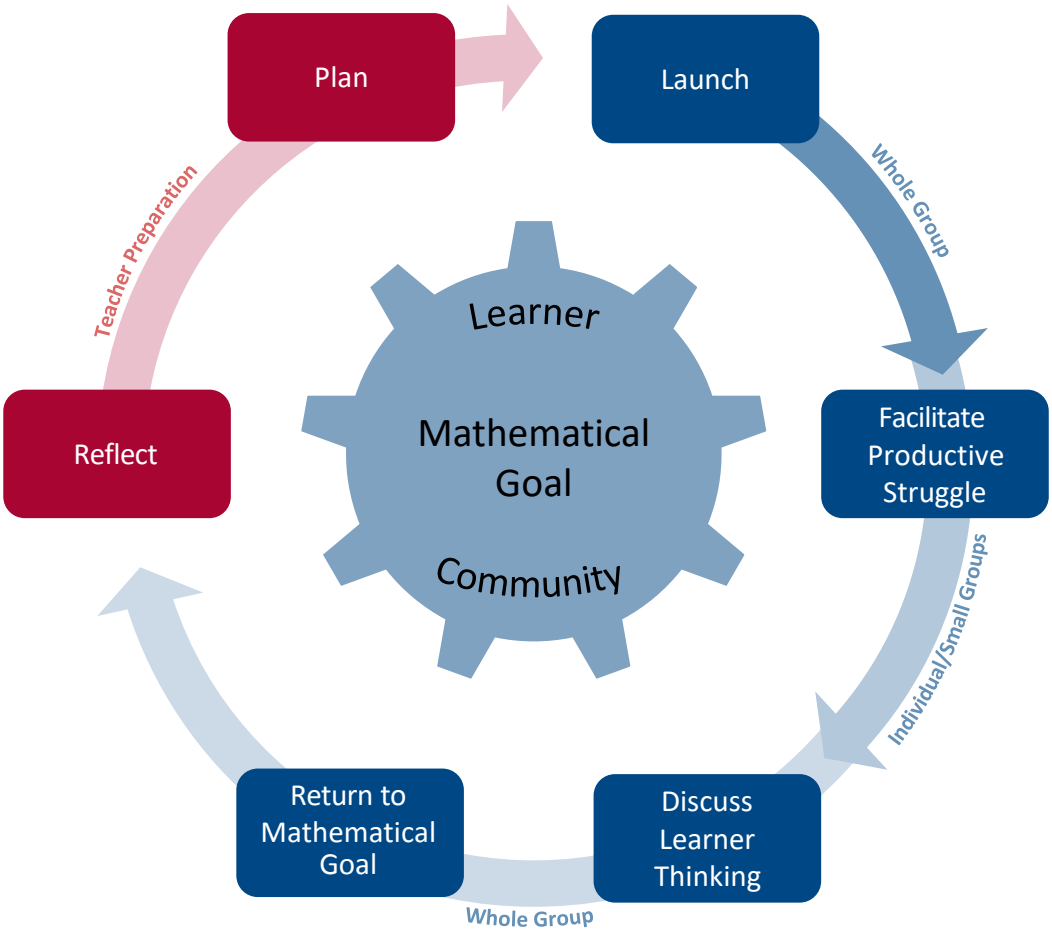
RESPONSIVE MATH TEACHING

RMT

PLANNING AND COACHING PROTOCOL



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Penn GSE
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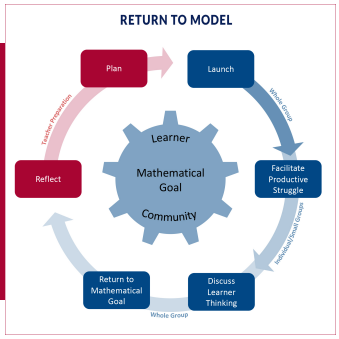
PLAN



