

# **Plasmonic and Metamaterial Cloaking: Physical Mechanisms and Potentials**

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## **Abstract**

Artificial materials, metamaterials and plasmonic media have recently received tremendous attention from the scientific communities, media and general public, following novel ideas and suggestions for their potential use in a variety of applications such as cloaking. Here we briefly review and highlight some of the available solutions for invisibility and cloaking that employ metamaterials and plasmonic materials at various frequencies. We briefly overview some of the different cloaking mechanisms recently proposed in the literature, such as plasmonic cloaking based on scattering cancellation, coordinate-transformation cloaking and anomalous localized resonances for cloaking, in particular providing some details for scattering-cancellation-based plasmonic cloaking. We mention the main analogies and differences among these various approaches, and we discuss some possible ideas for realizations and applications of these results, with particular attention to the physical phenomena involved.

## ***1. Introduction***

In the last decade, important technological advancements in micro- and nanomaterials have encouraged many researchers to concentrate their works on theoretical and experimental aspects of artificial materials formed by “atom-like” inclusions that would interact with electromagnetic waves in an anomalous fashion. Metamaterials, i.e., man-made materials with electromagnetic properties not readily available in nature and different from those of their constituents, have become a major topic of research and discussion in a growing number of scientific publications (see, e.g., [1]-[5] and references therein).

The attention of the media and the general public for these special materials has been attracted by some of their potentially breakthrough applications, like the possibility of sub-wavelength focusing and “perfect-lensing” [6], which may overcome some of the intrinsic limitations in optical imaging. This interest has been further fostered and enhanced after it was shown that metamaterials and plasmonic materials may pave the way to conceptually novel ways of making a given object “invisible” to the electromagnetic radiation [7]-[8], potentially overcoming some of the inherent limitations of some camouflaging techniques. This application of metamaterials has immediately attracted (some times with some exaggeration) the general interest of media and non-specialist public, associating with it the term “cloaking” [8]. Since 2005, interest in metamaterial cloaking has been steadily increasing in the technical literature, as various aspects and several different approaches and solutions to metamaterial cloaking and invisibility [9]-[52] have been offered. Each of these techniques has its own advantages and constraints, as we briefly discuss later in this review.

The interest of the electromagnetic and physics communities in invisibility and cloaking indeed dates back to several decades. Already at the beginning of the last century, specific and properly designed distributions of oscillating sources with no sensible radiation were predicted [53]-[56], and the concepts of “invisible” particles, sources and antennas have been investigated for several decades in a variety of scientific fields [57]-[69]. Clearly, these concepts are distinct from the more recent concepts of realistic engineering of passive cloaking devices, but the same issues of realizing a collection of oscillating sources that would not be detected in the far-field or of minimum-scattering antennas and particles have intrigued for decades the imagination of physicists and scientists.

The early interest in achieving plasmonic/metamaterial “invisibility” based on scattering cancellation exploiting the anomalous electromagnetic properties of metamaterials dates back to 2003 [70], where we have theoretically shown how an anomalous resonance between “complementary” metamaterials may produce wave tunneling, effectively making “invisible” the whole region of space occupied by the two material slabs. In a later contribution, two years later, we have described a more general mechanism for scattering cancellation that exploits the anomalous property of a local negative polarizability, which is inherent to plasmonic materials and metamaterials [7]. When these materials are juxtaposed to, or better when they surround, a dielectric or conducting object, the system may be designed such that the overall scattering may become extremely low, orders of magnitude lower than that of the isolated object itself. This effect relies on a scattering cancellation effect, for which effectively the wave scattered from the cloak may cancel the one from the object to be cloaked, leaving an external

observer with a very low residual scattering that makes the system approximately “invisible” at and around the design frequency. As discussed in details in [9] and in the following, this cancellation is distinct, and in some ways more robust, than other techniques, since it may, under certain conditions, be essentially independent of the form, direction, position and polarization of the illuminating source. Moreover, this invisibility cloaking technique is inherently non-resonant, and consequently it is less sensitive to variations in the shape, geometry and frequency of operation of the cloak and/or of the object to be covered. As we mention in the following, to achieve this phenomenon, even some naturally available plasmonic materials at THz, infrared and optical frequencies might be exploited, since it relies on isotropic and homogeneous covers and it does not require complicated requirements on the constituent materials. In [10] we have also proven how parallel-plate implants may be inserted in a suitably designed dielectric cover in order to achieve the required values of effective permittivity at microwave frequencies. This technique may allow achieving similar cloaking effects with a relatively simple artificial material at frequencies (not necessarily microwave, but also optical [14]) for which plasmonic materials may not be readily available in nature. This technique does of course have its own limitations, which we will discuss later in this paper.

While the first results on this problem [7]-[10] were focused on relatively small isolated dielectric or conducting objects at a single frequency of operation, which represent the easiest situation to apply this technique, in the subsequent papers we have extended these concepts to collections of particles and larger systems [11] and to multiple frequencies and somewhat broader bandwidths of operation [12]. The possibility of cloaking a given region of space, without necessarily isolating it from the external radiation, makes this

cloaking technique particularly interesting and intriguing, especially for other related aspects and concepts, such as applications in low-noise measurement and non-invasive probing. Other interesting theoretical papers have suggested the possibility of inducing isolated plasmonic resonances [38] or anomalous localized resonances [33] that may isolate and cloak a given object from the electromagnetic wave. These concepts have been developed in terms of cloaking devices in subsequent papers by Milton, Nicorovici, McPhedran and their coworkers [32]-[34], based on their earlier work on coated cylinders [36].

Of various interesting and exciting solutions in the field of metamaterial cloaking offered by various groups [16]-[52], the cloaking based on the coordinate-transformations, conformal mapping, and space distortions for tailoring and designing a metamaterial cloak capable of isolating a given region of space from the surrounding has attracted a great deal of attention [16]-[31]. This exciting transformation-based cloaking, together with the localized resonance approach [32]-[34], both depend on resonant cloaks (or cloaks made of metamaterials with resonant inclusions), which makes their effects highly dependent on frequency and on the geometrical and electromagnetic parameters of the cloak. Shortly after the theoretical discovery of transformation-based metamaterial cloaking in 2006, the first experimental verification of metamaterial cloaking has been presented, based on a transformation-based device operating at microwave frequencies for the 2-D scenario [17]. This experiment has attracted major attention from the news media and general public, opening up the way to metamaterial cloaking applications in several regimes. One may speculate that experimental verification of other available

cloaking techniques and the translation of these experiments into optical frequencies will follow soon.

The advantage of transformation-based and anomalous resonant cloaking reside in the fact that these designs in principle do not depend on the size, composition, and geometry of the object to be hidden, nor on the size of the region that the cloak is required to isolate. However, it should be mentioned that such methods might have their own limitations in practice due to their specific stringent requirements on material anisotropy and inhomogeneity profile when cloaking of large objects (many wavelength large) is desired, which perhaps might not be easily obtainable in practice with currently realistic metamaterials, particularly at the IR and optical frequencies. In the case of anomalous localized resonances for cloaking, the analyses associated with some of these techniques have been mostly performed under quasi-static approximations with less emphasis on material parameter requirements for possible realization of such cloaks.

In the following, we provide an overview of some of the proposed techniques that employ metamaterials and plasmonic materials for cloaking applications, showing their analogies and differences, underlining their inherent limitations and effective potentials and providing physical insights into the specific features and phenomena generally associated with these findings. This is not an exhaustive review, and it highlights only some of the features of some (but not all) of these techniques, but the interested reader may be referred to the reference list for further details. The cloaking phenomena may have exciting potential applications in various scientific and engineering fields, including camouflaging, low-noise measurements and non-invasive sensing and probing in medicine, biology and optics.

## ***2. Electromagnetic Cloaking: Background and Physical Remarks***

Making an object electromagnetically “invisible” to the impinging radiation is a topic of quite old debate in the technical literature. As pointed out in the introduction, the possibility of theoretically envisioning invisible bodies and non-radiating sources is a familiar concept to electromagnetic theorists, physicists and antenna designers [57]-[69]. The idea of applying these concepts to a real-life cloak capable of making objects of non negligible size, invisible at the frequency of interest, is however quite novel. Although some of the recent attempts in this sense may appear to be like simple extension of well established concepts, some new ideas have shown how breakthrough cloaking applications of special materials and metamaterials may indeed be envisioned. In the former class, the possibility of employing idealized active sources specifically designed to cancel the scattering from given objects under a specific illumination is a classic electromagnetic problem, which often becomes more of a mathematical challenge than a real engineering problem of practical interest, due to the complexity of the related optimization problem and the difficulty of obtaining a real-time effective response from the cloaking sources. Moreover, while the recent discussion about the ultimate possibility of realizing a perfectly “invisible” configuration of sources is indeed of theoretical interest, the practical implications of this problem may be limited and debatable.

However, the recent growing interest in metamaterial cloaking has brought these interesting theoretical problems to the attention of physicists and mathematicians. As recently shown in [39], for instance, the possibility of realizing a passive cloak capable of perfectly isolating a region of space from its surrounding without producing any

scattering [16], has some intrinsic physical limitations, which may be overcome only by using active materials. Similar limitations to “perfect cloaking” have been pointed out in [41]-[43].

The notion of non-radiating sources, which are “invisible” to an observer placed in the far-field, has also been the topic of a long debate in the technical literature [59]-[68] and it has been brought to a renewed attention in the field of metamaterial cloaking [43], [52]. From a more practical point of view, however, achieving a “perfect” and “idealized” cloaking device may not be necessarily needed, since inherent losses and technology limitations would at some point affect the ideal results predicted theoretically, as confirmed for instance in [42]. Rather, the main practical challenge may reside in achieving an overall cloaking effect by suppressing most part of the scattered wave produced by an object or by a system of objects, drastically reducing its overall visibility to an external observer positioned anywhere around it, in its near or far field. The way in which an electromagnetic detector (observer) may sense the presence of an object, is indeed based on measuring the scattered fields caused by the object. Any feasible way to drastically reduce this perturbation, by suppressing the dominant components of the scattering, and/or by rerouting the impinging radiation around the object itself, would make the system practically, if not ideally, invisible. Of major interest, of course, is realizing such a cloak with passive materials, without requiring active or properly adjusted sources, and in such a way that its design would not depend on the specific form of excitation, or on its direction of propagation. Otherwise, as recently proposed by researchers at the University of Tokyo [71]-[72], a simple possible camouflaging trick may be provided by a set of cameras projecting in real-time the image of the back of an



object onto its front. For an observer positioned in front of the system, such a simple device would work as a “cloak”, even though a change in the observer’s position, or in the form of illumination of the object, might easily reveal the object. Apart from this solution, well established techniques for reducing the visibility of an object are currently available, employing stealth devices, radar absorbing materials or active sources. The limitations of conventional non-metamaterial techniques for reduction of object visibility may include reduction of only portions of the scattered field, presence of the detectable shadows associated with wave absorption, possible need for active and complex devices, and dependence of the designs on specific form of the excitation and the position and nature of the observer. The novel research in metamaterials for cloaking applications aims, however, at overcoming some of intrinsic limitations of these current techniques.

The scattering cancellation technique [7]-[15], which we describe in the following section, is an example of a cloaking technique that may satisfy these requirements of passivity and general-purpose functionality. This idea generalizes the findings of some of the earlier contributions to the topic [57]-[58], [69], which showed how in the static limit a scatterer may become effectively “invisible” to an impinging plane wave, provided that its total electric polarizability is made sufficiently small by proper combination of different materials. In the past, these concepts had been overlooked as mere scattering curiosities limited to specific anomalous materials and strictly static considerations, and the problem was not investigated in further details and in the fully dynamic case. However, due to the recent technological advancements that may allow the realization and synthesis of plasmonic materials and metamaterials at the frequency of interest, we have been interested in analyzing the anomalous scattering features of combinations of

regular and special materials [73]. In that context, we have noticed how it may indeed be possible to achieve an identically zero dipolar scattering from a realistic moderately sized particle made of a combination of plasmonic and dielectric materials. This finding, which in the limit of electrically small particles is in agreement with the previously referred papers, brought us to investigate whether a suitably designed cover may effectively suppress the overall visibility of a given object, not necessarily small, within a full-wave dynamic analysis [7]. As reported in [7] and following papers, it turns out that it is indeed possible to “cancel” the dominant scattering orders (not necessarily the dipolar one) from a given dielectric or even conducting object with a suitable plasmonic cloak, and even though “perfect cloaking”, i.e., the simultaneous cancellation of all scattering harmonics, may not be strictly achievable, the reduction in visibility may reach orders of magnitude and make the particle practically and effectively undetectable.

