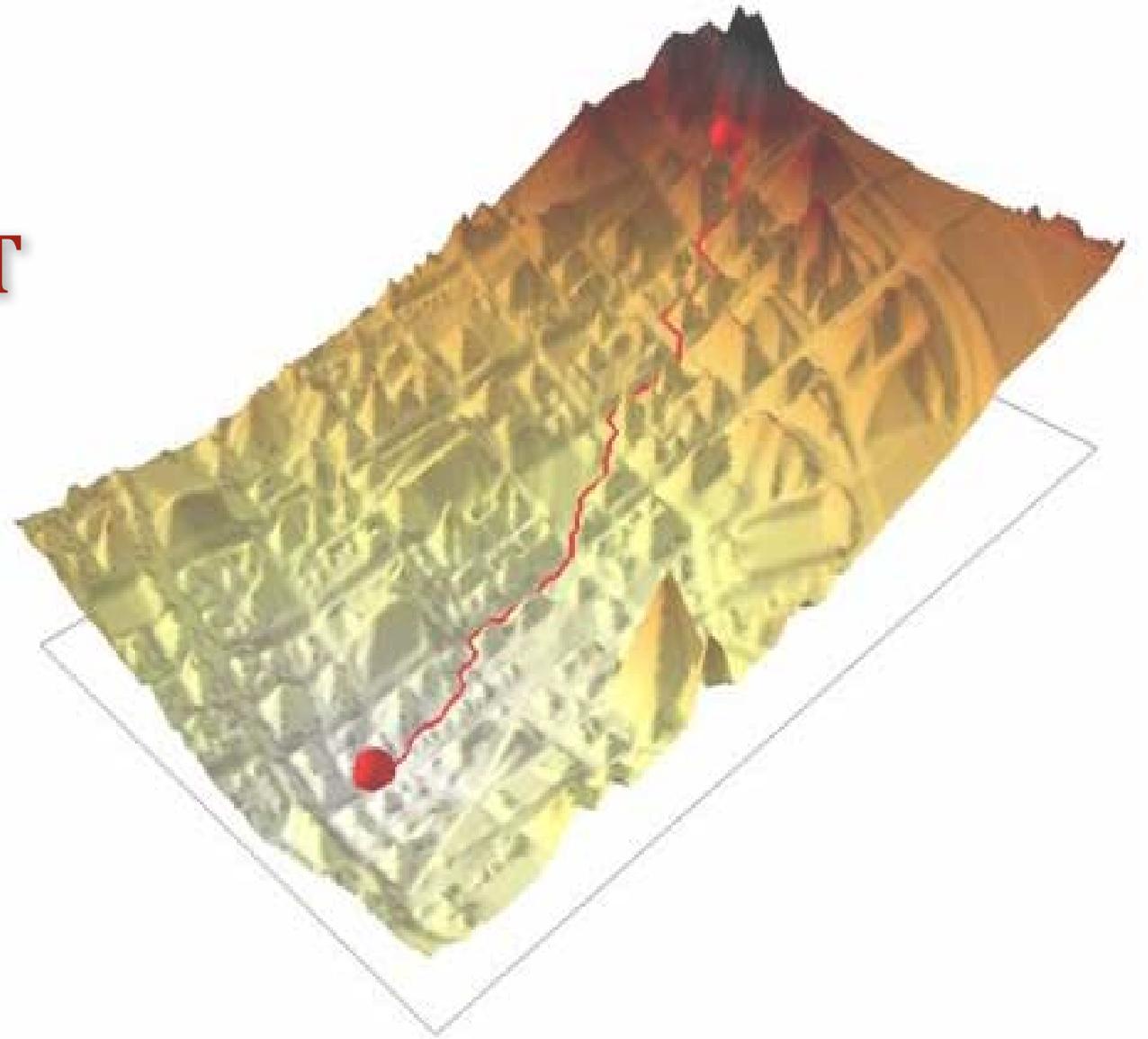


USING MAP ALGEBRA TO MODEL URBAN HEAT

analyzing hotspots



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CHAPTER FOUR

Hotspot. . .Though the term has been **around** for decades, it is one that seems especially well suited to the tongue of today’s popular media. Perhaps for that reason, this is also a term that now finds use in fields ranging from computer science to genetic theory and from veterinary medicine to digital cartography.

