

Reputation in Marketing Channels:

Repeated-Transactions Bargaining with Two-Sided Uncertainty

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Bargaining with Two-Sided Uncertainty

Technical Appendix: The Proofs

In this appendix we prove the lemmas and propositions stated in Appendix 2. The proofs focus primarily on the parties' *first period* strategies. This is because the second period strategies are so simple. Recall that in the second period the parties' optimal strategies are as outlined in §2 when both seller and buyer are myopic. That is, the buyer accepts any offer up to his valuation; if the seller's cost is low she offers v^+ if optimistic and v^- if pessimistic; and whatever the buyer's reputation, the seller offers v^+ if her cost is high.

PROOF OF PROPOSITION 1. The key to proving this result is demonstrating the effect of B 's updated belief, $b_1(b_0, p_1)$, about S 's cost on his expected payoff. This is key because, as the results of §2 show, S 's period 2 offer will vary depending on her cost. If S 's cost is high, then the second period offer will be v^+ . If S 's cost is low, then the second period offer will be v^- if $\beta_1(\cdot) \leq \beta^\circ$, and it will be v^+ if $\beta_1(\cdot) \geq \beta^\circ$. Therefore, on observing p_1 , B 's maximum expected payoff should he reject it is $s_1(0, p_1) = (v^+ - v^-)[1 - b_1(b_0, p_1)]\delta$, which means that accepting any p_1 such that

$$v^+ - p_1 \geq (v^+ - v^-)[1 - b_1(b_0, p_1)]\delta \quad (\text{A.9})$$

provides B with a payoff at least as large as any that he can expect should he reject it. Rearranging terms shows that any $p_1 \leq p^*$ satisfies Cdn. (A.9). Clearly, then, if $p_1 < p^*$ the (high valuation) buyer accepts, i.e., $y_1^*(p_1) = 1$ for $p_1 < p^*$. But suppose that $p_1 = p^*$ and that for this price $y_1(p_1) < 1$. Then S (in general) does better to reduce her price just slightly to exploit the fact that B certainly accepts the reduced price. This, of course, increases B 's payoff, making him better off. So it is clear that $y_1^*(p_1) \in [0, 1]$ for $p_1 = p^*$.^a Having established this result, we will henceforth ignore it for analytic convenience. With continuous prices S can set her price arbitrarily close to p^* , so the result has no qualitative impact here.^b Therefore, $y_1^*(p_1) = 1$ for all $p_1 \leq p^*$.

We now show that certain acceptance of any $p_1 > p^*$ is suboptimal. Suppose that $p_1 = p' > p^*$ and that $y_1^*(p_1) = 1$ for $p_1 = p'$. As $p' > p^* \geq v^-$, B 's acceptance reveals his type, which means that $p_2^*(\kappa, \beta_1(\cdot)) = v^+$, $\forall \kappa$. B 's payoff is then $s_1(1, p') = v^+ - p'$, which by the definition of p^* , is less than $(v^+ - v^-)[1 - b_1(b_0, p')]\delta$. This means that his payoff is less than the maximum that he can expect from rejecting p' . Recall that he expects this maximum iff $\beta_1(\cdot) \leq \beta^\circ \Leftrightarrow p_2^*(\kappa^-, \beta_1(\cdot)) = v^-$. If $y_1^*(p_1) = 1$ for $p_1 = p'$, then S 's posterior belief (i.e., B 's posterior reputation) is

^a Thanks to the Area Editor for pointing this out to us.

^b This does not mean, however, that this result is unimportant. In situations with discrete prices, especially if the price grid is coarse, this result can make a qualitative difference in the players' strategies.

$\beta_1(\beta_0, p', 1, a_1) = 0 < \beta^\circ$ if $a_1(p') = 0$, as she is then convinced that she faces a low valuation buyer. Then $p_2^*(\kappa^-, \beta_1(\cdot)) = v^-$, and B has increased his expected payoff by rejecting p' , a contradiction.

Finally, we show that if $p^\bullet$ is “large” (i.e., if $p^\bullet = v^+$), then B may *not* accept $p_1 = p^\bullet$ with certainty. Specifically, B does not accept $p_1 = p^\bullet = v^+$ with certainty if doing so induces a low cost S to offer v^+ . By the definition of $p^\bullet$ (see Eqn. (A.8) in Appendix 2), $p^\bullet = v^+$ means that an offer of v^+ is a foolproof signal to B that S ’s cost is high, i.e., $b_1(b_0, p_1) = 1$ for $p_1 = v^+$. Suppose that $y_1^*(p_1) = 1$ for $p_1 = p^\bullet = v^+$. Now suppose that B ’s certain acceptance of $p_1 = v^+$ induces a low cost S to offer v^+ . We see that B ’s strategy ruins the signal value of an offer of v^+ because the offer cannot prove to B that S ’s cost is high if a low cost seller also makes it, i.e., then $b_1(b_0, p_1) < 1$ for $p_1 = v^+$. Then $p^\bullet < v^+$, which implies that $y_1^*(p_1) < 1$ for $p_1 = v^+$, a contradiction. $\parallel$

PROOF OF COROLLARY 1. Take arbitrary $p_1 = p' > p^\bullet$ and suppose that $y_1^*(p_1) = y' > 0$ for $p_1 = p'$. Then, because $p_1 = p' > v^-$, the seller offers $p_2 = v^+$ if the period 1 offer is accepted whatever her cost, but if the first offer is rejected a pessimistic low cost seller offers $p_2 = v^-$. The buyer’s expected payoff is thus $s_1(y', p') = (v^+ - p')y' + (v^+ - v^-)(1 - b_1(\cdot))(1 - y')\delta$. Now suppose that $y_1^*(p_1) = 0$ for $p_1 = p'$. Then a pessimistic low cost seller offers $p_2 = v^-$ and the buyer’s expected payoff is $s_1(0, p') = (v^+ - v^-)(1 - b_1(\cdot))$. Comparing the payoffs, we find that $s_1(y', p') \geq s_1(0, p') \Rightarrow p' \leq v^+ - (v^+ - v^-)(1 - b_1(\cdot))\delta = p^\bullet$, a contradiction. Hence, when the seller is initially pessimistic, $y_1^*(p_1) = 0$ for $p_1 = p' > p^\bullet$. $\parallel$

PROOF OF LEMMA 1. (i) To prove that a high cost S offers no price below cost, we need to show that doing so cannot maximize her payoff. Consider any $p' \in (v^-, \kappa^+)$, a high cost S ’s payoff from which is

$$\pi_1(p', \kappa^+) = \{y_1(p')[p' - \kappa^+ + (v^+ - \kappa^+)\delta] + [1 - y_1(p')](v^+ - \kappa^+)\delta\}\beta_0 \text{ and any } p'' \in [\kappa^+, v^+], \text{ the payoff from which is}$$

$$\pi_1(p'', \kappa^+) = \{y_1(p'')[p'' - \kappa^+ + (v^+ - \kappa^+)\delta] + [1 - y_1(p'')](v^+ - \kappa^+)\delta\}\beta_0. \quad \pi_1(p', \kappa^+) \geq \pi_1(p'', \kappa^+) \text{ requires}$$

$$y_1(p') \leq \frac{y_1(p'')(p'' - \kappa^+)}{p' - \kappa^+}. \quad (\text{A.10})$$

As $p' - \kappa^+ < 0$, Cdn. (A.10) can be satisfied iff $y_1^*(p_1) = 0, \forall p_1 > v^-$, which can be satisfied iff $\beta_0 = 0$ or $b_1(b_0, p_1) = 0$, the former of which is ruled out by assumption and the latter is a contradiction (if S ’s cost is high B cannot *know* that her cost is low). Therefore, $\pi_1(p', \kappa^+) < \pi_1(p'', \kappa^+)$, which means offering a price below her cost is strictly dominated for a high cost seller.

We now consider $p^\circ > \kappa^+$ (we now know that we need not consider any $p_1 < \kappa^+$). To prove that a high cost S offers no price below p° , the key is to show that B will accept p° if a high cost S is willing to offer it because his posterior belief is at least as large as his prior belief.^c Assume that p° is not dominated by any higher first period

^c Thanks to an anonymous reviewer for calling this, the most direct route to proving this part of the result, to our attention.

offer for a high cost S . Then, if $p_1 = p^\circ$, B 's updated belief is at least as large as his prior, i.e., $b_1(b_0, p_1) \geq b_0$, *unless* p° is dominated by some *lower* price for a high cost S . But as $b_1(b_0, p_1) \geq b_0$ for $p_1 = p^\circ$ implies that $p_1 = p^\circ \leq p^\bullet$ (see Eqns. (5) and (A.8)), by Proposition 1 we know that B accepts $p_1 = p^\circ$. So, no lower price can dominate p° for a high cost S since no such price can be accepted with any larger probability by a high valuation B , the only type B with which a high cost S can trade. Hence, a high cost S never prices below p° and, having already shown that she never prices below cost, we have proved that she never prices below $\max\{p^\circ, \kappa^+\}$.