This is of course in full agreement with the theoretical limitations of passive cloaks and non-radiating sources [39], [43], [60]-[63]: the full-wave dynamic analysis predicts that only a limited number of scattering harmonics may be made exactly zero, and therefore a perfect cloaking system may never be achieved, thus a very minor residual component of higher-order harmonics may always be present in the scattering pattern. Still, this implies that under suitable conditions the major portion of the impinging field may be “tunneled” through or “re-routed” around the cloaked system, making it practically undetectable for an external observer. Also the other techniques recently proposed in the literature [16]-[37], involving passive metamaterial cloaks, are required to satisfy similar basic constraints and limitations.

In a certain sense, the scattering cancellation technique provides a method for invisibility cloaking of a given object by “matching” it to the background material, allowing the impinging electromagnetic wave to penetrate through it (or around it) without sensible scattering. Transformation-based cloaks [16]-[20], on the other hand, distort the space around a given region in order to re-route the impinging rays around the region, and to prevent them from penetrating into the region. Anomalous localized resonances [32]-[35] achieve a similar result by relying on the overall distortion and anomalous interaction of the impinging wave with a suitably designed set of plasmonic or metamaterial nanoresonators. Although based on different physical principles, these techniques all aim at substantially reducing the visibility of an object to the surrounding, and they may ideally achieve similar performances, but in distinct ways and with different features. In the following, we review and compare in details these different solutions.

### 3. *Plasmonic Cloaking Based on Scattering Cancellation*

#### *a) General Formulation*

Consider a general monochromatic  $e^{-i\omega t}$  electromagnetic wave propagating in a background material (not necessarily free-space) with permittivity  $\varepsilon_0$  and permeability  $\mu_0$ , impinging on a given object centered, without loss of generality, at the origin of a spherical coordinate system with coordinates  $r, \theta, \varphi$ . The impinging electric and magnetic fields may always be expanded in terms of a superposition of orthogonal spherical harmonics centered at the object center, as [74]:

$$\begin{aligned}
\mathbf{E}_i &= \sum_{n=1}^{\infty} \sum_{m=-n}^n a_{nm} \nabla \times \nabla \times (\mathbf{r} \psi_n^m) + i\omega\mu_0 \sum_{n=1}^{\infty} \sum_{m=-n}^n b_{nm} \nabla \times (\mathbf{r} \psi_n^m) \\
\mathbf{H}_i &= \sum_{n=1}^{\infty} \sum_{m=-n}^n b_{nm} \nabla \times \nabla \times (\mathbf{r} \psi_n^m) - i\omega\epsilon_0 \sum_{n=1}^{\infty} \sum_{m=-n}^n a_{nm} \nabla \times (\mathbf{r} \psi_n^m)
\end{aligned} \tag{1}$$

where  $\mathbf{r}$  is the radial vector and  $\psi_n^m$  are scalar spherical harmonics, which are the solutions of the Helmholtz equation in the spherical coordinate systems [74]. Each of these terms in the summations corresponds to a  $TM^r$  or  $TE^r$  spherical wave, which represent, respectively, the electric or magnetic impinging multipole of order  $(n, m)$  with amplitude  $a_{nm}$  or  $b_{nm}$ .

Owing to the orthogonality of such spherical harmonic waves and to the linearity of Maxwell equations, the scattering problem may be separately solved for each of the harmonics, and the field scattered by the object may therefore be represented by another sum of electric and magnetic scattering multipoles [75], written analogously to (1) as:

$$\begin{aligned}
\mathbf{E}_s &= \sum_{n=1}^{\infty} \sum_{m=-n}^n c_{nm}^{TM} a_{nm} \nabla \times \nabla \times (\mathbf{r} \psi_n^m) + i\omega\mu_0 \sum_{n=1}^{\infty} \sum_{m=-n}^n c_{nm}^{TE} b_{nm} \nabla \times (\mathbf{r} \psi_n^m) \\
\mathbf{H}_s &= \sum_{n=1}^{\infty} \sum_{m=-n}^n c_{nm}^{TE} b_{nm} \nabla \times \nabla \times (\mathbf{r} \psi_n^m) - i\omega\epsilon_0 \sum_{n=1}^{\infty} \sum_{m=-n}^n c_{nm}^{TM} a_{nm} \nabla \times (\mathbf{r} \psi_n^m)
\end{aligned} \tag{2}$$

The coefficients  $c$  in (2) relate the scattered fields to the impinging ones, and they depend on the geometry of the scatterer and on the frequency of operation. Depending on the size of the scatterer, only contributions up to a given order  $N$  are relevant, since for a regular scatterer the amplitude of the scattering coefficient  $(n, m)$  is of the order  $o(k_0 a)^{2n+1}$  where  $a$  is the scatterer characteristic size and  $k_0 = \omega\sqrt{\epsilon_0\mu_0}$  is the background wave number [73]. As derived in [73], for instance, for a core-shell spherical scatterer their expressions are given by:

$$c_{nm}^{TE} = -\frac{U_n^{TE}}{U_n^{TE} + iV_n^{TE}}, \quad c_{nm}^{TM} = -\frac{U_n^{TM}}{U_n^{TM} + iV_n^{TM}}, \quad (3)$$

with:

$$U_n^{TM} = \begin{vmatrix} j_n(ka) & j_n(k_c a) & y_n(k_c a) & 0 \\ [ka j_n(ka)]' / \varepsilon & [k_c a j_n(k_c a)]' / \varepsilon_c & [k_c a y_n(k_c a)]' / \varepsilon_c & 0 \\ 0 & j_n(k_c a_c) & y_n(k_c a_c) & j_n(k_0 a_c) \\ 0 & [k_c a_c j_n(k_c a_c)]' / \varepsilon_c & [k_c a_c y_n(k_c a_c)]' / \varepsilon_c & [k_0 a_c j_n(k_0 a_c)]' / \varepsilon_0 \end{vmatrix} \quad (4)$$

$$V_n^{TM} = \begin{vmatrix} j_n(ka) & j_n(k_c a) & y_n(k_c a) & 0 \\ [ka j_n(ka)]' / \varepsilon & [k_c a j_n(k_c a)]' / \varepsilon_c & [k_c a y_n(k_c a)]' / \varepsilon_c & 0 \\ 0 & j_n(k_c a_c) & y_n(k_c a_c) & y_n(k_0 a_c) \\ 0 & [k_c a_c j_n(k_c a_c)]' / \varepsilon_c & [k_c a_c y_n(k_c a_c)]' / \varepsilon_c & [k_0 a_c y_n(k_0 a_c)]' / \varepsilon_0 \end{vmatrix} \quad (5)$$

where  $j_n(\cdot)$ ,  $y_n(\cdot)$  are spherical Bessel functions [76],  $[\cdot]'$  denotes differentiation with respect to their argument,  $k \equiv \omega\sqrt{\varepsilon\mu}$  and  $k_c \equiv \omega\sqrt{\varepsilon_c\mu_c}$  are, respectively, the wave number in the inner sphere (with permittivity  $\varepsilon$ , permeability  $\mu$  and radius  $a$ ) and in the cover region (with parameters  $\varepsilon_c$ ,  $\mu_c$  and  $a_c$ ). Analogous expressions to (4)-(5) for the TE coefficients are obtained after substituting  $\varepsilon$  with  $\mu$ . We notice how, due to the symmetry of the scatterer in this specific geometry, Eq. (4)-(5) do not depend on the azimuthal order  $m$ . In general, however, they may also be a function of  $m$ .

The total scattering cross section (SCS) of an object, a quantitative measure of its overall visibility at the frequency of interest, is given in general by the formula [75]:

$$\sigma_s = \frac{2\pi}{|k_0|^2} \sum_{n=1}^{\infty} \sum_{m=-n}^n (2n+1) \left( |c_{nm}^{TE}|^2 + |c_{nm}^{TM}|^2 \right). \quad (6)$$

The possibility for an external observer, positioned in the near or far field of the object, to detect the presence of the object is fully determined by the amplitude of the scattering coefficients  $c$ , some of which may be of more or less importance as a function of its position and electromagnetic properties. However, if ideally all the scattering coefficients would be made negligibly small at the frequency of interest, then the impinging field would be totally unperturbed by the presence of the scatterer, and its presence would not be detectable for an external observer placed anywhere around it, i.e., the object would be effectively “invisible” at the frequency of interest. Indeed, the total scattering cross section in Eq. (6) would turn out to be identically zero in this case. The SCS (6), in other words, measures the overall visibility of the scatterer at the frequency of interest, independent of the specific position, or of the properties of the observer, nor on the direction and polarization of the excitation.

Apart from the trivial case of an object having the same constitutive parameters as the background material, however, the boundary conditions at the surface(s) of the object usually require the presence of a non-negligible scattered field, perturbing the original impinging radiation and making the presence of the object detectable. Although it may not be physically possible to completely cancel *all* the scattering coefficients with a suitably designed passive cloak, in order to achieve a practical “invisibility” it would be more than sufficient to make the relevant scattering terms sufficiently close to zero, resulting in a drastic reduction of  $\sigma_s$ .

Provided that the size of the scatterer is sufficiently small (usually up to the order of half-wavelength for its characteristic size), only the scattering multipoles of order  $n=1$  are significantly excited by the impinging wave. In this scenario, as we have first shown in

[7], it is possible to drastically lower the amplitude of  $\sigma_s$  by covering the object with a plasmonic metamaterial with low or negative permittivity at the frequency of interest. This effect is heuristically justified by considering the fact that the  $n=1$  electric multipole radiation is provided by the integral of the polarization vector  $\mathbf{P}_l = (\varepsilon_l - \varepsilon_0)\mathbf{E}_l$  over the scattering volume [75], with  $\varepsilon_l$  being the value of local permittivity and  $\mathbf{E}_l$  being the local electric field. A plasmonic material with low-positive or negative permittivity induces a local polarization vector 180 degrees out of phase with respect to the local electric field, providing the possibility of canceling part or all of the in-phase contribution given by the scattering object. This heuristic explanation may be extended to any higher-order TE or TM dominant multipole for the corresponding higher-order multipolar moment of the displacement current distribution, making possible, at least in principle, to design cloaks that may reduce any combination of scattering coefficients.

The exact analytical condition for total cancellation of a given multipolar order is given, following the previous discussion, by the condition for nulling the corresponding  $U_n$  term, consistent with the definition given in (3). It is interesting to underline that, as it was found in [73], the dual condition  $V_n = 0$  would induce a resonant scattering for the corresponding multipolar order, providing the opposite behavior for the scattering properties of the system: it would excite an enhanced scattering, dominated by the  $n$ -th spherical harmonic at resonance. A drastic difference is noticeable between these two dual conditions: the enhanced scattering is a highly resonant phenomenon, compared with the inherently non-resonant features of the scattering cancellation that we are interested in this section.

Considering the spherical core-shell system of formulas (4)-(5), in the quasi-static limit the condition for invisibility, of interest here, becomes [7]:

$$\frac{a}{a_c} = \sqrt[2n+1]{\frac{(\varepsilon_c - \varepsilon_0)[(n+1)\varepsilon_c + n\varepsilon]}{(\varepsilon_c - \varepsilon)[(n+1)\varepsilon_c + n\varepsilon_0]}}, \quad (7)$$

consistent with the previous analyses limited to static fields in [57]-[58], [69].

It is interesting to note that, although the dynamic condition  $U_n = 0$  may be achieved by any combination of materials, and even by a single homogeneous scatterer, such a condition may be achieved with regular materials only when the size of the scatterer is sufficiently large, which implies the presence of significant contributions from several other higher-order multipolar orders contributing to the overall scattering. For a small scatterer, on the other hand, for which the scattering is strongly dominated by the dipolar contributions, Eq. (7) with  $n = 1$  may hold, and such a condition, as discussed in [7], may be achieved only when one of the involved materials (the object, or more commonly the cover) has permittivity below that of the background material. In this case, it may be possible to suppress the dominant contribution to the scattering, without necessarily exciting higher-order multipoles. In this scenario, the visibility of the object may be dramatically reduced by a simple isotropic and homogeneous cover with properly tailored permittivity and thickness. In verifying experimentally this technique, the availability of plasmonic materials in nature may play a fundamental role, together with the possibility of using homogeneous and isotropic materials.

In [7] we have applied these concepts to cloak dielectric particles. As we have shown, by placing suitably designed simple plasmonic covers around them, the scattering may be reduced by over 99%, making the overall system (i.e., the cloaked particle) effectively



invisible. In particular, we have found that the most appealing frequency region for using a plasmonic cover is slightly above its plasma frequency, where the permittivity is low, but positive. This is because in this region, as discussed in [73], the transparency condition may be satisfied without necessarily inducing associated plasmonic resonances. Moreover, usually this frequency region is farther from material resonances, ensuring weaker frequency dispersion and lower absorption. These properties ensure relatively wider bandwidth and better performance of the cloaking phenomenon.

