Prospects for Social Security Reform

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Frontispiece: Special Treasury securities, stored in a federal government filing cabinet in West Virginia, represent $700 billion in Social Security Trust Fund assets. Photo: Jeff Baughan.

Chapter 9
Government Guarantees for Old Age Income

George G. Pennacchi

Many actual and proposed social security reforms seek to privatize a country's old-age obligations by requiring that individuals contribute to defined benefit or defined contribution pension plans. But when contributions to private pension plans are mandatory, individuals may be exposed to risks that they did not face in a government-sponsored defined benefit social security system. In the case of a privately-sponsored defined benefit plan, there is the risk that the plan sponsor might default on the pension benefits promised to participants. In the case of a defined contribution plan, the primary risk is that participants' investment returns may be lower than anticipated, leaving them with inadequate wealth during their retirement years.

To make privatization reforms more attractive to the public, governments have often provided guarantees that reduce peoples' exposure to default or investment risks. Thus, there is usually an important role left for governments in mandatory-contribution privatization schemes. If social security in the United States evolves to a system based primarily on defined contribution Personal Security Accounts (PSAs), as described in Chapter 1 of this volume, it is not unlikely that government guarantees on PSA returns would also be provided. The U.S. government already guarantees the retirement benefits of participants in privately-sponsored defined benefit pension plans through the Pension Benefit Guaranty Corporation (PBGC).

In this chapter, I illustrate how the costs of various forms of pension guarantees might be estimated. These estimates can help gauge the implicit subsidies associated with particular reforms and could be incorporated into budgetary measures of government spending. Estimates of these costs might also be used to set insurance premiums paid by guaranteed private pension plans that would reduce or eliminate any government subsidies. The values of pension guarantees made by governments throughout the world are analyzed using "contingent claims analysis" (CCA), also known as "option pricing theory." Following the seminal developments by Black and Scholes (1973) and Merton (1973), a large literature in CCA has emerged. This research applies the fundamental insights of valuing options to valuing more general claims whose payments are contingent on other asset prices. In particular, CCA has been used to value many different types of government guarantees and insurance contracts, such as loan guarantees, deposit insurance, and pension benefit guarantees.

An attractive feature of CCA is that relatively few assumptions are needed to value claims. Typically, valuation requires only the assumptions that there are negligible costs to trading assets and that the equilibrium prices of these assets do not allow for riskless arbitrage. Importantly, assumptions regarding investor preferences (risk-aversion) or assets' expected rates of return are not needed. Based on this preference-free feature of CCA, Cox and Ross (1976) showed how calculating contingent claims prices could be simplified. Their "risk-neutral" valuation method was generalized by Harrison and Kreps (1979) and has become known as the "martingale pricing" approach.

The accuracy of CCA measures to value pension guarantees depends on the validity of the model's assumptions regarding the absence of trading costs and arbitrage in assets held by pension funds. Since most pension fund assets are marketable securities, particularly when a fund is a defined contribution type, trading costs are likely to be low compared to those for non-marketable assets. The precision of CCA will improve with the degree of competition in securities markets and the lowering of securities' bid-ask spreads.

This chapter considers techniques for valuing guarantees on both defined benefit and defined contribution pensions. Previous research has focused on valuing defined benefit guarantees, perhaps because defined benefit pensions have been the dominant type of pension plan in developed countries. However, with the recent growth in social security privatization, especially in Latin America, guarantees of defined contribution pensions are becoming increasingly common.[1] The U.S. government might also decide to offer defined contribution guarantees should it adopt a PSA-based pension system. Therefore, we present a number of new results for valuing defined contribution guarantees. We also emphasizes that the martingale pricing approach can be a unifying framework for valuing all types of guarantees, both defined benefit and defined contribution. This approach may yield explicit formulas for guarantee values, or it can allow for valuation by Monte Carlo simulation. It is beyond the scope of this study to provide a detailed analysis of every possible type of government pension guarantee, but our methodology may be customized to handle specific cases.

When governments guarantee private contracts such as pension plans, adverse selection and moral hazard problems may arise. These incentive problems can be alleviated by properly structuring and pricing guarantees, and/or regulating the activities of the parties on whose behalf the guarantee is given. Discussions of these important issues can be found in a number of recent papers and, due to a lack of space, will not be repeated here.[2] Because my focus is on valuing guarantees, I often take the risk decisions of the participating parties as given. But it should be emphasized that these decisions are often linked to the guarantee's structure, pricing, or regulation.

In what follows, I first discuss the value of defined benefit pension guarantees and how the present value of premiums paid for these guarantees can be assessed. Some qualitative features of these guarantees are illustrated for typical defined benefit pension plans. Next, I discuss various types of defined contribution pension guarantees. Two types of rate of return guarantees are considered: one being a fixed rate of return guarantee and the other being a rate of return guarantee that is relative to the performance of other pension funds. I then examine a guarantee of a minimum pension level following participation in a mandatory defined contribution pension plan. Values for these rates of return and minimum pension guarantees are illustrated using typical parameter values.[3]

Defined Benefit Pension Guarantees

Defined benefit pension plans represent a majority of worldwide pension savings, and their dominance is likely to continue for the foreseeable future.[4] The liabilities of defined benefit plans more closely resemble debt claims, in contrast to defined contribution pension plans, whose liabilities are similar to those of equity claims on a security portfolio. Thus, valuing defined-benefit guarantees is similar to valuing guarantees on default-risky debt.

When a government insures a corporate sponsored pension plan, its net liability can be divided into two components: (1) its "gross" liability deriving from the loss that would occur if the sponsoring firm became bankrupt and its pension fund were underfunded; (2) the present value of the insurance premiums the government charges, as long as the firm avoids bankruptcy. We will consider the value of both components, so that by combining the two the government's net liability (subsidy) can be assessed.

In analyzing the gross value of defined benefit pension guarantees, it is useful to distinguish between pension plans that are collateralized by pension fund assets and those that are not. In most countries, defined benefit pensions are partially or fully funded by a separate portfolio of assets (a pension trust). But in some countries, notably France, Germany, and Japan, corporations need not segregate pension assets from other corporate assets for the sole purpose of backing promised retirement benefits. Rather, pension liabilities are combined with other corporate liabilities and backed only by the general assets of the corporation. Thus, for these non-segregated pension plans, pension guarantees can be valued using techniques developed to value default risky corporate liabilities.

