Characterizing the Role of Contextualized Problems in a Written and Enacted Algebra Unit

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Characterizing the Role of Contextualized Problems in a Written and Enacted Algebra Unit

Abstract
Numerous scholars propose that students can develop deep understandings of mathematical concepts through contextualized problem solving (e.g. Freudenthal, 1991), but little is known about how teachers view contextualized problems [CPs] and whether they implement CPs in ways that align with researchers’ recommendations (Chapman, 2006). The limited research available on the topic suggests that teacher beliefs may lead to practices that fail to realize CPs’ potential for building conceptual understanding (Gainsburg, 2008, 2009; Pierce & Stacey, 2006) and that teachers may lack a firm understanding of how CPs can support learning (Lee, 2012). This case study sought to characterize the role of contextualized problems in one algebra unit to provide insight into how CPs can support and constrain the learning of mathematics and to provide a better understanding of how teacher beliefs and practices mediate students’ experiences with a CP-based curriculum.

Qualitative methods were used to characterize the role of CPs in the written curriculum, the teacher’s plans, and the classroom enactment of the curriculum. Teacher interviews provided data on teacher beliefs and factors that influenced her decisions.

The role of CPs changed as the curriculum was transformed from the written page to classroom enactment. The teacher re-sequenced tasks in response to legitimate concerns; this adaptation compromised the progression from CPs to non-contextualized problems present in the written curriculum. As a result, students had fewer opportunities to leverage their experiences with CPs to make sense of analogous non-contextualized tasks, as intended by the curriculum developers. The teacher highlighted the intended mathematics during discussions of CPs by prompting students to reflect across contexts; this type of task was not present in the written curriculum. When the progression from CPs to non-contextualized problems was preserved, students’ experiences with CPs were rarely referenced during later work on non-contextualized tasks or during discussions of summative, generalizing tasks.

The analytical framework created to characterize the role of contextualized problems in the curriculum has the potential to guide research, curriculum development, and instruction. Findings around teacher adaptations and recommendations for leveraging the affordances of CPs in instruction have implications for teachers and curriculum developers.

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CHARACTERIZING THE ROLE OF CONTEXTUALIZED PROBLEMS IN A WRITTEN AND ENACTED ALGEBRA UNIT

Luke Reinke

A DISSERTATION

in

Education

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CHARACTERIZING THE ROLE OF CONTEXTUALIZED PROBLEMS IN A
WRITTEN AND ENACTED ALGEBRA UNIT

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CHARACTERIZING THE ROLE OF CONTEXTUALIZED PROBLEMS IN A WRITTEN AND ENACTED ALGEBRA UNIT

Luke Reinke
Janine Remillard

Numerous scholars propose that students can develop deep understandings of mathematical concepts through contextualized problem solving (e.g. Freudenthal, 1991), but little is known about how teachers view contextualized problems [CPs] and whether they implement CPs in ways that align with researchers’ recommendations (Chapman, 2006). The limited research available on the topic suggests that teacher beliefs may lead to practices that fail to realize CPs’ potential for building conceptual understanding (Gainsburg, 2008, 2009; Pierce & Stacey, 2006) and that teachers may lack a firm understanding of how CPs can support learning (Lee, 2012). This case study sought to characterize the role of contextualized problems in one algebra unit to provide insight into how CPs can support and constrain the learning of mathematics and to provide a better understanding of how teacher beliefs and practices mediate students’ experiences with a CP-based curriculum.
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# TABLE OF CONTENTS

**ACKNOWLEDGMENT** ............................................................................................................ III

**ABSTRACT** .......................................................................................................................... V

**LIST OF TABLES** ................................................................................................................ X

**LIST OF FIGURES** ............................................................................................................ XI

**CHAPTER 1: INTRODUCTION** ........................................................................................ 1

1. Theoretical Framework ........................................................................................................ 4
   1.1 Transformation of Curriculum .................................................................................... 4
   1.2 Contextualized Problem solving as a Basis for Learning Mathematics .................. 5

2. Research Questions ............................................................................................................ 8

3. Setting ................................................................................................................................ 10
   3.1 Core-Plus Mathematics: Contemporary Mathematics in Context ....................... 10
   3.2 Participating Teacher ................................................................................................. 11
   3.3 School Setting .......................................................................................................... 11

4. Data Set ............................................................................................................................ 12
   4.1 Document Analysis ................................................................................................. 12
   4.2 Observations and transcripts ..................................................................................... 13
   4.3 Interviews ................................................................................................................. 14

5. Summary .......................................................................................................................... 14

**CHAPTER 2: TOWARDS A FRAMEWORK FOR ANALYZING THE ROLE OF CONTEXTUALIZED PROBLEMS IN MATHEMATICS INSTRUCTION** ....15

1. Introduction ....................................................................................................................... 17

2. Theoretical Perspectives .................................................................................................. 21
   2.1 Perspectives on Conceptual Model Building ......................................................... 21
   2.2 Perspectives on the Use of Contextualized Problems in Mathematics Instruction .... 22
   2.3 Perspectives on Abstraction .................................................................................... 31
3. Empirical Testing ........................................................................................................ 35
  3.1 Setting and Participants....................................................................................... 35
  3.2 Data Sources......................................................................................................... 37
  3.3 Data Analysis ......................................................................................................... 37

4. A Framework for Analyzing the Role of Contextualized Problems in Mathematics Instruction ........................................................................................................ 40
  4.1 Activity Types in the CPMI Framework................................................................. 41
  4.2 Activity Types within the Contextualized Problem solving Category.................... 47

5. Application of the CPMI Framework ....................................................................... 52

6. Conclusions .................................................................................................................. 57

CHAPTER 3: THE ROLE OF CONTEXTUALIZED PROBLEMS IN INSTRUCTION: ONE TEACHERS’ TRANSFORMATION OF THE WRITTEN CURRICULUM ................................................................. 61

1. Introduction ............................................................................................................... 62

2. Theoretical Framework ............................................................................................. 67
  2.1 Teacher Intended Curriculum.................................................................................. 67
  2.2 Studying Teacher Practice....................................................................................... 70

3. Method ....................................................................................................................... 71
  3.1 Curriculum, Instructional Context, and Study Participant ....................................... 71
  3.2 Data sources............................................................................................................. 73
  3.3 Analysis .................................................................................................................... 74

4. Findings ..................................................................................................................... 84
  4.1 Modifications to the Progression from Contextualized Examples to Non-Contextualized Examples ........................................................................................................... 85
  4.2 Modifications to the Placement of Reflection on General Mathematical Principles Relative to Contextualized Examples .................................................................................... 92
  4.3 Supplementing with Connecting Activities............................................................. 98
  4.4 Summary of Findings........................................................................................... 103

5. Discussion .................................................................................................................. 105

6. Conclusions .................................................................................................................. 110
CHAPTER 4: “I WANTED THEM TO GET THE CONNECTION”: THE ROLE OF INSTRUCTIONAL COHERENCE IN PROMOTING GENERALIZATION FROM CONTEXTUALIZED PROBLEMS ............................................................. 114

1. Introduction ...................................................................................................... 115
2. Review of Relevant Literature .......................................................................... 117
   2.1 Rationale for Contextualized Problem Based Instruction ................................. 118
   2.2 Potential Obstacles to Student Learning from CP-based Instruction ............... 120
   2.3 Conceptual Framework ......................................................................................... 124
3. Research Design ............................................................................................... 126
   3.1 Setting and Participants ......................................................................................... 128
   3.2 Data ...................................................................................................................... 129
   3.3 Data Analysis ....................................................................................................... 132
4. Results .............................................................................................................. 137
   4.1 Episode 1: Illustrating Connections Between Problems Set in Different Contexts 137
   4.2 Episode 2: Attending to General Mathematical Principles during Contextualized Problem solving ...................................................................................................................... 146
   4.3 Episode 3: Illustrating the Absence of Explicit Connections Between Contextualized and Non-contextualized Problems ................................................................................. 152
   4.4 Episode 4: Illustrating Inconsistent Reference to Previous Contextualized Work While Summarizing Generalizing Mathematical Principles ................................. 161
   4.5 Summary of the Findings ...................................................................................... 164
5. Instructional Coherence Across Task Sequences ............................................... 167
6. Conclusions and Implications ........................................................................... 172

CHAPTER 5: CONCLUSION .................................................................................. 176

1. Summarizing the Findings ................................................................................ 177
2. Implications for Further Research ..................................................................... 179
   2.1 Profiling CP-based instructional approaches using the CPMI ............................... 179
   2.2 Digging Deeper into Student Responses to CP-based Instruction ................. 181
   2.3 Teachers’ Use of CP-based Curriculum ................................................................. 185
3. Conclusion ........................................................................................................ 186

BIBLIOGRAPHY ...................................................................................................... 188
LIST OF TABLES

The composition of Core-Plus Course 1, Unit 2: Patterns in Data .............................................. 13
The composition of Core-Plus Course 1, Unit 3: Linear Functions ............................................. 13
Frequency of Explicit References to Previously Worked Examples ......................................... 142
Frequency of Teacher Utterances Attending to General Mathematical Principles .................... 149
LIST OF FIGURES

Figure 1. A common graphical representation of the act of modeling (adapted from Lesh & Zawojewski, 2007; Niss, Blum, & Galbraith, 2007)........................................................... 23

Figure 2. A graphical representation of Gravemeijer’s (1997) distinction between translating and organizing approaches to modeling................................................................. 30

Figure 3. RME and MMP activity types ............................................................................. 31

Figure 4. Opportunities for abstraction in the various RME and MMP activity types ......... 34

Figure 5. The overall structure of the CPMI framework .................................................. 41

Figure 6. Sub-categories within contextualized problem solving .................................... 48

Figure 7. The Contextualized Problems in Mathematics Instruction (CPMI) framework .... 51

Figure 8. The traditional role of contextualized problems in mathematics instruction ........ 54

Figure 9. RME and MMP-aligned instruction emphasizing problem solving in “real-world” settings ....................................................................................................................... 55

Figure 10. Map of Unit 3, Lesson 1, Investigation 1 in the Core-Plus written curriculum .... 78

Figure 11. An excerpt from Core-Plus, Course 1, Unit 3, Lesson 1, Investigation 1 and the corresponding codes that were assigned to each TCT (p. 151)......................................................... 79

Figure 12. Map of Ms. Spence’s Intended Curriculum Unit 3, Lesson 1 ......................... 83

Figure 13. Ms. Spence’s intended curriculum for Course 1, Unit 3, Lesson 1, Investigation 1 and the corresponding codes that were assigned to each TCT ........................................ 83

Figure 14. Comparison of the Written and Teacher’s Intended Curriculum of Core-Plus Course 1, Unit 3, Lesson 2 ...................................................................................................... 88

Figure 15. Three types of instructional activity framing the study ........................................ 125

Figure 16. A record of which groups were recorded during each class session ............... 131

Figure 17: Examples of generalization statements and questions .................................... 134

Figure 18: Examples of references to prior contextualized examples and the codes that were assigned. .................................................................................................................. 136

Figure 19. A graph showing how Barry’s pay is related to the number of applications he sells. 138

Figure 20. Generalizations about rate of change from the Core-Plus curriculum ............. 151
Figure 21. Non-contextualized examples in Investigation 1 ..................................................... 153

Figure 22. A table showing how Cheri’s pay depends on the number of services sold ........... 183
CHAPTER 1: INTRODUCTION

Mathematics teachers and curriculum designers are frequently urged to connect instruction to students’ interests and to real-world applications that exist outside of the classroom. Recent policy documents in the US (Common Core State Standards Initiative, 2010; National Council of Teachers of Mathematics, 2014), for instance, highlight the need for students to be able to solve real-world problems. Numerous curriculum programs have been created to leverage the use of contextualized problems, or problems that refer to contexts that exist outside the realm of mathematics, including Core-Plus Mathematics: Contemporary Mathematics in Context (Hirsch, Fey, Hart, Schoen, & Watkins, 2008), Mathematics in Context (National Center for Research in Mathematical Sciences Education & Freudenthal Institute, 1998), and Everyday Mathematics (University of Chicago School Mathematics Project, 2004). Unlike curricula which position traditional word problems as application problems at the end of instructional sequences, these curricula tend to place contextualized problems at the beginning of instructional sequences, in hopes that students will invent various ways to solve problems for which solution strategies have not been provided. Despite recommendations for the use of contextualized problems in mathematics instruction and the development of curricula which emphasize contextualized problem solving, evidence suggests that many teachers do not frequently make connections to the real-world in the classroom (Banilower et al., 2013; Hiebert et al., 2003). Curricula like those cited above that emphasize learning through contextualized problem solving have failed to gain a
significant foothold in the market in middle and high school classrooms (Banilower et al., 2013).

The potential affordances of contextualized problems [CPs] are numerous. CPs are thought to aid in the transfer of mathematical knowledge to situations outside of the mathematics classroom (Boaler, 1993) and highlight the relevance of the mathematics being learned (Boaler, 1993; NCTM, 2000). Advocates argue that these problems also provide the opportunity to leverage students’ non-mathematical knowledge as a bridge towards mathematics understanding (Freudenthal, 1973; Moses & Cobb, 2001). Socially, culturally, and politically relevant contexts have been used to connect mathematics instruction to students’ out-of-school expertise and promote engagement (Civil, 2007; Gutstein, 2006; Moses & Cobb, 2001; Tate, 1995).

The instructional use of problems set in extra-mathematical contexts, or contexts that exist outside of the mathematical domain, does not come without complications. Critics of curriculum programs that rely heavily on CPs argue that an emphasis on the contextualization of mathematics in real-world settings often comes at the expense of opportunities for students to engage with formal mathematical language and conventions (Brantlinger, 2011; Dowling, 1998; Quirk, 2011; Wu, 1997). Teachers report difficulty identifying ideas that are relevant (Gainsburg, 2008, 2009; Nicol, 2002; Schmidt, 2011) and maintaining the contextual links when implementing CPs in the classroom (Nicol, 2002). They feel constrained by the class time needed to make real-world connections and a lack of material resources to support this type of teaching (Gainsburg, 2008, 2009; Schmidt, 2011). According to Gerofsky (2004), traditional word problems-- one form of CP-- require students to engage in hypothetical, fictional worlds where knowledge of the
real-world must be suspended and only the information given is relevant. In response, many teachers view non-mathematical contexts as wrappers that students needed to strip away to solve problems (Chapman, 2006).

A better understanding of how the potential affordances and complications inherent to contextualized problems play out in mathematics classrooms would provide important insight into how to maximize the benefits of CP-based curricula. Research into the use of CPs in actual classrooms would provide possible reasons why curriculum programs that emphasize CPs might not be implemented as intended, explanations for why these curricula are not used in more classrooms, and insight into how CP-based curricula could be improved.

In this study, I examine the implementation of curriculum program called Core-Plus Mathematics (Hirsch et al., 2008), which is a high school curriculum that emphasizes mathematical modeling and contextualized problem solving. By examining the written curriculum, a teacher’s plans for implementing Core-Plus lessons, and the resulting classroom instruction, I describe how the demands, complications and supports inherent to instruction based in contextualized problems play out in a high school classroom. My overarching research question was as follows: what is the role of extra-mathematical contexts in the implementation of one algebra unit of the Core-Plus Mathematics curriculum? I use my exploration of this question in a single classroom to identify curricular characteristics that either support or constrain students’ opportunities to learn; after identifying these characteristics I propose a number of recommendations for teachers and curriculum developers.
1. Theoretical Framework

1.1 Transformation of Curriculum

This study was designed on the assumption that the effect of a curriculum cannot be understood by simply examining student learning outcomes. To fully understand the impact of a curriculum, one must examine the process through which the written curriculum is transformed into actions. Stein, Remillard and Smith (2007) provide a framework for describing the temporal phases of curriculum as it is transformed from the written page (the *written* curriculum), to the teachers’ plans (the *intended* curriculum), and finally to the events that take place in the classroom (the *enacted* curriculum).

Through a detailed review, Stein et al. conclude that the written curriculum is only one of many components that influence what occurs in mathematics classrooms. To support this claim, the authors cite research demonstrating that written curriculum materials are implemented very differently by different teachers in terms of the amount of the curriculum that is covered (Tarr et al., 2008) and the manner in which the curriculum is implemented in the classroom (Collopy, 2003; Lloyd, 1999; Tarr et al., 2008). They also point to research demonstrating that curriculum implementation is mediated by teachers’ orientations towards curricula in general (M. W. Brown, 2009; Remillard & Bryans, 2004), interpretations and orientations towards the philosophy of the specific curriculum (Lambdin & Preston, 1995), knowledge and beliefs (Lloyd, 1999), strategies for reading the curriculum guide (Remillard, 1999, 2000; Sherin & Drake, 2009), and professional development experiences (Zhao & Cobb, 2006). Student interactions with the curriculum are mediated by classroom norms (Boaler & Staples, 2008) and the resources they bring as individuals, including their social and mathematical identities (Lubienski, 2000;
Martin, 2006). In this study, I aim to characterize the transformation that occurs as the curriculum is implemented and to provide insights into the factors that mediate this transformation. To achieve these aims, I follow one unit of the curriculum as it is transformed from the written page, to the teachers plans, and finally to enactment in the classroom.

1.2 Contextualized Problem solving as a Basis for Learning Mathematics

Three primary assumptions guided my examination of the Core-Plus curriculum. The first is that students can learn new mathematics as they solve novel problems, whether they are contextualized in an extra-mathematical setting or not. The second assumption complicates that notion, and introduces the need for students to be familiarized with mathematical conventions that are arbitrary and socially constructed. The third principle bridges the two, stating that the use of concrete, real-world contexts can provide support for developing students’ understanding of formal mathematical concepts, but also that these problem solving experiences must be supplemented by other types of mathematical activity for adequate learning to occur. In this subsection, I outline the theoretical basis for all of these assumptions.

According to the developers of Core-Plus, the design of the curriculum is founded on the principle that students can learn new mathematics as they engage in the act of problem solving (Schoen & Hirsch, 2003). The theoretical and empirical basis for this assumption draws on the work of Hiebert et al. (1996) and Schoenfeld (1992). Hiebert et al. highlight Dewey’s notion of reflective inquiry: the idea that students build knowledge by solving problems in a reflective manner (Dewey, 1910, 1929, 1933). They summarize
the process as follows: “(1) problems are identified [by students]; (2) problems are studied through active engagement; (3) conclusions are reached as problems are (at least partially) resolved” (Hiebert et al., 1996, p. 14). Hiebert et al. support Dewey’s claim by citing empirical research, much of which is focused at the elementary level, showing that students who learn mathematics through problem solving develop richer understandings of the number system than peers taught in a skills-based manner (Cobb et al., 1991; Hiebert & Wearne, 1993) and are able to flexibly solve new problems (Fennema, Franke, Carpenter, & Carey, 1993; Fuson & Briars, 1990; Hiebert & Wearne, 1993; Kamii & Joseph, 2004; Wearne & Hiebert, 1989).

The developers of Core-Plus also cite the work of Schoenfeld (1992), who offers a complementary view of ideal mathematics instruction. He argues that “students develop their sense of mathematics- and thus how they use mathematics- from their experiences with mathematics (largely in the classroom)” and proposes that “classroom mathematics must mirror this sense of mathematics as a sense-making activity, if students are to come to understand and use mathematics in meaningful ways” (p. 339). Schoenfelds’ literature review cites numerous examples at various educational levels of instruction that focus on developing students’ ability to solve novel problems, and he argues that the traditional textbook structure (the introduction of a mathematical technique, followed by an illustrative example then repetitive practice) does not offer students the opportunity to develop a mathematical, sense-making disposition. In light of the research cited by Schoenfeld (1992) and Hiebert et al. (1996), the proposed study, like the Core-Plus curriculum, is aligned with the assumption that students can and should learn new mathematics by solving novel problems, when possible.
The second principle the study was designed upon provides a complicating notion. Dowling (1998) contends that from a socio-cultural viewpoint, mathematics education must apprentice students into the formal practices and conventional language and understandings of the mathematically elite. He critiques instruction aligned with the reform movement, arguing that too narrow of a focus on student discovery and contextualization in “real-world” situations deemphasizes disciplinary conventions, many of which are arbitrary, socially-constructed, and not accessible by intuitive construction. In recognition of this critique, this study will consider the degree to which problem-based discovery leads to, or is balanced by the formalization of conventional mathematical practices.

The third principle involves the relationship between formal mathematics and the use of real-world problems. In their description of the design of the Core-Plus curricula, Schoen et al. (2010) explain that many Core-Plus instructional sequences begin with verbally presented problem settings set in concrete, familiar settings and that the use of formal mathematical language and symbols is intentionally delayed. This design characteristic is connected to a third assumption significant to the proposed study: students’ understanding of formal mathematics concepts can be anchored to their common sense understanding of real-world phenomena. This idea, popularized by Freudenthal (1973, 1991), also provides the theoretical foundation for the Dutch theory of realistic mathematics education (RME) (Gravemeijer, 1994). Although Core-Plus was not developed from RME principles, the developers note that the two approaches are similar in regard to the use of CPs as starting points for many of the mathematical
investigations (Hirsch, personal communication, 6/15/11; Fey, personal communication, 7/25/11).

2. Research Questions

To compare the role of contexts across the written, intended, and enacted curriculum, I first needed a consistent analytical lens that could be used in each of these three phases. In the first manuscript (Chapter 2), entitled Towards a framework for analyzing the role of contextualized problems in mathematics instruction, I describe the development of an analytical framework used to describe the role of contextualized problems in the Core-Plus curriculum. The development of the framework was guided by the following questions:

1. What are the critical problems of practice in instruction that emphasizes learning through contextualized problem solving, as identified in the literature and/or from the study of practice?

2. What sorts of tasks, questions and statements are possible (from teachers, students, and curriculum materials) within contextualized mathematics instruction?

3. How can these tasks, questions and statements be organized so as to clarify significant problems of practice and suggest potential ways to navigate effective solutions?

To answer these questions, I consulted literature on the use of CPs in mathematics instruction, performed a document analysis of the Core-Plus curriculum, observed as the curriculum was implemented by a teacher, and interviewed the teacher to gain insight into her instructional decisions.
In the second manuscript (Chapter 3), entitled *The role of contextualized problems in instruction: one teachers’ transformation of the written curriculum*, I use the framework to analyze how the role of CPs was transformed as the curriculum transformed from the written page to the teachers’ plan for enactment. This portion of the study was guided by the following questions:

1. What are the patterns in the way the teacher selected, sequenced, adapted, and supplemented the tasks offered by the textbook? How did her intended curriculum compare to the written curriculum, especially in regard to problems referring to extra-mathematical contexts?

2. What are the factors that influenced how the teacher transformed the CP-based curricula from the written curriculum to plans for enactment? What influenced how she selected, sequenced, adapted, and supplemented tasks?

Finally, to understand the role of extra-mathematical contexts in the resulting enactment of the curriculum, I performed a third study, described in the third manuscript, ‘*I wanted them to get the connection:* ’ the role of instructional coherence in promoting generalization from contextualized problems. This portion of the study was guided by the following questions:

1. To what extent and in what manner are generalizations made explicit during work on CPs?

2. To what extent and in what manner are CPs referenced during work on other CPs?

3. To what extent and in what manner are CPs explicitly referenced during non-contextualized work?

4. To what extent and in what manner are CPs referenced during work on generalizing mathematical principles?
3. Setting

In this section, I describe the Core-Plus curriculum, the participating teacher, and the school setting in which the study took place.

3.1 Core-Plus Mathematics: Contemporary Mathematics in Context

Fey and Hirsch (2007) describe the Core-Plus lesson sequence as a modified “launch-explore-summarize model”. In this model, a problem situation provides the stimulus for whole-class discussions during which the teacher can informally assess students’ prior understandings. Then, students are to work in small groups to solve a series of problems. The findings from these problem sessions are to be summarized in a final whole-class discussion. Schoen and Hirsch (2003) describe the “summarize” stage as a “teacher-moderated class discussion in which students share mathematical ideas developed by their groups and together construct a common understanding of important mathematical concepts, methods, and approaches” (p. 315).

In their description of a longitudinal study of the curriculum Schoen et al. (2010) briefly describe the curriculum level instructional sequence:

In order to accommodate the learning needs of all students, especially in Courses 1 through 3, most investigations and sequences of investigations are constructed so that the entry point is familiar and concrete to students – usually more verbal than symbolic. Development of robust concept images (Tall, 1991) precede formal definitions that often appear at the end of an investigation or lesson. Similarly, use of symbols is delayed and developed gradually, with the symbols and symbolic operations drawing meaning from the earlier, more concrete settings. (p. 9)

The Core-Plus text contains minimal exposition and few decontextualized practice problems, especially when compared to more traditional commercially-developed texts.
The student text reads like a series of tasks and questions that the students are intended to engage with, either as a whole class, in small group, or individually.

### 3.2 Participating Teacher

The teacher I partnered with on this project, whom I refer to with the pseudonym Brenda Spence, was a veteran teacher who enrolled in a course that I assisted with at the university. Through her participation in the course, the two of us came to know each other and began to discuss our mutual interest in researching the Core-Plus curriculum. She was an ideal participant in this study for a number of reasons. First, she was an experienced curriculum user. She had over 20 years of teaching experience, with 13 years of experience implementing Core-Plus. Furthermore, she had participated in a great deal of professional development specifically targeting Core-Plus implementation. Most importantly, she was excited about participating in this research because of the potential the project had to help her understand how her students experience the curriculum.

### 3.3 School Setting

The school in which the study took place was a comprehensive high school in a small school district on the east coast. The school was composed of approximately 2000 students; about half of the students were eligible for free or reduced lunch. Approximately half of the students were African-American, less than one percent were American Indian, 3% were Asian American, 15% were Hispanic/Latino, 30% were White, and less than 1% were identified as multi-racial. I observed as Ms. Spence
implemented the *Core-Plus* curriculum in two different settings. The first section was a general admission 9\textsuperscript{th} grade Integrated Math I class with no admittance restrictions. The other section, a 9\textsuperscript{th}/10\textsuperscript{th} grade Integrated Mathematics 1-2 split class, was available only to young men of color. This section was originally created to provide students who had not passed Integrated Mathematics 1 a chance to cover Integrated Mathematics 1 and 2 in one year. The purpose of the class had shifted, though, and during the year of the study, some of the students in the class were first time ninth graders who, upon completion of the class, would be on track to take AP math classes during their senior year. Although I observed both classes, the transcripts analyzed here are of the general admission class because I was unable to obtain consent from a sufficient number of students in the Integrated Math 1-2 split class.

4. Data Set

Data collection took place over the course of four months and consisted of document collection, teacher interviews, class observations and recordings of class sessions.

4.1 Document Analysis

I collected and coded two full units of *Core-Plus* textbook. In *Core-Plus*, units are segmented into lessons, which are segmented into investigations. I analyzed Unit 2, which focused on statistics, and Unit 3, which focused on linear functions. The statistics unit contained three lessons and was intended to span 20 instructional days, as shown in Table I.
Table I

*The composition of Core-Plus Course 1, Unit 2: Patterns in Data*

<table>
<thead>
<tr>
<th>Lesson</th>
<th>Number of Investigations</th>
<th>Recommended Pacing</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lesson 1</td>
<td>2 investigations</td>
<td>7 days</td>
</tr>
<tr>
<td>Lesson 2</td>
<td>5 investigations</td>
<td>11 days</td>
</tr>
<tr>
<td>Lesson 3</td>
<td>1 investigation</td>
<td>1 day</td>
</tr>
</tbody>
</table>

The algebra unit, focused on linear functions contained four lessons and was intended to span 26 instructional days, as shown in Table II.

Table II

*The composition of Core-Plus Course 1, Unit 3: Linear Functions*

<table>
<thead>
<tr>
<th>Lesson</th>
<th>Number of Investigations</th>
<th>Recommended Pacing</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lesson 1</td>
<td>3 investigations</td>
<td>9 days</td>
</tr>
<tr>
<td>Lesson 2</td>
<td>4 investigations</td>
<td>10 days</td>
</tr>
<tr>
<td>Lesson 3</td>
<td>2 investigations</td>
<td>5 days</td>
</tr>
<tr>
<td>Lesson 4</td>
<td>1 investigation</td>
<td>2 days</td>
</tr>
</tbody>
</table>

4.2 Observations and transcripts

I observed as Units 2 and 3 were implemented with two different sections, two to three times per week over the course of twelve weeks; in all, a total of 78 class periods were observed. I transcribed recordings of whole class discussions and selected partner groups from twenty days of instruction from Unit 3.
4.3 Interviews

I interviewed Ms. Spence once at the beginning of the study to gain an understanding of her conceptions about the use of contextualized problems in mathematics instruction and the Core-Plus curriculum. I also conducted twenty post-observation interviews to gain insight into her design decisions and her perception of how students responded to the lessons.

