Chapter 7

The UK Approach to Insuring Defined Benefit Pension Plans

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The UK government recently established a Pension Protection Fund (PPF) to protect members of private sector defined benefit (DB) occupational pension schemes[1] whose firms become insolvent (Pensions Act 2005). Some of the details of the way the fund will operate have still to be finalized. The purpose of this chapter is to identify and roughly quantify some of the main policy issues involved in the establishment of such a fund.

One key issue is the future solvency of the PPF, and possible claims on the public purse. The largest and best established exemplar of a Protection Fund is the Pension Benefit Guaranty Corporation (PBGC) of the USA. After a run of years of very low claims—claims over the period 1980–99 averaged $300 m/year—the PBGC has now faced some very large claims. In the period 2000–4, shortfalls amounted to some $21 billion in total. Its 2004 accounts show a deficit of $23.3 billion, taking account of probable claims from currently insured plans. The total underfunding of US pension plans covered by the PBGC increased from less than $30 billion in 1999 to more than $450 billion in 2004, as a result of interest rate changes and poor equity market performance (PBGC 2004). With premium income of $1.5 billion per year, and strong opposition in the Congress to raising premium levels substantially, it is questionable whether the PBGC will be able to meet its obligations without government support.

In this chapter, we model a generic plan to help analyze the extent to which these problems are inherent to a fund to protect DB pensions. Recognizing that corporate pensions are similar to corporate debt obligations, we show that the PPF is likely to face many years of low claims interspersed irregularly with periods of very large claims when prolonged weakness in equity markets coincide with widespread corporate insolvencies. We argue that it will not be possible to build up sufficient surpluses in the PPF in the good years to pay for the bad years. It will also be difficult to raise premiums sufficiently after a run of bad years to bring the PPF back to solvency. The government will not be able to let the PPF default, so it will be underwritten by the government whether the guarantee is recognized formally or not.

We consider, and reject, the argument that the problem can be mitigated by levying 'risk-based' premia.[2] They will have a limited impact on moral hazard. What they will do, however, is ensure that the burden of making good any deficit in the PPF will fall particularly on those schemes least able to bear it, making it more difficult to keep the PPF solvent, and increasing the likelihood of recourse to government. We also investigate the relation between the PPF and solvency requirements. The issue of pension default was the major focus of the Goode Report in 1994 set up by the British government following the theft by Robert Maxwell, Chairman of Mirror Group Newspapers, of the assets of its pension fund. The Goode Report considered, and rejected, the idea of a PPF. Instead, it recommended the introduction of a funding requirement to help ensure that there would be adequate assets in the pension fund to meet liabilities if the employer became insolvent or closed the fund for other reasons. This was subsequently introduced by the Pensions Act 1995 as the minimum funding requirement (MFR). Following criticism of its inflexibility and its distorting effect on pension fund investment, the government decided to withdraw the MFR. We argue that the need to protect the finances of the PPF will require constraints on scheme funding that are very similar to those imposed by a strong solvency-based MFR.

To address these issues, we develop a simple model of a pension plan. In its simplest form, company insolvency is a random (Poisson) event with a constant hazard rate. If the firm becomes insolvent, any deficit in the pension plan is picked up by the PPF. The contribution of the firm to the pension plan follows a simple smoothing rule that ensures that any deficits and surpluses are amortized over a number of years. Plan solvency varies because of the mismatch between the assets and liabilities; the assets are partly invested in equities, while the liabilities are bondlike. The investment policy and the contribution policy are exogenous. The model shows how the premium the PPF needs to charge to remain solvent depends on key parameters such as the investment policy of the pension plan, the contribution policy, the equity risk premium, and so on. The model is also used to dynamically simulate the behavior of claims over time.

We also develop a more sophisticated model in which the default rate is stochastic. Since a downturn in equity markets will not only increase pension fund deficits, but will also tend to be accompanied by an increase in insolvencies, the stochastic default model shows much greater volatility in the claims on the PPF. To model the default rate, we treat the PPF insurance as a guarantee of a corporate debt obligation, the firm's pension promise to its employees. We use a structural model of the firm, based on Collin-Dufresne and Goldstein (2001), where the firm's assets follow a stochastic process, and the firm defaults when its leverage ratio reaches a critical level. With defaults being correlated across firms (because of the positive correlation in asset values across firms), the claims process is much

more volatile than with Poisson default. With default being correlated with deficits in pension plans (because the assets of the firm are positively correlated with the assets of the pension plan), the claims level also becomes much larger.

The original paper on the topic of pension guarantees was by Marcus (1987), who also used an options framework to value pension insurance; in many respects, our model builds on his work. While he computes the value of insurance on a fixed portfolio of risks, which evolve with time in a nonstationary way, we compute the value of insurance for a steady-state population of firms. We choose to model firm funding policy in a way which ensures that firm assumptions about the equity risk premium enter into the steady-state risk-neutral density of firm solvency ratios. In common with other more recent work by Pennacchi and Lewis (1994) and Lewis and Cooperstein (1993), we assume that the pension protection fund does not receive the pension surplus if a firm declares bankruptcy and does not have any claim on the assets of a bankrupt firm.

The models we use take the firm's policy as exogenous. We also discuss how the existence of the PPF provides incentives may affect behavior—the moral hazard issue. We examine the consequences of varying premia according to the solvency of the pension fund, and the credit standing of the employer. For risk-based premia to have any significant impact on the future solvency of the PPF, they need to create a strong financial incentive on weak sponsors to fully fund their plans and to reduce the mismatch between assets and liabilities.

The Nature of Pension Liabilities and Claims on the PPF

In this section, we discuss the nature of the claims on the PPF in order to explain and motivate the model we will be using. Our main concern is with the factors determining the level of the premium to be charged and the pattern of claims over time. We model a representative firm and its pension plan. The investment policy of the plan and the contribution policy of the firm are exogenous; later, we consider how they may be affected by the existence of the PPF.

Firms offering DB pensions to their employees are obliged to fund their obligations. The adequacy of the pension fund is reviewed every three years by an independent actuary who recommends to the trustees the level of future contributions required to ensure that the fund is able to meet its liabilities on a continuing basis. The actuarial valuation of the fund is not related to solvency—ensuring that the assets of the plan exceed its liabilities—but rather to funding—setting a smooth path for contributions that will over the long term allow the plan to pay the promised pensions. In deciding whether a DB plan is adequately funded, the actuary must make judgments about future investment returns, though they are

irrelevant to solvency. So a DB scheme that is technically fully funded may actually be in substantial deficit.[3] That does not mean it will not meet its obligations, but it will need ongoing support from the employer to be sure of doing so.

   If the plan is underfunded, the actuary will recommend an increased level of contributions that will, assuming reasonable investment perform-ance, allow it to become fully funded in a number of years. The relation between the sponsoring firm's financial state and its contribution policy is complex. On the one hand, a firm facing financial distress may be particu-larly inclined to defer contributions; on the other hand, it is precisely in these cases where a rapid return to fund solvency is of greatest importance to pensioners. Recent evidence on the relationship between pension fund solvency and the financial status of sponsoring firms is difficult to find. In the USA, Bodie et al. (1985) reported a negative relationship between the credit rating of a firm and the solvency of its pension plan in weaker firms. Orszag (2004), however, found little evidence that weaker UK firms system-atically underfund their pension plans.