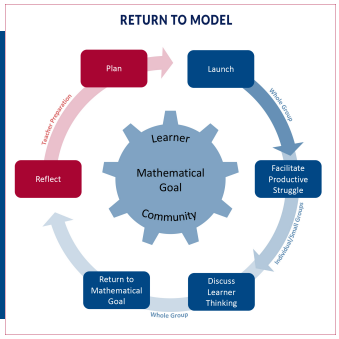
- **What is your mathematical goal?**

- **Do you need to make adjustments based on evidence from prior lessons?**

- **What task will you use? Does it need revisions?**
  - Is it low-floor, high-ceiling? Does it allow for multiple approaches? If not, how can you open the problem up?
  - Will it help get to your mathematical goal?
  - Is it accessible to your students? Possible revisions include:
    - changing the numbers
    - changing the context
    - changing the conditions

- **What do you anticipate students will think?**
  - What *strategies/ideas* will students use/have?
  - What *stumbling blocks* are they likely to encounter?
  - What *representations* could you use to help make

LAUNCH



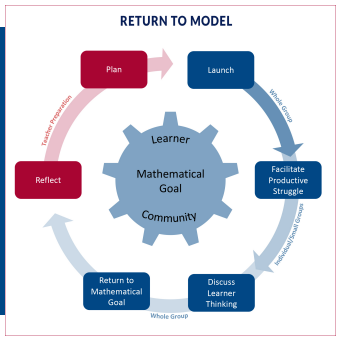
- How will you help learners begin to **make sense of the task individually**?

- How will you **establish collective understanding** of the task and context (without telling them how to do it)? Think about:
  - eliciting prior knowledge, clarifying context and vocabulary, and representing important information

- How will you **build a bridge to solving**? Think about:
  - communicating expectations for work, products, and tools
  - assessing individual and class readiness to engage in productive struggle

LAUNCH	
*How will you help learners begin to **make sense of the task individually**?*	
Give multiple ways to **process the task** (e.g. hearing it read out loud, reading silently, choral reading, different voices) When developmentally appropriate, encourage learners to **write notes** about their thinking	• *Listen carefully. I'm going to read it twice.* • *Your only job right now is to read the problem and try to make sense of it yourself.* • *Feel free to write on the problem to help you make sense of it.* • *Lizzy, will you read the problem out loud?*
Have learners describe task or problem **in their own words**	• *Who wants to try summarizing the problem in their own words?* • *What are we being asked to figure out?*
Ask learners to **visualize the problem** and describe what is happening in the situation	• *What is happening in this situation? What's going on here?*
*How will you **establish collective understanding** of the task and context (without telling them how to do it)?*	
Elicit and/or make connections to **prior knowledge and experience** and fill gaps as needed	• *Who here has cooked with a family member before? Did you use measuring cups? Can you share a personal story about your experience?* • *This problem uses the term regular polygon. We learned about them last month when we did the ____ activity. Talk to your partner about what it means to be a regular polygon.*
Provide opportunity for **clarification** of task and vocabulary (scaffolded or open-ended questions)	• *What do you think it means when it says a 'fair share'?* • *Is this the kind of problem that could have more than one solution, or are we looking for a single correct solution?*
Record group sense-making of problem	• *List important information/constraints the group agrees to on the board.*
If needed, ask **explicit questions** *without* specifying an equation, strategy, or operation for solving	• *It asks us to find the largest rectangle. Could the answer be a square? Why or why not?*
*How will you **build a bridge to solving**?*	
Set expectations for working (individuals, partners, groups, timing), products (solution AND justification), and available tools	• *Take a few minutes to start solving the problem yourself. In a few minutes I'll ask you to start talking with someone else about your ideas.* • *Think about using the counters or the base ten blocks up here if they might help you.* • *So you have two jobs. First--to find out how much money he started with. Second—to have some kind of work that shows your thinking and proves your solution.* • *In fifteen minutes, I'm going to ask each group to share what they are finding.*
Assess individual and class readiness to engage in productive struggle on the problem	• *Consider: Is there anyone who seems like they may need extra support understanding the problem or getting started?*

FACILITATE PRODUCTIVE STRUGGLE



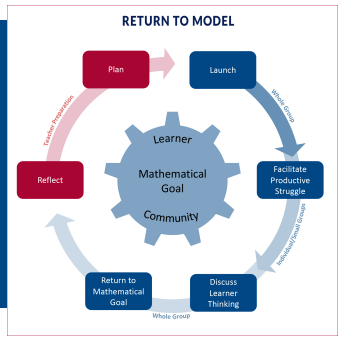
- What opportunities will learners have for **collaboration**?
  - Be specific—what will this look like?

