Pricing of Contingent Convertibles

HyeYoon Jung,
Wharton School,
University of Pennsylvania

Wharton Research Scholars Project*
Advisor: Dr. Philipp Illeditsch

April, 2012

*I am grateful to Professor Philipp Illeditsch for his mentorship and guidance on this project. I further want to thank Professor Martin Asher for his constant support for Wharton Research Scholars Program.

Abstract

This paper discusses the pricing of Contingent Convertible bonds (CoCos) with stock price triggers. CoCos are a new kind of hybrid securities that aim to provide a capital buffer for banks in times of financial distress. They are debt securities during periods of economic stability, but automatically convert into equity when a predetermined trigger is breached. Therefore, CoCos are attractive from a regulatory perspective, and several regulators have already shown an interest in using them to manage financial crisis. The fair values of CoCos are driven by their structures, and the goal of this paper is to price CoCos with stock price triggers that have varying structures in terms of the trigger level, conversion ratios, and their maturity.

This paper presents first the general form of the price and credit spread of CoCos without modeling stock price dynamics. Then, assuming the Black-Scholes model, we provide two explicit pricing formulas for CoCos. Because CoCos combine debt-like and equity-like features, they are priced using the credit derivatives (reduced form) and equity derivatives approaches. In addition to the analytical formulas presented herein, pricing by Monte Carlo simulation is also shown. In order to examine the suitability of the Black-Scholes assumptions, the formulas used in this study are applied to the CoCos issued by Credit Suisse. Because the market trigger, implied by the formulas, is associated with a constant accounting trigger, it is expected to be constant over time.

The comparative statics of the formulas show that the mathematical structures of the formulas explain the economic structure of CoCos. However, we find that the formula in the equity derivatives approach is more accurate than that in the credit derivatives approach because of its more realistic treatment of cash flow. Its accuracy is confirmed by Monte Carlo simulation, as the estimated confidence interval includes the price evaluated using the equity derivatives approach. If the interest rate is equal to the dividend yield, we find that the two analytical formulas provide the same price. The empirical analysis of the CoCos of Credit Suisse demonstrates that the Black-Scholes assumptions are empirically unreasonable for pricing CoCos, because the implied market trigger is volatile over time.

Given that the constant volatility assumption of the Black-Scholes model is empirically unreasonable, this paper suggests the stochastic volatility model (Heston model) to be a suitable alternative for modeling stock price dynamics, because it produces a more realistic fat-tail distribution of stock returns. Thus, the pricing under the Heston model is expected to show a constant implied trigger over time.

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1 Introduction

1.1 Motivation

Contingent Convertible bonds (CoCos) are debt securities that automatically convert into equity when a predetermined trigger is breached. They are hybrid securities that for the following reasons have emerged as a potential solution to the moral hazard problem and as an instrument to prevent financial institutions from becoming "too big to fail"[9]. If a certain trigger is breached, the automatic conversion of debt obligations into fresh equity allows the issuer to improve its capital structure. This increased level of equity helps issuers absorb future losses, while the reduced debt helps them raise private capital. In cases where issuers are not able to obtain private funding, the subordinated bondholders assume the liability for the losses, meaning that governments would not need to bail them out. With no anticipation of government bailouts, issuers would be incentivized to de-risk and de-leverage. For these reasons, several regulators have shown an interest in adding CoCos to their supervisory tools for crisis management [9]. For instance, CoCos or CoCo-like securities have been issued by the Lloyds Banking Group, Rabobank, and Credit Suisse.[1]

CoCos combine debt-like and equity-like features as do existing hybrid securities such as convertible bonds (CBs) and reverse convertible notes (RCNs). However, CoCos are a new kind of securities that are attractive from the regulators' perspective.

Conventional CBs give holders the right to exercise an option to convert, whereas the conversions of CoCos occur automatically once the predetermined trigger is breached. Hence, conventional CBs have limited downsides and unlimited upsides, whereas CoCos have limited upsides and unlimited downsides. The conversions of RCNs occur at the discretion of issuers on the maturity date, because RCNs pay holders the minimum value of the shares and the value of the debt at maturity. By contrast, the conversions of CoCos can occur anytime if and only if the trigger is breached.

Most existing hybrid capital instruments did not absorb losses as they were designed to do during the recent crisis because of the reluctance of banks to send negative signals, the forbearance of regulators, and/or capital injections by governments [9]. However, the automatic conversion feature with no fixed maturity of CoCos makes them serve as a regulatory capital buffer for issuers suffering financial stress.

1.2 Structure of CoCos

Designing CoCos involves deciding on the key structuring points including the type and level of the trigger, the conversion rate, and the maturity. These points all play a role in

[1] Lloyds Banking Group (LLOY) Plc sold 8.5 billion pounds of CoCos in November 2009 that would be converted to equity if core capital drops below 5 percent. Rabobank Nederland issued \$2 billion of the securities in January with an 8 percent equity capital ratio trigger. Rabobank paid an 8.375 percent coupon. Credit Suisse Group AG (CSGN), Switzerlands sold \$2 billion of Tier 2 CoCos with a 7 percent Core Tier 1 ratio trigger in February and announced plans to issue a further \$6.2 billion of Tier 1 securities in October 2013. Credit Suisse offered investors a 7.875 percent coupon on its Tier 2 securities and plans to offer 9 percent and 9.5 percent coupons on its Tier 1 CoCos.

determining the value of the security and in assessing how effective it meets its objectives.

The three types of triggers that have been issued or discussed in the previous literature are market-based triggers, regulator-based triggers, and capital-based triggers. Each has its own benefits and drawbacks.

- **Market-based Trigger**
  Market-based triggers use share prices or CDS spreads as the trigger variable. The transparency of such triggers is a significant benefit because it allows investors to model the conversion and monitor the movement of the trigger variable. Their main drawback is that market sentiment or market manipulation can cause unnecessary triggering.

- **Regulator-based Trigger**
  Under regulator-based triggers, regulators decide when and how to convert; therefore, such triggers offer regulators a certain degree of flexibility in the face of a financial crisis. However, regulator-based triggers can seem to be opaque and hard to model from an investor's perspective. Further, it is likely that regulators would want to delay the pronouncement of a conversion event if they judge that the prevailing problems will be short-lived.

- **Capital-based Trigger**
  Capital-based triggers use accounting ratios that can indicate the viability of a bank. Although it is possible for investors to model the likelihood of conversion based on disclosure, ratios such as Tier 1 capital ratios are not continuously available as stock prices are. In addition, financial statements do not always precisely reflect reality; for example, Bear Sterns, Lehman Brothers, Wachovia, and Merrill Lynch all had regulatory capital ratios far above the minimum level of 8%[10]. The benefits of capital-based triggers are that they are not subject to market manipulation as market-based triggers are and that they eliminate the uncertainty deriving from regulatory discretion.

In addition to the types of triggers, the levels of triggers determine the nature of the security. CoCos that have a high trigger (i.e., a trigger level that is close to the current level) are likely to be converted sooner than those are that have a low trigger. Therefore, high-trigger CoCos act as a preventative buffer, while low-trigger CoCos can be likened to insurance in the wake of a catastrophic systematic crisis. High-trigger CoCos are more expensive compared with low-trigger CoCos because their risk of conversion is higher.

Another factor that characterizes CoCos is their conversion ratio, which is the number of shares received per converted bond. A lower conversion rate reduces the incentive for stock price manipulation, because if the number of shares that investors receive at conversion is fixed, it is not worth influencing prices in order to provoke conversion artificially [7]. A high conversion ratio will thus incentivize shareholders and managers to prevent conversion because it will heavily dilute the stock.

In summary, the fair values of CoCos are driven by their structures. Thus, the goal of this paper is to price CoCos that have diverse structures.

1.3 Illustration of Pricing Problem

This paper covers the pricing of CoCos with a stock price trigger. Consider a zero-coupon CoCo with face value $\$D$ and maturity T. Define a trigger event as the issuer's stock price falling below $\$S^*$. If the stock price stays above $\$S^*$ over the entire lifetime of CoCo, the CoCo holders get $\$D$ at year T. If the stock price hits $\$S^*$ at τ, anytime before maturity T, CoCo holders get N shares of issuer's equity per one unit of bond. As the stock price on the day that trigger event happens will be $\$S^*$, the total value of equity shares to the holders is $\$S^* \times N$.

Suppose that the full amount of face value gets converted to the shares of equity with equivalent value. These CoCos are called "full CoCos."[2] For the full CoCos, the implied conversion price, P_c is determined as:

$$P_c = \frac{D}{N} \tag{1}$$

If either of N or P_c is given, the other can be determined from relationship (1). The cash flows of this CoCo are summarized in the following table:

	t	τ	T
No Trigger Event	-Cost		$+D$
Trigger is breached @ τ if the stock price falls below $\$S^*$	-Cost	+Value of Equity Shares $=N \times S^*$	

The goal is to find the credit spread CS, the premium over the risk-free interest rate r that investors require as a compensation for the risk of conversion. Therefore, the credit spread is defined as continuously compounded yield ρ of the CoCo minus the continuously compounded risk free rate:

$$CS = \rho - r$$

Here is the summary of common notations:
D : Face value of CoCo
S : Stock price
S^* : Trigger Price
N : Number of shares to CoCo holders at conversion
P_c : Implied conversion per share price at conversion=D/N
T : Maturity of CoCo
τ : Time of a trigger event
r : Risk free interest rate, continuously compounded
$CoCo$: Value of CoCo
CS : Credit Spread
ρ : Yield of CoCo, continuously compounded

[2] It is on debate whether CoCos should be "full CoCos", or only a fraction of the face value should get converted to equity shares [10].

1.4 Pricing Methodology

CoCos combine features of fixed income and equity, and thus can be valued by using the credit derivatives approach or the equity derivatives approach. In the credit derivatives approach, CoCos are considered as defaultable bonds, and the final payoff is treated as cash. In equity derivatives approach, CoCos are priced by finding the value of equity derivatives portfolio that replicates the final payoff, which is treated as shares of equity.[3] Using the two approaches, the explicit formulas for the price and the credit spread of CoCos can be derived.

The reduced form credit derivatives approach uses an abstract random process to match the behavior of credit derivatives prices. The credit spreads are determined by exogenously specified trigger probabilities and recovery rates. The equity derivatives approach prices a portfolio of knock-in-forward and bond that approximately replicates the price of CoCos.

In addition to the analytical formulas, the price of CoCos can be estimated by simulating the stock price paths that follow the geometric Brownian motion.

1.5 Outline of the Paper

The paper is organized as follows. In section 2, the general pricing formula for CoCo with no assumption on the stock price dynamics is presented. In section 3, under the Black-Scholes assumptions, CoCos are priced by using the credit derivatives approach and the equity derivatives approach. To derive the two explicit formulas, we assume that the cash flow of CoCo occurs at maturity T and not when conversion is triggered. Then, we relax this assumption and use a Monte Carlo simulation to find the credit spread. In section 4, we apply the pricing methods to the CoCo issued by Credit Suisse and examine if the Black-Scholes assumptions are empirically reasonable. In section 5, we suggest an alternative assumption for the stock price dynamics by employing Heston model.