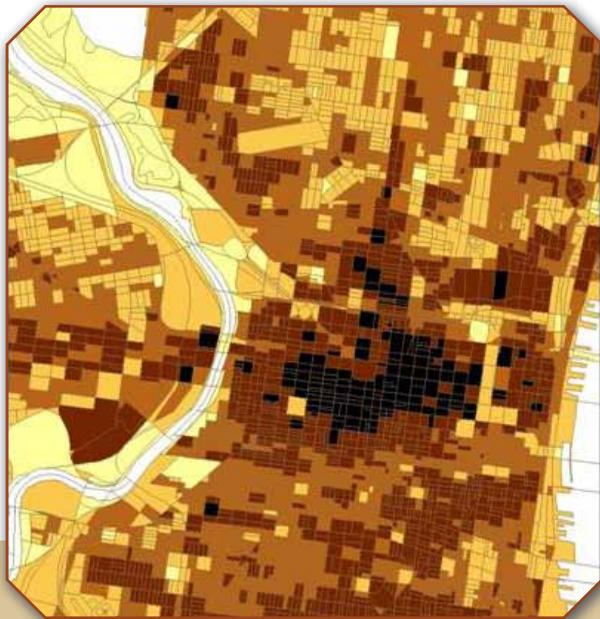
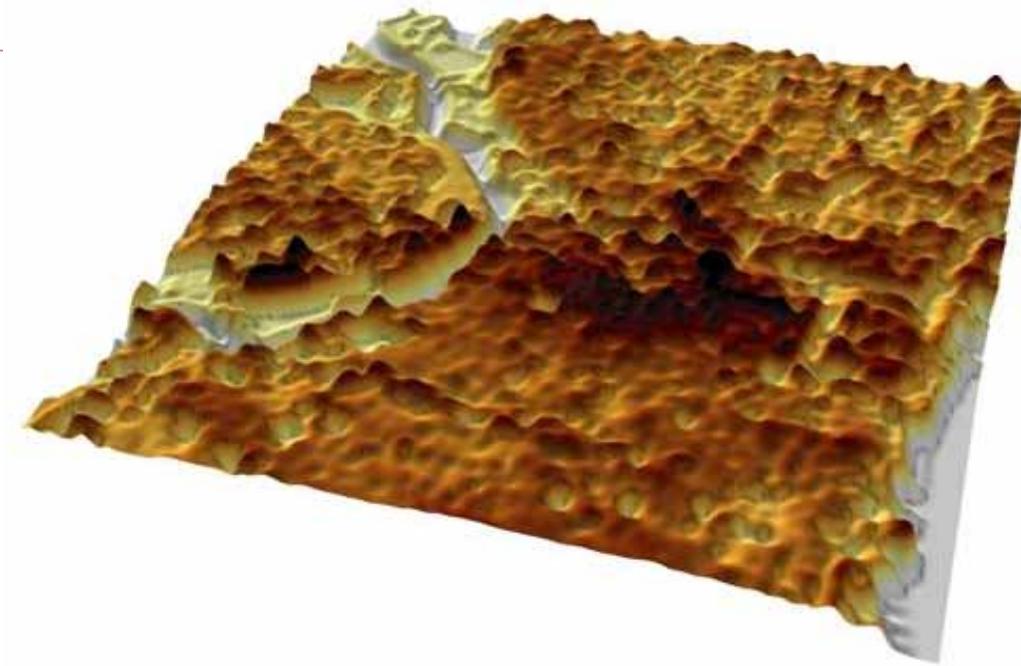
In modern cartographic parlance, a “hotspot” is simply a spatial concentration of some distinctive geographic condition. That condition could relate to anything from global politics to public health, from criminal behavior to real estate, or from natural disasters to nighttime entertainment. But it could also relate to physical heat in a very literal sense. While few of us are ever likely to experience hotspots of volcanic activity or those associated with forest fires, most of us are very familiar with normal outdoor temperatures. We know from everyday experience how much they

