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Perturbative and Non-Perturbative Aspects of Orientifold Compactifications

Abstract
This thesis mainly focuses on novel stringy instanton effects in orientifold compactifications. Such effects are known to generate superpotential couplings which are perturbatively forbidden due to U(1) selection rules arising from the Green-Schwarz mechanism. We discuss their gauge transformation behavior and zero mode structure and give an outline of how to compute these non-perturbative effects. Here we focus for concreteness on type IIA compactifications, where the instantons are given by E2-instantons, wrapping a three-cycle in the internal manifold and are pointlike in the four-dimensional spacetime.

Later we embed E2-instantons in local setups and give various examples of instanton induced superpotential couplings which are otherwise forbidden. In all these configurations non-perturbative effects give a natural explanation for hierarchies which are only poorly understood from the field theoretical point of view. Furthermore, we explicitly compute the instanton induced mass term of Majorana masses for right-handed neutrinos by using conformal field theory techniques.

Finally we investigate under what circumstances instantons provide the correct zero mode structure to induce superpotential corrections. We analyze two different strategies to lift additional undesired instanton zero modes. First we discuss the process of instanton recombination, and show that generically one needs additional instantons to generate superpotential couplings. With these multi-instantons we show that the class of superpotential contributing BPS instantons is much richer than mostly considered. Later we study the lifting of undesired zero modes in the type IIB framework with supersymmetric background flux. We show that indeed in certain situations the background fluxes lift the zero modes and the instanton gives rise to superpotential contributions.

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PERTURBATIVE AND NON-PERTURBATIVE ASPECTS OF ORIENTIFOLD COMPACTIFICATIONS

Robert Richter

A DISSERTATION

in

Physics and Astronomy

Presented to the Faculties of the University of Pennsylvania in Partial Fulfillment of the Requirements for the Degree of Doctor of Philosophy

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Mirjam Cvetič

Supervisor of dissertation

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Robert Richter

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ABSTRACT

This thesis mainly focuses on novel stringy instanton effects in orientifold compactifications. Such effects are known to generate superpotential couplings which are perturbatively forbidden due to $U(1)$ selection rules arising from the Green-Schwarz mechanism. We discuss their gauge transformation behavior and zero mode structure and give an outline of how to compute these non-perturbative effects. Here we focus for concreteness on type IIA compactifications, where the instantons are given by $E2$-instantons, wrapping a three-cycle in the internal manifold and are pointlike in the four-dimensional spacetime.

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Finally we investigate under what circumstances instantons provide the correct zero mode structure to induce superpotential corrections. We analyze two different strategies to lift additional undesired instanton zero modes. First we discuss the process of instanton recombination, and show that generically one needs additional instantons to generate superpotential couplings. With these multi-instantons we show that the class of superpotential contributing BPS instantons is much richer than mostly considered. Later we study the lifting of undesired zero modes in the type IIB framework with supersymmetric background flux. We show that indeed in certain situations the background fluxes lift the zero modes and the instanton gives rise to

superpotential contributions.

Apart from the non-perturbative effects, we analyze the proton decay via dimension six operators in supersymmetric SU(5)-Grand unified models based on intersecting D6-branes. We include in addition to $\mathbf{10}^{*}\mathbf{10}\mathbf{10}^{*}\mathbf{10}$ interactions also operators arising from $\bar{\mathbf{5}}^{*}\bar{\mathbf{5}}\mathbf{10}^{*}\mathbf{10}$ interactions. In the course of that analysis we provide a detailed construction of vertex operators for any massless string excitation in an arbitrary intersecting brane configuration. With the knowledge of the vertex operators for the $\mathbf{10}$ and $\bar{\mathbf{5}}$ chiral superfields, we explicitly calculate the string theory correlation functions for above operators. We show that for the most symmetric configuration, the stringy contribution to the proton life time is $\tau_p^{ST} = (0.6 - 2.6) \times 10^{36} years$, which could be up to a factor of two and a half shorter than that one predicted in field theory.

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Introduction

1.1 String theory

The standard model of fundamental interactions, containing the electro-weak and strong interactions, is a quantum field theory based on the gauge groups $SU(3) \times SU(2) \times U(1)$. Its spin-1 gauge bosons mediate the strong and electro-weak interactions, while the spin-$\frac{1}{2}$ particles describe the matter content: quarks and leptons. Furthermore it contains the Higgs boson, a spin-0 particle.

Despite its great experimental success the standard model suffers under two significant shortcomings. The first is that it has around twenty free parameters, such as couplings and masses of the observed particles. The second, even more striking shortcoming, is that it does not contain quantum gravity. In many circumstances it is sufficient to work with classical gravity coupled to the Standard model, but in order to study physics at times close to the Big Bang where quantum gravity kicks in it is necessary to have a fundamental theory unifying gravity with the other forces.

String theory is, at present time, the most promising candidate for such a unifying theory. As opposed to quantum field theory, it assumes that the elementary constituents are not point-like but rather one-dimensional objects. These one dimensional objects come in two different topologies, open and closed strings. Early on, it had been realized that the closed string sector gives rise to a spin-2 particle identified as the graviton. Thus, string theory naturally includes quantum gravity and has the potential to unify gravity and quantum field theory.

In 1995 it was realized that string theory contains not only strings but also involves higher dimensional objects called D-branes. These objects are $(p+1)$-dimensional hypersurfaces on which open strings can begin and end. The lowest open string excitations give rise to massless gauge fields. Furthermore, it was shown that chiral massless fermions appear at intersections of two D-branes. It is due to these two properties that intersecting brane worlds became a very popular playground for realistic model search.

While in the very beginning of orientifold compactifications one focussed mainly on perturbative effects, it is known that non-perturbative effects may have tremendous consequences for the stability of string vacua. In addition, instantons play a crucial role in moduli stabilization and can generate perturbatively forbidden superpotential terms, which are of phenomenological interest. Thus, it is of utmost importance to gain a better understanding of these effects. It is one of the main goals of this thesis to investigate such non-perturbative phenomena.

1.2 Outline

In chapter 2, we give a brief introduction to type IIA orientifold compactifications, also called intersecting brane worlds. We present all necessary principles and tools for model building in that corner of M-theory. Furthermore, we discuss the mechanism of D-brane recombination as a preparation for chapter 6, where we investigate instanton recombination as possible mechanism for lifting undesired zero modes.

In chapter 3, we investigate the proton decay via dimension six operators in a local SU(5)-GUT model based on intersecting D6-branes, which could be embedded in a huge class of globally consistent D-brane models. We extend a previous analysis of Klebanov and Witten by including in addition to the amplitude $\langle \mathbf{10}^* \mathbf{10} \mathbf{10}^* \mathbf{10} \rangle$ as well as the string amplitude computation of $\langle \bar{\mathbf{5}}^* \bar{\mathbf{5}} \mathbf{10}^* \mathbf{10} \rangle$. We show that in the most symmetric configuration the proton decay rate may receive a small stringy enhancement, which could result in an up to two and a half times shorter proton lifetime than that predicted in field theory.

In chapter 4 we set the stage for the main part of the thesis. We introduce novel stringy instanton effects, which do not have a analogue in field theory and thus are purely stringy. We show that instantons have the potential to generate couplings which are perturbatively forbidden due to various $U(1)$ selection rules. We give a detailed discussion on the instanton zero mode structure and provide the rules for computing instanton induced superpotential couplings.

In chapter 5, we present a detailed calculation of an instanton induced two fermion coupling and exemplify this analysis in a local setup in which the Majorana mass term for right-handed neutrinos is generated non-perturbatively. We present further examples in which perturbatively forbidden but desired couplings are induced non-perturbatively and we discuss the generation of higher fermionic F-terms à la Beasley and Witten.

In chapter 6, we present two mechanisms to lift undesired instanton zero modes. First, we study the lifting by instanton recombination and show that generically one needs to introduce additional instantons to give rise to non-vanishing superpotential terms. Furthermore, for such multi-instanton configurations we analyst global questions of holomorphicity of the superpotential. In the second part of chapter 6, we discuss the lifting via background fluxes.

Appendix A provides a detailed prescription for constructing vertex operators for any massless string excitation that arises for arbitrary D6-brane configurations in Type IIA toroidal backgrounds. These vertex operators will be needed in chapter 3 and 5, where we compute various string amplitudes. Appendix B is dedicated to the numerical analysis of the string amplitudes calculated in chapter 3. Lastly, in appendix C, we present a local multi-instanton setup on $T^6/\mathbb{Z}_2 \times \mathbb{Z}_2'$ which gives rise to non-perturbative superpotential contributions.

Chapter 2

Intersecting brane models

The purpose of this chapter is to make the reader familiar with the main principles and methods for model building in type IIA orientifolds, so called intersecting brane worlds. We are not aiming at a complete introduction to that subject but rather provide all necessary ingredients for topics investigated in 3 to 6. For more detailed background material we refer the reader to the textbooks [1, 2, 3, 4, 5, 6, 7, 8, 9, 10] and the reviews [11, 12, 13, 14, 15, 16, 17, 18].

Type IIA compactification, while dual to other corners of M-theory, have a very appealing geometrical interpretation. In such models the gauge groups appear on stacks of D6-branes, filling out the four-dimensional space-time and wrapping three-cycles in the internal compactification manifold. Chiral matter arises at intersections of stacks of branes wrapping different three-cycles in the internal manifold and their multiplicities are given by their topological intersection number. D6-branes carry charge under Ramond-Ramond (RR) and Neveu-Schwarz Neveu-Schwarz (NSNS) fields which need to be canceled in globally consistent string vacua leading to the

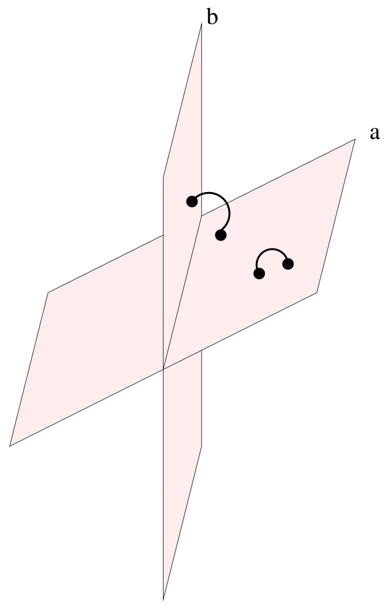

Figure 2.1: Gauge bosons and chiral matter fields in intersecting brane worlds.

so called RR Tadpole cancelation conditions. In addition one is interested in supersymmetric D-brane configurations at the string scale, since supersymmetry breaking on a much lower scale than string scale is more appealing from field theoretical point of view. Furthermore, in such models the NSNS charges are canceled automatically if RR charge is canceled. Requiring supersymmetry leads to further constraints on the 3-cycles the $D6$ brane can wrap.

In the following we will discuss all these issues in more detail, beginning with the introduction of orientifolds, turning to a detailed analysis of the massless spectrum of intersecting brane worlds and then moving on to a discussion of the constraints arising from Tadpole cancelation and supersymmetry. At the end we discuss the mechanism of D-brane recombination as preparation for chapter 6 where we study instanton induced superpotential contributions across lines of marginal stability.

2.1 Orientifold compactification

String theory[1] can be consistently formulated only in 10 dimensions, thus in order to connect it to the observable four-dimensional world one has to compactify six out of these ten dimensions. Requiring supersymmetry in $4D$ imposes constraints on the compactification manifold $\mathcal{M}$. To first order, without fluxes, it has to be a Calabi-Yau three-fold, a three dimensional complex, Ricci flat, Kähler manifold. In order to obtain $\mathcal{N} = 1$ supersymmetry in the closed string sector the Calabi-Yau manifold is accompanied with an orientifold action $\Omega\overline{\sigma}$. Here Ω is the world-sheet parity and in type IIA $\overline{\sigma}$ is the anti-holomorphic involution, $\overline{\sigma} : z \to \overline{z}$. Apart from truncating the closed string spectrum from $\mathcal{N} = 2$ down to $\mathcal{N} = 1$ the orientifold action introduces topological defects in the geometry, so called $O6$-planes. These non-dynamical objects are localized at the fixed point locus of $\overline{\sigma}$, π_{O6}, which is a three-cycle in $H_3(\mathcal{M}, \mathbb{Z})$ and carries tension and charge under the RR seven-form C_7. The internal manifold $\mathcal{M}$ is compact. Thus due to Gauss law, we need additional objects carrying opposite RR seven-form canceling the ones arising from the $O6$-planes.

The objects in question are $D6$-branes which are 7-dimensional hypersurfaces which wrap the whole four-dimensional space-time and a three-cycle $\pi \,\epsilon\, H_3(\mathcal{M}, \mathbb{Z})$ in the internal manifold. In contrast to $O6$-planes $D6$-branes are dynamical objects and fluctuations of it can be described by open strings attached to their hypersurface. A stack of N D-branes gives generically rise to a non-abelian $U(N)$ gauge factors in four dimensional space-time. The chiral matter is localized at intersections of stacks of D6-branes wrapping different three-cycles in the internal manifold and their number

[1] We are not dealing with non-critical string theory which can be formulated in less than 10 dimensions.

is given by topological data. In the next section we discuss the massless spectrum of a generic D-brane configuration in more detail.

2.2 Massless spectrum

As anticipated a stack of N_a $D6$-branes, wrapping a three-cycle π_a, non-invariant under the orientifold action, in the internal manifold gives rise to an $U(N_a)$ gauge theory in four dimensional space time. In order to ensure the symmetry $\Omega\overline{\sigma}$ we need to wrap also D6-branes on the orientifold image three-cycle π'_a. In case the $D6$-brane wraps a cycle invariant under $\Omega\overline{\sigma}$ the gauge indices, the so called Chan-Paton factors are subject to the orientifold action leading to $SO(2N_a)$ or $Sp(2N_a)$ gauge symmetry.

Let us introduce a natural basis for the space $H_3(\mathcal{M},\mathbb{Z})$ of all three-cycles in $\mathcal{M}$. The homology class $H_3(\mathcal{M},\mathbb{Z})$ splits under the orientifold action into an even and odd part H_3^+ and H_3^-, respectively. It is natural to choose for $H_3(\mathcal{M},\mathbb{Z})$ the symplectic basis (α_I,β_I) such that

$$\alpha_I \circ \alpha_J = 0 \qquad \alpha_I \circ \beta_J = \delta_{IJ} \qquad \beta_I \circ \beta_J = 0 \tag{2.1}$$

with $\alpha_I \,\epsilon\, H_3^+$ and $\beta_I \,\epsilon\, H_3^-$. Here I runs from 1 to $h_{21}+1$ with $h_{21}+1$ being the dimension of $H_3^+(\mathcal{M},\mathbb{Z})$ and $H_3^-(\mathcal{M},\mathbb{Z})$. Expanding the three-cycle π_a the stack of N_a D-brane one gets

$$\pi_a = \sum_{I=1}^{h_{21}+1} n_a^I \alpha_I + m_a^I \beta_I \tag{2.2}$$

while its orientifold image a' is given by

$$\pi'_a = \sum_{I=1}^{h_{21}+1} n_a^I \alpha_I - m_a^I \beta_I \ . \tag{2.3}$$

Strings located at intersections of two different stacks a and b wrapping the cycles π_a and π_b, respectively, transform as $(\overline{N}_a, N_b)$[2] and their index is given by the topological intersection number

$$\pi_a \circ \pi_b = \prod_{I=1}^{h_{21}+1} (n_a^I m_b^I - n_b^I m_a^I). \tag{2.4}$$

On the other hand states arising at intersections between stack a and the orientifold image of b, b', transform as $(\overline{N}_a, \overline{N}_b)$ and their multiplicity is given by

$$\pi_a \circ \pi'_b = \prod_{I=1}^{h_{21}+1} (n_a^I m_b^I + n_b^I m_a^I). \tag{2.5}$$

Finally there are chiral states at intersections between stack a and its orientifold image a'. String states localized at intersections on top of the orientifold are subject to the $\Omega\overline{\sigma}$ action and the symmetric ones under $U(N_a)$ get projected out. Thus the chiral spectrum in the aa' sector is given by

$$\mathbf{Sym}_a = \frac{1}{2}\left(\pi_{a'} \circ \pi_a - \pi_{O6} \circ \pi_a\right) \qquad \mathbf{Anti}_a = \frac{1}{2}\left(\pi_{a'} \circ \pi_a + \pi_{O6} \circ \pi_a\right) \tag{2.6}$$

Table 2.1 summarizes the chiral spectrum arising for intersecting D-brane. Note that generically there is additional non-chiral matter in the $\mathcal{N} = 2$ sector, whose

[2] This is true for positive topological intersection number, in case of negative topological intersection number the string transforms as $(N_a, \overline{N}_b)$.

Representation	Multiplicity
$\mathbf{Anti}_a$	$\frac{1}{2}\left(\pi'_a \circ \pi_a + \pi_{\mathrm{O6}} \circ \pi_a\right)$
$\mathbf{Sym}_a$	$\frac{1}{2}\left(\pi'_a \circ \pi_a - \pi_{\mathrm{O6}} \circ \pi_a\right)$
$(\overline{N}_a, N_b)$	$\pi_a \circ \pi_b$
$(\overline{N}_a, \overline{N}_b)$	$\pi'_a \circ \pi_b$

Table 2.1: Chiral spectrum for intersecting D6-branes.

multiplicity is not given by topological data but rather by geometrical intersection numbers,

$$
\begin{aligned}
(N_a, \overline{N}_b) + (\overline{N}_a, N_b) \quad &: \qquad \min\Big([\pi_a \cap \pi_b]^+ \,, [\pi_a \cap \pi_b]^- \Big) \\
(N_a, N_b) + (\overline{N}_a, \overline{N}_b) \quad &: \qquad \min\Big([\pi'_a \cap \pi_b]^+ \,, [\pi'_a \cap \pi_b]^- \Big) \\
\mathbf{Anti}_a + \overline{\mathbf{Anti}}_a \quad &: \qquad \frac{1}{2}\Big\{ (\min\Big([\pi'_a \cap \pi_a]^+ \,, [\pi'_a \cap \pi_a]^- \Big) \\
&\qquad\qquad + \min\Big([\pi_{O6} \cap \pi_a]^+ \,, [\pi_{O6} \cap \pi_a]^- \Big) \Big\} \\
\mathbf{Sym}_a + \overline{\mathbf{Sym}}_a \quad &: \qquad \frac{1}{2}\Big\{ \min\Big([\pi'_a \cap \pi_a]^+ \,, [\pi'_a \cap \pi_a]^- \Big) \\
&\qquad\qquad - \min\Big([\pi_{O6} \cap \pi_a]^+ \,, [\pi_{O6} \cap \pi_a]^- \Big) \Big\}
\end{aligned} \tag{2.7}
$$

In case the cycle π_a is non-rigid the aa sector exhibits apart from the gauge supermultiplets also complex bosonic modes c_I and $\overline{c}_I$ associated with deformations of the three-cycle π_a. Their number is given by $b_1(\pi_a)$ and they are accompanied by their fermionic super partners χ_I and $\overline{\chi}_I$, respectively.

2.3 Tadpole cancelation

As already mentioned earlier the $O6$ planes and the $D6$-branes are charged under the RR seven-forms and due to Gauss law they have to cancel in the internal manifold.

The action describing the coupling to RR seven-forms is given by the Chert-Simons action. For an $O6$-plane it is given by[3]

$$S_{CS}^{O6} = 4\frac{2\pi}{l_s^7}\int_{R^{1,3}\times\pi_{O6}} C_7 \tag{2.8}$$

and for a $D6$-brane by[4]

$$S_{CS}^{D6} = -\frac{2\pi}{l_s^7}\int_{R^{1,3}\times\pi_{D6}} C_7. \tag{2.9}$$

Note that $D6$-branes carry RR charge opposite to those of $O6$-planes. The whole action for the RR seven-form C_7 including the kinetic terms as well as the contributions arising from the mirror stacks a' takes the form

$$\begin{aligned} S &= -\frac{1}{4\kappa^2}\int_{R^{1,3}\times\mathcal{M}} dC_7\wedge\star dC_7 \qquad (2.10)\\ &\quad +\frac{2\pi}{l_s^7}\left(\sum_a^K N_a\left(\int_{R^{1,3}\times\pi_a} C_7+\int_{R^{1,3}\times\pi'_a} C_7\right)-4\int_{R^{1,3}\times\pi_{O6}} C_7\right), \end{aligned}$$

where $\kappa^2 = \frac{l_s^8}{4\pi}$ is the ten dimensional gravitational coupling and K denotes the number of stacks in the D-brane configuration. With (2.10) the equation of motion for C_7 computes to

$$d\star dC_7 = \kappa^2\frac{2\pi}{l_s^7}\left(\sum_a^K N_a(\delta(\pi_a)+\delta(\pi'_a)) - 4\delta(\pi_{O6})\right). \tag{2.11}$$

[3]Here we only display the part involving the seven-form C_7. There are also couplings to the three-form.

[4]Again we only display the part involving the seven-form C_7. In addition there are couplings to the three- and five-form.

Here $\delta(\pi)$ denotes the Poincaré dual of the three-cycle π. Integrating (2.11) over the internal manifold $\mathcal{M}$ gives then the RR-tadpole cancelation in terms of the homology classes the $D6$-branes wrap [19]

$$\sum_a^K N_a(\pi_a + \pi_a') - 4\pi_{O6} = 0 \tag{2.12}$$

Let us demonstrate that RR tadpole cancelation implies the vanishing of anomalies involving non-abelian gauge bosons. The anomaly contributions to states displayed in table 2.1 are

$$\begin{aligned} A^{aaa} &\sim \sum_{b\neq a} N_b\left[-\pi_a \circ \pi_b + \pi_a' \circ \pi_b\right] \\ &\quad + \frac{N_a-4}{2}\Big[\pi_a' \circ \pi_a + \pi_{O6} \circ \pi_a\Big] + \frac{N_a+4}{2}\Big[\pi_a' \circ \pi_a - \pi_{O6} \circ \pi_a\Big] \\ &= -\pi_a \circ \left[\sum_b N_b\Big[\pi_b + \pi_b'\Big] - 4\pi_{O6}\right] = 0 \end{aligned} \tag{2.13}$$

However note that using the spectrum in table 2.1 the pure abelian and mixed anomalies do not cancel among each other, but rather give the non-vanishing contribution

$$A^{abb} \sim N_a[-\pi_a + \pi_a'] \circ \pi_b \tag{2.14}$$

In section 4.1 we show that these anomalies can be canceled by the generalized Green-Schwarz mechanism, taking into account couplings to the RR three- and five-form.

2.4 Supersymmetry

By choosing the compactification manifold to be a Calabi-Yau three-fold accompanied with an orientifold action $\Omega\overline{\sigma}$ we ensure $\mathcal{N} = 1$ supersymmetry in the closed string sector. In order to obtain a D-brane configuration preserving also $\mathcal{N} = 1$ one has to require as a necessary condition that each D-brane by itself is a BPS-brane. This amounts to the two conditions

$$J|_{\pi_a} = 0 \qquad\qquad \Im(e^{i\phi_a}\Omega_3)|_{\pi_a} = 0 \ , \tag{2.15}$$

where J is the Kähler 2-form and Ω the holomorphic 3-form of the Calabi-Yau three-fold. In mathematical terms the first equation means that the D-brane has to wrap a Lagrangian submanifold of $\mathcal{M}$ while the second one implies that π_a is a special Lagrangian cycle. That allows one to compute the volume of this submanifold by the integral

$$Vol(\pi_a) = \left| \int_{\pi_a} \Re(e^{i\phi_a}\Omega_3) \right| \ . \tag{2.16}$$

Physically that means that a D-brane with a given homology class minimizes its volume and thus its energy, which one would naively expect from a BPS-object, where the first equation in (2.15) ensures F-flatness and the second one implies D-flatness. The parameter ϕ_a encodes which supersymmetry the brane preserves, thus to ensure supersymmetry for a set of $D6$-branes all branes have to be calibrated with respect to the same holomorphic three-form $\Re(e^{i\phi}\Omega_3)$. In order to make it compatible with the anti-holomorphic involution $\overline{\sigma}$, which acts Ω_3 and J as $\overline{\sigma}\Omega_3 = \overline{\Omega}_3$ and $\overline{J} = -J$, the phase ϕ has to be set to $\phi = 0$.

In the following we show that supersymmetry with the RR Tadpole condition implies the cancellation of NSNS charges in the internal manifold [19, 20]. Let us take a look at the Dirac-Born-Infeld action describing the coupling of the D6-brane to the to NSNS fields

$$S_{DBI}^{D6} = -\frac{2\pi}{l_s^7} \int d^7x \sqrt{G + \mathcal{F}}, \tag{2.17}$$

where G is the induced metric on the $D6$-brane world-volume, and $\mathcal{F} = B + 2\pi\alpha' F$ with B the induced 2-form and F the field strength of the gauge field living on the $D6$-brane. A similar action for the $O6$-planes exists

$$S_{DBI}^{O6} = 4\frac{2\pi}{l_s^7} \int d^7x \sqrt{G}. \tag{2.18}$$

Note that it carries again opposite charge than the D6-brane and also the absence of the field strength in 2.18 due to the fact that O6 planes are non-dynamical objects.

Integrating that action gives rise to the disk level scalar potential which is proportional to the volumes of the cycles the $D6$-branes and $O6$-planes wrap

$$\mathcal{V} \sim \sum_a^K N_a \left(\left| \int_{\pi_a} \Omega_3 \right| + \left| \int_{\pi_a'} \Omega_3 \right| \right) - 4 \left| \int_{\pi_{O6}} \Re(\Omega_3) \right| . \tag{2.19}$$

Note that in case all the branes are calibrated with respect to $\Re(\Omega)$ the integral simplifies to

$$\mathcal{V} \sim \int_{\sum_a^K N_a(\pi_a + \pi_a') - 4\pi_{O6}} \Re(\Omega_3) = 0 \tag{2.20}$$

which vanishes by using the RR-Tadpole cancellation condition (2.12). Thus as expected in a supersymmetric configuration the cancellation of RR-tadpoles is equivalent to the cancellation of NSNS-tadpoles.

2.5 Model building strategy

Right now we have all necessary ingredients to construct semi-realistic supersymmetric models. Let us briefly outline the strategy for finding realistic intersecting brane models.

First one chooses the six-dimensional compactification manifold $\mathcal{M}$, classifies all special Lagrangian three-cycles by their homological charges and determines the $\Omega\overline{\sigma}$ fixed locus, the orientifold planes π_{O6}. The next step is to select the stacks of matter D-branes according to what gauge symmetries one is interested to generate. Figure 2.2 shows for example a four stack quiver reproducing the standard model particle content. Ensuring the tadpole as well as the supersymmetry conditions usually requires the appearance of additional D6-branes. It has been shown that satisfying the tadpole conditions (2.12) is not enough to ensure the cancellation of all RR charges. This is due to the fact that the D-brane charges are classified by K-theory groups rather than homological groups [21]. Uncanceled K-theory charges would lead to discrete global anomalies in the low energy effective action. To ensure consistency of the model one has to require that any probe $Sp(2)$ brane leads to an even number of matter fields in the fundamental of $Sp(2)$ [22]. Once a complete set of supersymmetric D6-branes, which satisfy the tadpole cancellation condition (2.12) and the K-theory constraints, is found one computes the chiral spectrum according to table 2.1.

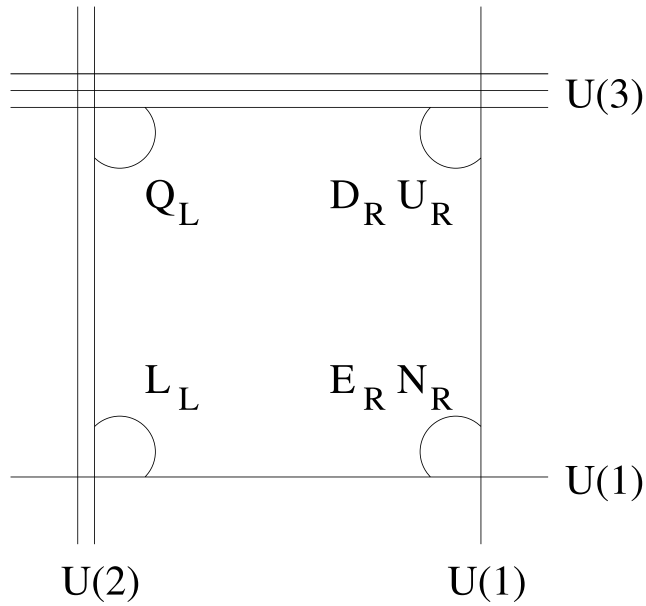

Figure 2.2: Standard model quiver.

Such a model search program has been performed on Orbifolds, six-tori modded out by some discrete symmetries. For these manifolds the class of special Lagrangians is known[5] and as shown in [23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35] one indeed gets semi-realistic supersymmetric three family models. Apart from the fact that all these models give rise to additional exotics which are not content of the standard model most of these backgrounds do not give rise to rigid three-cycles[6] and thus there are too many massless adjoint scalars corresponding to the deformations of the three-cycles. Therefore it is desirable to work on different backgrounds than these orbifolds. Unfortunately for other Calabi-Yau's only little is known about the special Lagrangians limiting the search for more realistic models in this corner of M-theory.

[5]Note that only the factorizable special Lagrangians are known.

[6]But see [36, 37] for backgrounds giving rise to rigid cycles

2.6 Fayet-Iliopoulos parameter

In preparation for section 2.7 we investigate what happens in case a brane a is calibrated with respect to a slightly different phase, such that $\phi_a \neq 0$, but still $\phi_a << 1$. This implies the violation of the second condition in (2.15) and induces a D-term potential which generically takes the form

$$\mathcal{V}_D = \frac{1}{2g_a^2}\left(\sum_i q_i \Phi_i + \xi_a\right)^2 . \tag{2.21}$$

Here g_a is the gauge coupling of $U(1)_a$ in 4 dimensions[7]

$$\frac{4\pi}{g_a^2} = \frac{g_s}{2\pi l_s^3}\left|\int_{\pi_a} \Re(e^{i\phi_a}\Omega_3)\right| , \tag{2.22}$$

Φ_i denote the scalar fields located at D-brane intersections, q_i their corresponding charge under the $U(1)_a$ and ξ_a is the Fayet-Iliopoulos parameter. For non-vanishing ξ_a the D-brane configuration moves to minimum of the D-term potential. Only if at the minimum the potential vanishes supersymmetry is restored, but note that as already expected from field theory the gauge symmetry is broken. This mechanism is called D-brane recombination and will be discussed in more detail in section 2.7.

For small deviations ϕ_a we expect the ξ_a^2 term in (2.22) to be equal with the now non-vanishing scalar potential (2.19). In first order of the imaginary part of

[7]That can be derived by expanding the Dirac-Born-Infeld action (2.17) in powers of the field strength and performing a dimensional reduction down to four dimensions.

$e^{i\phi_a}\Omega_3$ the volume of the $D6_a$ brane is given by

$$\left|\int_{\pi_a} e^{i\phi_a}\Omega_3\right| = \sqrt{\left(\int_{\pi_a} \Re(e^{i\phi_a}\Omega_3) + \int_{\pi_a} \Im(e^{i\phi_a}\Omega_3)\right)} = \frac{1}{2}\,\frac{\left(\int_{\pi_a} \Im(e^{i\phi_a})\Omega_3\right)^2}{\int_{\pi_a} \Re(e^{i\phi_a})\Omega_3^2} \quad (2.23)$$

Comparing the ξ_a^2 term in (2.22) with the scalar potential (2.19) with the use of (2.23) we can relate the FI-parameter to the phase ϕ_a

$$\xi_a^2 = \frac{4\pi^2}{l_s^4}\left(\frac{\int_{\pi_a} \Im(e^{i\phi_a})\Omega_3}{\int_{\pi_a} \Re(e^{i\phi_a})\Omega_3}\right)^2 = \frac{4\pi^2}{l_s^4}\phi_a^2 \quad (2.24)$$

Note that in case the cycle π_a is aligned to the orientifold-plane, e.g. $\phi_a = 0$, the FI-parameter is as expected vanishing as well.

2.7 D-brane recombination

Let us briefly summarize the previous results. In a configuration with several D-branes, a common $\mathcal{N} = 1$ supersymmetry is only preserved once all BPS phases are aligned. Here the orientifold plane singles out a preferred $\mathcal{N} = 1$ subalgebra, associated with the phase 0. The equation fixing the phase of BPS branes in agreement with the orientifold plane is related to the D-flatness conditions of the four-dimensional effective field theory supported by spacetime-filling branes. For small deviations from a supersymmetric configuration, $|\varphi| << 1$, the breaking of $\mathcal{N} = 1$ supersymmetry by non-aligned BPS states can be described as spontaneous D-term breaking within the usual two-derivative supergravity framework. Otherwise, higher derivative terms become non-negligible. As shown in the previous section in the above limit one can identify the BPS phase with the Fayet-Iliopoulos term of the diagonal $U(1)$ subgroup associated with the D-brane theory,

$$2\pi\alpha'\xi = \varphi. \tag{2.25}$$

In the next two subsections we discuss brane recombinations in absence and presence of additional F-terms arising from a non-vanishing superpotential. For the latter we exemplify that the presence of additional superpotential terms might forbid a stable BPS cycle of particular homological charge. This will be relevant in chapter 6.

2.7.1 Bound state decay in absence of F-term obstructions

Consider for simplicity the abelian low-energy effective theory of a pair of BPS D-branes π_1 and π_2 with n^+ and n^- chiral fields of positive and negative charge with respect to $U(1)_1 - U(1)_2$ [38]. Both BPS branes preserve the same $\mathcal{N} = 1$ supersymmetry provided the D-term

$$V_D = \frac{1}{2g_{YM}^2}\left(\sum_i |q_i||\phi_i^+|^2 - \sum_j |q_j||\phi_j^-|^2 - \xi\right)^2 \tag{2.26}$$

vanishes. For zero vacuum expectation values (VEVs) of the charged scalar fields, the supersymmetry condition $\xi = 0$ singles out a real codimension 1 hypersurface in complex structure moduli space which we will denote by $\mathcal{M}_0$ in the sequel. On this locus, there exists a BPS object with homological charges $[\pi_1]+[\pi_2]$, given by $\pi_1 \cup \pi_2$.

Deforming the respective moduli away from $\mathcal{M}_0$ generates an FI term ξ, and according to its sign we enter into the regions of moduli space denoted by $\mathcal{M}_-$ or $\mathcal{M}_+$. In $\mathcal{M}_+$ the fields ϕ_i^+, if present, are tachyonic and their condensation can trigger the formation of a bound state which we denote by $\pi_2\#\pi_1$. The existence of this bound state is guaranteed only in a small neighborhood away from $\mathcal{M}_0$. Likewise, in $\mathcal{M}_-$, condensation of ϕ_i^- can lead to formation of the BPS bound state $\pi_1\#\pi_2$. The charge of each of these bound states is again $[\pi_1] + [\pi_2]$. In the limit of sufficiently small deformations away from $\mathcal{M}_0$, the FI terms (or BPS phases) of the constituent objects add up linearly upon bound state formation.

We have to distinguish the following qualitatively different cases: If $n^+ = n^- \neq 0$, i.e. for vector-like intersections, BPS bound states exist on either side of $\mathcal{M}_0$, which should therefore be called, adopting the nomenclature of [39, 40], line of threshold stability. The same is true for chiral intersections where $0 \neq n^+ \neq n^- \neq 0$. By contrast, the interesting case of strictly chiral intersections with either $n^+ \neq 0$ or $n^- \neq 0$ leads to the genuine decay of a BPS object, say the bound state $\pi_2\#\pi_1$ in $\mathcal{M}_+$, as we pass the line of marginal stability, where the $\pi_1 \cup \pi_2$ is BPS. In general the representatives of a given homological charge can meet several lines of marginal and/or threshold stability in moduli space.

2.7.2 (Non-)BPS bound states and F-term obstructions

In more general situations, F-terms can destabilize otherwise BPS objects or obstruct the formation of BPS bound states. As an illustration consider the following simple system of 3 single spacetime-filling BPS-branes D_a, D_b, D_c which are taken to be suitable BPS branes, respectively. The associated field theory was considered before in [41, 42] as a model of supersymmetry breaking. If all three branes are calibrated with respect to the orientifold, the low-energy effective field theory is $\mathcal{N} = 1$ SYM with gauge group $U(1)_a \times U(1)_b \times U(1)_c$ (modulo one decoupled overall $U(1)$). We assume that the charged matter content of the system is given just by three chiral superfields $\Phi_{(-1_a,1_b)}$, $B_{(-1_b,1_c)}$ and $A_{(-1_c,1_a)}$.

Starting from the situation where all three branes preserve the same $\mathcal{N} = 1$ supersymmetry as the orientifold, we are interested in the behavior of the BPS-branes upon infinitesimal deformations of the complex structure. We are considering only such deformations for which the brane D_c continues to preserve the same $\mathcal{N} = 1$ supersymmetry as the orientifold, i.e. the FI-term associated with $U(1)_c$ vanishes, $\xi_c = 0$.

For sufficiently small deformations, the behavior of the system is captured by the scalar potential of the effective field theory,

$$V = V_D + V_F, \tag{2.27}$$

where

$$\begin{aligned} V_D &\simeq \frac{1}{2g_{YM}^2}\left((-|\phi|^2 + |A|^2 - \xi_a)^2 + (|\phi|^2 - |B|^2 - \xi_b)^2 + (|B|^2 - |A|^2)^2\right), \\ V_F &\simeq \lambda^2\,(|\Phi|^2|B|^2 + |\Phi|^2|A|^2 + |A|^2|B|^2). \end{aligned} \tag{2.28}$$

Here we consider, for simplicity, equal gauge couplings for all 3 branes. λ denotes the Yukawa coupling appearing in the superpotential $W = \lambda\,\Phi BA$, which we assume to be non-vanishing.

Unbroken SUSY is possible only for $-\xi_b = \xi_a = \xi \leq 0$, and the microscopic behavior of the branes in this regime is clear. Perturbing the system instead such that $-\xi_b = \xi_a = \xi > 0$, we have the following non-SUSY minimum for perturbational small values of $x = 2g_{YM}^2\,\lambda^2$:

$$|A| = |B| = \sqrt{\frac{2\xi}{2+x}}, \qquad \Phi = 0. \tag{2.29}$$

F- and D-flatness are both broken as the F-term prevents the system from recombining into a D-flat configuration corresponding to $|A| = |B| = \sqrt{\xi}$.

To understand this, we first consider the hypothetical BPS-bound state Ψ due to condensation of the tachyons A and B in absence of the F-term. It can be viewed as the result of first condensing A, leading to the intermediate state $Y = D_c \# D_a$, and its subsequent combination with D_b induced by the VEV of B,

$$\Psi = D_b \# D_c \# D_a. \tag{2.30}$$

Due to the described linearity of the FI-terms in the limit of small deformations, the would-be BPS bound state Ψ leads to a vanishing D-term, in agreement with the field theory analysis for $|A| = |B| = \sqrt{\xi}$. It still hosts a massless chiral multiplet Φ playing the role of a modulus, while the adjoint fields A, B have acquired D-term masses. But the F-term $W = \lambda\,\Phi B A$ before bound state formation indicates that the modulus Φ is actually 'obstructed' at linear order in that it suffers from a tadpole $W = \lambda\xi\Phi$. Together with the coupling $\lambda\sqrt{\xi}\,\Phi\,(\delta A + \delta B)$ to the massive fluctuations δA, δB this tadpole leads, in the scalar potential, to destabilizing terms linear in δA, δB. The bound state Ψ is driven into a truly non-BPS state $\widetilde{\Psi}$ of the same homological charge which breaks both D- and F-flatness while minimizing the total action.

Geometrically, it is not completely obvious in which sense $\widetilde{\Psi}$ violates the BPS condition. We would like to argue that it is not just a calibrated cycle preserving the wrong $\mathcal{N} = 1$ subalgebra, but rather not calibrated at all. After all we cannot form another BPS bound state in the same homology class as Ψ but with a different BPS phase. On the other hand, we see no indications that $\widetilde{\Psi}$ ceases to satisfy the topological brane, i.e. Lagrangian or holomorphicity, condition. Its violation should manifest itself in extra closed moduli dependent F-terms in the effective action (see e.g. [43]) in addition to the matter potential. We therefore propose that $\widetilde{\Psi}$ is a *non-calibrated brane*. The presence of the destabilizing superpotential terms for the hypothetical cycle Ψ reflects the fact that the geometry does actually not allow for a stable BPS cycle of this charge in this region of moduli space.

Chapter 3

Proton decay in intersecting brane models

Grand unified theories (GUT's) [44] not only give a neat and aesthetic description of our four dimensional world but also lead to an explanation of electric charge quantization and - with the aid of supersymmetry - predict the value of $\sin^2\theta_W$ in very good agreement with the experimental one. Moreover GUT's lead to Baryon number violating processes; in particular they predict proton decay [45] (for a recent review on proton decay see [46]).

In supersymmetric GUT field theories [47, 48] the proton decay can occur either by an exchange of a super heavy SUSY particle which corresponds to a decay via the dimension 5 operator $\int d^2\theta\, Q^3\, L$ or by a super heavy gauge boson exchange[1] a decay via the dimension 6 operator $\int d^4\theta\, Q^2\, \tilde{Q}^*\, \tilde{L}^*$. In the simplest supersymmetric GUT models, proton decay mediated via dimension 5 operators dominates and recent com-

[1]We forbid proton decay due to dimension four operators by introducing R-symmetry.

putations predict a lifetime for the proton, which is below the present experimental bounds [49, 50, 51], but [52, 53]. The fact that proton decay has not yet been observed, suggests the existence of some mechanism that suppresses or even forbids these dimension 5 operators, so that after all the proton decay via dimension six operators is the most dominant one.

In this chapter we investigate proton decay via dimension six operators in supersymmetric GUT models based on intersecting D6-brane constructions on type IIA string theory orientifolds. More precisely, we compute the string effects on the proton's decay into a pion and a positron ($p \to \pi^0 e^+$) for supersymmetric $SU(5)$-GUT-like models arising from intersecting D6-brane constructions. In $SU(5)$-GUT's there are two different amplitudes that contribute to this proton decay rate: $\langle \bar{\mathbf{5}}^* \bar{\mathbf{5}} \mathbf{10}^* \mathbf{10} \rangle$ and $\langle \mathbf{10}^* \mathbf{10} \mathbf{10}^* \mathbf{10} \rangle$, where $\bar{\mathbf{5}}$ and $\mathbf{10}$ denote the multiplets of the gauge group $SU(5)$. For intersecting D6-brane constructions with supersymmetric $SU(5)$-GUT's [24, 23, 25, 54, 55] , the latter amplitude was computed in [56], by explicitly calculating the string amplitude contribution to $\langle \mathbf{10}^* \mathbf{10} \mathbf{10}^* \mathbf{10} \rangle$ operator. However, even after pushing all the parameters to the limit, in order to maximize the proton on decay rate, the string contribution to the proton decay rate is at most a comparable to the field theory one. Here we include the additional contributions from the amplitude $\langle \bar{\mathbf{5}}^* \bar{\mathbf{5}} \mathbf{10}^* \mathbf{10} \rangle$ in the same class of models.

3.1 Setup

We analyze the proton decay occurring due to dimension 6 operators in a local intersecting D6-brane configuration. Thus, we consider scattering amplitudes of the form $\langle \bar{\mathbf{5}}^* \bar{\mathbf{5}} \mathbf{10}^* \mathbf{10} \rangle$ and $\langle \mathbf{10}^* \mathbf{10} \mathbf{10}^* \mathbf{10} \rangle$, where $\bar{\mathbf{5}}$ and $\mathbf{10}$ denote the multiplets of the gauge group $SU(5)$. While the latter amplitude was already examined in [56], we will determine the additional contribution to the proton decay arising from the amplitude $\langle \bar{\mathbf{5}}^* \bar{\mathbf{5}} \mathbf{10}^* \mathbf{10} \rangle$. Since we only consider scattering arising at the local intersection, the first step is to derive conditions on the angles so that we have at the local intersection matter fields in the $\bar{\mathbf{5}}$ and $\mathbf{10}$ representation, simultaneously. We will show that this condition is satisfied only for particular regions.

