Silent or Salient? Perks and Perils of Performance Posting

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Silent or Salient? Perks and Perils of Performance Posting

Abstract
Many firms in the U.S. spend more on their sales force than they do on other marketing activities. Thus, improving sales force performance is of paramount importance. A controversial way is to post performance (i.e., display everyone’s performance), now done with ease on social platforms due to advances in information technology. On one hand, posting performance encourages social comparison and competition. On the other hand, it may discourage low-end performers. Also, not posting performance may encourage greater effort from sales agents to push ahead or avoid falling behind, if they are unaware of how others are doing. The result of these opposing factors is, prima facie, unclear. I study the effectiveness of performance posting using theory and experiments. In a game-theoretic model of incomplete information about agents’ abilities, I allow a firm to control the precision of social comparison by choosing whether to post performance. Firstly, I find that a firm should not post performance when agents’ abilities are sufficiently homogenous, as this prevents a low-ability (high-ability) agent from being overly discouraged (overly complacent). In contrast, a firm should post performance when agents’ abilities are sufficiently heterogeneous, as a low-ability agent puts in more effort to avoid lagging by too much. Secondly, some social comparison or competitiveness helps performance posting but too much hurts, i.e., there is a non-monotonic relationship in its effectiveness of performance posting, due to tradeoffs between how much a high-ability and low-ability agent changes effort. Thirdly, I find that the firm’s profit from posting increases (decreases) when the financial compensation is unattractive (attractive). Said differently, firms that pay less are more likely to benefit from posting, and therefore, more likely to post. Next, I demonstrate the empirical validity of these propositions using a series of lab experiments and a field study. Together, my theoretical and empirical results provide guiding principles on when a firm can benefit from performance posting.

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SILENT OR SALIENT? PERKS AND PERILS OF PERFORMANCE POSTING

Cexun (Jeff) Cai

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Presented to the Faculties of the University of Pennsylvania

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University (The Stern School), Texas A&M University (Mays Business School), University of Notre Dame (Mendoza College of Business), University of South Carolina (Moore School of Business), University of Texas at Dallas (Naveen Jindal School of Management) and University of Virginia (Darden School of Business).

Many others ought to be mentioned – the list of which, I think, is lengthier than this dissertation. As such, I will have to confine myself to these important groups of people. My close friends and colleagues from the Wharton Marketing department, the Wharton Doctoral Programs, the Wharton Behavioral Lab, and the Wharton Risk Management and Decision Processes Center – all of you have been instrumental in providing the best possible research environment any one could possibly imagine.

Naturally, I leave the closing acknowledgement for my dear wife and lifelong partner, Kayla Chen, who is and has always been my strong pillar of support. Thank you for giving up a big portion of your life and its familiarities to make this journey possible, and for accompanying me on it.
Many firms in the U.S. spend more on their sales force than they do on other marketing activities. Thus, improving sales force performance is of paramount importance. A controversial way is to post performance (i.e., display everyone’s performance), now done with ease on social platforms due to advances in information technology. On one hand, posting performance encourages social comparison and competition. On the other hand, it may discourage low-end performers. Also, not posting performance may encourage greater effort from sales agents to push ahead or avoid falling behind, if they are unaware of how others are doing. The result of these opposing factors is, *prima facie*, unclear. I study the effectiveness of performance posting using theory and experiments. In a game-theoretic model of incomplete information about agents’ abilities, I allow a firm to control the precision of social comparison by choosing whether to post performance. Firstly, I find that a firm should not post performance when agents’ abilities are sufficiently homogenous, as this prevents a low-ability (high-ability) agent from being overly discouraged (overly com-
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Chapter 1

Introduction

What do people experience when they observe their own performance posted vis-à-vis similar others’ performance on the same task? Do people respond differently on a task when they know that their performance will be posted? Knowing how people respond, can a social planner strategically decide to post or not performance? These questions lie at the heart of my dissertation as I explore the topic of performance posting.

Performance posting is relevant to many disciplines. In marketing, the use of performance posting has posed a perennial puzzle within sales force management. Firms and sales managers today continue to wrestle with whether it is optimal to use performance posting. Furthermore, the growing ubiquity of social platforms for sales management amongst groups of sales employees has intensified the need for an answer. In health care, policy makers debate vigorously about whether performance posting of doctors and hospitals would hurt or help the quality of medical services\textsuperscript{1}. Likewise, public health officials explore the use of restaurant hygiene grade card posting to reduce the prevalence of foodborne illnesses\textsuperscript{2}. In education, a prolonged debate ensues on whether teacher ratings should be posted, and whether doing so would increase the quality of teaching – judges had

\textsuperscript{1}\url{http://www.intrahealth.org/page/motivating-providers-by-posting-performance-data}

\textsuperscript{2}\url{http://publichealth.lacounty.gov/eh/misc/ehpost.htm}
to deliberate hard on this decision\textsuperscript{3}.

Why is the desirability of performance posting difficult to assess? The benefits of performance posting appears to be, in the spirit of competition, seemingly obvious. And yet, it raises apprehension in the minds of social planners who, in addition to privacy concerns, recognize that performance posting as a practice may lead to a drop in morale and thus have negative effects on performance. The tension between these forces is not immediately clear, and their resolution is, at best, opaque. Perhaps this is why confusion and frustration underlie the ongoing debates about the effectiveness of performance posting. In my dissertation, I shed light on the outcomes of performance posting, and provide prescriptive guidance on when a social planner can use it.

Despite its breadth of applications, I take a modest first step by studying performance posting in the context of a sales force. There are two reasons for this. First, the sales force is a substantive area in marketing, and sales force expenditure is several times that of other marketing spends. Thus, the potential impact of performance posting on sales force productivity can be sizable in absolute terms. Second, the sales force within the firm is an ideal starting point for the study of performance posting, since there is little ambiguity to what performance means for the firm, and consequently, how performance posting translates to changes in the firm’s profit.

I begin the next chapter of my dissertation with the motivation and roadmap for the study of performance posting in the sales force. In the closing chapter, I will return to the prevalence and relevance of performance posting across different domains, and how I can apply my findings to generate insights in these areas.

\textsuperscript{3}http://articles.latimes.com/2013/aug/01/local/la-me-ln-teachers-ratings–20130801
Chapter 2

Motivation

Each year, firms in the U.S. spend $800 billion on sales force compensation and hire at least 20 million people to carry out their sales function (Zoltners et al. 2008). Given that many businesses spend more on their sales force than they do on any other marketing activity, improving the performance of the sales force is of paramount importance. Yet, managing the sales force is an often overlooked area in marketing research.

A firm has several levers to increase sales performance. The lever studied most in the marketing literature is sales force compensation. This broadly includes designing optimal compensation plans (e.g., Basu et al. 1985), accounting for heterogeneity in the sales force (e.g., Raju and Srinivasan 1996, Rao 1990, Misra, Nair and Daljord 2013), estimating the effectiveness of compensation plans (e.g., Chung, Steenburgh and Sudhir 2013, Misra and Nair 2011), and using sales contests in the form of rank order tournaments to motivate efforts (Kalra and Shi 2001). Another lever is territory allocation (e.g., Lodish 1975, Zoltners and Sinha 2005), i.e., how to allocate sales force across different geographies and industries. The interaction between sales compensation and territory allocation has also attracted attention (e.g., Caldieraro and Coughlan 2009).

More recently, progressive firms are leveraging social incentives to increase sales force performance. Specific levers include social pressure (Steenburgh and Ahearne 2012) and forced rankings (Ahearne et al. 2012). Another social lever that is intuitively important to
managers, relatively inexpensive to implement as far as out-of-pocket costs are concerned, potentially controversial, but not well-studied – posting performance in a social setting within a group. When is it profitable for firms to post performance of sales people versus not post performance? On one hand, posting performance might encourage social comparison and competition in the workplace. On the other hand, it might have the unintended effect of discouraging low-end performers. Also, not posting performance might yield better aggregate performance if being unaware of how others are doing translate to employees putting in more effort to push ahead or avoid falling behind.

Even though the question of displaying everyone’s performance (or not) is straightforward, the result of these opposing factors is unclear. There is also lack of guidance to this question. Existing literature has examined how relative performance affects compensation decisions (e.g., Kalra and Shi 2001, Lazear and Rosen 1981, Lim et al. 2009), but not when it is actually profitable for firms to make available information on relative performance when there exist information asymmetries with regards to the performance of others.

This question is not only important, but also increasingly relevant given recent advances in information technology. Enabled by the proliferation of social platforms, relatively low-cost software such as RepTivity, Hoopla and LevelEleven now integrate with customer relationship management databases provided by companies like salesforce.com. This allows for interfaces designed to share accurate information about the performance of sales agents in a group, as seen in Figure 2.1.

The increasing ease of posting performance has not, however, translated into an across-the-board adoption. Anecdotally, performance posting has received mixed reviews among sales agents and managers. Some find performance posting useful, citing it as “a public expression of progress, sales goals and objectives”, without which, sales agents “lack the
visibility into where they are in relation to the team and to their peers." Others decry performance posting as “childish” and “useless”. Given that performance posting affects the morale, effort and consequently, performance, we need more than anecdotal evidence of its value or lack thereof. More rigorous research can help firms assess the value of performance posting, and decide when to let sales agents know or not know their performance in a social setting, i.e., when to make it salient and when to remain silent about it.

How then should a firm decide when to post or not post performance? A firm whose objective is to maximize aggregate performance needs to consider how interdependencies among sales agents, specifically, how knowing one another’s performance, influences the collective outcome. To address this, I set up a stylized model of a single firm and two sales agents in a sequential game of incomplete information. I allow for agents' abilities to be private information that is drawn from a distribution which is common knowledge. In my model, the firm moves first and commits to performance posting (or not), and agents then simultaneously decide how much effort to exert in order to generate sales. I solve for a

\[ \text{http://www.mediasalestoday.com/should-managers-use-leaderboards-to-motivate-sales-teams/} \]
Perfect Bayesian equilibrium which arises from the strategic interdependency between a firm’s decision to post performance and the amount of effort agents put in.

Using the model, I develop several theoretical propositions on how performance posting affects a firm’s profit. First, I find that if agents’ abilities are sufficiently homogenous, a firm obtains higher profit by not posting performance. But if there is sufficient heterogeneity in agents’ abilities, a firm obtains higher profit by posting performance. Second, the extent of social comparison between the agents moderates the effectiveness of performance posting in a non-monotonic way. Specifically, social comparison has an inverted U-shaped effect on the effectiveness of performance posting. Some social comparison is good but too much is bad. By this, I refer to the degree of social comparison between agents that moderates the aggregate amount of utility both agents get from the simultaneous comparison of performances with each other, as will be made clear in the model chapter of this dissertation. Lastly, I find that the attractiveness of the financial compensation also affects the effectiveness of performance posting. In particular, I find that an attractive financial incentive reduces the differential profit between posting performance versus not. To test whether the propositions from my model hold up, I examine their empirical validity using a series of experiments, and find that there is good evidence. Taken together, my theoretical and empirical results suggest that firms need to carefully consider the interaction of multiple factors before making their decision on whether to carry out performance posting.

The remainder of this dissertation is as follows. Chapter 3 covers the various literatures related to the question I have posed. Chapter 4 presents a game-theoretic formulation of the problem and solves for a firm’s profit under the presence and absence of performance posting. Comparing the difference in profit across both cases, I outline the scenarios under which it is optimal for a firm to post performance, and explain why this is so. Chapter 5 presents a series of lab experiments designed to test the internal validity of my model, and to see whether my predictions hold up. I also run a field study to demonstrate the
ecological validity of my theoretical propositions. Chapter 6 concludes with a discussion of
the findings and managerial implications. For purposes of the dissertation, I have included
two additional exploratory chapters. Chapter 7 provides a perspective on issues which is
not only of personal interest, but serves as an outlet for the dimensions on which I can
advance future work on performance posting. I provide in Chapter 8 preliminary insights
from one such extension on “partial” performance posting.
Chapter 3

Literature Review

Performance posting relates to the social effects of information disclosure. Performance posting is, after all, a practice that informs one about the performance of others – itself an under-studied but important area in the sales force literature (Amaldoss et al. 2008) – and also informs others about one’s performance. In this chapter, I draw from several literatures (marketing, economics, psychology, neuroscience and education) and organize the findings into two main themes. In Section 3.1, I examine how social comparison matters and how it can affect behavior. In Section 3.2, I examine the literature on how a firm can provide social information to influence behavior, which, like performance posting, is a form of mechanism design. In Section 3.3, I explain the gap in our understanding of when performance posting is effective, and how my paper intends to bridge this.

3.1 Social Comparison Affects Behavior and Utility

The literature on social comparison often begins with Festinger (1954), whose seminal paper discusses the drive for self-evaluation within an individual that necessitates comparison
with other people, i.e., we look to others for information that will help us evaluate ourselves on dimensions relating to abilities or skills. The broader stream of literature relating to social motivation, however, goes back even further to the idea of social facilitation in Triplett (1898) who finds that bicycle racers had faster times when they raced with others than when they raced alone, children reeled in fishing lines faster when performing alongside another child as compared to when they were reeling on their own, and that the “bodily presence of another contest participating simultaneously” can “liberate latent energy not ordinarily available”, i.e., increase competitive motivation. These examples in Triplett (1898) are what Zajonc (1965) later categorizes as coaction effects (how behavior changes when other individuals are performing the same activity), versus another paradigm within social facilitation – audience effects (how behavior changes in the presence of passive spectators). In particular, Zajonc (1965) argues that a consequence of being in the presence of others is that is serves as arousal for organisms, and this heightened arousal can improve (hurt) an organism’s performance on familiar (unfamiliar) tasks.

Broadly, the informative aspect of social comparison (Festinger 1954) is analogous to what Deutsch and Gerard (1955) call informational social influence, which is that we “accept information obtained from another as evidence of reality”. Deutsch and Gerard (1955) also contrast this to normative social influence, which is that we want to “conform with the positive expectations” of “oneself, of a group, or of another person”. Both informational and normative social influence stand in contrast to the competitive aspect of social facilitation (Triplett 1898, Zajonc 1965), which recent literature in social comparison relates more to, and goes one step further by exploring how social comparison serves direct purposes such as self-enhancement (Wood 1989). This development is further complemented by recent papers in neuroscience, where evidence from functional magnetic resonance imaging (fMRI) show how social comparison triggers motivation-related processes in the brain during a rewards-related activity (Fliessbach et al. 2007), thus providing neurological support
for the commonly-observed psychological phenomena of people wanting to do well relative to others (Bunnk and Gibbons 2007). This brings us closer to this idea that emotions arising from social comparison may also be a primary source of utility: Dvash et al. (2010), also using fMRI, finds that participants who lost money expressed joy and gloating if the other player had lost more money, and participants who won money expressed envy if the other player had made more money. Said differently, even when a person loses money, merely adding information about another person’s greater loss may increase certain brain activations to a point similar to those of an actual gain. Taken together, this growing body of evidence provides motivation for economic models that allow for both absolute and relative measures to enter into a person’s primary utility function – an approach which economists are now using to resolve certain paradoxes in the economics of happiness (Clark et al. 2008).

### 3.2 Managing Social Comparison by the Strategic Firm

Given that social comparison affects behavior and utility, a firm can act in a manner that is optimal to its own objective if it knows how people engage in social comparison. This applies across a variety of settings in marketing. When a firm interfaces with multiple consumers, it can use social norms to influence consumers’ actions. For example, Goldstein, Cialdini and Griskevicius (2008) find that providing social norms in terms of towel reuse (e.g., a majority of other guests reuse towels as part of environmental conservation) increased guests’ towel reuse rates relative to traditional appeals focused solely on environmental conservation. Moreover, these were most effective when they “most closely matched individuals’ immediate situational circumstances” – evidence that perceived similarity, an important component of social comparison theory, mattered in influencing guests’
behavior. In another marketing context, knowledge about how individuals relate to others in shared goal-pursuit programs (e.g., alleviating uncertainty through similar others when participating in a Weight Watchers program) builds on understanding how and when social comparisons are made, and can help marketers “enhance consumer involvement” (Huang et al. 2015).

In managing these social comparisons, strategic behavior by the firm becomes most evident in the marketing literature when a firm is managing its sales force, and has to interface with multiple sales employees. In particular, Lim (2010) finds that when sales employees exhibit disutility of losing to others, a firm can obtain greater effort from its sales force by changing the proportion of winners to losers when designing its sales contest.

### 3.2.1 Performance Feedback in Other Literatures

Given that social comparison affects behavior and utility, and that a firm can manage social comparison in a strategic way, my research focus will be on a firm’s use of performance feedback as a way to influence social comparisons in a sales force. Currently, there is limited work in the marketing literature that relates to this question. Hence, I will draw from the performance feedback literature from other disciplines and then relate it to my research question.

Azmat and Iriberri (2010) study the effect of a high school providing performance feedback privately to students, and find that even though students are rewarded only for absolute performance, providing performance feedback about how they fared relative to the class average increases students’ grades by 5%. The authors attribute this improvement to the stimulating of competitive preferences. In another setting, Kuhnen and Tymula (2012) find that people put in more effort and expect to obtain a better rank when told that they
may learn about their ranking, suggesting that feedback about relative performance modifies one’s self-esteem. The findings from both of these studies are directionally consistent with Blanes i Vidal and Nossol (2011) who find that grocery packers at a warehouse who received private information about how they fared relative to others displayed a long-term increase in productivity. Here, the authors find that this is best explained by relative concerns stemming from social comparison.

Given that the above literature demonstrate the benefits of relative performance feedback, why then do we not observe every firm using it to boost performance? Can there be potential downsides to performance feedback? Barankay (2012) finds this to be the case, and offers contrasting evidence that there can be negative effects of relative performance feedback. In a randomized workplace experiment, he finds that removing private rank incentives actually increases sales performance – mostly that of male employees – by 11%, even though rankings do not convey any direct and additional financial benefit. Taken together, these studies show that relative performance feedback does affect performance, though it is not immediately clear whether this has a positive or negative effect.

Since relative performance feedback has an impact on performance even when feedback has no direct effect on financial incentives which are either fixed (i.e., payment is independent of production) or piece-rate (i.e., payment for each unit of individual production), it is no surprise then that relative performance feedback has an effect when it does affect financial incentives, as is the case with contests or tournament incentives. Hannan, Krishnan and Newman (2008) explore the effect of relative performance feedback on both piece-rate (i.e., payment for each unit of individual production) and tournament incentives (i.e., employees are ranked ordinally and compensated according to rank). They find that relative performance feedback improves the average performance of participants in a piece-rate incentive, regardless of the precision or content of the feedback. However, relative performance feedback hurts the average performance of participants in a tournament incentive
if the feedback is sufficiently precise, but not if the feedback is coarse. The authors find
that this drop in performance happens for participants whose performances are lagging, but
not for those whose performances are excelling, and that there are more of the former than
latter and therefore a net decrease in overall performance.

Freeman and Gelber (2010) examine the performance of participants solving paper-
and-pencil mazes under different tournament prize structures and the effect of information
about others’ performances. The authors find a main effect in tournament prize structure:
performance is lowest when incentives are independent of performance (everyone received
$5 regardless of performance), moderate when there is a single large prize (only the top
scorer receives $30), and highest when there are multiple, differentiated prizes (first prize
winner receives $15, second prize winner receives $7, so on and so forth). As for the
interaction of information and prize structure, the authors find that participants who knew
how their competitors did in a preliminary round perform even better under the multiple,
differentiated prizes treatment, compared to those who do not know. However, this is not
the case for a single, large prize as participants in the bottom half know that they stand little
chance of winning a single prize. This suggests that aggregate performance depends on the
interaction between the information and prize structure. Said differently, depending on the
prize structure, it may help a firm to provide or withhold such information if it elicits greater
effort from agents – a finding that resonates with work on interim performance evaluation
in dynamic tournaments (e.g., Ederer 2010).

3.2.2 Social Effects of Performance Posting

Performance posting, unlike relative performance feedback given in private, tends to be
social in nature. In other words, performance posting not only informs oneself about oth-
ers’ performance, it also informs others about one’s performance. To the extent that a sales person may care what others think about his or her performance when it is posted, performance posting may create social incentives (e.g., pride) or disincentives (e.g., social pressure, embarrassment) if people know that their performance is being observed or monitored by others. This is related to what Deutsch and Gerard (1955) previously describe as conforming to positive expectations, and in this case, more likely that of others’ expectations. Such social incentives may be especially relevant and useful in a group task when accountability to others is important given the lack of transparency: Lount and Wilk (2014) find that when individual performance was (not) publicly posted in the workplace, employees working in a group did better (worse) than when working alone. When it comes to individual tasks, however, the mechanism and effect of performance posting becomes less clear.

The social or public nature of performance posting is also related to the literature on peer effects and its influence on productivity. Falk and Ichino (2006) find that subjects who worked on the same task at the same time in the same room not only had smaller standard deviations in output (compared to those who worked alone) but also higher output. This was the case even though the financial incentive for a subject was the same and independent of output in both cases. Bandiera et al. (2005) find that workers produce higher output in piece-rate (i.e., payment for each unit of individual production) versus relative incentives (i.e., payment depends on the ratio of individual productivity to average productivity among all co-workers). This is because in the case of relative incentives, they internalize the negative externality of their actions (i.e., my working harder raises both my individual and also average productivity, which means much more pay for me and slightly less pay for you), especially when working amongst friends. Furthermore, the internalization of this externality happens when they can monitor others’ performances and when their own performance can be monitored, suggesting that accountability plays a role in the ef-
fectiveness of relative incentives. In another study, Mas and Moretti (2009) examine how the productivity of a focal worker in a supermarket chain goes up when highly productive coworkers are introduced, with the focal worker’s effort being positively related only to the productivity of other workers who see the focal worker, but not those who do not see the focal worker. Putting these findings together, peer effects exist and a firm may thus take this into consideration when designing its incentives. This is what Roels and Su (2014) does – social planners need to consider how ahead-seeking (or behind-averse) behavior of agents leads to output polarization (or clustering), and that one can mitigate these effects either by providing the full reference distribution of outputs or by assigning players into uniform rather than diverse reference groups.

