Coordination Of Two-Arm Pushing

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Abstract

Coordination of two manipulators performing the task of transporting objects is studied in this paper. Each manipulator is equipped with end effector — a flat surface palm. Grasping is achieved by the two palms pushing an object from two ends. The task requires simultaneous control of the object motion and the interaction force. The control of the interaction force is needed to ensure that the object is not dropped and to avoid excessive pressing. The motion and force control problem is further complicated by the presence of unilateral constraints since the manipulators can only push the object. This paper describes a control method which utilizes a state feedback to decouple position control and force control loops. A force control planning algorithm is also proposed which ensures the satisfaction of unilateral constraints. The effectiveness of the control method is verified by simulations.

1 Introduction

Coordinated control of two or multiple manipulators has been studied by many researchers including Nakamura et al. [1], Uchiyama et al. [2], Zheng and Luh [3], Hayati [4], Dauchez and Uchiyama [5], and Tarn et al. [6]. In most work, it is assumed that manipulators rigidly grasp the object so that both pushing and pulling are possible. In terms of modeling, equality constraints are considered only. This assumption requires that the object must be graspable by each hand/gripper. The potential of two cooperative manipulators is not fully utilized if they are restricted to manipulates objects graspable by a single hand. Two manipulators can grasp objects which are far beyond the capability of a single hand. For instance, two manipulators can easily transport a large (not necessarily heavy) cardboard box by pushing it from two ends. However, performing tasks by two-arm pushing imposes challenging control problems. Firstly, explicit control of interaction force is essential to avoid dropping the object and pressing it excessively. Secondly, the kinematic constraints are unilateral since manipulators can only push the object. In other words, the normal force applied to the object by the manipulator must be positive.

An excellent work on pushing operation is documented by Mason [7]. The closest to the present problem is the work by Kopf and Yabuta [8] and by Yoshikawa and Zheng [9]. Kopf and Yabuta conducted a comparison study of master/slave and hybrid two arm position/force control through an experiment in which two co-linear arms push an object. In Yoshikawa and Zheng's work, two arms move an object by inserting pins at arm tips into two holes on the object. The arms could pull (or push) the object. Once again, equality constraints are considered only. In this paper, two-arm pushing operations with explicit inequality constraints are studied. Dynamics of two-arm pushing is first represented in the state space. The output of the system consists the object position and the interaction force. A state feedback is then constructed to decouple the motion and force control loops. Finally, the developed control algorithm is verified by simulations.

2 Modeling of Two-Arm Pushing

2.1 Motion Equations

We consider the task of moving an object by two manipulators. Each manipulator has a flat palm as its end effector. The two manipulators grasp and move the object by pushing it from two opposites ends, as depicted in Figure 1. The discussion in this paper is restricted to the one dimensional case for thorough understanding of the problem. The object and palms in this discussion are assumed to be rigid. The one dimensional space under consideration is in the horizontal plane so that gravitational force will not play a role in the motion analysis.

The task for the two manipulators is to move the object, following a desired trajectory. It is a trivial modeling and control problem if forces applied to the object by each individual manipulators, F_1 and F_2, are not of concern and if the manipulators are allowed

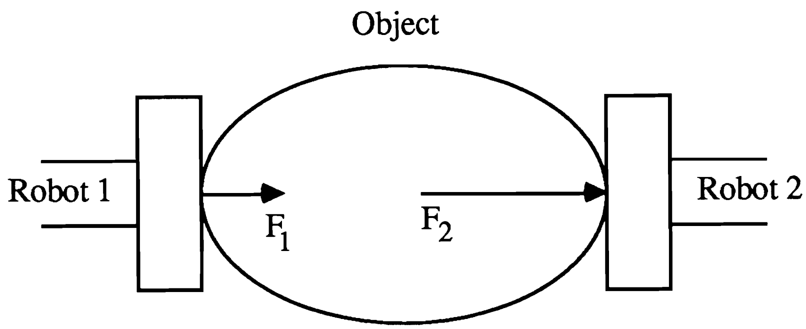

Figure 1: Two Manipulators Pushing an Object

to push and pull the object. The problem of this study is to perform the same task under the following requirement and constraint.

1. **Coordination requirement:** the forces applied to the object by the two manipulators must be coordinated to avoid unnecessary cancellation and to maintain a certain minimum required for grasping the object.

2. **Unilateral constraint:** the two manipulators can push, but can not pull, the object.