(ii) To prove that on seeing any $p_1 < \max\{p^\circ, \kappa^+\} = p^{\min}$ B concludes that S 's cost is low, we show that only a low cost seller is willing to offer such a price. We know by part (i) that for a high cost S any $p_1 < \kappa^+$ is dominated by any higher offer no matter how B responds to the lower or the higher offer and that it cannot benefit her to offer any $p_1 < p^\circ$ since B will accept p° if she is willing to offer it. We proceed by showing that a low cost S may be willing to offer $p_1 = v^- < p^{\min}$ even if a high valuation B is certain to accept $p^{\min}$. Then we show that if B is not certain to accept $p^{\min}$, which implies that even if B were certain to accept $p^{\min}$ it would be dominated by some higher offer for a high cost S (recall that, by part (i), B accepts $p^{\min}$ unless it is dominated by some higher offer for a high cost S), a low cost S may be willing to offer some $p_1 \in (v^-, p^{\min})$. A low cost S 's payoff from take-the-sure-thing, i.e., offer $p_1 = v^- < p^{\min}$, is $\pi_1(v^-, \kappa^-) = v^- - \kappa^- + (v^- - \kappa^-)\delta$, and her expected payoff from offering $p^{\min}$ is $\pi_1(p^{\min}, \kappa^-) = [p^{\min} - \kappa^- + (v^+ - \kappa^-)\delta]\beta_0 + (v^- - \kappa^-)(1 - \beta_0)\delta$ if B is certain to accept. We find that $\pi_1(v^-, \kappa^-) > \pi_1(p^{\min}, \kappa^-)$ if $\beta_0 < \frac{v^- - \kappa^-}{p^{\min} - \kappa^- + (v^+ - v^-)\delta} > 0$, which defines a set of admissible values of β_0 . Hence, for admissible values of S 's initial belief about the buyer, β_0 , a low cost S is willing to offer $p_1 = v^- < p^{\min}$ no matter how B responds to the higher offer. Now suppose that B does not accept $p^{\min}$ with certainty. A low cost S 's expected payoff from offering some $p' \in (v^-, p^{\min})$ is $\pi_1(p', \kappa^-) = y_1(p')[p' - \kappa^- + (v^+ - \kappa^-)\delta]\beta_0 + (v^- - \kappa^-)[1 - y_1(p')\beta_0]\delta$, where $y_1(p')$ is the probability with which B accepts p' . Comparing this payoff with a low cost S 's payoff from $p_1 = v^-$ (which is an offer that we have established her willingness to make) reveals that $\pi_1(p', \kappa^-) > \pi_1(v^-, \kappa^-)$ if $y_1(p')\beta_0 > \frac{v^- - \kappa^-}{p' - \kappa^- + (v^+ - v^-)\delta} < 1$, which defines a set of admissible values of the product $y_1(p')\beta_0$. Hence, for admissible values of S 's initial belief, β_0 , and B 's response, $y_1(p')$, to $p' \in (v^-, p^{\min})$, a low cost S is willing to offer such p' . We have thus shown that there exist conditions under which a low cost S will choose $p_1 = v^- < p^{\min}$ and conditions under which she will choose $p_1 \in (v^-, p^{\min})$. Having shown in part (i) that there are no conditions under which a high cost S offers any $p_1 < p^{\min}$, we have established that only a low cost S can benefit from making any such offer. The Intuitive Criterion is therefore invoked to assign zero weight to the likelihood that any $p_1 < p^{\min}$ is offered by a high cost S . Hence, on seeing any such offer, B concludes that S 's cost is low, i.e., $b_1(b_0, p_1) = 0$. $\parallel$

PROOF OF LEMMA 2. (i) Observe that $p^\bullet = p^-$ if $b_1(\cdot) = 0$ (see Eqns. (6) and (A.8)). Then, $p^- \leq p^\bullet$ for all values of $b_1(\cdot)$ and, by Proposition 1, the buyer optimally accepts.

(ii) Given that the buyer is certain to accept p^- , it is clear that no $p_1 \in (v^-, p^-)$ is optimal for either seller type whatever the buyer's reputation. Then, we need only compare an optimistic low cost seller's payoff from $p_1 = p^-$, $\pi_1(p^-, \kappa^-) = [p^- - \kappa^- + (v^+ - \kappa^-)\delta]\beta_0 + (v^- - \kappa^-)(1 - \beta_0)\delta$, with that from $p_1 = v^-$, $\pi_1(v^-, \kappa^-) = v^- - \kappa^- + (v^+ - \kappa^-)\delta\beta_0$. We find that $\pi_1(p^-, \kappa^-) \leq \pi_1(v^-, \kappa^-) \Rightarrow \beta_0 \leq \beta^e$, a contradiction if the seller is optimistic. $\parallel$

PROOF OF LEMMA 3. By Lemma 1, $b_1(b_0, p_1) = 0$ for $p_1 < p^{\min}$. This means that if $p_1 \in (p^-, p^{\min})$, then $p_1 > p^*$. By Corollary 1, we know that B rejects any such price when S is pessimistic.

(i) The condition that defines hyper pessimism is $\pi_1(v^-, \kappa^-) > \pi_1(p', \kappa^-)$, $\forall p' \in (v^-, v^+]$, irrespective of B 's response to p' . That is, a hyper pessimistic low cost S 's expected payoff from v^- exceeds that from any price that B will accept, even if (a high valuation) B is certain to accept v^+ . Then, by the Intuitive Criterion, we have $b_1(b_0, p_1) = 1$ for any $p_1 \in [p^{\min}, v^+]$ that B 's strategy makes optimal for a high cost S , which implies that $p_1 \leq p^* = v^+$ and, by Proposition 1, $y_1^*(p_1) = 1$ for any such price. By Proposition 1 we know that this means that B optimally accepts $p_1 = p^* = v^+$ with certainty, which makes v^+ the optimal offer for a hyper pessimistic high cost S .

Note that B 's certain acceptance of v^+ makes all $p_1 \in [p^{\min}, v^+)$ suboptimal for both S types whatever posterior beliefs and responses such offers induce. So the Intuitive Criterion has no bite for these prices. We assume that $b_1(b_0, p_1) = 1$ for any such price, which, by Proposition 1, implies that $y_1^*(p_1) = 1$ for all such prices (this has no effect on S 's strategy).

(ii) The condition that defines a very pessimistic S is $\pi_1(v^-, \kappa^-) > \pi_1(p', \kappa^-)$, $\forall p' \in (v^-, p^+]$, whatever B 's response to p' . That is, a very pessimistic low cost S 's expected payoff from v^- exceeds that from any price up to (though not beyond) p^+ . The key to proving this part of the lemma is showing that B 's threat of certain rejection of all $p_1 > p^+$ is credible when S is very pessimistic.

We first show that B 's best response to $p_1 = p^+$ is to accept with certainty. This is easy to see, as, given the above-stated relationship between the payoffs from v^- and p^+ , we have $b_1(b_0, p_1) = 1$ for $p_1 = p^+$ by the Intuitive Criterion. This means that $p_1 < p^* = v^+$ and, by Proposition 1, $y_1^*(p_1) = 1$ for $p_1 = p^+$.

Next, consider some $p_1 > p^+$. Of course, B would like to force S to avoid such prices, if he can. We show that his threat to reject all such offers does force S to avoid these prices if it is credible, and then we show that the threat is credible. First, note that no $p_1 > p^+$ is dominated for either S type if B accepts with large enough probability, as then, by the definition of p^+ , a very pessimistic low cost S (as well as a high cost S) is willing to offer such a price. Second, note that if B is certain to reject all $p_1 > p^+$, then all such prices are dominated by v^- for a low cost S . Last, note that B 's certain rejection of $p_1 > p^+$ makes any such price (at least weakly) dominated for a high cost S by all $p_1 \in [p^{\min}, p^+]$, as from any of the latter she gets trade with positive probability and non-negative profit. Hence, if credible, B 's threat to reject all $p_1 > p^+$ renders all such prices suboptimal for both S types and, therefore, zero probability prices. Since both S types will consider $p_1 > p^+$ if B accepts with large enough probability, we cannot

invoke the Intuitive Criterion to assign zero weight to either S type when $p_1 > p^+$. So for any such price $b_1(b_0, p_1) = b_0$, meaning that $p^* = p^\circ < p^+ < p_1$, and, by Corollary 1, $y_1^*(p_1) = 0$. B 's rejection threat is, thus, credible.

B 's certain acceptance of p^+ makes all $p_1 \in [p^{\min}, p^+)$ suboptimal for both S types whatever B 's posterior beliefs and responses. We assume that $b_1(b_0, p_1) = 1$ and, by Proposition 1, that $y_1^*(p_1) = 1$ for these zero probability prices (with no effect on S 's strategy).

(iii) The condition that defines moderate pessimism is $\pi_1(v^-, \kappa^-) < \pi_1(p', \kappa^-)$, $\forall p' \geq p^{\min}$ if B accepts p' with sufficiently large probability. That is, a moderately pessimistic low cost S is willing to set any price that a high cost S is willing to set, given that B accepts the price with large enough probability.

