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Propagation of Schedule Disturbances in Line-Haul Passenger Transportation

Abstract
Schedule disturbances in public transport operations have a tendency to intensify along the line and propagate to successive vehicles due to the uneven accumulation of passengers. This phenomenon, affecting efficiency and reliability of service, occurs frequently with surface services due to street congestion, as well as with rapid transit when it approaches capacity volumes. In recent years, considerable attention has been given to this problem.

Newell and Potts [1] (*), using a deterministic model, derived an expression for the behavior of delays both along the line and of subsequent vehicles at individual stations due to passenger accumulation. They gave a theoretical explanation of the phenomenon of pairing of buses, which later Potts and Tamlin tried to verify through observations of bus operations [2]. While they did observe the tendency for pairing of vehicles, their experiment indicated that numerous other factors in street operation (signals, traffic, etc.) make it difficult to distinguish individual causes of delays. Rapid transit is more convenient for these observations since passenger boarding is the dominant variable factor in operation. Tiercin [3] described a new method of schedule control tested by RATP in Paris for one of the principal « Metro » lines, and London Transport, in planning for « Victoria Line », used computer simulation of rapid transit operation at minimum intervals to derive operational measures to increase stability of service. This work was reported by Welding and Day (4) and in an unpublished Research Report [5]. Recently, Lehmann [6] and Sudmeyer [7] gave an interesting theoretical analysis of propagation of delays along the line; their discussion was followed by a paper by this author [8] which is incorporated and somewhat expanded here.

In this paper a theoretical analysis of the behavior of disturbances is extended to include the changes of disturbances with time (for subsequent vehicles at any given station). Practical implications are discussed and measures to minimize this phenomenon in public transport operations are suggested. A diagram for easy evaluation of stability of any service is also given here.

Disciplines
Electrical and Computer Engineering | Engineering | Systems Engineering

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Propagation of schedule disturbances in line-haul passenger transportation

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Schedule disturbances in public transport operations have a tendency to intensify along the line and propagate to successive vehicles due to the uneven accumulation of passengers. This phenomenon, affecting efficiency and reliability of service, occurs frequently with surface services due to street congestion, as well as with rapid transit when it approaches capacity volumes. In recent years, considerable attention has been given to this problem.

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In this paper a theoretical analysis of the behavior of disturbances is extended to include the changes of disturbances with time (for subsequent vehicles at any given station). Practical implications are discussed and measures to minimize this phenomenon in public transport operations are suggested. A diagram for easy evaluation of stability of any service is also given here.

The Model

A deterministic model based on the assumptions made by previous authors [1, 6, 7] will be used. The assumptions, given below, do represent simplification of actual situations. However, the critical comment in [7] that use of average values must lead to false conclusions is incorrect if the nature of the delay itself is not affected. Actually, deterministic model permits a clearer picture of the phenomenon of delay propagation and dissipation, although it does not allow exact assessment of the magnitude of the problem in actual operation.

Suppose that at all stations passengers arrive uniformly at a rate \( a \) and board the trains uniformly at a rate \( e \) [passengers/hour]. Naturally, the boarding rate must be greater than the arrival rate. It is also assumed that train stopped time at station, \( t_n \), is a linear function of the number of boarding passengers, and that each train takes all waiting passengers. Then the number of passengers accumulated during the interval between vehicles, \( t_x \), is equal to the number of those who board during \( t_x \) : \( a \cdot t_x = e \cdot t_x \), or

\[
\frac{a}{e} = \frac{t_x}{t_x} = 1
\]

(1)

We define:

\[
\gamma = \frac{t_x}{t_x - t_x} = \frac{a}{e - a} \quad (2)
\]

\( \alpha \) = initial (« external ») delay of a train;

(*) Numbers in square brackets refer to Bibliography at the end of the French version. All figures are also in the French text.
\( \Delta T_{\text{mn}} \) = time difference between scheduled and actual departures of train \( m \) from station \( n \). This time interval is designated as « delay », and it may be positive (running behind schedule) or negative (running ahead of schedule).

As long as the travel times of all trains on any one interstation section are the same, which is a realistic assumption, it is irrelevant what travel times on different interstation sections are; they may differ along the line.

The subject of this study is the behavior of delays \( \Delta T_{\text{mn}} \), caused by an initial delay \( \alpha \) of one train, along the line (stations 1, 2, ..., \( n \)) and for successive trains (1, 2, ..., \( m \)) due to accumulation of passengers at stations.

The Solution and Analysis

If train 1 is delayed prior to its arrival at station 1 by a time interval \( \alpha \), there are more passengers to board than usual, so that the delay increases by factor

\[
\frac{t_{\text{f1}}}{t_{\text{b1}}} = \gamma,
\]

as can be proved by figure 1 (explained later). This leads to a further increase of the delay at the next station, and the disturbance thus intensifies along the line. The mathematical expression, derived in [1, 6 and 7], is:

\[
\Delta T_{\text{m1}} = \alpha (1 + \gamma)^n. \tag{3}
\]

However, the phenomenon of two successive trains (or buses) getting together along the line, frequently observed in actual operation, is not to be attributed to this factor alone.