Guarantees of Pensions Backed by Corporate Assets

Let us first consider guarantees of non-segregated plans, and then we will discuss those for pension plans backed by a separate pension fund. Merton's (1974) seminal work provided the first detailed analysis for valuing default risky corporate debt and for valuing guarantees on this debt.[5] In these models, the rate of return on firm assets is typically assumed to follow a continuous time stochastic process (diffusion process) and firm assets may be depleted by firm payments for dividends or interest payments on existing debt. Should assets be insufficient to meet a payment of one of the corporation's liabilities or fall to a level that violates a covenant, then the firm is assumed to be liquidated and its assets are divided among the firm's security holders according to pre-existing seniority rules. Thus, the schedule of the firm's payments to various security holders, along with the uncertainty of asset returns, determines when bankruptcy occurs and the residual level of assets to be split among the firm's creditors. Applying this valuation technique to a corporation having pension liabilities backed only by corporate assets would require a detailed specification of the firm's other current and future liabilities, their terms, and their relative seniority. In general, this can be a rather complicated task.

Recently, Longstaff and Schwartz (1995) have extended the work of Black and Cox (1976) to provide a relatively simple but flexible model for valuing corporate liabilities that allows for interest rate risk as well as default risk. In this model, the value of firm assets at date t, V_t, has a rate of return that follows a constant volatility diffusion process. Should assets fall to a threshold level at date T, say $V_T = \delta$, then the firm is assumed to be unable to meet its financial obligations and a corporate reorganization (bankruptcy) occurs at date T. If bankruptcy occurs during the life of a security, the security holder is assumed to receive, with certainty, a fixed fraction of the security's promised payments at the dates when these payments were originally scheduled to be paid. In other words, the security holder is issued a new security, riskless in terms of default, but with payments equal to only a proportion, say $1 - w$, of the original security's promised payments.

This model offers a greatly simplified view of the timing of bankruptcy and the recovery rates of security holders, since it does not treat explicitly how they are affected by the terms and seniority structure of the firm's individual liabilities. Whether such a simplification is justified should be evaluated on a case-by-case basis. However, if this framework is adapted to a firm having pension liabilities, the pension guarantee could be valued using similar formulas. Should bankruptcy occur, the government insurer would be responsible for the proportional loss, w, of each future pension payment. The government's liability at bankruptcy would then be w times the value of the pension benefits at that date. Of course, the likelihood of bankruptcy as specified by the relative initial value of firm assets, V_t, versus the bankruptcy point, δ, is critical for determining the present value of this guarantee.

Guarantees of Pensions Backed by a Pension Fund

Let us now consider the value of pension guarantees when a pension fund, segregated from other corporate assets, collateralizes pension liabilities. This is the most common type of defined benefit plan, and can be found in countries such as Canada, the Netherlands, the United Kingdom, and the United States. A number of studies have applied contingent claims techniques to value this type of pension guarantee,[6] usually assuming that the term to maturity of the pension insurance is known. In other words, the models assume a non-random date at which the firm's pension plan could be terminated and, if it were underfunded, the insurer would experience a claim. This is a convenient assumption since it allows guarantees to be valued in a manner similar to that of a standard European put option (Merton 1997). In practice, however, a government insurer experiences a possible claim when the sponsoring firm enters bankruptcy, and this date is likely to be highly uncertain. Thus, in practice, it is clear that the value of a pension guarantee depends on both the financial condition of the insured pension fund, and the financial condition of the fund's sponsoring firm.

Research by Marcus (1987) departs from this literature by acknowledging that plan termination dates may be random. His formula for the value of a pension guarantee models the insurer as having a forward contract on the pension fund's assets, with the forward price equal to the fund's liabilities, and a maturity date contingent on the bankruptcy of the sponsoring firm. More recent work (Pennacchi and Lewis 1994; Lewis and Pennacchi 1997) has revised the Marcus analysis to model the guarantee as a contingent put option rather than a contingent forward contract, the difference being that, in practice, a government does not obtain a positive payment when a firm with an overfunded pension plan fails. The guarantee is then analogous to a put option on the pension fund's assets, with an exercise price equal to the pension fund's liabilities, and a contingent maturity date determined by the sponsoring firm's bankruptcy.

The general approach taken in this literature specifies continuous-time stochastic processes for the sponsoring firm's (corporate) assets, V_t, the sponsoring firm's (corporate) liabilities, D_t, the pension fund's assets, F_t, and the pension fund's accrued vested liabilities, A_t. In general, these four processes will be correlated, and their expected growth rates will be affected by the assumed behavior of the sponsoring firm and the demographic structure of the participants in the pension plan. For example, V_t depends not only on the random change in the return on the firm's assets but also on any net payouts from the corporation's assets made by the firm. Similarly, the process followed by F_t depends on the random rate of return on the pension fund's investments, and also on the firm's new contributions to the pension fund less any benefits paid to retirees. Finally, A_t depends on the random change in value of the fund's current vested benefits (due to changes in market interest rates), and also on the amount of new benefits granted to workers and retirees less any benefits paid to retirees (Marcus 1987).

Given this framework, we can now specify the payment required by a government guarantor should a pension claim arise. At date T, the government is assumed to be liable for the amount $L_T = \max(0, A_T - F_T)$, where T is defined as the first time that V_T falls to a level δD_T, and where $0<\delta\leq 1$. $1-\delta$ is the firm's level of negative net worth when bankruptcy occurs.[7] The contingent payment by the government has the structure of a put option, but with the maturity date T of the option being stochastic, coinciding with the bankruptcy of the sponsoring firm. The martingale pricing can be applied to calculate the present value of this government guarantee, L_t. This approach is equivalent to other CCA methods (Kocic 1996) and so requires only the assumption that equilibrium security prices not allow for arbitrage opportunities.[8]

The basic idea of the martingale approach is that the value of a government's guarantee can be computed as the expected payment made by the government, discounted at the risk-free rate of interest, where the expected payment is that which would occur if all assets had an expected rate of return equal to the risk- free rate.[9] Specifically, if we let r_s be the continuously compounded, short-maturity, risk-free interest rate at date s, then the present value of the government's guarantee can be calculated as $L_t = E_t^*[\exp(-\int_t^T r_s ds)\ L_T]$. Here $E_t^*[\bullet]$ is the date t expectation of the discounted payment assuming all assets have an expected rate of return at date s equal to the risk-free rate, r_s.[10] Note that a world where the average rate of return on all assets equaled the risk-free rate would be one where all investors were risk-neutral. Hence, the martingale approach often reduces to what is referred to as "risk-neutral pricing." However, it should be emphasized that this approach does not explicitly assume universal risk-neutrality. In fact, it does not require any specific assumption regarding investor preferences. The association with risk-neutrality is only a computational technique that leads to the unique arbitrage-free value for the government's guarantee.