2 General Pricing Formula

2.1 General Formula for the Price

The price of CoCo is the risk-neutral expectation of the sum of its discounted future cash flows. The payoff of CoCo would be equal to the value of debt if conversion never happens before maturity. If a trigger event occurs, then the payoff would be equal to the value of shares. Therefore, without making any assumption on the stock price dynamics, the price of CoCo at time t has the following form:

$$CoCo_t = E^Q[NS^* e^{-\int_t^{\tau} r(s)ds}\mathbf{1}\{\tau \leq T\} + De^{-\int_t^{T} r(s)ds}\mathbf{1}\{\tau > T\}] \tag{2}$$

[3] These two approaches were taken by Jan De Spiegeleer and Wim Schoutens 2011 [10].

If we assume that the interest rate is constant, it reduces to :

$$CoCo_t = E^Q[e^{-r(\tau-t)}NS^*\mathbf{1}\{\tau \le T\} + De^{-r(T-t)}\mathbf{1}\{\tau > T\}] \tag{3}$$

$$= NS^*E^Q[e^{-r(\tau-t)}\mathbf{1}\{\tau \le T\}] + De^{-r(T-t)}E^Q[\mathbf{1}\{\tau > T\}] \tag{4}$$

The term $e^{-r(\tau-t)}$ has to stay inside the expectation because $e^{-r(\tau-t)}$ and $\mathbf{1}\{\tau \le T\}$ are not independent. To avoid this complication, we will assume that the cash flow NS^* occurs at T for the derivation of the explicit formulas.[4] Hence, the price of CoCos can be expressed in terms of risk neutral probability of trigger event.

Denote the risk-neutral probability that a trigger event happens as:

$$P^Q = E^Q[\mathbf{1}\{\tau \le T\}].$$

Then, the price of CoCo is given by:

$$CoCo_t = e^{-r(T-t)}\left(NS^*P^Q + D(1-P^Q)\right) \tag{5}$$

Because N is equal to D/P_c for the full CoCos, (5) becomes:

$$= e^{-r(T-t)}\left(\frac{D}{P_c}S^*P^Q + D(1-P^Q)\right) \tag{6}$$

and hence:

$$\boxed{CoCo_t = De^{-r(T-t)}\left(1 - P^Q\left(1 - \frac{S^*}{P_c}\right)\right)} \tag{7}$$

We will refer to the equation (7) as the general price formula.

2.2 General Formula for the Credit Spread

The yield of CoCo is:

$$\rho = -\frac{1}{T-t}\ln\frac{CoCo_t}{D} \tag{8}$$

$$= -\frac{1}{T-t}\ln\frac{CoCo_t}{D}$$

Plugging the equation (7) into (8) and rearranging the terms, the credit spread $CS = \rho - r$ is expressed as:

$$\boxed{CS = -\frac{1}{T-t}\ln\left(1 - P^Q\left(1 - \frac{S^*}{P_c}\right)\right)} \tag{9}$$

[4]Later in section 3.3, Monte Carlo simulation, we will assume the cash flow occurs at τ.

We refer to this equation (9) as the general credit spread formula. By first order Taylor series approximation,

$$\ln(1-x) = \sum_{n=1}^{\infty} \frac{(-x)^n}{n} \approx -x,$$

for $|x| < 1$, (9) reduces to:

$$CS = \frac{1}{T-t} P^Q \left(1 - \frac{S^*}{P_c}\right) \tag{10}$$

Suppose the trigger τ is the first jump of a Poisson process with intensity parameter λ. Then, the probability of a trigger event happening in the next instant dt is λdt, and the probability that a trigger event happening in the next T years is $1 - e^{-\lambda T}$. Hence,

$$P^Q = 1 - e^{-\lambda(T-t)}$$

and the credit spread given in equation (10) becomes:

$$CS = \frac{1}{T-t}\left(1 - e^{-\lambda(T-t)}\right)\left(1 - \frac{S^*}{P_c}\right) \tag{11}$$

Using Taylor series approximation,

$$1 - e^{-\lambda h} \approx \lambda h,$$

we get an approximating formula for the credit spread:

$$\boxed{CS \approx \lambda \left(1 - \frac{S^*}{P_c}\right)} \tag{12}$$

We refer to the equation (12) as the approximating credit spread formula.

The approximating credit spread formula for CoCos (12) is analogous to that of a defaultable zero-coupon bond, if λ is default intensity and $\frac{S^*}{P_c}$ is recovery rate of the defaultable bond.[5] If a trigger event happens, the value to CoCo holders upon a trigger event is NS^*. Therefore, the loss is the difference between the debt value they are owed and the value of the shares:

$$\begin{aligned} \text{Loss} &= D - NS^* \\ &= D - \left(\frac{D}{P_c}\right) S^* \\ &= D\left(1 - \frac{S^*}{P_c}\right) \end{aligned}$$

Effectively, $\frac{S^*}{P_c}$ is the amount that CoCo holders are getting as a fraction of the amount they are owed, which is analogous to the recovery rate in case of defaultable bonds.

[5]This approach was taken by Jan De Spiegeleer and Wim Schoutens 2011

3 Pricing under the Black-Scholes Model

In this section, we price CoCos under the Black-Scholes assumptions. The Black-Scholes assumptions are that the underlying stock pays a constant known dividend yield q and follows the diffusion process:

$$\frac{dS}{S} = (r - q)dt + \sigma dB^Q$$

where B^Q is a Brownian motion under the risk-neutral measure. The volatility σ and the continuously compounded risk-free rate r are known and constant. Assume that there are no transaction costs or taxes, and it is possible to short-sell costlessly and borrow at the risk-free rate.

3.1 Credit Derivatives Approach

3.1.1 Explicit Formula

We model the trigger event as the first jump of a Poisson process with intensity parameter λ. Hence, the trigger probability is:

$$P^Q = 1 - e^{-\lambda(T-t)} \tag{13}$$

The probability of trigger event is the risk-neutral probability that stock price hits S^* during the life of the CoCo. This probability is given by the first-hitting-time equation (15) under Black-Scholes [10]. Therefore, using the general credit spread formula (9), the credit spread is:

$$CS = -\frac{1}{T-t} \ln\left(1 - P^Q \left(1 - \frac{S^*}{P_c}\right)\right) \tag{14}$$

where P^Q is specified as:

$$P^Q = \mathbf{N}\left(\frac{\ln(S^*/S) - \nu(T-t)}{\sigma\sqrt{T-t}}\right) + (S^*/S)^{2\nu/\sigma^2}\mathbf{N}\left(\frac{\ln(S^*/S) + \nu(T-t)}{\sigma\sqrt{T-t}}\right) \tag{15}$$

$$\nu = r - q - \sigma^2/2$$

3.1.2 Comparative Statics

Base case: Consider a 10-year ($T = 10$) zero coupon CoCo that converts into equity if stock price reaches \$50 ($S^* = 50$). The current stock price S is \$100 and the conversion ratio is 1 share per CoCo ($N = 1$). Hence the implied conversion price P_c is \$100. Suppose the volatility σ of the stock is 0.3, and the stock pays dividends at continuously compounded rate of 0.04 ($q = 0.04$). Assume that the risk free rate r is 0.04. The approximating credit spread formula (12) predicts the credit spread of base-case CoCo as

0.0493, and the exact credit spread formula (9) gives 0.0376:

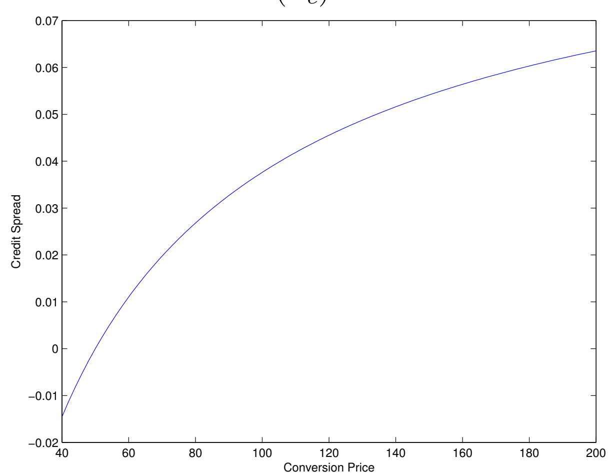
$$CS \approx \lambda(1 - \frac{S^*}{P_c}) = 0.0986(1 - \frac{50}{100}) = 4.93\%$$

$$CS = -\frac{1}{T-t}\ln\left(1 - P^Q(1 - \frac{S^*}{P_c})\right) = CS = -\frac{1}{10}\ln\left(1 - (0.4602)(1 - \frac{50}{100})\right) = 3.76\%$$

In the approximating formula (12), the credit spread is determined by the two factors, the intensity λ and the recovery rate $R = S^*/P_c$, and this simple formula provides an easy way to predict how credit spread would change as one of the parameters changes and all other parameters are fixed. Using the approximating formula (14) and the relationship between λ and P^Q, the credit spread is:

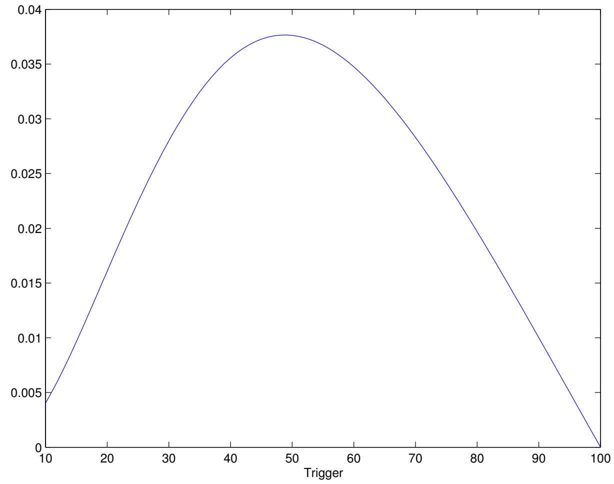
$$\begin{aligned} CS &\approx \lambda(1 - \frac{S^*}{P_c}) \\ &= -\frac{\ln(1-P^Q)}{T-t}(1 - \frac{S^*}{P_c}) \end{aligned} \tag{16}$$

- Credit Spread vs. Conversion Price (P_c)



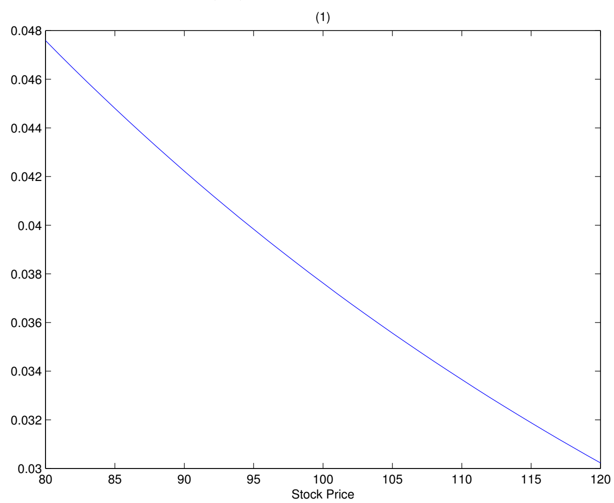
The conversion price affects only the recovery rate $R = S^*/P_c$. Mathematically, from equation (18), the credit spread is increasing function of P_c. Economically, if the ratio S^*/P_c is 1, that means CoCo holder will receive the amount they are owed regardless of a trigger event taking place. Therefore, the credit spread is zero. If the ratio S^*/P_c is less than 1, credit spread is negative. This implies that the conversion price cannot be set as being less than the stock price at conversion.