5. Summary

The aim of this study was to characterize the role of extra-mathematical contexts in the written, intended, and enacted Core-Plus curriculum. I analyzed curriculum materials to gain insight into the way these contexts were employed in the curriculum. I interviewed Ms. Spence, an experienced user of Core-Plus, in order to characterize her conceptions of the role of extra-mathematical contexts and the way these conceptions influenced her participation with the curriculum materials. And finally, I observed as Ms. Spence taught 78 lessons. I analyzed transcriptions of classroom audio recordings, field notes, and student interview data to gain insight into the way Ms. Spence and her students responded to CPs and the factors that mediated these responses. The three chapters that follow, written in the form of stand-alone papers, represent three different analytical approaches I took to characterize the role of contextualized problems in the written, intended, and enacted curriculum.
CHAPTER 2: TOWARDS A FRAMEWORK FOR ANALYZING THE ROLE OF CONTEXTUALIZED PROBLEMS IN MATHEMATICS INSTRUCTION

Abstract

Mathematics teachers are frequently urged to connect mathematics instruction to real-world problem settings, but research suggests that teachers do not consistently incorporate problems that refer to real-world contexts into instruction (Banilower et al., 2013; Hiebert et al., 2003). One potential explanation for this tendency is that teachers focus on the affective benefits of contextualized problems rather than the potential for these problems to help students construct an understanding of mathematical ideas (Pierce & Stacey, 2006); and, because of these beliefs, teachers might omit these problems when pressed for time (Gainsburg, 2008). Another is that teachers may lack a nuanced understanding of how contextualized problems can be used to develop new mathematical knowledge (Lee, 2012). To assist teachers, teacher educators, and the research community in developing a shared language for describing instruction that leverages real-world contexts, this article proposes an analytical framework, developed through synthesis of existing theory and analysis of the implementation of a Standards-based high school mathematics curriculum. The framework, which I call the Contextualized Problems in Mathematics Instruction (CPMI) framework, describes various types of tasks, questions and statements that are possible in a contextualized problem-based approach to instruction. The CPMI framework is organized along two dimensions:
particular/general and contextualized/non-contextualized. Grounded in theoretical and empirical research, the framework has the potential for use across the mathematics education community; perhaps most importantly, the framework can assist teachers in the work of developing students’ understanding of general mathematical principles through problem solving activity that references real-world settings.
1. Introduction

Mathematics teachers and curriculum designers are frequently urged to connect instruction to students’ interests and to real-world applications that exist outside of the classroom. This idea is not new (Dewey, 1902; Whitehead, 1929), but it plays a prominent role in reform efforts in a number of countries, as evidenced by published standards and curriculum documents (e.g. Common Core State Standards Initiative, 2010; Ministerie van Onderwijs, Cultuur en Wetenschappen (OCenW), 2004; Ministry of Education, 2007). One way of connecting to the world outside the mathematics classroom is through the use of contextualized problems (CPs), or problems that reference contexts that exist outside the mathematical domain. Research has shown that certain types of CP can be used to bring out students’ intuitions and informal strategies, and this informal work can then be leveraged to facilitate students’ learning of more conventional or formal mathematics (Carpenter, Fennema, & Franke, 1996; Gravemeijer, 1994; Greeno & The Middle-School Mathematics through Applications Project, 1997; Technology Group at Cognition and Technology Group at Vanderbilt, 1997). In light of this research, curriculum designers have developed numerous curriculum programs that emphasize CPs and position these problems at the beginning of instructional sequences. In the US, the development of these programs was funded by the National Science Foundation [NSF] (Senk & Thompson, 2003); the titles of many of these programs demonstrate an emphasis on context (e.g. Everyday Mathematics and Core-Plus: Contemporary Math in Context).

Despite widespread recommendations for teachers to anchor instruction to real-world contexts through the use of CPs, evidence suggests that many teachers do not do
so. In the 1999 TIMSS Video Study, investigators found that, with the exception of the Netherlands, the percentage of problems that were set up with the use of a real-life connection ranged from 9 to 27% (Hiebert et al., 2003). More recently, in a 2012 US survey, only 45% of elementary, 42% of middle school, and 29% of high school math teachers surveyed report heavily emphasizing real-life applications of mathematics through the use of CPs or otherwise (Banilower et al., 2013). The survey also showed that, of those who responded, only 25% of elementary, 11% of middle and less than 1% of high school classes use NSF-supported curricula.

Researchers have identified a number of factors that explain why teachers might infrequently use CPs or otherwise make connections between mathematics instruction and real-world contexts. Teachers have difficulty identifying or finding ideas that are relevant (Gainsburg, 2008, 2009; Nicol, 2002; Schmidt, 2011) and, once these contexts are identified, teachers have trouble maintaining the contextual link when attempting to bring these ideas to the classroom (Nicol, 2002). Teachers also feel constrained by the class time needed to make these connections and a lack of material resources to support this type of teaching (Gainsburg, 2008, 2009; Schmidt, 2011).

Furthermore, some teachers tend to view contextual settings as extras that students needed to strip away to solve problems, rather than as supports that can help students make sense of mathematics (Chapman, 2006). This tendency may be due to the fact that CPs, and more specifically word problems, have traditionally been positioned at the end of instructional sequences. By positioning this particular type of contextualized problem at this stage of instruction, students learn to ignore narrative elements of the problem, identify any necessary quantities and mathematical relationships, and solve the
problem according to the procedures practiced during the lesson (Gerofsky, 2004, p. 34). The belief that contextual settings are “extras” is a reasonable response to this traditional positioning of word problems.

Another reason that teachers would tend to not make connections to the world outside of mathematics is that some teachers tend to think of real-world connections primarily as tools to demonstrate the relevance of mathematics or to motivate students (Gainsburg, 2008, 2009; Pierce & Stacey, 2006). It is likely that teachers who do not see real-world contexts as potential scaffolds for the development of students’ mathematical understanding tend to sacrifice the use of contextualized problems when constrained by time (Gainsburg, 2008).

In other cases within the field where disparities have been observed between researchers recommendations and classroom practices, analytical frameworks have been developed to help teachers understand and enact recommendations from the research community. Specifically, researchers have developed analytical frameworks that provide a system of describing and categorizing various aspects of instruction, including the cognitive demand of tasks (Stein, Smith, Henningsen, & Silver, 2000), problem types within a particular mathematical domain (Carpenter et al., 1996), teacher question types (Boaler & Brodie, 2004), and student solution strategies (Carpenter et al.). These frameworks provide a common language to facilitate communication between teachers, teacher educators, researchers, and curriculum developers. Professional development around these frameworks has been shown to improve instruction (Boston & Smith, 2009; Carpenter et al.). In sum, frameworks are local theories that provide a technical vocabulary for describing conceptual distinctions (Niss, 2007) and have the potential to
bridge the significant divide (Heid et al., 2006) between research and practice (Smith, 2012).

In this article, I offer an analytical framework called the Contextualized Problems in Mathematics Instruction framework [CPMI], which is designed to classify instructional activity according to how that activity references extra-mathematical contexts,\(^1\) or contexts that exist outside the realm of mathematics. My intent is for this framework to be used by teachers, curriculum designers and researchers to analyze, understand, and communicate about instruction featuring CPs. The framework was developed through iterative cycles of theoretical and empirical research: ideas from the literature were tested as analytical categories for describing empirical observations; when these ideas were insufficient, further literature was consulted. For the sake of organization and readability, this article represents the process in a simplified, linear manner. In section two, I outline theoretical perspectives that significantly informed the resulting analytical framework. In the third section I explain the role of the empirical research in the development of the framework. The framework is presented in the fourth section. In the fifth section, I describe examples of applications of the framework; and, I conclude by discussing the implications and limitations for research and practice.

\(^1\) The term ‘real-world’ is often used to describe any statement, question or task that refers to a phenomenon existing outside the realm of mathematics. Researchers frequently critique the authenticity with which these phenomena and the related tasks are presented within mathematics instruction (e.g. Palm, 2006; Vos, 2011). I use the term ‘extra-mathematical’ to highlight the fact that the proposed framework applies to instruction that references any phenomenon that exists outside the realm of mathematics, regardless of whether the phenomenon is real or imaginary.
2. Theoretical Perspectives

2.1 Perspectives on Conceptual Model Building

Sloane and Gorard (2003) identify three stages in the process of building conceptual models like the analytical framework proposed here: formulation, estimation, and validation. This article reports on the first stage. My approach to formulating a conceptual model for analyzing instruction aligns strongly with the models and modeling perspective (MMP) described by Lesh and Zawojewski (2007). MMP theorists use the term model in multiple ways. Students build conceptual, mathematical models in response to problems; teachers and researchers create models for interpreting classroom activity. Lesh, English and Fennewald (2008) suggest that the latter type of conceptual models, like the one proposed here, are often developed first to understand local instructional contexts with the goal of producing powerful, generalizable analytical tools:

We do not expect realistic solutions to realistically complex problems to be solved by single research studies, nor even by single theories. Instead, what are most needed are models which are embodied in artifacts and tools that are designed to be powerful, sharable, and reusable. Such models also need to integrate ways of thinking drawn from a variety of practical and theoretical perspectives. (Lesh et al., 2008, p. 3)

The categories that compose the CPMI framework did not originate from a single theoretical source; instead, they were derived from a number of perspectives. Furthermore, the formulation process was iterative. I identified a set of significant ideas from the literature as a way of developing an eye, or theoretical sensitivity, for detecting distinctions between different types of activity around CPs. These distinctions informed a grounded approach to analyzing empirical data. When I identified activity types that
could not be described using the set of categories I had developed, I searched for literature that described the activity I observed. The framework was organized and re-organized through multiple iterations of this process; along the way, I identified, tested and ultimately discarded numerous ideas from the literature. The categories that make-up the CPMI framework were the most helpful for making sense of the classroom activity. As a consequence of this process, the resulting framework is grounded both in theory and empirical observation.

### 2.2 Perspectives on the Use of Contextualized Problems in Mathematics Instruction

A number of established theoretical perspectives informed the development of the framework; I describe these perspectives below. Because a primary goal for the CPMI framework is to provide a language for describing how CPs can be leveraged to help students learn new mathematical ideas, I first review two perspectives on mathematical modeling that specifically aim to develop students’ understanding of new mathematical concepts. Then, to provide additional insight into how students develop mathematical understanding through the types of activity recommended by those modeling theorists, I review multiple perspectives on how learners construct mathematical abstractions.

#### 2.2.1 Mathematical Modeling

A significant body of research describes the act of mathematical modeling, or the use of mathematics to solve problems set in extra-mathematical contexts. Lesh and Fennewald (2010) describe a model as “a system for describing (or explaining, or designing) another system(s) for some clearly specified purpose” (p. 7). A visual
representation frequently used to describe an idealized, simplified version of the mathematical modeling process is shown in Figure 1. In this description, the process of modeling consists of (1) developing a model of the context or situation in the physical world by identifying the aspects of the situation relevant to the question at hand, (2) representing this model in the mathematical realm by creating one or more mathematical models (represented by the upward arrow), (3) working with the mathematical model or models to obtain a mathematical result or solution, and (4) interpreting this result in terms of the extra-mathematical context or situation (represented by the downward arrow). The process is typically described as a cycle; if, during step four, the solution is not deemed viable, the four steps are repeated.

![Figure 1: A common graphical representation of the act of modeling (adapted from Lesh & Zawojewski, 2007; Niss, Blum, & Galbraith, 2007)](image)

Visual representations of the modeling process sometimes explicitly separate the steps of formulating the task, determining the relevant contextual information, and evaluating and refining models (e.g. Borromeo Ferri, 2006; Haines & Crouch, 2010).
Within the field of modeling, a number of perspectives have emerged, each with different aims and philosophical foundations (Kaiser & Sriraman, 2006). In particular, two goals for integrating modeling tasks within mathematics instruction are identified (Niss et al., 2007). One approach is for students to engage in modeling tasks once they have already been exposed to mathematical tools that could be used to solve the task. The primary goal for these sorts of tasks is for students to gain competency in the act of mathematical modeling. Another goal is for the act of modeling to serve development of new mathematical ideas; in this approach, modeling tasks are often positioned at the beginning of instructional sequences. Because the purpose of this paper is to develop a framework for describing how CPs can serve as a scaffold for the development of new mathematical understanding, I focused on the latter approach in my review of the literature.

Within the category of approaches that aim to develop students’ mathematical understanding, two perspectives are particularly prominent: the Models and Modeling Perspective (MMP) and the Dutch theory of Realistic Mathematics Education (RME). MMP and RME have different philosophical roots and their aims specific aims are different, but both include a set of design principles and a prescription for how instruction can be sequenced in order to develop students’ understandings of new mathematical ideas. In the next two sections, I review the proposed instructional sequences in MMP and RME; these sequences contain a number of significant ideas from which the proposed CPMI framework was built.
2.2.2 Models and Modeling Perspective

The MMP perspective described by Lesh and Doerr (2003a) is a theoretical perspective grounded in constructivism and American pragmatism. MMP theory is used to describe instructional activities, the ways teachers make sense of instruction, and an approach to educational research designed to solve complex problems (Lesh & Doerr, 2003c). Classroom instruction designed from the MMP often centers on a certain type of CP called *model-eliciting activities*: problems that allow for multiple strategies and prompt students to use informal knowledge to build models that help them solve problems. According to MMP advocates, cycles of modeling activities prompt students to develop powerful conceptual systems to describe foundational, elementary mathematical topics (e.g. ratio, linear relationships). For instance, one problem in the MMP literature requires students to create a kit that would help the user determine the height of a person from a footprint (Lesh & Doerr, 2003b). These model-eliciting situations are designed so that students can develop models of fundamental mathematical concepts that can later be used to understand a wide range of situations (e.g., models for proportional reasoning in the footprint example).

Model-eliciting activities differ from traditional word problems in two significant ways. Whereas traditional word problems are usually presented completely through text or mathematical symbols, model-eliciting activities are often presented through a variety of representational media including pictures or concrete models. Second, and more importantly, model-eliciting activities are meant to prompt students to construct powerful, generalizable ways of understanding problem situations, often relying on their understanding of the problem context. As stated previously, word problems are
traditionally placed at the end of an instructional sequence and students are intended to apply mathematical procedures they have already learned (Gerofsky, 2004).

Model-eliciting activities represent one stage within larger instructional sequences called *model development sequences* (Lesh, Cramer, Doerr, Post, & Zawojeski, 2003). Each sequence begins with a warm-up often centered on the real-world (e.g., a newspaper clipping) setting. The warm-up is used to familiarize students with the real-world context and provide space for the teacher to assess students’ understandings of the prerequisite concepts. Next, a *model-eliciting activity* is posed to the students, wherein they are expected to answer a question by developing models that describe their thinking. In these activities, students are meant to evaluate the appropriateness of their models and refine the models through iterative cycles of development and evaluation. In a subsequent *model-exploration* activity, the teacher leads the group towards a unified explanatory model, which is collectively refined. Here, the focus shifts from thinking *with* the model to thinking *about* the model. This unified model is then used in one or more *model-adaptation activities*. The contexts of the model-adaptation activities are structurally similar to the original problem, but usually contain some added complication. The students are meant to adapt their model to fit the new situation; this is a chance for the teacher to assess students’ understanding of the underlying mathematical concept. These activities are followed by a *discussion about the structural similarity*, wherein students compare and contrast the various models they have developed as well as the various contextual situations encountered during the model-eliciting activity and the model-adaptation activities. *Follow-up activities* link the model-development sequences to more traditional mathematics activities, often focused on traditional textbooks and tests.
2.2.3 Realistic Mathematics Education: modeling and reflection

The modeling approach described above shares a number of key characteristics with the Dutch theory of *Realistic Mathematics Education* [RME], although the historical origins and primary aims of RME and MMP are different. The theory of RME is founded in the work of Freudenthal (1973, 1991), who believed that students’ understanding of formal mathematics should be deeply connected to their common sense. Like MMP model development sequences, RME instructional sequences aim to develop students’ understanding of foundational mathematical ideas like proportional reasoning or linear relationships. The two perspectives differ, though, in that RME provides more detailed descriptions of how this activity can lead students to develop an understanding of formal conventions and procedures, which are less of a focus in MMP instructional sequences. By working with models that become increasingly formal, students are prompted to reinvent formal mathematical conventions, such as the long-division algorithm or fractional notation (Gravemeijer, 1994). The developers of RME refer to four distinct levels of activity, described below.

Instructional sequences based on RME typically begin with a CP based in a setting that is imaginable to the students (Gravemeijer, 1994).² For instance, in one frequently cited example, students are required to divide pizzas among a group

² Use of the word “realistic” is not meant to imply that the problem must reference a “real-world” setting, although it is often the case in RME. The term realistic in realistic mathematics education is actually closer to the word “imaginable” (van den Heuvel-Panhuizen, 2003). Students need to understand the context sufficiently to be able to place themselves as actors within the task setting. The context could be fanciful or even based in solely in mathematics, as long as it feels sufficiently real for students to make sense of it (Freudenthal, 1991).
(Streefland, 1991). In this first *situation*al level of the process, students work within the task setting, using only their common sense to solve the problem. The second level is called the *referential* level. Here, students’ activity in the task setting becomes the subject of their reflection. Students reflect on and discuss their own solutions to the problem. During this process, students are encouraged to draw or build various types of models to help them communicate and organize their thinking. The term model, in RME, refers to visual representations or descriptions of strategies—any way of describing activity in the situational level; in the pizza example, for instance, students might draw and divide circles to represent the act of dividing pizzas. Together, the first two stages of the RME levels align to a certain degree with the model-eliciting activities described by Lesh et al (2003). In the first stages of both perspectives, students create informal models in response to problems set in real-world contexts. (Figure 3 summarizes how the levels of RME and stages of the MMP model-development sequences align.)

In the third level of RME, the *general* level, students’ informal models are no longer tied to a specific situation; instead they function as entities that stand alone and represent mathematical relationships. In the pizza example, the act of drawing and dividing of circles, a “model of” the act of dividing the pizzas, becomes a “model for” reasoning about the formal idea of fractions and for solving decontextualized problems or problems set in different contexts (Gravemeijer, 1999, p. 160). In this stage, attention shifts to focus on the informal models and strategies rather than the initial problem situation, similar to the MMP model-exploration activities and model-adaptation activities described by Lesh et al (2003).
At the final stage of RME instructional sequences, students operate at the *formal* level where they reason with formal algorithms without the need for models. These formal algorithms are either developed by students or shared by teachers who connect them to the less formal strategies students developed in earlier stages. In the pizza example, students at the formal level would perform computations with fractions without drawing pictures. The process of building from common sense activity in a task setting to the re-invention of formal mathematics is, in its entirety, called “progressive mathematization” (Gravemeijer, 1994).

To further clarify the way contextualized problem solving leads to the abstraction of formal mathematics, Treffers (1987) describes processes of mathematizing both horizontally and vertically. *Horizontal mathematization* is the act of cropping and organizing the information one understands about the world into models that allow this information to be worked with; horizontal mathematization is most prominent at the first two levels described above: the situational and referential levels. *Vertical mathematization* involves reflecting on informal models and organizing the relationships present in these models to develop an understanding of formal mathematics; vertical mathematization is the emphasis of the general and formal levels.

Gravemeijer (1997) provides a nuanced explanation of how the RME approach to modeling relates to the common description of modeling represented in Figure 1. He describes two approaches to modeling, which I have represented visually in Figure 2. In a *translating* approach, students translate the elements of a problem situation to some known, formal mathematical model. RME, on the other hand, takes an *organizing* approach, where students organize the elements of the problem situation by inventing
informal models. These informal models become the basis for making sense of more formal models, consisting of conventional mathematical language, algorithms, and principles. The RME approach to modeling aligns significantly with the type of activity described in MMP; in both instructional design theories students are meant to develop their own informal models to solve contextual problems.

**Figure 2.** A graphical representation of Gravemeijer’s (1997) distinction between translating and organizing approaches to modeling

Figure 3 shows how the levels of RME and the stages of the model-development sequences from MMP align. Most significantly, in both approaches, the instructor guides students to reflect on their self-produced, informal models, and this leads to an understanding of formal mathematics through vertical mathematization. In the table, the transition from general activity to formal activity, as described in RME literature, aligns with the types of activity described in the MMP model exploration activities, model adaptation activities, discussions of structural similarities, and follow-up activities. The two instructional sequences represent two different ways of characterizing similar
processes. The RME levels distinguish between the types of models that are being used, and the MMP labels focus more on what students do with the models.

<table>
<thead>
<tr>
<th>RME levels of activity</th>
<th>MMP model development activity types</th>
<th>Description of activity</th>
</tr>
</thead>
<tbody>
<tr>
<td>situational</td>
<td>warm-up</td>
<td>making sense of the context</td>
</tr>
<tr>
<td></td>
<td>model eliciting</td>
<td>solving a CP using common sense and knowledge of the situation</td>
</tr>
<tr>
<td>referential</td>
<td></td>
<td>using models to explain thinking and solutions</td>
</tr>
<tr>
<td>general (informal models)</td>
<td>model exploration</td>
<td>shifting focus to the models themselves</td>
</tr>
<tr>
<td></td>
<td>model adaptation</td>
<td>using models to solve other problems</td>
</tr>
<tr>
<td>formal (conventional representations/algorithms)</td>
<td>discussion of structural similarities</td>
<td>reflecting across models and problem situations</td>
</tr>
<tr>
<td></td>
<td>follow-up activities</td>
<td>connecting to more traditional classroom activities</td>
</tr>
</tbody>
</table>

*Figure 3. RME and MMP activity types*

### 2.3 Perspectives on Abstraction

The instructional theories described in the previous section provide conceptual distinctions between types of instructional activity. To better understand how these activities might lead to formal mathematical understanding, and to classify certain types of activity around contextualized problems that I observed in the field that were not described by MMP or RME literature, I reviewed literature on how learners construct new mathematical ideas, or abstractions. Literature on abstraction provides a language for describing why the types of activities described in the framework might be important for fostering the construction of formal mathematical knowledge. These theories also
provide a different, more general kind of lens compared to the RME and MMP literature.

I review two perspectives on abstraction in this section: empirical and theoretical.

According to the classical, or Aristotelian view of abstraction, a learner begins the process of constructing a new concept by first identifying commonalities across a number of particulars; this process is often referred to as generalization (Dörfler, 1991; Ohlsson & Lehtinen, 1997). When an individual focuses on these commonalities while ignoring other properties or the particulars themselves, these common properties have been abstracted (Skemp, 1987). The product of this process, an understanding of the property at hand, is termed an abstraction. Skemp defines an abstraction as “some kind of lasting mental change, the result of abstracting, which enables us to recognize new experiences as having the similarities of an already formed class” (1987, p. 11). This view of abstraction is called empirical abstraction.

Some contemporary views on abstraction offer the perspective that empirical abstraction theories are insufficient for explaining how individuals develop scientific knowledge, including mathematics (Davydov, 1990; Ohlsson & Lehtinen, 1997). Such scholars prefer a more constructive characterization of abstraction that attends to the activity (often social) in which the abstraction is built. This form of abstraction, detailed by Davydov (1990), is often referred to as theoretical abstraction.

Hershkowitz, Schwartz and Dreyfus (2001), adopt Davydov’s theoretical abstraction perspective to construct a model of “abstraction in context.” They use this model, called the RBC+C model, to represent how students refine and crystallize undeveloped ideas into stable constructions through interactions with problem situations and other people (Hershkowitz, Hadas, Dreyfus, & Schwarz, 2007). In their model,
recognizing involves perceiving conditions related to a previously constructed abstraction and bringing the previous abstraction to mind. Building-with involves connecting two previously constructed abstractions. Both of these types of actions are nested within the larger action of constructing a new abstraction by assembling and organizing previously constructed abstractions into a new mental structure. At first, the stability of a students’ understanding of a concept might be fragile, demonstrated if they are easily talked out of an idea or slow to use the idea in another situation. Consolidation is said to have occurred if students can verbalize an idea or recognize and build-with a concept in subsequent situations. Hershkowitz et al. (2007) use the RBC+C model to characterize observed instances of vertical mathematization.

A number of scholars have used the RBC+C model to understand actions that bring about consolidation. Dreyfus and Tsamir (2004) propose that consolidation of a concept is prompted by three types of activity: building-with the concept in a different, yet structurally similar context (along the lines of MMP model-adapting activities); explicit reflection on building-with activity; and reflection on broader mathematical ideas related to the mathematics in the problem. Monaghan and Ozmantar (2006) highlight the importance of language and vocabulary for precipitating consolidation, concluding that instructors need to create opportunities for students to develop or learn precise language for the new ideas they construct. Both of these studies highlight the role that reflective discourse plays consolidation of mathematical ideas.

Figure 4 shows how the types of activity recommended in RME and MMP can be explained using these theoretical explanations of abstraction. First, the lower two levels of stages of RME and the corresponding stages of the model eliciting sequence are meant
to provide opportunities for students to recognize and build-with previously constructed abstractions in order to construct and begin to consolidate new ideas. At the general level of RME, attention shifts to the models themselves, providing the opportunity to reflect on building-with and to develop precise language around any newly constructed concepts, both of which are thought to foster consolidation (Dreyfus & Tsamir, 2004; Monaghan & Ozmantar, 2006). Applying the model to new situations provides a space for recognizing, and building-with to further promote consolidation. Discussions of structural similarity provide the opportunity for reflection on building-with and, subsequently, consolidation. These discussions also provide opportunity for identifying similarities between problem situations necessary for empirical abstraction.

<table>
<thead>
<tr>
<th>Description of activity</th>
<th>RME</th>
<th>MMP</th>
<th>Theoretical abstraction</th>
<th>Empirical abstraction</th>
</tr>
</thead>
<tbody>
<tr>
<td>Making sense of the context</td>
<td>Situational</td>
<td>Warm-up</td>
<td>Recognizing, Building-with, Constructing, Consolidating</td>
<td></td>
</tr>
<tr>
<td>Attempting to solve the problem using common sense</td>
<td>Referential</td>
<td>Model eliciting</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Using models to explain thinking and solutions</td>
<td>General Formal</td>
<td>Model exploration</td>
<td>Consolidating</td>
<td></td>
</tr>
<tr>
<td>Shifting focus to the models themselves</td>
<td>General Formal</td>
<td>Model adaptation</td>
<td>Recognizing, Building-with, Consolidating</td>
<td></td>
</tr>
<tr>
<td>Using models to solve other problems</td>
<td>General Formal</td>
<td>Discussion of structural similarity</td>
<td>Consolidating</td>
<td>Empirical abstraction</td>
</tr>
<tr>
<td>Reflecting across models and problem situations</td>
<td>General Formal</td>
<td>Follow-up activities</td>
<td>Recognizing</td>
<td></td>
</tr>
<tr>
<td>Using conventional algorithms to solve conventional tasks</td>
<td>General Formal</td>
<td>Recognizing</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

*Figure 4. Opportunities for abstraction in the various RME and MMP activity types*
The mapping of key ideas from RME and MMP literature resulted in seven distinct categories describing the types of activity that are recommended by advocates of these approaches. Theories of empirical and theoretical abstraction provide a language for describing how these activities might prompt the learner to abstract mathematical ideas. After synthesizing the literature in this way, the question remains: do these categories provide an effective lens for understanding non-RME or MMP instruction based on contextualized problem solving? Are these categories sufficient for understanding significant problems of practice faced in the design and implementation of CP-based instruction?

3. Empirical Testing

Armed with theoretical sensitivity towards significant terms, categories of activity, and the potential relationships between them described in the literature, I set out to determine how these distinctions might help me make sense of classroom instruction emphasizing CPs. My goal was not to determine whether the instructional theories outlined above were effective; rather, I set out to determine whether this a priori set of categories I developed from the literature were empirically viable. In the next section, I describe in the role of empirical observation in the development of the framework.

3.1 Setting and Participants

I partnered with a practicing high school teacher whom I refer to with the pseudonym Ms. Spence. Ms. Spence frequently used contextualized problems in
instruction but was not familiar with MMP or RME as theoretical models. I observed two different sections, two to three times per week over the course of twelve weeks, for a total of 78 observations. In addition to these observations, I conducted 20 post-instruction interviews with the teacher, to gain insight into the critical implementation issues that arise from practice (Heid et al., 2006). Ms. Spence taught from a curriculum called *Core-Plus Mathematics: Contemporary Mathematics in Context* [commonly called *Core-Plus*] (Hirsch et al., 2008), developed by an academic center in the US with funds provided by the National Science Foundation.

This setting was appropriate for the development of the framework for a number of reasons. First, Ms. Spence was an ideal partner in the research because she had 13 years of experience implementing *Core-Plus* and had participated in a great deal of professional development specifically targeting *Core-Plus* implementation. Because of this significant experience with the curriculum, she was able to consciously reflect on what she perceived as the strengths and weaknesses of the curriculum as well as the ways in which she adapted, supplemented, and selected from the curricular offerings to address critical issues of practice. Second, high school instruction is under-represented in the theoretical literature around the use of CPs for developing new mathematical understanding and in RME and MMP literature specifically, so a framework that can be used to describe CP-based instruction at this level would be valuable to the field. Finally, *Core-Plus* was a particularly appropriate curriculum to observe in combination with the theoretical perspectives described above because, although mathematical modeling is a unifying theme in the curriculum and an inspiration for its instructional design, the development of this curriculum was not explicitly guided by any single particular
instructional theory like RME or MMP model-development sequences (Fey, personal communication, July 25, 2011; Hirsch, personal communication, June 15, 2011). A framework that is grounded in these particular instructional theories and developed to make sense of a curriculum that was not designed from these theories has significant potential to be generalizable.

3.2 Data Sources

The primary dataset used for the development of the framework consisted of written Core-Plus lessons and transcriptions of a subset of the observed lessons. Two full units of Core-Plus textbook were analyzed: 26 instructional days of an algebra unit focused on linear relationships and 20 instructional days of a statistics unit focused on distribution and descriptions of center for one-dimensional data sets. I recorded, transcribed, and coded eight sessions of the algebra unit; each session was 1.5 hours long. During observations, I took field notes. I also engaged the teacher in 15 post-observation interviews. From the field notes and interviews, I identified types of tasks that were particularly meaningful to the teacher or to me as an observer.