(a) The situation here is that $p^\circ \geq \kappa^+$. The key to proving this part of the lemma is to show that B 's threatened certain rejection of all $p_1 > p^\circ$ is credible when S is moderately pessimistic and $p^\circ \geq \kappa^+$. Fortunately, it is easy to see that the arguments in support of the credibility of B 's threat are precisely the same as those expressed above in support of the credibility of his threat to reject all $p_1 > p^+$. Simply replace p^+ with p° , and the same results hold. Then, because a moderately pessimistic high cost S prefers p° to any higher price, we have $b_1(b_0, p_1) \geq b_0$ for $p_1 = p^\circ$, which means that $p^* \geq p_1 = p^\circ$ and, by Proposition 1, $y_1^*(p_1) = 1$ for $p_1 = p^\circ$. Then, by the definition of moderate pessimism, p° is also optimal for a low cost S , and $b_1(b_0, p_1) = b_0$, $p_1 = p^* = p^\circ$, and $y_1^*(p_1) = 1$ for $p_1 = p^\circ$, as required.^d We have established B 's beliefs on observing and best responses to lower prices in Lemmas 1 and 2.

(b) Here the situation is that $p^\circ < \kappa^+$. Proving this part of the lemma involves several steps. We must first show that B 's optimal strategy is such that no $p_1 \geq \kappa^+$ is optimal for a low cost S . Second, we must show that B 's strategy must make v^+ the optimal price for a high cost S but *not* for a low cost S , which imposes an upper bound on the probability with which B can accept v^+ . Last, we must show that the lower limit on the probability with which B optimally accepts v^+ is positive.

Suppose B 's optimal strategy is such that some $p_1 \geq \kappa^+$ is optimal for both S types. Then, for that price $b_1(b_0, p_1) = b_0 \Rightarrow p^* = p^\circ < \kappa^+ \leq p_1$. By Corollary 1, we know $y_1^*(p_1) = 0$ for any such price, a contradiction, as a pessimistic low cost S does better with v^- than she does with any price certain to be rejected. It is clear that the

^d Note, however, that if the seller's belief is at the boundary of moderate pessimism, i.e., if $\beta_0 = \beta^-$, then if her cost is low, the seller's payoff from take-the-sure-thing equals her expected payoff from learn-then-discriminate by offering p° . In such an instance, a low cost seller may randomize her first period offer, i.e., she may choose either v^- or p° , both with positive probability. Therefore, if $\beta_0 = \beta^-$, an offer of p° need not necessarily be a pooling offer, and the buyer's updated belief on observing it falls somewhere in the interval bounded below by the prior, b_0 (the buyer's updated belief if the offer is pooling), and bounded above by 1 (the buyer's updated belief if the offer is separating). But even if $\beta_0 = \beta^-$, the buyer's optimal strategy is unaffected. Because his updated belief is at least as large as his prior on observing p° , it is still optimal for him to accept. Most important, however, is that his updated belief on observing offers higher than p° is unaffected. That is, for $p_1 > p^\circ$, the updated buyer belief is the same as his prior because either type of seller would make such an offer if he accepts with sufficiently large probability. His updated belief on observing any $p_1 > p^\circ$ supports the optimality of rejecting any such offer. In any event, the case of $\beta_0 = \beta^-$ can be considered special, and as such we relegate its consideration to Proposition 4, which deals with all such special cases.

same argument holds for any strategy such that any $p_1 \geq \kappa^+$ is optimal for a low cost S but not for a high cost S , as then $b_1(b_0, p_1) = 0 \Rightarrow p^* = p^- < \kappa^+ \leq p_1 \Rightarrow y_1^*(p_1) = 0$, yielding the same contradiction. We have thus shown that if $p^\circ < \kappa^+$, moderate pessimism implies B 's optimal responses to all $p_1 \geq \kappa^+$ makes all such prices suboptimal for a low cost S .

We now show that B 's strategy must make v^+ the optimal price for a high cost S . Suppose that B 's best responses to $p_1 \geq \kappa^+$ are such that some $p_1 \in [\kappa^+, v^+)$ is optimal for a high cost S . Then, as all $p_1 \geq \kappa^+$ are suboptimal for a low cost S , for any such price we have $b_1(b_0, p_1) = 1$, implying that $p_1 < p^* = v^+$ and, by Proposition 1, that $y_1^*(p_1) = 1$ for any such price. This is a contradiction, as, by the definition of moderate pessimism, B 's certain acceptance of any price that a high cost S is willing to set makes such a price optimal for a low cost S as well, and we have already shown that no such price can be optimal for a low cost S . B 's optimal strategy must, therefore, render all $p_1 \in [\kappa^+, v^+)$ suboptimal for both seller types. Therefore, as some $p_1 \geq \kappa^+$ must be optimal for a high cost S , it must be the case that the optimal strategy for B must be such that $p_1^*(\kappa^+) = v^+$. Further, since a low cost S must not be tempted to offer this price, by the definition of y^- , we know that $y_1^*(p_1) < y^-$ for $p_1 = v^+$. Then, we have $b_1(b_0, p_1) = 1$ for $p_1 = v^+$, as required.

We conclude our examination of B 's optimal strategy by showing that $y_1^*(p_1) > 0$ for $p_1 = v^+$. Suppose that B optimally rejects $p_1 = v^+$ with certainty. Then a high cost S , anticipating his response, prices a bit lower than v^+ . This reveals to B that S 's cost is high since a moderately pessimistic low cost S chooses no such price, and we have $b_1(b_0, p_1) = 1$. Then $p_1 < p^* = v^+$, which implies that $y_1^*(p_1) = 1$, which we have already ruled out. Therefore, $y_1^*(p_1) > 0$ for $p_1 = v^+$. Together with the previous result, this means that $y_1^*(p_1) \in (0, y^-)$ for $p_1 = v^+$, as required.

Finally, we consider B 's posterior beliefs and responses on observing any $p_1 \in [\kappa^+, v^+)$. We have determined that all such prices must be suboptimal for both seller types. We also know that if B accepts with large enough probability, either moderately pessimistic S type is willing to choose prices from this range. So, we cannot use the Intuitive Criterion to place zero weight on either type if such a price is observed. We assume that any such price yields no information about S 's type, i.e., $b_1(b_0, p_1) = b_0$ for all $p_1 \in [\kappa^+, v^+)$.^c Therefore, they all exceed p^* and, by Corollary 1, $y_1^*(p_1) = 0$ for all such prices. This renders all of them suboptimal for both S types, as required. $\parallel$

PROOF OF COROLLARY 2. The fundamental logic of Corollary 2 is owed to Fudenberg and Tirole (1983, pp. 224 – 225) for $p^* < p_1 \leq v^+$. We extend their logic, however, for the case of $p_1 = p^* = v^+$.

First, note that by the definition of y° , if the buyer's response to any $p_1 > v^-$ is y° , then the seller's posterior belief is β° if the outcome is rejection. A low cost seller is then indifferent between offering v^- and v^+ in the second period. Next, note that for a mixed strategy to be optimal for the buyer, it is necessary that he be indifferent between accepting and rejecting the offer. So let $\eta_2(p_1)$ be the probability with which an indifferent low cost seller offers v^+ in period 2. The buyer is indifferent between accepting and rejecting in period 1 if

$$v^+ - p_1 = (v^+ - v^-)[1 - b_1(b_0, p_1)][1 - \eta_2(p_1)]\delta. \quad (\text{A.11})$$

And from Eqn. (A.11) we get the period 2 strategy of a low cost seller that yields buyer indifference in period 1.

Call that strategy $\eta_2^*(p_1)$.

$$\eta_2^*(p_1) = 1 - \frac{v^+ - p_1}{(v^+ - v^-)[1 - b_1(b_0, p_1)]\delta} \quad (\text{A.12})$$

Now take some $p_1 \in (p^*, v^+]$. Suppose that for such a price $y_1^*(p_1) > y^\circ$. Then, by Bayes' rule (see Eqn. (A.1)) $\beta_1(\cdot, 0) < \beta^\circ$. That is, if the observed outcome is a rejection of the price, S is made pessimistic and a low cost S then chooses $p_2 = v^-$, which, by Proposition 1, means that $y_1^*(p_1) = 0$, a contradiction. Hence, $y_1^*(p_1) \leq y^\circ$ for all $p_1 \in (p^*, v^+]$.

Now take some $p_1 \in (p^*, v^+)$ and suppose that for such a price $y_1^*(p_1) < y^\circ$. Then, by Bayes' rule, $\beta_1(\cdot) > \beta^\circ$. That is, whatever the observed outcome, S remains optimistic and $p_2 = v^+$ whatever her cost. As such, B should accept the first period price with certainty, a contradiction. Hence, $y_1^*(p_1) = y^\circ$ for any $p_1 \in (p^*, v^+)$.

But suppose that $p_1 = v^+ > p^*$. This means that a low cost S or both S types expect that pricing at v^+ is at least as profitable as any other strategy. So, given that $y_1^*(p_1) = y^\circ$ for any $p_1 \in (p^*, v^+)$, if B 's best response to $p_1 = v^+$ is some $y_1^*(p_1) < y^\circ$, S reduces her price ever so slightly to exploit the fact that B accepts any lower price with larger probability. Such a reduced price yields a positive surplus for B while not violating the conditions necessary for him to optimally play a mixed strategy (see Eqns. (A.11) and (A.12)). Accepting v^+ with probability y° yields the reservation utility, 0, for B , so it is clear that $y_1^*(p_1) \in [0, y^\circ)$ for $p_1 = v^+ > p^*$. Note that this situation is similar to the case of $p_1 = p^*$ and, for the same reason, we will ignore this result (i.e., S can price arbitrarily close to v^+). Therefore, $y_1^*(p_1) = y^\circ$ for all $p_1 \in (p^*, v^+]$.