   The distribution of pension liabilities and underfunding in the UK can be seen in Table 7-1. It shows the median funding ratio, the total pension

TABLE 7-1 Total UK DB Pension Liabilities for FTSE-350 Companies

S&P credit rating	Number of companies	UK pension liability (FRS17, £mil)	Unfunded UK pension liability (FRS17, £mil)	Median plan funding ratio
AA+	2	11,816	523	0.91
AA	5	21,184	3,349	0.87
AA−	5	14,743	3,267	0.76
A+	12	32,225	5,801	0.74
A	10	21,145	4,187	0.82
A−	12	55,230	14,539	0.78
BBB+	13	13,228	3,325	0.74
BBB	14	18,977	2,427	0.81
BBB−	7	12,760	1,730	0.84
BB+	2	3,784	453	0.85
BB	3	8,711	503	0.79
Not rated	163	63,886	14,180	0.70
Total	*248*	*277,689*	*54,285*	*0.73*

Source: Authors' calculations using Watson Wyatt Pension Risk Database on published accounts for the company financial year ending between June 2002 and May 2003.

Notes: Liability figures are in millions of pounds, calculated on the FRS17 basis reported in the accounts and include only UK liabilities. Figures include only those companies in the FTSE 350 which have DB plan liabilities. The credit rating is as reported by Standard and Poor's at the date of the accounts.

liability, and the total unfunded pension liability for all FTSE-350 companies with DB pensions. It is derived from the FRS17 disclosures in their accounts for fiscal year 2002/3.[4] Several patterns can be noted. First, the majority of pension liabilities (67 percent) and pension underfunding (69 percent) is with companies rated BBB or above, even making the conservative assumption that all nonrated companies have true credit ratings below BBB. The third column shows the median funding ratio in each rating category. There is no clear trend in funding as the credit strength of the sponsoring firm declines.

In the light of this, and to keep the model simple, we take the contribution policy to be independent of the firm's financial state, and to depend only on the plan's solvency level so that, over the long term, the assets equal the liabilities, although we later make allowance for the fact that pension plans may fund to a different standard.

The potential for a large deficit when an employer becomes insolvent depends on the investment policy of the pension plan. The plan's liabilities resemble a long-dated inflation-indexed bond. The assets of UK pension plans are typically at least 50 percent invested in equities. One might expect trustees of plans that are more precarious (larger deficits, weaker employers) to be more cautious about protecting their solvency, but the evidence does not bear this out. Table 7-2 shows the average equity proportion of a variety of types of fund and finds no strong relationship, except that plans that are less well funded appear to invest slightly more heavily in equities. In our model, we assume that the asset mix of the pension fund is constant.

It is clear that the level of claims on the PPF is heavily dependent on the mismatch between the assets and liabilities of pension plans, and the speed with which any over- or underfunding is corrected. The time profile of claims on the Fund will closely reflect the performance of the equity market, with a prolonged downturn leading to widespread underfunding, and large claims when firms become insolvent.

TABLE 7-2 Average Equity Proportion in Pension Fund Asset Portfolio for Different Pension Plan Types

	Below median	Above median
Pension plan assets/Pension plan FRS17 liabilities	0.72	0.58
Pension plan FRS17 liabilities/Company market capitalization	0.68	0.62
Book value of company debt/Company market capitalization	0.66	0.63
Company market capitalization/Book value of firm assets	0.65	0.65

Source: Authors' calculations using Watson Wyatt Pension Risk Database.

Notes: Each cell shows the proportion of the plans assets invested in equities for plans below and above the median value of each plan variable. Means differ as not all data are available for every company.

Modeling the Guarantee

In what follows we describe the model in outline; a detailed description appears in the Appendix. The PPF we model guarantees the pension liabilities of a set of firms. In this section, we assume that there is an infinite number of small, identical firms, and focus on one representative firm. The insolvency of a firm is modeled as a Poisson process, so a constant fraction of the firms becomes insolvent in each period. The present value of the accrued liabilities of a scheme but we assume that it is nonstochastic.

The assets of the pension plan comprise a riskless bond with constant interest rate, and an equity portfolio. We assume that each plan invests in the same market index fund. Equity returns are risky and attract a risk premium. The pension scheme receives contributions and pays out pensions. We assume a simple rule for determining the level of contributions that increases the level of contributions the greater the degree of underfunding in the scheme. If the firm becomes insolvent, and if the guaranteed liabilities of the plan at that time exceed the assets, the PPF pays the difference. The expenditure of the PPF in a period in the model is the equal to the sum of the deficits of the schemes whose sponsors become insolvent in the period.

So far as the income of the PPF is concerned, there are many possible ways of levying the premium. The PBGC uses a combination of a charge per member covered and a charge proportional to the dollar size of any deficit in the scheme. In the UK, the PPF is required to take account of other matters including the solvency of the scheme sponsor. We do not address the question of the optimum premium schedule directly. For the present, we assume that the premium is a constant proportion of the scheme's liabilities.

We wish to equate the present value of future claims on the PPF with the present value of its premium income in steady state. The simple market we model is complete—the only stochastic component in the revenue and expenditure of the PPF is the return on the equity market—so both assets and liabilities can be valued using standard contingent pricing arguments. We show in the Appendix that the fair premium level, measured in dollars per dollar of insured liabilities is a function of seven parameters:

- $\hat{\alpha}$, the market risk premium assumed by the scheme in determining contributions
- σ_m, the volatility of the market
- δ, the bankruptcy hazard rate of the sponsor company
- a^*, the maximum permitted funding ratio
- x, the equity proportion in the fund
- T, the time over which fund deficits are amortized
- λ, the proportion of liabilities that are guaranteed

Estimating the Model. For the purposes of our analysis we take $\hat{\alpha} = 6\%$ and $\sigma = 18\%$. The probability of the firm becoming insolvent, δ, is difficult to estimate. Using Moody's global database on long-term default rates by rating category for 1983–2003 (Hamilton et al. 2004, Exhibit 31), we apply it to the observed credit-rate distribution of UK pension liabilities. Table 7-1 implies a 10-year cumulative default rate of 2.95 percent, corresponding to an annual rate of 0.30 percent. This may be too high as a long-term estimate since it takes as its base ratings in 2002/3 when the corporate sector was in a financially weak state. Also, a firm that defaults on its debt may refinance and continue without defaulting on its pension obligations. On the other hand, the Moody's data apply only to rated companies; by contrast, the PPF insures plans of companies that are not rated, and the latter are likely to have, on average, higher probability of default. In the light of this, we take δ to be 0.25 percent per year. Consistent with the observed behavior of pension schemes, we take as our central case a maximum funding ratio a^* of 120 percent, an equity proportion of 2/3, a 10-year amortization period (T), and we assume that the PPF liabilities are 90 percent of liabilities assumed for funding purposes (so $\lambda = 0.9$). Since these parameters will vary between pension schemes, we also conduct sensitivity analysis.

