# AIR-FLUIDIZED GRAINS AS A MODEL SYSTEM: SELF-PROPELLING AND JAMMING 

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ABSTRACT<br>AIR-FLUIDIZED GRAINS AS A MODEL SYSTEM: SELF-PROPELLING AND JAMMING<br>Lynn J. Daniels<br>Douglas J. Durian

This thesis examines two concepts - self-propelling and jamming - that have been employed to unify disparate non-equilibrium systems, in the context of a monolayer of grains fluidized by a temporally and spatially homogeneous upflow of air. The first experiment examines the single particle dynamics of air-fluidized rods. For Brownian rods, equipartition of energy holds and rotational motion sets a timescale after which directional memory is lost. Air-fluidized rods no longer obey equipartion; they self-propel, moving preferentially along their long axis. We show that self-propelling can be treated phenomenologically as an enhanced memory effect causing directional memory to persist for times longer than expected for thermal systems. The second experiment studies dense collections of self-propelling air-fluidized rods. We observe collective propagating modes that give rise to anomalously large fluctuations in the local number density. We quantify these compression waves by calculating the dynamic structure factor and show that the wavespeed is weakly linear with increasing density. It has been suggested that the observed behavior might be explained using the framework put forth by Baskaran et al. [12]. The third experiment seeks to determine whether a force analogous to the critical Casimir force in fluids exists for a large sphere fluidized in the presence of a background of smaller spheres. The behavior of such a large sphere is fully characterized showing that, rather than behaving like a sphere driven by turbulence, the large ball self-propels. We also show that the background is responsible for the purely attractive, intermediate-ranged interaction force between two simultaneouslyfluidized large balls. The final experiment seeks to determine what parameters control the diverging relaxation timescale associated with the jamming transition. By tilting our apparatus, we quantify pressure, packing fraction, and temperature simultaneously with dynamics as we approach jamming. We obtain an equation of state that agrees well with simulation and free volume theory. We collapse the relaxation time by defining a time- and energy-scale using pressure, consistent with recent simulation [82]. These experiments are further confirmation of the universality of the concepts of self-propelling and jamming.

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## Introduction

Driven, non-equilibrium systems - encompassing a wide range of materials including colloids, foams, gels, human and animal traffic, and grains - pose a formidable research challenge in physics. These systems typically evolve with time and require a continuous injection of energy in order to maintain a steady state. Driven far from equilibrium, one must understand the microscopic dynamics in order to fully understand the macroscopic response to a perturbation. The difficulty in treating these systems arises because observed behavior is often dependent on the system details, driving mechanism, and history. Thus, there is no overall framework such as equilibrium statistical mechanics that can be invoked to characterize the behavior. As such, one avenue of research typically pursued when considering driven, non-equilibrium systems is to develop a common framework or a unifying concept for many of the disparate non-equilibrium systems. In other words, one must determine what behavior is universal or generic and what is dependent on the particulars of the system under study. For example, one can seek to determine when and why the behavior of a non-equilibrium system is well-described by equilibrium statistical mechanics.

This thesis will focus on granular material as an important example of a driven, non-equilibrium system. Not only are granular materials ubiquitous in nature, they represent an ideal non-thermal system that can be used as a proxy to study universal characteristics of non-equilibrium systems generally $[25,36,56]$. Granular systems are typically advantageous in this regard because the grains are large and easy to visualize with typical length- and time-scales able to be captured by standard video imaging. Unlike thermal systems, granular systems require a continuous input of energy in order to flow. This flow causes gas- or liquid-like behavior macroscopically and collisions and strong velocity fluctuations microscopically.

Characterizing these microscopic fluctuations is difficult and oftentimes dependent on the particulars of the system under study. Interestingly, despite the wide range of driving mechanisms, an effective temperature - determined from the relationship of these grain-scale fluctuations of a system and its macroscopic response to a perturbation - has become a unifying concept to characterize fluctuations in granular systems. A common definition of the effective temperature is the kinetic energy associated with velocity fluctuations. Examples of where this concept has been employed for granular materials range from dilute grains driven on a horizontal plane $[5,35,60,65]$, flowing granular liquids $[44,47,51]$, and vertically-vibrated granular gases [13, 23, 27, 61].

Understanding the macroscopic response of these granular systems requires determining what structural and dynamical signatures are universal and generic characteristics of driven, nonequilibrium systems. Self-propelling and jamming have arisen, like the effective temperature, as unifying concepts for disparate systems. The concept of self-propelling - that is, a particle converts energy into preferential motion that is dissipated by moving through its medium - attempts to unify diverse biological systems ranging from bacteria [16,24, 34, 72, 79], schooling fish [40], swarming insects [17], flocking birds [11, 48], and human and animal traffic [33, 37] under a common framework that ought to extend to granular systems as well. Similarly, the concept of a jamming transition - that is, when a system becomes stuck in a given packing configuration and is not observed to rearrange within the experimental window of measurement - has unified non-equilibrium systems ranging from supercooled liquids, colloids, foams, and granular materials [45, 46]. Under this avenue of research, the question arises: what behavior observed in a granular system where particles self-propel (approach jamming) is universal to all collections of self-propelling particles (particles approaching the jamming transition) and what behavior is dependent on choice of system itself?

The experiments carried out in this dissertation examine self-propelling and jamming in a monolayer of grains that are fluidized into motion by a spatially and temporally homogeneous upflow of air [43]. This particular method of driving the system affords some advantages over more conventional methods of shearing and vibration. In both sheared and shaken systems, the energy is injected only at the boundaries and the system may exhibit a frequency-dependent macroscopic response. With air fluidization, the input is uniform in space and time. These
methods of driving can also result in a wide range of interesting phenomena that are systemdependent: shear banding, pattern formation, and avalanches to name a few. Vibrated beds are prone to mechanical instabilities as well. Additionally, vertical vibration as a method of driving has several inherent disadvantages: the driving force is temporally inhomogeneous and the particle dynamics may vary depending on the phase of the oscillation cycle at the time the particle contacts the substrate. Further, for spherical particles on smooth plates, a lateral force is only generated by particle overlap out of plane. Thus, it is crucially important that other driving mechanisms, such as air fluidization, are explored.

The first class of experiments examines self-propelling as a unifying concept and extends previous work [3] that sought to test the robustness of a thermal analogy for particles fluidized by air. That is, spheres fluidized by air individually behave in exact analogy to a thermal particle undergoing Brownian motion [60]; the work of Ref. [3] asked what parameters could be varied to break the analogy. Two parameters were found for which this thermal analogy progressively failed: aspect ratio of the particle and the size ratio of two simultaneously fluidized spheres. In both cases, the behavior of the particles was not thoroughly studied; it was only demonstrated that the thermal analogy failed. Here, we are interested in more closely examining how elongated particles or spheres of different size behave when fluidized. During the course of these experiments, it was observed that rods move by propelling themselves along their long axis and a similar propelling mechanism exists for different-sized spheres. Similar self-propelling behavior has been observed for granular rods on a vertically-vibrated bed [54]. We naturally ask whether the framework developed to unify collections of biological self-propelling particles applies to a purely physical situation. Through comparison with vibrated-bed experiments we can begin to determine what observed behavior is due to self-propelling and what is due to the driving mechanism.

The second class of experiments explores the unifying concept of jamming and extends the use of the apparatus as a proxy for a colloidal, glassy system. Previous experiments by Adam Abate [4] found structural and dynamical signatures of the jamming transition for air-fluidized grains that are consistent with what has been seen in supercooled liquids, colloids, and foams, establishing our air-fluidized granular system as a model system with which to study the jamming transition. Establishing the universality of the jamming transition requires one to consider the
nature of the jamming transition and the diverging time- [4] and dynamical length-scale [39] associated with it. To this end, Liu and Nagel [46] proposed a jamming phase diagram with temperature, inverse density, and applied stress or shear as the axes. One outstanding question is what combination of these parameters control the diverging time- and length-scales as jamming is approached. This is crucially important because, depending on the details of a given system, the jamming transition will be approached by different trajectories in phase space. The exhaustive prior experiments by Abate were conducted by either varying airflow at fixed density or vice versa. For a given experiment, the density, effective temperature, and relaxation times were all well-defined and known. However, there was no access to information about the pressure exerted by the particles on the boundary. Here, we seek to measure all three control parameters - temperature, density, and pressure - simultaneously with the particle dynamics in order to address this question.

The remainder of this introduction will first review the thermal analogy in the gas-fluidized bed by discussing the behavior of a single ball in Section 0.1 and the perturbative testing of the thermal analogy in Section 0.2. In Section 0.3, I will review the theoretical framework for understanding collections of self-propelling particles as well as the robust macroscopic behavior predicted for the wide range of theoretical treatments. Lastly, I will discuss the structural and dynamical characteristics of the jamming transition generally in Section 0.4 and in the context of Ref. [4] in Section 0.5.

The remainder of the thesis is organized as follows. Self-propelling of air=fluidized rods will be examined in Chapters 1 and 2. The self-propelling behavior of and interaction between large balls fluidized with smaller balls is discussed in Chapter 3. The jamming experiment is covered in Chapter 4. Details of the apparatus as well as some programming notes can be found in Appendix A. For all experiments, actual video data is available online [1].

### 0.1 The Thermal Analogy and a Single Ball

In this Section, we will review the seminal experiment by Rajesh Ojha et al. [60] conducted using the gas-fluidized bed: a single ping-pong ball fluidized by an upflow of air. In the text, we
will refer parenthetically to figures in the original paper that are not reproduced here. The gasfluidized bed apparatus itself is described in Appendix A. The particle studied is a sphere of mass $m=4.2 \mathrm{~g}$, radius $R=1.9 \mathrm{~cm}$, and moment of inertia of a rolling, hollow sphere $I$. Because the fluidized sphere rolls without slipping, an effective mass can be defined as $m_{e f f}=m+I / R^{2}$. The particular system here has the advantage that it eliminates particle-particle interactions and, therefore, isolates gas-mediated interactions. Since the ball is large, microscopic details could be measured easily from video images, allowing direct contact with forces acting on the sphere. It was then demonstrated that statistical mechanics could be applied perfectly to this granular non-equilibrium system.

Ojha first showed that the probability distributions for the ball speed and radial position (Fig. 1(a) and 1(b) in Ref. [60]) have the same Gaussian form as a harmonically-bound Brownian particle, given by:

$$
\begin{align*}
& P(v)=\frac{2 v}{\left\langle v^{2}\right\rangle} e^{-v_{2} /\left\langle v^{2}\right\rangle}  \tag{1}\\
& P(r)=\frac{2 r}{\left\langle r^{2}\right\rangle} e^{-r_{2} /\left\langle r^{2}\right\rangle} \tag{2}
\end{align*}
$$

Similarly, it was confirmed that the equipartition theorem holds: $k T=m_{e f f}\left\langle v^{2}\right\rangle / 2$ and $K=$ $m_{e f f}\left\langle v^{2}\right\rangle /\left\langle r^{2}\right\rangle$ where $K$ is a spring constant. The effective temperature was verified by computing the distribution of the total energy $E=m_{e f f} v^{2} / 2+K r^{2} / 2$ (Fig. 1(c)) which agrees well with the Brownian expectation $P(E)=\left[E /(k T)^{2}\right] \exp (-E / k T)$.

To demonstrate the applicability of statistical mechanics, the single-ball dynamics were described by a Langevin equation and fluctuation-dissipation relation. The equation of translational motion can be written as

$$
\begin{equation*}
m_{e f f} \frac{d \mathbf{r}(t)}{d t}=-\nabla V(\mathbf{r})-m_{e f f} \int_{-\infty}^{t} d t^{\prime} \Gamma\left(t-t^{\prime}\right) \mathbf{v}\left(t^{\prime}\right)+\mathbf{F}_{r}(t) \tag{3}
\end{equation*}
$$

where the right-hand side is the sum of the forces acting on the sphere. The first term is the gradient of the effective harmonic potential in the apparatus equal to $-K \mathbf{r}(t)$. The integral term is the drag force; $\Gamma(t)$ is the memory kernel of the form

$$
\begin{equation*}
\Gamma(t)=\Gamma_{0} \gamma_{0} \exp \left(-\gamma_{0} t\right) \tag{4}
\end{equation*}
$$

where $1 / \gamma_{0}$ is the timescale over which memory is important. The final term is the random force, the components of which were shown to have Gaussian distributions and exponential temporal
autocorrelations, such that they obeyed a fluctuation-dissipation relation:

$$
\begin{equation*}
\left\langle\mathbf{F}_{r}\left(t^{\prime}\right) \cdot \mathbf{F}_{r}(t)\right\rangle=2 m_{e f f} k T \Gamma\left(t-t^{\prime}\right) . \tag{5}
\end{equation*}
$$

Thus, even though this is a far-from-equilibrium system, statistical mechanics holds perfectly with the caveat that the effective temperature is not a thermodynamic temperature.

To further characterize the system, Ojha et al. extended their initial work in Ref. [59] by quantifying the interactions of the single ball with the confining walls as well as with a second ball. These are crucial measurements that will be referred to many times in this thesis. Using statistical mechanics, the exact form of the interaction potential of a single ball with the confining walls can be deduced by noting that the probability to find a sphere in a ring of radius $r$ is proportional to the ring radius multiplied by a Boltzmann factor

$$
\begin{equation*}
P(r) \propto r \exp [-V(r) / k T] . \tag{6}
\end{equation*}
$$

From the radial position probability data, the potential is known to be nearly harmonic. However, using the form Eq. (6), the data can be converted to an exact interaction potential. The result (shown in Fig. 4 of Ref. [59]) is given by the same empirical form independent of airflow speed, ball size, or system size $R_{\text {sys }}$ as:

$$
\begin{equation*}
\frac{V(r)}{k T}=\frac{30\left(r / R_{s y s}\right)^{2}}{1+\left(r / R_{s y s}\right)^{3}} . \tag{7}
\end{equation*}
$$

Similarly, the interactions of two equal-sized balls can be deduced from the interaction potential $V_{s s}(\rho)$, where $\rho$ is the distance between the centers of the two spheres. When two spheres are fluidized simultaneously, the interaction is repulsive by visual inspection. The two balls will rarely collide, never stick together, and accelerate apart after close approaches. The velocity and position distributions (Fig. 5) remain radially symmetric and equal for each sphere. Both the rms speed and radial position increase with the addition of a second ball.

To deduce the interaction potential, Ojha et al. summed the Boltzmann factors for all possible configurations of arranging the spheres with the specified separation:

$$
\begin{gather*}
P(\rho) \propto \int d x d y d \theta \exp \left[-\frac{1}{2} K\left\{x^{2}+y^{2}+(x+\rho \cos \theta)^{2}+(y+\rho \sin \theta)^{2}\right\}\right] \exp \left[-V_{s s}(\rho) / k T\right] \\
\propto \exp \left[-\left(\frac{1}{4} K \rho^{2}+V_{s s}(\rho)\right) / k T\right] \tag{8}
\end{gather*}
$$

where $\theta$ is the angle between the ball centers-of-mass as measured from the horizontal axis. The separation probability is easily found from the video data (and shown in the top plot of Fig. 7). It rises abruptly from zero at a separation equal to a ball diameter, peaks near a separation of 7 cm , then decays to zero at large separations. The sphere-sphere interaction can then be deduced from Eq. (8). The actual force is then obtained by differentiating according to $F_{s s}(\rho)=-d V_{s s}(\rho) / d \rho$. The result (shown in the bottom plot of Fig. 7) showed hard-core repulsion followed by nearlyconstant repulsion over all length scales.

### 0.2 Breaking the Thermal Analogy

In the previous section, we reviewed two papers where the thermal behavior of a single fluidized sphere was established and ball-wall and ball-ball interactions were characterized. Here we will consider work by Adam Abate that systematically perturbed the system in order to determine parameters by which the thermal analogy would progressively break down [3]. It was shown that the equipartition theorem no longer held as two parameters were varied: size ratio of two different spheres and aspect ratio of a single particle.

When two spheres of different size are fluidized together, the resultant behavior is rather dramatic. Although two differently-sized spheres typically repel, if they approach closely enough, they form a lasting contact. This sudden loss of symmetry was proposed to cause airflow vortices to be shed preferentially along the line of centers, resulting in a net force in the direction of the smaller sphere. Thus, it appears as if the large sphere is chasing the small ball out of its vicinity. The quantitative result of this behavior is that the ratio of mean kinetic energies of the two balls is no longer equal to one (shown in Fig. 7(b) in Ref. [3]). For increasing radius ratio, the larger sphere becomes progressively more energetic than the smaller sphere. A second way to quantify this behavior is the zero-time velocity cross-correlation value (shown in Fig. 7(c)). This quantity will be zero for thermal behavior since degrees of freedom are independently populated but will be equal to one for chasing behavior. The quantity is shown to increase from zero as size ratio increases; however, beyond some large size ratio, the kinetic energy ratio decreases as the smaller sphere is too small for the large sphere to feel.

The spherical shape of a grain is special because it allows the grain to roll freely without
sliding and for the flowing air to be shed isotropically. If we consider elongated particles, this symmetry is broken and the thermal analogy breaks down. Long particles are observed to have little or no rotation about their long axis. More importantly, they can no longer roll without slipping, instead they must slide or scoot in order to translate or rotate. This is evident when the average kinetic energies for the different degrees of freedom are calculated. The translational kinetic energy is much greater than the rotational kinetic energy. Furthermore, as will be seen in Chapter 1, the translational degrees of freedom are unequal depending on the frame of reference used. This discrepancy between rotation and translation increases as airflow is increased, corresponding to motion perpendicular to the substrate. At low air speeds, the particle slides and rotates in contact with the surface. As air speed increases, the particle tilts out-of-plane and, depending on its smoothness and symmetry along its long axis, will either propel or oscillate back and forth along its length.

Both of these parameters by which the thermal analogy is broken are related by the phenomenon of self-propulsion. Self-propulsion is characterized by motion of a particle in a preferential direction that is chosen spontaneously. Typically, self-propulsion is treated phenomenologically; the microscopic details of how the particle self-propels are unimportant. Biological self-propelling particles have an internal mechanism - namely, digestion - that provides energy in order to propel. In the case of these granular experiments, the energy is provided by an upflow of air. The internal energy source is unimportant so long as the motion is characterized by some preferred broken-symmetry direction. In the next section, we will discuss the seminal simulation [78] detailing the macroscopic behavior of a collection of self-propelling particles. The simulation employed a simple minimal model based on a set of rules resulting in a dramatic broken symmetry kinetic phase transition. This behavior was further verified by more detailed microscopic theories which also treated self-propulsion phenomenologically. Thus, these predictions should apply to our granular system.

### 0.3 Collections of Self-Propelling Particles

In this section, we will review the seminal model put forward to understand the behavior of swarming or flocking particles. We will also briefly make note of the wealth of other theory and
experiment that has been done in this field.
The first rule-based model to describe the behavior of collections of self-propelling particles was put forward by Vicsek et al. [78]. The only rule of the model is that at every time step the particle, having constant absolute velocity, adjusts the orientation of its velocity to be equal to the average direction of motion of its neighboring particles plus some random kick. As such, the model is effectively a transport-related analogy to a ferromagnet. Here, instead of ferromagnetic interactions tending to align spins, the interaction requires that particles align directions of motion. Similarly, the random kick added to the direction is akin to the temperature.

The simulations utilized a square cell of size $L$ with periodic boundary conditions. The particles have no shape or volume; they are point particles. The simplest initial conditions were typically used: at time $t=0, N$ particles were randomly distributed across the cell with the same absolute velocity but with randomly distributed orientation. The position of each particle was determined at each time step $\Delta t$ according to

$$
\begin{equation*}
\mathbf{x}_{i}(t+1)=\mathbf{x}_{i}(t)+\mathbf{v}_{i}(t) \Delta t \tag{9}
\end{equation*}
$$

The direction of the velocity was obtained from a similar expression,

$$
\begin{equation*}
\theta(t+1)=\langle\theta(t)\rangle_{r}+\Delta \theta \tag{10}
\end{equation*}
$$

where $\langle\theta(t)\rangle_{r}$ is the average direction of the neighboring particles within some radius $r$. The noise term $\Delta \theta$ is a random number from the interval $[-\eta / 2, \eta / 2]$. Vicsek investigated the behavior of the collection by varying two basic parameters of the model: the noise $\eta$ and the density $\rho=N / L^{2}$. Although the absolute velocity could be varied, the simulation used a value $v=0.03$ so that particles always interact and that the overall configuration of particles changed after a few time-steps.

The simulation results can be characterized by the value of density and noise (shown in Fig. 1 of Ref. [78]). For small densities and noise, the particles form small localized groups that move coherently in random directions. For high densities and noise, the particles continue to move randomly but with correlated motion. For high densities but small noise, the particles order macroscopically and all particles tend to move in the same spontaneously chosen direction. There are two caveats to the observed behavior, though. First, the particles are point particles
and so they are able to occupy the same space, leading density to be a rather ill-defined quantity. Secondly, the particles are driven with an absolute constant velocity and, therefore, the net momentum of interacting particles is not conserved during a collision. Nevertheless, the kinetic phase transition was studied and subsequent, more detailed studies have confirmed many of the generic features of the behavior.

To study the transition, a dynamic order parameter was defined: the absolute value of the average normalized velocity

$$
\begin{equation*}
v_{a}=\frac{1}{N v}\left|\sum_{i=1}^{N} \mathbf{v}_{i}\right| \tag{11}
\end{equation*}
$$

This quantity is 0 if the direction of motion of the particles is randomly distributed; it is 1 if the particles are moving coherently. This quantity was measured for a system with fixed density as the noise level was lowered through the transition. The behavior of $v_{a}$ is very similar to that of order parameters of some equilibrium systems near a critical point. Namely, the dependence of $v_{a}$ on the distance to the critical point scales as a power law (as shown in Fig. 3):

$$
\begin{align*}
& v_{a} \sim\left[\eta_{c}-\eta\right]^{\beta}  \tag{12}\\
& v_{a} \sim\left[\rho-\rho_{c}\right]^{\delta} \tag{13}
\end{align*}
$$

where $\beta$ and $\delta$ are critical exponents and $\eta_{c}$ and $\rho_{c}$ are the critical noise and density.
Many other theoretical prescriptions have been put forward to model the behavior of selfpropelling particles. The theories generally have similar ingredients: particles attempt at all times to orient themselves with their neighbors; interactions are short-ranged within some radius of influence; stochastic noise causes the following behavior to be imperfect; the underlying system is completely rotationally symmetric a priori. The degree of detail varies as well, ranging from other rule-based models [30], to continuum hydrodynamic theories employing symmetry considerations [73-75], coarse-grained theories of a self-propelling nematic phase [20,67], and a microscopic theory of self-propelling particles taking into account their physical shape [12]. The particles themselves may be polar or apolar and may have cohesive or repulsive interactions. The results, despite the wide range of details, are fairly robust. Most theories generally predict a broken symmetry dynamic state - the size of which is much larger than the scale of particleparticle interactions - in which particles flock or swarm together coherently. The macroscopic
consequence of this behavior is that fluctuations in local number density diverge. This is to be expected since the flocks are long-lived and thus there are huge voids in the system coexisting with dense ordered flocks that persist for very long times. Similarly, it is predicted that density waves can propagate through the system [12,71,77] and swirling vortex motion can develop [8,9] for certain values of the input parameters.