Finally, we consider the situation when $p_1 = p^* = v^+$ and show that in this situation $y_1^*(p_1) \geq y^\circ$. By the definition of p^* , $p^* = v^+$ means that a high cost S finds v^+ an optimal price and a low cost S does not. Therefore, $b_1(b_0, p_1) = 1$ for $p_1 = v^+$. This implies that v^+ is dominated by some lower price for a low cost S when B 's response to v^+ is y° . Suppose that B 's optimal strategy is such that some $p_1 < v^+$ is optimal for a high cost S but not for a low cost S . Then, for such a price we have $b_1(b_0, p_1) = 1$, implying that $p^* = v^+ > p_1$, which means that $y_1^*(p_1) = 1$. Then, however, this price is also optimal for an optimistic low cost S , as an optimistic low cost S is willing to set any price that a high cost S is willing to set if B is certain to accept such a price, a contradiction. Hence, when S is optimistic B 's optimal strategy must be such that no $p_1 < v^+$ is optimal for a high cost S but not for a low cost S , i.e., $b_1(b_0, p_1) \leq b_0$ for any such price. Then $y_1^*(p_1) = y^\circ$ for, at minimum, any $p_1 \in (p^*, v^+)$ when S is optimistic. Therefore, it cannot be the case that $y_1^*(p_1) < y^\circ$ for $p_1 = v^+$, as such a response makes it optimal for a high cost S but not a low cost S to price

^e Such uninformed off the equilibrium path beliefs have been called "passive conjectures" (Fudenberg and Tirole [1983]), and we use passive conjectures frequently in this analysis. We should point out, however, that there are other ways of modeling such off the equilibrium path beliefs (interested readers should see Cho and Kreps [1987]; Fudenberg and Tirole [1991]).

just lower than v^+ , which we have ruled out. (If a low cost S is unwilling to price at v^+ when B 's response is y° , she cannot be willing to set any lower price when B 's response to that price is y°). $\parallel$

PROOF OF LEMMA 4. By Corollary 2 we know that when S is optimistic $y_1^*(p_1) \geq y^\circ$ for all $p_1 \leq v^+$ and that $y_1^*(p_1) = y^\circ$ for all $p_1 \in (p^\circ, v^+]$.

(i) The conditions that define hyper optimism are $\pi_1(p^-, \kappa^-) \leq \pi_1(v^+, \kappa^-)$ and $\pi_1(p^\circ, \kappa^+) \leq \pi_1(v^+, \kappa^+)$ when v^+ is accepted with probability y° , even if p° is certain to be accepted (recall that p^- is always accepted with certainty). To prove this part of the lemma we need to show that B 's optimal strategy must be such that no $p_1 \in (p^-, v^+)$ is optimal for either S type, from which it follows that $p_1 = v^+$ is optimal for both S types given the above-stated relationships between the payoffs. We will then show that B 's best response to S 's optimal price, v^+ , is unique.

We first establish that if $\beta_0 = \beta^+$ a high cost S is not indifferent between offering v^+ and p° . Let $\beta_0 = \beta^+$ so that a high cost S 's expected payoff from v^+ equals that from p° when the response to the former is y° and the latter is accepted with certainty. If she is indifferent between the two prices and if, in her indifference, she offers $p_1 = v^+$ with positive probability, then B 's posterior belief on observing $p_1 = p^\circ$ is less than his prior and he does not accept p° with certainty. But then a high cost S is not indifferent between these two prices; rather, she prefers $p_1 = v^+$. Because we already know, by Lemma 1, that all $p_1 < p^\circ$ are dominated for a high cost S , this result says that hyper optimism implies that all $p_1 \leq p^\circ$ are dominated for a high cost S . Then for any $p_1 \in (p^-, p^\circ]$ we have $h_1(b_0, p_1) = 0$, which means that $p^- = p^\circ < p_1$ and, by Corollary 2, $y_1^*(p_1) = y^\circ$ for any such price. But as $y_1^*(p_1) \geq y^\circ$ for all $p_1 \leq v^+$, it is immediately clear that no $p_1 \in (p^-, p^\circ]$ is optimal for either a low or a high cost hyper optimistic S .

Next consider $p_1 \in (p^\circ, v^+)$. Suppose that B 's optimal strategy is such that some such price is optimal for both S types. Then we have $h_1(b_0, p_1) = b_0$, from which it follows that $p^\circ = p^\circ < p_1$, which means that $y_1^*(p_1) = y^\circ$ for any such price. The contradiction is clear, as $y_1^*(p_1) \geq y^\circ$ for $p_1 = v^+$. Now suppose that B 's optimal strategy is such that some $p_1 \in (p^\circ, v^+)$ is optimal for a low cost S but not for a high cost S . Then we have $h_1(b_0, p_1) = 0$, implying that $p^- = p^\circ < p_1$, and therefore $y_1^*(p_1) = y^\circ$ for any such price, yielding the same contradiction. Now suppose that B 's optimal strategy is such that some $p_1 \in (p^\circ, v^+)$ is optimal for a high cost S but not for a low cost S . Then we have $h_1(b_0, p_1) = 1$, from which it follows that $v^+ = p^\circ > p_1$, meaning that $y_1^*(p_1) = 1$. However, B 's certain acceptance of such a price also induces an optimistic low cost S to set that price, which means that $h_1(b_0, p_1) = b_0$, and $p^\circ = p^\circ < p_1$, and $y_1^*(p_1) = y^\circ$ for any such price, and the familiar contradiction reappears. Hence, when S is hyper optimistic, B 's optimal strategy must be such that no $p_1 \in (p^\circ, v^+)$ is optimal for either S type.

We have shown that hyper optimism means that no $p_1 \in (p^-, v^+)$ can be optimal for either S type. From this result and Lemma 1, it immediately follows that $p_1^*(\kappa^+) = v^+$. We conclude our discussion of B 's optimal responses by showing that y° is a unique best response to $p_1 = v^+$. Recall that one of the conditions that define hyper optimism is $\pi_1(p^-, \kappa^-) \leq \pi_1(v^+, \kappa^-)$ when B accepts v^+ with probability y° . If, in fact, $\pi_1(p^-, \kappa^-) < \pi_1(v^+, \kappa^-)$ then it is clear that

$b_1(h_0, p_1) = b_0$ for $p_1 = v^+$, and $p^\circ = p^\bullet < p_1$. Then B 's optimal response, y° , is unique as is a low cost S 's optimal price, $p_1^*(\kappa^-) = v^+$. But suppose a low cost S is indifferent between $p_1 = v^+$ and $p_1 = p^-$ because $\pi_1(p^-, \kappa^-) = \pi_1(v^+, \kappa^-)$ when B 's response to v^+ is y° , i.e., $\beta_0 = \beta^{++}$.^f We know that the probability with which B must accept $p_1 = v^+$ is at least y° , but can he accept with any larger probability? It is easy to see that he cannot, for accepting v^+ with any larger probability yields $\pi_1(p^-, \kappa^-) < \pi_1(v^+, \kappa^-)$ and a low cost S is not indifferent. Therefore, B 's best response to v^+ is unique, y° . This also means that B 's optimal strategy will leave a hyper optimistic low cost S indifferent between v^+ and p^- , i.e., $p_1^*(\kappa^-) \in (p^-, v^+]$, when it happens that $\beta_0 = \beta^{++}$.^g

Finally, we consider B 's posterior beliefs on observing zero (prior) probability $p_1 \in (p^-, v^+)$ and his best responses to such prices. It is clear that any price lower than $p^\times$ is dominated for a high cost S , for even if B accepts such a price with certainty, her expected payoff is smaller than that she expects from v^+ when B accepts v^+ with probability y° . We invoke the Intuitive Criterion to set $b_1(h_0, p_1) = 0$ for any such price. Given this belief, we have $p^- = p^\bullet < p_1$, implying $y_1^*(p_1) = y^\circ$ for any $p_1 \in (p^-, p^\times)$, which makes any such price suboptimal for a low as well as a high cost S , as required. The Intuitive Criterion loses its bite, however, for prices at least as high as $p^\times$ because such prices are not dominated for either S type if B accepts with sufficiently large probability. We therefore assume that these prices are uninformative, i.e., $b_1(h_0, p_1) = b_0$ for all such prices. Given this belief, and since $p^\times > p^\circ$, we have $p^\circ = p^\bullet < p_1$, so $y_1^*(p_1) = y^\circ$ for any $p_1 \in [p^\times, v^+)$, which makes any such price suboptimal for both S types, as required.

(ii) The conditions that define a very optimistic S are $\pi_1(p^-, \kappa^-) > \pi_1(v^+, \kappa^-)$ and $\pi_1(p^\circ, \kappa^+) \leq \pi_1(v^+, \kappa^+)$ when v^+ is accepted with probability y° , even if p° is accepted with certainty. Proving this part of the lemma requires us to show that all $p_1 \in (p^-, v^+)$ must be suboptimal for both S types, from which it follows that $p_1 = v^+$ is optimal for a high cost S given the above-stated relationship between her payoffs from p° and v^+ . We must also show that B 's response to a high cost S 's price must be smaller than y^+ , making $p_1 = p^-$ the optimal price for a low cost S .