- How will you **support learner thinking without lowering the cognitive demand**?
  - Consider getting learners started
  - Consider getting learners "unstuck"
  - Consider pushing learners' ideas further
  - Consider tools/supplies/resources
  - Consider alternate versions of the task

- How will you **make decisions about what to do in the next parts of the lesson**?
  - Keep track of student strategies and misconceptions
  - Remind yourself to look for opportunities to assign competence

FACILITATE PRODUCTIVE STRUGGLE	
*What opportunities will students have for **collaboration**?*	
Provide opportunities for students to **work collaboratively**, in pairs or small groups	• *Talk with your partner about your ideas and their ideas. Are they thinking about the problem in a similar or different way?* • *You two have similar strategies. Why don't you see if you can help each other to find a solution?*
Scaffold collaboration between students	• *What do you think about what Keira just said?* • *That's a great question. Why don't you ask your partner that question and see what she thinks.*
*How will you **support student thinking without lowering the cognitive demand**?*	
Help students **understand the problem** by rereading the problem, focusing on understanding the situation and/or clarifying terms or contexts	• *Read the problem out loud to me.* • *What do we know?* • *Try explaining what's going on in your own words.*
Help students **get started or unstuck by:** • using models, diagrams or acting out the problem • thinking about a similar, simpler scenario • building on their informal and/or incomplete strategies • asking questions and/or providing just enough information (scaffolding) • pointing out an approach that has helped another group get started	• *Let's act it out* • *Can you draw a picture of what is happening?* • *What are you thinking about?* • *What are you trying to find out?* • *What's hard or standing in your way?* • *Does this remind you of anything we've done before?* • *Let's all come back together for a moment. Jamal has noticed an important pattern.*
Provide appropriate **tools, resources, and supplies** for students to engage with task	• *Remember there are counters if you would like to use them.* • *What are some materials you could use to help you solve this?*
For students that need it, provide **alternate versions** of the task (simpler/more complex) or follow-up tasks	• *Vary the numbers, the problem structure, or the context* • *Provide graphic organizers or sentence frames* • *Provide supports "just in time"—when a student demonstrates the need because the struggle is no longer productive*
Coach mathematical participation	• *What would happen if you used a different number? Let's try!* • *See if you can convince your groupmates that it works*
*How will you **make decisions about what to do in the next parts of the lesson**?*	
Circulate to visit, listen in on, and monitor	• *Try to check in with each group, asking questions to gauge understanding* • *Monitor time, progress, and group dynamics*
Determine what **strategies** students are using to solve the task and keep a mental or written record	• *Look for a range of student strategies you could share with the class (perhaps take a picture)* • *Look for common misconceptions that need addressing*
Look for opportunities to **assign competence,** particularly to low-status students**,** by giving praise that is public, specific, and meaningful to the task.	• *I noticed Eric was doing something really important—he was looking for patterns.* • *I love how Jada is trying out different combinations!*

DISCUSS LEARNER THINKING

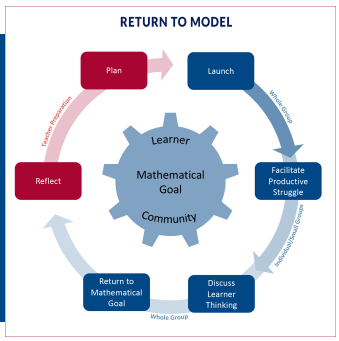


- How will you decide **what will be shared, by whom and in what order**?

- How will you **facilitate and support the sharing and discussion of work and strategies**?