Loosely speaking, the martingale approach gives the correct value for L_t because of two erroneous assumptions whose effects cancel each other out, resulting in a correct valuation. One incorrect assumption is that all assets have an average rate of return equal to the risk-free rate, that is, there are no "risk premia" in asset rates of return. This implies a "risk-neutral" expectation of L_T that differs from the "true" expectation of L_T, leading to the first error. The other incorrect assumption is that this risky payment should be discounted at the risk-free rate rather than a discount rate that includes a risk premium, leading to a second error. Because both the first and second "errors" involve a failure to account for risk premia, the first error "understates" the expected growth of L_T by the risk premia while the second error "overstates" the discount factor applied to L_T by the risk premia. Mathematically, these two errors cancel, leading to a correct valuation formula. Importantly, because this computational technique does not require specification of the actual risk premia of the assets in the economy, no assumptions regarding the signs or magnitudes of risk premia are needed.

Computing the expectation of the discounted value of the government's payment does require specific assumptions regarding the firm's rate of net payouts from corporate assets, the rate of net contributions to the pension fund, and the rate of net new pension benefits. An explicit solution can be derived for the case in which these rates are constants (Pennacchi and Lewis 1994; Lewis and Pennacchi 1997). Guarantee values for this case will be illustrated shortly. However, when firm payouts, pension fund contributions, or pension benefit growth rates are more general functions of the four variables, V_t, D_t, F_t, and A_t, the expected value of the government's discounted payment can be computed using a Monte Carlo technique. This involves simulating a large number of risk-adjusted sequences of the variables in a manner similar to Boyle (1977). The average value of the discounted payments generated by this Monte Carlo simulation will converge to the theoretical expected discounted payment of the government. When carrying out the Monte Carlo simulation it is easiest to assume that the firm's payouts, pension fund contributions, and net pension benefit increases occur at discrete dates, such as the end of every month or year (Cooperstein, Pennacchi, and Redburn 1995). Between these payout and contribution dates, the risk-adjusted processes for the variables are simulated excluding these payout or contribution effects.

We do not present estimates of defined benefit guarantees using such a simulation technique here, but it should be noted that prior work has used Monte Carlo simulation to value defined benefit guarantees. Estrella and Hirtle (1988) estimate the value of the U.S. PBGC guarantees by simulating stochastic processes for pension fund assets and the assets of the sponsoring firm. They do not value cashflows using a martingale approach, but instead assume that the expected real rate of return on pension fund assets is a constant 2.5 percent per year while the expected real rate of return on firm assets is 1 or 1.5 percent, depending on the firm's past growth performance. In addition, they assume a constant real interest rate and discount PBGC payments for terminated pension plans at a real 2.5 percent rate.

One could debate whether these expected return and discount rate assumptions are reasonable. Further, these assumptions are unnecessary if one values pension guarantees using martingale pricing, which requires only the no-arbitrage assumption. In addition, the martingale approach easily allows for stochastic interest rates, a potentially important consideration given the relatively long duration of pension fund liabilities. Thus, it is important to emphasize that ad hoc assumptions can usually be avoided and interest-rate uncertainty can be incorporated by following the martingale approach. This point is applicable to a simulation model currently being developed by the PBGC itself, known as the Pension Insurance Management System (PIMS). This is a highly detailed model which, in its current form, calculates the PBGC's expected future cashflows needed to resolve terminated pension plans. As with Estrella and Hirtle (1988), PIMS could be modified to incorporate stochastic interest rates and to calculate the present value of the PBGC's cashflows by applying martingale pricing techniques.

Valuing Premiums Paid for Pension Guarantees

We now consider how the present value of premiums received by a government pension insurer can be computed so that its net liability can be determined. While we consider the value of contingent insurance premiums given a rate structure similar to that paid by PBGC-insured pension plans, this valuation technique could be modified for other types of rate structures.

Valuing future premiums paid to the PBGC is a nontrivial problem because premiums are contingent on future levels of pension underfunding as well as the solvency of the sponsoring firm. Consider the premium to be received by a pension insurer in some future year, τ. If the sponsoring firm is solvent, this insurance premium is assumed to consist of a flat rate premium per current participant, denoted p_0, and a two-part variable premium equal to a proportion, p_1, of the amount of pension underfunding, and subject to a cap or maximum variable payment per participant of κ. For example, the PBGC sets a flat premium of p_0 = \$19 per participant plus a variable rate premium of p_1 = \$.009 per \$1 of current underfunding up to a maximum variable payment of κ = \$53 per participant. (This maximum variable payment, κ, is currently being phased out.)

Maintaining the notation A_τ and F_τ as the pension fund's accrued liabilities and assets in future year τ respectively, let us also define e_τ as the number of participants in the pension fund during year τ and $P_\tau(t)$ as the value at date t of the premium income to be received by the PBGC in year τ. Then, assuming the solvency of the sponsoring firm, the premium income received at date τ takes the form $P_\tau(\tau) = e_\tau p_0 + p_1 \max(0, A_\tau - F_\tau) - p_1 \max(0, A_\tau - F_\tau - e_\tau\kappa/p_1)$. The last two terms can be viewed as put options written on the pension fund assets, the first having an exercise price of A_τ and the second having an exercise price of $A_\tau - e_\tau\kappa/p_1$.

Given the four processes describing the pension fund's and the corporation's assets and liabilities, we again apply the martingale approach to value this contingent premium income. Similar to the previous analysis for valuing the government's gross liability, the current (date t) value of the premium income anticipated in future year τ can be computed as the "risk-neutral" expectation of the discounted premium received, that is, $P_\tau(t)$ =

$E_t^*[\exp(-\int_t^\tau r_s ds)\ P_\tau(\tau)]$. Summing this expression over all future years τ equals the value of all future premium income to be received by the government. For general assumptions regarding the firm's rate of net payouts from corporate assets, the rate of net contributions to the pension fund, and the rate of net new pension benefits, P_τ(t) can be calculated using the Monte Carlo simulation technique described in the previous section, where firm payouts, pension contributions, and new benefit increases occur at discrete dates. For the special case in which the payout rate, contribution rate, and the rate of new benefit increases are constants, Lewis and Pennacchi (1997) have derived an explicit formula for the value of this premium income. The next section illustrates the behavior of this premium value, along with the value of the government's gross liability for this particular case.