First, note that precisely the same result given above for the case of $\beta_0 = \beta^+$ also applies here, i.e., a high cost S strictly prefers $p_1 = v^+$ to $p_1 = p^\circ$ when the former is accepted with probability y° and the latter is accepted with certainty. Hence, all $p_1 \leq p^\circ$ are suboptimal for a very optimistic high cost S . So, as before, for any $p_1 \in (p^-, p^\circ]$ we have $b_1(h_0, p_1) = 0$, yielding $p^- = p^\bullet < p_1$, implying $y_1^*(p_1) = y^\circ$ for any such price, rendering all such prices suboptimal for both S types.

^f Comparing Eqns. (A.3) and (A.6) reveals that β^+ may take on a larger value than β^{++} . If $\beta^+ > \beta^{++}$, it is easy to show that the equilibria do not change. If $\beta^+ > \beta^{++}$, then S cannot be very optimistic and she is hyper optimistic if $\beta_0 \geq \beta^+$.

^g And in her indifference, a low cost seller may randomize between v^+ and p^- , which means that on observing a first period offer of v^+ , the buyer's updated belief need not be the same as his prior. Rather, it falls somewhere in the interval $[b_0, 1]$, depending upon the probability with which a low cost seller makes one or the other offer. But, as shown above, the buyer's strategy is unaffected by this. The case of $\beta_0 = \beta^{++}$ is a special one, and we handle it in Proposition 4.

Now consider $p_1 \in (p^\circ, v^+)$. Again, exactly the same arguments used to show that all prices in this interval must be suboptimal for both S types under hyper optimism also apply when S is very optimistic. Therefore, B 's optimal strategy is such that all $p_1 \in (p^\circ, v^+)$ are suboptimal for a very optimistic S , which, together with the previous result, means that all $p_1 \in (p^-, v^+)$ must be suboptimal for a very optimistic S .

Having established that when S is very optimistic B 's optimal strategy must be such that all $p_1 \in (p^-, v^+)$ are suboptimal for both S types, it is clear that it must be the case that $p_1^*(\kappa^+) = v^+$. How should B respond to this price? Let us suppose that his optimal response is such that v^+ is optimal for both S types (which means that $y_1 > y^\circ$, otherwise, by the definition of a very optimistic S , $\pi_1(p^-, \kappa^-) > \pi_1(v^+, \kappa^-)$ and v^+ is not optimal for a low cost S). But if v^+ is optimal for both S types, we have $b_1(b_0, p_1) = b_0$ for $p_1 = v^+$, implying that $p^\circ = p^* < p_1$, and therefore $y_1^*(p_1) = y^\circ$. This is a contradiction, as v^+ is suboptimal for a very optimistic low cost S when B 's response to this price is y° . Therefore, B 's response to v^+ must be such that this price is suboptimal for a low cost S , and by the definition of y^+ , this means that $y_1^*(p_1) < y^+$ for $p_1 = v^+$. Since we know that seller optimism means that $y_1^*(p_1) \geq y^\circ$ for $p_1 = v^+$, it must be the case that $y_1^*(p_1) \in [y^\circ, y^+)$ for this price. Given B 's best response, we have $b_1(b_0, p_1) = 1$ for $p_1 = v^+$, as required. It is clear that the interval $[y^\circ, y^+)$ always exists when S is very optimistic.

Finally, we consider B 's beliefs on observing and best responses to zero probability $p_1 \in (p^-, v^+)$. We again invoke the Intuitive Criterion to set $b_1(b_0, p_1) = 0$ for all $p_1 < p^*$, as all such prices are dominated by v^+ for a high cost S . And, again, as $p_1 \in [p^*, v^+)$ are not dominated for either S type if B accepts with large enough probability, these prices are uninformative and we set $b_1(b_0, p_1) = b_0$ for $p_1 \in [p^*, v^+)$. Given these beliefs, B optimally accepts any $p_1 \in (p^-, v^+)$ with probability y° , making all such prices suboptimal for a very optimistic S of either type, as required.

(iii) The condition that defines moderate optimism is $\pi_1(p^\circ, \kappa^+) > \pi_1(v^+, \kappa^+)$ when B accepts v^+ with probability y° and he accepts p° with certainty. It is easy to show that this condition also implies that $\pi_1(p^\circ, \kappa^-) > \pi_1(p', \kappa^-)$ for all $p' > p^\circ$. Therefore, $b_1(b_0, p_1) = b_0$ for $p_1 = p^\circ$, meaning that $p_1 = p^* = p^\circ$ and, by Proposition 1, that $y_1^*(p_1) = 1$ for $p_1 = p^\circ$ when S is moderately optimistic.

We now consider B 's beliefs on observing and best responses to zero probability prices. By Lemma 1 we know that $b_1(b_0, p_1) = 0$ for any $p_1 \in (p^-, p^\circ)$, and, therefore $y_1^*(p_1) = y^\circ$ for all such prices, by Corollary 2. Then B 's best response to any $p_1 \in (p^-, p^\circ)$ renders all such prices suboptimal for either S type, as required. Because either S type is willing to set $p_1 > p^\circ$ if B accepts with sufficiently large probability, we assume that any such price is uninformative and set $b_1(b_0, p_1) = b_0$ for any such price. Given this belief, we know by Corollary 2 that $y_1^*(p_1) = y^\circ$ for all $p_1 \in (p^\circ, v^+]$, which, by the definition of moderate optimism, makes all such prices suboptimal for both S types, as required. $\parallel$

The next three proofs are those of the equilibria given by Propositions 2, 3, and 4. These proofs focus on showing that deviations from the stated equilibria are not profitable for the seller. The logic underlying the buyer's strategies is stated by the results that precede these three proofs, and when one of those results is used in the proof of

one of the following propositions, we clearly reference it to help the reader understand why it applies in the current context.

PROOF OF PROPOSITION 2. (i) By definition, if the seller is moderately pessimistic and $\beta_0 > \beta^-$, then a low cost seller's expected payoff from offering any first period price that she would offer were her cost high, given that B is sufficiently likely to accept such a price, exceeds her payoff from take-the-sure-thing. That is, $\pi_1(v^-, \kappa^-) < \pi_1(p', \kappa^-)$, $\forall p' \geq p^{\min}$ if B accepts p' with sufficiently large probability. She is moderately optimistic when a high cost seller's expected payoff from offering p° exceeds her expected payoff from offering v^+ , i.e., $\pi_1(p^\circ, \kappa^+) > \pi_1(v^+, \kappa^+)$ if B accepts v^+ with probability y° and accepts p° with certainty.

Let us first consider the case when the seller is moderately pessimistic and $p^\circ \geq \kappa^+$. We know by Lemmas 1, 2, and 3 that under such conditions B 's optimal strategy is as follows. (1) Accept the first period price if it is p° , in which case B 's updated belief about the seller is the same as his prior, i.e., $b_1(\cdot) = b_0$ (see part iii.a of Lemma 3). (2) Reject if the first period price is higher than p° , in which case B 's updated belief is the same as his prior, i.e., $b_1(\cdot) = b_0$ (see part iii.a of Lemma 3). (3) Reject if the first period price is in the interval (p^-, p°) , in which case B concludes that the seller's cost is low, i.e., $b_1(\cdot) = 0$, because, by Lemma 1, we know that $p^\circ \geq \kappa^+$ means that $p^\circ = p^{\min}$, and that if the seller's cost is high she never offers a price below $p^{\min}$. (4) Accept if the first period price is less than or equal to p^- , which, by Lemma 2, we know B accepts whatever his belief.

Consider a deviation by the seller. Can she profit by deviating to a lower first period offer? By Lemma 1 we know that it is never profitable for the seller to offer any $p_1 < p^\circ = p^{\min}$ if her cost is high. Now consider whether the seller would profit from offering some $p_1 < p^\circ$ if her cost is low. Given B 's certain acceptance of p° (if his valuation is high), it is clear that the seller cannot profit from offering any $p_1 \in (v^-, p^\circ)$, as at any such price her margin is lower and the likelihood of its acceptance is no greater. And by the definition of moderate pessimism, if the seller's cost is low her expected payoff is larger if she offers any $p' \geq p^{\min} = p^\circ$ than it is if she offers v^- (or any price lower than v^- since no price is accepted with greater likelihood) when B accepts p' with sufficiently large probability. But here B is *certain* to accept $p' = p^\circ$. Therefore, whether her cost is low or high, it is not profitable for the seller to deviate to any $p_1 < p^\circ$.

Can the seller profit by deviating to a higher first period offer? To see that she cannot we note that the proof of Lemma 3 shows that B 's threat to reject any $p_1 > p^\circ$ is credible, so any such price is certain to be rejected. Now, observe that if B rejects such a price, then if a moderately pessimistic low cost seller makes such an offer her payoff is $0 + (v^- - \kappa^-)\delta$, and it is easy to show that this is smaller than $[p^\circ - \kappa^- + (v^+ - \kappa^-)\delta]\beta_0 + (v^- - \kappa^-)(1 - \beta_0)\delta$, the expected payoff that is hers if she learns-then-discriminates by offering p° . And it is even easier to see that B 's rejection of any $p_1 > p^\circ$ means that a high cost seller's expected payoff from offering such a price, $0 + (v^+ - \kappa^+)\delta$, can be no larger than her payoff from offering p° , which is $[p^\circ - \kappa^+ + (v^+ - \kappa^+)\delta]\beta_0$. Therefore, whether her cost is low or high, it is not profitable for the seller to deviate to any $p_1 > p^\circ$.

