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Abstract
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Keywords
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How does zero forward-scattering in magnetodielectric nanoparticles comply with the optical theorem?

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Abstract. A few decades ago, Kerker *et al.* [*J. Opt. Soc. Am.* **73**, 765-767 (1983)] theoretically pointed out the interesting possibility of conceiving small magnetodielectric spheres that may provide *zero scattering* in the forward direction, despite significantly larger scattering in any other direction. Recent experimental and theoretical papers on the topic have further discussed this possibility in more realistic scenarios. Inspecting some of their analyses, it seems indeed possible to conceive nanoparticles characterized by a scattering pattern with a sharp minimum, although not zero, in the forward direction. From a theoretical standpoint, however, it is well known that the total scattered power from any object has to be proportional to a portion of the scattered field in the forward direction, implying that very small or zero *forward* scattering should be synonymous to even smaller or zero *total* scattering, regardless of the nature of the object and of its design. Using analytical theory and an accurate scattering formulation, we clarify the nature of this apparent paradox and the limitations of this anomalous phenomenon in terms of particle size. In this way, we shed some new light on theoretical and experimental papers on the topic, identifying relevant missteps in some of their physical interpretation, and considering the general possibility of verifying these effects. This discussion may also be relevant to some cloaking applications using exotic artificial materials.

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1 INTRODUCTION

The scattering of electromagnetic waves has been the subject of interest in the scientific community for centuries. The exact formal solution of the scattering problem for a spherical object [1], explains, among other things, many optical phenomena that we experience every day, from the color of the sky, to the bright features of metallic nanoparticles, fascinatingly discussed in Professor Bohren's best-selling book [1]. Prof. Bohren's studies on scattering from small nanoparticles have spanned a large variety of topics, many of them characterized by renewed interest at present times, from optically active materials [2], to resonant [3] or anomalously low-scattering [1] nanoparticles. The recent progress in nanotechnology provides us with various exciting possibilities for verification of many of the theoretical predictions that Dr. Bohren and his colleagues have outlined in the past decades related to scattering from electrically small particles.

In particular, in the field of scattering cancellation and cloaking, the possibility to manufacture special materials with anomalous electromagnetic properties has led to various exciting possibilities in drastically reducing the overall scattering from a given object, by properly covering it with metamaterials or plasmonic materials [4-10]. Anomalous scattering from nanoparticles made of materials with exotic electromagnetic properties have been predicted since decades [1], and their verification is now within the framework of current

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nanofabrication possibilities. As another one of these anomalous scattering features, Kerker and his colleagues have analyzed the scattering from small magnetodielectric spheres [11], predicting that a small particle with diameter much smaller than the wavelength of operation may have a *zero forward scattering*, but a significantly larger (even orders of magnitude) scattering in all other directions, when the following condition is held:

$$\varepsilon = \varepsilon_{\min} = \frac{4-\mu}{2\mu+1} \ , \tag{1}$$

where ε is its relative permittivity and μ the corresponding relative permeability of the object at the frequency of interest. Although this condition was derived in the ideal quasi-static limit, it has been predicted that it may also hold for moderately sized magnetodielectric spheres in a fully dynamic scenario. In fact, renewed interest in this topic has been reported in recent theoretical [12-13] and experimental [14-16] papers on the topic.

One puzzling aspect of this anomalous scattering feature, surprisingly not addressed in all the aforementioned works, but raised in a recent comment [16], resides in the apparent violation of the fundamental theorem of optics (i.e., the *optical theorem*) that relates the total extinction cross section of an object σ_{ext} (sum of absorption and scattering cross sections) to the normalized scattering amplitude polarized in parallel with the impinging field in the forward direction $s_\theta\left(0,0\right)$:

$$\sigma_{ext} = \frac{\lambda_0^2}{\pi}\operatorname{Im}\left[s_\theta\left(0,0\right)\right], \tag{2}$$

where λ_0 is the wavelength of operation. This well-known identity, often addressed as the "optical theorem" for its generality, applies to the scattering from any object illuminated by a linearly polarized plane wave [17-18]. Clearly, Eq. (2) implies that near-zero *forward* scattering should be synonymous of near-zero *total* scattering, i.e., a nearly transparent object. Eq. (1), however, allows canceling only the *forward* scattering of the sphere in the quasi-static limit, implying that it may still be possible to achieve a significantly larger scattering in all other directions. This apparent incongruence was first outlined in a sentence of [16], in which the validity of the experimental evidence presented in [14] was seriously argued against.

In the following, we address this issue by reviewing the general theory of scattering from magnetodielectric particles, and we show that Kerker's original theory is indeed valid in the quasi-static limit, without necessarily violating the optical theorem. Moreover, we discuss the validity of the zero-forward scattering condition in the general case, and we show up to what limit on the electrical size of the particles "zero-forward scattering" may be practically achieved. By doing that, we will discuss the validity of recent papers on the topic, that have often misinterpreted the original findings from Kerker. Finally, we relate these concepts to recently proposed metamaterial cloaking techniques, discussing how this anomalous scattering properties of small magnetodielectric nanoparticles may be somehow related to the anomalous features of cloaked objects [6-7], sensors and antennas [28-29]. For simplicity, we focus our results on nanoparticles of spherical shape, consistent with the original work from Kerker [11], but it is clear that analogous results may be obtained with different shapes and geometries.