Table 7-3 explores the effects of varying the investment strategy (as measured by x) and the funding strategy (as measured by T) on the size of the premium. In the base case, the premium level is £0.50/£1,000 of liability.[5] The difficulty of estimating the mean default rate means that the absolute level of premium that we obtain from our model should be treated with great caution. But since the premium is directly proportional to the

TABLE 7-3 Premium with Poisson Default (£/year per £1,000 of liabilities)

	Equity proportion		
	1/3	*2/3*	*100%*
Base case	0.206	0.497	0.726
Higher solvency cap: $a^* = 200\%$ (120%)	0.206	0.494	0.716
Stricter solvency: $T = 4$ yrs (10)	0.044	0.191	0.339
No assumed risk premium: $\hat{\alpha} = 0\%$ (6%)	0.039	0.171	0.314
Partial guarantee: $\lambda = 80\%$ (90%)	0.062	0.297	0.510

Source: Authors' calculations.

Notes: The base case shows the unconditional fair value premium for guaranteeing a pension fund against default when the risk of default is 0.25% per annum, equities have an expected return of 6% in excess of the risk-free rate, deficits in the fund are made up over 10 years, and the fund value is not permitted to exceed 120% of liabilities. The premium is shown as a percentage of liabilities for different investment strategies. The other lines of the table show how the cost varies as each of the input parameters is varied. Base-case values are shown in parentheses.

default rate, the sensitivity of the premium to varying assumptions should be not be affected by the uncertainty in the default rate

The direction of the sensitivities is as one would expect: the higher the equity proportion, the larger the premium. Having a higher solvency cap does reduce the premium because the fund is allowed to build up large surpluses when the market does well. But the effect is small; raising the cap on assets from 120 to 200 percent of liabilities, even assuming 100 percent equity funding, reduces the premium by less than 2 percent.[6] Stricter solvency requirements, as modeled by amortizing deficits over 4 rather than 10 years, have a very substantial effect, cutting the premium by over 50 percent.

The assumed risk premium has a substantial impact, with a zero-risk premium cutting the insurance premium by nearly two-thirds in the central case. This can be interpreted in two ways. The first is as a measure of the importance of funding policy: if companies, in computing their contribution rate, assume that all their assets would just earn the risk-free interest rate, they would pay higher contributions for any given level of the solvency ratio, and so would on average achieve a higher solvency ratio. The burden on the Protection Fund would be lower because of the more conservative contribution policy, just as it would be with a more rapid amortization policy. A second interpretation is to see it as a measure of the importance of the price of risk in setting a fair insurance premium. The premium computed using a zero-risk premium is the same as the expected rate of claims (under the objective measure) when the true and assumed risk premium coincide. Taking the base case with 2/3 equity, the table shows that while the fair premium is 0.050 percent of liabilities each year, the (objective) expected rate of claims is about one-third of that level, at only 0.017 percent of liabilities each year. The difference between the two arises because claims on the Fund are most likely to occur when the market declines, and the cost of insuring against bad states of the world is higher than the objective probability of those states occurring.

The bottom line of Table 7-3 shows that restricting the guarantee to 80 rather than 90 percent of liabilities, while retaining the PPF's senior claim on all a pension plan's assets, also reduces the premium significantly. This is not only because the sum guaranteed is smaller, but because the first part of any deficit in the pension plan falls fully on the beneficiaries. With 2/3 equity, the effect of restricting the guarantee to 80 percent of liabilities reduces the premium per dollar of pension liabilities by 40 percent.

A Structural Model of Default Rates

Next, we extend the model to include a stochastic default rate. Allowing for variable default rates is important for pension plan guarantees, since the risk of default varies substantially over time and is correlated across firms.

It is also negatively correlated with the equity market. These facts have three important implications:

(1) A falling equity market increases both the probability of sponsor firms becoming insolvent and also the size of pension plan deficits. So stochastic default induces a positive correlation between the probability of a claim on the PPF and the size of the claim. This increases the fair premium.

(2) The correlation between default risk and equity returns means that default risk is priced. This will further increase the difference between the (objective) expected rate of claims on the Fund and the fair premium.

(3) The correlation of default risk across firms increases the skewness of the claims process.

To explore the practical significance of these issues, we need a model of default that captures correlations across firms and correlations with the equity market—phenomena not well-captured in the Poisson default model. We explore three strategies for modeling default: fitting the empirical evidence on default directly, fitting the behavior of corporate debt spreads, and structural models of the firm. Next, we explain why we prefer the structural model approach and why we choose mean reverting leverage à la Collin-Dufresne and Goldstein (2001). We then present premium calculations and claim simulations implied by this model.

Choice of Default Model. The simplest strategy for modeling default is to take historic default rates, postulate some functional form for their time series behavior, and estimate a relationship. The problem with this is the paucity of data. Defaults are rare—fewer than 1500 defaulted issuers are included in Moody's database between 1970 and 2003. As shown in Figure 7-1, default rates are highly autocorrelated over time. This is obviously important for modeling the PPF. But basing a model purely on the limited empirical data would be hard to do with any reliability. The peaks in 1990–91 and 2000–02 would drive results.

An alternative approach is to use information from the behavior of credit spreads. The empirical evidence does strongly support correlations in changes in credit spread across firms and strong negative correlation between credit spreads and the equity market. Pedrosa and Roll (1998) document the existence of strong common factors in credit spreads for portfolios of credits, where the sixty portfolios in question are characterized by broad industry group, credit rating category, and maturity. A more detailed analysis of the spreads on individual US industrial bonds is provided by Collin-Dufresne, Goldstein, and Martin (2001). They look at weekly changes in spreads against comparable Treasury bonds on a universe of 688 straight (not callable or convertible) bonds from 261 different issuers
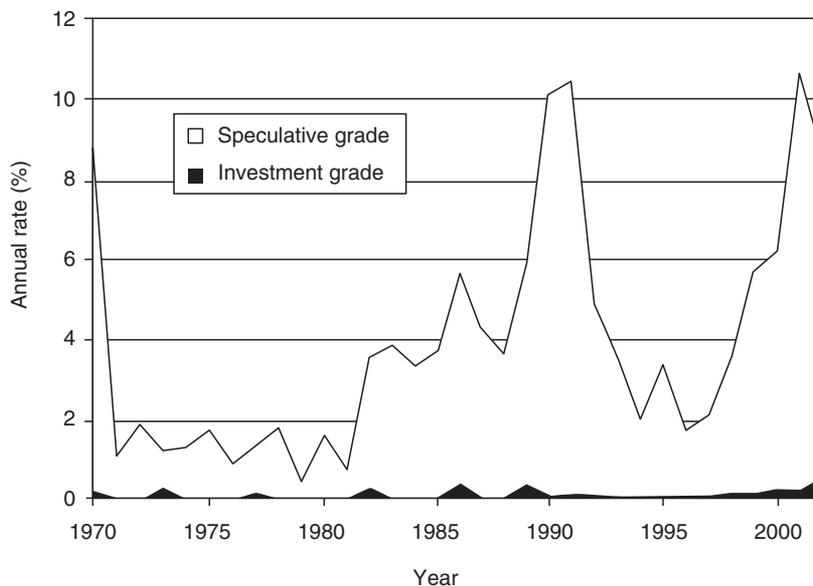
Figure 7-1. Global issuer-weighted default rates, 1970–2003.
Source: Hamilton et al. (2004).

over the period 1988–97. They regress the changes in a number of factors suggested by theory, including the firm leverage ratio, the level and slope of the government yield curve, the level and slope of implied volatility on the equity market, and the level of the equity market. They find that a 1 percent increase in the S&P500 index is associated with a credit-spread decrease of about 1.6 basis points. Their regressions explain about 25 percent of spread changes; by examining the residuals from the regression, they show that 75 percent of the unexplained change can be ascribed to a common factor that they fail to identify with any other macroeconomic variable. These results are based on US data, but similar results in the sterling Eurobond market are obtained by Manzoni (2002) where daily changes in the spread of the yield on the market index to UK Treasury yields are negatively correlated with returns on the UK stock market. Over the period 1991–99, a 1 percent increase in the FTSE 100 index is associated with a credit spread decrease of 2.1 to 3.5 basis points depending on the specification.