To simplify the analysis of the D-brane configuration we compactify the internal dimensions on a factorizable six-torus T^6. Later we assume that the compactification volume is larger than the string scale so that local effects dominate the amplitude and global ones can be neglected. This assumption also allows us to embed the local D-brane configuration, described below, into an arbitrary compactification manifold. The complex coordinates of the factorizable six-torus $T^6 = T^2 \times T^2 \times T^2$ are given by

$$z_1 = x^4 + \mathrm{i}x^5 \qquad z_2 = x^6 + \mathrm{i}x^7 \qquad z_1 = x^8 + \mathrm{i}x^9.$$

In order to construct an $SU(5)$ GUT model we take a stack b of M D6-branes oriented in the 0123468 directions that intersects with a stack a of 5 D6 branes along the 0123 directions. The dimensions 0123 have an interpretation as a $3+1$ dimensional intersecting brane world. Both types of D-branes are wrapped on the (n^I, m^I) cycle of the I^{th} torus. Obviously, the wrapping numbers of the stack b are given by

$$b: \ (n_b^1, m_b^1)(n_b^2, m_b^2)(n_b^3, m_b^3) = (1,0)(1,0)(1,0), \tag{3.1}$$

while the ones from stack a can take the general form

$$a: \quad (n_a^1, m_a^1)(n_a^2, m_a^2)(n_a^3, m_a^3). \tag{3.2}$$

Given the wrapping numbers, one can calculate the intersection angles which are in general given by (R_1^I,R_2^I denote the radii of the I^{th} torus)

$$\theta_{ab}^I = \theta_b^I - \theta_a^I = \arctan\left(\frac{m_b^I R_2^I}{n_b^I R_1^I}\right) - \arctan\left(\frac{m_a^I R_2^I}{n_a^I R_1^I}\right)$$

and in our case take the simple form (since $\theta_b = 0$)

$$\theta_{ab}^I = -\arctan\left(\frac{m_a^I R_2^I}{n_a^I R_1^I}\right). \tag{3.3}$$

In order to cancel the RR-tadpoles, we must introduce O6-planes and in particular the orientifold action $\Omega\overline{\sigma}$, where Ω is the world-sheet parity and $\overline{\sigma}$ acts by

$$\overline{\sigma} : (z_1, z_2, z_3) \longrightarrow (\bar{z}_1, \bar{z}_2, \bar{z}_3).$$

This orientifold action forces us to include stacks of image D-branes. While for stack b the orientifold action only effects the gauge group (for M coincident branes parallel to the $O6$-plane the $\Omega\overline{\sigma}$ projection leads to the gauge group $USp(2M)$) for the stack a we have to introduce an image stack a' of 5 D6-branes whose wrapping numbers are given by

$$a': \quad (n_a^1, -m_a^1)(n_a^2, -m_a^2)(n_a^3, -m_a^3)\ . \tag{3.4}$$

Fermions that arise from strings stretched between a and a' transform in the antisymmetric representation of $SU(5) \times SU(5)$. Depending on the sign of the intersection number these fermions transform as $\mathbf{10}$'s or $\overline{\mathbf{10}}$'s . Fermions in the ab and ab' sector transform in the bifundamental representation $(\mathbf{5}, \mathbf{M})$ or $(\bar{\mathbf{5}}, \mathbf{M})$ again depending on the sign of the intersection number. Here $\mathbf{M}$ denotes the representation of the gauge group $USp(M)$. In general, the intersection number for two intersecting D-branes a and b on a torus is given by

$$I_{ab} = \prod_{I=1}^{3} \left(n_a^I m_b^I - m_a^I n_b^I \right) \ . \tag{3.5}$$

Now we have all ingredients to determine the conditions the intersection angles θ_I have to satisfy in order to observe matter fields transforming as $\bar{\mathbf{5}}$ and $\mathbf{10}$ at the intersection. Using (3.1), (3.2), (3.4) and (3.5) we obtain for the intersection numbers I_{ab} and $I_{aa'}$

$$I_{ab} = (-1)^3 \prod_{I=1}^{3} m_a^I \qquad I_{aa'} = (-2)^3 \prod_{I=1}^{3} n_a^I m_a^I \ . \tag{3.6}$$

Obviously, the sign of the intersection number depends on the sign of the wrapping numbers. For every angle θ_I [2] we have to distinguish between four different cases

$$\begin{aligned}
&\bullet\ n_a^I,\, m_a^I > 0 \ \text{ which corresponds to an angle with } \ -\frac{\pi}{2} < \theta_I < 0 \\
&\bullet\ n_a^I > 0,\ m_a^I < 0 \ \text{ which corresponds to an angle with } \ 0 < \theta_I < \frac{\pi}{2} \\
&\bullet\ n_a^I < 0 \ \ m_a^I > 0 \ \text{ which corresponds to an angle with } \ -\frac{\pi}{2} < \theta_I < -\pi \\
&\bullet\ n_a^I,\, m_a^I < 0 \ \text{ which corresponds to an angle with } \ \frac{\pi}{2} < \theta_I < \pi \ .
\end{aligned} \tag{3.7}$$

In order to satisfy supersymmetry the intersection angles θ_I are subject to the constraint [57]

$$\theta_1 + \theta_2 + \theta_3 = 0 \qquad \mod 2\pi \ , \tag{3.8}$$

which restricts the choice of the angles. Let us start by investigating the case that the angles add up to 0, later on we also analyze configurations where the angles add up to 2π or -2π. If the sum is equal to 0 then one or two of the angles needs to be negative. In case only one angle is negative, let us assume without loss of generality that $\theta_3 < 0$. Since all angles $|\theta_I| \leq \pi$, we distinguish four different cases for which we obtain, by applying (3.6) and (3.7), the signs of the intersection numbers and thus their transformation behavior under the $SU(5)$

- $I_{ab} < 0$ and $I_{aa'} < 0$ for $\quad 0 < \theta_1 < \frac{\pi}{2} \qquad 0 < \theta_2 < \frac{\pi}{2} \qquad -\frac{\pi}{2} < \theta_3 < 0$
- $I_{ab} < 0$ and $I_{aa'} > 0$ for $\quad 0 < \theta_1 < \frac{\pi}{2} \qquad 0 < \theta_2 < \frac{\pi}{2} \qquad -\pi < \theta_3 < -\frac{\pi}{2}$

[2] From now on we denote θ_{ab}^I by θ_I where θ_{ab}^I is given by (3.3).

- $I_{ab} < 0$ and $I_{aa'} < 0$ for $\frac{\pi}{2} < \theta_1 < \pi \qquad 0 < \theta_2 < \frac{\pi}{2} \qquad -\pi < \theta_3 < -\frac{\pi}{2}$
- $I_{ab} < 0$ and $I_{aa'} < 0$ for $0 < \theta_1 < \frac{\pi}{2} \qquad \frac{\pi}{2} < \theta_2 < \pi \qquad -\pi < \theta_3 < -\frac{\pi}{2}$.

In case of exactly one negative angle we do not find fields transforming as $\bar{\mathbf{5}}$ and $\mathbf{10}$. Thus we do not observe a 4-point interaction of the form $\bar{\mathbf{5}}^*\bar{\mathbf{5}}\mathbf{10}^*\mathbf{10}$ at the intersection. Analyzing the case of two negative angles (without loss of generality we assume that θ_1 and θ_2 are negative) we again distinguish between four different cases

- $I_{ab} > 0$ and $I_{aa'} < 0$ for $-\frac{\pi}{2} < \theta_1 < 0 \qquad -\frac{\pi}{2} < \theta_2 < 0 \qquad 0 < \theta_1 < \frac{\pi}{2}$
- $I_{ab} > 0$ and $I_{aa'} < 0$ for $-\frac{\pi}{2} < \theta_1 < 0 \qquad -\frac{\pi}{2} < \theta_2 < 0 \qquad \frac{\pi}{2} < \theta_3 < \pi$
- $I_{ab} > 0$ and $I_{aa'} > 0$ for $-\pi < \theta_1 < -\frac{\pi}{2} \qquad -\frac{\pi}{2} < \theta_2 < 0 \qquad \frac{\pi}{2} < \theta_3 < \pi$
- $I_{ab} > 0$ and $I_{aa'} > 0$ for $-\frac{\pi}{2} < \theta_1 < 0 \qquad -\pi < \theta_2 < -\frac{\pi}{2} \qquad \frac{\pi}{2} < \theta_3 < \pi$.

Only in the region $-\frac{\pi}{2} < \theta_{1,2} < 0\,, \frac{\pi}{2} < \theta_3 < \pi$ we observe matter fields transforming as $\bar{\mathbf{5}}$ and $\mathbf{10}$, where strings stretched between the D-branes a and b transform as $\bar{\mathbf{5}}$ and strings stretched between a and a' transform as $\mathbf{10}$.

Let us now turn to the case in which the intersection angles θ_I add up to 2π. Then all the angles are positive and we have to distinguish between three different configurations (without loss of generality let us assume that θ_1 is always bigger than $\frac{\pi}{2}$)

- $I_{ab} > 0$ and $I_{aa'} > 0$ for $\frac{\pi}{2} < \theta_1 < \pi \qquad \frac{\pi}{2} < \theta_2 < \pi \qquad 0 < \theta_3 < \frac{\pi}{2}$
- $I_{ab} > 0$ and $I_{aa'} > 0$ for $\frac{\pi}{2} < \theta_1 < \pi \qquad 0 < \theta_2 < \frac{\pi}{2} \qquad \frac{\pi}{2} < \theta_3 < \pi$
- $I_{ab} > 0$ and $I_{aa'} < 0$ for $\frac{\pi}{2} < \theta_1 < \pi \qquad \frac{\pi}{2} < \theta_2 < \pi \qquad \frac{\pi}{2} < \theta_3 < \pi$.

Again only in one region, $\frac{\pi}{2} < \theta_{1,2,3} < \pi$, can we observe matter fields transforming as $\bar{\mathbf{5}}$ and $\mathbf{10}$, where strings stretched between the D-branes a and b transform as $\bar{\mathbf{5}}$ and strings stretched between a and a' transform as $\mathbf{10}$.

Finally, we examine the case in which the angles add up to -2π. Here all three angles have to be negative and again one has to distinguish between three different cases (without loss of generality we assume that θ_1 is smaller than $-\frac{\pi}{2}$)

- $I_{ab} < 0$ and $I_{aa'} < 0$ for $\quad -\pi < \theta_1 < \frac{\pi}{2} \qquad -\pi < \theta_2 < \frac{\pi}{2} \qquad -\frac{\pi}{2} < \theta_3 < 0$
- $I_{ab} < 0$ and $I_{aa'} < 0$ for $\quad -\pi < \theta_1 < \frac{\pi}{2} \qquad -\frac{\pi}{2} < \theta_2 < 0 \qquad -\pi < \theta_3 < -\frac{\pi}{2}$
- $I_{ab} < 0$ and $I_{aa'} > 0$ for $\quad -\pi < \theta_1 < -\frac{\pi}{2} \quad -\pi < \theta_2 < -\frac{\pi}{2} \qquad -\pi < \theta_3 < -\frac{\pi}{2}$.

As in the first case, the analysis shows that strings stretched between d-branes a and b transform as $\mathbf{5}$ under the $U(5)$ gauge group. Therefore, at the intersection we do not have any matter fields transforming as $\bar{\mathbf{5}}$.

Summarizing, we determined that only for the two regions $-\frac{\pi}{2} < \theta_{1,2} < 0\,, \frac{\pi}{2} < \theta_3 < \pi$ and $\frac{\pi}{2} < \theta_{1,2,3} < \pi$ we have matter fields transforming as $\bar{\mathbf{5}}$ and $\mathbf{10}$ at the intersection simultaneously. In addition to the amplitude $\langle \mathbf{10}^* \mathbf{10} \mathbf{10}^* \mathbf{10} \rangle$, we have for these two regions only, a non-suppressed contribution from $\langle \bar{\mathbf{5}}^* \bar{\mathbf{5}} \mathbf{10}^* \mathbf{10} \rangle$ to the proton decay rate. In order to compute these two amplitudes we need the corresponding vertex operators to the states $\bar{\mathbf{5}}$, and $\mathbf{10}$ in the respective configurations, which we determine in the next section.

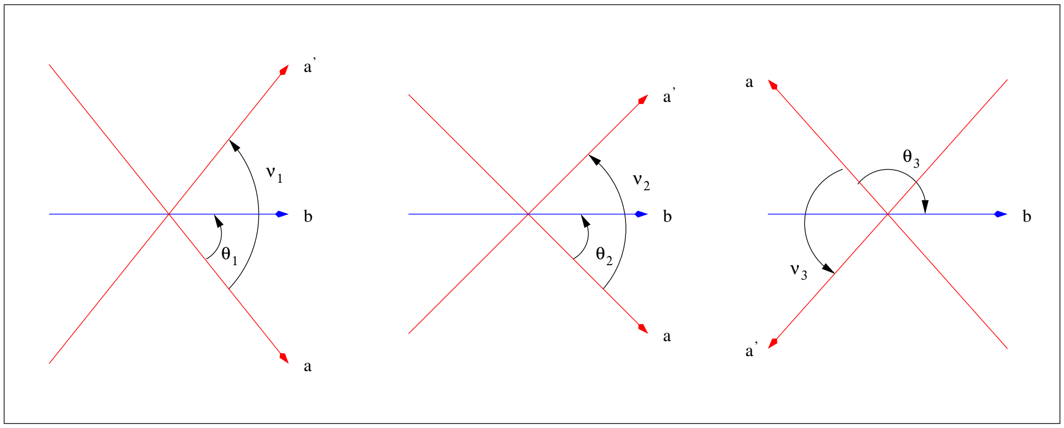

Figure 3.1: Intersection angles for the case $-\frac{1}{2} < \theta_1 < 0, -\frac{1}{2} < \theta_2 < 0, \frac{1}{2} < \theta_3 < 1$.

3.2 Vertex Operators

For different D-brane configurations we have different vacua and therefore different vertex operators. Knowing the D-brane configuration we can use the prescription given in appendix A to obtain the vertex operator for the massless fermion in the R-sector. In this way we can easily determine the vertex operators for $\bar{\mathbf{5}}$, arising from strings stretched between the stacks a and b. The vertex operator for $\mathbf{10}$ requires more effort. The simple approach just to replace the θ_I in the $\bar{\mathbf{5}}$ vertex operator by the double, $2\theta_I$ only works for $|\theta_I| < \frac{1}{2}$ [3], since in the expansion of the bosonic (A.2) and fermionic degrees of freedom (A.3) the shift number θ_I has to be in the interval $[-1, 1]$. Therefore if $|\theta_I| > \frac{1}{2}$ we need to find an expression ν_I which lies between 0 and 1 and describes the D-brane configuration aa'. Figure 3.1 which shows the D-brane configuration for the case $-\frac{1}{2} < \theta_{1,2} < 0\,, \frac{1}{2} < \theta_3 < 1$. The vertex operator in the $(-\frac{1}{2})$-ghost picture for the massless fermion, arising from a string stretched between

[3] From now on we replace θ_I by θ_I/π so that $\theta_I \,\epsilon\, [-1, 1]$.

D-branes a and b is given by (keep in mind that $\theta_{1,2}$ are negative)

$$V_{\bar{5}}^{-\frac{1}{2}}(z) = \Lambda_{\bar{5}}\, \mathrm{e}^{-\frac{\phi}{2}(z)}\, S^{\alpha}(z) \prod_{I=1}^{2} \sigma_{-\theta_I}(z)\, \mathrm{e}^{-\mathrm{i}\left(\theta_I+\frac{1}{2}\right)H_I(z)}\, \sigma_{1-\theta_3}(z)\, \mathrm{e}^{-\mathrm{i}\left(\theta_3-\frac{1}{2}\right)H_3(z)}\, \mathrm{e}^{\mathrm{i}k\cdot X(z)}. \tag{3.9}$$

Now we turn to the aa' sector in which the string state transforms as **10**. We see that the intersection angle in the third complex dimension is given by $\nu_3 = -2+2\theta_3$. Note that the intersection angle ν_3 is negative and lies between -1 and 0, since θ_3 takes a value between $\frac{1}{2}$ and 1 and therefore the corresponding vertex operator for the state **10** takes the form

$$V_{10}^{-\frac{1}{2}}(z) = \Lambda_{10}\, \mathrm{e}^{-\frac{\phi}{2}(z)}\, S^{\alpha}(z) \prod_{I=1}^{3} \sigma_{1+\nu_I}(z)\, \mathrm{e}^{\mathrm{i}\left(\nu_I+\frac{1}{2}\right)H_I(z)}\, \mathrm{e}^{\mathrm{i}k\cdot X(z)}, \tag{3.10}$$

where the angles ν_I are given by

$$\nu_1 = 2\theta_1 \qquad \nu_2 = 2\theta_2 \qquad \nu_3 = -2+2\theta_3\ .$$

Notice, that the angles ν_I add up to -2 so that the SUSY condition (3.8) is satisfied. In an analogous way (look at figure 3.2), we obtain for the other D-brane configuration.

- $\frac{1}{2} < \theta_1 < 1 \qquad \frac{1}{2} < \theta_2 < 1 \qquad \frac{1}{2} < \theta_3 < 1$

For this configuration the vertex operator that creates a string stretched be-

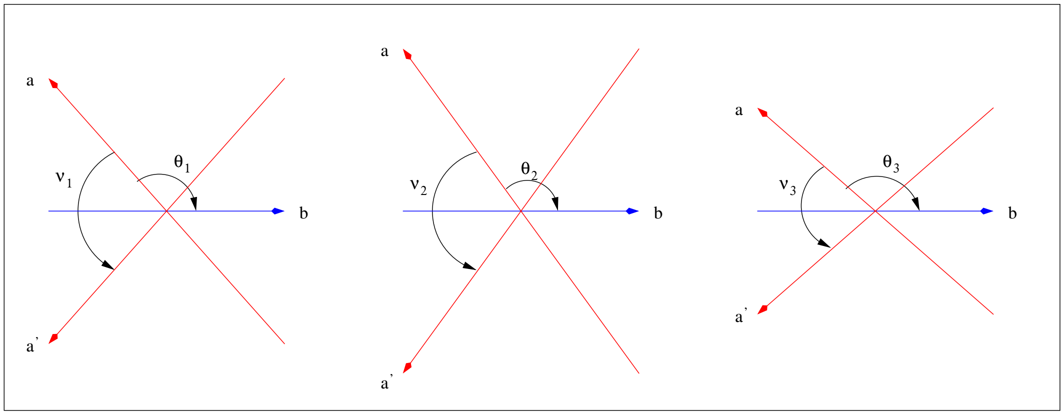

Figure 3.2: Intersection angles for the case $\frac{1}{2} < \theta_1 < 1, \frac{1}{2} < \theta_2 < 1, \frac{1}{2} < \theta_3 < 1$

tween a and b is

$$V_{\bar{5}}^{-\frac{1}{2}}(z) = \Lambda_{\bar{5}}\, \mathrm{e}^{-\frac{\phi}{2}(z)}\, S^{\alpha}(z) \prod_{I=1}^{3} \sigma_{1-\theta_I}(z)\, \mathrm{e}^{-\mathrm{i}\left(\theta_I - \frac{1}{2}\right) H_I(z)}\, \mathrm{e}^{\mathrm{i}k\cdot X(z)} \ . \tag{3.11}$$

The intersection angles ν_I are given by

$$\nu_1 = -2 + 2\theta_1 \qquad \nu_2 = -2 + 2\theta_2 \qquad \nu_3 = -2 + 2\theta_3 \ .$$

Obviously, they are all negative, so that the vertex operator which describes the massless aa'-string in the R-sector takes the form

$$V_{10}^{-\frac{1}{2}}(z) = \Lambda_{10}\, \mathrm{e}^{-\frac{\phi}{2}(z)}\, S^{\alpha}(z) \prod_{I=1}^{3} \sigma_{1+\nu_I}(z)\, \mathrm{e}^{\mathrm{i}\left(\nu_I + \frac{1}{2}\right) H_I(z)}\, \mathrm{e}^{\mathrm{i}k\cdot X(z)} \ . \tag{3.12}$$

Again the angles ν_I add up to -2.

In order to calculate scattering amplitudes we also need the vertex operators for the complex conjugated fields $\bar{\mathbf{5}}^*$ and $\mathbf{10}^*$. We obtain them by replacing the spin field by the spin field with opposite chirality and at the same time sending the angles θ_I and ν_I to $1-\theta_I$ and $1-\nu_I$, respectively (for negative angle we replace θ_I and ν_I by $-1-\theta_I$ and $-1-\nu_I$, respectively). For the two cases we obtain

- $-\frac{1}{2} < \theta_1 < 0 \qquad -\frac{1}{2} < \theta_2 < 0 \qquad \frac{1}{2} < \theta_3 < 1$

$$V_{\bar{5}^*}^{-\frac{1}{2}}(z) = \tilde{\Lambda}_{\bar{5}}\,\mathrm{e}^{-\frac{\phi}{2}(z)}\,\tilde{S}_{\dot{\alpha}}(z)\prod_{I=1}^{2}\sigma_{1+\theta_I}(z)\,\mathrm{e}^{\mathrm{i}\left(\theta_I+\frac{1}{2}\right)H_I(z)}\,\sigma_{\theta_3}(z)\,\mathrm{e}^{\mathrm{i}\left(\theta_3-\frac{1}{2}\right)H_3(z)}\,\mathrm{e}^{\mathrm{i}k\cdot X(z)} \tag{3.13}$$

for $\bar{\mathbf{5}}^*$ and

$$V_{10^*}^{-\frac{1}{2}}(z) = \tilde{\Lambda}_{10}\,\mathrm{e}^{-\frac{\phi}{2}(z)}\,\tilde{S}_{\dot{\alpha}}(z)\prod_{I=1}^{3}\sigma_{-\nu_I}(z)\,\mathrm{e}^{-\mathrm{i}\left(\nu_I+\frac{1}{2}\right)H_I(z)}\,\mathrm{e}^{\mathrm{i}k\cdot X(z)} \tag{3.14}$$

for $\mathbf{10}^*$.

- $\frac{1}{2} < \theta_1 < 1 \qquad \frac{1}{2} < \theta_2 < 1 \qquad \frac{1}{2} < \theta_3 < 1$

$$V_{\bar{5}^*}^{-\frac{1}{2}}(z) = \tilde{\Lambda}_{\bar{5}}\,\mathrm{e}^{-\frac{\phi}{2}(z)}\,\tilde{S}_{\dot{\alpha}}(z)\prod_{I=1}^{3}\sigma_{\theta_I}(z)\,\mathrm{e}^{-\mathrm{i}\left(\theta_I-\frac{1}{2}\right)H_I(z)}\,\mathrm{e}^{\mathrm{i}k\cdot X(z)} \tag{3.15}$$

for $\bar{\mathbf{5}}^*$ and

$$V_{10^*}^{-\frac{1}{2}}(z) = \tilde{\Lambda}_{10}\,\mathrm{e}^{-\frac{\phi}{2}(z)}\,S^{\alpha}(z)\prod_{I=1}^{3}\sigma_{-\nu_I}(z)\,\mathrm{e}^{-\mathrm{i}\left(\nu_I+\frac{1}{2}\right)H_I(z)}\,\mathrm{e}^{\mathrm{i}k\cdot X(z)} \tag{3.16}$$

for $\mathbf{10}^*$.

Finally, we will discuss the Chan-Paton factors. In a setup without orientifolds strings transform in the bifundamental of $U(N) \times U(M)$. As already mentioned above, the introduction of orientifolds changes the transformation behavior. The full orientifold action on the Chan-Paton factors takes the form

$$\Lambda = -\gamma_{\Omega R} \Lambda^T \gamma_{\Omega R}^{-1} ,$$

where $\gamma_{\Omega R}$ is given by [58]

$$\gamma_{\Omega R} = \begin{pmatrix} 0 & 1_N & 0 & 0 \\ 1_N & 0 & 0 & 0 \\ 0 & 0 & 0 & 1_M \\ 0 & 0 & 1_M & 0 \end{pmatrix} . \tag{3.17}$$

The choice of $N = 5$ leads to the following Chan-Paton factors for the $\mathbf{10}$'s

$$\Lambda_{10} = \begin{pmatrix} 0 & \lambda_{10} & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} , \tag{3.18}$$

where λ_{10} is an antisymmetric 5×5 matrix. For M we choose 1, which leads to a $Sp(2)$ gauge group on the D-brane b, which has two components in the fundamental representation. One component is associated with the matter field $\bar{\mathbf{5}}$ while the other corresponds to the Higgs particle. Their Chan-Paton factors take the form

$$\Lambda_{\bar{5}} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & \lambda_{\bar{5}} & 0 \\ 0 & 0 & 0 & 0 \\ -\lambda_{\bar{5}}^T & 0 & 0 & 0 \end{pmatrix} \qquad \Lambda_H = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & H \\ -H^T & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}. \tag{3.19}$$

Here $\lambda_{\bar{5}}$ and H are a 5×1 matrices. λ_{10} and $\lambda_{\bar{5}}$ denote the usual 10- and 5-dimensional representations of the $SU(5)$ gauge group and H is the 5 dimensional Higgs field in the gauge field theory.

3.3 String Amplitude

Having derived the vertex operators in the previous section, we have all the ingredients to compute the scattering amplitudes. Assuming that the compactification volume is larger than the string scale worldsheet instantons are suppressed and it is sufficient to compute just the quantum part of the amplitudes. First we will focus on $\langle {V^{\bar{5}}_{-\frac{1}{2}}}^* V^{\bar{5}}_{-\frac{1}{2}} {V^{10}_{-\frac{1}{2}}}^* V^{10}_{-\frac{1}{2}}\rangle$ and afterwards we will compute $\langle {V^{10}_{-\frac{1}{2}}}^* V^{10}_{-\frac{1}{2}} {V^{10}_{-\frac{1}{2}}}^* V^{10}_{-\frac{1}{2}}\rangle$, which was already examined in [56]

The amplitude $\langle {V^{\bar{5}}_{-\frac{1}{2}}}^* V^{\bar{5}}_{-\frac{1}{2}} {V^{10}_{-\frac{1}{2}}}^* V^{10}_{-\frac{1}{2}}\rangle$

We start with the region $-\frac{1}{2} < \theta_1 < 0\,, -\frac{1}{2} < \theta_2 < 0\,, \frac{1}{2} < \theta_3 < 1$ and calculate the amplitude

$$\int \prod_{i=1}^{4} \mathrm{d}z_i \; \langle V^{\bar{5}\,*}_{-\frac{1}{2}}(z_1)\, V^{\bar{5}}_{-\frac{1}{2}}(z_2)\, V^{10\,*}_{-\frac{1}{2}}(z_3) V^{10}_{-\frac{1}{2}}(z_4)\rangle \ ,$$

where the vertex operators are in the previous section. Note that all the vertex operators are in the $(-\frac{1}{2})$-ghost pictures, which guarantees a total charge of -2 on the disk. Plugging in the vertex operators we see that in order to calculate the amplitude we need the following correlators

$$\Big\langle \prod_{i=1}^{4} \mathrm{e}^{\mathrm{i}k_i\cdot X(z_i)} \Big\rangle = \prod_{\substack{i,j=1\\ i<j}}^{4} {z_{ij}}^{\alpha'\, k_i\cdot k_j} \qquad \Big\langle \mathrm{e}^{-\frac{\phi}{2}(z_1)}\, \mathrm{e}^{-\frac{\phi}{2}(z_2)}\, \mathrm{e}^{-\frac{\phi}{2}(z_3)}\, \mathrm{e}^{-\frac{\phi}{2}(z_4)} \Big\rangle = \prod_{\substack{i,j=1\\ i<j}}^{4} z_{ij}^{-\frac{1}{4}}$$

$$\bar{u}_1^{\dot\alpha}\, u_{\alpha\, 2}\, \bar{u}_3^{\dot\beta}\, u_{\beta\, 4}\, \langle \tilde{S}_{\dot\alpha}(z_1)\, S^{\alpha}(z_2) \tilde{S}_{\dot\beta}(z_3) S^{\beta}(z_4)\rangle = \bar{u}_1\gamma^{\mu}u_2\;\, \bar{u}_3\gamma_{\mu}u_4\, z_{13}^{-\frac{1}{2}}\, z_{24}^{-\frac{1}{2}} \ , \tag{3.20}$$

where z_{ij} denotes $z_i - z_j$. The correlator involving the four fermionic twist fields takes an easy form, since we can bosonize the spin fields

$$\Big\langle \prod_{i=1}^{4} \mathrm{e}^{\mathrm{i}\,\alpha_i H(z_i)} \Big\rangle = \prod_{\substack{i,j=1\\ i<j}}^{4} {z_{ij}}^{\alpha_i\cdot\alpha_j} \ . \tag{3.21}$$

The correlator for the bosonic twist fields is more involved. Using the stress energy tensor method, the quantum part of four bosonic twist fields with two independent angles evaluates to [59, 60, 61]

$$\langle \sigma_{1-\theta}(z_1)\, \sigma_\theta(z_2)\, \sigma_{1-\nu}(z_3)\, \sigma_\nu(z_4) \rangle = z_{12}^{-\theta(1-\theta)} z_{34}^{-\nu(1-\nu)} \left(\frac{z_{13}\, z_{24}}{z_{14}\, z_{23}} \right)^{\frac{1}{2}(\theta+\nu)-\theta\nu} I^{-\frac{1}{2}}(x) \ , \tag{3.22}$$

with $x = \frac{z_{12}z_{34}}{z_{13}z_{24}}$ and $I(x)$ is given by

$$I(x) = \frac{1}{2\pi} \left[B_1(\theta,\nu)\, \overline{G_2}(x) H_1(1-x) + B_2(\theta,\nu)\, G_1(x) \overline{H_2}(1-x) \right] \ ,$$

where

$$B_1(\theta,\nu) = \frac{\Gamma(\theta)\,\Gamma(1-\nu)}{\Gamma(1+\theta-\nu)} \qquad B_2(\theta,\nu) = \frac{\Gamma(\nu)\,\Gamma(1-\theta)}{\Gamma(1+\nu-\theta)}$$

$$G_1(x) = {}_2F_1[\theta, 1-\nu, 1; x] \qquad G_2(x) = {}_2F_1[1-\theta, \nu, 1; x]$$

$$H_1(x) = {}_2F_1[\theta, 1-\nu, 1+\theta-\nu; x] \qquad H_2(x) = {}_2F_1[1-\theta, \nu, 1-\theta+\nu; x] \ .$$

Applying the correlators, the amplitude becomes

$$A = \mathrm{i} Tr(\tilde{\Lambda}_1\, \Lambda_2\, \tilde{\Lambda}_3\, \Lambda_4)\, \bar{u}_1 \gamma^\mu u_2\, \bar{u}_3 \gamma_\mu u_4 (2\pi)^4 \delta^{(4)} \left(\sum_{i=1}^4 k_i \right)$$

$$\times \int \prod_{i=1}^4 \mathrm{d}z_i \frac{\left[I\left(-\theta_1, 1+\nu_1, x\right)\ I\left(-\theta_2, 1+\nu_2, x\right)\ I\left(1-\theta_3, 1+\nu_3, x\right) \right]^{-\frac{1}{2}}}{\left(z_{12}\, z_{34}\right)^{-\alpha' s-1} \left(z_{13}\, z_{24}\right)^{-\alpha' t} \left(z_{14}\, z_{23}\right)^{-\alpha' u-1}} \ ,$$

where s, t and u are the Mandelstam variables

$$s = -(k_1 + k_2)^2 \qquad t = -(k_1 + k_3)^2 \qquad u = -(k_1 + k_4)^2 \,.$$

The conformal Killing group can be used to fix three of the vertex operator positions. A convenient choice is

$$z_1 = 0 \qquad z_2 = x \qquad z_3 = 1 \qquad z_4 = z_\infty = \infty \ ,$$

which implies the c-ghost contribution

$$\langle \, c\,(0) \;\, c\,(1) \;\, c\,(z_\infty) \rangle = z_\infty^2 \ .$$

After fixing three positions, we are left with an integral over one worldsheet variable

$$A = \mathrm{i} C_A \, Tr(\tilde\Lambda_1 \, \Lambda_2 \, \tilde\Lambda_3 \, \Lambda_4) \, \bar u_1 \gamma^\mu u_2 \, \bar u_3 \gamma_\mu u_4 (2\pi)^4 \delta^{(4)} \left(\sum_{i=1}^{4} k_i \right) \int_0^1 \mathrm{d}x \, x^{-\alpha' \, s - 1} (1-x)^{-\alpha' \, u - 1}$$

$$\times \left[I\left(-\theta_1, 1+\nu_1, x\right) \, I\left(-\theta_2, 1+\nu_2, x\right) \, I\left(1-\theta_3, 1+\nu_3, x\right) \right]^{-\frac{1}{2}} .$$

In order to obtain the full amplitude we need to sum over all possible orderings

$$\begin{aligned} A_{total} = & \, \mathrm{C} \left(Tr(\tilde\Lambda_1 \, \Lambda_2 \, \Lambda_4 \, \tilde\Lambda_3 + Tr(\Lambda_2 \, \tilde\Lambda_1 \, \tilde\Lambda_3 \, \Lambda_4) \right) \int_{-\infty}^0 \mathrm{d}x \, U(x) \\ & + \mathrm{C} \left(Tr(\tilde\Lambda_1 \, \Lambda_2 \, \tilde\Lambda_3 \, \Lambda_4 - Tr(\Lambda_2 \, \tilde\Lambda_1 \, \Lambda_4 \, \tilde\Lambda_3) \right) \int_0^1 \mathrm{d}x \, U(x) \\ & - \mathrm{C} \left(Tr(\tilde\Lambda_1 \, \tilde\Lambda_3 \, \Lambda_2 \, \Lambda_4 + Tr(\tilde\Lambda_1 \, \Lambda_4 \, \Lambda_2 \, \tilde\Lambda_3) \right) \int_1^\infty \mathrm{d}x \, U(x) \ , \end{aligned}$$

with

$$\mathrm{C} = \mathrm{i}\, C_A\, \bar{u}_1 \gamma^\mu u_2\, \bar{u}_3 \gamma_\mu u_4 (2\pi)^4 \delta^{(4)} \left(\sum_{i=1}^{4} k_i \right)$$

and

$$U(x) = x^{-\alpha' s-1}(1-x)^{-\alpha' u-1} \left[I\left(-\theta_1, 1+\nu_1, x\right)\, I\left(-\theta_2, 1+\nu_2, x\right)\, I\left(1-\theta_3, 1+\nu_3, x\right) \right]^{-\frac{1}{2}} .$$

Explicit computation of the traces shows leads to the identities $Tr(\tilde{\Lambda}_1\, \Lambda_2\, \Lambda_4\, \tilde{\Lambda}_3) = Tr(\Lambda_2\, \tilde{\Lambda}_1\, \tilde{\Lambda}_3\, \Lambda_4)$ and all other vanish. Thus the amplitude takes the form

$$A_{total} = 2\mathrm{i} C_A\, \bar{u}_1 \gamma^\mu u_2\, \bar{u}_3 \gamma_\mu u_4 (2\pi)^4 \delta^{(4)} \left(\sum_{i=1}^{4} k_i \right)\; Tr(\tilde{\Lambda}_1\, \Lambda_2\, \Lambda_4\, \tilde{\Lambda}_3) T(\theta_1, \theta_2, \theta_3) \Big), \quad (3.23)$$

with

$$T(\theta_1, \theta_2, \theta_3) = \int_{-\infty}^{0} \mathrm{d}x\, U(x). \quad (3.24)$$

In field theory, the limit $x \to 0$ corresponds to proton decay via a gauge boson, while $x \to -\infty$ describes the proton decay mediated via a Higgs particle, arising from the Yukawa interaction $\mathbf{10}\, \bar{\mathbf{5}}\, \bar{\mathbf{5}}_{\mathbf{H}}$.

Finally we replace the ν's by the angles θ

$$\nu_1 = 2\theta_1 \qquad \nu_2 = 2\theta_2 \qquad \nu_3 = -2 + 2\theta_3$$

and obtain for U

$$U(x) = \frac{\left[I\,(-\theta_1, 1+2\theta_1, x)\,I\,(-\theta_2, 1+2\theta_2, x)\;\,I\,(1-\theta_3, -1+2\theta_3, x)\right]^{-\frac{1}{2}}}{x^{\alpha'\,s-1}(1-x)^{\alpha'\,u-1}}\;. \qquad (3.25)$$

Applying the same procedure for the other sector we obtain

- $\frac{1}{2} < \theta_1 < 1 \qquad \frac{1}{2} < \theta_2 < 1 \qquad \frac{1}{2} < \theta_3 < 1$

  The amplitude

$$\int \prod_{i=1}^{4} \mathrm{d}z_i \;\langle V^{\bar{5}\;*}_{-\frac{1}{2}}(z_1)\,V^{\bar{5}}_{-\frac{1}{2}}(z_2)\,V^{10\,*}_{-\frac{1}{2}}(z_3)V^{10}_{-\frac{1}{2}}(z_4)\rangle\;,$$

  takes the form

$$A_{total} = 2\mathrm{i}C_A\,\bar{u}_1\gamma^{\mu}u_2\,\bar{u}_3\gamma_{\mu}u_4\,(2\pi)^4\delta^{(4)}\left(\sum_{i=1}^{4}k_i\right)\;Tr(\tilde{\Lambda}_1\,\Lambda_2\,\Lambda_4\,\tilde{\Lambda}_3)T(\theta_1,\theta_2,\theta_3)\Big)\;, \qquad (3.26)$$

  T defined in (3.24) and U given by

$$U(x) = x^{-\alpha'\,s-1}(1-x)^{-\alpha'\,u-1}\prod_{I=1}^{3}[I(1-\theta_I, -1+2\theta_I, x)]^{-\frac{1}{2}}\;. \qquad (3.27)$$

The amplitude $\langle V^{10\,*}_{-\frac{1}{2}}\,V^{10}_{-\frac{1}{2}}\,V^{10\,*}_{-\frac{1}{2}}V^{10}_{-\frac{1}{2}}\rangle$

Note that in both cases, $\frac{1}{2} < \theta_{1,2} < 1\,, -\frac{1}{2} < \theta_3 < 0$ and $\frac{1}{2} < \theta_{1,2,3} < 1$, the vertex operators for the matter fields transforming as **10** take the same form. Thus the computation of the amplitude $\langle V^{10\,*}_{-\frac{1}{2}}\, V^{10}_{-\frac{1}{2}}\, V^{10\,*}_{-\frac{1}{2}} V^{10}_{-\frac{1}{2}}\rangle$ is identical for both cases. We use the same correlators stated above except for the one involving the bosonic twist fields, which takes a simpler form, since it involves only one independent angle [62]

$$\langle \sigma_{1-\theta}(z_1)\, \sigma_\theta(z_2)\, \sigma_{1-\theta}(z_3)\, \sigma_\theta(z_4)\rangle = \left(\frac{z_{13}\, z_{24}}{z_{12}\, z_{14}\, z_{23}\, z_{34}}\right)^{\theta(1-\theta)} L^{-\frac{1}{2}}(x) \qquad (3.28)$$

with

$$L(x) = \frac{1}{\sin(\pi\,\theta)}\, {}_2F_1[\theta, 1-\theta, 1, x]\, {}_2F_1[\theta, 1-\theta, 1, 1-x].$$

Plugging in all the correlators and fixing three vertex operator positions we obtain

$$A_{total} = \mathrm{i}\, C'_A \Big(Tr(\tilde{\Lambda}_1\, \Lambda_2\, \tilde{\Lambda}_3\, \Lambda_4) + Tr(\tilde{\Lambda}_1\, \Lambda_4\, \tilde{\Lambda}_3\, \Lambda_2)\Big)\, (2\pi)^4 \delta^{(4)} \left(\sum_{i=1}^{4} k_i\right)$$
$$\bar{u}_1 \gamma^\mu u_2\, \bar{u}_3 \gamma_\mu u_4 \int_0^1 \mathrm{d}x\, x^{-\alpha'\, s-1}\, (1-x)^{-\alpha'\, u-1} \prod_{I=1}^{3} L^{-\frac{1}{2}}(1+\nu_I, x)\ .$$

Finally we replace the ν_I by θ_I and obtain

- $-\frac{1}{2} < \theta_1 < 0 \qquad -\frac{1}{2} < \theta_2 < 0 \qquad \frac{1}{2} < \theta_3 < 1$

$$A_{total} = \mathrm{i} C'_A Tr\Big(\tilde{\Lambda}_1 \Lambda_2 \tilde{\Lambda}_3 \Lambda_4 + \tilde{\Lambda}_1 \Lambda_4 \tilde{\Lambda}_3 \Lambda_2\Big)(2\pi)^4 \delta^{(4)} \left(\sum_{i=1}^{4} k_i\right) \bar{u}_1 \gamma^\mu u_2\, \bar{u}_3 \gamma_\mu u_4\, M(\theta_1, \theta_2, \theta_3) \qquad (3.29)$$

with

$$M(\theta_1,\theta_2,\theta_3) = \int_0^1 \mathrm{d}x \, \frac{x^{-\alpha' s-1}\,(1-x)^{-\alpha' u-1}}{L^{\frac{1}{2}}(1+2\theta_1,x)\,L^{\frac{1}{2}}(1+2\theta_2,x)\,L^{\frac{1}{2}}(-1+2\theta_3,x)} \; . \qquad (3.30)$$

- $\frac{1}{2} < \theta_1 < 1 \qquad \frac{1}{2} < \theta_2 < 1 \qquad \frac{1}{2} < \theta_3 < 1$

$$A_{total} = \mathrm{i}C'_A Tr\left(\tilde{\Lambda}_1\Lambda_2\tilde{\Lambda}_3\Lambda_4 + \tilde{\Lambda}_1\Lambda_4\tilde{\Lambda}_3\Lambda_2\right)(2\pi)^4\delta^{(4)}\left(\sum_{i=1}^{4} k_i\right)\bar{u}_1\gamma^\mu u_2\,\bar{u}_3\gamma_\mu u_4\,M(\theta_1,\theta_2,\theta_3) \qquad (3.31)$$

with

$$M(\theta_1,\theta_2,\theta_3) = \int_0^1 \mathrm{d}x \, \frac{x^{-\alpha' s-1}\,(1-x)^{-\alpha' u-1}}{L^{\frac{1}{2}}(2\theta_1-1,x)\,L^{\frac{1}{2}}(2\theta_2-1,x)\,L^{\frac{1}{2}}(2\theta_3-1,x)} \; . \qquad (3.32)$$

The $\langle V^{10\,*}_{-\frac{1}{2}}\, V^{10}_{-\frac{1}{2}}\, V^{10\,*}_{-\frac{1}{2}} V^{10}_{-\frac{1}{2}}\rangle$ does not involve an Higgs exchange, since couplings of the form $\mathbf{10}\,\mathbf{10}\,\mathbf{5_H}$ are absent due to the $U(1)$ charge conversation [25].

Normalization

In this section we determine the two undetermined constants C_A and C'_A in the string amplitudes computed above. We will use the fact that even in the low energy limit the integrals (3.24), (3.30) and (3.32) are convergent in the limit $x \to 0$, which corresponds to a gauge boson exchange. Factorizing the amplitude into two three point functions, whose result we know, allows us to normalize it. We start with the amplitude $\langle V^{\bar{5}}_{-\frac{1}{2}}{}^{*} V^{\bar{5}}_{-\frac{1}{2}} V^{10}_{-\frac{1}{2}}{}^{*} V^{10}_{-\frac{1}{2}} \rangle$ and turn later to $\langle V^{10}_{-\frac{1}{2}}{}^{*} V^{10}_{-\frac{1}{2}} V^{10}_{-\frac{1}{2}}{}^{*} V^{10}_{-\frac{1}{2}} \rangle$.

The amplitude $\langle V^{\bar{5}}_{-\frac{1}{2}}{}^{*} V^{\bar{5}}_{-\frac{1}{2}} V^{10}_{-\frac{1}{2}}{}^{*} V^{10}_{-\frac{1}{2}} \rangle$

We first examine the limit $x \to 0$ and will see that even in the low energy limit the integral is convergent, due to the special kinematics of this problem.

Limit $x \to 0$

As $x \to 0$ the hypergeometric functions behave like

$$F(a,b,1,x) \to 1 \qquad F(a,b,a+b,1-x) \to \frac{\Gamma(a+b)}{\Gamma(a)\Gamma(b)} \ \ln\left(\frac{\kappa(a,b)}{x}\right) \ , \tag{3.33}$$

with

$$\ln \kappa(a,b) = 2\psi(1) - \psi(a) - \psi(b) \ .$$

Applying (3.33) I takes the form

$$\lim_{x\to 0} I(\theta,\nu,x) = \frac{1}{\sqrt{\pi}} \ \ln\left(\frac{\delta(\theta,\nu)}{x}\right) \ ,$$

where $\ln \delta(\theta,\nu)$ is given by

$$\ln \delta(\theta,\nu) = 2\psi(1) - \frac{1}{2}\psi(\theta) - \frac{1}{2}\psi(1-\theta) - \frac{1}{2}\psi(\nu) - \frac{1}{2}\psi(1-\nu) \ .$$

Therefore even for $s = t = 0$ we obtain for the integral (3.23) a convergent expression in the limit $x \longrightarrow 0$

$$\sim \pi^{3/2} \int_0 \frac{\mathrm{d}x}{x} \ln[1/x]^{-3/2} \ . \tag{3.34}$$

That allows us to normalize the amplitude by factorizing the amplitude in the limit $x \longrightarrow 0$, where it reduces to a product of two three-point functions

$$A_4(k_1, k_2, k_3, k_4) = \frac{\mathrm{i}}{2} \int \frac{\mathrm{d}^7 k \, \mathrm{d}^7 k'}{(2\pi)^7} \frac{\sum_{IJ\mu} A_j^{I\mu}(k_1, k_2, k) A_j^{I\mu}(k_3, k_4, k') \delta(k - k')}{k^2 - \mathrm{i}\epsilon} \ . \tag{3.35}$$

The unusual factor of $\frac{1}{2}$ is introduced to take into account the doubling in the Chan-Paton factors.

The three-point amplitudes describe the exchange of a gauge boson and are given by

$$A^\mu(k_1, k_2, k_3) = \mathrm{i}\, g_{D_6}\, (2\pi)^4 \delta^{(4)} \left(\sum_{i=1}^{3} k_i \right) \bar{u}_1 \gamma^\mu u_2 Tr(\tilde{\Lambda}_1 \, \Lambda_2 \, \Lambda_A) \ . \tag{3.36}$$

Here μ corresponds to the polarization and Λ_A denote the Chan Paton factors of the gauge boson. The latter takes the form

$$\Lambda_A = \begin{pmatrix} \lambda_a & 0 & 0 & 0 \\ 0 & \lambda_a & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} , \tag{3.37}$$

where the λ_a's are the gauge bosons of $U(5)$ which satisfy $Tr(\lambda_a \, \lambda_b) = \frac{1}{2}\delta_{ab}$. The intermediate state is a massless $a - a$ string, which is a gauge boson, that can carry arbitrary momentum p along the directions of the D-brane a orthogonal to the intersection. In these directions we have to integrate over

$$\int \mathrm{d}^3 q \int_0 \mathrm{d}x\, x^{\alpha' q^2 - \alpha' s - 1} = \pi^{3/2} (\alpha')^{-3/2} \int_0 \mathrm{d}x\, x^{-\alpha' s - 1} [\ln(1/x)]^{-3/2}$$

which tells us that the replacement, going from effective field theory in four dimensions to the form of the string integrand near $x = 0$ is no longer

$$\frac{1}{s} \to \alpha' \int_0 \mathrm{d}x\, x^{-\alpha' s - 1} ,$$

but

$$\int \frac{\mathrm{d}^3 q}{q^2 - s} \to \pi^{3/2} (\alpha')^{-1/2} \int_0 \mathrm{d}x\, x^{-\alpha' s - 1} [\ln(1/x)]^{-3/2} . \tag{3.38}$$

Performing the integral on the right hand side of (3.35) and using the replacement (3.38) we obtain

$$\mathrm{i} \frac{g_{D_6}^2 \pi^{5/2}}{2\alpha'^{1/2}} Tr(\tilde{\Lambda}_1 \Lambda_2 \tilde{\Lambda}_3 \Lambda_4) \bar{u}_1 \gamma^\mu u_2 \bar{u}_3 \gamma_\mu u_4 \delta^{(4)} \left(\sum_{i=1}^4 k_i \right) \int_0 \mathrm{d}x\, x^{-\alpha' s - 1} [\ln(1/x)]^{-3/2} . \tag{3.39}$$

This needs to be the same as (3.23) in the limit $x \to 0$

$$2\mathrm{i}\, C_A\, Tr(\tilde{\Lambda}_1 \Lambda_2 \tilde{\Lambda}_3 \Lambda_4) \bar{u}_1 \gamma^\mu u_2 \bar{u}_3 \gamma_\mu u_4 (2\pi)^4 \pi^{3/2} \delta^{(4)} \left(\sum_{i=1}^4 k_i \right) \int_0 \mathrm{d}x\, x^{-s-1} [\ln(1/x)]^{-3/2} ,$$

which leads us with $g_{D_6}^2 = (2\pi)^4 \alpha'^{3/2} g_s$ to the normalization constant C_A

$$C_A = \frac{\pi}{2}\, g_s\, \alpha' \ . \tag{3.40}$$

For the second amplitude one obtains, following the same procedure, the same normalization constant.