- Credit Spread vs. Trigger Price (S^*)



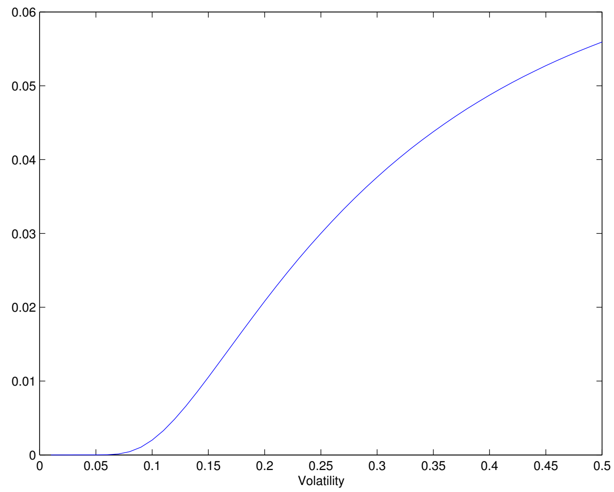
If the trigger price is very low, it is unlikely that the stock price falls below the very low S^*, and this leads to a low spread. On the other hand, if S^* is low, the investors receive shares with low value, it increases the spread. The credit spread is maximized when both the probability of trigger event happening and the loss when trigger event occurs are significant.

- Credit Spread vs. Stock Price (S)



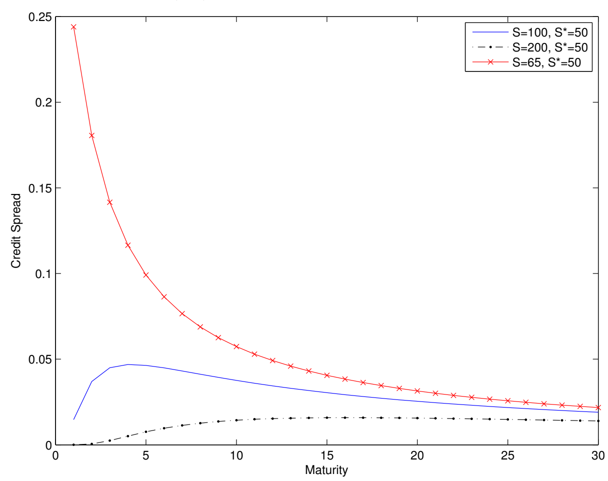
The stock price affects λ. As the stock price goes up, it is less likely that it falls below S^*. Consequently, the credit spread goes down monotonically as the stock price goes up. Since the stock price never goes below zero, the lower bound of credit spread is zero.

- Credit Spread vs. Stock Price Volatility (σ)



As the volatility of stock price goes up, there is more chance that the stock price falls below S^*. Therefore, credit spread increases monotonically with respect to the volatility.

- Credit Spread vs. Maturity (T)



Mathematically, the time to maturity affects λ, and the shape of the term structure comes from the function,

$$\lambda = -\frac{\ln(1 - P^Q)}{T - t},$$

where P^Q is a monotonically increasing function with respect to the maturity.

Economically, the term structure of credit spread depends on how far the current stock price is from the trigger. The ratio of S^*/S could represent the relative distance. If a company is performing well, (low S^*/S), the overall credit spread would be low, but it increases with respect to the maturity because the longer the time, the more the uncertainty and the risk of conversion. If a company is on the edge of conversion (high S^*/S), the credit spread monotonically decreases with respect to maturity because the more time company has, the more chance of reducing the risk it has. When S^*/S is 1/2, for the short-term CoCos, it is less likely to be triggered in a shorter period; therefore the term structure is upward sloping. However, for the long-term CoCos, it is less likely to be triggered in the longer horizon as there is more chance that the company improves its performance in the longer time horizon.

In summary, the comparative statics shows that the mathematical structure of formula in the credit derivatives (16) match the economic structure of CoCos.

3.2 Equity Derivatives Approach

In equity derivatives approach, we price CoCos by finding a portfolio of equity derivatives that replicates the cash flow of CoCo. If the replicating portfolio produces the same cash flows as the CoCos', then the cost of the replicating portfolio must be the price of CoCo.

3.2.1 Explicit Formula

The payoff of CoCo in case of conversion resembles that of knock-in-forwards (KIFs) with barrier S^*. However, the two payoffs do not match perfectly. If the trigger is breached, the investors of KIFs are getting the forwards (with maturity T) at τ, whereas the investors of CoCos are getting the shares with per share value of $S_\tau = S^*$ at τ. Therefore, by entering into a forward contract instead of getting the shares immediately at τ, the value to the KIF investors is the future value of shares discounted at the cost of carry q. If a trigger event occurs long time before the maturity (i.e. $T - \tau$ is large) or if the cost of carry q is large, the error would be pronounced. Because the issuer is unlikely to pay out dividends in case of conversion, the payoff of CoCo can be approximated by the cost of portfolio with KIFs.

Assuming that the CoCo holders are receiving the forwards at conversion, the cash flow of CoCo at T is NS_T, if the trigger event occurs at $\tau < T$, and D if trigger event does not occur.

$$CoCo_T = \begin{cases} NS_T & \text{if } min(S_t)_{t\leq T} \leq S^* \\ D & \text{otherwise} \end{cases}$$

The cash flow can be rewritten as:

$$\begin{aligned} CoCo_T &= D + (DS_T/P_c - D)\mathbf{1}\{\min(S_t)_{t\leq T} \leq S^*\} \\ &= D + N(S_T - D/N)\mathbf{1}\{\min(S_t)_{t\leq T} \leq S^*\} \\ &= D + N(S_T - P_c)\mathbf{1}\{\min(S_t)_{t\leq T} \leq S^*\} \end{aligned}$$

The second term is zero if the minimum stock price during the lifetime of CoCo falls below the pre-specified trigger price S^*, and therefore it is equivalent to a long position in N knock-in-forwards with strike P_c and barrier S^*. Since the cash flow of portfolio consists of a long position in bond and a long position in N KIFs replicates the cash flow of CoCo, the price of CoCo at t must be approximately equal to the value of the portfolio at t.

$$CoCo_t = \text{value(Bond+KIFs)}_t$$

A long position in KIF is equivalent to a long position in knock-in-call (KIC) and a short position in knock-in-put (KIP):

$$KIF_T = KIC_T - KIP_T$$

where

$$KIC_T = \begin{cases} S_T - P_c & \text{if } S_T > P_c,\, min(S_t)_{,t \leq T} \leq S^* \\ 0 & \text{otherwise} \end{cases}$$

$$KIP_T = \begin{cases} P_c - S_T & \text{if } S_T < P_c,\, min(S_t)_{,t \leq T} \leq S^* \\ 0 & \text{otherwise} \end{cases}$$

First, the value of down-and-in call (KIC) can be computed as the following.
Define x as:

$$x = \begin{cases} 1 & \text{if } S_T > P_c,\, min(S_t)_{t \leq T} \leq S^* \\ 0 & \text{otherwise} \end{cases}$$

Then the value of the down-and-in call at maturity is $xS^* - xP_c$, and the value at time t is:

$$e^{-q(T-t)} S_0 Pr^V(x = 1) - e^{-r(T-t)} P_c Pr^R(x = 1)$$

where $V(t) = e^{qt}S(t)$ and $R(t) = e^{rt}$.

$$Pr^V(S_T \leq L) = \mathbf{N}(d_1)$$

$$Pr^R(S_T \leq L) = \mathbf{N}(d_2)$$

$$d_1 = \frac{\ln(S_t/S^*) + (r - q + \sigma^2/2)T}{\sigma\sqrt{(T-t)}} \tag{17}$$

$$d_2 = d_1 - \sigma\sqrt{(T-t)} \tag{18}$$

$$Pr^V(x = 1) = \left(\frac{S^*}{S_t}\right)^{2(r-q+\frac{1}{2}\sigma^2)/\sigma^2} \mathbf{N}(d_1')$$

$$Pr^R(x = 1) = \left(\frac{S^*}{S_t}\right)^{2(r-q-\frac{1}{2}\sigma^2)/\sigma^2} \mathbf{N}(d_2')$$

$$d_1' = \frac{\ln(S^*/S_t) + (r - q + \sigma^2/2)T}{\sigma\sqrt{(T-t)}} \tag{19}$$

$$d_2' = d_1' - \sigma\sqrt{(T-t)} \tag{20}$$

Therefore, the value of down-an-in call (KIC) at time t is:

$$KIC_t = S_t e^{-q(T-t)} \left(\frac{S^*}{S_t}\right)^{2(r-q+\frac{1}{2}\sigma^2)/\sigma^2} \mathbf{N}(d_1') - P_c e^{-r(T-t)} \left(\frac{S^*}{S_t}\right)^{2(r-q-\frac{1}{2}\sigma^2)/\sigma^2} \mathbf{N}(d_2') \quad (21)$$

In a similar way, the value of down-and-in put (KIP) is:

$$KIP_t = P_c e^{-r(T-t)} \mathbf{N}(-d_2) - S_t e^{-q(T-t)} \mathbf{N}(-d_1) \quad (22)$$

As a result, the value of CoCo is:

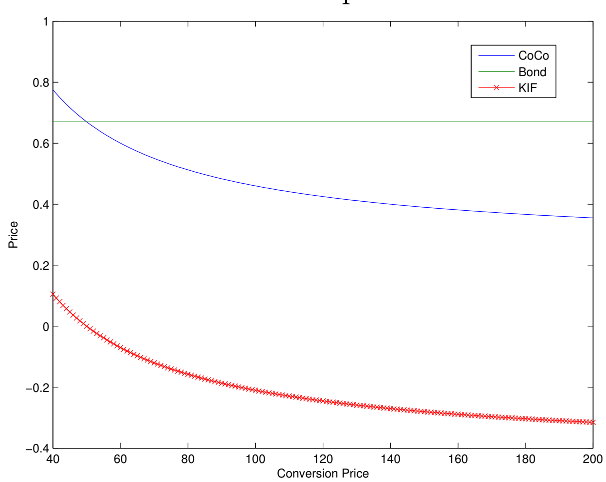
$$\boxed{CoCo_t = e^{-r(T-t)} D + N \times (KIC_t - KIP_t)} \quad (23)$$

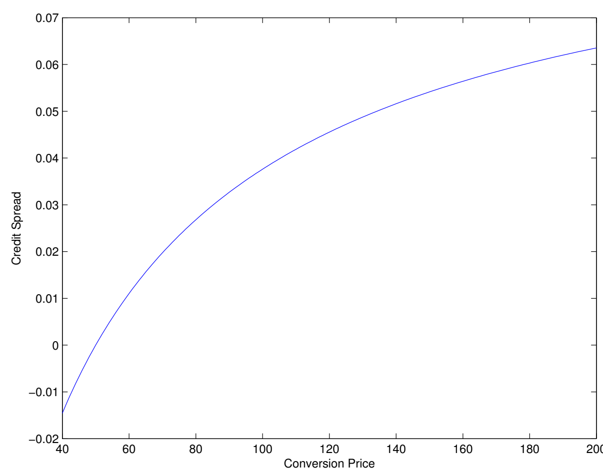
3.2.2 Comparative Statics

Base case: The pricing formula in equity derivatives approach (23) gives the credit spread on the base-case CoCo ($T = 10$, $S^* = 50$, $S = 100$, $N = 1$, \$100, P_c=100, $\sigma = 0.3$, $q = 0.04$, $r = 0.04$) of 0.0376.