3.3 Contribution to the Literature

Having gone through several literature streams that relate to my research question, I will now briefly describe my research contribution. Previous papers have skipped over the distinction between the effects of social comparison in the presence versus absence of knowing how others perform. Also, the literature that recognizes this distinction has assumed that social comparison does not take place in the absence of information about how others perform (e.g., Kuhnen and Tymula 2012) – this is somewhat peculiar given the voluminous research on social comparison as a prevalent phenomena, whether in the presence or absence of such information.

Using a game-theoretic model, I will make a distinction between the types and effects of social comparison depending on the availability of information, thereby disentangling these two constructs. Said differently, I model the effects of social comparison – both in the presence and absence of performance information. To be clear, I assume that if the firm provides performance information, an agent compares himself or herself to the realized
outcome of the other agent’s performance; if not an agent’s comparison is to an “average”
agent’s performance. This will become clearer when I specify the model in the next chapter.
Through my model, I can study when it is optimal for a firm to use performance posting to
elicit the greatest amount of collective effort from a sales force who works individually on
a sales task but functions broadly as part of a sales team.

In my model, I build on the findings from the above literature in the following ways.
First, people care not only about absolute incentives but also about relative incentives in
a primary and direct way. I thus enrich the psychology of the agents to include social
interdependencies via my utility specification. This extends previous work (Bandiera et al.
2005) which examined the separate effects of absolute and relative incentives on behavior.
Second, firms – knowing that people do care about both absolute and relative incentives –
can take certain actions to fulfill its objective of profit maximization. I explore the context
of performance posting as a relevant firm action and how it affects the effort levels which
agents exert. Third, it is not clear whether information about how others do relative to
themselves helps or hurts both the firm and the agents; different studies provide mixed
evidence and also contingencies. I attempt to reconcile the mixed evidence using my model
by showing when posting does work versus when it does not work.

In the next chapter, I lay out a game-theoretic model that incorporates factors related to
performance posting. Specifically, I consider the potential heterogeneity in the abilities of
agents, the extent of social comparison or competitiveness among agents, and the structure
of the financial incentive.
Chapter 4

Model

The road map for this chapter is as follows. I first set up my theoretical model in Section 4.1 and then present the results for when performance posting yields higher profit for the firm in Section 4.2. Within Section 4.1, I lay out the players, how performance is determined, the objectives of the players, and the sequence of the game. I then solve for the equilibrium that arises in this game, and lay out in Section 4.2 the comparative statics for the equilibrium, followed by the outcomes when the firm chooses not to post performance versus when it chooses to post performance. Table 4.1 lists all parameters and decision variables that appear in my model. Figure 4.1 in Section 4.1.5 shows how the dynamic game unfolds.

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P$</td>
<td>Exogenous price per unit of sales to firm</td>
</tr>
<tr>
<td>$w$</td>
<td>Commission rate per unit of sales chosen by firm and received by agents</td>
</tr>
<tr>
<td>$s$</td>
<td>Decision of firm whether to post performance ($s = 1$) or not ($s = 0$)</td>
</tr>
<tr>
<td>$a_i$</td>
<td>Ability of agent $i$</td>
</tr>
<tr>
<td>$a_j$</td>
<td>Ability of agent $j$</td>
</tr>
<tr>
<td>$a_H$</td>
<td>Level of high ability in the population</td>
</tr>
<tr>
<td>$a_L$</td>
<td>Level of low ability in the population</td>
</tr>
<tr>
<td>$p$</td>
<td>Probability that an agent is of high ability</td>
</tr>
<tr>
<td>$e_i$</td>
<td>Effort put in by agent $i$</td>
</tr>
<tr>
<td>$e_j$</td>
<td>Effort put in by agent $j$</td>
</tr>
<tr>
<td>$k$</td>
<td>Extent of social comparison or competitiveness between agents</td>
</tr>
</tbody>
</table>

Table 4.1: Parameters and Decision Variables
4.1 Game Set-Up

4.1.1 Players

In business-to-business settings, it is common practice for a firm to hire agents to obtain sales for the firm. I begin with a simple model comprising of three players: a single firm and two sales agents, $i$ and $j$. Assume that $i$ and $j$ work on a similar and comparable task for the firm. Going forward, I will use $i$ as the focal agent and a similar case always applies to $j$.

I assume that each agent has an inherent ability parameter ($a_i > 0, a_j > 0$) which is private information, i.e., known only to himself or herself, and not known to either the other agent or to the firm. That said, the distribution of agents’ abilities is common knowledge: an agent is of high ability ($a_H$) with probability $p$ and low ability ($a_L$) with probability $(1 - p)$. This gives rise to four possible scenarios: \{\(a_i = a_H, a_j = a_H\}\}, \{\(a_i = a_H, a_j = a_L\}\}, \{\(a_i = a_L, a_j = a_H\}\} and \{\(a_i = a_L, a_j = a_L\}\} with respective probabilities of $p^2$, $p(1 - p)$, $(1 - p)p$ and $(1 - p)^2$.

4.1.2 Performance of Agents

Agent $i$’s performance is a function of his or her ability parameter $a_i$ and effort decision $e_i \geq 0$. In addition to ability being unobservable to the other agent and the firm, the same applies for an agent’s effort decision, i.e., effort is not observable to the other agent and the firm. The firm observes, however, the performances of both agents that transpire from each agent’s combination of ability and effort. I allow for substitutability between ability and effort in which performance is the sum of $i$’s ability and effort, $a_i + e_i$. This is a specification commonly used in the literature (e.g., Ederer 2010, Kuhnen and Tymula 2012).

A firm, in order to achieve high levels of performance, would ideally like to hire high-
ability agents who also put in high levels of effort. However, neither ability nor effort is observable by the firm. That said, the firm does observe performance that transpires from the additive combination of ability and effort, in terms of the sales numbers that get realized by each agent. That firms do not observe ability is a realistic assumption if the agents are newly hired employees, or if there is a high turnover of employees (oftentimes the case in a sales setting) such that there is limited learning of agents’ abilities by the firm. That firms do not observe effort is a common assumption in many principal-agent settings, in which the principal only observes the “outcome”, which is an imperfect signal of the action taken, primarily because supervisory measures are extremely costly (Salanié 2005). This is an especially relevant assumption in the sales force setting, given that agents spend most of their time in the field getting deals done, and very little of such effort is observed by the firm.

Furthermore, on the abilities of agents, I assume that these are independent draws from a known distribution that reflect the inherently random nature of abilities, despite best efforts by a firm to screen for ability. A firm, in order to maximize profit, seeks to maximize agents’ aggregate performance and specifically chooses the action (whether to post performance or not) in order to maximize agents’ aggregate effort. I will now specify the firm’s objective.

### 4.1.3 Firm’s Objective

The firm’s objective is to maximize the following profit function:

$$\pi = (P - w) \cdot [(a_i + e_i) + (a_j + e_j)]$$  \hspace{1cm} (4.1)

In equation 4.1, $P$ is an exogenous price net of variable costs which the firm obtains per unit of sales, and $w$ is an exogenous commission paid by the firm to the agents per unit of
sales performance achieved. \((P - w)\) is thus the margin per unit of sales for the firm, where \(P > w\).

I assume that the performances of \(i\) and \(j\) sum to give the aggregate performance of the firm, \((a_i + e_i) + (a_j + e_j)\). In other words, there is no direct substitutability or complementarity between the performances of both agents. The firm’s objective is to maximize profit by choosing whether to post performance \((s = 1)\) or not \((s = 0)\). To maximize profit given exogenous margins \((P - w)\), a firm’s objective is then to maximize aggregate performance \((a_i + e_i) + (a_j + e_j)\). Ideally, a firm would like to have agents with high abilities putting in high levels of effort. That said, the ability of an agent is determined by a random draw from the distribution of abilities, and is therefore not within control of the firm. In essence, then, the firm’s objective is to maximize, conditional on expected abilities, the aggregate effort \(e_i + e_j\), and seeks to do so by deciding whether to post or not post performance. I will elaborate on this decision of the firm, \(s\), shortly.

### 4.1.4 Agents’ Utility Function

I now turn to the agents’ utility functions. I enrich the psychology of agents to include social interdependencies, i.e., I assume that agents are motivated not only by financial incentives, but also obtain utility from social comparison. This builds not only on decades of research on social comparison in social psychology (e.g., Bunkk and Gibbons 2007, Festinger 1954, Wood 1989) but also a growing body of literature in neuroscience (e.g., Dvash et al. 2010, Fliessbach et al. 2007) which show the direct effects of social comparison on reward processing in the brain. Assuming that the individual rationality constraints for the agents are satisfied and that the agents remain in the firm, i.e., there is no attrition (or hiring for that matter), then agent \(i\) chooses effort level \(e_i\) to maximize his or her utility given by:
\[ U_i(e_i) = w \cdot (a_i + e_i) + k \cdot \frac{a_i + e_i}{(a_i + e_i) + (a_j + e_j)} - \frac{e_i^2}{2} \] (4.2)

Equation 4.2 shows \( i \)'s utility being made up of three additively separable terms. The first two represent utility arising from income ("financial utility") and social comparison ("social utility") respectively, and the third represents disutility arising from cost of effort.

Based on \( i \)'s performance, an agent receives a financial compensation of \( w \cdot (a_i + e_i) \) from the firm. As seen in the mathematical expression, financial compensation is the product of the commission rate per unit of performance and performance. Social comparison is specified as \( k \cdot \frac{a_i + e_i}{(a_i + e_i) + (a_j + e_j)} \) and comprises of two components. The latter component \( \frac{a_i + e_i}{(a_i + e_j) + (a_j + e_j)} \) represents the performance share of \( i \). It can also represent the remuneration share of \( i \) because commission rate \( w \) is constant per unit of performance, and will factor out in the numerator and denominator. I specifically chose this specification for two reasons. First, because of its concavity, it has the desirable property that marginal gains are smaller in magnitude than marginal losses of the same magnitude — in line with behavioral literature across many domains, that gains are good, but losses of the same magnitude oftentimes feel worse. Second, this specification is continuous and thus gives me greater tractability\(^1\). Furthermore, the added benefit of this specification is that it bears similarity to a sales contest setting, in which the proportion of an agent’s effort determines the expected probability of winning the sales contest prize, or simply the expected "financial utility"; here, the proportion of performances determines the expected "social utility" from social comparison. In using this specification, therefore, I am able to link the intuitions of my propositions below with the findings of the literature on sales contests.

I allow performance share to be moderated by a former term, the parameter \( k \geq 0 \), which represents the extent of social comparison or a “culture of competitiveness” (Garcia, Tor and Schiff 2013) between the two agents. In my setup, a higher \( k \) represents a higher

\(^1\)Compare this to an alternative social comparison function with a discontinuous reference point.
level of social comparison or competition between both agents. Notice here that I do not have an agent-specific subscript for $k$: my intent is to let $k$ capture a common level of social comparison or competition within a firm. For example, there are certain firms or countries (e.g., those with a low level of collectivism) which may have a predominant culture of comparison and competition (i.e., higher $k$), more so than others, where this is a need – either because of self-selection of agents into a firm, or inculcated through experience – to compare oneself to peers (and in turn, be compared to). That said, while I model $k$ as a parameter common to both agents in my model, the magnitude of this parameter can be influenced by the firm\(^2\). This will be clearer once I examine the comparative statics in my propositions.

Finally, I operationalize the assumption of convex costs using a quadratic cost function, $e_i^2$, which is a common assumption in this nature of work and captures the notion that effort is increasingly costly.

**Firm Does Not Post Performance.** Since this is a game of incomplete information, a firm not posting performance ($s = 0$) implies that the focal agent will not know about the performance of the other agent. In this case, I am going to assume that the focal agent compares himself or herself to an agent of average ability, and how an agent of average ability will perform. Specifically, the expected utility of $i$ depends on social comparison with the performance of an agent of average ability, $E_i[a_j]$, putting in an average-ability type level of effort, $e_j|E_i[a_j]$, as seen in the following:

$$\text{EU}_i(e_i|s = 0) = w \cdot (a_i + e_i) + k \cdot \frac{a_i + e_i}{(a_i + e_i) + (E_i[a_j] + e_j|E_i[a_j])} - \frac{e_i^2}{2} \quad (4.3)$$

**Firm Posts Performance.** On the other hand, if the firm posts performance ($s = 1$), this

\(^2\) If $k$ is a reflection of firm culture, then conventional wisdom suggests that this often takes a long time to build and to change.
means that with probability $p$, $i$’s utility will be:

$$U_i(e_i|s = 1) = w \cdot (a_i + e_i) + k \cdot \frac{a_i + e_i}{(a_i + e_i) + (a_j = H + e_{j = H})} - \frac{e_i^2}{2}$$  (4.4)

And with probability $1 - p$, $i$’s utility will be:

$$U_i(e_i|s = 1) = w \cdot (a_i + e_i) + k \cdot \frac{a_i + e_i}{(a_i + e_i) + (a_j = L + e_{j = L})} - \frac{e_i^2}{2}$$  (4.5)

Thus, the expected utility of $i$ will be:

$$EU_i(e_i|s = 1) = w \cdot (a_i + e_i) + p \cdot k \left[ \frac{a_i + e_i}{(a_i + e_i) + (a_j = H + e_{j = H})} \right] + (1 - p) \cdot k \left[ \frac{a_i + e_i}{(a_i + e_i) + (a_j = L + e_{j = L})} \right] - \frac{e_i^2}{2}$$  (4.6)

### 4.1.5 Sequence of Events

I now describe the sequence of events in my multi-stage game of incomplete information shown in Figure 4.1. Throughout the game, I assume that the following are common knowledge: (i) the distribution of agents’ abilities; (ii) commission rate $w$; and (iii) the extent of social comparison or competitiveness between the agents as represented by $k$. All of these are exogenous parameters in my model.

In stage one, agents receive private information from nature about their own ability types (either $a_H$ or $a_L$), which are independent draws from the ability distribution. In stage two, the firm maximizes profit, with the margin per unit of sales remaining constant at $(1 - w)$, i.e., the firm maximizes profits by maximizing aggregate sales $(a_i + e_i) + (a_j + e_j)$ and does so by committing to post or not post performance. In stage three, agents choose efforts to maximize utilities based on their private information about their own abilities and also on the firm’s decision $s$. In stage four, the firm observes each agent’s performance after
the agent chooses effort, regardless of whether it carries out performance posting. But if the firm had committed in stage two to post performance ($s = 1$), each agent also learns precisely about the other agent’s sales performance; if the firm commits in stage two to not post performance, then each agent makes the best possible inference that the other agent is of average ability with a corresponding effort and performance level. In other words, $i$ thinks that $j$’s performance is what an average agent would achieve and $j$ thinks likewise of $i$’s performance in the absence of performance posting. I then solve for the unique equilibrium in this game of incomplete information.

It is worth highlighting that in my model, regardless whether the firm posts performance or not, each agent is inclined to compare his or her performance to the other agent’s performance as moderated by $k$, the competitiveness between the agents. In other words, I disentangle social comparison – a key aspect of human nature – from the availability of information about others’ performance. This allows us to understand the effects of social comparison on effort when a firm can control the availability or precision of information about others’ performance.

I model this as a non-repeated “one-shot” game to understand the steady-state equilibrium which arises from the firm’s decision on whether to post or not post performance. More specifically, my setup allows me to examine the difference in firm’s profit between two scenarios: one in which the firm decides to post performance versus the other in which the firm decides not to post performance. For example, a firm may decide on the first day of work whether to post or not post performance for a new batch of agents. My model predicts the subsequent steady-state difference in profits some time down the road based on these two decisions. To be clear, I do not model the potential learning that can happen.

---

3Here is where I depart from Kuhnen and Tymula (2012) who assume that the absence of information about the other agent means that social comparison does not happen. In my model, I disentangle the existence of social comparison from the availability of information: the only difference between both cases is the precision with which agents are able to make an inference about the other agent’s performance.
if this one-shot game is repeated. It is possible that if the game is repeated, information revealed in stage four of the first game then narrows the asymmetry of information because the firm receives more information about agents’ abilities and each agent knows the other agent’s ability a bit better.

To complicate matters in a repeated game, agents might strategically withhold effort in the first game to influence the learning that will happen in between the games, so as to strategically influence the outcome of the next game. This is analogous to the ratcheting effect in dynamic principal-agent settings: agents strategically cut back on effort in the first game given rational anticipation that the firm will raise the performance bar for the next game based on the performance in the first game, though Charness, Kuhn and Villeval (2010) finds that the presence of competition eliminates ratcheting effects. Such an extension is of interest for future work. For now, I employ a one-shot game to cleanly understand the performance posting decision by the firm and its influence on agents’ efforts.
4.2 Theoretical Results

From the firm’s perspective, there are four possible scenarios that can arise regarding the ability distribution of both agents. The first scenario is that both agents are of high ability: \(\{a_i = a_H, a_j = a_H\}\). The second and third scenarios are that one agent is of high ability while the other agent is of low ability: \(\{a_i = a_H, a_j = a_L\}\), \(\{a_i = a_L, a_j = a_H\}\). The fourth and last scenario is that both agents are of low ability: \(\{a_i = a_L, a_j = a_L\}\). Given that the ability draws from the distribution are independent, each of these scenarios then have respective probabilities of \(p^2\), \(p(1 - p)\), \((1 - p)p\) and \((1 - p)^2\).

Agents of different ability types respond differently when the firm posts or not posts performance. To understand how this is so, I first examine in Section 4.2.1 the unique non-negative Nash equilibrium effort of an agent in a general setting. Next, I examine the effort levels of agents when the firm does not post performance (Section 4.2.2) versus when the firm posts performance (Section 4.2.3), and compare how the effort levels are different in both cases (Section 4.2.4). After providing some intuition about performance for both types of agents in Section 4.2.5, I obtain aggregate performance from the firm’s perspective, and compare expected profit under both cases in Section 4.2.6 to derive the cases under which a firm does better by posting performance.

4.2.1 Equilibrium Effort of an Agent

The amount of effort that \(i\) decides to put in against \(j\) depends on his or her belief about \(a_j\). Given \(i\)’s belief about \(a_j\), there exists a unique Nash equilibrium in non-negative efforts given by:

\[
\begin{align*}
e^*_i(a_i, a_j, k, w) &= \frac{(a_i^2 + a_ia_j + 2k + 3a_iw + a_jw + 2w^2)\sqrt{\lambda_i} + (w - a_i)\lambda_i}{2\lambda_i} \\
\end{align*}
\]

(4.7)
where $\lambda_i = a_i^2 + a_j^2 + 4a_jw + 2a_i(a_j + 2w) + 4(k + w^2)$.

When $i$ decides how much effort to put in, it is a function of $i$’s ability $a_i$, $i$’s beliefs about the other agent’s ability $a_j$, the degree of competitiveness or social comparison $k$, and the commission rate $w$. There is, however, uncertainty in $i$’s belief about $a_j$. Depending on the firm’s decision to post or not post performance, this uncertainty about $a_j$, and consequently $j$’s performance, affects $i$’s utility differently in both cases. I will elaborate on this further in Sections 4.2.2 and 4.2.3.

As for the firm, it seeks to maximize profit. In my model setup, a firm maximizes profit by maximizing agents’ aggregate performance. Since ability and effort combine additively to generate performance – and ability draws for the agents are assumed to be exogenous and not within a firm’s control – the firm essentially decides whether to post or not performance with the objective of maximizing agents’ aggregate effort, conditional on the ability draws for the agents. I lay out this decision in Section 4.2.6.

To understand how a firm makes this decision, I first study how each agent’s optimal effort varies with the parameters in my model. The lemmas below are based on comparative statics of the optimal effort in equation 4.7. Details are provided in Appendix A.

**Lemma 1A: When agents are symmetric in abilities, an agent’s effort $e_i$ is weakly decreasing in ability.**

Based on the draws of ability distribution, agents can both be of high ability $a_H$ or of low ability $a_L$. These are two out of four scenarios in which agents are symmetric in abilities for these, I take the first derivative of $e_i^*$ with respect to $a = a_i = a_j$. I find that $\frac{\partial e_i^*}{\partial a} \leq 0$ for $k \geq 0$, and $\frac{\partial e_i^*}{\partial a} < 0$ for $k > 0$, i.e., $i$’s optimal effort is weakly (strictly) decreasing in ability if $k$ is weakly (strictly) positive. When agents are symmetric in abilities, and correspondingly, symmetric in performance since there is a one-to-one mapping between ability and optimal effort, an agent’s optimal effort decreases as ability increases. This is because ability and
effort are substitutes in the production of sales performance. In addition, \( \frac{\partial^2 e_i^*}{\partial a_i^2} \geq 0 \) for \( k \geq 0 \) and \( \frac{\partial^2 e_i^*}{\partial a_i^2} > 0 \) for \( k > 0 \). This means that for every unit increase in \( i \)'s ability, the corresponding decrease in \( i \)'s effort is proportionately smaller.