The amplitude $\langle V^{10\,*}_{-\frac{1}{2}}\, V^{10}_{-\frac{1}{2}}\, V^{10\,*}_{-\frac{1}{2}} V^{10}_{-\frac{1}{2}} \rangle$

Note that the amplitude is invariant under the exchange of x and $1-x$ if one simultaneously interchanges s and u. Therefore we obtain similar limits for $x \rightarrow 0$ and $x \rightarrow 1$. That is not too surprising taking into account that we expect an exchange of a gauge boson in both limits.

Limit $x \rightarrow 0$ *and* $x \rightarrow 1$

Using (3.33) and taking the low energy limit $s, t \rightarrow 0$ we get for $x \rightarrow 0$

$$\sim \pi^{3/2} \int_0 \frac{\mathrm{d}x}{x} \ \ln[1/x]^{-3/2} \tag{3.41}$$

and a similar result for $x \rightarrow 1$

$$\sim \pi^{3/2} \int^1 \frac{\mathrm{d}x}{1-x} \ \ln[1/(1-x)]^{-3/2} \ . \tag{3.42}$$

Following the same procedure as in the case of the amplitude $\langle {V^{\bar{5}}_{-\frac{1}{2}}}^* V^{\bar{5}}_{-\frac{1}{2}} {V^{10}_{-\frac{1}{2}}}^* V^{10}_{-\frac{1}{2}} \rangle$ we obtain for normalization constant ${C_A}'$

$$C'_A = \pi g_s \, \alpha' \ . \tag{3.43}$$

3.4 Numerical analysis

We want to compute the contribution of the amplitude which arises from the four-Fermi interaction in the low energy effective theory. That means that we take the low energy limit ($s, t, u \to 0$ while we still satisfy $s + t + u = 0$) and subtract the s, t and u poles, if present. For the s-channel we expect a massless gauge boson exchange, which would be indicated by a s-pole. We saw that the integral does not diverge at the s-pole, since we neglected global effects coming from the internal space. Locally, the internal dimensions look like a flat space with infinite volume which leads to a vanishing gauge coupling in four dimensions

$${g_{YM}}^2 \sim \frac{1}{V_{int}} \ . \tag{3.44}$$

Here V_{int} denotes the internal volume and g_{YM} is the gauge coupling in four dimensions. Thus even if we observe a gauge boson exchange, we do not see an s-pole in our effective low energy theory. In the limit $x \to -\infty$, which corresponds to a t-pole, the integral is divergent and in order to obtain the four-Fermi interaction we have to subtract this pole. A detailed discussion of the numerical analysis of the integrals T and M in the amplitudes (3.23),(3.26), (3.29) and (3.31) can be found in appendix

B, where for simplification we set $\theta_1 = \theta_2 = \theta$. Table 3.1 shows the contribution

$-\frac{1}{2} < \theta_1 < 0$ $-\frac{1}{2} < \theta_2 < 0$ $\frac{1}{2} < \theta_3 < 1$			$\frac{1}{2} < \theta_1 < 1$ $\frac{1}{2} < \theta_2 < 1$ $\frac{1}{2} < \theta_3 < 1$		
θ	T	M	θ	T	M
-.40	5.4	10.3	.505	1.5	2.5
-.42	5.1	9.4	.51	2.1	3.5
-.44	4.6	8.3	.52	2.9	4.9
-.46	4.0	6.9	.54	4.0	6.9
-.48	2.9	4.9	.56	4.6	8.3
-.49	2.1	3.5	.58	5.1	9.4
-.495	1.5	2.5	.60	5.4	10.3

Table 3.1: Contribution to T and M for different angles θ

M for the string amplitude $\langle V^{10\,*}_{-\frac{1}{2}} V^{10}_{-\frac{1}{2}} V^{10\,*}_{-\frac{1}{2}} V^{10}_{-\frac{1}{2}} \rangle$ and the contribution T arising from $\langle V^{\bar{5}\,*}_{-\frac{1}{2}} V^{\bar{5}}_{-\frac{1}{2}} V^{10\,*}_{-\frac{1}{2}} V^{10}_{-\frac{1}{2}} \rangle$ for different angles θ. For $\theta = -1/3$ and $\theta = 2/3$ we observe a second massless fermion which indicates that we now have $N = 2$ supersymmetry. Since our world is chiral we choose θ in the ranges, given in table 3.1.

Note, that going from the first sector $-\frac{1}{2} < \theta_1 < 0\,, -\frac{1}{2} < \theta_2 < 0\,, \frac{1}{2} < \theta_3 < 1$ to the second one $\frac{1}{2} < \theta_1 < 1\,, \frac{1}{2} < \theta_2 < 1\,, \frac{1}{2} < \theta_3 < 1$ and replacing θ by $1-\theta$,simultaneously leads to the same results for T and M. This is not too surprising, since the respective vertex operators correspond to the same states if you interchange θ with $1 - \theta$.

3.5 Comparison to Four-Dimensional Field theory

In this section we want to compare the amplitude obtained due to massive string states in string theory with the amplitude on the field theory side. Therefore, we would like to replace all the string theory parameters such as the string coupling g_s or the gauge coupling g_{D6} by appropriate expressions using quantities about which we have some knowledge of, such as M_{GUT} and α_{GUT}. We follow closely the analysis of [56].

The action for the gauge fields living on the $D6$-branes is

$$\frac{1}{4g_{D_6}^2}\int \mathrm{d}^7x\, \mathrm{Tr} F_{ij}F^{ij}.$$

where the F_{ij} is the Yang-Mills field strength and Tr denotes the trace in the fundamental representation of $U(N)$. After compactification on $R \times Q$ the action becomes

$$\frac{V_Q}{4g_{D_6}^2}\int \mathrm{d}^4x\, \mathrm{Tr} F_{ij}F^{ij},$$

where V_Q is the volume of Q. Keeping in mind the usual convention $Tr(Q_aQ_b) = \frac{1}{2}\delta_{ab}$ we finally obtain for the action

$$\frac{V_Q}{8g_{D_6}^2}\int \mathrm{d}^4x\, \mathrm{Tr} F_{ij}F^{ij}. \tag{3.45}$$

On the other hand, the GUT action is given by

$$\frac{1}{4g_{GUT}^2}\int \mathrm{d}^4x\, \mathrm{Tr} F_{ij}F^{ij} \ , \tag{3.46}$$

where g_{GUT} is the GUT coupling. Comparing (3.45) and (3.46), along with $g_{D_6}^2 = (2\pi)^4 g_s \alpha'^{3/2}$ [5] and $\alpha_{_{GUT}} = g_{_{GUT}}^2/(4\pi)$, leads to the identification

$$\alpha' = \left(\frac{\alpha_{_{GUT}}\, V_Q}{(2\pi)^3\, g_s}\right)^{2/3} \ . \tag{3.47}$$

The volume V_Q enters into the running of the $SU(3) \times SU(2) \times U(1)$ gauge coupling from high energies to low energies. Approximately, one can say that $V_Q^{-1/3}$ plays the role of the mass scale unification M_{GUT} in four dimensions. In order to obtain the exact relation between them one needs to compute the one loop threshold correction to the gauge coupling, which was done for M-theory on a manifold of G_2 holonomy [63][4]

$$V_Q = \frac{L(Q)}{M_{GUT}^3} \ , \tag{3.48}$$

where $L(Q)$ is a topological invariant, the Ray-Singer torsion. In [56] it is argued that this relation holds true in Type IIA string theory and thus we finally obtain

$$\alpha' = \left(\frac{\alpha_{GUT}\, L(Q)}{(2\pi)^3\, g_s\, M_{GUT}^3} \right)^{2/3} \ . \tag{3.49}$$

We would like to replace all the string parameters in the amplitudes (3.23) and (3.23) in terms of four dimensional field theory quantities. Unfortunately, equation (3.49) still includes two string parameters $L(Q)$ and g_s. The Ray-Singer torsion depends crucially on the compact space and takes for simple lens spaces values around 8 [63]. In order to neglect higher order loop amplitudes the string coupling g_s is better smaller than 1. On the other hand we are interested in the largest possible contribution to the enhancement and set therefore g_s approximately to 1.

Field theory amplitude

[4]An explicit computation for the one loop threshold correction in type IIA string theory was performed in [64], which leads in the limit $g_s \to 1$ to an equivalent relation.

After relating the string parameters to four dimensional field theory constants, of which we have some experimental knowledge, we now recall the analysis of proton decay in the $SU(5)$ GUT model [5]. This treatment closely follows [45].

The kinetic energy for an $SU(5)$ gauge theory, involving the gauge field A, the fermionic field $\psi_{\bar{5}}$, which transforms as $\bar{\mathbf{5}}$, and the fermionic field ψ_{10} transforming as $\mathbf{10}$ under the $SU(5)$ takes the form

$$T = \frac{1}{4g^2_{GUT}} Tr(F^2(A)) + \mathrm{i}\bar{\psi}_{\bar{5}}\gamma^\mu D_\mu \psi_{\bar{5}} + \mathrm{i}\bar{\psi}_{10}\gamma^\mu D'_\mu \psi_{10} \tag{3.50}$$

with

$$D_\mu {\psi_{\bar{5}}}^a = \partial_\mu {\psi_{\bar{5}}}^a - \frac{\mathrm{i}\, g_{GUT}}{\sqrt{2}} (A_\mu)^a_b \, {\psi_{\bar{5}}}^b$$

and

$$D'_\mu {\psi_{10}}^{ab} = \partial_\mu {\psi_{10}}^{ab} - \frac{\mathrm{i}\, g_{GUT}}{\sqrt{2}} (A_\mu)^a_c \, {\psi_{10}}^{cb} - \frac{\mathrm{i}\, g_{GUT}}{\sqrt{2}} (A_\mu)^b_d \, {\psi_{10}}^{ad} \; .$$

By explicitly using the antisymmetry of ψ_{10}, the latter can be simplified to

$$D'_\mu {\psi_{10}}^{ab} = \partial_\mu {\psi_{10}}^{ab} - \frac{\mathrm{i}\, 2 g_{GUT}}{\sqrt{2}} (A_\mu)^a_c \, {\psi_{10}}^{cb}.$$

[5] As done usually we neglect because of the weakness of the Yukawa couplings to light fermions the Higgs mediated Proton decay.

The gauge field A can be displayed as a 5×5 matrix

$$A_\mu = \left(\begin{array}{ccc|cc} & & & X^C_{1\ \mu} & Y^C_{1\ \mu} \\ & \frac{1}{\sqrt{2}} \sum_a G^a_\mu \lambda^a & & X^C_{2\ \mu} & Y^C_{2\ \mu} \\ & & & X^C_{3\ \mu} & Y^C_{3\ \mu} \\ --- & --- & --- & --- & --- \\ X_{1\mu} & X_{2\mu} & X_{3\mu} & \frac{W^3_\mu}{\sqrt{2}} & W^+_\mu \\ Y_{1\mu} & Y_{2\mu} & Y_{3\mu} & W^-_\mu & -\frac{W^3_\mu}{\sqrt{2}} \end{array} \right) + \frac{B_\mu}{\sqrt{30}} \begin{pmatrix} -2 & & & & \\ & -2 & & & \\ & & -2 & & \\ & & & 3 & \\ & & & & 3 \end{pmatrix},$$

where the λ^a are the Gell-Mann matrices, the G^a_μ denote the gluon fields of $SU(3)$ and W^+_μ, W^-_μ, W^3_μ, B_μ are the bosons of the $SU(2) \times U(1)$. The X and Y are the new gauge bosons that are contained in $SU(5)$ and do not occur in the standard model. The exchange of these new gauge bosons leads to Baryon-Lepton number violating processes and therefore allows proton decay.

To make contact to the standard model the $SU(5)$ needs to be broken, which will be achieved by giving the Higgs field, which transforms under the 24-dimensional adjoint representation of $SU(5)$ an expectation value. This generates a mass M_X of order of 10^{16} Gev for the gauge bosons X and Y.

From (3.50) one can easily deduce the effective four-Fermi interactions which lead to proton decay. Ignoring mixing effects as well as second and third families one obtains for the

$$L_{eff} = \frac{g_{GUT}^2}{2\,M_X^2} \Big(\varepsilon_{\alpha\,\beta\,\gamma}\, \bar{u}_L^{C\gamma}\, \gamma^\mu\, u_L^\beta \Big) \Big(2\bar{e}_L^+\, \gamma_\mu\, d_L^\alpha + \bar{e}_R^+\, \gamma_\mu\, d_R^\alpha \Big) \;, \tag{3.51}$$

where the first factor arises from a $\mathbf{10}^*\,\mathbf{10}\,\mathbf{10}^*\,\mathbf{10}$ interaction and the second factor from a $\bar{\mathbf{5}}^*\,\bar{\mathbf{5}}\,\mathbf{10}^*\,\mathbf{10}$ interaction .

Comparison

This result (3.51) we want to compare with the string theory contribution. In order to do that we turn on Wilson lines, that break the $SU(5)$ gauge group into the standard model ones. Assuming such a mechanism of symmetry breaking exist we compute the traces of (3.23) and (3.29) only for entries which lead to proton decay. One obtains for (3.23) and (3.26)

$$A_{total}^{\bar{5}\bar{5}1010} = \mathrm{i}\,(2\pi)^4 \delta^{(4)} \left(\sum_{i=1}^{4} k_i \right) \, \pi g_s \alpha' \Big(\varepsilon_{\alpha\,\beta\,\gamma}\, \bar{u}_L^{C\gamma}\, \gamma^\mu\, u_L^\beta \Big) \Big(\bar{e}_R^+\, \gamma_\mu\, d_R^\alpha \Big) T(\theta) \tag{3.52}$$

and for (3.29) and (3.31)

$$A_{total}^{10101010} = 2\mathrm{i}\,(2\pi)^4 \delta^{(4)} \left(\sum_{i=1}^{4} k_i \right) \, \pi g_s\, \alpha' \Big(\varepsilon_{\alpha\,\beta\,\gamma}\, \bar{u}_L^{C\gamma}\, \gamma^\mu\, u_L^\beta \Big) \Big(\bar{e}_L^+\, \gamma_\mu\, d_L^\alpha \Big)\, M(\theta)\,. \tag{3.53}$$

Comparing the string theory proton decay rate with the one from four dimensional gauge theory one obtains

$$\frac{\Gamma_{ST}(p \to \pi^0\, e^+)}{\Gamma_{FT}(p \to \pi^0\, e^+)} = \left(\frac{g_s^{1/3}\, L(Q)^{2/3}}{8\pi^2 \alpha_{GUT}^{1/3}} \right)^2 \left(\frac{M_X}{M_{GUT}} \right)^4 \left(\frac{T^2 + 4M^2}{4} \right) . \tag{3.54}$$

Most recent calculations [51] for the proton decay mediated via gauge bosons in an $SU(5)$-GUT model gave the lifetime τ_p^{FT} in dependence of gauge boson mass M_X and α_{GUT}

$$\tau_p^{FT} = 1.6 \times 10^{36} years \left(\frac{.04}{\alpha_{GUT}} \right)^2 \left(\frac{M_X}{2 \times 10^{16} GeV} \right)^4 . \tag{3.55}$$

This leads with the values $M_X = M_{GUT} = 2 \times 10^{16} GeV$ and $\alpha_{GUT} = 0.04$ to a proton lifetime of $1.6 \times 10^{36} years$. The present lower bound on the proton lifetime for $p \to \pi^0\, e^+$ is $1.6 \times 10^{33} years$ [65] and even the next generation proton decay experiments, based on underground water Cherenkov detectors will reach a lower bound close to $10^{35} years$ [66]. Therefore in the near future, unless there is an enhancement to the proton decay amplitude, we will not observe the proton decay via gauge boson exchange. Using (3.54) and (3.55) the proton lifetime in the considered type IIA string models is

$$\tau_p^{ST} \approx 1.6 \times 10^{36} years \frac{54^2}{L^{4/3}(Q)\, g_s^{2/3}\, (T^2 + 4M^2)} \left(\frac{.04}{\alpha_{GUT}} \right)^{4/3} \left(\frac{M_{GUT}}{2 \times 10^{16} GeV} \right)^4 , \tag{3.56}$$

where $\frac{54^2}{L^{4/3}(Q)\, g_s^{2/3}\, (T^2+4M^2)}$ is the string enhancement factor. Note that in (3.56) the heavy gauge boson mass M_X, which is model dependent, is absent and the proton lifetime depends only on M_{GUT}. We also observe an anomalous power of α_{GUT} in (3.56) indicating the stringy nature of the enhancement.

Let us examine the enhancement factor $\frac{54^2}{L^{4/3}(Q)\, g_s^{2/3}\,(T^2+4M^2)}$. As already mentioned earlier the Ray-Singer torsion is around 8 for lens spaces with small fundamental group. The string coupling takes values between 0 and 1, but in order to obtain the largest possible enhancement to the proton decay amplitude we assume it is approximately 1. Table 3.1 shows that M ranges between 5 and 10, while $T \approx 0.6 \times M$, leading with the numerical four-dimensional $SU(5)$ supersymmetric values $M_{GUT} = 2 \times 10^{16} GeV$ and $\alpha_{GUT} = 0.04$ to a proton lifetime $\tau_p^{ST} = (0.6 - 2.6) \times 10^{36} years$. We see that although there is in addition to the contribution to the four-Fermi interaction which in field theory are due to gauge boson exchange, there is also a contribution due to terms that in field theory arise from Higgs particle exchange, the total string contribution is not large enough to lead to a considerable enhancement in the proton decay rate.

The dimension six operators $\mathbf{\bar{5}}^*\mathbf{\bar{5}10}^*\mathbf{10}$ have in contrast to the operators $\mathbf{10}^*\mathbf{1010}^*\mathbf{10}$ a second proton decay mode; they lead in addition to the decay mode $p \rightarrow \pi^0\, e^+$ also to $p \rightarrow \pi^+ \bar{\nu}$. Plugging in the respective entries in (3.23) leading to the mode $p \rightarrow \pi^+ \bar{\nu}$ one obtains

$$A_{total}^{\bar{5}\bar{5}1010} = \mathrm{i}\, (2\pi)^4 \delta^{(4)} \left(\sum_{i=1}^{4} k_i \right) \pi g_s \alpha' \Big(\varepsilon_{\alpha\,\beta\,\gamma}\, \bar{u}_L^{C\gamma}\, \gamma^\mu\, d_L^\beta \Big) \Big(\bar{\nu}_R^C\, \gamma_\mu\, d_R^\alpha \Big) T(\theta) \ . \tag{3.57}$$

Within the field theory the effective interaction

$$L_{eff} = \frac{g_{GUT}^2}{2\, M_X^2} \Big(\varepsilon_{\alpha\,\beta\,\gamma}\, \bar{u}_L^{C\gamma}\, \gamma^\mu\, d_L^\beta \Big) \Big(\bar{\nu}_R^C\, \gamma_\mu\, d_R^\alpha \Big) \ , \tag{3.58}$$

the ratio between the proton decay rates is given by

$$\frac{\Gamma_{ST}(p \to \pi^+ \bar{\nu})}{\Gamma_{FT}(p \to \pi^+ \bar{\nu})} = \left(\frac{g_s^{1/3} \, L(Q)^{2/3}}{16\pi^2 \alpha_{GUT}^{1/3}} \right)^2 \left(\frac{M_X}{M_{GUT}} \right)^4 T^2 \ . \tag{3.59}$$

For this decay mode the string enhancement to the proton decay rate is even smaller than for the mode $p \to \pi^0 \, e^+$ due to the absence of the $\mathbf{10}^* \mathbf{10} \mathbf{10}^* \mathbf{10}$ interaction term. For the same choice of parameter as above (in addition we assume that $M_X = M_{GUT}$) the ratio (3.59) takes values between 0.2 and 0.8.

3.6 Summary

In this chapter we computed the local, string contribution to the proton decay rate for supersymmetric SU(5) GUT's of D6-brane models on Type IIA orientifolds by calculating explicitly string amplitude contribution to the dimension six operators. If the compactification volume is larger than the string scale, world-sheet instanton effects are negligible and the local contribution is the dominant one. In the computation presented, we assumed that the matter fields $\bar{\mathbf{5}}$ and $\mathbf{10}$ are located at the same intersections on top of each other, and thus the leading string amplitude contributions have no suppressions from area factors. In this case the amplitudes give the largest possible contribution to the proton decay rate. In contrast to the authors [56], who only considered the amplitude $\langle \mathbf{10}^* \mathbf{10} \mathbf{10}^* \mathbf{10} \rangle$, we also included the explicit calculation of the string amplitude for $\langle \bar{\mathbf{5}}^* \bar{\mathbf{5}} \mathbf{10}^* \mathbf{10} \rangle$ operators.

After relating the string theory result to the field theory computations we obtain for the proton lifetime in type IIA string theory models

$$\tau_p^{ST} \approx 1.6 \times 10^{36} years \frac{54^2}{L^{4/3}(Q)\, g_s^{2/3}\, (T^2 + 4M^2)} \left(\frac{.04}{\alpha_{GUT}}\right)^{4/3} \left(\frac{M_{GUT}}{2 \times 10^{16} GeV}\right)^4 , \tag{3.60}$$

which has an anomalous power of α_{GUT} indicating the string effects. The string enhancement factor depends on the Ray-Singer torsion, the string coupling g_s and the numerical quantities M and T (see table 3.1). Choosing common values for $L(Q)$, assuming that the string coupling g_s is approximately 1 and plugging in the computed numerical quantities M and T the proton lifetime (3.60) is $\tau_p^{ST} = (0.6 - 2.6) \times 10^{36} years$.

Chapter 4

Novel stringy instanton effects

During the last two years there has been some progress towards a better understanding of non-perturbative effects in supersymmetric four-dimensional string compactifications on Calabi-Yau orientifolds. As was realized in [67, 68, 69, 70], D-brane instantons can induce couplings between open string fields which are perturbatively forbidden because they violate global $U(1)$ selection rules. These effects are intrinsically stringy in that they cannot be described by conventional gauge instantons. For type IIA orientifolds with intersecting D6-branes the relevant class of instantons is given by Euclidean $D2$-brane instantons, short $E2$ instantons, wrapping special Lagrangian three-cycles of the internal Calabi-Yau space [67, 69].

In general the gauge group $U(N_a)$, carried by N_a coincident D-branes, contains an anomalous $U(1)_a$ which becomes massive via the generalized Green-Schwarz mechanism and survives as a global perturbative symmetry. It is due to this $U(1)$ selection rule that particular phenomenologically important couplings are absent in intersecting brane worlds. These include Majorana masses for the right-handed neutrino, μ-terms for the MSSM Higgs sector or Yukawa-couplings of type $\mathbf{10}\cdot\mathbf{10}\cdot\mathbf{5}_H$ in $SU(5)$-like GUT models. On the other hand, in the dual strongly coupled description in terms of M-theory compactified on G_2-manifolds the $U(1)_a$ decouples completely. There is therefore no associated selection rule and no obstruction for the above mentioned couplings to exist. The resolution to this puzzle is given by $U(1)_a$ breaking non-perturbative terms in the type IIA picture.

Indeed, from the axionic shift symmetries under the abelian symmetries induced by the Chern-Simons couplings of the N_a $D6_a$-branes one finds that an instanton could transforms under the $U(1)_a$ in such a way that the full coupling

$$\prod_i \Phi_{a_i b_i}\, e^{-S_{E2}} \tag{4.1}$$

is invariant again. In this way an instanton with the appropriate zero mode structure has the potential to generate perturbatively forbidden couplings.

In the first part of this chapter we review the generalized Green-Schwarz mechanism and demonstrate that the anomalous $U(1)$ symmetries become massive and survive as a global symmetry. Furthermore we show that an $E2$-instanton can be charged under these $U(1)$'s and thus induce non-perturbative couplings of the form (4.1). In section 4.3 we give a detailed account of the zero mode structure of generic $E2$-instantons where we also present their respective vertex operators needed later in chapter 5. We conclude by giving a general prescription of how to compute instanton induced effective couplings focussing on non-perturbative superpotential corrections, while relegating a concrete calculation to chapter 5.

4.1 Green-Schwarz mechanism

In section 2.3 we have shown that the non-abelian gauge anomalies of all $SU(N_a)$ factors vanish while the mixed anomalies with abelian and non-abelian gauge fields naively do not cancel among the charged fields displayed in table 2.1. They give rise to an anomaly between stack a and b of the form

$$\mathcal{A}^{abb} \sim N_a[-\pi_a + \pi_a'] \circ \pi_b. \tag{4.2}$$

It has been shown in [71] that these anomalies can be cancelled by the generalized Green-Schwarz mechanism, taking into account couplings to the RR three- and five-form

$$\int_{R^{3,1}\times\pi_a} C_3 \wedge \mathrm{Tr}(F_a \wedge F_a) \qquad\qquad \int_{R^{3,1}\times\pi_a} C_5 \wedge \mathrm{Tr}(F_a). \tag{4.3}$$

Here C_3 and C_5 denote the RR three- and five-form, respectively, and F_a denotes the gauge field living on the D6 brane.

Let us introduce the Poincaré dual (B^I, A^I) to the symplectic basis (α_I, β_I) introduced in (2.1)

$$\int_{\alpha_I} A^J = \delta_{IJ} \qquad\qquad \int_{\beta_I} B^J = -\delta_{IJ} \tag{4.4}$$

and expand the RR three- and five-form in terms of (B_I, A_I)

$$C_5 = (X_I^2 A^I - Y_I^2 B^I) \qquad C_3 = (X_I^0 A^I - Y_I^0 B^I)\ . \tag{4.5}$$

Here X_I^0, Y_I^0 are the four-dimensional axions and X_I^2, Y_I^2 denote the 2-forms. Note that in four dimensions (X_I^2, Y_I^2) is Hodge dual to $(-Y_I^0, X_I^0)$ due to the duality relation between C_5 and C_3.

Integrating (4.3) over the internal space and making use of (2.2), (2.3) and (4.5) one obtains

$$\int_{R^{3,1}\times\pi_a} C_3 \wedge \mathrm{Tr}(F_a \wedge F_a) = \int_{R^{3,1}} \left(n_I^a\, X_I^0 + m_I^a\, Y_I^0\right) \wedge \mathrm{Tr}(F_a \wedge F_a) \tag{4.6}$$

and

$$\int_{R^{3,1}\times\pi_a} C_5 \wedge \mathrm{Tr}(F_a) = N_a \int_{R^{3,1}} \left(n_I^a\, X_I^2 + m_I^a\, Y_I^2\right) \wedge \mathrm{Tr}(F_a). \tag{4.7}$$

The factor N_a in (4.7) is due to the normalization of the $U(1)_a$ generator. An additional contribution arises from the orientifold image a'

$$\int_{R^{3,1}\times\pi'_a} C_3 \wedge \mathrm{Tr}(F_a \wedge F_a) = \int_{R^{3,1}} \left(n_I^a\, X_I^0 - m_I^a\, Y_I^0\right) \wedge \mathrm{Tr}(F_a \wedge F_a) \tag{4.8}$$

and

$$\int_{R^{3,1}\times\pi_a} C_5 \wedge \mathrm{Tr}(F_a) = -N_a \int_{R^{3,1}} \left(n_I^a\, X_I^2 + m_I^a\, Y_I^2\right) \wedge \mathrm{Tr}(F_a). \tag{4.9}$$

Here we used that under the orientifold action the $U(1)_a$ field strength transforms $F_a \to -F_a$. Thus the total dimensional reduced CS-coupling to the RR three- and

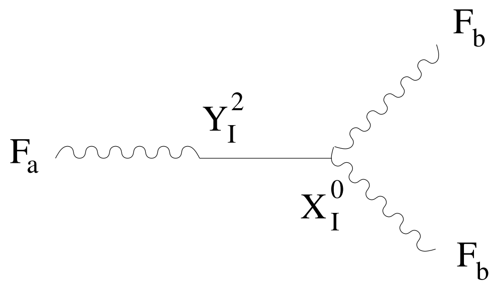

Figure 4.1: Green-Schwarz mechanism.

five-form takes the form

$$\int_{D6_a} C_3 \wedge \mathrm{Tr}(F_a \wedge F_a) = \int_{R^{3,1}} 2n_I^a \, X_I^0 \wedge \mathrm{Tr}(F_a \wedge F_a) \tag{4.10}$$

and

$$\int_{D6_a} C_5 \wedge \mathrm{Tr}(F_a) = N_a \int_{R^{3,1}} 2m_I^a \, Y_I^2 \wedge \mathrm{Tr}(F_a) \ . \tag{4.11}$$

As depicted in figure 4.1 these two couplings (4.10) and (4.11) give contributions to the mixed anomaly

$$\mathcal{A}^{abb} \quad \sim \quad 4N_a n_I^b \, m_I^a \sim N_a[\pi_a - \pi_a'] \circ \pi_b \tag{4.12}$$

and indeed have the required structure to cancel the anomaly (4.2).

Note that an important side effect of the Green-Schwarz mechanism is that most of the U(1) gauge fields become massive via the coupling (4.11). The surviving massless anomaly free $U(1)$ gauge fields are given by the kernel of the $K \times (h_{21} + 1)$

matrix

$$M_{aI} = 2\, N_a\, m_I^a = N_a\, (\pi_a - \pi'_a) \ \ . \tag{4.13}$$

Here K denotes the number of stacks in the D-brane configuration. The massive $U(1)$ symmetries still survive as global symmetries in the low energy effective action and thus put severe constraints on the allowed couplings. Due to these $U(1)$ selection rules that particular desired couplings, such as Majorana-terms for right-handed neutrinos are perturbatively forbidden.

Secondly the Green-Schwarz mechanism implies that under the gauge transformation $A_a^\mu \rightarrow A_a^\mu + \partial^\mu \Lambda_a$ the axions transform as

$$X_I^0 \rightarrow X_I^0 + 2\pi\, N_a\, 2m_a^I\, \Lambda_a \ . \tag{4.14}$$

Let us try to motivate that in a little more detail. Look at the action of some scalar field b^0 which is given by the standard kinetic energy plus a coupling to an abelian field[1]

$$\int_{R^{3,1}} \partial_\mu b^0\, \partial^\mu b^0 + Q_m b^0 \partial_\mu A_m^\mu \tag{4.15}$$

In order to leave the action (4.15) invariant under the transformation of $A_m^\mu \rightarrow A_m^\mu + \partial^\mu \lambda_m$ the scalar field has to transform as

$$b^0 \rightarrow b^0 + \frac{Q_m}{2} \lambda_m \ . \tag{4.16}$$

[1]Note that the coupling term is exactly of the structure of (4.7) after applying Hodge duality.

Thus in case the $U(1)$ gauge symmetry gets massive via the generalized Green-Schwarz mechanism, and survives just as global symmetry the continuous axionic shift symmetry of RR axions (see equation (4.14)) is broken to a discrete one. In the next section we will see an $E2$ instanton with the right charge under the global U(1) symmetries can induce perturbative forbidden couplings.

4.2 E2-instanton

Let us briefly summarize the results of the previous section. We have shown that the anomalies due to the chiral spectrum in table 2.1 get cancelled by additional couplings of the field strength to the RR three- and five-forms. Via these couplings $U(1)$ gauge fields become massive and $U(1)$ gauge symmetry is broken to a global one. On the perturbative level couplings have to obey these global $U(1)$ selection rules, forbidding various desired couplings. Furthermore, we have shown that the RR axions transform under these $U(1)$'s. In the sequel we show that under specific circumstances an instanton can generate perturbatively forbidden couplings.

In type IIA the instantons in questions are so-called $E2$ instantons wrapping a three-cycle in the internal manifold and are point-like in spacetime. The Euclidian action of such an object includes the Dirac-Born-Infeld- as well as the Chern-Simons action[2]

$$S^{cl}_{E2} = \frac{2\pi}{l_s^3}\left(\frac{1}{g_s} Vol_{\Xi} - i\int_{\Xi} C_3\right). \tag{4.17}$$

Note that in the path integral the instanton action appears exponentially and thus

[2]The appearance of the minus sign in front of the CS term will be explained in section 4.3.

with the transformation behavior (4.14) under a $U(1)_a$ gauge symmetry we obtain

$$e^{-S_{E2}^{Cl}} \to e^{iQ_a(E2)}\, e^{-S_{E2}^{Cl}} \tag{4.18}$$

where Q_a is given by

$$Q_a(E2) = \Xi \circ [\pi_a - \pi_a'] \ . \tag{4.19}$$

Let us exemplify the instantonic generation of a perturbatively forbidden coupling. Assume we have a superfield positively charged under the global $U(1)_a$. While the combination $\Phi_{1_a}\,\Phi_{1_a}$ is neutral under all other gauge symmetries it is forbidden since it breaks the global $U(1)_a$. A single instanton wrapping the three-cycle Ξ with charge

$$Q_a = -2 = \Xi \circ [\pi_a - \pi_a'] \tag{4.20}$$

compensates for the $U(1)_a$ charges arising from the matter fields Φ_{1_a}. Thus it has the potential to induce the coupling

$$e^{-S_{E2}^{Cl}}\, \Phi_{1_a}\, \Phi_{1_a} \tag{4.21}$$

which due to the first factor in (4.17) depends on the complex structure moduli.

Generalizing this, an instanton with a right charge under all the global $U(1)$'s can generate superpotential couplings of the form

$$e^{-S_{E2}^{Cl}} \prod_i \Phi_{a_i\,b_i}. \tag{4.22}$$

Let us emphasize that due to the suppression factor in (4.22) these non-perturbative effects can give a natural stringy explanation for hierarchies which from a field theory point of view are only poorly understood.

So far we only gave selection rules for generating instanton induced couplings based on abelian invariance. These turn out to be just necessary conditions. Further restrictions whether an instanton gives corrections to the effective action or not requires a genuine instanton computation, crucially depending on the instanton zero mode structure. These will be subject to the next section.

4.3 Instanton zero modes

As we will see in section 4.4 the computation of D-brane instanton effects hinges upon a careful analysis of the instanton zero modes. In the following we perform that analysis, where we assume that the $E2$-instanton wraps a special Lagrangian three-cycle Ξ in the internal manifold, so that it is point-like in four-dimensional space-time. In general, if Ξ is not invariant under the orientifold action we have to consider also the image $E2$ instanton wrapping Ξ'.

For this topological sector to correspond to a local minimum of the full string action, Ξ has to be volume minimizing in its homology class, i.e. special Lagrangian. Due to its localization in the external four dimensions, Ξ respects at most two of the four supercharges $Q_\alpha, \overline{Q}_{\dot{\alpha}}$ of the $\mathcal{N}=1$ SUSY algebra preserved by the orientifold and the $D6$-branes. This is the situation we are interested in when computing corrections to the superpotential.

Recall that the concrete $\mathcal{N}=1$ subalgebra preserved by a $D6$-brane wrapping the sLag π is determined schematically by the phase ϕ appearing in

$$\mathrm{Im}(e^{i\pi\phi}\Omega|_\pi) = 0. \tag{4.23}$$

For orientifolds we require $\theta = 0$ mod2 since this is the value corresponding to the $\mathcal{N}=1$ algebra preserved by the orientifold planes. Standard arguments taking into account the localization of the $E2$-brane in the external dimensions show that if it wraps a cycle Ξ with $\phi_\Xi = 0$ it preserves the supercharges Q_α, $\overline{Q}'_{\dot{\alpha}}$ and breaks Q'_α, $\overline{Q}_{\dot{\alpha}}$ where the unprimed and primed quantities Q_α, $\overline{Q}_{\dot{\alpha}}$ and Q'_α, $\overline{Q}'_{\dot{\alpha}}$ generate the $\mathcal{N}=1$ subalgebra preserved by the orientifold and the one orthogonal to it, respectively.

$\mathcal{N}=1$	$\mathcal{N}=1'$
θ^α	τ^α
$\overline{\theta}^{\dot{\alpha}}$	$\overline{\tau}^{\dot{\alpha}}$

Table 4.1: Universal fermionic zero modes $\theta^\alpha, \overline{\tau}^{\dot{\alpha}}$ $(\tau^\alpha, \overline{\theta}^{\dot{\alpha}})$ of an (anti-)instanton associated with the breaking of the $\mathcal{N}=1$ SUSY algebra preserved by the orientifold and its orthogonal complement $\mathcal{N}=1'$.

To restore supersymmetry in the topological sector containing the $E2$-brane we have to integrate all amplitudes over the corresponding Goldstone fermions $\overline{\theta}_{\dot{\alpha}}$ and τ_α associated with the violation of $\overline{Q}_{\dot{\alpha}}$ and Q'_α, see table 4.1. Depending on the details of the orientifold action (see section 5.1) the τ^α can be projected out provided the cycle Ξ is invariant, $\Xi = \Xi'$. In this case the associated topological sector contributes to the anti-holomorphic superpotential involving anti-chiral superfields. We call the corresponding sector the anti-instanton sector. By contrast, the instanton, given by $\theta_\Xi = 1$ mod2, preserves the anti-chiral supercharges $\overline{Q}_{\dot\alpha}$ and violates Q_α, thus contributing to the holomorphic superpotential.

In the sequel it will be useful to consider only 'aligned' cycles Ξ with $\phi_\Xi = 0$ mod2 wrapped by the (anti-)instanton. At the CFT level, the fact that the instanton is actually 'anti-aligned' w.r.t. the $D6$-branes internally is taken into account by projecting the spectrum between the E2 and the $D6$-branes of the model onto its GSO-odd part. From the worldvolume perspective, the two objects clearly carry opposite charge under the RR three-form C_3 coupling to the worldvolume. The classical part of the Euclidean (anti-)instanton action appearing as $e^{-S_{E2}}$ in corresponding F-term couplings reads

$$S_{E2}^{Cl} = \frac{2\pi}{\ell_s^3}\left(\frac{1}{g_s}\mathrm{Vol}_\Xi \mp i\int_\Xi C_3\right), \tag{4.24}$$

with the (lower) upper sign corresponding to (anti-)instantons[3].

The zero modes of the (anti-)instanton can be computed in setups where the $\mathcal{N} = (2,2)$ CFT describing the internal sector is known exactly. The general form of the various vertex operators can be found in [67]. For the sake of concreteness and as preparation for our explicit computations in chapter 5.2, we specialize here to the case that the $D6$- and the $E2$-branes wrap factorizable three-cycles of toroidal orientifolds[4].

[3]We define the string length as $\ell_s = \sqrt{2\pi\alpha'}$.

[4]A detailed summary of the covariant open string quantization between two $D6$-branes in this

One distinguishes between two kinds of instanton zero modes corresponding to whether or not they are charged under the gauge groups on the $D6$-branes.

We start by analyzing the uncharged $E2-E2$ sector. It always comprise the universal four bosonic Goldstone zero modes x^μ due to the breakdown of four-dimensional Poincaré invariance. Their vertex operator (in the (-1)-ghost picture) is given by

$$V_{x_E^\mu}(z) = \Omega_{E2E2}\, x_E^\mu\, \frac{1}{\sqrt{2}} \psi_\mu(z)\, e^{-\varphi(z)}. \tag{4.25}$$

Here Ω_{E2E2} denotes the Chan Paton factor. The polarization x_E^μ carries no mass dimension, corresponding to a field in $d=0$ dimensions. It is related to the position x_0^μ of the instanton in external spacetime via $x_E^\mu = x_0^\mu/\ell_s$. The factor $\frac{1}{\sqrt{2}}$ accounts for the fact that $\psi_\mu(z)$ are real fields. In this and all following vertex operators we absorb the open string coupling into the polarization (see section 4.4 for a detailed discussion of this point).

Generically, for instantons away from the orientifold fixed plane, these come with four fermionic zero modes θ^α and $\overline{\tau}^{\dot\alpha}$ [73, 74, 75, 76] . As mentioned earlier this reflects the fact that the instanton breaks half of the eight supercharges preserved by the Calabi-Yau manifold away from the orientifold fixed plane. Due to its localization in the four external dimensions, an instanton breaks one half of the $\mathcal{N}=1$ supersymmetry preserved by the orientifold and one half of its orthogonal complement inside the $\mathcal{N}=2$ supersymmetry algebra respected by the Calabi-Yau. As displayed in table 4.1, the θ^α are the Goldstinos associated with the breakdown of the first $\mathcal{N}=1$

context can be found, e.g., in [72] and in A.

supersymmetry, while the $\overline{\tau}^{\dot{\alpha}}$ are associated with the orthogonal $\mathcal{N} = 1'$ algebra. Their vertex operator (in the $(-1/2)$-ghost picture) is given by the

$$\begin{aligned} V_\theta(z) &= \Omega_{E2E2}\, \theta_\alpha\, S^\alpha(z) \prod_{I=1}^{3} e^{\frac{i}{2} H_I(z)} e^{-\varphi(z)/2}, \qquad q = 3/2, \\ V_{\overline{\tau}}(z) &= \Omega_{E2E2}\, \overline{\tau}_{\dot{\alpha}}\, S^{\dot{\alpha}}(z) \prod_{I=1}^{3} e^{-\frac{i}{2} H_I(z)} e^{-\varphi(z)/2}, \qquad q = -3/2. \end{aligned} \tag{4.26}$$

Here $H_I(z)$ denotes the bosonization of the complexified internal fermions Ψ^I and S^α ($S^{\dot{\alpha}}$) are the left-(right-)handed four-dimensional spin fields. We have also included the worldsheet charge q. Clearly, these states are even under the usual GSO-projection given in the covariant formulation by

$$R: (-1)^F = (-i)\ exp(i\pi \sum_{i=0}^{4} s_i), \qquad NS: (-1)^F = (-1)\ exp(i\pi \sum_{i=0}^{4} s_i), \tag{4.27}$$

with $s_i = \pm 1/2$ and ± 1, respectively.

In general the $E2 - E2$ sector also comprises $b_1(\Xi)$ chiral superfields corresponding to the position moduli of the $E2$-brane. On toroidal backgrounds, they are associated with the moduli along those two-tori in which the $E2$-brane is not fixed. For completeness, we display the vertex operators for the chiral component fields corresponding to the position moduli in the, say, first torus,

$$V_c(z) = \Omega_{E2E2}\, c\, e^{iH_1(z)} e^{-\varphi(z)}, \tag{4.28}$$

$$V_{\chi_\alpha} = \Omega_{E2E2}\, \chi_\alpha S^\alpha(z)\, e^{\frac{i}{2} H_1(z)} \prod_{I=2,3} e^{-\frac{i}{2} H_I(z)}, \tag{4.29}$$

to be supplemented by their anti-chiral counterparts.

Furthermore there arise zero modes at non-trivial intersections of the instanton $E2$ with its orientifold image $E2'$. Intersections away from the orientifold give rise to a chiral supermultiplet (m, μ^{α}) and its anti-chiral counterpart. We relegate their discussion to chapter 6.

Additional zero modes arise at the intersection of the $E2$-brane with the various $D6$-branes from open strings localized at the intersection point (or the overlap manifold). Open strings in this sector are subject to Dirichlet-Neumann boundary conditions in the extended four dimensions and to mixed DN boundary conditions internally, depending on the concrete intersection angles. The external DN conditions shift the oscillator moding in these directions by 1/2. In the Ramond sector, the zero point energy is still vanishing and we find massless fermions. The novelty as compared to the case of spacetime filling branes at angles is that the degeneracy of states is lifted in that the four-dimensional spin fields S_{α} or $S_{\dot{\alpha}}$ are no longer present. This also affects the details of the GSO-projection. In the NS sector, the vacuum energy is zero only for completely parallel branes, which is the only situation with bosonic zero modes in the non-singular geometric phase.

We first consider the case of non-trivial intersection of an instanton (anti-instanton) wrapping Ξ and a stack of N_a $D6$-branes wrapping Π_a in all three two-tori. It gives rise to $[\Xi \cap \Pi]^+$ fermionic zero modes in the $(\mathbf{N_a}, -1_E)$ $((1_E, \overline{\mathbf{N_a}}))$ and $[\Xi \cap \Pi]^-$ fermionic zero modes in the respective conjugate representation [67].

To see this the concrete form of the vertex operators needed. For actual computations it is indispensable to carefully distinguish between positive and negative intersection angles in the three two-tori. Generically, the intersection number between factorizable three-cycles Π_a and Π_b are given by

$$I_{ab} = \prod_{I=1}^{3} I_{ab}^I \ , \tag{4.30}$$

where I_{ab}^I denotes the intersection number in the I-th torus. Here positive (negative) intersection number I_{ab}^I corresponds to positive (negative) angle θ_{ab}^I and it is understood that $|\theta_{ab}^I| < 1$.

Given the total intersection number, say $I_{ab} > 0$, one distinguishes four different cases, three cases where one has negative intersection number I_{ab}^I in two internal tori and positive in the left one and the symmetric one in which the intersection numbers in all three internal two-tori are positive. In supersymmetric configurations the intersection angles add up to 2 for the latter choice, while for the other three their sum is 0.

Consider now an instanton wrapping the cycle Ξ such that all intersection angles θ_{E2a}^I are positive for some cycle Π_a wrapped by a $D6$-brane. Upon projection onto states odd under the GSO-operator (4.27), the vertex operators for the fermionic zero mode λ_a at the intersection $E2 - a$ is given by

$$V_{\lambda_a} = \Omega_{aE2}\, \lambda_a\, \Sigma(z) \prod_{I=1}^{3} \sigma_{1-\theta_{E2a}^I}(z)\, e^{-i(\theta_{E2a}^I - \frac{1}{2})H_I(z)}\, e^{-\varphi(z)/2}. \tag{4.31}$$

Here $\Sigma(z)$ denotes the bosonic twist field ensuring Dirichlet-Neumann boundary conditions in spacetime. The λ_a are Grassmannian variables and represent the polarization of the fermionic zero mode, normalized again as a field in $D = 0$ dimensions. Note that the GSO-projection forces us to keep only the state in the sector starting from the $D6$-brane and ending on the $E2$ and projects out the state with the reversed orientation. The relevant intersection angles are therefore negative and lead to the above form of the vertex (see e.g. [72]) carrying worldsheet charge $q = -1/2$. As indicated by the CP indices, it transforms as $(\mathbf{N_a}, -1_E)$. For the case the CP indices transform differently the analysis is analogously and we will refer to states negatively charged under $U(1)_a$ as $\overline{\lambda}_a$.