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2 THEORETICAL FORMULATION

2.1 General scattering solution

The scattering problem of a homogeneous magnetodielectric sphere of radius a, relative permittivity ε and relative permeability μ may be approached using the rigorous Mie expansion in spherical harmonics [1]. In particular, using the compact formalism that we have used in Ref. 19, it is easy to show that the scattered electric field for plane wave incidence $\mathbf{E}_{inc} = \hat{\mathbf{x}} E_0 e^{2\pi iz/\lambda_0}$ may be expressed as a superposition of spherical harmonics:

$$\mathbf{E}_s = E_0 \left(\sum_{n=1}^{\infty} c_n^{TM} \nabla \times \nabla \times \left(\mathbf{r} \psi_n^1 \right) + 2\pi i f \mu_0 \sum_{n=1}^{\infty} c_n^{TE} \nabla \times \left(\mathbf{r} \psi_n^1 \right) \right), \tag{3}$$

where μ_0 are the free-space permeability and ψ_n^m are scalar spherical harmonics, solutions of Helmholtz equation in the spherical coordinate system (r, θ, ϕ), and we have assumed an $e^{-i\omega t}$ time dependence. Using the notation introduced in Ref. 19, the Mie scattering coefficients c_n may be compactly written as:

$$c_n^{TM} = -\frac{U_n^{TM}}{U_n^{TM} + iV_n^{TM}}, \quad c_n^{TE} = -\frac{U_n^{TE}}{U_n^{TE} + iV_n^{TE}}, \tag{4}$$

with:

$$\begin{aligned}
U_n^{TM} &= \begin{vmatrix} j_n(x) & j_n(x_0) \\ \left[x j_n(x) \right]' / \varepsilon & \left[x_0 j_n(x_0) \right]' \end{vmatrix}, \quad U_n^{TE} = \begin{vmatrix} j_n(x) & j_n(x_0) \\ \left[x j_n(x) \right]' / \mu & \left[x_0 j_n(x_0) \right]' \end{vmatrix}, \\
V_n^{TM} &= \begin{vmatrix} j_n(x) & y_n(x_0) \\ \left[x j_n(x) \right]' / \varepsilon & \left[x_0 y_n(x_0) \right]' \end{vmatrix}, \quad V_n^{TE} = \begin{vmatrix} j_n(x) & y_n(x_0) \\ \left[x j_n(x) \right]' / \mu & \left[x_0 y_n(x_0) \right]' \end{vmatrix},
\end{aligned} \tag{5}$$

where $j_n(x)$ and $y_n(x)$ are the spherical Bessel functions of order n, $x_0 = 2\pi a / \lambda_0$ and $x = 2\pi a \sqrt{\varepsilon \mu} / \lambda_0$. It should be noted that this notation is different from that used in [1], where the scattering coefficients (4) are indicated by a_n and b_n, respectively.

In the far-field of the sphere, using the well known approximation of Hankel functions $\lim_{z \to \infty} \left[j_n(z) + i y_n(z) \right] = i^{-n-1} \frac{e^{iz}}{z}$, the two non-zero components of the scattered electric field (3) may be written in simplified form as:

$$\begin{aligned}
S_\theta(\theta, \varphi) &= iE_0 \cos\varphi \frac{\lambda_0 e^{ik_0 r}}{2\pi r} \sum_{n=1}^{\infty} \frac{2n+1}{n(n+1)} \left(c_n^{TM} \frac{dP_n^1(\cos\theta)}{d\theta} + c_n^{TE} \frac{P_n^1(\cos\theta)}{\sin\theta} \right) \\
S_\varphi(\theta, \varphi) &= -iE_0 \sin\varphi \frac{\lambda_0 e^{ik_0 r}}{2\pi r} \sum_{n=1}^{\infty} \frac{2n+1}{n(n+1)} \left(c_n^{TM} \frac{P_n^1(\cos\theta)}{\sin\theta} + c_n^{TE} \frac{dP_n^1(\cos\theta)}{d\theta} \right)
\end{aligned} \tag{6}$$

where $P_n^m(x)$ are the associated Legendre functions.

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This implies that the differential scattering cross section $\sigma_{scat}^{d}(\theta,\phi)$, defined as the ratio of the scattered power in the angular sector $r^2 d\theta d\phi$ over the impinging power flux density may be written as:

$$\sigma_{scat}^{d}(\theta,\phi)=\frac{\lambda_0^2}{4\pi^2}\left[\begin{array}{l}\sin^2\varphi\left|\sum_{n=1}^{\infty}\frac{2n+1}{n(n+1)}\left(c_n^{TM}\,\frac{P_n^1(\cos\theta)}{\sin\theta}+c_n^{TE}\,\frac{dP_n^1(\cos\theta)}{d\theta}\right)\right|^2+\\ +\cos^2\varphi\left|\sum_{n=1}^{\infty}\frac{2n+1}{n(n+1)}\left(c_n^{TM}\,\frac{dP_n^1(\cos\theta)}{d\theta}+c_n^{TE}\,\frac{P_n^1(\cos\theta)}{\sin\theta}\right)\right|^2\end{array}\right]. \tag{7}$$

Integrating over the visible angles and exploiting the orthogonality of Legendre polynomials yields the well-known result for the total scattering cross section [1]:

$$\sigma_{scat}=\oint_{\Omega}\sigma_{scat}^{d}(\theta,\phi)\,d\Omega=\frac{\lambda_0^2}{2\pi}\sum_{n=1}^{\infty}(2n+1)\left(\left|c_n^{TE}\right|^2+\left|c_n^{TM}\right|^2\right), \tag{8}$$

defined as the ratio of the total scattered power over the impinging plane-wave power flux density.