DISCUSS LEARNER THINKING	
*How will you decide **what will be shared, by whom and in what order**?*	
Consider the **type of mathematical goa**l you have identified	• *Build or reinforce a new conceptual understanding* • *Connect a conceptual understanding to a procedure* • *Develop procedural fluency (with understanding)*
Using this knowledge, strategically **select** solution **approaches** to make visible to the class	• *Related and/or contrasting strategies* • *Common misconceptions* • *Useful notation or organization*
Intentionally **sequence or organize** solutions to lead to the mathematical goal.	• *Accessible/concrete to efficient/abstract* • *Common to unconventional* • *Exhibition protocols*
Attend to **equitable participation, identities, and status**. Consider who will represent work: the facilitator, a student or multiple students .	• *Whose work will be represented and why?* • *Whose work and/or voices are not represented?* • *Allow everyone's work to be shown? (e.g., gallery walks)*
*How will you **facilitate and support the sharing and discussion of work and strategies**?*	
Coach students **on how** to present their ideas and solutions to peers	• *Turn and face the class so that we can hear and see you.* • *Could you stand to the side so we can see your drawing?* • *Could you say that again so everyone can hear your ideas?*
Represent learner strategies visually by projecting or recording learner solutions using pictures, equations, and/or words and orienting learners to parts of solutions	• *Use visual models (number lines and area models) to make strategies accessible.* • *Use colors, circling, arrows, etc. to highlight parts of solutions.*
Cultivate **rich learner explanations** by providing wait time, probing, and pressing learner thinking • follow up with "why" and "how" • ask learners to unpack a strategy, provide evidence or counterexamples, and/or explain why a strategy works	• *Can you say more about that?* • *Why do you think Tiffany's method works?* • *Tell us how your strategy works.* • *I'm wondering if this method will always work?* • *I want to hear from someone else who hasn't spoken yet*
Engage learners in **making sense of each other's thinking** by: • revoicing for clarity • prompting and scaffolding participation from learners • asking learners to rephrase, agree or disagree, or add on to another learner's response	• *I think what you're saying is _____. Did I get your idea right?* • *Do you agree with Joy's idea? Or have a question for her?* • *Who can try to explain her idea in your own words?* • *What do you think he means when he says the bigger one?* • *Compliment or question*
Intentionally make space for and **assign competence** to marginalized and/or low-status learner contributions by: • acknowledging mathematical practices • attributing an idea to a specific learner • treating errors as an opportunity for learning	• *That mistake helped us all to better understand this!* • *Rakeem's method is really important for us to think about.* • *Lila, you are trying different approaches like a mathematician!* • *Let's see if we can build on Lila's great idea.*
Introduce standard **language, notation, and/or models** in response to learner ideas and questions	• *Here is one way to visualize what Anna is doing. [Teacher draws a model.] How do you see Anna's strategy in this picture?* • *How could we write that with equations?* • *Did you notice how in John's pattern we keep multiplying numbers by themselves? We call these square numbers. Mathematicians have a special notation for this.*

RETURN TO MATHEMATICAL GOAL

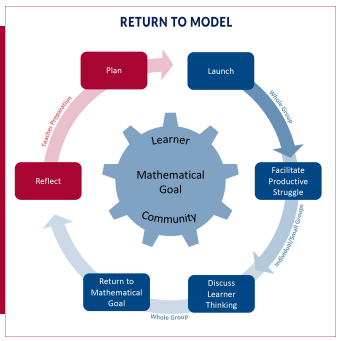


- What are the connections you need learners to make to **explicitly bridge their thinking and strategies to the important mathematical ideas**? What is your plan for helping learners to make these **connections**?

- How will you **help learners to solidify connections and consolidate their understanding**? How will you formatively assess learners' understanding?