In these theories, self-propelling is treated phenomenologically. As such, it is of no import how a particle self-propels so long as it does. For this reason, the predicted behavior should extend to driven granular systems so long as the particles self-propel and a mechanism exists to orient the particles with their neighbors. Despite the abundance of models and simulations, however, little experiment on physical systems has been conducted, with what has been done focusing on vertically-vibrated granular systems [9, 15, 29, 42, 53, 54]. Such granular systems afford an advantage over biological systems: there is control over the microscopics and energy input, and a steady state can typically be reached provided the system does not age. Vibrated-bed experiments have observed swirling vortices [9, 15], pattern formation [29], and shape-dependent long-range ordering [53]. Recently, giant number fluctuations were observed in a verticallyvibrated monolayer of nematic-ordered granular rods [54]. Large number fluctuations were also reported for a collection of vibrated spheres [7], fueling some debate [55] as to whether the large density fluctuations are due to the self-propelling nature of the particles, inelastic collisions, or a heaping instability [15] observed in vibrated media. Our experiments seek to contribute to these observations by using flowing air as the fluidization mechanism.

### 0.4 Jamming

Jamming, unlike self-propelling, is more difficult to define precisely. A material is said to be jammed if the constituent particles become stuck or frozen in a given configuration and are not observed to rearrange within the experimental observation window. The concept of jamming is inherently 'fuzzy' because one can always argue that the experimentalist could have simply waited longer to observe a rearrangement. Stated more quantitatively, jamming is when the stress relaxation timescale diverges. One way to think of this is that a particle becomes trapped in a 'cage' of its nearest neighbors and is unable to move unless its neighbors also move. Most of the
athermal non-equilibrium systems discussed in this introduction exhibit some sort of jamming. Granular material flowing out of a silo is one example; a foam can be considered jammed because it will not flow unless a shear is applied. Not surprisingly, the approach to jamming for all of these systems is quite different.

As noted above, a foam will unjam under application of a shearing force. Granular material will jam if the driving force is reduced and the system is no longer able to flow. In these types of systems, the transition is kinetic in nature. In colloidal suspensions, the particles may jam if a pressure is applied or if the density is high enough to prevent diffusion. Here, the transition is related to the amount of free volume accessible to the particles. Molecular systems experience jamming as temperature is lowered; this transition is thus thermal in nature. The concept of the 'cage' is helpful here. When a particle becomes trapped, it needs to move cooperatively with its neighbors in order to rearrange. Thus, there is an energy barrier that must be overcome. The energy to overcome this barrier can come either from increased thermal energy (raising the temperature), injecting energy into the system (applying shear, pressure, or driving), or increasing the free volume available to the system (decreasing density). There are thus three parameters that can be varied to control the approach to the jamming transition, depending on the type of system being studied - applied load, temperature, and density. This led to the introduction of the jamming phase diagram $[57,58]$ for which these parameters are the axes. Each of these systems - colloids, foams, grains, and supercooled liquids - will follow a different trajectory in the phase space as jamming is approached but all three parameters are important to each system.

What is clear is that all of these systems jam; what is unclear is whether there is a connection between the transitions and whether it is a true phase transition. Interestingly, as the jamming transition is approached in each of these systems, they exhibit remarkably similar characteristics. As jamming is approached, there is no structural change to the system. Because the system remains disordered, there is no order parameter that can be defined associated with structural changes that develop.

However, both a diverging dynamical length scale and time scale have been observed in these systems. The dynamical length scale is characterized by long strings of particles that are moving cooperatively in order to facilitate a structural rearrangement. These chains of particles get longer and longer, diverging as jamming is approached. These cooperatively-moving particle strings are
indicative of kinetic heterogeneity in these systems. In other words, some regions of the sample will move very quickly and other regions will be nearly static. The diverging timescale $\tau_{r}$ that arises on approach to jamming is similarly because particles become trapped in their 'cages' for longer times. The presence of energy barriers suggests that this timescale should follow an Arrhenius form $\tau_{r} \propto \exp ^{-E_{a} / T}$ where $E_{a}$ is an activation energy. Experimental evidence of these systems shows that, in fact, these variables increase more rapidly than the Arrhenius form. This is the Vogel-Fulcher form, $\tau_{r} \propto \exp ^{-E /\left(x-x_{0}\right)}$, where $x$ is the relevant control parameter for the system.

That these diverging length- and time-scales are observed in many different systems is by no means dispositive but it is suggestive of an underlying connection between the diverse systems. Thus, one outstanding question concerns the relationship of these parameters as the jamming transition is approached.

### 0.5 Jamming in a Granular Monolayer

In this section, we will review earlier experiments conducted by Adam Abate [4] using the gasfluidized bed apparatus as a proxy to study the jamming transition. As before, references to plots will be to figures appearing in Ref. [4]. In so doing, we will enumerate the structural and dynamical signatures of the jamming transition observed in an air-fluidized granular monolayer. We will primarily focus on those characteristics that are germane to the experiments of Chapter 4.

The system under study was a monolayer of bidisperse steel ball bearings - diameters $d_{b i g}=$ 0.873 cm and $d_{\text {small }}=0.635 \mathrm{~cm}$ and masses 2.72 and 1.05 g , respectively - fluidized in the apparatus as detailed in Appendix A. The use of two grain sizes is to prevent crystallization of the monolayer and enforce homogeneous disorder. The jamming transition was approached in two ways: increasing the packing fraction from $0.487 \%$ to $0.826 \%$ and decreasing the grain speeds - reducing their effective temperature - by lowering the fluidizing airflow. The beads are never observed to crystallize, segregate according to size, or stick together in clusters. The system is in a steady-state and ergodic with the beads able to explore many different packing configurations and explore the entire system during the experimental runs.

A typical measure of structure used in amorphous systems such as glasses is the pair distribution function $g(r)$. This quantity is essentially the probability of finding another grain a distance $r$ away from a given grain. For a bidisperse monolayer, there are four different distributions to calculate: between big beads, between small beads, between big and small beads, and between any two beads. The distributions, plotted against radial distance $r$ scaled by the sum of the bead radii $d_{i j}$ (shown in Figure 5 of Ref. [4]) are normalized so that, at large $r$, they approach one, indicating homogeneity at large length scales. Because of hard-core contacts, all distributions show a large peak at $r / d_{i j}=1$. As density increases, oscillations develop at integer values of $r / d_{i j}$, with peaks narrowing and increasing in height. Additionally, the second peak at $r / d_{i j}=2$ bifurcates. The growth of the hard-core peak and the splitting of the second peak have been determined to be structural characteristics of the glass transition [70, 80].

As the jamming transition is approached, the average kinetic energies - the effective temperature - decrease nearly linearly towards zero. Unlike in a thermal system however, the big and small beads have different temperatures. The ratio of kinetic energies (shown in Fig. 9(c)) is roughly constant before decreasing towards the ratio of the particle masses at about $74 \%$ area fraction. Interestingly, despite the change in dynamics at this point, there is no discernable change in the structure of the system. Thus, as jamming is approached, the particles are approaching the same mean-squared speeds. This can be thought of in terms of the 'cage' effect. That is, at very high densities, a particle is unable to move unless its neighbors also move out of the way at the same speed.

The most common quantity to measure in order to characterize the dynamics of a complex system is the mean-squared displacement (MSD) as a function of delay time interval $\tau$. For simple systems, at short delay times, the motion is ballistic - that is $\left\langle\Delta r^{2}(\tau)\right\rangle=\left\langle v^{2}\right\rangle \tau^{2}$. At very long delay times, the motion is diffusive, $\left\langle\Delta r^{2}(\tau)\right\rangle=4 D \tau$ where $D$ is the diffusion coefficient.

In simple systems, there is only this single crossover timescale. In supercooled and glassy systems, there are two additional timescales associated with the emergence of a subdiffusive plateau [49] in the MSDs between the ballistic and diffusive regimes. The time at which ballistic behavior ends and the plateau begins is called the $\beta$ relaxation time; the beginning of diffusion and end of the plateau is termed the $\alpha$ relaxation time. This sequence of dynamics is readily seen in the data for the air-fluidized system (shown for large and small balls in Figure 7(a) and 7(b),
respectively). The closer to the jamming transition, the larger the $\alpha$ relaxation time as the onset of diffusion occurs later.

One outstanding question is what combination of control parameters control the diverging relaxation timescale as jamming is approached. In other words, can we find a way to collapse relaxation time data for different systems such that, given temperature, pressure, and density, we know the relaxation time regardless of system specifications? For the above experiments, jamming was approached along two distinct trajectories for the above experiments: increasing density or decreasing driving which effectively lowered the temperature. Only two control parameters were available to measure. In Chapter 4, we extend this experiment by modifying the apparatus so that we are able to measure all three parameters - pressure, packing fraction, temperature - as well as relaxation time simultaneously.

## Chapter 1

## Self-Propelling Gas-Fluidized Rods

This first experiment characterizes the dynamics of individual air-fluidized rods. As discussed in Section 0.2, particles with aspect ratio greater than one do not satisfy a thermal analogy by violating equipartition of energy. It was qualitatively observed that rod-shaped particles can self-propel. This previous experiment [3], however, did not fully characterize the behavior of these elongated particles. As reviewed in Section 0.3, collections of self-propelling rods exhibit fascinating emergent collective phenomena. Our observation, then, is of great interest because there is little experimental verification of these theories. In this chapter, we will fully characterize the dynamics of rods fluidized by an upflow of air and establish that they do self-propel. In Section 1.1, we present the apparatus and system details. The observed behavior of fluidized rods will be qualitatively discussed in Section 1.2. Short-time behavior - both self-propulsion and particle-particle collisions - are quantified in Sections 1.3 and 1.4. In Sections 1.5-1.7, we analyze intermediate- and long-time behavior and compare to that of a Brownian particle. Lastly in Section 1.8, we phenomenologically model the behavior by encoding self-propulsion as nonthermal noise terms using a Langevin formalism. We propose that self-propelling can be thought of as an enhancement of a particle's memory of its prior orientation.

### 1.1 Experimental Details

The system we study, depicted in Fig. 1.1, consists of a monolayer of 435 cylindrical plastic dowel pins - length $L=0.95 \mathrm{~cm}$, diameter $d=0.24 \mathrm{~cm}$, and mass $m=0.055 \mathrm{~g}$ - fluidized


Figure 1.1: A monolayer of gas-fluidized rods. The diameter of the system is 17 cm . This is a frame capture of the system from the video data after thresholding. The particles are plastic dowel pins - length 0.95 cm , width 0.24 cm , and mass 0.055 g - occupying an area fraction of $42 \%$. The call-out shows an actual particle. The highlighted particle's trace is shown in Fig. 1.2.
by an upflow of air. The particular area fraction $\phi=42 \%$ ensures that the particles are uniformly distributed across the system. The particle aspect ratio, $L / d=4$, is chosen to prevent any long-range ordering effects on particle dynamics [21,62]. Although an effective harmonic potential exists due to interactions with the confining walls [59, 60], volume exclusion interactions and the turbulent, chaotic mixing of individual particle wakes overwhelms interactions with the boundaries and eliminates this external potential. These choices will be further justified in Section 1.2.

In-plane motion of the rods is excited by a spatially and temporally uniform upflow of air at speed $U=370 \mathrm{~cm} / \mathrm{s}$ as measured by a hot-wire anemometer. The flow rate is uniform within $\pm 10 \mathrm{~cm} / \mathrm{s}$ and on a timescale of 0.5 s . This is lower than the terminal free-fall speed for a horizontal rod, so the rods do not levitate or fully lift off the plane. This upflow of air induces random short-time motion. Since the Reynolds number based on rod length and air speed is large, $\sim 10^{3}$, there is turbulence in the form of irregular wakes shed by the rods. The shedding frequency $f_{v}$ is given by a universal value of the Strouhal number: St $=f_{v} L / U=0.18$; a new wake is generated every time the air flows a distance of about $5 L$ [66]. Therefore, the rods experience a corresponding random kicking force that fluctuates on a time scale $\tau_{v}=1 / f_{v} \approx$ 0.018 s [60].

The apparatus, fluidization method, and lighting setup are identical to those of Refs. [59, 60] and are detailed in Appendix A. The apparatus is a rectangular windbox, $1.5 \times 1.5 \times 4 \mathrm{ft}^{3}$, positioned upright. A circular brass testing sieve with mesh size $150 \mu \mathrm{~m}$ and diameter 30.5 cm rests horizontally on top. To reduce alignment of the particles with the wall, we place the particles in a free-standing cylindrical insert, inner diameter of 17 cm and thickness of 0.32 cm , at the center of the larger bed. This eliminates any stray horizontal component of airflow at the boundaries which tends to radially align the rods. A blower attached to the windbox base provides vertical airflow perpendicular to the sieve. The upper and lower halves of the windbox are separated by a 1-inch-thick foam filter between two perforated metal sheets to eliminate largescale structures in the airflow. Raw video data of the fluidized particles is captured for 10 minutes at 120 frames per second by a digital camera mounted above the apparatus. Post-processing of the video data is accomplished using LabVIEW.

Figure 1.2 shows a sample time-trace, $\{x(t), y(t), \theta(t)\}$, of a single particle. The color code
denotes the instantaneous orientation $\theta(t)$ of the particle with respect to the horizontal axis. The motion appears heterogeneous: we note several long stretches where the particle is preferentially moving parallel to its long axis. Apart from these, the wandering of the particle is like that of an isotropic particle undergoing a random walk. To obtain this time-trace, we convert each frame of the raw data to binary as it is saved to video from buffer. Using LabVIEW's "IMAQ Particle Analysis" algorithm, we locate the centroid of each particle in the thresholded image as well as the orientation with respect to a fixed horizontal axis. We impose an upper bound on the allowable area of a single particle; any particles identified above the area bound consist of two or more particles that have collided. In order to distinguish the individual particles, a series of image processing steps known as erosions, similar to Ref. [3], is carried out. Figure 1.1(a) shows the result of the erosion process for one frame in which all particles have been separated. Additional detail on the tracking program can be found in Appendix A.

Next, we link together particle positions and orientations from frame to frame by finding the minimum displacement between two center-of-mass positions in subsequent frames. To ensure correct matches, we constrain this with a maximum allowable displacement and rotation. The resulting time-traces are then smoothed by a running average. We estimate error in the position and angle data as $\sigma_{r m s} / \sqrt{N}$, where $\sigma_{r m s}$ is the rms deviation of the raw data from the smoothed data and $N$ is the number of frames in the smoothing window. This yields errors of $18 \mu \mathrm{~m}$ and 3.3 mrads in the position and angle data, respectively. Finally, to further minimize any wall effects, we include only those segments for which the particles are at least three particle diameters away from the wall.

### 1.2 Fluidized Rod Behavior

Although we seek to characterize single-particle dynamics, the system we study is semi-dilute, having a finite area fraction of $42 \%$. The reason for choosing a dilute system is due to how a single rod behaves when fluidized by itself. Firstly, the presence of the confining walls creates a harmonic potential which causes a single particle to spend most of its time near the center of the system. Oftentimes, this results in the particle propelling itself back and forth with little or


Figure 1.2: A time-trace of one particle's motion (highlighted in Fig. 1.1) in the presence of other particles. The color code corresponds to the instantaneous orientation with respect to the horizontal axis of the image.
no rotation. We wanted to eliminate this potential and characterize the behavior of an essentially freely-moving rod in the absence of an external potential. By using a system at finite area fraction, volume exclusion interactions prevent the particles from clustering at the center and the rods uniformly sample the entire system, effectively eliminating the harmonic well. The rods will have a tendency to order as the density increases, so we use particles with an aspect ratio of 4 in order to minimize the emergence of long-range order and the subsequent effects on the dynamics.

The behavior of the rods themselves can be grouped into three basic categories. For low and intermediate airflows, the rods will either propel or 'whirl', that is persistently rotate in one direction. Self-propulsion is a general characteristic for elongated particles provided the particle surface is smooth. Because rods have a higher coeffiction of friction associated with motion perpendicular to their long axis, we rarely see side-to-side rolling motion. Instead, when fluidized, one end of a rod will tilt slightly out-of-plane and the rod will propel itself in the direction of the tilt parallel to its long axis.

On the other hand, the 'whirling' phenomenon appears to be caused by a small manufacturing defect found on some of the rods used. The presence of other particles and subsequent collisions has the additional effect of eliminating much of this long-time persistent rotation. We note that some particles were still observed to persistently rotate chirally as can be seen in Fig. 1.3. Here, we plot the mean angular speed $\langle\omega\rangle$ versus the root-mean-square angular speed $\sqrt{\left\langle\omega^{2}\right\rangle}$. If a particle has a very large mean angular speed, this indicates persistent unidirectional 'whirling'. These particles are indicated by red data points in Fig. 1.3. We verified that the dynamics do not substantially change when these particles are ignored. As such, due to the relative arbitrariness of defining 'whirlers', we did not exclude them from the subsequent analysis.

The third category of behavior is 'chattering'; that is, as airflow increases, the the ends of the rod alternatively lift off the substrate. This motion perpendicular to the sieve dramatically inhibits in-plane rotation and translation. As such, the failure of equipartition of energy for aspherical grains in Ref. [3] was attributed to this increased out-of-plane motion. This type of behavior can be avoided by using a low enough airflow. Thus, we have chosen the system parameters in such a way to minimize 'whirling' and 'chattering'. In the next section, we focus on quantifying the self-propelling behavior of the rods.


Figure 1.3: Mean angular speed $\langle\omega\rangle$ versus root-mean-square angular speed $\sqrt{\left\langle\omega^{2}\right\rangle}$ of airfluidized rods. Each point represents the values for an individual rod averaged over its entire time-trace. Red points are those particles considered to be 'whirlers'.

### 1.3 Self-propulsion

As noted in Section 1.1, the sample time-trace, Fig. 1.2, shows long stretches where particle orientation is aligned with the direction of motion, indicating scooting motion. To quantify this self-propelling behavior, we obtain the distributions of kinetic energies for motion parallel and perpendicular to the rod's long axis and for the rod's orientation, shown in Fig. 1.4. We immediately see that equipartition of energy does not hold: at large energies, the distribution of parallel kinetic energies is much greater than the perpendicular distribution. The parallel component, with average value $\left\langle K_{\|}\right\rangle=1 / 2 m\left\langle v_{\|}^{2}\right\rangle \sim 0.021$ ergs, is roughly twice as energetic as the perpendicular component, with $\left\langle K_{\perp}\right\rangle=1 / 2 m\left\langle v_{\perp}^{2}\right\rangle \sim 0.012$ ergs. The rotational kinetic energy splits the difference between the two. Its average value is roughly the average of these two: $\left\langle K_{\theta}\right\rangle=1 / 2 I\left\langle\omega^{2}\right\rangle \sim 0.016$ ergs, where we have used $I=(m / 12)\left(3(d / 2)^{2}+L^{2}\right)$. This shows that the gas-fluidized rods convert energy provided by the upflow of air into in-plane motion, preferentially parallel to the long axis. They do so at the expense of motion perpendicular to the rod's long axis. Such microscopic dynamical anisotropy - to which the emergence of
collective macroscopic behavior has been attributed [53] - is a universal feature of theories and simulations of self-propelled particles, even when the anisotropy is not specified as a part of the model a priori. We can rightly consider our gas-fluidized rods in the context of self-propelled particles.

We also obtain the average self-propelling velocity, $v_{\|} \sim 0.87 \mathrm{~cm} / \mathrm{s}$, from $\left\langle K_{\|}\right\rangle$. From this and the lab frame diffusion coefficients $D_{x}$ and $D_{\theta}$ to be discussed in Section 4.3, we calculate $\alpha=v_{\|}^{2} /\left(4 D_{x} D_{\theta}\right)=6.8$, a dimensionless parameter that determines the extent to which self-propulsion dominates over stochastic fluctuations [12]. For values $\alpha>1$, self-propulsion dominates. Thus, we are clearly in a regime where self-propulsion effects will be readily observable. Furthermore, we can quantify the spatial and temporal extent of the 'scooting' behavior by calculating an alignment order parameter $m(t)=\cos [\theta(t)-\phi(t)]$, the cosine of the angle between the particle's instantaneous orientation, $\theta(t)$, and the direction of its instantaneous velocity, $\phi(t)$. This quantity is 1 if a particle moves in the same direction that it is pointing and 0 if it moves perpendicular to its orientation. Here, $\langle m\rangle=0.46$ and $\sqrt{\left\langle m^{2}\right\rangle}=0.75$. From the autocorrelation $\langle m(t) \cdot m(t+\tau)\rangle$ shown in Fig. 1.5, we extract a correlation time of 6.4 s . We quantify the spatial extent of the stretches from the value of the mean square parallel displacement at the correlation time. This gives a displacement of 1.6 cm , roughly one particle length per correlation time.

Self-propulsion is a strictly non-thermal phenomenon. The shape of the energy distributions in Fig. 1.4 further indicates the non-thermal character of our system. All of the distributions deviate sharply from an exponential at small energies. This sharp maximum has been observed for a single rod bouncing on a vibrated surface [81]. This is due to a strong correlation mentioned earlier between translation and rotation. Specifically, when the particle tilts out-of-plane and selfpropels, its in-plane rotation is significantly reduced, and vice versa. The non-exponential form indicates that the velocity distributions are non-Gaussian, as seen in Fig. 1.6. Here, we have plotted the displacement distributions for the smallest time-step possible, $1 / 120 \mathrm{~s}$. The inset plots the same distribution versus the displacement squared. The solid lines show a Gaussian fit and emphasize the large tails of the distributions. We also calculate the kurtosis excess $\left\langle v^{4}\right\rangle /\left\langle v^{2}\right\rangle^{2}-3$, which equals 0 for Gaussian distributions - of the velocity distributions. With kurtosis values of 0.6 for the angle, 0.5 for the parallel, and 0.4 for the perpendicular, none of the


Figure 1.4: Distributions of kinetic energies, $\mathrm{K}_{\|}$and $\mathrm{K}_{\perp}$, for motion parallel to and perpendicular to the particle's long axis and $\mathrm{K}_{\theta}$, the rotational kinetic energy. The average kinetic energy for each components is $\left\langle K_{\|}\right\rangle=0.021$ ergs, $\left\langle K_{\perp}\right\rangle=0.012 \mathrm{ergs}$, and $\left\langle K_{\theta}\right\rangle=0.016$ ergs. The kurtosis excess of the velocity distributions are 0.5 for parallel, 0.4 for perpendicular, and 0.6 for angle. A Gaussian distribution has a kurtosis excess of 0.
velocity distributions are Gaussian.