For dielectric particles of relatively small size, as those considered in [7], the quasi-static condition (7) for the electric dipole term, which is more relevant for the previous considerations, reads:

$$a = \sqrt[3]{\frac{(\varepsilon_c - \varepsilon_0)[2\varepsilon_c + \varepsilon]}{(\varepsilon_c - \varepsilon)[2\varepsilon_c + \varepsilon_0]}} a_c. \quad (8)$$

As a special case, one may also consider the limit in which the particle becomes impenetrable, which may describe the case of a conducting material or of a metal with sufficiently negative permittivity, as it was considered in [9]. In this case, condition (8) is further simplified into:

$$a_c = \sqrt[3]{\frac{2\varepsilon_c + \varepsilon_0}{\varepsilon_0 - \varepsilon_c}} a, \quad (9)$$

which holds only for plasmonic materials with  $0 < \varepsilon_c < \varepsilon_0$ .

The general quasi-static condition for achieving cancellation of a given multipolar order for a spherical core-shell system, as expressed in Eq. (7), may be represented in a contour plot as in Fig. 1, as a function of the permittivities of the inner core and of the outer cloaking shell. This plot shows in a schematic way the concepts outlined above: it is required that the two involved materials (the object and the cloak) should be

characterized by oppositely-signed local polarizabilities. Fig. 1 may serve as a “design chart” to pick the correct value of permittivity for the cloak, depending on the material of the object to be covered.

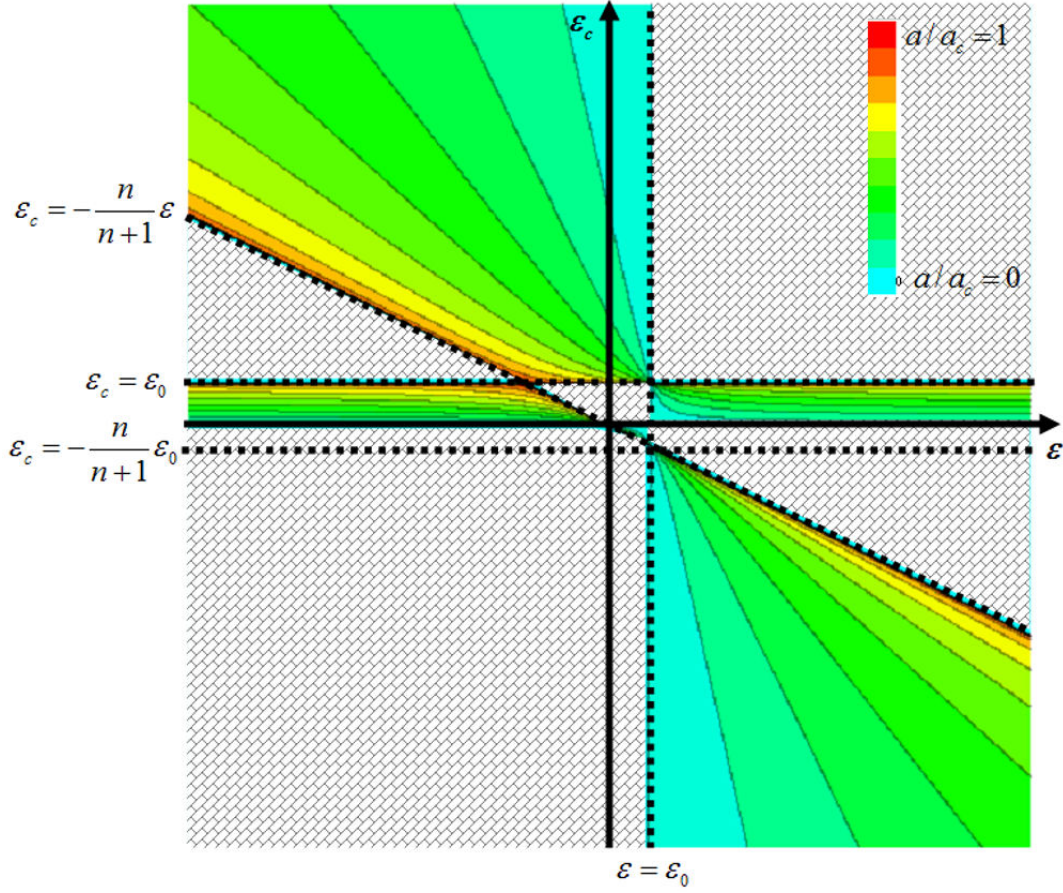


Fig. 1. (From [7], A. Alù, and N. Engheta, “Achieving transparency with plasmonic and metamaterial coatings,” *Phys. Rev. E* 72, 016623 (2005)) Regions for which the quasi-static condition (7) is satisfied, for the corresponding value of  $a/a_c$  between zero and unity. The “forbidden” regions, indicated by the “brick” symbols, correspond to values of  $\epsilon$  and  $\epsilon_c$  for which the quasi-static condition (7) for the TM case may not be fulfilled. Copyright (2005) by the American Physical Society.

#### b) Cloaking Isolated Dielectric or Impenetrable Particles

Examples of the potentials of this technique for cloaking isolated dielectric or conducting particles of relatively small size have been provided in [7]-[9]. Here we report some new numerical results for the field distributions and scattering patterns calculated using full-wave Mie theory simulations for two significant geometries of the object to be cloaked: a dielectric (with  $\varepsilon = 5\varepsilon_0$ ) and a conducting spherical particle, both with diameter  $2a = \lambda_0/5$ , with  $\lambda_0$  being the free-space wavelength. In both cases, due to the relatively small electrical dimensions of the scatterer the scattering from the bare objects is dominated by the first TM harmonic, corresponding to the electric dipole radiation, with a residual, but non-negligible, scattering produced by the magnetic dipole contribution, which is of larger amplitude for the conducting particle. For each case, we consider four different designs for the spherical shell used as the cloak: a) a cover designed to satisfy the condition  $U_1^{TM} = 0$  with permittivity  $\varepsilon = \varepsilon_0/10$  at the frequency of interest; b) a cover designed to satisfy the same condition, but with  $\varepsilon = \varepsilon_0/20$ ; c) a cover made of the background material, corresponding to the case of an uncovered particle; d) a cover made of the same material as the particle, with thickness as in case (a), corresponding to a larger homogeneous particle. These last two cases are added for the sake of comparison. Figure 2, as a first example, reports the total scattering cross section (in logarithmic scale, normalized to  $\lambda_0^2$ ), as a function of the ratio  $a_c/a$  for the first geometry considered here, i.e., for which the particle is made of dielectric material, with a cover with different thicknesses. It is evident how employing plasmonic materials, despite an increase in the cover thickness, generates an anomalous reduction in the visibility of the system, consistent with the findings in [7]. A minimum is achieved when the condition  $U_1^{TM} = 0$

is reached, as previously derived. The position of the minimum obviously changes with the cover material properties, and a material with lower permittivity, being characterized by a larger “negative” polarizability, and therefore being able to cancel a larger dipole moment for the same volume, in general requires a smaller thickness. It can be seen, however, that both minima for the two different covers reach a similar quantitative value, which corresponds to the residual scattering from the higher-order harmonics of the particle, once the dipolar term has been totally canceled by the cover. In this case, owing to the relatively small size of the systems, the residual scattering orders (after cancellation of the electric dipolar term) are dominated by the magnetic dipole contribution. The SCS may be reduced by more than 20 dB, corresponding to more than 99% scattering cancellation with respect to the uncovered particle.

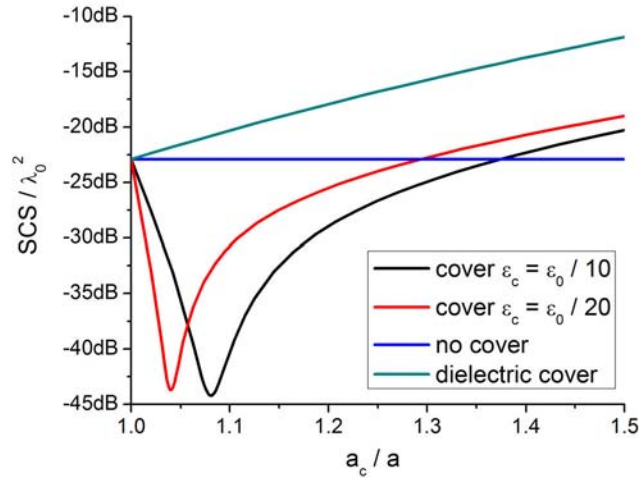


Fig. 2. Total scattering cross section versus cover thickness for a dielectric particle with  $\epsilon = 5\epsilon_0$  and

$$a = \lambda_0 / 10 .$$

Three features may be underlined in Fig. 2: (1) the relatively smooth transition through the minima of the two lines, (2) the relatively large “radial bandwidth” (in this case in

terms of sensitivity to geometry variation, but similar concepts apply also when frequency variations are considered), both features that are related to the inherent non-resonant properties of this cloaking mechanism, and (3) the non-zero value of the scattering minima, due to the inherent presence of a residual (very low) scattering from higher-order multipoles. This latter aspect, as already discussed, is to be expected due to the dynamic nature of the problem and the passivity of the constituent materials, while we notice however that an overall effective invisibility is anyhow achieved around the minima of the black and red curves in Fig. 2. For comparison, the other two curves in Fig. 2 show the reference cases of an uncovered particle (blue line) and of a cover made of the same material as the particle (grey) line. In this last case, as it is usually expectable, an increase in the particle size results in a corresponding increase in the total scattering cross section. We note that there is a wide range of values for the cover thickness that allows significant reduction of the overall scattering from the system, ensuring that the design is fairly robust to changes and variations in its different parameters [9].

Figure 3 reports the magnitude of the total magnetic field (the component orthogonal to the E plane of polarization) for the four cases of Fig. 2. In particular, Fig. 3a corresponds to the geometry of the black line in Fig. 2 for a cover thickness  $a_c = 1.08a$ , which corresponds to the minimum scattering for this configuration. Analogously, Fig. 3b corresponds to the geometry of the red line in Fig. 2 for  $a_c = 1.04a$ . Fig. 3c and Fig. 3d relate to dielectric particles with radius  $a$  (no cover, blue line in Fig. 2) and  $a_c = 1.08a$  (with dielectric cover, green line in Fig. 2), respectively. The black lines in the figures depict the corresponding geometries and all the plots use the same color scale for ease of comparison. Consistent with the axes in the figure, the excitation consists of a

plane wave with electric field polarized linearly from left to right and propagating from bottom to top in all the panels. It is evident how the amplitude of the magnetic field, orthogonal to the plane of the figures, is very much perturbed by the presence of the scatterer in the cases without cloak (Fig. 3c and 3d). However, when the cloak is suitably designed (Fig. 3a and 3b) the amplitude of the magnetic field outside the covered particle is barely perturbed by the presence of the scatterer and the field is nearly uniform outside the cloak.

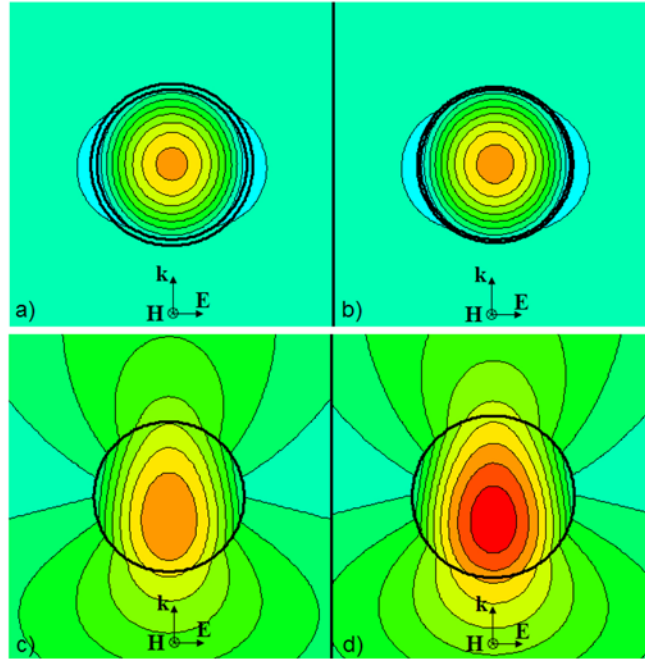


Fig. 3. Total magnetic field distribution (amplitude) perpendicular to the E plane for the four cases of Fig.