For a generic instanton cycle Ξ away from the orientifold, this gives rise to the charged zero mode spectrum summarized in table 4.2. As a result, the instanton

zero modes	$\text{Reps}_{Q_{ws}}$	number
$\lambda_{a,I}$	$(-1_E, \Box_a)_{-1/2}$	$I = 1, \ldots, [\Xi \cap \Pi_a]^+$
$\overline{\lambda}_{a,I}$	$(1_E, \overline{\Box}_a)_{-1/2}$	$I = 1, \ldots, [\Xi \cap \Pi_a]^-$
$\lambda_{a',I}$	$(-1_E, \overline{\Box}_a)_{-1/2}$	$I = 1, \ldots, [\Xi \cap \Pi'_a]^+$
$\overline{\lambda}_{a',I}$	$(1_E, \Box_a)_{-1/2}$	$I = 1, \ldots, [\Xi \cap \Pi'_a]^-$

Table 4.2: Zero modes at chiral $E2$ -$D6$ intersections.

carries the charge [67, 69].

$$Q_a(E2) = N_a \; \Xi \circ (\pi_a - \pi'_a) \tag{4.32}$$

under the gauge group $U(1)_a$, in agreement with the previous analysis, see (4.19).

For completeness, we briefly discuss non-chiral intersections between the E2- and the $D6$-branes. Consider first the supersymmetric situation that the corresponding cycles are parallel in one torus such that, say, $\theta^1_{E2a} > 0$, $\theta^2_{E2a} = -\theta^1_{E2a}$, $\theta^3_{E2a} = 0$. For instantons, we find one chiral mode in the $E2 \to D6_a$ sector with vertex

$$V_{\overline{\lambda}_a} = \Omega_{E2a} \, \overline{\lambda}_a \, \Sigma(z) \, \sigma_{\theta^1_{E2a}}(z) e^{i(\theta^1_{E2a} - \frac{1}{2}) H_1(z)} \sigma_{1-\theta^1_{E2a}}(z) e^{i(-\theta^1_{E2a} + \frac{1}{2}) H_2(z)} \, e^{-\frac{i}{2} H_3(z)} \, e^{-\frac{\varphi(z)}{2}}$$

and one in the $D6_a \to E2$ sector with vertex

$$V_{\lambda_a} = \Omega_{aE2}\, \lambda_a\, \Sigma(z)\, \sigma_{1-\theta^1_{E2a}}(z) e^{i(-\theta^1_{E2a}+\frac{1}{2})H_1(z)} \sigma_{\theta^1_{E2a}}(z) e^{i(\theta^1_{E2a}-\frac{1}{2})H_2(z)}\, e^{-\frac{i}{2}H_3(z)}\, e^{-\frac{\varphi(z)}{2}}.$$

Note that both zero modes carry worldsheet charge $q = -1/2$, i.e. are 'chiral' from the worldsheet point of view. The corresponding modes for anti-instantons should be clear.

If finally the $E2$- and $D6$-brane are completely parallel internally, the fermionic instanton zero mode sector for non-rigid cycles comprises simply the four states

$$V_{\lambda_a} = \Omega_{aE2}\, \lambda_a\, \Sigma(z) \prod_{I=1}^{3} e^{is_I H_I(z)}\, e^{-\varphi(z)/2} \tag{4.33}$$

with worldsheet charge $q = \sum_I s_I = 3/2$ or $-1/2$ and likewise four $\overline{\lambda}_a$ in the $E2 \to D6_a$ sector. Note that for completely rigid branes only the two states $q = 3/2$ are present. Since the zero point energy vanishes for completely trivial intersections, the lowest lying bosons are now also massless. In both the $E2 \to D6_a$ and the $D6_a \to E2$ sector the GSO-projection removes 2 out of the 4 spinorial groundstates from the external dimensions, giving for instantons

$$V_{w_{\dot\alpha}} = \Omega_{aE2}\, w_{\dot\alpha} S^{\dot\alpha}(z) \Sigma(z)\, e^{-\varphi(z)} \tag{4.34}$$

(plus the orientation reversed one), whereas the anti-instanton carries anti-chiral modes.

4.4 Instanton calculus

In this section we present the general rules for the computation of an instanton induced coupling. Later we will discuss the simplifications which arise if one is only interested in non-perturbative superpotential corrections.

Due to the lack of knowledge of string field theory we are not able to derive the rules from first principles but rather make some educated guesswork, where we use the experience gained from field theory. The goal is to determine couplings of the form

$$\prod_i \Phi_{a_i,b_i} e^{-S_{E2}} \tag{4.35}$$

where $\Phi_{a_i,b_i} = \phi_{a_i,b_i} + \theta\,\psi_{a_i,b_i}$ are the superfields arising at intersections between the stacks a_i and b_i. The measure of the path integral in field theory involves all instanton modes, on-shell (massless) as well as off-shell (massive) ones. The off-shell ones do not appear in the on-shell instanton action S_{E2} and one can integrate them out

$$\int [Da'^i] e^{-S_{E2}} = \int [Da^i] e^{-S_{E2}}\, e^Z \ . \tag{4.36}$$

Here $[a'_i]$ denotes all instanton modes while $[a_i]$ are only the massless ones. The factor e^Z are contributions from (multiple) vacuum loop diagrams, which correspond to n-particle irreducible diagrams in field theory.

We move on to determine the instanton action S_{E2} which includes the classical part (4.24) charged under the global $U(1)$'s. Apart from this generic term there might, in the presence of charged zero modes λ, be additional interaction terms of the form

$$S_{E2}^{int} = C_{ab}^{Aik} \left(\overline{\lambda}_a^i \, \phi^A \, \overline{\lambda}_b^k + \overline{\theta}^\alpha \, \lambda_a^i \, \psi_\alpha^A \, \overline{\lambda}_b^k \right) + ... \tag{4.37}$$

Here the dots indicate that there can be extra couplings involving more or none matter fields and/or interactions with more than just two λ zero modes and even couplings which include additional uncharged instanton zero modes. Furthermore there are also contributions from loop diagrams involving an arbitrary number of matter fields Φ as well as instanton zero modes. In principle all these couplings can be computed by standard CFT methods.

Once this action is determined we can perform the path integral over the instanton zero modes a_i. Due to the Grassmannian nature of the fermionic zero modes their integration is relatively simple and results in the fact that each fermionic zero mode has to be pulled down exactly ones in the path integral. It is due to this restriction that most instantons do not give rise to a non-vanishing contribution in the effective action.

In the sequel we focus on superpotential contribution. This simplifies the analysis due to the non-renormalization theorem. Note that the dilaton $e^\phi = g_s$ is the real part of a closed string supermultiplet. Its complex counterpart is given by the axion $\int C_3$, thus in order to ensure holomorphicity of the superpotential they can only appear in form of (4.24). All couplings carrying any g_s factors in S_{E2}^{int} are not contributions to the superpotential but rather indicate non-perturbative corrections to the Kählerpotential and in the sequel we ignore these.

Let us discuss how we determine the g_s scaling of the respective terms in (4.37).

As already said in principle all these couplings can be computed by applying standard conformal field theory tools. We work in the frame where the vertex operators do not carry any explicit factors of g_s. Note that instanton zero modes appearing in the measure are not necessarily the ones appearing as polarizations in the vertex operators, and thus may carry an implicit g_s scaling.

Indeed as shown in [77] one finds the following g_s scaling between the instanton zero modes $\tilde{\lambda}$ and the polarization appearing in the vertex operator λ the scaling

$$\lambda \sim \sqrt{g_s}\tilde{\lambda}. \tag{4.38}$$

There, the authors investigate instantons wrapping the same cycle as one of the D-branes leading to gauge instantons in four dimensional field theory. The corresponding instanton action S^{int}_{E2}, see equation (4.37), should match the ADHM action, known from field theory, in the low energy limit. By comparing the string computed action (4.37) with the ADHM action the authors were able to find the relation (4.38) between the polarizations in the vertices and the ones appearing in the measure.

Once this relation is given one computes all couplings by calculating string amplitudes involving instanton zero modes and/or matter fields. Note that only disk diagrams and 1-loop amplitudes give g_s^0 contributions in S^{int}_{E2}. Disk diagrams are normalized by $\frac{1}{g_s}$ and thus in order to give g_s^0 terms they need to involve vertices with implicit g_s scaling. By contrast for one-loop amplitudes the loop normalization factor is g_s independent. Thus we do not allow for any vertices with implicit g_s scaling. All higher loop amplitudes depend on g_s which cannot be cancelled by any implicit g_s scaling of the instanton zero modes and thus do not contribute to the superpotential.

Furthermore, focussing on superpotential contributions restricts also the e^Z factor. Any regularized higher loop contributions are accompanied with powers of g_s due to their normalization and thus cannot give contributions to the superpotential. Thus a superpotential[5] contribution takes the form

$$Y = \int D[a_i] e^{S^{Cl}_{E2} + S'^{int}_{E2}} \, e^{Z_0} \tag{4.39}$$

where S^{cl}_{E2} is given by (4.24), S'^{int}_{E2} is the part of (4.37) after performing the limit $g_s \to 0$ and Z_0 are the regularized instanton 1-loop contributions.

The latter contain the vacuum annulus diagrams with at least one boundary on the E2 instanton $Z^A(E2, E2)$, $Z^A(E2, E2')$, $Z^A(E2, D6)$ and $Z^A(E2, D6')$ and the Möbius strip diagram $Z^M(E2, O6)$. In case the instanton wraps a supersymmetric cycle we expect $Z^A(E2, E2)$ and $Z^A(E2, E2')$ to vanish. On the other hand $Z^A(E2, D6)$, $Z^A(E2, D6')$ and $Z^M(E2, O6)$ are not Bose-Fermi degenerate and do give non-vanishing contribution. Thus the whole instanton induced coupling is given by (4.39) with

$$Z_0 = \sum_a \left[Z^A(E2, D6_a) + Z^A(E2, D6'_a) \right] + Z^M(E2, O6) \ . \tag{4.40}$$

Keep in mind that this coupling is not the pure superpotential, but rather an effective coupling accompanied with the perturbative Kählerpotential.

In the sequel let us take a look at a concrete setup, in which the instanton exhibits the uncharged zero modes x^μ and θ^α [6] and in addition four charged zero

[5] We still compute rather the physical coupling than the pure superpotential contribution.

[6] As we will see in chapter 5 an $O(1)$ instanton gives rise to exactly such an uncharged zero mode structure.

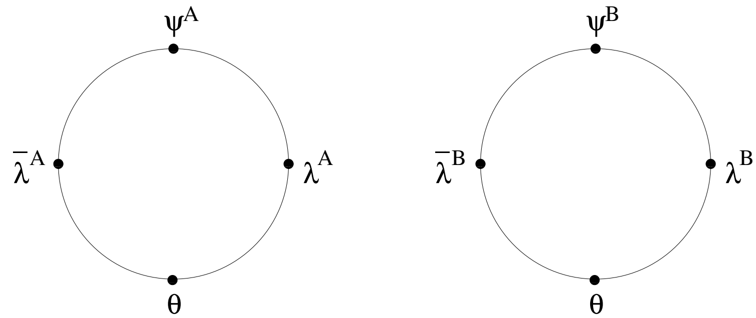

Figure 4.2: Gauge bosons and chiral matter fields in intersecting brane worlds.

modes, λ^A, $\overline{\lambda}^A$, λ^B and $\overline{\lambda}^B$ such that S^{int}_{E2} is given by

$$S'^{int}_{E2} = C_A\ \theta^\alpha \overline{\lambda}^A \psi^A_\alpha \lambda^A + C_A\ \theta^\alpha \overline{\lambda}^B \psi^B_\alpha \lambda^B. \tag{4.41}$$

Then the instanton induced mass term $\psi^A\,\psi^B$ in momentum space is given by

$$\psi^A\,\psi^B = \int d^4x d^2\theta \delta^4 \lambda^i < \theta\,\overline{\lambda}^A \psi^A \lambda^A >< \theta\,\overline{\lambda}^B \psi^B \lambda^B > e^{Z_0} \tag{4.42}$$

and amounts into computing two disk diagrams with the insertions of two fermionic λ modes depicted in figure 4.2. In the next chapter we will compute this instanton induced two fermion coupling in more detail. Note that in case an instanton exhibits the uncharged zero mode structure x^μ and θ^α one may interpret these zero modes as the $\mathcal{N} = 1$ superspace coordinates.

Chapter 5

Phenomenological implications

In this chapter we present a few phenomenological applications of the previously discussed instanton effects. We focus on the non-perturbative generation of Majorana mass terms for right-handed neutrinos, particular Yukawa couplings in $SU(5)$ GUT-like models, mass terms for exotics and higher fermionic F-terms a la Beasley and Witten. For simplicity we assume the superpotential generating instanton in the local and global setups to be an $O(1)$ instanton, a single instanton wrapping an orientifold invariant cycle. As we discuss in section 5.1 in detail such an instanton exhibits the correct zero mode structure to give rise to the superpotential contributions. Later, in chapter 6 we will discuss multi-instanton configurations or instantons in setups with background fluxes, which give non-perturbative corrections to the superpotential. All results presented here can be easily promoted to these setups and are in this respect general.

We start by a detailed analysis of single instantons wrapping cycles which are invariant under $\Omega\overline{\sigma}$. In this case the zero modes are subject to the orientifold action and we show that the $\overline{\tau}^{\dot{\alpha}}$ gets projected out leaving an instanton with the desired zero mode structure. In section 5.2 we perform a detailed computation of an instanton induced fermion two point coupling by using conformal field theoretic tools [78]. We discuss the proper g_s normalization of the instanton amplitude and give further constraints on the family structure due to the Grassmannian integration. This computation can be easily generalized to higher point coupling. Then we exemplify this analysis in a local setup in which the Majorana mass term for right-handed neutrinos is induced by an $E2$ instanton[67, 78, 69, 76, 79]. Together with perturbatively realized Dirac neutrino masses, these non-perturbative couplings give rise to the see-saw mechanism. We further study instanton generated, perturbatively forbidden Yukawa couplings in $SU(5)$ GUT like models [80] and show in a globally consistent model that non-perturbative effects can serve as a mechanism to decouple exotic matter [80]. At the end of the chapter we investigate instantons which do have apart x^{μ} and θ^{α} additional zero modes and demonstrate that they generically give rise to higher fermionic F-terms [81].

Finally, let us remark that non-perturbative effects have various further phenomenological interesting applications such as in moduli stabilization [82, 83], generating dynamical supersymmetry breaking terms [84, 85, 86], the Affleck-Dine-Seiberg (ADS) [87, 88, 89], various other superpotential terms [73, 90, 91, 92, 93]. Further aspects of instanton induced couplings are subject in [91, 94, 74, 75, 95, 96, 97, 98, 99, 100, 101]. We will not discuss these subjects in this work and refer the interested reader to the literature.

5.1 $O(1)$ instanton

In section 4.3 we saw that an instanton $E2$ generically comprises the four universal bosonic zero modes x^μ as well as the four fermionic zero modes θ^α and $\overline{\tau}^{\dot\alpha}$ [1]. In order to give contributions to the superpotential the two fermionic zero modes $\overline{\tau}^{\dot\alpha}$ need to be lifted or projected out. While it has been suggested to lift them via background fluxes the most intriguing way to achieve the absence of these zero modes is if the instanton wraps an orientifold invariant cycle. Zero modes of such an instanton are in addition subject to the orientifold action and as we will see in the sequel that under particular circumstances the instanton has indeed the right zero mode structure to contribute to the superpotential.

In the following we describe explicitly the orientifold action $\Omega\overline{\sigma}$ on the zero modes of an $E2$-instanton wrapping the orientifold invariant cycle Ξ. The orientifold action on the bosonic and fermionic instanton zero modes in the $E2-E2$ sector can be deduced from the action on spacetime-filling $D6$-branes wrapping the same internal cycle Ξ as follows:

(1) The orientifold action on the internal oscillator part of the vertex operators agrees in the $D6$ and $E2$ case. The only difference in the $E2$ case is that the external 4D space is orthogonal to the $E2$-brane and thus counts as transverse when applying the usual rules for representing $\Omega\overline{\sigma}$. This entails the inclusion of an additional minus sign for bosonic excitations in the external 4D space and the inclusion of a factor $e^{i\pi(s_0+s_1)}$ for all fermionic zero modes. Here $e^{i\pi(s_0+s_1)}$ acts on the (anti-)chiral 4D spin fields S^α ($S^{\dot\alpha}$) as $e^{i\pi(s_0+s_1)}S^\alpha=-1$ ($e^{i\pi(s_0+s_1)}S^{\dot\alpha}=1$).

[1] In addition to these they generically admit also zero modes at their intersection with its orientifold image $E2'$.

(2) Let $\gamma_{\Omega\overline{\sigma},D6}$ denote the matrix representing the orientifold action on the CP factors of the $D6$-brane modes. Then the corresponding matrix for the $E2$-instanton $\gamma_{\Omega\overline{\sigma},E2}$ enjoys

$$\gamma_{\Omega\overline{\sigma},D6} = \pm\gamma^T_{\Omega\overline{\sigma},D6} \Longleftrightarrow \gamma_{\Omega\overline{\sigma},E2} = \mp\gamma^T_{\Omega\overline{\sigma},E2}. \tag{5.1}$$

The + and - cases for the projection relevant for D6-branes are referred to as orthogonal (SO) and symplectic (SP) projections, respectively, because for invariant D6-branes they yield gauge bosons in the adjoint of the respective gauge groups. In the latter case, invariant cycles have to be wrapped by an even number of D6-branes.

Finally, the relation (5.1) follows via T-duality from the D9-D5 system analyzed by Gimon-Polchinski [102].

It is straightforward to apply these rules to the universal zero modes for an instanton wrapping an orientifold invariant cycle. One obtains for the universal zero modes x^μ and $\theta^\alpha, \overline{\tau}^{\dot{\alpha}}$

$$\Omega_{x^\mu} = \gamma_{E2}\Omega^T_{x^\mu}\gamma^{-1}_{E2}, \tag{5.2}$$

$$\Omega_{\theta^\alpha} = \gamma_{E2}\Omega^T_{\theta^\alpha}\gamma^{-1}_{E2}, \qquad \Omega_{\overline{\tau}^{\dot{\alpha}}} = -\gamma_{E2}\Omega^T_{\overline{\tau}^{\dot{\alpha}}}\gamma^{-1}_{E2}, \tag{5.3}$$

where for x^μ and θ^α the minus sign due to the excitation gets cancelled by the minus sign due to rule (1). Thus for a single instanton subject to the projection $\gamma_{E2} = \gamma^T_{E2}$, only x^μ and θ^α survive.

Generically the $E2 - E2$ sector exhibits $b_1(\Xi)$ complex bosonic zero modes c_I associated with special Lagrangian deformations of the $E2$-instanton. For an instanton wrapping an $\Omega\overline{\sigma}$ invariant cycle $c_I, \overline{c}_I$ and $\overline{\chi}_I^{\dot{\alpha}}$ are symmetrized while χ_I^{α} are anti-symmetrized or vice versa, depending on the type of cycle the instanton wraps. In the sequel we refer to them as deformations of the first and second kind, respectively. In the T-dual Type I picture they correspond to the position and Wilson line moduli of an $E1$-instanton wrapping a holomorphic curve.

Let us briefly state the results of this section. An instanton wrapping an orientifold invariant rigid cycle comprises the correct zero mode structure to contribute to the superpotential. In case the orientifold invariant cycle is non-rigid one distinguishes between two different scenarios, either $c_I, \overline{c}_I$ and $\overline{\chi}_I^{\dot{\alpha}}$ survive the $\Omega\overline{\sigma}$ action corresponding to deformations of the first kind or χ_I^{α} survives, which is usually referred to deformations of the second kind. The latter give non-perturbative corrections to the gauge kinetic function. In section 5.6 we show that O(1)instantons of the second type generically give rise to higher fermionic F-terms of Beasley-Witten type.

5.2 General two point amplitude

In this section we provide details on the computation of instanton corrections from Euclidean D2-branes, called $E2$-branes. While the general framework has been described in chapter 4.4 here we present a detailed CFT computation of a prototype of instanton induced two point coupling.

5.2.1 Amplitudes - Generalities and normalization

E2-instantons of the above kind can induce F-term couplings involving the open string superfields Φ_{ab} between the $D6$-branes present in the model. Of particular interest are those couplings which are absent perturbatively since they violate some of the global abelian symmetries which are the remnants of the $U(1)$ gauge symmetries on the $D6$-branes broken by Stückelberg-type couplings to the RR-forms of the background. The exponential suppression factor $e^{-S_{E2}}$ characteristic for instantonic couplings transforms under the global $U(1)$ symmetries in such a way that the full coupling

$$W_{np} = \prod_i \Phi_{a_i b_i} \, e^{-S_{E2}} \tag{5.4}$$

is invariant again. More precisely, from the axionic shift symmetries under these abelian symmetries induced by the Chern-Simons couplings of the N_a $D6_a$-branes one finds the $U(1)_a$ transformation of the instanton [67].

$$e^{-S_{E2}} = \exp\left[\frac{2\pi}{\ell_s^3}\left(-\frac{1}{g_s}\mathrm{Vol}_{\Xi} + i\int_{\Xi} C^{(3)}\right)\right] \longrightarrow e^{i\,Q_a(E2)\,\Lambda_a}\; e^{-S_{E2}} \tag{5.5}$$

with

$$Q_a(E2) = N_a\; \Xi \circ (\Pi_a - \Pi'_a). \tag{5.6}$$

Indeed this charge is exactly the amount of $U(1)_a$ charge carried by the fermionic zero modes between the $E2$ and the $D6_a$, which serves as an important check that our identification of the instanton vs. anti-instanton and the associated choice of

GSO-projection is correct.

The general procedure for the computation of the instanton induced physical M-point couplings involving the canonically normalized fields of the four-dimensional effective action has been outlined in section 4.4(see also in [67]). In momentum space it is given, after some refinements, by

$$\begin{aligned}
&\langle \Phi_{a_1,b_1}(p_1)\cdot \ldots \cdot \Phi_{a_M,b_M}(p_M)\rangle_{E2-\text{inst}} = \\
&-\frac{1}{M!}\int d^4\tilde{x}_E\, d^2\tilde{\theta}\, \sum_{\text{conf.}} \prod_a \big(\prod_{i=1}^{[\Xi\cap\Pi_a]^+} d\tilde{\lambda}_a^i\big)\, \big(\prod_{i=1}^{[\Xi\cap\Pi_a]^-} d\tilde{\overline{\lambda}}_a^i\big)\;\; e^{-S_{E2}} \times e^{Z_0'} \\
&\quad \times \langle \widehat{\Phi}_{a_1,b_1}[\vec{x}_1]\rangle_{\lambda_{a_1},\overline{\lambda}_{b_1}} \cdot \ldots \cdot \langle \widehat{\Phi}_{a_L,b_L}[\vec{x}_L]\rangle_{\lambda_{a_L},\overline{\lambda}_{b_L}} \times \prod_k \langle \widehat{\Phi}_{c_k,c_k}[\vec{x}_k]\rangle^{\text{loop}}_{A(E2,D6_{c_k})}.
\end{aligned}$$

Its basics building blocks are disk and annulus diagrams with insertion of an appropriate product of boundary changing vertex operators, denoted schematically by $\widehat{\Phi}_{a_1,b_1}[\vec{x}_1]$. It is understood that either precisely two of these diagrams carry one θ-mode each or one of them carries both. Each disk carries two of the fermionic modes λ_a from the E2-D6 sector, whereas the annulus diagrams are uncharged in that they are free of λ_a-insertions. The instanton suppression factor $e^{-S_{inst}}$ arises from exponentiation of tree-level disks with no matter insertion and is corrected by the exponentiated regularized one-loop amplitude $e^{Z'}$ with

$$Z' = \sum_a \left[\mathcal{A}(D6_a, E2) + \mathcal{A}(D6'_a, E2)\right] + \mathcal{M}(E2, O6) \tag{5.7}$$

in terms of the annulus and Möbius amplitudes $\mathcal{A}$ and $\mathcal{M}$ (for details see [67]). This one-loop factor has been computed in [94, 87, 95] and is related to the regularized threshold correction to the gauge coupling of a $D6$-brane wrapping the same internal cycle Ξ as the instanton. The latter have been determined in [64, 103] for toroidal orientifolds. The prefactor $-\frac{1}{M!}$ arises from expansion of $e^{-S_{mod}}$ to order M, where the instanton moduli action S_{mod} contains the tree-level and annulus coupling terms.

After this review we clarify the proper g_s-normalization of the instanton amplitude. It is convenient to work in the frame where all vertex operators (including the ones for fields between two $D6$-branes) carry no explicit factors of g_s. A disk with boundary only on one type of Dp-brane carries the normalization factor

$$C_p = \frac{2\pi}{g_s \ell_s^{p+1}}. \tag{5.8}$$

Consequently, all kinetic and tree-level perturbative coupling terms arise formally at order g_s^{-1}. This is therefore the tree-level order in g_s to which non-perturbative couplings are to be compared.

The disks appearing in the above expression (5.7) are bounded partly by the $D6$-branes a and b and the $E2$-instanton. In such a case, the amplitude has to be normalized with respect to the dimension of the overlap of the branes involved and therefore carries a factor (see also [87])

$$C = \frac{2\pi}{g_s}. \tag{5.9}$$

Consider now the normalization of the instanton moduli measure. As noted, the integration over the four-dimensional supermoduli $\int d^4\tilde{x}_E\, d^2\tilde{\theta}$ restores Poincaré invariance and $\mathcal{N}=1$ supersymmetry. The inclusion of the charged zero modes $\tilde{\lambda}_a$ in the measure can be understood as the process of integrating these modes out since they would result in a zero in the Pfaffian $e^{Z'}$ [104]. While the Grassmannian integral is trivial and merely results in a combinatorical factor, the integration over $d^4\tilde{x}_E$ will ensure momentum conservation of the M-point amplitude (see equ. (5.25)). The tilde indicates that we have to integrate over the properly normalized zero modes corresponding to the instanton moduli in the ADHM action in the limit where the $E2$-brane wraps the same cycle as one of the $D6$-branes and therefore represents a gauge instanton (or its stringy generalization for $\ell_s \neq 0$.) The resulting Jacobian in the transition from the polarizations appearing in our vertex operators takes care of the proper normalization procedure in the more familiar case of field theory instantons (see e.g. the review [105] for details). The situation of parallel $E2$ and $D6$-branes is T-dual to the $D(3)-D(-1)$ system in Type IIB theory. Adapting the CFT analysis of [77] to our case[2], we find the following relation between the polarization in the vertices and the ones to appear in the measure,

$$\tilde{x}_E^\mu = \frac{x_E^\mu}{2}\sqrt{\frac{2\pi v_{E2}}{g_s}}, \qquad \tilde{\theta}^\alpha = \theta^\alpha\sqrt{\frac{2\pi v_{E2}}{g_s}} \qquad \tilde{\lambda} = \lambda\sqrt{\frac{2\pi}{g_s}}, \tag{5.10}$$

where $v_{E2} = Vol_{E2}/\ell_s^3$. Most importantly, the contribution from two λ-modes[3] cancels the g_s-dependent topological normalization of the disk (5.9).

If indeed precisely 2 λ-modes are inserted per disk (and none on the annulus

[2]Unlike [77] we do not assign four-dimensional canonical mass dimensions to the instanton moduli but treat them as dimensionless fields in zero dimensions. The disk normalization between parallel $E2$ and $D6$-branes is $2\pi v_{E2}/g_s$. The resulting amplitudes and effective moduli action before rescaling therefore differ by a power of v_{E2}/ℓ_s^4 as compared to the ones in [77]. Our rescaling for the case of parallel $E2$ and $D6$ systems is otherwise identical upon replacing $g_0 \to \sqrt{g_s\, v_{E2}/\pi}$. Finally, for $E2$ and $D6$-branes at angles, the rescaling of the λ-modes does not contain any v_{E2}, in agreement with (5.9).

[3]Recall that $d(a\psi) = a^{-1}d\psi$ for a Grassmann field ψ.

diagrams carrying an additional factor of g_s in their normalization), then the induced M-point amplitude is proportional to $\frac{2\pi v_{E2}}{g_s}$ due to the remaining normalization factors from $\int d^4\tilde{x}_E\, d^2\tilde{\theta}$ (times the exponential dependence, of course). It therefore arises at 'string tree-level' as compared to the perturbative terms.

5.2.2 Amplitudes - CFT details

A phenomenologically interesting application is the computation of instanton-induced $U(1)$ charge violating 2-point couplings. These can be thought of as Majorana masses for right-handed neutrinos or as μ-terms in the MSSM [67, 69, 106]. We will now compute such 2-point couplings in a general setup, which can then be adapted to concrete examples.

Consider the superfield Φ^A_{ab} at the intersection A between two $D6$-branes wrapping the cycles Π_a and Π_b. We would like to generate couplings of the form $<\psi^A_{ab}\psi^B_{ab}>_{E2}$. The zero mode structure of the instanton has to allow for a compensation of the excess of $U(1)_a$ and $U(1)_b$ charge. This requires

$$\begin{array}{llll} [\Pi_{E2}\cap\Pi_a]^+ = 2 & [\Pi_{E2}\cap\Pi_b]^- = 2 & \text{for} & I_{ab} > 0 \\ [\Pi_{E2}\cap\Pi_a]^- = 2 & [\Pi_{E2}\cap\Pi_b]^+ = 2 & \text{for} & I_{ab} < 0 \end{array} \tag{5.11}$$

and the intersection between the $E2$ and all other $D6$-branes has to vanish[4]. We reiterate that the absence of additional reparametrization and other uncharged zero

[4]Strictly speaking, this is only true if the $E2$ lies away from the orientifold brane. In case $E2 = E2'$, the $E2 - a$ and $a' - E2$ sector are identified.

modes in the $E2 - E2'$ sector necessitates the $E2$-brane to be rigid and to satisfy $[\Pi_{E2} \cap \Pi_{E2'}]^{\pm} = 0$. The four zero modes are denoted by λ_a^i and $\overline{\lambda}_b^k$ for $i, k = 1, 2$. Since the CFT computation depends on the concrete form of the vertex operators, we have to make a definite choice of angles and intersection numbers. Consider e.g. the simple situation corresponding to $I_{ab} > 0$ such that[5]

$$\begin{aligned} &\theta_{ab}^I > 0, \quad \theta_{E2a}^I > 0, \quad \theta_{E2b}^I < 0, \\ &\sum_{I=1}^3 \theta_{ab}^I = \sum_{I=1}^3 \theta_{E2a}^I = 2 = -\sum_{I=1}^3 \theta_{E2b}^I. \end{aligned} \tag{5.12}$$

With this choice of angles the vertex operator for ψ_{ab}^A takes the form

$$V_{\psi_{ab}^A} = \ell_s^{\frac{3}{2}}\, \Omega_{ba}\, \psi_\alpha^A\, S^\alpha(z) \prod_{I=1}^3 \sigma_{1-\theta_{ab}^I}(z)\, e^{-i(\theta_{ab}^I - 1/2) H_I(z)}\, e^{ik_\mu^A X^\mu(z)}\, e^{-\varphi(z)/2}, \tag{5.13}$$

where ψ_α^A carries canonical mass dimensions. The vertex for the zero mode at the intersection of $E2$ and $D6_a$ has been given in (4.31), and the one between $E2$ and $D6_b$ reads

$$V_{\overline{\lambda}_b^k} = \Omega_{E2b}\, \overline{\lambda}_b^k\, \Sigma(z) \prod_{I=1}^3 \sigma_{1+\theta_{E2b}^I}(z)\, e^{i(\theta_{E2b}^I + 1/2) H_I(z)}\, e^{-\varphi(z)/2}\ . \tag{5.14}$$

We then have to compute

[5] Note that in most concrete realizations including our example given in section 5.3 the angles will be less symmetric, but this can easily be dealt with.

$$
\begin{aligned}
<\psi^A_{ab}\psi^B_{ab}>_{E2} \; &= \; -\frac{1}{2!}\,\frac{v_{E2}}{16}\,\frac{g_s}{2\pi}\,\int d^4x_E \int d^2\theta \int d^2\lambda_a \int d^2\overline{\lambda}_b\; e^{-S_{inst.}}\; e^{Z'} \\
&\sum_{i,j,k,l} < V^{-\frac{1}{2}}_{\Theta^\alpha} V^{-\frac{1}{2}}_{\overline{\lambda}^k_b} V^{-\frac{1}{2}}_{\psi^A_\alpha} V^{-\frac{1}{2}}_{\lambda^i_a} >< V^{-\frac{1}{2}}_{\Theta^\beta} V^{-\frac{1}{2}}_{\overline{\lambda}^l_b} V^{-\frac{1}{2}}_{\psi^B_\beta} V^{-\frac{1}{2}}_{\lambda^j_a} > . \qquad (5.15)
\end{aligned}
$$

This already includes the rescaling (5.10). It is understood that the summation is only over those combinations of family indices with non-trivial disk diagrams. This important point has to be studied in concrete examples.

The disk amplitudes appearing in (5.15) can be evaluated using standard CFT methods. The computation of the four-point function $<\theta\,\overline{\lambda}\psi\,\lambda>$ involving the vertex operators (4.26,4.31,5.13,5.14) requires the following correlators

$$
\begin{aligned}
<\prod_{i=1}^{4} e^{-\varphi/2(z_i)}> = \prod_{i=1}^{4} z_{ij}^{-\frac{1}{4}}, \qquad\qquad & < S^\alpha(z_1)\, S^\beta(z_2) > = \epsilon^{\alpha\beta} z_{12}^{-\frac{1}{2}}, \\
< e^{i\alpha H_I(z_1)}\, e^{i\beta H_I(z_2)}\, e^{i\gamma H_I(z_3)}\, e^{i\delta H_I(z_4)} > \; &= \; z_{12}^{\alpha\,\beta}\, z_{13}^{\alpha\,\gamma}\, z_{14}^{\alpha\,\delta}\, z_{23}^{\beta\,\gamma}\, z_{24}^{\beta\,\delta}\, z_{34}^{\gamma\,\delta} \qquad (5.16) \\
< \Sigma(z_1)\, e^{ik_\mu X^\mu(z_2)}\, \Sigma(z_3) > \; &= \; e^{ik_\mu x_0^\mu}\, z_{13}^{-\frac{1}{2}}.
\end{aligned}
$$

Here z_{ij} denotes $z_i - z_j$ and x_0 is the position of the E2-instanton in spacetime. The most involved ingredient is the correlator of the three bosonic twist fields. In general, it reads [59]

$$
< \sigma_\alpha(z_1)\sigma_\beta(z_2)\sigma_\gamma(z_3) > = (4\pi\,\Gamma_{\alpha,\,\beta,\,\gamma})^{\frac{1}{4}}\; z_{12}^{-\alpha\,\beta} z_{13}^{-\alpha\,\gamma} z_{23}^{-\beta\,\gamma} \sum_m e^{-\mathcal{A}(m)}, \qquad (5.17)
$$

where $\Gamma_{\alpha,\beta,\gamma}$ is given by

$$\Gamma_{\alpha,\beta,\gamma} = \frac{\Gamma(1-\alpha)\,\Gamma(1-\beta)\,\Gamma(1-\gamma)}{\Gamma(\alpha)\Gamma(\beta)\Gamma(\gamma)} \tag{5.18}$$

and $\mathcal{A}(m) = A(m)/(2\pi\alpha')$ is the area in string units of the triangle formed by the three intersecting branes. The correlator (5.17) vanishes if the angles α, β, γ do not add up to an integer. The normalization $(4\pi)^{\frac{1}{4}}$ was determined in [59] by factorizing the four-point amplitude involving four bosonic twist fields in the limit corresponding to a gauge boson exchange. Putting everything together, including the disk normalization factor $\frac{2\pi}{g_s}$ and using the supersymmetry conditions (5.12) result in

$$< \theta^\alpha \overline{\lambda}_b^k \psi_\alpha^A \, \lambda_a^i \ >= \frac{2\pi\, \ell_s^{\frac{3}{2}}}{g_s} Tr\left(\Omega_{E2E2}\Omega_{E2b}\Omega_{ba}\Omega_{aE2}\right) \theta^\alpha \, \overline{\lambda}_b^k \psi_\alpha^A \, \lambda_a^i \tag{5.19}$$

$$\times \prod_{I=1}^{3} \left[4\pi\Gamma_{1-\theta^I_{ab},\,1-\theta^I_{E2a},\,1+\theta^I_{E2b}}\right]^{\frac{1}{4}} \sum_{m_I} e^{-\mathcal{A}^A_{ik}(m_I)} \int \frac{\prod_{i=1}^4 dz_i}{V_{CKG}} \, z_{13}^{-\frac{1}{2}} \, z_{24}^{-\frac{1}{2}} \prod_{i,j=1}^{4} z_{ij}^{-\frac{1}{2}} \; e^{-ik^A_\mu x_0^\mu}.$$

After we fix the vertex operator positions to[6]

$$z_1 = 0, \qquad z_2 = x, \qquad z_3 = 1, \qquad z_4 = \infty \tag{5.20}$$

and add the c-ghost part

$$< c(z_1)\, c(z_3)\, c(z_4) >= z_{13}\, z_{14}\, z_{34} \tag{5.21}$$

[6]We need to include the other cyclic order as well.

the amplitude computes to[7]

$$< \theta^\alpha \overline{\lambda}_b^k \psi_\alpha^A \lambda_a^i > = \frac{2\pi}{g_s} \ell_s^{\frac{3}{2}} C_{ik}^A e^{-ik_\mu^A x_0^\mu} \quad (\theta^\alpha \overline{\lambda}_b^k \psi_\alpha^A \lambda_a^i) \tag{5.22}$$

with

$$C_{ik}^A = \pi \prod_{I=1}^{3} \left[4\pi \Gamma_{1-\theta_{ab}^I, 1-\theta_{E2a}^I, 1+\theta_{E2b}^I} \right]^{\frac{1}{4}} \sum_{m_I} e^{-\mathcal{A}_{ik}^A(m_I)}. \tag{5.23}$$

Here we omit the trivial trace structure and use

$$\int_0^1 dx \, [x(1-x)]^{-\frac{1}{2}} = \pi. \tag{5.24}$$

In order to obtain the coupling $< \psi_{ab}^A \psi_{ab}^B >_{E2}$ we plug (5.22) into (5.15) and perform the integrals over all fermionic and bosonic zero modes. In doing so, we make use of the integral representation of the δ-function (recall that $x_0^\mu = \ell_s x_E^\mu$),

$$\int d^4 x_E \, e^{-ik_\mu x_0^\mu} = \frac{(2\pi)^4}{\ell_s^4} \delta^4(\, k \,) \;, \tag{5.25}$$

and find

$$\begin{aligned} < \psi_{ab}^A \psi_{ab}^B >_{E2} \quad = \quad & -\frac{v_{E2}\, \pi}{16\, g_s} M_s \, e^{-S_{inst.}} e^{Z'} \; \psi_\alpha^A \epsilon^{\alpha\beta} \psi_\beta^B \\ & \times (2\pi)^4 \, \delta^4 \left(k^A + k^B \right) \sum_{i,j,k,l} \epsilon^{ij} \epsilon^{kl} \, C_{ik}^A C_{jl}^B . \end{aligned} \tag{5.26}$$

The overall sign can always be absorbed into phases of the fermions. Note that due to

[7] Note that even after taking into account the other cyclic order the only non vanishing trace is $Tr\,(\Omega_{E2E2}\Omega_{E2b}\Omega_{ba}\Omega_{aE2})$ and therefore we need to integrate from 0 to 1.

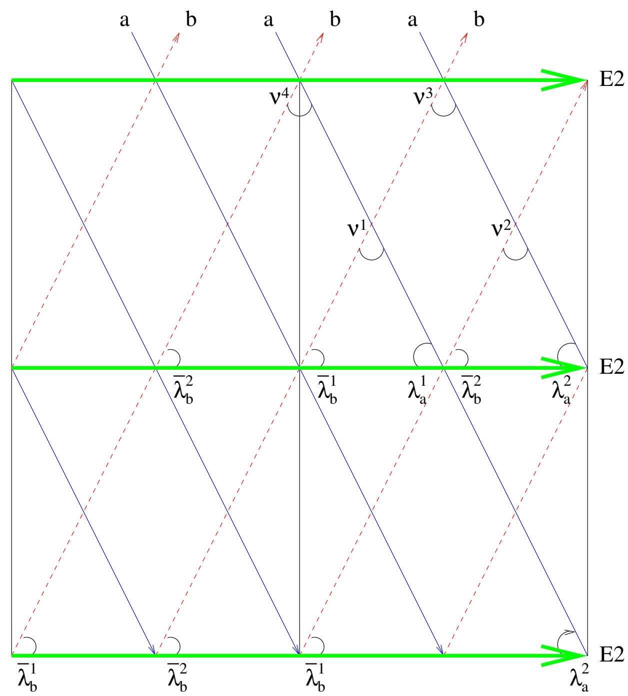

Figure 5.1: Example of family structure yielding diagonal and off-diagonal fermion bilinears.

the Grassmannian integral, non-vanishing mass terms occur only for a suitable family structure such that indeed $\sum_{i,j,k,l} \epsilon^{ij} \epsilon^{kl} \, C^A_{ik} C^B_{jl} \neq 0$.

5.2.3 Family structure for (off-)diagonal bilinears in a simple example

To appreciate this latter point, consider the family structure depicted schematically in figure 5.1. We can think of it as representing one of the three two-tori of a toroidal orientifold model with factorizable $D6$-branes wrapping a one-cycle on each torus. In this case, branes a and b correspond to wrapping numbers $(1,-2)$ and $(1,2)$, respectively. For simplicity we assume here that the complete family replication is due to multiple intersections on just the depicted torus, though more general situations leading to non-vanishing terms on a factorizable T^6 are possible[8]. Consider now the instanton wrapping the cycle $(1,0)$.

Straightforward inspection of the possible triangles connecting the various intersection points reveals that the coupling terms in the (zero-dimensional) moduli action of the instanton are proportional to

$$\nu^1\theta\,\overline{\lambda}_b^1\,\lambda_a^1\;e^{-\mathcal{A}_{11}^1}+\nu^1\theta\,\overline{\lambda}_b^2\;\lambda_a^2\,e^{-\mathcal{A}_{22}^1}+\nu^2\theta\,\overline{\lambda}_b^2\,\lambda_a^2\;e^{-\mathcal{A}_{22}^2}+\nu^2\theta\,\overline{\lambda}_b^1\,\lambda_a^1\;e^{-\mathcal{A}_{11}^2}+$$
$$\nu^3\theta\,\overline{\lambda}_b^2\,\lambda_a^1\;e^{-\mathcal{A}_{21}^3}+\nu^3\theta\,\overline{\lambda}_b^1\,\lambda_a^2\;e^{-\mathcal{A}_{12}^3}+\nu^4\theta\,\overline{\lambda}_b^1\,\lambda_a^2\;e^{-\mathcal{A}_{12}^4}+\nu^4\theta\,\overline{\lambda}_b^2\,\lambda_a^1\;e^{-\mathcal{A}_{21}^4}. \quad (5.27)$$

Note that we have only given the leading area suppression due to the smallest possible triangles. The Grassmann integration now dictates the possible combination of fermion bilinears appearing the amplitude. This results in the following fermion bilinears in the four-dimensional effective action,

$$S_{non-pert.}=-\int d^4x\;\frac{2\pi}{g_s}\,M_s\,e^{-S_{inst}}e^{Z'}\;(\nu^1\nu^2\nu^3\nu^4)\;\mathcal{M}\;(\nu^1\nu^2\nu^3\nu^4)^T \quad (5.28)$$

with the 4×4 matrix

$$\mathcal{M}=\begin{pmatrix} A & 0 \\ 0 & B \end{pmatrix}$$

given in terms of

$$A=\frac{\pi^2}{16}(4\pi\Gamma)^{1/2}\begin{pmatrix} e^{-(\alpha+\beta)} & \frac{1}{2}(e^{-2\alpha}+e^{-2\beta}) \\ \frac{1}{2}(e^{-2\alpha}+e^{-2\beta}) & e^{-(\alpha+\beta)} \end{pmatrix},$$

[8] In fact, our local model discussed in the next section is more general in this respect.

$$B = \frac{\pi^2}{16}(4\pi\Gamma)^{1/2} \begin{pmatrix} e^{-(\gamma+\delta)} & \frac{1}{2}(e^{-2\gamma} + e^{-2\delta}) \\ \frac{1}{2}(e^{-2\gamma} + e^{-2\delta}) & e^{-(\gamma+\delta)} \end{pmatrix}.$$

Here we have defined

$$\begin{aligned} \mathcal{A}^1_{11} = \mathcal{A}^2_{22} = \alpha, \qquad & \mathcal{A}^1_{22} = \mathcal{A}^2_{11} = \beta, \\ \mathcal{A}^3_{21} = \mathcal{A}^4_{12} = \gamma, \qquad & \mathcal{A}^3_{12} = \mathcal{A}^4_{21} = \delta \end{aligned} \tag{5.29}$$

and we have omitted the $(2\pi)^4\delta(\sum k)$ in going from momentum to position-space. We have also made use of our freedom to absorb a phase $e^{i\pi/2}$ into the fields ν^1 and ν^2 to adjust the signs of the Majorana mass terms.

As a result, we have found both diagonal couplings $\nu^i\nu^i$ and the off-diagonal ones $\nu^1\nu^2$ and $\nu^3\nu^4$. The overall scale of these terms is governed by the exponential suppression factor $e^{-S_{inst}}$, whereas the relative size of the various couplings is set by the ratio of the triangles involved. These depend on the concrete Kähler and open string moduli. For example, for a particular choice of brane positions we can set one of the areas, say α, to zero, in which case the off-diagonal coupling $\nu^1\nu^2$ would dominate over the diagonal ones $\nu^1\nu^1$, $\nu^2\nu^2$.

Finally we point out that the above non-perturbative couplings in this example are allowed since not all possible intersection points are connected by worldsheet instantons, i.e. disk triangles. As observed already in [107], this is a generic consequence of the fact that the three intersection numbers I_{Ea}, I_{Eb}, I_{ab} are not coprime.

If, by contrast, in addition to the couplings (5.27), also the combination, say,

$$\nu^1\theta\,\overline{\lambda}_a^1\,\lambda_b^2\;e^{-\widetilde{\mathcal{A}}_{12}^1}+\nu^1\theta\,\overline{\lambda}_b^2\lambda_a^1 e^{-\widetilde{\mathcal{A}}_{21}^1} \tag{5.30}$$

were present, the Grassmann integral would give zero for the coupling $\nu^1\nu^1$ whenever $\widetilde{\mathcal{A}}_{11}^1+\widetilde{\mathcal{A}}_{22}^1=\mathcal{A}_{12}^1+\mathcal{A}_{21}^1$. This results in yet another important constraint on the architecture of concrete models exhibiting $E2$-instanton effects, as has also been appreciated in [69].