Train 2 follows train 1, which is falling progressively behind its schedule. Thus the interval between the two trains at station 1 is reduced to \( t_{\text{b2}} - \alpha (1 + \gamma) < t_{\text{b1}} \), so that the number of waiting passengers is less than normal. The stopped time of train 2 is therefore reduced and it begins to travel ahead of its schedule. Its « negative delay » is:

\[
\Delta T_{\text{2,1}} = -\alpha (1 + \gamma) \frac{t_{\text{b1}}}{t_{\text{b} \text{-} t_{\text{b}}} = -\alpha \gamma (1 + \gamma). \tag{4}
\]

Thus, as train 1 falls further behind its schedule, train 2 progressively gets ahead of its schedule; train 3, due to the same effect of changed passenger volume, gets behind, and train 4 ahead of their respective schedules. The result is that there is a phenomenon of delay transfer through time, causing oscillation of intervals and pairing of trains. Figure 1 illustrates this phenomenon graphically: accumulation of passengers during the interval \( t_{\text{b} - t_{\text{b}}} \), and boarding in the interval \( t_{\text{b}} \), is shown for three successive trains at three stations for regular (solid lines) and for disturbed (dashed lines) schedules (the abscissa for each station is plotted from the departure time of the train < 0, i.e. the one preceding train 1). The time-distance diagram of the same operations is shown in Figure 2.

The mathematical expression for delay of any train \( m \) at station \( n \) has been derived by Newell and Potts [1]. Adapted and simplified through the model used here, that expression is:

\[
\Delta T_{\text{mn}} = \alpha \frac{(m + n - 2)!}{(m-1)! (n-1)!} (-\gamma)^{n-1} (1 + \gamma)^n. \tag{5}
\]

Equation (5) indicates that for any individual train the delay intensifies along the line: for any \( m \), \( \Delta T_{\text{mn}} \) increases with \( n \). However, it is positive (running behind schedule) for odd-numbered trains, and negative (ahead of schedule) for even-numbered trains, thus reflecting the pairing effect.

The question not explored in depth so far is the behavior of delays through time. In other words: do the absolute values of delays at any one station increase with subsequent trains (unstable situation), or do they dissipate with time and the schedule normalizes by itself (stable situation)? This question can also be examined through equation (5). For any station \( n \) the expression \( \alpha \frac{(1 + \gamma)^n}{(n-1)!} \) is constant and the delays through the time (with increasing \( m \)) will depend on the expression \( \frac{(m + n - 2)!}{(m-1)!} (-\gamma)^{n-1} \).

Since the absolute values of the delay are analyzed, the negative sign may be disregarded for a moment. Clearly, for \( \gamma > 1 \), \( \Delta T_{\text{mn}} \) increases with \( m \) since both members of (6) increase. However, for \( \gamma < 1 \) the question of stability of service is not so simple, since for \( n > 1 \) the first member of (6) increases, while the second member decreases with \( m \). The trend of the whole expression (6) can be explored by examining the magnitudes of subsequent delays. If the absolute value of a delay, \( |\Delta T_{\text{mn}}| \), has a maximum, it will be for \( m = M \):

\[
|\Delta T_{\text{m-1}}| < |\Delta T_{\text{m}}| > |\Delta T_{\text{m+1}}|. \tag{7}
\]

Introducing these three values for \( m \) into (6), one derives that the delay is maximum for \( M \):

\[
\left( \frac{n-1}{1-\gamma} \right) \leq M \leq \left( \frac{n-1}{1-\gamma} \right) + 1. \tag{8}
\]

square brackets representing integer values. When two integer values satisfy (8), then delays of two subsequent trains at that station are equal and maximal.

Equation (5) and the above analysis show that stability of service depends on the value of \( \gamma \): the smaller it is, the greater stability is. Equation (8), however, indicates that stability changes with \( m \) and \( n \); delays decrease when
consequently, stability is the greatest at the station of initial delay (n = 1); thereafter the delays are smaller for each subsequent train (if, as mentioned, \( \gamma < 1 \)). For other stations (n > 1), the delay increases for several subsequent trains, reaches a maximum for train M defined by (8), and then decreases and dissipates with time. For station stability \( \gamma \) must be smaller when n is greater.