3.3 Data Analysis

The framework was developed through an iterative data analysis process. First, I consulted relevant literature to develop theoretical sensitivity and an initial set of conceptual categories. Each of the four levels of RME, for example, represented a potential category of activity. Next, I analyzed instructional activity in both the written
curriculum and transcripts of classroom observations. I use the term *activity* as an umbrella term that includes tasks, questions and statements. In order to assign codes to sections of the textbook and class transcripts, I segmented the written curriculum and enacted curriculum into *topically contained turns* [TCT], a variation on Mehan’s (1979) *topically related sets*. In discourse analysis, a turn is the speech unit that begins when one person speaks and ends when another person speaks. Mehan defines a topically related set as a group of these turns focused on a single subject (e.g. the spelling of a word, the solution to a particular instructional exercise). To adapt this idea for the purposes of coding the text, I decided that, in a textbook, a “turn” ends when the reader is signaled to perform some sort of task or to answer a question. A transition to a new topic or section also indicates a new turn. Each problem or task denoted by a letter or number is its own topically contained turn. Similarly, teacher utterances in the transcriptions of the enacted lessons were segmented into topically contained turns. A new TCT was indicated when a student was expected to perform some sort of task or respond in some way. A single utterance was segmented into multiple TCTs if a new topic was clearly indicated.

I analyzed the written and enacted curriculum using the constant comparative method (Glaser & Strauss, 1967). As I read each TCT, I assigned a code, or label, describing what type of activity was present. As I continued coding, if a particular activity matched the type assigned to a previous activity, it was coded with that label. If it did not fit, an existing code was modified or a new code was created. Over the course of the coding, the definitions of these codes stabilized. These codes were then categorized and organized according to the interpretive ideas from the literature and problems of
practice identified in my observation field notes and teacher interviews. This
categorization was guided by the following questions: what are the dimensions or
organizing concepts that connect these categories? How do they relate to each other?
What types of activity are contained inside each category?

When ideas from the literature were not sufficient for describing or organizing the
empirical data, I performed another round of literature review targeting any activity types
that did not fit into my existing codes. Existing categories were adjusted or new
categories were established; existing codes were re-categorized. In the end, a set of 105
activity-type codes were identified and categorized into the ten categories found in the
resulting framework. For example, the code “answer a context question given a
mathematical representation” was placed into a broader category called “interpret
model”; the code “explain the connection between two different types of representations”
was placed into a category called “focus on model”.

After the framework was established, a coding manual was developed that
described the ten categories, including the various types of activity within the categories.
The ten categories were validated through a test of inter-rater reliability. I trained an
advanced mathematics education graduate student on the use of the coding manual, and
she coded two of the Core-Plus lessons, assigning one of the ten category codes to each
TCT. Comparing her codes to my own, I found the joint probability of agreement to be
85%. The resulting framework is grounded in empirical data, connected to prior theory
and empirical research, and also empirically validated.
4. A Framework for Analyzing the Role of Contextualized Problems in Mathematics

Instruction

Because my aim was to define, organize, and integrate the ideas from the literature and empirical observation into a cohesive framework, I looked for properties that could be used to characterize the various types of activity I encountered in my literature review and field observations. I identified two dimensions that were particularly salient: contextualized/non-contextualized and particular/general. I organized the categories around these two dimensions; the result was the overall structure of the resulting framework as displayed in Figure 5. The contextualized/non-contextualized dimension is represented horizontally in the figure. Contextualized activity, found on the left, refers to some extra-mathematical situation or situations. Non-contextualized activity, found on the right, refers only to realm of mathematics. The particular/general dimension is represented vertically. Activity represented toward the bottom references a particular problem or example. Activity at the top references general mathematical ideas, meaning universal principles that apply to many problem situations; this type of activity does not reference particular problems or examples at all. In the vertical dimension, activity that references multiple examples is located in the middle layer. This activity is more general than that which references a single example; but, because these examples are explicitly addressed, it is less general than activity that does not reference specific examples at all. Use of this organization scheme resulted in seven categories of activity, which are described with examples, in the following paragraphs.
4.1 Activity Types in the CPMI Framework

Beginning in the lower left corner, activity in the contextualized problem solving domain is situated in some extra-mathematical setting. In an example from Core-Plus, the algebra unit I observed begins by introducing a fictional character named Barry, who is employed by a credit card company. Barry’s job is to recruit people to fill out credit card applications. The details of Barry’s compensation are presented via a graph showing that the pay Barry receives increases linearly in relation to the quantity of applications he
collects. The authors ask, “How does Barry’s daily pay change as the number of applications he collects increases?” (p. 151).

Research suggests that certain types of contextualized problem solving have the potential to serve as foundational activities for mathematical learning. In both RME and MMP perspectives, for example, students are meant to use their understanding of a problem situation to build informal models that can serve as conceptual anchors for formal mathematical conventions and reasoning. But, the term contextualized problem solving is meant to encompass a wide variety of tasks, not only those that would fit in RME or MMP instructional sequences. This category also refers to more traditional word problems intended to provide an opportunity to apply known procedures and examples like that from Core-Plus above which are different tab RME or MMP tasks but are still aimed at developing students’ understanding of new mathematical concepts. In section 4.2, I describe sub-categories of activity types within the contextualized problem solving category, and in section 5, I further explain how the framework can be used distinguish between traditional, Core-Plus, MMP, and RME approaches.

In the visual model of the framework in Figure 5, the contextualized problem solving category is represented by a number of stacked boxes. This is meant to emphasize the possibility and importance of multiple examples. Dreyfus and Tsamir (2004) note that students often consolidate abstractions through recognition and application of the abstraction to new situations. Applying models to structurally similar situations is also the focus of model-adaptation activities in MMP.

Moving to the right, activity in the non-contextualized problem solving domain requires students to work or reflect on a “bare” or “naked” mathematical situation that
does not contain any extra-mathematical elements. An example comes from the same
Core-Plus investigation cited above:

**Draw a graph for each function on a separate set of coordinate axes.**

\[ a. \quad y = 1 + \frac{2}{3}x \quad b. \quad y = 2x \quad c. \quad y = 2x - 3 \quad d. \quad y = 2 - \frac{1}{2}x \]

(p. 155)

These equations do not refer to any extra-mathematical situation. Each sub-task, a, b, c, and d, is considered a separate example because it refers to a different mathematical relationship. Non-contextualized problem solving falls within general level of RME; here, students are meant to recognize the applicability of the models previously constructed in the referential level. In terms of the RBC+C model, the acts of recalling models developed in response to contextualized problems and applying them to non-contextualized examples could foster the consolidation of constructed abstractions.

In the middle row, beginning from the left, reflection across contextualized examples involves the identification of similarities and relationships between problems (or the solutions to problems) set in two or more distinct extra-mathematical contexts. An example from Core-Plus asks students to look back over a number of tasks in a previously encountered section: “What features of expressions like those in the “Applications” tasks suggest that the graph of the function defined by that expression will be a line?” (p. 228). Reflection across contexts provides an opportunity for empirical abstraction as described by Skemp (1987). Lesh et al (2003) note the importance of this type of activity in their description of discussions of structural similarity: “Isolated problem-solving activities are seldom enough to produce the kinds of results we seek.”
Sequences of structurally related activities are needed, and discussions and explorations are needed to focus on structural similarities among related activities” (p. 44).

Moving to the right, the next category describes activity that refers to particular contextualized and non-contextualized settings. This type of activity is not explicitly mentioned in the descriptions of the RME levels or MMP model-development sequences; rather, it is included in the framework as a result of my empirical analysis. This category of activity emerged as a solution to a problem of practice identified by Ms. Spence during a post-interview. She consistently referred to a lack of connection between students’ activity in contextualized problem-situations and their work with non-contextualized examples. This was particularly evident during one particular investigation that took place over a number of class sessions. Students had spent three class sessions working on CPs to determine rates of change from tables, graphs and symbolic rules, including the example featuring Barry described above. Then, the lesson transitioned to non-contextualized problem solving: students were asked to graph the line representing the equation \( y = 1 + \frac{2}{3}x \) and the other non-contextualized equations cited above on a coordinate plane and to calculate the slope between two points for each equation. In the post-observation interview, Ms. Spence remarked,

I think that they’re not, they’re not making the connection between looking at the rule now and identifying the slope and the y-intercept as clearly as I thought they did last week when it was in context... I feel like I’m going back and I’m having to grab and pull those pieces together. (10/31/11)

Connecting contextualized problem solving with non-contextualized work was clearly a problem of practice. The category of reflection across contextualized and non-contextualized examples emerged during data analysis as a potential solution. Upon
looking for this type of activity within the data, I found an example of this type of activity: Ms. Spence created a task asking students to come up with a situation that could be described by the equation \( y = 5 + 4x \). Here, the task refers to a non-contextualized example, and a solution would need to refer to a contextualized example that could be described by the equation. Activity in this category provides the opportunity for empirical abstraction across these two problem types and could play an important role in connecting students’ contextualized and non-contextualized problem solving experiences. Descriptions of this sort of activity can be found in RME literature (e.g. Treffers, 1987) but is not emphasized in the four levels. I found no mention of this sort of activity in descriptions of MMP instructional sequences. The emphasis given to this activity type in the proposed framework marks one fundamental way the CPMI framework extends past the levels of RME or model-developments sequences in MMP.

The next category, represented on the right in the middle row, is reflection across multiple non-contextualized examples. In an example from Core-Plus, the authors provide a number of non-contextualized expressions:

*The following four expressions look very similar:* \( 2 - x - 5, 2 - (x - 5), 2 - (x + 5), 2 - 5 - x \... Which of the above expressions will always have the same value when you substitute the same number for \( x \) in the expressions? Explain. (p. 212).

Again, this type of question provides the opportunity for empirical abstraction, this time across non-contextualized examples.
Moving to the top of the figure, activity that involves reflection on general mathematical principles explicitly addresses generalizable procedures, concepts, representations, definitions or relationships without referring to particular examples. This domain includes tasks prompting students to describe procedures or algorithms or explain the meaning of abstract mathematical concepts. Activity in this category is frequently at the end of Core-Plus lessons. For example, the authors ask: “How can you determine whether a function between two variables is linear by inspecting: (i) the table relating two variables? (ii.) the graph of the function? (iii.) a symbolic rule relating the two variables?” (p. 156). Notice that this question, and its response, does not refer at all to particular contextualized or non-contextualized examples. The task refers to concepts (functions, linearity), procedures (how to determine linearity) and various representations (tables, graphs, and rules). This sort of activity is not described explicitly in RME or MMP; however, it could be found at various points in the RME or MMP sequence if students make generalizations across models or representations, most likely in the general/model-exploration stage or in discussions of structural similarity. It is in these types of activity that the focus is on models themselves, rather than on how the model relates to the context or particular examples. Dreyfus and Tsamir (2004) describe this sort of activity as reflection on broader mathematical ideas and note its potential to enhance students’ consolidation of mathematical ideas.

Ellis (2007) provides a taxonomy of “reflection generalizations”. This taxonomy contains four subcategories under “reflection on general principles”: reflections on a (1) rule, (2) pattern, (3) strategy or procedure, or (4) global rule. The first two sub-categories refer to reflections that pertain to a single example or situation. The latter two are more universal or global and extend “beyond a specific case”. In the CPMI framework, I have chosen the term “reflection on general mathematical principles” to describe reflections that would fall in the latter two sub-categories, meaning they are global, or universal, principles that apply broadly to many cases of a particular kind.
4.2 Activity Types within the Contextualized Problem solving Category

The activity types presented to this point describe important conceptual distinctions that can be used to analyze and understand instruction emphasizing contextualized problems. But, to this point, the framework does not provide detailed language for classifying different types of activity within the category of contextualized problem solving. To address this issue and to gain further resolution in this category, I used conceptual distinctions present in literature on mathematical modeling (including RME and MMP) and empirical observation to identify five subcategories of activity possible within the contextualized problem solving category (see Figure 6). I describe the relationship between these subcategories using an adaptation of the common representation of modeling shown in Figure 1.

To capture the distinction between translation and organization approaches to modeling as described by Gravemeijer (1997), I distinguish between activity focused on informal models and activity featuring more conventional or formal mathematical models or representations. The dotted line is meant to signify the possibility of activity in the informal level rising to formal activity through the process of vertical mathematization, as described by Gravemeijer (1994).
The *Focus on context* box at the bottom of Figure 6 represents activity that focuses particularly on the situation that the problem refers to rather than on a mathematical model. This category includes activity wherein students and their teacher discuss a context before a particular problem is posed, assess which data are important to a particular problem, or collect data. An example from *Core-Plus* occurs on page 198: “Do you or someone you know use the Internet? For what purposes?” This category is also where the warm-up activities in MMP model-development sequences would be classified.

The acts of translating and organizing elements from a problem context into a mathematical model comprise the *Produce model* category. These tasks or questions are focused on the act of producing a mathematical model, given a particular situation or data.
set. An example of this type of activity from Core-Plus occurs on page 102: “Use the words NOW and NEXT to write a rule that shows how to use the price of the item in one year to find the price of the item in the next year.” This would be an example of what Gravemeijer (1997) would describe as a translating activity; one could distinguish between a translating activity and an organizing activity more in line with an RME approach within this category.

Activity in the focus on model category pertains to one or more informal or formal mathematical models and is not as concerned with the relationship between the model and the context. This category includes questions and tasks that ask students to compare solution strategies, translate from one type of representation to another, match different types of representations, compare or explain the relationship between two different types of representations, or work within one model to produce a model of a particular mathematical relationship (e.g. find the mean, standard deviation, slope or rate of change, y-intercept, etc.). The importance of this category cannot be overstated: this is the key domain for activity related to vertical mathematization in RME, as this is where informal models are organized, combined and formalized. This shift to the focus on the model aligns with the general level of RME and the model exploration activities in MMP. In reference to the RBC+C model, this is where activity involving reflection on building-with lower level abstractions (Dreyfus & Tsamir, 2004) could lead to consolidation. This is also where formal language for describing mathematical concepts could be introduced (Monaghan & Ozmantar, 2006).

Tasks in the Interpret model category involve the use of a mathematical model to answer a contextualized question of some sort. This category includes questions or tasks
that (a) ask students to summarize what a particular model says about the context, (b) answer a specific question about the context by looking at a single model or comparing a number of models, (c) explain how to use a model to answer a question, or (d) assess a hypothetical response to a question. An example from *Core-Plus* occurs on page 174: “Using the linear model, estimate the median income of women in 1983 and 2007.”

These four subcategories describe tasks that specifically prompt students to engage with specific steps within the modeling process. However, to solve complex modeling tasks like those described in MMP literature, students would engage with the modeling process in its entirety, and often more than once, in response to a single task.

Now that the contextualized problem solving category has been further decomposed, these four subcategories can be placed into the broader organizational scheme, as shown in Figure 7, and the framework is complete.
Figure 7. The Contextualized Problems in Mathematics Instruction (CPMI) framework

I have included both informal and formal models in the non-contextualized frame to emphasize that students can reason with informal models in non-contextualized settings. This allows for the description of activities that leverage students’ informal reasoning but do not specifically refer to contextualized problem situations.
With these additions, this framework represents a synthesis of significant concepts from two prominent modeling-based instructional theories (RME and MMP model development sequences) and important activity types that emerged from empirical observation of the implementation of two units from the Core-Plus curriculum. The importance of the various categories toward abstracting mathematical ideas is supported by research on empirical and theoretical abstraction. I have added the additional category of reflecting across contextualized and non-contextualized examples, neither of which is explicitly identified in RME or MMP instructional sequences. This category emerged as a potential solution to a significant problem practice: a lack of coherence between students’ contextualized problem solving and non-contextualized problem solving.

5. Application of the CPMI Framework

Now that the framework has been described, the question remains: what is it good for? What can examining mathematics curriculum or classroom practice through the lens of this framework tell us? There are three primary ways in which the framework can be used to interpret mathematics curriculum. First, the framework can be used to analyze written and enacted curriculum for the presence or omission of each of the types of activity described in the framework and particularly those that research tells us are important for student learning. For instance, Boaler (2002) as well as Jackson, Shahan, Gibbons and Cobb (2012) argue that one key to increasing the potential of instruction based on contextualized problems is to make sure that students understand the contextual features of contextualized problem situations. To do this, they recommend that teachers
devote a portion of instructional time to identifying important contextual elements, clarifying vocabulary words, and verbalizing relationships that might be obvious only to those who are familiar with the setting. In terms of the CPMI framework, this sort of discussion would include statements or questions that would be classified in the Focus on context subdomain within the contextualized problem solving category. A different category of instructional activity emphasized in the RME and MMP literature falls within the Focus on model subdomain within the contextualized problem solving category: a shift of attention from the problem situation at hand to the mathematical models that students used to solve the problems. Finally, White and Mitchelmore (2010) and Lesh et. al (2003), argue that activity that would be classified as Reflection across contextualized examples represents a critical step in the process of abstracting fundamental mathematical concepts from contextualized problem solving. By analyzing textbooks or enacted lessons using the framework, teachers, instructional leaders, curriculum developers or researchers could detect the presence or absence of each of these recommended varieties of statements or questions in instruction.

In addition to being used as a tool for detecting the presence or absence of particular categories of instructional activity, the framework can be used to characterize the sequencing of instructional activity. In many classrooms mathematical procedures are first explained through a non-contextualized problem, the steps for the procedure are generalized (reflection on general mathematical principles), and students practice the procedure on similar examples (a return to non-contextualized problem solving). Towards the end of the lesson, students are often asked to apply the procedure to a word problem involving some extra-mathematical setting (contextualized problem solving).
Streefland (1991) terms this a *mechanistic* approach to mathematics instruction. This traditional positioning of contextualized problems within math lessons can be described using the framework as shown in Figure 8; arrows describe the progression of activity.

*Figure 8.* The traditional role of contextualized problems in mathematics instruction

Lessons aligned with the RME and MMP instructional sequences would tend to look very different. A common sequence from these approaches is represented in Figure 9.
RME and MMP approaches argue that students’ understanding of the physical world can often be used as the starting point for instructional sequences. Lessons from a curriculum programs aligning with these perspectives would often begin with students working on tasks in the *Contextualized problem solving* domain. After time to work in groups, students might be asked to share their strategies. Next, the teacher might facilitate a discussion in which strategies, procedures and/or ideas are generalized (*reflection on general mathematical principles*). Following this, the students might apply this procedure to a different contextualized problem and eventually to tasks situated in the *non-contextualized problem solving* domain. In this form of instruction, contextualized problems do not serve only as settings for the application of known math; instead, students construct new math knowledge through contextualized problem solving. By characterizing the sequencing of lessons in this way, practitioners and researchers would
gain insight into the role that contextualized problems play in particular written or enacted lessons.

The sub-categories within contextualized problem solving domain in Figure 6 could even be used to identify distinctions between different approaches that begin with contextualized problem solving tasks. For instance, RME instructional sequences are designed so that students create informal models, which subsequently are connected to conventional, formal models through vertical mathematization. This vertical mathematization would show up as a transition from informal to formal activity within the focus on model sub-category. RME sequences also would contain a transition to non-contextualized problem solving. MMP instructional sequences, on the other hand are meant to develop foundational understanding of important mathematical concepts through multiple trips around the modeling cycle; these sequences not explicitly concerned with progressing to formal models, so activity at the more formal level would be less emphasized. Multiple trips around the modeling cycle within a single problem situation would be observed. Also, non-contextualized problem solving is not described explicitly in MMP model development sequences.

The framework can also be used to clarify differences between Core-Plus instructional sequences observed for this study and RME or MMP inspired instructional sequences. Rather than emphasizing the creation of informal models through tasks that would be classified as produce model activities, the Core-Plus sequences in the units analyzed here frequently begin from formal models, which students are asked to interpret or translate to another formal models. For instance, in the unit featuring Barry described throughout this article, students are first presented with a formal graph and a symbolic
function rule, which they are asked to interpret in terms of the extra-mathematical situation, a task that would be classified in the interpret category. This type of activity would not show up at the beginning of RME or MMP sequences because the students did not produce these models on their own.

In addition to providing a way to consider the presence, absence and sequencing of various activity types, the framework can be used to understand the extent to which instructional activity in the various categories is explicitly connected. Instruction could be analyzed, for instance, to determine the extent to which students, the teacher, or curriculum materials explicitly reference prior contextualized problem solving during the discussion of tasks or questions categorized as reflection on generalizable mathematics. Connecting across instructional tasks in these ways would likely increase the potential that students’ understanding of formal, mathematical connections would be connected to their common sense understandings of the world around them, as described by Freudenthal (1973) and other RME theorists.

6. Conclusions

Analytical frameworks like the one proposed here can contribute significantly to the field of mathematics education in a variety of ways. They provide a way of connecting research to practice, by providing a common language that both practitioners and researchers can use to understand instructional events. Frameworks help practitioners like educators and instructional designers to think and work systematically. When they are composed of critical ideas from theory and empirical research, frameworks help
practitioners act in ways that align with scholars’ recommendations. They provide a lens for researchers to use in developing and testing new instructional theories, and a way of communicating those theories that show promise. The framework described here has the potential to help teachers, curriculum developers, teacher developers, and researchers in a variety of ways.

For teachers, the framework offers a lens for understanding the types of tasks present in their curriculum materials and the ways these tasks are sequenced. A nuanced understanding of these sequences could help teachers implement curriculum programs more effectively, by providing criteria to use when selecting from, adapting, or supplementing the provided tasks. For curriculum developers, the framework could be used as a tool for developing new instructional materials that match particular instructional goals and philosophies. Once curricular programs are developed, the framework could be used to transmit the specifics of the instructional design to teachers. This would give teachers a clear idea of where a particular task was headed mathematically so that they would be better equipped to guide the classroom enactment of the task, especially when complications arose. Teacher educators could use the framework to prepare teachers to teach from the many different programs on the market, an issue of great importance to the field.

The CPMI framework offers a host of applications to researchers of written and enacted curricula. The framework could be used to determine whether, for instance, non-contextualized representations of mathematical concepts are generally introduced before contextual applications within the corpus of mathematics textbooks. Comparative analyses of curriculum programs are also possible using this framework; for instance, the
framework could enable a researcher to differentiate between a translation approach and more of an RME inspired organizing approach (Gravemeijer, 1997). Elsewhere, I demonstrate how the framework can be used to characterize how teachers’ transform written curriculum into plans for enactment and the ways in which teachers’ beliefs and instructional context affect this transformation (Reinke, Chapter 3 of the dissertation). I have also used the framework to identify patterns in regard to how students and teachers respond to the different types of tasks and questions during instruction (Reinke, Chapter 4 of this dissertation).

The framework can be adapted, depending on the experience of the audience and the goals of the users. For instance, the degree of resolution in Figure 5 would suffice for a teacher developer helping teachers think about how to connect work in contextualized examples to other types of tasks and instruction. Scholars interested in comparing the types of contextualized tasks included in various curricular materials, on the other hand, might be more apt to focus only on the details of the contextualized problem solving domain in Figure 6.

Although the CPMI framework was designed to be generalizable, the phases of the curriculum development cycle that involved empirical observation were based primarily on observations of two units of a single curriculum enacted by a single teacher. As such, this first iteration of the framework is highly situated. Further research is needed to understand the way that problems of practice that arise from other curriculum programs and other instructional settings can or cannot be characterized, and potentially navigated, using this framework.
Given the continued emphasis on contextualized problems in standards documents and curriculum materials, there is an immediate need for a technical language distinguishing different types of activity within CPs and a way of describing how CPs relate to other instructional activity. By providing such a vocabulary, the CPMI framework has the potential to improve the way CPs are used within mathematical instruction.
CHAPTER 3: THE ROLE OF CONTEXTUALIZED PROBLEMS IN INSTRUCTION: ONE TEACHERS’ TRANSFORMATION OF THE WRITTEN CURRICULUM

Abstract

Research has shown that students can develop deep understanding of mathematical concepts by solving problems referring to contexts that exist outside the realm of mathematics (e.g. Gravemeijer, 1999), but some teachers view contextualized problems (CPs) in more affective terms (Gainsburg, 2008, 2009; Pierce & Stacey, 2006). This study sought to determine how teacher beliefs about CPs and other factors influence a teacher’s implementation of CP-based curriculum materials. The participating teacher felt that some students learned best if concepts were introduced in context and other students learned best if the mathematics was first presented more abstractly. The teacher re-sequenced the curriculum, and the CP-first order present in the written curriculum was sometimes compromised as a result. A school-wide lesson-planning template also influenced the way the teacher transformed the written curriculum. In response to the template, the teacher included tasks that prompted students to generalize about underlying mathematical principles earlier and more frequently than prescribed by the written curriculum. These findings have implications for curriculum developers, who are urged to provide explanations about the design of instructional sequences for teachers, particularly if they intend for students to leverage prior experiences while working on instructional tasks.
1. Introduction

Mathematics problems that reference some real or imagined social or cultural situation are sometimes called contextualized problems (CPs) (Chapman, 2006). Traditionally, mathematics lessons have incorporated CPs toward the end of the instructional sequence, as settings for the application of particular concepts or skills that were introduced and developed through non-contextualized examples. Over the past twenty years, though, advocates of mathematics education reform in a number of countries have suggested that students can and should learn new mathematical concepts by solving particular types of contextualized problems (e.g. NCTM, 1989; Common Core State Standards Initiative, 2010; Ministerie van Onderwijs, Cultuur en Wetenschappen (OCenW), 2004; Ministry of Education, 2007). These recommendations are backed by research showing that strategies students develop to solve some contextualized problems can serve as a foundation for understanding the conventions of formal mathematics (Carpenter et al., 1996; Gravemeijer, 1994; Greeno & The Middle-School Mathematics through Applications Project, 1997; Technology Group at Cognition and Technology Group at Vanderbilt, 1997). In these research programs, instructional sequences often begin with open-middle contextualized problems, meaning problems that have a single starting point and a single answer but can be approached and solved in a multitude of ways. Students use their understanding of contextual situations to develop intuitive, informal strategies that can be connected to formal mathematics. Freudenthal (1993) argued that through this type of CP-first instruction, students’ understandings of abstract
mathematical notions are connected to their common sense understanding of the world around them.

In the US, a number of curriculum programs were developed to help teachers respond to recommendations for the use of CPs as settings for new mathematical learning. The titles of a number of these programs communicate an emphasis on context (e.g. *Everyday Mathematics* (University of Chicago School Mathematics Project, 2004), *Mathematics in Context* (National Center for Research in Mathematical Sciences Education & Freudenthal Institute, 1998), and *Contemporary Math in Context* (Coxford et al., 2003)); in this paper, I refer to these types of curricula as CP-based. These curriculum programs make up the majority of curriculum programs that fall within a larger category of mathematics curriculum often referred to as Standards-based curricula (Senk & Thompson, 2003). Standards-based programs are so-called because they were developed to be closely aligned with the numerous recommendations for reform contained within the standards documents published by the National Council of Teachers of Mathematics (1989; 2000).

Research into Standards-based curriculum programs indicates that the way any given curriculum is implemented varies significantly from classroom to classroom (Stein, Remillard & Smith, 2007). Teachers beliefs about the nature of mathematics and mathematics pedagogy (Lambdin & Preston, 1995), their orientations towards curriculum materials (Lloyd, 1999; Remillard & Bryans, 2004), and the contexts in which they teach (McClain, Zhao, Visnovska, & Bowen, 2008) have been shown to influence the way teachers transform the written curriculum as they plan and implement classroom activities. In some cases, teachers implement reform-oriented curricula in ways that do
not align with the Standards (Tarr et al., 2008) or the developers’ intent (S. A. Brown, Pitvorec, Ditto, & Kelso, 2009).

Research suggests that some teacher beliefs about CPs may contribute to patterns of implementation that do not align with reform recommendations or curriculum developers’ intentions. For instance, some teachers primarily see CPs in affective terms, as motivators, as opposed to scaffolds for conceptual development (Chapman, 2006; Pierce & Stacey, 2006). This view is understandable given the traditional role that word problems have played within mathematical instruction: as applications of previously taught mathematics that require students only to ignore the narrative elements and extract the necessary mathematical information (Gerofsky, 2004). In cases where CPs are intended to foster students’ development of new mathematical ideas, however, this view could make it less likely that teachers leverage the contexts in ways that scaffold learning. Also, some teachers believe that mathematical concepts should be taught through non-contextualized examples first and then practiced through CPs (Chapman, 2006). This second view directly contradicts the recommendation of scholars such as Freudenthal (1973, 1991) who argue that if instructional sequences begin with carefully designed CPs, students can leverage their understanding of real-world phenomena to develop mathematical understanding. Given these findings related to teacher beliefs about CPs, it is possible that these beliefs could influence how teachers choose from and implement problems in CP-based curricula. Because the sequencing of problems from contextually-based to more formal and abstract is central to the design of many CP curricula, implementation decisions that conflict with this aspect of designers’ intents are likely to be consequential. If teachers fail to leverage the potential benefits of CPs in
Standards-based curricula, the effectiveness of these programs could be significantly compromised.

An important component of a CP-first approach, specified by Freudenthal (1973) and promoted in the NCTM Standards (2000), is the view that, when possible, students can and should learn formal mathematics by solving novel problems then comparing across solution strategies to generalize mathematical principles. There is evidence that some teachers doubt whether students will develop an understanding the intended mathematics through this active problem solving approach using particular Standards-based materials; this belief leads to patterns of implementation that contradict reform recommendations. Lloyd (1999) and Wilson and Lloyd (2000), studied teachers implementing a context-based high school curriculum, and found that a number of the teachers in the study were concerned that their students had difficulty grasping the mathematical point of the explorations. Because of these concerns, one of the teachers in Lloyd’s (1999) study modeled solution strategies for students rather than giving students the chance to develop strategies themselves. A teacher in Wilson and Lloyd’s (2000) study spent more time in whole-class format than recommended by the curriculum designers because she was concerned “whether or not [students] would make appropriate connections on their own, without her explicit intervention and explanation” (p. 158). Another teacher provided explicit direction about important mathematical connections as students worked in small groups because of similar concerns. Wilson and Lloyd conclude

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4 This view is central to, but not limited to, a CP-first approach. Neither Freudenthal (1973, 1991) nor the authors of the NCTM Standards (2000) assert that instructional sequences should always begin with CPs or that learning can occur through active problem solving only if the problems are contextualized.
that it is difficult for “some teachers to accept that students can make important connections without direct teacher explanation” (p. 167); it is important to point out that these teachers were responding to a particular curriculum and were not directly asked whether students were capable of making these types of connections at all. Regardless of the teachers’ more global beliefs, the fact remains that students were not given the opportunity to make these connections on their own, as reformers (Common Core State Standards Initiative, 2010; NCTM, 2000) recommend. Taken together, this evidence suggests that if teachers believe that students using CP-based curriculum programs cannot make these connections without direct intervention, they will be less likely to implement these programs in ways that align with reformers recommendations.