Building a model of default that is calibrated to bond prices is attractive because of the large amount of high quality data on the behavior of bond yield spreads. But it faces a serious obstacle. There is mounting evidence (Elton et al. 2001; Huang and Huang 2003) that credit risk accounts for only a part—according to Huang and Huang, in the case of investment grade bonds, less than a quarter—of the yield spread. In the absence of any

generally accepted explanation of why the risk-adjusted expected return on corporate bonds is higher than on default free bonds, the credibility of a model would be in doubt if it incorporates the whole yield spread in valuing the pension fund guarantee.

Accordingly, we model the default process from fundamentals, using a structural model of the firm. Structural models originate with Merton (1974) who describes a risky bond as a portfolio consisting of riskless bond and a short position in a put option on the assets of the firm. This simple idea has been developed by many other authors (Duffie and Single-ton 2003 offer an overview). Structural models are widely used as a basis for pricing credit sensitive instruments though they do not appear to capture yield spreads on corporate bonds with great accuracy. Nevertheless, Huang and Huang (2003) show that structural models, when suitably calibrated, do fit the empirical data on default rather well. For our specific purpose, structural models have three other advantages: the correlation between corporate default and the behavior of the equity market arises naturally within the model; the correlation in default rates across firms arises naturally in the model from the correlation in firms' asset values; and, unlike models based on the yield spread, the price of default risk can be computed within the model, without the need to make any assumptions about the behavior of recovery rates.

Previously we had a stationary process for pension plan deficits that allowed us to compute an unconditionally fair insurance premium that is a constant proportion of the value of insured liabilities. To retain this feature, we need a structural model of default that is also stationary. The natural candidate is Collin-Dufresne and Goldstein (2001, hereafter CDG) who have a model with mean-reverting leverage ratios. As in other structural Merton-type models, debt is a claim on the firm's assets V. The assets follow a diffusion process with constant volatility σ_v, and the firm's leverage varies accordingly. But CDG argue that firms tend to adjust their leverage over time through their financing strategy. This causes the leverage ratio to revert to some target level. The key variable is the log leverage ratio of the firm, l. The leverage ratio is defined as the ratio of the critical asset level at which default will occur to the current asset level. CDG model the dynamics of l as a first-order autoregressive process where l reverts to some long run mean level $\bar{l}$ with speed κ and volatility σ_v. The correlation between changes in l and equity market returns is a constant, ρ.

The log leverage ratio l is strictly negative so long as the firm is solvent; if it hits zero, the firm defaults. Two additional elements complete the specification of the model. First, we need to specify the correlation structure of firm asset returns. We assume that each firm's return is the market return plus a noise term that is identically and independently distributed across firms. Second, we assume that idiosyncratic risk is unpriced. Starting with a portfolio of firms with the same leverage and the same pension

funding, the pension funding level varies over time with the equity market but remains the same across firms, while leverage ratios disperse because of firm idiosyncratic risk. We have now fully specified the processes governing the claims on the PPF; a more formal presentation appears in the Appendix.

With no new firms being born, the steady-state joint probability function of the scheme solvency level a and the firm leverage level l is $g(a,l)e^{-\delta t}$, where δ is now the steady-state default rate driven by the condition that $l = 0$ is an absorbing barrier. The results would be unaltered if there is a steady entry of new firms into the portfolio provided that their distribution in (a, l) space is the same as the steady-state distribution.

Estimating the Model. To estimate the model, we generally follow Huang and Huang (2003); their estimates are broadly consistent with CDG. Since those estimates vary slightly according to the credit rating of the bond in question, we take their results for an A-rated issuer (Moody's or S & P's). In particular, we take the mean reversion parameter κ to be 0.2, the asset volatility σ_v to be 24.5 percent and the asset risk premium 4.89 percent. Huang and Huang show this is consistent with an equity premium for the firm of 5.99 percent. Taking the equity β to be one, the market risk premium is also 5.99 percent, and the asset β is 0.82. Using an equity market volatility σ_m of 18 percent, the correlation between the change in firm asset value and the equity market return is $\rho = \beta\frac{\sigma_m}{\sigma_v} = 0.60$.

Using Huang and Huang's estimate of the long-term average leverage ratio of 38 percent gives a long-run average default rate of 0.75 percent per year. For the reasons already discussed, this looks very high, so we use an average leverage ratio of 31.7 percent; this gives a long-run default rate of 0.25 percent per year. We compute the steady-state joint density of the solvency ratio a and the leverage ratio l using a two-dimensional binomial tree with births and deaths, and iterate forward in time until the default rate and rate of claims on the fund converge to their limiting values. In all the iterations, we use a time step of 0.1 year.

Table 7-4 shows the premium and expected claims rate for a variety of parameter values. Using the same base-case parameters as before (2/3 of the pension fund invested in equity, 120 percent ceiling on overfunding, 10-year deficit amortization period, 90 percent of liabilities guaranteed), the average rate of claims is calculated to be £0.68/£1,000 of liabilities per year. This compares with a claims rate of £0.17/£1,000 in Table 7-3 with a Poisson default process. This fourfold increase in claims is entirely attributable to adding a correlation between corporate defaults and pension underfunding in the structural model.

The impact of the structural default model on the premium is still greater. With Poisson default, the fair premium in Table 7-3 was £0.50/£1,000. In the structural default model, it is more than seven times as high, at £3.90/£1,000. The other two rows of the table show that the level of

TABLE 7-4 Premium and Average Claims with Structural Default (£/year per £1,000 of liabilities)

Equity proportion (x):	2/3		100%	
	Premium	Claim	Premium	Claim
Poisson default	0.50	0.17	0.73	0.31
Structural default:				
Base case	3.90	0.68	5.23	1.01
$\lambda = 80\%$ (90%)	2.86	0.45	4.19	0.77
$T = 4$ (10)	2.34	0.43	3.50	0.70

Source: Authors' calculations.

Notes: The Poisson default case is from Table 7-3. The Structural Default model base case has the same dynamics for the solvency ratio as the Poisson model; the two also have the same expected default rate (0.245%). The first variant on the base case has only 80% of liabilities guaranteed by the PPF, and the second has an amortization period for pension fund deficits of 4 years rather than 10. The other parameters of the models are: $a^* = 120\%$, $\sigma_m = 18\%$, $\sigma_v = 24.5\%$, $\bar{l} = -1.15$, $\kappa = 0.2$, and $\rho = 0.6$.

premiums, and the average rate of claim, can be reduced significantly by limiting the proportion of liabilities guaranteed (with the PPF retaining first claim on all the assets of the pension fund), and by stricter pension fund solvency requirements.