5.3 Majorana mass term for right-handed Neutrinos

In this section we present a local brane configuration on the orientifold $T^6/\mathbb{Z}_2\times\mathbb{Z}_2'$ which serves as a toy model for realizing the see-saw mechanism for neutrino masses. While our ultimate object of desire are globally consistent MSSM-like string vacua satisfying all tadpole- and K-theory constraints, we content ourselves for the time being with a local model with GUT gauge group. Apart from demonstrating the CFT techniques developed in the previous section, our primary aim is two-fold: First to show that rigid cycles meeting the strong requirements for the generation of 2-point couplings exist even on toroidal backgrounds; and second to demonstrate that the resulting $E2$-instanton effects do have the potential to yield Majorana mass terms for the right-handed neutrinos within the range $10^8 GeV - 10^{15} GeV$.

5.3.1 Background on the $T^6/\mathbb{Z}_2 \times \mathbb{Z}_2'$ orientifold

Consider the orientifold $T^6/\mathbb{Z}_2 \times \mathbb{Z}_2'$ with Hodge numbers $(h_{11}, h_{12}) = (3, 51)$. We stick to the notation of [36], to which we refer for details of the geometry and the construction of rigid cycles. The orbifold group is generated by θ and θ' acting as reflection in the first and last two tori, respectively.

This background exhibits two types of factorizable special Lagrangian three-cycles. The first class is given by the usual non-rigid bulk cycles

$$\Pi_a^B = 4 \bigotimes_{I=1}^{3} (n_a^I[a^I] + \widetilde{m}_a^I[b^I]), \tag{5.31}$$

defined in terms of the fundamental one-cycles $[a^I], [b^I]$ of the I-th T^2 and the corresponding wrapping numbers n_a^I and $\widetilde{m}_a^I = m_a^I + \beta^I n_a^I$. Here $\beta^I = 0, 1/2$ for rectangular and tilted tori, respectively.

In addition there exist so-called g-twisted three-cycles

$$\Pi_{ij}^g = n^{I_g}[\alpha_{ij,n}^g] + \widetilde{m}^{I_g}[\alpha_{ij,m}^g], \tag{5.32}$$

where $i, j \in \{1, 2, 3, 4\} \times \{1, 2, 3, 4\}$ labels one of the 16 blown-up fixed points of the orbifold element $g = \theta, \theta', \theta\theta' \in \mathbb{Z}_2 \times \mathbb{Z}_2'$. The cycles $[\alpha_{ij,n}^g]$ ($[\alpha_{ij,m}^g]$) can be understood as twice the product of the corresponding $\mathbb{P}_1$ and the one-cycle $[a]^{I_g}$ ($[b]^{I_g}$) in the I_g-th T^2 invariant under g. Here $I_g = 3, 1, 2$ for $g = \theta, \theta', \theta\theta'$, respectively.

These twisted cycles are the building blocks for certain fractional cycles Π^F charged under all three twisted sectors. They are rigid and will serve as candidates for $E2$-branes contributing to the superpotential. The general expression for Π^F is

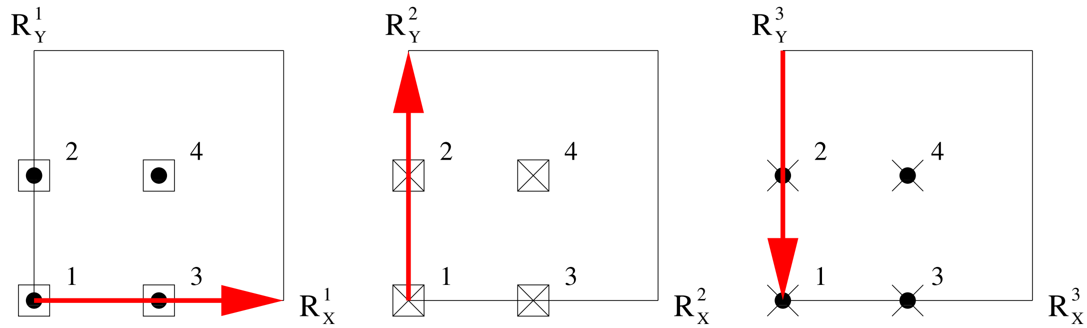

Figure 5.2: $T^6/\mathbb{Z}_2 \times \mathbb{Z}_2'$ with $\beta^1 = \beta^2 = \beta^3 = 0$.

given by

$$\Pi^F = \frac{1}{4}\Pi^B + \frac{1}{4}\Big(\sum_{i,j\in S_\theta} \epsilon^{\theta}_{ij}\Pi^{\theta}_{ij}\Big) + \frac{1}{4}\Big(\sum_{j,k\in S_{\theta'}} \epsilon^{\theta'}_{jk}\Pi^{\theta'}_{jk}\Big) + \frac{1}{4}\Big(\sum_{i,k\in S_{\theta\theta'}} \epsilon^{\theta\theta'}_{ik}\Pi^{\theta\theta'}_{ik}\Big). \qquad (5.33)$$

The sets S_g denote the four different fixed points in the g-twisted sector compatible with the bulk wrapping numbers and the concrete position of the brane, as detailed in [36]. A given set of bulk wrapping numbers allows for a choice of $2 \times 2 \times 2 = 8$ inequivalent positions of the fractional brane. Each of these branes is further specified by the signs ϵ^g_{ij}, corresponding to the orientation with which the various $\mathbb{P}_1$ are wrapped in the twisted sector. They are subject to various consistency conditions [36] such that for each choice of position of the fractional brane, there are only 8 inequivalent choices of ϵ^g_{ij}.

The orientifold action $\Omega\mathcal{R}$ on the untwisted cycles follows from

$$\Omega\mathcal{R}:\ [a_I] \to [a_I] \qquad \Omega\mathcal{R}:\ [b_I] \to -[b_I], \qquad (5.34)$$

whereas the twisted cycles transform as

$$\Omega\mathcal{R}:\ \alpha^g_{ij,n} \to -\eta_{\Omega\mathcal{R}}\,\eta_{\Omega\mathcal{R}g}\,\alpha^g_{\mathcal{R}(i)\mathcal{R}(j),n}, \qquad \Omega\mathcal{R}:\ \alpha^g_{ij,m} \to \eta_{\Omega\mathcal{R}}\,\eta_{\Omega\mathcal{R}g}\,\alpha^g_{\mathcal{R}(i)\mathcal{R}(j),m}. \tag{5.35}$$

Here the reflection $\mathcal{R}$ leaves all fixed points of an untilted two-torus invariant and acts on the fixed points in a tilted two-torus as

$$\mathcal{R}(1) = 1 \qquad \mathcal{R}(2) = 2 \qquad \mathcal{R}(3) = 4 \qquad \mathcal{R}(4) = 3. \tag{5.36}$$

The signs $\eta_{\Omega\mathcal{R}g} = \pm 1$ defining the orientifold action are subject to the constraint

$$\eta_{\Omega\mathcal{R}}\,\eta_{\Omega\mathcal{R}\theta}\,\eta_{\Omega\mathcal{R}\theta'}\,\eta_{\Omega\mathcal{R}\theta\theta'} = -1. \tag{5.37}$$

In our subsequent example we choose for simplicity all tori to be untilted and

$$\eta_{\Omega\mathcal{R}} = \eta_{\Omega\mathcal{R}\theta} = \eta_{\Omega\mathcal{R}\theta\theta'} = -\eta_{\Omega\mathcal{R}\theta'} = 1. \tag{5.38}$$

In this case, the orientifold image of the cycle Π^F in equ. (5.33) is given by Π'^F,

$$\Pi'^F = \frac{1}{4}\widehat{\Pi}^B - \frac{1}{4}\Big(\sum_{i,j\in S_\theta} \epsilon^\theta_{ij}\widehat{\Pi}^\theta_{ij}\Big) + \frac{1}{4}\Big(\sum_{j,k\in S_{\theta'}} \epsilon^{\theta'}_{jk}\widehat{\Pi}^{\theta'}_{jk}\Big) - \frac{1}{4}\Big(\sum_{i,k\in S_{\theta\theta'}} \epsilon^{\theta\theta'}_{ik}\widehat{\Pi}^{\theta\theta'}_{ik}\Big),$$

where the $\widehat{\ }$ denotes the substitution $m^I \to -m^I$ (see also [37]).

The fixed point locus sets expressed in terms of the toroidal cycles take the form

$$\pi_{O6} = 2\,[a_1]\,[a_2]\,[a_3] - 2\,[b_1]\,[b_2]\,[a_3] + 2\,[a_1]\,[b_2]\,[b_3] - 2\,[b_1]\,[a_2]\,[b_3]. \tag{5.39}$$

We also recall the topological intersection number I_{ab} of two bulk branes Π_a^B and Π_b^B,

$$I_{ab} = 4 \prod_{i=1}^{3} (n_a^i\, m_b^i - n_b^i\, m_a^i). \tag{5.40}$$

Since our conventions are such that a stack of N_a coincident branes away from the orientifold carries gauge group $U(N_a/2)$ upon taking the $\mathbb{Z}_2$-projection on the Chan-Paton factors into account, the quantity (5.40) counts the number of chiral multiplets in the bifundamental of the gauge group $(\overline{\frac{\mathbf{N_a}}{\mathbf{2}}}, \frac{\mathbf{N_b}}{\mathbf{2}})$ living at the intersection of two stacks of N_a and N_b bulk cycles a and b, respectively. The number of chiral multiplets transforming as antisymmetric and symmetric representations under $U(N_a/2)$ is

$$I^{Anti} = \frac{1}{2}(I_{a'a} + I_{O6a}), \qquad\qquad I^{Sym} = \frac{1}{2}(I_{a'a} - I_{O6a}), \tag{5.41}$$

where

$$\begin{aligned} I_{aa'} &= -32\, n_a^1\, m_a^1\, n_a^2\, m_a^2\, n_a^3\, m_a^3, \\ I_{O6a} &= 8\; m_a^1\, m_a^2\, m_a^3 - 8\; n_a^1\, n_a^2\, m_a^3 + 8\; m_a^1\, n_a^2\, n_a^3 - 8\; n_a^1\, m_a^2\, n_a^3\; . \end{aligned}$$

In our applications the $E2$-instanton will wrap a rigid cycle Ξ. Its intersection with a bulk brane Π_a^B and its image $(\Pi_a^B)'$ is independent of the twisted charge of Ξ,

$$I_{\Xi a} = \prod_{i=1}^{3} (n_\Xi^i\, m_a^i - n_a^i\, m_\Xi^i), \qquad I_{\Xi a'} = -\prod_{i=1}^{3} (n_\Xi^i\, m_a^i + n_a^i\, m_\Xi^i). \tag{5.42}$$

5.3.2 Wrapping numbers and spectrum of a local toy model

We proceed with the construction of a local $SU(5)$ GUT-like model. In this approach[9], the Standard Model arises from a stack of 10 coincident $D6$-branes carrying gauge group $U(5) = SU(5) \times U(1)$, where the abelian part is massive due to the Green-Schwarz mechanism. For simplicity, we choose the GUT stack to be given by non-rigid bulk branes so that the GUT group can be broken down to the Standard Model gauge group by invoking brane-splitting, i.e. by giving suitable VEVs to the GUT Higgs fields in the adjoint of $SU(5)$. Right-handed neutrinos are localized at the intersection of two more stacks a and b of D-branes such that they are indeed singlets under the GUT $SU(5)$. We choose a and b to be likewise given by bulk cycles. The actual "Standard Model" spectrum arises, upon GUT breaking, from 4 chiral generations in the $\mathbf{10}$ of $SU(5)$ as well as chiral multiplets transforming as $\overline{\mathbf{5}}$ localized at the intersection of c and a. The electroweak Higgs fields $\mathbf{5}_H$ arise from the intersection between stack c and b. In table 5.4 we display the wrapping numbers of the stacks a, b and c of D-branes in a particular realization of the described local set-up. This table also contains the intersection numbers of the stacks and their image stacks with the $E2$-instanton to be defined later in equ. (5.48). Table 5.2 gives the multiplicities of the "Standard Model" spectrum. In addition, there is chiral exotic matter which we do not make explicit.

Note that the charges $(-1_a, 1_b)$ of N_R^c under the global symmetries $U(1)_a$ and $U(1)_b$ indeed forbid perturbative Majorana masses. We therefore seek to generate such terms non-perturbatively. In order for the potential instanton-induced Majorana mass terms to yield, via the standard see-saw mechanism, hierarchically small masses for

[9] See e.g. [25, 108] for global constructions of a similar type.

stack	N	$(n^1, m^1) \times (n^2, m^2) \times (n^3, \tilde{m}^3)$	I_{E2x}
E_2	1	$(1,0) \times (0,1) \times (0,-1)$	
a	2	$(1,2) \times (1,1) \times (-1,-1)$	2
b	2	$(-3,-2) \times (1,-1) \times (-1,0)$	-2
c	10	$(1,0) \times (-1,2) \times (-2,-3)$	0

Table 5.1: Wrapping numbers of the local setup.

the neutrino mass eigenstates, the toy model has to allow for perturbatively generated Dirac neutrino masses.

sector	I_{xy}	representation	matter
(c, c')	4	Anti	$\mathbf{10}$
(c, a)	24	$(\overline{c}, a)$	$\overline{\mathbf{5}}$
(c, b)	-24	$(c, \overline{b})$	$\mathbf{5}_H$
(a, b)	32	$(\overline{a}, b)$	N_R^c

Table 5.2: Matter spectrum of the local setup.

This feature is indeed realized, as can be seen from the concrete intersection pattern in table 5.2. The Dirac mass terms are encoded schematically in the coupling

$$H\, L_L\, N_R^c = \mathbf{5}_H\ \overline{\mathbf{5}}\ \mathbf{1}. \tag{5.43}$$

The size of the Dirac mass is suppressed exponentially by the area of the triangle formed by the intersecting branes and thus depends on the Kähler and open string moduli of the background in the way found in [107, 59].

In order for the model to be supersymmetric each stack of branes has to satisfy the two conditions [36]

$$m_x^1\, m_x^2\, m_x^3 - \sum_{I\neq J\neq K} \frac{n_x^I\, n_x^J\, m_x^K}{U^I\, U^J} = 0 \tag{5.44}$$

and

$$n_x^1\, n_x^2\, n_x^3 - \sum_{I\neq J\neq K} m_x^I\, m_x^J\, n_x^K\, U^I\, U^J > 0\ , \tag{5.45}$$

where U^I denotes the complex structure modulus $U^I = R_Y^I/R_X^I$ of the $I-th$ torus with radii R_X^I, R_Y^I. The toy model satisfies the equations above for the following choice of complex structure moduli U^I,

$$U^1 = \sqrt{3}\ , \qquad\qquad U^2 = \frac{2}{\sqrt{3}}\ , \qquad\qquad U^3 = \frac{8}{3\sqrt{3}}\ . \tag{5.46}$$

We stress once more that the brane configuration of table 5.4 as such does not satisfy all of the tadpole cancelation conditions ensuring global consistency and therefore only represents a local model.

5.3.3 The $E2$-instanton

We are now in a position to analyze the $E2$-instanton sector of the toy model defined in the previous section. We are particularly interested in fermion bilinears of type (5.26) for the right-handed neutrinos. As described in detail, they are due to $E2$-instantons wrapping a rigid supersymmetric cycle Ξ which do not give rise to zero modes in the $\Xi - \Xi'$ sector and which are subject to (5.11). Our analysis therefore consists in two steps: First classify the rigid cycles Ξ with no $\Xi - \Xi'$-modes and then distinguish them according to their charged zero modes structure.

The only type of sLags under technical control corresponds to the class of factorizable three-cycles described in section 5.3.1. Even though a complete analysis of the instanton sector should take into account all possible sLags our analysis is therefore forced to content itself with this special class. A closer look reveals that the constraint $\Xi \cap \Xi' = 0$ is extremely restrictive and can be met (at best) by two different types of rigid cycles. Either Ξ and its orientifold image Ξ' are parallel, but separated in at least one of the three tori. In that case the vector-like fermionic zero modes in the $\Xi - \Xi'$ sector carry a mass proportional to the distance between Ξ and Ξ'. Alternatively, one can consider those rigid cycles with the property $\Xi = \Xi'$. For such invariant cycles, the massless modes in the $E2 - E2'$ sector are identical to the geometric moduli of the cycle. Rigidity then guarantees the absence of open string moduli and $E2 - E2'$ zero modes at the same time.

As is immediately clear, the explicit form of the orientifold action on the fixed points in the $\mathbb{Z}_2 \times \mathbb{Z}_2'$ background at hand excludes the first type of cycles. Potential candidate cycles for the second class have to lie on top of one of the four orientifold planes to ensure that the bulk part is indeed mapped to itself under $\Omega\mathcal{R}$. In addition, we have to take into account the non-trivial orientifold action on the g-twisted sector encoded in (5.35). Depending on the choice of $\eta_{\Omega\mathcal{R}g}$, a certain combination of twisted charges of Ξ may also be Ω invariant such that $\Xi = \Xi'$. With our given choice (5.38) for $\eta_{\Omega\mathcal{R}g}$, only those rigid cycles parallel to the x^1-, y^2- and y^3-axis have a chance to be invariant. This can be seen e.g. from the fact that the $\alpha^g_{ij,m}$ are invariant only for $g = \theta$ and $g = \theta\theta'$, cf. equ. (5.35). Due to the additional minus sign in the orientifold projection resulting from the external DD boundary conditions, the non-dynamical gauge group on a stack of N such invariant instantons is $SO(N)$[10].

To completely specify a cycle of this type we have to choose the explicit values for the bulk wrapping numbers, the actual position of the brane and thus the fixed points S_g to be wrapped in the twisted sector and finally the ϵ^g signs.

We start with the bulk wrapping numbers. From (5.11) we need intersection numbers $I_{E2a} = 2$ and $I_{E2b} = -2$ in order for the instanton to exhibit abelian charges $Q_a = 2$ and $Q_b = -2$. Since $E2 = E2'$, the zero modes in the $E2 - a$ and $a' - E2$ are identified and do therefore not count as independent. This uniquely determines the bulk wrapping numbers of the fractional cycle to

$$\Pi^B_{\Xi} \; : \; [(1,0)\,(0,1)\,(0,-1)]. \tag{5.47}$$

We note in passing that by this analysis there exist no $E2$-instantons leading to dangerous open string tadpoles of the form $\Phi\, e^{-S_{inst}}$ for matter between the stacks a, b or c. For perturbatively well-defined string vacua such tadpoles would spoil stability at the quantum level.

Given the bulk wrapping numbers (5.47), we have the following options for the twisted sector: The fractional brane can run through the fixed points $(1,3)$ or $(2,4)$ in the first torus and through $(1,2)$ or $(3,4)$ in the second and third torus (see figure 5.2). Thus, we have 8 different positions for the invariant cycle, together with the mentioned 8 inequivalent sign choices ϵ^g for each position. One example of these 64

[10]Recall that for $D6$-branes wrapping invariant cycles, the gauge group was determined in [36] to be $Sp(2N)$.

different cycles takes the form

$$\Pi_{\Xi} = \frac{1}{4}\Pi_{\Xi}^{B} - \frac{1}{4}\sum_{i,j\epsilon(13)\times(12)} \alpha^{\theta}_{ij,m} + \frac{1}{4}\sum_{j,k\epsilon(12)\times(12)} \alpha^{\theta'}_{jk,n} + \frac{1}{4}\sum_{i,k\epsilon(13)\times(12)} \alpha^{\theta\theta'}_{ik,m}. \qquad (5.48)$$

It corresponds to an $E2$ passing through the origin in each of the three tori and the choice $\epsilon^{g}_{ij} = 1$ in all sectors, as depicted in figure 5.2. The remaining 63 instanton cycles are obvious modifications of this one. One may convince oneself that the choice (5.38) indeed yields $\Xi = \Xi'$, thus qualifying Ξ as an $E2$-instanton cycle relevant for the superpotential.

In the sequel, when analyzing the single $E2$-instanton sector relevant for the Majorana mass terms, we have to consider each of these inequivalent choices of the twisted sector. The final result for the non-perturbative coupling will be the sum of the contribution from each sector. Our explicit computation will be for instanton (5.48) and we will discuss the remaining contributions at the end of section 5.3.4.

It is crucial for the generation of fermion bilinears that there exist no *chiral* zero modes from strings stretching between the $E2$-instanton and the stack c since $I_{\Xi c} = 0$. However, since Ξ and c share the same bulk wrapping numbers in the first torus, there exist vector-like pairs at the intersection of Ξ and c in the second and third torus with mass proportional to twice the distance between Ξ and c in the first torus. In order to avoid *massless* vector-like pairs, we have to assume that the latter stack is separated from the instanton in the second torus by a non-zero distance. In the absence of effects stabilizing the open string moduli, we can freely move along the corresponding flat direction in moduli space.

To summarize, the zero mode structure meets the required constraints to give rise to

Majorana mass terms for the right-handed neutrinos ν_R^c sitting in the superfields N_R^c at the intersection of branes (a,b) carrying abelian charge $(-1_a, 1_b)$ (see table 5.2). One may check that $U(1)_a$ and $U(1)_b$ are indeed both broken as a gauge symmetry since the corresponding vector potentials acquire a Stückelberg-type mass. Recall that this is the *conditio sine qua non* for the instanton to off-set the abelian charge violation of the open string operator in the non-perturbative coupling.

5.3.4 Computation of the Majorana masses

We finally apply the results of section 5.2.2 and obtain the neutrino Majorana mass terms by evaluating the two-point correlator

$$\begin{aligned} < \nu^A \nu^B >_{E2} &= -\frac{1}{2!}\frac{v_{E2}}{16}\frac{g_s}{2\pi}\int d^4x_E \int d^2\theta \int d^2\lambda_a \int d^2\overline{\lambda}_b \, e^{-S_{inst.}} \, e^{Z'} \\ & \sum_{i,j,k,l} < V_{\Theta^\alpha}^{-\frac{1}{2}} V_{\overline{\lambda}_b^k}^{-\frac{1}{2}} V_{\nu_\alpha^A}^{-\frac{1}{2}} V_{\lambda_a^i}^{-\frac{1}{2}} >< V_{\Theta^\beta}^{-\frac{1}{2}} V_{\overline{\lambda}_b^l}^{-\frac{1}{2}} V_{\nu_\beta^B}^{-\frac{1}{2}} V_{\lambda_a^j}^{-\frac{1}{2}} > . \end{aligned} \tag{5.49}$$

For the concrete intersection anglese

$$\begin{aligned} \theta_{ab}^1 &= 0.86, & \theta_{ab}^2 &= -0.54, & \theta_{ab}^3 &= -0.32, \\ \theta_{E2a}^1 &= 0.41, & \theta_{E2a}^2 &= -0.23, & \theta_{E2a}^3 &= -0.18, \\ \theta_{E2b}^1 &= -0.73, & \theta_{E2b}^2 &= -0.77, & \theta_{E2b}^3 &= -0.50 \end{aligned} \tag{5.50}$$

the vertex operators read

$$\begin{aligned}
V_{\nu} &= \ell_s^{\frac{3}{2}}\, \Omega_{ba}\, \nu_{\alpha}\, S^{\alpha}(z)\, \sigma_{1-\theta^1_{ab}}(z)\, e^{-i(\theta^1_{ab}-\frac{1}{2})H_1(z)} \\
&\qquad \times \prod_{I=2}^{3} \sigma_{-\theta^I_{ab}}(z)\, e^{-i(\theta^I_{ab}+\frac{1}{2})H_I(z)}\, e^{ik_\mu X^\mu(z)}\, e^{-\varphi(z)/2}, \\
V_{\lambda_a} &= \Omega_{aE2}\, \lambda_a\, \Sigma(z)\; \sigma_{1-\theta^1_{E2a}}(z)\, e^{-i(\theta^1_{E2a}-\frac{1}{2})H_1(z)} \\
&\qquad \times \prod_{I=2}^{3} \sigma_{-\theta^I_{E2a}}(z)\, e^{-i(\theta^I_{E2a}+\frac{1}{2})H_I(z)}\, e^{-\varphi(z)/2}, \\
V_{\overline{\lambda}_b} &= \Omega_{E2b}\, \overline{\lambda}_b\, \Sigma(z) \prod_{I=1}^{3} \sigma_{1+\theta^I_{E2b}}(z)\, e^{i(\theta^I_{E2b}+\frac{1}{2})H_I(z)}\, e^{-\varphi(z)/2}.
\end{aligned} \tag{5.51}$$

It follows that the angle dependence of the disk amplitude

$$< V_{\Theta^\alpha}^{-\frac{1}{2}} V_{\overline{\lambda}_b^k}^{-\frac{1}{2}} V_{\nu_\alpha^A}^{-\frac{1}{2}} V_{\lambda_a^i}^{-\frac{1}{2}} > = \frac{2\pi}{g_s}\, \ell_s^{\frac{3}{2}}\, C^A_{ik}\; e^{-ik^A_\mu x^\mu_0} \quad (\theta^\alpha \overline{\lambda}^k_b \nu^A_\alpha\, \lambda^i_a) \tag{5.52}$$

is given by

$$C^A_{ik} = \pi \left[4\pi\, \Gamma_{1-\theta^1_{ab},1-\theta^1_{E2a},1+\theta^1_{E2b}} \prod_{I=2}^{3} 4\pi\, \Gamma_{-\theta^I_{ab},-\theta^I_{E2a},1+\theta^I_{E2b}} \right]^{\frac{1}{4}} \sum_{m_j} e^{-\mathcal{A}^A_{ik}(m_j)} \tag{5.53}$$

for index combinations with non-vanishing diagrams. Before turning to this question, we first investigate the instanton suppression factor

$$e^{-S_{inst}} = e^{-\frac{2\pi}{\ell_s^3\, g_s} \mathrm{Vol}_{E2}} = e^{-\frac{2\pi}{\alpha_{\mathrm{GUT}}} \frac{\mathrm{Vol}_{E2}}{\mathrm{Vol}_{\Pi_c}}}, \tag{5.54}$$

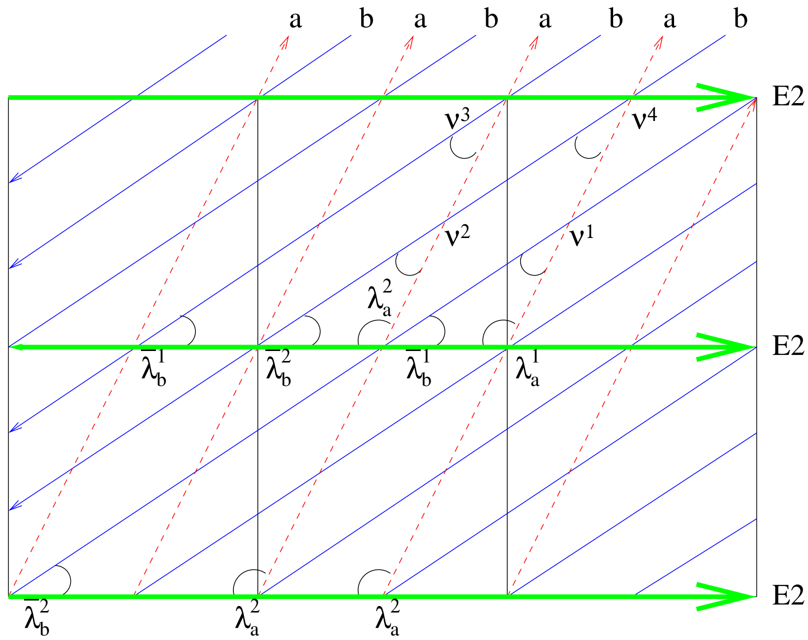

Figure 5.3: Intersection pattern in first torus.

where the last equation uses the standard relation (see e.g.[56])

$$\alpha_{GUT} = g_s \frac{\mathrm{Vol}_{\Pi_c}}{\ell_s^3} \tag{5.55}$$

for α_{GUT} in terms of the volume of the GUT stack c. Given the geometric data of our concrete string vacuum, the ratio $\frac{\mathrm{Vol}_{E2}}{\mathrm{Vol}_{\Pi_a}}$ can easily be computed and is determined entirely by the wrapping numbers in table 5.4 and the complex structure moduli (C.17),

$$\frac{\mathrm{Vol}_{E2}}{\mathrm{Vol}_{\Pi_c}} = \Big(\prod_I \frac{(n_{E2}^I)^2 + (\widetilde{m}_{E2}^I)^2 U_I^2}{(n_c^I)^2 + (\widetilde{m}_c^I)^2 U_I^2}\Big)^{1/2} = \frac{8}{57} \ . \tag{5.56}$$

As discussed in section 5.2.3, for C_{ik}^A to be non-vanishing in each torus the

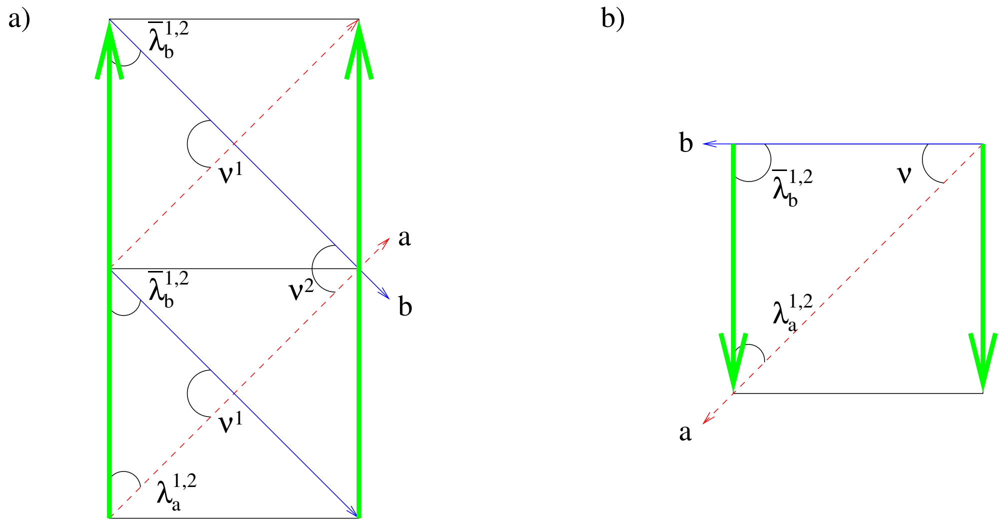

Figure 5.4: Intersection pattern in second and third torus.

modes λ_a^i, $\overline{\lambda}_b^k$ and ν^A have to form a triangle. Let us analyze this nontrivial constraint for our setup. Figure 5.3 displays the intersection in the first torus. One can easily read off that the combinations of λ_a^i, $\overline{\lambda}_b^k$ and ν^A with triangles in the first torus have the same structure as in the example in section 5.2.3.

The remaining two tori are depicted in figure 5.4. While a and b intersect twice in the second torus there is only one intersection in the third one. Most importantly, the replication of λ_a and $\overline{\lambda}_b$ modes is entirely due to multiple intersections in the first torus.

The complete location of a neutrino $\nu^{i,j}$ is described by two upper indices i and j, where i denotes the position in the first torus while j gives the location in the second[11]. Ignoring all higher worldsheet instanton effects we obtain

$$< \nu^A \nu^B >_{E2} \;=\; \frac{2\pi\, v_{E2}}{g_s}\; \overrightarrow{v}^T\, \mathcal{M}\, \overrightarrow{v}\; (2\pi)^4 \delta^4(k^A + k^B)\ . \tag{5.57}$$

Here $\overrightarrow{v}$ is defined as

$$\overrightarrow{v}^T = (\nu^{1,1},\, \nu^{2,1},\, \nu^{3,1},\, \nu^{4,1},\, \nu^{1,2},\, \nu^{2,2},\, \nu^{3,2},\, \nu^{4,2}) \tag{5.58}$$

and the 8×8 matrix $\mathcal{M}$ takes the form

$$\mathcal{M} = x\, M_s\, e^{-\frac{16\pi}{57\alpha_{\rm GUT}}} \begin{pmatrix} A & 0 & B & 0 \\ 0 & C & 0 & D \\ B & 0 & E & 0 \\ 0 & D & 0 & F \end{pmatrix}\ , \tag{5.59}$$

where x is given by

$$x = \frac{\pi^2}{16} \left[4\pi\, \Gamma_{1-\theta^1_{ab},1-\theta^1_{E2a},1+\theta^1_{E2b}} \prod_{I=2}^{3} 4\pi\, \Gamma_{-\theta^I_{ab},-\theta^I_{E2a},1+\theta^I_{E2b}} \right]^{\frac{1}{2}}\; e^{Z'}\ . \tag{5.60}$$

At 'tree-level', i.e. ignoring the corrections due to the one-loop determinant $e^{Z'}$, the numerical factor is approximately $x \approx 0.87$. The building blocks of $\mathcal{M}$ in (5.59) take

[11] Recall that a and b are bulk branes and so that each intersection point gives rise to 4 right-handed neutrinos $\nu_R^c \equiv \nu$. This yields the overall $4 \times 8 = 32$ of them. We leave the additional factor of 4 implicit.

a similar form as in the simpler example in section 5.2.3,

$$A = \begin{pmatrix} e^{-(\alpha+\beta+2\kappa+2\tau)} & \frac{1}{2}\left(e^{-2(\alpha+\kappa+\tau)} + e^{-2(\beta+\kappa+\tau)}\right) \\ \frac{1}{2}\left(e^{-2(\alpha+\kappa+\tau)} + e^{-2(\beta+\kappa+\tau)}\right) & e^{-(\alpha+\beta+2\kappa+2\tau)} \end{pmatrix} \tag{5.61}$$

$$B = \begin{pmatrix} e^{-(\alpha+\beta+\kappa+\mu+2\tau)} & \frac{1}{2}\left(e^{-(2\alpha+\kappa+\mu+2\tau)} + e^{-(2\beta+\kappa+\mu+2\tau)}\right) \\ \frac{1}{2}\left(e^{-(2\alpha+\kappa+\mu+2\tau)} + e^{-(2\beta+\kappa+\mu+2\tau)}\right) & e^{-(\alpha+\beta+\kappa+\mu+2\tau)} \end{pmatrix}, \tag{5.62}$$

where α, β, γ and δ are defined in (5.29), κ (μ) denotes the area of the triangle spanned by $\nu^{i,1}$ ($\nu^{i,2}$), λ_a and $\overline{\lambda}_b$ in the second torus, whereas τ is the area in the third torus. The other 4 building blocks can be easily obtained in the following manner. Replacing α and β in A (B) by γ and δ yields C (D), replacing in addition also κ by μ one obtains F. In order to get E we just substitute in A κ by μ.

The suppression due to world sheet instantons depends crucially on the open string moduli. For particular choices of those the area the triangles form vanishes and there is no suppression at all, e.g. for the choice in figure (5.3, 5.4) there is no suppression for the coupling $\nu^{3,2}\,\nu^{4,2}$. With $M_s = 1.2 \times 10^{18} GeV$ and for α_{GUT} within the range of $\frac{1}{24}$ and $\frac{1}{20}$ [12] the factor in (5.59) takes values in the range of $(0.1 - 1) \times 10^{10} GeV$.

As discussed, the above coupling is the contribution of just 1 out of 8×8 rigid factorizable sLags with the required zero-mode structure to yield Majorana mass terms. The first factor is due to the two different positions of the $E2$-brane per two-torus, corresponding to which of the fixed points it passes through (see figure

[12] Note that the familiar value $\alpha_{\mathrm{GUT}} = \frac{1}{24}$ refers to the exact MSSM spectrum. Given the large amount of exotic matter of the toy model, we work, just for illustration, with a bigger value for α_{GUT}.

5.2). Clearly, each of these 8 choices comes with different areas of the worldsheet triangles and therefore relative suppression factors between families. For each geometric position we have to sum in addition over 8 inequivalent choices of signs of the twisted charges ϵ^g_{ij}. In our example with all other $D6$-branes of the local model wrapping bulk cycles, the result for the two-point coupling is independent of these twisted charges. In particular, this is true for the one-loop determinant $e^{Z'}$ (5.7) since the twist part of the $E2$ boundary state is orthogonal to the boundary states of the bulk $D6$-branes and the cross-cap. It follows that each of the 8 geometrically distinct sectors just contributes with an additional factor of 8, i.e. the various contributions from the factorizable rigid $E2$-instantons with appropriate zero modes do add up to a non-vanishing result. This is a fortunate result since in principle, one might have feared non-trivial cancelations. Indeed, for heterotic $(0,2)$ models explicit examples of such cancelations are known for special constructions such as (half-)linear sigma models [109], even though they do not correspond to the generic situation. Of course, a complete classification of instanton effects would require control over all special Lagrangian manifolds and seems out of reach even on toroidal backgrounds.

5.4 Yukawa couplings in $SU(5)$ GUT-like models

Grand Unified $SU(5)$-like models based on intersecting $D6$-branes generically suffer from the absence of the important Yukawa coupling $\mathbf{10}\cdot\mathbf{10}\cdot\mathbf{5}_H$ and are therefore so far not considered realistic. Such models were first generally proposed in [110] and explicitly constructed for intersecting D6-branes in [111, 112, 25, 54, 113, 114].

The minimal intersecting $D6$-brane model realizing $SU(5)$ GUT is shown in table 5.3. Such a model involves only two stacks a and b of branes giving rise to a

sector	number	$U(5)_a \times U(1)_b$ reps.	$U(1)_X$
(a', a)	$3 + (1, 1)$	$\mathbf{10}_{(2,0)}$	$\frac{1}{2}$
(a, b)	3	$\overline{\mathbf{5}}_{(-1,1)}$	$-\frac{3}{2}$
(b', b)	3	$\mathbf{1}_{(0,-2)}$	$\frac{5}{2}$
(a', b)	1	$\mathbf{5}^H_{(1,1)} + \overline{\mathbf{5}}^H_{(-1,-1)}$	$(-1) + (1)$

Table 5.3: GUT $SU(5)$ intersecting D6-brane model, $U(1)_X = \frac{1}{4}U(1)_a - \frac{5}{4}U(1)_b$. The multiplet $\mathbf{10}_{(2,0)}$ also contains the GUT Higgs field which should appear as a vector-like pair.

$U(5)_a \times U(1)_b$ gauge symmetry. The $U(5)_a$ splits into $SU(5)_a \times U(1)_a$, so that there are two abelian gauge groups $U(1)_a \times U(1)_b$. One linear combination of these is anomalous and becomes massive via the generalized Green-Schwarz mechanism. However, it survives as a global symmetry in the effective action. Matter fields transforming as $\mathbf{10}$ under $SU(5)_a$ arises at the intersections of stack a with its image a' and the matter fields transforming as $\overline{\mathbf{5}}$ as well as Higgs fields $\mathbf{5_H}$ and $\overline{\mathbf{5}}_\mathbf{H}$ are located at intersections of stack a with b and b'. For a globally consistent model the concrete wrapping numbers decide if the anomaly free combination $U(1)_X$ of the abelian groups really remains massless. If not, the model is of the usual Georgi-Glashow type, while in the presence of a massless $U(1)_X$ it represents a flipped $SU(5)$ model.

From the $U(1)_{a,b}$ charges it is clear that perturbatively the two Yukawa couplings

$$\langle \mathbf{10}_{(2,0)}\, \overline{\mathbf{5}}_{(-1,1)}\, \overline{\mathbf{5}}^H_{(-1,-1)} \rangle, \qquad \langle \overline{\mathbf{5}}_{(-1,1)}\, \mathbf{1}_{(0,-2)}\, \mathbf{5}^H_{(1,1)} \rangle \tag{5.63}$$

are present. Focussing for concreteness on flipped $SU(5)$, these give masses to the heavy (u,c,t)-quarks and the leptons. However, the Yukawa couplings for the light (d,s,b)-quarks

$$\langle \mathbf{10}_{(2,0)}\, \mathbf{10}_{(2,0)}\, \mathbf{5}^H_{(1,1)} \rangle \tag{5.64}$$

are not invariant under the two $U(1)$s. Note that this interaction is also of key importance for the solution of the doublet-triplet splitting problem for flipped $SU(5)$. For a non-zero VEV of the Standard Model singlet component in $\mathbf{10} + \overline{\mathbf{10}}$ there is no partner for the weak Higgs doublet to pair up with. For Georgi-Glashow $SU(5)$ models, the role of (u,c,t) and (d,s,b)-quarks has to be interchanged and the GUT Higgs field is usually in the adjoint representation of $SU(5)$.

Our main result is that the coupling (5.64) can be generated by an E2-instanton of suitable zero mode structure. Concretely, the instanton has to wrap a rigid three-cycle Ξ invariant under the orientifold projection $\Omega\overline{\sigma}$ and carrying gauge group $O(1)$ [74, 75, 76]. This guarantees that the uncharged part of the instanton measure only contains the four bosonic and two fermionic modes x^μ, θ^α required for superpotential contributions. Now, from the arguments in [67, 70, 69] the coupling (5.64) requires in addition charged fermionic zero modes at intersections between Ξ and the D6-branes. These are responsible for an effective $U(1)$ charge of the instanton which can compensate for the excess of $U(1)$ charge of the operator (5.64). For intersection numbers w we get five zero modes $\overline{\lambda}^i_{[5]}$ from the intersection of the instanton with $D6_a$ and one zero mode $\overline{\nu}_{[-1]}$ from the intersection with $D6_b$. The computation of the resulting couplings can be performed following the prescription proposed in [67] and exemplified for a concrete local model in [78]. Since the instanton lies in an $\Omega\overline{\sigma}$ invariant position, one can absorb these six matter zero modes with the three disc diagrams depicted in figure 5.5. All charge selection rules are satisfied. Each

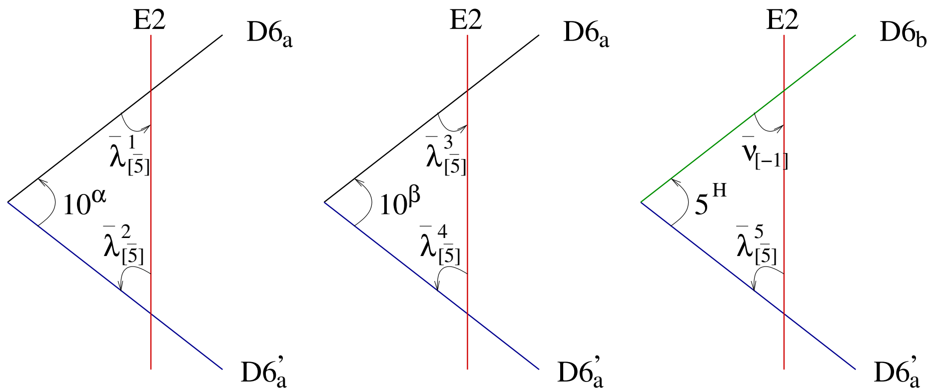

Figure 5.5: Absorption of zero modes

tree-level coupling is by itself a sum over world-sheet instantons connecting the three intersection points in the disc diagrams like $10^{\alpha}_{[ij]}\,\overline{\lambda}^i\overline{\lambda}^j$, where $\alpha = 1, 2, 3$ denotes the generation index. These discs induce open string dependent terms in the instanton moduli action of type

$$\exp\left(-S_{mod}\right) = \exp\left(C^{10}_{\alpha}\, 10^{\alpha}_{[ij]}\,\overline{\lambda}^i\overline{\lambda}^j + C^5\, 5_m\,\overline{\lambda}^m\overline{\nu}\right) \tag{5.65}$$

which are integrated over the charged fermionic measure $\int d^5\overline{\lambda}\, d\overline{\nu}$. Due to its Grassmannian nature, the index structure of the Yukawa coupling is

$$W_Y = Y^{\alpha\beta}_{\langle \mathbf{10}\,\mathbf{10}\,\mathbf{5}_H\rangle}\,\epsilon_{ijklm}\;\mathbf{10}^{\alpha}_{ij}\,\mathbf{10}^{\beta}_{kl}\,\mathbf{5}^H_m\; e^{-S_{E2}}\; e^{Z'}\;\;, \tag{5.66}$$

where the instanton action can be written as $S_{E2} = \frac{2\pi}{\alpha_{GUT}}\frac{\mathrm{Vol}_{E2}}{\mathrm{Vol}_{D6}}$. Here we have used that the volume of the $D6_a$-brane determines the gauge coupling at the GUT scale. Note that the ratio $\mathrm{Vol}_{E2}/\mathrm{Vol}_{D6}$ depends only on the complex structure moduli, which are known to be constrained by the D-term supersymmetry conditions for the D6-branes. The superpotential coupling W_Y also depends on the holomorphic part of the one-loop determinant $e^{Z'}$ arising from the annulus and Möbius diagrams ending on the instanton and the D6-branes or O-plane, respectively [67]. As shown in [94, 87, 95], these are related to one-loop gauge threshold corrections [64, 103].

One observes that the replication of the zero modes $\overline{\lambda}_i$ is entirely due to Chan-Paton indices so that each of the discs in figure 5.5 depends only on the family index and not on the pair of zero modes to which the open string operator couples. Therefore the final instanton generated Yukawa coupling factorizes into

$$Y^{\alpha\beta}_{\langle \mathbf{10}\,\mathbf{10}\,\mathbf{5}_H \rangle} = Y^{\alpha}\, Y^{\beta} \tag{5.67}$$

and the induced mass matrix for the quarks is always of unit rank. In order to exhibit non-perturbative masses for all three generations the model therefore has to possess three independent E2-instanton sectors.

Concerning the suppression scale of the instanton generated Yukawa coupling, for $\alpha_{GUT} = 1/24$, $\mathrm{Vol}_{E2}/\mathrm{Vol}_{D6} = (R_{E2}/R_{D6})^3$ with the moderate suppression $R_{D6} = \frac{7}{2}\, R_{E2}$, the main instanton suppression factor is $\exp\left(-S_{E2}\right) \simeq 3 \cdot 10^{-2}$. Since the E2-instanton lies in a $\overline{\sigma}$ invariant position it seems natural that the length scale of the internal volume is smaller than that of the $U(5)$ stack of D6-branes.

To summarize, we find that D-brane instantons can generate the $\langle \mathbf{10}\,\mathbf{10}\,\mathbf{5}_H \rangle$ Yukawa coupling. The described mechanism works both for Georgi-Glashow as well as flipped $SU(5)$ models. It is particularly attractive for the case of flipped SU(5): Here the E2-instanton not only generates the desired couplings, but the complex structure dependent exponential suppression $\exp(-S_{E2})$ can explain, as a bonus, the hierarchy between the (u, c, t) quarks and the (d, s, b) quarks.

5.5 Lifting of undesired chiral matter

Most semi-realistic string models constructed so far come with exotic vector-like states. For the phenomenological features of such models it is important to know whether these states can become massive. To date mostly perturbative mechanism have been discussed in the literature for generating such mass terms. In this section we demonstrate for a concrete globally consistent model that E2-instantons can also generate such mass terms. We are working with a Type IIA orientifold background which serves as a simple model based on $U(4)$ gauge symmetry with a certain number of matter fields in the anti-symmetric representation of $U(4)$. For a similar global model in Type I theory see [91, 75].