**Lemma 1B:** When agents are asymmetric in abilities, an agent with higher ability will put in effort that is weakly increasing in the other agent’s ability. An agent with lower ability will put in effort that is weakly decreasing in the other agent’s ability. Furthermore, an agent’s effort \( e_i \) is convex in \( a_j \) only if the following conditions are jointly satisfied: \( 0 < a_i < \frac{a_j}{2}, \ 0 < k < \frac{1}{2}(-2a_i^2 - a_ia_j + a_j^2 - 5a_iw + a_jw - 2w^2) \) and \( 0 < w < a_j - 2a_i \).

For agents with asymmetric abilities, \( \frac{\partial e_i^*}{\partial a_j} \geq 0 \) if \( a_i \geq a_j \), or \( \frac{\partial e_i^*}{\partial a_j} > 0 \) if \( a_i > a_j \) and \( k > 0 \). However, \( \frac{\partial e_i^*}{\partial a_j} < 0 \) if \( 0 < a_i < a_j \) and \( k > 0 \). Thus, if an agent has higher (lower) ability, then his or her effort is weakly increasingly (decreasing) in the other agent’s ability. Said differently, if \( i \) is the agent with higher (lower) ability, his or her optimal action is to scale up (down) effort as \( j \)'s ability increases. This result is similar to a common finding in contest theory amongst heterogeneous players, in which a weaker player (in this case, the agent with lower ability) experiences “discouragement” and will cut back on costly effort as it becomes relatively unprofitable to beat the stronger player (in this case, the agent with higher ability), while the stronger player will be more passive or “complacent” unless s/he faces another player of similar strength (Dechenaux et al. 2012).

Furthermore, \( \frac{\partial^2 e_i^*}{\partial a_j^2} > 0 \) if certain conditions are jointly satisfied. First, \( i \)'s ability needs to be less than half of the other agent’s ability. This can happen if \( i \) is the weaker agent and there is sufficient difference in ability levels, or if \( i \) is the stronger agent and there is limited difference in ability levels. The latter is ruled out by the condition on the commission rate, which needs to be positive, which means \( i \) needs to be the weaker agent. In addition, the level of competitiveness and the commission rate both need to be sufficiently small.
If so, then for every unit increase in \( j \)'s ability, the corresponding decrease in \( i \)'s effort is proportionately smaller for a weaker agent. As we will examine later, the convexity in the other agent’s ability corresponds to the case in which \( i \) is “discouraged” by his or her lower ability, but \( i \) is not “overly discouraged” if the above conditions are jointly satisfied. The intuition here is that when the level of competitiveness and commission rate are both sufficiently low, the effort levels of agents will be low, and the difference in effort levels between the stronger and weaker agent will not be large.

**Lemma 2:** An agent’s effort \( e_i \) is always increasing in social comparison or competitiveness \( k \), and is convex in \( k \) only if the following conditions are jointly satisfied:

\[
a_i > 2a_j, \quad 0 < k < \frac{1}{2}(a_i^2 - a_ia_j - 2a_j^2 + a_iw - 5a_jw - 2w^2), \quad \text{and} \quad 0 < w < a_i - 2a_j.
\]

Next, \( \frac{\partial e_i}{\partial k} > 0 \) for the entire range of the parameter space and is convex, i.e., \( \frac{\partial^2 e_i}{\partial k^2} > 0 \), only if the following conditions are jointly satisfied: \( i \) has higher ability and there is sufficient difference in ability levels between both agents, if competitiveness is lower than a threshold level, and if the commission rate is sufficiently small. Otherwise, effort is concave in \( k \) and there will be decreasing returns to effort as \( k \) increases. From the firm’s perspective, a higher \( k \) seems to be better because it yields higher effort from agents. Furthermore, a higher \( k \) does not increase the firm’s cost. But as we will examine later, there may exist tradeoffs between regions of concavity and convexity in effort such that depending on the circumstances, a firm can do better by posting or not posting performance.

Also, notice the similarity in the convexity conditions for effort in Lemma 2 compared to that of Lemma 1B. The only difference between both conditions is that the perspective of \( i \) in Lemma 2 is one in which \( i \) is the stronger agent, while the perspective of \( i \) in Lemma 1B is one in which \( i \) is the weaker agent. The similarity in the convexity conditions comes about because \( k \) and \( a_j \) both affect agent \( i \)'s utility via the numerator and denominator of the
social utility component respectively. The implication is that if there is sufficient difference in ability levels between both agents, there cannot be convexity in both the stronger and weaker agent’s effort in regions where both the stronger agent’s ability and the level of competitiveness are increasing at the same time.

Lemma 3: An agent’s effort \( e_i \) is always increasing in commission rate \( w \), and is convex in \( w \) under any one of these three conditions: (i) \( 0 < a_i < 2a_j \) and \( k > 0 \); or (ii) \( a_i > 2a_j, \ k > \frac{1}{2}(a_i^2 - a_i a_j - 2a_j^2 + a_i w - 5a_j w - 2w^2) \), and \( 0 < w < a_i - 2a_j \); or (iii) \( a_i > 2a_j, \ k > 0 \) and \( w > a_i - 2a_j \).

As for the commission rate, I observe that \( \frac{\partial e_i^*}{\partial w} > 0 \) for the entire parameter space. This means that the firm can increase the efforts of both agents as long as it increases the commission rate. This is intuitive, as agents put in more effort when they are rewarded with a greater financial incentive to do so. Also, I observe that \( \frac{\partial^2 e_i^*}{\partial w^2} > 0 \), i.e., there is convexity in effort with respect to the commission rate under any one of these three conditions: (i) given any presence of competitiveness \( k \), either \( i \) is the weaker agent, or if \( i \) is the stronger agent and there is limited difference in both agents’ abilities; or (ii) \( i \) has substantially higher ability \( a_i \) such that there is sufficient difference in ability levels between agents, competitiveness \( k \) is higher than some threshold level, and the commission rate \( w \) is small; or (iii) \( i \) has substantially higher ability \( a_i \) such that there is sufficient difference in ability levels between agents, there is some presence of competitiveness \( k \), and the commission rate \( w \) is sufficiently large. Otherwise, effort is increasing and concave in the commission rate \( w \).

Notice that the convexity conditions under which \( \frac{\partial^2 e_i^*}{\partial k^2} > 0 \) (from Lemma 2) and \( \frac{\partial^2 e_i^*}{\partial w^2} > 0 \) (from Lemma 3) are mutually exclusive. This means that the stronger agent’s effort can only be convex in either \( k \) or \( w \), but not both at the same time.
4.2.2 If Performance is Not Posted

If performance is not posted, both agents will not know the realized performance of the other agent at stage four of the game. In this case, the only inference for social comparison that agent $i$ can make is that s/he is facing an agent of average ability, $E_i[a_j] = pa_{H} + (1 - p)a_{L}$, who puts in average-type level of efforts as denoted by $e_j|E_i[a_j]$. In this case, $i$’s optimal effort is given by:

$$e_i^*(a_i, E_i[a_j], k, w|s = 0) = \frac{(w - a_i)\lambda_i + (a_i^2 + a_iA + 2k + 3a_iw + Aw + 2w^2)\sqrt{\lambda_i}}{2\lambda_i} \quad (4.8)$$

where $A = pa_{H} + (1 - p)a_{L}$, $\lambda_i = a_i^2 + A^2 + 4Aw + 2a_i(A + 2w) + 4(k + w^2)$.

4.2.3 If Performance is Posted

Given that the expected utility of $i$ is linearly separable in the scenarios of the other agent being either of high or low ability, and weighted by $p$ and $(1 - p)$ respectively, I first solve for the case in which $i$ is of high ability, and anticipates $j$ to be of high ability as well. If so, $i$’s optimal effort is:

$$e_i^*(a_i = a_{H}, a_j = a_{H}, k, w|s = 1) = \frac{F + 4G^{3/2}}{8G}$$

where $F = -4a_{H}^3 - 4a_{H}k - 4a_{H}^2w + 4kw + 4a_{H}w^2 + 4w^3$,

$G = a_{H}^2 + k + 2a_{H}w + w^2$.

If $i$ is of high ability, and anticipates $j$ to be of low ability, then $i$’s optimal effort is:

$$e_i^*(a_i = a_{H}, a_j = a_{L}, k, w|s = 1) = \frac{H + J\sqrt{M}}{2M} \quad (4.10)$$
where \( H = -a_H^3 - 2a_H^2a_L - a_Ha_L^2 - 4a_Hk - 3a_H^2w - 2a_Ha_Lw + a_L^2w + 4kw + 4a_Lw^2 + 4w^3, \)
\( J = a_H^2 + a_Ha_L + 2k + 3a_Hw + a_Lw + 2w^2, \)
\( M = a_H^2 + 2a_Ha_L + a_L^2 + 4k + 4a_Hw + 4a_Lw + 4w^2. \)

Putting both equations 4.9 and 4.10 together, I obtain \( i \)'s optimal effort if \( i \) is of high ability:

\[
e_i^*(a_i = a_H, k, w|s = 1) = p \left( \frac{F + 4G^{3/2}}{8G} \right) + (1 - p) \left( \frac{H + J\sqrt{M}}{2M} \right) \tag{4.11}
\]

I do the same, assuming \( i \) is of low ability and anticipates \( j \) to be of high ability:

\[
e_i^*(a_i = a_L, a_j = a_H, k, w|s = 1) = \frac{Q + R\sqrt{M}}{2M} \tag{4.12}
\]

where \( M = a_H^2 + 2a_Ha_L + a_L^2 + 4k + 4a_Hw + 4a_Lw + 4w^2, \)
\( Q = -a_H^2a_L - 2a_Ha_L^2 - a_L^3 - 4a_Lk - a_H^2w - 2a_Ha_Lw - 3a_L^2w + 4kw + 4a_Hw^2 + 4w^3, \)
\( R = a_Ha_L + a_L^2 + 2k + a_Hw + 3a_Lw + 2w^2. \)

And if \( i \) is of low ability and anticipates \( j \) to be of low ability:

\[
e_i^*(a_i = a_L, a_j = a_L, k, w|s = 1) = \frac{T + 4U^{3/2}}{8U} \tag{4.13}
\]

where \( T = -4a_L^3 - 4a_Lk - 4a_L^2w + 4kw + 4a_Lw^2 + 4w^3, \)
\( U = a_L^2 + k + 2a_Lw + w^2. \)

Putting both equations 4.12 and 4.13 together, I obtain \( i \)'s optimal effort if \( i \) is of lowability:

\[
e_i^*(a_i = a_L, k, w|s = 1) = p \left( \frac{Q + R\sqrt{M}}{2M} \right) + (1 - p) \left( \frac{T + 4U^{3/2}}{8U} \right) \tag{4.14}
\]
4.2.4 Comparing Effort Levels of Agents for No Posting versus Posting

**Varying Ability Levels.** Figure 4.2 compares the effort levels of the high-ability agent and low-ability agent under the firm’s decision of no posting (dotted line) versus posting (solid line), when there is a small difference in ability levels ($a_H > a_L$). These are shown for different values of parameter $p$ on the abscissa, which is the probability that an agent is of high ability. Holding ability levels $a_L$ and $a_H$ constant, a higher $p$ increases the average ability of agents.

Notice that when there is a small difference in ability levels, both types of agents put in more effort under no posting than under posting. This is explained by Lemma 1B, in which effort is never convex in the other agent’s ability for the low-ability agent given a limited difference in ability levels, and effort is never convex in the other agent’s ability for the high-ability agent for all conditions. In other words, when there is limited difference in ability levels, effort of the high-ability and low-ability agents are both concave in the other agent’s ability. The implication of having a limited difference in ability levels is that not posting performance prevents the high-ability agent from being overly complacent and this increases the high-ability agent’s effort level. Also, not posting performance prevents the low-ability agent from being overly discouraged, and this also increases the low-ability agent’s effort level.

Figure 4.3 shows a similar case when there is now a sufficient difference in ability levels ($a_H \gg a_L$). As before, the high-ability agent puts in more effort under no posting than s/he would have under posting, because the high-ability agent’s effort is always concave in the other agent’s ability. Furthermore, the difference in effort levels between no posting and posting *increases* for the high-ability agent when the difference in ability levels increases. This is because not posting prevents the high-ability agent from being overly complacent,
and as the difference in ability levels increases, the effect of preventing over-complacency of the high-ability agent increases. Thus, not posting becomes even more effective in yielding higher effort levels from the high-ability agent compared to the case of posting.

For the low-ability agent, given that there is sufficient difference in ability levels, posting now yields higher effort levels compared to the case of not posting. According to
Lemma 1B, when there is a sufficient difference in ability levels, and assuming also that the conditions for $k$ and $w$ are satisfied (this is the case for Figure 4.3), the low-ability agent’s effort is convex in the other agent’s ability. This means that the low-ability agent puts in less effort under the case of no posting than in the case of posting. Intuitively, when there is sufficient difference in ability levels, the effect of preventing over-discouragement when performance is not posted is overridden by the motivation of the low-ability agent not to be too far behind when performance is posted.

Given the differential impact of posting performance for both high-ability and low-ability agents, this now sets up a tension for a firm: the positive difference in effort from the high-ability agent under the case of no posting needs to be weighed against the positive difference in effort from the low-ability agent under the case of posting. It is also worth pointing out, and this is always the case, that there is never a difference in effort levels at $p = 0$ or $p = 1$. This is because when $p = 0$ or $p = 1$, there is complete information in agents’ abilities, i.e., agents are always of low ability or always of high ability respectively, and therefore, there is no value in information from the posting.

**Varying Levels of Social Comparison or Competitiveness.** Figure 4.4 compares the effort levels of the high-ability agent and low-ability agent under the firm’s decision of no posting (dotted line) versus posting (solid line), for varying levels of $k$. First, notice that the high-ability agent always puts in more effort under no posting than posting for all levels of $k$. This is the same argument as before – according to Lemma 1B, the high-ability agent’s effort is always concave in the other agent’s ability, and no posting prevents the high-ability agent from being overly complacent. That said, for a higher $k$, the difference between the effort of the high-ability agent under no posting and posting increases. This comes about because according to Lemma 2, the effort of the high-ability agent is convex under lower levels of $k$, but concave under higher levels of $k$, and not posting enlarges the region of $k$ under which the effort of the high-ability agent is convex. Said differently, higher levels of
$k$ positively moderate the effectiveness of no posting in preventing the high-ability agent from being overly complacent.

For the low-ability agent, as $k$ increases, the regions in which not posting does better shifts to the right, as seen in Figure 4.4. When $k$ is lower, not posting yields more effort at lower values of $p$, while posting yields more effort at higher values of $p$. When $k$ is higher, not posting yields more effort from the low-ability agent for a larger range of low-valued $p$’s. As $k$ continues to increase, this trend continues until no posting does better for $0 < p < 1$. The basic intuition is as follows. At lower levels of $k$, there is less weight on the social utility, and therefore, limited effect of no posting in mitigating over discouragement of the low-ability agent; on the contrary, posting yields higher effort by keeping the low-ability agent on his or her toes. As $k$ increases, the weight on social utility increases, and no posting being more effective for the former, and therefore, a greater region in which no posting yields greater effort form the low-ability agent.

![Graphs showing effort levels of agents for varying levels of $k$.](image)

**Figure 4.4: Effort Levels of Agents for Varying Levels of $k$ (No Posting is Shown in Dotted Line. Posting is Shown in Solid Line)**
**Varying Levels of Commission Rates.** Figure 4.4 compares the effort levels of the high-ability agent and low-ability agent under the firm’s decision of no posting (dotted line) versus posting (solid line), for varying levels of $w$, when there is limited difference in ability levels. Notice that when $w$ is higher, the difference in effort levels between no posting and posting decreases, both for the high-ability agent and the low-ability agent.

Figure 4.5: Effort Levels of Agents for Varying Levels of $w$ (No Posting is Shown in Dotted Line, while Posting is Shown in Solid Line)

Lemma 3 lends some insight into this, although the mechanisms differ depending on the parameter space. The first condition of Lemma 3 states that there is always convexity in effort in $w$ for the agent with lower ability. If there is also a limited difference in ability levels, then there is also convexity in effort in $w$ for the high-ability agent, though the increase in effort by the high-ability agent is larger than that of the low-ability agent. This leads to a growing difference in effort levels as $w$ increases when there is limited difference in ability levels, and thus, larger differences in social utility between both types of agents. This also means that there is greater effectiveness for the no posting condition to bridge
this growing difference in social utility between the high-ability agent and the low-ability agent, by preventing over complacency of the high-ability agent, and over discouragement of the low-ability agent.

If, however, there is sufficient difference in ability levels, then effort is convex in $w$ – but only for the high-ability agent. At lower $w$, this happens only if $k$ is sufficiently large, as in the second condition of Lemma 3. At higher $w$, this happens for all positive values of $k$, as in the third condition of Lemma 3. Here, the convexity in the increase in effort by the high-ability agent, coupled with the concavity in the increase in effort by the low-ability agent, results in an even greater difference in social utility between the high-ability agent and low-ability agent than before. Therefore, there is an even greater effectiveness than before for the no posting condition to bridge this growing difference in social utility between the high-ability agent and the low-ability agent, and the same argument holds as before.

The basic intuition is that a higher commission rate shifts the relative weight of the utility function away from the social utility towards the financial utility. As a result, this diminishes the effect of social comparison, and therefore, the effect of posting versus not posting.

**Aggregation of Optimal Efforts under No Posting.** From the firm’s perspective, there are four possible scenarios regarding the realized abilities of the agents: \{${a_i = a_H, a_j = a_H}$\}, \{${a_i = a_H, a_j = a_L}$\}, \{${a_i = a_L, a_j = a_H}$\}, and \{${a_i = a_H, a_j = a_H}$\}. Each of these scenarios come with the respective probabilities of $p^2$, $p(1 - p)$, $(1 - p)p$ and $(1 - p)^2$. Under no performance posting, the aggregate effort in equilibrium is given by $e_{s=0}^* = e_i^*(a_i, E_i[a_j], k, w | s = 0) + e_j^*(E_j[a_i], a_j, k, w | s = 0)$, which is given by:
((1 - p)^2(D + B(2k + a_L^2(2-p) + w(pa_H + 2w) + a_L(pa_H + (4-p)w)))) / 
(4k + a_L^2(2-p)^2 + 2a_L(2-p)(pa_H + 2w) + (pa_H + 2w)^2) + (p^2(EC)) / 
\begin{align*}
\epsilon_{s=0}^* &= (a_L^2(1-p)^2 + a_H^2(1+p)^2 + 2a_H(1+p)(a_L(1-p) + 2w) + 4a_L(1-p)w + 4(k+w^2)) + \\
&((1-p)p(D + B(2k + a_L^2(2-p) + w(pa_H + 2w) + a_L(pa_H + (4-p)w)))) / \\
&\frac{(4k + a_L^2(2-p)^2 + 2a_L(2-p)(pa_H + 2w) + EC)}{C^2}
\end{align*}

\text{(4.15)}

where \(A = pa_H + (1-p)a_L\),
\(B = \sqrt{4k + a_L^2(2-p)^2 + 2a_L(2-p)(pa_H + 2w) + (pa_H + 2w)^2}\),
\(C = \sqrt{a_L^2(1-p)^2 + a_H^2(p+1)^2 + 2a_H(1+p)(a_L(1-p) + 2w) + 4a_L(1-p)w + 4(k+w^2)}\),
\(D = -a_L^3 - 4a_Lk - 2a_L^2A - a_L,A^2 - 3a_L^2w + 4kw - 2a_LAw + A^2w + 4Aw^2 + 4w^3\),
\(E = -a_H^3 - 4a_Hk - 2a_H^2A - a_H,A^2 - 3a_H^2w + 4kw - 2a_HAw + A^2w + 4Aw^2 + 4w^3 + (2k + a_H^2(1+p) + w(a_L - pa_L + 2w) + a_H(a_L - pa_L + (3+p)w))\).

Given the aggregate effort in equilibrium, the firm’s profit when it does not post performance is given by:

\[\pi_{s=0}^* = (P - w)\epsilon_{s=0}^*\] \text{(4.16)}

\textbf{Aggregation of Optimal Efforts under Posting.} From the firm’s perspective, again, there are four possible scenarios with regards to the realized abilities of the agents: \(\{a_i = a_H, a_j = a_H\}, \{a_i = a_H, a_j = a_L\}, \{a_i = a_L, a_j = a_H\}, \text{ and } \{a_i = a_H, a_j = a_H\}\). Each of these scenarios come with the respective probabilities of \(p^2\), \(p(1-p)\), \((1-p)p\) and \((1-p)^2\). Solving for the aggregate effort in equilibrium, \(\epsilon_{s=1}^* = \epsilon_{i}^*(a_i,a_j,k,w|s=1) + \epsilon_{j}^*(a_i,a_j,k,w|s=1)\) can be expressed as:
\[ e^*_s = w - V + p^2X + Y - 2pY + p^2Y + pZ - p^2Z \]  
(4.17)

where 
\[ V = pa_H + (1 - p)a_L, \]
\[ X = \sqrt{a_H^2 + 2a_Hw + k + w^2}, \]
\[ Y = \sqrt{a_L^2 + 2a_Lw + k + w^2}, \]
\[ Z = \sqrt{a_H^2 + a_L^2 + 4(k + a_L + w^2) + 2a_H(a_L + 2w)}. \]

Given the aggregate effort in equilibrium, the firm’s profit when it posts performance is given by:

\[ \pi^*_s = (P - w)e^*_s. \]  
(4.18)

### 4.2.5 Performance for Both Types of Agents

Figure 4.6 summarizes the intuition from the comparative statics laid out in Sections 4.2.2 and 4.2.3, holding \( k \) and \( w \) constant. This figure represents the directional change (but not magnitude) of performance that \( i \) obtains against \( j \) under the various scenarios of ability draws, as we compare between the cases of a firm not posting versus posting performance.