Evidently, fair premium levels are substantially higher than those envisaged for the PPF. It is difficult to compare our calculated premia with actual premia charged by the US PBGC, as the latter depend on actual pension underfunding while our calculations assume a steady-state distribution of funding and firm leverage. In any event, in fiscal 2004, the PBGC collected $1,481 million in premia on its single-employer program. Guaranteed liabilities amounted to $1.35 trillion in 2001 (the latest date for which figures are available; PBGC 2003) and the premium amounted to $1.10 per $1,000 of liability. Our calculated premium is thus more than three times greater than the PBGC premium, yet our expected claims are roughly half. We caution that it would be wrong to attach too much importance to the absolute numbers, as they are sensitive to model parameters, in particular to the assumptions concerning the long-run average leverage ratio. Using Huang and Huang's estimate of 38 percent, rather than the value we have used of 31.65 percent, would imply fair premia that are more than twice as high.

Claims Distribution. Thus far we have established the average level of claims in the long run, reflecting the average long-run claims experience of the PPF. Of course the variation in the claims level is also a matter of considerable concern. To investigate the variation in the claims level, we

simulate the claims process, and ask: how high a claims rate can one reasonably expect over a period of say thirty years? To address this point, simulations are carried out with the same base case as Table 7-4, using the structural default model, and an equity proportion of 2/3. As in Table 7-4, the fair premium is £3.90/£1,000 of liabilities, while the expected level of claims is £0.68/£1,000.

Table 7-5 shows the distribution of the thirty-year worst case, using objective probabilities; it is based on 1,000 simulations, with a time step of one-tenth of a year. The simulations start with the steady-state distribution of firm leverage and pension fund solvency. A path for the equity market is then simulated. The liabilities of schemes grow at a constant rate equal to the average rate of insolvency, so ensuring that the level of insured liabilities is stationary. Since the pension assets of all firms are perfectly correlated, and deficits are corrected by adjusting contribution policy, the initial dispersion in pension funding levels across firms quickly narrows. Firm asset value is subject to idiosyncratic risk, so while there is comovement, there is also substantial dispersion. In running the simulations, the first seventy years are used as a conditioning period, and the following thirty years are then used as the sample period. The conditioning period is needed to ensure that the start of the sample period is suitably randomized. For comparison, we also show comparable figures for the Poisson default case. The claims are expressed as a percentage of the average size of liabilities over the thirty-year period.

The table shows how the structural model of default not only increases the magnitude of average claims, but also greatly increases their skewness. In the Poisson model, the level of claims in the worst year in thirty was just over three times the average claim level in the median case. By contrast, the

TABLE 7-5 Claims/£1,000 in Worst Period in 30 Years (simulation)

	Structural default		Poisson default	
Fair premium	3.90		0.50	
Average claim	0.68		0.17	
	1 year	*5 years*	*1 year*	*5 years*
Median	4.1	6.9	0.59	2.1
Top quartile	11.5	19.2	0.74	2.9
Top decile	24.1	42.8	0.87	3.6

Source: Authors' calculations.

Notes: The table is based on 1,000 simulations of the evolution of the distribution of firm leverage and solvency level for the population of insured firms, and shows the average and peak annual claim level over each 30-year period. The parameter values for the base case are: $a^* = 120\%$, $T = 10$, $\lambda = 90\%$, $\beta = 1$, $\sigma_m = 18\%$, $\sigma_v = 24.5\%$, $\bar{l} = -1.15$, $\kappa = 0.2$, and $\rho = 0.6$. The Poisson default case is identical except that $\rho = 0$.

structural default model has a ratio in excess of six. In the worst decile of 30-year period, the contrast is even starker: with Poisson default, the ratio is about 5, while with structural default the ratio is well over 30. The effect is strongly visible even looking at five-year periods, with the worst five-year period being comparable to twice the worst single-year experience.

While it would be wrong to attach much precision to the numbers—these are rare and extreme events—the results of the simulation do illustrate the extent to which correlated defaults across firms, and the correlation between the mean default rate and the equity market, may create considerable skewness in claims experience. This has important implications for the setting of premia. If the PPF seeks to build up reserves sufficient to meet claims in the worst year in 30 years with 90 percent probability, Table 7-5 suggests it would need to have reserves in excess of 24 years of average claims, or roughly 2.5 percent of insured liabilities. It is difficult to believe that agreement could be reached on setting the level of premiums necessary to build up such a high level of reserves.

In the absence of such reserves and of any support from government, the PPF would need to borrow to pay claims, using its future premium income as collateral. But this alternative looks barely more palatable, since it would require premia to be raised very substantially. If, for example, there were claims equal to 2.5 percent of liabilities in 1 year, and they were met by borrowing that had to be repaid over 10 years, then additional premia equal to nearly four times the normal average claims level would need to be charged to repay the debt, ignoring any real interest due on the debt. This high premium would have to be charged at a time when, by assumption, the solvent firms that remain are heavily leveraged, and themselves have pension funds in substantial deficit.

If the PPF cannot weather extreme events either by way of reserves or by way of borrowing backed by increased premia, then that leaves two alternatives: default or some form of government involvement. The PPF will have powers to reduce the amount guaranteed under extreme circumstances, but this is a route that is fraught with problems. The very name of the fund, and the fact that the government has frequently stated that it has acted to restore confidence in the pensions promise[7] means that it will be very difficult politically for a government to allow the PPF to significantly reduce its commitment. It is hard to avoid the conclusion that the government will be left as the final guarantor of DB pensions.

Cross Subsidy and Moral Hazard

In our base case, the fair insurance premium is £3.90/£1,000 of insured liabilities per year, and the expected claim level is £0.68 per year. This is not a deadweight loss to pension plans. The total cost of the premia to all pension plans is exactly matched by the gains to pensioners in failed

pension plans. But for individual firms, the benefits and costs will not match. For pensioners of firms with high credit ratings, the probability of default is very small, and for them (or their employers), the PPF represents, in effect, a significant tax from which they derive little benefit. Since it is a tax just on DB pensions, it will tend to discourage the provision of such pensions.

As yet, we have assumed that the existence of the PPF and the way that its premiums are set has no effect on pension fund investment strategy and plan sponsors' contribution/benefit policy. But there is reason to doubt these assumptions. The difference between the costs and the benefits of the PPF insurance at the fund level creates incentives for firms to maximize the net value of their own PPF cover, with potential consequences for the solvency of the PPF as a whole. For instance, members and firms receive the benefits of a risky investment strategy, but the costs may be paid by the PPF. Therefore, trustees have an incentive to follow a riskier investment strategy than they would otherwise. Weaker firms might find that an underfunded pension plan—effectively a loan from employees guaranteed by the PPF— is cheaper than a loan from the markets and has no consequence for pension fund members because it is guaranteed by the PPF. The PPF could thus become a source of subsidized financing for unscrupulous firms. Firms may also collude with employees to increase pension benefits— guaranteed by the PPF—in lieu of current wages, effectively a joint raid on the PPF. Firms may also alter the relative funding of their pension plans to take into account benefit limits on the PPF. The list of opportunities for dishonest behavior is limited only by the imagination of firms and their advisors, as pointed out by two ex-PBGC Executive Directors (Kandarian 2003; Utgoff 1993).