Concretely, we consider the orientifold $T^6/\mathbb{Z}_2\times\mathbb{Z}_2'$ with Hodge numbers $(h_{11}, h_{12}) = (3, 51)$. We employ the notation of [36], to which we refer for details of the geometry and the construction of rigid cycles (see also [115]). The orbifold group is generated by θ and θ' acting as reflection in the first and last two tori, respectively.

Table 5.4 displays the wrapping numbers of the simplest globally consistent, supersymmetric model for the choice that the $O6$-plane lying parallel to the instanton is an $O6^+$-plane with the other three being $O6^-$-planes.

It involves only one stack of four bulk $D6$-branes (and its orientifold image) carrying $U(4)$ gauge group with three superfields in the adjoint representation. One can easily check that all consistency conditions are indeed satisfied and supersymme-

stack	N	$(n^1, m^1) \times (n^2, m^2) \times (n^3, m^3)$	I_{E2x}
$U(4)$	8	$(1, -1) \times (1, 1) \times (1, 1)$	1
E_2	1	$(1, 0) \times (0, 1) \times (0, -1)$	

Table 5.4: Wrapping numbers of $U(4)$ global model.

try fixes the complex structure moduli to the sublocus $U^1 - U^2 - U^3 = U^1 U^2 U^3$. The model has also 32 chiral superfields in the conjugate anti-symmetric representation $\bar{\mathbf{6}}$ of $U(4)$. Note that the $\mathbf{6}$ of $SU(4)$ is a real representation so that these states are chiral only with respect to the diagonal $U(1) \subset U(4)$. Since $U(1)$'s can be broken by instantons, there is a chance that mass terms are non-perturbatively generated.

As shown in [78], to which we refer for more details, the background of this model exhibits one class of rigid $O(1)$ instantons, whose bulk part is also shown in Table 5.4. The intersection number of these $E2$-instantons with the bulk $D6$ branes is exactly one. Taking into account the Chan-Paton label of the gauge group $U(4)$, there are four fermionic zero modes localized at the intersection of $E2$ and the matter branes. As shown in figure 5.6, these four fermionic zero modes λ can be saturated via two disc diagrams thereby generating mass terms for the matter fields in the $\bar{\mathbf{6}}$ representation.

We denote the 32 matter superfields as $\Phi_I^A = \phi_I^A + \theta \psi_I^A$, where the lower index $I = 1, \ldots, 8$ refers to the various intersections on T^6 and $A = 1, \ldots, 4$ counts the different orbifold images.

As in the previous section, we can compute the disc diagrams in figure 5.6. Taking also the Grassmannian nature of the fermionic zero modes into account, the

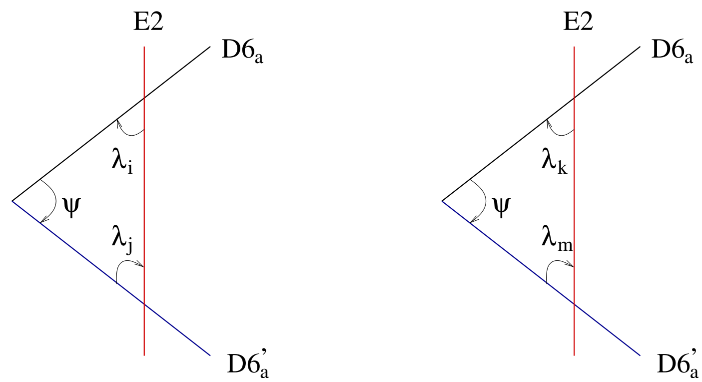

Figure 5.6: Absorption of the zero modes

overall structure of the generated mass terms is

$$\mathcal{L}_{\text{mass}} = C' M_s \, e^{-S_{E2}} \; \epsilon_{ijkl} \, M^{I,J}_{A,B} \, \left(\psi^A_I\right)_{ij} \, \left(\psi^B_J\right)_{kl} \tag{5.68}$$

with the instanton action $S_{E2} = \frac{2\pi}{\alpha_{SU(4)}} \frac{V_{E2}}{V_{D6}}$. Moreover, C' includes all angle dependent constants due to the CFT-computation as well as due to integration over all bosonic and fermionic zero modes [78, 95]. Since the four instanton zero modes arise from the Chan-Paton factors of $U(4)$, the mass matrix factorizes into

$$M^{I,J}_{A,B} = h^I_A \, h^J_B, \tag{5.69}$$

where these factors are essentially the disc amplitudes in figure 5.6 containing a sum over world-sheet instantons. Due to the factorized form (5.69), one linear combination of the 32 matter fields receives a mass. This exemplifies that string instantons can also generate mass terms for exotic matter fields.

5.6 Higher fermionic F-terms

Our discussion has hitherto focussed on $O(1)$ instantons which are either rigid such that they give rise to genuine superpotential terms. Alternatively, there are situations where additional zero modes induce so-called higher fermionic F-term couplings in the effective action. In the dual Type I/heterotic model this effect was first described in [116][13]. There it arises for $E1/$ worldsheet-instantons moving in a family.

Beasley and Witten found that such instantons can generate higher fermionic couplings for the closed string fields [116]. In superspace notation, these are encapsulated in interactions of the form

$$S = \int d^4x\, d^2\theta\ w_{\overline{i}\overline{j}}\,(\Phi)\overline{\mathcal{D}}^{\dot{\alpha}}\overline{\Phi}^{\overline{i}}\overline{\mathcal{D}}_{\dot{\alpha}}\overline{\Phi}^{\overline{j}} \qquad (5.70)$$

for the simplest case that the instanton moves in a one-dimensional moduli space. Note that supersymmetry requires a holomorphic dependence of $w_{\overline{i}\overline{j}}\,(\Phi)$ on the superfields Φ.

On the type IIA side, these instantons correspond to non-rigid $O(1)$ instantons such that the chiral reparametrization modulini $\chi_I^{\alpha}, I = 1, \ldots, b_1(\Xi)$ are anti-symmetrised and therefore projected out under the orientifold action. First reproduce Beasley' and Witten's result and show that it extends to open string superfields as well. Later we show that rigid $U(1)$ instantons generically give derivative corrections to the complex structure moduli.

[13] For another example in the context of heterotic M-theory see [117].

5.6.1 Non-rigid $O(1)$ instantons

We start with a non-rigid $O(1)$ instanton of first kind wrapping a cycle Ξ with $b_1(\Xi) = 1$, such that $c\,\overline{c}$ and $\overline{\chi}^{\dot{\alpha}}$ survive the orientifold action. The resulting uncharged part of the measure takes the form

$$\int d^4x\, d^2\theta \prod_I c\,\overline{c}\,\overline{\chi}^{\dot{\alpha}}. \tag{5.71}$$

Let us assume for time being no further charged zero modes in the $E2 - D6$ sector. Denoting by $\mathcal{T} = T + \theta^\alpha t_\alpha$ the $\mathcal{N} = 1$ chiral superfield associated with the Kähler moduli, we can absorb the instanton modulini by pulling down from the moduli action two copies of the schematic anti-holomorphic coupling $\overline{\chi}^{\dot{\alpha}}\overline{t}_{\dot{\alpha}}$.

In general the open-closed amplitude $\langle\overline{\chi}^{\dot{\alpha}}\overline{t}_{\dot{\alpha}}\rangle$ does not violate any obvious selection rule of the $\mathcal{N} = (2,2)$ worldsheet theory and is therefore expected to induce the above coupling[14].

Similarly, the two θ-modes can be soaked up by the holomorphic coupling $\theta^\alpha u_\alpha$ involving the fermionic partners of the complex structure moduli encoded in the superfield $\mathcal{U} = U + \theta^\alpha u_\alpha$. This results in a four-fermion interaction of the schematic form $e^{-S_{E2}}\, u u\, \overline{t}\overline{t}$. Note that the coupling of the complex and Kähler structure modulini only to the universal and reparametrization zero modes, respectively, is a consequence of $U(1)$ worldsheet charges of the associated vertex operators.

[14] In particular, the total U(1) worldsheet charge is conserved. Still there might be situations, such as factorizable 3-cycles on $(T^2)^3$, where some of the separate U(1) charges are violated by this coupling.

Alternatively, we can absorb one pair of $\theta^\alpha \overline{\chi}^{\dot\alpha}$ in a coupling of the form

$$\theta \sigma^\mu \overline{\chi}\, \partial_\mu \overline{T} \tag{5.72}$$

which follows from evaluating the amplitude $\langle \theta^\alpha\ \overline{\chi}^{\dot\alpha}\ \overline{T} \rangle$. Integrating out two copies of this term yields the derivative superpartner to the above four-fermi term. This can be summarized in superspace notation by writing

$$S = \int d^4x\, d^2\theta\ e^{-\mathcal{U}(\Xi)}\ f_{\bar{i},\bar{j}} \left(e^{-\mathcal{T}_i}, e^{-\Delta_i} \right) \overline{\mathcal{D}}^{\dot\alpha} \overline{\mathcal{T}}^{\bar{i}} \overline{\mathcal{D}}_{\dot\alpha} \overline{\mathcal{T}}^{\bar{j}}, \tag{5.73}$$

where $\mathcal{U}(\Xi)$ is associated with the specific combination of complex structure moduli appearing in the classical instanton action and the holomorphic function $f_{\bar{i},\bar{j}}$ depends in general on the Kähler and open string moduli of the $D6$-branes Δ_i.

In the presence of a suitable number of charged λ zero-modes there exist, in addition to these closed string couplings, terms which generate higher fermi-couplings also for the matter fields. Consider again for simplicity the case $b_1(\Xi) = 1$. If the Chan-Paton factors and worldsheet selection rules only allow the λ modes to couple holomorphically to the chiral open string superfields, as for the generation of a superpotential, the instanton induces an interaction as in (5.73), but with $e^{-\mathcal{U}(\Xi)}$ simply replaced by $e^{-\mathcal{U}(\Xi)} \prod_{a_i,b_i} \Phi_{a_i,b_i}$ (and modified coupling $f_{\bar{i},\bar{j}}$).

For suitable configurations, the action can also pick up derivative terms directly involving the open string fields. For this to happen the instanton moduli action has to contain couplings of the form[15]

$$\overline{\chi}^{\dot{\alpha}}_{1/2}\lambda^{a}_{-1/2}\ (\overline{\psi}_{1/2})_{\dot{\alpha}}\ \overline{\lambda}^{b}_{-1/2}, \tag{5.74}$$

where the fermionic matter field $\overline{\psi}^{\dot{\alpha}}_{1/2}$ lives at the intersection $D6_a - D6_b$ and lies in the anti-chiral superfield $\overline{\Phi} = \overline{\phi} + \overline{\tau}\overline{\psi}$.

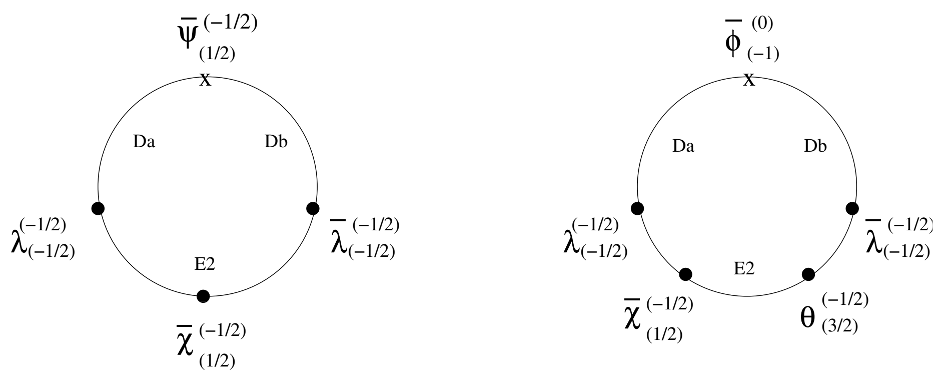

Figure 5.7: Absorbtion of θ and $\bar{\chi}$-modes leading to F-terms.

Integrating out two copies of this interaction term brings down the fermion bilinear $\overline{\psi}_{1/2}\overline{\psi}_{1/2}$. In addition, the two θ^{α} modes again pull down a bilinear of *chiral* fermions u^{α} or, in the presence of more λ modes, ψ^{α}_{ab}, as in the case of superpotential contributions. This induces again a four-fermi coupling. Alternatively, we can absorb one pair of $\theta^{\alpha}\overline{\chi}^{\dot{\alpha}}$ in a coupling of the form

$$\theta^{\alpha}_{3/2}\ \overline{\chi}^{\dot{\alpha}}_{1/2}\ \lambda^{a}_{-1/2}\ \overline{\phi}_{-1}\ \overline{\lambda}^{b}_{-1/2}. \tag{5.75}$$

After bringing the $\overline{\phi}_{-1}$ into the zero ghost picture this clearly generates a

[15]The subscripts denote the worldsheet $U(1)$-charges.

derivative coupling of the form $\theta\sigma^\mu\overline{\chi}\,\lambda^a\,\partial_\mu\overline{\phi}\,\overline{\lambda}^b$. Integrating out two copies of this term yields the derivative superpartner to the above four-fermi term.

5.6.2 Rigid $U(1)$ instanton

In this section we are interested in the induced couplings if the uncharged measure merely takes the form

$$\int d^4x\, d^2\theta\, d^2\overline{\tau} \tag{5.76}$$

in the first place. Consider therefore a rigid $U(1)$ instanton with the geometric intersection numbers

$$\Xi' \cap \Xi = 0 = \Pi_{O6} \cap \Xi. \tag{5.77}$$

The uncharged zero mode measure (5.76) is to be supplemented by additional charged zero modes λ if present. Since there are no zero modes in the $E2 - E2$' sector which would be sensitive to the orientifold action, we might expect this type of instantons to be describable in terms of half-BPS instantons of the underlying $\mathcal{N} = 2$ supersymmetry preserved by the internal Calabi-Yau before orientifolding. The correction to the complex structure moduli space metric by $E2$-instantons in type IIA Calabi-Yau compactification has been discussed recently in [118][16]. Following this logic, we would anticipate the generation of $E2$-corrections to the complex structure Kähler potential

[16] Recall that the local factorization of the moduli space describing the vector and hypermultiplets in general $\mathcal{N} = 2$ compactifications [119] forbids corrections to the Kähler moduli since the dilaton sits in a hypermulitplet.

by the $U(1)$ instanton described by (5.76).

However, while the chiral Goldstino modes θ are indeed associated with the breakdown of the $\mathcal{N} = 1$ subalgebra of this $\mathcal{N} = 2$ symmetry which is preserved by the orientifold, their anti-chiral partners $\overline{\tau}$ correspond to the orthogonal $\mathcal{N} = 1$ subalgebra. The above measure (5.76) does therefore not cover the full $\mathcal{N} = 1$ superspace as required for the generation of a Kähler potential. Rather, the integral is only over half of the $\mathcal{N} = 1$ superspace. While this calls for the generation of an F-term as opposed to a D-term, the additional fermionic zero modes $\overline{\tau}$ will result in higher fermionic couplings of Beasley-Witten type.

An important difference to the F-terms discussed previously is that now only the complex structure moduli receive derivative corrections. Denote by w and a the scalar and axionic parts of the scalar component $U = w - ia$ of a complex structure superfield. Then evaluation of the amplitudes $\langle \theta\, \overline{w}\, \overline{\tau} \rangle$ and $\langle \theta\, \overline{a}\, \overline{\tau} \rangle$ gives rise to the terms

$$\theta\, \sigma^{\mu}\, \overline{\tau}\; \partial_{\mu}\, \overline{w}^{i}, \qquad \theta\, \sigma^{\mu}\, \overline{\tau}\; \partial_{\mu}\, \overline{a}^{i}, \tag{5.78}$$

in the moduli action. The absence of analogous terms for the Kähler moduli is a consequence of $U(1)$ worldsheet charge conservation. Integrating out two copies thereof indeed generates a derivative coupling of the form $e^{-S_{E2}}\, \partial \overline{U} \partial \overline{U}$. Together with their fermionic partners, the derivative F-terms can be summarized by

$$S = \int d^4x\, d^2\theta\; e^{-\mathcal{U}(\Xi)}\; f_{\bar{i},\bar{j}}\left(e^{-\mathcal{T}_i}, e^{\Delta_i}\right)\; \overline{\mathcal{D}}^{\dot{\alpha}} \overline{\mathcal{U}}^{\bar{i}}\, \overline{\mathcal{D}}_{\dot{\alpha}} \overline{\mathcal{U}}^{\bar{j}} + \;\; h.c., \tag{5.79}$$

where the complex conjugate part is due to the anti-instanton contribution. In the presence of charged zero modes λ these F-term corrections for the complex structure moduli involve appropriate powers of charged open string fields required to soak up the λ modes. This amounts to replacing $e^{-\mathcal{U}(\Xi)} \rightarrow e^{-\mathcal{U}(\Xi)} \prod_i \Phi_{a_i,b_i}$.

Chapter 6

Lifting of D-Instanton Zero Modes

The main obstacle for finding appealing global string vacua exhibiting a non-perturbative superpotential of the described type are the severe restrictions on the zero mode structure of the instanton. First, in the absence of other mechanisms to lift the fermionic zero modes associated with deformations of the cycle, the instanton has to be rigid. Secondly, the generic $U(1)$-instanton exhibits four instead of two Goldstino modes, required for the generation of a superpotential. If the instanton lies on top of an appropriate orientifold plane, the two extra modes $\overline{\tau}^{\dot{\alpha}}$ are projected out and the instanton can generate a superpotential. Given its significance for the topography of the landscape of string vacua, it is obviously quite important to investigate if this is actually the only configuration of D-brane instantons which corrects the superpotential.

The key point is to decide if there exists a way to lift the two extra Goldstinos $\overline{\tau}^{\dot{\alpha}}$. Generally speaking, this requires contact terms in the instanton moduli action involving the zero modes in question such that they can be soaked up in the path integral without giving rise to higher derivative or higher fermionic terms in the non-perturbative couplings.

We investigate two different strategies to achieve this. In section 6.1, we analyze couplings of the $\overline{\tau}^{\dot{\alpha}}$ modes to massless states in the $E2 - E2'$ sector, which likewise have to be absorbed. As a consequence of the D-term constraints for the bosonic zero modes a non-zero result requires the presence of a non-vanishing Fayet-Iliopoulos term. The latter arises after slightly deforming the background such that the $\Xi - \Xi'$ pair of instantonic branes recombines.

In section 4.3 we describe in detail the zero mode structure of the $U(1)$ instantons and how it changes by the process of condensation of the bosonic modes. We find that for chiral $\Xi - \Xi'$ recombination, due to charge conservation the recombined object always contains extra fermionic zero modes which cannot be absorbed perturbatively by pulling down either closed string or matter fields [81]. Alternatively we show in section 6.1.4 that including additional instantons might lift these zero modes giving a non-vanishing contribution to the superpotential [120].

Before investigating such multi-instanton superpotential contributions we discuss the recombination of non-chiral $\Xi - \Xi'$ instantons. In that case one obtains after recombination an $O(1)$ instanton with deformations. In the Type I dual model it corresponds to an $E1$-instanton which wraps a holomorphic curve moving in a family as discussed by Beasley-Witten [116]. Such instantons can generate multi-fermion couplings also for matter field superpotentials and under certain circumstances can also contribute to the superpotential [81, 121].

An alternative mechanism, speculated upon already in the literature [74, 76, 96], is the lifting of the $\overline{\tau}^{\dot{\alpha}}$ modes after including supersymmetric background fluxes.

The hope would be that in their presence the instanton does not feel the full $\mathcal{N} = 2$ supersymmetry algebra preserved locally away from the orientifold, but only the $\mathcal{N} = 1$ subalgebra preserved by the fluxes. This should then result in only two as opposed to four Goldstinos.

The lifting of reparametrization zero modes of $M5$-brane or Type IIB $D3$-brane instantons has been studied in detail [122, 123, 124, 125, 126, 127] (see also [128, 129, 130]). The analysis consists in determining the bilinear couplings of the fermionic zero modes to the background fluxes responsible for their lifting. In section 6.2 we recall, building upon the expressions for the fermion bilinears derived in [124, 125], that in Type IIB orientifolds a lifting of the $\overline{\tau}^{\dot{\alpha}}$ of $E3$-instantons is not possible as long as one sticks to supersymmetric three-form flux. As we then show, this generically continues to hold even for $E3$-instantons with gauge flux which are mirror symmetric to Type IIA $U(1)$ instantons at general angles. A possible exception are compactifications with divisors allowing for anti-invariant two-cycles.

6.1 Instanton recombination

So far we have only considered instantons not intersecting their orientifold image, i.e those without uncharged zero modes arising from the $E2 - E2'$ sector. For more generic instantons exhibiting such zero modes to give contributions to the superpotential the fermionic zero modes arising at the intersections of $E2$ and $E2'$ need to be lifted in addition to the $\overline{\tau}^{\dot{\alpha}}$ modes [81]. A reason to expect that this is possible is the following: For $D6$-branes it is known that under certain circumstances a pair of $D6$-$D6'$ branes can recombine into a new sLag D6-brane which obviously wraps an

$\Omega\overline{\sigma}$ invariant three-cycle. If a similar story also applies to pairs of $E2 - E2'$ instantonic branes, the recombined objects would be candidates for new $O(1)$-instantons contributing to the superpotential. Consequently, also the disjoint sum of $E2$ and $E2'$ prior to recombination should yield a superpotential contribution.

6.1.1 Zero mode structure on $U(1)$ instantons

Consider a $U(1)$-instanton wrapping a general rigid cycle $\Xi \neq \Xi'$. From the $E2 - E2$ and $E2' - E2'$ sectors we now have the zero mode measure

$$\int d^4x\, d^2\theta\, d^2\overline{\tau}. \tag{6.1}$$

If such an instanton also intersects the $D6$-branes present in the model, this yields the fermionic zero-modes listed in table 4.2. From there, the overall $U(1)_E$ charge of these matter zero modes can be read off,

$$\begin{aligned}\sum_i Q_E(\lambda^i) &= \sum_a N_a \left(-(\Xi\cap\Pi_a)^+ + (\Xi\cap\Pi_a)^- - (\Xi\cap\Pi_{a'})^+ + (\Xi\cap\Pi_{a'})^-\right)\\ &= -\sum_a N_a\, \Xi\circ(\Pi_a+\Pi_{a'}) = -4\;\Xi\circ\Pi_{O6}.\end{aligned} \tag{6.2}$$

In the last line we have used the tadpole cancellation condition[1]. This shows that in a globally consistent model the total $U(1)_E$ charge of all matter zero modes is proportional to the chiral intersection between the instanton and the orientifold plane. For an $\Omega\overline{\sigma}$ invariant instanton this last quantity vanishes, whereas for a generic $U(1)$

[1]Notice that Π_{O6} denotes the total homological charge of all orientifold fixed planes present in the background. In what follows we will always refer to the effective orientifold projection which arises after taking into account the contribution from all different sectors, which may of different types individually.

instanton it does not.

If $\Xi \circ \Pi_{O6} \neq 0$, there must be additional charged zero modes in order for the zero mode measure to be $U(1)_E$ invariant. Indeed there are also zero modes from the $E2 - E2'$ intersection. This is the open string sector which is invariant under $\Omega\overline{\sigma}$ and gets symmetrized or anti-symmetrized. Following the rules spit out in chapter 5.1 for a single $U(1)$ instanton we get the zero modes shown in Table 6.1.

zero mode	$(Q_E)_{Q_{ws}}$	Multiplicity
$m, \overline{m}$	$(2)_1$,$(-2)_{-1}$	$\frac{1}{2}(\Xi' \circ \Xi + \Pi_{O6} \circ \Xi)$
$\overline{\mu}^{\dot{\alpha}}$	$(-2)_{1/2}$	$\frac{1}{2}(\Xi' \circ \Xi + \Pi_{O6} \circ \Xi)$
μ^{α}	$(2)_{-1/2}$	$\frac{1}{2}(\Xi' \circ \Xi - \Pi_{O6} \circ \Xi)$

Table 6.1: Charged zero modes on $E2 - E2'$ intersection

6.1.2 Recombination of chiral $E2 - E2'$ instantons

For concreteness we consider from now on the simplest non-trivial case with the intersection numbers

$$\Xi' \circ \Xi = \Pi_{O6} \circ \Xi = 1. \tag{6.3}$$

We get two additional bosonic zero modes m and $\overline{m}$ and two additional fermionic ones $\overline{\mu}^{\dot{\alpha}}$ from the $E2 - E2'$ sector. Ensuring global consistency (6.2) the instanton exhibits an excess of additional four charged zero modes $\overline{\lambda}$ giving rise to the zero mode measure[2],

$$\int d\mathcal{M}_I = \int d^4x\, d^2\theta d^2\,\overline{\tau}\; dm\; d\overline{m}\; \underbrace{d^2\overline{\mu}^{\dot\alpha}}_{Q_E=4}\; \underbrace{\prod_a d\lambda_a \prod_b d\overline{\lambda}_b}_{Q_E=-4} \tag{6.4}$$

We first show that the uncharged modes in the $E2 - E2'$ sector and the extra universal $\overline{\tau}$ can successfully be lifted upon taking into account the interaction of the instanton with its orientifold image [81]. The two crucial couplings in the instanton effective action are

$$S_{E2} = (2m\,\overline{m} - \xi)^2 + m\,\overline{\tau}_{\dot\alpha}\,\overline{\mu}^{\dot\alpha}, \tag{6.5}$$

where the Fayet-Iliopoulos ξ depends on the complex structure moduli. It is proportional to the angle modulo 2 between the cycle Ξ and its image Ξ' and vanishes for supersymmetric configurations. For ξ positive the bosonic modes m become tachyonic and thus condense. This results in a new instanton wrapping a new cycle with homology class equal to $[\Xi]+[\Xi']$. The instanton computation is performed for $\xi = 0$, for which the CFT description of the effective action is valid. Integrating out two copies of the second term in (6.5) saturates the extra uncharged zero modes in the instanton measure. Upon performing the path integral over the bosonic modes this results in the measure

$$\int d\mathcal{M}_I = \int d^4x\, d^2\theta\, dm d\overline{m} \underbrace{\prod_a d\lambda_a \prod_b d\overline{\lambda}_b}_{Q_E=-4} m^2 \exp(-(2m\,\overline{m} - \xi)^2). \tag{6.6}$$

[2]Note the inverse scaling behavior of the Grassmann numbers.

This is encouraging as with the $\overline{\tau}$-modes dropping out everything seems to point towards a superpotential contribution. It only remains to absorb the matter zero modes λ_a, $\overline{\lambda}_b$ which were forced upon us by $U(1)_E$ invariance of the zero mode measure. Recall that the sum of all charges of these fields is $Q_E = \sum_a Q_E(\lambda_a) + \sum_b Q_E(\overline{\lambda}_b) = 4$. It is clear that pairs of such zero modes with opposite $U(1)_E$ charge can generate the usual matter field couplings of the type

$$\lambda_a \, \phi_{ab} \, \overline{\lambda}_b \tag{6.7}$$

but there will always be the surplus of four zero modes of type $\overline{\lambda}_b$.

As shown in figure 6.1[3], due to the $U(1)_E$ charge the only way to absorb these extra $\overline{\lambda}$ zero modes is by couplings of the type

$$\overline{m}^{-1} \, \overline{\lambda}'^{-1/2}_b \prod \phi^1_{b_i c_i} \, \overline{\lambda}^{-1/2}_c \tag{6.8}$$

always involving the field $\overline{m}$. In (6.8) the upper index indicates the world-sheet charge Q_{ws}. Since all the fields except $\overline{m}$ are chiral (in the sense of the $\mathcal{N} = 2$ world-sheet supersymmetry) and $\overline{m}$ itself is anti-chiral, the chiral ring structure tells us that all couplings of type (6.8) are *vanishing*: When we apply the Picture changing operator (PCO) on $\overline{m}^{-1}$ we do not pick up the right pole structure for a non-zero amplitude [131]. In case of no additional matter field ϕ in (6.8) the amplitude is vanishing due to the violation of the $U(1)$ world-sheet charge.

Therefore, we conclude that in contrast to naive expectations, the recombined $E2' - E2$ instanton cannot contribute to the superpotential. There always remain

[3]Figure 6.1 displays the case with no additional matter field, namely $< \bar{m} \, \bar{\lambda} \, \bar{\lambda} >$.

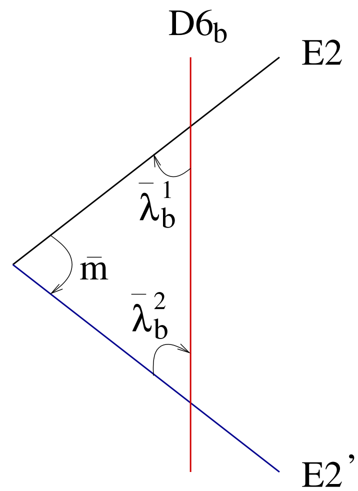

Figure 6.1: Absorption of $\overline{\lambda}_b$ zero modes

four fermionic matter zero modes which cannot be absorbed in a chiral manner. Later in section 6.1.4 we will see that including additional instantons we induce couplings that in principle soak up all excess $\overline{\lambda}_b$ modes.

6.1.3 Recombination of non-chiral $E2 - E2'$ instantons

The deeper reason why chiral $E2 - E2'$ intersecting instantons as in case I do not lead, after brane recombination, to $O(1)$ instantons seems to be that this $E2 - E2'$ system carries charge along the "directions" of the orientifold $O6^-$ plane. In the Type IIB dual situation this means that the magnetized $E5 - E5'$ system carries $E5$-brane charge.

Consequently, it may be more promising to start with a magnetized $E3 - \overline{E3}'$ system which after brane-recombination only contains $E1$-charge. Such a system necessarily has $E3 \circ \overline{E}3' = 0$ and can only support vector-like zero modes on the intersection. This immediately implies that there are no $U(1)_E$ charged matter zero modes necessary to reconcile the $U(1)_E$ invariance of the zero mode measure.

The simplest non-trivial case involves one vector-like pair of zero modes, i.e.

$$[\Xi' \cap \Xi]^+ = [\Xi' \cap \Xi]^- = 1, \quad [\Pi_{O6} \cap \Xi]^+ = [\Pi_{O6} \cap \Xi]^- = 1. \tag{6.9}$$

Therefore, for an $O6^-$-plane we have the zero modes shown in figure 6.2.

zero mode	$(Q_E)_{Q_{ws}}$
$m, \overline{m}$ $\overline{\rho}^{\dot\alpha}$	$(2)_1$,$(-2)_{-1}$ $(-2)_{1/2}$
$n, \overline{n}$ $\overline{\nu}^{\dot\alpha}$	$(-2)_1$,$(2)_{-1}$ $(2)_{1/2}$

Table 6.2: Charged zero modes on non-chiral $E2 - E2'$ intersection with $O6^-$ plane

There is still the fermionic coupling

$$S_{E2} = \overline{\tau}_{\dot\alpha} \left(m\, \overline{\rho}^{\dot\alpha} - n\, \overline{\nu}^{\dot\alpha} \right) \tag{6.10}$$

so that the $\overline{\tau}^{\dot\alpha}$ modes absorb one linear combination of the fermionic zero modes. In addition the single real bosonic D-term constraint[4]

$$m_1^2\, \overline{m}_{-1}^{-2} - n_1^{-2}\, \overline{n}_{-1}^{2} = 0 \tag{6.11}$$

fixes

$$m\, \overline{m} = n\, \overline{n}, \tag{6.12}$$

[4]One might expect that similar to the ADHM construction of gauge instantons one has three D-term constraints. But from the $U(1)_E$ and $U(1)_{ws}$ charges in Table 6.2 it is clear that one can build only the neutral combination in eq. (6.11).

he lower index denotes the $U(1)_{ws}$ charge while the upper one refers to the $U(1)_E$. For initially rigid instantons, i.e. in the absence of $E2$-reparametrization moduli, there exist no F-term constraints which would prevent a non-vanishing VEV $m\,\overline{m} = n\,\overline{n} \neq 0$ corresponding to brane recombination.

As in the analogous process for chiral intersections, recombination breaks the $U(1)_E$. The associated gauge degree of freedom can be used to set

$$m = n \tag{6.13}$$

as opposed to merely (6.12). Integrating out the $\overline{\tau}$ modes together with the linear combination $\overline{\mu} = \overline{\rho} - \overline{\nu}$ of fermionic zero modes as appearing in (6.10) brings down a factor of m^2.

After recombination, one is left with the measure

$$\int d\mathcal{M}_{III} \;\; = \;\; \int d^4x\, d^2\theta\, d^2\overline{\widetilde{\mu}}^{\dot{\alpha}}_{1/2}\, dm_1\, d\overline{m}_{-1}\, m_1^2, \tag{6.14}$$

where again the lower index denotes the $U(1)_{ws}$ charge in the canonical ghost picture and $\overline{\widetilde{\mu}}^{\dot{\alpha}}_{1/2} = \overline{\rho} + \overline{\nu}$ stands for the remaining linear combination of fermionic zero modes. In addition, there can of course be charged zero modes λ_a, $\overline{\lambda}_b$.

Ignoring the additional factor of m_1^2 for the moment, this zero mode structure is precisely that of an $O(1)$ instanton with one deformation $b_1(\Xi) = 1$ of the first type (as defined in [95]). From our discussion in section 5.6 we expect this configuration to generate higher fermionic F-terms of Beasley-Witten type.

In certain situations there may be additional quartic couplings in the instanton effective action which allow one to integrate out also the net deformation modes $\overline{\overline{\mu}}^{\dot{\alpha}}_{1/2}$ [121]. In this case, the $E2 - E2'$ pair contributes to the superpotential. Whether or not these terms are present can be read off uniquely from the dimension of the moduli space of the deformed cycle upon crossing the line of marginal stability. E.g. for non-chiral instanton recombination on toroidal orbifolds they are absent, as can be verified by a direct CFT computation.

6.1.4 Non-perturbative lifting of charged zero modes

As we saw in chapter 6.1.2 recombination of chiral $U(1)$ instantons do not yield to superpotential contribution due to additional excess zero modes required by global consistency. While the extra Goldstino modes $\overline{\tau}^{\dot{\alpha}}$ can be lifted by the three point coupling $m\overline{\mu}^{\dot{\alpha}}\overline{\tau}_{\dot{\alpha}}$ there is no way to soak up the additional excess λ^i-modes. Thus the whole path integral vanishes and there is no contribution to the superpotential.

In the following we investigate the possibility, that the charged excess modes are lifted through the interaction with other D-brane instantons. In fact, D-brane instantons can induce superpotential couplings in the worldvolume theory of other D6-branes which are forbidden perturbatively [67, 68, 69, 70]. The solution to the above problem would then be to invoke such couplings involving the excess modes λ^i in the instanton effective action. The result will be a multi-instanton contribution to the superpotential. A related discussion of multi-instanton effects in non-chiral configurations has been given in [121, 132]; for a recent treatment of different aspects of multi-instantons see [133] and also [134].

In order to avoid the generation of even more charged excess modes we consider the possible lifting via extra $O(1)$ as opposed to $U(1)$ instantons. As will become apparent, the simplest possible such situation involves *two* more $O(1)$ instantons $\widetilde{E}_1$ and $\widetilde{E}_2$ wrapping the invariant cycles $\widetilde{\Xi}_1$ and $\widetilde{\Xi}_2$, respectively, with non-vanishing intersections being precisely

$$[\widetilde{\Xi}_1 \cap \Pi_a]^+ = 2 = [\widetilde{\Xi}_2 \cap \Pi_a]^+, \qquad [\Xi \cap \widetilde{\Xi}_1]^+ = 1 = [\Xi \cap \widetilde{\Xi}_2]^+. \tag{6.15}$$

The situation is depicted in figure 6.2. In Appendix C we construct an explicit example of such a multi-instanton configuration on the toroidal orbifold $T^6/\mathbb{Z}_2 \times \mathbb{Z}_2'$. Each of the $O(1)$ instantons contributes, in the universal sector, the Goldstone modes $\widetilde{x}_i^\mu$ and $\widetilde{\theta}_i^\alpha$, and to avoid extra deformation modes we assume the wrapped cycles are rigid. The $\widetilde{E}_1 - D6_a$ and $\widetilde{E}_2 - D6_a$ sectors yield two charged fermionic zero modes each, $\widetilde{\lambda}_1^i$ and $\widetilde{\lambda}_2^i$. Given the nature of the cycles $\widetilde{\Xi}_1, \widetilde{\Xi}_2, \Pi_a$ as invariant cycles, the intersection is actually vector-like, but half the modes are projected out, leaving us again with a chiral spectrum.

There are also modes between the $U(1)$ instanton and the two $O(1)$ instantons, given by (k_1, κ_1^α) and their charge conjugate $(\overline{k}_1, \overline{\kappa}_1^{\dot{\alpha}})$, and similarly for $\widetilde{E}_2$. Note that, in contrast to the $E - E'$ sector, both the chiral and anti-chiral bosonic and fermionic fields survive the orientifold projection here as this sector is not invariant under $\Omega\overline{\sigma}$.

We can now analyze the combined instanton effective action involving these fields. In this section we start on the hypersurface in complex structure moduli space where the $U(1)$ instanton E is supersymmetric with respect to the orientifold plane. On this locus, the bosonic modes are massless. The relevant parts of the effective action of the multi-instanton effective action first include the couplings

zero mode	sector	repr.	multiplicity
m	$E - E'$	(2_E)	$[\Xi' \cap \Xi]^+ = 1$
$\overline{m}$, $\overline{\mu}^{\dot\alpha}$	$E - E'$	(-2_E)	$[\Xi' \cap \Xi]^+ = 1$
k_1, κ_1^α	$\widetilde{E}_1 - E$	$(1_{\widetilde{E}_1}, -1_E)$	$[\Xi \cap \widetilde{\Xi}_1]^+ = 1$
$\overline{k}_1$, $\overline{\kappa}_1^{\dot\alpha}$	$\widetilde{E}_1 - E$	$(1_{\widetilde{E}_1}, 1_E)$	$[\Xi \cap \widetilde{\Xi}_1]^+ = 1$
k_2, κ_2^α	$\widetilde{E}_2 - E$	$(1_{\widetilde{E}_2}, -1_E)$	$[\Xi \cap \widetilde{\Xi}_2]^+ = 1$
$\overline{k}_2$, $\overline{\kappa}_2^{\dot\alpha}$	$\widetilde{E}_2 - E$	$(1_{\widetilde{E}_2}, 1_E)$	$[\Xi \cap \widetilde{\Xi}_2]^+ = 1$
λ^i	$E - D6_a$	$(1_E, -1_a)$	$[\Pi_a \cap \Xi]^+ = 4$
$\widetilde{\lambda}_1^i$	$\widetilde{E}_1 - D6_a$	$(1_{\widetilde{E}_1}, 1_a)$	$[\widetilde{\Xi}_1 \cap \Pi_a]^+ = 2$
$\widetilde{\lambda}_2^i$	$\widetilde{E}_2 - D6_a$	$(1_{\widetilde{E}_2}, 1_a)$	$[\widetilde{\Xi}_2 \cap \Pi_a]^+ = 2$

Table 6.3: Summary of boundary changing zero modes.

$$S_1 = Y_{1ij} \left(\kappa_1^\alpha \widetilde{\theta}_{1\alpha} \, \widetilde{\lambda}_1^i \, \lambda^j + k_1 \widetilde{\lambda}_1^i \, \lambda^j \right) \qquad + \qquad (1 \leftrightarrow 2) \tag{6.16}$$

involving the charged modes λ^i which we are trying to lift. For their computation see [78].

A second class of couplings can be understood as coming from F-terms of the type

$$W \simeq \sum_{i=1}^{2} M \, K_i \, K_i, \tag{6.17}$$

where K_i formally denotes the superfield associated with the zero modes (k_i, κ_i^α) and similarly for M [5]. In components the fermionic terms are

$$S_2 = \overline{L}_1 \left(\overline{\mu}^{\dot\alpha} \, \overline{\kappa}_{1\dot\alpha} \, \overline{k}_1 + \overline{m} \, \overline{\kappa}_1^{\dot\alpha} \, \overline{\kappa}_{1\dot\alpha} \right) + L_1 m \, \kappa_1^\alpha \, \kappa_{1\alpha} \qquad + \qquad (1 \leftrightarrow 2), \tag{6.18}$$

[5] Recall, however, that the chiral fermion μ^α is projected out.

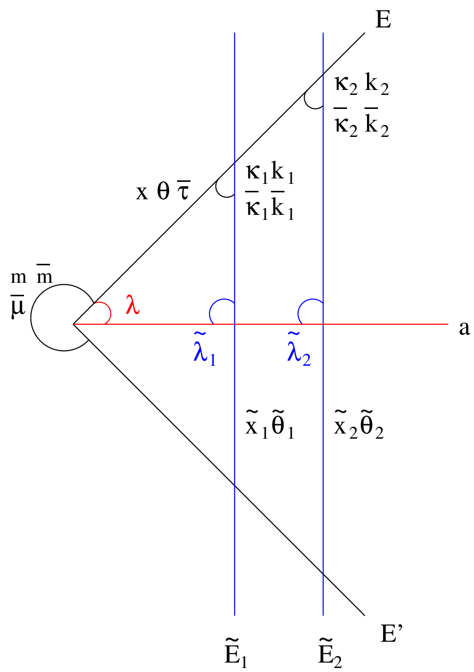

Figure 6.2: Multi-instanton configuration involving two O(1) instantons.

where we introduced the physical coupling constants L_1, L_2. These are related to the holomorphic coupling constants l_1 and l_2 via (C.22) described in appendix C.

A third class of interactions consists of the couplings [81]

$$S_3 = C_m\,(m\,\overline{\mu}^{\dot{\alpha}}\,\overline{\tau}_{\dot{\alpha}}) + C_{k_1}\,(\overline{\kappa}_1^{\dot{\alpha}}\,\overline{\tau}_{\dot{\alpha}}\,k_1) + C_{k_2}\,(\overline{\kappa}_2^{\dot{\alpha}}\,\overline{\tau}_{\dot{\alpha}}\,k_2). \tag{6.19}$$

The bosonic fields furthermore enter the D-term for $U(1)_E$ in the usual way as

$$S_D = \frac{1}{2g_E^2}\,(2m\overline{m} - k_1\overline{k}_1 - k_2\overline{k}_2 - \xi)^2, \tag{6.20}$$

where the gauge coupling of the instanton theory $\frac{1}{g_E^2} = \frac{1}{g_s}\frac{\mathrm{Vol}_{E2}}{\ell_s^3}$ induces an inverse scaling with g_s, as will become crucial later on[6]. Besides, the F-term potential associated with the above trilinear couplings reads

$$S_F = l_1^2 |m\, k_1|^2 + l_2^2 |m\, k_2|^2 + |l_1 k_1^2 + l_2 k_2^2|^2. \tag{6.21}$$

With the help of the above coupling terms we can indeed saturate all fermionic zero modes other than the universal θ^α required for superpotential contributions of E. Concretely, we pull down

$$Y_{1ij}\, Y_{2kl} \;\times\; (\kappa_1^\alpha \widetilde{\theta}_{1\alpha}\, \widetilde{\lambda}_1^i\, \lambda^j)\; (\kappa_2^\alpha \widetilde{\theta}_{2\alpha}\, \widetilde{\lambda}_2^k\, \lambda^l) \tag{6.22}$$

with $i \neq k$ and $j \neq l$ in the instanton path integral. The remaining fermionic modes can be absorbed by the product

$$C_{k_1}\, C_{k_2}\, \overline{L}_1\, \overline{L}_2\; (k_1\, \overline{\kappa}_1\, \overline{\tau})\; (\overline{\mu}\, \overline{\kappa}_1\, \overline{k}_1)\; (k_2\, \overline{\kappa}_2\, \overline{\tau})\; (\overline{\mu}\, \overline{\kappa}_2\, \overline{k}_2). \tag{6.23}$$

Schematically, we are left with the nonvanishing, finite bosonic integral

$$Y_{1ij}^2\, Y_{2kl}^2\, \overline{L}_1\, \overline{L}_2 \int dk_1\, d\overline{k}_1\; dk_2\, d\overline{k}_2\; dm\, d\overline{m}\;\; |k_1|^2\, |k_2|^2\;\; exp(-S_D - S_F). \tag{6.24}$$

Instead of (6.23) we can also saturate the remaining fermionic modes by

$$(C_m)^2\, \overline{L}_1\, \overline{L}_2\; (m\, \overline{\mu}\, \overline{\tau})^2\; \overline{m}\, \overline{\kappa}_1\, \overline{\kappa}_1\; \overline{m}\, \overline{\kappa}_2\, \overline{\kappa}_2, \tag{6.25}$$

[6]The normalization of the D-term is chosen such that the kinetic terms for all instanton modes scale as $\frac{1}{2g_E^2}$. For conventions and their consequences for the vertex operators see [78].

which leads to the non-vanishing bosonic integral

$$Y_{1ij}^2\, Y_{2kl}^2\, \overline{L}_1\, \overline{L}_2 \int dk_1\, d\overline{k}_1\;\, dk_2\, d\overline{k}_2\;\, dm\, d\overline{m}\quad |m|^4\quad exp(-S_D - S_F). \tag{6.26}$$

As a result of summing up all different channels, the multi-instanton BPS configuration produces a non-vanishing contribution to the superpotential. The scale of this contribution is set by the exponentiated classical instanton action,

$$W \simeq exp\left(-\frac{2\pi}{\ell_s^3}(\int_{\Xi}\frac{1}{g_s}\Omega + iC_3 + \int_{\widetilde{\Xi}_1}\frac{1}{g_s}\Omega + iC_3 + \int_{\widetilde{\Xi}_2}\frac{1}{g_s}\Omega + iC_3)\right). \tag{6.27}$$

As in single instanton computations, this classical suppression factor is multiplied by the exponentiated sum over all one-loop annulus diagrams with one end on the instantons and one end on the D6-branes of the model, $\sum_b Z'_A(E2, D6_b)$, together with the Möbius amplitudes $M'(E2, O6)$ [67]. Here $E2 = E, \widetilde{E}_1, \widetilde{E}_2$ and the massless modes are excluded. As an important consistency check, holomorphicity of the generated superpotential is ensured by the cancellation of the non-holomorphicities in the physical couplings $\overline{L}_i$, Y_{1jk}, Y_{2jk} appearing in (6.22), (6.23), (6.25), partially among one another and partially with the non-holomorphic part of these one-loop amplitudes. More details are given in the context of our concrete example at the end of appendix C.

Before proceeding we would like to notice that the simpler configuration consisting of the $U(1)$ instanton pair and only one $O(1)$ instanton does not induce a superpotential. While for suitable intersection numbers the resulting effective action may contain the couplings required to saturate all extra fermionic zero modes, the complex integral over the bosonic modes contains now a monomial in $m^a k^b$ and not in $|m|^a |k|^b$. It vanishes as a result of the uncanceled relative phase. We will come back to this point at the end of the next section.