If \( i \) is of low ability \( a_L \), and does not know whether s/he is going up against a low- or high-ability agent in \( j \), for the case in which performance is not posted, then the performance that s/he obtains (two units) will lie between the cases in which performance is posted and the realized performance of \( j \) is either that of a low-ability agent (three units, due to more intense competition) or a high-ability agent (one unit, due to discouragement).
If \( i \) is of high ability \( a_H \), and does not know whether s/he is going up against a low- or high-ability agent in \( j \), for the case in which performance is not posted, then the performance that s/he obtains (five units) will lie between the cases in which performance is posted and the realized performance of \( j \) is either that of a low-ability agent (four units due to complacency) or a high-ability agent (six units due to more intense competition).

From the firm’s perspective, it considers the performance each type of agent will put in under the four possible scenarios \( \{a_i = a_H, a_j = a_H\} \), \( \{a_i = a_H, a_j = a_L\} \), \( \{a_i = a_L, a_j = a_H\} \) and \( \{a_i = a_L, a_j = a_L\} \), and integrates it out across these scenarios with respective probabilities of \( p^2 \), \( p(1-p) \), \( (1-p)p \) and \( (1-p)^2 \). It does this for each case of no posting versus posting, and then compares the profits. I will now lay out the key propositions.

\[
\begin{array}{|c|c|c|}
\hline
\text{Self (i) / Other (j)} & a_l & a_H \\
\hline
a_l & $$ & $$ \\
\hline
a_H & $$ & $$ \\
\hline
\end{array}
\]

No Posting

\[
\begin{array}{|c|c|c|}
\hline
\text{Self (i) / Other (j)} & a_l & a_H \\
\hline
a_l & $$$ & $ \\
\hline
a_H & $$$ & $$ $$ $$ $$ $$ \\
\hline
\end{array}
\]

Figure 4.6: How Performance Levels of Agent \( i \) Compare Across No Posting versus Posting

### 4.2.6 Firm’s Decision

The three propositions I present in this section stem from the firm’s comparison of its profit under the decision to not post performance (\( \pi_{s=0}^* \)) and to post performance (\( \pi_{s=1}^* \)).

In Proposition 1, I describe how the firm’s decision to post performance depends on the heterogeneity of abilities – a function of not only \( a_L \) and \( a_H \), but also \( p \). Propositions 2 and
3 then examine how the effectiveness of performance posting vary as a function of the level of social comparison $k$ and the commission rate $w$.

**Proposition 1**: Performance posting does better for the firm when there is sufficient heterogeneity in agents’ abilities.

In Figure 4.7, I show the difference in profit between posting versus not posting and how it varies across different values of $p$. First, notice in the case of limited heterogeneity (labeled $a_H > a_L$ in the figure) that performance posting hurts for all values of $p$. The intuition is that when there is limited heterogeneity, one dimension of which is the difference in the ability levels of a high-ability versus low-ability agent, a firm does better by not posting performance. This is because it prevents a low-ability agent from being overly discouraged, and at the same time, ensures a high-ability agent from being overly complacent. As a result, this yields more effort from both types of agents compared to the case when a firm posts performance – but only if the difference in agents’ abilities is sufficiently small.

When there is sufficient heterogeneity, again simply in terms of the difference in ability levels of a high-ability versus low-ability agent ($a_H \gg a_L$), a firm can begin to do better by posting performance. This is because the low-ability agent will now put in more effort to avoid being too far behind when performance is posted. But there is another dimension to heterogeneity, beyond just the difference in $a_H$ versus $a_L$ that I discussed so far. Heterogeneity is not only influenced by the difference in ability levels, but also by the probability that an agent is of high ability, $p$. When $p$ is close to 0 (or 1), chances are both agents are of low (or high) ability, and therefore, there is again limited heterogeneity even though difference in ability levels are sufficiently big. As a result, notice in Figure 4.7 that a firm does worse by posting performance when $p$ is close to 0 or 1. To be clear, when $p$ is close to 0, it is likely that both agents are of low ability; similarly, when $p$ is close to 1, it is likely that
both agents are of high ability. In both cases, there is limited heterogeneity regardless of how different $a_H$ is from $a_L$, and as shown in Figure 4.7, performance posting does worse in these regions. This goes back to Proposition 1 in which there needs to be sufficient heterogeneity for performance posting to yield higher profit, and to reiterate, heterogeneity is defined not only by the difference in ability levels between a high- and low-ability agent, but also the probability that agents have similar abilities.

As an aside, notice in the extremes – when $p$ is 0 or 1 – that there is no longer any difference in profit between posting and not posting performance. This is because in either case, there is complete information for the firm and both agents in these two extremes: it is common knowledge that both agents are always either of low ($p = 0$) or high ability ($p = 1$), and therefore there is no information gained whether the firm posts or does not post performance.

![Figure 4.7: Heterogeneity in Abilities Leads to Effectiveness of Posting](image)

Proposition 2: As the extent of social comparison or competitiveness between agents increases, there is a non-monotonic effect on the effectiveness of performance posting.
Figure 4.8 depicts graphically what happens as I increase the extent of social comparison or competitiveness $k$ between agents using comparative statics. This is conditional on a firm deciding to post performance if there is sufficient heterogeneity – the figure on the left for the case of low comparison is the same case as $a_H \gg a_L$ in Proposition 1. First, as $k$ increases from a low to moderate level, performance posting does better for moderate values of $p$. The reason for this is that under moderate levels of social comparison or competitiveness, the amount of incremental effort a low-ability agent puts in under performance posting will more than compensate for the decline in effort by a high-ability agent, thus making aggregate profit higher. However, when $k$ increases from moderate to high, notice that not only does the peak in the profit difference drop, but there are also now larger regions of $p$ in which performance posting does worse. The reason for this is that when $k$ is high, the low-ability agent gets overly discouraged such that not posting performance does better. More specifically, whatever additional effort the low-ability agent puts in under performance posting will no longer sufficiently compensate for the decline in effort by a high-ability agent, such that not posting performance actually does better. As a result, aggregate profit for the firm becomes lower when a firm posts performance.

![Figure 4.8: Social Comparison Helps Posting, But Only Up to a Certain Point](image)

As an aside, let me comment briefly about my assumption that $k$ is an exogenous parameter across the cases of posting versus no posting, compared to a case in which $k$ is not inde-
pendent of the firm’s decision to post performance. It is conceivable in the real world that a firm’s decision to carry out performance posting actually *heightens* the level of social comparison between agents. If so, \( k \) now becomes endogenous such that \( k(s = 1) > k(s = 0) \). If such an asymmetry in \( k \) exists between the cases of posting versus no posting, then one potential implication may be that some regions of the parameter space in this analysis become less relevant than other regions. For example, it may be that a moderate level of social comparison is harder to achieve because the firm’s decision to post performance may naturally lead to a high comparison, and thus mitigate the effectiveness of performance posting. This will be an interesting extension to explore.

**Proposition 3:** As commission rate increases, there is a smaller difference in profit between performance posting and not.

Next, I explore the effect of a change in the commission rate, \( w \), and how that affects my results. Figure 4.9 depicts graphically what happens as I increase \( w \), again conditional on a firm deciding to post performance if there is sufficient heterogeneity. Notice that when the commission rate increases, the peaks and troughs of the difference in profit between posting and not posting gets compressed towards zero. In other words, the stakes for this decision to post or not post are lower when the firm is paying well. The intuition for this is that when a firm is paying well, an agent will be motivated to put in more effort, simply because the financial incentive is relatively more attractive. Put differently, the effect of social comparison matters less when the money is good. As a result, the effectiveness of performance posting via social comparison gets reduced, and therefore the difference in profit between posting and not posting performance becomes smaller.

Another angle of looking at this proposition is that the stakes for this decision become higher when the firm is not paying well in a low commission setting. The implication for
this is that firms that have narrow margins (and therefore less leeway to increase the commission rate) may find performance posting to have potentially more perks (or more perils). Along the same lines, there is also some anecdotal evidence that during recessionary environments or when sales are slow-moving, firms are more inclined to consider posting of performance in order to motivate employees to work harder⁴. My model shows that firms are indeed thinking in the right direction. That said, it becomes even more important to ensure that the firm is making an optimal decision with regards to performance posting, as the stakes for getting this decision right become much higher (i.e., the gains or losses in profit become amplified).

Figure 4.9: If a Firm is Not Paying Well, the Stakes are Higher

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⁴http://www.insightsquared.com/2014/05/25-reasons-your-sales-results-are-struggling/
Chapter 5

Empirics: Lab Experiment and Field Study

Having established several propositions using a theoretical model, I now test the empirical validity of these propositions using a series of lab experiments and also a field study. Heretofore, I have motivated my research question in the context of the sales force within the field of marketing. Going forward, I hope to demonstrate the broader applicability of my propositions in an educational setting which similarly involves ability and effort as the building blocks of performance. Specifically, I employ a real-effort task undertaken by university undergraduates in a lab setting – solving double-digit multiplication problems (e.g., 89 x 73, 56 x 39) for bonus payments. The parameters and variables in my sales force model are readily translatable to my experimental design, which I will later describe. A similar task had also been used in previous literature to elicit real effort from participants (e.g., Brüggen and Strobel 2007). Let me first describe the setup of the experiment in Section 5.1 before I present the results from a series of experiments in Section 5.2.
5.1 Description of Lab Experiment

Members of a university subject pool were recruited to participate in a web-based computer task, and received $10 for an hour’s worth of time in the lab. The number of participants was determined in advance based on the number of subjects signed up to participate in a single week-long lab session afforded to me given the lab schedule. Throughout the task, participants identified themselves only via their lab ID, and performance posting in the treatment condition was done via their lab IDs. In other words, the identity of participants are not revealed to one another\footnote{One would imagine that the social effects of performance posting in the treatment condition would be stronger if I used actual names (e.g., Lim 2010). Thus, the results of performance posting in my experiment were possibly a lower bound.}.

This experiment adhered to the general guidelines of experimental economics: the task itself was incentive-compatible with real monetary payoffs (participants received an attractive bonus payoff ranging from $0.10 to $1.00 for each problem answered correctly) and there was no use of deception. In my experiment, participants first went through a practice round and then an actual round of 10 questions. There were a total of 20 problems, and they were incentivized for both practice and actual rounds. Each session was designed either as a treatment condition with performance posting of scores from every participant in the lab, or a control condition where there was no performance posting.

Participants first did the practice round in which they solved 10 randomly-generated double-digit multiplication problems with a time limit per problem. Before the start of the practice round, participants were informed about the distribution of practice round scores from previous participants who had done the same task in a pre-test. Participants then went on to do one multiplication problem at a time. Please see Figure 5.1 for a screen-shot of each multiplication problem. A timer was provided in the experimental task to inform participants how much remaining time they had for each question. Once the time limit was
up for each question, the program proceeded to the next question regardless of whether an answer was provided. Participants needed to key in the correct answer by the time limit in order to receive a point for the problem. After finishing all 10 multiplication problems, the program provided a score (out of 10) at the top of the screen, and a recap of which questions the participants answered correct or incorrectly.

At this point (i.e., in between the practice and actual round), depending on whether the participants were in the treatment (or control) condition, participants were informed (not informed) that there will be performance posting of their actual round results by lab ID. Participants then proceeded to the actual round. If participants were in the treatment condition (i.e., performance posting), the lab assistant – blind to the purpose of the study and unaware of the theoretical propositions that are being tested as hypotheses – obtained the scores of all participants and proceeded to list the ranked scores of every participant with the lab ID as the identifier. This was done on the blackboard or whiteboard that was visible to all participants in the lab. Furthermore, the lab assistant was also instructed to verbally announce the lab ID and the ranked scores as s/he wrote them on the board. This process took around five minutes. If participants were in the control condition (no performance posting), they remained in their seats for five minutes. Finally, participants receive their financial payout for their combined score across both practice and actual rounds (consisting of both the initial participation fee of $10 and bonus earnings from this task).

5.2 Treatments and Results for Lab Experiment

A series of lab experiments were designed to test the propositions from the theoretical model. In Section 5.2.1, using a 2x2 between-subjects design, I test the interaction of

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2An alternative approach is to randomly pair up participants, and to post performance across each dyad.
heterogeneity (lesser versus greater) in participants’ abilities and performance posting (no posting versus posting), as described in Proposition 1. Having shown that performance posting does better when there is greater heterogeneity in participants’ abilities, I test Proposition 2 using a 3x2 between-subjects design, and manipulate the degree of competitiveness (low versus moderate versus high) to examine the difference between the no posting versus posting conditions\(^3\). Following that, I test Proposition 3 using a 2x2 between-subjects design, and manipulate the bonus payment (low versus high) to examine the difference between the no posting versus posting conditions\(^4\).

5.2.1 Testing the Effects of Heterogeneity

130 members of a university subject pool were recruited to test Proposition 1. Here, I needed to manipulate the heterogeneity of participants’ abilities, and examine if perfor-

\(^3\)There is an overlap of two cells between the test of Proposition 1 and 2: the condition with greater heterogeneity in abilities for the low degree of competitiveness in the test of Proposition 2 is the same as the condition with greater heterogeneity in abilities in the test of Proposition 1.

\(^4\)Again, there is an overlap of two cells between the test of Proposition 1 and 3: the condition with greater heterogeneity in abilities for the high bonus in the test of Proposition 3 is the same as the condition with greater heterogeneity in abilities in the test of Proposition 1.
mance posting resulted in a greater increase in performance during the actual round (over the practice round) for the case in which there was a greater heterogeneity in abilities. Given that participants in each session are undergraduate students from the same institution, the distribution of abilities should remain fairly constant across different sessions. Since it is challenging to manipulate the abilities of participants coming from the same institution, I manipulated the difficulty of the task instead. In the experiment described above, I set up two types of tasks – a difficult task (for which the time limit to solve each question is 15 seconds) and an easy task (for which the time limit to solve each question is 18 seconds). These two time limits were pre-tested on a small pool of students to ensure that different degrees of heterogeneity in performance were obtained. To be clear, an assumption here is that heterogeneity in ability to solve math questions under different time constraints is a factor that gives rise to heterogeneity in performance.

Figure 5.2 shows the distribution in performance of the practice round, i.e., before the effects of the treatment and control were in place. There was greater heterogeneity in performance \(n = 66\), \(\text{mean} = 5.25\), \(\text{s.d.} = 3.24\), \(\text{s.d./mean} = 0.617\) in the practice round of the easy task compared to the performance of the practice round of the difficult task \(n = 64\), \(\text{mean} = 3.20\), \(\text{s.d.} = 2.66\), \(\text{s.d./mean} = 0.831\), with the standard deviation of the performance in the easy task being higher than that of performance in the difficult task. Doing the Levene’s test for equality of variances, I find that they are statistically different for the easy versus the difficult task \(W = 4.17, p = 0.043\). I also do the Kolmogorv-Smirnov test and find that both distributions are statistically different \(D = 0.299, p = 0.006\).

Based on Proposition 1, my prediction is that performance posting will result in better

\[5\] As a next step, I would like to run this task across different universities (or different disciplines within a university) where there may be inherently different distributions of abilities to do such math questions under time constraints. This may be a more natural way to obtain heterogeneity. Also, other types of tasks can be considered. For example, simple tasks such as translating numbers to words might yield less heterogeneity in ability (though boredom might result in a high heterogeneity in effort). Other potential tasks that have been used in past literature include solving anagrams or paper-and-pencil mazes.
performance for the easy task with greater heterogeneity, relative to the case of no performance posting. However, not posting performance will result in better performance for the difficult task with less heterogeneity, relative to the case of performance posting. To control for baseline differences in scores for participants in different treatments, the analysis was done using the difference in scores of participants from the practice to the actual round. As observed in Figure 5.3, for the easy task with a more heterogeneous distribution, participants in aggregate improved by 3.1% without performance posting (an improvement in score of 0.18 from practice to actual round). In comparison, participants in aggregate improved by 18.1% with performance posting (an improvement in score of 0.85 from practice to actual round). On the other hand, for the difficult task with a less heterogeneous distribution, participants in aggregate improved by 17.4% without performance posting (an improvement in score of 0.64 from practice to actual round), and by 9.7% when performance was posted (an improvement in score of 0.26 from practice to actual round).

For the easy task with greater heterogeneity, the simple main effect of this improvement is statistically significant ($F = 5.18, p = 0.024$), while for the difficult task with less
heterogeneity, the simple main effect of this improvement is not statistically significant \((F = 1.62, p = 0.204)\). Most importantly, the crossover interaction between the degree of heterogeneity and the presence or absence of performance posting is statistically significant \((F = 6.27, p = 0.013)\), giving empirical validity for Proposition 1.

![Figure 5.3: Posting Does Better Only for Heterogeneous Distribution](image)

**Decomposing Changes in Performance to Ability versus Effort.** At both easy and difficult tasks, participants on average improved from the practice to actual round. How much of this improvement in performance comes from a change in ability, and how much of it comes from a change in effort? A direct measure of observable effort would have allowed for a decomposition of this improvement into its two sources. A possible proxy for effort might have been the amount of scratch work that was done using pen and paper. Alternatively, participants may be asked to assess perceived effort on the math task. In the absence of a direct measure for effort, however, I am assuming in my interpretation of the experimental results that changes in performance stem from changes in effort, and that math ability of participants remained constant across both rounds. This is highly plausible if one assumes that math ability should not vary from a math task that amounted to less than
five minutes. If this assumption is reasonable, then the experiment is in line with the setup in the theoretical model whereby ability is a constant parameter and effort is a variable. That said, participants may have improved from the practice to the actual round because of greater task familiarity, which is neither related to math ability nor effort, but more of a gain in productivity or efficiency. This can be in terms of quickness in typing the answer on the computer interface (e.g., strategies to position the cursor on the right spot before working out the math questions), or some form of test-taking strategy in optimizing usage of time for a question (e.g., knowing that if time is running out for a question, it is better to give up and wait patiently for the next question than to beat the clock and risk missing out the first few seconds of the next question). Such productivity or efficiency gains happened across all treatments, and are controlled for in the between-subjects design of the experiment.

5.2.2 Testing the Effects of Social Comparison Levels \((k)\)

I tested Proposition 2 in the following way. To examine how social comparison or competitiveness \(k\) affects optimal efforts and profit under performance posting, I keep task difficulty constant and manipulate social comparison or competitiveness using a message screen that appears just before the start of the actual round in the math task. The purpose of this message screen is to increase social comparison through a general sense of competitiveness amongst participants. There was no set time for which the participants saw this message screen. In order to proceed to the actual round, participants had to view this message screen and then click the forward arrow at the bottom right of the screen to proceed to the math task. To be clear, participants in both posting and no posting treatments saw this message screen.
To evoke different levels of competitiveness through the message screen, I considered variation of this message using different font sizes and colors. In a pre-test, I tried out four different types of font combinations: two types of colors (red or black), and two sizes (size 14 or size 24), an example of which is shown in Figure 5.4. To ensure that these manipulations were indeed perceived to yield different levels of competitiveness (and also for me to understand the differences in magnitudes of these perceptions), I used a between-subjects design of 110 MTurk participants to pre-test the effectiveness in evoking competitiveness through these messages. In the task, participants are asked to assess “how competitive do you feel after seeing this message?” on a scale of 0 to 100. The means of the four combinations are as follows: $k_{black,14} = 59.4$; $k_{black,24} = 73.0$; $k_{red,14} = 70.3$; $k_{red,24} = 72.1$. Performing a two-tailed t-test, the only difference that was statistically significant was between $k_{black,14}$ and any of the remaining three (all differences were significant at $p < 0.01$). Interestingly, while the color red was perceived to be more competitive, as was the larger font size in black, the combination of the color red and the larger font size was not perceived to be more competitive than either on its own.

The competition is heating up! Are you up for the challenge?

Figure 5.4: Example of Manipulating Competitiveness in Math Task

For this portion of the analysis, I used a subset of my results from the test of Proposition 1 – the heterogeneous (i.e., easy) task where an absence of a competitive message represented the case for a low level of social comparison – as my benchmark. I then operationalized a moderate level of social comparison using $k_{black,14}$ and a high level of social comparison using $k_{black,24}$ for the high level of social comparison, given that the latter had

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6 The use of the color red has been shown to elevate heart rates and provoke a more intense reaction (Alter 2013).

7 To screen MTurk participants, I used a criteria of HIT approval rate ≥ 97% and number of hits approved ≥ 1000, and provided a $0.02 payment for a task that took about 10 seconds, i.e., about a $7/hour payment.
the highest mean amongst the three (even though the differences across the three are not statistically significant). Arguably, the choice of these three levels of social comparison are somewhat arbitrary, and perhaps less so if there was an ex ante measure of $k$ in the absence of a competitive message.

In my test of Proposition 2, I had a total of 143 participants do the heterogeneous (i.e., easy) task in another 2 (moderate versus high level of social comparison) x 2 (post versus not post performance) between-subjects factorial design. Again, all participants received the standard bonus of $0.25 per correct answer for both practice and actual round. The purpose of this experiment was to examine if, in line with Proposition 2, the difference between posting and not posting was non-monotonic, i.e., whether it was largest for the moderate level of social comparison, relative to the low and high levels of social comparison. As described above, I used the previous results from the heterogeneous (i.e., easy task) in Proposition 1 as one of the three cases, in which the absence of a competitive message served as the case for the low level of social comparison. Reiterating the result for the low level of social comparison, participants’ average score improved by 0.18 in the case of no posting, versus an improvement of 0.85 in the case of posting. Comparing the difference in improvement between the no posting and posting cases for the low level of social comparison, there is a difference in mean improvement of scores by 0.67 in favor of posting.