### 1.4 Collisions

The anisotropy of particle shape gives rise to self-propelled behavior and also alters the excluded volume interaction between particles. As a first step toward understanding these steric interactions, we compile binary collision statistics. We generate distributions of free times and free paths between collisions, shown in Fig. 1.7, using a graphical approach. We create a particle template that is the size of a grain had it not undergone any erosions or thresholding during data postprocessing. Then, we reconstruct each frame of the video by overlaying the template at the center-of-mass of each actual particle and rotating it by the particle's orientation. By detecting particle overlap, we determine whether a given particle has collided with any neighbors in a given frame. We then extract the time increment, $\Delta t$, and displacement, $\Delta r$, between collisions for a given particle and compile them as probability distributions of free times, Fig. 1.7(a), and


Figure 1.5: Autocorrelation function for the alignment order parameter, $m(t)=\cos [\theta(t)-\phi(t)]$, where $\theta(t)$ is the particle orientation and $\phi(t)$ is the direction of the instantaneous velocity. The autocorrelation decays from a value of $\left\langle m^{2}(t)\right\rangle=0.56$ to $\langle m(t)\rangle^{2}=0.21$ on a timescale of approximately 6 s .
free paths, Fig. 1.7(b).
Although the tails of the distributions in Fig. 1.7 behave exponentially, there is substantial deviation at short times and paths. As such, we obtain well-defined mean values by taking the average of all the data rather than from an exponential fit. This yields a mean free time between collisions, $\tau_{c} \sim 0.063 \mathrm{~s}$, and a mean free path, $r_{c} \sim 0.042 \mathrm{~cm}$, marked as the dashed vertical lines in the figure. The ratio, $r_{c} / \tau_{c}=0.67 \mathrm{~cm} / \mathrm{s}$, is near the rms velocity, $0.76 \mathrm{~cm} / \mathrm{s}$. On subsequent dynamics plots, we mark the collision time with an orange dashed vertical line.

For thermal, isotropic systems, collisions are independent events; we expect exponential distributions for free paths and free times. The deviation of the distributions in Fig. 1.7 from exponential behavior at short times and paths further indicates non-thermal behavior. Distributions similar to ours were observed for spherical grains vibrated on an inclined plane [14]. There, the sharp maximum was attributed to inelastic clustering. Here, it may be attributable to local alignment - which may make a subsequent collision dependent on prior collisions - or to dynamical anisotropy. That is, a particle may be more likely to collide at short times as it propels through


Figure 1.6: Displacement distributions for the body frame components. The delay time for the displacement is $1 / 120 \mathrm{~s}$. The solid curves are fits to a Gaussian. The inset shows the same distributions versus displacement-squared, emphasizing the long tails of the distributions.


Figure 1.7: Probability distribution of a) times between collisions and b) displacements between collisions. The mean free time, $\tau_{c} \sim 0.063 \mathrm{~s}$, and mean free path, $r_{c} \sim 0.042 \mathrm{~cm}$, are labeled.
its environment.

### 1.5 Lab Frame Dynamics

Despite being driven far from equilibrium, one or two gas-fluidized spheres act as Brownian particles in a harmonic trap [60]. However, in Section 1.3, we showed that a rod moves more energetically parallel to its long axis and this thermal analogy subsequently fails. Here, we further ask how does self-propulsion alter the dynamics of a gas-fluidized rod? We begin by calculating single-particle dynamics in the lab frame where particle motion is characterized by both angular and translational displacements. The lab frame axes, $x$ and $y$, correspond to the horizontal and vertical axes of the raw images; angular orientation is measured counterclockwise with respect to $+x$. Recall from Section 1.1 that we only analyze data for which a particle is within 3 particle diameters from the wall, effectively breaking a single time-trace into many shorter time-traces. A typical particle moves across the system in less than a minute. Therefore, dynamical quantities, such as $\mathrm{MSD}_{\theta}$, are truncated for delay times greater than 40 s .

Figure $1.8(a)$ shows the rotational mean square displacement, $\left\langle[\theta(t+\tau)-\theta(t)]^{2}\right\rangle$, as a function of delay time $\tau$. We observe ballistic behavior $\left(\propto \tau^{2}\right)$ at the shortest time scales and a crossover to diffusive behavior $(\propto \tau)$ at long times.

Rotational diffusion, characterized by the rotational diffusion coefficient $D_{\theta}(\tau)=\left\langle\Delta \theta^{2}(\tau)\right\rangle /(2 \tau)$ shown in Fig. 1.8(b), sets a 'directional memory' timescale $\tau_{\theta}=\left(2 D_{\theta}\right)^{-1}$. At times greater than $\tau_{\theta}$, a particle will have forgotten its initial direction and all directions become equal. The longtime value of the angular diffusion coefficient, $D_{\theta}=0.22 \mathrm{~s}^{-1}$, is obtained from the plateau in Fig. 1.8(b), giving the value $\tau_{\theta} \sim 2.27 \mathrm{~s}$, shown as the vertical (green) dashed line in Fig. 1.8.

Chiral particles subject to external forces, such as those arising from the vibration of a substrate [76] or from air flowing past them as in the current experiment, spin in a preferred direction determined by the sign of their chirality. Small manufacturing defects impart chirality with a random sign and magnitude to some of the rods in this experiment. Some of the long-time rise in $D_{\theta}(\tau)$ is due to the spinning of some particles throughout the entire data set. If these particles are removed, an increase in slope is still detected, indicating heterogeneous dynamics for nonwhirling particles. Just as the time-trace in Fig. 1.1(b) shows regions where particle scooting is


Figure 1.8: Dynamics of rod orientation: a) mean square angular displacement, b) angular diffusion coefficient $D_{\theta}=\left\langle[\theta(t+\tau)-\theta(t)]^{2}\right\rangle /(2 \tau)$, and c) angular velocity autocorrelation function. The line with slope 1 in a) corresponds to late-time diffusive motion; the line with slope 2 shows short-time ballistic motion. The horizontal line in b ) is the rotational diffusion coefficient, $D_{\theta}=$ $0.22 \mathrm{~s}^{-1}$. The dashed vertical line (orange) marks the collision time, $\tau_{c}$; the dash-dot line (green) marks the directional memory time, $\tau_{\theta}=\left(2 D_{\theta}\right)^{-1}$. The solid curves are a Langevin model using a non-thermal noise source, given by Eqs. (1.9) and (1.10).
intermittent with thermal-like wiggling, plots of $\theta(t)$ show regions of fluctuating motion intermittent with rapid rotation. We stress that, if we exclude whirlers, the long-time value of $D_{\theta}$, and thus the timescale $\tau_{\theta}$, does not change significantly.

The angular velocity autocorrelation, $W_{\theta}(\tau)=\langle\omega(t+\tau) \cdot \omega(t)\rangle$, plotted in Fig. 1.8(c), shows a two-step decay, characterized by a large, positive rebound near $\tau_{c}$ due to collisions, followed by small oscillations at longer times before noise dominates. The behavior of $W_{\theta}(\tau)$ at $\tau_{c}$ allows us to deduce the effect that collisions have on the particle. Typically, a collision results in a negative rebound: the particle recoils in a direction opposite to its incident direction. The large, positive rebound in $W_{\theta}(\tau)$ suggests that the nature of our collisions is to re-align the particle as to its initial direction.

We compute the translational dynamics in the lab frame, shown in Fig. 1.9. There is no discernable difference between data along $x$ and $y$; only data along $x$ is plotted. Figure 1.9 (a) shows the lab frame mean square displacement, $\left\langle[x(t+\tau)-x(t)]^{2}\right\rangle$. The behavior is ballistic at short times and becomes diffusive at longer times. This is confirmed in Fig. 1.9(b): the lab frame diffusion coefficient, $D_{x}(\tau)=\left\langle\Delta x^{2}(\tau)\right\rangle /(2 \tau)$, approaches its long-time value, $D_{x}=0.128$ $\mathrm{cm} / \mathrm{s}$, at about 10 s . The lab frame velocity autocorrelation function, $W_{x}(\tau)$, shown in Fig. 1.9(c), has slower-than-exponential decay with a small wiggle at $\tau_{c}$. Long-time statistics are poorer and the rebound at $\tau_{\theta}$ may or may not be real.

Unexpectedly, equipartition of energy holds between the lab frame and the angle. From $W_{x}(0)$ and $W_{\theta}(0)$, we find that $(1 / 2) m\left\langle v_{x}^{2}\right\rangle \sim 0.017$ ergs, and $(1 / 2) I\left\langle\omega^{2}\right\rangle \sim 0.016$ ergs. That is, both the rotational and translational degrees of freedom in the lab frame have the same kinetic energy. Remarkably, despite the self-propelled nature of the particles, if one averages over all possible orientations, the resulting bulk behavior appears thermal. This is consistent with the results in Fig. 1.4; the distribution of kinetic energies in the lab frame is nearly identical to that of the angle and the average of the parallel and perpendicular distributions.

### 1.6 Coupling Rotation and Translation

In this section, we ask how do rotation and translation couple for a self-propelling particle? To explicitly visualize rotational-translational coupling, we calculate the dynamics in a 'fixed-angle'


Figure 1.9: Dynamics of rod position in the lab frame: a) mean square displacement, b) diffusion coefficient $D_{x}=\left\langle[x(t+\tau)-x(t)]^{2}\right\rangle /(2 \tau)$, and c) velocity autocorrelation function. The horizontal line in b ) shows the long-time value of the diffusion coefficient, $D_{x}=0.128 \mathrm{~cm} \cdot \mathrm{~s}^{-1}$. The dashed vertical line (orange) marks the collision time, $\tau_{c}$; the dash-dot line (green) marks the directional memory time, $\tau_{\theta}=\left(2 D_{\theta}\right)^{-1}$. The solid curves are a Langevin model using a non-thermal noise source, given by the first term in Eq. 1.15.
lab frame with axes $\bar{x}$ and $\bar{y}$. We construct this frame by rotating the coordinates of an entire time trace by the initial particle orientation so that $\bar{x}$ and $\bar{y}$ are, respectively, parallel and perpendicular to the initial direction of the long axis of the particle. The axes then remain fixed in time. This is equivalent to setting the initial orientation of all particles to $\theta_{0}=0$.

For comparison purposes, it is instructive to review how rotation and translation couple for a Brownian particle $[31,63,64]$. If a particle is not allowed to rotate, translational motion is characterized by anisotropic diffusion - with two diffusion coefficients, $D_{\|}$and $D_{\perp}$ - for displacements parallel and perpendicular to the particle's long axis. The two components, it should be noted, become diffusive on the same timescale. If the particle is allowed to rotate, this anisotropic diffusion regime will cross over to isotropic diffusion characterized by a single diffusion coefficient, $D_{x}=(1 / 2)\left(D_{\|}+D_{\perp}\right)$. The crossover timescale is the same 'directional memory' timescale discussed earlier in Section 4.3, $\tau_{\theta}=\left(2 D_{\theta}\right)^{-1}$. Thus, at times longer than $\tau_{\theta}$, the fixed-angle lab frame axes will become random and the dynamics along $\bar{x}$ and $\bar{y}$ will become equivalent to the conventional lab frame, $x$ and $y$.

The fixed-angle lab frame mean square displacements $\left\langle\Delta \bar{x}^{2}(\tau)\right\rangle$ and $\left\langle\Delta \bar{y}^{2}(\tau)\right\rangle$, Fig. 1.10(a), are both ballistic at short times. This short-time behavior confirms the discussion in Section 1.3 that equipartition of energy does not hold, with $\left\langle\Delta \bar{x}^{2}(\tau)\right\rangle$ twice as large as $\left\langle\Delta \bar{y}^{2}(\tau)\right\rangle$. After $\tau_{\theta}$, the fixed-angle MSDs become diffusive and eventually converge, showing that the coordinate axes are randomized and the particle has forgotten its initial direction. This indicates coupling between rotation and translation in our system. However, examining Fig. 1.10(b), we do not observe a fully-developed anisotropic diffusion regime. Instead, following the initial anisotropic ballistic regime, $\bar{D}_{x}$ indicates diffusion at approximately 6 s but $\bar{D}_{y}$ does not become diffusive until about 20 s . Isotropic diffusion occurs near 20 s , when the two diffusion coefficients converge.

The means by which the two components converge is a point of interest. We calculate the logarithmic derivative of the fixed-angle MSDs,

$$
\lambda(\tau)=\frac{\partial \ln \left[\left\langle(\Delta r(\tau))^{2}\right\rangle\right]}{\partial \ln (\tau)}
$$

shown in Fig. 1.11. For ballistic behavior, the value of $\lambda(\tau)$ is 2 ; for diffusive, the value is 1 .


Figure 1.10: Dynamics of rod position in the fixed-angle lab frame: a) mean square displacement, b) diffusion coefficient, and c ) velocity autocorrelation function. $\mathrm{At} \mathrm{t}=0, \tilde{x}$ is aligned parallel to the particle's long axis. The dashed vertical line (orange) marks the collision time, $\tau_{c}$; the dashdot line (green) marks the directional memory time, $\tau_{\theta}=\left(2 D_{\theta}\right)^{-1}$. The solid curves are a Langevin model using non-thermal noise sources, given by Eq. (1.15).

Both components are ballistic at short time. At roughly the collision time, $\bar{y}$ reaches a superdiffusive plateau while $\bar{x}$ continues to decrease monotonically toward diffusion. This allows the $\bar{y}$ component to 'catch up' to the more energetic $\bar{x}$ component. The slopes of the two components then converge at the directional memory timescale.

Although long-time statistics are poor, Fig. 1.10(b) shows that the two diffusion coefficients characterizing the anisotropic diffusion regime, $D_{\|}$and $D_{\perp}$, become equivalent at long time. Since $D_{\|}$is greater than $D_{\perp}, D_{\bar{x}}$ must decrease while $D_{\bar{y}}$ increases. They converge at the value of the isotropic diffusion coefficient, $D_{x}=0.128 \mathrm{~cm} \cdot \mathrm{~s}^{-1}$, equivalent to the lab-frame value. In order for this convergence to occur, the relation,

$$
\begin{equation*}
D(t)=\int_{0}^{t} d \tau W(\tau) \tag{1.1}
\end{equation*}
$$

implies that $W_{\bar{x}}(\tau)$ must be negatively correlated at long times while $W_{\bar{y}}(\tau)$ is positively correlated. We see this in Fig. 1.10(c). At $\tau_{\theta}, W_{\bar{x}}(\tau)$ becomes negative and slowly approaches zero from below; $W_{\bar{y}}(\tau)$ approaches zero from above.

We see that coupling for self-propelled particles is different than Brownian in two significant ways. First, the intermediate-time dynamics show that the two components become diffusive on different timescales. Recalling the ballistic-to-diffusive timescale for the conventional orientation-averaged lab frame, $\bar{x}$ becomes diffusive sooner and $\bar{y}$ later. This is most likely because the parallel component is more energetic than the perpendicular component; thus, it takes longer for the $\bar{y}$ component to 'catch up' and become equal to $\bar{x}$. Secondly, isotropic diffusion occurs at a timescale an order of magnitude larger than $\tau_{\theta}$. Thus, we think of self-propulsion as a 'memory enhancement' effect. Isotropic diffusion occurs much later because self-propulsion, in effect, allows the particle to remember its initial direction for a longer time.

### 1.7 Body Frame Dynamics

While the fixed-angle lab frame is useful for illustrating the effects of self-propulsion at short and intermediate times, it is useful to consider yet another reference frame to capture the effects at longer times. The body frame is a set of axes, $\tilde{x}$ and $\tilde{y}$, which are re-oriented at each time step to coincide with the long and short dimensions of each rod, respectively. The individual


Figure 1.11: Logarithmic derivative of the fixed-angle frame mean square displacements. The dashed vertical lines mark the collision time (orange), $\tau_{c}$, and the directional memory time (green), $\tau_{\theta}$. A value of 2 corresponds to ballistic motion; a value of 1 is diffusion.
displacements are then summed up successively to form a set of time-traces, $\{\tilde{x}(t), \tilde{y}(t)\}$. Here, $\theta(t)=0$ at all times and there is no coupling in this frame.

At short times, the body frame mean square displacements in Fig. 1.12(a) are identical to the fixed-angle lab frame mean square displacements of Fig. 1.10(a). At long times, $\left\langle\Delta \tilde{y}_{\perp}^{2}(\tau)\right\rangle$ crosses over to diffusive behavior whereas $\left\langle\Delta \tilde{x}_{\|}^{2}(\tau)\right\rangle$ becomes superdiffusive $\left(\propto \tau^{5 / 4}\right)$. Selfpropulsion gives rise to enhanced diffusion for translations along the particle's long axis. As seen in Fig. 1.12(b), the perpendicular diffusion coefficient, $D_{\perp}$, has reached its long-time value while $D_{\|}$continues to increase.

The body-frame velocity autocorrelation functions, $W_{\|}(\tau)$ and $W_{\perp}(\tau)$, shown in Fig. 1.12(c), reveal the long memory of the particle motion. The perpendicular component has a slow decay that exhibits the same rebounding features at $\tau_{c}$ and $\tau_{\theta}$ as seen in $W_{\theta}(\tau)$ and $W_{x}(\tau)$. The parallel component shows a smooth, slower algebraic decay for the entire run indicating remarkably long-lived velocity correlations. This is consistent with the long, unbroken stretches of scooting motion seen in the time-trace image, Fig. 1.1(b) and is another dramatic indicator of the non-thermal nature of the rod's behavior.

One final comparison to thermal behavior is the shape of the displacement probability distribution functions (PDFs) for the various reference frames over all delay times. To measure how


Figure 1.12: Dynamics of rod position in the body frame: a) mean square displacement $\mathrm{MSD}_{b}$, b) diffusion coefficient, and c) velocity autocorrelation function. The co-rotating body frame axes are always either parallel or perpendicular to the particle's long axis. The line with slope $5 / 4$ in a) corresponds to superdiffusive motion. The dashed vertical line (orange) marks the collision time, $\tau_{c}$; the dash-dot line (green) marks the directional memory time, $\tau_{\theta}=\left(2 D_{\theta}\right)^{-1}$. The solid curves are a Langevin model using a non-thermal noise source, given by Eqs. (1.12) and (1.13).

Gaussian the PDFs are, we calculate the kurtosis excess,

$$
\beta(\tau)=\frac{\left\langle(\Delta r(\tau))^{4}\right\rangle}{\left\langle(\Delta r(\tau))^{2}\right\rangle^{2}}-3
$$

for all values of $\tau$. A Gaussian distribution will have $\beta(\tau)=0$ for all $\tau$.
Because in the body frame $\theta(t)=0$ at all times, rotational motion and body frame translations are independent of one another. If there is no external torque and noise is thermal, angular displacement PDFs will obey Gaussian statistics at all times. Similarly, for no external forces, the body frame displacement PDFs will be Gaussian. Therefore, the kurtosis for the body frame and angle, provided noise is Gaussian-distributed, will be zero for all times.

The noise in the fixed-angle frame will be a coupled, nonlinear function of the body frame and angular noise functions. This was calculated and verified in Ref. [31]; the exact form of this coupling is made explicit in Section 1.8. We expect that, at short times, the body frame and fixed-angle displacement PDFs will be identical. For a thermal system, at intermediate times, the fixed-angle PDFs will deviate from Gaussian behavior, reaching a maximum at $\tau_{\theta}$. The lab frame PDFs - averaged over all initial angles - will be non-Gaussian even at short times [31]. For all frames, the PDFs should tend toward a Gaussian profile at long times. This is because, at times greater than $\tau_{\theta}$, rotation and translation are no longer coupled. Therefore, all displacements are independent of one another and a Gaussian distribution is expected by the central limit theorem. The calculated kurtoses, $\beta(\tau)$, for all reference frames are shown in Fig. 1.13.

At short times, the kurtosis is non-zero for each frame. Considering the displacement distributions in Fig. 1.6, we know that short-time statistics will not be Gaussian. This indicates that none of the noise sources in our system can be assumed to be Gaussian. Our previous studies with gas-fluidized spheres [4] also found non-zero kurtosis at short times. In both cases, this is due to the non-thermal, far-from-equilibrium nature of the system.

Except for the angle, all kurtoses tend towards a more Gaussian profile at long times, as expected. The long-time increase in the angle kurtosis is more evidence of 'whirling' that caused a long-time increase in $\operatorname{MSD}_{\theta}(\tau)$. The 'whirling' particles have very large long-time displacements which cause the angle PDFs to have extremely long tails, resulting in a steady increase in the value of $\beta(\tau)$.

The kurtosis for the body frame and the fixed-angle frame are initially equal; this is expected


Figure 1.13: Time evolution of the kurtosis excess for displacement PDFs in all frames for all time delays. A Gaussian distribution has a kurtosis excess of 0 . The dashed vertical lines mark the collision time (orange), $\tau_{c}$, and the directional memory time (green), $\tau_{\theta}$.
since the PDFs are identical. At the collision time, $\tau_{c}$, the two distributions become distinct. At $\tau_{\theta}$, the kurtoses of the fixed-angle frame components become equivalent to one another and to the lab frame. This confirms that the fixed-angle frame axes become randomized at $\tau_{\theta}$. Once again, we see that $\tau_{\theta}$ plays a very important role in determining the dynamics for our system. The directional timescale corresponds roughly to local maxima in many of the frames - when PDFs are least Gaussian. Moreover, it is the timescale after which displacements become independent of one another as evidenced by the collapse of the lab frame and fixed-angle frame curves and their subsequent approach to zero.

### 1.8 Model

Collections of self-propelled particles are typically modeled in one of two ways: establishing a minimal set of rules [30,78] or writing hydrodynamic equations, including all terms consistent with relevant symmetries [73-75]. Self-propulsion is usually included as a phenomenological
parameter. In keeping with such minimal models, we describe our system with a Langevin formalism constructed along the lines of that for an anisotropic Brownian particle. We make a single modification: rather than write a self-propelling force, we implicitly include self-propulsion with non-thermal noise terms. We begin by constructing Langevin equations for the three independent degrees of freedom - the angle and the two body frame components.

If we ignore inertial effects, the Langevin equation for the orientation of the $\operatorname{rod} \theta(t)$ with Hamiltonian $H$ is

$$
\begin{equation*}
\frac{d \theta}{d t} \equiv \omega(t)=-\int d t^{\prime} \Gamma_{\theta}\left(t-t^{\prime}\right) \frac{\partial H}{\partial \theta}\left(t^{\prime}\right)+\zeta_{\theta}(t) \tag{1.2}
\end{equation*}
$$

where $\Gamma\left(t-t^{\prime}\right)$ is the friction coefficient with memory and $\zeta_{\theta}(t)$ is Gaussian angular noise with zero average and variance

$$
\begin{equation*}
\left\langle\zeta_{\theta}(t) \zeta_{\theta}\left(t^{\prime}\right)\right\rangle=\left\langle\omega(t) \omega\left(t^{\prime}\right)\right\rangle \equiv W_{\theta}(\tau) . \tag{1.3}
\end{equation*}
$$

The effective harmonic potential of the bed has been eliminated and there are no externally imposed forces or torques. The remaining forces - arising from interparticle collisions, interactions with the substrate, and hydrodynamic interactions with wakes - can be considered as noise. Thus, our Langevin equations simply relate time derivatives of angle and displacement to random-noise torques and forces. The equation for the orientation of the rod then reduces to

$$
\begin{equation*}
\frac{d \theta}{d t} \equiv \omega(t)=\zeta_{\theta}(t) . \tag{1.4}
\end{equation*}
$$

From Eq. (1.4), $W_{\theta}(\tau)$ is the velocity autocorrelation function of Fig. 1.8(c).
The equations for displacement can be expressed in the lab frame $\mathbf{x}_{l a b}(t)=\{x(t), y(t)\}$ or in the body frame, $\tilde{x}(t)=\{\tilde{x}(t), \tilde{x}(t)\}$. In the body frame, $\tilde{x}(\mathrm{t})$ and $\tilde{y}(\mathrm{t})$ decouple:

$$
\begin{align*}
& \frac{d \tilde{x}}{d t} \equiv \tilde{v}_{x}(t)=\zeta_{\|}(t),  \tag{1.5}\\
& \frac{d \tilde{y}}{d t} \equiv \tilde{v}_{y}(t)=\zeta_{\perp}(t), \tag{1.6}
\end{align*}
$$

where $\zeta_{\|}(t)$ and $\zeta_{\perp}(t)$ are Gaussian random noises with zero mean and variance

$$
\begin{equation*}
\left\langle\zeta_{\|}(t) \zeta_{\|}\left(t^{\prime}\right)\right\rangle=\left\langle v_{\|}(t) v_{\|}\left(t^{\prime}\right)\right\rangle \equiv W_{\|}(\tau) \tag{1.7}
\end{equation*}
$$

$$
\begin{equation*}
\left\langle\zeta_{\perp}(t) \zeta_{\perp}\left(t^{\prime}\right)\right\rangle=\left\langle v_{\perp}(t) v_{\perp}\left(t^{\prime}\right)\right\rangle \equiv W_{\perp}(\tau) \tag{1.8}
\end{equation*}
$$

The noises $W_{\perp}(\tau)$ and $W_{\|}(\tau)$ are the body frame velocity autocorrelation functions of Fig. 1.12(c).
The Langevin equations derived above apply to both equilibrium and non-equilibrium systems. In equilibrium systems, the noise fluctuations of Eqs. (1.3), (1.7), and (1.8) are determined by the fluctuation-dissipation theorem. They adopt white-noise forms when rotational and translational friction coefficients do not exhibit memory effects. Non-equilibrium systems are not restricted by the fluctuation-dissipation theorem. Here, we use the experimental forms of the angular and translational velocity autocorrelation functions to set $W_{\theta}(\tau), W_{\perp}(\tau)$, and $W_{\|}(\tau)$. Since these quantities contain all the information about self-propulsion, we are able to implicitly include self-propulsion in our model via non-thermal noise. We will then be able to test whether modeling self-propulsion as non-thermal noise is sufficient in lieu of specifying an actual force acting along the long axis of the particle.