Comparative Statics for Defined Benefit Guarantees

For parameter estimates characterizing typical U.S. pension plans, Figures 1 and 2 graph the value of the government's gross liability as a percentage of pension liabilities, $100\times L_t/A_{p}$, and its net liability (assuming no cap on premiums) as a percentage of pension liabilities, $100\times[L_t-P_\tau(t)]/A_t$.[11] Figure 1 graphs these values as a function of the firm's net worth for the case of a 30 percent underfunded plan and for the case of a 30 percent overfunded plan. In both cases, the difference between gross and net liabilities rises with firm net worth, reflecting the higher present value of premiums to be paid by the more solvent firms. As expected, gross and net liabilities are much less for the overfunded plan that the underfunded plan, and government liability for underfunded plans rises as the firm's net worth falls. Interestingly, however, the government's liability for overfunded plans falls as the firm's net worth falls. The intuition for this result is that should a firm with an overfunded plan fail, the government will bear no loss and have no further exposure to this pension plan.

Figure 2 shows insurer gross and net liabilities as a function of pension fund's funding ratio, for a firm whose net worth is 10 percent of its corporate liabilities, and for a firm with a 100 percent net worth-liability ratio. In both cases, liabilities rise as pension funding falls. In addition, the difference between gross and net liabilities rises with declines in pension funding, reflecting the assumed premium rate structure that charges higher premiums to firms with greater underfunding. However, note that government liabilities are higher for low-net-worth firms when pensions are underfunded, but the reverse occurs when pensions are overfunded. The intuition is similar to that reflected in Figure 1: a firm with high net worth having a moderately overfunded plan may pose a larger liability than would a firm on the brink of bankruptcy with the same pension overfunding. The high net worth firm is likely to remain in operation longer, increasing the probability that its pension fund will become underfunded in the future.

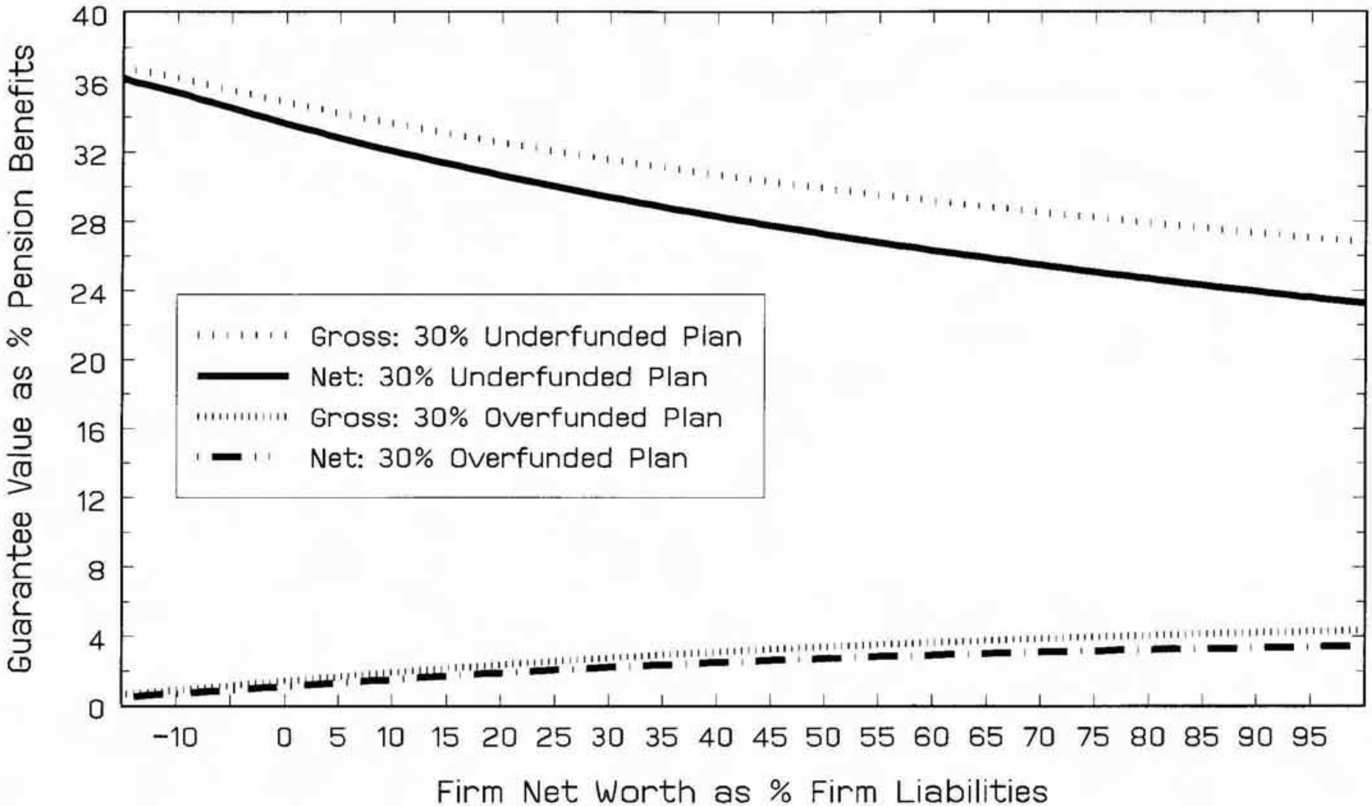

Figure 1. Value of gross and net pension guarantees by firm net worth. Source: Author's calculations.