In the case of the moderate level of social comparison ($k_{black,14}$), participants’ average score improved by 0.69 in the case of no posting, versus an improvement of 1.65 in the case of posting. Comparing the difference in improvement between the no posting and posting cases for the moderate level of social comparison, there is a difference in improvement of scores by 0.96 in favor of posting. The difference in improvements of these two treatments was statistically significant ($p = 0.032$). I also compared the scores of participants across the low versus moderate levels of social comparison, both within the no posting and posting
treatments: the improvement for the no posting treatment (0.69 in moderate versus 0.18 in low) was not statistically significant ($p = 0.347$), while the improvement for the posting treatment (1.65 in moderate versus 0.85 in low) was marginally significant ($p = 0.0936$). This provides us some evidence that this increase in improvement of scores for the moderate level of social comparison (0.96 in moderate versus 0.67 in low) came more from the posting than the no posting treatment.

In the case of the high level of social comparison ($k_{black,24}$), participants’ average score improved by 0.85 in the case of no posting, versus an improvement of 0.69 in the case of posting. Comparing the difference in improvement between the no posting and posting cases for the moderate level of social comparison, there is a difference in improvement of scores by 0.16 in favor of no posting. The difference between the absolute differences of these two treatments was not statistically significant ($p = 0.727$), and neither were the statistical tests for low versus high levels of social comparison within the no posting ($p = 0.224$) and posting ($p = 0.708$) treatments.

Putting the above together, my results provide directional evidence for Proposition 2, in which a moderate amount of social comparison increases the effectiveness of performance posting, relative to the baselines in which there was a low or high level of social comparison. Next, I will assess where the improvement in scores is coming from, i.e., how did participants in the top versus bottom half of the distribution for the practice round vary in their improvement for the posting versus no posting treatments.

In the low level of social comparison (from the easy/heterogeneous experiment for the test of Proposition 1), the average score for the bottom half of participants in the no posting treatment improved by 0.76, while the average score for the top half of participants decreased by 0.44. As for the posting treatment, the average score for the bottom half of participants improved by 1.35, while the average score for the top half of participants increased by 0.31. Thus, we observe that posting in the low level of social comparison
yields an improvement for both bottom and top half of participants, and that this increase in average scores relative to the no posting treatment was comparable (0.59 for the bottom half and 0.75 for the top half).

In the moderate level of social comparison, the average score for the bottom half of participants in the no posting treatment improved by 1.38, compared to no improvement for the top half of participants. As for the posting treatment, the average score for both bottom and top half participants improved by 1.65. Thus, we observe that the posting (versus no posting) treatment in the moderate level of social comparison again brings about an improvement across both bottom and top half of participants, but this time round, the increase in average scores relative to the no posting treatment was more prominent for the top half of participants (difference of 0.27 for the bottom half versus 1.65 for the top half). This gives us some evidence that the top half of participants were improving more in the posting condition under a moderate level of social comparison, and that the improvement was smaller for the bottom half of participants.

Finally, in the high level of social comparison, the average score for the bottom half of participants in the no posting treatment improved by 1.20, compared to an improvement of 0.47 for the top half of participants. As for the posting treatment, the average score for the bottom half of participants improved by 0.89 while the top half of participants improved by 0.53. Here, we observe that the posting (versus no posting) treatment in the high level of social comparison brings about a smaller improvement for the bottom half of participants, and a slightly larger improvement for the top half of participants (difference of -0.31 for the bottom half versus 0.06 for the top half). This gives us some evidence that the bottom half of participants may have been discouraged in the high level of social comparison when performance was posted (notice that this improvement of 0.89 for the bottom half is also smaller compared to the improvement of 1.35 for the bottom half of the low level of social comparison for the posting treatment).
<table>
<thead>
<tr>
<th></th>
<th>No Posting</th>
<th>Posting</th>
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</thead>
<tbody>
<tr>
<td>Low level of social comparison</td>
<td>0.18</td>
<td>0.85</td>
</tr>
<tr>
<td>Moderate level of social comparison ($k_{black,14}$)</td>
<td>0.69</td>
<td>1.65</td>
</tr>
<tr>
<td>High level of social comparison ($k_{black,24}$)</td>
<td>0.85</td>
<td>0.68</td>
</tr>
</tbody>
</table>

Table 5.1: Mean Improvement in Absolute Scores from Practice to Actual Round for No Posting versus Posting

<table>
<thead>
<tr>
<th></th>
<th>Bottom Half</th>
<th>Top Half</th>
</tr>
</thead>
<tbody>
<tr>
<td>Low level of social comparison</td>
<td>0.76</td>
<td>-0.44</td>
</tr>
<tr>
<td>Moderate level of social comparison ($k_{black,14}$)</td>
<td>1.38</td>
<td>0</td>
</tr>
<tr>
<td>High level of social comparison ($k_{black,24}$)</td>
<td>1.2</td>
<td>0.47</td>
</tr>
</tbody>
</table>

Table 5.2: Mean Improvement in Absolute Scores from Practice to Actual Round for Bottom and Top Half in No Posting

<table>
<thead>
<tr>
<th></th>
<th>Bottom Half</th>
<th>Top Half</th>
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</thead>
<tbody>
<tr>
<td>Low level of social comparison</td>
<td>1.35</td>
<td>0.31</td>
</tr>
<tr>
<td>Moderate level of social comparison ($k_{black,14}$)</td>
<td>1.65</td>
<td>1.65</td>
</tr>
<tr>
<td>High level of social comparison ($k_{black,24}$)</td>
<td>0.89</td>
<td>0.53</td>
</tr>
</tbody>
</table>

Table 5.3: Mean Improvement in Absolute Scores from Practice to Actual Round for Bottom and Top Half in Posting

### 5.2.3 Testing the Effect of Incentives ($w$)

To test Proposition 3, I vary the size of the bonus payment per correct answer. According to my model, Proposition 3 predicts that as I increase the size of the bonus payment, there will be a smaller difference between posting and not posting performance. As described earlier, the intuition here is that with a higher bonus payment, participants will be motivated by the higher financial incentive to put in more effort than before, whether they are in the posting or no posting treatment. This relative shift in focus towards the attractive financial incentive then mitigates the effect of social comparison, and consequently, reduces the effect of performance posting.

Similar to the test of Proposition 2, I again used a subset of my earlier results – the
heterogeneous (i.e., easy) task in the test of Proposition 1, with the standard bonus payment of $0.25 per correct answer – as the benchmark. I tested Proposition 3 twice, and I will explain why I had done so. In the first test of this proposition, I increased the size of the bonus payment to $1 per correct answer for 45 participants across both no posting and posting treatments. Here, I found that increasing the size of the bonus payment did result in a smaller difference between the mean improvement in scores for posting and no posting (a difference in mean improvement of 0.36 for $1 per correct answer versus 0.67 for $0.25 per correct answer), as seen in Table 5.4. This provided empirical evidence for Proposition 3. But I also found that aggregate performance for the $1 bonus treatment was unexpectedly lower than that of the performance for the $0.25 bonus payment. To me, this was an anomaly as I had expected (according to the assumption in my model) a monotonically increasing relationship between incentives and aggregate performance. To uncover why this was the case, I examined the contents of an exit survey that participants used to provide feedback on the task.

<table>
<thead>
<tr>
<th></th>
<th>No Posting</th>
<th>Posting</th>
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<tbody>
<tr>
<td>Bonus payment of $0.25 per correct answer</td>
<td>0.18</td>
<td>0.85</td>
</tr>
<tr>
<td>Bonus payment of $1 per correct answer</td>
<td>-0.09</td>
<td>0.27</td>
</tr>
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</table>

Table 5.4: Mean Improvement in Absolute Scores from Practice to Actual Round for No Posting versus Posting ($0.25 bonus versus $1 bonus)

In this exit survey, one of the questions asked was whether the bonus payment was attractive. For the bonus payment of $1 per correct answer, 98% of participants said that this was attractive, though 71% of participants also mentioned that they felt anxious during the task given the high stakes of the experiment, especially when they were unable to finish an earlier question in time. I compared this with the exit survey for the bonus payment of $0.25 per correct answer, in which 92% of participants thought that the bonus payment was attractive, while only 3% of participants expressed any form of anxiety. This
phenomenon is similar to the findings of Ariely et al. (2009), where excessive rewards can result in decreased performance, and is also consistent with the Yerkes-Dodson law (Yerkes and Dodson 1908) which states that beyond an optimal level of arousal for tasks, further increases in arousal can result in a decrease in performance.

To ensure that anxiety stemming from an overly attractive bonus payment was not affecting my results, I tested Proposition 3 another time (64 participants across both posting and no posting treatments), this time with a bonus payment of $0.10 per correct answer. The rationale for lowering the bonus payment is that even for a $0.25 bonus payment, 92% of participants had found it attractive, and so, I could afford to lower the bonus payment: in the $0.10 bonus payment, 69% of participants thought that the bonus payment was attractive, and there were no participants who expressed any form of anxiety.

Table 5.5 shows the result for these treatments. For the bonus payment of $0.10, there were no issues with the aggregate performance being vastly different from that of the $0.25 bonus payment. Here, participants in the no posting treatment improved by 0.71 on average, while participants in the posting treatment improved by 1.82 on average. Comparing this to the bonus payment of $0.25, the improvement for the no posting treatment (0.71 versus 0.18) was not statistically significant ($p = 0.33$), while the improvement for the posting treatment (1.82 versus 0.85) was statistically significant ($p = 0.042$). More importantly, reducing the size of the bonus payment did result in a larger difference between the mean improvement in scores for posting and no posting (a difference in mean improvement of 1.09 for $0.10 per correct answer versus 0.67 for $0.25 per correct answer, both in favor of posting). This is in line with Proposition 3: in the case of a higher bonus payment (the treatment which pays $0.25 per correct answer), participants would be more motivated by the higher financial incentive relative to the smaller bonus payment ($0.10 per correct answer), whether they are in the posting or no posting treatment. This relative shift in focus towards the attractive financial incentive mitigates the effect of social comparison,
and consequently, reduces the effect of performance posting.

<table>
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<td>1.82</td>
</tr>
<tr>
<td>Bonus payment of $0.25 per correct answer</td>
<td>0.18</td>
<td>0.85</td>
</tr>
</tbody>
</table>

Table 5.5: Mean Improvement in Absolute Scores from Practice to Actual Round for No Posting versus Posting ($0.10 bonus vs $0.25 bonus)

5.3 Field Study

With the assistance of a faculty member teaching in a large public university located in the southeastern United States, I examined the effect of performance posting using a field study. The faculty member was teaching two groups of a marketing course, in which the grades of the students were determined primarily by the students’ aggregate performance on three exams. Going forward, I will call these groups A ($n_1 = 42$) and B ($n_2 = 17$). Upon agreement with the faculty member and grader at the start of the semester, who were blind to the propositions above, performance posting would be administered to both groups when graded scripts were returned for the penultimate exam (and not the first exam because that was early in the semester before the composition of the class was stable). Students would receive back their exam scripts and scores, and then the faculty member would put up a powerpoint slide showing an anonymous scatterplot of scores. This form of performance posting was similar to that administered for the lab experiment. Thus, students would know how they did on the exam, and where they stood amongst the distribution within the group. The purpose of the field study was to see if the change in scores for the last exam would be different based on the predictions of the model. The caveat for this field study is that there might be interdependence of performances across different exams within an individual, i.e.,
a student is more incentivized to do well on the next exam if s/he did not do well previously.

For the exam in which performance was posted, the scores of group A had a higher standard deviation than group B that was marginally significant ($p = 0.08$) – the coefficient of variation was 0.180 for group A and 0.107 for group B. In between the second and third exam, a survey containing the social comparison orientation scale (Gibbons and Bunnk 1990) was taken by the students for extra course credit (survey participation rate was 100% for both groups). The purpose of the survey was to (i) get a measure of the aggregate level of social comparison using individual propensities to engage in social comparison as a proxy, (ii) to assess the level of social comparison for the two groups (i.e., low, moderate, or high), and (iii) to see if there was any difference in social comparison orientation between both groups. I found that both groups had moderate levels of social comparison as proxied by their social comparison orientation scores of 3.32 (group A) and 3.45 (group B) out of 5. Also, there was no difference in their mean social comparison orientation scores ($p = 0.42$).

Based on the theoretical parameters of interest, group A had greater heterogeneity in their scores for the second exam compared to group B. Furthermore, both groups had moderate levels of social comparison. Returning to the theoretical propositions, and basing my prediction on a combination of Proposition 1 (for heterogeneity) and Proposition 2 (for levels of social comparison), the effect of performance posting should be more effective for group A than group B, i.e., I was expecting a greater improvement for group A than for group B.

Here are the results: the mean score for group A was initially 10.3% lower than that of group B in the second exam; after performance posting of scores for the second exam, the mean score on the third and final exam for group A was only 4.4% lower than the mean score for group B. This improvement in mean scores was statistically significant ($p = 0.007$), i.e., students in group A improved more than the students in group B. To see where this improvement came from, I partitioned the group into the bottom half and top
half of performers based on the students’ scores in the second exam. This improvement came primarily from the bottom half of the students in group A, who were initially 16.1% lower than the bottom half of group B for the second exam, but eventually closed the gap to be only 4.2% lower than the bottom half of group B for the third and final exam. As for the top half of the students in group A, they were initially 6.3% lower than the top half of group B for the second exam, and then 4.9% lower for the third and final exam. Thus, conditional on sufficient heterogeneity which was the case in group A, performance posting was effective for both bottom and top half of students, but especially so for the bottom half of students.
Chapter 6

Discussion and Managerial Implications

6.1 Discussion

The sales force is one of the most important functions in marketing. Many businesses spend more on their sales force than they do on any other marketing function. Annual sales force expenditures in the U.S. amount to several times that of other marketing spends. More than 20 million people are employed in sales just in the U.S. alone. Increasing the performance of the sales force thus promises substantial profits for firms.

Motivating and managing the performance of sales force is one of the key factors. I focus on performance posting – a practice used by firms but under-studied in the literature despite its potential importance. Enabled by advances in information technology, performance posting is now relatively costless to implement, though not readily adopted across industries. I explore potential tradeoffs of performance posting, and prescribe circumstances under which performance posting can have its perks or perils.

To solve this research question, I use a blend of theory and empirics. Motivated by a growing body of evidence in psychology, economics and neuroscience, I enrich the psy-
ology of the agents by specifying a utility function that contains both financial payoffs and social interdependencies in performance. Using a game-theoretic model of incomplete information consisting of two agents, I allow each agent’s ability to be private information, unknown to the firm and other agent. I first examine how optimal effort of each agent changes with or without performance posting, and solve for the profit under these two different policies. I then compare the difference in profit for the firm, and derive propositions on when performance posting does better for the firm.

I develop three key propositions. First, if agents’ abilities are sufficiently homogeneous, a firm does better by not posting performance. On the contrary, if agents’ abilities are sufficiently heterogeneous, a firm does better with performance posting. The intuition for this proposition is that when agents’ abilities are sufficiently homogenous, not posting performance prevents a low-ability agent from being overly discouraged, and at the same time, ensures a high-ability agent from being overly complacent. As a result, this yields more effort from both types of agents compared to the case when a firm posts performance. Second, I find that the extent of social comparison or competitiveness of the agents has a non-monotonic effect on the firm’s profit if a firm decides to post performance. Specifically, a firm’s profitability from performance posting increases when there is a moderate (versus low) amount of social comparison, such that the incremental amount of effort that a low-ability agent puts in will more than compensate for the decline in effort by a high-ability agent. However, when there is a high (versus moderate) amount of social comparison, the decline in effort by a low-ability agent from discouragement becomes too large and aggregate effort decreases. Lastly, I find that a more attractive financial incentive reduces the difference in profits between the presence and absence of performance posting, as the attractive financial incentive shifts the relative focus of the agent away from the utility of social comparison, and consequently, reduces the effect of performance posting. Using a series of lab experiments and a field study, I demonstrate the empirical validity of my
propositions, and in doing so, also show the generalizability of my model to contexts outside of the sales force.

My contribution to the research is twofold. To date, social comparison has been modeled as non-existent in the absence of information about others’ performance, but this runs counter to a growing body of evidence in social psychology that social comparison is a part of human nature. My first contribution is to methodologically disentangle social comparison from the availability of information about others’ performance. This allows us to understand the effects of social comparison when a firm can control the precision of information about others’ performance.

My other contribution is to answer a substantive question that is of managerial interest and importance, which hitherto had not been examined in past literature. Based on my analytical and empirical results, we now have guiding principles on when a firm benefits from performance posting. Going one step further beyond sales force management in marketing, the same set of guidelines can be applied to other domains such as education and healthcare which are currently considering when to use performance posting to increase aggregate performance.

6.2 Managerial Implications

6.2.1 Prescriptive

While I focus on performance posting within the context of the sales force, there are broader uses and applications of my research findings. First, performance posting is used not only in sales and marketing, but also in other domains like education (e.g., teacher posting student scores, department chairs posting teaching ratings of faculty members), health care (e.g.,
success rates of medical operations for doctors or hospitals), public health (Jin and Leslie 2003) and even the design of online reputation systems (Dellarocas 2010). I hope that the combination of theory and empirics in my paper help provide a clear set of guidelines for managers considering the use of performance posting, and when it can potentially help or hurt the firm.

6.2.2 Descriptive

In addition to my work being prescriptive, my model and its propositions can also explain some instances of empirical phenomena in various industries. Let me illustrate with a couple of examples. The first example is in restaurant hygiene, where Jin and Leslie (2003) find support for the use of restaurant hygiene grade cards (equivalent to “performance posting”) in improving overall hygiene standards in Los Angeles County. I searched out other sources to see if there was sufficient heterogeneity (Proposition 1) in restaurant hygiene within the county. According to a commentary, Jin and Leslie (2005) report that there was dispersion of restaurant ratings across the three grades: 25% of restaurants were rated A, and the rest of the restaurants being rated either B or C. In another city where there was less heterogeneity in restaurant hygiene (say in Philadelphia), the use of restaurant hygiene grade cards may be less effective, perhaps explaining why they are not used.

The second example arises from a natural experiment in the case of online ratings, and I will explain how my findings apply also to an e-commerce phenomenon. Amazon, because of its one-way seller rating evaluation (i.e., buyers rate sellers, sellers do not rate buyers), tends to yield greater heterogeneity in seller ratings relative to eBay, which because of its reciprocal evaluation (i.e., buyers and sellers both rate each other for every transaction), has more uniformity in seller ratings since there are direct repercussions on a buyer’s own
rating if s/he gives the seller a bad rating. At the same time, for both platforms, seller ratings are made available (equivalent to the performance of sellers being “posted” online). Over time, it was clear that there was a quality increase over time in the service level of Amazon sellers, but not so for eBay sellers. This is again in line with performance posting being more effective in increasing aggregate performance when there is sufficient heterogeneity (Proposition 1).

6.2.3 Other Similar Managerial Issues

While I have examined performance posting, I hope that my results are a first step towards understanding the efficacy of other similar practices such as an open salary policy in which employees can look up anyone’s salary or bonus. For example, Whole Foods has an open salary policy\(^1\), not just within a same hierarchy of employees, but across all hierarchies including the CEO level. To properly address the question of whether firms should adopt an open salary policy, social incentives such as equity and fairness will also need to be considered in the theoretical model, as these would presumably have an impact on utility. In addition, there needs to be greater clarity on the proper reference set for social comparison. For example, are employees comparing themselves “horizontally” to their peers, or are they comparing themselves “upward” in view of a promotion? These are challenging issues for both researchers and practitioners, with substantial implications for productivity and welfare. This is a topic that I will like to examine in my future stream of research.

\(^1\) http://www.businessinsider.com/whole-foods-employees-have-open-salaries-2014-3)
In this chapter, I lay out consideration of other issues which I think is worth pondering upon in the dissertation, which may also be of interest to the reader. The purpose of this chapter is twofold. First, I take this opportunity to provide more exploratory thinking on certain constructs and assumptions. Second, this chapter serves to provide an outlet for dimensions of future research, and discusses several extensions I will like to consider. I organize the myriad of issues by section. As an example of such future research, I carry out an extension in a separate chapter (Chapter 8), and see whether a firm can do better in profits by engaging in “partial” performance posting.

### 7.1 Other Firm Actions

While I assumed that commission rate $w$ is exogenous in my model, a firm can jointly optimize profits by simultaneously choosing $w$ and the performance posting decision ($s$). According to Proposition 3, I would intuit that firms which are contemplating whether they can increase profits by reducing $w$ should also be more divided about the use of per-
formance posting – profits can be much more positive or negative depending on $p$ in the model.

Another possible firm action is in the design of sales compensation incentives to allow agents of different abilities to self-select into different compensation schemes (e.g., Rao 1990) – this can be done by a combination of quotas and different commission rates to allow for a separating equilibrium between both types of agents. The question then becomes whether such an incentive design can outperform the use of performance posting, or alternatively, whether both can be combined to obtain even higher profits. This is an interesting angle for future work.

7.2 Social Comparison versus Competition

In my dissertation, I have assumed that social comparison and competitiveness are interchangeable constructs. That said, a distinction can be drawn between social comparison and competition: there cannot be competition without social comparison, though there can be social comparison without competition; a similar idea is expressed in Garcia, Tor and Schiff (2013) who argue that social comparison is an “important source of competitive behavior”. There are individual and situational factors that influence social comparison, and consequently, competitive behavior. Examples of individual factors include personal factors (e.g., personality, dimensions that are relevant to the self) and relational factors (e.g., how similar and close the target is prompts more comparison), while examples of situational factors include incentive structures (e.g., a zero-sum naturally increases comparison and competition), proximity to some standard (e.g., more comparison near the top or bottom in rankings when the standard is meaningful), and number of competitors.