The form of the angular noise $\zeta_{\theta}(t)$ can be determined from the two-step decay of $W_{\theta}(\tau)$, Fig. 1.8(c). This suggests that the simplest functional form is the sum of two exponentials:

$$
\begin{equation*}
W_{\theta}(\tau)=D_{\theta}\left(\frac{a}{\tau_{1}} e^{-|\tau| / \tau_{1}}+\frac{1-a}{\tau_{2}} e^{-|\tau| / \tau_{2}}\right) \tag{1.9}
\end{equation*}
$$

where $a$ is a real number between 0 and 1. The fit to Eq. (1.9), shown as the solid curve in Fig. 1.8(c), was made by constraining $D_{\theta}$ to equal its long-time value, $0.22 \mathrm{~s}^{-1}$, rather than using it as a fitting parameter. Although unable to capture the sharp rebound caused by collisions, Eq. (1.9) provides a good fit to $W_{\theta}(\tau)$. We stress that this form is not a prediction of our model but rather the simplest functional form that describes the data well. We extract two correlation times: $\tau_{1}=0.018 \pm 0.005 \mathrm{~s}$ and $\tau_{2}=0.11 \pm 0.01 \mathrm{~s}$. The smaller correlation time is identical to the vortex shedding timescale $\tau_{\nu}$ calculated in Section 1.1. The value of $\tau_{2}$ is roughly the collision time.

We also calculate the analytical form of the mean square angular displacement:

$$
\begin{equation*}
\left\langle(\Delta \theta)^{2}\right\rangle=2 D_{\theta}\left[a S\left(t-t^{\prime}, \tau_{1}\right)+(1-a) S\left(t-t^{\prime}, \tau_{2}\right)\right] \tag{1.10}
\end{equation*}
$$

where we define

$$
\begin{equation*}
S(t, \tau)=|t|-\tau\left(1-e^{-|t| / \tau}\right) \tag{1.11}
\end{equation*}
$$

Using the fit values from Eq. (1.9), we plot Eq. (1.10) as the solid curve in Fig. 1.8(a). As a consequence of constraining the value of $D_{\theta}$ when fitting to Eq. (1.9), the result overestimates the value of the MSD at short times.

The form of the body frame noise, $\zeta_{\|}(t)$ and $\zeta_{\perp}(t)$, is obtained from $W_{\|}(\tau)$ and $W_{\perp}(\tau)$, Fig. 1.12(c), respectively. The slow decay suggests a power law form:

$$
\begin{equation*}
W_{i}(\tau)=\frac{A_{i}}{\left(1+b_{i} \tau\right)^{\alpha_{i}}} \tag{1.12}
\end{equation*}
$$

The solid curves in Fig. 1.12(c) are fits to Eq. (1.12), yielding exponents of $\alpha_{\perp}=0.99 \pm 0.04$ and $\alpha_{\|}=0.73 \pm 0.02$. These terms implicitly include the two non-thermal effects of self-propulsion. First, the magnitudes, $A_{i}$, contain information about the energy gap between the two components. Secondly, the power law exponents incorporate the extended memory effect of self-propulsion. We note that a power law decay, $\tau^{-d / 2}$, is expected for particles suspended in a fluid due to diffusive transport of momentum through the surrounding fluid [6]. Although this is not the case for our study, we highlight it as a potential analogy: particles in a viscous medium and selfpropelling particles both exhibit velocity autocorrelations with extended memory effects. This form is also able to capture the superdiffusive behavior of $\left\langle\Delta \tilde{x}_{\|}^{2}(\tau)\right\rangle$. We obtain the following expression for the body frame mean square displacements:

$$
\begin{equation*}
\left\langle\left(\Delta \tilde{x}_{i}\right)^{2}\right\rangle=\frac{2 A_{i}}{\left(1-\alpha_{i}\right)\left(2-\alpha_{i}\right) b_{i}^{2}}\left[\left(a_{i}+b_{i} \tau\right)^{2-\alpha_{i}}-a_{i}^{2-\alpha_{i}}\right]-\frac{2 A_{i}}{\left(1-\alpha_{i}\right) b_{i}} a_{i}^{1-\alpha_{i}} \tau \tag{1.13}
\end{equation*}
$$

This form shows a crossover from $\tau^{2}$ at short times to $\tau^{2-\alpha_{i}}$ at long times. Using the parameters obtained from the fit to Eq. (1.12), we plot the functional forms given by Eq. (1.13) as the solid lines in Fig. 1.12(a).

We now have enough information to construct the coupled fixed-angle lab frame dynamics. Writing the velocity in the fixed-angle lab frame in terms of the body frame velocity,

$$
\begin{equation*}
\bar{v}_{k}(t)=R_{k l}^{-1}(\theta(t)) \tilde{v}_{l}(t) \tag{1.14}
\end{equation*}
$$

where $R_{k l}^{-1}(\theta(t))$ is the rotation matrix, we obtain for the fixed-angle lab frame velocity autocorrelation function:

$$
\begin{align*}
& W_{f}\left(t, t^{\prime}\right)=\frac{1}{2}\left[W_{\|}(\tau)+W_{\perp}(\tau)\right] e^{-\left\langle(\Delta \theta)^{2}\right\rangle / 2} \delta_{i j} \\
&+\frac{1}{2}\left[W_{\|}(\tau)-W_{\perp}(\tau)\right] M_{i j}(\theta(0)) e^{-\left\langle\left(\theta\left(t^{\prime}\right)+\theta(t)\right)^{2}\right\rangle / 2}, \tag{1.15}
\end{align*}
$$

where $\theta(0)=0$ and

$$
M_{i j}=\left(\begin{array}{cc}
\cos 2 \theta(t) & \sin 2 \theta(t)  \tag{1.16}\\
\sin 2 \theta(t) & -\cos 2 \theta(t)
\end{array}\right)
$$

We use the fitting parameters obtained from our fits to Eqs. (1.9) and (1.12) to generate Eq. (1.15), plotted as the solid curves in Fig. 1.10(c). The model shows that $W_{\tilde{x}}(\tau)$ and $W_{\tilde{y}}(\tau)$ converge at $\tau_{\theta}$, consistent with a crossover in the data. The discrepancy between the model prediction and the data at long times may indicate that memory effects persist even longer than expected.

We then numerically integrate $W_{f}\left(\mathrm{t}, \mathrm{t}^{\prime}\right)$ according to

$$
\begin{equation*}
\left\langle[\Delta \bar{x}(t)]^{2}\right\rangle=\int_{0}^{t} d t_{1} \int_{0}^{t} d t_{2} W_{f}\left(t_{1}, t_{2}\right) \tag{1.17}
\end{equation*}
$$

to obtain the solid curves in Fig. 1.10(a). The components of the model converge on the same timescale as the data. We see that modelling self-propulsion as an external noise source with long-lived correlations is sufficient to reproduce rotational-translational coupling. Our model suggests that a phenomenological way to think about collections of self-propelling particles is in terms of enhanced memory effects rather than explicitly detailing novel forces and torques in microscopic equations.

The model also reproduces the angle-averaged lab frame dynamics well (Fig. 1.9). Averaging our expressions over all initial angles eliminates the second term in Eq. (1.15); the result is plotted in Fig. 1.9(c) as the solid curve. We integrate according to Eq. (1.17) to obtain the solid curve in Fig. 1.9(a). The model describes the angle-averaged velocity correlations very well out to 1 s , after which the data falls off more rapidly. The good agreement with the fit here confirms the bulk thermal behavior of the collection of self-propelling rods.

### 1.9 Conclusion

We have investigated the dynamics of air-fluidized rods. Particle shape anisotropy leads to dynamical anisotropy, characterized by preferential motion parallel to the particle's long axis. Ours is a model system - with the advantage of a temporally and spatially homogeneous driving method - to further investigate universal phenomena predicted for collections of self-propelled particles.

In this chapter, we compared the coupling of rotation and translation for a self-propelled particle to that of an anisotropic Brownian particle. A modified Langevin formalism implicitly specifying self-propulsion via non-thermal noise describes the dynamics data well, capturing rotational-translational coupling at the correct timescale. Despite the energy gap between the parallel and perpendicular components, the model was able to reproduce the loss of directional memory at long times. Furthermore, despite the non-thermal behavior of individual particles, the bulk angle-averaged behavior is nearly thermal.

This provides us with a stepping stone to Chapter 2 where we will continue to explore phase space in the interest of observing collective behavior and spontaneous symmetry breaking for denser collections of self-propelled particles. We hope that, through comparison with theoretical models [12,20,30,67,71,73-75,77,78] and recent vibrated-bed experiments [9, 15, 29, 42,53,54], our system will further shed light on universal behavior of collections of self-propelled particles.

## Chapter 2

## Propagating Wave Instability

In this chapter, we extend our previous experiments to denser collections of self-propelling airfluidized rods. In Chapter 1, we studied a dilute monolayer of rods and showed that an individual rod self-propels along its long axis with root-mean-square speed $\sim 1 \mathrm{~cm} / \mathrm{s}$. This observation is of importance because, as discussed in Section 0.3, collections of self-propelling particles have been predicted to exhibit fascinating collective phenomena - such as flocking, propagating waves, and anomalously large fluctuations in local number density - from minimal models typically including only particle geometry and self-propulsion. However, such emergent phenomena is typically only expected in dense systems where ordering plays an important role. In Section 2.1, we will outline the apparatus and system details. In Section 2.2, we will explore and map out the phase behavior of air-fluidized rods as a function of the packing density and the fluidizing airflow. Although no broken-symmetry state is ever observed, we do observe an instability for compression waves to propagate. One remarkable feature of the compression waves is that they give rise to anomalously large number fluctuations, calculated in Section 2.3. In order to assess the role of self-propulsion, we quantify the waves in Section 2.4 by obtaining a dynamic structure factor. From this, we are able to extract a wavespeed and its dependence on density and airflow. In Section 2.5, we discuss potential theoretical explanations for our observations and current progress in this regard. Interestingly, our system is most like the high density, high noise case of Ref. [78] where no flocking is observed but particles have strongly correlated motion.

### 2.1 Experimental Details

We study a monolayer of cylindrical plastic dowel rods - length 1.27 cm , diameter 0.24 cm , and mass 0.076 g - fluidized by a spatially and temporally uniform upflow of air. Except when otherwise noted, we analyze a model system of 1353 rods occupying area fraction $67 \%$, fluidized at air speed $220 \mathrm{~cm} / \mathrm{s}$ as measured by a hot-wire anemometer. The fluidizing air speed is low enough that rods do not overlap out-of-plane and the system remains a monolayer. In order to emphasize the role that shape plays here, we analyze a companion system of 838 bidisperse plastic spheres - diameters 0.64 cm and 0.95 cm - at the same area fraction and fluidization speed.

The apparatus, fluidization method, and lighting setup is identical to that of Ref. [22] and Chapter 1, as detailed in Appendix A. The apparatus is a rectangular windbox, $1.5 \times 1.5 \times$ $4 \mathrm{ft}^{3}$, positioned upright. A circular brass testing sieve with mesh size $150 \mu \mathrm{~m}$ and diameter 30.5 cm rests horizontally on top. Unlike in Chapter 1 where we placed the rods in an insert to reduce the system size, here we use the entire area of the sieve. Because we are exploring higher densities, particle-particle interactions eliminate the harmonic potential in the bed and the use of an insert was no longer necessary. However, in order to prevent particles from becoming trapped in a small groove around the edge of the sieve, we place a $3 / 8$ "-diameter norprene tube around its inside edge. A blower attached to the windbox base provides vertical airflow perpendicular to the sieve. Raw video data of the fluidized particles is captured for 10 minutes at 120 frames per second by a digital camera mounted above the apparatus. Post-processing of the video data is accomplished in LabVIEW, using the same tracking programs used in Chapter 1 and Appendix A when appropriate. For the analysis of the waves in Section 2.4, we treat the video as a binary density map $\rho(x, y, t)$ rather than individually track particles.

### 2.2 Phase Behavior

The first step we took was to map out the phase behavior of fluidized rods as a function of packing fraction and fluidizing airflow. We explored a range of densities from a single particle up to $75 \%$ packing fraction and fluidization speeds from $150 \mathrm{~cm} / \mathrm{s}$ to $500 \mathrm{~cm} / \mathrm{s}$. Because of


Figure 2.1: Behavior of fluidized rods as a function of fluidizing airflow and packing fraction. The diagram in a) is for a large system with diameter 28.6 cm ; b) is for a smaller system with diameter 17 cm . The light blue cross in a) indicates the conditions for the initial analysis of Section 2.4. The orange cross in b) represents the conditions for the experiment of Ref. [22] and Chapter 1. The vertical dashed line in both plots corresponds to the density above which the rods are uniformly distributed across the system.
the long processing times needed to analyze the data, the phase diagram shown in Fig. 2.1 is obtained qualitatively. We note that there are strong particle-wall interactions making the exact shape of the phase boundaries sytem-size-dependent. As such, we show a phase diagram for the full system size - diameter 28.6 cm - in Fig. 2.1(a) and the system size used in Ref. [22] diameter 17 cm - in Fig. 2.1(b). These wall effects are most significant at lower densities where they cause the particles to cluster toward the center of the system. As such, area fraction is not well-defined until the particles are uniformly distributed across the system. The area fraction at which the rods are uniformly dense is indicated as a vertical dashed line in both Figs. 2.1(a) and (b).

We first place the desired number of rods into the system with no air flowing. We then turn on the air to $150 \mathrm{~cm} / \mathrm{s}$. We then increase the airspeed slowly by increments of roughly $10 \mathrm{~cm} / \mathrm{s}$. After each increase in the airflow, we wait 1 minute to ensure the system has reached a steady state and then characterize the behavior observed before increasing airflow once more.

For all packing fractions, there is a threshold airflow below which the rods do not have enough energy to tilt out-of-plane or rotate in-plane and no motion is observed. As we increase the airflow, we observe two distinct behaviors depending on the density of the rods. For more dilute systems, the rods self-propel with very little out-of-plane motion. The experimental conditions for our previous work [22] in Chapter 1 are shown as the solid orange cross in Fig. 2.1(b), within this self-propelling regime. At higher densities, the system becomes unstable to the existence of propagating compression waves. The waves appear to arise due to particle-particle collisions. At sufficiently high densities, a rod may attempt to self-propel but is confined to a limited area due to the presence of other rods. When it collides with another particle, the other particle is able to overcome friction perpendicular to its long axis and slide, colliding with yet another particle. This cascade of particle collisions then results in a propagating wave. At the highest airflows, we continue to observe waves, however the particles begin to overlap out-of-plane. Thus, the waves gain a transverse component and the system is no longer a monolayer. It should be noted that, by visual inspection, the rods do appear to have strongly correlated motion in these high density regimes. However, long-range ordering or a spontaneously-broken-symmetry state are never observed to emerge.

A time series of images, each separated by 0.09 s from top to bottom, shown in Figure 2.2


Figure 2.2: A propagating wave in a monolayer of gas-fluidized rods. The system has diameter 28.6 cm . The rods occupy $67 \%$ area fraction and are fluidized at $220 \mathrm{~cm} / \mathrm{s}$. The time between images, moving top to bottom, is 0.09 s . The red arc moves at constant velocity and serves as a guide to the eyes.
depicts one such propagating compression wave for a system at $67 \%$ area fraction. The experimental conditions for this time series, as well as the initial analysis of Section 2.4, are marked as a light blue cross in Fig. 2.1(a). The red arc moves at constant velocity with the wavefront and serves as a guide to the eye. When particles are compressed together, they reflect more light back to the camera. As such, the wavefront is seen as a brighter region in the images and the less dense rarefaction zones are darker. Within this region of the phase diagram, the particles do not overlap out-of-plane and the waves are purely compressional.

We note that the phase behavior of bidisperse spheres is markedly different than that of rods. For all packing fractions below some threshold airflow, spheres are unable to overcome rolling friction and we observe no motion. As airflow is increased, we observe smooth rolling behavior for all densities. Above some airflow, the spheres gain enough to energy to leave the plane and the system is no longer a monolayer. Although we do not observe compression waves for spheres, at high densities and very high airflows, we observe strong transverse waves similar to a sheet of paper buckling.

### 2.3 Number Fluctuations

Although we have explored very dense collections of self-propelling rods, we do not observe either a dynamic broken-symmetry state or the development of long-range order. The emergence of both is suppressed by the propagation of compression waves which disrupts the local ordering and any coherent motion developing between neighboring particles. As can be seen in the video data [1], the waves travel much faster than the particles themselves. To estimate the wavespeed, we can look at a spacetime plot of a line of pixels taken from the video data as shown in Fig. 2.3. In the figure, time increases along the horizontal axis from 0 to 4.2 seconds; the vertical axis is 11.7 cm tall. As a guide to the eye, we place diagonal lines along several wavefronts. Although each diagonal line is only a projection of the wave onto this particular line of pixels, we can obtain an order of magnitude estimate of the wavespeed from the slopes of these lines. Doing so, we obtain wavespeed values ranging from $\sim 10 \mathrm{~cm} / \mathrm{s}$ to $\sim 30 \mathrm{~cm} / \mathrm{s}$. By simultaneously tracking the particles in the raw video data, we calculate the root-mean-square particle speed of $\sim 1 \mathrm{~cm} / \mathrm{s}$. Thus, the waves move an order of magnitude faster than the particles themselves and an order of


Figure 2.3: A spacetime plot showing propagating waves in a monolayer of gas-fluidized rods. Time increases along the horizontal axis from 0 to 4.2 s . The vertical axis is 11.7 cm tall. Solid diagonal lines (green) have been placed as guides to the eye to highlight several waves.


Figure 2.4: Magnitude of the number fluctuations normalized by $\sqrt{N}$ versus average number of particles in a subregion of the system. The circles (green) are for a bidisperse collection of spheres. The squares (blue) are for a collection of rods. Both systems occupy $55 \%$ area fraction and are fluidized at $220 \mathrm{~cm} / \mathrm{s}$. The vertical dashed lines correspond to the total number of particles in each system, respectively.
magnitude slower than the fluidizing airflow.
This separation of timescales has an interesting consequence. As a wave propagates across the system, a compression front builds up leaving a dilute rarefaction zone in its wake. Because the waves move much faster than the particles, the subsequent relaxation of the compression and rarefaction zones occurs an order of magnitude more slowly than the compression process itself. Thus, the separation of timescales between the individual particle motion and the collective wave motion causes compressed and dilute regions to coexist for periods of time long compared to the time it takes the waves to propagate. Such persistent large fluctuations in the density suggest the presence of anomalously large fluctuations in the local number density for these systems.

To quantify number fluctuations, we take a 10 -minute video of the system at a given area fraction and airflow. We select a square subregion of interest with side length $l$ and count the number of particles within that region for each frame. From this time series, we calculate the mean value $\langle N\rangle$ and standard deviation $\Delta N$. We then repeat this procedure for systematically larger regions of interest, ranging from a single pixel up to roughly half of the system size.

For a thermal system, the quantity $\Delta N / \sqrt{N}$ should be a constant. Results for a system at $55 \%$ area fraction and fluidized at $220 \mathrm{~cm} / \mathrm{s}$ are shown in Figure 2.4, plotting $\Delta N / \sqrt{N}$ versus


Figure 2.5: Number fluctuations autocorrelation function for various subregion sizes, $l \times l$, where the value of $l$ is noted in the legend.
$\langle N\rangle$ for a particular subregion. Here, a horizontal line indicates "thermal" behavior and so-called giant number fluctuations will be characterized by a non-zero slope and magnitude larger than 1 . The number fluctuations for bidisperse spheres show "thermal" behavior over the entire range of subregion sizes. Rods, however, show number fluctuations with both an exponent and magnitude larger than thermal expectations, indicating giant fluctuations in local number density. The large $\langle N\rangle$ fall-off is a finite size effect; the vertical dashed lines in Fig. 2.4 indicate the total number of particles in each system.

Although the maximum in the number fluctuations is due to finite size effects, it serves as a useful benchmark to quantify the spatial extent of local number fluctuations. We accomplish this by converting the value of $\Delta N$ into an effective range of area fractions. The value of $N+\Delta N$ at the maximum corresponds to a range of area fractions $\phi \sim 55 \% \pm 20 \%$. Thus, as the wave propagates through the system, particles are compressed by the front up to $75 \%$ area fraction while the rarefaction zones are diluted to approximately $35 \%$.

Figure 2.4 is strikingly similar to the number fluctuations obtained for rods fluidized by vertical vibrations [54]. In that experiment though, the large voids responsible for giant number fluctuations persisted for very long times, indicated by a logarithmic decay in the number density fluctuation autocorrelation function $\langle\delta N(t) \cdot \delta N(t+\tau)\rangle$, where $\delta N=N(t)-\langle N\rangle$ is the

| Area Fraction | $(\Delta N / \sqrt{N})_{\max }$ | $\alpha$ |
| :--- | :---: | :---: |
| $0.52 \%$ | 4.13 | 0.617 |
| $0.57 \%$ | 3.85 | 0.58 |
| $0.595 \%$ | 3.65 | 0.611 |
| $0.62 \%$ | 3.54 | 0.638 |
| $0.645 \%$ | 2.12 | 0.47 |

Table 2.1: Table summarizing how density fluctuations change with packing fraction, as listed in the first column. The second columns is the maximum magnitude of number fluctuations when normalized by $\sqrt{N}$. The final column is the exponent $\alpha$ of the power law fit $\Delta N=A\langle N\rangle^{\alpha}$.
fluctuation from the average. We calculate the same autocorrelation as in Ref. [54], shown in Fig. 2.1, for different subregion sizes. At the smallest wavelength 0.67 cm , there is an initial dropoff and a slow decay to zero. As we increase the wavelength, the timescale of the initial dropoff increases and a strong rebound feature develops, largest at an intermediate wavelength of 8.7 cm , followed by strong oscillations. The oscillations are most coherent at a wavelength of 15.4 cm , representing about half the system size. At higher wavelengths, the rebound and oscillations die out due to the finite system size. The oscillations and rebounds are consistent with the presence of propagating waves. We note that the decay is very rapid in stark contrast to the long-lived fluctuations in Ref. [54]. The autocorrelation function is related by Fourier transform to the dynamic structure factor which will be calculated in Section 2.4, at which point we will discuss the connection further.