can vary under different conditions relating to factors such as vegetation, water bodies, paving material, and shade. At broader scales, these variations give rise to what have come to be called “urban heat islands,” situations in which the outdoor temperatures of urban enclaves rise well above those of their suburban or rural surroundings.

In Figure 1 is a map depicting urban heat for a portion of central Philadelphia. Here, census blocks are represented by colors that range from light to dark as their relative

temperatures range from cool to warm. Figure 2 uses the same range of color to depict the same range of temperatures but does so in a manner that now indicates more local variations within each census block. Such local variations could also be visualized by equating temperature with position in a third dimension perpendicular to the cartographic plane. In Figure 3, for example, the darker “hills” and the lighter “valleys” are respectively associated with warmer and cooler locations. It is the hills shown here that correspond to islands of urban heat.

These data on Philadelphia’s urban heat were generated not by direct observation in the field but, rather, by drawing inferences from digital maps of urban land cover. This relationship is illustrated in Figure 4, where the HeatNearby map shown in **Figures 2 and 3** has been superimposed onto an aerial photograph and presented in a view that focuses on part of the University of Pennsylvania campus in the west central part of that map. Note



ABOVE LEFT Figure 1 - HEATBYBLOCK, a map layer indicating levels of urban heat associated with census blocks in central Philadelphia

ABOVE RIGHT Figure 2 - HEATNEARBY, a map layer indicating levels of urban heat associated with finer locations in central Philadelphia

TOP Figure 3 - HEATNEARBY, a map layer indicating levels of urban heat depicted as “hills and valleys” of higher and lower temperatures

here that the darker hotspots tend to be associated with large building masses, while the lighter areas are associated with broad expanses of open water or green space. The **HEATNEARBY** map shown in Figure 4 was created from the **HEATONSITE** map shown in Figure 5, which also shows the **SHADE** and **HEATBYMATERIAL** maps from which **HEATONSITE** was created.

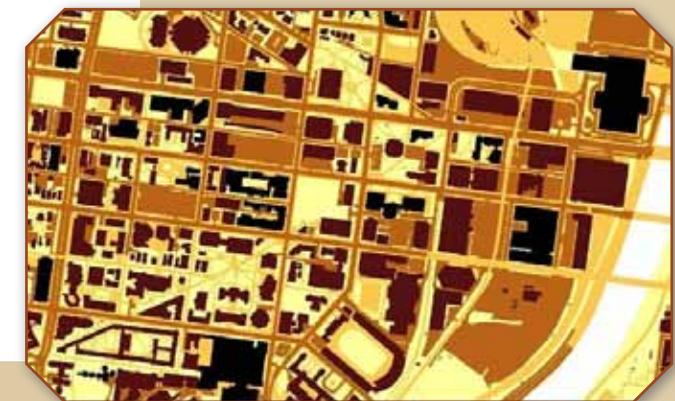
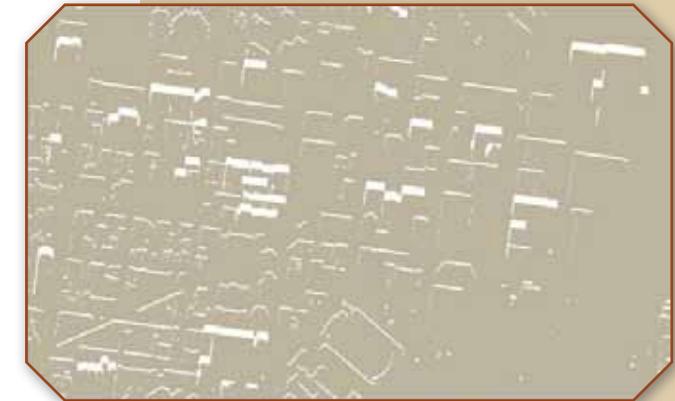
In Figure 6 is a diagram of these several steps that were taken to generate the **HEATBYBLOCK** map shown in Figure 1. These constitute what is generally referred to as a “cartographic model,” and this particular model is one that has been contrived in order to demonstrate what has come to be called “Map Algebra.” Map Algebra (Tomlin, 1990) is a general-purpose computing language that attempts to encompass a large and powerful set of geospatial analysis capabilities. It does this by decomposing both data and data-processing capabilities into elemental units that can then be recombined with relative ease and with great flexibility. The result is not unlike standard algebra in which:

- numerical values are represented in general form by variables;
- those variables can be subjected to mathematical operations;
- these operations can be combined by using the output from one as input to another;
- both variables and operations are specified by way of particular notation; and
- the use of this language tends to engender a distinctive analytical perspective.

What follows is a brief introduction to this language in which Philadelphia’s urban heat is used to generate examples of Map Algebra’s variables, operations, notation, and perspective.



ABOVE Figure 4 -
The relationship between HEATNEARBY and the urban site conditions



MAP ALGEBRAIC VARIABLES

Each of the rectangles shown in Figure 6 represents what has now been referred to several times as a map but which should more properly be called a map “layer.” A layer is like a conventional map in that it represents the geometric configuration of selected conditions within a bounded geographic area,

RIGHT Figure 5 -
HEATONSITE (top), created by combining
SHADE (middle) and HEATBYMATERIAL
(bottom)

and it does so by projecting geographic space onto a cartographic plane. Unlike most conventional maps, however, a map algebraic layer characterizes each of its mapped locations in terms of no more (and no less) than one a single characteristic. The roads, buildings, water bodies, and ground elevations that might all appear on a common topographic map, for example, would here be organized as a set of separate layers: one depicting different types of roads, another recording the heights of buildings, and so on. Layers may also be associated with less tangible characteristics such as proximity to the nearest road or residential building density. In any case, it is the layer that serves as the primary form of variable in Map Algebra.

The different geographic conditions represented by any given layer are referred to as the layer's "zones." On a layer of road types, for example, one zone might represent all interstate highways, while another is associated with local streets, a third zone encompasses non-road areas, and so on.

Layers are typically presented such that each zone is either distinguished from or combined with others through the use of graphic symbolism and descriptive text that can be modified at will. Regardless of the manner in which zones are portrayed, however, each is represented internally by way of a numerical "value" that distinguishes the zone from all others within the same layer.

That layer of road types, for example, might be encoded such that 0 represents non-road areas, while local roads are set to 1, highways to 2, and so on. In the case of building heights or ground elevations, on the other hand, each zone's value might well relate to a measure of vertical distance.