We now proceed to model the manipulators-object system. Let x_o be the position of a point on the object and m_o be the mass of the object. From the Newton's law, the motion equation of the object is

$$m_o \ddot{x}_o = F_1 + F_2, \qquad F_1 \geq 0, \qquad F_2 \leq 0 \tag{1}$$

The two inequalities in the above are from the unilateral constraints. Assuming that both manipulators are one dimensional. Their motion equations can be described as follows

$$m_1 \ddot{x}_1 + b_1 \dot{x}_1 + c_1 x_1 = \tau_1 - F_1 \tag{2}$$
$$m_2 \ddot{x}_2 + b_2 \dot{x}_2 + c_2 x_2 = \tau_2 - F_2 \tag{3}$$

where x_1 and x_2 are the position of palms 1 and 2, respectively, τ_1 and τ_2 are the actuator forces, and m_i, b_i, and c_i are the effective mass, damping and spring constants of the manipulators. As long as the manipulators are in contact with the object, we may properly choose the coordinates of palm 1, palm 2, and the object in such a way that

$$\begin{aligned} x_1 &= x_o \\ x_2 &= x_o \end{aligned}$$

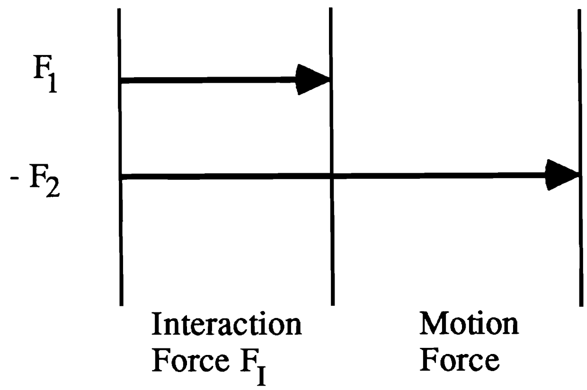

Figure 2: Interaction Force and Motion Force

It follows that velocities and accelerations of the two palms and the object during contacts are governed by

$$\begin{aligned} \dot{x}_1 &= \dot{x}_2 = \dot{x}_o \\ \ddot{x}_1 &= \ddot{x}_2 = \ddot{x}_o \end{aligned}$$

2.2 Interaction Force

Since the two manipulators can only push the object, F_1 is always nonnegative and F_2 is always nonpositive. Further, to secure the object between the palms, F_1 and F_2 can not be zero. We define *interaction force* as the minimum of magnitudes of F_1 and F_2 (see Figure 2),

$$F_I = \min\{F_1, -F_2\} = \frac{F_1 - F_2 - \mid F_1 + F_2 \mid}{2} \tag{4}$$

The interaction force F_I does not generate motion. It is needed for grasping the object. The amount of the interaction force is determined by the task to be performed. On the one hand, F_I must be as small as possible to avoid unnecessary cancellation due to coordination requirement. On the other hand, F_I must be sufficiently large so that the tangential friction force is able to balance the gravity force of the object. The minimal amount of F_I is then determined by the weight of the object and the coefficient of friction between the object and palms. In this paper, the desired value of F_I, denoted by $F_I^d(t)$, is assumed to be given by the task planner. The present problem is to maintain F_I while the object is in motion, that is, to design a controller which regulates both the motion of the object and the interaction force.

2.3 State Space Representation

We are dealing with a system whose inputs are clearly the actuator forces τ_1 and τ_2. To control the motion of the object and the interaction force, the outputs of the system should be related to x_o and F_I. To completely describe the system, a set of state variables must be selected and state equations must be established.

Since $x_o = x_1 = x_2$ during the contact, adding Equations (1), (2), and (3) together to eliminate F_1 and F_2, we obtain

$$m\ddot{x}_o + b\dot{x}_o + cx_o = \tau_1 + \tau_2 \tag{5}$$

where $m = m_1 + m_2 + m_o$, $b = b_1 + b_2$, and $c = c_1 + c_2$. Equation (5) will be the basis of the state equation. We now derive a representation for F_1. Substituting Equation (5) into Equation (2) and collecting terms, we get

$$\begin{aligned} F_1 &= \tau_1 - m_1\ddot{x}_o - b_1\dot{x}_o - c_1x_o \\ &= \tau_1 - \frac{m_1}{m}(\tau_1 + \tau_2 - b\dot{x}_o - cx_o) - b_1\dot{x}_o - c_1x_o \\ &= (1 - \frac{m_1}{m})\tau_1 - \frac{m_1}{m}\tau_2 + (\frac{m_1b}{m} - b_1)\dot{x}_o + (\frac{m_1c}{m} - c_1)x_o \end{aligned} \tag{6}$$