Another difference between our observations and those of Ref. [54] are how number fluctuations change with increasing area fraction. In the vibrated-bed experiment, as the system area fraction was increased, nematic ordering increased and the magnitude and slope of the number fluctuations increased. Similarly, for theoretical treatments of self-propelling particles, as density increases, broken-symmetry states are more likely to emerge and maintain coherence. In our experiment, we see the opposite trend as shown in Table 2.1. The first column lists the various packing fractions for which we calculated number fluctuations. As the area fraction increases, the magnitude $\Delta N / \sqrt{N}$ of the number fluctuations decreases, as seen in the second column. The scaling exponent $\alpha$ listed in the third column - obtained from a fit to $\Delta N=A\langle N\rangle^{\alpha}$ - shows no trend until very high densities. At the highest area fractions for rods, the number fluctuations
appear "thermal", having zero slope with no maximum over the entire range of subregions. In our system, as area fraction is increased, we never observe nematic ordering or emergent behavior that would facilitate these trends. Rather, particles are confined more strongly by their neighbors and so, as waves propagate, the compressed and rarefied regions are smaller and relax more quickly.

We have shown that the fluctuations in local number density are anomalously large for our system in the absence of either long-range nematic order or a broken-symmetry state such as a coherently-moving flock. In our system, the only ingredient we have is self-propulsion. However, the nature of density fluctuations in our system is exactly opposite to that expected: the magnitude of the fluctuations and slope both decrease with increasing packing fraction. This leaves open the question: what role does self-propulsion play here? Unfortunately, the high density, high noise situation for collections of self-propelling rods most applicable to our system has not been extensively studied. Most of the predictions concerning propagating waves and giant number fluctuations are for situations in which a flock exists. Thus, we have few theoretical benchmarks to test our system against. The theory with the most potential applicability is that of Baskaran and Marchetti [12] for frictional, self-propelling rods interacting via volume exclusion. In this theory, despite the lack of a broken-symmetry state, propagating waves are predicted. For comparison with the work of Baskaran et al. and anticipation of future theoretical investigation of these high density, high noise cases, we now concern ourselves with quantifying how the waves themselves change with the packing fraction and airflow.

### 2.4 Dynamic Structure Factor

Propagating waves are of interest because they have been predicted for collections of selfpropelling particles interacting via volume exclusion with no emergent broken-symmetry state [12]. Quantitatively, the wave speed is predicted to be set by the root-mean-square speed of the particle, $v_{r m s}$. For self-propelling particles, $v_{r m s}$ is the self-propelling speed. We have already seen that the wavespeed in our experiment is an order of magnitude larger than the particle speed. In order to assess the applicability of this theory, we need to consider how wavespeed and particle speed change with packing fraction and airflow. To do so, we obtain more detailed quantitative


Figure 2.6: Dynamic structure factor $S(\omega,|k|)$ for a) a collection of rods and b) a collection of bidisperse spheres, both occupying $67 \%$ area fraction and fluidized at $220 \mathrm{~cm} / \mathrm{s}$. Each curve corresponds to a different wave-vector value as shown by the color legend in a). The arrows in a) serve to highlight the most important feature, the location of a peak.
information about the waves by calculating the dynamic structure factor $S(\omega,|k|)$ - the spatial and temporal Fourier transform of the density $\rho(x, y, t)$ - where $\omega$ is frequency and $|k|$ is wave-vector.

We calculate $S(\omega,|k|)$ using the same videos that were analyzed to obtain number fluctuation information. We treat the video data as a density map, a function of position and time $\rho(x, y, t)$. Using LABView's Vision package, we first obtain the spatial Fourier transform of each frame in the video. Because there is no long-range order, the spatial Fourier transforms of the rods data show two annuli at $|k| / 2 \pi \simeq 0.79 \mathrm{~cm}^{-1}$ and $4.17 \mathrm{~cm}^{-1}$, corresponding to the long and short dimensions of the rod. The annuli are isotropic with respect to the polar angle indicating that there is no long-range order in the system. We next calculate the temporal power spectrum for each pixel within an annulus at fixed $|k|=\sqrt{k_{x}^{2}+k_{y}^{2}}$. We average the temporal power spectrum over all pixels within a given annulus to obtain the dynamic structure factor $S(\omega,|k|)$ for the particular value of $|k|$. More details on and intermediate steps of the program used to calculate $S(\omega,|k|)$ can be found in Appendix A.

We then plot slices of $S(\omega,|k|)$ for fixed $|k|$ as a function of $\omega$. Our results are shown in Fig. 2.6(a) for rods and (b) for spheres, both at $67 \%$ area fraction and $220 \mathrm{~cm} / \mathrm{s}$ airflow. We are most interested in the long-wavelength, long-time hydrodynamic limit as both $k$ and $\omega$ approach zero. The range of wave-vectors shown correspond to one-fourth the system size to approximately one particle length.

Since the integral $\int S(\omega,|k|) d \omega=\left\langle\rho(0)^{2}\right\rangle$, the magnitude of the dynamic structure factor as $\omega \rightarrow 0$ is proportional to the density of the particles. Because a sphere appears to the camera as a small dot at its center whereas rods reflect light along their entire length, the video images of spheres have fewer white pixels and subsequently the dynamic structure factor for spheres has a much smaller zero-frequency magnitude than rods. The most salient feature of $S(\omega,|k|)$ is a single peak in the dynamic structure factor at a given $|k|$ for rods, indicating a traveling excitation. For larger wave-vectors outside of the hydrodynamic limit, the peak is no longer observed. Recalling that $S(\omega,|k|)$ is the power spectrum of the number fluctuation autocorrelation function $\langle\delta N(t) \cdot \delta N(t+\tau)\rangle$ in Fig. 2.1, we confirm that the two are consistent. Oscillations were coherent and large for larger wavelengths, hence the peaks are sharpest at smaller $|k|$. As wavelength is decreased, oscillations die out; similarly, at larger $|k|$, the peak is no longer observed.


Figure 2.7: Contour plot of the dynamic structure factor for rods occupying $67 \%$ area fraction and fluidized at $220 \mathrm{~cm} / \mathrm{s}$. The locations of the peaks (circles) form the dispersion relationship for the propagating waves. The slope of the linear region is the wave speed $c=20.8 \mathrm{~cm} / \mathrm{s}$.

In contrast, the same analysis for spheres shows $1 / \omega^{2}$ decay for small frequencies. This behavior suggests that local density fluctuations decay diffusively, consistent with the "thermal" number fluctuations observed for spheres. The roll-off seen at approximately $10 \mathrm{~s}^{-1}$ in both plots is also observed for videos of still particles where the only "motion" is due to pixel noise. As such, we attribute this feature to pixel noise and do not obtain useful information from it.

By plotting the location of the peaks as a function of $\omega$ and $|k|$, we can construct the dispersion relationship for the waves, shown in Fig. 2.7. Here, we show the same data as in Fig. 2.6(a) but as a top-down contour plot with the peak locations marked by solid circles. In the hydrodynamic limit, the dispersion is linear and we can extract a wave speed $c=\omega /|k|$ from the slope. For this particular example, $c=20.8 \mathrm{~cm} / \mathrm{s}$. This is consistent with the video data which show


Figure 2.8: Wavespeed (blue circles), $\mathrm{c}=\omega / k$, and root-mean-square parallel (red squares) and perpendicular (black triangles) particle speeds as a function of area fraction for rods fluidized at $280 \mathrm{~cm} / \mathrm{s}$. The shaded green region is where out-of-plane motion becomes substantial.
the waves propagating across the system in 1-2 seconds as well as the value obtained from the spacetime plot, Fig. 2.3. Again, we note that the wave speed is an order of magnitude larger than the self-propelling speed of the particles, $v_{r m s} \sim 1 \mathrm{~cm} / \mathrm{s}$, and an order of magnitude smaller than the fluidizing airflow $\sim 200 \mathrm{~cm} / \mathrm{s}$.

Now, we are in a position to calculate the wavespeed accurately as a function of density and airspeed. By determining what sets the wavespeed we can better understand whether to model our monolayer as an elastic medium - where the wavespeed depends only on the elastic moduli and the density $\propto \sqrt{1 / \rho}-$ or in terms of collections of self-propelling particle - where the wavespeed is set by the self-propelling velocity. We repeat the dynamic structure factor analysis for a fixed airflow of $280 \mathrm{~cm} / \mathrm{s}$ while increasing the density of the system over the range where we observe waves. We simultaneously calculate the average parallel and perpendicular speeds of the individual particles.

The results of our analysis are shown in Figure 2.8. At fixed air speed, the wave speed increases slightly as we increase the packing fraction of the system. The increase appears to be

| Airspeed | $0.52 \%$ | $0.57 \%$ | $0.595 \%$ | $0.62 \%$ | $0.645 \%$ | $0.67 \%$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $220 \mathrm{~cm} / \mathrm{s}$ | - | - | - | - | 20.0 | 20.8 |
| $250 \mathrm{~cm} / \mathrm{s}$ | - | - | 17.8 | 18.8 | 19.7 | 20.55 |
| $280 \mathrm{~cm} / \mathrm{s}$ | - | 17.0 | 17.9 | 18.5 | 19.6 | 20.6 |
| $300 \mathrm{~cm} / \mathrm{s}$ | 16.7 | 17.3 | 18.1 | 18.9 | 19.7 | - |
| $320 \mathrm{~cm} / \mathrm{s}$ | 16.6 | 17.2 | 18.0 | - | - | - |

Table 2.2: The value of the wavespeed, $\omega /|k|$, in $\mathrm{cm} / \mathrm{s}$ for different values of airspeed, as listed in the first column, and area fraction, as listed across the top row. When no value is recorded, there was either no particle motion or the particles overlapped out-of-plane.
linear, although the dynamic range of our data is limited. This linear trend is seen for all other fixed airflows that were analyzed as seen by reading across each row in Table 2.2. By contrast, if we fix the area fraction and increase the fluidizing airflow, the wavespeed shows no dependence on airflow. This is seen by reading down each column in Table 2.2. We can conclude that the wavespeed is slightly linearly dependent on the density but independent of how strongly the system is driven.

For most densities, the parallel self-propelling speed is slightly larger than the transverse speed, although the effect is smaller than that observed for dilute systems [22]. Although the rods are self-propelling, we see that the side-to-side motion induced by the propagating waves causes $v_{\perp}^{r m s}$ to be nearly equal to $v_{\|}^{r m s}$. As area fraction is increased at fixed airflow, both $v_{\perp}^{r m s}$ and $v_{\|}^{r m s}$ increase linearly with increasing density. For all densities and airspeeds analyzed, the wavespeed remains an order of magnitude larger than the particle speed.

We now need to examine what our observations mean in the context of self-propelling particles. Like vibrated-bed granular experiments as well as simulation and theory, we observe giant number fluctuations in a collection of self-propelling air-fluidized rods. The mechanism in each case is quite different. In the vibrated-bed experiments, the fluctuations arise due to the establishment of long-range nematic order and the emergence of large voids that persist for very long times. In simulation and theory, typically the giant fluctuations arise due to the emergence of a coherent flock. In our experiment, the fluctuations are caused by propagating compression waves and are very short-lived. Despite these quantitative differences, we stress that the systems all observe similar qualitative behavior and that self-propelling is the common link between all
of these cases. If we recall the phase behavior examined by Vicsek et al. [78] in Section 0.3, the emergence of a coherent flock only occurs below a critical noise threshold. Otherwise, at high densities, neither long-range order nor a dynamic broken symmetry are observed. What is observed is correlated motion and propagating waves, both of which we find in our experiment. Our experiment is conducted at high density but the driving is turbulent and, thus, the noise level in our system is very high. Therefore, considering the results of Vicsek, we would not expect to observe long-range order or a dynamic broken symmetry. Thus, our results are entirely consistent with theory and simulation. This suggests that the noise level is the crucial parameter and it may be possible to observe a dynamic broken symmetry state in a physical system so long as the noise is negligible.

### 2.5 Conclusion

We have investigated the behavior of dense collections of self-propelling rods fluidized by an upflow of air. Despite our system possessing the minimal ingredients needed in order for a dynamic broken-symmetry state to arise, we do not observe the spontaneous emergence of coherent motion or an ordered phase. Rather, for a large range of densities and airflows, the monolayer of self-propelling rods becomes unstable to the propagation of compression waves. The waves travel at a speed an order of magnitude larger than the root-mean-square speed of the particles themselves. As the compressed and rarefied regions relax after a wave's passage, this difference in timescales gives rise to anomalously large fluctuations in the local number density. Unlike giant number fluctuations expected for swarms and seen in vibrated-bed experiments, number fluctuations in our system are short-lived and become more "thermal" as the system packing fraction is increased.

To ascertain the role that self-propulsion plays in the collective wave behavior we observe, we quantified the waves by calculating the dynamic structure factor for the rods. From the position of peaks in this function, we extracted a dispersion relationship and, in the hydrodynamic limit, the wavespeed. Repeating our analysis for systems of increasing density and constant airflow, we determined that both the wavespeed and the particle root-mean-square speed are roughly linear with particle density.

The existence of propagating waves in the absence of a coherent flock is predicted theoretically for a collection of self-propelling rods interacting via volume exclusion interactions. Recently, this theory has been shown to predict a linear dependence of the wavespeed on particle density [2]. We are optimistic that this theory which closely models our own system in terms of 'ingredients' will be able to adequately describe our observations. Interestingly, the behavior we have observed is most close to that of the high density, high noise regime in the seminal simulations by Vicsek et al. [78]. Recall that, in that simulation, the emergent flocking behavior occurred as the noise in the system was slowly lowered through some critical noise value. Systems in which the noise level was high, such as ours, were not observed to form a dynamics spontaneously-broken symmetry state. These broad similarities to simulation and results from vibrated granular experiments suggest that self-propelling is emerging as a successful unifying concept.

## Chapter 3

## Self-Propelling of a Sphere in a Thermal Bath

Previous experiments [3] have identified two ways in which the thermal analogy can be broken for grains fluidized by an upflow of air. The first method - particles with an aspect ratio greater than one - was investigated in Chapters 1 and 2. The second method is to fluidize simultaneously two spheres of appreciably different size. Not only do to the two spheres reach different effective temperatures but we often see a 'follow-the-leader' behavior in which the two spheres stick together and move in the direction of the smaller ball.

In this chapter, we return to the concept of the thermal analogy for spheres of different size. Individual spheres behave like Brownian particles in a harmonic trap when fluidized in an empty sieve [60]. Dense bidisperse mixtures of spheres do not behave thermally: the two ball species come to slightly different effective temperatures [4]. Provided that the collection is dilute and the balls are close to each other in size, the temperatures are not appreciably different and we can treat a monolayer of bidisperse spheres as a thermal bath. The question we pose here is how does a very large sphere behave when fluidized in the presence of a "thermal bath" background of smaller spheres? For example, does the sphere come to thermal equilibrium with the background? Or, does the self-propelling 'follow-the-leader' behavior arise?

Similarly, the interaction between two equal-sized spheres in an empty sieve has previously been characterized [3], showing a hard-core repulsive interaction at short distances and weak
repulsion at all other length scales. How does the presence of a thermal background change this ball-ball interaction? For example, we might expect a short-range depletion attraction due to free area considerations [10,28,50,52,68] as well as a Casimir-type long-range attraction [19] such that the large balls seek to minimize the chaotic turbulence of the background separating them $[18,32,41,69]$. In Section 3.1, we introduce the apparatus and the system that we study. In Section 3.2, we qualitatively discuss the observed behavior of the large ball as well as the mechanism responsible, focusing on the role that the background balls play. In Section 3.3, we characterize the behavior of a single fluidized large ball by examining the statistics of various time-independent quantities; in Section 4.3, we discuss the time-dependent single ball dynamics. Next, in Section 3.5, we fully characterize the forces acting on a single large ball using a Langevin formalism. Lastly, in Sections 3.6 and 3.7, we examine the behavior of two balls fluidized together and discuss current progress in characterizing their interaction.

### 3.1 Experimental Details

The principal system that we investigate, shown in Fig. 3.1(a), is a monolayer of a bidisperse mixture of plastic spheres - having radii 0.477 cm and 0.397 cm and masses 0.21 g and 0.165 g , respectively - fluidized by an upflow of air. This monolayer constitutes the "thermal bath" background and occupies an area fraction of $55 \%$ in the absence of larger fluidized balls. This particular area fraction was chosen so that the background is uniformly distributed across the entire system. The larger balls fluidized in the presence of this background are ping-pong balls - radius $a=1.9 \mathrm{~cm}$ and mass $m=2.7 \mathrm{~g}$ - that have been spray-painted chrome to aid in visualization. The size ratio of the large balls to the average size of the background is 4.4 . We have taken and analyzed data for other size ratios; except where noted, the data analyzed in this chapter is for size ratio 4.4.

The system is fluidized by an upflow of air at $300 \mathrm{~cm} / \mathrm{s}$, spatially and temporally homogeneous within $\pm 10 \mathrm{~cm} / \mathrm{s}$ and 0.5 s , as measured by a hot-wire anemometer. The Reynolds number is $10^{4}$ such that the motion of the balls is driven stochastically by turbulence. The airflow is low enough that the balls maintain contact with the sieve and move by rolling without slipping.

The apparatus, fluidization method, and lighting setup are identical to those of Ref. [59], as


Figure 3.1: a) A large sphere, radius 1.9 cm and mass 2.7 g , fluidized in the presence of a bidisperse background of smaller spheres - radii 0.477 cm and 0.397 cm and masses 0.21 g and 0.165 g , respectively. The size ratio of the large ball to the background is 4.4 . The radius of the system is 15.3 cm . The vectors define position $\vec{r}$, velocity $\vec{v}$, as well as the polar angle $\theta$ as measured from the horizontal axis. b) A one-minute long trace of the position of the large ball shown in a). To emphasize the observed circular motion, the trace is colored red for clockwise motion, blue for counterclockwise.
detailed in Appendix A. The apparatus is a rectangular windbox, $1.5 \times 1.5 \times 4 \mathrm{ft}^{3}$, positioned upright. A blower attached to the windbox base provides vertical airflow perpendicular to a circular brass testing sieve with mesh size $150 \mu \mathrm{~m}$ and radius 15.3 cm that rests horizontally on top. To prevent background spheres from getting caught in a small groove at the inner edge of the sieve, we place a 0.953 cm -diameter norprene tube around its inside edge. Thus, the system has an actual radius, $R=14.3 \mathrm{~cm}$.

The particles are illuminated from above by six 100 W incandescent bulbs arranged in a 1-foot diameter ring positioned 3 feet above the sieve. A digital CCD camera placed at the center of this ring captures the raw video data, typically for 4 hours at a time at 120 frames per second. Because we are able to obtain good statistics on the background balls from very short videos, we threshold these long videos as they save to buffer so that only the highly-reflective large ball is seen. Post-processing of the video data is accomplished using LabVIEW, using the same tracking programs of previous chapters as detailed in Appendix A. From the center-ofmass position obtained from the video data, we determine velocity and acceleration by fitting the position data to a third-order polynomial over a window of $\pm 4$ frames. The window is Gaussianweighted, vanishing at the window edge, to ensure the continuity of the derivatives. This gives us an estimate of the error in the position data of $18 \mu \mathrm{~m}$.

### 3.2 Background Fluctuations

We had intuitively supposed that the large ball, when fluidized in the presence of a bidisperse "thermal" background, would behave like a Brownian particle but with reduced interactions with the wall. The turbulent mixing of wakes from the smaller background would prevent the large ball from detecting the wall and, at the same time, provide thermal kicks to the large ball. The observed behavior is dramatically different than our expectations.

As soon as the large ball is placed in the background, it begins to self-propel ballistically across the system. Typically, it will propel in a straight line until it reaches the boundary or the background balls jam in front of the large ball, at which point it is forced to change direction. When this occurs at the boundary, the particle begins to propel along the boundary edge resulting in circular motion at some stable orbit position. We emphasize the observed circular behavior
in Figure 3.1(b). Here, we show a one-minute long time-trace of the large ball center-of-mass position. Whenever the ball is moving clockwise, we color the trace red; counterclockwise motion is blue. Several circular orbits of both type can be seen in the figure. This orbiting behavior is largely dependent on the size ratio of the large ball to the background. For very small background balls, the particle is observed to almost always circularly orbit. The background does not have enough 'stopping power', so to speak, to change the direction of the large ball. As the size of the background increases, the background balls are large enough that they are able to 'kick' the large ball in other directions. In the case of Fig. 3.1(b), the size ratio is 4.4 and we see that the orbits are not well-developed, with the large ball traversing the center of the system often. For smaller background balls, the time-traces show very circular orbits with very few forays into the center of the system.

The mechanism by which the large ball propels is analogous to a previous experiment that established the size ratio of two balls as a parameter by which to one can progressively break the thermal analogy [3]. The two differently-sized balls will not obey equipartition of energy; they equilibrate to different effective temperatures. It was also observed that, occasionally, the two different-sized balls would receive a random kick that was strong enough to overcome mutual repulsion and bring them into close contact with one another. Unlike two balls of equal size in which short-range interactions are brief and quickly followed by an immediate repulsion, the two balls of very different size will stick together and then propel in the direction of the smaller ball. We believe the same mechanism is at work in the current case of a large ball fluidized in a background of smaller balls.

In this experiment, the large ball receives a kick from the airflow or collides with a background ball and begins to move in some direction. The background balls in front of the large ball become compressed whereas the region behind the large ball becomes dilute. This is because as will be quantified in Section 4.3 - the large ball moves much faster than the small balls and the background is unable to rearrange itself quickly enough to fill in the void behind the large ball as it propels through the system. The dilute wake behind the ball is easily visible in Fig. 3.1(a). The compressed region in front of the large ball behaves just like the smaller ball in the earlier two-ball experiment. The large ball moves in the direction of this compressed region and gains speed. Additionally, as the wake becomes larger, air is able to escape more easily through the
void. Thus, the air behind the ball is moving faster than the air in front of it, further propelling it forward. This cycle acts as a feedback mechanism allowing the large ball to move faster as the compressed region grows. Only when the compressed regions jams or the ball reaches the boundary does it change direction.

Considering this mechanism, the local density of the background balls and the subsequent force that they exert on the large ball will be dependent on the speed of the large ball. When the large ball is nearly stationary, the background should be isotropically distributed. As the large ball gains speed, both the dilute wake behind it and the compressed region in front of it should increase in size. To test the accuracy of this hypothesis, we take a 20-minute video in which both the background and large ball are visible.