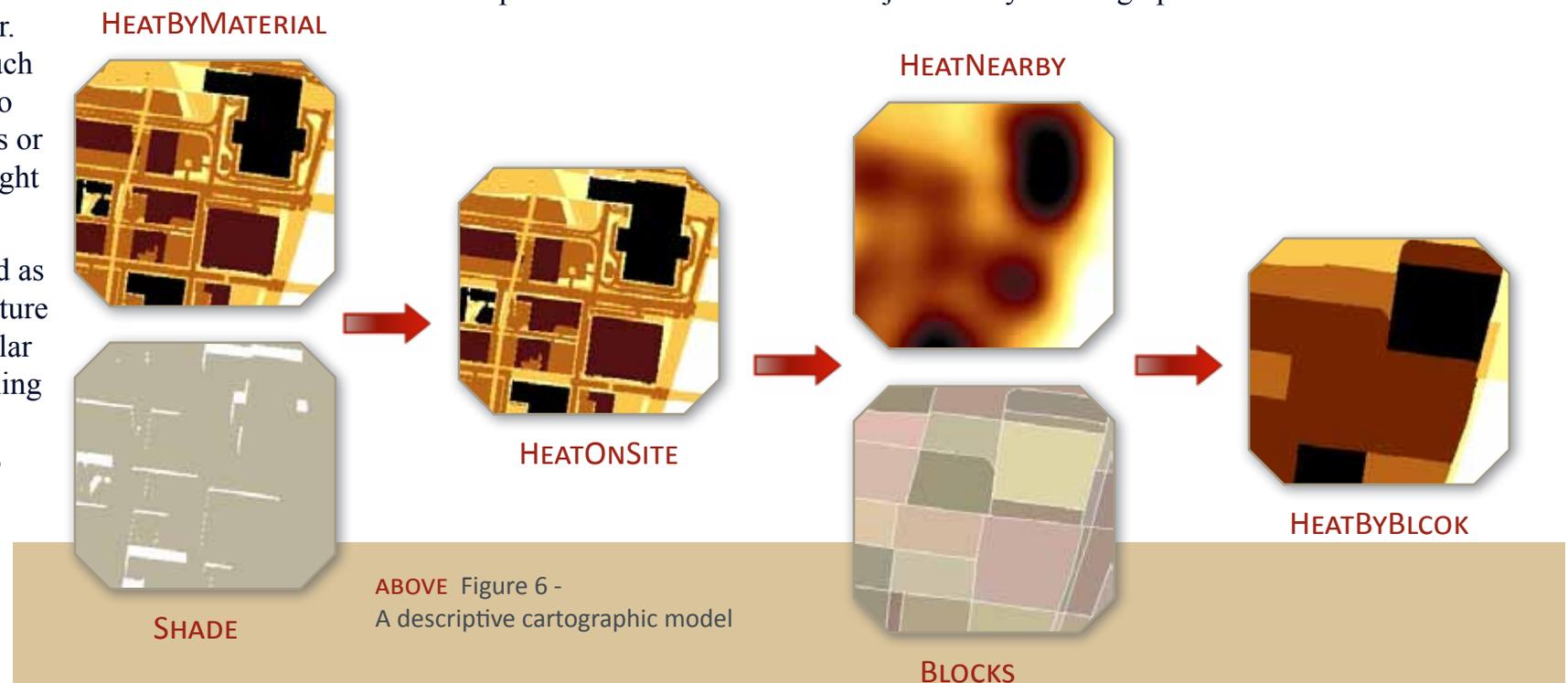
The set of locations associated with a zone may be encoded as a set of points, lines, polygons, or "pixels." Pixels (or "picture elements") are distinct locations defined by the perpendicular intersections of equally-spaced rows and columns partitioning the cartographic plane. An example is shown in Figure 7, where a portion of the **HEATONSITE** layer shown in Figure 5 has been enlarged.

MAP ALGEBRAIC OPERATIONS

In algebra, variables represent numbers, and relationships among variables are expressed by way of numerical operations such as addition and subtraction. In Map Algebra, the variables represent map layers, and relationships among these variables are expressed by way of cartographic operations. As a computational language, Map Algebra also regards its operations as more than declarative assertions. Each is in fact an imperative instruction calling for the generation of a new map layer from one or more of those that already exist.

The fact that all Map Algebraic operations accept input and generate output in one common format is actually quite significant. Because of this, the output from any one operation can be used as input to any other. And since operations can be combined in this manner, no one operation ever needs to be very complex. Just as elementary operations such as addition and subtraction can be used to construct complex algebraic equations, so can a basic set of Map Algebraic operations be used to construct an unlimited variety of cartographic models.