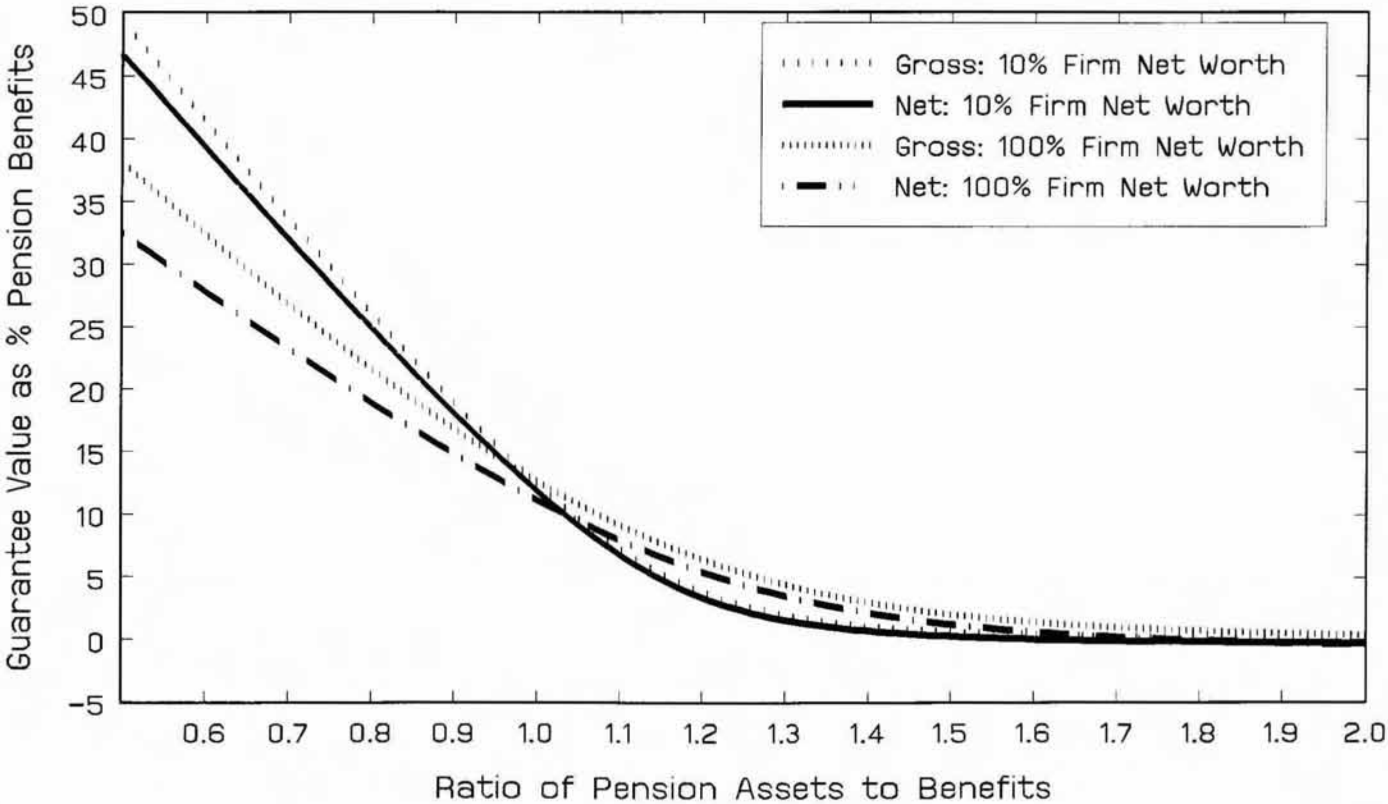

Figure 2. Value of gross and net pension guarantees by pension funding level. Source: Author's calculations.

Defined Contribution Pension Guarantees

Defined contribution pension plans are sometimes afforded government guarantees on rates of return. These guarantees can be valued by recognizing their similarity to so-called "exotic" options such as "forward start options," "options to exchange one asset for another," and "options on the minimum of two risky assets."[12] We begin by considering a relatively simple fixed minimum rate of return guarantee, similar to one provided by Uruguay. We then consider a minimum rate of return guarantee that is a function of the average rate of return earned by all pension funds, such as that provided by the government of Chile.

A Minimum Fixed Rate of Return Guarantee

Uruguay permits both private and public pension funds, known as "Asociaciones de Fondos de Ahorro Previsional" (AFAP).[13] In the case of public AFAPs (but not the private AFAPs), the government guarantees to pension fund participants a minimum annual real rate of return of 2 percent. Thus, a public AFAP which earns less than 2 percent during a given year would require a government transfer to make up the difference.

Applying martingale pricing methods, Pennacchi (1997) obtains an explicit formula for the value of these annual rates of return guarantees, making use of the similarity between these guarantees and an annual series of forward start options. A forward start option is an option that is paid for now, but whose exercise price is set equal to the contemporaneous value of the underlying asset at some future date prior to the maturity date of the option.[14] The analogy between a rate of return guarantee and a forward start option is that a (continuously compounded) rate of return on an asset over some future interval, say from date t_1 to time t_2, needs to be computed based on two future asset values: $\log[F(t_2)] - \log[F(t_1)]$, where $F(t)$ is the value of the (pension fund) asset at date t. In general, the t_1 beginning date of the rate of return is in the future, so $F(t_1)$ is unknown and analogous to the unknown beginning exercise price of the forward start option.

Valuing the government's rate of return guarantee makes of a weighted annual series of "at-the-money" Black and Scholes type (1973) put options, where the weights are proportional to the assumed growth in net new contributions to the pension fund (Pennacchi 1997). If an individual annual guarantee has any value and the real growth rate of the pension fund is nonnegative, the value of the annual series of guarantees can be shown to grow without bound as the number of future years for which this guarantee is made increases. Clearly governments should be very cautious in providing such a guarantee, particularly to funds anticipated to grow substantially.

A Minimum Relative Rate of Return Guarantee

In Chile, private pension funds known as "Administradora de Fondos de Pensiones" (AFPs) are required to earn an annual real rate of return that is a function of the average annual real rate of return of all private pension funds. If R_a is the (ex-post) average annual rate of return earned by all AFPs, then each AFP must earn at least $\min(R_a-\alpha, \beta R_a)$ where $\alpha = .02$ and $\beta = \frac{1}{2}$. Thus, if R_a turns out to be ≥ 4 percent, each AFP must earn at least $\frac{1}{2} R_a$, while if R_a turns out to be ≤ 4 percent, each AFP must earn at least $R_a - 2$ percent. All AFPs are required to hold capital (a guarantee fund) of at least 1 percent of the value of its pension portfolio, invested in the same security portfolio as that of its pension fund. If the fund's return is less than $\min(R_a-\alpha, \beta R_a)$, it must make up the difference from its capital and replenish its capital within 15 days. The AFP's license will be revoked if it fails to do so. Thus, given an AFP capital ratio of $c = .01$, the government is exposed to loss following an AFP that earns less than $\min(R_a-\alpha, \beta R_a) - c = \min(R_a-\alpha-c, \beta R_a-c)$.

The Chilean-style government guarantee for an individual AFP is analogous to an annual series of options to exchange the individual AFP's pension assets for the minimum of two other risky assets. A formula for the value of this guarantee takes the form of an annual series of bivariate normal distribution functions.[15] As one might expect, the value of this relative rate of return guarantee is sensitive to the standard deviation of the individual AFP's rate of return as well as the correlation between the individual AFP's return and the average return of all AFPs.