We first track the large ball to obtain its velocity in each frame. Then, each frame is first shifted so that the large ball is positioned at the center and then rotated so that the large ball is always moving to the right along the horizontal axis. As such, the compressed regions in front of the ball, which are brighter due to having more reflective surfaces, will be on the right side of the image. Dilute regions, appearing darker, will be to the left. Since we suggest that the density is dependent on the large ball speed, we bin these centered-and-rotated images according to the speed of the large ball and then average over all images within each speed bin. The result, for a selection of large ball speeds, is shown in Fig. 3.2. The images have been color-coded by linearly interpolating between $0 \%$ area fraction, shown as black, and jammed particles at $84 \%$, shown as white.

The effect is very dramatic. In the top image, the large ball - outlined by a white circle - is moving very slowly and the compressed and dilute regions are relatively small. As we increase speed from top to bottom, we observe that both the compressed region in front of the ball and the dilute wake behind the ball become larger. The slight asymmetry in the image is due to the ball circulating in the counterclockwise direction more often than clockwise for the video analyzed.

To quantify the difference in density between the compressed and rarefied regions, we obtain the average packing fraction within a hemisphere of radius 9.4 cm both in front of, $\phi_{\text {ahead }}$, and behind, $\phi_{\text {behind }}$, the large ball. We then plot the difference between these packing fractions $\Delta \lambda=\phi_{\text {ahead }}-\phi_{\text {behind }}$ as a function of the large ball speed, as shown in Fig. 3.3. We see that the relative size of the dilute and compressed regions, and thus the local density and interaction


Figure 3.2: Time-averaged density of the background for different values of the large ball speed. The large ball (outlined in white), radius 1.9 cm , is always moving horizontally to the right. The top plot is for the slowest large ball speeds, with the large ball speed increasing from top to bottom. The density scale linearly interpolates between $0 \%$ area fraction (black) and a jammed region at $84 \%$ (white).


Figure 3.3: The difference in packing fraction between compressed regions in front of and rarefied regions behind a large ball fluidized in a background of smaller balls - as seen in Fig. 3.2, as a function of the speed of the large ball.
of the large ball with the background, is dependent on the speed of the large ball.

### 3.3 Single Ball: Time-Independent Statistics

Because our system reaches a steady state very quickly, the simplest way in which to characterize it is by compiling statistics of time-independent quantities. Intuitively, we thought that the presence of the turbulent background would serve to overwhelm interaction of the large ball with the bounding walls. In other words, the large ball would behave as a Brownian particle but would sample all of the system space. In this case, the radial probability distribution $P(r)$ would be linear from the origin $r=0$ to the reduced radius of the system $R^{\prime}=R-a=12.4 \mathrm{~cm}$, at which point $P(r)$ would discontinuously vanish.

The observed $P(r)$ for our system is shown in Fig. 3.4. We see that, at small $r$ values close to the center of the sieve, $P(r)$ is linear, suggesting that, far from the walls, there is no external potential. However, there is a large peak at an intermediate radius as the large ball begins to detect the wall, showing that the large ball is repelled from the wall. This peak radius roughly corresponds to the radial position at which the particle prefers to orbit circularly.


Figure 3.4: Radial probability distribution for a single large sphere fluidized in a bidisperse background.

For a thermal particle obeying equipartition of energy, we would expect the velocity distribution $P(\mathrm{v})$ to be Gaussian, symmetric about zero, and that each velocity component would be equivalent. The compiled $P(\mathrm{v})$ for our system is shown in Fig. 3.5. Here, we have decomposed the velocity into radial and polar components to emphasize the circular motion observed for this system. The radial velocity distribution is roughly Gaussian, although having shorter tails. However, the polar velocity exhibits two peaks, consistent with circular behavior. The peaks are symmetric about the origin, showing that the particle does not preferentially orbit in any particular direction.

### 3.4 Single Ball: Dynamics

As discussed at the beginning of Section 3.2, the large ball propels itself ballistically across the system. This is directly observable in the mean-square displacement (MSD) for the large ball, shown in Fig. 3.6(a). Here, at short times, we see ballistic motion ( $\propto \tau^{2}$ ) characterized by a root-mean-square speed of $\sim 4 \mathrm{~cm} / \mathrm{s}$. If our system was infinite in size, we believe the particle would


Figure 3.5: Radial (solid) and polar (dashed) velocity distributions for a single large sphere fluidized in a bidisperse background. The dotted line is a fit of $P\left(v_{r}\right)$ to a Gaussian.
continue to move in a more-or-less straight line rather than in circular motion. Random kicks from the background balls would serve to change the direction of the large ball and, at some long time scale, we would expect to see diffusive motion $(\propto \tau)$ consistent with such random walks. However, the system is too small for us to see any indication of a crossover to diffusive behavior. The MSD saturates within about 2 seconds at the optimum orbital radius, followed by oscillations about this value.

To get a sense of the characteristic timescales in our system, we consider the velocity and acceleration autocorrelation functions as shown in Figs. 3.6(b) and (c). The velocity autocorrelation has a long plateau that rolls off after approximately 1 s and then oscillates as the function decays to zero. This timescale corresponds roughly to the relaxation time of the large ball $a / v_{r m s} \sim$ 0.95 s . The strong oscillations are indicative of the circular motion exhibited by the ball. The acceleration autocorrelation decorrelates much more quickly than the velocity does, having one small oscillation at 0.08 s , before decaying to zero at approximately 0.3 s . The small oscillation is most likely due to the background in front of the large ball compressing and then exerting a force on the large ball as it propels forward.

The other remaining timescale that is important is related to the behavior of the background


Figure 3.6: Dynamics of a large ball fluidized in a bidisperse background. a) Mean squared displacement. The line with slope 2 indicates ballistic behavior. The horizontal dashed line indicates the reduced system radius squared $(R-a)^{2}$. b) Velocity autocorrelation function. c) Acceleration autocorrelation function.


Figure 3.7: Velocity and speed (inset) distribution of the bidisperse background balls.
balls. In Fig. 3.7, we plot the velocity and speed (inset) distributions for the background balls. The distribution is very peaked, showing that the background is indeed not thermal. However, except for the peak at zero velocity, the distribution does not significantly differ from a Gaussian and we continue to approximate the background as "thermal". From the speed distribution, we obtain a root-mean-square speed of the background of $1.18 \mathrm{~cm} / \mathrm{s}$. Thus, we see that the large ball has a root-mean-square speed that is roughly 4 times the speed of the background balls.

### 3.5 Background-Mediated Forces on a Single Ball

In this section, we fully characterize the forces acting on a single large ball fluidized in the presence of a "thermal background" of smaller particles. We begin with the Langevin formalism used for a single ball fluidized in an empty sieve with the addition of a term $\vec{F}_{\text {background }}$ that will characterize the force on the large ball due to the background. Using the effective mass for a hollow spherical shell $m_{e f f}=(5 / 3) m$, the Langevin equation for the translational motion of
the large ball can be written as:

$$
\begin{equation*}
m_{e f f} \vec{a}=-\vec{\nabla} V(r)+\int_{-\infty}^{t} d t^{\prime} \Gamma\left(t-t^{\prime}\right) \vec{v}\left(t^{\prime}\right)+\vec{F}_{\text {background }}+\vec{\zeta}(t) \tag{3.1}
\end{equation*}
$$

The form of $P(r)$ in Fig. 3.4 shows that there is a position-dependent force due to interactions with the walls. With that consideration and the circular symmetry of the apparatus, we include a radially-dependent force due to an external potential, $C(r)=-\vec{\nabla} V(r)$. Similarly, we have frictional interaction with the substrate so we include a drag term characterized by a memory kernel $\Gamma\left(t-t^{\prime}\right)$. Lastly, interactions with the turbulent airflow as well as particle collisions are included via a stochastic force $\vec{\zeta}(t)$ with zero mean, assumed to be independent of position and velocity.

The force of the background balls acting on the large ball $\vec{F}_{\text {background }}$ will be dependent on the local configuration of the background balls. We saw in Section 3.2 that the local density of the background could be determined by the speed of the large ball. As such, we can assume that $\vec{F}_{\text {background }}$ is a function only of the large ball speed. Further, we showed in Section 4.3 that the velocity changes slowly compared to the force. As such, we can write the memory term as simply a velocity-dependent linear drag $-\gamma v$. We can then combine the background force and drag as a single speed-dependent term $D(v) \hat{v}$. Thus, we can write the Langevin equation as the sum of a radial force, a speed-dependent force, and a stochastic force:

$$
\begin{equation*}
m_{e f f} \vec{a}=-\vec{\nabla} V(r)+D(v) \hat{v}+\vec{\zeta}(t) \tag{3.2}
\end{equation*}
$$

In order to identify the forces acting on the large ball, we pursue a dynamical approach. First, we isolate the radial force by taking the cross-product of Eq. 3.2 and $\hat{v}$, the instantaneous direction of the large ball velocity. This results in

$$
\begin{equation*}
C(r)-\left[\frac{\zeta \times \hat{v}}{\hat{r} \times \hat{v}}\right]=\left(\frac{\vec{a} \times \hat{v}}{\hat{r} \times \hat{v}}\right) m_{e f f} \tag{3.3}
\end{equation*}
$$

We eliminate the term in brackets by averaging over time for each value of position. The resulting central force is shown in Fig. 3.8. We see that there is essentially zero force at small radii, consistent with the observation of a linear $P(r)$ suggesting a "free" particle. At increasing radius as the ball approaches the bounding walls, the force becomes repulsive. The data suggest a slight attraction very close to the wall consistent with what has been seen for single balls in


Figure 3.8: Radially-dependent central force as calculated from Eq. 3.3. The solid line is a fit to $C_{f i t}(r)=A r^{3} /\left(r-R^{\prime}\right)^{2}$ where $R^{\prime}=R-a=12.4 \mathrm{~cm}$ is the reduced radius of the system. The dashed line is the radial force obtained via a thermal approximation according to $C^{\underline{t h}}(r)=-\frac{d}{d r} k T_{\text {eff }} \log (-P(r) / r)$.
an empty sieve [59]. The solid line in the figure is a fit to $C(r)=A r^{3} /\left(r-R^{\prime}\right)^{2}$, where $R^{\prime}=R-a=12.4 \mathrm{~cm}$ is the reduced radius of the system, the largest radius that the large ball center-of-mass can reach. As a comparison, the dashed curve is the radial force obtained if we had approximated the system as thermal and calculated the radial force using the data for $P(r)$, according to $C(r)=-\frac{d}{d r} k T_{\text {eff }} \log (-P(r) / r)$.

The next term we characterize is the speed-dependent force $D(v)$. We isolate this term by taking the cross-product of Eq. 3.2 and $\hat{r}$ :

$$
\begin{equation*}
D(v)-\left[\frac{\zeta \times \hat{r}}{\hat{v} \times \hat{r}}\right]=\left(\frac{\vec{a} \times \hat{r}}{\hat{v} \times \hat{r}}\right) m_{e f f} . \tag{3.4}
\end{equation*}
$$

Again, we average over time for each speed $v$ to eliminate the term in brackets. The calculated force is shown in Fig. 3.9. For speeds between $0 \mathrm{~cm} / \mathrm{s}$ and roughly $10 \mathrm{~cm} / \mathrm{s}$, the speed-dependent force is positive, causing the large ball to increase its speed. For speeds larger than about 10 $\mathrm{cm} / \mathrm{s}$, the force is negative thus slowing the particle. This is consistent with our observations of the system. Once the ball receives a kick and begins to move, it compresses a region of the background in front of it. This creates a feedback mechanism that causes the particle to


Figure 3.9: Speed-dependent force as calculated from Eq. 3.4. The solid curve is a trendline to guide the eye.
continue gaining speed. However, if the large ball moves too quickly, the compressed region becomes jammed and is able to slow the large ball down. The polar velocity distributions in Fig. 3.5 suggest that there is a stable polar speed of approximately $10 \mathrm{~cm} / \mathrm{s}$ at which the particle orbits the system in circular motion. This is consistent with the speed at which $D(v)$ crosses the horizontal axis - the stable speed at which there is no speed-dependent force.

Lastly, we isolate the stochastic force. Since the behavior of the background in the direction of the large ball's motion is very different from that perpendicular to it, we characterize the stochastic force with respect to the direction of the large ball motion. To do so, we subtract off the central force $C(r)$ and take the dot- and cross-product of the resultant equation with $\hat{v}$ :

$$
\begin{gather*}
{\left[m_{e f f} \vec{a}-C(r) \hat{r}\right] \cdot \hat{v}-D(v)=\zeta_{\|}(t)}  \tag{3.5}\\
{\left[m_{e f f} \vec{a}-C(r) \hat{r}\right] \times \hat{v}=\zeta_{\perp}(t) .} \tag{3.6}
\end{gather*}
$$

We then compile probability distributions for $\zeta_{\|}(t)$ and $\zeta_{\perp}(t)$, as shown in Fig. 3.10 for the particular speed of $4.5 \mathrm{~cm} / \mathrm{s}$. The perpendicular stochastic force, shown as the dashed curved, is roughly Gaussian with somewhat long tails. By contrast, the distribution of the parallel stochastic force is significantly skewed, having much larger probability to provide a kick in the direction


Figure 3.10: Probability distribution of the stochastic force parallel to (solid) and perpendicular to (dashed) the large ball velocity, for speed of $4.5 \mathrm{~cm} / \mathrm{s}$, as defined by Eqs. 3.5 and 3.6 respectively. The dotted curve is a fit of $P\left(\zeta_{\|}\right)$to a Gaussian.
of motion. This shows that our assumption that the stochastic force is independent of the speed was incorrect. The rough shape of these distributions does not significantly change, though, depending on the magnitude of the large ball speed. This can be seen in Fig. 3.11 where we plot the standard deviations $\sigma$ of the probability distributions $P\left(\zeta_{\|}(t)\right)$ and $P\left(\zeta_{\perp}(t)\right)$ as a function of the large ball speed. Both distributions broaden and then remain roughly constant width until 8 $\mathrm{cm} / \mathrm{s}$, roughly the stable speed for the large ball, at which point the distributions narrow.

This directional dependence of the stochastic force has been seen before. When we characterized the behavior of fluidized rods in Chapter 1, we modeled the autocorrelation of the stochastic force as a power law rather than a decreasing exponential in order to encode self-propulsion in the stochastic force as an extended memory effect. Using this formalism, the power law exponent was different depending on the direction the particle moved with respect to its long axis. Since we have effective self-propelling in the instant experiment, it is not surprising that the stochastic force is directionally-dependent.


Figure 3.11: The standard deviation of the stochastic force probability distributions parallel to (solid) and perpendicular to (dashed) the large ball velocity, as a function of the large ball speed.

### 3.6 Two Balls: Individual Behavior

In this section, we investigate the interaction of two large balls when fluidized in the presence of "thermal" background of smaller balls. When fluidized in an empty sieve, the force between two balls is characterized by a strong short-range hardcore repulsion and a persistent repulsive force over all separation distances [59]. With the inclusion of a background, we might expect a short-range depletion interaction that would attract the two balls together as well as a long-range Casimir-type force arising in order to reduce the turbulence between the two balls when they are separated by a large distance.

Contrary to our intuition, the observed behavior suggests a strong intermediate-range attraction. Both particles propel ballistically and orbit the system in circular motion. Often, the two large balls will become trapped in one another's wake and travel as a pair for long periods of time. However, the two balls rarely come into contact with one another. When cooperatively traveling together, the two balls are always some small distance apart. Interestingly, if we decrease the size of the background balls, we get a distinct long-range interaction. The two balls will often circulate around the system diametrically opposite one another. However, for the subsequent


Figure 3.12: Radial probability distributions for one (dashed) and two (solid) large balls fluidized in a background of smaller balls.
analysis, we consider size ratio of large ball to background of 4.4.
We begin by characterizing the behavior of the individual balls in the same way that we did for a single fluidized ball. The radial probability distribution is shown in Fig. 3.12; the radial and polar velocity distributions are shown in Fig. 3.13. We see that the statistical probabilities are essentially unchanged when a second large ball is added to the system. The $P(r)$ for both systems are essentially identical with only the peak position slightly shifted to large radius for the case of two fluidized large balls. The two-ball velocity distributions both appear to be slightly broader and less peaked than their single ball counterparts. However, the peak orbital speed of the polar velocity is identical.

The dynamics of the two balls are also essentially identical. However, the velocity crosscorrelation $\left\langle v_{1}(t) \cdot v_{2}(0)\right\rangle$, shown in Fig. 3.14, emphasizes how strongly correlated the motion of the large balls is. For a thermal system in which statistical mechanics is applicable, the velocity cross-correlation must vanish as delay time goes to zero. Here, there is a finite non-zero value of the cross-correlated velocity, approximately $-0.25 \mathrm{~cm} / \mathrm{s}$. This speed is only about $6 \%$ of the root-mean-square speed and suggests that statistical mechanics may still be applicable to the two-ball situation. The strong oscillations at longer time, which slowly decay to zero at very long times, are indicative of the cooperative circling often seen.


Figure 3.13: a) Radial velocity distributions and b) polar velocity distributions for one (dashed) and two (solid) large balls fluidized in a background of smaller balls.


Figure 3.14: Velocity cross-correlation for two large balls fluidized in a background of smaller balls.

### 3.7 Two Balls: Interaction

The first step in deducing the interaction between the two large balls is to compile the probability of their separation distance $P\left(\left|r_{1}-r_{2}\right|\right)=P(\rho)$. This is shown as the solid black line in Fig. 3.15. The distribution is zero for separations less than a particle diameter 3.8 cm , as expected for volume exclusion in hard spheres. At small separations, the probability reaches a maximum showing that the balls prefer strongly to stay near one another. This immediately suggests that the nature of the interaction between the two balls has dramatically changed and become attractive. However, we note that depletion interactions act on a length scale of the size of the small balls $\sim 0.87 \mathrm{~cm}$. The broadness of the peak indicates that the interaction is over a length scale of approximately 5 cm and cannot be attributed to depletion interactions. In fact, the absence of a strong peak within one small-ball diameter from the large ball diameter suggests that depletion interactions play no role here.

To further stress the contribution due to the background, we run a Monte Carlo simulation that is consistent with the interactions felt by a single large ball fluidized in a background. That is, the simulation randomly chooses two sets of position data such that the $P(r)$ obtained from the simulation for each data set is identical to that of one ball, as shown in Fig. 3.4. As such, any difference between our data and the simulation is due to interactions mediated by the background.

The simulation result is shown as a dashed red line in Fig. 3.15. The small-separation peak does not appear in the simulation and is entirely attributable to the background. The tails of the distribution match fairly well, suggesting that there is no long-range Casimir-type interaction for this size ratio.

The shape of this distribution is fairly robust for most values of the size ratio of the large balls to the background. However, for very small background balls, a second peak is observed to arise in $P(s)$ at large separation distances. As mentioned in Section 3.6, this second peak is due to the particles orbiting circularly in tandem, diametrically across the system from one another.

To emphasize that the ball-ball interaction is mediated by the background, we took a data set in which one of the large balls was stationary and placed at the center of the system. We accomplished this by drilling a hole in the hollow ball and placing a steel ball bearing inside. The ball was then held in place by a magnet placed underneath the sieve. Since all the spheres are plastic and the magnet cross-section was much smaller than the large ball, its presence did not affect the ball behavior or airflow. We then computed the separation probability between the freely-moving large ball and the stationary one. The result was nearly identical to the singleball $P(r)$ except that it was zero for separations less than one ball diameter. This proved that the strong attraction of the two balls was entirely mediated by the background and strongly dependent on the rarefaction and compression zones that arise due to the large ball motion. Since the background configuration is dependent on the large ball speed, any interaction force between the two large balls must also be dependent on the speeds of both large balls.

Because our system is clearly non-thermal, we are loath to apply statistical mechanics in order to back out an interaction force from the separation probability. Rather, here, we seek to phenomenologically model what the interaction force might be and then compile the separation probability that would be obtained if that force were correct. Here, we must assume that the position probability density in polar coordinates is of the form

$$
\begin{equation*}
P\left(r_{1}, \rho, \theta\right) \propto r_{1} \rho \exp \left(-\frac{V\left(r_{1}\right)+v\left(r_{2}\right)+V(\rho)}{T_{e f f}}\right) \tag{3.7}
\end{equation*}
$$

where $r_{2}=\sqrt{r_{1}^{2}+\rho^{2}+2 r_{1} \rho \cos (\theta)}$, and that the interaction potential results from a force, according to:

$$
\begin{equation*}
V(\rho)=-\int_{0}^{\rho} F\left(\rho^{\prime}\right) \tag{3.8}
\end{equation*}
$$



Figure 3.15: Probability of the separation of two large balls fluidized in the presence of a background (solid black). The dashed red line is the result of a Monte Carlo simulation consistent with the interactions felt by a single fluidized large ball, characterized by $\mathrm{P}(\mathrm{r})$ in Fig. 3.4.

Then, we can obtain the separation probability distribution by numerically integrating $P\left(r_{1}, \rho, \theta\right)$ over both $r_{1}$ and $\theta$.

We have proposed four different trial interactions forces, shown in Fig. 3.16(b). The first is simple hard-core repulsion $F_{H C}(\rho)$, shown as small dashes. The second is hard-core repulsion with the depletion interaction $F_{d e p}(\rho)$ put forward by Melby et al. [50], shown as dash-dot. Third, we consider a long-range exponentially decaying attractive force $F_{\text {long }} \propto \exp \left(-(\rho-2 a) / \rho_{0}\right)$, shown as long dashes. Lastly, we consider an attractive force that is peaked at some intermediate separation $F_{\text {intermediate }} \propto(\rho-2 a)^{\alpha} \exp \left(-(\rho-2 a) / \rho_{0}\right)$, shown as a solid line.

The results of numerically integrating Eq. 3.7 are shown in Fig. 3.16(a). The actual data from Fig. 3.15 is reproduced as gray squares. As expected, both a hard-core repulsion (small dash) and depletion interaction (dash-dot) cannot accurately describe the data. The peak in the separation probability is too broad and extends over too large a length-scale for a depletion attraction. The long-range exponentially decaying force (long dash) adequately fits the tail of the separation probability distribution but cannot capture the shorter-range attraction. The peaked attractive force (solid) however, nearly perfectly matches the data, confirming that the the interaction mediated by the background is neither short- nor long-ranged but intermediate-ranged and


Figure 3.16: a) Separation probability distribution for trial interaction forces shown in b). Actual data, also shown in Fig. 3.15, are gray squares. b) Trial interaction forces to model interaction between two large balls fluidized in a background of smaller balls: hard-core repulsion (small dash); hard-core repulsion with depletion attraction (dash-dot); exponentially-decaying longrange force (long dash); attractive intermediate-range force (solid). The form of the forces is explicitly given in the text.
attractive over all length scales.

### 3.8 Conclusion

We have characterized the behavior of and forces acting on a large sphere fluidized in a bidisperse background of smaller balls. The large ball self-propels ballistically through the medium, strongly perturbing the local density of the background. This propelling behavior is reminiscent of behavior observed for two different-sized spheres fluidized simultaneously in an empty sieve. Partly due to the small system size and circular boundaries, the large ball typically orbits the system at a stable radius approximately one ball diameter from the wall.