Using (7), we may also define the scattering cross section in specific directions of interest. For instance, noticing that:

$$\lim_{\theta\to\pi}\frac{P_n^1(\cos\theta)}{\sin\theta}=-\lim_{\theta\to\pi}\frac{dP_n^1(\cos\theta)}{d\theta}=(-1)^{n+1}\,\frac{n(n+1)}{2}, \tag{9}$$

the backscattering cross section yields the known formula:

$$\sigma_{bw}=\frac{\lambda_0^2}{4\pi}\left|\sum_{n=1}^{\infty}(-1)^n(2n+1)\left(c_n^{TM}-c_n^{TE}\right)\right|^2. \tag{10}$$

Similarly, we may define the forward scattering cross section, noticing that:

$$\lim_{\theta\to 0}\frac{P_n^1(\cos\theta)}{\sin\theta}=\lim_{\theta\to 0}\frac{dP_n^1(\cos\theta)}{d\theta}=-\frac{n(n+1)}{2}, \tag{11}$$

yielding the result:

$$\sigma_{fw}=\frac{\lambda_0^2}{4\pi}\left|\sum_{n=1}^{\infty}(2n+1)\left(c_n^{TE}+c_n^{TM}\right)\right|^2. \tag{12}$$

Moreover, applying the optical theorem (2) with normalized scattered field $s=S/\dfrac{\lambda_0 E_0 e^{ik_0 r}}{2\pi r}$ and using the identity (11), the extinction cross section may be written in the well-known form [1]:

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$$\sigma_{ext} = -\frac{\lambda_0^2}{2\pi}\sum_{n=1}^{\infty}(2n+1)\,\mathrm{Re}\left[c_n^{TE} + c_n^{TM}\right]. \tag{13}$$

The absorption cross section $\sigma_{abs} = \sigma_{ext} - \sigma_{scat}$ is zero for real ε and μ (limit of zero losses), and in this case it is expected that Eq. (8) and (13) yield the same value. All these properties are valid for any choice of the sphere parameters a, ε, μ.

2.2 Quasi-static limit and the zero-forward scattering condition

In the case of electrically small spheres $x_0 \ll 1$, the scattering coefficients usually tend to zero as $o\left(x_0^{2n+1}\right)$, implying that the $n=1$ coefficients and the associated dipolar fields dominate the scattering. In this case, these scattering coefficients are usually approximated by their first-order Taylor expansion [11-16]:

$$\lim_{x_0 \to 0} c_1^{TM} = \frac{2i}{3}x_0^3\frac{\varepsilon-1}{\varepsilon+2}, \quad \lim_{x_0 \to 0} c_1^{TE} = \frac{2i}{3}x_0^3\frac{\mu-1}{\mu+2}, \tag{14}$$

whereas the higher-order terms may be safely neglected, implying that:

$$\begin{aligned} \lim_{x_0 \to 0} S_\theta(\theta,\varphi) &= E_0 x_0^3 \cos\varphi \frac{\lambda_0 e^{ik_0 r}}{2\pi r}\left(\frac{\varepsilon-1}{\varepsilon+2}\cos\theta + \frac{\mu-1}{\mu+2}\right) \\ \lim_{x_0 \to 0} S_\varphi(\theta,\varphi) &= -E_0 x_0^3 \sin\varphi \frac{\lambda_0 e^{ik_0 r}}{2\pi r}\left(\frac{\varepsilon-1}{\varepsilon+2} + \frac{\mu-1}{\mu+2}\cos\theta\right) \end{aligned}, \tag{15}$$

and in particular for the forward scattering cross section:

$$\sigma_{fw} = \frac{\lambda_0^2}{\pi}x_0^6\left|\frac{\varepsilon-1}{\varepsilon+2} + \frac{\mu-1}{\mu+2}\right|^2. \tag{16}$$

It is evident that in this limit the forward scattering tends to zero as x_0^6 and, as predicted by Kerker [11], it may be possible to achieve *zero* forward-scattering by applying condition (1), without necessarily implying zero scattering in all other directions [see Eq. (15)]. Notice that using Eq. (1) implicitly assumes that ε and μ are real quantities, i.e., in what follows we will neglect absorption losses. In practice, Kerker's condition aims at canceling the forward scattering by achieving $c_n^{TE} = -c_n^{TM}$, without necessarily making both of them zero, which is indeed possible using (14) in combination with (1). This ensures that the overall scattering may be significantly different from zero, despite its zero in the forward direction.

However, since the scattered fields are non-zero for all other values of θ in Eq. (15), this seems to contradict the optical theorem (13), that indeed would yield zero extinction cross section in this same limit. Indeed, in this limit Eq. (8) yields:

$$\sigma_{scat} = \frac{2\lambda_0^2}{3\pi}x_0^6\left(\left|\frac{\varepsilon-1}{\varepsilon+2}\right|^2 + \left|\frac{\mu-1}{\mu+2}\right|^2\right), \tag{17}$$

which, when condition (1) is applied, provides a non-zero scattering cross section:

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$$\sigma_{scat}^{(1)} = \frac{4\lambda_0^2}{3\pi} x_0^6 \left| \frac{\mu - 1}{\mu + 2} \right|^2 . \tag{18}$$

Also the total scattering cross-section tends to zero as x_0^6 in this quasi-static limit, but it may be evidently made much larger (even orders of magnitude larger) than the forward scattering cross-section when condition (1) is satisfied. On the other hand, Eq. (13) provides, in this limit for which $c_n^{TE} = -c_n^{TM}$:

$$\sigma_{ext}^{(1)} = \sigma_{fw}^{(1)} = 0 . \tag{19}$$

It is noticed, as an aside, that the total scattering cross section $\sigma_{scat}^{(1)}$ in (18) is actually twice as large as σ_{scat} for a regular sphere of same size with same value of μ, but $\varepsilon = 1$ [see Eq. (17)]. In other words, the proper choice of permittivity following Eq. (1) may *drastically reduce the forward scattering of the sphere*, but also *double the total scattering for all other angles, compared to a magnetic sphere with same size and same permeability*! The only possibility this may hold within power balance considerations is the trivial case for which $\varepsilon = \mu = 1$, for which zero forward scattering obviously coincides with zero total scattering. In all other circumstances, the total scattering appears to be larger than the total extinction of the particle, yielding an evident inconsistency in the power balance and in the application of the optical theorem to this special situation.