Our model can be used to assess the size of incentives for bad behavior if we assume that the PPF charges our constant base-case premium of £3.90/ £1,000 of liability to all firms. In our model, a firm that invested its pension assets entirely in bonds would never be underfunded and so would derive no value from the PPF insurance. It would therefore suffer a loss of £3.90/ £1,000 of liability per year. A firm which invested its assets entirely in equities, on the other hand, would receive pension insurance with a value of £5.23/£1,000 per year, but would pay only £3.90, hence receiving an annual windfall of £1.33/£1,000 of liability. Similarly, a firm which maintained full pension funding at all times would derive no value from pension insurance but would pay £3.90/£1,000 of liability. A firm which amortized any surpluses and deficits over four instead of ten years—a stricter funding standard than the base case—would value the PPF insurance at only £2.34/ £1,000 per year, but would pay £3.90/£1,000—a loss of £1.44/£1,000 of liability per year. Similarly, firms with an average leverage ratio of 38 percent (these firms would be financially weaker than our base-case firms, which have an average leverage ratio of 31.6 percent), would value

insurance at £8.20/£1,000 per year and pay £3.90—and hence would pay less than half the fair premium.

One way of reducing these transfers would be to set premia for guaranteeing pensions that more closely reflect their risk level. The PPF is required by law to ensure that 80 percent of the premium is 'risk-rated', related to factors such as the degree of pension underfunding, pension investment policy, and the strength of the corporate sponsor. Although risk-rating with these factors would remove some of the moral hazards associated with the PPF, it would probably do little to control the extreme lumpiness of the claims process pointed out in the previous section. We have already seen that, in the absence of the PPF, schemes are heavily invested in equities and are often seriously underfunded. As Table 7-4 shows, the value of the PPF to a sponsor and the beneficiaries are greater, the more the equity investment and the less strict the funding. Consequently, considerable risk-rating in setting the premium would be necessary just to offset these benefits.

The lumpiness of the claims process can only be mitigated by forcing financially weak sponsors to ensure their schemes are fully funded. The premiums required to do this are large. In the absence of premium penalties, underfunding a pension scheme is similar to borrowing from the scheme at the riskless rate to fund the business. To induce a sponsor to put additional money into a scheme, the penalty on maintaining a deficit needs to be of the same order as the borrowing spread the sponsor would pay. Similarly, in order to induce firms to switch from equities into bonds, the penalty on equity investment would need to be of the same order as the equity risk premium—up to 6 percent per annum of the amount invested in equities.

So risk-related premiums may improve fairness by ensuring that those schemes benefiting most from the PPF pay more toward its cost, but they will do little to reduce the probability of very high claims unless they lead to radical changes in the level of contributions and investment policy. Indeed, risk-related premia may make it more likely that a run of bad years could force government intervention. For if, as we have argued, the PPF will be unable to build up large reserves, and if it is unlikely in practice to cut back benefits, then the only way it can react to a run of bad years is to raise premiums. But the constraint on raising premiums is the damage it does to companies and to employment. The pressure to raise premiums will be particularly acute if premia bear more heavily on the highest risk sponsors, since these are precisely the companies where raising premiums is most likely to cause financial distress.

In order for the PPF to work effectively, something other than risk-rating may well be required. A strong MFR on a transparent basis will effectively control underfunding, and hence claims on the PPF. The PPF itself will have to lay down precise rules for computing the solvency ratio, and could

not allow pension funds leeway in making their own assumptions. Further, the PPF could cut back on the level of guaranteed benefits without changing the funding process. Under current legislation, the PPF guarantees only 90 percent of deferred pensions, there is a cap on the amount of each pension that is protected, and some pension increases are not covered by the PPF. Lowering the level of guaranteed benefits will have a significant effect on the cost of insurance to pension funds and, eventually, to the taxpayer but will not reduce the volatility of claims.

Conclusions

Our analysis illustrates some of the problems that may be faced by PPF, and we offer way to adapt the design of such a guarantee fund to rectify these problems. Although failure of pension plans to pay people their entitlements have been unusual in the UK, it would be dangerous and wrong to conclude that failures will be rare and small in the future. The way that pension schemes are funded, and the way that funds are invested, imply that a deep and prolonged decline in financial markets could readily lead to widespread failure. An inherent feature of the claims process facing the PPF is likely to be that many years of small claims will be interspersed with rare and unpredictable periods of exceedingly large claims. These periods will coincide with periods when the stability of the whole of the financial sector is under maximum strain.

Though we do not claim to have a very accurate or even a practical method of determining fair premiums, our models imply that the magnitude of the claims in these unstable periods will be so large that it will not be politically feasible or economically sensible to build up reserves to meet them. When such a crisis does occur, it may well be impossible to meet claims by a steep increase in the levy on employers since they will simultaneously be facing heavy financial demands to rebuild their own depleted pension funds. There may be little alternative to having the government step in, even though in the UK case, the government has repeatedly made clear that it will not guarantee the PPF. Consequently, a substantial part of the cost of the scheme will actually fall to the taxpayer. Further, the PPF will necessarily involve large transfers from companies that are unlikely to default to companies that may well default. These transfers are inefficient, and create opportunities for moral hazard.

To minimize the cost of the insurance and to keep down the level of cross-subsidy, the government has argued that the PPF must risk-rate its premiums. We argue that risk-rated premia will need to be sufficiently steep to alter the current investment and funding policy of UK pension plans if they are to have a significant impact. Premium risk-rating may need to be implemented in tandem with a strong minimum funding ratio, to reduce the potential cost of the PPF to future UK taxpayers.

Our model is necessarily simplistic, but we note that the assumptions we make tend to underplay the nature of the problem. That is, we model the liabilities as a continuum of small plans, which therefore ignores the lumpiness in claims that comes from a large plan failing. We also assume zero correlation in the idiosyncratic risk of companies, and so we take no account of whole industries facing financial distress. We have set aside the problems that might arise if the PPF fails to match the assets and liabilities of the defaulted plans it is managing. If the fund were to invest in equities, the volatility in the PPF's net worth would be further increased. Finally, we assume that the only systemic risk affecting the sector is equity market risk; other risks, such as unpredicted changes in interest rates and longevity, could further increase volatility of claims on the PPF. Integrating these elements is a task for future research.

Appendix: Modeling the Pension Guarantee

Our model assumes that the PPF guarantees the pension liabilities of a continuum of small, identical firms, and focuses on one representative firm.

The Poisson Case

The insolvency of a firm is first taken to follow a Poisson process with hazard rate δ. With δ being constant and default risk uncorrelated across firms, each firm faces a constant and equal probability of default in each time period. With an infinite number of firms, a constant fraction δdt of the firms become insolvent in each period dt. The present value of the accrued liabilities of the firm's pension plan at time t, denoted by L_t, may vary over time, but is assumed to be nonstochastic. The assets of the plan have value A_t. If the firm becomes insolvent at time t, and if the guaranteed liabilities of the plan exceed the assets, the PPF pays $L_t - A_t$.