Figure 3 plots the annual cost of this Chilean-style guarantee as a percentage of the current value of the pension fund assets, $100 \times L_t/F_t$. This is done for different assumed correlations between individual AFP and average AFP returns. The guarantee value is shown for three cases: when the individual AFP standard deviation equals, is twice, or is one-half that of the average of AFPs. As would be expected, the value of the guarantee falls as the correlation rises. Interestingly, when the standard deviation of the individual AFP's return exceeds that of the average of AFPs (which should be the case for the typical AFP since individual risk is diversified by averaging), then even when the correlation is perfect, the guarantee has positive value. Currently no premium is charged to cover this government insurance in Chile.

Valuing Minimum Pension Guarantees for Defined Contribution Plans

This section considers the value of a minimum pension guarantee for a participant in a mandatory defined contribution pension system, where a fixed proportion of a worker's wage is assumed to be contributed to a pen-

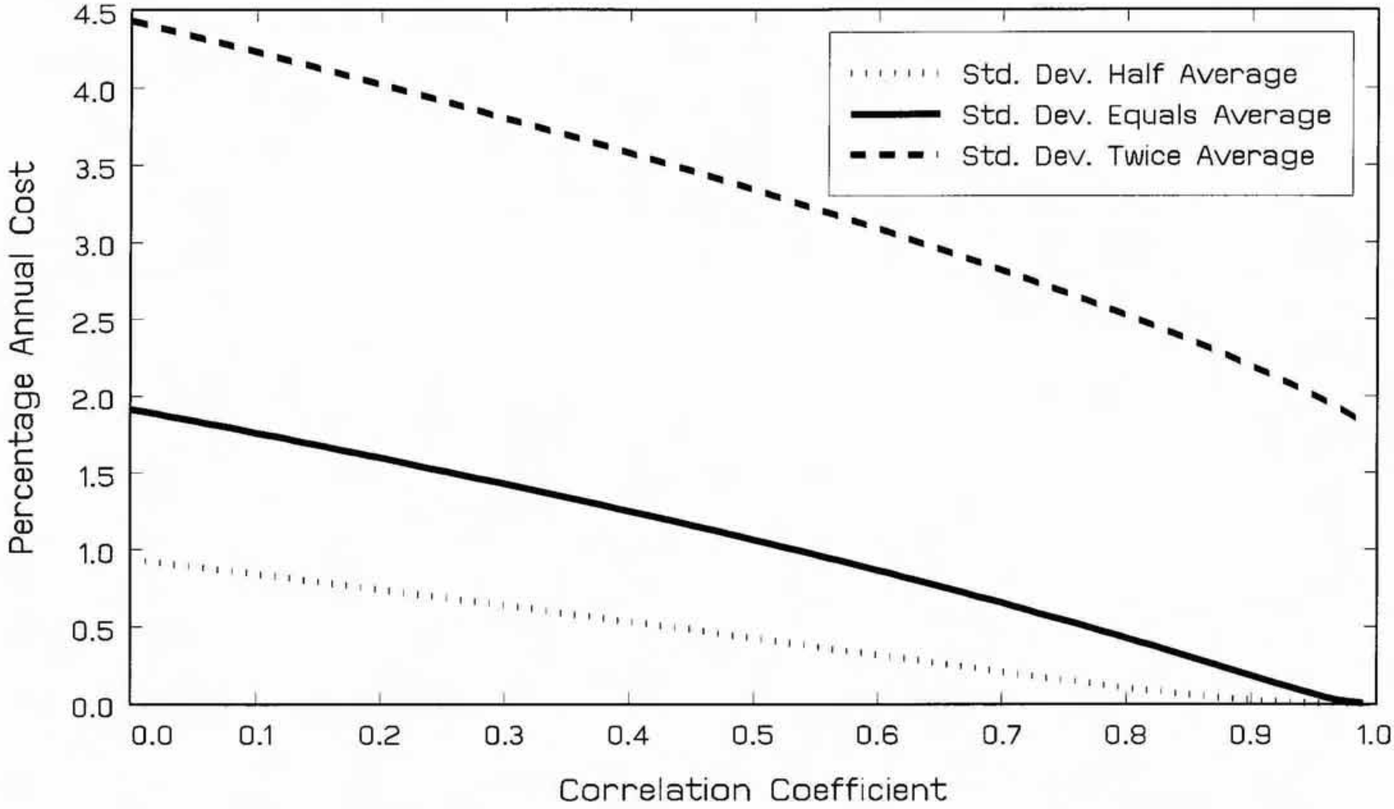

Figure 3. Value of relative rate of return guarantee by correlation between individual and average AFP. Source: Author's calculations.

sion fund that earns risky returns. Previous studies estimating the value of this guarantee for the case of Chile include work by Wagner (1991) and Zarita (1994). Wagner values this guarantee by simulating its annual cost when the demographics and maturity of the pension system are at their steady state values. The model calculates this cost under different assumptions regarding the real rate of return on pension fund assets and the level of the minimum pension guarantee. Contingent claims techniques are used by Zarita with a model that explicitly allows for a stochastic rate of return on pension fund assets, so that a worker's accumulated pension savings at retirement is random. When a worker's saving at retirement is less than the cost of an annuity providing the minimum pension, the government is assumed to make a payment to cover the difference. The risk-neutral expected value of this government payment is calculated using a Monte Carlo simulation of the worker's risky pension investment assuming a deterministic level of wage contributions each period and a constant real interest rate.

Our approach is similar to that of Zarita (1994) but includes three important extensions. First, in addition to allowing pension returns to be stochastic, we also allow a worker's real wage, and thus his monthly pension contribution, to follow a random process. The evolution of real wages is also assumed to influence the minimum pension set by the government when the worker retires. Second, real interest rates are assumed to follow a stochastic process. This is potentially important since retirement annuity values are a function of real interest rates. Also valuing the government's guarantee requires that real interest rates discount the government's guarantee payments and, in general, these payments are systematically related not only to asset returns and wage levels, but also to the real interest rate. Third, we model the government's payments for a minimum pension in a more realistic manner. Upon reaching retirement, a retiree may have a choice regarding his benefit payments. If he has sufficient pension savings, he may choose to close his pension account and use his savings to purchase a lifetime annuity that provides a benefit at or above the minimum pension. Alternatively, he can maintain his pension account and receive benefits by a scheduled withdrawal of funds from his account. For a retiree with an account balance insufficient to purchase a minimum pension annuity, a scheduled withdrawal of funds is required. The maximum amount that a retiree can withdraw each year is determined by a government schedule that depends on the retiree's current pension account balance and the value of a lifetime annuity, where this annuity is calculated using the government's "technical" interest rate. If and when a retiree's pension account balance is exhausted, the government guarantees that it will pay him the minimum monthly pension for the remainder of his life.