No studies have specifically examined how teachers’ beliefs about contextualized problems influence their implementation of CP-based curriculum. To address this gap in the research literature and to identify other factors influencing teachers’ use of CP-based curriculum, I examined one teacher’s implementation of Core-Plus Mathematics: Contemporary Mathematics in Context (Hirsch et al., 2008), a high school curriculum program that emphasizes learning through contextualized problem solving. I examined the teachers’ beliefs about CPs and how these beliefs impacted how she chose, sequenced, and supplemented tasks from the curriculum. Findings from this analysis offer insight into how teachers might transform a curriculum in order to mitigate potential obstacles to learning inherent to a CP-based approach.
2. Theoretical Framework

This study, which examines a teacher’s implementation of a CP-based curriculum program, draws on two perspectives on the study of mathematics instruction. The first perspective, proposed by Stein, Remillard and Smith (2007) and Remillard (2005), provides a framework for considering the role of the teacher in curriculum implementation and highlights the significance of the teacher as an object of study within the larger field of curriculum studies. The second perspective, introduced by Simon and Tzur (1999), describes the stance I take in studying an individual teacher’s instructional decisions and the factors that influence them. In the following section, I describe the way these theoretical perspectives informed the methodology of the study.

2.1 Teacher Intended Curriculum

In their review of research on teachers’ use of curriculum, Stein et al. (2007) offer a framework for interpreting research on how mathematics curricula influence student learning. The framework consists of three phases; the authors contend that any published curriculum program proceeds through these phases as it is transformed from the written page to enactment. First, teachers transform curriculum from the written page into their own intended curriculum, or their plans for enactment. This is sometimes called the teacher intended curriculum (Remillard & Heck, 2014), to distinguish the teacher intended curriculum from the author intended curriculum or the official curriculum outlined in policy documents. The teacher intended curriculum is important to understand as step in the transformation process because it is in this phase that the teacher decides
which instructional tasks students will be exposed to and the order in which they will encounter these tasks. The study reported here focuses on this part of the curriculum transformation process, examining the way one teacher transformed the written curriculum to her intended plans for enactment. Like Stein et al., I argue that examining this step is significant in order to understand students’ opportunities to learn. By focusing on this phase of curriculum, I aim to determine the factors that mediated the teachers’ decisions about which tasks to use and when. Elsewhere, I describe the way the teacher and students in this study interacted to create the enacted curriculum (Reinke, Chapter 4 of this dissertation).

Remillard (2005) organizes studies of teachers’ curriculum use into four categories. The first category consists of studies that conceive of written curricula as a set of instructions. In these studies, curriculum use is conceptualized as following or subverting those instructions. Studies in the second category focus on the enacted curriculum and describe how teachers draw on curriculum materials as resources. A third category consists of studies that focus on how teachers interpret or make meaning of written curricula; these studies assume that fidelity to a written curriculum is impossible because the text has no meaning apart from the readers’ interpretation. Finally, a fourth set of studies focus analysis on the participatory relationship between the teacher and the written curriculum. These studies examine the relationship between the teacher and the curriculum materials, the factors that mediate this relationship, and the effect of the relationship on both the teacher and the way the curriculum is implemented. The methodological approach of the study reported here aligns most strongly with this last category because I study the teacher’s relationship with the curriculum resources, the
factors that influence the relationship, and the effect of the relationship on how the
teacher transforms the written curriculum into plans for enactment. Furthermore, like the
third group of studies that position use as interpretation, I acknowledge that curriculum
materials do not represent a set of comprehensive instructions that can be followed or not
followed. Isomorphism between the physical enactment of a curriculum and the words
on the page is not possible; enactment requires interpretation and adaptation in response
to in-the-moment classroom events. However, I acknowledge that there are components
of the curriculum, like the sequence of tasks or the use of particular tasks, for instance,
that can be followed or subverted. Researchers can determine whether a teacher follows
or subverts these aspects of the curriculum, even if other aspects require interpretation. In
sum, I study the way the teacher interacts with the curriculum to create plans for
enactment, and I compare and contrast these plans with the curriculum as written. Unlike
many fidelity studies that examine whether teachers follow or subvert curriculum
materials, I do not assume that effective curriculum use means necessarily following
those aspects of the curriculum that are indeed follow-able. Teaching is a complex
process in which teachers pursue multiple goals; to best suit the needs of the students in a
particular classroom, adapting a curriculum that is designed for many classrooms may, in
fact, enhance students’ opportunities to learn. This stance is supported by Brown et al.
(2009), who found that lessons which demonstrate low fidelity to the written curriculum
can in fact, produce the types of opportunities for learning intended by the authors of the
curriculum. In the next section, I describe my stance toward studying an individual
teacher’s decisions.
2.2 Studying Teacher Practice

As I describe a teachers’ intended curriculum and her reasons for transforming the written curriculum in the ways that I observed, I attempt to do so in a way that portrays the teacher’s decisions as inherently sensible. Some studies of teachers’ use of curriculum identify deficits in teacher practice compared to reform recommendations. Others attempt to characterize teachers’ understandings of their own practice. In contrast with either approach, this study is designed to align with what Simon and Tzur (1999) would describe as an account of a teacher’s practice described from the researcher perspective. This sort of account analyzes teacher actions through conceptual lenses that differ from that which the teacher uses; but, as opposed to deficit-based accounts, studies from this perspective “arrive[s] at an appropriate… articulation of the teacher’s current practice in a way that portrays the reasonableness of all the teacher’s observed actions” (p.256). The conceptual lens I use to describe patterns in the way the teacher transforms the written curriculum is not one she used to guide her practice. But I do examine and describe the rationale for her instructional decisions and the many factors that are involved. In my description, I hope to make clear that at each decision moment, she considered the information available to her and made the choice that she believed would maximize her students’ learning. Accordingly, her decisions were necessarily reasonable and sensible, given her understanding of the complex factors at hand.

Overall, the study aims to address the following research questions:

1. What are the patterns in the way the teacher selected, sequenced, adapted, and supplemented the tasks offered by the textbook? How did her intended
curriculum compare to the written curriculum, especially in regard to problems referring to extra-mathematical contexts?

2. What are the factors that influenced how the teacher transformed CP-based curricula from the written curriculum to plans for enactment? What influenced how she selected, sequenced, adapted, and supplemented tasks?

In the next section, I describe in detail the methods I used to answer these questions.

3. Method

To understand the ways in which the teacher transformed the CP-based curriculum program into her intended plans for enactment, I analyzed the written curriculum and her lesson plans. To gain insight into her beliefs and the factors she weighed when making instructional decisions, I interviewed the teacher multiple times over the course of the study. In the following section, I describe the data set and the analysis process in detail.

3.1 Curriculum, Instructional Context, and Study Participant

I observed the implementation of one algebra unit of a Standards-based high school curriculum called Core-Plus Mathematics: Contemporary Mathematics in Context (Hirsch et al., 2008), commonly known as Core-Plus. According to the developers, the curriculum is developed so that “investigations of real-life contexts lead to (re)invention of important mathematics that makes sense to students” (Schoen & Hirsch, 2003, p. 314).
Accordingly, many lessons in the curriculum are designed so that students first encounter mathematical topics through CPs that are often introduced through verbal descriptions. Later on in instructional sequences, mathematical concepts are formalized and students are provided the opportunity to work with more abstract, symbolic examples (Schoen et al., 2010). According to one of the primary authors of the *Core-Plus* algebra units, the development team “wanted students to become engaged with the mathematics through work on mathematics in context and to have those contextual concept images to think with and remember in the future” (Fey, personal communication, 7/25/11).

This study focuses on Unit 3 of the first course of the curriculum, entitled “Linear Functions”. The unit is divided into 4 lessons. Each lesson is made up of three to four investigations, each of which are intended to take two to three class periods. Unit 3 is the second unit in which students examine linear relationships. In Unit 1, students encountered situations that could be described with different types of functions, including linear functions. They are expected to have learned how to write symbolic rules (in the form of algebraic equations) to describe contextual situations and to create tables and graphs from these rules.

The setting for the study was a public high school located just outside a midsized city on the east coast of the US. The participating teacher, whom I refer to with the pseudonym Ms. Spence, was the department chair. She was one of the teachers who piloted *Core-Plus* for the district and had used *Core-Plus* for 13 of her 20 years of teaching mathematics. This study focuses on her preparations for two sections: a 9th grade Integrated Math I class and a 9th/10th grade Integrated I/II combined class. Both sections were using *Course 1* of the *Core-Plus* curriculum and, during the period during
which this study took place, Ms. Spence taught the sections in parallel, using the same plans for both sections.

3.2 Data sources

Data collected for this study includes interviews and two types of curriculum documents: the written Core-Plus curriculum, and Ms. Spence’s lesson plans.

3.2.1 Interviews

To understand Ms. Spence’s conceptions of CPs and other factors that influenced her decisions around the implementation of the curriculum, I engaged her in an introductory interview in which I asked about her thoughts about CPs, Core-Plus, and the process through which the curriculum leads students towards learning new mathematics. I asked questions including, “why do you think the designers chose to include real world contexts?”, “how does Core-Plus compare with your sensibilities as far as the way students learn math?” and “how well do you think the students are doing with these textbooks… how well do you think it’s working?”

Following the introductory interview, I observed Ms. Spence as she taught Unit 3 of the Core-Plus curriculum; I was present for twenty-nine out of thirty-six days of instruction. I engaged Ms. Spence in post-observation interviews after fifteen of these observations. These post-observation interviews lasted between seven and 27 minutes (median 16.5 minutes). At the beginning of these interviews, I asked, “what did you think?” then followed up with probing questions when her reflections were relevant to the research questions. I also asked questions about her intended curriculum, often related to
her reasoning behind particular instructional decisions. For instance, when she rearranged the sequence of the tasks presented in the textbook, I asked about her rationale for this decision. When she created supplementary tasks, I asked her to describe the purpose of the task she created and why she felt the need to supplement the offerings from the textbook. During these interviews, I also inquired about her plans for the upcoming class sessions and her rationale for the decisions she described as she shared her plans.

3.2.2 Curriculum documents
To characterize the way Ms. Spence transformed the curriculum from the written page to her intended plans for enactment, I analyzed the Core-Plus student text as well as the teacher’s guide. I compared the contents of the written curriculum with the slides Ms. Spence created as she prepared for class. These slides contained the tasks she selected for use from the textbook in the order in which she planned to use them in class along with adaptations and supplementations she made to the tasks. I also collected any handouts she created to supplement the textbook.

3.3 Analysis

3.3.1 Analytical framework
My aim in analyzing the curriculum documents was to compare Ms. Spence’s intended curriculum with the written Core-Plus curriculum. I analyzed the written Core-
Plus student text and the slideshows and supplemental tasks created by Ms. Spence using an analytical framework grounded in theory on the use of contexts in mathematics instruction. The development of the framework is described elsewhere (Reinke, Chapter 2 of this dissertation). The framework describes various categories into which instructional activity emphasizing CPs can be classified. For this study, I focus on five of these categories. Contextualized examples are tasks or statements that refer to some sort of social or cultural context. Non-contextualized examples are tasks, statements or questions that refer to a particular mathematical example but do not reference any social or cultural contexts. Another type of activity involves reflection across contextualized examples set in different contexts. Activity involving reflection across contextualized and non-contextualized examples refers to both types of example. And finally, tasks or statements that involve reflection on general mathematical principles explicitly address generalizable procedures, concepts, representations or relationships and do not necessarily refer to any specific examples. I use these five categories to characterize the sequencing of the tasks in the teachers’ intended curriculum and compare this sequence to the written curriculum.

To compare the written and intended curricula, I needed to segment both phases of the curriculum into units of analysis that could be systematically categorized. I defined the unit of analysis for coding the curriculum documents as a topically contained turn. This construct combines the idea of a turn and a topically related set (Mehan, 1979), both of which are used as units of analysis in discourse analysis. In the discourse analysis literature, a turn begins when one person starts speaking and ends when another person responds. A topically related set is a collection of turns related to the same topic.
In this study, I conceive of the use of curriculum documents (those from Core-Plus and those produced by the teacher) as an interaction between the text and the reader, so these constructs are applicable. A turn for the curriculum document begins when the reader begins to read. At the point at which the reader is meant to pause from reading while they work on a particular task or answer a question, the curriculum document’s “turn” ends, and the reader takes her “turn” in the interaction. Accordingly, I segmented the text into turns, or collections of text placed between those moments when the reader is meant to pause reading and work. An additional segmenting step was necessary, though, because this method resulted in turns that contained text focused on multiple topics. This happened because at the transition from one section to the next, there was no intended “pause” for the audience to work a task. To achieve a unit of analysis that focused only on a single topic, I chunked each turn into what I call a **topically contained turn (TCT)**. Each TCT ends when the student is signaled to perform some sort of task or when the text transitions to a new topic or section as indicated by a change in formatting.

### 3.3.2 Analysis of the written curriculum

To look for patterns in the way Ms. Spence transformed the various investigations in the unit into plans for enactment, I created lessons maps that show how each TCT was coded. I have included sample lesson maps in Figures 10 and 12. The lesson map in Figure 10 shows the results of coding Unit 3, Lesson 1, Investigation 1 of the written Core-Plus curriculum; the map in Figure 12 shows Ms. Spence’s plans for the beginning of the same investigation. In Figure 10, the columns represent the sequence of TCT’s
contained in the text. Each row represents a different category of activity as described by
the analytical framework. Activity referencing individual examples is represented by the
bottom two rows: *contextualized examples* and *non-contextualized examples*. Activity
referencing multiple examples is represented in the next two rows: *reflection across
contextualized examples* and *reflection across contextualized and non-contextualized
examples*. Activity involving *reflection on general mathematical principles* is represented
by the top row. Closed circles indicate instructional statements; these are cases in which
textbook authors provide expository text or explanations. Open circles indicate tasks or
questions to which the authors expect the students to respond. TCTs in which the authors
provide narration about the unit, lesson or investigation without providing instruction or
explanation, are marked with “n”. TCTs were sometimes coded as containing more than
one type of activity, as will be demonstrated in the example below.
Figure 10. Map of Unit 3, Lesson 1, Investigation 1 in the Core-Plus written curriculum
Overall, the table in Figure 10 shows that, apart from narration and isolated statements of general mathematical principles, the investigation begins with contextualized examples, proceeds to non-contextualized examples, and ends with a series of tasks through which students are meant to generalize mathematical principles.

To illustrate my analysis and the resulting lesson maps in a more detailed manner, Figure 11 shows the first five TCTs from the investigation and the codes they were assigned.

<table>
<thead>
<tr>
<th>TCT label</th>
<th>Quotation</th>
<th>Code</th>
</tr>
</thead>
<tbody>
<tr>
<td>U3L1 intro</td>
<td>“In the [previous] unit, you studied a variety of relationships between quantitative variables. Among the most common were linear functions-- those with straight-line graphs, data patterns showing constant rate of change in the dependent variable, and rules like $y=a + bx$. For example, Barry represents a credit card company on college campuses... The graph on the next page shows the relationship between Barry’s daily pay and the number of credit card applications he collects. The graph suggests that <em>daily pay</em> is a linear function of <em>number of applications</em>”</td>
<td>statement: reflection on general mathematical principles statement: contextualized example</td>
</tr>
<tr>
<td>TATS intro</td>
<td>“Think about the connections among graphs, data patterns, function rules, and problem conditions for linear relationships.”</td>
<td></td>
</tr>
<tr>
<td>TATSa</td>
<td>“How does Barry’s daily pay change as the number of applications he collects increases? How is that pattern of change shown in the graph?”</td>
<td></td>
</tr>
<tr>
<td>TATSb</td>
<td>“If the linear pattern shown by the graph holds for other (number of applications, daily pay) pairs, how much would you expect Barry to earn for a day during which he collects just 1 application? For a day he collects 13 applications? For a day he collects 25 applications?”</td>
<td></td>
</tr>
<tr>
<td>TATSc</td>
<td>“What information from the graph might you use to write a rule showing how to calculate daily pay for any number of applications?”</td>
<td></td>
</tr>
</tbody>
</table>

*Figure 11. An excerpt from *Core-Plus*, Course 1, Unit 3, Lesson 1, Investigation 1 and the corresponding codes that were assigned to each TCT (p. 151)*
Lesson 1 begins with a paragraph containing sentences referring to both general mathematical principles and contextualized examples ("U3L1 intro"), indicated with closed circles at the reflection on general mathematical principles level and the contextualized example level. In this paragraph, the authors explain first explain what linear functions are through a sentence referencing only general mathematical principles. In the next sentence in the same TCT, the text sets up the context for the first contextualized example, a situation involving Barry, a credit card salesperson. The authors explain that “Barry’s daily pay is a linear function of number of applications” (p. 150). This is the statement represented by the closed circle at the contextualized example level.

The next set of TCTs, from the section called “Think about the Situation”, come from the textbook excerpt in Figure 11. The introductory sentence in the excerpt, labeled “TATS intro”, is marked with an “n” at the general level in the lesson map. The three tasks related to the graph of Barry’s daily pay that follow (“TATSa”, “TATSb”, and “TATSc”) are each marked with an open circle (representing a task) at the contextualized example level.

The instructional sequence proceeds through a number of contextualized example tasks (Questions 1-5), with occasional explanatory paragraphs containing statements of general mathematical principles and contextualized examples.

At Question 6, the nature of the tasks changes significantly. This question references non-contextualized examples, which are marked as open circles at the non-
contextualized example level. These tasks ask students to graph and identify the slope of four linear functions, including $y = 1 + \frac{2}{3}x$.

The investigation proceeds to the *Summarize the Mathematics (STM)* section, which contains questions about general mathematical principles, marked by open circles at the general level. Question ai, for instance, asks “How can you determine whether a function is linear by inspecting a table of (x,y) values? (p. 156). Finally, the Check Your Understanding (CYU) section features contextualized examples, involving data from an experiment with weights being hung from a spring.

### 3.3.3 Analysis of Ms. Spence’s intended curriculum

Tasks, questions, and statements in Ms. Spence’s lesson plans were analyzed in the same manner in which the written curriculum was analyzed. Figure 12 shows Ms. Spence’s intended curriculum for the first four days of the unit, as determined from her PowerPoint presentations and supplemental written tasks. Supplemental activity created by Ms. Spence and not contained in the textbook is shaded; activity from the textbook is not shaded. Figure 13 shows the first five TCTs of Ms. Spence’s intended curriculum.

On 10/20, the first day of the unit, Ms. Spence planned to introduce the Unit Essential Question (“UEQ”), which was focused on a general mathematical principle: “How can you represent, interpret, and manipulate a given situation that has a constant rate of change?” Because this question is not from the text, the column is shaded. Because Ms. Spence used the question to narrate what the unit was about, rather than having students answer the question, the TCT is marked with an “n” at the general level. For the next TCT, (“LEQ”) she planned to ask students to respond to the general Lesson
Essential Question (LEQ): “what makes a function linear?” This is marked with an open circle. Following this, she planned to introduce, but not have students respond to, the assessment prompts: “Use a table to determine if a function is linear”; and, “match a table to a graph for a linear situation.” These are marked by "n"s at the general level. Next, Ms. Spence planned a supplemental task referring to a contextual example, asking students to observe what they notice and what they wonder about the graph in which Barry’s daily pay and the number of credit card applications he collects are displayed. Because this task was not from the book, these columns are shaded and the tasks are marked with open circles at the contextualized example level. Following this activity, students were to work through the Think about the Situation (TATS) questions in the textbook (“TATS intro”-“TATS c”). Because these tasks come directly from the written tasks, they are the first TCTs that are shaded in the figure.

After coding each TCT from the written and intended curricula and creating tables to visualize the data more easily, I looked across the tables for patterns related to what types of activity Ms. Spence used or omitted from the written curriculum. I also identified patterns in the way activities were sequenced in the written curriculum and Ms. Spence’s plans, to determine whether patterns present in the written curriculum (proceeding from contextualized examples to non-contextualized examples, for instance) were maintained or changed in Ms. Spence’s intended curriculum. Finally, I identified patterns in the type of supplemental tasks Ms. Spence planned that were not from the Core-Plus curriculum and identified the points in the instructional sequences at which she chose to supplement.
**Figure 12.** Map of Ms. Spence’s Intended Curriculum Unit 3, Lesson 1

<table>
<thead>
<tr>
<th>TCT label</th>
<th>TCT</th>
<th>code</th>
</tr>
</thead>
<tbody>
<tr>
<td>UEQ</td>
<td>“How can you represent, interpret, and manipulate a given situation that has a constant rate of change?”</td>
<td>narration- reflection on general mathematical principles</td>
</tr>
<tr>
<td>LEQ</td>
<td>“What makes a function linear?”</td>
<td>task- reflection on general mathematical principles</td>
</tr>
<tr>
<td>Assessment prompts</td>
<td>“Use a table to determine if a function is linear”; “match a table to a graph for a linear situation.”</td>
<td>narration- reflection on general mathematical principles</td>
</tr>
<tr>
<td>notice</td>
<td>What do you notice [about the graph]?</td>
<td>task- contextualized example</td>
</tr>
<tr>
<td>wonder</td>
<td>What do you wonder [about the graph]?</td>
<td>task- contextualized example</td>
</tr>
</tbody>
</table>

**Figure 13.** Ms. Spence’s intended curriculum for Course 1, Unit 3, Lesson 1, Investigation 1 and the corresponding codes that were assigned to each TCT
3.3.4 Interview analysis

To analyze transcriptions of the interviews with Ms. Spence, I first identified the factors known to influence the transformation from written to enacted curricula identified in the research literature; I used these ideas to create an initial set of codes. Examples of these a priori codes included “beliefs about contextualized problems”, “orientation toward the curriculum (Core-Plus)”, and “orientation toward curriculum in general.” Within these categories, I used an open coding process to identify themes that emerged as Ms. Spence described her beliefs about the curriculum and how students learn mathematics. I also identified significant contextual factors that influenced her instructional decisions, such as a lesson-planning template mandated by the school administration and her participation in a professional learning community.

After identifying patterns in the way Ms. Spence transformed the written curriculum into her intended plans for enactment, I searched the interview data for the rationale Ms. Spence provided for these modifications. In the following section, general themes from the interviews as well as specific rationale Ms. Spence provided are used to explain her thought process around the curriculum transformation.

4. Findings

A central focus of my analysis was the role of contextualized problems in the written and intended curriculum. After analyzing the lesson maps for patterns in the way Ms. Spence used, omitted, adapted, re-sequenced, and supplemented the written curriculum, three significant patterns emerged. First, the progression from contextualized
examples to non-contextualized examples in the written curriculum was not preserved in Ms. Spence’s intended curriculum in two of the three lessons. In other words, contextualized examples were not positioned as initial, foundational instructional activities in the intended curriculum in the same way they were in the written curriculum. Second, as Ms. Spence transformed the curriculum into plans for enactment, she modified the way tasks focused on reflection on general mathematical principles were positioned relative to contextualized examples. She incorporated this general activity throughout students’ contextualized problem solving experiences to a greater extent than the written curriculum. Lastly, Ms. Spence supplemented the written curriculum by including activity that referenced multiple contextualized examples; this type of activity was not present in the written curriculum.

In the following section, I describe these three patterns describing how the curriculum was transformed in detail. I also explain how these transformations mediated students’ opportunities to interact with the written curriculum. To provide insight into Ms. Spence’s thought process as she planned her lessons, I draw on the interview data to explain the factors that influenced these curricular transformations.

4.1 Modifications to the Progression from Contextualized Examples to Non-Contextualized Examples

For me as a kid, personally, I need you to teach me the concept first, and then you could throw it in the context. Because then, I would get it. And I have kids who are like that. And then I have the other kids, "well give it to me in a context because it needs to make sense to me first and then I can maybe do the
In this excerpt from the introductory interview, Ms. Spence describes her belief that some students learn best when mathematics concepts are first presented without a context and others learn best when these mathematical concepts are presented first through contextualized examples. In the Core-Plus curriculum, developers often present mathematical ideas first in contexts that are “concrete and familiar to students” to make the ideas more accessible (Schoen et al., 2010, p. 9). Then, the sequences proceed to analogous non-contextualized, symbolic examples. In the map of the written Investigation 1 of Unit 3 shown in Figure 10, this progression is shown by the open circles at the contextualized example level, followed by open circles at the non-contextualized example level. According to the developers, the later non-contextualized symbolic representations draw “meaning from the earlier, more concrete settings” (Schoen et al., 2010, p. 9). Ms. Spence’s ambivalence about this context-first approach was one factor that influenced her decision-making around the sequencing of the instructional tasks. She re-sequenced the curriculum in ways that did not consistently preserve the context-first sequences in the unit.

In the first lesson entitled “Modeling Linear Relationships, Investigations 1 and 2 both contain a CP-first sequence. This sequence was preserved in Investigation 1 as Ms. Spence transformed the suggestions on the written page into a plan for enactment. In Investigation 2, the sequence was not preserved; she chose to postpone assigning questions 5 and 6, the non-contextualized examples, until later in the unit because she felt they fit better mathematically during a different lesson.
During Lesson 3: Equivalent Expressions, which Ms. Spence taught immediately after Lesson 1, she taught only from the second investigation in the lesson, which featured only non-contextualized examples. In Investigation 2, students are expected to write equivalent expressions using algebraic properties including the distributive property, the commutative property, and the associative property. Ms. Spence did not teach from the first investigation, in which “context clues guide the development of equivalent expressions” (p. T214). Notably, she did supplement the second investigation with a contextual anchor: a video relating the distributive property to the prices charged by a skateboard company. When I asked about this supplementation, she explained that she was surprised that Investigation 2 did not begin in context, and she did not identify that Investigation 1 provided a contextual anchor:

They made sense of that [video]... And when we went from that over to the Core[-Plus], they were... it just fell apart. It just fell apart and they couldn't make the connection from there to here. Which would be almost surprising to me that they didn't do a better job than this. And it seems like what I found on that clip should have been their [the developers’] approach. So, like all these other times you have the kids in context in terms of how we apply it and then you get to this particular stretch and it just... it fell apart to me. (11/29/11)

According to Ms. Spence, students struggled during the non-contextualized examples and could not successfully leverage ideas from the video.

The CP-first progression was also not preserved in Ms. Spence’s plans for Lesson 2, which she taught last. Figure 14 shows a comparison of the written curriculum and Ms. Spence’s plans for this lesson.
### Written Core-Plus curriculum

<table>
<thead>
<tr>
<th>Mathematical Topic</th>
<th>Investigation 1</th>
<th>Investigation 2</th>
<th>Investigation 3</th>
<th>Investigation 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>equations and inequalities</td>
<td>equations</td>
<td>inequalities</td>
<td>Systems of equations</td>
<td></td>
</tr>
<tr>
<td>Representation</td>
<td>tables and graphs</td>
<td>symbolic</td>
<td>symbolic</td>
<td>symbolic</td>
</tr>
<tr>
<td>contextualized (CP) or non-CP</td>
<td>CP</td>
<td>non-CP, then CP</td>
<td>non-CP</td>
<td>CP</td>
</tr>
</tbody>
</table>

### Ms. Spence’s intended curriculum

<table>
<thead>
<tr>
<th>Mathematical Topic</th>
<th>12/12-12/15</th>
<th>12/15-12/19</th>
<th>12/20</th>
<th>12/21</th>
</tr>
</thead>
<tbody>
<tr>
<td>Investigation 2</td>
<td>Investigation 4</td>
<td>Investigation 1</td>
<td>Investigation 3</td>
<td></td>
</tr>
<tr>
<td>equations</td>
<td>systems of equations</td>
<td>equations and inequalities</td>
<td>inequalities</td>
<td></td>
</tr>
<tr>
<td>Representation</td>
<td>symbolic</td>
<td>symbolic</td>
<td>tables and graphs</td>
<td>symbolic</td>
</tr>
<tr>
<td>contextualized (CP) or non-CP</td>
<td>non-CP then CP</td>
<td>CP</td>
<td>CP</td>
<td>non-CP</td>
</tr>
</tbody>
</table>

**Figure 14.** Comparison of the Written and Teacher’s Intended Curriculum of Core-Plus Course 1, Unit 3, Lesson 2

In the written curriculum, the lesson progresses from contextualized examples of equations and inequalities presented as graphs and tables (Investigation 1) to non-contextualized and contextualized examples of equations presented symbolically (Investigation 2), to non-contextualized inequalities presented symbolically (Investigation 3), and ends with contextualized systems of equations presented symbolically (Investigation 4). The progression in Ms. Spence’s intended curriculum was sequenced differently. She did not begin with Investigation 1, in which students are prompted to write equations to describe questions about a contextualized example. Instead, she re-sequenced the lesson to begin with Investigation 2, which begins with non-contextualized equations. Her intended plans then proceed to contextualized systems of equations (Investigation 4). Next, she assigned Investigation 1, featuring contextualized equations.
and inequalities. Her plans for the lesson conclude with linear inequalities presented symbolically (Investigation 3). Ms. Spence’s re-sequencing of the investigations within this lesson resulted in a lesson sequence that no longer progressed from contextualized to non-contextualized examples or from tabular or graphical representations to symbolic representations, as intended by the curriculum developers.