Now consider the case when the seller is moderately optimistic. We know by Lemmas 1, 2, and 4 that under this condition B 's optimal strategy is as follows. (1) Accept the first period price if it is p° , in which case B 's updated belief about the seller is the same as his prior, i.e., $b_1(\cdot) = b_0$. (2) Accept with probability y° if the first period price is in the interval (p^-, p°) , in which case B 's updated belief is $b_1(\cdot) = 0$ because moderate optimism does not exist unless $p^{\min} = p^\circ$, and, by Lemma 1, we know that if the seller's cost is high she never offers a price below $p^{\min}$.^h (3) Accept with probability y° if the first period price is in the interval $(p^\circ, v^+]$, in which case B 's updated belief is the same as his prior (see part iii of Lemma 4). (4) Accept if the first period price is less than or equal to p^- , which, by Lemma 2, we know B accepts whatever his belief.

Consider a deviation to a lower first period offer. Again, by Lemma 1 we know that no $p_1 < p^\circ = p^{\min}$ can be profitable for the seller if her cost is high. If the seller's cost is low, then, again, as B is certain to accept p° (if his valuation is high), deviating to any $p_1 \in [p^-, p^\circ)$ is not profitable, as at any such price her margin is lower and the likelihood of its acceptance is no greater. And, by Lemma 2.ii, we know that if the seller's cost is low, optimism (of any degree) implies that her expected payoff from offering p^- exceeds that from offering any lower price; thus, if no $p_1 \in [p^-, p^\circ)$ is a profitable deviation, neither can be any $p_1 < p^-$.

Consider a deviation to a higher period 1 offer. By the definition of moderate pessimism, we know that if the seller's cost is high her expected payoff from offering p° exceeds that from offering v^+ when B is certain to accept the former and accepts the latter with probability y° , as is the case here. Further, because here B accepts any price in $(p^\circ, v^+]$ with probability y° , it is clear that if the seller's cost is high she cannot profit from deviating to any price higher than p° .

Next we show that the seller does not profit by deviating to a higher first period offer if her cost is low. To do so, we need to show that if a high cost seller's payoff from p° is larger than it is from v^+ (and, hence, any offer in $(p^\circ, v^+]$), then the same is true about a low cost seller's payoff. If the seller's cost is low, then straightforward algebra shows that her expected payoff from playing learn-then-discriminate by offering p° exceeds that from gambling by offering v^+ (when B is certain to accept p° and accepts v^+ with probability y°) if

$$y^\circ < \frac{[p^\circ - \kappa^- - (v^- - \kappa^-)\delta]\beta_0 + (v^- - \kappa^-)\delta}{(v^+ - \kappa^-)\beta_0} \equiv y'^-. \text{ By comparison, if the seller's cost is high the same sort of}$$

comparison reveals that her expected payoff is higher from offering p° than it is from offering v^+ if

$$y^\circ < \frac{p^\circ - \kappa^+}{v^+ - \kappa^+} \equiv y'^+. \text{ Now observe that if } y'^-, \text{ the value of } y^\circ \text{ below which a low cost seller's payoff is larger from}$$

p° , is larger than y'^+ , a high cost seller's threshold, it means that a low cost seller's payoff from p° exceeds that from v^+ for all values of y° where the same holds for a high cost seller. Notice that y'^- is a function of β_0 ,

^h The seller is moderately optimistic when $\beta^\circ < \beta_0 < \beta^+$. But if $p^{\min} \neq p^\circ \Leftrightarrow p^{\min} = \kappa^+$, then Eqn. (A.3) shows that $\beta^+ = \beta^\circ$, so the seller cannot be moderately optimistic.

while y^{+} is not. It is easy to show that y^{-} decreases in β_0 . Thus, the smallest value of y^{-} is its value when β_0 approaches 1, in which case y^{-} approaches $\frac{p^{\circ} - \kappa^{-}}{v^{+} - \kappa^{-}}$, its infimum, and $\inf y^{-} > y^{+}$ implies that all y^{-} exceed y^{+} . Straightforward algebra shows that $\inf y^{-} \leq y^{+}$ requires that $v^{+} \leq p^{\circ}$, which requires that either $b_0 = 1$ or $\delta = 0$ or $v^{-} = v^{+}$, all of which are ruled out by assumption. Therefore, $y^{-} > y^{+}$, which implies that a low cost seller does not deviate to a first period offer higher than p° if a high cost seller does not, and we have already established that a moderately optimistic high cost seller does not.

(ii) By the definition of hyper optimism, the seller is hyper optimistic when $\beta_0 \geq \beta^{++}$, and both of the following conditions hold. (1) A low cost seller's expected payoff from playing learn-then-discriminate by offering p^{-} , which B is certain to accept, is no more than is her expected payoff from gambling by offering v^{+} , when B accepts v^{+} with probability y° . (2) A high cost seller's expected payoff from offering v^{+} , which B accepts with probability y° , is at least as large as is her expected payoff from offering p° if B accepts with certainty. Observe that because y° increases in β_0 , it follows that the strict inequality, $\beta_0 > \beta^{++}$, implies that a low cost seller's payoff from gambling by offering v^{+} strictly exceeds her payoff from learn-then-discriminate with an offer of p^{-} .

By Lemma 4.i we know that B optimally accepts all $p_1 \in (p^{-}, v^{+}]$ with probability y° when the seller is hyper optimistic, in which case his updated belief about the seller is the same as his prior, i.e., $b_1(\cdot) = b_0$. And, whatever his or the seller's belief, we know by Lemma 2 that B is certain to accept any $p_1 \leq p^{-}$.

Consider a deviation by the seller to a lower first period offer. It is clear that no deviation to any $p_1 \in (p^{-}, v^{+})$ can be profitable for the seller whether her cost is low or high since no such offer is accepted with any greater probability than is v^{+} , and all yield smaller margins. B is certain to accept an offer of p^{-} , yet the expected payoff for a low cost seller is less than that she expects from an offer of v^{+} , and as $p^{-} < \max\{p^{\circ}, \kappa^{+}\} = p^{\min}$, we know by Lemma 1 that a high cost seller does not profitably deviate to p^{-} (or any lower offer). And, by Lemma 2, we know that, when optimistic, a low cost seller's expected payoff from p^{-} exceeds that from any lower offer, implying that the payoff from all prices lower than p^{-} are smaller than is that from v^{+} . Hence, whatever her cost, a hyper optimistic seller does not profitably deviate to a first period offer lower than v^{+} .

It is clear that a deviation to a price higher than v^{+} cannot be profitable since at any such price there is certain rejection, while $p_1 = v^{+}$ is accepted with probability $y^{\circ} > 0$. $\parallel$

PROOF OF PROPOSITION 3. (i) By definition the seller is hyper pessimistic if a low cost seller's payoff from take-the-sure-thing (i.e., making a first period offer of v^{-}) exceeds that from any offer up to and including v^{+} , whatever B 's response to the higher offer. We know by Lemma 3 that when the seller is hyper pessimistic B 's optimal strategy is as follows. (1) Accept the first period price if it is in the interval $[p^{\min}, v^{+}]$, in which case B 's updated belief about the seller is $b_1(\cdot) = 1$ (i.e., B takes any such offer as conclusive evidence that the seller's cost is high). (2) Reject if $p_1 \in (p^{-}, p^{\min})$, in which case B 's updated belief is $b_1(\cdot) = 0$ (i.e., B takes all such offers as proof that

the seller's cost is low because they are lower than $p^{\min}$, the lowest offer that a high cost seller would ever make).

(3) Accept if $p_1 \leq p^-$, which, by Lemma 2, we know B accepts whatever his belief.

Consider a deviation by the seller if her cost is high. It is clear that it is not profitable for her to deviate to any $p_1 \in [p^{\min}, v^+)$, as no such price is accepted with any greater likelihood than is v^+ and all yield lower margins. And because we know by Lemma 1 that she does not profitably deviate to any $p_1 < p^{\min}$, we can conclude that the seller does not profit from a deviation to a price lower than v^+ if her cost is high. It is also clear that deviating to a price higher than v^+ does not pay, for at any such price there is no chance of trade, while at v^+ there is trade yielding a positive margin with positive probability. Therefore, a hyper optimistic high cost seller does not profitably deviate from offering v^+ .

Now consider a deviation by the seller if her cost is low. As no offer is accepted with any greater likelihood than is v^- , offering a lower price cannot be more profitable. It is easy to show that, when hyper pessimistic, take-the-sure-thing, the payoff from which is $v^- - \kappa^- + (v^- - \kappa^-)\delta$, dominates offering any $p_1 > v^+$, the payoff from which is $0 + (v^- - \kappa^-)\delta$, and at least weakly dominates offering any $p' \in [p^{\min}, v^+]$, the payoff from which is $[p' - \kappa^- + (v^+ - \kappa^-)\delta]\beta_0 + (v^- - \kappa^-)(1 - \beta_0)\delta$. Hence, when hyper pessimistic, a low cost seller does not optimally deviate from take-the-sure-thing.