As discussed in Turner and Wantanabe (1995) and Smalhout (1996), a worker who attains retirement age with a pension balance slightly above or equal to the price of a minimum pension annuity will have an incentive to not purchase an annuity, but will instead choose the scheduled withdrawal option. By choosing this scheduled withdrawal, he will receive free longevity insurance at the government's expense. Should he live longer than expected, the government provides him with a minimum pension. If, instead, he dies sooner than expected, his heirs will inherit the balance of his pension account. Thus, in some states of the world, he received a government subsidy that would not occur if he had immediately purchased a lifetime annuity. Hence, for someone reaching retirement with moderate to small pension savings, who is the individual most likely to require minimum pension assistance, it is more realistic to assume a scheduled withdrawal of pension funds. Unlike Zarita (1994), our model explicitly considers the scheduled withdrawal option.

The full model, detailed in Pennacchi (1997), is based on three random processes: the rate of return on pension fund assets, the growth in real wages, and the change in the short term real interest rate. These three processes may be correlated. The short-term real interest rate determines the term structure of real yields (Vasicek 1977). An additional minor source of uncertainty is the individual's mortality. The probability of death at each age is assumed to be uncorrelated with economic variables and is taken from Chile's official life table. A hypothetical male worker is assumed to begin making pension contributions at age 20 and, should he live until the retirement age of 65, begin a scheduled withdrawal of his pension savings at the maximum level allowed by law. The worker's mandatory monthly contribution equals 10 percent of his randomly evolving wage and is invested in his pension fund earning a random rate of return.

At retirement, the maximum that can be withdrawn each month is calculated following the actual Chilean government formula, described in Diamond and Valdés-Prieto:

> Every twelve months, the fixed real amount that will be withdrawn in each of the following twelve months is calculated. This amount is P = F/UC, where F is the current balance in the individual account and UC is calculated from the official life table and a technical interest rate (TR), and it is essentially the reserve needed to finance an annuity that pays $1 a month when investments yield TR. The return TR in turn is calculated according to a formula fixed by law. This formula specifies that, for AFP i, TR_i for year $t = 0.2 *$ (average of past real returns of Fund i during past five years) + $0.8 *$ (average of implicit rates of return on all real annuities sold in calendar year $t-1$). (1994: 290)

Our model follows this formula exactly, except that in calculating TR we approximate "the implicit rate of return on all real annuities sold in calendar year $t-1$" with the date t real yield on a nine-year zero-coupon indexed bond, since Diamond and Valdés-Prieto (1994) report that the duration of newly issued annuities is approximately nine years. Thus, during the individual's retirement period, the amount withdrawn is a function of the last five years' returns of the individual's AFP (affecting TR), the current randomly evolving real yield on a nine-year bond (affecting TR), the individual's age (affecting UC), and the individual's pension fund balance (which is affected by past withdrawals and pension fund asset returns).

The above formula's "maximum" withdrawal is, however, truly the maximum *only* if it exceeds the government's minimum pension level. If not, the amount withdrawn is equal to the minimum pension. This occurs until the retiree's pension account is exhausted, should he live that long. After the account balance is exhausted, the government pays the minimum pension until the end of the retiree's life.

The minimum pension is set at the discretion of the government, and it depends on a number of political and economic factors. For simplicity, our model assumes that the minimum pension at the beginning of an individual's retirement follows the formula: minimum pension = ¼*(average wage at start of individual's working life)*(growth in the individual's real wages over his working life)*½. This assumed formula reflects the likelihood that the government will tend to raise the minimum pension should real wages (and the standard of living) rise. Since Turner and Wantanabe (1995) report that the minimum pension is approximately 25 percent of the average wage and because our model assumes that the individual's real wage will almost double over his 45 years of work (1.5 percent average annual growth), the formula represents a minimum replacement rate of approximately 25 percent.[16]

To value this guarantee, the martingale approach is used to transform the model's three random processes into risk-adjusted counterparts, so that the value of the guarantee can be computed as the expectation of the government's discounted minimum pension payments. This expectation is calculated using a Monte Carlo simulation, where contributions or withdrawals from the individual's pension fund account occur each month. Parameter values typical of Chile were selected (see Pennacchi 1997). Guarantee values are calculated for the use of a 20-year-old male beginning wage earner starting with a zero pension fund balance. Mortality is based on the Chilean life tables for male annuitants. Normalizing an average Chilean monthly real wage at 100 as of the time this individual begins work, we find that the average level of the minimum pension set by the government (according to the formula discussed above) at the worker's retirement date was 44.7.

Figure 4 graphs the present values of the minimum pension guarantee for this 20-year-old worker, using alternative initial monthly wages ranging between 10 and 100.[17] The value of this guarantee ranges from 251.8 for an individual with an initial monthly wage of 10, to 5.8 for an individual with an initial monthly wage of 100. The shape of the relationship is convex, as one might expect given the put option-like nature of this guarantee. Also plotted in Figure 4 is the individual's age at which his pension fund account would be depleted, should he live that long. This ranges from age 72.1 for an initial