2. The four panels correspond to: (a)  $\varepsilon_c = \varepsilon_0 / 10$  and  $a_c = 1.08a$ ; (b)  $\varepsilon_c = \varepsilon_0 / 20$  and  $a_c = 1.04a$ ; (c)  $\varepsilon_c = \varepsilon_0$ ,  $a_c = a$ , (d)  $\varepsilon_c = 5\varepsilon_0$ ,  $a_c = 1.08a$ . Here and in the following figures, brighter colors correspond to larger values of the field. The axes in the figure depict the direction of the impinging plane wave and of the field vectors. For better comparison, the color scale is the same in the four plots. The geometry of the four spheres is depicted in black in the figures.

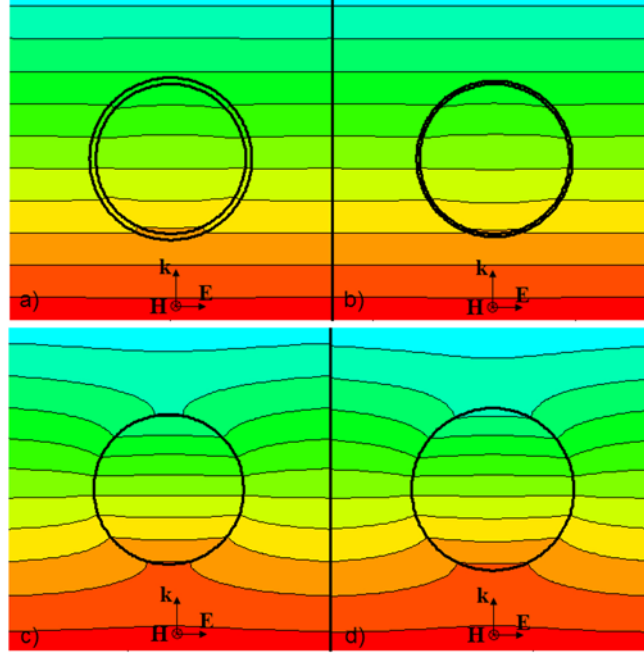


Fig. 4. Total magnetic field distribution (phase) in the E plane for the four cases of Fig. 3.

Figure 4 reports the phase of the total magnetic field distributions on the same plane of polarization. Here it is even more evident how the cloak (Fig. 4a and 4b) manages to restore the planar phase fronts of the impinging plane wave, which would be strongly deformed by the presence of the uncloaked dielectric particle (Fig. 4c and 4d). It is interesting to observe the complete absence of a shadow in the cloaked geometry, implying that an observer placed on the back of a sphere would see “through” and “behind” the cloaked sphere, as if the sphere were not placed between him and the source of illumination. This phenomenon is even more striking when impenetrable (metallic) particles are considered, as considered in the following.

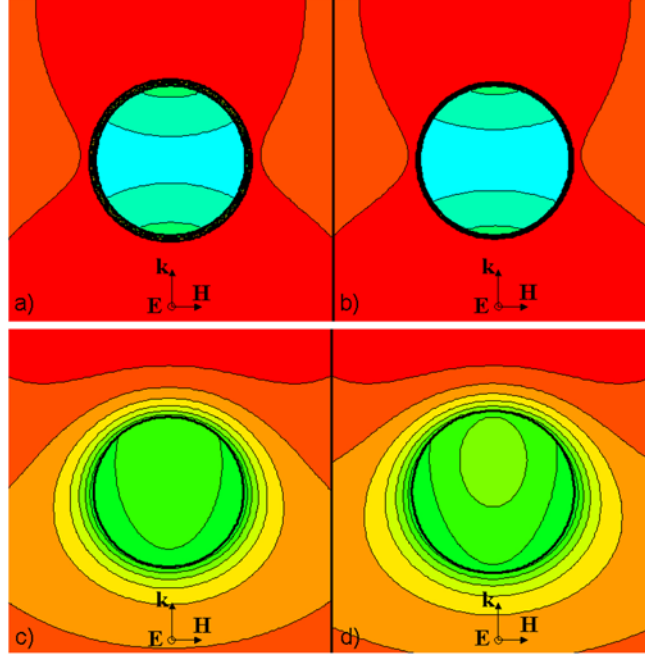


Fig. 5. Total electric field distribution (amplitude) perpendicular to the H plane for the four cases of Fig. 3.

Figures 5 and 6 report the total electric field distribution, its amplitude and phase respectively, for the H plane of polarization, i.e., the plane orthogonal to the impinging electric field vector. We note how the scattered field in the E plane is dominated by the magnetic dipole contribution for the scattered field, whereas the scattered field in the H plane is dominated by the electric dipole contribution. This implies that, for the geometries considered here, the maximum residual scattering, and correspondingly, for what discussed above, the better scattering cancellation, is achieved on this H plane.



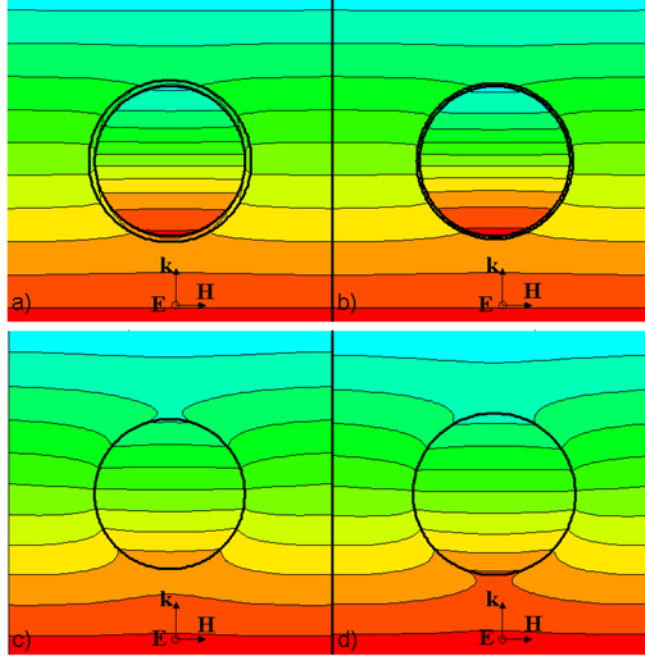


Fig. 6. Total electric field distribution (phase) in the H plane for the four cases of Fig. 3.

This is noticeable in these figures, where the field disturbance strongly affects the impinging uniform amplitude (Fig. 5) and planar phase fronts (Fig. 6) in the cases without cloak, whereas they are consistently restored by the presence of the plasmonic covers.

Figures 3-6 ensure that the near-field configuration, when plasmonic cloaks are suitably designed, makes the systems effectively invisible to an external observer. This is also clearly reflected in the far-field patterns, as we report in Figure 7. The figure reports the scattering patterns in the far-field for the four cases of Fig. 3. It is shown how the scattering pattern is dramatically reduced by the presence of suitably designed covers, consistently with the near-field results of the previous figures. Moreover, the original pattern shape of uncloaked particle (Fig. 7c and 7d), donut-shape due to the electric dipole radiation of the uncovered particles, is very much affected by the presence of the

cloak (Fig. 7a and 7b). In this case, the residual scattering is indeed still donut-shape, but oriented according to the magnetic field polarization, due to the residual magnetic dipole scattering from the particles. The field levels (in arbitrary units in the figure), however, are reduced by over two orders of magnitude with respect to the uncovered case. Suppression of the residual magnetic dipole scattering, which may be of interest for larger and/or for conducting particles for which this residual scattering might still be large, may be achieved by varying the magnetic permeability of the cloak, as we have discussed in details in [9].

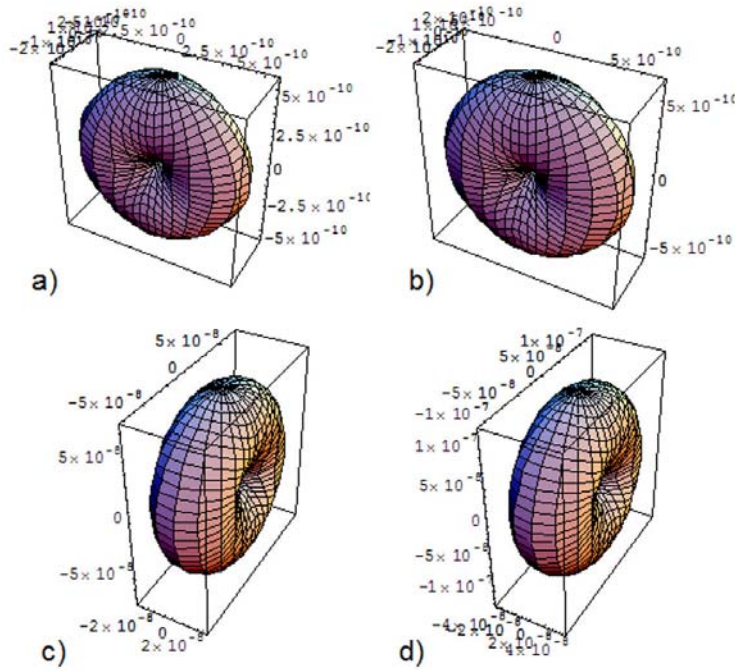


Fig. 7. Scattering patterns in the far-field for the four cases of Fig. 3. The reported values are in arbitrary units, but all the plots have been obtained for the same amplitude of incident excitation. The scales in the cloaked scenarios (top row) are two orders of magnitude lower than those for the uncloaked case (bottom row).

Figure 8-10 present analogous results for the case of an impenetrable/conducting sphere. In this case the required covers to achieve the minimum scattering, with a cover permittivity value of  $\varepsilon_0/10$  and  $\varepsilon_0/20$ , have thickness  $a_c = 1.1a$  and  $a_c = 1.05a$ , respectively. Compared with the previous geometry, we note that here slightly thicker covers are required in order to obtain dipolar scattering cancellation, due to the larger scattering that an impenetrable sphere causes when compared with a dielectric sphere of the same size. Once again, a plasmonic material with permittivity closer to zero requires a lower volume for obtaining the scattering cancellation effect. The overall minimum scattering achievable for the two geometries (represented by the residual scattering from higher-order harmonics) is comparable.

From Fig. 8 we can see how in this case the residual magnetic dipole contribution is somewhat larger than in the previous case of dielectric sphere, leaving a slightly larger residual scattering at the “invisibility” condition. Even in this case, however, the scattering reduction is over 10 dB, i.e., larger than 90%. Better cloaking may be achieved in this configuration by increasing the cover permeability (consistent with [9]) and/or adding an extra layer to the cover (consistent with [12]), in order to add extra degrees of freedom to the cloak that may allow simultaneous cancellation of multiple spherical harmonics. Even with this simple cover design the bandwidth over which scattering reduction is achieved with respect to the uncovered case is relatively wide.

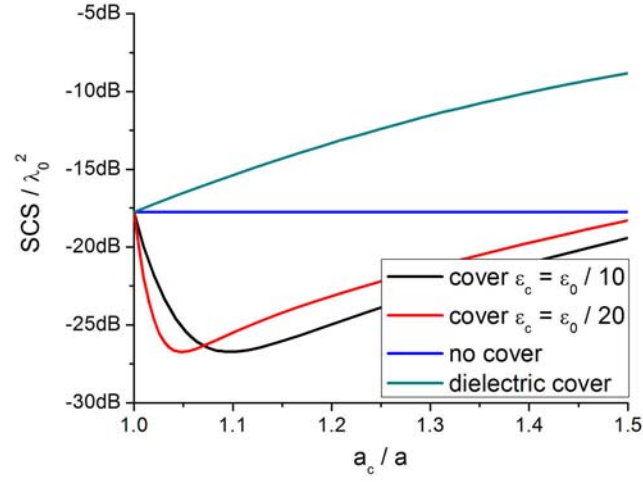


Fig. 8. Total scattering cross section versus cover thickness for a conducting or impenetrable particle with radius  $a = \lambda_0 / 10$ .

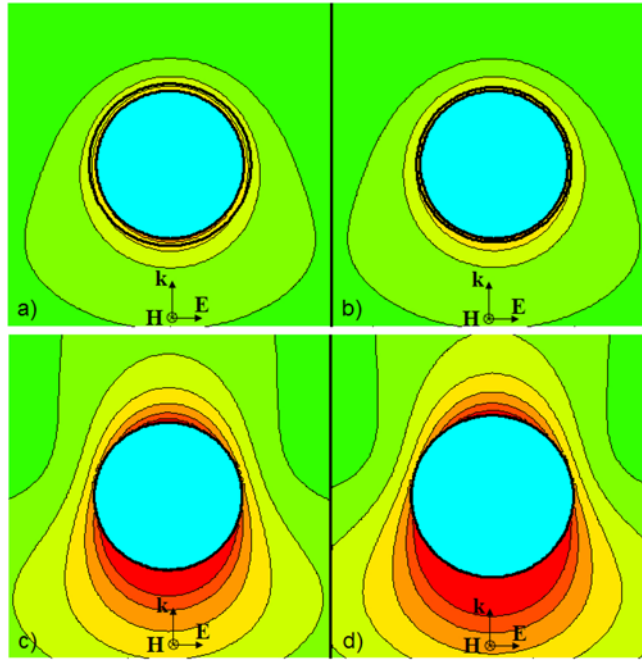


Fig. 9. Total magnetic field distribution (amplitude) perpendicular to the E plane for the four cases of Fig.