2.3 A self-consistent quasi-static solution

The solution of this incongruence in Eqs. (18) and (19) may be found by improving the approximation of the scattering coefficients represented by Eq. (14). Although this assumption is generally used when $x_0 \to 0$ [1], [3], [11-16], it should be realized that this approximation does not comply with power conservation requirements [20-24]. A purely imaginary scattering coefficient would indeed necessarily imply an effective polarization current in quadrature with the excitation field, which in turn would imply identically zero extracted power [from which the zero extinction cross section in Eq. (19)]. A correct, complete expression for the quasi-static scattering coefficients (14), which is consistent with power conservation, was originally suggested in Ref. 20, and it is commonly identified as the *radiative correction* [21]. This is given by the following expression within the present notation [24]:

$$\lim_{x_0 \to 0} c_n^{TM} = \left(-1 - \frac{3i}{2} x_0^{-3} \frac{\varepsilon + 2}{\varepsilon - 1} \right)^{-1}, \ \lim_{x_0 \to 0} c_n^{TE} = \left(-1 - \frac{3i}{2} x_0^{-3} \frac{\mu + 2}{\mu - 1} \right)^{-1} . \tag{20}$$

It is noticeable that, for practical purposes, when $x_0 \to 0$ Eq. (20) tends to Eq. (14), but neglecting the relevant real part of the scattering coefficients [which is equivalent to neglecting U_n in the denominators of Eq. (4)] may affect the overall power balance, as in the present case. Of course, this choice is also consistent with the more general unitarity condition introduced in Ref. 22 to preserve the power balance in quasi-static scattering problems.

Using (20) in combination with condition (1), we find that the forward scattering $S_\theta(0,0)$ is now non-zero even in the limit of condition (1) and its residual value is:

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$$\lim_{x_0 \to 0} S_\theta(0,0) = -iE_0 \frac{\lambda_0 e^{ik_0 r}}{2\pi r} \frac{4x_0^6}{3} \left| \frac{\mu - 1}{\mu + 2} \right|^2 , \tag{21}$$

in whose derivation we have implicitly assumed that $|\mu - 2| \gg 2x_0^3 |\mu - 1| / 3$. This is usually the case in this quasi-static limit, since $x_0 \to 0$, unless we are very close to the special resonant condition of such magnetodielectric nanosphere, for which $\varepsilon \simeq \mu \simeq -2$ [still supported by condition (1)]. As first noticed in Ref. 12, this special resonant condition represents an exception to the zero-forward scattering theorem. In the following, we concentrate on all the other pairs of (ε, μ) values that satisfy (1) and support Eq. (21).

The forward scattering cross section in this limit reads, using Eq. (21):

$$\sigma_{fw}^{(1)} = \frac{16\lambda_0^2}{27\pi} x_0^{12} \left| \frac{\mu - 1}{\mu + 2} \right|^4 , \tag{22}$$

which is indeed orders of magnitude smaller compared to the scattering cross section (18), but it is *not* identically zero.

The total scattering cross section is still well approximated by Eq. (18), but the total extinction cross section, which was zero in Sec. 2.2 simply because the scattering coefficients were assumed purely imaginary, now is consistent with the optical theorem and it has the same value as the scattering cross section, as expected:

$$\sigma_{ext}^{(1)} = \frac{\lambda_0^2}{\pi} \mathrm{Im}\left[s_\theta(0,0) \right] = \frac{4\lambda_0^2}{3\pi} x_0^6 \left| \frac{\mu - 1}{\mu + 2} \right|^2 . \tag{23}$$

It is worth noticing that indeed Eq. (22) still ensures that, under Kerker's original condition $\varepsilon = \frac{4 - \mu}{2\mu + 1}$, the forward-scattering may be made extremely small, orders of magnitude smaller than the scattering in all other directions, which is paradoxical, if read in conjunction with the usual interpretation of the optical theorem. We have outlined here, however, how this anomalous scattering feature may indeed fully satisfy the optical constraints of passivity and energy conservation. Moreover, we notice that under this condition the residual scattering in the forward direction (21) is purely imaginary, since its dominant real part, represented by (15), is *identically* zero under condition (1). This small residual imaginary term is the one that effectively contributes to the optical theorem, consistent with Eq. (2), and it cannot be neglected in this quasi-static limit, if not at the cost of violating the power conservation requirements. Indeed, a purely real scattered field in the forward direction, as the one calculated in (15) would imply zero extinction power.