A representation for F_2 can be similarly obtained. We choose the following state variables.

$$x = [x_1 \quad x_2 \quad x_3 \quad x_4]^T = [x_o \quad \dot{x}_o \quad \tau_1 \quad \tau_2]^T$$

The state equation of the system is established by rewriting Equation (5) in terms of the state variables.

$$\dot{x} = \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \end{bmatrix} = \begin{bmatrix} x_2 \\ -\frac{b}{m}x_2 - \frac{c}{m}x_1 + \frac{1}{m}(x_3 + x_4) \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} \tag{7}$$

The above state variables and state equation deserve some explanation. It is noted that we have included τ_1 and τ_2 in the state variables. Since we explicitly control the interaction force, F_I would be part of the output equations. However, F_I is algebraically related to τ_1 and τ_2 through F_1 and F_2. By enlarging the state space to include τ_1 and τ_2 and adding an integrator on each input channel ($\dot{\tau}_1 = u_1$ and $\dot{\tau}_2 = u_2$), we will be able to formulate the present coordinated control problem as a control problem of an affine nonlinear system $\dot{x} = f(x) + g(x)u, y = h(x)$, in which the output y is a function of the state x only instead of a function of both the state and inputs.

As stated early, the fulfillment of the task requires simultaneous control of the object motion and the interaction force. Thus, the outputs of the system would consist of x_o and F_I, i.e.,

$$y = \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} h_1(x) \\ h_2(x) \end{bmatrix} = \begin{bmatrix} x_o \\ F_I \end{bmatrix} \tag{8}$$

It is clear that $h_1(x) = C_1x = x_1$ where $C_1 = [1 \quad 0 \quad 0 \quad 0]$. $h_2(x)$ is obtained by substituting F_1 (Equation (6)) and F_2 into Equation (4). From Equation (4), F_I is not differentiable with respect to the state variables. A non-differentiable output function will prevent us from using powerful design techniques such as differential geometric control theory. Specifically for this example of one dimensional case, the state equations are linear. The output equations would be nonlinear as well as non-differentiable if F_I is part of the outputs. An alternative is to control something else while providing the stability of F_I. We will replace F_I in the output equations by F_1. To make this possible, we must establish a relationship between errors in F_I and F_1, and a planning rule for F_1 based on the desired values of F_I which is specified by the task. We defer the discussion on error bounds and force control planning to Subsections 3.2 and 3.3. With F_1 replacing F_I in Equation (8), the output equations become differentiable and linear in state x.

$$y = \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} h_1(x) \\ h_2(x) \end{bmatrix} = \begin{bmatrix} x_o \\ F_1 \end{bmatrix} = Cx \tag{9}$$

where

$$C = \begin{bmatrix} C_1 \\ C_2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ \frac{m_1 c}{m} - c_1 & \frac{m_1 b}{m} - b_1 & 1 - \frac{m_1}{m} & -\frac{m_1}{m} \end{bmatrix}$$

For the present one dimensional case, both state equations and output equations are linear. For general multi-dimensional case, the system representing two arm pushing will be nonlinear.

3 Coordinated Control of Two-Arm Pushing

In the preceding section, we have characterized two-arm pushing as a dynamic system in the state space. The focus of the this section is to design a controller to achieve the task of moving the object in a coordinated fashion.

3.1 Feedback Decoupling

As noted early, the system representing 1-D two-arm pushing is linear. Nevertheless, the inputs and outputs of the system are coupled. In terms of physics, the effect of actuator forces on the object motion and F_1 is coupled. In this subsection, we derive a state feedback which will decouple the force control subsystem from the motion control subsystem.