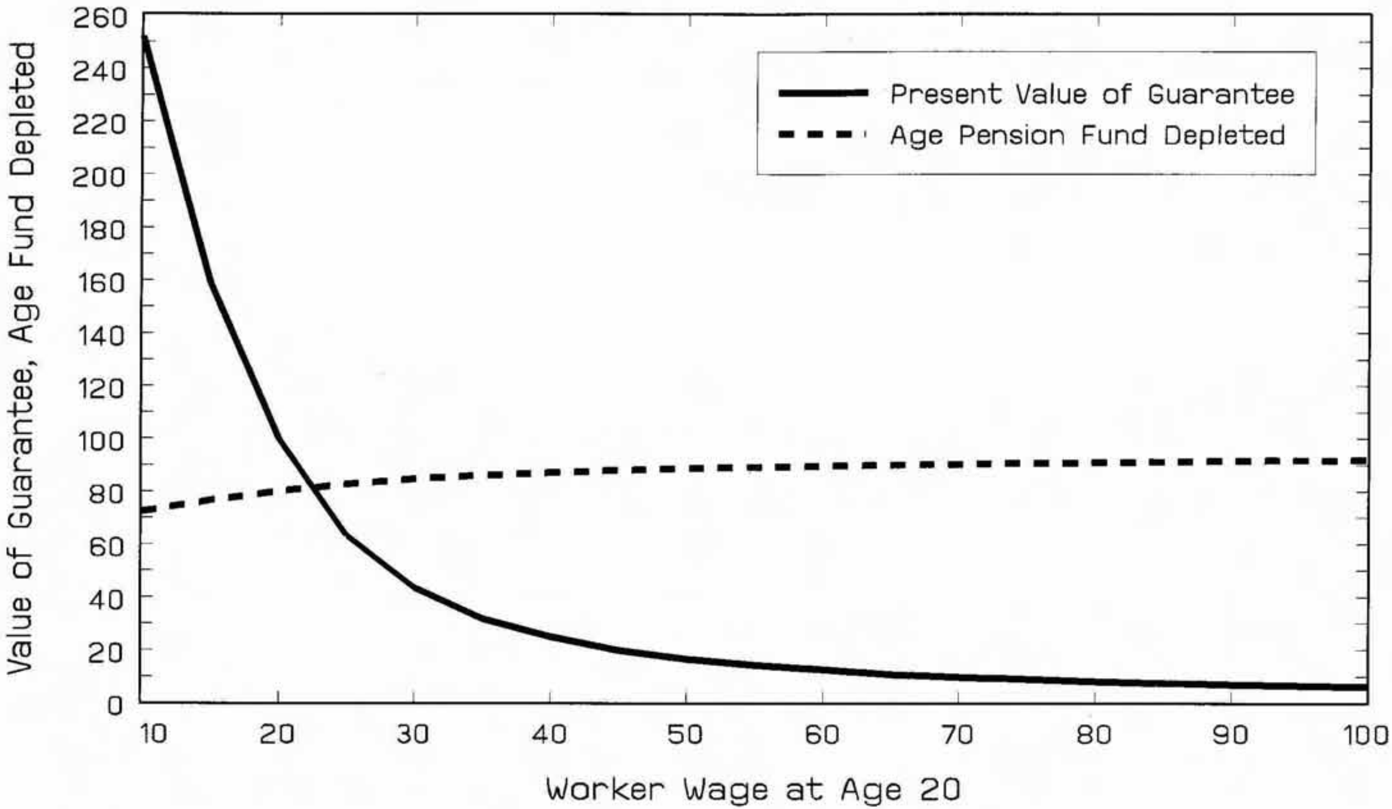

Figure 4. Minimum pension guaranteed by initial wage level. Source: Author's calculations.

wage of 10, to age 91.8 for an initial wage of 100. Note that this age profile has a concave shape: higher initial wage increase the time before the pension account is depleted, but less than proportionally. While higher initial wages tend to result in proportionally higher accumulated pension savings at retirement, the government's scheduled withdrawal formula allows greater pension withdrawals for individuals with higher savings. Thus the withdrawal schedule tends to dampen the effect that greater retirement savings have on the age at which pension funds are depleted.

Conclusion

Social security privatization programs frequently require that workers contribute to defined benefit and/or defined contribution pension plans. Relative to government-sponsored defined benefit social security systems, these privately-sponsored pension plans subject individuals to default risk (in the case of defined benefit plans) or investment risk (in the case of defined contribution plans). To make privatization reforms politically attractive to the public, governments have typically offered guarantees that reduce individuals' exposure to default or investment risks. Should the United States privatize a substantial portion of its pension obligations using a PSA-type approach, government guarantees of pension plan performance might be a real possibility.

Our analysis illustrates how the martingale pricing approach, also known as the risk-neutral valuation method, can be applied to value such pension guarantees. This methodology then permits the computation of risk-based insurance (guarantee) premiums. Requiring that riskier pension funds, and possibly riskier individuals, pay higher insurance premiums could help control adverse selection and moral hazard behavior. It would reduce the subsidies and the economic distortions associated with government guarantees. The potential for reducing such distortions through risk-based premiums may ultimately change the type of pension system that a government chooses to adopt.

The ability to price guarantees can also allow government budgets to be measured on a market-value basis. A government's total liability from providing guarantees can be calculated by aggregating the values of individual guarantees. This aggregation requires detailed data on the economy's individual pension plans, pension funds, and/or worker demographics. Such an exercise is beyond the scope of this chapter, but the analysis presented here provides a foundation for obtaining a more accurate indicator of government fiscal policy.

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Notes

1. Mitchell and Barreto (1997) and Queisser (1995) describe Latin American pension reforms. More general discussions of pension systems are given in Mitchell (1994), Davis (1996), and Turner and Wantanabe (1995).

2. See Bodie and Merton (1993), Pesando (1996), and Smalhout (1996).

3. To simplify the exposition, details of the methods and formulas for valuing guarantees are not presented, but these can be found in Pennacchi (1997).

4. For example, several governments, including those of Great Britain and Italy, are currently promoting private pensions as a solution to their over-dependence on government-provided retirement benefits. Their reform proposals presume that private pensions will be mainly of the defined benefit type.

5. An excellent review of this and subsequent work appears in Merton (1990).

6. A partial list includes Treynor (1977), Langetieg, Findlay, and da Motta (1982), Marcus (1987), Bodie and Merton (1993), and Hsieh, Chen, and Ferris (1994). Lewis and Cooperstein (1993) use the approach in Marcus (1987) to estimate the current exposure of the PBGC.

7. Note that this modeling of bankruptcy is similar to that of Longstaff and Schwartz (1995). In general, the analysis is valid when δ is any positive number, though it is best interpreted as being less than or equal to 1.

8. A more detailed and technical discussion of martingale pricing can be found in Duffie (1996).

9. This risk-free rate of interest can be stochastic, so that interest rate risk can be explicitly modeled. See Lewis and Pennacchi (1997) or Pennacchi (1997) for details.

10. The risk-free discount factor is given by $\exp(-\int_t^T r_s ds)$, where $\exp(\bullet)$ is the exponential function.

11. This analysis draws on Lewis and Pennacchi (1997).

12. For a description and analysis of these exotic options, see Hull (1997).

13. See Mitchell (1996) for a discussion of pension system reform in Uruguay.

14. In other words, the option's exercise price is set so that it is "at-the-money" at some prespecified future date prior to the option's maturity date.

15. See Pennacchi (1997), who builds on results in Margrabe (1978), Stulz (1982), and Johnson (1987).

16. One component of the lifetime growth of an individual's real wage is likely to reflect increased (economy-wide) average productivity, while another component should reflect the individual's increased productivity due to greater experience and seniority. Thus, it is reasonable to expect that an individual's lifetime real wage growth will exceed the economy-wide average. For this reason, the formula includes a final factor of one-half. The result is that our simulations give an average minimum pension at the individual's retirement date equal to 44.7 percent of the initial average real wage, implying that, on average, there is a slightly less than doubling (from 25 percent) of the minimum pension.

17. A GAUSS program that calculates the guarantee values in Figure 4 is available from the author upon request.

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