Another way of describing this solution to the previous inconsistency is that it is not possible to achieve $c_n^{TE} = -c_n^{TM}$ with passive materials, unless in the special case of a transparent material ($\varepsilon = \mu = 1$, $c_n^{TE} = -c_n^{TM} = 0$). Under Kerker's condition one can obtain $\mathrm{Im}\left[c_n^{TE}\right] = -\mathrm{Im}\left[c_n^{TM}\right]$, which can drastically reduce the forward scattering, but not completely suppress it. In fact, using Eq. (20) one should expect a residual $\mathrm{Re}\left[c_n^{TE}\right] \simeq \mathrm{Re}\left[c_n^{TM}\right]$, which sum up in phase in the forward direction. This residual contribution to the quadrature component of the forward scattered wave is indeed responsible for the power balance due to scattering in all other directions.

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From a physical standpoint, for a small scatterer the radiated spherical wave is indeed dominated by the contribution in phase with the impinging field, as is the induced dipole moment. In fact, the main shadow we usually experience in the forward direction from a regular scatterer is formed by the interference of the impinging fields and the scattered fields radiated with opposite phase. However, as discussed above and in Ref. 24, this portion of scattered field does not contribute to power extraction from the impinging wave, and it is necessary to consider the non-zero component in quadrature to the impinging fields to ensure power balance. In the zero-forward scattering limit, as in condition (1), we are effectively canceling the dominant in-phase contribution to the scattering in the forward direction, which does not contribute to the optical theorem. However, the residual small component of the scattered wave (21) in quadrature with the impinging wave, that cannot be canceled, ensures power balance, and the satisfaction of the optical theorem.

It is evident from this discussion that it is indeed possible to conceive the design of a small magnetodielectric nanosphere that, although creating a very limited (almost zero) shadow in the forward direction, may still scatter a significantly larger field in all the other directions, as originally predicted by Kerker [11]. This anomalous particle would still satisfy the optical theorem, since the overall scattering from this particle is indeed low and the residual quadrature component of the forward scattering take into account of the extinction from the sphere. The apparent inconsistency outlined above is therefore explained. However, it is easy to realize that for larger sizes the forward scattering may be hardly made close to zero, since in such cases the scattering in all directions is expected to be significant and the corresponding quadrature component of the forward scattering may not be sufficiently small any longer. This implies that a zero-forward scattering particle is necessarily small compared to the wavelength, different from what presented in Ref. 14. In the following section, we provide some numerical examples that confirm and verify these theoretical findings and discuss the size dependency of this effect, in part consistent with Ref. 13. We will show in particular that the experimental results presented in Ref. 14 cannot be attributed to this effect, since the particles considered there are electrically too large, as anticipated in the comment [15].

2.4 Relationship with scattering cancellation and cloaking of sensors

The recent interest in metamaterial cloaking and scattering cancellation from objects of various sizes [6]-[10] has revived the interest for anomalous scattering properties. It is relevant to stress that the forward scattering cancellation outlined here is drastically different from cloaking, and in a sense more challenging. The "nearly-zero" forward scattering particles analyzed here indeed present, on purpose, a significantly larger scattering in all other directions, and are therefore perfectly detectable from any observer not placed directly in the back of the object with respect to the source position. However, near absence of forward scattering implies the cancellation of shadow from an obstacle, which is the most difficult attribute to achieve also in total scattering cancellation and in cloaking. It is indeed less challenging to suppress, for instance, the backscattering from an object, which may be relatively easily achieved with anti-reflection coatings, stealth technology, or simply considering a matched object of any electrical size, since, when $\varepsilon = \mu$, duality requires $c_n^{TE} = c_n^{TM}$, and therefore $\sigma_{bw} = 0$ in Eq. (10).

The optical theorem is not necessarily an issue in cloaking problems: if the cloaking effect is ideal, i.e., no scattering and no absorption, the absence of scattering and absorption implies zero extinction, which is consistent with zero forward scattering. Therefore, cloaking and scattering cancellation in principle would not suffer from the power restrictions highlighted in the previous sections. It is still arguable whether it may be possible to completely suppress the scattering from an object, due to the uniqueness in inverse scattering problems [25-27], but this is clearly beyond the interests and scope of the present paper. However, there is a relevant

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connection between cloaking phenomena and the forward scattering theorem in the recently introduced concept of *cloaked sensors* or receiving antennas [28-29]. As we have recently shown, the scattering cancellation mechanism that some plasmonic covers offer may be applied to sensing devices (e.g., antennas) and absorbing particles, conceiving a system that may absorb a portion of the impinging wave without necessarily creating a sensible scattering in its surroundings. Clearly, in this case power balance is a relevant factor, due to non-zero absorption in the system, and it should be properly taken into account as discussed below.

In the general case, the extinction power may be expressed as the power associated with the cross-coupled interference between the impinging and scattered fields [1]:

$$P_{ext} = -\frac{1}{2}\mathrm{Re}\left[\oint_S \left(\mathbf{E}_{inc}\times\mathbf{H}_s^* + \mathbf{E}_s\times\mathbf{H}_{inc}^*\right)\right], \tag{24}$$

where $\left(\mathbf{E}_{inc},\mathbf{H}_{inc}\right)$ are the impinging electric and magnetic fields, $\left(\mathbf{E}_s,\mathbf{H}_s\right)$ are the scattered electric and magnetic fields and S is any given surface surrounding the cloaked sensor. This implies that the power absorbed by a cloaked sensing device is necessarily associated with the interference between its own scattered fields and the impinging fields. If zero scattering is not possible in the case of absorption, we may expect from the previous analysis that the significant contribution to the scattering for power balance is the one scattered in the forward-direction in quadrature with the impinging fields. It is not surprising, therefore, that one may achieve drastic reduction of the overall scattered fields by using an external plasmonic cloak [28]. Similar to what we have shown in the previous section, a good portion of the scattered fields do not contribute to the power balance, and they may be canceled with proper design. Of course, this does not mean that one may achieve identically zero scattering in this situation in which absorption is desirable, since a non-zero quadrature scattering component in the forward direction is always necessary to satisfy power balance and the optical theorem, similar to what we have discussed in the previous sections. This is consistent with the general requirement of minimum-scattering receiving antennas to produce a directive scattering pattern in the forward direction [30]. It is interesting how these seemingly unrelated concepts are all connected by common power balance considerations and the relation between different phase components of the scattered and incident fields.