To construct the feedback for input-output decoupling, we may use Wonham's geometric approach for linear multivariable systems [10], or differential geometric approach for nonlinear systems [11]. We will use the later approach since it provides insight into the general nonlinear case of multi-dimensional two-arm coordination. For this purpose, we rewrite state Equation (7) and output Equation (9) together as follows:

$$\begin{aligned} \dot{x} &= f(x) + g(x)u && (10) \\ y &= h(x) && (11) \end{aligned}$$

To construct the feedback for input-output decoupling, it is necessary to compute the decoupling matrix [11], which in turn requires the following Lie derivatives.

$$
\begin{aligned}
L_g h_1 &= \frac{\partial h_1}{\partial x} g = C_1 g = [0 \quad 0] \\
L_f h_1 &= \frac{\partial h_1}{\partial x} f = C_1 f = x_2 \\
L_g L_f h_1 &= \frac{\partial L_f h_1}{\partial x} g = [0 \quad 0] \\
L_f^2 h_1 &= \frac{\partial L_f h_1}{\partial x} f = -\frac{b}{m} x_2 - \frac{c}{m} x_1 + \frac{1}{m}(x_3 + x_4) \\
L_g L_f^2 h_1 &= \frac{\partial L_f^2 h_1}{\partial x} g = [\frac{1}{m} \quad \frac{1}{m}] \neq 0 \\
L_f^3 h_1 &= \frac{\partial L_f^2 h_1}{\partial x} f = -\frac{c}{m} x_2 + \frac{b}{m}[\frac{b}{m} x_2 + \frac{c}{m} x_1 - \frac{1}{m}(x_3 + x_4)] \\
L_g h_2 &= \frac{\partial h_2}{\partial x} g = C_2 g = [1 - \frac{m_1}{m} \quad -\frac{m_1}{m}] \neq 0 \\
L_f h_2 &= \frac{\partial h_2}{\partial x} f = C_2 f = \frac{m_1 c - m c_1}{m} x_2 - \frac{m_1 b - m b_1}{m}[\frac{b}{m} x_2 + \frac{c}{m} x_1 - \frac{1}{m}(x_3 + x_4)]
\end{aligned}
$$

It follows that the decoupling matrix is

$$
\Phi(x) = \begin{bmatrix} L_g L_f^2 h_1 \\ L_g h_2 \end{bmatrix} = \begin{bmatrix} \frac{1}{m} & \frac{1}{m} \\ 1 - \frac{m_1}{m} & -\frac{m_1}{m} \end{bmatrix} \tag{12}
$$

The determinant of the decoupling matrix is

$$
\det(\Phi(x)) = -\frac{1}{m} \neq 0
$$

which implies that the system will be decoupled in the entire state space.

Having obtained the above Lie derivatives and the decoupling matrix, the state space transformation and state feedback for input-output decoupling are given as follows [11]. The state transformation is

$$
z = [z_1 \quad z_2 \quad z_3 \quad z_4]^T = T(x) = [h_1 \quad L_f h_1 \quad L_f^2 h_1 \quad h_2]^T = T_* x \tag{13}
$$

where the differential T_* is given by

$$
T_* = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ -\frac{c}{m} & -\frac{b}{m} & \frac{1}{m} & \frac{1}{m} \\ \frac{m_1 c}{m} - c_1 & \frac{m_1 b}{m} - b_1 & 1 - \frac{m_1}{m} & -\frac{m_1}{m} \end{bmatrix}
$$

The state feedback is

$$
u = \alpha + \beta v \tag{14}
$$

where α and β satisfy the following matrix equations

$$\begin{aligned} \Phi\alpha &= -\begin{bmatrix} L_f^3 h_1 \\ L_f h_2 \end{bmatrix} \\ \Phi\beta &= I \end{aligned}$$

Since Φ is nonsingular and its inverse is

$$\Phi^{-1} = \begin{bmatrix} m_1 & 1 \\ m - m_1 & -1 \end{bmatrix},$$

the state feedback is then given by

$$u = \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} = \begin{bmatrix} m_1 & 1 \\ m - m_1 & -1 \end{bmatrix} \left(\begin{bmatrix} v_1 \\ v_2 \end{bmatrix} - \begin{bmatrix} -\frac{c}{m}x_2 + \frac{b}{m}[\frac{b}{m}x_2 + \frac{c}{m}x_1 - \frac{1}{m}(x_3 + x_4)] \\ \frac{m_1 c - m c_1}{m}x_2 - \frac{m_1 b - m b_1}{m}[\frac{b}{m}x_2 + \frac{c}{m}x_1 \end{bmatrix} \right) \tag{15}$$