8. The four panels correspond to: (a)  $\varepsilon_c = \varepsilon_0 / 10$  and  $a_c = 1.1a$ ; (b)  $\varepsilon_c = \varepsilon_0 / 20$  and  $a_c = 1.05a$ ; (c)

$\varepsilon_c = \varepsilon_0$ , (d)  $\varepsilon_c = -j\infty$  (i.e., PEC) and  $a_c = 1.08a$ .

The near-field plots on the E and H planes (Fig. 9 and Fig. 10, respectively) confirm the previous results for this geometry, showing the drastic reduction in visibility of the object on both planes (As expected, better performance is obtained in the H plane where the electric dipole contribution to the scattering is maximum). (Here for sake of brevity, we report only the amplitude plots in Figs. 9 and 10). This effect is particularly striking in this configuration, since the power flow cannot actually tunnel inside the impenetrable region as in the dielectric case, and therefore a rerouting of energy is required to squeeze a substantial amount of power in the tiny plasmonic cover and therefore to suppress the shadow effect in the back of the particle. Movies and further simulations for a similar geometry related to this effect have been reported in [9]. This effect is somehow connected to the other striking properties of materials with permittivity near zero that our group has recently analyzed [77]-[79].

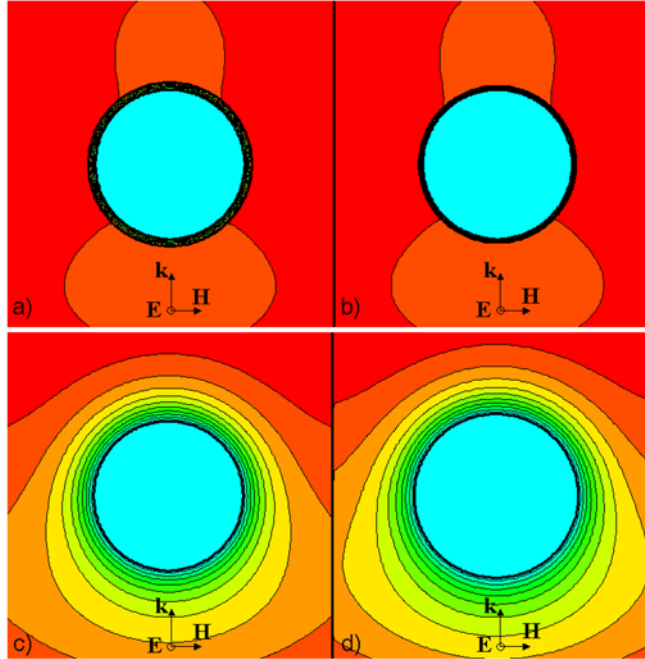


Fig. 10. Total electric field distribution (amplitude) in the H plane for the four cases of Fig. 9.

In order to further highlight the functionality of this cloaking technique as experienced by an external observer, we have reported in Fig. 11 the total electric field magnitude projected on a square screen with a side dimension of  $20\lambda_0$  ( $\lambda_0$  being the free-space wavelength) placed at a distance of  $10\lambda_0$  from the center of the object on its back, for the four cases analyzed in Fig. 3 (dielectric particle). It is evident how at the back of the particle a relevant shadow is measured when the particle is uncloaked, together with strong interference fields. The presence of the covers drastically reduces the scattered field that an observer may pick up, suppressing the particle shadow and field distortion and letting the observer “see behind and through” the cloaked system.

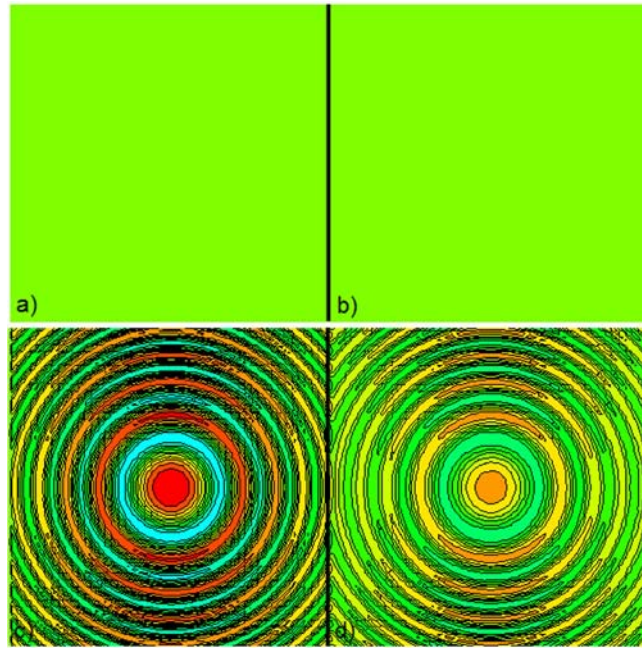


Fig. 11. Total electric field distribution (amplitude) on a screen with dimensions  $20\lambda_0 \times 20\lambda_0$  and located at  $10\lambda_0$  behind the sphere along the direction of propagation of the impinging plane wave, in the four cases of

Fig. 3.

*c) Bandwidth and robustness to design variations and losses*

An interesting property of the plasmonic cloaking technique that we aim to stress in this section consists of its inherent non-resonant features. This results in an intrinsic robustness to some frequency variations and to modifications of the geometrical and electromagnetic parameters of the system, as well as to the presence of absorption and material losses. This is consistent with Fig. 2 and 8, in which the scattering minima do not exhibit sharp dips, but rather smooth transitions, with relatively wide regions in which the scattering is substantially reduced with respect to the uncovered case.

In [9], we have analyzed in detail the intrinsic robustness of this cloaking technique to different variations in the geometrical parameters of the object to be cloaked and of the cover, in the electromagnetic parameters of the cloaking material and in the frequency of operation, also considering the intrinsic material dispersion and losses associated with plasmonic materials. This property may suggest an advantage of this technique over other resonant techniques for metamaterial cloaking.

In Fig. 12 we report the variation of the SCS in the cases of the conducting particle considered above using materials with a Drude frequency dispersion as the covers. Since this cancellation technique is an integrative non-resonant effect, its main dependence on frequency comes from the inherent dispersion of plasmonic materials. In this specific case, the cloak permittivity has been assumed to follow the dispersion model

$$\varepsilon_c = \left(1 - \frac{f_p^2}{f_0^2}\right) \varepsilon_0, \text{ with } f_0 \text{ being the design frequency and } f_p \text{ the plasma frequency,}$$

respectively. For the left and right panels in Fig. 12, we choose  $f_p = 0.95f_0$  and  $f_p = 0.975f_0$ , respectively, which correspond to  $\varepsilon_c = 0.1\varepsilon_0$  and  $\varepsilon_c = 0.05\varepsilon_0$  at the frequency  $f_0$ , consistent with the previous examples.

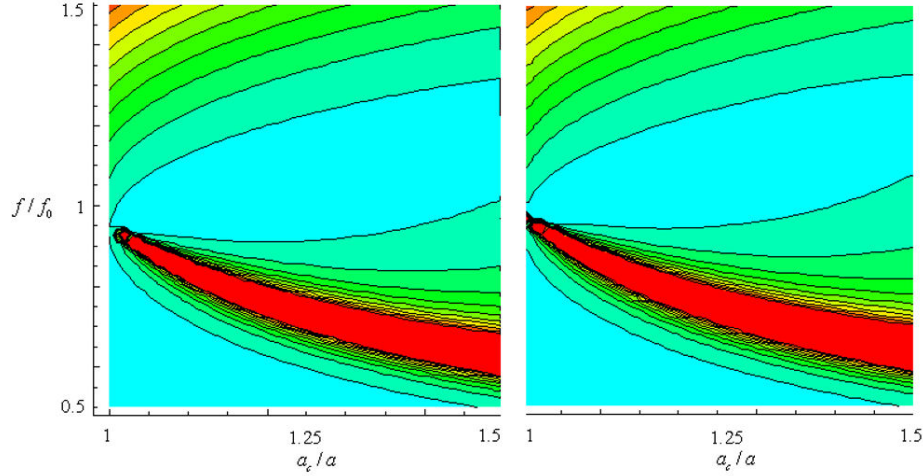


Fig. 12. (From [9], A. Alù, and N. Engheta, “Plasmonic materials in transparency and cloaking problems: mechanism, robustness, and physical insights,” *Optics Express* 15, 3318-3332 (2007)). Variation of the SCS for an impenetrable particle as in panel a and b of Fig. 11, varying the frequency of operation and the cover thickness. We assume here a Drude dispersion for the plasmonic cover, with  $f_p = 0.95f_0$  (left) and  $f_p = 0.975f_0$  (right). Copyright (2007) by the Optical Society of America.

The figure shows a relatively large bandwidth over which  $\sigma_s$  may be substantially reduced (light blue regions), and the object is practically invisible to its surroundings. As we have shown in [9], the non-resonant features of this phenomenon ensure robustness also in terms of ohmic losses in the material cover, variations in the cloaked object in terms of cuts, dimples or bumps in its surface. These considerations equally apply to dielectric and conducting/impenetrable objects.

#### *d) Collection of objects and electrically larger volumes*

The discussion in the previous sections has been focused on single isolated objects of moderate size, for which the dipolar fields represent the dominant contribution to the



overall scattering. Under these conditions, as we have shown above, a drastic reduction of visibility may be obtained with a simple homogeneous plasmonic cover.

The question of whether the presence of multiple objects placed in close proximity of one another may affect the previous results and whether it may worsen the cloaking performance has been rigorously addressed in [11]. Considering the generality of Eq. (1)-(2) and of the multipole expansion of the impinging field, the condition  $U_n = 0$  for making a given multipolar order  $n$  transparent remains indeed valid for any shape of the impinging wave front, even when it is not represented by a simple plane wave, as it was assumed in the previous subsections. In order to show the independence of this transparency phenomenon on the form of excitation, in [9] we have indeed also examined the case in which the isolated cloaked particle is excited by a short electric dipole placed in close proximity of its surface. The results are analogous to the plane wave case, and they show drastic reduction of the scattered fields, leaving the dipolar fields essentially unperturbed by the presence of the system, even right outside the cloak.

By similar arguments, it may be shown that the coupling between closely spaced objects, each of them individually cloaked, is essentially cancelled, since the reduction or suppression of the multipolar scattering holds in the near as well as in the far field. It should be realized in this context that the interaction among closely spaced particles is based on higher-order spherical harmonics that, although decaying faster in the far-field with respect to the lower-order contributions, may become dominant in the very near field of the object. This interaction, however, can still be represented by the general form of excitation given by Eq. (1) for each one of the particles, ensuring that the transparency phenomenon holds even in this situation.

Following these concepts, in [11] we have presented extensive numerical simulations confirming the fact that collections of multiple cloaked objects, even when closely spaced, show similar performance in terms of cloaking as for the isolated particles. Without performing any optimization procedure, but rather designing the cover as if each individual object were isolated, in [11] we have also analyzed the extreme situations in which the covers are “merged” together, or even for which the objects themselves are joined to constitute a single object of large electrical size. Interestingly, even though the results are slightly deteriorated by such drastic modification in shape, still a significant reduction of scattering is obtained for such systems around the design frequency.

These results indeed represent an important extension of this cloaking technique, showing that it may possibly consider collections and systems of objects occupying volumes larger than the wavelength of operation, and therefore typically characterized by several orders of multipolar scattering. The number of multipoles  $N$  contributing to the overall SCS of electrically large systems indeed increases rapidly with the total occupied volume, and the simple technique of using a single homogeneous plasmonic cover may not in principle be as effective for drastically reducing the total SCS of large systems as it is for relatively small particles. Rather, this technique may be effective in just canceling one or two of the dominant scattering orders.

One way to overcome this limitation may consist in adding extra degrees of freedom to the cover design, in terms of extra layers in the cloak [12] or adjustments in its permeability, when possible. Another, and in some senses more appealing, technique might be represented by the concepts discussed in this section, namely, larger and more

complex objects may be cloaked by subdividing them judiciously in electrically smaller subunits, when possible, and then cloaking each of them individually.