To do this effectively, those basic operations must encompass a wide range of capabilities. To do so efficiently, they must also be well organized. Map Algebra attempts to achieve both of these objectives by defining operations from what can be



envisioned as a “worm’s eye” perspective. Though every operation does what it does on a layer-by-layer basis, it also does what it does by processing all of a layer’s pixels in a similar manner. Thus, each operation can be fully defined in terms of its effect on a single, typical pixel.

It is from this perspective that Map Algebraic operations are classified as “local,” “zonal,” or “focal.” A local operation is one that computes each pixel’s new value as a function of the existing values that are associated with that pixel’s particular location on one or more specified layers. In the case of zonal operations, each pixel’s new value is computed from the existing values of one specified layer that lie within that pixel’s zone on another specified layer. And focal operations compute each pixel’s new value as function of the existing values, distances, and/or directions of neighboring pixels on a specified layer.

Examples of these three types of operation have already been presented. The **HEATONSITE** layer shown in Figure 5 was created by applying an operation given as **LOCALMEAN** to the layers **HEATBYMATERIAL** and **SHADE**. The **HEATNEARBY** layer shown in **Figure 2** was created by applying **FOCALMEAN** to the **HEATONSITE** layer of Figure 5. And the **HEATBYBLOCK** layer presented in Figure 1 was created from the **HEATNEARBY** layer shown in **Figure 2** by using a **ZONALMEAN** operation to aggregate **HEATNEARBY** values within each of the zones of a layer of census blocks. In each case, output values are computed by averaging input values.

If the values of pixels can be averaged on a local, zonal, or focal basis, they can also be subjected to other mathematical, statistical, or logical functions by way of operations with names like **LOCALSUM**, **ZONALMAJORITY**, and **FOCALCOMBINATION**. In the case of focal operations, these functions may also be applied to neighborhoods that are defined in terms of travel costs or lines-of sight rather than physical distance. A more specialized set of focal operations is sometimes referred to as “incremental.” By comparing the values of adjacent pixels, these operations are able to measure characteristics such as length, perimeter, area, volume, direction, inclination, and curvature at each of the individual pixels comprising a one-, two-, or three-dimensional cartographic form.

MAP ALGEBRAIC NOTATION

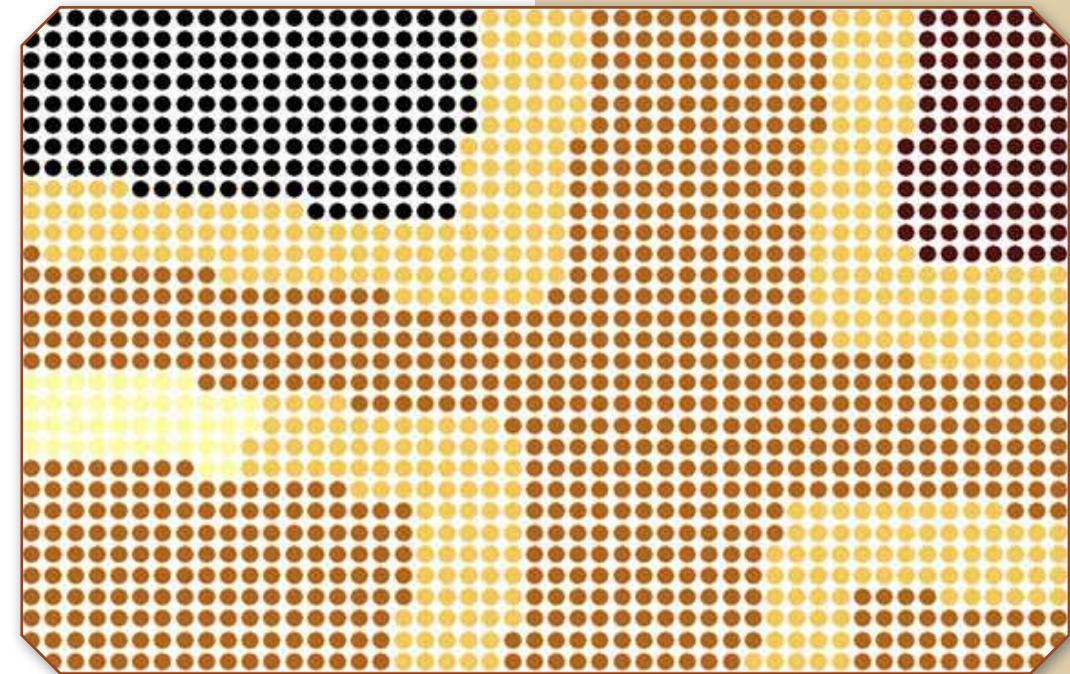
Map Algebraic capabilities have been incorporated into a number of geographic information systems, each employing its own particular form of verbal or graphic expression. In the original Map Algebraic language, operations and variables are specified by way of imperative statements that resemble a conversational form of algebraic notation. The sequence of operations represented by Figures 6-11, for example, would be expressed in this form as follows.