In practice, pension plan liabilities are measured in several different ways. For the purpose of this model, L_t should be interpreted as the cost at time t of buying out the guaranteed liabilities of the pension plan at that time, and A_t is the market value of the assets of the plan, after allowing for any costs of winding up. Implicitly, we are assuming that if the firm becomes insolvent, the PPF has full access to the assets of the pension plan, at least so far as they do not exceed the guaranteed liabilities, but no access to the assets of the firm itself. By topping up the pension plan's assets to equal its liabilities, the PPF can ensure that there is no further claim on the PPF from that pension plan.

The assets of the pension plan comprise a riskless bond with constant interest rate r, and an equity portfolio. We assume that each plan invests in

the same market index fund, which may be assumed to be the portfolio of all available equities, weighted in proportion to their market capitalization. We assume that the instantaneous return on the market portfolio, dS/S, follows an Ito process:

$$\frac{dS}{S} \ = \ (r + \alpha)\,dt + \sigma_m dz_m, \tag{1}$$

where z_m is a standard Brownian process, α the market risk premium, and σ_m the volatility of the market.

We wish to compute the present value of future claims on the PPF. The claims are stochastic, and depend on future stock market performance. Rather than compute the expected level of future claims, and taking their present value by discounting back to the present at a suitably chosen discount rate that reflects the riskiness of the cash flows, we use the risk-neutral methodology that is standard in the finance literature. Where, as in our model, all risks are hedgeable,[8] present values can be obtained by projecting future outcomes using a pricing or risk-neutral pricing measure Q in place of the objective probability measure, and discounting the expected claims using the risk-free interest rate. The risk-neutral probability measure is that measure under which all the assets have an expected return equal to the risk-free rate. Hence, Equation (1) can be rewritten as

$$\frac{dS}{S} \ = \ rdt + \sigma_m dz_m^Q, \tag{2}$$

where dz_m^Q is a standard Brownian motion process under measure Q. Setting the right-hand sides of Equations (1) and (2) equal, we can derive an expression for dz_m^Q in terms of dz_m:

$$dz_m^Q \ = \ \frac{\alpha}{\sigma_m}\,dt + dz_m. \tag{3}$$

The expected value of dz_m^Q under measure Q is 0, so taking expectations of both sides under the risk-neutral measure Q gives

$$E^Q[dz_m] \ = \ -\frac{\alpha}{\sigma_m}\,dt \tag{4}$$

I_t is an indicator function that takes the value 1 if the firm is still solvent at time t, and 0 otherwise. If the firm becomes insolvent, the pension plan is closed. If the firm becomes insolvent at time t (so $dI_t \ = \ -1$) and if the pension fund is in surplus at that time ($L_t \leq A_t$) then the pension plan is able to pay pensions due in full, and no liability falls on the Protection Fund.[9] If there is a deficit in the pension plan when the firm becomes insolvent, the Protection Fund takes over both the assets and the liabilities. The cost to the Fund at the time the firm becomes insolvent is thus:

$$-\int_t^\infty [L_u - A_u]^+ \mathrm{d}I_u \quad \text{where } [x]^+ \equiv Max\,(x,0). \tag{5}$$

Determining the Premium. The firm pays an insurance premium P_t to the PPF. From the Fund's perspective, insuring the plan has a present value equal to the expected value of the premiums paid by the firm when it is solvent, less the expected value of the payments that the PPF will have to make if a firm defaults, both discounted at the risk-free rate. We take expectations using the risk-neutral measure Q to ensure that the value obtained takes proper account of the risk in the Fund's premiums and its liabilities:

$$E^Q\left[\int_t^\infty P_u I_u e^{-r(u-t)}\,\mathrm{d}u + \int_t^\infty [L_u - A_u]^+ e^{-r(u-t)}\,\mathrm{d}I_u\right]. \tag{6}$$

If the PPF is to be able to cover the cost of claims from its premium income then, ignoring administrative costs, the present value of premium income less claims must be 0. Hence, any premium must satisfy the condition:

$$E^Q\left[\int_t^\infty P_u I_u e^{-r(u-t)}\,\mathrm{d}u + \int_t^\infty [L_u - A_u]^+ e^{-r(u-t)}\,\mathrm{d}I_u\right] = 0 \tag{7}$$

In principle, there are many ways of levying the premium. We do not address the question of the optimum premium schedule directly. For the present, we assume that the premium is a constant proportion of the scheme's liabilities.

If the premium is levied at rate p:

$$P_t = pL_t. \tag{8}$$

From (7), using the Poisson default rate process and the nonstochastic nature of the liabilities, the rate p is given by:

$$p = \frac{\int_t^\infty E_t^Q[\delta(1 - A_u/L_u)]^+ L_u e^{-(r+\delta)(u-t)}\,\mathrm{d}u}{\int_t^\infty L_u e^{-(r+\delta)(u-t)}\,\mathrm{d}u}. \tag{9}$$

Modeling default as a Poisson event among a continuum of atomistic schemes with uncorrelated default rates ensures that a constant proportion $\delta\,du$ of plans become insolvent in each time period du. This gives rise to the δ inside the expectation term. The δ in the discount factor reflects the fact that the number of plans is declining at rate δ because of insolvencies. The premium rate is a weighted average of the expected claim rate into the future. It depends on the current solvency level of the scheme. The main focus of this chapter is on the impact of different contribution schedules, investment policies, and guarantee arrangements on the level of the premium. To abstract from variations caused by initial conditions, we look at

processes that generate stationary distributions of insolvency rates and deficit levels, and take unconditional expectations.

With unconditional expectations, Equation (9) simplifies to:

$$p = E^Q[\delta(1 - \frac{A_u}{L_u})]^+. \tag{10}$$

The Dynamics of Scheme Solvency. In order to evaluate the expectation in Equation (10), we need to specify the dynamics of the scheme solvency ratio A_u/L_u under the risk-neutral measure Q. The dynamics of A depend on the return on the portfolio, outflows to pensioners, and inflows from contributions. Again, written as an Ito process, we have:

$$dA = [(r + x\alpha)A + (\kappa_t - \pi_t)]dt + x\sigma_m A dz_m, \tag{11}$$

where x is the (fixed) proportion of the assets held as equity, κ the contribution rate, and π the rate of pay out to pensioners. The first component of the dt term states that the expected rate of return on the assets is the risk-free rate plus the equity risk premium on the equities held by the plan. The second component shows that the assets increase at rate κ because of contributions to the fund and decrease at rate π because of payments to pensioners. As before, the dz term has zero expected value and models how the value of the assets changes as a result of random fluctuations in the value of the equities held by the fund.