By linking social comparison to a single “culture of competitiveness”, I am assuming that there is a certain degree of homogeneity in the way these factors influence social
comparison, and consequently competitiveness. Naturally, this has greater validity when agents themselves are fairly homogenous in personality and other characteristics, either as a function of self-selection or screening by the firm. This may be especially relevant in a sales force (think about Al Pacino’s and Alec Baldwin’s characters in the film Glengarry Glen Ross). More importantly, by showing that the level of social comparison moderates effectiveness of performance posting (Proposition 2), firms can then find ways to influence the level of social comparison, and thereby increase or decrease competitiveness. Applying the findings of Garcia, Tor and Schiff (2013), a firm can increase the level of social comparison by enhancing similarity or closeness of the sales force, or change the design of incentive structures (e.g., having a contest prize structure to promote the zero-sum nature of the payoffs), amongst other actions.

7.3 Risk Aversion of Agents

In my analysis, I have assumed that the firm is risk neutral, and so are both agents in wealth, i.e., there is no concavity in the monetary (i.e., salary) component of utility for agents. This differs from the conventional assumption in a principal-agent setting where agents are risk averse and the principal is risk neutral. I will address two natural questions: (i) why I assume that agents are risk neutral, and (ii) how my results will change if agents are risk averse.

The answer to the first question revolves around the issue of moral hazard. First, the objectives of the firm and the agent(s) are different. Second, efforts by the agents are not directly observable by the firm, these efforts influence performances, and performances have different consequences for firm and agents. In a first-best situation when the firm can perfectly monitor agents’ efforts, this removes the issue of moral hazard: the firm can
recommend that the agent chooses the most efficient effort level, or else impose a fine on the agent. But if perfect monitoring of efforts by the firm is not possible, a second-best situation ensures: performances now become imperfect signals of efforts (because abilities of agents are uncertain). In such cases, moral hazard is present, and this requires that the firm offers agents contracts that require trade-offs between risk sharing and incentives. If agents are risk averse, these trade-offs affect equilibrium effort as greater compensation is needed for the agents to take on more risk. But if agents are risk neutral, such trade-offs are nonexistent because there is no need for risk sharing, and the contract in my model (firm gets a fixed price $P$ for every unit of sale, and agent receives from the firm a fixed commission $w$ for every unit of sale) is an optimal contract, i.e., the firm cannot do better under unobservable effort than observable effort.

For the second question, if agents are indeed risk averse and effort is unobservable, then the setup becomes more complicated. This is because the firm needs to design a contract and condition the agent’s wage on the performance outcome in a manner that ensures optimality given the agents’ risk aversion, while at the same time, being cognizant of the effects of social comparison in the agents’ utilities. This goes beyond adjusting the agent’s utility function to allow for concavity in wealth, for example, $U_i(e_i) = \sqrt{w \cdot (a_i + e_i) + k \cdot \frac{a_i + e_i}{(a_i + e_i) + (a_j + e_j)}} - \frac{e_i^2}{2}$, and analyzing how equilibrium efforts of agents change as a result of such a specification (which arguably would reduce the high-ability agent’s effort level more so than the low-ability agent’s effort level due to concavity, though the difference between posting and not posting remains to be examined). The larger issue here is that more than just the focal decision of performance posting by the firm, an additional stage in the game is required in which the firm has to first solve for the optimal compensation plan. According to Basu et al. (1985), a firm’s optimal compensation plan should be a convex increasing function of sales as agents become increasingly risk averse. Another type of compensation plan design involves the setting of sales quotas, which the
firm can undertake as a form of a screening mechanism to encourage the agent with high ability to self-select into the higher quota plan (Rao 1990). The interaction of compensation design with the effects of performance posting are interesting extensions worth exploring, and will require an augmentation of both model setup and structure of the game.

7.4 From Two to $N$ Agents

I will now briefly describe how I can extend my model from the case of two to $N$ agents – a more common setting for a sales team, and also what we are more likely to observe in the real world when performance is posted, as seen in Figure 2.1. This is also the setting for my experiment.

In the theoretical model, I can obtain the distribution of abilities for $N$ agents by increasing the number of draws in a binomial distribution from two to $N$. Each draw yields an outcome of success (high-ability agent) or failure (low-ability agent).

I will outline two approaches on how to extend my utility specification in the case of $N$ agents. The first approach is to generalize $j$ from the case of one other single agent to the sum of all other agents. Assuming agents’ abilities are known, the utility function of agent $i$ now becomes:

$$U_i(e_i) = w \cdot (a_i + e_i) + k \cdot \frac{a_i + e_i}{(a_i + e_i) + \sum_{j \neq i}^{N} (a_j + e_j)} - \frac{e_i^2}{2}$$  \hspace{1cm} (7.1)

The key difference here is how the presence of multiple agents enters into the utility function via the denominator of the social comparison function. Holding all else constant, the effect of increasing the number of agents increases the denominator of the social comparison component from summing over a greater number of agents’ abilities and efforts.
However, due to an agent’s share of performance becoming a smaller fraction as the number of agents increases, this has the same effect as shifting agents into a region with greater concavity arising from the social comparison function. Thus, while having more agents dilutes the effect of the social comparison, it increases the marginal payoff of an agent’s effort. This formulation can, interestingly, be compared to Garcia and Tor (2009) who find that increasing the number of competitors can decrease an individual’s competitive motivation because there are now fewer comparison concerns and also, people lose interest in comparison information as $N$ increases. If this is the case, I can let $k$ be a monotonically decreasing function in $N$.

A second approach is to allow for multiple pairwise comparisons while keeping the structure of the social comparison function. This preserves the intent of the original model, as the single pairwise social comparison for two agents is now replaced by the arithmetic mean of all pairwise social comparisons.

$$U_i(e_i) = w \cdot (a_i + e_i) + \frac{k}{N-1} \cdot \sum_{j \neq i}^{N} \frac{a_i + e_i}{(a_i + e_i) + (a_j + e_j)} - \frac{e_i^2}{2} \quad (7.2)$$

Alternatively, a geometric mean of all pairwise social comparisons is also feasible:

$$U_i(e_i) = w \cdot (a_i + e_i) + k \cdot \left[ \prod_{j \neq i}^{N} \frac{a_i + e_i}{(a_i + e_i) + (a_j + e_j)} \right]^{\frac{1}{N-1}} - \frac{e_i^2}{2} \quad (7.3)$$

Both approaches introduce multiple agents in a non-linear manner through the social comparison function, and this makes mathematical simplification less straightforward. Thus, it may be difficult to obtain analytical tractability; numerical analysis may be required to study the effects of increasing the number of agents. It will be interesting to check the robustness of the result across different group sizes.
7.5 Team Sales

In my dissertation, my focus was on the individual “production” of sales through a combination of one’s ability and effort. Another type of setting in the sales force is the use of a sales team by a firm to achieve an aggregate sales objective. Here, I outline how my model can be extended to a team sales setting. There are two approaches. The first approach is to specify a production function that comprises solely of team sales; the other approach is to specify a production function that is made up of both individual and team sales. Both cases are observed in the real world, and may have different implications. For both cases, I introduce an interaction term between the performances of both agents and weight it by a parameter $t$ that is either negative or positive to respectively allow for substitutability or complementarity between the production functions of agents. Said differently, I am introducing interdependencies between the performance of agents in determining the aggregate sales for the firm.

For the first approach, the firm’s objective in a team setting is to maximize the following profit function:

$$\pi = (P - w) \cdot [t(a_i + e_i)(a_j + e_j)] \quad (7.4)$$

As for the second approach, the firm’s objective in a team setting is to maximize the following profit function:

$$\pi = (P - w) \cdot [(a_i + e_i) + (a_j + e_j) + t(a_i + e_i)(a_j + e_j)] \quad (7.5)$$

As observed in equation 7.4 for the first approach, aggregate sales in a team setting depend on the interaction of each agent’s performance, moderated by $t$. In the second approach, as observed in equation 7.5, aggregate sales in a team setting depend not only on
the sum of each agent’s individual performance, but also the interaction of each agent’s performance, moderated by $t$. In both cases, the firm’s objective is to maximize profit by choosing whether to post performance ($s = 1$) or not ($s = 0$), though the interesting issues to consider in this question is (i) whether the firm can attribute team performance to the individual agents, and depending on this, (ii) how the firm designs its team compensation plan.

7.6 Structural Estimation of Parameters from Experiment

To test my theoretical propositions, I used a between-subjects factorial design in experiments catered specifically for each of the propositions. Alternatively, I can collect experimental data with the intent of performing structural estimation of the parameters from my theoretical model. The value of obtaining estimates of the structural parameters is that I can make out-of-sample predictions about how the outcomes would change based on varying the magnitude of the parameters, and also calculate welfare changes since my model is utility-based. If structural estimation is the objective, I will also consider employing a more flexible specification that allows for individual-specific parameters in the following way:

$$U_i(e_i) = w \cdot (a_i + e_i) + k \cdot \kappa_i \cdot \frac{a_i + e_i}{(a_i + e_i) + (a_j + e_j)} - \gamma \frac{e_i^2}{2} \quad (7.6)$$

First, by introducing $\kappa_i$, I allow for heterogeneity in individuals’ propensity to engage in social comparison, which then moderates the utility s/he receives from social comparison. This is certainly a richer specification, as individuals are likely to vary in the way they care about social comparison, and this may be a function of personality traits. Thus, while
there is still $k$ that captures the “culture of competitiveness” that agents face in a firm, some individuals are more susceptible to this than others. In addition, I will allow for an additional parameter $\gamma_i$ that captures individual-specific slopes for cost of effort.

In my experimental design, I will have repeated rounds of the same task that I used. Within-subject variation of $w$ will help me identify $\gamma_i$. In addition, I can proxy for $\kappa_i$ using a survey of participants’ social comparison orientation scale (Gibbons and Bunnk 1990).

Another point worth noting is that my utility specification nests the typical model used in the traditional economics literature – by setting $k$ to 0, I can recover the restricted model. With this design though, I can estimate whether the effect size of $k$ is significantly different from zero, and to assess if the “standard models fail” (Ho, Lim and Camerer 2006). As a final note, given that my utility specification is non-linear (and difficult to linearize), I will use maximum likelihood estimation as my estimation routine.

### 7.7 Effect of Biased Inferences

For future work on this topic, I will like to explore the effect of biased inferences on performance posting. This is important because there is a growing body of literature on how people are biased not only in their inferences, but also in the way they update their inferences depending on the type of information (e.g., Mobius et al. 2013). To give a sense of what this entails, Eil and Rao (2011) find that participants receiving negative feedback on objective information they care about (intelligence and beauty) do not respect the strength of the signals, are less predictable in updating, and are averse to such new information; on the other hand, when the feedback is positive, participants conform more to Bayes’ rule in the updating of beliefs. Yet another type of biased inference is how people consistently misestimate their performance relative to others (e.g., Kruger 1999).
Biased inferences can thus stem from information type (e.g., good or bad news) as well as in the comparison with others (e.g., overestimating or underestimating others’ performance). Both will require different model specifications, and should have interesting implications as to when a firm should use performance posting.
Chapter 8

An Extension: Partial Performance Posting

8.1 Motivation for this Extension

In my dissertation, I explored the effects of performance posting on the amount of effort agents exerted in a sales task, and their resulting performances. I contrasted two extreme cases – the firm choosing whether to post or not post performance with the intention of maximizing aggregate performance. However, casual observation of performance posting in the real world suggest a myriad of intermediate cases. For example, a firm may choose only to provide the average performance of the group, and maybe along with this, some measure of dispersion (e.g., standard deviation or the maximum/minimum performance). Sometimes, a firm may display rank order of the agents without showing performance. At other times, a firm may display performance in broad bands (e.g., Cat A/B/C or Tier 1/2/3), or improvize on this by revealing the top performers with full granularity and keeping the rest of the performers in broad bands, or even simply leaving out the bottom performers altogether. The intuition here is that the firm may want to encourage those at the top to
work harder and to differentiate themselves, while at the same time, preventing those at the bottom from being discouraged.

There may be several reasons for the existence and prevalence of these intermediate cases. These reasons are oftentimes context-specific, though an underlying theme for not revealing too much information is to preserve the privacy of agents, which is a fundamental human right that is highly valued in certain societies and cultures\(^1\). In such situations, a firm has to balance the potentially positive influence of social comparison in engaging agents to put in more effort to obtain better performance, while at the same time, maintaining anonymity of the agents. A common way of doing so is for a firm to show some overall measure like the aggregate or average performance. This is what I term “partial” performance posting – an intermediate of the two extreme cases that I studied in the main part of my dissertation. In this extension, I will study the specific case of the firm providing an overall performance measure via the aggregate performance, though average performance is simply the aggregate performance divided by the number of agents. With this extension, I hope to examine the circumstances under which such a practice can be optimal\(^2\).

### 8.2 Providing an Overall Performance Measure (“Partial” Posting)

In this extension, I largely preserve the setup and structure of my original game. The only difference is the enlarged strategy space of the firm to include partial performance posting \((s = p)\), in addition to complete posting \((s = 1)\) or not posting \((s = 0)\).

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\(^1\)http://en.wikipedia.org/wiki/Right_to_privacy

\(^2\)A firm can also engage in another form of partial posting by not displaying names and faces during performance posting. I argue that this resembles my setup, and that such a form of privacy simply reduces the magnitude of social comparison \((k)\).
Here, partial performance posting by the firm means providing only a measure of aggregate performance[^3], which both agents will know at stage four of the game. I argue, from a psychological viewpoint of social comparison, that this is a different case from (i) not posting, in which the comparison is with the performance of an agent of average ability, as determined by the distribution of abilities, putting in average-ability type level of efforts or (ii) complete posting, in which the expected utility comes from a probabilistic integration of the scenarios in which the other agent is of high or low ability. In this intermediate case, the social comparison that agent $i$ makes is expressed by the ratio of his or her performance to the overall performance – the overall performance is posted by the firm, part of which comprises of his or her performance:

$$EU_i(e_i|s = p) = w \cdot (a_i + e_i) + k \cdot \frac{a_i + e_i}{(a_i + e_i) + E_i[a_j + e_j]} - \frac{e_i^2}{2}$$ (8.1)

Alternatively, this can be expressed as:

$$EU_i(e_i|s = p) = w \cdot (a_i + e_i) + k \cdot \left[ \frac{a_i + e_i}{(a_i + e_i) + p(a_j = H + e_j = H) + (1 - p)(a_j = L + e_j = L)} \right] - \frac{e_i^2}{2}$$ (8.2)

Notice that in this intermediate case, the integrating of uncertainty across the scenarios is now in the denominator of the social comparison term. Mathematically, this added complexity makes it difficult to obtain closed-form solutions of the equilibrium. To overcome this analytical intractability, I solve for this extension numerically, and compare the aggregate performance under partial posting to the aggregate performances of both the posting and no posting cases. This leads me to the following proposition:

[^3]: I assume this to be the sum of both agents’ performance in this model, though more realistically, it is the average performance as the number of agents increases. If the benchmark is the average performance, and an agent compares his or her performance to the average performance in the denominator, then I can rescale $k$ to account for the number of agents.
Proposition 4: Given sufficient heterogeneity in ability levels, partial performance posting can do better than both posting and no posting, if the extent of social comparison is sufficiently low.

In Proposition 1, I find that a firm does better by posting performance when there is sufficient heterogeneity in agents’ abilities. This comes primarily from the low-ability agent putting in more effort to avoid being too far behind when performance is posted. In Proposition 2, I explained how the effectiveness of performance posting increases from a low to moderate level of social comparison, and decreases from a moderate to high level of social comparison. Proposition 4 builds on these two propositions, and provides insight on how partial performance posting can outperform posting and no posting. Specifically, when there is sufficient heterogeneity in ability levels such that posting does better than not posting, I find that partial performance posting can do even better than both under certain circumstances – when the extent of social comparison $k$ is small and approaches zero.

The reason is that when $k$ is small, the amount of “social utility” from social comparison is negligible not only relative to the “financial utility”, but also more likely to be reined in by the convexity in the cost of effort, i.e., it is unlikely to be worthwhile to invest that extra amount of effort to gain an incremental amount of “social utility”. This happens for both the high- and low-ability agents, but more so for the high-ability agent because it is on a relatively “less concave” portion of the social comparison function. Thus, the difference in performance from the high-ability agent becomes minimal across the cases of posting versus no posting – and the same notion applies to the high-ability agent when there is partial performance posting. Moreover, in the case of partial performance posting, there is a “disadvantageous” marginal change in the proportion of performances in the social comparison for partial performance posting. To elaborate, an agent’s effort gets amplified in the denominator by an additional factor of $p$ for the high-ability agent or $1 - p$ for the low-ability agent, thus reducing the rate of change from an incremental unit of effort, which is
not present in both cases of posting or not posting. The combination of these factors means
that the high-ability agent’s social comparison is disproportionately muted in the case of
partial performance posting, more so than any other cases, thus giving an opportunity for
the low-ability agent to gain by scaling up effort without much resistance in terms of the
high-ability agent responding in equilibrium. This results in partial performance posting
yielding better performance when there is sufficient heterogeneity in ability and when the
level of social comparison $k$ is low.

This proposition explains why sometimes, a sales manager or a professor may prefer
to announce an aggregate performance measure on a particular task rather than post per-
formance (even if privacy is not the primacy concern), in the general spirit of maintaining
minimal social comparison or competition (low $k$), but at the same time, letting those who
are behind the curve know that they should buck up, and perhaps giving a slight pat on the
back for those who are doing well. As explained above, this practice prevents the high-
ability agent from “over-exerting” effort since social comparison is muted, and at the same
time, allows the low-ability agent to play catch-up.
Chapter 9

Conclusion

I opened this dissertation with general questions about performance posting – what do people experience under performance posting, how differently they would respond, and whether posting can be strategically used to elicit better performance. I chose to answer these questions in the specific context of a sales force because it is an important substantive area in marketing and the potential for impact is sizable. Since the effects of performance posting are prima facie unclear, I first set up a game-theoretic model to analyze the strategic interaction between a firm’s decision to post performance, and how multiple agents would respond in effort on the sales task. By examining the differences in profit for the firm under the decision to post versus not post, I provided situations under which posting does better and when it does worse. Evidence from a series of lab experiments and a field study yielded empirical validation for my theoretical findings, thereby providing confidence for prescription.

A general consensus in the definition of performance within the sales force offered an ideal starting point to understand the effects of performance posting. However, this does not limit the applicability of my research to other domains. By seeking to abstract the economic fundamentals of behavior and understanding their interplay, an astute reader can apply a more liberal interpretation of the terms in my theoretical framework to topics of personal interest. Furthermore, my model can be modified with ease to capture the

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nuances of different settings across different disciplines. It is this flexibility, I believe, that allowed me to first demonstrate the empirical relevance of my theoretical propositions through experiments unrelated to the sales force, and second, to postulate explanations for relevant field observations in public health and online reputation systems. Exploratory chapters towards the end of my dissertation discussed ways in which to further generalize the theoretical model, and hopefully, enlarge the avenues of investigation. In sum, I hope that my analytical and empirical findings in this dissertation provide a modest first step towards understanding the effectiveness of performance posting.
Appendix

A.1 Comparative Statics for Symmetric Agents

We begin with the case in which agents have same abilities. With probability $p^2$, both agents will have high ability $a_i = a_j = a_H$, and equation 4.7 simplifies to the following:

$$e^*(a_H, k, w) = \sqrt{\frac{a_H^2 + k + 2a_Hw + w^2 + w - a_H}{2}}$$

(9.1)

Likewise, with probability $(1 - p)^2$, both agents will have low ability $a_i = a_j = a_L$, and equation 4.7 then simplifies to the following:

$$e^*(a_L, k, w) = \sqrt{\frac{a_L^2 + k + 2a_Lw + w^2 + w - a_L}{2}}$$

(9.2)

In either case, let the optimal effort for symmetric agents be given by $e^*(a, k, w) = \sqrt{\frac{a^2 + k + 2aw + w^2 + w - a}{2}}$, where $a$ can represent $a_H$ or $a_L$.

Taking the first derivative of $e^*$ with respect to $a$, we obtain:

$$\frac{\partial e^*}{\partial a} = \frac{w + a}{2\sqrt{a^2 + k + 2aw + w^2}} - \frac{1}{2}$$

(9.3)

Since $k > 0$, $w + a$ can never be greater than $\sqrt{a^2 + k + 2aw + w^2}$ since $\sqrt{a^2 + k + 2aw + w^2} = \frac{\sqrt{a^2 + k + 2aw + w^2}}{2}$.
\( \sqrt{(a+w)^2+k} \), i.e., \( \frac{w+a}{2\sqrt{a^2+k+2aw+w^2}} \leq \frac{1}{2} \). \( \frac{\partial e^*}{\partial w} \leq 0. \)

Taking the first derivative of \( e^* \) with respect to \( w \), we obtain:

\[
\frac{\partial e^*}{\partial w} = \frac{w + a}{2\sqrt{a^2 + k + 2aw + w^2}} + \frac{1}{2} \tag{9.4}
\]

Again, since \( k \geq 0, \frac{w+a}{2\sqrt{a^2+k+2aw+w^2}} \leq \frac{1}{2} \). \( \therefore \frac{\partial e^*}{\partial w} \geq 0. \)

Taking the first derivative of \( e^* \) with respect to \( k \), we obtain:

\[
\frac{\partial e^*}{\partial k} = \frac{1}{4\sqrt{a^2+k+2aw+w^2}} \tag{9.5}
\]

Again, since \( k \geq 0, w > 0 \) and \( a > 0, \sqrt{a^2+k+2aw+w^2} \) has to be positive and real. \( \therefore \)
\( \frac{\partial e^*}{\partial k} > 0. \)
A.2 Comparative Statics for Asymmetric Agents

Taking the first derivative of $e_i^*$ with respect to $a_i$, we obtain:

\[
\frac{\partial e_i^*}{\partial a_i} = \frac{(a_i^3 + a_j^3 + a_j^2(3a_j + 6w - \sqrt{\lambda}) - a_j^2(-6w + \sqrt{\lambda}) - 4(k + w^2)(-2w + \sqrt{\lambda}) + a_j(3a_j^2 + 6k + 12a_jw + 12w^2 - 2a_j\sqrt{\lambda} - 4w\sqrt{\lambda}) + 2a_j(k + 6w^2 - 2w\sqrt{\lambda}))}{2\lambda_i^{3/2}}
\]

where $\lambda_i = a_i^2 + a_j^2 + 4a_jw + 2a_i(a_j + 2w) + 4(k + w^2)$.