The central harmonic potential due to the bounding walls is significantly reduced by the presence of the background balls. The presence of the background not only modifies the forces felt by a fluidized ball moving in an empty sieve but also mediates a novel speed-dependent force. The speed-dependent force acts to keep the large ball propelling itself at a stable speed of approximately $10 \mathrm{~cm} / \mathrm{s}$. Furthermore, the stochastic force is directionally-dependent and preferentially kicks the large ball in its direction of motion.

When two large balls are fluidized simultaneously, the background completely eliminates air-mediated repulsive ball-ball interactions that were present when they were fluidized in an empty sieve. Rather, a trial interaction force suggests that the ball-ball interaction is attractive over all length scales and is greatest at an intermediate length scale.

## Chapter 4

## Controlling the Relaxation Time in an Air-Fluidized Monolayer on Approach

## to Jamming

In this chapter, we extend previous experiments [4] in which the structure and dynamics of a monolayer of air-fluidized grains was studied on approach to the jamming transition. In those experiments, point $\mathbf{J}$ [46] was approached by increasing the density at fixed airflow or by decreasing the airflow at fixed density. As such, packing fraction and effective temperature were well-defined for each experiment. However, there was no access to information about the other possible control parameter: the pressure felt by the particles in the system. Without information about the presssure, we cannot determine the appropriate control parameters for the dynamics in our system. In the current experiment, we modify the apparatus so that packing fraction, temperature, pressure, and relaxation time may be measured simultaneously. In Section 4.1, we present the system and apparatus that we study. Pressure is obtained by tilting the entire apparatus, introducing a component of gravity in the plane of the monolayer. We discuss potential pitfalls of this approach in Section 4.2. We examine the particle dynamics and measure the relaxation times in Section 4.3 before calculating the dependence on depth of various quantities in Section 4.4. We obtain an equation of state and compare it to simulation and theory in Section 4.5. Lastly in Section 4.6, we collapse our relaxation time data by defining a time- and energy-scale using the
measured pressure.

### 4.1 Experimental Details

The principal system under investigation, shown in Fig. 4.1, is a monolayer of a bidisperse mixture of steel ball bearings - with radii $\sigma / 2=r_{\text {small }}=0.1984 \mathrm{~cm}$ and $1.4(\sigma / 2)=r_{b i g}=$ 0.2778 cm and masses $m_{\text {small }}=0.255 \mathrm{~g}$ and $m_{\text {big }}=0.7 \mathrm{~g}$, respectively - fluidized by an upflow of air. For most of our data analysis, the system is initially filled so that grains occupy $67 \%$ area fraction, corresponding to 398 of each bead size. This particular value is chosen so that the grains are uniformly distributed across the cell when it is flat. Ball motion is stochastically driven by turbulent air flowing at $700 \mathrm{~cm} / \mathrm{s}$, spatially and temporally homogeneous within $\pm 10 \mathrm{~cm} / \mathrm{s}$ and 0.5 s , as measured by a hot-wire anemometer. The airflow is low enough that the balls maintain contact with the sieve at all times and move by rolling without slipping. We define an effective mass $m_{\text {eff }}=m+I / r^{2}$ where $I$ is the moment of inertia for a solid sphere rolling about its center-of-mass.

The apparatus, fluidization method, and lighting setup are identical to those of Ref. [59], detailed in Appendix A. The apparatus is a rectangular windbox, $1.5 \times 1.5 \times 4 \mathrm{ft}^{3}$, positioned upright. A blower attached to the windbox base provides vertical airflow perpendicular to a circular brass testing sieve with mesh size $150 \mu \mathrm{~m}$ and radius 15.3 cm that rests horizontally on top. The particles are placed in a square insert, sides with length 14.9 cm , that is oriented such that two sides of the insert lie parallel to the tilt axis of the apparatus. We approach the jamming transition by tilting the entire apparatus through a range of angles $\theta$ from 0 to 0.9 degrees by placing spacers underneath one side of the apparatus. The particles thus feel a net force down the plane $m g \sin (\theta)$, where $g$ is the acceleration due to gravity $9.8 \mathrm{~cm} / \mathrm{s}^{2}$. Thus, the packing fraction increases as a function of depth $z$ down the plane and, for sufficiently high tilt angles and initial filling fractions, we approach jamming at the bottom of the tilted monolayer. The top of the system is liquid-like with particles moving rapidly. The dynamics slow down greatly with increasing depth. The particles at the bottom of the plane are nearly static with only occasional rearrangements. It is truly like a phase diagram on a plate.


Figure 4.1: A tilted monolayer of bidisperse steel beads fluidized by an upflow of air. The tilt angle for this image is 0.72 degrees. The red horizontal bar is an example of the typical strip size into which we divide the system in order to calculate depth-dependent quantities. We only show the region of our system for which we analyze data.

The particles are illuminated from above by six 100 W incandescent bulbs arranged in a 1foot diameter ring positioned 3 feet above the sieve. A digital CCD camera placed at the center of this ring captures the raw video data, typically for 20 minutes at a time at 120 frames per second. The steel beads are highly reflective; light is reflected from the tops of the balls so that they appear as small dots in the video data. The dots for the two sizes of balls are different enough in size that big and small balls are easily distinguished. Post-processing of the video data is accomplished using LabVIEW, using the same tracking programs of previous chapters as detailed in Appendix A. The resulting position data $\{x(t), y(t)\}$ for each of the 796 beads is smoothed using a running average, yielding estimates of the error in the position data of $18 \mu \mathrm{~m}$.

### 4.2 Segregation

Care must be taken that the square insert is aligned with the tilt axis and the system is level parallel to the tilt axis. If either of these are slightly off, strong convection currents arise and dominate the dynamics of the particles. We are restricted in our range of tilt angles because, as we tilt the apparatus more, the small and large beads begin to segregate strongly. In typically less than a minute, at angles greater than roughly 0.9 degrees, most of the small balls move to the top of the system whereas the large balls sink to the bottom. Because the small balls are less massive, they behave as a liquid at the top; the large balls begin to form nearly static crystalline domains at the bottom

To determine that our system is in a steady state and that segregation effects are not influencing the behavior of the system, we define an order parameter to quantify how bidisperse the system is as a function of depth and time. The quantity $m=N_{b} /\left(N_{b}+N_{s}\right)$, where $N_{b}$ is the number of big beads and $N_{s}$ is the number of small beads, will have a value of 1 if only big beads are present, 0.5 if the system is bidisperse, and 0 if only small beads are present. We graphically show this in Fig. 4.2 as a spacetime plot for a tilt angle of 0.18 degrees, where the grayscale value corresponds to the value of $m$. White corresponds to only small beads present, black only big beads, with grayscale interpolated between these two extremal values. Since a depth of zero corresponds to the top of the system, we can verify that the system remains bidisperse at all depths over the entire course of the experiment.


Figure 4.2: Spacetime plot of how bidisperse the system is as a function of depth and time. White regions are where only small beads are present; black areas are only big beads. The grayscale values are interpolated between these two extremal values.

Similarly, if convection cells arise, they are easily detectable by looking at a parametric plot of the particle's $x$ and $y$ position. When strong convection currents are present, this time-trace appears as a large orbit that the particle continues to re-trace over time. In the absence of such flows, the time-traces appear as random walks with the particle sampling the system evenly. We have verified that convection does not play a role in the data analyzed in later sections.

### 4.3 Depth-Dependent Dynamics

Once we have the position of each particle versus time, we are able to compute various quantities as a function of depth $z$. Because we have an additional force acting down the plane, we only use position data for horizontal motion across the plane to calculate temperature and dynamics. We first divide the system into strips, typically of width two large particle diameters, and then we break the time-traces of each particle into smaller segments according to which strip the particle is in at a given time. An example of a strip is shown in red in the middle of the image in Fig. 4.1. We also eliminate traces in which a grain is within 5-6 particle diameters from the bounding walls parallel to the tilt axis into order to eliminate wall and convection effects. Typically, a particle will meander down the plane and, upon reaching the bottom, will be swept up the sides


Figure 4.3: Mean squared displacement for a) big beads and b) small beads as a function of packing fraction down the plane of a tilted monolayer of air-fluidized beads. Here, the tilt angle is 0.18 degrees. Each curve is taken at a different depth down the plane as shown in the legend. The dashed horizontal lines are the diameter-squared for each bead species from which the relaxation time is obtained.
of the system and re-emerge at the top. Similarly, there appears to be an increased repulsion from the upper corners of the system causing the particle interface to curve away from the wall.

Once we have the position versus time data binned according to depth, the simplest quantity we can calculate to characterize dynamics is the mean squared displacement (MSD) $\langle[x(t+$ $\left.\tau)-x(t)]^{2}\right\rangle$ as a function of delay time $\tau$. Again, here, we use only position data for horizontal motion across the monolayer. Since we have binned the particle position data according to their depth, we can use the segmented time-traces to calculate the MSDs as a function of depth. For tilt angle of $\theta=0.18$ degrees, we show MSDs as a function of depth for big and small beads in Fig. 4.3(a) and (b), respectively. Each curve is time-averaged over all particle trace segments within the strip at the given depth. The ragged ends of the individual curves are because the range of timescales that we can achieve depends on whether or not we find any particles that remain in a given strip for a certain amount of time. Taking longer data sets may improve our
range of measurable timescales. In the interest of getting better relaxation time data - defined as the time it takes for a particle to move its own diameter, shown as horizontal dashed lines in Fig. 4.3 - longer data runs would increase the chance that we track a particle that stays in a given strip long enough to move its own diameter horizontally. Additionally, towards the bottom of the monolayer for large tilt angles, the mean squared displacement reaches a roughly constant plateau and does not show diffusive behavior at all during the entire data run. A longer data set might enable us to measure these diverging relaxation times, but there is a tradeoff given the amount of time it takes to process even a 20 minute long video. We believe the data is sufficient for our purposes.

Even at this very shallow angle, the data in Fig. 4.3 shows all of the characteristics associated with dynamics on approach to jamming. At the shortest time scales, we observe ballistic motion $\left(\propto \tau^{2}\right)$. At long time scales, we observe diffusive motion $(\propto \tau)$. There is just this single ballistic-to-diffusive crossover timescale for the shallow depth data. As we move down the plane, approaching the jamming transition as packing fraction increases toward random close packing, we see the progressive development of a subdiffusive plateau that is associated with 'cage' motion. That is, a particle and its neighbors are frozen in a given configuration and can only move out of this 'cage' if there is cooperative motion among the neighborhood. After this, the particles break out of the cage and enter a new configuration. At long enough timescales, these successive cage breakouts show diffusive behavior.

### 4.4 Depth-Dependent Pressure

Aside from the dynamics, we can also examine time-averaged quantities as a function of depth as shown in Fig. 4.4. New to this experiment, we can define a depth-dependent average pressure as

$$
\begin{equation*}
P(z)=\frac{m_{>}(z)}{L} g \sin (\theta) \tag{4.1}
\end{equation*}
$$

where $m_{>}(z)$ is the average total mass of all balls above depth $z$ and $L$ is the length of the strip, averaged over all times. We always ensure that there is a free interface at the top of the system where the pressure can be defined as zero. Since we know the number of particles in each strip at all times, we calculate the depth-dependent average area fraction $\phi$. These two quantities
are shown as functions of depth down the plane in Fig. 4.4. The pressure, Fig. 4.4(a), is an increasing function of depth. At lower depths, it increases nearly linearly. This is also where the packing fraction, Fig. 4.4(b), becomes nearly constant. This is consistent with the linear increase in pressure with depth in a fluid with constant density. Since each system has the same amount of mass, the pressure at each tilt angle systematically increases according to the value of $\sin \theta$. There is no observable trend with the packing fraction, except that all systems seem to approach a roughly constant density - the two-dimensional random close packing value of $84 \%$ - at the bottom of the monolayer. The plot of packing fraction versus depth shows how difficult it is to obtain low density data. When the tilt angle is increased, the beads do not distribute themselves across the entire free area of the system. Rather, there is typically an unfilled region at the top of the monolayer and then the packing fraction jumps precipitously at the interface.

By taking the derivative of the time-traces, we obtain the particle velocity and can quantify a depth-dependent effective temperature, defined as $k T=m_{e f f}\left\langle v^{2}\right\rangle / 2$. As observed in Ref. [4], the two bead species come to different temperatures with the large beads having a greater temperature. We define an effective temperature for the system by the taking the average of the small and large bead temperatures. We plot the average temperature versus depth in Fig. 4.4(c). Towards the top of the monolayer where the behavior is like a liquid, the temperature is roughly constant with depth. As depth increases, the temperature slowly decreases before vanishing rapidly as point J is approached at the bottom of the monolayer. Generally, the temperature systematically increases with increasing tilt angle. The effective temperature data at the higher tilt angles begins to show the effects of segregation, in which the small and large beads separate into distinct populations. Since the small beads move to the top, they would be more energetic than otherwise expected; the large beads at the bottom would be less so.

We can define a relaxation time $\tau_{r}$ as the time it takes for a particle to have moved one small-ball diameter $\sigma$, shown in both Fig. 4.3(a) and (b) as the horizontal dashed line. Thus, we can readily obtain relaxation time as a function of depth. For larger tilt angles, the MSDs at lower depths will often not reach the value of $\sigma^{2}$. For these, we fit the data to a constant plus a linear term in order to extract a rough estimate for the relaxation time. The relaxation time for different tilt angles for small balls is shown in Fig. 4.4(d). The relaxation time slowly increases with increasing depth before diverging over 5 orders of magnitude as we approach random close


Figure 4.4: a) Pressure, b) packing fraction, c) effective temperature, and d) relaxation time as a function of depth down the plane of a tilted monolayer of air-fluidized beads for different tilt angles $\theta$. Pressure is defined as $P(z)=\left(m_{>}(z) / L\right) g \sin (\theta)$ where $m_{>}(z) / L$ is the total mass per unit length above depth $z$ and $g$ is $9.8 \mathrm{~cm} / \mathrm{s}^{2}$. Relaxation time is defined as the time it takes a ball to move one small ball diameter.
packing. This behavior of $\tau_{r}$ is consistent with the expectation for a jamming system. As a system approaches jamming, its dynamics slow down and particles become locked into a given configuration. The closer to jamming one is, the longer it takes for the particles to rearrange. Thus, the relaxation time diverges as expected. Now that we have pressure, temperature, and packing fraction as functions of depth, we are able to plot these quantities against one another.

### 4.5 Equation of State

To ensure that our pressure data is healthy before we look at scaling the relaxation time data, we calculate the equation of state for our system and compare our results to simulation and free volume theory. This is shown in Fig. 4.5. We scale the equation of state data by temperature according to the ideal gas expectation, plotting $P \sigma^{2} / k T$ where the small ball diameter $\sigma$ is introduced to make the quantity dimensionless. As such, ideal gas behavior at very low densities would be linear with packing fraction. The solid line is a two-dimensional simulation of disks interacting via hard-core repulsion. Our data collapse very well and agree with the simulation for packing fractions from $20 \%$ to approximately $70 \%$. The data, however, diverge much more rapidly on approach to jamming than the simulation does. This may be because, as jamming is approached, it takes longer for the simulation to equilibrate and the values of temperature and pressure obtained for very high packing fraction may not be correct. We were unable to obtain much data at low packing fractions.

We can also fit to the theoretical expectation of free volume theory near the jamming transition $P=A \phi /\left(1-\left(\phi / \phi_{c}\right)^{2}\right)$ [38]. Although we do not expect this theory to work for all packing fractions since it does not allow for collective rearrangements, it is a useful benchmark for our data. There is a single fitting parameter, the proportionality constant, which equals 1 in our case. The functional form fits the data well near jamming. The collapse and agreement with simulation and theory further underscore how remarkably thermal this system is despite being driven far from equilibrium. This also gives us confidence that our pressure data is healthy. We can now move forward to determine which of these parameters - density, temperature, and pressure - control the diverging relaxation timescale on approach to jamming for our system.


Figure 4.5: Equation of state for air-fluidized beads. The dashed vertical line denotes random close packing in two dimensions. The dashed black curve is a two-dimensional simulation of disks interacting via hard-core repulsion. The solid black curve is a fit to the free volume expectation.

### 4.6 Relaxation Time Scaling

When discussing activated dynamics in which particles must overcomes energy barriers in order to rearrange, we typically think in terms of temperature as the important parameter. The data for relaxation time versus temperature do not collapse, as seen in Fig. 4.7(a), demonstrating that relaxation time is a function of another control parameter in addition to temperature. To plot our relaxation time data, we make both axes dimensionless by defining an energy-scale $P \sigma^{2}$ and a time-scale $\sqrt{m / P}$ using the pressure. The result for our data is shown in Fig. 4.6. As a function of this combination of control parameters, the data collapse remarkably well. This suggests that, for our system, temperature and pressure are equally important in controlling the diverging relaxation timescale. There are no other parameters that can be tuned in our system so this also implies that the dimensionless timescale $\tau_{r} \sqrt{P / m}$ is a function of $T / P \sigma^{2}$ only.

We can deduce more information from our data by fitting it to some conventional forms used when considering activated or glassy dynamics. The Vogel-Fulcher form, shown as the red curve


Figure 4.6: Scaled relaxation time $\tau_{\text {rel }} \sqrt{m / P}$ versus scaled temperature $T / P \sigma^{2}$ for a tilted monolayer of air-fluidized grains.
in Fig. 4.6, is a super-Arrhenius form

$$
\begin{equation*}
\tau \sqrt{P / m}=C \exp \left[A /\left(x-x_{0}\right]\right. \tag{4.2}
\end{equation*}
$$

where $x=T / P \sigma^{2}$. This form diverges at finite temperature. In the figure, we have fit to the low-temperature data, in which case this form fits the high-temperature data poorly. If we choose the fit the high-T data, then the Vogel-Fulcher form misses the divergence. In either case, the form does not fit our data well. Interestingly, our data indicates that the relaxation time diverges at $T / P \sigma^{2}=0$ rather than non-zero temperature. In fact, in the fit to the Vogel-Fulcher form at low T, the value of $x_{0}$ is zero. This indicates that there is no thermodynamic glass transition on the approach to point J , the hard sphere jamming limit as $T / P \sigma^{2} \rightarrow 0$. A second fitting form

$$
\begin{equation*}
\tau \sqrt{P / m}=C \exp \left[A^{2}\left(\frac{1}{x}-\frac{1}{x_{0}}\right)^{2}\right] \tag{4.3}
\end{equation*}
$$

where $x=T / P \sigma^{2}$, proposed by Elmatad et al. [26], shown as the green curve in Fig. 4.6, is nearly indistinguishable from the Vogel-Fulcher form.

Interestingly, the data is very well-described by a simple stretched exponential $\tau \sqrt{P / m}=$ $C \exp \left[A(1 / x)^{\beta}\right]$ with stretching exponent $\beta=1 / 3$, shown as the black curve in Fig. 4.6. We are uncertain whether or not the particular value of the exponent has physical meaning. What


Figure 4.7: a) Effective temperature and b) packing fraction plotted parametrically against pressure for several tilt angles $\theta$.
this form may indicate is that there is a range of activation energy barriers available for a bead to overcome and then rearrange. This may be because, at a given depth, the cage of nearest neighbors consists of some particles higher in the plane at lower density and higher temperature as well as some particles lower in the plane at higher density and lower temperature.

One important question to ask is whether simply varying the tilt angle is sufficient for the system to experience different conditions and whether the collapse seen in Fig. 4.5 and 4.6 is real. In other words, do beads experiencing pressure $P$ at one tilt angle have the same local packing fraction and temperatures as beads at $P$ at a different tilt angle. To demonstrate that the data do not, in fact, collapse except as a function of both temperature and pressure, we have plotted packing fraction and temperature parametrically versus pressure for the different tilt angles in Fig. 4.7. Taking vertical slices on these plots, we can demonstrate that beads at the same pressure experience completely different conditions simply by adjusting the tilt angle of the apparatus.

### 4.7 Conclusion

We have investigated the parameters that control the diverging relaxation time in a quasi-twodimensional monolayer of air-fluidized steel beads. By tilting the apparatus on its side, we are able to define a depth-dependent pressure. Similarly, by tracking the particles and binning their position time-traces by depth, we are able to measure temperature, density, and dynamics simultaneously with depth. By plotting pressure parametrically versus density, we obtain an equation of state. The equation of state diverges as the density approaches random close packing, as expected. The form agrees well with both molecular dynamics simulation and the free volume theory expectation near jamming. This reinforced the remarkably thermal character of the system.

We extract a relaxation time from the dynamics data as the time it takes a particle to move one small-ball diameter. We plot the relaxation time against the temperature as is typical for glass-forming systems where energy barriers play an important role. The data appears to diverge only at zero temperature. By defining a time- and energy-scale using the pressure, we are able to collapse the data for all tilt angle onto a single curve. The form of the data is not well-fit by conventional functional forms. Rather, a stretched exponential with stretching exponent $1 / 3$ fits the data very well. This suggests the presence of a range of energy barriers present in the system.

This experiment marks the first time that the three control parameters for the jamming transition have been measured simultaneously with the dynamics. The collapse of the relaxation time data suggests that, for the range of parameters measured in this experiment, temperature and pressure are equally important in setting the behavior of the relaxation time. Furthermore, the collapse is identical to that obtained recently for three-dimensional simulations [82] suggesting the universality of this finding.

## Chapter 5

## Conclusion

This thesis examined two concepts that are employed to unify a wide range of non-equilibrium systems - self-propelling and jamming - in the context of a monolayer of grains fluidized by an upflow of air.

The concept of self-propelling - a particle converting energy into motion in a spontaneouslychosen preferred direction - seeks to provide a simple, rules-based framework to model the collective flocking and swarming behavior seen in a wide range of biological systems. The framework should apply to purely physical systems so long as the particles self-propel and there is a mechanism by which the particles may order. Our first experiment in Chapter 1 calculated the single-particle dynamics of air-fluidized rods. We first showed that rods fluidized in this manner self-propel. Our analysis showed that self-propulsion can be phenomenologically thought of as an enhanced directional memory effect. Our subsequent experiment in Chapter 2 examined dense collections of self-propelling rods in pursuit of the fascinating collective phenomena predicted by theory. We do not observe spontaneous emergence of a broken-symmetry state or longrange order. We do observe propagating compression waves, however, that are not seen in a comparable system of spheres. We quantified the waves in two ways. First, we showed that fluctuations in the local number density are anomalously large. The mechanism at work here is the separation of timescales between the fluidizing airflow and particle motion. The particles are unable to rearrange quickly enough to fill rarefaction voids left behind the waves as they propagate. This coexistence of compressed and dilute regions gives rise to number fluctuations
that are proportional to roughly $N^{3 / 4}$ rather than $\sqrt{N}$. Secondly, we calculated the dynamic structure factor from which we obtained the dispersion relationship for the waves. We showed a weakly linear relationship of the wavespeed with increasing density. It has been suggested that our observations may be explained by the framework put forth by Baskaran et al. [2]. Our results, combined with vibrated bed experiments [54] and simulation [78], suggest the utility of the concept of self-propelling. We observe many of the predicted behaviors for collections of self-propelling particles - propagating waves and giant number fluctuations - in a system where fluctuations and noise are quite strong. The work of Vicsek et al. [78] showed that flocking only existed below some critical noise threshold but that other phenomena are robust to the noise level. We hope that more experiment will be conducted over a wide range of noise levels in the hope of extending self-propelling as a truly unifying concept.