The firm's contribution to the pension plan has two components: the first maintains the current solvency level after allowing for payments to pensioners, any change in net liabilities, and the expected return on the assets of the plan. The second component is designed to eliminate any surplus or deficit in the plan over a specified period of T years. The lower the level of T, the faster any deficit is eliminated and the lower the potential claim on the PPF. The simplest formulation that achieves this is:

$$\kappa_t = \left(\pi_t + \frac{dL_t}{dt}\frac{A_t}{L_t} - (r + x\hat{\alpha})A_t\right) + \left(\frac{L_t - A_t}{T}\right). \tag{12}$$

$\hat{\alpha}$ is the excess return on equities assumed by the firm in setting its contribution rate; it may be identical with the true α, but is not necessarily so. Define the solvency ratio of the fund a as:

$$a_t = \frac{A_t}{L_t}. \tag{13}$$

Then, using Ito's lemma, we can calculate the stochastic differential equation governing the evolution of the solvency ratio as follows:

$$da = dA/L - \frac{adL}{L}$$

$$= \left[\left(r + x\alpha - \frac{dL}{Ldt} \right) a + (\kappa_t - \pi_t)/L \right] dt + x\sigma_m a dz_m \tag{14}$$

$$= \left(\frac{1-a}{T} + x(\alpha - \hat{\alpha}) a \right) dt + x\sigma_m a dz_m.$$

We can express this equation in terms of the risk-neutral probability measure Q by substituting Equation (3) to give:

$$da = \left(\frac{1-a}{T} - \hat{\alpha}ax \right) dt + x\sigma_m a dz_m^Q \tag{15}$$

Given the investment policy and the contribution policy, the solvency ratio follows a stationary stochastic process that is independent of the behavior of liabilities. We can derive the unconditional distribution of a at time t under the risk-neutral measure by stating the condition that the distribution is stationary using Equation (15) to derive a differential equation. Formula (10) then gives the fair premium rate p (expressed as a proportion of the liabilities of the pension plan) as:

$$p = \delta \int_0^1 (1-a) g^Q(a) \, da. \tag{16}$$

Note that the true equity risk premium, α, does not enter into Equation (14), and hence will not affect the risk neutral density function g^Q or the premium rate p. A higher equity premium raises the expected future solvency level of pension schemes, but this is offset by the increase in discount rates used for valuing the PPFs. However the equity premium assumed by the scheme ($\hat{\alpha}$) does enter into the premium; the higher the assumed premium, the lower the contribution rate and the greater the expected claim on the Fund.

The premium can be compared with the unconditional objective expectation of the rate of claims as a proportion of liabilities, c, which we calculate in a similar way, using Equation (14) instead of Equation (15). The resulting expected cost of claims is:

$$c = \delta \int_0^1 (1-a) g^P(a) \, da \quad \text{where } g^P \text{satisfies:}$$

$$\frac{1}{2} \frac{d^2}{da^2} \left(x^2 \sigma_m^2 a^2 g^P(a) \right) - \frac{d}{da} \left(\left(\frac{1-a}{T} + x(\alpha - \hat{\alpha}) a \right) g^P(a) \right) = 0.$$

The differential equation expresses the condition that the distribution of the solvency ratio is stationary.

Extending the Model. One element of unrealism in our model is that the solvency ratio of the pension fund is not bounded above. There are limits on the degree to which the pension fund can hold assets in excess of its liabilities, imposed largely to prevent the sponsor company using the pension fund as a tax avoidance device. We can readily impose the condition in our model that a is not permitted to exceed some limit a^*. Whenever a does exceed the limit, the contribution rate is constrained to force a below the limit. a^* acts as a reflecting barrier.

We assume that firms are able to reclaim investment surpluses from their pension plans over the same time horizon over which deficits are amortized. In practice, firms may struggle to reclaim investment surpluses because they face pressure to improve benefits or because they do not wish to be seen 'raiding' the pension plan of their employees.

We have also assumed that the liabilities that are guaranteed by the PPF are *the* same as those used to determine the firm's pension contribution. In practice these two measures of liability may well differ substantially, and in either direction. Not all accrued liabilities are guaranteed; there is a cap on the level of wages on which the pension is guaranteed; the PPF only guarantees 90 percent of deferred pensions, and certain pension increases are not guaranteed. In addition, the definition of liabilities used by actuaries in computing funding levels generally takes account of future wage growth in computing the pension liability arriving from past service. Finally, the actuarial valuation may also use a higher discount rate in valuing liabilities than the rate at which the liabilities can be bought out in the market.

The model can readily be adapted to distinguish between the liabilities used for funding requirements and those that are guaranteed by the PPF if we assume that the ratio of guaranteed liabilities to the actuary's measure of liabilities is constant. Denote the ratio by λ. Assume also that the PPF retains a prior claim on all the assets of the fund if the firm becomes insolvent. Maintain the definition of a as the ratio of plan assets to the cost of meeting the liabilities guaranteed by the PPF. Then a mean reverts to $1/\lambda$ rather than to 1. The adjustments to the model are obvious. For example Equation (14) becomes:

$$da = \left(\frac{1/\lambda - a}{T} + x(\alpha - \hat{\alpha})a\right)dt + x\sigma_m a dz_m. \tag{17}$$

The Stochastic Default Approach

With stochastic default, the log-leverage ratio l^i of firm i follows the stochastic process:

$$dl^i = \kappa(\bar{l} - l^i)dt + \sigma_v dz_v^i \tag{18}$$

and the firm defaults when the leverage ratio hits an absorbing barrier at 0. The correlation between innovations in the log-leverage ratio and innovations in the market index is constant, and the idiosyncratic risk is uncorrelated across firms so:

$$E[dz_v^i \cdot dz_m] = \rho dt;$$
$$E[dz_v^i \cdot dz_v^j] = \rho^2 dt \ (i \neq j). \tag{19}$$

Endnotes

1. For brevity, this chapter uses the word 'pension' to mean a private-sector DB occupational pension.
2. Legislation provides that at least 80 percent of the premium should be risk-based, tied to scheme solvency, sponsor credit rating, investment policy, and other factors relevant to the likelihood of a claim.
3. We use the term 'deficit' to mean the difference between the market value of assets and the cost of buying out accrued liabilities. According to a forthcoming report by an Institute and Faculty of Actuaries working party, of 685 actuarial valuations surveyed in 2001 and 2002, the average valuation discount rate was approximately 140 b.p's above gilt rates (Institute and Faculty of Actuaries 2004).
4. These figures should be treated with caution inasmuch as the valuations for each firm are at the balance sheet date which varies across firms.
5. This figure appears comparable with the premium rates initially proposed for the PPF, which is expected to raise £300 m per year in revenues. While it is not easy to give a precise estimate of the insured liabilities, it is worth noting that the DB liabilities of FTSE-350 companies in Table 7-1 amount to nearly £300 billion, and the pension liabilities of non UK-based companies, including UK subsidiaries of overseas companies may be of the same order.
6. The reason that raising the cap has such a small effect is that the risk-adjusted probability of reaching 120 percent solvency is rather small, so the cap does not greatly affect contribution levels.
7. For example: 'We will make sure that in future individuals in final salary schemes will never again face the injustice of saving throughout their lives only to have their hard-earned pension slashed just before they retire. The Pension Protection Fund will allow individuals to save with confidence.' (Smith 2004)
8. The level of liabilities faced by the PPF is perfectly correlated with the level of the equity market. This means that the PPF could, in principle, reduce or even eliminate credit risk by selling equities. While this hedging might not be desirable or even practicable, it does provide a price for the risks to which the PPF is exposed.
9. We are implicitly assuming that the investment policy of a closed fund precludes the trustees from investing in risky assets and putting the solvency of the fund at risk.

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