There were a number of factors that contributed to this pattern of curriculum re-sequencing. First, Ms. Spence’s orientation toward the curriculum materials and the autonomy afforded to her by the administration combined in such a way that she felt empowered to modify the way tasks were sequenced in the curriculum. Ms. Spence saw Core-Plus as her primary resource for instructional material, but ultimately she viewed herself as the instructional authority in the classroom. She explained this to be her overall orientation towards curriculum materials, saying, “We, as teachers, know what works for our kids. So I don’t think there’s a perfect textbook anyway. I think that I would still arrange stuff” (Introductory Interview, 9/30/11) Accordingly, Ms. Spence sequenced the tasks in the way that she felt best suited her students.

Ms. Spence’s ambivalence about following a CP-first approach also contributed to a lack of consistency in regard to the placement of contextualized problems within the instructional sequence. Her ambivalence is demonstrated in the opening quotation, where she explained that different students learn best in different ways. She notes her personal preference: to be first taught the mathematical ideas free from contextualized situations, then to apply them after. On the other hand, she frequently cited instances where contextualized problems provided accessible entry points to the mathematics. In her words, students often “get it” [understand the mathematics] when they work “in context”.

89
A major struggle, in her mind, is for students to apply what they did in contextualized problems to non-contextualized examples. In the following example, she explains students’ difficulty applying their work on a contextualized example to subsequent non-contextualized examples of linear equations.

They’re not making the connection between looking at the rule now [as they work on non-contextualized examples] and identifying the slope and the y-intercept as clearly as I thought they did last week when it was in context. They got that when they had... a situation in context, they get if Emily has a four hundred eighty dollar TV that the rate of change is that she's spending twenty dollars. And when you give it to them in letters that make sense to them, like \( m \) stands for the number of months so obviously every month she's paying down something. But I think, what I found out today in both of the classes is there are some kids who get that this rate of change does have a variable with it. There are others who claim to get it but then they don't get it. Like as soon as they got on their own, and they really.. it's not there yet for them. (10/31/11)

When I asked specifically about whether she felt the curriculum supported students in making these connections between contextualized and non-contextualized examples, she attributed students’ struggles with non-contextualized examples to a lack of connection in the written curriculum.

LR: What do you think about the way the book does this?

Ms. Spence: Somehow to me they could have integrated it a little better when they did the part with Emily and tied it into that... I think they scattered it a little bit. I'm thinking that from where I sit, it's scattered, because when I had them doing that one part table from last week...why couldn't you actually have another column that represented that as a change in your “whatever” and then relate that to being your change in y. Because maybe with that connection right there for the kids, then if.. I feel like I'm going back and I'm having to grab and pull those pieces together because I know the Core-Plus assumes that these kids are going to be able to make those connections themselves.

(10/31/11)
Here, she recommended that developers tie the non-contextualized language of “change in y” back to the work that students had done in context. To summarize, Ms. Spence saw that students had trouble connecting their work out of context to their initial work in context. They did not appear to “think with and remember” the contextual problems as the developers hoped (Fey, personal communication 7/25/11). Because she did not observe students using the contexts to make sense of non-contextualized tasks, she made changes that did not preserve this aspect of the curriculum design when other factors seemed more important.

Another major factor contributing to Ms. Spence’s re-sequencing of the curriculum was institutional in nature rather than individual; some of the decisions regarding the omission or re-sequencing of units, lessons, and investigations were made by professional learning communities (PLCs). At the school in which Ms. Spence worked, the term PLC was used to refer to a required participation in regular meetings focused on particular courses. There was one PLC for each course (Integrated Mathematics 1, 2, 3 and 4) and each teacher was required to choose one PLC in which to participate. In these PLCs, teachers determined which lessons in the curriculum addressed the content that was required by the state standards. The PLC decided which portions of the curriculum they would teach and how they would sequence the curriculum. According to Ms. Spence, the decision to change the sequence of Unit 3 Lesson 2 was made at the PLC level; teachers in the PLC felt the Core-Plus approach did not make sense in terms of the order in which the mathematical concepts were presented. Ms. Spence strongly agreed with this observation; she criticized the Core-Plus approach of introducing inequalities along with equations on four occasions during interviews.
the following quotation, she uses the term “we” to mark her participation in the PLC in regard to this decision:

This is why we ended up changing it up... they come out of equations and... they hop to inequalities. But the way they present the inequalities is like using the lines and if the kids don't really have a firm grasp of this it just totally like.. it just blows them away. (10/27/11)

Here, and in each of the other four instances where she discussed this change, she noted difficulties her previous students have had understanding the graphical presentation of inequalities before spending a substantial amount of instructional time solving equations.

To summarize, Ms. Spence’s ambivalence about the CP-first approach, the agency granted to her at the institutional level, and the PLC’s reorganization of one of the lessons to reflect what was determined to be a more logical sequence of mathematical content led to an intended curriculum that did not preserve the designed curriculum sequence from contextualized to non-contextualized examples.

4.2 Modifications to the Placement of Reflection on General Mathematical Principles

Relative to Contextualized Examples

...by the time you go through all this stuff and you have these kids who are already at risk in terms of learning... By the time they get here, they forgot the point of the whole investigation. And it's like you keep trying to tie it all together and say well the reason why you did this, this, this to get here is because you wanted to... you know. And so sometimes I think Core-Plus can take too long to get to the point. And I think for some kids who are beginning to understand, that if you take that long, you frustrate them. And they want to throw their hands back up in despair. (Introductory Interview, 9/30/11)

In the Core-Plus investigations in this study, students were only asked to answer questions aimed at general mathematical principles toward the end of each investigation,
in a section called *Summarize the Mathematics (STM)*. According to the developers, the *STM* tasks are to be worked as a group then discussed as a class so that students can “construct a shared understanding of important concepts, methods, and approaches” (Hirsch, Fey, Hart, Schoen, & Watkins, 2011, p. 12). Ms. Spence used the *STM* sections at the end of investigation only once during the entire unit, despite her concern that students struggled to identify exactly what general mathematical principles they were meant to understand from working the *Core-Plus* sequences. In one investigation, she used the *STM* questions at the beginning of an investigation, because she felt that the questions fit better there as a review of the previous investigation. In the remainder of the investigations, she omitted the *STM* questions altogether, sometimes replacing them with supplemental summative activities.

When I asked why she tended to not use the *STM* questions as a summarizing activity at the end of each investigation, Ms. Spence indicated that she understood how the authors intended for the *STM* questions to be used but sometimes chose to position them elsewhere simply to attempt a better fit:

> Summarize the Math is for them to go back and reflect back across the investigation and go for those big concepts. And then the Check Your Understanding ends up being a practice problem for them to try. I don’t always use the Summarize the Math like the way they would have me use it at the end of the investigation. I think it depends on where it fits. And it may fit somewhere else. (Introductory Interview, 9/30/11)

In addition to critiquing where the tasks were placed within the lesson sequences, she also noted two other difficulties students had with the *STM* questions. In the following quotation, she explains that her students had trouble with the literacy demands presented
by the STM and she describes difficulties they had connecting these questions to activities that may have been done a few days before:

I think that they look at it and they just shut down. It’s too many words on the page. I hate to say that for kids that are in ninth and tenth grade but I think in how it’s all put together, they just still have trouble reading here, here, here, here [pointing to various tasks] and just connecting them. (11/02/11)

Instead of using the Summarize the Mathematics sections as intended, Ms. Spence frequently supplemented the written curriculum with tasks at the general level during the portions of the instructional sequences comprised of contextualized examples. The lesson map in Figure 12 illustrates the addition of activity at the reflection on general mathematical principles level. In the columns representing tasks posed to students on 10/27, Ms. Spence supplements their contextualized problem solving experiences with activity at the reflection on general mathematical principles level. This supplementation shows up in Figure 12 as both open and closed circles at the reflection on general mathematical principles level in shaded regions (teacher supplements) surrounded by non-shaded regions (tasks from the written text) containing tasks at the contextualized example level.

Some of the supplements at the level of reflection on general mathematical principles were the result of a significant institutional factor: the school’s use of a school-wide lesson plan template. This template contained a number of elements, including essential questions, “activation strategies” to motivate lessons and activate prior knowledge, and an expansive library of graphic organizers. The template was meant to provide students with a consistent experience across classrooms and to make explicit for students the content that they were meant to learn. Ms. Spence supplemented the written
curriculum with additional tasks meant to address the required elements of the template; often, these tasks addressed general mathematical principles. As a result, the supplementation of these elements increased the extent to which general mathematical principles were discussed throughout the lesson.

Additional activity at the reflection on general mathematical principles level also occurred due to the presence of “essential questions” as recommended by the instructional template. The essential questions that Ms. Spence used to fulfill the requirement in the instructional template were determined by the PLC. These questions were written to align with the *Core-Plus* curriculum but were not taken directly from the textbook. Ms. Spence introduced essential questions addressing general mathematical principles at the beginning of each lesson. Twice, out of six times she introduced new essential questions, she planned to go beyond simply stating the essential question for students. In these instances, she planned to have students discuss the answers to these questions. This action resulted in students engaging with activity at the general mathematics principles level earlier than would have been the case if she did not supplement the textbook in this way.

Additionally, Ms. Spence used graphic organizers provided by the school-wide instructional program to help students consolidate general mathematical principles. In the lesson-planning template, the graphic organizers were positioned at the beginning of instruction. Ms. Spence planned to use the organizers at various points in instructional sequences. In one investigation, she used the graphic organizer at the beginning of the instructional sequence, which resulted in the introduction of general mathematical principles before students had worked with the contextual examples. In another
investigation, she planned to introduce the organizer at the beginning of an investigation and to have students fill out only information they already were assumed to know; then, she planned have students complete the graphic organizer as a summative activity. In two other investigations, she planned to introduce the graphic organizers midway through the investigation, to consolidate general mathematical principles that students had engaged with during the earlier part of the task sequence. Taken together, the use of graphic organizers, as recommended by the lesson plan template, resulted in activity at the general mathematics level earlier and more frequently than would have been the case if she had not supplemented the written curriculum.

Ms. Spence also added activities targeting general mathematical principles that were not the result of following the lesson plan template. She tended to supplement the curriculum with tasks or questions targeting general mathematical principles after the book introduced a general mathematical principle. For instance, after students read a paragraph describing the concept of rate of change, Ms. Spence added a task asking students to write a brief description of what rate of change means and what it can be used to do. This task, and others like it, prompted students to actively engage with generalizing mathematical concepts earlier and more often than the text required.

A major factor influencing Ms. Spence’s insertion of supplemental general activity throughout the instructional sequence was her belief that the “discovery” aspect of the Core-Plus design was not working for some of her students, as demonstrated in the following quotation:

I see it more as Core-Plus wants the kids to go through the exercise and discover what's happening as they go through it on their own. I think that having taught this so long that it doesn't.. personally.. I don't see it working the way wanted..
authors intended for it to work... because I think that for the most part [to align with the authors’ intent] it should be more student-centered and I should be more a facilitator than a leader. And I think I end up leading more than I facilitate. But I think I also do that because I think that the kids sometimes have trouble interpreting what is meant by what they're asking and also I think sometimes there's so much that they want the kids to figure out on their own I think the point of the investigations sometimes gets lost if I don't find a way to lead them where they need to go. (Introductory Interview, 9/30/11)

Ms. Spence felt that the mathematical ideas were not made explicit enough and took too long to develop in the text. Because of this, she felt her students were distracted from the mathematics they were meant to learn. In other interviews, Ms. Spence distinguished between how the curriculum worked for different students in her classes, particularly in regard to the “discovery learning”. She felt that students who were well-prepared were able to make the necessary connections. But for a majority of the students who had been less academically successful in the past, she felt that she needed to take an active role in mediating students’ experiences with the written curriculum. To mitigate what she felt were mismatches between the curriculum as written and her particular students, Ms. Spence intentionally took on a more directive role that she understood to be in conflict with the authors’ intentions.

In summary, Ms. Spence saw the value in a discovery approach, but she also believed that her students were not prepared to make these discoveries without additional support. As a result, she supplemented the textbook with activity focused on general mathematical principles throughout the task sequences, rather than at the end, in order to compensate for a curriculum that did not adequately match the level of preparedness possessed by her students.
4.3 Supplementing with Connecting Activities

LR: Is there anything else that they had trouble with that surprised you.. or not surprise you but you noticed they had trouble with?

Ms. Spence: It's not so much something that surprises me, it's something that I've grown to expect that they don't really make connections well.

LR: You mentioned that last time I was here.

Ms. Spence: They don't make connections well even in terms of the work that they just did.

(10/27/11)

Ms. Spence frequently noted that students had trouble making connections from one activity to the next. To promote these connections, she supplemented with three types of connective instructional activity that were not present in the Core-Plus unit. Although she did not supplement with each type described below on a consistent basis, taken cumulatively, these insertions represent a pattern of inserting activities that offered the potential to bridge between various tasks presented by the curriculum developers.

4.3.1 Reflecting across contextualized examples

On three occasions during the unit, Ms. Spence planned to supplement the written curriculum with tasks that asked students to reflect across contexts. This type of activity is not present in the Core-Plus investigations, a pattern illustrated by the absence of circles in the “reflect across contextualized examples” in the lesson map in Figure 10 and in the other lesson maps I created. Ms. Spence supplemented with this connective type of activity twice in the first investigation of the unit. After the terms dependent and
independent variable were introduced, Ms. Spence planned to ask students to students “look back over the two preceding contextualized examples to identify the independent and dependent variables” (slideshow for 10/27/11). This is indicated by the TCT labeled “look back dependent” and “look back independent” on 10/27 in the map in Figure 12.

The next day, Ms. Spence planned an “exit slip” activity asking students to look across three different contextualized examples: “Write down what you notice between questions 1, 3 and 4 in terms of what we talked about today.” In another investigation, she planned an activity where students were to answer the question: “What is the same about questions 1 and 2?” and “What is different about questions 1 and 2?” (slideshow for 11/8/11). Questions that ask students to identify similarities across contexts have the potential to prompt students to shift their attention from the specific examples to the general mathematical principles that underlie the examples; and, questions targeting differences across contextualized tasks have the potential to prompt students to attend to how mathematical ideas are developing along the instructional sequence.

4.3.2 Reflecting across contextualized and non-contextualized examples

In the Core-Plus curriculum, contextualized examples are meant to provide students access to more abstract, non-contextualized examples. With this in mind, it is notable that, in the lessons analyzed in this study, there were no tasks in the Core-Plus unit that asked students explicitly consider contextualized and non-contextualized examples simultaneously. The map in Figure 10 is an illustrative example of this trend; there are no circles in the “reflect across contextualized and non-contextualized” row.
The absence of this type of question left the students (and teacher) without support for making connections between these two types of activity. And although Ms. Spence frequently noted students’ struggles connecting these two types of activity, she did not frequently supplement the curriculum with tasks that asked students to simultaneously consider both types of example. However, she did supplement with such a task once, at the end of Lesson 1, Investigation 2. After grading students’ responses to a quiz, she planned to begin class with a review in which she posed two tasks:

- For the linear function below, identify the rate of change and the y-intercept.
  \[ y = 5 + 4x \]
- Write a situation that this linear function could represent.

The second task asked students to develop a context around a non-contextualized example. This task stands out amongst all the other tasks in the written and intended curricula as an illustrative example of a type of activity that asks students to reflect across both contextualized and non-contextualized examples.

### 4.3.3 Connecting general principles to contextualized examples

Ms. Spence also supplemented the curriculum with tasks that involved both general mathematical principles and contextualized examples. This sort of task has the potential to help students make connections between these two different types of instructional activity involves activity. One instance of this involved a graphic organizer used to show students four ways that linear functions can be represented: as a verbal description, a symbolic rule, a table, and a graph. Ms. Spence planned to have students copy the table and symbolic representation of the function from the example involving
Barry, the representative from the credit card company. When I asked why she referred back to this example, she explained that students understood the example and were comfortable with it:

LR: I'm wondering why you decided to use Barry's situation in that graphic organizer.

Ms. Spence: Because they understood that one.

LR: okay

Ms. Spence: it made sense to them. I mean pretty much it was flushed out when we did the notice wonder [on the first day of the unit]... They started with the graph, they were able to read and interpret the graph. Based on that graph they were able… we came up with an equation even though they didn't really understand writing equations at that point but they did understand the idea that n represented the number of applications and five dollars per application they got that. And they did understand what did he get twenty dollars or ten dollars or something like that.

LR: yeah it was twenty

Ms. Spence: So they understood that and to me it made it relevant for them because they already had that, the graph. Then we were able to make the table because that was part of question one I think it was, then they had the rule. It was there. And I thought it was something that was easy enough to understand if they went back to look at their graphic organizer than that was something they were already comfortable with.

(1/17/13)

Through this graphic organizer, Ms. Spence explicitly connected the generalized mathematical principles to students’ previous work with a contextualized example.

An instance of supplemental activity that contained a contextualized example, non-contextualized example, and reflection on general principles is a video that Ms. Spence played as an activating strategy. The video began with an explanation of how to
draw best-fit lines to determine whether the correlation between two variables is positive or negative. Instruction began at the reflection on general mathematical principles level, proceeded to non-contextualized examples of positive and negative correlations, and concluded with a contextualized example: drawing a best fit line to represent age and weight data. Following the video, Ms. Spence planned a series of questions asking students to make generalizations about best-fit lines and correlations.

A third example of connective activity was manifested in a supplemental general activity that Ms. Spence created as a replacement for a Summarize the Mathematics section of the book. At the end of Lesson 1 Investigation 3, Ms. Spence planned to ask students to answer the essential questions of the lesson as a consolidating activity. Her written prompt asked students to “use examples to explain.” Like the activity asking students to write a context that matches the non-contextualized example, this activity prompted students to develop an example, leaving the possibility for students to use previously encountered contextualized example or developing a new one. The Summarize the Mathematics sections of Core-Plus examined in the study do not contain any explicit cue to provide or consider examples.

A primary factor influencing the supplementation of these sorts of connective activities was Ms. Spence’s assessment that her students had difficulty making the connections that she felt the developers of Core-Plus intended her to make. On sixteen occasions across nine interviews, Ms. Spence noted connections that students struggled to make. This perception, combined with her belief about her duty to supplement the curriculum to fit the needs of her students, contributed to the pattern of transformation described above.
4.4 Summary of Findings

The purpose of this study was to examine the way Ms. Spence transformed the Core-Plus curriculum into plans for enactment and to identify factors mediating this transformation. In particular, I sought to understand her beliefs about contextualized problems and how these beliefs might have influenced the role of contextualized problems in her intended plans.

Over the course of the unit, Ms. Spence consistently made several kinds of modifications to the Core-Plus curriculum. First, she re-sequenced the instructional sequence at the task, investigation, and lesson levels. Second, within investigations, she supplemented activity aimed at general mathematical principles within the body of the investigations. Lastly, she supplemented with activity referencing multiple contextual examples and activity that addressed general mathematical principles while referencing contextual examples. Taken together, these modifications represent significant adaptations to the written curriculum.

Several factors influenced these changes; a number of these factors were features of the school setting. First, Ms. Spence was granted the autonomy to supplement, omit, and re-sequence tasks, investigations, and lessons as she saw fit; this institutional norm opened the door for significant modification of the curriculum. Additionally, her efforts to conform to school-wide instructional template resulted in supplemental activity that altered the sequencing of task types present in the written curriculum. Lastly, the teachers in the professional learning community charged with planning the schools’
implementation of Course 1 rearranged the instructional tasks to align with their understanding of the ideal progression through which the mathematical concepts should be presented and to align with the content mandated by state requirements.

Finally, Ms. Spence’s own beliefs and orientations influenced her transformation of the curriculum. Her orientation toward curricula in general was such that she saw herself as the expert on her students’ needs, and she made changes to the curriculum in response to her perception of those needs. Her beliefs about contextualized problems and “discovery learning”, and her ambivalence about the progression from contextualized examples, to non-contextualized tasks, to reflection on general mathematical principles found in written curriculum in this unit, opened the door for her to resequence and supplement the tasks provided in the written curriculum. Like a number of teachers observed by Wilson and Lloyd (2000), Ms. Spence worried that the curriculum did not adequately support her students in attending to the mathematical principles underlying the CPs in the text. In response, Ms. Spence planned tasks that prompted students to reflect across contexts, and she supplemented the written curriculum with additional tasks targeting general mathematical principles.

There were also characteristics of the curriculum that may have contributed to the degree to which Ms. Spence re-ordered the instructional tasks. The significance of the progression from contextualized to non-contextualized was not strongly messaged in the curriculum materials. And the way in which this progression was manifested in the unit was not always consistent or explicitly described. In some lessons, these progressions took place within investigations, other times the progression took place over a number of investigations. This variation may have made it difficult to identify what effect re-
sequencing investigations would have on this progression. Finally, a lack of explicit references across instructional tasks resulted in Ms. Spence feeling that her students were not making the connections that designers intended but left implicit.

5. Discussion

This design of this study assumes that teachers adapt the curriculum as they transform the written text into a plan for enactment (Remillard, 2005). My analysis of these adaptations and the factors influencing them provides a number of insights into the implementation of CP-based curriculum and teachers’ adaptations of written curricula in general. First, this case provides evidence that teachers’ beliefs about CPs can lead to transformations that modify the role that CPs play in the curriculum. Also, school-wide instructional initiatives have the potential to mediate how CP-based curricula, or any curricula, are implemented. More generally, it is clear from this analysis that some teacher adaptations might be supportive of curricular designs, while others work against them. Adaptations that do not align with the design of a curriculum program may occur as a teacher pursues other goals that, although in conflict with the design of the curriculum, may be equally worthwhile.

Ms. Spence’s re-sequencing of the curriculum, and her beliefs about CPs, provide evidence supporting the hypothesis that particular beliefs about CPs have the potential to contribute to patterns of implementation that conflict with the design of CP-based curricula. Many of the Core-Plus instructional sequences in the unit studied here were ordered such that students introduced to mathematical concepts through CPs. The
developers intended for students to leverage their experience with verbally presented, contextualized tasks during their encounters with more symbolic, non-contextualized tasks (Fey, personal communication, 7/25/11; Schoen et al., 2010). Ms. Spence, though, was ambivalent about whether concepts should be introduced contextually; consequently, she did not see the CP-first sequence as essential. As Ms. Spence and the PLC planned the schedule, the CP-first sequencing of the tasks was not always preserved; this adaptation limited students’ opportunity to use their contextualized problem solving experiences to help them interpret formal mathematical conventions, as intended by the developers. This finding provides evidence that teachers’ beliefs about CPs can open the door for adaptations that transform the role of CPs as intended by the curriculum design. This finding is particularly significant in light of Chapman’s (2006) finding that, unlike Ms. Spence, some teachers strongly believe that instruction should not begin with contextualized problems. These teachers may be even more likely to modify intentionally planned instructional sequences in CP-based curriculum.

This case also sheds light on the complexity of the institutional contexts that act as factors influencing teachers’ decisions in developing their intended curriculum. In particular, this case demonstrates that school-wide instructional initiatives have the potential to significantly influence teachers’ decisions as they transform curriculum. Stein et al. (2007) note that institutional factors potentially influence teachers’ decision-making around curriculum materials. Factors that have been identified include professional development opportunities (Stein & Kim, 2008), the degree of autonomy offered to teachers (McClain et al., 2008), and support from the administration (Fullan, 1991; Manouchehri & Goodman, 1998). In this study, a school wide instructional
program played a significant role in how Ms. Spence transformed the curriculum. Ms. Spence needed to transform the Core-Plus mathematics curriculum in a way that aligned with the instructional template required by the school administrators. The mere presence of such an initiative has the potential to increase the degree to which teachers actually modify the curriculum (by re-sequencing, supplementing or omitting) because teachers may feel that they need to pick and choose only those tasks that fit into the fields present in the template; they also might re-sequence those tasks to fit the order dictated by those fields. In the absence of an instructional template or similar influence, teachers may be more likely to use the tasks as written in the order in which they are presented in the text. Often, well thought-out and researched initiatives that have the potential to substantially improve instruction are introduced in schools. But these initiatives do not exist independently, and there is the potential for these initiatives to undermine curricular approaches.

After considering the re-sequencing of tasks and other ways Ms. Spence adapted the curriculum, it is clear that some teacher adaptations support the curriculum in accomplishing the authors’ intended opportunities for learning and others conflict with the design of the curriculum. Davis et al. (2011) identified patterns of adaptation that two science teachers exhibited when implementing curriculum and found that one teacher made modifications to the curriculum materials that aligned with the learning goals stated in the curriculum materials. Another teacher made changes that did not align with the goals of the curriculum materials; these changes were in pursuit of other learning goals. Similar analyses can be performed on the adaptations that mathematics teachers make, as demonstrated in this study. In introducing “reflecting across” questions, Ms. Spence
supplemented the curriculum in a way that highly aligned with the instructional principles described by the *Core-Plus* developers who write that “students should be regularly involved in mathematical activities like searching for and explaining patterns, making and verifying conjectures, generalizing, applying, proving and reflecting on the process” (Schoen et al, 2010). Ms. Spence’s “reflecting across” questions provided opportunities for students to generalize across tasks and to “reflect on the process” that were not present in the written curriculum. The re-sequencing of tasks and investigations in curriculum, on the other hand, represents a violation of a principle upon which the unit was designed. Two aspects of the instructional progression present in the written curriculum were compromised: the progression from contextualized problem solving to non-contextualized problem solving and the progression from tabular and graphical representations of functions to more abstract symbolic representations. Although it is not clear how the transformations Ms. Spence engaged in altered the effectiveness of the curriculum in this study, transformations that compromise the curriculum design principles of a curriculum program do represent risks because they potentially limit students’ opportunities to learn in the way intended by the developers.

After analyzing the interview data, it becomes clear that when teachers transform the curriculum in ways that compromise a particular design principle, they may do so in pursuit of another, equally worthwhile goal. Ms. Spence’s experience implementing the curriculum, and the expertise she developed as a result, allowed her to reasonably critique the way the design principles are operationalized in the curriculum. She described difficulties students had in making connections between contextualized and non-contextualized tasks; she also pointed to challenges her students faced when trying to
understand linear inequalities before significant work with equations. Ms. Spence understood that the developers introduced concepts using contextualized problems in this unit; but, at times, she chose to sequence tasks otherwise because she deemed other considerations, including mathematical complexity, to be more important. As Lampert (1985) points out, teachers often have to navigate competing aims. Each of the design principles a curriculum is developed from represents a different aim to be considered among countless others during the enactment of a lesson.

The finding that Ms. Spence critiqued the way the curriculum philosophy was operationalized introduces an important consideration in teachers’ use of curriculum: teachers’ expertise gained from prior experiences implementing a given curriculum program. A number of studies describe instances in which teachers’ philosophies align or do not align with the philosophy upon which the curriculum is designed (e.g. Collopy, 2003; Lloyd, 1999; Remillard, 1999; Remillard & Bryans, 2004). These studies typically examine teachers who are transitioning to using a new style of curriculum. By examining a teacher with prolonged experience teaching from the curriculum, I introduce another possible consideration. Teachers may have a certain orientation toward the philosophy on which a curriculum is developed, yet they may hold a very different opinion of the way the philosophy is operationalized at particular points in the curriculum. Ms. Spence critiqued the lack of support for making connections between various tasks present in the curriculum, and this critique contributed to the degree to which she modified the curriculum by supplementing and re-sequencing tasks.
6. Conclusions

In many curriculum programs, tasks and lessons are designed and sequenced around key design principles (see Senk & Thompson, 2003). The intentional design of a curriculum may be compromised during implementation unless four conditions are fulfilled. First, teachers must recognize and understand the design principles. Second, teachers must value these principles as essential. Even if teachers understand a principle and appreciate it, they may implement the curriculum in ways that compromise that principle in pursuit of other aims, unless they see the principle as essential for curriculum effectiveness. Third, teachers must see clearly how design principles are manifested within the curriculum. Without this sort of transparency, teachers will be more likely to compromise the principle through re-sequencing or other curriculum transformations. Finally, teachers must believe that the design principles are effectively operationalized through the design of the curriculum. Even if a teacher values a given principle and understands how the principle is manifested in the curriculum, she may decide to pursue other aims if she or her students struggle to enact that principle due to curricular constraints or lack of support.

These four criteria are particularly significant in light of the development of the Common Core State Standards and curriculum programs designed to align with “research-based learning progressions detailing what is known today about how students’ mathematical knowledge, skill, and understanding develop over time” (Common Core State Standards Initiative, 2010, p. 4). High-stakes tests and other considerations often force teachers and other decision makers to make difficult decisions about what curricular content should be presented and in what order. If teachers and other stakeholders do not
value and understand how curriculum sequences leverage these progressions, they may be inclined to re-sequence the curriculum in pursuit of other goals.

There are three major practical implications of the four conditions described above. First, curriculum designers should be explicit about design characteristics that are essential for effective implementation. The rationale for these design characteristics must be presented carefully, so that teachers and other stakeholders are convinced of their importance. Once teachers and other stakeholders are aware of these essential design characteristics, they should carefully consider any decisions that might compromise these principles. Teacher developers should help teachers attend to design principles during planning.