Applying the above state feedback, the system is decoupled into two sybsystems in the transformed state space z. The first subsystem is the one which controls the motion of the object and is described by

$$\begin{bmatrix} \dot{z}_1 \\ \dot{z}_2 \\ \dot{z}_3 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} z_1 \\ z_2 \\ z_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} v_1 \tag{16}$$

$$y_1 = [1 \quad 0 \quad 0] \begin{bmatrix} z_1 \\ z_2 \\ z_3 \end{bmatrix} \tag{17}$$

and the second subsystem controls the force F_1 and is described by a first-order system

$$\dot{z}_4 = [0]z_4 + [1]v_2 \tag{18}$$

$$y_2 = [1]z_4 \tag{19}$$

Now we have two decoupled subsystems. A feedback can be easily designed for each subsystem which stabilizes it by placing the poles at any desired locations.

3.2 Force Control Planning

In this subsection, we address the problem of force control planning. From the task specification, a desired motion trajectory $x_o^d(t)$ of the object as well as a force trajectory $F_I^d(t)$ of the interaction force will be planned based on factors such as collision avoidance and holding the object while not excessively squeezing it. Due to the difficulty of directly controlling the interaction force, we have argued in section 2.3 to control F_1 instead. The problem of force control planning in this context is to generate a desired trajectory of F_1 based on that of F_I.

From Equation (4), we may obtain the difference between F_1 and F_2

$$F_1 - F_2 = 2F_I + \mid F_1 + F_2 \mid$$

Replacing $(F_1 + F_2)$ by $m_o \ddot{x}_o$ (from Equation (1)), we have

$$F_1 - F_2 = 2F_I + m_o \mid \ddot{x}_o \mid$$

Now adding the above equation and the motion equation of the object (Equation (1)) and dividing the result by 2, we get

$$F_1 = \frac{1}{2} m_o (\ddot{x}_o + \mid \ddot{x}_o \mid) + F_I \tag{20}$$

Given a desired motion trajectory $x_o^d(t)$ and a force trajectory $F_I^d(t)$, Equation (20) in the above provides a dynamic force control planner to calculate the desired trajectory of F_1, i.e.,

$$F_1^d(t) = \frac{1}{2} m_o (\ddot{x}_o^d(t) + \mid \ddot{x}_o^d(t) \mid) + F_I^d(t) \tag{21}$$

This is the planning rule for F_1 in the ideal case. As we will observe in Section 4 of simulations, in the presence of large position errors, this planning rule may command one of the manipulators to pull, which is definitely undesirable. A solution is to replace $x^d(t)$ by the actual motion trajectory. A detailed discussion on this issue in conjunction with simulations is in Section 4.

3.3 Error Bounds

In Section 2.3, we have replaced F_I by F_1 in the output equations to simplify the controller design. To make this replacement valid, we must establish an error bound for F_I.

We define the position error as follows

$$e_x(t) = x_o^d(t) - x_o^m(t) \tag{22}$$

where $x_o^m(t)$ is the actual value of x_o. Similarly, the errors in F_1 and F_I are defined by

$$\begin{aligned} e_1 &= F_1^d - F_1^m \\ e_I &= F_I^d - F_I^m \end{aligned}$$

Since the output equations are composed of x_o and F_1, $e_x(t)$ and $e_1(t)$ are directly compensated by the controller, whereas e_I is left uncompensated. The measured interaction force may be expressed in terms of the measurement of F_1 and F_2, i.e.,

$$F_I^m = \frac{F_1^m - F_2^m - \mid F_1^m + F_2^m \mid}{2}$$

Using the above equation, the error in the interaction force can be written as

$$e_I = F_I^d - F_I^m = F_I^d - \frac{F_{n1}^m - F_{n2}^m - \mid F_{n1}^m + F_{n2}^m \mid}{2}$$

Let $\tilde{m}_o = m_o + \Delta m$ be the actual mass of the object. Using Equation (21) and the motion equation of the object, we obtain

$$e_I = F_1^d - \frac{1}{2}m_o(\ddot{x}_o^d(t) + \mid \ddot{x}_o^d(t) \mid) - \frac{1}{2}(2F_1^m - \tilde{m}_o\ddot{x}_o^m(t) - \tilde{m}_o \mid \ddot{x}_o^m(t) \mid)$$