In equity derivatives approach, the price of CoCo can be decomposed into the value of bond and the value of KIFs. Since the value of bond would not change with respect to the parameters, P_c, S^*, S, and σ, the change in CoCo price depends solely on the value of KIFs. The price of KIFs depend on NF, where F represents the forward price and the probability that the stock price hits barrier S^* during the lifetime of CoCo.

- Price and Credit Spread vs. Conversion Price (P_c)

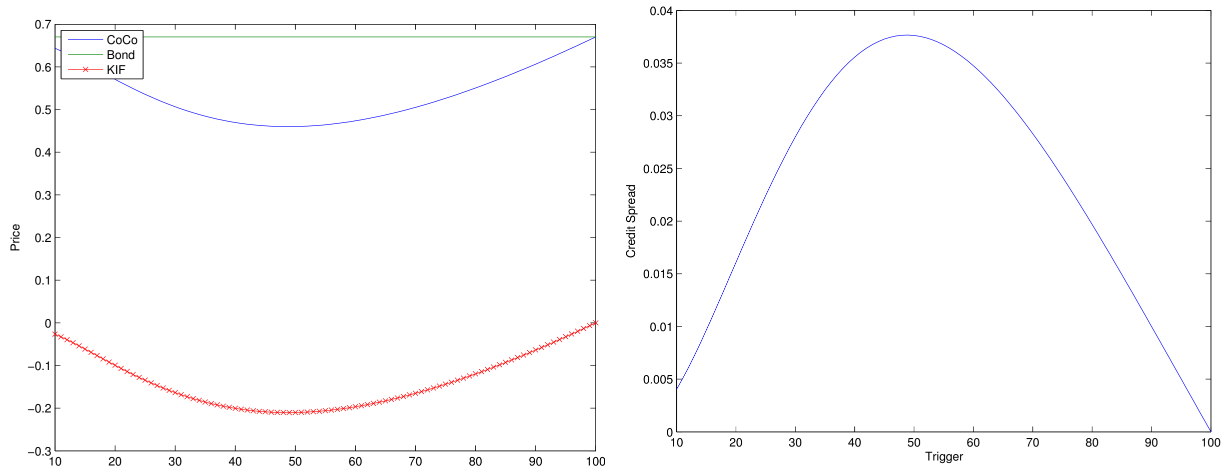




The conversion price determines the number of KIFs in the replicating portfolio. As the conversion price goes up, CoCo holders will get less number of KIFs, and therefore, the price of CoCo monotonically decreases as conversion price increases.

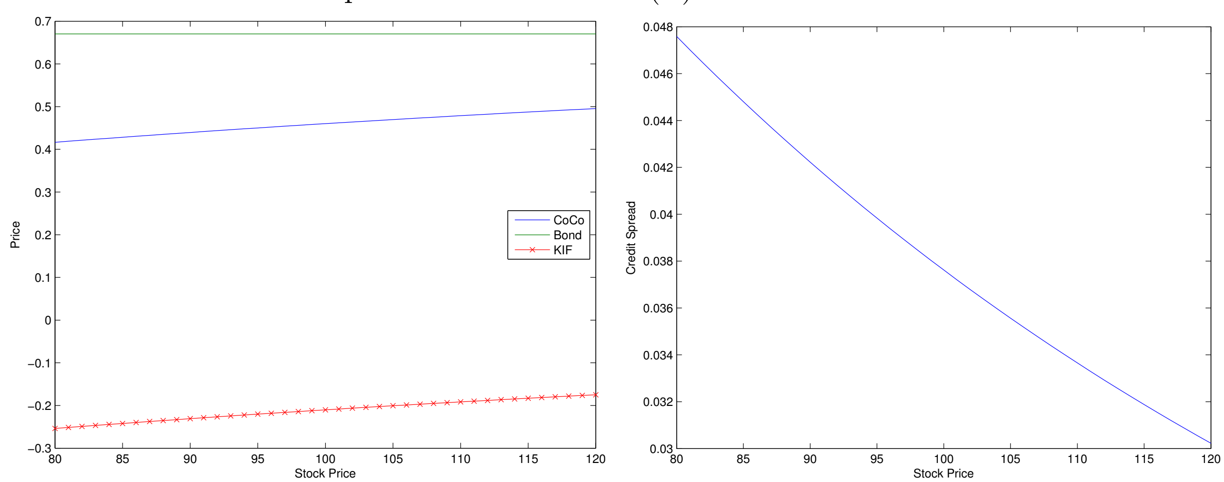
If the conversion price is very low, the investors get a large number of shares and it makes the price of CoCo exceed the price of the bond.

- Price and Credit Spread vs. Trigger Price (S^*)



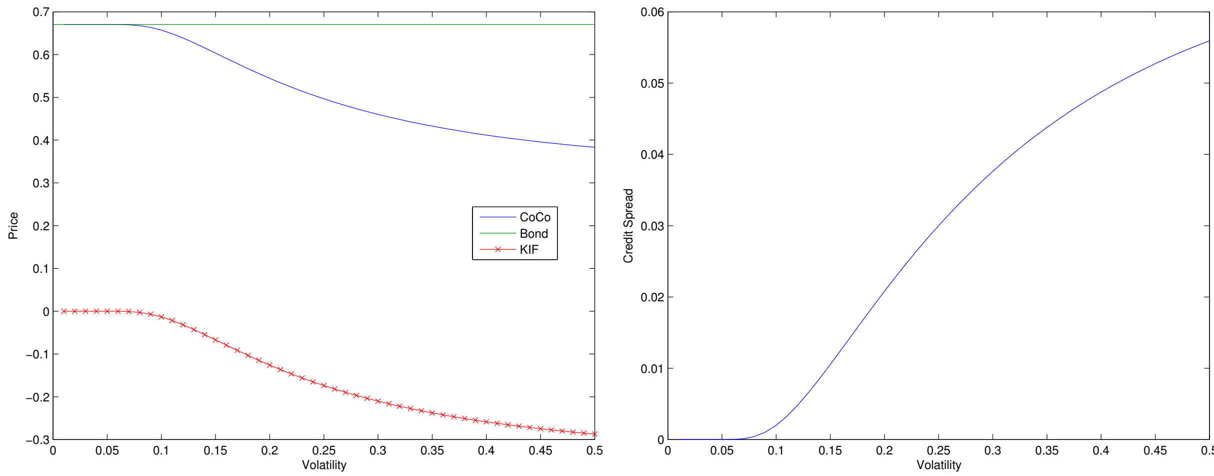
The trigger price determines the barrier of KIFs, and it affects the probability of getting knocked in and the forward price. If the trigger is high, then it is more likely to receive the forwards but the forwards will have low price.

- Price and Credit Spread vs. Stock Price (S)



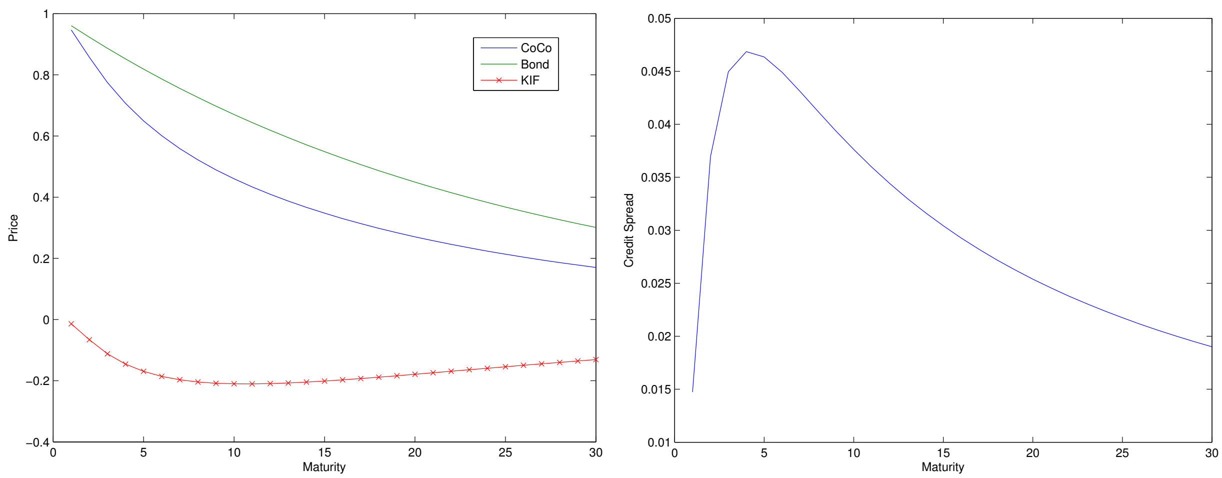
The stock price affects the probability of knock-in. As the stock price rises, the probability of knock-in declines. Therefore, the price of CoCo falls and the credit spread rises as the stock price goes up.

- Price and Credit Spread vs. Stock Price Volatility (σ)



As the volatility increases, the probability of knock-in goes up, and the value of KIF becomes more negative.

- Price and Credit Spread vs. Maturity (T)



The price of bond is downward sloping due to the flat term structure of interest rate. The hump shape of the credit spread is produced by the value of the KIFs with a medium level of the ratio $S^*/S = 1/2$. For the short-term KIFs, the stock price is more likely to hit the barrier in the longer time horizon; for the long-term KIFs, the probability of hitting the barrier decreases as the company has more time to improve its performance.

In short, the comparative statics analysis demonstrates that the formula in equity derivatives approach explains the economics of CoCos.

3.3 Model Comparison

Base case: The following table summarizes the prices of base-case CoCo ($T = 10$, $S^* = 50$, $S = 100$, $N = 1$, \$100, P_c=100, $\sigma = 0.3$, $q = 0.04$, $r = 0.04$) implied by the two explicit formulas.

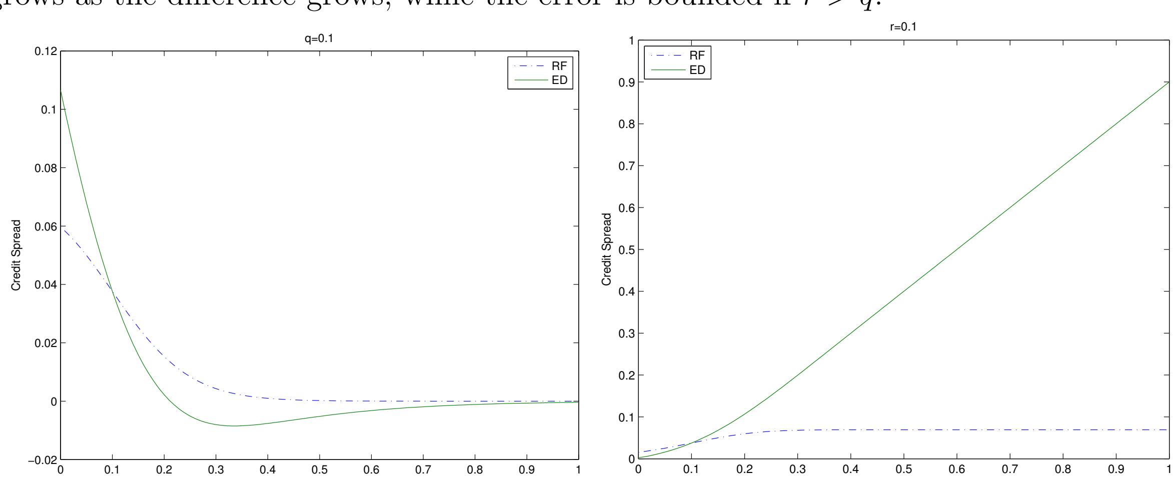
	Credit Derivatives	Equity Derivatives
Credit Spread	0.0376	0.0376
Price	0.4602	0.4602

If we assume that the company does not paye dividend ($q = 0$), the two approaches price the same CoCo differently.