Second, transparency in how instructional principles are manifested in the design of a curriculum is crucial. Curriculum designers should provide descriptions of the role of various sections of the curriculum in terms of the overall instructional sequence. They should consider what is flexible within a curriculum’s design and make teachers aware of this flexibility. Instances of this type of communication are present within Core-Plus. In the overview to the first unit in Course 1, for instance, the developers state: “Spreadsheets and computer algebra system symbol manipulation are introduced in Lessons 2 and 3. This content can be omitted without jeopardizing future work” (Hirsch et al., 2008, p. T1). This sort of description has the potential to support teachers in using their expertise to adapt or customize curriculum for contextual realities while maintaining alignment to the core principles implicit in the curriculum design.

Finally, teachers and students must be supported in enacting curriculum in ways that leverage the sequencing of curriculum materials. Curriculum designers should
include clear indications of how students and teachers could explicitly leverage work performed early in the sequence during later work. In the case of CP-first curriculum, designers should make clear how teachers and students might refer to previous contextualized work during work with non-contextualized problems. Messaging within prompts provided to students could highlight for teachers and students how previous work is relevant, even in cases where teachers do not closely read the curriculum guide.

There are a number of research questions that arise from this exploratory study. First, how do teachers’ view the role of CPs and are trends the same or different for teachers who teach from CP-based curriculum materials? A better understanding of teachers’ beliefs around CPs would provide valuable information for curriculum developers and teacher development programs. The findings describing how Ms. Spence re-sequenced the curriculum suggest that teachers’ beliefs about CPs matter, but it is unclear whether Ms. Spence’s ambivalence about a CP-first approach is an exception or the norm for teachers using Standards-based curricula.

More generally, this study demonstrates the limitations of current approaches to analyzing fidelity to written curriculum. Brown et al. (2009) broadened the scope of fidelity studies by proposing that authors’ intentions and characteristics of the written curriculum should be taken into account. But this study serves as a reminder that to simply examine fidelity to a written curriculum or even the authors’ intent tells only one part of a complex story. The reasons teachers have for making significant curriculum adaptations are also important factor that deserve consideration. In this study, unique insights emerged because Ms. Spence had prolonged experience implementing Core-Plus. More research is needed to understand the adaptations of teachers who have gained
implementation expertise and to determine the extent to which curriculum materials can be modified yet still maintain their demonstrated effectiveness. Ultimately, this case highlights a significant tension between the curriculum developers and teacher in terms of who acts authority for determining the tasks that are presented to students and the order in which they are presented.

This tension is reflected in another question that is particularly significant as curriculum materials are developed around research-based learning progressions: to what degree do teachers or other stakeholders re-sequence curriculum materials as they create pacing guides and lesson plans? Large-scale studies (Banilower et al., 2013; Tarr et al., 2008) describe amount of coverage or the degree to which materials are used as a primary resource, but these reports do not tend to provide information about re-sequencing. This information, like the conclusions from the study reported here, would be of vital importance as curriculum designers and teacher developers prepare teachers to teach from the next generation of curriculum materials.
Abstract

Scholars suggest that students’ understanding of formal mathematical concepts can emerge from experience solving contextualized problems, or problems referring to contexts that exist outside of mathematics (e.g. Freudenthal, 1991). Curriculum materials have been created to align with this recommendation (Robinson, Robinson, & Maceli, 2000). But critics of these materials argue that these materials do not adequately expose students to formal disciplinary language and ways of thinking (e.g. Wu, 1997). Previous research suggests other possible complications: students may have difficulty identifying the mathematics they are meant to learn from contextualized problem solving; and they may not perceive mathematical coherence across tasks set in various extra-mathematical contexts, including non-contextualized mathematics. To understand the ways in which these potential obstacles to learning are reinforced or mitigated through classroom discourse, transcripts of instruction from one algebra classroom were analyzed for the presence of explicit references to general mathematical principles and to previous problem solving experiences. The teacher explicitly pointed students’ attention to formal mathematical principles during discourse around contextualized problem solving, but students’ contextualized problem solving experiences were not leveraged once
instructional sequences proceeded to non-contextualized problems or summative tasks focused on generalizing mathematical principles. This finding supports the recommendation that contextualized problem-based curriculum materials explicitly support teachers and students in making connections between instructional tasks.

1. Introduction

Over the past twenty-five years, documents guiding a reform movement in mathematics education (NCTM 1989, 2000, 2014) have emphasized that students can and should build mathematical understanding by actively solving challenging problems. These documents, along with the Common Core State Standards (2010), also stress that students should have the opportunity to solve problems that reference contexts from the “real world” and “everyday life” (Common Core State Standards Initiative, 2010, pp. 7, 28). In combination, these recommendations align with a significant body of research demonstrating that problems set in rich contexts can prompt students to develop informal solution strategies, which can subsequently be connected to formal mathematical ideas (Carpenter et al., 1996; Gravemeijer, 1994; Technology Group at Cognition and Technology Group at Vanderbilt, 1997).

In the US, the National Science Foundation provided funding for the development of a number of curriculum programs that align with the vision promoted by the NCTM; these curriculum programs are often referred to as Standards-based (Senk & Thompson, 2003). The developers of many of these programs intend for students to develop an
understanding of new mathematical concepts by solving and discussing contextualized problems (CPs), or problems that reference contexts that exist outside of the realm of mathematics (Mokros, 2003; Ridgway, Zawojewski, Hoover, & Lambdin, 2003; Schoen & Hirsch, 2003). Despite strong support from many mathematics educators, a national survey study of US teachers found that these NSF-funded programs were used by only 25% of the elementary school teachers, 11% of the middle school teachers, and less than 1% of the high school teachers surveyed (Banilower et al., 2013).

Some authors have criticized the curriculum materials developed to align with the reform movement. Quirk (2011), for example, wonders how students develop mathematical understanding by solving problems: “[proponents of the reform oriented curricula] say first attempt to solve problems and math knowledge will emerge. Emerge from where?” (Chapter 4). He and other critics argue that these programs are based on the assumption that mathematics can be “discovered” by students as they solve problems based in settings that exist outside of mathematics, and they point out that much of mathematics is socially-constructed and arbitrary and thus cannot be “discovered” (Brantlinger, 2011; Dowling, 1998; Wu, 1997). These authors also assert that an overemphasis on CPs limits students’ access to socially-constructed mathematical principles and language. Dowling, for instance, argues that problems set in the physical world are limited in the pedagogical value they provide:

Whilst the physical world may provide starting points, the mathematical interpretation of these starting points must be made explicit by the only person in the classroom who is able to make them explicit: the teacher... the detail of mathematical knowledge is essentially a sociocultural arbitrary. Sooner or later, if someone is going to learn mathematics, someone else is going to have to tell them about it. (1998, p. 44)
Wu (1997) similarly critiques what he considers a lack of explicit attention to formal mathematical conventions and ways of reasoning within reform oriented curriculum:

> An application-oriented curriculum can furnish a valid mathematics education provided enough attention is given to mathematical closure. Tools developed for the purpose of solving a practical problem should be put in the proper mathematical context, and abstract ideas distilled from such solutions should preferably be applied to completely different situations to demonstrate the fundamental role of abstraction in mathematics. Unfortunately, mathematical closure is hardly ever applied in the reform. (p. 948)

These critics do not take issue with the inclusion of real-world contexts in mathematics instruction. But they argue that the reform-oriented curricula described above do not go far enough in making mathematical principles underlying the CPs explicit.

The aim of this study is to inform the discussion about CP-based approaches by taking a detailed look into a single high school classroom enacting a contextually-based mathematics curriculum program called Core-Plus Mathematics (Hirsch et al., 2008), commonly called Core-Plus. After more fully describing a number of potential benefits and obstacles to learning inherent to a CP-based approach identified in the literature, I explain how I analyzed classroom discourse to look for evidence of these potential benefits and obstacles to learning and for aspects of the curriculum and teachers’ instruction that mediated students’ opportunities to learn.

### 2. Review of Relevant Literature

In this section, I frame the study by describing two bodies of literature that informed the research design. I first describe literature describing the potential benefits of positioning CPs as starting points for instructional sequences. Then, I review literature
identifying challenges inherent to a CP-based approach to instruction. I conclude the
section by describing how ideas from both sets of literature are synthesized in the
conceptual framework for this study.

2.1 Rationale for Contextualized Problem Based Instruction

Traditionally, CPs, in the form of word problems, have been positioned at the end
of instructional sequences, after concepts or procedures have already been introduced and
practiced. Many reform-oriented curricula, including Core-Plus, place CPs throughout
instructional sequences, including at the beginning (Robinson et al., 2000). To align with
the recommendations in the NCTM Standards, these curriculum programs feature CPs
that tend to require more cognitive demand on the part of students than traditional
application-style word problems (Stein et al., 2007). Instead of functioning only as
application problems, the types of CPs found in Standards-based materials are intended
to provide opportunities for students to engage in mathematical sense-making and to
develop new mathematical ideas (Robinson et al., 2000). The developers of Core-Plus
state that “major ideas are developed through student investigations of rich applied
problem situations” (Schoen & Hirsch, 2003, p. 315) and “investigations of real-life
contexts lead to (re)invention of important mathematics that makes sense to students and
that, in turn, enables them to make sense of new situations and problems” (Schoen &
Hirsch, 2003, p. 314). Concepts are usually presented first in context and often through
verbal descriptions, graphs and tables. These concepts are later developed using more
symbolic representations and non-contextualized problems, either within the same
investigation or through a sequence of investigations (Schoen et al., 2010). The
developers intended for students to “become engaged with the mathematics through work
on mathematics in context and to have those contextual concept images to think with and
remember in the future” (Fey, personal communication, 7/25/11).

In their descriptions of the curriculum (Fey & Hirsch, 2007; Hirsch et al., 2011;
Schoen & Hirsch, 2003; Schoen et al., 2010), the developers of Core-Plus do not refer to
formal theory on how CPs in particular can be used build mathematical understanding;
however, their descriptions of the role of CPs in Core-Plus align in some ways with
descriptions of the Dutch theory of realistic mathematics education, or RME
(Gravemeijer, 1994). RME grew out of the work of Dutch mathematician Hans
Freudenthal, who argued that students often fail to learn mathematics with understanding
through traditional instructional sequences, which present abstract symbolic
manipulations before students are prepared to make sense of them (1973, 1991). In order
to cope with this form of instruction, Freudenthal argues, students take to memorizing
procedures they do not understand. The developers of RME assert that the starting point
for mathematics instruction should be problem situations that feel real to students, and
that students will naturally re-invent solutions that correspond to established
mathematical conventions. By proceeding through intentional sequences of increasingly
formal representations, students develop an understanding of formal mathematical
conventions that is rooted in their common sense solutions to contextualized problems
(Freudenthal, 1991). Stated another way, “formal mathematics grows out of the students’
Panhuizen (2003) stresses that at any point in RME instruction, students should be able to
revert back to less formal solution strategies, including those developed in response to the initial contextualized problems presented at the beginning of an instructional sequences. RME was developed first through work at the primary mathematics level, but has been the basis for instructional design throughout the school curriculum, including at the university level (Gravemeijer & Doorman, 1999; Rasmussen & King, 2000; Stephan & Rasmussen, 2002).

The metaphors used to describe the role of contextualized problems in RME provide one lens for analyzing contextualized-problem based instruction from Core-Plus. In RME, mathematical understanding “grows” out of roots embedded in students’ common sense understanding of the world around them. In this study, I examined the enactment of one unit of the Core-Plus curriculum materials to look for evidence in the classroom discourse of formal mathematics extending from roots developed from work with CPs.

2.2 Potential Obstacles to Student Learning from CP-based Instruction

What is meant by formal mathematics? In this paper, I define formal mathematics as established, generalizable, abstract mathematical principles including conventional algorithms, procedures, representations, relationships, concepts, language, symbols, and ways of reasoning and arguing—important outcomes of what Wu (1997) calls “mathematical closure” (p. 194). Proponents of the Standards argue that students can abstract or generalize many of these mathematical principles through work in problem situations and that teachers and the curriculum can provide established conventions like symbols and vocabulary.
Research on generalization in mathematics education suggests that the process of generalizing mathematical principles from work on CPs might not be straightforward. Davis (2007), for instance, describes students’ difficulties in understanding the meaning of a \( y \)-intercept while using a context-based Standards-based curriculum. He found that students invented terminology to describe contextual instances of \( y \)-intercept; but, as instruction proceeded from work on contextualized problems to the formal introduction of the term “\( y \)-intercept”, the instructor failed to connect students’ invented terminology to the formal term. As a result, many students demonstrated an incomplete understanding of the concept. Lubienski (2000) in a study focused on variation between students being taught from a Standards-based curriculum program, found that students from families with low-SES tended to have greater difficulty generalizing from their work on contextualized problems than students from families with higher SES. She also found that students from families with low-SES more often complained that they did not see the connection between different contextualized tasks and did not understand what they were supposed to be learning.

Lubienski’s (2000) findings point to an issue particular to contextualized-problem based instruction: if multiple problems set in multiple contexts are used, students may not necessarily see the mathematical similarities linking the problems. Laboratory studies directly targeting this issue provide additional evidence that students struggle to make the desired connections between different problem situations. Herbert & Pierce (2011), for example, found that students did not necessarily transfer successful solutions strategies developed in one context to problems set in other contexts. Gick and Holyoak (1983) found that when different problem contexts are used, students are unlikely to use solution
strategies from a previous context unless they are explicitly reminded of the previous problem.

In addition to difficulties in transferring strategies developed in one context to another contextualized problem situation, research also suggests that students might have difficulty transferring strategies that they developed in response to CPs to non-contextualized problems, or mathematical tasks that are not situated in any contextual setting. Walkington, Sherman and Petrosino (2012) found that students were more likely to attempt story problems compared to analogous, non-contextualized symbolic problems in a laboratory setting; they also pointed out that students brought informal, arithmetic strategies to solve story problems but did not easily coordinate these strategies with more formal, algebraic strategies involving symbolic manipulation. The authors concluded that, “if contexts are to promote access to central concepts, they ultimately should give meaning to abstract representational systems. Whether this can be achieved by traditional story problems within the system of school algebra, and whether it is likely without strong support for such coordination, is more complex than everyday notions of the benefits of contextualization would suggest” (Walkington et al., 2012, p. 198). Lobato and Ellis (2002) noted classroom disconnects between contextualized and non-contextualized problems in a class using Standards-based curriculum; in that case, the teacher failed to maintain a focus on the relationship between two variables in a linear function once the instruction moved from contextualized to non-contextualized problems.

To mitigate difficulties students have with generalizing intended mathematical ideas and seeing inherent connections between instructional tasks, researchers suggest practices that might prompt students to make the intended generalizations from
contextualized tasks. Jurow (2004) documents students’ generalizations working from a Standards-based, contextualized curriculum sequence and concludes that, in addition to opportunities to work on CPs, students “need guided reflection and multiple scaffolded opportunities to talk about, write about, and otherwise represent what is general in and across situations” (p. 296). Davis (2007) recommends that teachers and curriculum designers need to carefully attend to possible disconnects between informal language and strategies that might arise from contextualized problems and analogous formal language and experiences with non-contextualized problems.

Together, these studies highlight an instructional practice that is an essential for facilitating the development of mathematical generalizations from work on particular examples. This practice is what Mason (1996) describes as supporting students’ “shift of attention” from the particular to what is general about the particular. Teachers, Mason notes, are of prime importance: “The presence of someone whose attention is differently structured, whose awareness is broader and multiply-leveled, who can direct or attract pupil attention appropriately to important features, is essential” (p. 71). There is reason to believe, though, that this shift of attention might not be occurring to the extent necessary for productive learning in many classrooms, particularly in the US. Boaler and Brodie (2004) found a great deal of variance in the frequency with which teachers asked questions that engage students in attending to underlying relationships and meanings, i.e. the formal mathematical generalizations that are the goals of lessons. They found these questions were used frequently in only one of the four classrooms they observed using Standards-based curriculum. Furthermore, international comparative studies have consistently found that US teachers were are less likely to engage in statements of
mathematical summary than their counterparts in nations that perform higher on comparative assessments (Hiebert et al., 2003; Stigler, Gonzales, Kawanaka, Knoll, & Serrano, 1999).

It follows from research described above that one way teachers can promote generalization from contextualized tasks is to prompt students to see and articulate what is similar among various CPs and their analogous non-contextualized analogs. The degree to which prior and future tasks are referenced during instruction is one component of a construct called *instructional coherence* in mathematics education research, another focus of international comparative studies (Cai, Ding, & Wang, 2014; Stigler & Hiebert, 1999). Here again, there is reason to believe that in some countries, including the US, high levels of coherence may not be the norm. After comparing lessons taught in three countries, researchers found that in Japan, references to past and future activity were made significantly more frequently than in the US or Germany (Stigler et al., 1999). Like researchers who study generalization, scholars describing instructional coherence stress that the connections within lessons should be made explicit, to ensure that students have an understanding of the sequence of instruction and how they can leverage prior work toward learning.

### 2.3 Conceptual Framework

Taken as a whole, these bodies of literature highlight the need for teachers and students to make connections between three key types of classroom activity: *contextualized problem solving, non-contextualized problem solving, and reflection on general mathematical principles*; these three types of activity form the basis for the
conceptual framework used in this study, illustrated in Figure 15. I use the term *contextualized problem solving* to refer to engagement with problems or tasks situated in some setting from outside the world of mathematics; this category is located on the bottom left of the figure. The three, stacked rectangles represent multiple problems set in distinct contexts. As discussed earlier, studies by Lubienski (2000), Gick and Holyoak (1983), and Herbert and Pierce (2011) suggest the potential for a lack of connection between students’ activity on problems set in different contexts. Studies by Walkington et al. (2012) and Lobato and Ellis (2002) suggest a potential lack of connection between students’ work on contextualized problems and their work on *non-contextualized problem solving*, shown to the right on the bottom row. Finally, the rectangle at the top of the figure represents activity through which the students and/or teacher engage in what I call *reflection on general mathematical principles*; this type of activity allows for “mathematical closure”, or a shift of attention to what students are meant to generalize from a particular example or set of examples.

*Figure 15. Three types of instructional activity framing the study*
Literature on the use of contextualized-problems in mathematics instruction emphasizes that students’ understanding of general mathematical principles should arise from, and maintain connections with, their work on contextualized problems. Generalization literature stresses the need for students to have the opportunity to reflect across various instances and tasks, including across contextualized and non-contextualized. The importance of coherence is significant, particularly in light of the possibility that a high degree of coherence may not consistently be found in many mathematics classrooms.

3. Research Design

To study the way the students and teacher in this study made meaning of mathematical generalizations through work on contextualized problems, I adopt the interactionist perspective developed by Bauersfeld, Krummheuer, and Voigt (1988) for understanding the construction of meaning in mathematics classroom. Researchers adopting a constructivist perspective assume that learning is enabled by participation in cultural and social processes in the classroom. This perspective can be contrasted with psychological constructivism, which attends to the mind of the individual, and sociocultural analyses, which attends to individuals’ participation in cultural practices at the societal level. Researchers adopting an interactionist approach limit their focus to the classroom micro-culture and emphasize individuals’ contributions to this culture. The relationship between students’ mathematical activity and their participation in classroom interactions is seen as reflexive: “students are seen as actively contributing to the
development of both classroom mathematical practices and the encompassing microculture, and these both enable and constrain their mathematical activities” (Cobb & Bauersfeld, 1995, p. 9). Because students learn through interactions, these interactions are the focus of research, rather than inferences about what might be going on in students’ minds. In this study, the interactions within the group (be it small group or whole class) and the explicitly shared generalizations that arise as a result of these interactions, will be the subjects of my analysis, rather than individual students’ understandings. Following other interactionist analyses (Bauersfeld et al., 1988) I aim to identify normalized patterns of interaction that arise as similar stimuli occur in the classroom.

I examine the interactions between students and teacher as they participate in three types of mathematical activity: contextualized problem solving, non-contextualized problem solving, and reflection on generalized mathematical principles. The study is designed to determine the extent to which generalized mathematical principles are made explicit by the student and the teacher through instruction from a Standards-based curriculum and the degree to which these generalizations are connected to students work on contextualized problems. Specifically, the design is guided by research questions around these three types of activity. First, I address the question: to what extent, and in what manner, do the teacher, students, and written curriculum address general mathematical principles during work on specific contextualized problems? Second, in light of research that suggests the importance of coherence for the development of generalizations, I address a second question: to what extent, and in what manner, do the students, teacher and written curriculum make explicit connections between
contextualized problems and (a) other contextualized problems (b) non-contextualized problems, and (c) activity focused on generalizing mathematical principles.

It is important to point out that this type of analysis is not based on the assumption that explicit connections between formal mathematical principles and contextualized problem solving must be made in order to say that formal mathematical understanding is emerging from contextualized problem solving for individual students; but, I argue that in order for all students to have the opportunity to leverage these connections, even those students who do not spontaneously identify the connections between these different types of instructional activity on their own, these connections need to appear in classroom discourse.

### 3.1 Setting and Participants

In the study, I examine the role that CPs serve in a high-school classroom implementing one algebra unit from Core-Plus (Hirsch et al., 2008). A typical Core-Plus instructional sequence involves an introductory activity in which an extra-mathematical context is introduced. Students continue the investigation in groups, working on tasks related to the initial task and often moving to other contexts. Finally, the investigation proceeds to the Summarize the Mathematics tasks. After students work on these tasks, the teacher is to facilitate a discussion during which the students share their results and summarize mathematical ideas (Hirsch et al., 2011).

The study was set in public high school located outside a midsized city on the east coast of the US. The participating teacher, whom I refer to with the pseudonym Ms.
Spence, was the chair of the mathematics department in her school; she had 20 years of teaching experience and 13 years of experience implementing Core-Plus. The school was a comprehensive high school in a small school district on the east coast. At the time of the study, the school enrolled approximately 2000 students; about half of the students were eligible for free or reduced lunch. About half of the students in the school were African-American, less than one percent were American Indian, 3% were Asian American, 15% were Hispanic/Latino, 30% were White, and less than 1% were identified as multi-racial. The class observed in the study was a general admission 9th grade Integrated Math I class. Twenty of the 30 students in the class agreed to participate in the study.

3.2 Data

The dataset included observations of twenty instructional days; each class session lasted approximately 1.5 hours. During this time, the teacher taught from Core-Plus Level One, Unit 3, Lesson 1. The pace was slower than recommended in the teachers’ guide, which suggested that the entire lesson, composed of three separate investigations, should take nine days, or roughly three days per investigation. This discrepancy was due primarily to the amount of time spent on the first investigation (seven days) and a number of supplemental activities that Ms. Spence added. Investigations 2 and 3 took three and four days respectively, which is in line with the recommended pacing. The first investigation focused on the concepts of rate of change and y-intercept and how these concepts are manifested in symbolic rules, tables, and graphs. The second investigation
asked students to use data from graphs, tables, or verbal descriptions to write symbolic rules relating two variables. The third investigation focused on the use of best-fit lines to model approximately linear samples of data.

Two voice recorders were used to record group work and whole class discussions; these recorders were placed on the desk of participating students. Generally two groups were recorded at a time. Because the partner groups in the class shifted frequently, I was not able to record the same group each day. One partner group was relatively stable; I recorded this group as frequently as possible, during 11 out of the 20 sessions, in an attempt to track the development of mathematical generalizations over time. I placed the other recorder with different groups, in an attempt to capture variation of possible interactions among students. Figure 16 indicates data collected for each student.

I interviewed the teacher once at the beginning of the study to gain an understanding of her conceptions about the use of contextualized problems in mathematics instruction and the Core-Plus curriculum. I also interviewed her after eight of the observations, for an average of twenty minutes each time, to understand her perceptions of the curriculum and instruction. Recordings of the classroom observations and the interviews were transcribed and entered into qualitative data analysis software.
|       | Oct | 20 | 24 | 27 | 28 | 31 | 1  | 2  | 3  | 4  | 7  | 8  | 9  | 10 | 14 | 15 | 16 | 17 | 18 | 21 | 22 |
|-------|-----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|
| Marcus | A   | A  | A  | A  | A  | A  | A  | A  | A  | A  | A  | A  | A  | A  | A  | A  | A  | A  | A  | A  |
| TJ     | A   | A  | A  | A  | A  | A  | A  | A  | A  | A  | A  | A  | A  | A  | A  | A  | A  | A  | A  | A  |
| Miesha | B   | B  | A  | A  | A  | A  | A  | A  | A  | A  | A  | A  | A  | A  | A  | A  | A  | A  | A  | A  |
| Patricia| A   | A  | A  | A  | A  | A  | A  | A  | A  | A  | A  | A  | A  | A  | A  | A  | A  | A  | A  | A  |
| Tom    | A   | A  | A  | A  | A  | A  | A  | A  | A  | A  | A  | A  | A  | A  | A  | A  | A  | A  | A  | A  |
| Andre  | B   | B  | A  | A  | A  | A  | A  | A  | A  | A  | A  | A  | A  | A  | A  | A  | A  | A  | A  | A  |
| Javier | B   | A  | A  | A  | A  | A  | A  | A  | A  | A  | A  | A  | A  | A  | A  | A  | A  | A  | A  | A  |
| Daniel | B   | B  | A  | A  | A  | A  | A  | A  | A  | A  | A  | A  | A  | A  | A  | A  | A  | A  | A  | A  |
| Jon    | B   | B  | A  | A  | A  | A  | A  | A  | A  | A  | A  | A  | A  | A  | A  | A  | A  | A  | A  | A  |
| Nick   | B   | B  | A  | A  | A  | A  | A  | A  | A  | A  | A  | A  | A  | A  | A  | A  | A  | A  | A  | A  |
| India  | B   | B  | A  | A  | A  | A  | A  | A  | A  | A  | A  | A  | A  | A  | A  | A  | A  | A  | A  | A  |

A- Group was recorded with recorder A  
B- Group was recorded with recorder B

*Figure 16.* A record of which groups were recorded during each class session
3.3 Data Analysis

To analyze the data, I first classified each sentence in the written curriculum according to activity type, using the three types of activity defined in the conceptual framework. For example, the first investigation in the written curriculum opens with a problem that references Barry, a fictional credit card salesman. Barry’s compensation is presented via a graph showing that the pay Barry receives increases linearly in relation to the quantity of applications he collects. The authors ask, “How does Barry’s daily pay change as the number of applications he collects increases?” (Hirsch et al., 2011, p. 151). This question was coded as contextualized problem solving. Later in the investigation, students encounter a problem asking them to graph and determine the slope of the function \( y = 1 + \frac{2}{3}x \) (p. 155). This was coded as non-contextualized problem solving. In the Summarize the Mathematics section at the end of the investigation, students are asked “How can you determine whether a function between two variables is linear by inspecting: (i) the table relating two variables?” (p. 156). This task was coded as reflection on general mathematical principles. I similarly classified the written tasks that the teacher added to supplement the Core-Plus curriculum.

Then, I analyzed the transcripts of the classroom dialogue. To understand the extent to which generalizations arose and were made explicit during students’ work with contextualized problems and other types of activity, I coded each utterance for the presence of what Ellis (2007) classifies as statements of general principles: statements of rules, patterns, strategies, procedures or global rules that are not limited to specific cases. I separated these instances into five categories: cases in which (1) the teacher made
statements, (2) the teacher asked questions, (3) a student answered the teacher’s question, (4) a student asked a question of his/her own, or (5) a student made a spontaneous statement of generalization. Figure 17 provides two excerpts that contain each of these five types of generalization codes that were applied. To identify at what point during instruction these generalizations were made, I classified the utterances as occurring at the beginning of a lesson, at the end of a class session, or in discourse around tasks that were coded as contextualized problem solving, non-contextualized problem solving, or reflection on general mathematical principles. I also classified whether these generalizations occurred when problems were being introduced, as students worked in groups, or as the solutions were debriefed.
Examples of Utterances Containing Generalization Statements or Questions

<table>
<thead>
<tr>
<th>Utterance</th>
<th>Codes Applied</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ms. Spence: So all of your rate of changes there should be two. <strong>That should make sense because if it’s a linear pattern they’re all the same…</strong> … If I got one that was different, what would that tell me about my function?”</td>
<td>generalization; teacher statement; debriefing contextualized problem solving task</td>
</tr>
<tr>
<td>Michael: <strong>That it wasn’t linear.</strong></td>
<td>generalization: student answer; contextualized problem solving task</td>
</tr>
<tr>
<td>Rosie: What do they mean by the rule?</td>
<td>generalization; spontaneous student statement; working contextualized example</td>
</tr>
<tr>
<td>Beth: <strong>That’s like when you do the y equals something plus whatever x.</strong></td>
<td>generalization; spontaneous student statement; working contextualized example</td>
</tr>
<tr>
<td>Rosie: <strong>How do you find the x or whatever?</strong></td>
<td>generalization; spontaneous student question; working contextualized example</td>
</tr>
</tbody>
</table>

*Figure 17: Examples of generalization statements and questions*

The results of this analysis will be explained in detail in the next section.