Taking the absolute value on the both sides, we have the following inequality

$$\begin{aligned} \mid e_I(t) \mid \;\; &\leq \;\; \mid e_1(t) \mid + \frac{1}{2}m_o \mid \ddot{x}_o^d - \ddot{x}_o^m(t) \mid + \frac{1}{2}m_o \mid \; \mid \ddot{x}_o^d(t) \mid - \mid \ddot{x}_o^m(t) \mid \; \mid \\ &\quad + \frac{1}{2}\Delta m \ddot{x}_o^m(t) + \frac{1}{2}\Delta m \mid \ddot{x}_o^m(t) \mid \\ &\leq \;\; \mid e_1(t) \mid + m_o \mid \ddot{e}_x(t) \mid + \mid \Delta m \mid\mid \ddot{x}_o^m(t) \mid \end{aligned}$$

which establishes a bound on e_I in terms of that of e_1 and e_x.

4 Simulations

The dynamic model and control algorithm developed in the previous sections have been verified through simulations. The desired motion trajectory of the object is chosen as

$$x_o^d(t) = \sin(\omega t) \tag{23}$$

and the desired interaction force $F_I = 5.0$ N-m. The initial values of F_1 and F_2 are assumed to be zero, so is F_I. Figure 3 shows the trajectories of F_1, F_2, and F_I. We observe the following:

1. The unilateral constraint is maintained at every instance since F_1 is always positive and F_2 is always negative.

2. The two manipulators interchange the roles they play. While one manipulator pushes hard to generate the required motion, the other merely pushes back to maintain the desired interaction force. It becomes clear that this force planning and control algorithm is fundamentally different from other force control algorithms in which two manipulators simply share the force needed for generating motions.

3. Even though the original system is linear, its output is nonlinear and non-differentiable, which is the desired result for this task and is achieved by the proper force control planner.

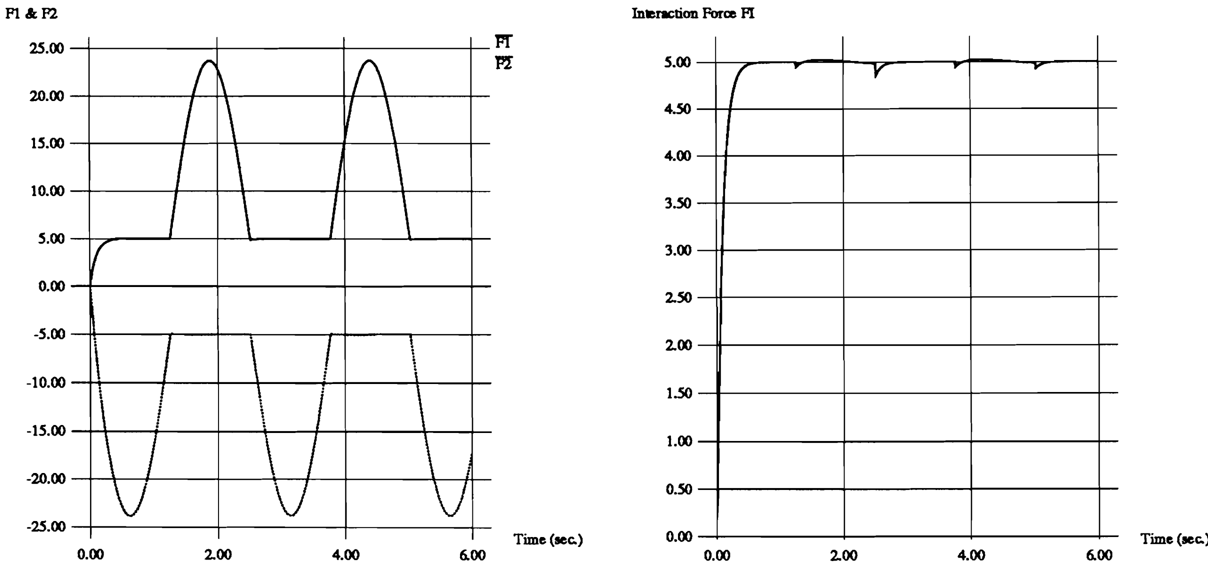

Figure 3: Trajectories of Forces F_1 and F_2 (left) and Interaction Force F_I (right)