(ii) By definition the seller is very pessimistic if a low cost seller's payoff from take-the-sure-thing exceeds that from offering any price in the interval $(v^-, p^+]$, no matter how B responds to such a price. By Lemma 3 we know that when the seller is very pessimistic B 's optimal strategy is as follows. (1) Accept the first period price if it is in the interval $[p^{\min}, p^+]$, in which case B 's updated belief about the seller is $h_1(\cdot) = 1$ (i.e., B takes any such offer as proof that the seller's cost is high). (2) Reject if $p_1 > p^+$, in which case B 's updated belief is the same as his prior. (3) Reject if $p_1 \in (p^-, p^{\min})$, in which case B 's updated belief is $h_1(\cdot) = 0$ (i.e., B takes all such offers as proof that the seller's cost is low because they are lower than $p^{\min}$). (4) Accept if $p_1 \leq p^-$.

Consider a deviation by the seller if her cost is high. Because no $p_1 \in [p^{\min}, p^+)$ is accepted with any greater likelihood than is p^+ , and all yield smaller margins, it is clear that a high cost seller does not profit by deviating to any such price. And by Lemma 1 we know that she does not profitably to any $p_1 < p^{\min}$. Hence, a very pessimistic high cost seller makes no first period offer lower than p^+ . Might she make a higher first period offer? Lemma 3 shows that B 's threat to reject any $p_1 > p^+$ is credible when the seller is very pessimistic. Therefore, offering any $p_1 \in [p^{\min}, p^+]$ (at least weakly) dominates offering any $p_1 > p^+$, as at any of the former there is trade yielding non-negative margins with positive probability while at any of the latter there is no chance of trade.

Consider a deviation by the seller if her cost is low. It is by now clear that she does not profitably deviate to any offer lower than v^- . It is easy to show that, when very pessimistic, take-the-sure-thing, which pays her $v^- - \kappa^- + (v^- - \kappa^-)\delta$, dominates offering any $p_1 > p^+$, which pays $0 + (v^- - \kappa^-)\delta$, and at least weakly dominates offering any $p' \in [p^{\min}, p^+]$, which pays her $[p' - \kappa^- + (v^+ - \kappa^-)\delta]\beta_0 + (v^- - \kappa^-)(1 - \beta_0)\delta$. Hence, a very pessimistic low cost seller does not profitably deviate from take-the-sure-thing.

(iii) First, recall that moderate pessimism is the condition under which $\pi_1(v^-, \kappa^-) < \pi_1(p', \kappa^-)$, $\forall p' \geq p^{\min}$ if B accepts p' with sufficiently large probability. Second, recall that $p^\circ < \kappa^+$ means that $p^{\min} = \kappa^+$. Together, these mean that if the seller's cost is high, she is willing to consider first period offers in the interval $[\kappa^+, v^+]$, and if her cost is low, she will also consider making offers from the same interval if B accepts with large enough probability. Therefore, any offer that the seller would make if her cost is high will be a pooling offer if B accepts with sufficiently large probability. But notice that the highest pooling offer that B is certain to accept, p° , is unacceptable to the seller if her cost is high because here $p^{\min} = \kappa^+$ and $p^\circ < \kappa^+$. Therefore, there is no price that a high cost seller is willing to offer that B can optimally accept with certainty (see Lemma 3.iii.b for details). B 's optimal strategy is as follows. (1) Accept v^+ with any probability, y_1 , such that $0 < y_1 < y^-$, where y^- is the acceptance probability at which the seller is induced to gamble with an offer of v^+ if her cost is low, but below which her payoff from gambling is smaller than it is from take-the-sure-thing. Thus, if $p_1 = v^+$, then B 's updated belief is $h_1(\cdot) = 1$. (2) Reject if $p_1 \in [\kappa^+, v^+]$, in which case B 's updated belief is the same as his prior, i.e., $h_1(\cdot) = h_0$, because his strategy makes any such offer suboptimal for both seller types, yet both types would consider such an offer if B were to accept with sufficiently large probability. (3) Reject if $p_1 \in (p^-, \kappa^+)$, in which case $h_1(\cdot) = 0$ because, by Lemma 1, a high cost seller never offers such a price. (3) Accept if $p_1 \leq p^-$.

Consider a deviation by the seller if her cost is high. First, it is clear that she cannot profit by deviating to a higher offer because there is no chance for trade at any price higher than v^+ , while at v^+ there is a positive probability of trade with a positive margin. It is also clear that it cannot profit the seller to deviate to any $p_1 \in [\kappa^+, v^+]$, as the buyer is certain to reject any such offer. And by Lemma 1, we know that the seller does not profitably offer any $p_1 < p^{\min} = \kappa^+$ if her cost is high. Hence, the seller does not profitably deviate from v^+ if her cost is high.

Next consider a deviation by the seller if her cost is low. It is clear that she does not profit by deviating to any $p_1 \in (p^-, v^+)$ because her payoff from any such offer is $0 + (v^- - \kappa^-)\delta$, which is less than is her payoff from take-the-sure-thing. By Lemma 2 we know that she does not deviate to learn-then-discriminate by offering $p_1 = p^-$ because the expected payoff therefrom is smaller than is that from take-the-sure-thing when she is pessimistic. Further, because no $p_1 \in (v^-, p^-)$ is accepted with any greater likelihood than is p^- , but all yield smaller margins, no such deviation is profitable. It is easy to show that when moderately pessimistic, a low cost seller's expected payoff from gambling with an offer of v^+ , which is $y_1(v^+) [v^+ - \kappa^- + (v^+ - \kappa^-)\delta] \beta_0 + (v^- - \kappa^-) [1 - y_1(v^+) \beta_0] \delta$, is smaller than is her payoff from take-the-sure-thing when $y_1(v^+) < y^-$, as is the case here. Hence, the seller does not profitably deviate from take-the-sure-thing if her cost is low. Note that B 's response here is not unique, as any response to v^+ such that $0 < y_1(v^+) < y^-$ is an equilibrium response.

Finally, observe that because B 's response to a high cost seller's offer is not unique, neither is a high cost seller's updated belief about the buyer if her first period offer, v^+ , is rejected. It is important, however, that we establish the boundaries on her updated belief in the event that her offer is rejected. In particular, we must show that the lower boundary of $\beta_1(\cdot)$ is positive, for if it is zero, a high cost seller withdraws from the interaction under the belief that she cannot trade with the buyer. First, because B accepts with positive probability, we know by Bayes'

rule that her updated belief must be smaller than her prior, i.e., $\beta_1(\cdot) < \beta_0$, if her offer is rejected. More important, however, is to show that if a high cost seller's first period offer is rejected, her updated belief is positive. Let $\dot{\beta}$ be the infimum of a high cost seller's updated belief if her first period offer of v^+ is rejected. We must show that $\dot{\beta} > 0$. By Eqn. (A.1) in Appendix 1, we know that a moderately pessimistic high cost seller's updated belief in the event of a rejection is given by $\beta_1(\cdot) = (1 - y_1)\beta_0 / (1 - y_1\beta_0)$, which is a decreasing function of y_1 . And because B accepts her offer with $y_1 < y^-$, the infimum of her updated belief in the event of rejection, $\dot{\beta}$, is the value of $\beta_1(\cdot)$ when $y_1 = y^-$. Substituting, we find that $\dot{\beta} = \frac{[v^+ - \kappa^- + (v^+ - v^-)\delta]\beta_0 - v^- + \kappa^-}{v^+ - v^- + (v^+ - v^-)\delta}$, which is not positive if

$$\beta_0 \leq \frac{v^- - \kappa^-}{v^+ - \kappa^- + (v^+ - v^-)\delta}. \quad (\text{A.13})$$

But moderate pessimism means $\beta_0 \geq \beta^-$, and $p^{\min} = \kappa^+$ means $\beta^- = \frac{v^- - \kappa^-}{\kappa^+ - \kappa^- + (v^+ - v^-)\delta} > \frac{v^- - \kappa^-}{v^+ - \kappa^- + (v^+ - v^-)\delta}$, which means that Cdn. (A.13) cannot hold if the seller is moderately pessimistic. Therefore, $\dot{\beta} > 0$.

(iv) Recall that the seller is very optimistic when the following conditions hold. (1) A low cost seller's expected payoff from playing learn-then-discriminate by offering p^- , which B is certain to accept, exceeds that from gambling by offering v^+ , if B accepts v^+ with probability y° . And (2) a high cost seller's expected payoff from offering v^+ is at least that from offering p° if B accepts the former with probability y° and the latter with certainty. By Lemma 4.ii we know that when the seller is very optimistic, B 's optimal strategy is as follows. (1) Accept v^+ with any probability, $y_1(v^+)$, such that $y^\circ \leq y_1(v^+) < y^+$, where y^+ is the acceptance probability at which a low cost seller's expected payoff from gambling equals that from playing learn-then-discriminate by offering p^- , but below which learn-then-discriminate by offering p^- dominates gambling. Thus, on seeing that the offer is v^+ , B 's updated belief is $h_1(\cdot) = 1$. (2) Accept all $p_1 \in (p^-, v^+)$ with probability y° , in which case B 's updated belief is the same as his prior. (3) Accept if $p_1 \leq p^-$.