6.1.5 (Non-)BPS bound states and contributions to the superpotential

I.) $\xi > 0$

Now we deform the complex structure of the Calabi-Yau manifold away from the line of marginal stability $\mathcal{M}_0$ determined by $\xi = 0$ for the cycles Ξ and Ξ'. For simplicity we assume we can take a path in complex moduli space along which the calibration of the other D-branes remains unchanged.

Due to the strictly chiral nature of the intersection of the cycle Ξ with its image Ξ', (6.32), it is possible only for deformations into $\mathcal{M}_+$ where $\xi > 0$ that Ξ and Ξ' combine into a new special Lagrangian cycle

$$Y = E' \# E \tag{6.28}$$

with homological charge $[E] + [E']$ and which preserves the same $\mathcal{N} = 1$ supersymmetry as the orientifold. The bound state Y disappears from the spectrum of BPS branes on the other side in complex moduli space, i.e in $\mathcal{M}_-$. It is therefore an interesting question how the instanton-induced superpotential behaves as the line of marginal stability is crossed.

Let us begin with small deformations leading to formation of the BPS bound state Y. From the effective field theory point of view, the Fayet-Iliopoulos parameter ξ for $U(1)_E$ becomes positive and renders the bosons $m, \overline{m}$ tachyonic. At the end of the recombination process $m, \overline{m}$ have acquired a VEV such that D-flatness is preserved. The fluctuation modes δm and $\delta\overline{m}$ become massive via the D-term, and so do the fermions $\overline{\mu}^{\dot\alpha}$ and $\overline{\tau}^{\dot\alpha}$ through the coupling $\langle m\rangle\,\overline{\mu}\,\overline{\tau}$. The VEV for m and $\overline{m}$ likewise induces a mass term for the bosonic modes $k_i, \overline{k}_i$ and the fermions κ_i^α and $\overline{\kappa}_i^{\dot\alpha}$.

The only massless modes of the multi-instanton system (besides x^μ, θ^α and $\widetilde{x}_i^\mu$) are the charged modes λ^i, $\widetilde{\lambda}^j$ together with $\widetilde{\theta}_1^\alpha, \widetilde{\theta}_2^\alpha$. From general $\mathcal{N}=2$ worldsheet arguments there should exist the six-point couplings

$$\langle \widetilde{\theta}_1^\alpha\, \widetilde{\theta}_1^\beta\; \widetilde{\lambda}^i \lambda^j \widetilde{\lambda}^k \lambda^l \rangle + 1 \leftrightarrow 2. \tag{6.29}$$

The easiest way to see this is to put the vertex operators for the respective zero modes in the following pictures,

$$\langle V_{1/2}^{1/2}(\widetilde{\theta}_1^\alpha)\; V_{3/2}^{-1/2}(\widetilde{\theta}_1^\beta)\;\; V_{-1/2}^{-1/2}(\widetilde{\lambda}^i)\;\; V_{-1/2}^{-1/2}(\lambda^j)\;\; V_{-1/2}^{-1/2}(\widetilde{\lambda}^k)\;\; V_{-1/2}^{-1/2}(\lambda^l)\rangle, \tag{6.30}$$

where the superscript denotes the ghost picture and the subscript the worldsheet $U(1)$ charge. Pulling down these two couplings therefore saturates all extra fermionic modes, and the instanton bound state $(E'\#E) \cup \widetilde{E}_1 \cup \widetilde{E}_2$ contributes to the superpotential.

There is an alternative way to describe the system by thinking of the instantons wrapping the individual cycles $\Xi, \Xi', \widetilde{\Xi}_1$ and $\widetilde{\Xi}_2$ before formation of the bound state Y in the following way: As Ξ and Ξ' are at non-supersymmetric angles, the open string excitations describing the bosons $m, \overline{m}$ are tachyonic, while the ones corresponding to $k_i, \overline{k}_i$ acquire positive $(\text{mass})^2$. From the quantization of the open string modes it is furthermore clear that in this picture, i.e. prior to condensation of $m, \overline{m}$, all fermionic modes remain massless. The instanton effective action for this system is obtained by integrating out the bosonic non-zero modes and keeping only the couplings involving the fermionic zero modes. In fact, non-zero instanton modes are strictly off-shell as it is not possible, in absence of four-dimensional momentum, to write down a consistent vertex operator for massive excitations. This is reflected in the usual procedure to allow for the non-zero modes to appear only in the one-loop amplitudes.

The effective coupling replacing the interactions (6.18) and (6.19) upon integrating out k_i and $\overline{k}_i$ become

$$S' = \overline{\mu}\,\overline{\kappa}_1\,\overline{\kappa}_1\,\overline{\tau} + \overline{\mu}\,\overline{\kappa}_2\,\overline{\kappa}_2\,\overline{\tau}. \tag{6.31}$$

These terms allow us to saturate all extra fermionic zero modes, reproducing the conclusion that the instanton system contributes to the superpotential.

II.) $\xi < 0$

Now we deform the complex structure such as to enter the region $\mathcal{M}_-$ of moduli space where the special Lagrangian Y ceases to exist. However, as encoded already in the D-term potential (6.33), E can recombine instead with the $O(1)$ instantons on the cycles $\widetilde{\Xi}_1$ or $\widetilde{\Xi}_2$. The D-term only fixes the combination $|k_1|^2 - |k_2|^2$ and leaves us with one complex bosonic modulus consisting of the orthogonal combination as well as the relative phase between the complex fields k_1 and k_2. Both are fixed by the F-term in a D- and F-flat manner. The non-zero VEV for k_1 and k_2 also renders the boson m massive. All extra fermionic modes $\widetilde{\lambda}_i$, λ_j, κ_k and $\overline{\kappa}_l$ acquire a mass via their couplings to k_i.

This shows how holomorphicity of the D-brane instanton induced superpotential is maintained even in situations where specific BPS instantons disappear across lines of marginal stability. In $\mathcal{M}_+$ the superpotential is corrected by instantons wrapping the BPS configuration $(E'\#E) \cup \widetilde{E}_1 \cup \widetilde{E}_2$. It is a multi-instanton configuration with constituents $\widetilde{E}_1$, $\widetilde{E}_2$ and the BPS bound state $Y = E'\#E$. Along $\mathcal{M}_0$ this bound state Y meets a line of marginal stability, but the multi-instanton $E \cup E' \cup \widetilde{E}_1 \cup \widetilde{E}_2$ is still BPS and contributes to the superpotential. In $\mathcal{M}_-$ the former BPS state $Y = E'\#E$ has disappeared, but there exists a new BPS state $\Psi = \widetilde{E}_1\#(E \cup E')\#\widetilde{E}_2$ with charge $[E]+[E']+[\widetilde{E}_1]+[\widetilde{E}_1]$. The two additional instantons $\widetilde{E}_1$ and $\widetilde{E}_2$ required to lift the fermionic zero modes for $\xi = 0$ conspire such that the number of BPS states of total charge $[E]+[E']+[\widetilde{E}_1]+[\widetilde{E}_1]$ does not jump across the line of marginal stability.

To illustrate this connection further, it is instructive to analyze how a jump in the BPS spectrum is correlated with a microscopic obstruction to a superpotential contribution already at threshold. The simplest example would of course be just the $U(1)$ instanton and its image which cannot contribute due to extra charged modes. But there are even more subtle obstructions to superpotential contributions in agreement with a discontinuous BPS spectrum.

Consider the lifting of the charged zero modes by a single $O(1)$ instanton wrapping the cycle $\widetilde{\Xi}$. In order to lift the additional 4 charged zero modes λ we require e.g. [7]

$$[\widetilde{\Xi}_1 \cap \Pi_a]^+ = 4, \qquad [\Xi \cap \widetilde{\Xi}_1]^+ = 1. \tag{6.32}$$

Such a setup is depicted in figure 6.3. The massless spectrum comprises 4 additional charged zero modes $\tilde{\lambda}$ the bosonic and fermionic zero modes k and κ^α and their conjugates and finally the universal zero modes $\tilde{x}$ and $\tilde{\theta}^\alpha$ of the $O(1)$ instanton.

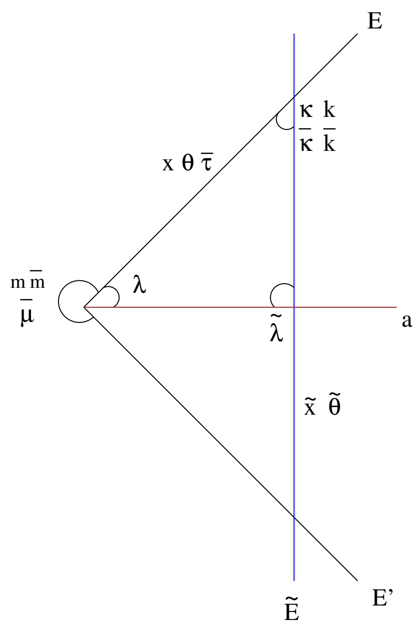

Figure 6.3: Multi-instanton configuration involving a single O(1) instanton.

One observes the same couplings as in (6.16), (6.18) and (6.19), but the D-term

[7]Our results hold also true for different intersections in the $E - \widetilde{E}$ sector.

and the F-term now take the form

$$S_D = \frac{1}{2g_E^2}\,(2m\overline{m} - k\overline{k} - \xi)^2, \qquad S_F = l_1^2\left((k\,\overline{k})^2 + |m\,k|^2\right). \tag{6.33}$$

As before we can saturate all charged zero modes λ and $\tilde{\lambda}$ by pulling down

$$Y_{ij}\,Y_{kl}\ \times \quad (\kappa^\alpha \widetilde{\theta}_\alpha\,\widetilde{\lambda}^i\,\lambda^j)\ \ (k\,\widetilde{\lambda}^k\,\lambda^l), \tag{6.34}$$

while the remaining fermionic zero modes can be absorbed by

$$C_k\,C_m\,\overline{L}\ (k\,\overline{\kappa}\,\overline{\tau})\ (\overline{\mu}\,\overline{\kappa}\,\overline{k})\ (\overline{\mu}\,m\,\overline{\tau}). \tag{6.35}$$

This leaves us with the bosonic integral

$$Y_{ij}^4\,C_k\,\overline{L}\int dk\,d\overline{k}\ dm\,d\overline{m}\ \ |k|^2\,k^2\,m\ \ exp(-S_D - S_F). \tag{6.36}$$

Unlike in the previous case this vanishes after integrating over the relative phase between m and k. Alternatively one can saturate all fermionic zero modes via the couplings

$$C_m^2\,\overline{L}\ (\overline{m}\,\overline{\kappa}\,\overline{\kappa})\ \ (\overline{\mu}\,m\,\overline{\tau})^2 \tag{6.37}$$

leading to the bosonic integral

$$Y_{ij}^4\,C_m^2\,\overline{L}\int dk\,d\overline{k}\ dm\,d\overline{m}\ \ |m|^2\,k^2\,m\ \ exp(-S_D - S_F). \tag{6.38}$$

Again this integral vanishes and there are no superpotential contributions.

This is consistent with the behavior of such an instanton configuration for a small deformation of the complex structure. The configuration is very similar to the D-brane setup discussed in section 2.7.2. For sufficiently small deformations of the complex structure the new stable geometric object is described by condensation of the bosonic modes such that the potential $V = S_D + S_F$ is minimized. For $\xi < 0$, this happens at

$$|k| = \sqrt{-\frac{\xi}{1+a}}, \qquad m = 0, \tag{6.39}$$

where $a = 2g_E^2\, l << 1$. Note that this minmum breaks both D-flatness and F-flatness. It corresponds to an instanton wrapping the bound state $\widetilde{\Psi}$ of the cycle $\Xi \cup \Xi'$ with $\widetilde{\Xi}$.

This new multi-bound state $\widetilde{\Psi}$ is truly non-BPS. As in section 2.7.2 a possible way to think about $\widetilde{\Psi}$ is as a deformation of the sLag Ψ defined as the would-be BPS bound state formed by $\widetilde{\Xi}$, Ξ and Ξ' if the superpotential (6.17) were absent, i.e. $l = 0$. From the field theory point of view, the tachyon k would condense as $|k| = \sqrt{-\xi}$ and the excitation mode δk around this vacuum expectation value would be massive. Now by switching on $l \neq 0$, m acquires a mass. The F-terms also induce a term linear in the massive fields δk. This indicates that the system is unstable towards formation of the metastable non-BPS state $\widetilde{\Psi}$. In the spirit of the discussion at the end of section 2.7.2, $\widetilde{\Psi}$ is a *non-calibrated Lagrangian three-cycle.*

Such a non-supersymmetric state is not expected to contribute to the superpotential, thus on the line of margin stability $\xi = 0$ there should not be any contributions either. This is in complete agreement with our previous analysis.

Extrapolating from the CFT of the $E2-E2'$ sector before recombination, the relevant couplings after recombination are inherited from

$$(\overline{m}_{-1}\overline{\nu}^{\dot{\alpha}}_{1/2}+\overline{n}_{-1}\overline{\rho}^{\dot{\alpha}}_{1/2})\ \lambda^a_{-1/2}\ (\overline{\psi}_{1/2})_{\dot{\alpha}}\ \overline{\lambda}^b_{-1/2}\longrightarrow \overline{m}_{-1}\overline{\widetilde{\mu}}^{\dot{\alpha}}_{1/2}\lambda^a_{-1/2}\ (\overline{\psi}_{1/2})_{\dot{\alpha}}\ \overline{\lambda}^b_{-1/2}, \quad (6.40)$$

where the fermionic matter field $\overline{\psi}^{\dot{\alpha}}_{1/2}$ lives at the intersection $D6_a-D6_b$ and lies in the anti-chiral superfield $\overline{\Phi}=\overline{\phi}+\overline{\tau}\overline{\psi}$. Note that the above coupling does not violate any of the general $\mathcal{N}=2$ SCFT selection rules so that even without a direct computation we expect it to be present for sufficiently generic backgrounds. Integrating out two copies of this interaction term brings down the fermion bilinear $\overline{\psi}_{1/2}\overline{\psi}_{1/2}$ characteristic for the higher fermionic terms described in [116] as well as a factor of $\overline{m}^2_{-1}$. The bosonic measure can then be brought into standard form by a simple change of variables with $\widetilde{m}=m^3$ and we are left with

$$\int d\mathcal{M}_{III} \quad = \quad \int d^4x\, d^2\theta\, d\widetilde{m}_1\, d\overline{\widetilde{m}}_{-1}\ \overline{\psi}_{1/2}\overline{\psi}_{1/2}. \quad (6.41)$$

Together with the chiral fermion bilinear pulled by the two θ^α modes this results in the four-fermi terms as discussed in section 5.6.

Its bosonic derivative superpartner involves absorbing one pair of $\theta\overline{\mu}$ in a coupling of the form (after recombination)

$$\overline{m}_{-1}\,(\overline{\tau}_{\dot{\alpha}})_{3/2}\ \overline{\widetilde{\mu}}^{\dot{\alpha}}_{1/2}\ \lambda^a_{-1/2}\ \overline{\phi}_{-1}\ \overline{\lambda}^b_{-1/2}. \quad (6.42)$$

With $\overline{\phi}_{-1}$ and $\overline{m}_{-1}$ in the zero ghost picture[8] this generates a derivative for the boson $\overline{\phi}_{-1}$. Bringing down two copies of this term indeed yields the derivative superpartner to the above four-fermi term, again in agreement with [116].

6.2 Flux-induced lifting of zero modes

The additional two zero modes $\overline{\tau}^{\dot{\alpha}}$ which, if present, prevent the generation of a superpotential by the instanton, are a consequence of the underlying $\mathcal{N} = 2$ supersymmetry preserved in the bulk of the Calabi-Yau away from the orientifold plane in the way described in section 4.3. It has therefore been speculated in the literature [74, 76, 96] that these Goldstinos might be lifted in the presence of suitable background fluxes. An intuitive reason why this could be the case is that under appropriate circumstances the instanton is expected to feel only the $\mathcal{N} = 1$ supersymmetry preserved by the flux in the bulk. In such situations the $\overline{\tau}$ modes are not protected as the Goldstinos of the orthogonal $\mathcal{N} = 1$ supersymmetry and it might be possible that indeed only the two θ^{α} modes remain massless in the universal zero modes sector.

While our previous presentation has focused on D-brane instantons in Type IIA orientifolds, the natural arena to study the effects of background fluxes is the framework of Type IIB compactifications, where we can take advantage of the by now quite mature understanding of a fully consistent incorporation of supersymmetric three-form flux (for references see e.g. [135, 136]). The lifting of fermionic zero modes by supersymmetric three-form flux has been analyzed in special cases in [124, 125, 126, 127] in the context of $E3$-instantons wrapping a holomorphic divisor of the internal

[8]Note that for $\overline{m}_{-1}$ the PCO can only act non-trivially in the internal part since its vertex does not carry any momentum.

(conformal) Calabi-Yau. The most general such situation involves the presence also of supersymmetric gauge flux on the worldvolume $E3$-brane. This corresponds on the Type IIA side to $E2$-instantons at general angles with the $O6$-plane and is the configuration we are primarily interested in. Before addressing the more general case, we review first the situation of vanishing gauge flux.

6.2.1 Zero mode lifting for unmagnetized E3-instantons

In the spirit of [137], we consider Type IIB orientifold compactifications with an $\mathcal{N} = 1$ supersymmetric combination $G = F - \tau H$ of RR and NS flux $F = dC_2$ and $H = dB$ such that the complexified dilaton $\tau = C_0 + ie^{-\phi}$ is constant. The internal manifold is therefore conformally Calabi-Yau with constant warp factor. In order to preserve supersymmetry, the flux has to be of (2,1) type[9] and satisfy the primitivity condition $J \wedge G = 0$ in terms of the Kähler form J. We consider an $E3$-brane wrapping a holomorphic divisor Γ. Since our interest here focuses on the lifting of $\overline{\tau}$-modes, we assume that Γ is not invariant under the holomorphic involution σ defining the orientifold action $\Omega(-1)^{F_L}\sigma$.

The part in the $E3$-brane worldvolume action describes the coupling of such three-form flux to the (uncharged) zero modes ω[10] reads [138, 125]

$$S = \int_\Gamma d^4\zeta \sqrt{\det g}\, \omega \, \left(e^{-\phi}\, \Gamma^{\tilde{m}} \nabla_{\tilde{m}} + \frac{1}{8}\, \widetilde{G}_{\tilde{m}\tilde{n}p}\, \Gamma^{\tilde{m}\tilde{n}p} \right) \omega. \tag{6.43}$$

The combination $\widetilde{G}_{\tilde{m}\tilde{n}p}$ appearing above is defined as $\widetilde{G}_{\tilde{m}\tilde{n}p} = e^{-\phi} H_{\tilde{m}\tilde{n}p} + iF'_{\tilde{m}\tilde{n}p}\gamma_5$ in

[9]In the presence of a non-perturbative superpotential this condition is relaxed to include also (0,3) components [127].

[10]The corresponding objects in [125] are called θ, see eq. (4.1.). Recall that we reserve the notation θ and $\overline{\tau}$ for the four-dimensional spinor associated with the universal zero modes.

terms of $F'_{\tilde{m}\tilde{n}p} = F_{\tilde{m}\tilde{n}p} - C_0 H_{\tilde{m}\tilde{n}p}$ and the four-dimensional matrix γ_5. The indices $\tilde{m}, \tilde{n}$ are along the four-cycle Γ and p is transverse to it. While the above action was derived in [138, 125] entirely with the help of supergravity methods, one could of course likewise determine it by analyzing the CFT coupling of the closed string fields in the bulk to the boundary [139, 60, 140, 141].

The Euclidean action (6.43) uses a particular gauge fixing condition to eliminate the unmusical degrees of freedom due to κ-symmetry (cf. eq. 4.9 of [125]). As a result, the spinor ω is a sixteen-component Weyl spinor. Locally, we can choose complex coordinates $a, b = 1, 2$ along Γ and $z, \overline{z}$ for the transverse direction. It is convenient to use the standard definition of the Clifford vacuum $|\Omega\rangle$,

$$\Gamma^z|\Omega\rangle = 0, \quad \Gamma^a|\Omega\rangle = 0 \tag{6.44}$$

and to decompose the spinor ω into its external and internal part. The latter can be grouped according to its chirality along the normal bundle of the divisor as

$$\begin{aligned} \epsilon_+ &= \phi|\Omega\rangle + \phi_{\overline{a}}\Gamma^{\overline{a}}|\Omega\rangle + \phi_{\overline{a}\overline{b}}\Gamma^{\overline{a}\overline{b}}|\Omega\rangle, \\ \epsilon_- &= \phi_{\overline{z}}\Gamma^{\overline{z}}|\Omega\rangle + \phi_{\overline{a}\overline{z}}\Gamma^{\overline{a}\overline{z}}|\Omega\rangle + \phi_{\overline{a}\overline{b}\overline{z}}\Gamma^{\overline{a}\overline{b}\overline{z}}|\Omega\rangle. \end{aligned} \tag{6.45}$$

In this language we can immediately identify the universal fermionic zero modes with four-dimensional polarization θ^α and $\overline{\tau}^{\dot{\alpha}}$ as given by

$$\omega_0^{(1)} = \theta \otimes \phi|\Omega\rangle, \qquad \omega_0^{(2)} = \overline{\tau} \otimes \phi_{\overline{a}\overline{b}\overline{z}}\Gamma^{\overline{a}\overline{b}\overline{z}}|\Omega\rangle. \tag{6.46}$$

The remaining components in (6.45) are associated with the reparametrization modulini and Wilson line fermions of the four-cycle counted by $H^{(0,2)}(\Gamma)$ and $H^{(0,1)}(\Gamma)$, respectively.

Starting from the above action, i.e. in the absence of gauge flux, [125] computed the remaining zero modes in the presence of primitive (2,1) three-form flux. In particular, their analysis shows that the four universal zero modes (6.46) are not lifted in such a situation. In fact, one can easily convince oneself that the zero mode $\omega_0^{(2)}$ does not couple to primitive (2,1) flux. E.g.

$$\begin{aligned}\widetilde{G}_{\overline{a}bz}\Gamma^{\overline{a}bz}\Gamma^{\overline{1}}\Gamma^{\overline{2}}\Gamma^{\overline{3}}|\Omega\rangle &= \widetilde{G}_{\overline{a}bz}g^{b\overline{1}}g^{z\overline{3}}\Gamma^{\overline{a}}\Gamma^{\overline{2}}|\Omega\rangle - \widetilde{G}_{\overline{a}bz}g^{b\overline{2}}g^{z\overline{3}}\Gamma^{\overline{a}}\Gamma^{\overline{1}}|\Omega\rangle \\ &= \widetilde{G}_{\overline{1}bz}g^{b\overline{1}}g^{z\overline{3}}\Gamma^{\overline{1}}\Gamma^{\overline{2}}|\Omega\rangle + \widetilde{G}_{\overline{2}bz}g^{b\overline{2}}g^{z\overline{3}}\Gamma^{\overline{1}}\Gamma^{\overline{2}}|\Omega\rangle \\ &= \widetilde{G}_{\overline{a}bz}g^{b\overline{a}}g^{z\overline{3}}\Gamma^{\overline{1}}\Gamma^{\overline{2}}|\Omega\rangle = 0. \end{aligned} \tag{6.47}$$

The last equation follows from the identity [125]

$$\widetilde{G}|\Omega\rangle = i\,G|\Omega\rangle \tag{6.48}$$

together with primitivity of G,

$$g^{c\overline{c}'}G_{bc\overline{c}'} = 0. \tag{6.49}$$

Likewise, potential $(0,3)$ components of G-flux can be shown not to couple to the universal modes. This type of flux is allowed by the equations of motion and supersymmetric once the non-perturbative superpotential is taken into account in the analysis of the gravitino variation [127].

6.2.2 Zero mode lifting for magnetized E3-instantons

We are now ready to address our main question, the inclusion of non-trivial gauge flux on the instanton. The worldvolume action of the $E3$-instanton contains, in addition to (6.43), two pieces linear and quadratic in the gauge invariant combination $\mathcal{F} = F_{\text{gauge}} - B$ of the worldvolume gauge field and Neveu-Schwarz two-form. Since we are considering an orientifold, we have to add the contributions from the $E3$-instanton together with its image under $\Omega(-1)^{F_L}\sigma$.

As described in [142], since $\mathcal{F}$ is anti-invariant under Ω, the linear terms in the action survive only for the components of $\mathcal{F}$ along elements of $H^-_{(1,1)}(\Gamma)$, while those quadratic in $\mathcal{F}$ survive along elements of $H^+_{(1,1)}(\Gamma)$. Before orientifolding, the relevant part of the quadratic term is the sum of the two terms[11] [124]

$$\begin{aligned} S_{\text{DBI}} &= -\frac{\mu}{48}\int_\Gamma d^4\zeta\sqrt{\det g}\,\omega\,\Gamma^{\tilde{m}\tilde{n}p}\,\omega\; e^{-\phi}H_{\tilde{m}\tilde{n}p}\left(\frac{1}{4}\,\mathcal{F}^2\right), \\ S_{\text{WZ}} &= -\frac{\mu}{48}\int_\Gamma \,\omega\,\Gamma^{\tilde{m}\tilde{n}p}\,\omega\;(iF'_{\tilde{m}\tilde{n}p})\left(\frac{1}{2}\,\mathcal{F}\wedge\mathcal{F}\right). \end{aligned} \tag{6.50}$$

In four Euclidean dimensions, solutions to the field equations and Bianchi identity can be taken to satisfy the self-duality constraint $\mathcal{F} = \star\mathcal{F}$. Together with $\int\sqrt{\det g}\left(\frac{1}{4}\,\mathcal{F}^2\right) = \int\frac{1}{2}\mathcal{F}\wedge\star\mathcal{F}$ we find that the relevant couplings combine into

$$-\frac{\mu}{48}\int_\Gamma d^4\zeta\sqrt{\det g}\,\omega\;G_{\tilde{m}\tilde{n}p}\,\Gamma^{\tilde{m}\tilde{n}p}\,\omega\;\left(\frac{1}{4}\,\mathcal{F}^2\right). \tag{6.51}$$

[11]Note that for simplicity, we are using here the gauge of [124], which is different from the one in which (6.43) is written. As emphasized in [125] the gauge fixing condition and the orientifold projection have to be compatible for branes invariant under the orientifold. Since we are interested in the more general situation of non-invariant branes or instantons, it suffices for our purposes to work in the gauge of [124].

By the same reasoning as above, this interaction does not induce any mass terms for the universal zero modes provided we stick to supersymmetric (2,1) (or even (0,3)) flux.

Let us now discuss if the term linear in $\mathcal{F}$ saves the day, given in the upstairs geometry by [124]

$$\begin{aligned} S &= \frac{\mu}{16}\int_\Gamma d^4\zeta\sqrt{\det g}\left(\mathcal{F}_{\tilde{i}\tilde{k}}\,\omega\,\Gamma^{\tilde{k}pq}\,e^{-\phi}\,H^{\tilde{i}}{}_{pq}\,\omega - \frac{i}{2}\epsilon^{\widetilde{ijkl}}\,\mathcal{F}_{\widetilde{ij}}\,\omega\Gamma_{\widetilde{k}}{}^{pq}\,(F')_{\widetilde{l}pq}\,\omega\right) \\ &= -i\,\frac{\mu}{16}\int_\Gamma d^4\zeta\sqrt{\det g}\,\mathcal{F}_{\widetilde{ij}}\,\omega\Gamma^{\widetilde{i}pq}\omega\;g^{\widetilde{jk}}\;G_{\widetilde{k}pq}. \end{aligned} \tag{6.52}$$

Again self-duality of the gauge flux, $\frac{1}{2}\epsilon^{\widetilde{ijkl}}\mathcal{F}_{\widetilde{ij}} = \mathcal{F}^{\widetilde{kl}}$, is used and a tilde denotes indices parallel to the worldvolume, whereas p, q are general internal indices.

While the index structure of the matrices Γ is still of type $(2,1)$ due to contraction with the hermitian metric, the above action may in principle induce non-vanishing couplings involving the universal modes. After all, the vanishing of such couplings in the absence of gauge flux rested also upon primitivity of G_3, which is not necessarily satisfied by the combination of $\mathcal{F}$ and G_3 contracted with the Γ in (6.52). As we stressed, these couplings, being linear in $\mathcal{F}$, only survive the orientifold action in the presence of anti-invariant two-cycles on the divisor. We will illustrate this issue in more detail in the next subsection.

On the other hand, in the absence of such cycles, as e.g. for the $T^6/\mathbb{Z}_2$ example studied in [143], the $\overline{\tau}$-modes remain massless even after taking into account the backreaction of the three-form flux on the instanton moduli action. While this may seem counter-intuitive because they are no longer protected as Goldstinos in the presence of three-form flux, this is just an example of the familiar fact even though all symmetries broken by the instanton result in associated zero modes, the converse need not be true.

6.2.3 A simple example with linear gauge fields

In the presence of suitable three-form flux, the linear term in $\mathcal{F}$ leads to a coupling of the zero mode $\omega_0^{(2)}$ proportional to

$$G_{\overline{a}bz}\mathcal{F}^{b\overline{a}}g^{z\overline{3}}\Gamma^{\overline{1}}\Gamma^{\overline{2}}|\Omega\rangle. \tag{6.53}$$

As stated above, this does not vanish directly due the primitivity condition for G-flux and the hermitian Yang-Mills equation for the gauge flux $\mathcal{F}$. It would therefore lead to a coupling between $\omega_0^{(2)}$ to the mode $\phi_{\overline{a}\overline{b}}\Gamma^{\overline{a}\overline{b}}|\Omega\rangle$. If present, integrating out both types of zero modes would lift $\omega_0^{(2)}$. However, under the orientifold projection the flux components $\mathcal{F} \in H^{+}_{1,1}(\Gamma)$ are mapped to $-\mathcal{F}$ and therefore (6.53) vanishes trivially. But for the components $F \in H^{-}_{1,1}(\Gamma)$ there is a chance that the zero mode $\omega_2^{(0)}$ becomes massive.

Let us discuss a simple local example on a toroidal orientifold. We compactify Type IIB on T^6 with metric

$$ds^2 = \sum_I dz_I \, d\overline{z}_I \tag{6.54}$$

and mod out by the orientifold projection $\Omega\sigma(-1)^{F_L}$ with $\sigma : z_2 \to -z_2$. Ignoring the resulting tadpole cancellation conditions for the moment, we now turn on $\Omega\sigma(-1)^{F_L}$ invariant G_3-form flux. Let us consider a magnetized $E3$-brane in this background which wraps the first two T^2s and is pointlike on the third one. On this $E3$-brane we also turn on a constant gauge flux of type

$$\mathcal{F}^{\bar{1}2} \in H^{-}_{1,1}(\Gamma), \tag{6.55}$$

which in invariant under the orientifold projection. Apparently, this flux satisfies the HYM equation. Consistently, this brane couples to the two-form $(C_2)_{\bar{1},2}$ which is also invariant under the orientifold projection. Since for vanishing Wilson line along the first T^2 the instanton is invariant under the orientifold projection it is of type $O(1)$.

Then the coupling of the zero mode $\omega_0^{(2)}$ on the instanton is proportional to

$$G_{\bar{a}bz}\mathcal{F}^{\bar{a}b} = G_{\bar{1}23}\mathcal{F}^{\bar{1}2}, \tag{6.56}$$

which can be non-vanishing. Indeed the flux component $G_{\bar{1}23}$ is invariant under the orientifold projection. This simple example shows that, ignoring tadpole constraints, it is possible that the $\omega_0^{(2)}$ modes decouple for non-vanishing G_3 form flux.

However, when it comes to satisfying the tadpole constraints, we have to introduce both further $D7$-branes to cancel the $O7$-plane tadpole and an $O3$-plane to cancel the tadpole induced by the G_3-form. The easiest way to get the $O3$-plane is to also mod out the model by the $\mathbb{Z}_2$ action $z_{1,3} \to -z_{1,3}$, essentially turning the configuration into the fluxed $K3 \times \frac{T^2}{\mathbb{Z}_2}$ model studied in [144]. However, in this case the $E3$ is not invariant under this $\mathbb{Z}_2$, but mapped to an $E3$ brane with opposite gauge flux $-\mathcal{F}^{\bar{1},2}$. Therefore, the coupling of the $\omega_0^{(2)}$ modes again trivially vanishes.

Chapter 7

Summary

Let us briefly summarize the results of chapter 4 to chapter 6:

Anomalous $U(1)$ gauge fields become massive via the generalized Green-Schwarz mechanism. While the $U(1)$ gauge symmetries are broken they nevertheless survive as global symmetry. On a perturbative level all couplings have to obey these global symmetries. Thus various desired couplings, such as Majorana masses for right-handed neutrinos, μ-terms and particular Yukawa couplings in SU(5) GUT-like models, are forbidden. E2-instantons can carry charge under these global $U(1)$'s. Thus they can compensate for the U(1) charge excess arising from the matter fields and induce the coupling

$$\prod_i \Phi_i \, e^{-S_{E2}} \tag{7.1}$$

Apart from carrying the correct charge under all global $U(1)$'s, an instanton has to satisfy additional constraints in order to generate a coupling of the form (7.1).

These are very restrictive constraints on the instanton zero mode structure. We demonstrate in section 5.1 that an instanton, wrapping an $\Omega\overline{\sigma}$ invariant cycle, exhibits the right zero mode structure to give rise to superpotential terms. Such an instanton is called an $O(1)$ instanton. We exemplify, in various local setups, that perturbatively forbidden couplings indeed can be induced non-perturbatively. In addition, these stringy non-perturbative effects give a natural explanation for hierarchies which are only poorly understood from the field theory perspective. Furthermore, we show that generic $U(1)$ instantons induce higher fermionic F-terms à la Beasley and Witten.

A global embedding of these stringy instanton effects is very challenging due to the very restrictive constraints on the instanton zero mode structure. Therefore, it is worthwhile to investigate under what circumstances undesired zero modes can be lifted. This is probed in chapter 6, where we investigate two different strategies: instanton recombination and lifting by background fluxes.

In the latter case, the analysis is performed in the type IIB framework with supersymmetric background flux. Here the instantons in question are E3 instantons. In case the E3 instanton does not carry any magnetic flux, the universal $\overline{\tau}^{\dot{\alpha}}$ modes do not get lifted. Thus, such an E3 instanton does not give rise to purely superpotential terms. On the other hand, we show that for E3 instantons with magnetic flux on their world volume a lifting might be possible. However, this can occur only in situations where the divisor, wrapped by the instanton, contains non-trivial two-cycles which are anti-invariant under the orientifold action. It would be very interesting to find a global realization of such an effect. Furthermore, it would be desirable to gain a comparable understanding in the T-dual picture, i.e. effects of type IIA background fluxes on E2 instanton zero modes.

The second strategy for lifting undesired zero modes is instanton recombination. We show that in certain situations, for non-chiral intersections the instanton can generate superpotential contributions but generically gives rise to higher fermionic F-terms. In case the instanton intersects chirally with its image, additional instantons are needed to lift the charged zero modes. We demonstrate that such a multi-instanton configuration indeed exhibits the right zero mode structure to contribute to the superpotential.

Let us stress that non-perturbative effects play an important role in (de)-stabilizing the string vacua. It is therefore essential to account for all possible instanton contributions. Thus it is desirable to find tractable criteria to differentiate the contributing instantons from the non-contributing ones. We leave this for future work.

Appendix A

Vertex operators for intersecting D-branes

In this appendix we discuss the vertex operators of bosonic and fermionic string states arising at intersections of two D6-branes. Let us start by introducing the setup we investigate. We will consider D6-branes in flat, non-compact Minkowski space that fill out the first four dimensions (our actual spacetime) and intersect in the 3rd, 4th and 5th complex plane. Strings that are stretched between these D-branes have to satisfy special boundary conditions in the internal dimensions which leads to intersection angle dependent non-integer mode expansions for the degrees of freedom. In the vertex operators for the corresponding string configuration one introduces bosonic and fermionic twist fields to take into account these non-integer mode excitations. In the sequel we present a prescription for constructing such vertex operators arising from strings stretched between intersecting D-branes.

As a first step we deduce the mode expansions for the bosonic and fermionic degrees of freedom. We start with the Neveu-Schwarz (NS)-sector, where strings stretched between the intersecting D-branes correspond to massive scalars in the four-dimensional space-time. Later we will also deal with strings in the Ramond (R)-sector and show that in this sector we always have a massless fermion, independent of the choice of the intersection angles. After deriving the mode expansions we quantize the string, impose the condition for physical states, and obtain the mass formula. In the NS-sector we will see that the scalars become massless only for particular choices of angles that match with the supersymmetry condition. In order to derive the structure of the vertex operators take we examine the operator product expansions (OPE's) of the bosonic and fermionic fields with specific string excitations. These OPE's show the same behavior as the OPE's of the twist fields in orbifold theories [145]. Therefore the vertex operators for strings stretched between intersecting D-branes will involve bosonic and fermionic twist fields, σ_θ and s_θ in the internal dimensions. The exact knowledge of the OPE's of the bosonic and fermionic fields with the string states allows us to write the vertex operators for the string states in arbitrary intersecting D-brane configurations.

An open string stretched between two D-branes at an angle $\pi\theta_I$ has to fulfill the boundary conditions [146, 147]

$$\begin{aligned} \partial_\sigma X^p(\tau,0) = 0 = X^{p+1}(\tau,0) \\ \partial_\sigma X_p(\tau,\pi) + \tan(\pi\theta_I)\ \partial_\sigma X_{p+1}(\tau,\pi) = 0 \\ X_{p+1}(\tau,\pi) - \tan(\pi\theta_I)\ X_p(\tau,\pi) = 0\ . \end{aligned} \tag{A.1}$$

Given these boundary conditions, we can deduce the mode expansion for Z^I (we use complex coordinates $Z^I = X^{2I+2} + \mathrm{i}X^{2I+3}$) to

$$\left.\begin{aligned} Z^I(z,\bar{z}) &= \sum_n \frac{\alpha^I_{n-\theta_I}}{(n-\theta_I)}\ z^{-n+\theta_I} + \sum_n \frac{\alpha^I_{n+\theta_I}}{(n+\theta_I)}\ \bar{z}^{-n-\theta_I} \\ \bar{Z}^I(z,\bar{z}) &= \sum_n \frac{\alpha^I_{n+\theta_I}}{(n+\theta_I)}\ z^{-n-\theta_I} + \sum_n \frac{\alpha^I_{n-\theta_I}}{(n-\theta_I)}\ \bar{z}^{-n+\theta_I} \end{aligned}\right\} \text{for } I = 1,2,3\ . \tag{A.2}$$

Upon quantization the only nonvanishing commutator is

$$[\alpha^I_{n\pm\theta}, \alpha^{I'}_{m\mp\theta}] = \pm m\,\delta_{n+m}\,\delta^{II'}.$$

World-sheet supersymmetry

$$\delta X^p = \bar{\epsilon}\psi^p$$

leads to the same modding for the complexified worldsheet fermions[1]

$$\Psi^I(z) = \sum_{r+\frac{1}{2}} \ \psi^I_{r-\theta_I}\, z^{-r-\frac{1}{2}+\theta_I} \qquad \bar{\Psi}^I(z) = \sum_{r+\frac{1}{2}} \ \psi^I_{r+\theta_I}\, \bar{z}^{-r-\frac{1}{2}-\theta_I} \quad . \tag{A.3}$$

Notice that we consider the NS-sector where the fermions are half integer modded. The only nonvanishing anti-commutator is given by

$$\{\psi^I_{r-\theta_I}, \psi^I_{q+\theta_I}\} = -\delta_{r,q}\ .$$

[1]Here we already used the doubling trick.

For positve θ_I $(0 < \theta_I < 1)$ the vacuum in the internal dimensions is defined by

$$\begin{aligned} &\alpha^I_{m-\theta_I}|\,0\rangle = 0 \qquad m \geq 1 \qquad\qquad \psi^I_{r-\theta_I}|\,0\rangle = 0 \qquad r \geq \frac{1}{2} \\ &\alpha^I_{m+\theta_I}|\,0\rangle = 0 \qquad m \geq 0 \qquad\qquad \psi^I_{r+\theta_I}|\,0\rangle = 0 \qquad r \geq \frac{1}{2}\ . \end{aligned} \tag{A.4}$$

The physical state constraint requires annihilation with all the positive modes of the Virasoro generators L_n, in particular with L_0, which takes the form

$$\begin{aligned} L_0 = &\sum_{\mu=0}^{3}\left\{\sum_{n\,\epsilon Z} :\alpha^\mu_{-n}\,\alpha^\mu_n: + \sum_{r\,\epsilon Z} r\ :\psi^\mu_{-r}\,\psi^\mu_r:\right\} \\ &+ \sum_{I=1}^{3}\left\{\sum_{m\epsilon Z} :\alpha^I_{-m+\theta_I}\alpha^I_{m-\theta_I}: + \sum_{m\epsilon Z}(q-\theta_I) :\psi^I_{-q+\theta_I}\psi^I_{q-\theta_I}:\right\} + \epsilon_0\ . \end{aligned} \tag{A.5}$$

Here α^μ_n and ψ^μ_r denote the excitations in space-time and ϵ_0 is the zero point energy. Using the fact that the zero mode α^μ_0 represents the momentum of the string we manipulate equation (A.5) and obtain a mass formula for the open string in the twisted sector

$$\begin{aligned} M^2 = &\sum_{\mu=0}^{3}\left\{\sum_{n\epsilon Zn} :\alpha^\mu_{-n}\,\alpha^\mu_n: + \sum_{n\epsilon Z} r\ :\psi^\mu_{-r}\,\psi^\mu_r:\right\} \\ &+ \sum_{I=1}^{3}\left\{\sum_{m\epsilon Z} :\alpha^I_{-m+\theta_I}\alpha^I_{m-\theta_I}: + \sum_{m\epsilon Z}(q-\theta_I) :\psi^I_{-q+\theta_I}\psi^I_{q-\theta_I}:\right\} + \epsilon_0\ . \end{aligned} \tag{A.6}$$

The zero point energy can be computed from the ζ-function regularization, as we demonstrate in the following (for one internal dimension only)

$$\begin{aligned}\epsilon_0^I &= \sum_{m=-\infty}^{0} [\alpha_{-m+\theta_I}, \alpha_{m-\theta_I}] + \sum_{m=-\infty}^{-1/2} (q-\theta_I)\,\{\psi_{-q+\theta_I}, \psi_{q-\theta_I}\} \\ &= \zeta[-1,\theta_I] - \zeta[-1, 1/2+\theta_I] = -\frac{1}{8} + \frac{1}{2}\,\theta_I\ .\end{aligned} \tag{A.7}$$

To get an expression for the vertex operators we need to determine the OPE's of Ψ^I and $\bar{\Psi}^I$ with some particular excitations. First we examine the vacuum state $|\,0\rangle$

$$\Psi^I(z)\,|\,0\rangle = \sum_{r=-\infty}^{\infty} z^{-r-\frac{1}{2}+\theta_I}\psi_{r-\theta_I}|\,0\rangle = \sum_{r=-\infty}^{-\frac{1}{2}} z^{-r-\frac{1}{2}+\theta_I}\psi_{r-\theta_I}|\,0\rangle \to z^{\theta_I}\,t_I(0)\ ,$$

where $t_I(0)$ denotes the excited twist field at the intersection. Similarly we obtain for $\bar{\Psi}^I(z)\,|\,0\rangle$

$$\bar{\Psi}^I(z)\,|\,0\rangle\ \to\ z^{-\theta_I}\,t'_I(0)\ .$$

Using the same procedure, the OPE of Ψ and $\bar{\Psi}$ with the state $\psi_{-\frac{1}{2}+\theta_I}\,|\,0\rangle$ is

$$\Psi^I(z)\,\psi_{-\frac{1}{2}+\theta_I}\,|\,0\rangle\ \to\ z^{\theta_I-1}\,t_I(0) \qquad\qquad \bar{\Psi}^I(z)\,\psi_{-\frac{1}{2}+\theta_I}\,|\,0\rangle\ \to\ z^{1-\theta_I}\,t'_I(0)\ .$$

Considering a negative angle θ_I $(-1<\theta_I<0)$ leads to a different definition of the vacuum

$$\begin{aligned} &\alpha^I_{m-\theta_I}|\,0\rangle = 0 \qquad m\geq 0 \qquad\qquad \psi^I_{r-\theta_I}|\,0\rangle = 0 \qquad r\geq\frac{1}{2} \\ &\alpha^I_{m+\theta_I}|\,0\rangle = 0 \qquad m\geq 1 \qquad\qquad \psi^I_{r+\theta_I}|\,0\rangle = 0 \qquad r\geq\frac{1}{2}\end{aligned} \tag{A.8}$$

and the zero point energy, calculated in the same way as above, takes the form

$$\epsilon_0^I = -\frac{1}{8} - \frac{1}{2}\,\theta_I \tag{A.9}$$

(keep in mind, that the angle θ_I is negative). Again we examine the OPE's of some special physical states with the fermionic fields $\Psi(z)$ and $\bar{\Psi}(z)$. For $|\,0\rangle$ we get

$$\Psi^I(z)\,|\,0\rangle \;\rightarrow\; z^{\theta_I}\,t_I(0) \qquad\qquad \bar{\Psi}^I(z)\,|\,0\rangle \;\rightarrow\; z^{-\theta_I}\,t_I'(0)$$

and for $\psi_{-\frac{1}{2}-\theta_I}\,|\,0\rangle$

$$\Psi^I(z)\,\psi_{-\frac{1}{2}-\theta_I}\,|\,0\rangle \;\rightarrow\; z^{1+\theta_I}\,t_I(0) \qquad\qquad \bar{\Psi}^I(z)\,\psi_{-\frac{1}{2}-\theta_I}\,|\,0\rangle \;\rightarrow\; z^{-1-\theta_I}\,t_I'(0)\;.$$

Before formulating the vertex operators for particular states we also need the OPE's with the bosonic fields

$$\partial\,Z^I(z)\,|\,0\rangle = z^{-(1-\theta_I)}\,\tau_I(0) \qquad\qquad \bar{\partial}\,Z^I(\bar{z})\,|\,0\rangle = z^{-\theta_I}\,\tau_I(0)$$
$$\partial\,\bar{Z}^I(z)\,|\,0\rangle = z^{-\theta_I}\,\tau_I(0) \qquad\qquad \bar{\partial}\,\bar{Z}^I(z)\,|\,0\rangle = z^{-(1-\theta_I)}\,\tau_I(0)\;.$$

For negative angle, we replace θ_I by $\alpha_I = 1+\theta_I$.