Before concluding this section that deals with electrically larger systems, we should point out that a related application of plasmonic cloaks effective for electrically larger objects has been suggested in [45]. In this paper, it was found that an electrically large metallic sphere may be cloaked by employing a two-layer plasmonic-dielectric cloak that creates a band-gap in the series of plasmonic resonances of a relatively large particle. It is interesting to underline how the addition of the extra layer in the configuration suggested in [45] does not provide major advantages with respect to our technique described in this section. This provides a scattering reduction comparable to that obtainable with a simpler homogeneous plasmonic cover as described above. To show this concept, in Fig. 13 we have reported the SCS calculated for a silver particle with radius  $a = 500\text{nm}$  (where the silver permittivity has been modeled following the same Drude model used in [45]), consistent with the object cloaked in [45]. Moreover, in the same figure we have plotted the variation of SCS for the same particle when covered by a simple plasmonic cover

with  $\epsilon_c = \left(1 - \frac{f_p^2}{f_0^2}\right)\epsilon_0$ ,  $f_p = 593\text{THz}$  and  $a_c = 507\text{nm}$ , and the SCS for a dielectric cloak

with  $\epsilon_c = 20\epsilon_0$  and  $a_c = 510\text{nm}$ , both designed and optimized to suppress the dominant scattering harmonics at the frequency of  $500\text{THz}$ . It is interesting to note that the scattering reduction achieved with these simple homogeneous covers is quantitatively similar to the simulations reported in [45] at the same frequency, where it was used an arguably more complex cloaking design consisting of a multilayered cloak. It is noticed that in this specific example for which the object has a negative permittivity at the

frequency of interest and the dominant contribution to the scattering is not dipolar, both a plasmonic and a dielectric cloak may be effective in reducing the overall scattering. As can be noticed when comparing Fig. 13 with the results in [45], the reduction in SCS, although clearly not complete in both cases due to the large size of the particle, is consistent with what obtained with the multilayered “optimized” cloak in [45] in the same range of frequencies, showing how the plasmonic cloaking technique, in its simplicity, may turn out to be useful in different applications even for cloaking larger systems.

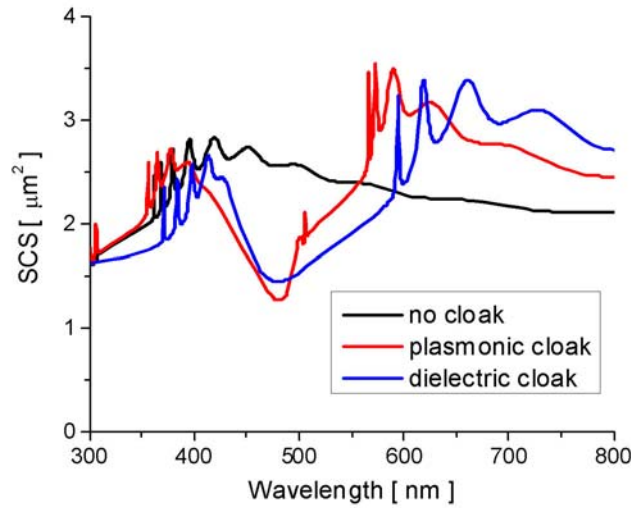


Fig. 13. Variation of SCS for a relatively large silver particle (diameter of  $1\ \mu m$ ) at optical frequencies, covered with a plasmonic shell (or a dielectric shell) designed to suppress the dominant scattering orders.

*e) Towards a recipe for an experimental realization of cloaking based on scattering cancellation*

The results of the previous sections have been obtained assuming the availability of plasmonic materials with the required value of permittivity. As already noticed, materials with low values of permittivity are indeed available in nature at THz, infrared and optical

frequencies. Therefore, as a first step towards proving the concepts outlined in the previous sections, one may consider using plasmonic materials already available in nature, and combine them with dielectrics. To this end, we have numerically simulated several setups employing silver and gold covers combined with dielectric nanoparticles in order to induce a cloaking effect at optical frequencies. Silver and gold, in fact, belong to the class of noble metals that possess naturally negative permittivity and a relatively low level of losses at optical frequencies [69].

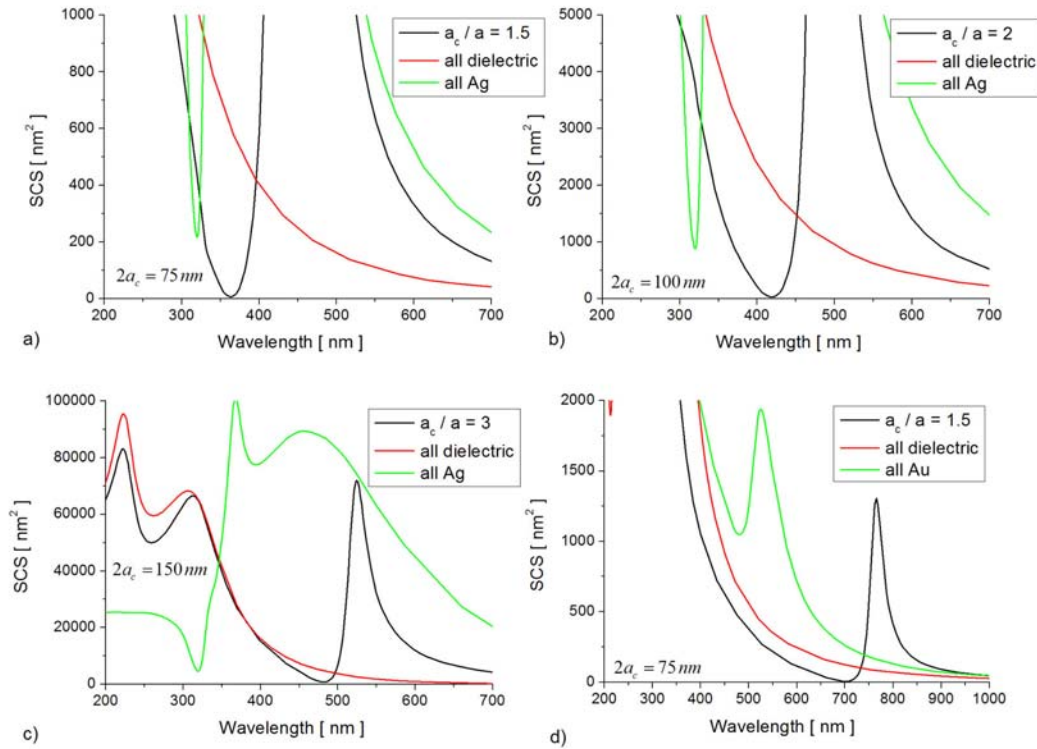


Fig. 14. Variation of SCS versus wavelength of operation for a spherical nanoparticle with an inner core made of silver (a, b, c) or gold (d) and an outer shell made of dielectric materials ( $\epsilon_c = 4\epsilon_0$  for a, b, c;  $\epsilon_c = 17.9\epsilon_0$  for d). The size of the particles is indicated in the insets. The curves are compared with the case of homogeneous particles made of the same plasmonic or dielectric materials with same size.

As an example, Fig. 14 shows the SCS comparison, for different geometries, of a particle made of a plasmonic material (silver for Fig. 14a, b, c, and gold for Fig. 14d, using experimental values of permittivity [80]) combined with a dielectric material as a cover, and particles of the same size made of only plasmonic or dielectric materials. Figs. 14a, b and c compare three different sizes of particles ( $2a_c = 75\text{ nm}$ ,  $2a_c = 100\text{ nm}$ ,  $2a_c = 150\text{ nm}$ , respectively) with same sized particles made of silver only or all of a material with  $\varepsilon = 4\varepsilon_0$  only. Fig. 14d refers to a nanoparticle with diameter  $2a_c = 75\text{ nm}$  made of a gold inner core surrounded by a silicon shell ( $\varepsilon_c = 17.9\varepsilon_0$ ). In all these scenarios, it is evident how a judicious choice of the filling ratio (i.e., ratio of radii) between the materials allows a drastic reduction of SCS with respect to nanoparticles of the same size. Also in this case, the cloaking mechanism is evidently achieved over a wide range of frequencies and despite the realistic absorption of the particles. It is worth mentioning, following the discussion of the previous section, that for an easier experimental verification of these results it may be appealing to test collections of such coated nanoparticles, ensuring a collective response more easily detectable.

Although the previous simulations may already show an interesting recipe for a proof of concept of the scattering cancellation cloaking technique at optical frequencies using silver or gold, general plasmonic materials with the required values of permittivity may not be readily available at any given frequency of interest. In order to overcome this limitation, and to extend these concepts to the lower frequencies, where plasmonic materials may not be naturally available, we have recently proposed the use of simple metamaterial geometries to create the required isotropic plasmonic behavior at microwave frequencies. In particular, in [10] we have presented an extensive study for

the design of a suitable parallel-plate metamaterial, composed of radial metallic implants/inclusions creating radial waveguides slightly below cut-off in a cylindrical cloak, designed to have an overall negative effective permittivity at the frequency of interest, capable of hiding a dielectric cylinder with permittivity  $\varepsilon = 3\varepsilon_0$ . We have demonstrated that, if the parallel-plate metamaterial shell is properly designed, the behavior and effect of the cover are completely consistent with the theoretical results derived in the previous sections, and in particular such metamaterial covers may provide scattering reduction of some orders of magnitude for the combined system. These results may be generalized to the case in which metallic implants in the cover are not perfectly conducting, or in which they are made of plasmonic materials, in order to tailor the response of a metamaterial cloak at infrared and optical frequencies. This is currently under investigation by our group.

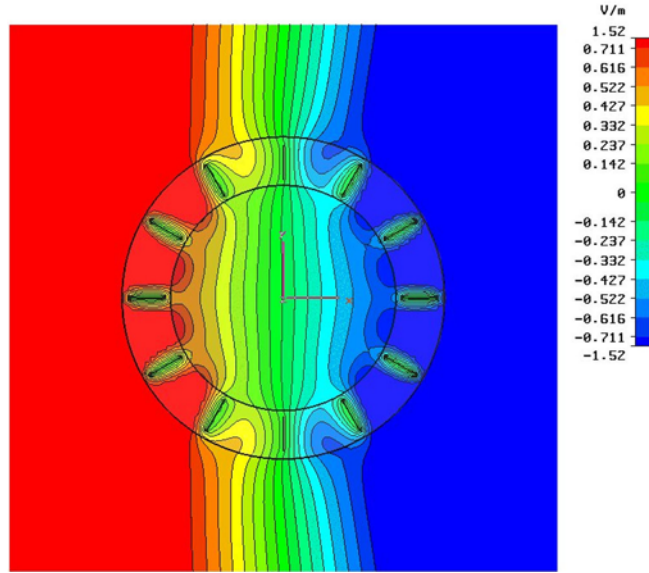


Fig. 15. Snapshot in time of the electric field distribution (logarithmic scale) for the geometry considered in [10] at the frequency for which the inner cylinder core has a permittivity  $\varepsilon = 3\varepsilon_0$  and a diameter

$2a = 0.27\lambda_0$ . The design parameters of the cloak are described in details in [10].

In Fig. 15 we have reported a snapshot in time of the electric field distribution for the cloaked system designed in [10]. The figure shows how the planar phase fronts and uniform amplitude may be restored by the presence of the conducting implants in the cloak region and, despite the presence of the scatterer and of the cloak, the overall system is nearly invisible to an external observer even in the very near-field of the structure.

#### **4. *Metamaterial Cloaking***

In the last couple of years, other interesting techniques that utilize metamaterials for cloaking applications have been suggested [16]-[52], resulting in significant increase in the general interest in these topics. In the following we briefly review the main concepts underlying two of these exciting methods for the cloaking problem.

##### *a) Coordinate-transformation technique*

The coordinate-transformation technique, as one of such interesting novel proposals, has been analyzed in different theoretical and experimental papers in recent times [16]-[23], [39]-[42]. It is based on a conformal transformation of coordinates that allows “distorting” the ray paths by suitably designing the cloak material, in such a way to isolate a given region of space from its surrounding. This is schematically sketched in Fig. 16, as described in detail in [16]. In some sense, this effect is similar to a mirage effect or optical illusion: as in a mirage in which the image is produced by the distortion of optical rays by the temperature difference in the air above the ground and ahead of the observer, here a proper choice of the material constitutive parameters in the cloak region



allows bending the optical path of the rays around the cloaked region, causing the electromagnetic wave to tunnel through the system without distortion and scattering.