Our third experiment in Chapter 3 sought to find an force analogous to the critical Casimir force in a fluid for a large sphere fluidized in the presence of a background sea of smaller spheres. Contrary to our intuition, the large sphere does not behave like a particle in a turbulent fluid but rather it self-propels across the system. The mechanism appears to be due to our method of driving. Interestingly, when a second large ball is added to the system, the interaction between the two balls is dramatically different than when they are fluidized in an empty bed. We showed that the background mediates a purely attractive, intermediate-ranged interaction force in stark contrast to the purely repulsive, short-ranged force between two balls fluidized in an empty sieve.

The concept of jamming - that a system is stuck in a given configuration effectively forever - has been used very successfully to unify a broad assortment of non-equilibrium systems. Our fourth experiment in Chapter 4 sought to determine the relevant parameters that control the diverging relaxation time as the jamming transition is approached. We approached the jamming system by tilting the apparatus on its side such that density and pressure increased and temperature decreased as depth down the plane increased. In this way, we were able to measure the control parameters - density, pressure, and temperature - simultaneously with the particle dynamics as a function of depth. We calculated an equation of state for our system of air-fluidized spheres which agreed well with simulation and free volume theory, further confirming the thermal nature of our system. Lastly, we plotted relaxation time versus temperature, defining a time- and energy-scale using the pressure in order to make the plot dimensionless. Plotting the quantities
in this manner resulted in excellent collapse for data at different tilt angles. The data appeared to diverge at zero temperature, suggesting that the jamming transition occurs only at point J. Conventional fitting forms such as the super-Arrhenius Vogel-Fulcher form were unable to fit the data whereas a stretched exponential provided a good fit. Thus, although we have demonstrated that temperature and pressure are equally important on approach to jamming, why these forms do not fit the data remains to be answered. We believe that our results here will provide further insight into the nature of the jamming transition.

The work of this thesis has demonstrated the utility of an air-fluidized granular system to study a broad range of fascinating phenomena. The similarities in observed phenomena between our system and a vast array of other non-equilibrium systems are remarkable. We believe that the results of these experiments provide further evidence for the use of unifying concepts - selfpropelling, a thermal analogy, and jamming - as common frameworks to study a broad range of driven, non-equilibrium systems.

## Appendix A

## Technical Appendix

This Appendix details some technical aspects of the experiments in more detail than they were explained in the text. In Section A.1, we detail the components of the apparatus and lighting fixture as well as how videos are saved. Because rods are more difficult to track than spheres, we outline several tricks we employ to resolve and track individual rods in Section A.2. Lastly in Section A.3, we show more explicitly several intermediate steps in the calculation of the dynamic structure factor used in Chapter 2.

## A. 1 Gas-Fluidized Bed

The gas-fluidized bed apparatus was built by an earlier graduate student, Rajesh Ojha. For details on its construction, gas-fluidization technology generally, and any other quantitative information such as pressure drop values, refer to his thesis. In this section, we will examine each component of the apparatus in somewhat more detail than in the text.

The apparatus itself is a $1.5 \times 1.5 \times 4 \mathrm{ft}^{3}$ box made of compressed wood having a hole cut out of its side near the base for air to enter and a second hole - diameter 12 " - cut out of the top for air to exit. We connect an air blower to the lower hole via a cloth sock and place brass testing sieves in the top, where we fluidize grains. The seams of the box are held together by epoxy so as to prevent any air from escaping. Inside of the box, foam air filters are positioned between perforated metal sheets. Because the air must make a 90 -degree turn as it enters the box, these filters are used to straighten the airflow by breaking up large-scale vortices or nonuniformities
that might arise. We also must ensure that the interior is debris and defect free as these can greatly affect the airflow and subsequent behavior of the fluidized particles.

The air blower is a two-speed carpet dryer that is controlled through a variac autotransformer. In this way, we control the voltage applied to the blower and thus the airspeed over a range from $0 \mathrm{~cm} / \mathrm{s}$ to roughly $1000 \mathrm{~cm} / \mathrm{s}$ with a resolution of $10 \mathrm{~cm} / \mathrm{s}$. All airspeed measurements are verified by a hot-wire anemometer that uses a thin wire and resistor to measure the airflow. The anemometer resolution is $10 \mathrm{~cm} / \mathrm{s}$ on a timescale of 0.5 s . Although possible, we do not calibrate the airspeed as a function of voltage and interpolate to get finer resolution. We use many different inserts - free-standing, so that air may flow around, and flush with the sieve, in order to increase airspeed - and the calibration is only accurate for a given cross-section of the apparatus.

The sieves are 12 "-diameter brass testing sieves purchased from McMaster-Carr, matching the size of the hole in the top of our apparatus. Typically, we choose a mesh size on order $100 \mu \mathrm{~m}$ so that the fluidized grains are not influenced by the regular array of holes. For example, when we fluidized very thin rods with a diameter of $1 / 64$ " using $100 \mu \mathrm{~m}$ mesh, we noticed that they would tend to self-propel exactly along the array of airholes in the mesh. Similarly, care must be taken to ensure that the sieve surface remains flat and that there are no indentations or scrapes. Even small dents or marks can significantly alter the airflow at that point and cause convection currents to arise.

As noted earlier, we have used several inserts in the course of our experiments. One type of insert is a hollow plastic tube, also purchased from McMaster-Carr, with some desired radius. These tubes are free-standing, meaning the particles are fluidized inside of the tube but there is still empty sieve outside of the tube for airflow to pass through. As such, we chose the tubes to have very thin walls, typically $3 / 8$ " thick. Thus, the airflow direction and magnitude are not appreciably altered at the insert boundary. The second type of insert is made from pressed wood and requires machining. These inserts have a circular outer boundary but have been cut to have circular, oval, hexagonal, and square inner boundaries. These inserts are thick and, when placed in the sieve, are flush against the sieve wall. Thus, they not only effectively reduce the system size but also the cross-sectional area through which the air can flow. This in turn causes the airspeed to increase greatly, essential to fluidize steel spheres. In order to avoid horizontal components of the air at the insert boundary, we place one above and one below the sieve mesh.

The final part of the apparatus is the base which includes a platform that is able to tilt and oscillate. The base itself sits on three adjustable feet which are used to level the base or tilt the entire apparatus up to approximately 2 degrees. For the experiments detailed in this thesis, we did not use the motor to oscillate the apparatus and will not detail it here. In order to tilt the apparatus in Chapter 4, we used aluminum spacers placed underneath one side of the base rather than adjust the feet.

Above the apparatus sits the lighting fixture on a scaffold with the camera. The setup is identical to that used by previous students and was optimized for use in visualizing and tracking spherical particles. Spheres, especially reflective ones, will reflect back light only from a small region at their top and, as such, appear as a small bright spot to the camera. Rods, on the other hand, are particularly difficult to visualize because they reflect light along their entire length. Further complicating the picture, the rods self-propel by tilting out-of-plane. Thus, one end of the rod is closer to the camera and appears brighter than the lower end. Thus, when thresholding the image, care must be taken to prevent the center-of-mass from shifting up the rod's length. Reflective rods are not ideal for just this reason as they tend to exacerbate this center-of-mass shift. As such, we use opaque white plastic rods. The primary difficulty with rods is that they reflect along their entire surface so that, when two of them collide, specular reflection off the interface cause the collision interface to be brighter than the rods themselves. This also occurs for opaque spheres but, because spheres are isotropic, we can use an image analysis technique known as an erosion to separate the two particles. This technique is not as useful for rods because it preferentially removes pixels from the ends of the rods rather than their long sides. As such, it is very difficult to discern two rods that have collided. The way in which we handle these instances is detailed in Section A.2.

The lighting consists of six incandescent $65-\mathrm{W}$ bulbs placed 3 feet above the surface of the sieve, arranged in a ring. At the center of the ring sits the camera. The camera is a Pulnix 6710, 120 frames per second, 8 -bit CCD camera. The settings for the camera are changed using "Measurement and Automation Explorer Window' in LABView. Typically, videos are taken in inverse logarithmic or binary so that the brighter particles are accentuated over the sieve in the background.

To capture the images and record video data, we write a series of images to buffer using
a LABView program written by Adam Abate. The video is saved as an AVI movie using a "lossless" codec "Microsoft RLE". The upside of this codec is that it is "lossless" and allows the computer to stream at the full frame rate of the camera. The downside is that it has poor compression and the output videos are typically dozens of gigabytes in size. As noted, we use a buffer to allocate system memory to store images to prevent frames from being dropped if the computer cannot process quickly enough. It is typically necessary to close all other open processes when recording a movie. Otherwise, the system will run out of memory and the output video will have glitches where it drops frames or speeds up.

## A. 2 Tracking Rods

Once we have the video data recorded, we must begin the process of tracking all of the particles over the duration of the data run. As a cautionary tale, we stress that it is most important that the video data itself is as clean as possible and that any visualization problems be solved by the optics rather than through post-processing. The first step is to locate every particle in each frame. For reflective and opaque spheres as in Chapter 4 and very dilute rods, this is exceptionally simple. We input the video into a program that does some preliminary image processing. Each raw frame is converted to grayscale and has a user-determined threshold applied to convert the image to binary. The LABView program "Particle Analysis" then recognizes any continuous grouping of non-zero pixels as a particle and can output most any necessary quantitative information about the particle, e.g. center-of-mass, orientation, or area. If there is concern that collided particles will not be discernable, we will introduce an erosion step to separate them. This is fairly straightforward for spheres since we can identify such composite particles by their total area, which will be twice that of a typical single particle.

For two rods that have collided, the process of resolving constituent particles is more difficult. We note that the following procedure only works for the case of two rods. If we identify three or more rods in a collision, we will drop all of them from that frame and recover them at a later frame. The process by which we allow for this dropping in and out is detailed later in this section. For the collision of exactly two long particles that cannot be separated via erosion, we are able to deduce their individual positions and orientations by using the parallel axis theorem and some


Figure A.1: The procedure for resolving the positions and orientations of two rods that have collided. Row A shows the initial composite particle (left image) as extracted from the image of the full system and the result (right image) of employing the parallel axis theorem to determine the individual particle centers-of-mass, shown as red crosses. Row B shows the process of applying the inner disk (light blue) and outer annulus (green) to the image, centered at the respective centers-of-mass, in order to determine the orientation of the constituent particles. Row C shows the result of the annulus procedure, with a red line indicating the orientation output by the procedure.
extra image processing steps. Once we have identified a composite pair of rods, we extract it from the larger image, e.g. Figure 1.1, as shown in the left-hand image in row A of Fig. A.1.

We assume that the centers-of-mass of the individual rods are collinear with the center-ofmass of the composite pair, lying along the pair's axis of lowest moment of inertia, $\hat{\theta}$. This assumption proves to be quite accurate except in the instance of rods colliding along their long edges. We determine the center-of-mass coordinates and $\hat{\theta}$ for the composite particle and then calculate the moment of inertia, $I_{C M}=\sum_{i} m_{i} r_{i}^{2}$, about the composite particle's center-ofmass, where the sum is over all pixels in the extracted image, $m_{i}$ is either 0 or 1 , and $r_{i}$ is measured with respect to the composite center-of-mass. We assume that the moment of inertia about the center of an individual rod is that of a cylinder: $I_{\text {rod }}=\frac{1}{12} m\left(3 r^{2}+l^{2}\right)$, where $r$ is the radius, $l$ the length, and $m$ the area of an individual rod. We then use the parallel axis
theorem to solve for the distance $d$ along $\hat{\theta}$ where the individual centers-of-mass are located: $d= \pm\left[\left(I_{\text {rod }}^{\text {parallel }}-I_{z z}\right) / A\right]^{1 / 2}$, where $A=\sum m_{i}$ is the area of the composite particle. Knowing $d$ and $\hat{\theta}$, we are able to obtain the coordinates of the constituent particles by moving a distance $\pm d$ from the composite center-of-mass along $\hat{\theta}$. The result of this procedure is shown as the right-hand image in row A of Fig. A.1.

Now that we know the center-of-mass position of the constituent particles, we need to know their orientation. To do so, we employ a graphical technique that takes the center-of-mass coordinates for one of the individual particles and overlays a black, filled circle - diameter half the length of the rod - centered at that position onto the image. This can be seen in row B of Fig. A.1; for ease of visualization, the circle is colored light blue. We then overlay a black annulus - diameter one rod length - shown as the outer green circle in each image of row B of Fig. A. 1 and then fill in the image outside of this outer annulus with black. The net result of applying the interior circle and exterior annulus is shown in row C of Fig. A.1. In this manner, the conjoined particle is split into either two or three subparticles. Two of these are the "endcaps" of the desired particle and the third is the residue of the other particle. Sometimes, as is the case in row C, the third subparticle does not appear. We then locate the centers-of-mass of these two or three subparticles. For three particles, we calculate the distances between each subparticle and choose the distance closest to the radius of the inner overlaid circle. Knowing which subparticles are separated by roughly the inner circle separation, we quickly compute the angle of a line drawn between their centers-of-mass. This is output as the orientation of that particle, shown as the red lines in row C of Fig. A.1. The process runs twice in order to find the orientation for both of the constituent particles.

We show a sampling of other types of collisions and the results of this procedure in Fig. A.2. Each row shows the journey a unique composite particle made as it was processed. The first column A shows the original extracted particle. Column B shows the result of the parallel axis theorem in determining the constituent centers-of-mass. Note that particles that collide together along their long side, an example is shown in the bottom row, are not accurately parsed. These collision types are the only ones that have difficulty with our method. Finally, in column C, we see the result of overlaying the circles on the plot and the determination of the particle orientations. Despite the difficulty of the method for the particle in the third row, the output orientations
and centers-of-mass are fairly close to their actual values.
Now we are armed with the centers-of-mass and orientations for nearly all of the particles in every frame. The next step is to link up these positions to form time-traces for individual rods over the course of the entire data run, a process we term 'unwinding'. As noted earlier, if three or more particles collide, we drop them entirely from the frame. It is here, in the unwinding process, that we accommodate the dropping in and out of particles. The most versatile of our unwinding programs allows for particles not appearing in the initial frame, particles being lost due either to erosion, rotating in the out-of- image direction, or three or more particles colliding and particles returning in a later frame. Others may have a more expedient method to do this but what follows is the procedure used in our experiments with rods. It is rather slow and uses a great deal of memory but works sufficiently well for the relatively short data sets that we analyzed. The explanation will be technical and requires some knowledge of LABView and, more generally, concepts such as 'for loops' and 'shift-registering'.

The input to this program is an array of particle positions and orientations in each frame. Within a given frame, the particles are jumbled so that a particle's index in that row of the array is not the same in subsequent frames. The output is an array in which each column is the position and orientation data for a single particle in all frames.

The unwinding process begins by inputting into a for loop - which runs as many times as there are frames in the video - the position and orientation data array $O_{k l}$ where $k$ is is the frame and $l$ the particle index in a given frame. To each array element we attach a default Boolean of 'FALSE'. We then create an initially blank companion array $C_{k l}$ that always has $i+1$ rows, where $i$ is the current frame being analyzed. As the program puts the particle data in order, rows 0 to $i$ of $C_{k l}$ will be ordered - that is, each column will be the position and orientation of a single particle from time 0 up to the current frame - and row $i+1$ will be set to all zeroes.

The first iteration of the program, i.e. the first frame in the video, is a special case but all other iterations are the same. We begin by extracting the row $O_{i+1}$ corresponding to all particle positions and orientations in the $i+1$ th frame of the video. The companion array $C_{i j}$ is shiftregistered into the for loop. All Boolean values attached to both $O_{i}$ and $C_{i}$ are set to 'FALSE'. The program then searches for null elements in $O_{i+1}$ - those in which the centers-of-mass and orientation all equal zero - and set these to TRUE. These null elements correspond to the particle


Figure A.2: A sampling of different composite particles and the result of attempting to obtain their constituent centers-of-mass and orientations. Each row is a different composite particle. Column A is the original extracted image. Column B is the result of applying the parallel axis to determine the center-of-mass locations (red crosses). Column C is the result of overlaying circles to determine the particle orientation, as indicated by red lines.
indices where no particle was found in the instant frame but particles were found in other frames. Thus, the null elements correspond to dropped particles. Then, the program loops, one element at a time, through $C_{i}$, considering only those elements whose Boolean is 'FALSE'. It then compares each element, one-by-one, with every element in $O_{i+1}$ that also has a FALSE Boolean. It finds the distance between each of these points and creates a new array of distances. It then searches this array for a minimum within a specified threshold. If a match is found, this means that the particle $C_{i l}$ has found itself in the next frame $O_{i+1, l}$. The program then replaces the value of $C_{i+1, l}$ with the value at $O_{i+1, l}$ and switches the Boolean value of $O_{i+1, l}$ to 'TRUE' so that it cannot be chosen again as a future particle's position. If no match is found, the value of the Boolean in $C_{i l}$ is changed to 'TRUE' and the loop continues. At the end of this loop, the row $C_{i+1}$ has been written and the Boolean values in $C_{i}$ and $O_{i+1}$ have been modified. Every particle that has a match in the next frame has been identified.

We now search through $O_{i+1}$ for 'FALSE'-valued Boolean elements. These are particles that were not found in the previous frame. We then insert a new blank row into the companion array and replace the element $C_{i+1, l}$ with the value in $O_{i l}$. After this, we search through $C_{i}$ for 'TRUE'-valued Booleans. These are particles that could not find a match in the next frame and will have their columns stopped. To do so, we write a null element in $C_{i+1, l}$ for each value of $l$ that has stopped. We then shift-register the companion array to begin the loop again, adding a new blank row and loading in the next row - i.e. the data from the next frame - from $O_{k l}$. The output is a two-dimensional array - each element is a cluster of the particle center-of-mass, orientation, and the Boolean value - of the following form:

| $x$ | $x$ | $x$ | $x$ | $x$ | 0 | 0 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| $x$ | $x$ | $x$ | $x$ | 0 | 0 | 0 |
| $x$ | 0 | $x$ | $x$ | 0 | 0 | 0 |
| $x$ | 0 | $x$ | $x$ | 0 | $x$ | 0 |
| $x$ | 0 | $x$ | $x$ | 0 | $x$ | $x$ |

Each column is a particle; each row is a particular frame. Here, a particular element containing data is marked as an ' $x$ ', whereas a null element in which no particle is recorded is a 0 . As we move down each column, if the trace suddenly goes from ' $x$ ' to 0 , it means that the particle could not be found and its trace was terminated. Appended to the end of the array are columns with
long series of 0 elements preceding non-null ' $x$ ' elements. These are particles that 'popped in' that weren't found in a previous frame. It is then be possible to link up and linearly interpolate all traces. In the example shown above, the second column terminates in the third frame - meaning that particle was lost afterwards - but could possibly then be one of the particles that reappears in either the sixth or seventh columns. Of course, whichever is correct must be decided according to the distance between each of these elements.

As noted, this is much more complicated than when no particles vanish or suddenly appear. For very clean data, it is sufficient to compute the distance between a given element and every element in the subsequent frame and determine the smallest distance moved without specification of a threshold. Once we have this array, we save either the entire array or each individual column as "datalog" files with the extension .dat, unique to LABView but readable by other programs. The exact type of data file differs from experiment to experiment but we can easily save data even in complicated formats such as arrays of clusters in this way. Typically, due to memory considerations, the data files are split apart into smaller portions so that, when a program needs to read information into memory, it does not have to read all of the data entirely.

## A. 3 Dynamic Structure Factor

In this section, we will give some more detail and show intermediates steps in calculating the dynamic structure factor for videos of rods. Recall from Chapter 2 that we treat the video data as a density map, a function of position and time $\rho(x, y, t)$. Using LABView's Vision package, we first obtain the spatial Fourier transform of each frame in the video. The resultant image is arranged in Cartesian coordinates with the zero wavevector DC-component placed at the center of the Fourier transform image. An example of the Fourier transform of an image of rods with propagating waves - like the images used in Figure 2.2 - is shown in Fig. A.3. The image displays the power spectrum, the magnitude of the Fourier transform, with magnitude increasing in grayscale from black to white. The axes are Cartesian: the horizontal axis of the image is the x-component of the wavevector $k_{x}$; the vertical axis is the y-component $k_{y}$. Both axes range from $0 \mathrm{~cm}^{-1}$ at the center of the image to $15.4 \mathrm{~cm}^{-1}$ at the edge. As noted in the text, the spatial Fourier transform of the rods data shows two annuli at $|k| / 2 \pi \simeq 0.79 \mathrm{~cm}^{-1}$ and $4.17 \mathrm{~cm}^{-1}$,


Figure A.3: The spatial power spectrum of an image of a collection of rods with propagating waaves, like those of Figure 2.2. The horizontal axis of the image is the x -component of the wavevector $k_{x}$; the vertical axis is the y-component $k_{y}$. The zero wavevector DC-component is at the center of the image. The axes range from $0 \mathrm{~cm}^{-1}$ at the center of the image to $\pm 15.4 \mathrm{~cm}^{-1}$ at the edges.
corresponding to the long and short dimensions of the rod. The smaller annulus is difficult to visualize as it is found within only a dozen pixels from the center of the image. The annuli are isotropic with respect to the polar angle indicating that there is no long-range order in the system.

The next step is to calculate the temporal power spectrum for fixed wavevector $|k|=\sqrt{k_{x}^{2}+k_{y}^{2}}$. Since the image is arranged in a typical Cartesian coordinate system, all points in an annulus of radius $|k|$ have the same magnitude of wavevector. Further, since the spatial power spectrum is isotropic with respect to direction, we can average over all pixels within a given annulus.

As an example, we choose the wavevector $|k|=0.42 \mathrm{~cm}^{-1}$. This corresponds to an annulus


Figure A.4: Time-trace of the spatial power spectrum for wavevector components $k_{x}=0.35$ $\mathrm{cm}^{-1}$ and $k_{y}=0.232 \mathrm{~cm}^{-1}$ - such that $\sqrt{k_{x}^{2}+k_{y}^{2}}=|k|=0.42 \mathrm{~cm}^{-1}-$ versus time.
approximately 12 pixels from the center of the image shown in Fig. A.3. The first step is to obtain a time-trace of the value of the spatial power spectrum for each pixel in that annulus for all time. One such time-trace is shown in Fig. A. 4 for the particular values $k_{x}=0.35 \mathrm{~cm}^{-1}$ and $k_{y}=0.232 \mathrm{~cm}^{-1}$.

The next step is to obtain the temporal power spectrum of the time-trace for each pixel in the annulus of constant wavevector. The temporal power spectrum of the trace shown in Fig. A. 4 is shown in Fig. A.5. As seen in the figure, the spectrum of an individual pixel is very noisy and the signal of the peak is greatly increased when we average over all pixels in the annulus. The final result, of course, is one single curve in the plot shown in Figure 2.6(a). The remaining curves are obtained by repeating this procedure with incrementally increasing radius of the $k$-space annulus.


Figure A.5: Temporal power spectrum for wavevector components $k_{x}=0.35 \mathrm{~cm}^{-1}$ and $k_{y}=$ $0.232 \mathrm{~cm}^{-1}$ - such that $\sqrt{k_{x}^{2}+k_{y}^{2}}=|k|=0.42 \mathrm{~cm}^{-1}$.

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