This behavior of delays can be explained physically as follows. Delay of train 1 intensifies along the line due to the increasing accumulation of passengers since the departure of the preceding train, which is running on schedule. Train 2 (and all subsequent trains) does not have an initial delay (\( \Delta T_{M0} = 0 \)); its delay at station 1 is created only by the delay of train 1, and it is therefore smaller than \( \alpha \) (exponentially decreasing with \( m \), assuming \( \gamma < 1 \)). However, that delay of train 2 increases not only due to its own intensification, but also due to the constantly decreasing (or increasing for odd-numbered trains) gap between the trains 2 and 1, created by the delay of train 1. As a result of these two effects, the delays of subsequent trains in leaving station 1 are progressively smaller, but they grow faster along the line.

For example: passengers arrive at a rapid transit station at the rate of 3,600 persons per hour and board at the rate of 4 persons/sec. Then \( \gamma = 0.333 \). Delays \( \Delta T_{Mn} \) for this case for selected stations and trains are given in Table 1 in terms of the initial delay \( \alpha \). The absolute values of factors of increase of delay \( \alpha \) for \( m = 1, \ldots, 7 \) and \( n = 1, \ldots, 7 \) and plotted on diagrams in Figures 3 and 4. Fast propagation of the delay (due to a relatively high value of \( \gamma \)) is apparent.

### Conclusions and Possible Corrective Measures

1. Delays in public transport line-haul services intensify with distance (along the line), and are propagated through time (subsequent trains). They cause running of trains in pairs, since some are delayed and others speeded up.
2. The speed of delay intensification and propagation depends on the ratio of the rates of passenger arrivals at the stations and their boarding of trains, in the form \( \gamma = \frac{t_a}{(t_b - t_e)} \). The lower ratio \( \gamma \) is, the smaller is the increase of disturbances in distance and time.
3. Delay propagation can be most efficiently reduced by providing fast boarding. This can be done by such measures as faster fare collection (automatic ticket

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**TABLE 1: Values of \( \Delta T_{Mn} \) for the Example (\( \gamma = 0.333 \))**

<table>
<thead>
<tr>
<th>Bahnhof</th>
<th>Zug</th>
<th>1</th>
<th>4</th>
<th>7</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1,333a</td>
<td>3,361a</td>
<td>7,492a</td>
<td>17,758a</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>-0,049a</td>
<td>-2,241a</td>
<td>-43,309a</td>
<td>-44,704a</td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>0,002a</td>
<td>0,354a</td>
<td>9,637a</td>
<td>111,240a</td>
<td></td>
</tr>
<tr>
<td>10</td>
<td>-0,006a</td>
<td>-0,036a</td>
<td>-1,905a</td>
<td>-63,860a</td>
<td></td>
</tr>
</tbody>
</table>

Bahnhof = station; Zug = train.

Another example points out an interesting fact. Suppose that passengers arrive at a bus stop at a rate of 150 per hour and that the average boarding time is 6 seconds per passenger. For this case again \( \gamma = 0.333 \), so that the same tendency for delay propagation as in the previous example will take place. It follows that since the delay intensification and propagation are not dependent on frequency of service (or \( t_b \)) alone, but on \( \gamma \), the problem of instability of service may be experienced even on the lines with relatively low frequency of service if boarding rates are low. Naturally, if service intervals are short, additional disturbance may occur due to mutual interference of vehicles on the line, but that factor will not be discussed here.

### Practical Meaning of the Results

The major practical conclusion of the above analyses is that since stability of service depends on \( \gamma \), which is defined by equation (2), both passenger arrival and passenger boarding rates are important factors. More precisely, stability of service through time depends on the ratio of the passenger arrival rate (\( a \)) and rate of reduction of the accumulated queue of passengers (\( e \)). The operation is definitely unstable through time (delays increase with each subsequent train at all stations) when \( \gamma > 1 \), while it is stable for

\[
\gamma < \frac{m}{m + n - 1}
\]
sale, monthly passes, etc.), improved design of doors and efficient vehicle departure control. In some cases delays can be reduced by skipping stops or stopping only to drop off passengers; this is possible only when frequency of service is high, so that waiting passengers are not delayed significantly.

4. When boarding cannot be accelerated (e.g. London Underground cannot reduce standing times during peak hours much below 30 seconds [5]), control of schedules should be made as frequently as possible and running ahead of schedule (which drivers often try to achieve to increase terminal layover time) must be prevented as much as falling behind it.

5. Delay propagation may be reduced in some cases by slowing down the train preceding the delayed train; tests made by London Transport [5] showed that this reduces delay propagation, though it increases somewhat the average headway. This measure requires some special train control devices, as has been described in [3].

6. Delays are often created on sections used by several lines, where propagation is fast. This problem is common, for example, with buses operating at short intervals, due to their slow boarding rate. This effect may be reduced by allowing overtaking, which is seldom used. In general, vehicle schedules should be controlled particularly carefully prior to the merging points of different lines to minimize mutual interference on common sections.

Most of these phenomena and corrective measures have been intuitively known and applied in practice. However, with the increasing importance of reliability of public transport services and increasing possibilities to control schedules by radio communications and use of computers, a better understanding of the nature of operational phenomena is becoming a necessity.