3 NUMERICAL RESULTS

In order to validate the previous results, consider the scattering from a lossless magnetodielectric nanosphere with $\mu = 3$ and $a = \lambda_0/100$. With very good approximation, this size falls within the quasi-static limit, and therefore we may expect to see the results of the previous sections, and those reported by Kerker [11] to be confirmed. Figure 1(a) (black solid line) reports the variation of the forward scattering efficiency, defined as the ratio of the forward scattering cross section and the physical cross section of the sphere, $\eta_{fw} = \sigma_{fw}/\left(\pi a^2\right)$, versus its permittitivity ε. Correspondingly, Fig. 1(b) reports the total scattering efficiency $\eta_{scat} = \sigma_{scat}/\left(\pi a^2\right)$. It is seen that at the value of permittivity predicted by (1), in this case $\varepsilon \simeq 0.143$, the forward scattering cross section is drastically reduced, without necessarily producing a dip in the total scattering efficiency. Indeed, as predicted in Sec. 2.2, the total scattering efficiency for this value of permittivity is actually 2 times as large as η_{scat} for $\varepsilon = 1$, despite the dramatic reduction of forward scattering (over 60 dB). Power conservation and the optical theorem, however, are indeed verified, consistent with Sec. 2.3, by the small residual component of forward scattering, fully in quadrature with the excitation. The two dominant scattering coefficients indeed satisfy, for $\varepsilon \simeq 0.143$, the relationships:

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$$
\begin{aligned}
\mathrm{Im}\left[c_1^{TM}\right] &\simeq -\mathrm{Im}\left[c_1^{TE}\right] \\
\mathrm{Re}\left[c_1^{TM}\right] &\simeq \mathrm{Re}\left[c_1^{TE}\right]
\end{aligned}, \tag{25}
$$

derived in the previous section, and ensure drastically reduced forward scattering [otherwise dominated by the relatively large imaginary parts of Eq. (25)], but also satisfaction of the power conservation requirements, associated with their much smaller residual real parts.

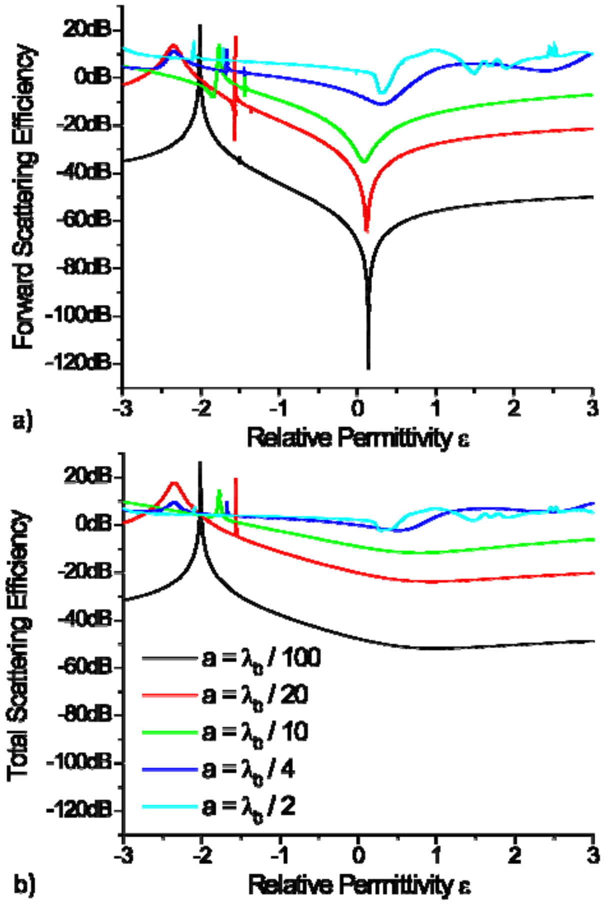

Fig. 1. Variation of: (a) the forward scattering efficiency η_{fw}, (b) the total scattering efficiency η_{scat}, as a function of the relative permittivity of a magnetodielectric particle with relative permeability $\mu = 3$.

When a is increased (different curves in Fig. 1), a minimum may still be obtained for values of permittivity near the one predicted by the quasi-static condition (1), although with some deviation associated with the retardation of the fields for larger spheres. Even when the size of the sphere is as large as one free-space wavelength, it is possible to somewhat reduce the forward scattering by some dBs by properly tuning the permittivity below that of the free-space. Of course, forward scattering cannot be as small as that in the smaller spheres, due to the relevant scattering in other directions, but proper reduction may be obtained by canceling

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$\mathrm{Re}\left[s_\theta(0,0)\right]$, which does not contribute to the extinction power. The corresponding scattering cross sections, reported in Fig. 1(b), confirm that the total scattering is not drastically affected by the choice of permittivity near $\varepsilon_{\min}$. The scattering peaks for negative values of permittivity are clearly associated with plasmonic scattering resonances [1], which are not of interest here.