RETURN TO MATHEMATICAL GOAL	
*What are the connections you need learners to make to **explicitly bridge their thinking and strategies to the important mathematical ideas**? What is your plan for helping learners to make these **connections**?*	
Facilitate a **whole class debrief**	*Try to surface and highlight:* • *an understanding of the main mathematical ideas of the lesson* • *any remaining questions or uncertainties* • *connections to other concepts* • *real-world applications*
Help learners to **formalize** their ideas by generalizing patterns and relationships, articulating theories and/or proving or disproving conjectures	• *Do you see any patterns?* • *Will that strategy always work? How do you know?* • *Can you think of an example where that wouldn't be true?* • *Why do you think that makes sense?* • *Let's look at the table we made. What did we discover about the relationship between the value of a digit when it is in the tens place, and the value of that same digit when it is in the hundreds place?*
Connect **procedures to concepts**	• *Let's take a look at Chris' drawing and at Jamir's equation. How are they connected to each other?* • *Is there a more efficient way to record those steps?* • *Now I'm going to show you how we can write that a different way.*
*How will you **help learners to solidify connections and consolidate their understanding**? How will you formatively assess learners' understanding? Choose one or more of the options below.*	
Give learners the opportunity to **revise their thinking** on the task	*Revisions to rough draft thinking might include:* • *Making the work clear and understandable* • *Making a convincing argument* • *Changing the answer, or adding additional solutions* • *Trying a different strategy to get to the solution* • *Looking for patterns or generalizing a rule*
Engage learners in **written reflection**	• *General Feedback: What is something you learned today? What do you still have questions about?* • *Focused on the mathematical goal: In your own words, what makes a relationship proportional?* • *Focused on identity: How do you feel about being called on to share your thinking? What's it like to listen to other people's strategies?* • *Focused on group process: How well did your group work together?*
Collaboratively **create a chart** to anchor problem solving	*Clearly record things like:* • *steps to a procedure discovered through learner's strategy (e.g. Martha's Method of Factoring)* • *definitions, theorems or mathematical truths discovered and clarified through the course of the lesson* • *a list of different kinds of strategies that could be used to solve a certain kind of problem based on the class's work for the day*
Select and administer an **exit slip**	*Characteristics of a good exit slip:* • *can be solved quickly, in about 5 minutes* • *provides an opportunity for learners to use different strategies* • *requires mathematical justification or explanation* • *is not exactly the same kind of problem they just solved*

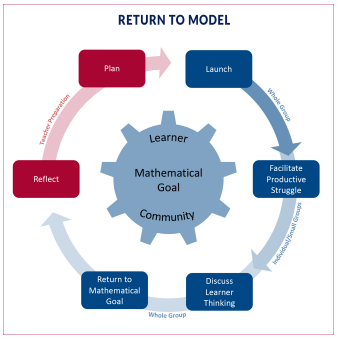
REFLECT



- How will you **use the evidence you collected to make decisions** about instructional next steps?

- **What options do you have for follow-up work** to build from and expand learners' current understanding?
  - A mini lesson to the whole class
  - A small group lesson on an identified area of need
  - Practice problems or activity
  - Adding in a new math routine
  - A new task
  - Something else?

REFLECT	
*How will you **use the evidence you collected to make decisions** about instructional next steps?*	
Look for evidence of developing understanding or underlying issues in your students' work	• *What is the good news that I can build upon?* • *What issues or concepts do I want to make sure to address before moving forward?*
Analyze evidence of learner participation, identity, and/or group processes	• *Is there evidence that my students are engaging in problem solving and discussion?* • *Are all students developing positive mathematical identities?* • *How could I help Amira develop more confidence in her own thinking?*
Review video and/or observation notes in relation to professional learning goals	• *Identify a moment that went well – what facilitation moves did you use and how did they support your learners?* • *Identify a moment that didn't go as well as you hoped – what could you have done differently and what can you learn from this?* • *Set goals for your next lesson to build upon*
***What options do you have for follow-up work** to build from and expand learners' current understanding?*	
Does the whole group need more instruction? Teach or plan to teach a **mini-lesson** on an identified need to the whole class	• *If you notice that learners are not labeling their answers, you might do a mini-lesson on identifying the meanings of the quantities in word problems* • *A mini-lesson on equipartitioning area models might follow a task where learners had difficulty visually representing a whole with equally sized fractional units (i.e. sixths)*
Does a small group have a specific need? Provide **differentiated support or instruction to a small group**	• *Work with a small group on an identified area of need, using a different approach, model, or activity* • *Intentionally pair learners based on identified strengths/needs*
Would students benefit from more practice? Choose **additional problems or an activity** to give students more opportunities to utilize a new math skill or concept	• *Introduce a mathematical routine (e.g. number talks, Which One Doesn't Belong, Quick Images)* • *Have students play a math game to reinforce and practice a specific skill* • *Give students a small number of problems to work on with a partner/small group*



RESPONSIVE MATH TEACHING RMT

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