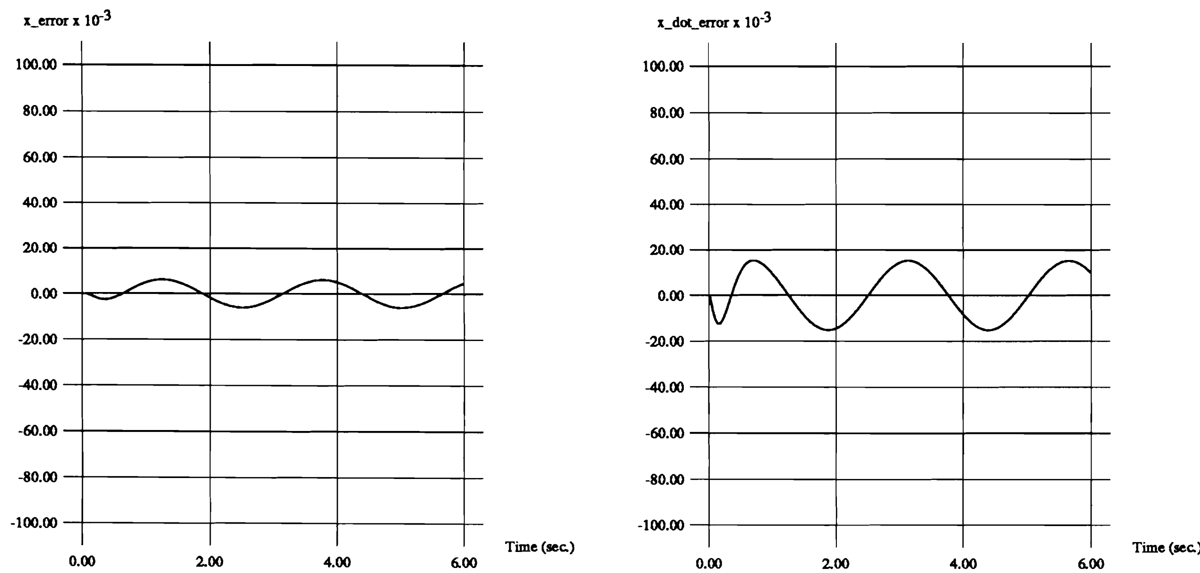

Figure 4: Plots of Errors of $x(t)$ (left) and $\dot{x}(t)$ (right)

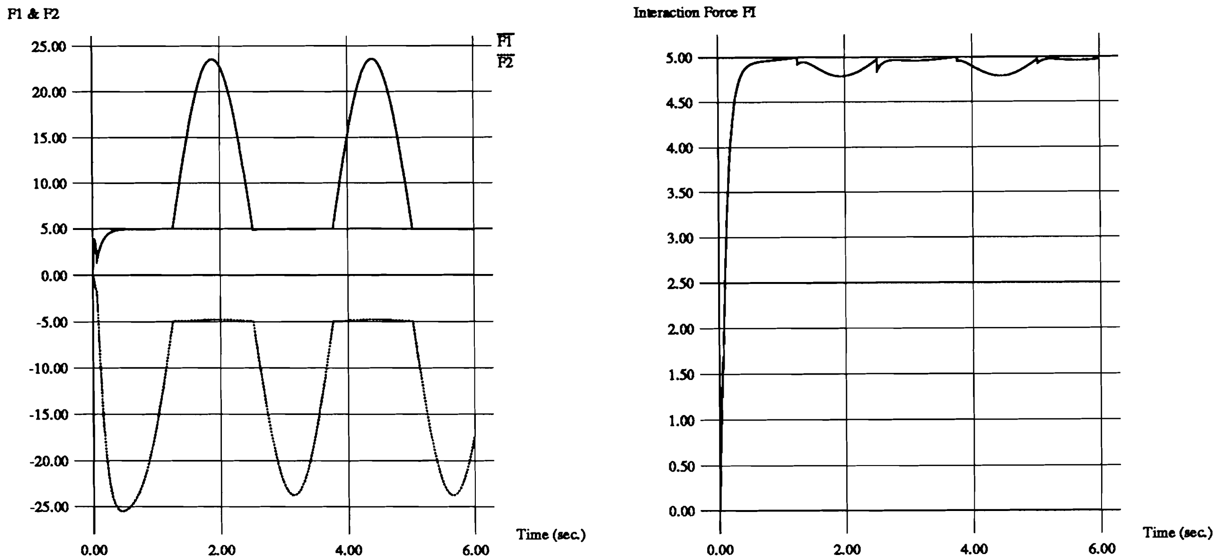

Figure 5: Trajectories of Forces F_1 and F_2 (left) and Interaction Force F_I (right) with Initial Position and Velocity Errors

4. F_I converges to its desired trajectory following a first-order system behavior. Since F_I is not directly controlled, there is noticeable amount (less than 4%) of errors at the instances when F_1 and F_2 switch the roles.