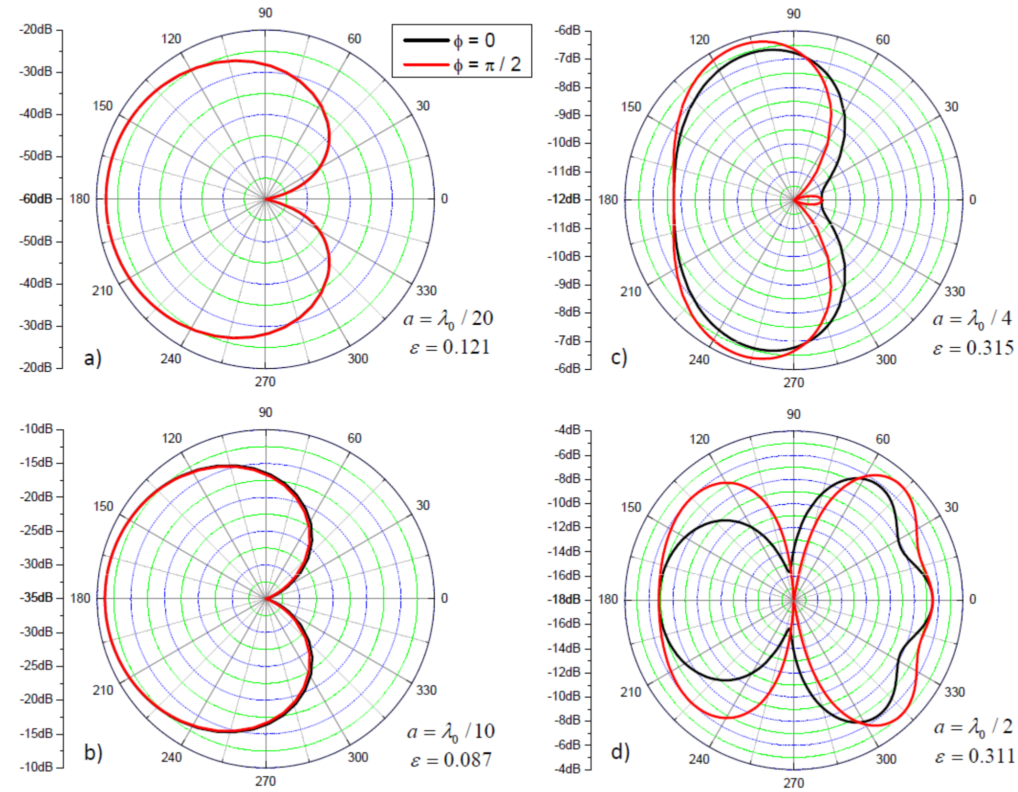

Fig. 2. Scattering pattern $\eta^d_{scat}(\theta,\phi)$ in the E and H planes for different sphere sizes for the minimum forward scattering condition derived as in Fig. 1.

Figure 2 reports the patterns of the differential scattering efficiency $\eta^d_{scat}(\theta,\phi)=\sigma^d_{scat}(\theta,\phi)/(\pi a^2)$ for different sphere sizes. The sphere radius and the corresponding value of ε are reported in each panel. For very small particles, as in Fig. 2(a), the scattering patterns in the E and H planes are identical, since the two scattering coefficients have same amplitude, satisfying Eq. (25). It is seen that the cancellation of $\mathrm{Re}\left[s_\theta(0,0)\right]$ implies a drastic reduction of forward scattering, producing almost zero shadow in the forward direction ($\theta = 0$). It is interesting, however, that the scattering in other directions is increased, since the total scattering cross section is actually twice the one obtained for $\varepsilon = 1$. Even for $a = \lambda_0/10$ [Fig. 2(b)] the situation is quite similar, although small differences in the two scattering planes start to arise. For $a = \lambda_0/4$ [Fig. 2(c)] the scattering pattern is still shifted towards the backscattering direction of the sphere, but it is seen that a non-negligible forward scattering is now necessary to sustain the total extinction from such larger particle. Indeed, in the forward direction a local angular maximum is visible in this case, produced by

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$\mathrm{Im}\left[s_\theta(0,0)\right]$, despite this value being the absolute minimum achievable in the forward direction (see Fig. 1). Finally, for $a=\lambda_0/2$ the forward scattering, although minimized, is very pronounced in both planes of polarization, and evidently the zero forward scattering condition does not hold any more in terms of the scattering pattern. As an aside, the interested reader may be referred to Ref. 13 for a series of additional numerical simulations regarding the dependence of the zero-forward scattering condition on the particle size.

It is worth noticing in these plots that for larger particles zero forward scattering is not achievable, implying that the interpretation of the experimental evidence presented in Ref. 14 may be questionable. In that paper, the authors consider a suspension of magnetite nanoparticles of few μm in diameter inside a ferrofluid background. Such suspension is well known to have tunable birefringent properties by varying the applied biasing magnetic field [14]. The authors have measured the optical transmission at $\lambda_0 = 633\,nm$ through such collection of magnetic nanoparticles, verifying absence of transmission for a specific level of biasing magnetic field. They have attributed this effect to zero-forward scattering from the magnetic nanoparticles. However, their argument and interpretation is evidently flawed in the following ways: (a) zero forward scattering implies absence of shadow and therefore total transmission through a collection of particles. This is the opposite of what the authors have measured, i.e., no transmission; (b) the particles are few wavelengths large; as evident from our Figs. 2 and 3 and from the comment [15], zero forward scattering is not achievable for such large particles; (c) the magnetic effects of these particles are only available at much lower frequencies, and are not obtainable at optical frequencies, where it is well known that all natural materials are characterized by a permeability very close to unity [31]. This implies that zero forward-scattering condition might not be attainable in the visible (if not in the trivial condition $\varepsilon=\mu=1$ for which the whole sample would become transparent). It is evident that the claim of having verified experimentally the zero-forward scattering concepts in Ref. 14 is not consistent with the previous theoretical results, and future experimental attempts should consider particles with smaller sizes and lower frequencies of operation, where magnetic effects are naturally available.