Consider a deviation by the seller if her cost is high. Again, we know that it does not benefit her to offer any price higher than v^+ . Consider a lower offer. It cannot benefit her to offer any $p_1 \in [p^{\min}, v^+)$, where $p^{\min} = \max\{p^\circ, \kappa^+\} > p^-$, as no such price is accepted with greater likelihood and all yield smaller margins. And, by Lemma 1, we know that it cannot benefit the seller to deviate to any $p_1 < p^{\min}$ if her cost is high. Hence, when very optimistic, the seller does not profitably deviate from offering v^+ if her cost is high.

If the seller's cost is low, we know by Lemma 2 that her expected payoff from playing learn-then-discriminate by offering p^- in the first period exceeds the payoff that is hers if she takes-the-sure-thing by offering v^- (or any lower price) when she is optimistic. And because no $p_1 \in (v^-, p^-)$ is accepted with greater likelihood than is p^- , but all yield smaller margins, she does not profitably deviate to any such offer. Thus, when very optimistic, a low cost seller does not profitably deviate to any price lower than p^- . Now let us consider the possibility that she profits from offering a higher price. First, it is clear that it is not profitable for her to offer any $p_1 > v^+$ since at any

such price there is no chance of trade. Second, it is easy to show that when very optimistic, a low cost seller's expected payoff from offering p^- , which is $[p^- - \kappa^- + (v^+ - \kappa^-)\delta]\beta_0 + (v^- - \kappa^-)(1 - \beta_0)\delta$, exceeds that from gambling with an offer of v^+ , which is at most $y_1(v^+)[v^+ - \kappa^- + (v^+ - \kappa^-)\delta]\beta_0 + (v^- - \kappa^-)[1 - y_1(v^+)\beta_0]\delta$ when $y^\circ \leq y_1(v^+) < y^+$, as is the case here. And because no $p_1 \in (p^-, v^+)$ is accepted with any greater likelihood than is v^+ , but all yield smaller margins, it is not profitable for her to deviate to any such price. Therefore, if the seller's cost is low, she does not profitably deviate from learn-then-discriminate with an offer of p^- when she is very optimistic. Note here that, as in part (iii) of this result, the buyer's response to a high cost seller's offer is not unique, as any response to v^+ such that $y^\circ \leq y_1(v^+) < y^+$ is an equilibrium response.

Finally, because B 's response to a high cost seller's offer is not unique, a high cost seller's updated belief about the buyer is likewise not unique if her first period offer, v^+ , is rejected. We can nonetheless establish the boundaries within which her updated belief, $\beta_1(\cdot)$, must fall. Most important is to show that the lower boundary is positive since if $\beta_1(\cdot)$ is not, then a high cost seller withdraws from the interaction. First, however, we establish the upper boundary. Recall that $\beta_1(\cdot)$ is a decreasing function of y_1 , the probability with which B accepts the offer (see Eqn. (A.1) in Appendix 1). Then, because here y_1 is at least as large as y° and y° is the response such that $\beta_1(\cdot) = \beta^\circ$ when the outcome is rejection, it is easy to see that $\beta_1(\cdot)$ can be no larger than β° if a high cost seller's first period offer is rejected. To show that a high cost seller's updated belief is positive if her first period offer is rejected, we will show that $\tilde{\beta}$, the infimum of $\beta_1(\cdot)$ when the seller is very optimistic, is positive. Because here y_1 is smaller than y^+ , we derive $\tilde{\beta}$ by substituting y^+ for y_1 in Eqn. (A.1). After some algebra, we find that

$$\tilde{\beta} = \frac{(v^+ - v^-)\delta\beta_0}{(v^+ - \kappa^-)(1 - \beta_0) + (v^+ - v^-)\delta} > 0. \quad \parallel$$

PROOF OF PROPOSITION 4. (i) Recall that β^- is the initial buyer reputation (equivalently, the initial seller belief) at which a low cost seller's expected payoff from offering $p^{\min}$ equals that from take-the-sure-thing if B is certain to accept $p^{\min}$. Also, recall that $p^\circ \geq \kappa^+$ means that $p^{\min} = p^\circ$. Hence, if B is certain to accept p° , then a low cost seller's expected payoff from offering this price is the same as it is from take-the-sure-thing. Finally, recall that when the seller is moderately pessimistic and $p^\circ \geq \kappa^+$, B 's optimal strategy is described in the proof of Proposition 2.i.

Given that B 's strategy is precisely the same as is given in the proof of Proposition 2.i, for the same reasons that the seller does not profitably deviate if her cost is high there, she does not deviate here if her cost is high. If the seller's cost is low, deviating to a higher price is not profitable for the same reasons given in the proof of Proposition 2.i. But, for a moderately pessimistic low cost seller, $\beta_0 = \beta^-$ implies that the expected payoff from playing learn-then-discriminate by offering p° , which is $[p^\circ - \kappa^- + (v^+ - \kappa^-)\delta]\beta_0 + (v^- - \kappa^-)(1 - \beta_0)\delta$, equals the payoff from take-the-sure-thing, which is $v^- - \kappa^- + (v^- - \kappa^-)\delta$. She is therefore completely indifferent between these two strategies, and any $\lambda_1 \in [0, 1]$ is optimal. Note also that the fact that any $\lambda_1 \in [0, 1]$ is optimal for the seller if her cost is low affects B 's updated belief, $h_1(\cdot)$, on observing an offer of p° (specifically, as λ_1 increases, so does $h_1(\cdot)$ if the offer is p°). And

because there is no unique optimal value of λ_1 when $\beta_0 = \beta^*$, there is no unique value of $h_1(\cdot)$ if the offer is p° . We can, however, establish the boundaries on the buyer's updated belief on observing a first period offer of p° . Of particular importance is the lower boundary of $h_1(\cdot)$, for B will accept p° if, but only if, $h_1(\cdot) \geq b_0$. Fortunately, it is easy to see that for any value of $\lambda_1 \in [0,1]$, the buyer's updated belief, $h_1(\cdot)$, is at least as large as his prior, b_0 . By Bayes' rule, if the observed offer is p° , then $h_1(\cdot)$ takes its smallest value if a low cost seller pools with a high cost one and offers p° with certainty, i.e., if $\lambda_1 = 0$. And by definition, if p° is a pooling offer, then the buyer's updated belief is the same as his prior on observing it. In other words, if the first period offer is p° , then the smallest possible value of $h_1(\cdot)$ is b_0 . Hence, B accepts p° . It is also important to note that none of this affects B 's updated beliefs upon observing offers higher than p° . Since either seller type is willing to make such an offer if B accepts with sufficiently large probability, on observing any such offer the buyer's updated belief is still the same as his prior. His optimal response to any $p_1 > p^\circ$ is likewise unchanged: he rejects them. As a result, if the seller's cost is high, the unique optimal strategy for her is to offer p° .

(ii) Recall that β^{++} is the initial buyer reputation at which a low cost seller's expected payoff from gambling by offering v^+ , which is $[v^+ - \kappa^- + (v^+ - \kappa^-)\delta]y^\circ\beta_0 + (v^- - \kappa^-)(1 - y^\circ\beta_0)\delta$, equals that from playing learn-then-discriminate by offering p^- , which is $[p^- - \kappa^- + (v^+ - \kappa^-)\delta]\beta_0 + (v^- - \kappa^-)(1 - \beta_0)\delta$, if B accepts the former with probability y° and the latter with certainty. Also note that when the seller is hyper optimistic, B 's optimal strategy is as described in the proof of Proposition 2.ii.

Given that B 's strategy is the same as in Proposition 2.ii, the same reasons that make it unprofitable for the seller to deviate from offering v^+ if her cost is high also apply here. If the seller's cost is low, we use the same arguments presented in Proposition 2.ii to show that she does not profitably deviate to any offer higher than v^+ , nor any in the interval $(p^- - v^+)$. But, as $\beta_0 = \beta^{++}$ means that her expected payoff from gambling by offering v^+ equals that from learn-then-discriminate by offering p^- , a low cost seller is completely indifferent between these two strategies. Therefore, any $\eta_1 \in [0,1]$ is optimal. Note that because any $\eta_1 \in [0,1]$ is an optimal strategy, B 's updated belief, $h_1(\cdot)$, on observing an offer of v^+ is affected. Specifically, because there is no unique optimal value of η_1 , there is also no unique value of $h_1(\cdot)$. But there are boundaries. At one extreme, if $\eta_1 = 1$, i.e., if a low cost seller were certain to pool with a high cost one, then $h_1(\cdot) = b_0$. At the other extreme, if $\eta_1 = 0$, i.e., if a low cost seller were certain to separate from a high cost one, then $h_1(\cdot) = 1$. Intermediate values of η_1 yield updated buyer beliefs between b_0 and 1. Therefore, if the first period offer is v^+ , we know that $h_1(\cdot) \in [b_0, 1]$. This, however, has no effect on B 's strategy, for if he accepts v^+ with any larger probability than y° , a low cost seller is no longer indifferent between v^+ and p^- , and we encounter the contradictions described in the proof of Corollary 2. And if he accepts v^+ with any smaller probability than y° , a high cost seller reduces her offer slightly, and again we encounter the contradictions described in the proof of Corollary 2. $\parallel$