Figure 4 depicts the errors of the motion trajectory and velocity trajectory. The maximal position error is about 1% while the maximal velocity error is about 2%.

We also simulated the effect of the initial position and velocity errors on the performance of the controller. With large initial position errors, the second manipulator tends to pull the object in order to catch up with the desired motion trajectory. A solution to this problem is to replace the desired motion trajectory with the actual one in the force control planner. However, this has a negative effect of degenerating the force control performance at the steady state (see the plot on the right in Figure 5). From Figure 6, the position and velocity trajectories converge to the desired ones in about 0.5 seconds.

5 Conclusion

An approach to the coordinated control of two-arm pushing is presented. Two-arm pushing operations have the potential of grasping and manipulating large objects, such as cardboard boxes, which are not graspable by a single arm/hand. Unlike other two-arm cooperative operations in which the deviation of the interaction force may affect the degree of performance, the success of two-arm pushing operations is critically up to the precise control of the interaction force. Furthermore, the pushing forces must obey a

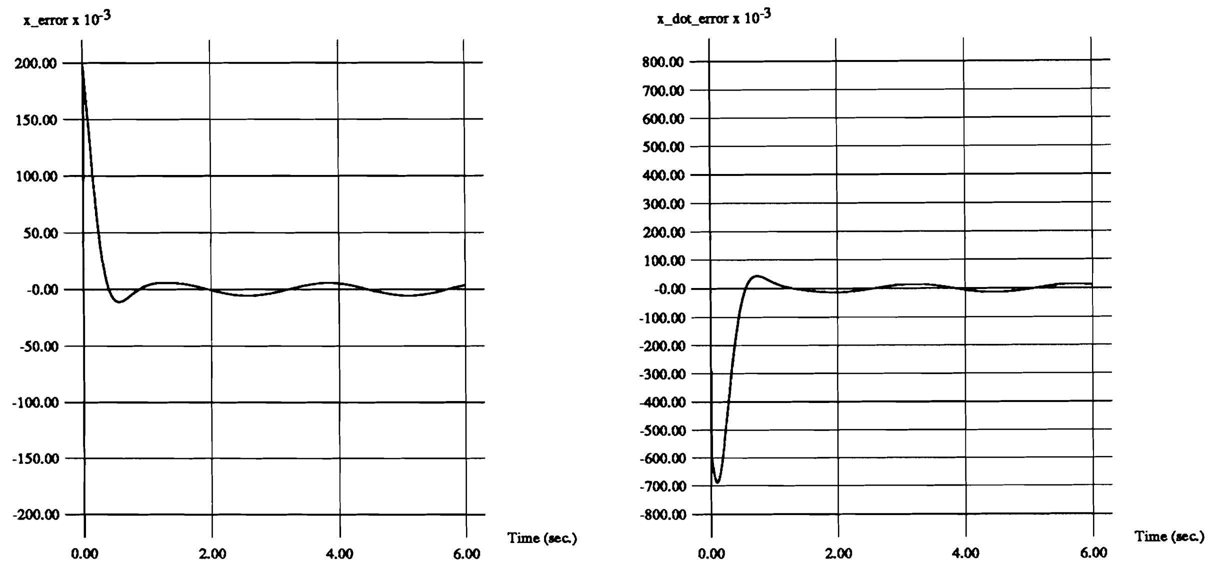

Figure 6: Plots of Errors of $x(t)$ (left) and $\dot{x}(t)$ (right) with Initial Position and Velocity Errors

set of unilateral constraints. Those constraints are modeled as inequalities, rather than equalities, which are in general difficult to deal with.

Represented in the state space, one dimensional two-arm pushing is modeled as a standard linear system by properly choosing output equations. A state feedback is constructed which decouples position control and force control. The stability and performance is accomplished by another feedback applied to each individual motion or force control subsystem. A force control planning algorithm is derived which makes it possible to establish the desired force trajectory directly the task specification. An analytic error bound on the interaction force in relation to the system output errors and model parameter error is also derived. Simulations not only confirm the correctness of the control algorithm but also illustrate that the algorithm is insensitive to model parameter errors.

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