	Credit Derivatives	Equity Derivatives
Credit Spread	0.0276	0.0196
Price	0.5086	0.5512

The two formulas give different prices because they treat the terminal cash flow differently. The credit derivatives (reduced form) approach assumes that if a trigger event occurs, CoCo holders get cash amount equal to NS^* at T, while the equity derivatives approach assumes that the CoCo holders get forwards on the share which mature at T. The difference between the two approaches can be interpreted in a way that the former discounts the cash flow at the risk free rate r, while the latter tails the number of shares at the dividend yield q. Therefore, both approaches give the same answer if $r = q$. If $r > q$, which was the case for the base-case CoCo, the reduced form approach over-discounts the cash flow and it leads to the lower price (and the higher credit spread) of CoCo. The left figure below shows the credit spreads implied by the two approaches, with respect to r, holding the dividend yield constant at 0.1. It demonstrates that the two approaches give the same credit spread if $r = q$, and the reduced form approach underestimates the spread if $r < q$, and it overestimates the spread if $r > q$, compared to the equity derivatives approach.

The right plot shows the credit spreads as a function of q, where r is fixed at 0.1. The non-symmetry of the two plots implies that even if $|r - q|$ are the same, if $q > r$, the error grows as the difference grows, while the error is bounded if $r > q$.



More precisely speaking, the source of the difference comes from the fact that the risk-neutral measure of the trigger probability for the stock and that of the bond are different. The two approaches coincide if:

$$S^* e^{-r(T-t)} p^* = S_t e^{-q(T-t)} q^*$$

where p^*, q^* are the probabilities such that satisfy:

$$KIF = S_t e^{-q(T-t)} q^* - P_c e^{-r(T-t)} p^*$$

The detailed computation is in appendix A.

The equity derivatives approach, which treats the cash flow as the value of the shares, not just cash amount, is more close to the realistic CoCo. The credit derivatives approach makes sense only if the investors cash out the shares immediately when they receive them on the trigger event. Furthermore, in case of a trigger event taking place, q is likely to be zero, and the error of approximation in equity derivatives approach would be insignificant.

3.4 Monte Carlo Simulation

3.4.1 Simulation by Euler Discretization

In this section, we will relax the assumption that the cash flow occurs at T. That is, the cash flow to a CoCo holder will occur at τ if trigger event happens.

In order to generate paths of stock prices following the Ito's process:

$$dS_t = (r - q)S_t dt + \sigma S_t dB_t^Q$$

$$d \log S_t = (r - q - \sigma^2/2)dt + \sigma dB_t^Q$$

with risk-neutral measure B_t^Q, we use Euler discretization. The evolution of $X = \log S$ can be approximated by:

$$X(t_j) - X(t_{j-1}) = \mu \Delta t + \sigma \Delta B(t_j)$$

where $\mu = r - q - \sigma^2/2$, Δt is a small time step increment, $t_j = j\Delta t$ and

$$\Delta B(t_j) = B(t_j) - B(t_{j-1}) \sim N(0, \sqrt{\Delta t}).$$

Based on the simulated paths of daily stock prices using the risk-neutral measure, the price of CoCo can be calculated by taking expected value of payoff discounted to the present value:

$$CoCo_t = E^Q[e^{-r\tau}(NS^*)\mathbf{1}\{\tau \leq T\} + e^{-rT}\mathbf{1}\{\tau > T\}] \tag{24}$$

The MATLAB code for the simulation is included in Appendix B.1.3.

3.4.2 Comparative Statics

Base case: The table below compares the estimated prices and the credit spreads of the base-case CoCo with those implied by the explicit formulas developed in the previous sections. For Monte Carlo simulation, the step size of 1/252 (daily) and the number of paths generated of 5000 were used.

	Credit Derivatives	**Equity Derivatives**	**Simulation**
Credit Spread	0.0376	0.0376	0.0258 (0.0249, 0.0267)
Price	0.4602	0.4602	0.5179 (0.5134, 0.5224)

While the two analytical formulas are based on the assumption that the cash flow occurs at maturity T, the Monte Carlo simulation is based on the cash flow occurring at τ. Consequently, the analytical formulas underestimate the value of CoCo.

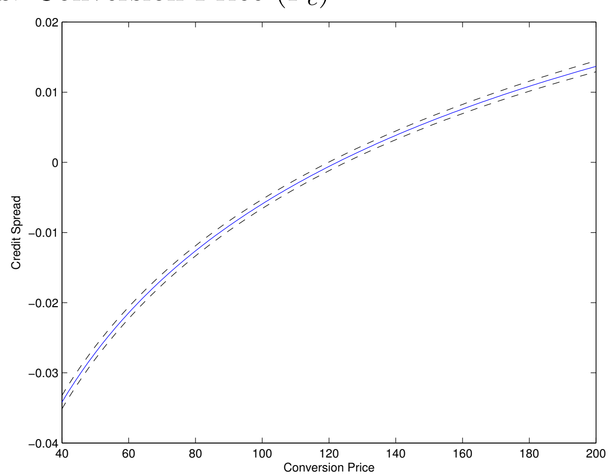
If the dividend rate is assumed to be zero ($q = 0$), the CoCo is priced as follows:

	Credit Derivatives	**Equity Derivatives**	**Simulation**
Credit Spread	0.0276	0.0196	0.0192 (0.0201, 0.0184)
Price	0.5086	0.5512	0.5562 (0.5454, 0.5576)

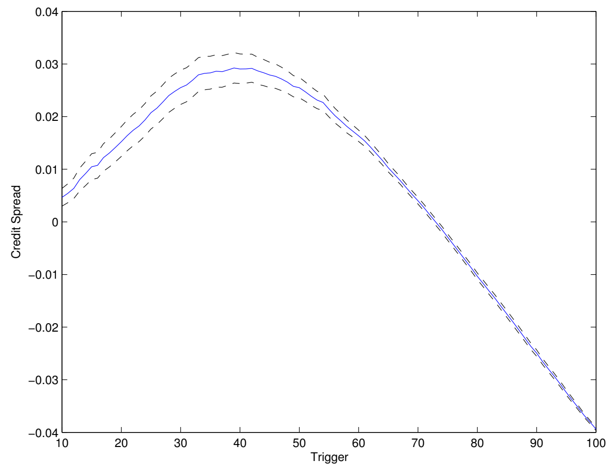
The price implied by the equity derivatives approach mostly lies within the confidence interval estimated in the simulation. This implies that the timing of the cash flow does not affect the price significantly when $q = 0$. In case of coupon CoCos, because the to-be-paid coupon stream is lost once the trigger gets pulled, the earlier the trigger event happens, the greater the approximation error would be.

The comparative statics plots below show that the Monte Carlo simulation results explain the economics of CoCos appropriately.

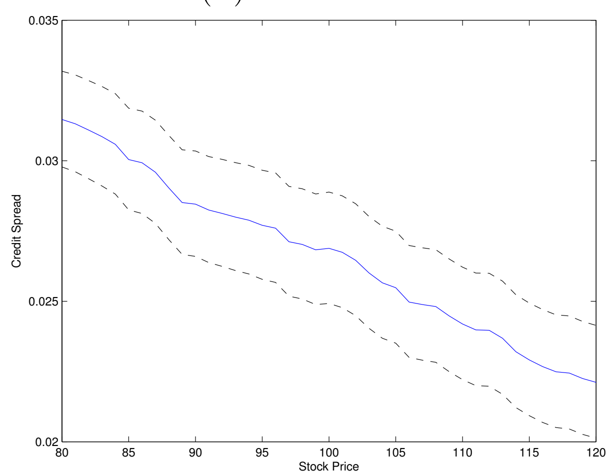
- Credit Spread vs. Conversion Price (P_c)



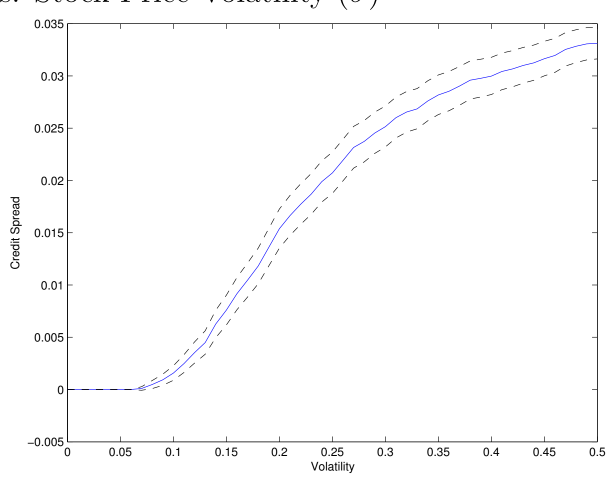
- Credit Spread vs. Trigger Price (S^*)



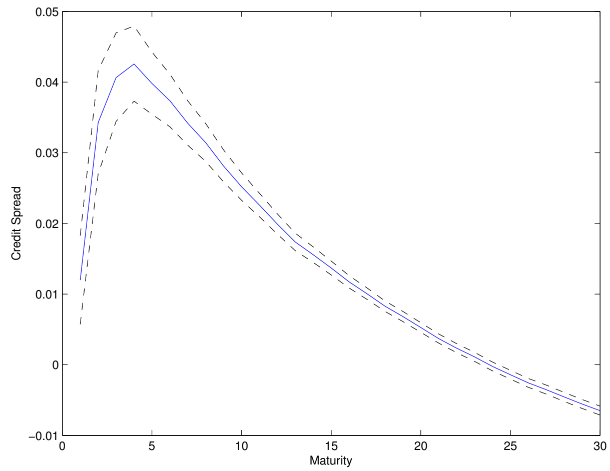
- Credit Spread vs. Stock Price (S)



- Credit Spread vs. Stock Price Volatility (σ)



- Credit Spread vs. Maturity (T)



4 Empirical Analysis

The purpose of empirical analysis is to examine if the Black-Scholes assumptions are empirically reasonable in pricing CoCos. Since Credit Suisse has an accounting trigger, not only volatility but also the market trigger is not observable. If we assume that the accounting trigger can be associated with a market trigger [10], we expect the implied market trigger to be constant over the time. As such, we construct the historical time series of implied trigger.