HEATONSITE = **LOCALMEAN** of **HEATBYMATERIAL** and **SHADE**

HEATNEARBY = **FOCALMEAN** of **HEATONSITE**

HEATBYBLOCK = **ZONALMEAN** of **HEATNEARBY** by **BLOCKS**

BELOW Figure 7 -
Pixels of a map layer



One of the more promising forms of Map Algebraic expression is exemplified by **Figure 6** itself. Some geographic information systems are able to draw their instructions directly from such model diagrams

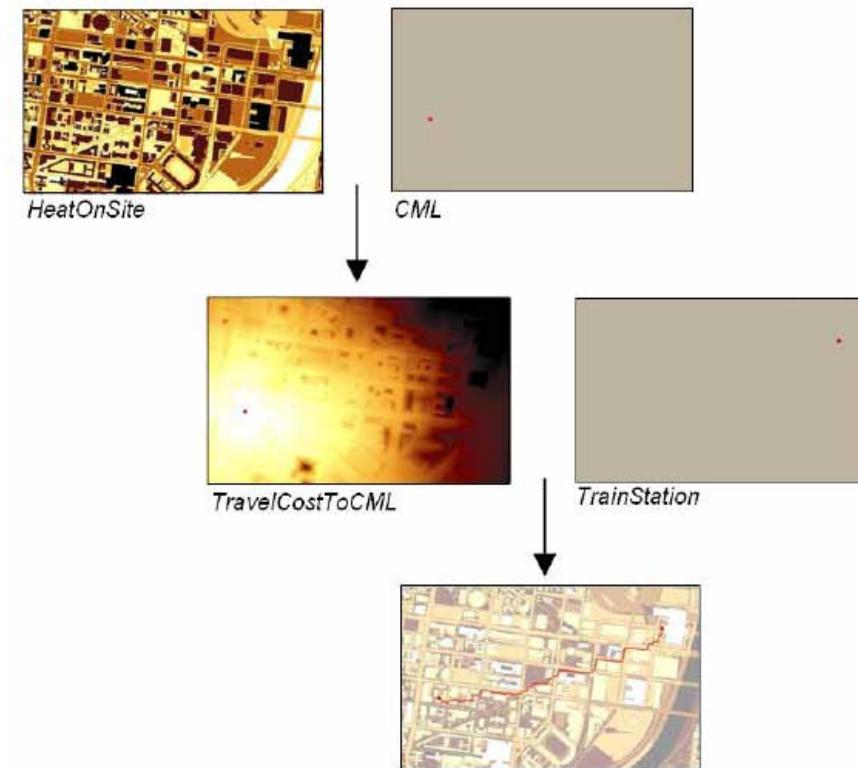
MAP ALGEBRAIC PERSPECTIVE

In **Figure 8** is the diagram of another cartographic model, this one intended to give a sense of the more general problem-solving philosophy associated with Map Algebra. The model allocates a walking route through West Philadelphia that attempts to minimize midday heat along the way.