In the seminal paper introducing this technique [16], Pendry *et al.* proposed that the permittivity and permeability tensors for an idealized cloak can be explicitly derived for a spherical cloak as:

$$\begin{aligned}\epsilon_{rr} = \mu_{rr} &= \frac{a_c}{a_c - a} \frac{(r-a)^2}{r} \\ \epsilon_{\theta\theta} = \mu_{\theta\theta} &= \frac{a_c}{a_c - a} \\ \epsilon_{\phi\phi} = \mu_{\phi\phi} &= \frac{a_c}{a_c - a}\end{aligned}, \quad (10)$$

with  $r$  being the radial coordinate. It should be noticed how the requirements on the permittivity and permeability tensors are quite specific, in the form of both anisotropy and inhomogeneity.

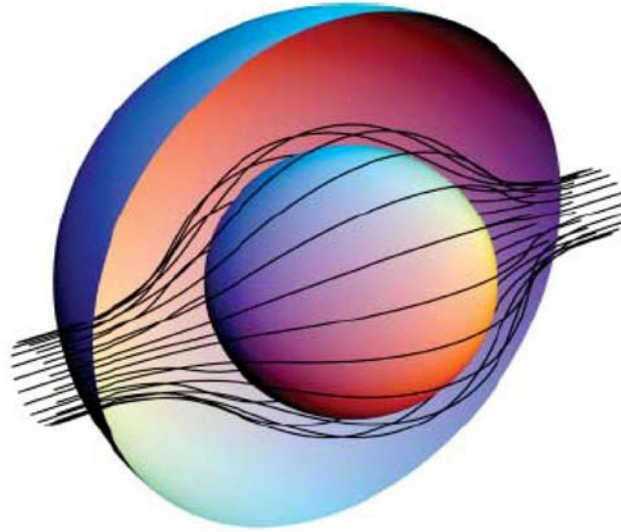


Fig. 16. (From [16]. J. B. Pendry, D. Schurig, and D. R. Smith, “Controlling electromagnetic fields,” *Science* 312, 1780-1782 (2006), Reprinted with permission from AAAS) A schematic that shows the principle on which coordinate-transformation cloaking schemes work: the electromagnetic rays are bent to

pass through the cloak without entering the cloaked region, making the whole system effectively “invisible”, independent of the specific object placed inside the cloak.

A simplified version of the previous formulas, relying on linear, rather than parabolic, variations of the inhomogeneity profile, has been suggested in [17], providing an imperfect, but easier to realize, cloak, which has been successfully tested experimentally. Simplified or approximate realizations of (10), however, causes limited cloaking capabilities, as pointed out in recent papers on the topic [40]-[42]. With an optimized design the reduction in cloaking effectiveness may be accompanied by an increase in bandwidth of operation [40].

The main advantage of this technique over the plasmonic cloaking discussed in the previous section consists in the fact that, in principle, it may be applied to any size of the region to be cloaked, without any restriction on its total electrical size and on its geometry. However, this property would require a perfect realization of the ideal anisotropy and inhomogeneity profiles (10), and electrically large cloaks would be highly sensitive to technological imperfections, presence of losses and narrow bandwidth of operation. In practice, when technological limitations and losses are considered, this technique might also have some limitation in size. In addition, most of the theoretical analysis and experiments using this technique have so far been limited to 2D geometries and to specific field polarization, which simplifies the cloak design. The simulated and experimental results based on metamaterial cloaks synthesizing the simplified version of the constitutive parameters in (10) that have been recently presented in the literature [17]-[23] do not exhibit a “perfect” cloaking, as expected, but rather possess some residual

scattering from the system. These limitations may understandably be related to the approximating (10) and the realistic inclusion-made metamaterials.

An exciting proposal for extending these concepts to the optical frequencies has been recently put forward in [23], using optical metallic inclusions to tailor a proper permittivity distribution in the cloak. Although still the cloaking effect is not perfect and the setup is limited to 2D, this result may represent an interesting venue for possible realization in the visible domain.

A somewhat similar transformation and distortion of spatial coordinates has been proposed independently by Leonhardt in recent papers [28]-[31].

An important property of these techniques, which makes them quite distinct from the plasmonic cloaking discussed in the previous section, consists in the total electromagnetic isolation of the core region induced by the cloak. It is evident in Eq. (10), that indeed  $\varepsilon_{rr} = \mu_{rr} = 0$  for  $r = a$  avoid field penetration in the cloaked region. This makes the cloaking design, at least in its ideal version, totally independent of the specific objects to be cloaked. This is often mentioned as an important advantage of these techniques over the plasmonic cloaking discussed in the previous version. However, it should be realized that, as shown in the previous section, the plasmonic cloak may also be effective over an impenetrable cloak, and therefore the combination of a plasmonic cloak and an impenetrable cavity would produce similar independence property over any object placed inside a cavity. Designing an impenetrable shield at  $r = a$  is equivalent to having a conducting wall in the inner surface of the cloak. On the other hand, we point out that the plasmonic cloaking has the possibility of letting the designer choose whether to design a cloak available for any object, by combining the plasmonic cloak with an impenetrable

cavity, or tailoring the cloak design for the specific object to be cloaked, which may be dielectric and therefore allow better performance in terms of bandwidth and robustness to losses, associated with the fact that it allows the wave to penetrate also in the cloaked region. Not to mention that, since the plasmonic cloaking setup may allow the wave to penetrate the cloaked region without producing a sensible scattering, it may also be employed to capture and observe the fields that penetrate the cloaked region without being perceived by the surrounding. In this way, an observer (sensor or detector) placed in the cloaked region may capture the outside fields without its presence being possibly detected.

Another advantage of the plasmonic cloaking technique, apart from its inherent simplicity of design, which usually requires also a much thinner cloak, relies in its non-resonant features, as discussed in the previous section, which makes it arguably more robust to variations in the geometry and electromagnetic design, to the presence of losses and absorption and it may provide an overall larger bandwidth of operation.

#### *b) Anomalous localized resonances*

As an alternative way of employing metamaterials and plasmonic materials for cloaking applications, Milton, Nicorovici, McPhedron and their coworkers have proposed the use of cloaks operating based on localized anomalous resonances, which may allow hiding a region of space from its surrounding [32]-[35], leaving the impinging field unperturbed. In contrast with the coordinate-transformation methods, as pointed out in [35], the cloaking is achieved in the region external to the cloak, exploiting the metamaterial resonances between the cloak and the background. Figure 17 reports an example of such

cloaking phenomenon (from [34]), designed in this case to cloak a polarizable dipole. The fields outside the cloaking region and inside the core are not perturbed by the presence of the source, due to the anomalous localized resonances induced in the shell regions and in the cloak.

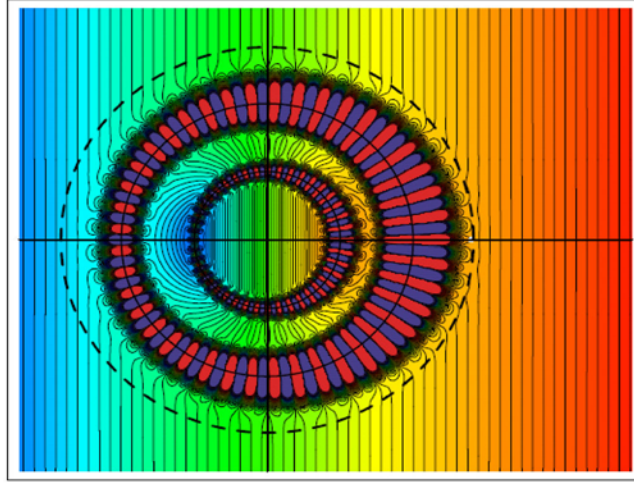


Fig. 17. (From [35] N. A. P. Nicorovici, G. W. Milton, R. C. McPhedran, and L. C. Botten, “Quasistatic cloaking of two-dimensional polarizable discrete systems by anomalous resonance,” *Optics Express* 15, 6314-6323 (2007), reprinted with permission from the Optical Society of America) Potential distribution induced by a single polarizable dipole in the cloaking system described in [35]. The concentric circles delimit the core region (cloaked), the shell region and the cloaking region (dashed line). Copyright (2007) by the Optical Society of America.

The strong resonances associated with this technique, however, may produce strong sensitivity to losses, frequency and other design variations, which have not been fully explored yet. Moreover, the theoretical papers describing this technique are based on quasi-static approximations and analyze 2D geometries and specific field polarizations. However, interesting quasi-static simulation results showing how such resonances may

indeed cloak an arbitrary region of space illuminated by an arbitrary excitation have been recently presented [32]-[35].

There are several other cloaking techniques developed by various groups, and the interested reader is referred to the list of references here for further information on these methods.

## **5. *Conclusions***

To conclude, in this review we have outlined and reviewed some of the recent proposals and techniques that employ metamaterials for cloaking applications. We have presented simulation results on the plasmonic cloaking technique based on scattering cancellation, highlighting its main features and providing physical insights into the involved mechanisms. We have also briefly reviewed two other exciting metamaterial techniques recently proposed in the literature to design metamaterial cloaks, highlighting the main analogies, differences and advantages among these various approaches.

In general, it may be challenging to provide a full and quantitative comparison among the different available approaches to metamaterial cloaking, since the involved problems are quite complex. In an ideal scenario, in which one may have access to the perfectly ideal (and lossless) cloaks suggested in these different theoretical contributions that we have outlined here, it may be possible to quantitatively compare the different performances in terms of size of the regions to be cloaked and bandwidth of operation. In this sense, for larger objects at the specific design frequency the coordinate-transformation-based cloaks may outperform other cloaking techniques, even if bandwidth would be more of an issue with respect to plasmonic cloaking. However, in practical scenarios, as the recent experimental realizations for the coordinate-transformation cloaking have shown,

feasibility and realizability of the stringent metamaterial profiles may limit the cloaking performances of some of these methods, even more at infrared and optical frequencies. In this sense, the sensitivity of these designs to practical issues, such as losses, frequency variations and imperfections are crucial points to take into account in future quantitative comparison among the different methods. From our analysis, one may suggest that the plasmonic cloaks may be arguably easier to realize since they do not require specific inhomogeneity and anisotropy profiles for the constitutive tensors, and they may be less sensitive to imperfections in the design.

We hope that this review may provide the reader with some road maps for a comparison among the different techniques and an insight into the future of metamaterial cloaking. We believe that metamaterial invisibility and cloaking may become of major interest for various exciting applications in various scenarios including camouflaging, non-invasive probing and sensing in medical, biological applications. Prospects and suggestions for cloaking at optical frequencies have also been mentioned here, and we believe that the recent progresses in nanotechnology may soon open up possibilities for confirming these concepts in realistic experimental setups at optical frequencies.

## ***References***

- [1] C. Caloz, and T. Itoh, *Electromagnetic Metamaterials: Transmission Line Theory and Microwave Applications*, Wiley-IEEE Press, 2005.
- [2] G. V. Eleftheriades, and K. G. Balmain (editors), *Negative Refraction Metamaterials: Fundamental Properties and Applications*, IEEE Press, John Wiley & Sons Inc., Hoboken, New Jersey, 2005.

- [3] N. Engheta, and R. W. Ziolkowski (editors), *Electromagnetic Metamaterials: Physics and Engineering Explorations*, John Wiley and Sons, New York, 2006.
- [4] A. K. Sarychev, and V. M. Shalaev, *Electrodynamics of Metamaterials*, World Scientific Publishing Company, 2007.
- [5] R. Marqués, F. Martín, and M. Sorolla, *Metamaterials with Negative Parameters: Theory, Design and Microwave Applications*, Wiley Series in Microwave and Optical Engineering, New York, 2008.
- [6] J. B. Pendry, “Negative refraction makes a perfect lens,” *Physical Review Letters*, vol. 85, no. 18, pp. 3966-3969, 2000.
- [7] A. Alù, and N. Engheta, “Achieving transparency with plasmonic and metamaterial coatings,” *Phys. Rev. E* 72, 016623 (2005).
- [8] P. Ball, “Engineers devise invisibility shield,” *Nature News*, Feb. 28, 2005, online at: <http://emboj.org/news/2005/050228/full/news050228-1.html>.
- [9] A. Alù, and N. Engheta, “Plasmonic materials in transparency and cloaking problems: mechanism, robustness, and physical insights,” *Optics Express* 15, 3318-3332 (2007).
- [10] M. G. Silveirinha, A. Alù, and N. Engheta, “Parallel-plate metamaterials for cloaking structures,” *Phys. Rev. E* 75, 036603 (2007).
- [11] A. Alù, and N. Engheta, “Cloaking and transparency for collections of particles with metamaterial and plasmonic covers,” *Optics Express* 15, 7578-7590 (2007).
- [12] A. Alù, and N. Engheta, “Multi-frequency optical invisibility cloaks with layered plasmonic shells,” *Phys. Rev. Lett.* 100, 113901 (2008).