4.1 CoCo Issued by Credit Suisse

The summarized information on Prospectus[6] is as the following:

- Name: CS Group (Guernsey) I Limited 7.875% Tier 2 Buffer Capital Notes due 2041
- Issue Size: USD 2 billion
- Issue Date: Feb 24, 2011
- Maturity: 30 Year ($T = 30$)
- Callable[7] after 5 years and 6 months
- Coupon is reset to a fixed rate that is 522bp over mid market swap rate after 5 years, and it will thereafter reset every five years, at the same basis over swaps.[8]

[6] `https://www.credit-suisse.com/investors/doc/buffer_capital_notes_information_memorandum.pdf`

[7] The call feature has no effect on the pricing, as we assume that the interest rate is constant.

[8] We assume that the coupon rate will be fixed at 7.875%.

- Coupon 7.875%, Semi-annual
- Trigger Event: Core Tier 1 Ratio < 7% or the Swiss regulator determines that the CS Group requires public sector support to prevent it from becoming insolvent.
- Conversion Price (P_c)= $\max(\text{USD } 20, \text{CHF } 20, S^*)$

4.2 Pricing of Coupon CoCos

CoCo issued by Credit Suisse has a coupon stream. We can formulate the pricing formula for coupon CoCos by adding the value of coupon stream and subtracting the value of the remaining coupon stream if trigger is breached.

4.2.1 Equity Derivatives Approach

When trigger is breached, CoCo holder loses remaining coupon stream. Each coupon payment in the lost stream of coupons can be replicated by short positions in binary down-and-in call with maturities corresponding to each of the coupon dates [10]. Consequently, the price of coupon CoCo will be reduced by the sum of the values of binary down-and-in calls.

$$CoCo_{coupon_t} = CoCo_{zero_t} - \sum(\text{Binary Down-and-In Calls})$$

The value of each binary option is:

$$\text{Binary Down-and-In Call}_i =$$

$$\text{Coupon}_i \times e^{-r(T-t)} \times \Big(\mathbf{N}(-d1_i + \sigma\sqrt{(T_i - t)}) + (S/S^*)^{2(r-q-\frac{1}{2}\sigma^2)/\sigma^2}\mathbf{N}(d1'_i - \sigma\sqrt{(T_i - t)})\Big)$$

where Ti's are the coupon dates of CoCo. Each binary option with maturity Ti will match each coupon payment on date Ti.

4.2.2 Monte Carlo Simulation

Whereas the equity derivatives approach in the previous section assumes that the cash flow in case of conversion takes place at T, Monte Carlo simulation accounts for the precise timing of the cash flow. After generating the stock prices that follow the Geometric Brownian Motion, the value of CoCo for each realized path is calculated as:

$$CoCo_t = E^Q[(e^{-r(T-t)}D + C_T)\mathbf{1}\{\tau > T\} + e^{-r(\tau-t)}(NS^* + C_\tau)\mathbf{1}\{\tau \leq T\}]$$

where C_T represents the coupon stream up to T, and C_τ represents the coupon stream up to τ.

$$C_T = \sum_{i \in \text{Time to Coupon Dates before } T} Ce^{-ri}$$

$$C_\tau = \sum_{i \in \text{Time to Coupon Dates before } \tau} Ce^{-ri}$$

The MATLAB code is attached in appendix B.1.3.

4.3 The Implied Trigger on a Single Date

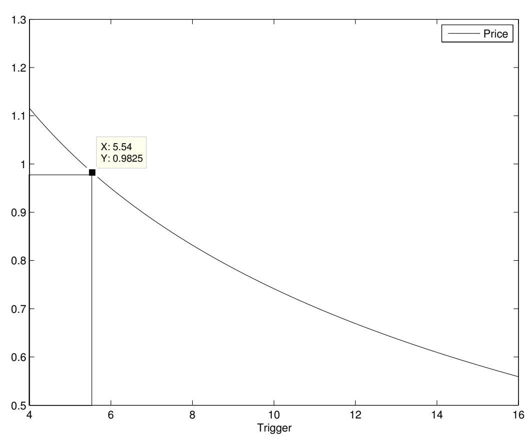
The following information was used in order to compute an implied market-trigger:

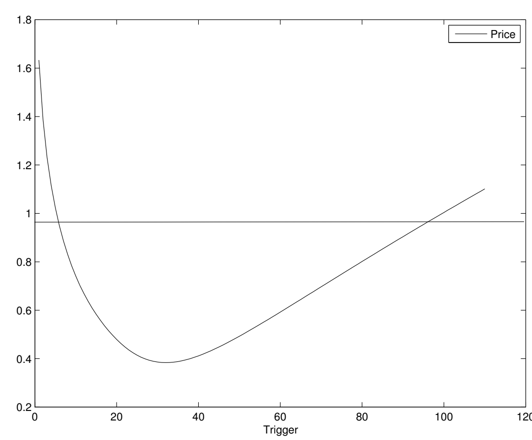
- Valuation Date: 02/24/2012
- 1 CHF = 1.1166 USD
- P_c=max(USD 20, CHF 20, S^*) = max(20, 22.332, S^*)
- Assume that S^*/P_c is 1. If $S^* > 22.332$, the number of shares (N) to CoCo holders at conversion is $1000/S^*$, and if $S^* \leq 21.6$, N is fixed to $1000/22.332 = 44.7788$.

Valuation Parameters:

- $r = 0.0188$
- $q = 0.0388$
- $\sigma = 0.42$
- $S = 27.31$
- Observed Market Value of CoCo = 98.2390

The analytic formula in section 4.2 was used to calculate the implied trigger price.

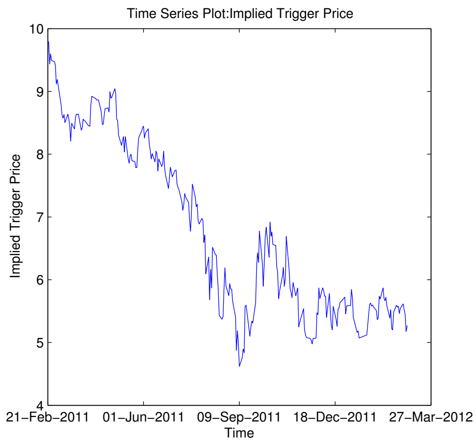


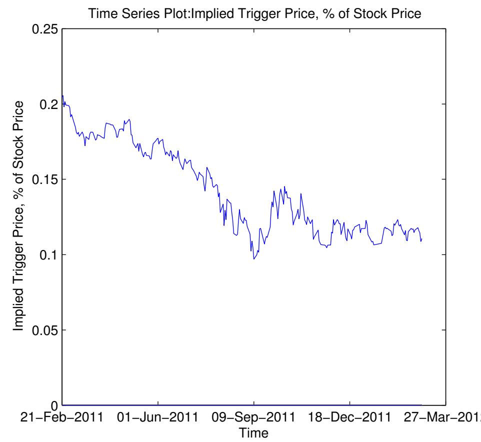


As the left figure shows, based on an observed market value of CoCo, an implied trigger price could be computed using the pricing formula shown in section 3.4. Using MATLAB function, `fzero`, the estimated implied trigger price was computed as 5.5408. Since CoCo price as a function of trigger price is not monotonically increasing or decreasing, technically, there are two solutions as the right plot indicates. However, in this case, since the trigger event has not taken place as of valuation date, the implied trigger must be lower than the current stock price of 27.31.

4.4 Time Series of the Implied Trigger

By repeating the single-date computation of implied trigger in the previous section since the issuance date of CoCo, we obtained the time series of implied trigger price. Since Black-Scholes Model assumes constant volatility, volatility of 0.42 was used for all data points. Using daily closing market price of CoCo and share price of Credit Suisse, the implied trigger price was computed for every trading day.





The left plot above demonstrates that the implied market trigger price, corresponding to the accounting trigger is not constant, and the implied market trigger price decreases as the share price decreases. The right plot shows that the implied trigger price as a fraction of share price is less volatile, and it is not constant as well. The non-constant implied trigger indicates that the pricing under the Black-Scholes assumptions are not empirically reasonable for pricing CoCos. As such, in section 5, we propose the Heston model as an alternative model for the stock price dynamics.

5 Pricing under the Heston Model

Black-Scholes model is widely employed as a useful approximation, because it is easy to calculate from the explicit closed-form formula. However, empirical evidences suggest that the key assumption, constant volatility, of Black-Scholes model is not reasonable. The distribution of market stock prices tends to have fatter tails than the distribution of prices that geometric brownian motion implies. That is, Black-Scholes model underestimates the risk of extreme events. Stochastic volatility models are based on the assumption that the volatility of stock price follows itself a stochastic process, and the most popular stochastic volatility model is the Heston model. (Heston 1993) proposed the following model [5]:

$$dS_t = S_t(r-q)dt + S_t\sqrt{v}_t dB$$

$$dv = \kappa(\bar{v} - v_t)dt + \sqrt{v}_t \sigma_v dB_v$$

κ : mean reversion coefficient
$\bar{v}$: long-run mean reversion level

σ_v : volatility of volatility, “volvol”
ρ : correlation between stock price and volatility

If κ is positive, the variance rate v mean reverts toward the long run level of $\bar{v}$. The larger κ, the quicker the reversion. The correlation ρ between the log-returns and the volatility determines the skewness of the return distribution. If ρ is negative, then volatility increases as the stock return decreases; as a result, it produces fat left-tailed distribution. The volvol affects the kurtosis of the return distribution. If volvol is zero, the stock price follows geometric Brownian motion. Positive values of volvol create fat tails in both sides. The higher volvol, the more prominent the volatility smile.

The parameters, $\kappa, \bar{v}, \sigma_v$ are positive constants and ρ is negative, which addresses to the empirical fact that negative shocks on stock prices have a greater impact on the future volatility than do positive shocks.