One of the two termini for this walking route will be the University of Pennsylvania's Cartographic Modeling Lab (CML). In **Figure 9** is a layer depicting distance from the CML, where distance is measured "as the crow flies" to surrounding locations. In contrast to this is the layer of distance depicted in **Figure 10**. Here, distance from the CML has been measured "as the crow walks" to surrounding locations through conditions that impede such travel to varying degrees. The pun here is fully intended, as the degrees involved do indeed relate to temperature. It is, in fact, the **HEATONSITE** layer shown in **Figure 5** that was used here to define impedance. As a result, walking distance to the CML is now as shown in **Figure 10**.

Figure 11 also depicts this layer of travel-cost distance from the CML. It does so, however, such that greater distances are now equated with higher elevations on a three-dimensional surface. In **Figure 3**, a topographic surface of this sort is used for purposes that are primarily cosmetic. Here, however, the notion of physical topography serves an analytical purpose as well. Note the red line. It begins at a one of the entrances to Philadelphia's 30th Street train station and then does just what droplet of water would do on a physical model of this travel-cost topography. It follows a downstream path that drains its way over that surface of travel-cost elevations and ultimately comes to a stop at the CML.

The operation used to trace this path is in fact one that might just as well be used to simulate hydrological flow. In this case, however, it is used to trace what amounts to the path of least resistance. As indicated in **Figure 12**, it is the walking route between the CML and 30th Street Station that minimizes urban heat.



LEFT Figure 8 -
A descriptive
cartographic model

RIGHT Figure 9 -
A map layer indicating distance to a selected point
in terms of physical separation

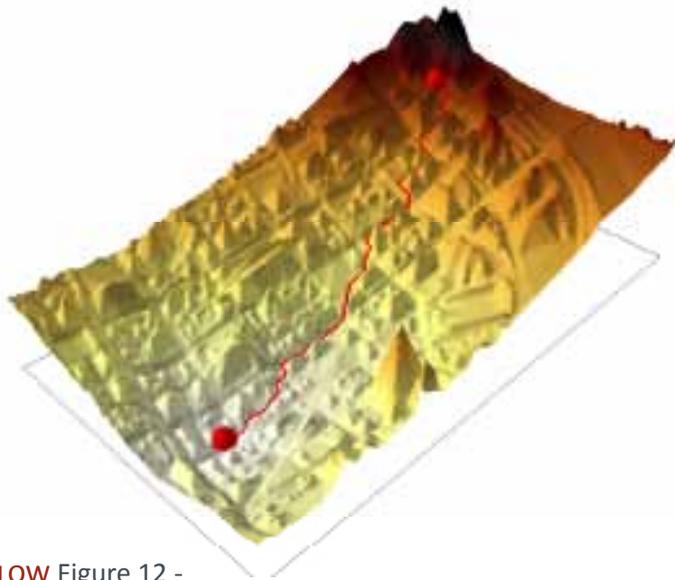
RIGHT Figure 10 -
A map layer indicating distance to a selected point
in terms of travel cost

CONCLUSION

It was almost a century ago that noted landscape architect Warren H. Manning was first making use of overlaid maps for environmental analysis (Manning 1913). It was almost four decades ago that such techniques were refined and promulgated through the work of Penn professor Ian L. McHarg (McHarg 1969). And it has been almost 25 years now since the digital implementation of these techniques was first proposed in terms of a fully-articulated Map Algebra (Tomlin 1983).

Today, Map Algebraic conventions and capabilities are employed by most of the world's geographic information systems, and recent advances in Web-based mapping have raised the prospects for much broader access to this technology. As that happens, the value of a common language is likely to become more evident. It is also likely to become more apparent that techniques for mapping very different types of hotspot are really not so different after all.

LEFT Figure 11 -
A map layer depicting travel-cost distance as a three-dimensional surface on which greater distances are at higher elevations



BELOW Figure 12 -
The optimal path between two points in terms of travel cost elevations