To understand the ways in which the connections between the various instructional tasks were made explicit in classroom discourse, I coded each utterance in the data set for the presence of any explicit reference to a prior contextualized or non-contextualized problem or task. In the case where multiple, adjacent utterances referred to the same problem or task, I grouped the utterance as a single instance. I looked across instances where prior contextualized problems were referenced to understand similarities...
and differences between the individual cases. I separated these instances into seven categories: cases in which references were made (1) at the beginning of class session as the teacher and students reflected on previous work, (2) as a new contextualized task was being introduced, (3) as students worked on a contextualized task, (4) as a subsequent contextualized task was debriefed, (5) within discourse around a subsequent non-contextualized task, (6) within discourse specifically about general mathematical principles and (7) at other times during instruction. Figure 18 provides sample quotations that contain references to prior contextualized problem solving tasks and the codes that were assigned to these quotations. By categorizing the instances in which the teacher and students made reference to previous examples, I was able to compare the relative frequencies with which connections to prior work were made by the teacher and students during these different portions of the instructional sequence. These results of the analysis are summarized in Table I and will be explained in detail in the following section.
<table>
<thead>
<tr>
<th>Quotations Containing Reference to Prior Contextualized example</th>
<th>Assigned codes</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ms. Spence: I want you to look at question five on page one fifty five and question five is having you match the equation to the particular graph just like what we did I believe on question three.</td>
<td>reference to prior contextualized example; during introduction of subsequent contextualized problem solving task</td>
</tr>
<tr>
<td>Ms. Spence: Can we make a rule for Cheri’s pay like we did for Barry?</td>
<td>reference to prior contextualized example; during debrief of subsequent contextualized problem solving task</td>
</tr>
<tr>
<td>Ms. Spence: [after asking students to write a story problem to match the equation $y = 5 + 4x$] The reason I gave you these questions because it was very similar to a question you had on your quiz on Friday. I believe the situation there was that you had a girl that was participating in a walk… not sure what the variables were. It was something like e equals eight plus two d.</td>
<td>reference to prior contextualized example; during discourse around a subsequent non-contextualized problem solving task</td>
</tr>
<tr>
<td>Ms. Spence: Let’s go back to Barry’s real quick. If you look back to Barry’s situation, which do you think is the dependent variable?</td>
<td>reference to prior contextualized example; during activity focused on general mathematical principle</td>
</tr>
</tbody>
</table>

*Figure 18:* Examples of references to prior contextualized examples and the codes that were assigned.
4. Results

The following section is organized around four classroom episodes; each episode exemplifies a particular set of patterns that emerged from the data. The first episode and the findings that follow illustrate how Ms. Spence pointed students’ attention to connections between contextualized tasks; in this subsection I also note how infrequently students made spontaneous explicit connections between tasks set in different contexts. The second episode illustrates the ways in which Ms. Spence pointed students’ attention to the generalizable mathematical principles they were meant to take away from work in contextualized problem settings. The third set of findings, illustrated by the third episode, point to a lack of explicit connections made between work on contextualized examples and subsequent work on non-contextualized examples. The final episode typifies classroom interactions around the *Summarize the Mathematics* sections and the lack of references to prior contextualized problem solving experiences in particular.

4.1 Episode 1: Illustrating Connections Between Problems Set in Different Contexts

To determine the ways in which explicit connections were made between various CPs in Ms. Spence’s classroom, I analyzed the classroom discourse around CPs for the presence of references to previous contextualized tasks. Episode 1 exemplifies the ways in which Ms. Spence explicitly made connections between problems set in different contexts. This narrative also contains a rare instance of a student explicitly referring to prior contextualized problem solving. The episode begins on the first day of a new unit.
Ms. Spence: Here's your unit essential question. This is for the whole unit and by the time we're done you should be able to answer this question. So write it down, the unit essential question. Today's date is October 20th. Your unit essential question is, "How can you represent, interpret, and manipulate a given situation that has a constant rate of change?"

After introducing the new unit, Ms. Spence asked students to share what they noticed about the graph shown in Figure 19. This graph shows how the daily pay earned by Barry, an employee of a credit card company, is related to the number of applications he solicits.

![Figure 19](image)


Marcus, a vocal and academically successful young man, stated his observation:
Marcus: The dots are going over one up one. It has a slope of one.

Ms. Spence recorded this and other students’ observations on the board at the front of the room. After the other students have shared, she returned to Marcus’s statement:

Ms. Spence: [Marcus] said that every time I move over a block I seem to go up one. But here’s my question. When I’m moving over one block am I moving over one?

Marcus: No.

Ms. Spence: I'm moving over one block but does that mean the same thing as moving over one.

Marcus: No.

Ms. Spence: No. What happens when I'm moving over one block.

Through a series of questions, Ms. Spence led Marcus to determine that the slope is “five over one”. During this class period and the next, students worked through a series of questions which ask them to identify how Barry’s daily pay and his pay per application are represented in the graph, in a symbolic rule, and in a table.

The next question in the book introduced a character named Cheri, who works for the same credit card company; Cheri sells “additional services” over the phone. As the class discusses this new situation, Ms. Spence explicitly connected back to question 1 featuring Barry:

Ms. Spence: Does [Cheri] appear to have a linear function for her job as well?

Students: Yes.

Marcus: She appears to.

Ms. Spence: Yes. She appears to. So what would it be? What's the pattern that you see in your graph? What happens now? How is this different than question 1? What’s the pattern for this one?
After the students completed the tasks involving Cheri, the context shifted to involve a character named Emily; Emily bought a $480 TV with a charge card and paid $20 per month to gradually reduce her balance. The book prompted students to create a table and a graph to show how Emily’s balance changes each month. Andre shared his graph, and Ms. Spence asked the other students what they noticed about the graph. Ms. Spence called on Daniel, who was corrected by Marcus:

Daniel: The slope is.. it’s down one over two.

Marcus: It’s not down one over two. Nuh huh.

Ms. Spence: Talk about it. Go ahead figure it out. Talk about it.

Marcus: What? Nuh huh ‘cause last time she said it’s really… it’s really… it’s over for every one it goes down twenty dollars or for every month she pays off twenty dollars.

During this episode, in which the concept of rate of change was developed over a series of contextualized tasks, two events occurred that illustrate the types of connections that were made across contexts in Ms. Spence’s classroom. First, Ms. Spence made an explicit mathematical connection between the first problem, which featured Barry, and the second, involving Cheri; Ms. Spence occasionally made these sorts of connections over twenty instructional days observed. Second, Marcus recalled a previous classroom event that occurred during contextualized problem solving activity; this was a rare instance of a student spontaneously making a connection between two contextualized tasks.
Table I shows the frequency with which the teacher and students made reference to prior contextualized and non-contextualized work over the course of the twenty instructional days of the unit.
Table I

*Frequency of Explicit References to Previously Worked Examples*

<table>
<thead>
<tr>
<th>Reference</th>
<th>Within Discourse Around a Subsequent Contextualized Example</th>
<th>With-in Discourse Around a Subsequent Non-contextualized Example</th>
<th>Within Discourse Around General Mathematical Principles</th>
<th>Within Other Discourse</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Previous Contextualized Example</td>
<td>6</td>
<td>0</td>
<td>5</td>
<td>1</td>
<td>13</td>
</tr>
<tr>
<td>Previous Non-contextualized Example</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>2</td>
</tr>
<tr>
<td>Previous Contextualized Example</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>2</td>
</tr>
<tr>
<td>Previous Non-contextualized Example</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>3</td>
</tr>
</tbody>
</table>
As shown in Table 1, Ms. Spence referenced prior contextualized problems eleven times in discourse around a different contextualized task. These eleven references occurred on eight different instructional days in response to eight separate tasks. On two other occasions, Ms. Spence assigned tasks that explicitly asked students compare and contrast previous contextualized work; these were not counted in the table because they were tasks rather than part of the classroom discourse. In interviews, Ms. Spence frequently stated that her students struggled to make connections between different instructional tasks in the textbook. To alleviate this struggle, and as a way of emphasizing the mathematical point of the investigations, she worked to make connections between problems explicit. Two questions in Episode 1, “How is this different than question 1?” and “What’s the pattern for this one?”, represent one way she prompted students to make these connections: by prompting students to reflect across various contextualized tasks. On five occasions, Ms. Spence asked this type of question as students shared their solutions, as was the case here. On six occasions, she referred to previous contextualized problems when introducing new problems, as she did during the second session when she asked, “Can we make a rule for Cheri’s pay like we did for Barry’s?” Given Ms. Spence’s inclination to help students see the connections between instructional tasks, it is notable that each of these examples took place during whole-class discussion. There were no instances in which Ms. Spence referred students to their work on previous problems as they worked on tasks in their groups. The absence this sort of connection during the group-work phase of instruction is notable particularly because of how she attended to these connections before and after students engaged with the problems in groups. It should also be pointed out, though, that only two groups were
recorded at a time, and it is possible that Ms. Spence made explicit connections between contextualized tasks with other groups during these twenty days of instructions.

Episode 1 also contains a rare example of a student making a spontaneous connection to prior contextualized problem solving. After Daniel observed that the slope of Andre’s graph was “one over two”, Marcus reminded him of previous work, saying “last time she said it’s really, it’s really... for every one it goes down twenty dollars...” Although the referent of his statement “last time she said” is somewhat ambiguous, Ms. Spence took up Marcus’ reference to “last time” as a reference to the first session of the unit, when she had probed his assertion that the slope of the graph showing Barry’s daily pay was “1 over 1”. Marcus’s statement, which seemed to refer to previous contextualized work, represents an exception rather than the rule. As shown in Table 1, this occurrence was one of only two times I observed a student spontaneously and explicitly reflect back on previous contextualized work while engaging with a subsequent contextualized problem, without being prompted to by the teacher. Once again, it is important to point out that not all student groups were recorded, so these types of explicit connections between problems could have been made in groups that were not recorded.

In addition to these two explicit references to previous contextualized problems made by students, there were also a number of implicit references to prior contextualized work. On seven occasions, students transferred contextual elements of contextualized problems to a later problem without explicitly noting that the element came from an earlier problem. On five of these seven occasions, the contextual elements students transferred from one problem to another did not fit with the latter problem. Students carried the idea of “selling applications” from the Barry problem over to the problem
featuring Cheri three times, despite the fact that Cheri was actually selling “additional services” (p. 152). The transfer of this contextual element was not disruptive and may have even helped students to leverage solution strategies developed in the Barry situation to solve problems based in the new but similar situation. On two other occasions, Marcus transferred elements from the Barry problem to other, unrelated problems. When Marcus first encountered the problem based around Emily and her credit card-enabled purchase of a television, for example, he initially transferred an element from Barry’s context, telling his partner, “You gotta end up finding the starting pay.” But in the situation involving Emily, there was no starting pay; the $-intercept represented the $480 cost of the television and the rate of change was negative because Emily’s account balance was the dependent variable. Here, the transfer of the contextual elements of an earlier problem may have interfered with Marcus’s ability to solve the problem. It was not until Ms. Spence intervened before they realized that the situation was significantly different and the rate of change was negative.

In the written curriculum, mathematical connections between contextualized problems were not explicitly made in any of these three investigations, and students were not explicitly prompted to reflect across multiple situations. According to the *Core-Plus* implementation guide (Hirsch et al., 2011), students are meant to reflect back on their work during the “Summarize the Math” portion. The extent to which Ms. Spence and her students explicitly reflected on their work on contextualized problems during the “Summarize the Math” portion is addressed after Episode 3.
4.2 Episode 2: Attending to General Mathematical Principles during Contextualized Problem solving

To understand the ways in which generalizations emerged from students’ work in contextualized problems, I analyzed classroom discourse around contextualized problems for the presence of explicit statements addressing general mathematical principles. My analysis showed that Ms. Spence frequently emphasized the mathematical principles underlying various contextualized tasks by asking questions and making statements that drew attention to these principles. The episode below provides a descriptive example of how she pointed students to the mathematical point of Investigation 1: how to identify the slope and intercept of linear functions by examining tables, graphs and symbolic rules.

In this episode, students examined graphs representing the account balances of three different customers who used charge cards to purchase electronics. They were asked to match the graphs to symbolic rules. After working in groups, Ms. Spence asked students to share their answers with the whole class.

Ms. Spence: Which line is parallel to Philicia’s line?
Saby: Darryl’s.
Ms. Spence: How can you tell? How do you know Darryl's is?
Marcus: They got the same slope.
Ms. Spence: Because they have the same slope or the same rate of change. What was their rate of change?
Students: Forty.
Ms. Spence: It was forty. Alright, now. What's more important in addition to being able to match them up is you should be able to recognize the starting point. How do I know the starting point versus the rate of
change? How can I tell which one's which? What's important that you see from looking at the equations?

Marcus: The letter after the \[ trails \ off \]

Ms. Spence: You always have a letter associated with what?

Marcus: slope

Ms. Spence: With the slope. Alright? And if you notice that starting point is a free standing constant it doesn't have a letter associated with it. But if you look every time that we talked about our rate of changes, they always have a variable. So here, my rate of change was twenty. But it has a variable with it. [\textit{She circles the 20 on the board, then does this for the rate of change for all three customers}] My rate of change here is negative forty. I should be saying they're negative. And I also have a negative forty there. Alright. But they always have a variable with them.

In the introductory interview, Ms. Spence described her students’ struggles with identifying the important mathematics as they worked through contextualized problems in \textit{Core-Plus}, saying “I think sometimes there's so much that they want the kids to figure out on their own, I think the point of the investigations sometimes gets lost if I don't find a way to lead them where they need to go”. To mitigate what she perceived as students’ difficulties attending to the point of the lesson, she frequently pushed students to generalize \textit{during} discussion around on these problems. In the example above, Ms. Spence attempted to shift their attention from the particular example to the general principle at hand. First, she emphasized that generalization is the goal, saying “What's more important in addition to being able to match them up is you should be able to recognize the starting point.” Then, she prompted students to do the work of generalizing by asking for the general rule: “How do I know the starting point versus the rate of change? How can I tell which one's which?” To support students in making the
generalization, she pointed them to the particulars from which the general conclusion can be drawn, asking, “what's important that you see from looking at the equations?”

The case presented here illustrates a pattern observed in Ms. Spence’s practice. Table 2 shows the different ways general mathematical principles were made explicit in the classroom discourse. Certainly, one would expect to the teacher make statements and ask questions targeting general mathematical principles as she and her students engaged with tasks targeting these principles directly; in these 20 days, this occurred 84 times. But Ms. Spence also consistently stated general mathematical principles and asked questions aimed at such generalizations in discussions of contextualized problems: in 42 utterances (median of 2 per class session)\(^5\), she made statements of general mathematical principles while discussing contextualized tasks; and, in 50 utterances (median of 2 per class session), she asked students questions targeting general mathematical principles while discussing contextualized tasks. The overwhelming majority of these utterances (79/92) came as the class debriefed solutions to contextualized problems. On seven other occasions, Ms. Spence supplemented the written curriculum by adding tasks focused on general mathematical principles during portions of Core-Plus task sequences that involved only CPs; these did not show up in the table because they were not utterances, but they did contribute to the extent to which reflection on general mathematical principles occurred in the midst of contextualized problem solving.

Apart from utterances occurring during discussion of contextualized problem solving tasks and those explicitly targeting general principles, the other significant portion of her

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\(^5\) Only class sessions in which CPs were discussed were included in this calculation
statements and questions containing generalizations came at the beginning of class sessions as she summarized work from previous day; this occurred 77 times.

Table II

*Frequency of Teacher Utterances Attending to General Mathematical Principles*

<table>
<thead>
<tr>
<th></th>
<th>Statements</th>
<th>Questions</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Beginning of Lesson or Session</td>
<td>26</td>
<td>51</td>
<td>77</td>
</tr>
<tr>
<td>Discourse Around Contextualized Problem Solving</td>
<td>42</td>
<td>50</td>
<td>92</td>
</tr>
<tr>
<td>Discourse Around Non-contextualized Problem Solving</td>
<td>14</td>
<td>22</td>
<td>36</td>
</tr>
<tr>
<td>Discourse Around Tasks Targeting General Mathematical Principles</td>
<td>25</td>
<td>59</td>
<td>84</td>
</tr>
<tr>
<td>Summarizing at the end of a Class Session</td>
<td>2</td>
<td>3</td>
<td>5</td>
</tr>
</tbody>
</table>

Spontaneous student generalizations, meaning those unprompted by the teacher, were far less commonplace. Classroom microphones only recorded 16 instances of students spontaneously generalizing about broad mathematical principles in discourse around contextualized tasks (compared to ten generalizations during non-contextualized activity). Eleven of these generalizations occurred as students used previously established general ideas as justifications or explanations of how to find the answer to a contextualized problem. For instance, during group work, Jon explained to students in another group that the “y-intercept doesn’t have an x-variable” to justify why they did not
need to take into account the rate of change when finding the y-intercept from a symbolic rule. The other five spontaneous generalizations represent windows into students’ initial attempts at generalizing global rules. Two of these spontaneous generalizations occurred as Marcus verbalized out loud, but seemingly to himself, about how to calculate the rate of change from a table. After working on a horizontal table, he noted, “So it’s the bottom one divided by the top.” Then, after working with a vertical table, he said “Basically you divide the second column by the first column... I got you.” Although these comments occurred during class discussion, Marcus did not have the attention of the teacher or other students at the time. These seem to be examples of Marcus thinking out loud, verbalizing generalizations as he perceived them. In the other two instances in which spontaneous student generalizations were recorded, Saby asked questions of Ms. Spence to figure out whether the y-variable was used as the independent variable during discussion of contextualized examples. The fact that these “shifts in attention” toward general mathematical principles were exhibited by Marcus and Saby is notable because Ms. Spence considered these two to be among the strongest students in the class. It may be that some students possess a natural tendency to attend to generalities through contextualized problem solving, and these students in particular may be more likely to benefit from CP-based instruction.

The textbook did not include questions prompting students to generalize about underlying mathematical principles within the CPs in these investigations. However, statements about general mathematical principles were sometimes made between contextualized problem prompts, or, more often, directly before or after contextualized
problems. This is the case for the generalizations in the excerpt presented in Figure 20.

This excerpt is located directly after the contextualized problems in Investigation 1.


In this investigation, the generalization serves as the transition from contextualized to non-contextualized problems.

Overall, the data show that the contextualized problems did serve as settings from which generalizations emerged. Ms. Spence consistently made or prompted these generalizations; but, in isolated cases, students asked questions or made statements involving general mathematical principles spontaneously. The book sometimes provided statements of generalizations before, within, or after contextualized problems, but did not
prompt students to generalize until the *Summarize the Mathematics* section at the end of each investigation.

### 4.3 Episode 3: Illustrating the Absence of Explicit Connections Between Contextualized and Non-contextualized Problems

To understand the extent to which connections between contextualized and non-contextualized activity were made explicit in Ms. Spence’s class, I analyzed teacher and student utterances during discussion of non-contextualized tasks for the presence of explicit references to previously worked contextualized tasks.

Ms. Spence assigned non-contextualized exercises toward the end of Investigations 1 and 3, after related work on contextualized examples. My analysis of the portions of the transcripts pertaining to these non-contextualized examples indicated that neither Ms. Spence nor the students tended to make reference to prior contextualized examples during non-contextualized work. Table 1 shows that Ms. Spence made this sort of reference only once, and there were no recorded examples of students making this type of explicit connection during the twenty days of instruction. The following episode, which takes place during discussion of the non-contextualized examples at the end of Investigation 1 (Figure 21), is representative of the discourse around non-contextualized tasks.
Ms. Spence began by modeling how to solve question 6b on the board.

Ms. Spence: Alright. Look at question six b with me. We’re going to do that together. And it says what is the slope and what is the rate of change. Can anyone look at that and tell me. What is the slope for this particular equation

Marcus: [not waiting until Ms. Spence is finished] Two x. Two x.

Ms. Spence: and how do I know what it is?

Marcus: Two x. Two x.

Ms. Spence: What's the slope?

Marcus: Zero

Ms. Spence: The slope is the?

Students: Two.

Ms. Spence: Two. How do you know it's the two? Kristen?
Kristen: I just remembered it's the y equals slope x from last year
Marcus: You hype because you don’t know how to explain it
Ms. Spence: Because it's always with the?
Kristen: x.

In this case, neither the teacher, nor the students, nor the written curriculum explicitly referenced previous work on contextualized problems during discussion of the non-contextualized problems. Instead, Kristen referenced work from the previous year, and Ms. Spence referenced a previously established generalization, that “the slope is always with the x”.

In the entire data set, I found only one instance in which Ms. Spence explicitly referred to previously-worked contextualized problems as students worked or discussed non-contextualized examples. This excerpt, presented below, provides an image of what this sort of explicit connection looks like in practice. Ms. Spence directed students’ attention to the similarities between the non-contextualized equation, \( y = 5x + 4 \), and an analogous contextualized equation, \( E = 8 + 2d \), that they had encountered on an assessment:

Ms. Spence: They gave it to you in a real life context because you wanted to be able to apply the mathematics you learned. And what they told you, I forgot what her name was, I think it was Emily and her uncle, and the rule there was a little different. Not sure what the variables were. It was something like e equals eight plus two d. How is that problem different than what you just did?
Marcus: y changed to e
Ms. Spence: The y changed to e and the
Marcus: and x changed to d
Ms. Spence: The x changed to d. But did the situation change? Do I not still have slope and y-intercept?

In this instance, Ms. Spence pointed students’ attention to the specific elements that are different between the two equations and then noted a commonality they share: that both linear functions possess a slope and a y-intercept.

The interview data provides insight into Ms. Spence’s thoughts about the relationship between contextualized and non-contextualized problems in the curriculum. In a post-observation interview after Episode 3, Ms. Spence identified the difficulty students had making connections between contextualized and non-contextualized examples, saying “Somehow to me they could have integrated it a little better when they did the part with Emily and tied it into that”. She also emphasized her own role in helping them make those connections, saying, “I feel like I'm going back and I'm having to grab and pull those pieces together because I know the Core-Plus assumes that these kids are gonna be able to make those connections themselves.”

I observed three ways in which Ms. Spence tried to help students make connections between contextualized and non-contextualized problem solving over the course of the study. First, at points of transition from contextualized to non-contextualized problem solving, Ms. Spence referred back to previous work. An example occurred immediately prior to Episode 3. In the book, the transition from contextualized problem solving (problems 1-5) to non-contextualized problem solving (problem 6) is marked by the paragraph in Figure 20, containing exposition of general principles of slope. After students read that paragraph, Ms. Spence prompted students to reflect back over their contextualized work on problems 1-5 to make sense of the formula. In this
way, Ms. Spence pointed students’ attention back to their prior work as they transitioned to non-contextualized examples. But, it is important to point out that, because the textbook section that Ms. Spence were referring to at that point was focused on general mathematical principles, this activity did not explicitly connect the non-contextualized problems to the contextualized examples. In order for this connection to have been made explicit, Ms. Spence, the students, or the textbook would have needed to referred to the contextualized problem solving while discussing the non-contextualized examples that followed the transitional text.

Second, I observed one instance of Ms. Spence connecting contextualized and non-contextualized problem solving by asking students to contextualize a non-contextual example. After many students did poorly on an assessment during Investigation 2, Ms. Spence attempted to help them make the connection between contextualized and non-contextualized examples by asking them to develop a context that could be described by the linear function $y = 5 + 4x$. This type of task represents a potential bridge connecting students’ work in these two domains that does not involve explicitly referring to previously-worked contextualized examples. In this particular instance, asking students this question did prompt at least one student to recall a previous example: Marcus recalled the situation involving Barry, saying, “I was selling credit card applications... If I didn’t sell any I got 5 dollars and for every one, I got four dollars.” The fact that Marcus recalled this example demonstrates the degree to which he had internalized the “Barry” context; clearly this was an accessible example to which he could refer in order to make sense of later examples.
The third way that Ms. Spence prompted students to recall contextualized problems to support their work on non-contextualized examples was more indirect. As students worked the non-contextualized examples, Ms. Spence prompted many student groups to refer to graphic organizers they had created. These graphic organizers included the four ways that linear functions had been represented (symbolic rule, graph, table and verbal description), the two components of a linear function ($y$-intercept and rate of change), and descriptions of how to identify these components in each of the four ways of representing linear functions. The graphic organizer also included an example of each of these general mathematical principles using the contextual and mathematical elements from the Barry example. As Ms. Spence prompted students to use their graphic organizer to help them remember how to solve the problem, she indirectly referred them back to their work with Barry.

There were no instances where students explicitly referred to previous contextualized problems during their work on non-contextualized examples I observed. Instead, on three occasions students referred to instruction from previous years, as Kristen did in Episode 3. In that episode, Kristen cited work done the previous year despite the fact that the class had identified the rate of change in symbolic rules numerous times during the previous contextualized examples over the previous four instructional days. During the same activity, Marcus recalled writing a symbolic rule to fit a graph “last year” and Kavita referenced a previous teacher teaching her to graph a symbolic rule. These references are striking. Students seemed more likely to reference non-contextualized work done months prior than analogous contextualized work done just days before, a point that will be taken up in the discussion.
Upon analysis, there were a number of superficial differences between the contextualized and non-contextualized representations that might have obscured the similarity between the two types of tasks for students, making students less likely to refer to contextualized problems they worked on previously. For example, in many of the contextualized tasks, students calculated the rate of change from horizontal tables; but, when working with non-contextualized examples, the tables were either presented vertically or the data were presented in ordered pairs. And, the graphs of contextualized problems frequently displayed only Quadrant I of the coordinate plane, yet the graph paper Ms. Spence gave students to use to graph non-contextualized problems contained all four quadrants. These differences, though superficial, may have obscured the connections between contextualized and non-contextualized examples.

For its part, the textbook did not refer to previous contextualized examples within any of the non-contextualized examples in Investigations 1-3, as was the case in the excerpt in Figure 21. There were also no tasks that explicitly asked students to reflect across work on both types of problem settings. Instead, the text connected contextualized and non-contextualized work in these three investigations through transitional paragraphs, such as the one at the beginning of Figure 20. In that paragraph, the beginning of the description of general principles begins with a brief, implicit reference to students’ previous work in context “When studying linear functions, it helps to think about real contexts. However, the connections among graphs, tables, and symbolic rules are the same for linear function relating any two variables” (page 155). Because the non-contextualized examples directly follow this paragraph, this sentence could be seen as a
cue that the contextualized and non-contextualized examples were linked by the
generalizations described in the paragraph, but the connection is not made explicit.

The lack of cohesion between contextualized and non-contextualized problem
solving in the written and enacted curriculum was a significant factor that influenced
another important pattern in Ms. Spence’s instruction around non-contextualized
examples. At both points in the Core-Plus instructional sequence when students
encountered non-contextualized problems, Ms. Spence modeled for students how to do
the problems by first going over one example with the class and asking them step-by-step
how to complete the exercise. This practice was not aligned with the developers’ intent;
from the Core-Plus implementation guide, it is clear that students are intended to develop
solutions to tasks in the investigations in groups: “As students collaborate in pairs or
small groups, the teacher circulates among student providing guidance and support,
clarifying or asking questions, giving hints, providing encouragement, and drawing group
members into the discussion to help groups collaborate more effectively” (Hirsch et al.,
2011, p. 12).

Ms. Spence chose to directly model strategies for solving the non-contextualized
problems because she believed her students were inadequately prepared to successfully
develop strategies to solve these problems. Episode 3 provides an illustration when Ms.
Spence told students that they would do question 6b together. The fact that Ms. Spence
modeled how to do these exercises is particularly significant because she did not
similarly model CPs. In an interview, she noted how modeling the example conflicted
with the developers’ intentions. She went on to explain that she modeled one of the
exercises in question 6 (y = 2x) before letting students work in groups because she was
afraid that students would not know what to do when they got to that problem. She pointed out that they had not encountered a symbolic rule without a $y$-intercept of zero in the previous contextualized work. Because these were non-contextualized examples, students could not draw on any context to help them think about this new case.

To summarize, the students and teacher rarely reflected back on work with contextualized examples as they worked on non-contextualized examples. Two factors in particular may have contributed to the absence of this particular type of connection. First, the non-contextualized tasks contained elements that students had not encountered in previous contextualized examples. Second, the textbook did not support students or the teacher in connecting non-contextualized work to their previous contextualized work. At these points in the instructional sequences, the previous contextualized problem solving experiences no longer served as a reference point for class discussions. Although some students may have made the connections implicitly, the opportunity for all students to actively make sense of non-contextualized work through the lens of their previous contextualized problem solving experiences was not present. The transition to a focus on achieving fluency with conventional mathematical exercises, a part of what Wu (1997) calls “mathematical closure”, seems to have come at the cost of a lost connection to the contextual anchors described by the Core-Plus developers and RME theorists.
4.4 Episode 4: Illustrating Inconsistent Reference to Previous Contextualized Work While Summarizing Generalizing Mathematical Principles

Each Investigation in the Core-Plus textbook contained a section called Summarize the Mathematics; through the tasks in this section, students are meant to reflect on the mathematical ideas underlying the tasks they encountered in the investigation to “construct a shared understanding of important concepts, methods and approaches” (Hirsch et al., 2011, p. 12). Ms. Spence assigned the Summarize the Mathematics section in Investigations 1 and 2 and created her own summative activity targeting general principles for Investigation 3. To determine the extent to which the discourse during these tasks was explicitly connected to prior contextualized work, I analyzed student and teacher interactions around these sections for the presence of explicit references to previous contextualized problems. The following excerpt is from a class discussion about the Summarize the Mathematics section in Investigation 1. In the excerpt, there is no reference to previous, contextualized problem solving, which is representative of discourse that occurred around summative tasks addressing general mathematical principles in this unit.

Question b of the Summarize the Mathematics section of Investigation 1 asks the following:

How can the rate of change or the slope of the graph for a linear function be found from a

1. table of (x, y) values?
2. graph of the function?
3. Symbolic rule relating y to x?
4. NOW-NEXT rule. (p. 156)

After giving students time to work, Ms. Spence asked students to share their answers.
Ms. Spence: Now part b we’ve done several times today. I’m not gonna do it another fifty times but I am gonna show you and talk about it real quick. It says how can you find the rate of change or slope by looking at a table? How can I find the rate of change by looking at a table? Marcus explained it very nicely a couple times today. How do you find a rate of change when you look at a table? Javier, do you know?

Javier: Huh?

Ms. Spence: How do you find the rate of change by looking at a table?