5.1 Monte Carlo Simulation

Heston model can be simulated by discretizing the stochastic differential equation for the log of the stock price $X = \ln S$ and the variance v using Euler scheme:

$$X(t_j) = X(t_{j-1}) + \left(r - q - \frac{1}{2}v(t_{j-1})\right)\Delta t + \sqrt{v(t_{j-1})}\Delta B(t_j),$$

$$v(t_j) = |v(t_{j-1}) + \kappa\left(\bar{v} - v(t_{j-1})\right)\Delta t + \sqrt{v(t_{j-1})}\sigma_v \Delta B_v(t_j)|$$

where value of X and $\Delta B(t_j)$ and $\Delta B_v(t_j)$ are jointly normally distributed under risk neutral measure with mean 0, variance Δt and correlation ρ. Based on the paths generated in a way described above, the value of CoCo with coupon can be computed as:

$$CoCo_t = E^Q[(e^{-r(T-t)}D + C_T)\mathbf{1}\{\tau > T\} + e^{-r(\tau-t)}(NS^* + C_\tau)\mathbf{1}\{\tau \leq T\}]$$

$$C_T = \sum_{i \in \text{Time to Coupon Dates before } T} Ce^{-ri}$$

$$C_\tau = \sum_{i \in \text{Time to Coupon Dates before } \tau} Ce^{-ri}$$

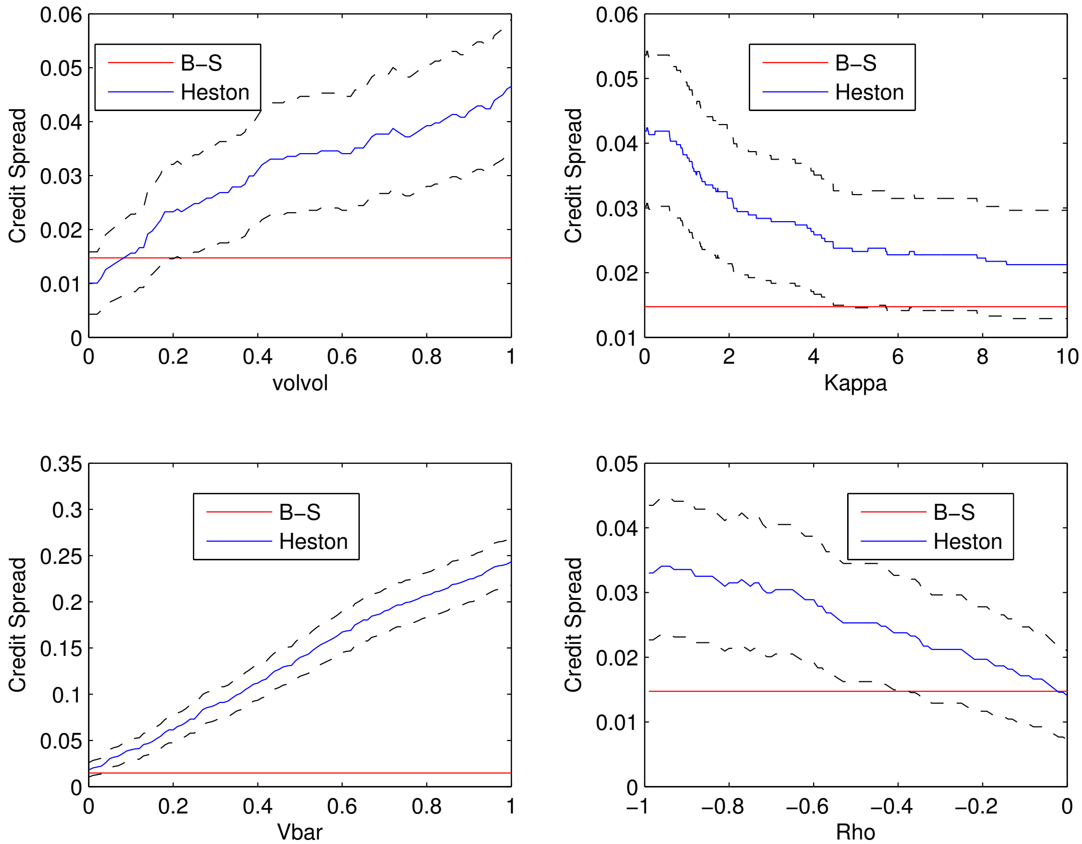
The MATLAB code for the simulation is included in Appendix B.3.1. The pricing of base-case security is 0.5072 with number of paths generated of 5000 and the step size of 1/252 (daily). Assumed Heston parameters were $\sigma_v = 0.44$, $\kappa = 1.62$, $\rho = -0.76$ [9], and $\bar{v} = 0.09$ =variance of base-case CoCo's stock. As a table below shows, positive $\sigma, \kappa, \bar{v} = \sigma^2$ and negative ρ result in a higher credit spread and a lower price.

	Black-Scholes	**Heston**
Credit Spread	0.0258 (0.0249, 0.0267)	0.0365 (0.0352, 0.0378)
Price	0.5179 (0.5134, 0.5224)	0.4654 (0.4594, 0.4714)

[9] This parameters were taken from (Bakshi, Cao, and Chen 1997)[2]

5.2 Comparative Statics with respect to Heston Parameters

In order to understand how the parameters in the Heston model affects credit spread of CoCo, the credit spreads with respect to varying levels of the parameters were estimated using Monte Carlo simulation. Assumed CoCo parameters were $T = 1$, $S^* = 50$, $S = 100$, $N = 1$, \$100, P_c=100, $\sigma = 0.3$, $q = 0.04$, $r = 0.04$.



The relationships between the credit spread and the Heston parameters are explained as the following:

- Higher volvol means that the volatility is more volatile. Since there is more chance for the extreme movements in the stock prices, the credit spread is monotonically increasing function of the volvol.
- The higher mean reversion coefficient κ, the faster the volatility is reverted to the mean. Therefore, the risk decreases as κ goes up.
- The higher the long run mean level of volatility $\bar{v}$, the greater the risk of conversion.
- If the correlation ρ is strongly negative, the volatility increases as the stock price falls. Consequently, strongly negative ρ pushes up the credit spread.

6 Conclusion and Implication

The present study has demonstrated that the explicit formulas used in the credit derivatives and equity derivatives approaches, as well as the Monte Carlo simulation under the Black-Scholes settings, all explain the economics of CoCos appropriately. However, when we applied them to the CoCos issued by Credit Suisse, the implied market trigger, associated with a constant Core Tier 1 ratio of 7%, was volatile and showed a trend. This finding implies that the Black-Scholes assumptions are empirically unreasonable for pricing CoCos. By contrast, the Heston model can be considered to be a suitable alternative because it produces a more realistic fat-tail distribution of stock returns. Thus, the pricing under the Heston model is expected to show a constant implied trigger over time.

A Reconciliation of Reduced form approach and Equity derivatives approach

A.1 CoCo Price in Reduced Form Approach

In section 3.2.1, we matched $P^Q = E^Q[\mathbf{1}\{\tau \le t\}]$ with the first-hitting-time probability under the assumption that the stock price follows the Geometric Brownian Motion. As a result,

$$\begin{aligned} CoCo_t &= NS^* e^{-r(T-t)} E^Q[\mathbf{1}\{\tau \le t\}] + De^{-r(T-t)} E^Q[\mathbf{1}\{\tau > t\}] \\ &= NS^* e^{-r(T-t)} P^Q + De^{-r(T-t)}(1 - P^Q) \\ &\boxed{= e^{-r(T-t)} \left(P^Q + NS^* P^Q + D(1 - P^Q)\right)} \end{aligned} \tag{25}$$

where

$$P^Q = \mathbf{N}\left(\frac{\ln(S^*/S) - \nu(T-t)}{\sigma\sqrt{T-t}}\right) + (S^*/S)^{2\nu/\sigma^2}\mathbf{N}\left(\frac{\ln(S^*/S) + \nu(T-t)}{\sigma\sqrt{T-t}}\right)$$

$$\nu = r - q - \sigma^2/2$$

Using the notations in (19)-(22),

$$P^Q = \left(\frac{S^*}{S_t}\right)^{2\lambda-2}\mathbf{N}(d_2') + \mathbf{N}(-d_2) \tag{26}$$

where

$$\lambda = \frac{r - q + \sigma^2/2}{\sigma^2}$$

A.2 CoCo Price in Equity Derivatives Approach

In section 3.2.1, we showed that with replication argument, the price of CoCo is:

$$CoCo_t = e^{-r(T-t)}D + N \times (KIC_t - KIP_t)$$

where

$$KIC_t = S_t e^{-q(T-t)}(\frac{S^*}{S_t})^{2\lambda}\mathbf{N}(d_1') - P_c e^{-r(T-t)}(\frac{S^*}{S_t})^{2\lambda-2}\mathbf{N}(d_2')$$

$$KIP_t = P_c e^{-r(T-t)}\mathbf{N}(-d_2) - S_t e^{-q(T-t)}\mathbf{N}(-d_1)$$

Expressing explicitly, the price of Knock-in-forward is written as:

$$KIF_t = S_t e^{-q(T-t)}\left(\left(\frac{S^*}{S_t}\right)^{2\lambda}\mathbf{N}(d_1') + \mathbf{N}(-d_1)\right) - P_c e^{-r(T-t)}\left(\mathbf{N}(-d_2) + \left(\frac{S^*}{S_t}\right)^{2\lambda-2}\mathbf{N}(d_2')\right)$$

$$= S_t e^{-q(T-t)} \left(\left(\frac{S^*}{S_t} \right)^{2\lambda} \mathbf{N}(d_1') + \mathbf{N}(-d_1) \right) - P_c e^{-r(T-t)} P^Q$$

Denoting P^Q as p^* and $\left(\left(\frac{S^*}{S_t} \right)^{2\lambda} \mathbf{N}(d_1') + \mathbf{N}(-d_1) \right)$ as q^*, the expression is simplified to:

$$KIF_t = S_t e^{-q(T-t)} q^* - P_c e^{-r(T-t)} p^*$$

As a consequence, the price of CoCo in equity derivatives approach is:

$$\boxed{CoCo_t = e^{-r(T-t)} D + N(S_t e^{-q(T-t)} q^* - P_c e^{-r(T-t)} p^*)} \tag{27}$$

The expression (22) and (24) are equal if and only if:

$$e^{-r(T-t)} \left(N S^* p^* + D(1-p^*) \right) = e^{-r(T-t)} D + N(S_t e^{-q(T-t)} q^* - P_c e^{-r(T-t)} p^*)$$

$$\iff S^* e^{-r(T-t)} p^* = S_t e^{-q(T-t)} q^*$$

$$\iff S_t \mathbf{N}(d_2') + S^* \mathbf{N}(d_2) = S^* \mathbf{N}(d_1') + S_t \mathbf{N}(d_1)$$

If $r = q$, the condition above is met, and therefore, the reduced form approach and the equity derivatives approach give the same price of CoCo.

B MATLAB Implementations

B.1 Zero Coupon CoCos

B.1.1 Reduced Form

```
function[cs price pstar lambda RR]=reducedform_exact(S,K,T,r,q,sigma,Cp)
mu=r-q-sigma^2/2;
a=(log(K/S)-mu*T)/(sigma*sqrt(T));
b=(log(K/S)+mu*T)/(sigma*sqrt(T));
Na=normcdf(a,0,1);
Nb=normcdf(b,0,1);
pstar=Na+(K/S)^(2*mu/sigma^2)*Nb;
lambda=-log(1-pstar)/T;
RR=K/Cp;
cs=(1/T)*log(1/(1-(1-exp(-lambda*(T)))*(1-RR)));
price=exp(-r*T)*(1-pstar*(1-RR));
```

B.1.2 Equity Derivatives

```
function[price yield spread]=ed_zeroc(N,S,K,T,r,q,sigma,Cp)
lambda=(r-q+0.5*sigma^2)/sigma^2;
Cr=N/Cp;
d1prime=(log(K/S)/(sigma*sqrt(T)))+lambda*sigma*sqrt(T);
d2prime= d1prime-sigma*sqrt(T);
d1=(log(S/K)/(sigma*sqrt(T)))+lambda*sigma*sqrt(T);
d2= d1-sigma*sqrt(T);
% Discounted Face value
Bond=exp(-r*T)*N;
% KIC: Down-and-in Call
KIC= exp(-q*T)*S*(K/S)^(2*lambda)*normcdf(d1prime)...
-exp(-r*T)*Cp*(K/S)^(2*lambda-2)*normcdf(d2prime);
% KIP: Down-and-in Put
KIP= Cp*exp(-r*T)*normcdf(-d2,0,1)-S*exp(-q*T)*normcdf(-d1,0,1);
%CoCo Price= Bond + Cr*( KIC - KIP)
price= Bond+ Cr*(KIC-KIP);
yield= (1/T)*log(N/price);
spread= yield-r;
```