- [13] A. Alù, and N. Engheta, “Robustness in design and background variations in metamaterial/plasmonic cloaking,” *Radio Science* 43, RS4S01 (2008).
- [14] M. G. Silveirinha, A. Alù, and N. Engheta, “IR and optical cloaking with plasmonic implants,” *Phys. Rev. B*, in press (2008).
- [15] A. Alù, and N. Engheta, “Dispersion characteristics of metamaterial cloaking structures,” *Electromagnetics*, in press (2008).
- [16] J. B. Pendry, D. Schurig, and D. R. Smith, “Controlling electromagnetic fields,” *Science* 312, 1780-1782 (2006).
- [17] D. Schurig, J. J. Mock, B. J. Justice, S. A. Cummer, J. B. Pendry, A. F. Starr, and D. R. Smith, “Metamaterial electromagnetic cloak at microwave frequencies,” *Science* 314, 977-980 (2006).
- [18] S. A. Cummer, B. I. Popa, D. Schurig, D. R. Smith, and J. B. Pendry, “Full-wave simulations of electromagnetic cloaking structures,” *Phys. Rev. E* 74, 036621 (2006).
- [19] D. Schurig, J. B. Pendry, and D. R. Smith, “Calculation of material properties and ray tracing in transformation media,” *Optics Express* 14, 9794-9804 (2006).
- [20] S. A. Cummer, and D. R. Schurig, “One path to acoustic cloaking,” *New Journal of Physics* 9, 45 (2007).
- [21] D. Torrent, and J. Sánchez-Dehes, *New Journal of Physics* **10**, 063015 (2008).
- [22] F. Zolla, S. Guenneau, A. Nicolet, and J. B. Pendry, “Electromagnetic analysis of cylindrical invisibility cloaks and the mirage effect,” *Optics Letters* 32, 1069-1071 (2007).

- [23] W. Cai, U. K. Chettiar, A. V. Kildishev, and V. M. Shalaev, "Optical cloaking with metamaterials," *Nature Photonics* 1, 224-227 (2007).
- [24] W. Cai, U. K. Chettiar, A. V. Kildishev, V. M. Shalaev, *Opt. Expr.* 16, 5444 (2008).
- [25] W. Cai, U. K. Chettiar, A. V. Kildishev, G. W. Milton, and V. M. Shalaev, "Nonmagnetic cloak with minimized scattering," *Appl. Phys. Lett.* 91, 111105 (2007).
- [26] Z. Jacob and E. E. Narimanov, "Semiclassical description of non magnetic cloaking," *Opt. Express* 16, 4597-4604 (2008)
- [27] A. V. Kildishev, and V. M. Shalaev, "Engineering space for light via transformation optics," *Opt. Lett.* 33, 43 (2008)
- [28] U. Leonhardt, "Optical conformal mapping," *Science* 312, 1777-1780 (2006).
- [29] A. Hendi, J. Henn, and U. Leonhardt, "Ambiguities in the scattering tomography for central potentials," *Phys. Rev. Lett.* 97, 073902 (2006).
- [30] U. Leonhardt, "Notes on conformal invisibility devices," *New Journal of Physics* 8, 118 (2006).
- [31] U. Leonhardt, "General relativity in electrical engineering," *New Journal of Physics* 8, 247 (2006).
- [32] G. W. Milton, and N. A. Nicorovici, "On the cloaking effects associated with anomalous localized resonance," *Proc. R. Soc. Lond. A: Math. Phys. Sci.* 462, 3027-3059 (2006).
- [33] G. W. Milton, N. A. Nicorovici, R. C. McPhedran, and V. A. Podolskiy, "A proof of superlensing in the quasistatic regime, and limitations of superlenses in this

- regime due to anomalous localized resonance,” *Proc. Roy. Soc. Lond.* **461**, 3999-4034 (2005).
- [34] G. W. Milton, M. Briane, and J. R. Willis, “On cloaking for elasticity and physical equations with a transformation invariant form,” *New Journal of Physics* **8**, 248 (2006).
  - [35] N. A. P. Nicorovici, G. W. Milton, R. C. McPhedran, and L. C. Botten, “Quasistatic cloaking of two-dimensional polarizable discrete systems by anomalous resonance,” *Optics Express* **15**, 6314-6323 (2007).
  - [36] N. A. P. Nicorovici, R. C. McPhedran, and G. W. Milton, *Phys. Rev. B*, Vol. 49, No. 12, pp. 8479-8482, March 1994
  - [37] X. Zhou and G. Hu, “Design for electromagnetic wave transparency with metamaterials,” *Phys. Rev. E* **74**, 026607 (2006).
  - [38] M. Bashevoy, V. Fedotov, and N. Zheludev, “Optical whirlpool on an absorbing metallic nanoparticle,” *Optics Express* **13**, 8372-8379 (2005).
  - [39] A. Greenleaf, Y. Kurylev, M. Lassas, and G. Uhlmann, “Full-wave invisibility of active devices at all frequencies,” *Comm. in Math. Phys.* **275**, 749-789 (2007).
  - [40] H. Chen, X. Jiang, and C. T. Chan, “Extending the bandwidth of electromagnetic cloaks,” *Phys. Rev. B* **76**, 241104 (2007).
  - [41] M. Yan, Z. Ruan, and M. Qiu, “Cylindrical invisibility cloak with simplified material parameters is inherently visible,” *Phys. Rev. Lett.* **99**, 233901 (2007).
  - [42] Z. Ruan, M. Yan, C. W. Neff, and M. Qiu, “Ideal cylindrical cloak: perfect but sensitive to tiny perturbations,” *Phys. Rev. Lett.* **99**, 113903 (2007).
  - [43] D. A. B. Miller, “On perfect cloaking,” *Optics Express* **14**, 12457-12466 (2006).

- [44] A. Hakansson, "Cloaking of objects from electromagnetic fields by inverse design of scattering optical elements," *Optics Express* 15, 4328-4334 (2007).
- [45] C. Rohde, K. Hasegawa, and M. Deutsch, "Plasmon-assisted transparency in metal–dielectric microspheres," *Optics Letters* 32, 415-417 (2007).
- [46] D. Torrent and J. Sánchez-Dehesa, "Acoustic Cloaking in two dimensions: a feasible approach," *New J. Phys.* 10, 063015 (2008)
- [47] A. Yaghjian, and S. Maci, "Alternative derivation of electromagnetic cloaks and concentrators," online at: *arXiv:0710.2933v4*.
- [48] P. Alitalo, O. Luukkonen, L. Jylhä, J. Venermo, and S.A. Tretyakov, "Transmission-line networks cloaking objects from electromagnetic fields," *IEEE Trans. Antennas Propagation* 56, 416-424 (2008).
- [49] V. A. Fedotov, P. L. Mladyonov, S. L. Prosvirnin, and N. I. Zheludev, "Planar electromagnetic metamaterial with a fish scale structure," *Phys. Rev. E* 72, 056613 (2005).
- [50] Y. Zhao, C. Argyropoulos, and Y. Hao, "Full-wave finite-difference time-domain simulation of electromagnetic cloaking structures," online at: *arXiv:0803.2063v1*.
- [51] D. H. Kwon, and D. H. Werner, "Restoration of antenna parameters in scattering environments using electromagnetic cloaking," *Appl. Phys. Lett.* 92, 113507 (2008).
- [52] R. C. Rumpf, M. A. Fiddy, and M. E. Testorf, "Design of generalized invisible scatterers," *Optics Express* 15, 4735-4744 (2007).
- [53] P. Hertz, "Die Bewegung eines Elektrons unter dem Einflusse einer stets gleich gerichteten Kraft," *Math. Ann.* 65, 1 (1908).

- [54] G. A. Schott, *Phil. Mag. Suppl.* 7, 15, 752 (1933).
- [55] D. Bohm and M. Weinstein, "The self-oscillations of a charged particle," *Phys. Rev.* 74, 1789 (1948).
- [56] G. H. Goedecke, "Classically radiationless motions and possible implications for quantum theory," *Phys. Rev.* 135, 282-288 (1964).
- [57] M. Kerker, "Invisible bodies," *J. Opt. Soc. Am.* 65, 376-379 (1975).
- [58] H. Chew, and M. Kerker, "Abnormally low electromagnetic scattering cross sections," *J. Opt. Soc. Am.* 66, 445-449 (1976).
- [59] W. K. Kahn, and H. Kurss, "Minimum-scattering antennas," *IEEE Trans. Antennas Propagat.* Vol. 13, pp. 671-675, Sept. 1965.
- [60] N. Bleistein, and J. K. Cohen, "Nonuniqueness in the inverse source problem in acoustics and electromagnetics," *J. Mathem. Phys.* 18, 194-201 (1977).
- [61] N. G. Alexopoulos, and N. K. Uzunoglu, "Electromagnetic scattering from active objects: invisible scatterers," *Applied Optics* 17, 235-239 (1978).
- [62] A. J. Devaney, "Nonuniqueness in the inverse scattering problems," *J. Math. Phys.* 19, 1526-1531 (1978).
- [63] A. J. Devaney, and G. C. Sherman, "Nonuniqueness in inverse source and scattering problems," *IEEE Trans. Antennas Propag.* 30, 1034-1042 (1982).
- [64] A. Greenleaf, M. Lassas, and G. Uhlmann, "On nonuniqueness for Calderons inverse problem," *Math. Res. Let.* 10, 685-693 (2003).
- [65] A. Greenleaf, M. Lassas, and G. Uhlman, "Anisotropic conductivities that cannot be detected by EIT," *Physiol. Meas.* 24, 413 (2003)

- [66] P. S. Kildal, A. A. Kishk, and A. Tengs, "Reduction of forward scattering from cylindrical objects using hard surfaces," *IEEE Trans. Antennas Propagat.* 44, 1509-1520 (1996).
- [67] B. J. Hoenders, "Existence of invisible nonscattering objects and nonradiating sources," *J. Opt. Soc. Am. A* 14, 262-266 (1997).
- [68] A. D. Boardman, K. Marinov, N. Zheludev, and V. A. Fedotov, "Dispersion properties of nonradiating configurations: Finite-difference time-domain modeling," *Phys. Rev. E* 72, 036603 (2005).
- [69] C. F. Bohren, and D. R. Huffman, *Absorption and Scattering of Light by Small Particles*, Wiley, New York, 1983.
- [70] A. Alù, and N. Engheta, "Pairing an epsilon-negative slab with a mu-negative slab: anomalous tunneling and transparency," *IEEE Trans. Antennas Propagat.* 51, 2558 (2003).
- [71] S. Tachi, "Telexistence and Retro-reflective Projection Technology (RPT)," Proceedings of the 5th Virtual Reality International Conference (VRIC2003) pp.69/1-69/9, Laval Virtual 2003, France, May 13-18, 2003.
- [72] M. Inami, N. Kawakami, Y. Yanagida, T. Maeda and S. Tachi, "Method and Device for Providing Information," US Patent no. 6,341,869, 2002.
- [73] A. Alù, and N. Engheta, "Polarizabilities and effective parameters for collections of spherical nano-particles formed by pairs of concentric double-negative (DNG), single-negative (SNG) and/or double-positive (DPS) metamaterial layers," *Journal of Applied Physics* 97, 094310 (2005) [erratum in *Journal of Applied Physics* 99, 069901 (2006)].

- [74] C. H. Papas, Theory of Electromagnetic Wave Propagation (Dover Publications, New York, U.S.A., 1988).
- [75] J. D. Jackson, Classical Electrodynamics (Wiley, New York, U.S.A., 1998).
- [76] M. Abramowitz, and I. A. Stegun (editors), Handbook of Mathematical Functions with Formulas, Graphs, and Mathematical Tables (Dover, New York, U.S.A., 1972).
- [77] A. Alù, M. G. Silveirinha, A. Salandrino, and N. Engheta, “Epsilon-near-zero metamaterials and electromagnetic sources: tailoring the radiation phase pattern,” Physical Review B, Vol. 75, 155410 (13 pages), April 11, 2007.
- [78] A. Alù, F. Bilotti, N. Engheta, and L. Vegni, “Metamaterial Covers over a Small Aperture,” IEEE Transactions on Antennas and Propagation, Vol. AP-54, No. 6, pp. 1632-1643, June 2006.
- [79] M. Silveirinha, N. Engheta, Phys. Rev. Lett. 97, 157403, (2006).
- [80] P. B. Johnson, and R. W. Christy, Phys. Rev. B 6, 4370 (1972).