Javier: You have to... [trails off]

Ms. Spence: TJ?

TJ: by looking at the y and x?

Ms. Spence: Okay so what would I do with the y's?

TJ: You would um... I forgot. Saby?

Saby: find the difference in the numbers of the y’s and x.

Ms. Spence: Find the difference in the y's find the difference in the x's. And in this case it would give me the change in y would be three over the change in x, which would be negative one. So the rate of change would be negative three. [writes $\Delta y \over \Delta x$ equals 3 over negative 1 equals negative 3]

This example is typical of discussion of the summative tasks for two reasons. First, in this episode, both Javier and TJ struggled to articulate the desired generalization; students tended to demonstrate difficulty understanding and answering questions aimed at having them articulate generalizations. Second, neither Ms. Spence nor the students referenced prior work on contextualized problems.

As shown in Table 1, Ms. Spence referred students back to a previous contextualized problem only once throughout their discussions of these summative type
tasks. Students never referred back to previous contextualized examples during these summative tasks, although on one occasion a student created a contextualized example involving hourly pay (similar, but not identical to Barry’s situation). In comparison, on three occasions, students referred to non-contextualized examples worked during the same class period in order to make sense of tasks focused on general mathematical principles.

There were a number of reasons why explicit references to previous contextualized activity might not have occurred during these portions. First, because the task progression went from contextualized to non-contextualized work, these summary activities did not tend to take place on the same day as students’ work with contextualized examples. There is evidence that proximity matters: students referenced non-contextualized work during these summaries only when the non-contextualized work was done on the same day. Secondly, neither the textbook nor the teacher explicitly pointed students to recall their previous contextualized work during this section.

Although Ms. Spence did not explicitly encourage students to reflect back on their previously contextualized work while discussing tasks that asked students to articulate generalizations, there were thirteen instances in which she referenced contextualized examples during other instructional activity focused on general principles, as shown in Table 1. Eight of these instances occurred during the creation of the graphic organizer around the Barry example, mentioned in the previous subsection. Although this activity was summative, like the Summarize the Mathematics section, this activity was different in that it was teacher directed instruction rather than a task. Students were not meant to articulate the generalizations on their own.
Other examples of explicit connections between activity focused on general principles and previous work on contextualized examples occurred at earlier points in the instructional sequence. As described in the previous section, Ms. Spence had students read the paragraph in Figure 20 and prompted them to reflect back over their contextualized work to make sense of the formula for slope. On three other occasions, she pointed to previous contextualized work when asking students questions about general mathematical principles during her class introductions.

In sum, when activity focused on generalized mathematical principles occurred in non-summative portions of the instructional sequence, Ms. Spence encouraged students to make sense of mathematical generalizations by recalling the contextual work from which it emerged. When discussing summative tasks in which students were asked to articulate generalizations, however, neither Ms. Spence nor the students tended to refer to contextualized examples. Similarly, the textbook did not contain references to prior contextualized examples either in these portions. During the push for mathematical closure at the end of the investigations, the contextual anchors were not explicitly leveraged.

4.5 Summary of the Findings

The purpose of this study was to examine the implementation of one Core-Plus lesson in one classroom to characterize the extent to which students’ experiences engaging with formal mathematical principles and conventions arose from and were rooted in students’ initial experiences with contextualized problems. Using the
conceptual framework presented in Figure 15, I characterized the types of connections the students and teacher did and did not make explicit between contextualized problems and non-contextualized problem solving and reflection on generalized mathematical principles. After identifying these patterns, I examined patterns in the teachers’ practices and the textbook design to seek explanations for the connections that students frequently made or did not make.

I examined how mathematical generalizations arose from students’ work on contextualized problems and found that discussion of generalized mathematical principles occurred frequently around contextualized problems, primarily because Ms. Spence explicitly pointed students’ attention toward the underlying mathematical principles. These generalizations arose spontaneously from students only occasionally. Statements of generalizations were sometimes present in the text before, within, or immediately after contextualized problems.

I also examined the extent to which students’ work throughout instructional sequences was connected back to their earlier experiences with contextualized problems. Students rarely made explicit connections back to contextualized problems during non-contextualized exercises or questions that asked them to reflect on general mathematical principles. There was evidence that for some students, contextualized activity functioned as a support for later work in tasks; Marcus, for instance, explicitly and implicitly referred back to a problem featuring Barry, the credit card company employee, when working on other contextualized tasks. But the findings suggest that, at the point in the instructional sequences where tasks shifted from contextualized to non-contextualized examples, students did not reference their work on previous contextualized tasks to
identify solution strategies. From this point on in the investigation, students’ mathematical understandings may have been more anchored to work with non-contextualized tasks from a previous year or to the direct instruction provided by the teacher rather than to related contextualized examples. Furthermore, because non-contextualized problems were more likely to be referenced during summative tasks intended to prompt the articulation of general mathematical principles, these mathematical generalizations may not have been built firmly on the conceptual foundation laid during students’ work with contextualized problems.

Finally, I looked for interactions between the students, teacher and text that facilitated or impeded connections between the various types of mathematical activity. By examining Ms. Spence’s practices through the lens of the conceptual framework shown in Figure 15, I identified two patterns in her practice that had the potential to facilitate connections across contexts and problem types. Ms. Spence occasionally pointed students’ attention to connections between contextualized tasks by asking them to identify the similarities and differences across problems set in different contexts, even those encountered during previous class sessions. During contextualized work, she attended to the vertical bridge between contextualized problems solving and reflection on general mathematical principles. However, during summative work generalizing mathematical principles, Ms. Spence did not consistently help students connect downward to contextualized examples. She also did not explicitly make horizontal connections between contextualized and non-contextualized examples.

In the three investigations observed in this study, the textbook did not support students and teachers in making connections between various contextualized problems;
students were not explicitly asked to reflect across different contexts. Similarly, the
students and teacher were not explicitly supported in leveraging students’ work on
contextualized problems to help them solve non-contextualized problems. Finally, during
the Summarize the Mathematics portion, students were not explicitly pointed back to
contextualized examples.

5. Instructional Coherence Across Task Sequences

The above findings reveal two significant complications involved in
implementing curricula that require students to make connections across different types
of problems and contexts. First, students do not seem to naturally reflect back on prior
work. Second, at later points in instructional sequences beginning with CPs, a focus on
mathematical closure can undermine the goal of anchoring students’ understanding of
formal mathematics to their initial contextualized problem solving experiences.

The finding that students did not spontaneously refer back to previous
contextualized work, particularly when working on non-contextualized exercises,
provides data supporting the claim that students are not necessarily disposed to attend to
the connection between instructional tasks, as noted by Lubienski (2000), Herbert and
Pierce (2011), and Gick and Holyoak (1983). Given that, in this study, investigations
took place over several days, and mathematics class was one of many classes and extra-
curricular activities students engaged in each day, this tendency is perhaps not surprising.
Put simply, the student experience of multi-session investigations may not be as cohesive
as textbook authors intend. The absence of spontaneous connections on the part of
students in this study may have been exacerbated by the fact that Ms. Spence’s class took longer to do Investigation 1 than recommended by the text, increasing the time elapsed between examples; but, even during Investigations 2 and 3, these connections were not present. From the suggested pacing guides, it is clear that the designers did not intend for the investigations to begin and end in the same class period. Moreover, work on later investigations is intended to build on students’ previous work. The finding that students were not inclined to reference related instructional tasks suggests that such connections may not be apparent to many students without explicit supports.

It may simply be the case that many students simply are not inclined to attend to mathematical connections between tasks that look different from one another. Research documenting the difference between the way experts and novices interpret and organize information provides potential insight into this possibility. Chi, Feltovich and Glaser (1981) showed that, when asked to sort physics problems, experts sorted problems according to the abstract physics principles that the problems addressed, which the authors describe as the “deep structure” of the problems. Novices, on the other hand, sorted the problems according to superficial, “surface structure” features such as the contextual elements involved. Silver (1979) demonstrated related findings amongst mathematics students; students who were more proficient in solving algebra word problems were more likely to identify deep structure similarities between verbal problems than their less proficient counter parts. These studies, which are limited to contextualized problems, suggest the need to support students in noticing similarities in the “deep structure” of related contextualized examples.
Chi et al.’s (1981) observation that novices tended to attend to superficial features of problems sheds light on a particular challenge students experienced transitioning between contextualized and non-contextualized tasks. Ms. Spence’s students explicitly recalled non-contextualized tasks worked in prior years, but they did not make explicit connections to mathematically relevant contextualized tasks they had completed as recently as the day before. Findings from the problem sorting studies described above suggest that students may have attended to superficial, surface structure similarities to other non-contextualized problems from past years. The non-contextualized examples students encountered in this unit may have looked more like the non-contextualized exercises they experienced in previous years because they were presented symbolically; contextualized examples in these Core-Plus investigations were presented mostly through text, horizontal tables, and graphs. When equations were used in contextualized examples, upper-case letters that clearly represented elements from the context were used, rather than the conventional lower-case $x$’s and $y$’s found in non-contextualized exercises. Superficial differences between contextualized and non-contextualized exercises such as these may play a significant role in determining whether students experience a sequence of tasks as a coherent whole.

Some students may be naturally inclined to internalize contextualized problem solving experiences and to see mathematically significant connections across tasks, as illustrated by Marcus in this study. But if these connections are not made explicit within the discourse of the classroom, access to a coherent experience is limited only to those who perceive the connection on their own. If the tendency to see connections between tasks somehow correlates in some way to students’ family backgrounds as Lubienski
(2000) suggests, then it is even more important that these connections are made explicit so that students from various backgrounds are provided with equal opportunities to learn.

Furthermore, if students are not inclined to spontaneously notice mathematical connections between tasks, the role of the teacher in helping them attend to general mathematical principles underlying multiple examples is crucial, as Mason (1996) suggests. Ms. Spence prompted students to shift their attention toward the generalized principles as they worked on contextualized problems. But she did not similarly call attention to the connections between contextualized problems and their non-contextualized analogs in this study. And like the teacher described by Davis (2007), Ms. Spence did not tend to reference previous contextualized tasks when engaging the class in discussion of summative tasks meant to elicit student generalizations.

One way to understand Ms. Spence’s decisions is to examine her beliefs through the lens of the conceptual framework in Figure 15. Like proponents of the Standards, Ms. Spence deeply valued contextualized problem solving for their ability to prompt students to make sense of mathematical ideas. But, like Wu (1997), she was deeply concerned with mathematical closure, and she worried that students would not make the connections intended by the authors. So, Ms. Spence prompted students to reflect across contexts and she frequently prompted students to attend to general mathematical principles as they worked and discussed contextualized problems. Her focus on mathematical closure continued, and even increased, as instructional sequences proceeded to non-contextualized problem solving and reflection on generalizing mathematical principles. Perhaps because of this focus, she did not tend to point students’ attention back toward previous work during these types of tasks. Instead, she
and her students tended to refer to the general principles that they had discussed previously. Ms. Spence also resorted to modeling non-contextualized tasks, even she knew this contradicted the intent of the curriculum authors. The fact that she did not point students’ attention back to contextualized problems during non-contextualized tasks is particularly striking given her desire for students to see the connection between different types of mathematical tasks.

Examination of Ms. Spence’s beliefs, and the practices that she adopted as a result, suggest a possible tension between the desire for mathematical closure and the desire for students’ mathematical understandings to remain anchored to their common sense understanding of the world around them. An important question emerges: is this tension the result of a conflict between two mutually exclusive aims?

After close analysis of Ms. Spence’s instruction, through the lens of the conceptual framework, I argue that these aims are not at conflict with one another. Both aims could be achieved if students were reminded of their contextualized work while they engaged in non-contextualized problem solving or reflection on generalized principles. Particularly in light of how different these types of problems appear on the surface, at the level at which students notice similarities, these reminders seem essential. Reminding students of the contextualized problems in this way would likely increase the opportunity for students to “fall back” on the common sense understandings from which the generalizations arose, as described by van den Heuvel-Panhuizen (2003).

Curriculum materials play an important role in helping teachers and students achieve mathematical closure while maintaining connections to students’ initial sense-making experiences. The developers of Core-Plus intend for students’ work with non-
contextualized mathematical tasks to draw meaning from their work with contextualized problems. The fact that neither Ms. Spence nor her students referred back to contextualized examples during work on non-contextualized tasks or summarizing activities suggest that, if students are not explicitly supported in reflecting back on previous contextualized work, the developers’ intent may not be realized for all students.

6. Conclusions and Implications

This study is limited to a single classroom, but the analysis provides significant insight into the challenges inherent to the implementation of curriculum programs based on CPs. It appears that problems set in extra-mathematical contexts can provide students access to mathematical ideas and, with the help of teachers, these problems can act as settings from which mathematical formalizations and generalizations emerge. But, initial contextualized problem solving experiences may not provide the anchoring role that curriculum authors intend. If students’ understanding of formal mathematical concepts are to remain rooted in their common sense, students, teachers, and curricular materials need to maintain a reflective eye back toward initial sense making experiences, especially as attention turns toward mathematical closure.

There are a number of ways that teachers can help to strengthen the ties between formal mathematical ideas and initial contextualized problem solving experiences. Like Ms. Spence, they can focus students’ attention on the desired mathematics during contextualized work by attending to the connections between contextualized tasks and prompting students to reflect on the mathematical principles underlying the tasks. When
the sequence progresses other types of tasks, teachers can explicitly prompt students to recall previous problems at the transition point and while they work tasks. If consistently reminded to review prior work, students may learn this practice as a beneficial strategy for gaining insight on new tasks. During summarizing work, teachers can direct students to reflect on their previous contextualized problem solving experiences and explicitly identify the particular tasks from which general principles emerged.

In order for these recommendations to be realized, teachers would need to have opportunities to reflect on the ways in which they do and do not promote instructional cohesion in their classroom. Furthermore, these practices require an understanding of the relationship between different types of instructional tasks and the roles these tasks play in instruction. The analytical framework used in this study has the potential to be used with teachers and possibly even students to help them develop such an understanding. Examining teachers’ use of such a framework would be a fruitful site for further research.

The above analysis also leads to several recommendations for curriculum developers. In 1996, Ball and Cohen noted that teachers’ guides did little to “help teachers think about the temporal dimensions of curriculum construction” (1996, p. 7) and they argued that curriculum materials could help teachers make connections across lessons and units. Sleep (2012) argues that making these connections within and across lessons is one crucial aspect of what she calls attending to the mathematical purpose of a lesson. There are a number of ways that curriculum designers could support teachers and students in making connections across lessons and, possibly more importantly, within contextualized-problem based lessons. First, curriculum guides could direct teachers’ attention to the particular tasks from which specific general principles are meant to arise.
If the teacher is aware of which particular lesson objective or mathematical concept each task targets, they might be more likely to help students attend to that generalization during those particular tasks. Second, the curriculum could explicitly prompt students to look back at their previous work when responding to summarizing tasks. In the lessons analyzed here, this sort of prompt was not present; however, in other Core-Plus investigations, the designers do refer explicitly to previously worked problems. Consistent reminders to reflect back could foster intentional reflection as a normative practice. Finally, when students encounter non-contextualized examples, the curriculum could explicitly identify connections to previous contextualized work. There are examples of this at other points in the Core-Plus curriculum. For instance, in the first investigation in Course 1, Unit 3, Lesson 3, students work in contextualized examples to write equivalent symbolic algebraic expressions. The next investigation, Investigation 2, represents a transition to non-contextualized work. The prompt for the first question in Investigation 2 explicitly refers students to previous problem situations: “These expressions might represent the profit for a given number of sales. Using your thinking from Investigation 1 as a guide, write at least two different but equivalent expressions for each” (p. 19). This sort of explicit cue may increase the likelihood that students leverage prior work.

As many curriculum programs transition from print to digital presentation, more options for how to make these types of cues and connections will be available. Hyperlinks and pop-up windows, for instance, offer two ways to increase cohesion without disrupting the flow through instructional sequences. As these new instructional delivery systems are produced, it is imperative that designers pay close attention to
instructional cohesion and intentionally work to help students see the connections between instructional tasks. Research on how students and teachers take up these types of supports would add valuable insight for curriculum developers, particularly as new technology evolves.
CHAPTER 5: CONCLUSION

In 1973, Hans Freudenthal proposed that instruction beginning with contextualized problems can prompt students to re-invent mathematical ideas and that learning mathematics in this way results in deep understanding rooted to the learner’s common sense understandings of the world around them (1973). The theory of realistic mathematics education, developed by Freudenthal and others at the IOWO in the Netherlands, guided the direction of curriculum development in that country for decades.

In the US in the 1980’s and 90’s, a wave of reform set the stage for similarly-minded mathematics educators to design curriculum programs that, like RME, emphasized learning through active contextualized problem solving. Although some of these curriculum programs were developed intentionally from RME principles, others, like Core-Plus (Hirsch et al., 2011), were developed independently but arrived at a similar approach.

Inspired by the RME theory and puzzled by the reticence with which contextually-based curriculum programs have been taken up in the US, I observed the implementation of Core-Plus in a single classroom to find out what instruction from a contextual problem-based curriculum looks like in a reality. I wondered how students would engage with the CPs and whether their understanding of formal mathematics would emerge from their responses to those problems.
1. Summarizing the Findings

After spending two to three mornings a week for four months in a single classroom, I have gained a deep appreciation for the complexity of CP-based instruction. I remain optimistic about the promise of this form of instruction; however, I am more aware of how challenging it is to effectively leverage CPs toward the development of formal mathematical understanding.

The first, and possibly the most important contribution this study provides to the mathematics education community is the Contextualized Problems in Mathematics Instruction (CPMI) framework: an analytical framework that allowed me to systematically make-sense of the way the curriculum transformed as it passed from the written page, to Ms. Spence’s intended plans, to transcripts of the classroom dialog. The CPMI framework has the potential to inform teachers, curriculum designers, and future research into the way contextualized problems are used in mathematics instruction.

Using the CPMI framework to identify and characterize the types of questions and tasks that were present in the written and enacted curriculum, I found that Ms. Spence supplemented the written curriculum with tasks and questions that explicitly prompted students to reflect across contexts. This type of activity, which was largely absent from the written curriculum, has the potential to greatly increase the cohesion of contextually-based mathematics instruction.

Using the CPMI framework to compare between the written Core-Plus curriculum and Ms. Spence’s intended curriculum, I found that the sequence from CPs to non-contextualized problems present in the written Core-Plus curriculum was not consistently preserved in Ms. Spence’s plans. The reasons for this were numerous,
including her ambivalence about CP-first instructional sequences, a desire to order tasks in terms of mathematical complexity, and a focus on covering the material required by the state standards. The finding that these sequences were not preserved is significant, because students had fewer opportunities to make sense of non-contextualized exercises by referring back to previous contextualized work, as intended by the Core-Plus developers.

By using the framework to examine the connections or lack of connections between the various types of instructional activity, I found that students were not inclined to reference prior tasks, and that they were not supported in doing so once the instructional sequences transitioned from CPs to problems that asked them to engage with mathematics presented in the abstract, even when the CP to non-CP to generalizing mathematical principles sequences were preserved. This finding is significant because it shows that opportunities for students to make sense of formal mathematics through the lens of their contextualized problem solving experiences may not be leveraged in classrooms to the extent that the curriculum authors anticipate.

Together these findings lead to numerous recommendations for curriculum development and further research. Most notably, in the third and fourth chapters, I propose a number of ways that curriculum designers can support students and teachers in fully leveraging work on contextualized problems toward the development of formal mathematical understanding.
2. Implications for Further Research

In each of the manuscripts, I argue that more research into the use of CPs in instruction is necessary to further our understanding of the supports and constraints that contextually-based instruction offers. In the next section, I will describe in detail my recommendations for further research. I begin by describing possible research into existing curricular materials. I then describe potential research into student responses to CP-based instruction. I conclude by describing potential research into teachers’ implementation of CP’s.

2.1 Profiling CP-based instructional approaches using the CPMI

In the first manuscript of this dissertation, I described in detail the CPMI framework, which is designed to characterize instructional activity around CPs. In the two articles that followed, I described how the role of CPs changed as the curriculum transformed from the written page into enacted curriculum, especially in relation to other types of mathematical activity. But the scope of the study did not permit me to analyze in detail the activity within the contextualized problem solving category as I originally intended. In developing the CPMI framework, I used a common representation of mathematical modeling to describe four categories of instructional activity that were possible around contextualized-problems: focus on context, produce model, focus on model, and interpret model. The framework also distinguishes between formal models and informal models, which are often invented by students. Although there was not space to analyze the intended or enacted curriculum in terms of these types of activity, I did analyze the written curriculum using these codes. I found that few questions in this
Core-Plus unit were directed at the contextual settings themselves. In the curriculum guide, the authors recommend that teachers supplement the written curriculum with questions that orient students to the contexts if necessary. The other three types of activity within the contextualized example domain are consistently asked in the written curriculum, and there is much to consider about the ordering and types of tasks within these categories. In the Core-Plus units I analyzed, for instance, approximately half of the CPs provide the students with ready-made models and then ask students to interpret those models. The remaining half of the tasks ask students to work in the opposite direction in terms of the framework; they contain descriptions of contextual situations and students are prompted to create a model. The curriculum also seems to take what Gravemeijer (1994) describes as an organizing approach to modeling: the authors prompt students to produce formal models, rather than posing questions that prompt students to create informal models. I look forward to analyzing the classroom transcripts further to examine the degree to which students produce informal models or strategies to answer contextualized problems.

I also plan to use the CPMI framework to compare across curriculum programs. Comparative analysis will reveal similarities and differences in the types of tasks that are included in contextualized problems and how CP’s are used relative to other types of activity. In the units of the written Core-Plus curriculum analyzed here, for instance, instructional sequences tend to begin with contextualized problems, transition to non-contextualized examples, then end with reflection on generalized principles. But, in the enacted curriculum, Ms. Spence tended to prompt students to generalize while debriefing contextualized problems, before students encountered non-contextualized problems. She
referred to these generalizations, rather than to previously worked CPs as she modeled how to do non-contextualized problems. I wonder whether other curriculum programs take an approach similar to the one Ms. Spence used or whether this approach is used elsewhere in Core-Plus. RME and MMP approaches both seem to suggest that students should reflect on the models they create in response to contextualized problems before applying to non-contextualized problems. Comparing student responses to different types of sequences would no doubt provide much needed insight into the challenges and benefits of both approaches.

2.2 Digging Deeper into Student Responses to CP-based Instruction

The second potential direction for future research involves student responses to CP’s. At the outset of the dissertation, I hoped to identify patterns related to whether students took ownership of, engaged with, or resisted the various types of tasks and questions. There was not room to address these questions, so I plan to analyze the data further to look for patterns in how students in Ms. Spence’s class reacted to the different categories of questions described in the CPMI. Did they more frequently ask for help during one type or another? Did some students engage mathematically with contextualized tasks but immediately ask for help from non-contextualized tasks? Did they engage in fewer off task behaviors when the mathematics was contextualized? Without rigorously analyzing the data, it does seem that students struggled more with non-contextualized examples and tasks that asked them to reflect on general mathematical principles than they did with contextualized problems. Also, some students
seemed to show more engagement when they could use a context to help them understand a problem. During work non-contextualized problems, or problems when the context did not actually help students understand the mathematics, these students were more likely to spend time off task.

During my observations, I also became very interested in another student response to CPs, one that I describe in my field notes as “when the context becomes real… or at least when it means something vs. when [the context is] irrelevant or almost non-existent”. Some contexts seemed to prompt students to legitimately engage students; students seemed genuinely interested in the setting. For instance, in the unit on statistics, one student looked at a box plot and questioned whether it made sense, saying “Naw, ‘cause when I was his age I was five foot seven. I guess I was in the eightieth percentile?” It is unclear whether this genuine engagement led to deeper engagement with the mathematics, but this authentic engagement seems important to research. I am particularly interested in understanding what teacher moves might have prompted authentic interest and whether there was something about the contexts that promoted this sort of reaction that was different than those contexts that did not prompt this sort of engagement.

While transcribing classroom transcripts, I also became interested in the resources students used to solve problems. Some students seem to lean on the context of the problem to make sense of the mathematics. Other students, though not many, seemed more inclined to draw on previous problems. And finally, others seemed to draw on generalizations that the class had made, like the statement that the “slope is the one with the x” in a symbolic rule. An example from a partner interaction will help to clarify this
point. Two students in Ms. Spence’s second section, the section I did not analyze for the papers in this dissertation, were working on a Core-Plus problem involving Cheri, who sells “services” for the credit care company. Her pay structure was similar to that of Barry, who sells credit card applications in the previous problem.


The two students drew on different resources to make sense of the problem. Kenny leveraged the context and Greg looked back at the previous problem.

Kenny: what about these two? [pointing to the cells under 100 and 101 services sold.] That would be like two something right?

Greg: No. It'd be more than that. See look this is how you find the rule yo. [rapping] You find the rule by lookin’ at this and lookin’ at this. See how this change from this to make the rule that. In other words you gotta figure out why this much equals that much. To get this much. You understand?

Kenny: So basically it's like, you get paid forty dollars for no applications automatically. And then for ten he gets...

Notice that Greg does not use any contextualized language. His description involves looking for a pattern in the values. Kenny, though, leans on the context of the problem, using contextualized language to try to describe the situation. As the interaction
continues, Greg uses the previous problem as a resource, but, again he does not use contextualized language.

Greg: Yeah just like the other one. You know that the rule was plus twenty times five n. So when n was zero, twenty. So for this one it's plus forty.

Kenny: And so for every five.. so for every five you get ten. Basically.

Kenny, once again, seems to be contextualizing the situation, saying “for every five you get ten”. Although he doesn’t refer to dollars, applications, or services, he says “you get ten” which could imply that for every five hours you work, you get ten dollars. This seemed to be a pattern in their interactions. Greg was one of the rare students who explicitly referred to previous problems as he worked. Kenny, on the other hand, seemed to draw heavily from the contexts the problems were in. Kenny tended to express frustration when working non-contextualized problems, because this support was no longer available.

Another question arises when considering students like Greg, who did seem to draw on previous work. In the study of the enacted curriculum, I examined the degree to which students made explicit connections between various instructional tasks. But, some may argue that students likely make these connections implicitly. One approach to analysis that I began, but was not able to complete due to the limitations of the data set, was to identify whether students transferred strategies learned from contextualized problems implicitly to their work on non-contextualized analogs. As I explored the possibility, the RBC+C framework (Hershkowitz et al., 2007, 2001), described in chapter 2 showed a great deal of promise as an analytical tool for this task. The RBC+C
framework describes actions learners perform to construct abstractions: recognizing, building with, constructing, and consolidating. Using this framework, a researcher could track how students created abstractions during contextualized problems, then attempt to determine whether they recognized and built-with these abstractions during work on non-contextualized tasks. This sort of analysis, across students, would provide insight into how frequently students implicitly recognize abstractions constructed in response to previously-worked contextualized problems.

Perhaps the most promising research question that has emerged from this study came in response to an intervention I used when assisting students who were not participating in the study. When working on non-contextualized tasks, I experimented with reminding these students of their prior contextualized work. Often, these cues helped students tap into their previous experiences to use as supports for solving these problems. On more than one occasion, students seemed surprised to find out that the strategies used to understand contextualized problems actually applied to non-contextualized tasks. In Chapters 3 and 4, I recommend that teachers and curriculum designers cue students to remember previously worked problems. It would be valuable to study the effect that this intervention has on students’ ability to solve non-contextualized analogs to previously-worked contextualized problems.

2.3 Teachers’ Use of CP-based Curriculum

One final potential research strand that emerges from this study involves teacher responses to CPs. In this dissertation, I observed only one teacher. A similar analysis, across a number of teachers, would no doubt provide further insights about instructional
strategies that can be used to promote cohesion and increase the degree to which CP’s are leveraged toward formal mathematical understanding. The research community’s understanding of how teachers use CPs would be greatly enhanced by analyzing a study that examines two contrasting groups of teachers use of CP-based curriculum: those who more strongly believed that instruction should begin in contextualized problems whenever possible, and those who believe strongly that the CP-first approach is detrimental to student learning.

In each of the manuscripts I propose that the CPMI framework has the potential to serve as a valuable tool for professional development. At the outset of this study, I hoped to share the framework with the teacher midway through, and to observe how the framework informed her subsequent instruction. Unfortunately, the development of the framework took much longer than I anticipated, and this was not possible. After the study was over, I shared the framework with Ms. Spence and she and I discussed my recommendations for helping students more fully leverage their contextualized experiences. I was not able to return to her classroom to collect data on the results of this discussion, but she indicated that she found the framework to be a helpful lens for reflecting upon her own practice. Research into how teachers can use the framework to understand and transform their own practice will hopefully follow.

3. Conclusion

As I wrote this conclusion, I received an email that the president of the National Council for Teachers of Mathematics, Diane Briars, sent to all NCTM members (Briars, 2014). The email lists thirteen “Top Lessons Learned” about effective curriculum
materials evaluation. In the email, Briars recommends that those who evaluate curricula should look for “the use of applications to introduce new content, as well as to apply concepts and skills after initial instruction.” This email serves as an indication that the use of CPs at the beginning of instructional sequences will continue to be emphasized, as more and more curriculum materials are developed to align with the Common Core State Standards. Another indicator of the continued prevalence of contextually-based instruction comes from my own work on a curriculum development project, which aims to utilize increased access to technology to provide media-rich lessons for students. A digital environment lends itself to rich, contextualized problems presented in ways that do not require extensive textual descriptions. Our instructional sequences often begin with extra-mathematical contexts. The findings of this dissertation have significantly informed my practice in this endeavor, and it is my hope that these articles and my future research using the CPMI framework will inform others as well.
BIBLIOGRAPHY