B.1.3 Monte Carlo Simulation

```
function SPaths=AssetPaths(S0,mu,sigma,T,NSteps,NRep1)
dt=T/NSteps;
nudt=(mu-0.5*sigma^2)*dt;
sidt=sigma*sqrt(dt);
Increments=nudt+sidt*randn(NRep1,NSteps);
LogPaths=cumsum([log(S0)*ones(NRep1,1),Increments],2);
SPaths=exp(LogPaths);
Spaths(:,1)=S0;
```

```
function[price LCIp UCIp yield spread LCIs ...
    UCIs]=mc_zeroc(N,S,K,T,r,q,sigma,Cp,NRep)
Cr=N/Cp;
%Simulation Parameters
NSteps=252*T;
dt=T/NSteps;
%Generate stock price paths
paths=AssetPaths(S,r-q,sigma,T,NSteps,NRep);
%Simulate tau
count=0;
tau=[];
for i=1:NRep
    if min(paths(i,:)) ≤K
        index=find(paths(i,:)≤K);
        tau(i)=min(index*(dt));
```

```
        count=count+1;
    else
        tau(i)=T;
    end
end
prob=count/NRep;
%Compute Payoff V(T)=E[e^(-r*tau)*(Payoff at tau)+e^(-r*T)*(Payoff ...
    at T)]
V=[];
for j=1:NRep
    if tau(j)==T
        V(j)=N*exp(-r*T);
    else
        V(j)=(Cr*K)*exp(-r*tau(j));
    end
end

alpha=0.01; % 90%CI
[price, sigmahat,CI] = normfit(V,alpha);
yield=(1/T)*log(N/price);
LCIp=CI(1);
UCIp=CI(2);
CIy=CI;
CIy(1)=(1/T)*log(N/CI(1));
CIy(2)=(1/T)*log(N/CI(2));
spread= yield-r;
CIs=CI;
LCIs=CIy(1)-r;
UCIs=CIy(2)-r;
```

B.2 Coupon CoCos

B.2.1 Equity Derivatives

```
function[price yield spread]=ed_c(N,S,K,T,r,q,sigma,Cp,tvec,coupon)
lambda=(r-q+0.5*sigma.^2)./sigma.^2;
Cr=N/Cp;
d1primef= @(K,S,sigma,lambda,T) ...
    (log(K./S)./(sigma.*sqrt(T)))+(lambda.*sigma).*sqrt(T);
d2primef= @(d1prime,sigma,T) d1prime-sigma.*sqrt(T);
d1f= @(K,S,sigma,lambda,T) ...
    (log(S./K)./(sigma.*sqrt(T)))+(lambda.*sigma).*sqrt(T);
d2f= @(d1,sigma,T) d1-sigma.*sqrt(T);
d1prime= d1primef(K,S,sigma,lambda,T);
d2prime= d2primef(d1prime,sigma,T);
d1= d1f(K,S,sigma,lambda,T);
d2= d2f(d1,sigma,T);
% Discounted Future Cash Flow
Bond=exp(-r.*T).*N+sum(N.*coupon.*exp(-r.*tvec));
% KIC: Down-and-in Call
```

```
KIC= exp(-q.*T).*S.*(K./S)^(2*lambda).*normcdf(d1prime)...
-exp(-r.*T).*Cp.*(K./S)^(2*lambda-2).*normcdf(d2prime);
% KIP: Down-and-in Put
KIP= Cp.*exp(-r*T)*normcdf(-d2,0,1)-S.*exp(-q*T).*normcdf(-d1,0,1);
% Lost coupon
d1i=d1f(K,S,sigma,lambda,tvec);
d1primei=d1primef(K,S,sigma,lambda,tvec);
Undiscounted_Lost_coupon=normcdf(-d1i+sigma.*sqrt(tvec))...
+(K/S)^(2*lambda-2)*normcdf(d1primei-sigma.*sqrt(tvec));
Lost_coupon=sum((N*coupon).*exp(-r*tvec).*Undiscounted_Lost_coupon);
%CoCo Price= Bond + Cr*( KIC - KIP) - Lost coupon
price= Bond+ Cr.*(KIC-KIP)-Lost_coupon;
yield= (1/T).*log(N./price);
spread= yield-r;
```

B.2.2 Monte Carlo Simulation

```
function[price LCIp UCIp yield spread LCIs ...
    UCIs]=mc_c(N,S,K,T,r,q,sigma,Cp,tvec,coupon,NRep)
Cr=N/Cp;

%Simulation Parameters
NSteps=252*T;
dt=T/NSteps;
%Generate stock price paths
paths=AssetPaths(S,r-q,sigma,T,NSteps,NRep);
%Simulate tau
count=0;
k=1;
tau=[];
for i=1:NRep
    if min(paths(i,:)) ≤K
        index=find(paths(i,:)≤K);
        tau(i)=min(index*(dt));
        count=count+1;
    else
        p=0;
        tau(i)=T;
    end
    k=k+1;
end
prob=count/NRep;
%Compute Payoff V(T)=E[e^(-r*tau)*(Payoff at tau)+e^(-r*T)*(Payoff ...
    at T)]
V=[];
for j=1:NRep
    if tau(j)==T %no trigger event
        V(j)=N*exp(-r*T)+sum(N*coupon.*exp(-r*tvec));
    else %trigger event
        tvecnew=tvec-tau(j);
```

```
        index=find(tvecnew>0);
        if isempty(index)==false
            tvecnew=tvecnew(index(1):index(end)); %lost coupon stream
            V(j)=(Cr*K)*exp(-r*tau(j)) ...
            +sum(N*coupon.*exp(-r*tvec))-sum(N*coupon.*exp(-r*tvecnew));
        else
            V(j)=(Cr*K)*exp(-r*tau(j))+sum(N*coupon.*exp(-r*tvec));
        end

    end
end
alpha=0.01; % 90%CI
[price, sigmahat,CI] = normfit(V,alpha);
yield=(1/T)*log(N/price);
LCIp=CI(1);
UCIp=CI(2);
CIy=CI;
CIy(1)=(1/T)*log(N/CI(1));
CIy(2)=(1/T)*log(N/CI(2));
spread= yield-r;
CIs=CI;
LCIs=CIy(1)-r;
UCIs=CIy(2)-r;
```

B.3 Extensions

B.3.1 Heston Monte Carlo Simulation

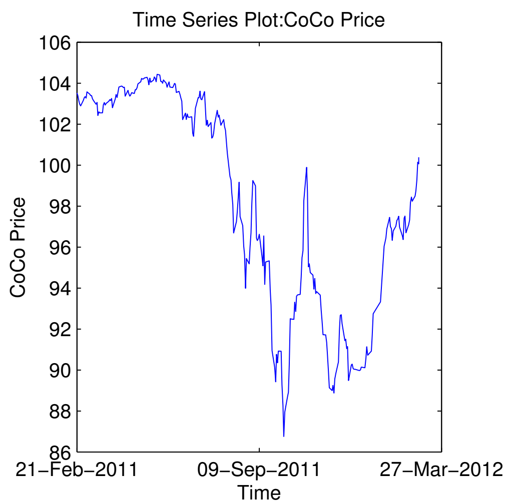
```
function [ S ] = HestonPath(S,T_,sigma,k_,vbar_,sigmav_,rho_,dt_,r)
v0_=sigma^2;
length = floor(T_/dt_)+1;
X = zeros(length,1);
X(1) = log(S);
vt = v0_;
MU = [0 0];
SIGMA = [dt_, dt_*rho_; dt_*rho_, dt_];
dwt = mvnrnd(MU,SIGMA,length);
for i = 2:length
    X(i) = X(i-1) + (r-q-0.5*vt)*dt_ + sqrt(vt)*dwt(i-1,1);
    vt = abs(vt + k_*(vbar_ - vt)*dt_ + sqrt(vt)*sigmav_*dwt(i-1,2));
end
S = exp(X);
end
```

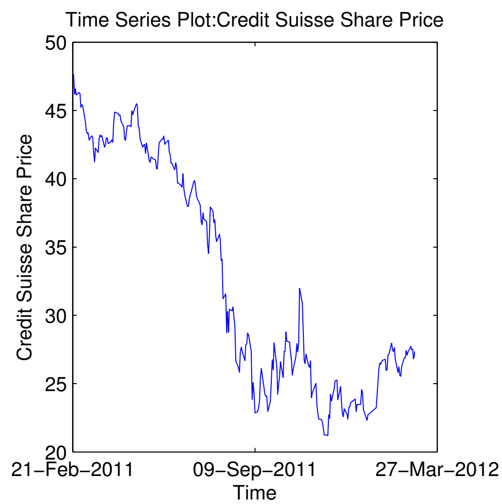
```
function[price LCIp UCIp yield spread LCIs ...
    UCIs]=mc_heston_c(N,S,K,T,r,q,sigma,Cp,tvec,coupon,NRep)
Cr=N/Cp;

```

```
%Simulation Parameters
NSteps=252*T;
dt=T/NSteps;
%Generate stock price paths under Heston
paths=zeros(NRep,NSteps+1);
for i=1:NRep
    paths(i,:)=HestonPath(S,T,sigma,k,vbar,sigmav,rho,dt,r-q);
end
%Simulate tau
count=0;
k=1;
tau=[];
for i=1:NRep
    if min(paths(i,:)) <=K
        index=find(paths(i,:)<=K);
        tau(i)=min(index*(dt));
        count=count+1;
    else
        p=0;
        tau(i)=T;
    end
    k=k+1;
end

prob=count/NRep;

%Compute Payoff  %V(T)=E[e^(-r*tau)*(Payoff at tau)+e^(-r*T)*(Payoff at T)]
V=[];
for j=1:NRep
    if tau(j)==T %no trigger event
        V(j)=N*exp(-r*T)+sum(N*coupon.*exp(-r*tvec));
    else %trigger event
        tvecnew=tvec-tau(j);
        index=find(tvecnew>0);
        if isempty(index)==false
            tvecnew=tvecnew(index(1):index(end)); %lost coupon stream
            V(j)=(Cr*K)*exp(-r*tau(j))...
            +sum(N*coupon.*exp(-r*tvec))-sum(N*coupon.*exp(-r*tvecnew));
        else
            V(j)=(Cr*K)*exp(-r*tau(j))+sum(N*coupon.*exp(-r*tvec));
        end

    end
end

alpha=0.01; % 90%CI
[price, sigmahat,CI] = normfit(V,alpha);
yield=(1/T)*log(N/price);
LCIp=CI(1);
UCIp=CI(2);
CIy=CI;
```

```
CIy(1)=(1/T)*log(N/CI(1));
CIy(2)=(1/T)*log(N/CI(2));
spread= yield-r;
CIs=CI;
LCIs=CIy(1)-r;
UCIs=CIy(2)-r;
```

C Time Series Data of CoCo Price and Credit Suisse Share Price





Summary Statistics (for 260 days, from 02/21/2011 to 03/02/2012):

	CoCo Price	**Share Price**
Mean	98.3311	33.0298
Stdev	5.1605	8.3042
95% CI	(97.7008, 98.9613)	(32.0157, 34.0440)
Min	86.7560	21.2000
Max	104.4350	47.6300

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