Taking the second derivative of $e_i^*$ with respect to $a_i$, we obtain:

\[
\frac{\partial^2 e_i^*}{\partial a_i^2} = \frac{(-6a_i - 4a_j - 6w - \frac{4(a_i + 2a_j + 4w)^2\mu_i}{4\lambda_i^{3/2}} + \frac{(2a_i + 2a_j + 3w)(2a_i + 2a_j + 4w)\mu_i}{2\lambda_i^{3/2}} + 2\sqrt{\lambda_i})}{2\lambda_i - \xi_i + \mu_i\sqrt{\lambda_i}/\lambda_i^2} - \frac{(2a_i + a_j + 3w)\sqrt{\lambda_i})}{\lambda_i^2} + \frac{(2a_i + 2a_j + 4w)^2(\xi_i + \mu_i\sqrt{\lambda_i})}{\lambda_i^3 - \xi_i + \mu_i\sqrt{\lambda_i}/\lambda_i^2}
\]

where $\lambda_i = a_i^2 + a_j^2 + 4a_jw + 2a_i(a_j + 2w) + 4(k + w^2)$,
$\mu_i = a_i^3 + a_i a_j + 2k + 3a_i w + a_j w + 2w^2$,
$\xi_i = -a_i^3 - 2a_i a_j - a_i a_j^2 - 4a_i k - 3a_i^2 w - 2a_i a_j w + a_j^2 w + 4k w + 4a_j w^2 + 4w^3$.

Taking the first derivative of $e_i^*$ with respect to $a_j$, we obtain:

\[
\frac{\partial e_i^*}{\partial a_j} = \frac{(a_i - a_j)k}{(a_i^2 + a_j^2 + 4a_jw + 2a_i(a_j + 2w) + 4(k + w^2))^{3/2}}
\]

Since the denominator is always positive, the sign of $\frac{\partial e_i^*}{\partial a_j}$ depends on the numerator. If $a_i \geq a_j$, then $\frac{\partial e_i^*}{\partial a_j} \geq 0$.

Taking the second derivative of $e_i^*$ with respect to $a_j$, we obtain:
\[
\frac{\partial^2 e^*_i}{\partial a^*_j} = -\frac{2k(2a_i^2 - a_j^2 - a_iw + a_i(a_j + 5w) + 2(k + w^2))}{(a_i^2 + a_j^2 + 4a_jw + 2a_i(a_j + 2w) + 4(k + w^2))^{5/2}}
\]  
(9.9)

Conditions under which \( \frac{\partial^2 e^*_i}{\partial a^*_j} > 0 \):

(i) \( 0 < a_i < \frac{a_j}{2} \), and \( 0 < w < a_j - 2a_i \), and \( 0 < k < \frac{1}{2}(-2a_i^2 - a_ia_j + a_j^2 - 5a_jw + a_jw - 2w^2) \).

Taking the first derivative of \( e^*_i \) with respect to \( w \), we obtain:

\[
\frac{\partial e^*_i}{\partial w} = \frac{(a_i^3 + a_j^3 + 4(k + w^2)(2w + \sqrt{\lambda_i}) + 4a_jw(3w + \sqrt{\lambda_i}) + a_j^2(6w + \sqrt{\lambda_i}) + a_i(3a_j^2 + 8k + 12a_jw + 12w^2 + 2a_j\sqrt{\lambda_i} + 4w\sqrt{\lambda_i})}{2\lambda_i^{3/2}}
\]  
(9.10)

where \( \lambda_i = a_i^2 + a_j^2 + 4a_jw + 2a_i(a_j + 2w) + 4(k + w^2) \).

Taking the second derivative of \( e^*_i \) with respect to \( w \), we obtain:

\[
\frac{\partial^2 e^*_i}{\partial w^2} = \frac{8k \left( -a_i^2 + 2a_j^2 + a_i(a_j - w) + 5a_jw + 2(k + w^2) \right)}{\lambda_i^{5/2}}
\]  
(9.11)

where \( \lambda_i = a_i^2 + a_j^2 + 4a_jw + 2a_i(a_j + 2w) + 4(k + w^2) \).

Conditions under which \( \frac{\partial^2 e^*_i}{\partial w^2} > 0 \):

(i) \( 0 < a_i < 2a_j \) and \( k > 0 \); or

(ii) \( a_i > 2a_j \) and \( 0 < w < a_i - 2a_j \) and \( k > \frac{1}{2}(a_i^2 - a_ia_j - 2a_j^2 + a_iw - 5a_jw - 2w^2) \); or

(iii) \( a_i > 2a_j \) and \( w > a_i - 2a_j \) and \( k > 0 \).

Taking the first derivative of \( e^*_i \) with respect to \( k \), we obtain:
\[
\frac{\partial e^*_i}{\partial k} = \left(4(w - a_i) + \frac{2\mu_i}{\sqrt{\lambda_i}} + 2\sqrt{\lambda_i}\right) / 2\lambda_i - (2(-a_i^3 - 2a_i^2a_j - a_i^2a_j - 4a_ik - 3a_i^2w - 2a_iaw + a_j^2w + 4kw + 4a_jw^2 + 4w^3 + \\
\mu_i\sqrt{\lambda_i})/\lambda_i^2)
\]

where \(\lambda_i = a_i^2 + a_j^2 + 4a_jw + 2a_i(a_j + 2w) + 4(k + w^2)\),
\(\mu_i = a_i^2 + a_i a_j + 2k + 3aiw + ajw + 2w^2\).

Taking the second derivative of \(e^*_i\) with respect to \(k\), we obtain:

\[
\frac{\partial^2 e^*_i}{\partial k^2} = \frac{\frac{-4\mu_i}{\lambda_i^{3/2}} + \frac{8}{\lambda_i}}{2\lambda_i} - \frac{4(4w - 4a_i + \frac{2\mu_i}{\sqrt{\lambda_i}} + 2\sqrt{\lambda_i})}{\lambda_i^2} + (16(-a_i^3 - 2a_i^2a_j - a_i^2a_j - 4a_i k - 3a_i^2w - 2a_iaw + a_j^2w + 4kw + 4a_jw^2 + 4w^3 + \\
\mu_i\sqrt{\lambda_i})/\lambda_i^3)\)
\]

where \(\lambda_i = a_i^2 + a_j^2 + 4a_jw + 2a_i(a_j + 2w) + 4(k + w^2)\),
\(\mu_i = a_i^2 + a_i a_j + 2k + 3aiw + ajw + 2w^2\).

Conditions under which \(\frac{\partial^2 e^*_i}{\partial k^2} > 0\):

(i) \(a_i > 2a_j, 0 < w < a_i - 2a_j\) and \(0 \leq k \leq \frac{1}{2}(a_i^2 - a_ia_j - 2a_j^2 - aiw - 5a_jw - 2w^2)\)
A.3 Comparative Statics for the Case where Performance is Not Posted

A.3.1 Agent with Low Ability

\[
\frac{\partial e_i^*(a_i = a_L, E_i[a_j], k, w|s = 0)}{\partial a_H} =
\]

\[
\frac{(-2a_L^2p - 2a_Lp(a_L(1 - p) + a_H)p) - 2a_Lpw + 2p(a_L(1 - p) + a_H)p)w + 4pw^2 + (\mu \xi / 2\sqrt{\lambda}) + (a_Lp + pw)\sqrt{\lambda})}{(2\lambda) - (\mu(-a_L^2 - 4a_Lk - 2a_L^2(a_L(1 - p) + a_H)p) - a_L(a_L(1 - p) + a_H)p)^2 - 3a_L^2w + 4kw - 2a_L(a_L(1 - p) + a_H)p)w + (a_L(1 - p) + a_H)p)^2w + 4(a_L(1 - p) + a_H)p)w^2 + 4w^3 + \xi \sqrt{\lambda})/(2\lambda^2)}
\]

(9.14)

where \(\lambda = a_L^2 + 4k + 2a_L(a_L(1 - p) + a_H)p + (a_L(1 - p) + a_H)p)^2 + 4a_Lw + 4(a_L(1 - p) + a_H)p)w + 4w^2,
\]
\(\mu = 2a_Lp + 2p(a_L(1 - p) + a_H)p + 4pw,
\]
\(\xi = a_L^2 + 2k + a_L(a_L(1 - p) + a_H)p + 3a_Lw + (a_L(1 - p) + a_H)p)w + 2w^2.
\]

\[
\frac{\partial^2 e_i^*(a_i = a_L, E_i[a_j], k, w|s = 0)}{\partial a_H^2} =
\]

\[
\frac{2kp^2(-2k + a_H^2p^2 + a_L^2(-2 - p + p^2) + a_Hpw - 2w^2 + a_H(p(1 + 2p) + (4 + p)w))}{(4k + a_H^2(-2 + p)^2 - 2a_L(-2 + p)(a_Hp + 2w) + (a_Hp + 2w)^2)^5/2}
\]

(9.15)

Conditions under which \(\frac{\partial^2 e_i^*(a_i = a_L, E_i[a_j], k, w|s = 0)}{\partial a_H^2} > 0:\)

(i) \(a_H > 2a_L + w, \) and \(-\frac{a_L - w}{a_H + a_L} < p \leq 1, \) and

(ii) \(0 < k < \frac{1}{2}(-2a_L^2 + a_Ha_Lp - a_L^2p^2 + a_H^2p^2 - 2a_Ha_Lp^2 + a_L^2p^2 - 4a_Lw + a_Hpw - a_Lpw - 2w^2).\)

\[
\frac{\partial e_i^*(a_i = a_L, E_i[a_j], k, w|s = 0)}{\partial a_L} =
\]
\[-3a_2^2 - 4k - 2a_2^2(1 - p) - 4a_L(a_L(1 - p) + a_Hp) - 2a_L(1 - p)(a_L(1 - p) + a_Hp) - (a_L(1 - p) + a_Hp)^2 - 6a_Lw - 2a_L(1 - p)w - 2(a_L(1 - p) + a_Hp)w + 2(1 - p)(a_L(1 - p) + a_Hp) + 2) + 4w(1 - p)w\xi) / (2\sqrt{\lambda})(2L + 2a_L(1 - p) + a_Hp + 3w + (1 - p)w)\sqrt{\lambda} / (2\lambda) - ((2a_L + 2a_L(1 - p) + 2(a_L(1 - p) + a_Hp) + 2(1 - p)(a_L(1 - p) + a_Hp) + 4w(1 - p)w) (2a_L + 2a_L(1 - p) + 2(a_L(1 - p) + a_Hp) + 2(1 - p)(a_L(1 - p) + a_Hp) + 4w(1 - p)w) (2a_L + 2a_L(1 - p) + 2(a_L(1 - p) + a_Hp) + 2(1 - p)(a_L(1 - p) + a_Hp) + 4w(1 - p)w) (2a_L + 2a_L(1 - p) + 2(a_L(1 - p) + a_Hp) + 2(1 - p)(a_L(1 - p) + a_Hp) + 4w(1 - p)w) (2a_L + 2a_L(1 - p) + 2(a_L(1 - p) + a_Hp) + 2(1 - p)(a_L(1 - p) + a_Hp) + 4w(1 - p)w) (2a_L + 2a_L(1 - p) + 2(a_L(1 - p) + a_Hp)) + 3a_Lw + (a_L(1 - p) + a_Hp)w + 2w^2.\]

where \(\lambda = a_2^2 + 4k + 2a_L(a_L(1 - p) + a_Hp) + (a_L(1 - p) + a_Hp)^2 + 4a_Lw + 4(a_L(1 - p) + a_Hp)w + 4w^2.\)

\[\xi = a_2^2 + 2k + a_L(a_L(1 - p) + a_Hp) + 3a_Lw + (a_L(1 - p) + a_Hp)w + 2w^2.\]

\[
\frac{\partial^2 e^s_i(a_i = a_L, E_i[a_j], k,w|s = 0)}{\partial a_2^2} = \frac{(2k(1 - p)(a_2^2(-2 + p)^2(-1 + p) - 4a_2^2p^2 + a_2^2p^3 - 2k(1 - p) - 10a_Hpw + a_Hp^2w - 4w^2 - 2pw^2 - a_L(-2 + p)(a_Hp(5 + 2p) + (-4 + p)w)) / (4k + a_2^2(-2 + p)^2 - 2a_L(-2 + p)(a_Hp + 2w) + (a_Hp + 2w)^2)^{5/2}}{\partial k} = \frac{(-4a_L + 4w + 2\xi / \sqrt{\lambda} + 2\sqrt{\lambda}) / (2\lambda)}{(2a_L^3 + 4a_L(1 - p) + a_Hp) - a_L(a_L(1 - p) + a_Hp)^2 - 3a_Lw + 4k - 2a_L(a_L(1 - p) + a_Hp)w + (a_L(1 - p) + a_Hp)^2w + 4(a_L(1 - p) + a_Hp)w^2 + 4w^3 + \xi \sqrt{\lambda}) / (2\lambda)}
\]

where \(\lambda = a_2^2 + 4k + 2a_L(a_L(1 - p) + a_Hp) + (a_L(1 - p) + a_Hp)^2 + 4a_Lw + 4(a_L(1 - p) + a_Hp)w + 4w^2.\)

\[\xi = a_2^2 + 2k + a_L(a_L(1 - p) + a_Hp) + 3a_Lw + (a_L(1 - p) + a_Hp)w + 2w^2.\]
\[
\frac{\partial^2 e_i^*(a_i = a_L, E_i[a_j], k, w|s = 0)}{\partial k^2} = \\
-2(2k + 2a_H^2 p^2 + a_L^2 (2 - 5p + 2p^2) + 5a_H p w + 2w^2 + a_L(a_H(5 - 4p)p + (4 - 5p)w)) \\
(4k + a_L^2(-2 + p)^2 - 2a_L(-2 + p)(a_H p + 2w) + (a_H p + 2w)^2)^{5/2}
\]
(9.19)

\[
\frac{\partial e_i^*(a_i = a_L, E_i[a_j], k, w|s = 0)}{\partial w} = \\
(-3a_L^2 + 4k - 2a_L(a_L(1-p) + a_H p) + (a_L(1-p) + a_H p)^2 + \\
8(a_L(1-p) + a_H p)w + 12w^2 + ((4a_L + 4(a_L(1-p) + a_H p) + 8w)\xi)/(2\sqrt{\lambda}) + \\
(3a_L + a_L(1-p) + a_H p + 4w)\sqrt{\lambda})/(2\lambda) - \\
((4a_L + 4(a_L(1-p) + a_H p) + 8w)(-a_L^3 - 2a_L k - 2a_L^2(a_L(1-p) + a_H p) - \\
a_L(a_L(1-p) + a_H p)^2 - 3a_L^3 w + 4kw - 2a_L(a_L(1-p) + a_H p)w + \\
\xi \sqrt{\lambda}))/(2\lambda^2) \\
\]
(9.20)

where \(\lambda = a_L^2 + 4k + 2a_L(a_L(1-p) + a_H p) + (a_L(1-p) + a_H p)^2 + 4a_L w + 4(a_L(1-p) + \\
a_H p)w + 4w^2\), \\
\(\xi = a_L^2 + 2k + a_L(a_L(1-p) + a_H p) + 3a_L w + (a_L(1-p) + a_H p)w + 2w^2\).

\[
\frac{\partial^2 e_i^*(a_i = a_L, E_i[a_j], k, w|s = 0)}{\partial w^2} = \\
\frac{8k(2k + 2a_H^2 p^2 + a_L^2 (2 - 5p + 2p^2) + 5a_H p w + 2w^2 + a_L(a_H(5 - 4p)p + (4 - 5p)w))}{(4k + a_L^2(-2 + p)^2 - 2a_L(-2 + p)(a_H p + 2w) + (a_H p + 2w)^2)^{5/2}}
\]
(9.21)

\[
\frac{\partial e_i^*(a_i = a_L, E_i[a_j], k, w|s = 0)}{\partial p} = \\

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\[
(-2(a_H - a_L)a_L^2 - 2(a_H - a_L)a_L(a_L(1 - p) + a_H p) - \\
2(a_H - a_L)a_L w + 2(a_H - a_L)(a_L(1 - p) + a_H p)w + \\
4(a_H - a_L)w^2 + ((2(a_H - a_L)a_L + 2(a_H - a_L)(a_L(1 - p) + a_H p) + 4(a_H - a_L)w)\xi)/(2\sqrt{\lambda}) + \\
((a_H - a_L)a_L + (a_H - a_L)w)\sqrt{\lambda}/(2\lambda) - \\
((2(A_H - a_L)a_L + 2(a_H - a_L)(a_L(1 - p) + a_H p) + 4(a_H - a_L)w) \\
(-a_L^3 - 4a_L k - 2k a_L (1 - p) + a_H p) - a_L (a_L(1 - p) + a_H p)^2 - 3a_H^2 w + 4kw - \\
2a_L(a_L(1 - p) + a_H p)w + (a_L(1 - p) + a_H p)^2 w + 4(a_L(1 - p) + a_H p)^2 w^2 + 4w^3 + \\
\xi \sqrt{\lambda})/(2\lambda^2)
\]

(9.22)

where \(\lambda = a_L^2 + 4k + 2a_L(a_L(1 - p) + a_H p) + (a_L(1 - p) + a_H p)^2 + 4a_L w + 4(a_L(1 - p) + a_H p)w + 4w^2\),
\[\xi = a_L^2 + 2k + a_L (a_L(1 - p) + a_H p) + 3a_L w + (a_L(1 - p) + a_H p)w + 2w^2.\]

\[
\frac{\partial^2 e_i^*}{\partial p^2}(a_i = a_L, E_i[a_j], k, w|s = 0)
\]

\[
\frac{2(a_H - a_L)^2 k(-2k + a_H^2 p^2 + a_L^2 (-2 - p + p^2) + a_H p w - 2w^2 - a_L(a_H p(-1 + 2p) + (4 + p)w))}{(4k + a_L^2 (-2 - p)^2 - 2a_L(-2 + p)(a_H p + 2w) + (a_H p + 2w)^2)^{3/2}}
\]

(9.23)

Conditions under which \(\frac{\partial^2 e_i^*}{\partial p^2}(a_i = a_L, E_i[a_j], k, w|s = 0) > 0\):
(i) \(a_H > 2a_L + w\) and \(\frac{-a_L - w}{a_H + a_L} < p \leq 1\) and
(ii) \(0 < k < \frac{1}{2}(-2a_L^2 + a_H a_L p - a_H^2 p + a_H^2 p^2 - 2a_H a_L p^2 + a_L^2 p^2 - 4a_L w + a_H p w - a_L p w - 2w^2)\).
A.3.2 Agent with High Ability

\[
\frac{\partial e_i^s(a_i = a_H, E_i[a_j], k, w|s = 0)}{\partial a_H} =
\]

\[
( -3a_H^2 - 4k - 2a_H^2 - 4a_H(a_L(1 - p) + a_H p) - \\
2a_H p(a_L(1 - p) + a_H p) - (a_L(1 - p) + a_H p)^2 - \\
6a_H w - 2a_H p w - 2(a_L(1 - p) + a_H p) w \\
+ 2p(a_L(1 - p) + a_H p) w + 4p w^2 + \\
((2a_H + 2a_H p + 2(a_L(1 - p) + a_H p) \\
+ 2p(a_L(1 - p) + a_H p) + 4w + 4p w) \\
( a_H^2 + 2k + a_H(a_L(1 - p) + a_H p) + 3a_H w + \\
( a_L(1 - p) + a_H p) w + 2w^2 ) ) / (2\sqrt{\lambda}) + \\
( 2a_H + a_L(1 - p) + 2a_H p + 3w + pw ) \sqrt{\lambda} / (2\lambda) - \\
((2a_H + 2a_H p + 2(a_L(1 - p) + a_H p) \\
+ 2p(a_L(1 - p) + a_H p) + 4w + 4p w) \\
( -a_H^3 - 4a_H k - 2a_H^2 (a_L(1 - p) + a_H p) \\
- a_H(a_L(1 - p) + a_H p)^2 - 3a_H^2 w + 4k w - \\
2a_H(a_L(1 - p) + a_H p) w + (a_L(1 - p) + a_H p)^2 w + \\
4(a_L(1 - p) + a_H p) w^2 + 4w^3 + \\
( a_H^2 + 2k + a_H(a_L(1 - p) + a_H p) + 3a_H w + \\
(a_L(1 - p) + a_H p) w + 2w^2 ) \sqrt{\lambda} ) / (2\lambda^2) \\
\]

where \( \lambda = a_H^2 + 4k + 2a_H(a_L(1 - p) + a_H p) + (a_L(1 - p) + a_H p)^2 + 4a_H w + 4(a_L(1 - p) + a_H p) w + 4w^2 \)

\[
\frac{\partial^2 e_i^s(a_i = a_H, E_i[a_j], k, w|s = 0)}{\partial a_H^2} =
\]

\[
( 2k(1 + p) (a_H^2 p(1 + p)^2 + a_L^2(-1 + p)^2(3 + p) \\
- a_L(-0 + 8p + p^2) w - \\
a_H(1 + p)(a_L(-3 + p + 2p^2) - (3 + p) w \\
- 2(-3 + p)(k + w^2)) ) / \\
( a_L^2(-1 + p)^2 + a_H^2(1 + p)^2 - 2a_H(1 + p)(a_L(-1 + p) - 2w \\
- 4a_L(-1 + p) w + 4(k + w^2))^{5/2} \\
\]

\[
\frac{\partial e_i^s(a_i = a_H, E_i[a_j], k, w|s = 0)}{\partial a_L} =
\]