Now we can start to construct the vertex operators for the respective states. First we consider the state $\chi = \psi_{-\frac{1}{2}+\theta_3}\,|\,0\rangle$, where θ_1,θ_2 are positive and θ_3 is negative, which means that the string starts at D-brane a and ends at D-brane b. The mass of this state is given by

$$M^2 = -\frac{1}{2} + \frac{1}{2}\,\theta_1 + \frac{1}{2}\,\theta_2 - \frac{1}{2}\,\theta_3 + \frac{1}{2} + \theta_3 = \sum_{I=1}^{3} \theta_I \ .$$

The scalar χ becomes massless when the sum of the angles adds up to zero. This is in agreement with the supersymmetry condition. The corresponding vertex operator in the (-1)-ghost picture takes the form

$$V_{\chi}^{-1}(z) = \mathrm{e}^{-\phi(z)} \prod_{I=1}^{2} \sigma_{\theta_I}(z)\, \mathrm{e}^{\mathrm{i}\theta_I\, H_I(z)}\, \sigma_{1+\theta_3}(z)\, \mathrm{e}^{\mathrm{i}(1+\theta_3)H_3(z)}\, \mathrm{e}^{\mathrm{i}k\cdot X(z)} \ , \tag{A.10}$$

where the H_I's denote the bosonized worldsheet fermion Ψ^I. Notice that in the case of supersymmetry, when the state becomes massless ($k^2 = 0$), the conformal weight of the vertex operator adds up, as required, to one.

The corresponding complex conjugate state χ^* is represented by the same excitation as above but oriented from brane b to brane a. That means that the intersection angles $\theta'_I = -\theta_I$ take the opposite sign as before and therefore the vertex operator is given by

$$V_{\chi^*}^{-1}(z) = \mathrm{e}^{-\phi(z)} \prod_{I=1}^{2} \sigma_{1-\theta_I}(z)\, \mathrm{e}^{-\mathrm{i}\theta_I\, H_I(z)}\, \sigma_{-\theta_3}(z)\, \mathrm{e}^{-\mathrm{i}(1+\theta_3)H_3(z)}\, \mathrm{e}^{\mathrm{i}k\cdot X(z)} \ , \tag{A.11}$$

Let us take a closer look at the vertex operators in the case of supersymmetry, when they carry a $N = 2$ world sheet charge $H = \sum_{I=1}^{3} H_I$. The chiral superfield χ has $N = 2$ world sheet charge +1, while the charge for the complex conjugate partner χ^* is -1, which is in agreement with [148].

Next, we examine the state $\chi^* = \psi_{-\frac{1}{2}+\theta_1}\, \psi_{-\frac{1}{2}+\theta_2}\, \psi_{-\frac{1}{2}+\theta_3} | 0\rangle$, where $0 < \theta_I < 1$ for all

I. Again the string is oriented from brane a to brane b (see figure (3.2)). Why we denote the state by χ^* rather than χ becomes clear later. The mass of χ^* is given by

$$M^2 = -\frac{1}{2} + \frac{1}{2}\sum_{I=1}^{3}\theta_I - \sum_{I=1}^{3}\left(-\frac{1}{2} + \theta_I\right) = 1 - \frac{1}{2}\sum_{I=1}^{3}\theta_I \tag{A.12}$$

and becomes massless, when the sum of the angles is equal to two, again in agreement with the supersymmetry condition. The vertex operator in the (-1)-ghost picture corresponding to this state takes the form

$$V_{\chi^*}^{(-1)}(z) = \mathrm{e}^{-\phi(z)}\prod_{I=1}^{3}\sigma_{\theta_I}(z)\,\mathrm{e}^{\mathrm{i}(\theta_I-1)H_I(z)}\,\mathrm{e}^{\mathrm{i}k\cdot X(z)}\ , \tag{A.13}$$

and as above the requirement that the vertex operator has conformal weight one is satisfied. The corresponding complex conjugated state χ is stretched from brane b to brane a and the intersection angles $\theta'_I = -\theta_I$ are all negative. Therefore the vertex operator is given by

$$V_{\chi}^{(-1)}(z) = \mathrm{e}^{-\phi(z)}\prod_{I=1}^{3}\sigma_{1-\theta_I}(z)\,\mathrm{e}^{-\mathrm{i}(\theta_I-1)H_I(z)}\,\mathrm{e}^{\mathrm{i}k\cdot X(z)}\ . \tag{A.14}$$

A look at the N=2 world sheet charge in the case of supersymmetry ($\sum_{I=1}^{3}\theta_I = 2$) explains the notation since χ^* carries charge -1 while χ carries +1.

We now turn to the Ramond sector, in which the string excitations between two intersecting D-branes correspond to space-time fermions. The mode expansion for the fermionic degrees of freedom takes the same form as for the NS sector, but now we sum over integers instead of half integers

$$\Psi^I(z) = \sum_n \psi^I_{r-\theta_I}\, z^{-r-\frac{1}{2}+\theta_I} \qquad \bar{\Psi}^I(z) = \sum_n \psi^I_{r+\theta_I}\, \bar{z}^{-r-\frac{1}{2}-\theta_I} \ . \tag{A.15}$$

Nothing changes for the bosonic world sheet fields $Z(z,\bar{z})$ and $\bar{Z}(z,\bar{z})$. The vacuum is defined by $(0 < \theta_I < 1)$

$$\begin{aligned} \alpha^I_{m-\theta_I}|\,0\rangle = 0 \qquad m \geq 1 \qquad\qquad \psi^I_{r-\theta_I}|\,0\rangle = 0 \qquad r \geq 1 \\ \alpha^I_{m+\theta_I}|\,0\rangle = 0 \qquad m \geq 0 \qquad\qquad \psi^I_{r+\theta_I}|\,0\rangle = 0 \qquad r \geq 0 \ . \end{aligned} \tag{A.16}$$

With this definition the zero point energy is independently of the choice of angles given by

$$\epsilon^I_0 = 0 \ , \tag{A.17}$$

and therefore independently of the choice of the angles there exists a massless fermion at the intersection. On the other hand the vertex operator for the vacuum $|0\rangle$ depends crucially on the choice of angles. Let us therefore examine the OPE's of worldsheet fermions with $|0\rangle$ for the two different situations that we have positive and negative intersection angles. We obtain for $0 < \theta_I < 1$

$$\Psi^I(z)\,|\,0\rangle \to z^{-\frac{1}{2}+\theta_I}\, t_I(0) \qquad\qquad \bar{\Psi}^I(z)\,|\,0\rangle \to z^{\frac{1}{2}-\theta_I}\, t_I(0) \ .$$

For negative angles we must change the definition of the vacuum to

$$\begin{aligned} \alpha^I_{m-\theta_I}|\,0\rangle &= 0 \qquad m \geq 0 \qquad\qquad \psi^I_{r-\theta_I}|\,0\rangle = 0 \qquad r \geq 0 \\ \alpha^I_{m+\theta_I}|\,0\rangle &= 0 \qquad m \geq 1 \qquad\qquad \psi^I_{r+\theta_I}|\,0\rangle = 0 \qquad r \geq 1\ . \end{aligned} \tag{A.18}$$

The zero point energy is still zero. But now we obtain different OPE's for the vacuum $|\,0\rangle$

$$\Psi^I(z)\,|\,0\rangle \to z^{\frac{1}{2}+\theta_I}\,t_I(0) \qquad\qquad \bar{\Psi}^I(z)\,|\,0\rangle \to z^{-\frac{1}{2}-\theta_I}\,t_I(0)\ .$$

As before for the NS-sector we present for particular states the vertex operators. The first state we consider is the vacuum state $\chi = |\,a\rangle$, whose mass is independent of the choice of angles equal to zero. Assuming, that the intersection angles $\theta_1, \theta_2 > 0$ and $\theta_3 < 0$, the vertex operator takes the form

$$V_\chi^{-\frac{1}{2}}(z) = \mathrm{e}^{-\frac{\phi}{2}(z)}\,S^\alpha(z)\prod_{I=1}^{2}\sigma_{\theta_I}(z)\,\mathrm{e}^{\mathrm{i}\left(\theta_I-\frac{1}{2}\right)H_I(z)}\,\sigma_{1+\theta_3}(z)\,\mathrm{e}^{\mathrm{i}\left(\theta_3+\frac{1}{2}\right)H_3(z)}\,\mathrm{e}^{\mathrm{i}k\cdot X(z)}\ , \tag{A.19}$$

where $S^\alpha = \mathrm{e}^{\pm\frac{1}{2}\mathcal{H}_1\pm\frac{1}{2}\mathcal{H}_2}$ denotes the spin field with positive chirality [2]. As for the NS-sector the corresponding vertex operator for the complex conjugated state χ^* is simply given by orientation reversal, so that the intersection angles are $\theta'_I = -\theta_I$. Thus the vertex operator in $(-\frac{1}{2})$-ghost picture has the form

$$V_{\chi^*}^{-\frac{1}{2}}(z) = \mathrm{e}^{-\frac{\phi}{2}(z)}\,\tilde{S}_{\dot\alpha}(z)\prod_{I=1}^{2}\sigma_{1-\theta_I}(z)\,\mathrm{e}^{-\mathrm{i}\left(\theta_I-\frac{1}{2}\right)H_I(z)}\,\sigma_{-\theta_3}(z)\,\mathrm{e}^{-\mathrm{i}\left(\theta_3+\frac{1}{2}\right)H_3(z)}\,\mathrm{e}^{\mathrm{i}k\cdot X(z)}\ , \tag{A.20}$$

[2] $\mathrm{e}^{\mathcal{H}_{1,2}}$ are the bosonized world sheet fermions Ψ^a with a the four dimensional complexified indices

where $\tilde{S}_{\dot\alpha} = \mathrm{e}^{\pm\frac{1}{2}\mathcal{H}_1 \mp \frac{1}{2}\mathcal{H}_2}$ represents the spin field with opposite chirality as S^α. Notice that independent of the choice of angles the vertex operator has as expected conformal weight one. In case of supersymmetry ($\sum_{I=3}^3 \theta_I = 0$) the vertex operators χ and χ^* carry N=2 world sheet charge $-\frac{1}{2}$ and $\frac{1}{2}$, respectively, which is again in agreement with [148].

Finally let us assume that all the intersecting angles θ_I are positive. In that case the vertex operator for the vacuum state χ^* takes a very symmetric form

$$V_{\chi^*}^{-\frac{1}{2}}(z) = \mathrm{e}^{-\frac{\phi}{2}(z)}\, \tilde{S}_{\dot\alpha}(z) \prod_{I=1}^{3} \sigma_{\theta_I}(z)\, \mathrm{e}^{\mathrm{i}\left(\theta_I - \frac{1}{2}\right) H_I(z)}\, \mathrm{e}^{\mathrm{i}k\cdot X(z)} \ . \tag{A.21}$$

For a similar reason as in the NS-sector we call this vacuum state rather χ^* than χ,since in case of supersymmetry ($\sum_{I=1}^3 \theta_I = 2$) it carries $\frac{1}{2}$ N=2 world sheet charge. Following the procedure described above we obtain for χ

$$V_{\chi}^{-\frac{1}{2}}(z) = \mathrm{e}^{-\frac{\phi}{2}(z)}\, S^{\alpha}(z) \prod_{I=1}^{3} \sigma_{1-\theta_I}(z)\, \mathrm{e}^{-\mathrm{i}\left(\theta_I - \frac{1}{2}\right) H_I(z)}\, \mathrm{e}^{\mathrm{i}k\cdot X(z)} \ . \tag{A.22}$$

One can easily check that in case supersymmetry the vertex operator carries as expected N=2 world sheet charge

Appendix B

Numerical analysis

Before we extract the low energy limit of the amplitudes, given above, let us take a look at the three different limits, namely $x \to 0$, $x \to 1$ and $x \to -\infty$. The first one we observe in both amplitudes and it corresponds in field theory to a gauge boson exchange. The limit $x \to 1$ appears only in the amplitude $\langle V^{10}_{-\frac{1}{2}}{}^* V^{10}_{-\frac{1}{2}} V^{10}_{-\frac{1}{2}}{}^* V^{10}_{-\frac{1}{2}} \rangle$ and again corresponds to a gauge boson exchange. The amplitude $\langle V^{\bar{5}}_{-\frac{1}{2}}{}^* V^{\bar{5}}_{-\frac{1}{2}} V^{10}_{-\frac{1}{2}}{}^* V^{10}_{-\frac{1}{2}} \rangle$ factorizes in the limit $x \to -\infty$, indicating a Higgs particle exchange, due to the coupling $\mathbf{10}\,\bar{\mathbf{5}}\,\bar{\mathbf{5}}_{\mathbf{H}}$. We start by analyzing all those limits[1] and numerically compute the amplitudes 3.23, 3.26, 3.29 and 3.31.

$x \to 0$

The limit $x \to 0$ was already explored in section 4 in order to normalize the amplitude. Here we just state the result for the case that $\theta_1 = \theta_2 = \theta$

- $-\frac{1}{2} < \theta_1 < 0 \qquad -\frac{1}{2} < \theta_2 < 0 \qquad \frac{1}{2} < \theta_3 < 1$

[1] The amplitude $< \mathbf{10}^*\mathbf{10}\mathbf{10}^*\mathbf{10} >$ is invariant under $x \to 1$, thius it is sufficient to analyze the limit $x \to 0$.

In the limit $x \to 0$ $K(\theta, \theta, -2\theta)$ behaves like

$$\sim \pi^{3/2} \int_0 \frac{\mathrm{dx}}{x} \left[\left(\ln \left(\frac{\delta(\theta, 1+2\theta)}{x} \right) \right)^2 \ln \left(\frac{\delta(1+2\theta, -1-4\theta)}{x} \right) \right]^{-1} , \qquad \text{(B.1)}$$

where $\ln \delta(\theta, \nu)$ is given by

$$\ln \delta(\theta, \nu) = 2\psi(1) - \frac{1}{2}\psi(\theta) - \frac{1}{2}\psi(1-\theta) - \frac{1}{2}\psi(\nu) - \frac{1}{2}\psi(1-\nu) \; . \qquad \text{(B.2)}$$

while $M(\theta, \theta, -2\theta)$ scales like

$$\pi^{3/2} \int_T^\infty \mathrm{d}t \; (t + \ln \delta(1+2\theta, 1+2\theta))^{-1} \, (t + \ln \delta(-1-4\theta, -1-4\theta))^{-\frac{1}{2}} \qquad \text{(B.3)}$$

- $\frac{1}{2} < \theta_1 < 1 \qquad \frac{1}{2} < \theta_2 < 1 \qquad \frac{1}{2} < \theta_3 < 1$

For that choice of angles $K(\theta, \theta, 2-2\theta)$ scales like

$$\sim \pi^{3/2} \int_0 \frac{\mathrm{dx}}{x} \left[\left(\ln \left(\frac{\delta(-\theta, -1+2\theta)}{x} \right) \right)^2 \ln \left(\frac{\delta(-1+2\theta, 3-4\theta)}{x} \right) \right]^{-1} \qquad \text{(B.4)}$$

and $M(\theta, \theta, -2\theta)$ like

$$\pi^{3/2} \int_T^\infty \mathrm{d}t \; (t + \ln \delta(2\theta-1, 2\theta-1))^{-1} \, (t + \ln \delta(3-4\theta, 3-4\theta))^{-\frac{1}{2}} \qquad \text{(B.5)}$$

Here $\delta(\theta, \nu)$ takes the same form as above.

$x \to -\infty$

Let us now turn to the limit $x \to -\infty$. As mentioned earlier this limit in (3.23) and (3.26) corresponds to the Higgs particle exchange and thus we expect in this limit a pole indicating the exchange of a massless particle.

The hypergeometric functions behave in the limit $x \to -\infty$

$$\lim_{x\to-\infty} F(a,b,c,x) = \frac{\Gamma(c)\,\Gamma(b-a)}{\Gamma(b)\,\Gamma(c-a)}\, x^{-a} + \frac{\Gamma(c)\,\Gamma(a-b)}{\Gamma(a)\,\Gamma(c-b)}\, x^{-b}$$

$$\lim_{x\to-\infty} F(a,b,c,1-x) = \mathrm{e}^{-\mathrm{i}\pi a}\,\frac{\Gamma(c)\,\Gamma(b-a)}{\Gamma(b)\,\Gamma(c-a)}\, x^{-a} + \mathrm{e}^{-\mathrm{i}\pi b}\,\frac{\Gamma(c)\,\Gamma(a-b)}{\Gamma(a)\,\Gamma(c-b)}\, x^{-b}\ .$$

Hence $I(a,b,x)$ for $x \to \infty$ takes the form

$$\lim_{x\to\infty} \frac{1}{2\pi} I_j(a,b,x)^{-1} \longrightarrow \begin{cases} (-1)^{a-b}\, x^{a+b}\, \Gamma_{a,b} & 0 < a+b < 1 \\ -(-1)^{a-b}\, x^{2-a-b}\, \Gamma_{1-a,1-b} & 1 < a+b < 2 \end{cases}, \tag{B.6}$$

with

$$\Gamma_{a,b} = \frac{\Gamma(1-a)\,\Gamma(1-b)\,\Gamma(a+b)}{\Gamma(a)\,\Gamma(b)\,\Gamma(1-a-b)}\ . \tag{B.7}$$

Using (B.6) the amplitude (3.23) becomes in the limit $x \to \infty$

$$\sim (2\pi)^{\frac{3}{2}}\, \Gamma^{\frac{1}{2}}_{\theta_1,1-2\theta_1}\, \Gamma^{\frac{1}{2}}_{\theta_2,1-2\theta_2}\, \Gamma^{\frac{1}{2}}_{1+\theta_3,-1-2\theta_3} \int_{-\infty} \mathrm{d}x\ x^{-\alpha' t-1}\ . \tag{B.8}$$

Thus, we observe an exchange of a massless particle, which we identify as the Higgs-particle. Note that the prefactor in (B.8) is the expected relative factor between the

Yukawa couplings in string and field theory basis [59, 60] .

Applying the limit for our second amplitude (3.26) we obtain

$$\sim (2\pi)^{\frac{3}{2}} \prod_{I=1}^{3} \Gamma^{\frac{1}{2}}_{1+\theta_I,-1-2\theta_I} \int_{-\infty} \mathrm{d}x \; x^{-\alpha' t-1} \tag{B.9}$$

and again we can observe a massless Higgs exchange in this limit.

The amplitude $\langle V^{\bar{5}}_{-\frac{1}{2}}{}^{*} V^{\bar{5}}_{-\frac{1}{2}} V^{10}_{-\frac{1}{2}}{}^{*} V^{10}_{-\frac{1}{2}} \rangle$

The analysis for both amplitudes, (3.23) and (3.26) is similar, so that we will describe the steps for the first one and apply these later for the second amplitude.

$T(\theta_1, \theta_2, \theta_3)$

In the following we compute the four fermi coupling $\bar{\mathbf{5}}^{*}\,\bar{\mathbf{5}}\,\mathbf{10}^{*}\,\mathbf{10}$. In order to get just the four fermi term we need to subtract all appearing poles. While due to the special kinematics of the setup the integral $T(\theta_1, \theta_2, \theta_3)$ is not divergent in the limit $x \to 0$ we do need to subtract the pole arising due to the Higgs exchange $x \to -\infty$.

We start by splitting the integral (3.24) into two parts (again we assume that

$\theta_1 = \theta_2 = \theta)$

$$\int_{-\infty}^{L} \mathrm{d}x\, x^{-\alpha' s-1} (1-x)^{-\alpha' u-1} \left[I^2(-\theta, 1+2\theta, x)\ I(1+2\theta, -1-4\theta, x)\right]^{-\frac{1}{2}}$$
$$+ \int_{L}^{0} \mathrm{d}x\, x^{-\alpha' s-1} (1-x)^{-\alpha' u-1} \left[I^2(-\theta, 1+2\theta, x)\ I(1+2\theta, -1-4\theta, x)\right]^{-\frac{1}{2}} \quad . \tag{B.10}$$

We replace x by $1-e^z$ in the first summand and in the second by $\frac{1}{1-e^z}$

$$\int_{\ln(1-L)}^{\infty} \mathrm{d}z \frac{\left[I^2(-\theta, 1+2\theta, 1-e^z)\ I(1+2\theta, -1-4\theta, 1-e^z)\right]^{-\frac{1}{2}}}{(e^z)^{\alpha' u} (1-e^z)^{\alpha' s+1}} \tag{B.11}$$
$$+ \int_{\ln(1-\frac{1}{L})}^{\infty} \mathrm{d}z \frac{\left[I^2\left(-\theta, 1+2\theta, \frac{1}{1-e^z}\right)\ I\left(1+2\theta, -1-4\theta, \frac{1}{1-e^z}\right)\right]^{-\frac{1}{2}}}{(e^z)^{\alpha' u} (1-e^z)^{\alpha' t}} \quad .$$

To simplify the computation, we break up both terms into two parts

$$\int_{\ln(1-L)}^{T_1} \mathrm{d}z \frac{\left[I^2(-\theta, 1+2\theta, 1-e^z)\ I(1+2\theta, -1-4\theta, 1-e^z)\right]^{-\frac{1}{2}}}{(e^z)^{\alpha' u} (1-e^z)^{\alpha' s+1}}$$
$$+ (2\pi)^{\frac{3}{2}}\, \Gamma_{-\theta, 1+2\theta}\, \Gamma^{\frac{1}{2}}_{1+2\theta, -1-4\theta} \int_{T1}^{\infty} \mathrm{d}z\, (e^z)^{-\alpha' u+1} (1-e^z)^{-\alpha' s-1}$$
$$+ \int_{\ln(1-\frac{1}{L})}^{T_2} \mathrm{d}z\, \frac{\left[I^2\left(-\theta, 1+2\theta, \frac{1}{1-e^z}\right)\ I\left(1+2\theta, -1-4\theta, \frac{1}{1-e^z}\right)\right]^{-\frac{1}{2}}}{(e^z)^{\alpha' u} (1-e^z)^{\alpha' t}}$$
$$+ \pi^{\frac{3}{2}} \int_{T_2}^{\infty} \mathrm{d}z \frac{(z+\ln\delta(-\theta, 1+2\theta))^{-1} (z+\ln\delta(1+2\theta, -1-4\theta))^{-\frac{1}{2}}}{(e^z)^{\alpha' u} (1-e^z)^{\alpha' t}}$$

Here we replaced the hypergeometric expressions by their respective limits in the range from T_1 to ∞ and T_2 to ∞ . As mentioned above in order to get the four-Fermi interaction contribution, we need to subtract the $\frac{1}{\alpha' t}$ pole and take the low energy limit

$$T(\theta) = \lim_{s,t,u\to 0} \Bigg\{ \int_{\ln(1-L)}^{T_1} \mathrm{d}z \, \frac{\left[I^2\left(-\theta, 1+2\theta, 1-e^z\right) \, I\left(1+2\theta, -1-4\theta, 1-e^z\right)\right]^{-\frac{1}{2}}}{\left(e^z\right)^{\alpha' u}\left(1-e^z\right)^{\alpha' s+1}}$$
$$+ \left((2\pi)^{\frac{3}{2}} \, \Gamma_{-\theta,1+2\theta} \, \Gamma^{\frac{1}{2}}_{1+2\theta,-1-4\theta} \int_{T1}^{\infty} \mathrm{d}z \, \left(e^z\right)^{-\alpha' u+1}\left(1-e^z\right)^{-\alpha' s-1} - \frac{1}{\alpha' t}\right)$$
$$+ \int_{\ln(1-\frac{1}{L})}^{T_2} \mathrm{d}z \, \frac{\left[I^2\left(-\theta, 1+2\theta, \frac{1}{1-e^z}\right) \, I\left(1+2\theta, -1-4\theta, \frac{1}{1-e^z}\right)\right]^{-\frac{1}{2}}}{\left(e^z\right)^{\alpha' u}\left(1-e^z\right)^{\alpha' t}}$$
$$+ \pi^{\frac{3}{2}} \int_{T_2}^{\infty} \mathrm{d}z \, \frac{\left(z+\ln\delta(-\theta,1+2\theta)\right)^{-1}\left(z+\ln\delta(1+2\theta,-1-4\theta)\right)^{-\frac{1}{2}}}{\left(e^z\right)^{\alpha' u}\left(1-e^z\right)^{\alpha' t}} \Bigg\} \, .$$

For the second region $\frac{1}{2} < \theta_1 < 1$, $\frac{1}{2} < \theta_2 < 1$ and $\frac{1}{2} < \theta_3 < 1$, $T(\theta)$ takes the form

$$T(\theta) = \lim_{s,t,u\to 0} \Bigg\{ \int_{\ln(1-L)}^{T_1} \mathrm{d}z \, \frac{\left[I^2\left(1-\theta, -1+2\theta, 1-e^z\right) \, I\left(-1+2\theta, 3-4\theta, 1-e^z\right)\right]^{-\frac{1}{2}}}{\left(e^z\right)^{\alpha' u}\left(1-e^z\right)^{\alpha' s+1}}$$
$$+ \left((2\pi)^{\frac{3}{2}} \, \Gamma_{1-\theta,-1+2\theta} \, \Gamma^{\frac{1}{2}}_{-1+2\theta,3-4\theta} \int_{T1}^{\infty} \mathrm{d}z \, \left(e^z\right)^{-\alpha' u+1}\left(1-e^z\right)^{-\alpha' s-1} - \frac{1}{\alpha' t}\right)$$
$$+ \int_{\ln(1-\frac{1}{L})}^{T_2} \mathrm{d}z \, \frac{\left[I^2\left(1-\theta, -1+2\theta, \frac{1}{1-e^z}\right) \, I\left(-1+2\theta, 3-4\theta, \frac{1}{1-e^z}\right)\right]^{-\frac{1}{2}}}{\left(e^z\right)^{\alpha' u}\left(1-e^z\right)^{\alpha' t}}$$
$$+ \pi^{\frac{3}{2}} \int_{T_2}^{\infty} \mathrm{d}z \, \frac{\left(z+\ln\delta(1-\theta,-1+2\theta)\right)^{-1}\left(z+\ln\delta(-1+2\theta,3-4\theta)\right)^{-\frac{1}{2}}}{\left(e^z\right)^{\alpha' u}\left(1-e^z\right)^{\alpha' t}} \Bigg\} \, .$$

Mathematica is not able to take that limit, however by plugging in different small values for the Mandelstam variables s, t and u we get a stable contribution for $T(\theta)$.

The amplitude $\langle V^{10\,*}_{-\frac{1}{2}} \, V^{10}_{-\frac{1}{2}} \, V^{10\,*}_{-\frac{1}{2}} V^{10}_{-\frac{1}{2}} \rangle$

The amplitude is finite even in the low energy limit, thus i n opposite to the amplitude $\langle V^{\bar{5}}_{-\frac{1}{2}}{}^{*} \, V^{\bar{5}}_{-\frac{1}{2}} \, V^{10\,*}_{-\frac{1}{2}} V^{10}_{-\frac{1}{2}} \rangle$ we do not have to subtract any poles. After taking the low energy limit we are left with

$$M = \int_0^1 \mathrm{d}x \frac{\sqrt{\sin[\pi(1+\nu_I)]}\ L^{-\frac{1}{2}}(1+\nu_I, x)}{x(1-x)} \tag{B.12}$$

We split the integral by using the expression

$$\frac{1}{x(1-x)} = \frac{1}{x} + \frac{1}{1-x}\ . \tag{B.13}$$

and note that both summands are equal so that after substituting e^{-t} for x we obtain

$$M = 2\int_0^\infty \mathrm{d}t \prod_{I=1}^{3} \sqrt{\sin[\pi(1+\nu_I)]}\ L^{-\frac{1}{2}}(1+\nu_I). \tag{B.14}$$

Mathematica is not able to evaluate this expression numerically since it is hard to maintain numerical precision for large t. Therefore we will split integral (B.14) into the range from 0 to T and from T to ∞. For the computation of the first region we will use Mathematica to evaluate it numerically, while for the second region we replace the hypergeometric functions by their asymptotic behavior given in (B.1)

$$M = 2\int_0^T \mathrm{d}t \prod_{I=1}^{3} \sqrt{\sin[\pi(1+\nu_I)]}\ L^{-\frac{1}{2}}(1+\nu_I) + \pi^{3/2} \int_T^\infty \mathrm{d}t\ \prod_{I=1}^{3} (t + \ln\delta(1+\nu_I, 1+\nu_I))^{-\frac{1}{2}}$$

Replacing ν_I by θ_I and assuming that $\theta_1 = \theta_2$ we get for

- $-\frac{1}{2} < \theta_1 < 0 \qquad -\frac{1}{2} < \theta_2 < 0 \qquad \frac{1}{2} < \theta_3 < 1$

$$M = 2\int_0^T \mathrm{d}t\ \sin[\pi(1+2\theta)]\sqrt{\sin[\pi(-1-4\theta)]}\ L^{-1}(1+2\theta)\,L^{-\frac{1}{2}}(-1-4\theta)$$

$$+\,\pi^{3/2}\int_T^\infty \mathrm{d}t\ (t+\ln\delta(1+2\theta,1+2\theta))^{-1}\,(t+\ln\delta(-1-4\theta,-1-4\theta))^{-\frac{1}{2}}\ , \qquad \text{(B.15)}$$

and for

- $\frac{1}{2}<\theta_1<1 \qquad \frac{1}{2}<\theta_2<1 \qquad \frac{1}{2}<\theta_3<1$

$$M = 2\int_0^T \mathrm{d}t\ \sin[\pi(2\theta-1)]\sqrt{\sin[\pi(3-4\theta)]}\ L^{-1}(2\theta-1)\,L^{-\frac{1}{2}}(3-4\theta) \qquad \text{(B.16)}$$

$$+\,\pi^{3/2}\int_T^\infty \mathrm{d}t\ (t+\ln\delta(2\theta-1,2\theta-1))^{-1}\,(t+\ln\delta(3-4\theta,3-4\theta))^{-\frac{1}{2}}\ .$$

Appendix C

Local multi-instanton setup on $T^6/\mathbb{Z}_2 \times \mathbb{Z}_2'$

In this appendix we present a local realization of the multi-instanton effect discussed in the section 6.1.4 in a Type IIA compactification. As compactification manifold we choose $T^6/\mathbb{Z}_2 \times \mathbb{Z}_2'$ orientifold with Hodge numbers $(h_{11}, h_{12}) = (3, 51)$ [36] which is known to exhibit rigid cycles[1]. We adopt the notation of [36], where further details can be found. The orbifold group is generated by θ and θ' acting as reflection in the first and last two tori, respectively.

Each sector, θ, θ' and $\theta\theta'$ exhibits 16 fixed points which after blowing up give rise to additional two-cycles with the topology of $\mathbb{P}_1$. Apart from the usual non-rigid bulk cycles

$$\Pi_a^B = 4 \bigotimes_{I=1}^{3} (n_a^I [a^I] + \widetilde{m}_a^I [b^I]), \tag{C.1}$$

[1]For a different orientifold background based on shift orbifolds giving rise to rigid cycles see [37].

defined in terms of the fundamental one-cycles $[a^I], [b^I]$ of the I-th T^2 and the corresponding wrapping numbers n_a^I and $\widetilde{m}_a^I = m_a^I + \beta^I n_a^I$ where $\beta^I = 0, 1/2$ for rectangular and tilted tori, respectively, the background also contains so-called g-twisted cycles

$$\Pi_{ij}^g = [\alpha_{ij}^g] \times [(n^{I_g}, \widetilde{m}^{I_g})]. \tag{C.2}$$

Here $i, j \in \{1,2,3,4\} \times \{1,2,3,4\}$ labels one of the 16 blown-up fixed points of the orbifold element $g = \theta, \theta', \theta\theta' \in \mathbb{Z}_2 \times \mathbb{Z}_2'$. These cycles are basically twice the product of the two cycles of the corresponding $\mathbb{P}_1$ and the I_g invariant one cycle $[(n^{I_g}, \widetilde{m}^{I_g})]$, where $I_g = 3, 1, 2$ for $g = \theta, \theta', \theta\theta'$.

Cycles which are charged under all three twisted sectors are rigid and take the form

$$\Pi^F = \frac{1}{4}\Pi^B + \frac{1}{4}\Big(\sum_{i,j \in S_\theta} \epsilon_{ij}^\theta \Pi_{ij}^\theta\Big) + \frac{1}{4}\Big(\sum_{j,k \in S_{\theta'}} \epsilon_{jk}^{\theta'} \Pi_{jk}^{\theta'}\Big) + \frac{1}{4}\Big(\sum_{i,k \in S_{\theta\theta'}} \epsilon_{ik}^{\theta\theta'} \Pi_{ik}^{\theta\theta'}\Big). \tag{C.3}$$

Here S_g denotes the set of fixed points that the rigid brane runs through in the g-twisted sector. The $\epsilon_{ij}^g = \pm 1$ correspond to the two different orientation the brane can wrap the $\mathbb{P}_1$ and have to satisfy various consistency conditions [36].

The orientifold action $\Omega\mathcal{R}$ on untwisted cycles takes the usual form

$$\Omega\mathcal{R} : [(n_1, \widetilde{m}_1)(n_2, \widetilde{m}_2)(n_3, \widetilde{m}_3)] \to [(n_1, -\widetilde{m}_1)(n_2, -\widetilde{m}_2)(n_3, -\widetilde{m}_3)] \tag{C.4}$$

whereas the twisted cycles transform as

$$\Omega\mathcal{R} :\ \alpha_{ij}^g[(n^{I_g}, \widetilde{m}^{I_g})] \to -\eta_{\Omega\mathcal{R}}\, \eta_{\Omega\mathcal{R}g}\, \alpha_{\mathcal{R}(i)\mathcal{R}(j)}^g[(-n^{I_g}, \widetilde{m}^{I_g})], \tag{C.5}$$

where the reflection $\mathcal{R}$ leaves all fixed points of an untilted two-torus invariant and acts on the fixed points in a tilted two-torus as

$$\mathcal{R}(1) = 1, \qquad \mathcal{R}(2) = 2, \qquad \mathcal{R}(3) = 4, \qquad \mathcal{R}(4) = 3. \tag{C.6}$$

The orientifold charges $\eta_{\Omega\mathcal{R}g} = \pm 1$ are subject to the constraint

$$\eta_{\Omega\mathcal{R}}\, \eta_{\Omega\mathcal{R}\theta}\, \eta_{\Omega\mathcal{R}\theta'}\, \eta_{\Omega\mathcal{R}\theta\theta'} = -1. \tag{C.7}$$

In our subsequent local setup we choose them to be

$$-\eta_{\Omega\mathcal{R}} = \eta_{\Omega\mathcal{R}\theta} = \eta_{\Omega\mathcal{R}\theta\theta'} = \eta_{\Omega\mathcal{R}\theta'} = 1. \tag{C.8}$$

In addition we assume all three tori to be tilted such that the orientifold planes are given by

$$\Pi_{O6} = -[(2,\tilde{0})(2,\tilde{0})(2,\tilde{0})] - [(2,\tilde{0})(0,\tilde{1})(0,\tilde{1})] - 2[(0,\tilde{1})(2,\tilde{0})(0,\tilde{1})] - [(0,\tilde{1})(0,\tilde{1})(2,\tilde{0})].$$

The $U(1)$-instanton E wraps a bulk cycle of the form

$$\Pi^B_\Xi = [(-1,0)(-1,0)(-1,0)] = [(-1,-\frac{\tilde{1}}{2})(-1,-\frac{\tilde{1}}{2})(-1,-\frac{\tilde{1}}{2})] \tag{C.9}$$

and passes through the origin in all three tori. Thus its whole homology class Ξ is given by

$$\Pi^F_{\Xi} = \frac{1}{4}\Pi^B_{\Xi} + \frac{1}{4}\sum_{i,j\epsilon(13)\times(13)} \varepsilon^{\theta}_{ij}\Pi^{\theta}_{\Xi} + \frac{1}{4}\sum_{j,k\epsilon(13)\times(13)} \varepsilon^{\theta'}_{jk}\Pi^{\theta'}_{\Xi} + \frac{1}{4}\sum_{i,k\epsilon(13)\times(13)} \varepsilon^{\theta\theta'}_{ik}\Pi^{\theta\theta'}_{\Xi},$$

$$\Pi^{\theta}_{\Xi} = [((-1,-\frac{\tilde{1}}{2})], \qquad \Pi^{\theta'}_{\Xi} = [(-1,-\frac{\tilde{1}}{2})], \qquad \Pi^{\theta\theta'}_{\Xi} = [(-1,-\frac{\tilde{1}}{2})]. \tag{C.10}$$

Its orientifold image takes the form

$$\Pi^F_{\Xi'} = \frac{1}{4}\Pi^B_{\Xi} + \frac{1}{4}\sum_{i,j\epsilon(14)\times(14)} \varepsilon^{\theta}_{ij}\Pi^{\theta}_{\Xi'} + \frac{1}{4}\sum_{j,k\epsilon(14)\times(14)} \varepsilon^{\theta'}_{jk}\Pi^{\theta'}_{\Xi'} + \frac{1}{4}\sum_{i,k\epsilon(14)\times(14)} \varepsilon^{\theta\theta'}_{ik}\Pi^{\theta\theta'}_{\Xi'},$$

$$\Pi^B_{\Xi'} = [(-1,\frac{\tilde{1}}{2})(-1,\frac{\tilde{1}}{2})(-1,\frac{\tilde{1}}{2})], \tag{C.11}$$

$$\Pi^{\theta}_{\Xi'} = [((-1,\frac{\tilde{1}}{2})], \qquad \Pi^{\theta'}_{\Xi'} = [(-1,\frac{\tilde{1}}{2})], \qquad \Pi^{\theta\theta'}_{\Xi'} = [(-1,\frac{\tilde{1}}{2})].$$

With the intersection formulae

$$\Pi^B_a \circ \Pi^B_b = 4\prod_{i=1}^{3}(n^i_a \widetilde{m}^i_b - n^i_b \widetilde{m}^i_a),$$

$$\Pi^g_{ij} \circ \Pi^h_{kl} = 4\delta^{gh}\,\delta^{ik}\,\delta^{jl}(n^{I_g}_a \widetilde{m}^{I_h}_b - n^{I_h}_b \widetilde{m}^{I_g}_a) \tag{C.12}$$

it is easy to show that

$$\Pi_{\Xi'} \circ \Pi_{\Xi} = 1, \qquad \Pi_{O6} \circ \Pi_{\Xi} = 1. \tag{C.13}$$

As discussed in section 6.1.4 we need 4 additional charged zero modes between E and some D-brane a. We choose the brane a wrapping the cycle

$$\Pi_a^F = \frac{1}{4}\Pi_a^B + \frac{1}{4}\sum_{i,j\epsilon(12)\times(34)} \varepsilon_{ij}^{\theta}\Pi_a^{\theta} + \frac{1}{4}\sum_{j,k\epsilon(12)\times(34)} \varepsilon_{jk}^{\theta'}\Pi_a^{\theta'} + \frac{1}{4}\sum_{i,k\epsilon(34)\times(34)} \varepsilon_{ik}^{\theta\theta'}\Pi_a^{\theta\theta'} \quad (C.14)$$

with

$$\Pi_a^B = [(0,1)(-4,3)(-4,3)] = [(0,\tilde{1})(-4,\tilde{1})(-4,\tilde{1})], \quad (C.15)$$

$$\Pi_a^{\theta} = [(-4,\tilde{1})], \qquad \Pi_a^{\theta'} = [(0,\tilde{1})], \qquad \Pi_a^{\theta\theta'} = [(-4,\tilde{1})]. \quad (C.16)$$

Note that in contrast to section 6.1.4 we choose the D-brane to be not invariant under the orientifold action. Thus we additionally have to ensure $\Pi_{\Xi} \circ \Pi_a' = 0$ for its orientifold image a', which is indeed satisfied. In order to satisfy supersymmetry we choose the complex structure moduli U^I to be

$$U^1 = \frac{8}{3}\ , \qquad U^2 = 4\ , \qquad U^3 = 4\ . \quad (C.17)$$

As described in [81] the $\overline{\tau}_{\dot{\alpha}}$- and $\overline{\mu}_{\dot{\alpha}}$-modes can be soaked up by the coupling $m\overline{\tau}_{\dot{\alpha}}\overline{\mu}^{\dot{\alpha}}$ but there is no way to absorb the charged zero modes λ unless we take into account additional $O(1)$-instantons. Indeed there are two instantons satisfying the constraints (6.32). Their homology classes are given by

$$\Pi^F_{\widetilde{\Xi}_1} = \frac{1}{4}\Pi^B_{\widetilde{\Xi}_1} + \frac{1}{4}\sum_{i,j\epsilon(12)\times(12)} \varepsilon^{\theta}_{ij}\Pi^{\theta}_{\widetilde{\Xi}_1} + \frac{1}{4}\sum_{j,k\epsilon(12)\times(12)} \varepsilon^{\theta'}_{jk}\Pi^{\theta'}_{\widetilde{\Xi}_1} + \frac{1}{4}\sum_{i,k\epsilon(12)\times(12)} \varepsilon^{\theta\theta'}_{ik}\Pi^{\theta\theta'}_{\widetilde{\Xi}_1}, \tag{C.18}$$

$$\Pi^F_{\widetilde{\Xi}_2} = \frac{1}{4}\Pi^B_{\widetilde{\Xi}_2} + \frac{1}{4}\sum_{i,j\epsilon(34)\times(12)} \varepsilon^{\theta}_{ij}\Pi^{\theta}_{\widetilde{\Xi}_2} + \frac{1}{4}\sum_{j,k\epsilon(12)\times(12)} \varepsilon^{\theta'}_{jk}\Pi^{\theta'}_{\widetilde{\Xi}_2} + \frac{1}{4}\sum_{i,k\epsilon(34)\times(12)} \varepsilon^{\theta\theta'}_{ik}\Pi^{\theta\theta'}_{\widetilde{\Xi}_2} \tag{C.19}$$

with

$$\Pi^B_{\widetilde{\Xi}_1} = \Pi^B_{\widetilde{\Xi}_2} = [(2,-1)(2,-1)(2,-1)] = [(2,\tilde{0})(2,\tilde{0})(2,\tilde{0})], \tag{C.20}$$

$$\Pi^{\theta}_{\widetilde{\Xi}_1} = \Pi^{\theta}_{\widetilde{\Xi}_2} = [(2,\tilde{0})], \qquad \Pi^{\theta'}_{\widetilde{\Xi}_1} = \Pi^{\theta'}_{\widetilde{\Xi}_2} = [(2,\tilde{0})], \qquad \Pi^{\theta\theta'}_{\widetilde{\Xi}_1} = \Pi^{\theta\theta'}_{\widetilde{\Xi}_2} = [(2,\tilde{0})]. \tag{C.21}$$

Note that both cycles are invariant under the orientifold action and are separated in the first torus ensuring that the additional zero modes appearing in the $\widetilde{E}_1 - \widetilde{E}_2$ sector become massive. Now it is possible to soak up all the zero modes via the couplings (6.16), (6.18) and (6.19).

Let us briefly discuss the holomorphicity of the superpotential based on this example. The Yukawa couplings $\overline{L}_i$ and Y_i in (6.16) and (6.18), respectively take the form

$$Y_{kij} = y_{kij} \prod_{I=1}^{3} \Gamma^{\frac{1}{4}}_{1+\phi^I_{Ea},1-\phi^I_{E\tilde{E}_k},1-\phi^I_{\tilde{E}_k a}}, \quad \overline{L}_k = l_k \prod_{I=1}^{3} \Gamma^{\frac{1}{4}}_{-\phi^I_{EE'},\phi^I_{E\tilde{E}_k},\phi^I_{E\tilde{E}_k}}, \tag{C.22}$$

where ϕ^I_{ij} denotes the intersection angle between instanton i and brane or instanton

j, respectively and

$$\Gamma_{\alpha,\beta,\gamma} = \frac{\Gamma(1-\alpha)\,\Gamma(1-\beta)\,\Gamma(1-\gamma)}{\Gamma(\alpha)\Gamma(\beta)\Gamma(\gamma)}. \tag{C.23}$$

The lowercase letters denote the holomorphic part of the Yukawa couplings, which essentially are given by the world sheet contributions. Note that the dependence on $\phi_{E\tilde{E}_k}$ in (6.24) and (6.26) drops out due to the inverse dependence of Y^2_{ikl} to $\overline{L}_i$. In addition there are also non-holomorphic contributions from the annulus diagrams $A(E,a)$ and $A(\tilde{E}_k,a)$ as well as the Möbius diagram $M(E,O6)$ [67]. In our example they are given by [95, 103, 149]

$$\begin{aligned}
\exp(A^{n.h.}(E,a)) &= \left(\frac{\Gamma(1+\phi^1_{Ea})\Gamma(1+\phi^2_{Ea})\Gamma(1+\phi^3_{Ea})}{\Gamma(-\phi^1_{Ea})\Gamma(-\phi^2_{Ea})\Gamma(-\phi^3_{Ea})}\right), \\
\exp(A^{n.h.}(\tilde{E}_k,a)) &= \left(\frac{\Gamma(\phi^1_{\tilde{E}_k a})\Gamma(\phi^2_{\tilde{E}_k a})\Gamma(\phi^3_{\tilde{E}_k a})}{\Gamma(1-\phi^1_{\tilde{E}_k a})\Gamma(1-\phi^2_{\tilde{E}_k a})\Gamma(1-\phi^3_{\tilde{E}_k a})}\right)^{\frac{1}{2}}, \\
\exp(M^{n.h.}(E,O6)) &= \left(\frac{\Gamma(-\phi^1_{E'E})\Gamma(-\phi^2_{E'E})\Gamma(-\phi^3_{E'E})}{\Gamma(1+\phi^1_{EE'})\Gamma(1+\phi^2_{EE'})\Gamma(1+\phi^3_{EE'})}\right)^{\frac{1}{2}}.
\end{aligned} \tag{C.24}$$

Indeed, after plugging (C.22) and (C.24) into (6.24) or (6.26) all angle dependence cancels and one is left with a holomorphic expression for the superpotential.

Let us deform the complex structure in the first torus away from the line of marginal stability. Note that under deformation of the complex structure U^1, while keeping the complex structures in the other two tori fixed, the brane a remains supersymmetric. For $U^1 > 8/3$ we induce a positive Fayet-Iliopoulus parameter ξ for the $U(1)_E$ and as described in section 6.1.5 the cycles Ξ and Ξ' combine into a new special Lagrangian $Y = \Xi\#\Xi'$ preserving the same $\mathcal{N}=1$ supersymmetry as the orientifold. The whole multi-instanton configuration is then given by $(\Xi\#\Xi')\cup$

$\widetilde{\Xi}_1 \cup \widetilde{\Xi}_2$. For $U^1 < 8/3$ we induce a negative ξ for the $U(1)_E$ and the multi-instanton configuration recombines into the new BPS state $\widetilde{\Xi}_1 \# ((\Xi \cup \Xi')) \# \widetilde{\Xi}_2$.

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