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\[-2a_H^2(1-p) - 2a_H(1-p)(a_L(1-p) + a_Hp) - 2a_H(1-p)w \\
+ 2(1-p)(a_L(1-p) + a_Hp)w + 4(1-p)w^2 + \\
(2a_H(1-p) + 2(1-p)(a_L(1-p) + a_Hp) + 4(1-p)w) \\
(a_H^2 + 2k + a_H(a_L(1-p) + a_Hp) + 3a_Hw + \\
(a_L(1-p) + a_Hp)w + 2w^2)\right)/(2\sqrt{\lambda}) + \\
(a_H(1-p) + (1-p)w)\sqrt{\lambda}/
\\(2(a_H^2 + 4k + 2a_H(a_L(1-p) + a_Hp) + \\
\quad a_L(1-p) + a_Hp)^2 + 4a_Hw + \\
4(a_L(1-p) + a_Hp)w + 4w^2)) - \\
(2a_H(1-p) + 2(1-p)(a_L(1-p) + a_Hp) + 4(1-p)w) \\
(-a_H^3 - 4a_Hk - 2a_H^2(a_L(1-p) + a_Hp) - \\
a_H(a_L(1-p) + a_Hp)^2 - 3a_H^2w + 4kw- \\
2a_H(a_L(1-p) + a_Hp)w + (a_L(1-p) + a_Hp)^2w + \\
4(a_L(1-p) + a_Hp)w^2 + 4w^3 + \\
(a_H^2 + 2k + a_H(a_L(1-p) + a_Hp) + 3a_Hw + \\
(a_L(1-p) + a_Hp)w + 2w^2)\sqrt{\lambda})/(2\lambda^2)
\] 

where \( \lambda = a_H^2 + 4k + 2a_H(a_L(1-p) + a_Hp) + (a_L(1-p) + a_Hp)^2 + 4a_Hw + 4(a_L(1-p) + a_Hp)w + 4w^2 \)

\[
\frac{\partial^2 e^*_i(a_i = a_H, E_i[a_j], k, w | s = 0)}{\partial a_L^2} = -(2k(-1 + p)^2(-a_L^2(-1 + p)^2 + a_H^2(2 + p - p^2) + a_L(-1 + p)w + \\
a_H(a_L - 3a_Hp + 2a_Hp^2 + 5w - pw) + 2(k + w^2))/ \\
(a_L^2(-1 + p)^2 + a_H^2(1 + p)^2 - 2a_H(1 + p)(a_L(-1 + p) - 2w) - \\
4a_L(-1 + p)w + 4(k + w^2))/2
\]

\[
\frac{\partial e^*_i(a_i = a_H, E_i[a_j], k, w | s = 0)}{\partial k} = 
\]

\[
(-4a_H + 4w + (2a_H^2 + 2k + a_H(a_L(1-p) + a_Hp)) + 3a_Hw + \\
(a_L(1-p) + a_Hp)w + 2w^2))\sqrt{\lambda} + 2\sqrt{\lambda})/(2\lambda) - \\
(2(-a_H^3 - 4a_Hk - 2a_H^2(a_L(1-p) + a_Hp) - a_H(a_L(1-p) + a_Hp)^2 - 3a_H^2w + 4kw- \\
2a_H(a_L(1-p) + a_Hp)w + (a_L(1-p) + a_Hp)^2w + 4(a_L(1-p) + a_Hp)w^2 + 4w^3 + \\
(a_H^2 + 2k + a_H(a_L(1-p) + a_Hp) + 3a_Hw + (a_L(1-p) + a_Hp)w + 2w^2)\sqrt{\lambda})/\lambda^2
\]
where $\lambda = a_H^2 + 4k + 2a_H(a_L(1 - p) + a_Hp) + (a_L(1 - p) + a_Hp)^2 + 4a_Hw + 4(a_L(1 - p) + a_Hp)w + 4w^2$

$$\frac{\partial^2 e_i^*(a_i = a_H, E_i[a_j], k, w|s = 0)}{\partial k^2} =$$

$$-2(2a_H^2(-1 + p)^2 + a_H^2(-1 + p + 2p^2) - 5a_L(-1 + p)w + a_H(a_L + 3a_Lp - 4a_Lp^2 - w + 5pw + 2(k + w^2)))/$$

$$\frac{(a_L^2(-1 + p)^2 + a_H^2(1 + p)^2 - 2a_H(1 + p)(a_L(1 - p) - 2w) - 4a_L(-1 + p)w + 4(k + w^2))^{3/2}}{\partial k^2}$$

Conditions under which $\frac{\partial^2 e_i^*(a_i = a_H, E_i[a_j], k, w|s = 0)}{\partial k^2} > 0$:
(i) $a_H > w$ and $0 < a_L < \frac{a_H - w}{2}$ and $0 < p < \frac{a_H - 2a_L - w}{2a_H - 2a_L}$ and

$$0 \leq k < \frac{1}{2}(a_H^2 - a_Ha_L - 2a_H^2 - a_H^2p - 3a_Ha_Lp + 4a_L^2p - 2a_H^2p^2 + 4a_Ha_Lp^2 - 2a_L^2p^2 + a_Hw - 5a_Lw - 5a_Hpw + 5a_Lpw - 2w^2).$$

$$\frac{\partial e_i^*(a_i = a_H, E_i[a_j], k, w|s = 0)}{\partial w} =$$

$$(-3a_H^2 + 4k - 2a_H(a_L(1 - p) + a_Hp) + (a_L(1 - p) + a_Hp)^2 + 8(a_L(1 - p) + a_Hp)w + 12w^2 + ((4a_H + 4(a_L(1 - p) + a_Hp) + 8w)$$

$$(a_H^2 + 2k + a_H(a_L(1 - p) + a_Hp) + 3a_Hw + (a_L(1 - p) + a_Hp)w + 2w^2))/(2\sqrt{\lambda}) + (3a_H + a_L(1 - p) + a_Hp + 4w)/\sqrt{\lambda}) - ((4a_H + 4(a_L(1 - p) + a_Hp) + 8w)(-a_H^3 - 4a_Hk - 2a_H(a_L(1 - p) + a_Hp) - a_H(a_L(1 - p) + a_Hp)^2 + 3a_Hw + 4kw - 2a_H(a_L(1 - p) + a_Hp)w +$$

$$(a_L(1 - p) + a_Hp)^2w + 4(a_L(1 - p) + a_Hp)w^2 + 4w^3 + (a_H^2 + 2k + a_H(a_L(1 - p) + a_Hp) + 3a_Hw + (a_L(1 - p) + a_Hp)w + 2w^2)/\sqrt{\lambda}))/(2\lambda)$$

$$\frac{\partial^2 e_i^*(a_i = a_H, E_i[a_j], k, w|s = 0)}{\partial w^2} =$$

where $\lambda = a_H^2 + 4k + 2a_H(a_L(1 - p) + a_Hp) + (a_L(1 - p) + a_Hp)^2 + 4a_Hw + 4(a_L(1 - p) + a_Hp)w + 4w^2$
\[
\frac{\partial^2 e^*_i(a_i = a_H, E_i[a_j], k, w|s = 0)}{\partial p^2} = \frac{(8k(2a_L^2(-1+p)^2 + a_H^2(-1+p+2p^2)) - 6a_L(-1+p)\lambda + a_H(a_L + 3a_Lp - 4a_Lp^2 - w + 5pw) + 2(k + w^2)))}{(a_L^2(-1+p)^2 + a_H^2(1+p)^2 - 2a_H(1+p)(a_L(-1+p) - 2w) - 4a_L(-1+p)w + 4(k + w^2))^{5/2}}
\]

Conditions under which \(\frac{\partial^2 e^*_i(a_i = a_H, E_i[a_j], k, w|s = 0)}{\partial w^2} > 0\):

(i) \(0 < a_H \leq w\) and \(k > 0\), or

(ii) \(a_H > w\) and \(0 < a_L < \frac{a_H - w}{2}\) and \(0 \leq p < \frac{a_H - 2a_L - w}{2a_H - 2a_L}\) and \(k > \frac{1}{2}(a_H^2 - a_H a_L - 2a_L^2 - 2a_H a_L - 3a_H a_L p + 4a_L^2 p - 2a_H^2 p^2 + 4a_H a_L p^2 - 2a_H^2 p^2 + a_H w - 5a_L w - 5a_H p w + 5a_L p w - 2w^2)\), or

(iii) \(\frac{a_H - w}{2a_H - 2a_L} \leq p \leq 1\) and \(k > 0\), or

(iv) \(\frac{a_H - w}{2a_H - 2a_L} \leq a_L < a_H\) and \(k > 0\).

\[
\frac{\partial e^*_i}{\partial p}(a_i = a_H, E_i[a_j], k, w|s = 0) = \]

\[
\frac{(-2a_L^2(a_L - a_L) - 2a_H(a_L - a_L)(a_L(1-p) + a_H p) - 2a_H(a_L - a_L)w + 2(a_L - a_L)(a_L(1-p) + a_H p)w + 4(a_H - a_L)w^2 + ((2a_H a_L - a_L) + 2(a_H - a_L)(a_L(1-p) + a_H p)w + 3a_H w + (a_L(1-p) + a_H p)w + 2w^2) / (2\lambda) + (a_H(a_L - a_L) + (a_H - a_L)w) / (2\lambda) - ((2a_H(a_L - a_L) + 2(a_H - a_L)(a_L(1-p) + a_H p)) + 4(a_H - a_L)w(-a_H^3 - 4a_H k - 2a_H^2(a_L(1-p) + a_H p) - a_H(a_L(1-p) + a_H p)^2 - 3a_H^2 w + 4kw - 2a_H(a_L(1-p) + a_H p)w + (a_L(1-p) + a_H p)^2 w + 4(a_L(1-p) + a_H p)w^2 + 4w^2 + (a_L^2 + 2k + a_H(a_L(1-p) + a_H p) + 3a_H w + (a_L(1-p) + a_H p)w + 2w^2) / (2\lambda^2)) / (2\lambda^2)}{(a_L^2(-1+p)^2 + a_H^2(1+p)^2 - 2a_H(1+p)(a_L(-1+p) - 2w) - 4a_L(-1+p)w + 4(k + w^2))^{5/2}}
\]

where \(\lambda = a_H^2 + 4k + 2a_H(a_L(1-p) + a_H p) + (a_L(1-p) + a_H p)^2 + 4a_H w + 4(a_L(1-p) + a_H p)w + 4w^2\).
\[
\frac{(2(a_H - a_L)^2 k (a_L^2 (-1 + p)^2 + a_H^2 (-2 - p + p^2) + a_L (w - pw) - a_H (a_L - 3 a_L p + 2 a_L p^2 + 5w - pw) - 2(k + w^2)))}{(a_L^2 (-1 + p)^2 + a_H^2 (1 + p)^2 - 2a_H (1 + p) (a_L (-1 + p) - 2w) - 4a_L (-1 + p) w + 4(k + w^2))^{5/2}}
\]
A.4 Comparative Statics for the Case where Performance is Posted

A.4.1 Agent with Low Ability

\[
\frac{\partial e^*_i(a_i = a_L, k, w|s = 1)}{\partial a_H} = \frac{(-a_H + a_L)kp}{\mu^{3/2}}
\]  

(9.34)

where \( \mu = a_H^2 + a_L^2 + 4a_Lw + 2a_H(a_L + 2w) + 4(k + w^2) \).

\[
\frac{\partial e^*_i(a_i = a_L, k, w|s = 1)}{\partial a_L} =
\]

\[
\frac{1}{2}(2(-1 + p)(a_L + w)(-a_L + w + \sqrt{\lambda})/\lambda +
\]

\[
(1 - p)(-3a_L^2 - k - 2a_Lw + w^2 + 3(a_L + w)\sqrt{\lambda})/\lambda +
\]

\[
(p(-a_H^2 - 4a_Ha_L - 3a_L^2 - 4k - 2a_Hw - 6a_Lw +
\]

\[
(a_H + a_L + 2w)(a_L^2 + 3a_Lw + a_H(a_L + w) + 2(k + w^2))/\sqrt{\mu} +
\]

\[
(a_H + 2a_L + 3w)\sqrt{\mu}))/\mu - (2p(a_H + a_L + 2w)
\]

\[
(-a_H^2a_L - 2a_Ha_L^2 - a_L^3 - 4a_Lk - a_H^2w - 2a_Ha_Lw + 3a_L^2w + 4kw + 4a_Hw^2 + 4w^3 +
\]

\[
(a_L^2 + 3a_Lw + a_H(a_L + w) + 2(k + w^2))/\mu^2)
\]

(9.35)

where \( \lambda = a_L^2 + k + 2a_Lw + w^2 \),

\( \mu = a_H^2 + a_L^2 + 4a_Lw + 2a_H(a_L + 2w) + 4(k + w^2) \).

\[
\frac{\partial e^*_i(a_i = a_L, k, w|s = 1)}{\partial k} =
\]

\[
\frac{1}{2}((-1 + p)(a_L + w + \sqrt{\lambda})/\lambda +
\]

\[
(-1 + p)(3a_L^2 + 3k + 6a_Lw + 3w^2 - 2a_L\sqrt{\lambda} + 2w\sqrt{\lambda})/2\lambda^{3/2} +
\]

\[
2p(-2a_L + 2w + a_L^2 + a_H(a_L + a_H + 2(k + w^2))/\sqrt{\mu})/\mu -
\]

\[
4p(-a_H^4a_L - 2a_Ha_L^2 - a_L^3 - 4a_Lk + a_H^2w - 2a_Ha_Lw - 3a_L^2w + 4kw + 4a_Hw^2 + 4w^3 +
\]

\[
(a_L^2 + 3a_Lw + a_H(a_L + w) + 2(k + w^2))/\mu^2)
\]

(9.36)

where \( \lambda = a_L^2 + k + 2a_Lw + w^2 \),

\( \mu = a_H^2 + a_L^2 + 4a_Lw + 2a_H(a_L + 2w) + 4(k + w^2) \).

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\[
\frac{\partial e_i^*(a_i = a_L, k, w | s = 1)}{\partial w} =
\]
\[
\frac{1}{2}(2(-1 + p)(a_L + w)(-a_L + w + \sqrt{\lambda})/\lambda +
(1 - p)(-a_L^2 + k + 2a_Lw + 3w^2 + 3(a_L + w)\sqrt{\lambda})/\lambda +
p(a_H^2 - 2a_H a_L - 3a_L^2 + 4k + 8a_Hw + 12w^2 +
2(a_H + a_L + 2w)(a_L^2 + 3a_Lw + a_H(a_L + w) + 2(k + w^2)) +
\[
\frac{(a_H + 3a_Lw + a_H(a_L + w) + 2(k + w^2))\sqrt{\mu}}{\mu}
\]}
\]  
\(9.37\)

where \(\lambda = a_L^2 + k + 2a_Lw + w^2,\)
\(\mu = a_H^2 + a_L^2 + 4a_Lw + 2a_H(a_L + 2w) + 4(k + w^2).\)

\[
\frac{\partial e_i^*(a_i = a_L, k, w | s = 1)}{\partial p} =
\]
\[
\frac{1}{2}(a_L - w - \sqrt{\lambda} +
(-a_H^2a_L - 2a_Ha_L^2 - a_L^2 - 4a_Lk + a_H^2w - 2a_Ha_Lw - 3a_L^2w + 4kw + 4a_Hw^2 + 4w^3 +
(a_L^2 + 3a_Lw + a_H(a_L + w) + 2(k + w^2))\sqrt{\mu})/\mu)
\]}
\]  
\(9.38\)

where \(\lambda = a_L^2 + k + 2a_Lw + w^2,\)
\(\mu = a_H^2 + a_L^2 + 4a_Lw + 2a_H(a_L + 2w) + 4(k + w^2).\)
A.4.2 Agent with High Ability

\[
\frac{\partial e^*_i(a_i = a_{H,k},w \mid s = 1)}{\partial a_H} = \\
\frac{1}{2}(-2p(a_H + w)(-a_H + w + \sqrt{\lambda})) / \lambda + \\
p\left(-3a_H^2 - k - 2a_Hw + w^2 + 3(a_H + w)\sqrt{\lambda}\right) / \lambda + \\
((1-p)(-3a_H^2 - 4a_Ha_L - a_L^2 - 4k - 6a_Hw - 2a_Lw + \\
(a_H + a_L + 2w)(a_H^2 + 2k + w(L + 2w) + a_H(a_L + 3w)) / \sqrt{\mu} + \\
(2a_H + a_L + 3w)\sqrt{\mu}) / \mu^2 \\
(-a_H^2 - 2a_H^2a_L - a_Ha_L^2 - 4a_Hk - 4a_Ha_Lw + a_L^2w + 4kw + 4a_Lw^2 + 4w^2 + \\
(a_H^2 + 2k + w(a_L + 2w) + a_H(a_L + 3w))\sqrt{\mu}) / \mu^2)
\]

(9.39)

where \( \lambda = a_H^2 + k + 2a_Hw + w^2 \),
\( \mu = a_H^2 + a_L^2 + 4a_Lw + 2a_H(a_L + 2w) + 4(k + w^2) \).

\[
\frac{\partial e^*_i(a_i = a_{H,k},w \mid s = 1)}{\partial a_L} = -\frac{(a_H - a_L)(-1 + p)}{\mu^{3/2}}
\]

(9.40)

where \( \mu = a_H^2 + a_L^2 + 4a_Lw + 2a_H(a_L + 2w) + 4(k + w^2) \).

\[
\frac{\partial e^*_i(a_i = a_{H,k},w \mid s = 1)}{\partial k} = \\
(a_H^2p\sqrt{\mu} - 4(k + w^2)(-2\sqrt{\lambda} + 2p\sqrt{\lambda} - p\sqrt{\mu}) + \\
a_L^2(4\sqrt{\lambda} - 4p\sqrt{\lambda} + p\sqrt{\mu}) + \\
4a_Lw(3\sqrt{\lambda} - 3p\sqrt{\lambda} + p\sqrt{\mu}) + \\
a_H(4w(\sqrt{\lambda} - p\sqrt{\lambda} + p\sqrt{\mu}) + a_L(4\sqrt{\lambda} - 4p\sqrt{\lambda} + 2p\sqrt{\mu}))) / \\
(4\mu^{3/2}\sqrt{\lambda})
\]

(9.41)

where \( \lambda = a_H^2 + k + 2a_Hw + w^2 \),
\( \mu = a_H^2 + a_L^2 + 4a_Lw + 2a_H(a_L + 2w) + 4(k + w^2) \).

\[
\frac{\partial e^*_i(a_i = a_{H,k},w \mid s = 1)}{\partial w} = 
\]

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\[
\frac{1}{2}(-2p(a_H + w)(-a_H + w + \sqrt{\lambda})/\lambda + \\
p(-a_H^2 + k + 2a_Hw + 3w^2 + 3(a_H + w)\sqrt{\lambda})/\lambda + \\
((1 - p)(-3a_H^2 - 2a_Ha_L + a_L^2 + 4k + 8a_Lw + 12w^2 + \\
2(a_H + a_L + 2w)(a_H^2 + 2k + w(a_L + 2w) + a_H(a_L + 3w))/\sqrt{\mu} + \\
(3a_H + a_L + 4w)/\sqrt{\mu}))/
\]

\[
\mu - (4(1 - p)(a_H + A_L + 2w) \\
(-a_H^2 - 2a_Ha_L - a_Ha_L^2 - 4a_Hk - 3a_H^2 w - 2a_Ha_Lw + a_L^2w + 4kw + 4a_Lw^2 + 4w^3 + \\
(a_H^2 + 2k + w(a_L + 2w) + a_H(a_L + 3w))(\sqrt{\mu}))/\mu^2)
\]

(9.42)

where \( \lambda = a_H^2 + k + 2a_Hw + w^2 \),
\( \mu = a_H^2 + a_L^2 + 4a_Lw + 2a_H(a_L + 2w) + 4(k + w^2) \).

\[
\frac{\partial e^*_i(a_i = a_H, k, w | s = 1)}{\partial p} =
\]

\[
\frac{1}{2}(-a_H + w + \sqrt{\lambda} - \\
(-a_H^2 - 2a_Ha_L - a_Ha_L^2 - 4a_Hk - 3a_H^2 w - 2a_Ha_Lw + a_L^2w + 4kw + 4a_Lw^2 + 4w^3 + \\
(a_H^2 + 2k + w(a_L + 2w) + a_H(a_L + 3w))(\sqrt{\mu}))/\mu)
\]

(9.43)

where \( \lambda = a_H^2 + k + 2a_Hw + w^2 \),
\( \mu = a_H^2 + a_L^2 + 4a_Lw + 2a_H(a_L + 2w) + 4(k + w^2) \).