Figure 3 reports various dispersion plots for the variation of the minimum forward scattering condition when the size of the sphere is increased, for different values of its permeability. Fig. 3(a) refers to the variation of the required permittivity $\varepsilon_{\min}$ to achieve the minimum forward scattering. It can be seen that this value tends to Eq. (1) for very small spheres, but it varies in the region $-1<\varepsilon<1$ for larger sizes. A lower value of permittivity is required for larger magnetic effects, as expected from (1). Fig. 3(b) reports the dispersion of η_{fw} in the minimum forward scattering condition ($\varepsilon=\varepsilon_{\min}$, consistent with Fig. 3(a), solid lines) and in the regular case of $\varepsilon=1$ (dashed lines). It is noted that a proper choice for the permittivity, following the curves in Fig. 3(a), may provide a substantial reduction of forward scattering compared to the regular case, even for relatively larger particles. This effect is present, despite the expected non-zero scattering for larger particles. Fig. 3(c) reports similar curves, but for the total scattering efficiency η_{scat}. It is seen that, as predicted in the previous sections, for small spheres the total scattering is larger in the minimum forward-scattering condition than for a simple magnetic nanoparticle with $\varepsilon=1$. However, for larger spheres the situation changes, and as expected the reduction of forward scattering coincides with a reduction of total scattering. Indeed, in this case the required permittivity tends to low positive values, that have been proven to provide the best cloaking performance in the scattering cancellation technique scenario [6-7]. Evidently, the cloaking in this case is not ideal, since we are not using a different layer as done in the plasmonic cloaking scenario, but we are simply varying the permittivity of the same magnetic sphere. However, it is interesting to notice how low positive permittivities in this scenario may also partially suppress the forward

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and overall scattering from a relatively large sphere. Somehow, this connects once again the zero-forward scattering condition with the plasmonic cloaking technique for larger spheres.

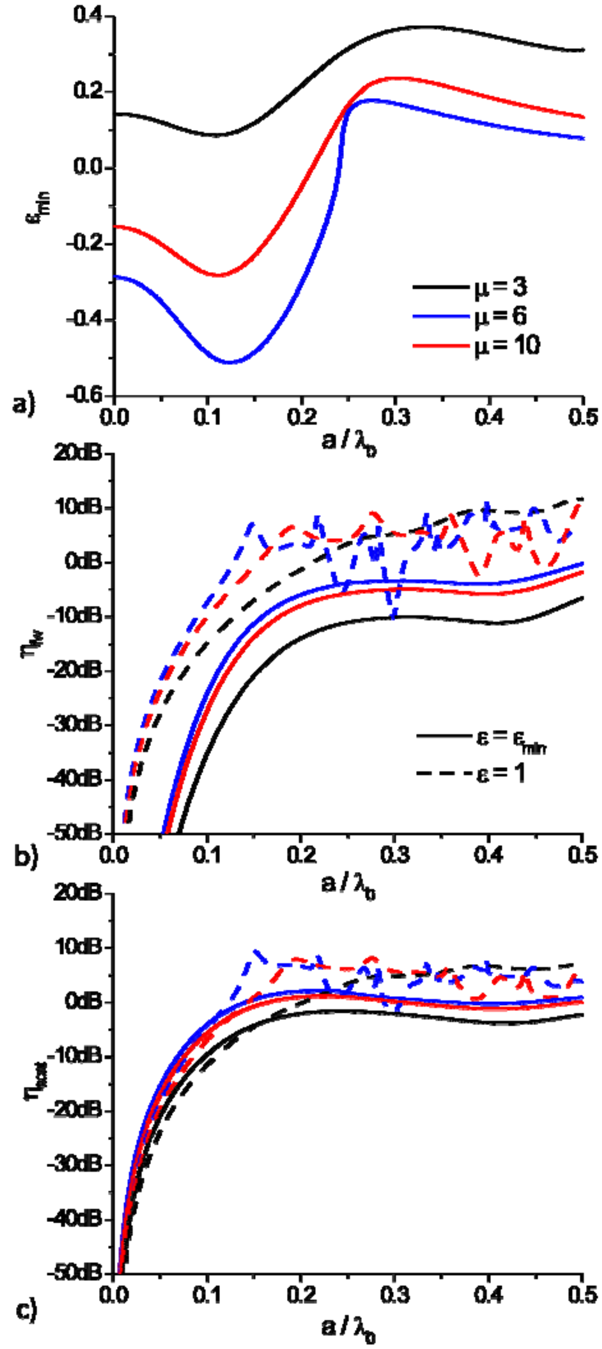

Fig. 3. Variation of: (a) the permittivity required for minimum forward-scattering, (b) η_{fw}, and (c) η_{scat} as a function of sphere normalized radius. Solid lines refer to the minimum-forward scattering condition, dashed lines to magnetic spheres with $\varepsilon = 1$.

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Figure 4 reports similar plots considering the presence of realistic losses in the permittivity, as $\varepsilon = \varepsilon_r + i\varepsilon_i$, for $\mu = 3$. In particular, Fig. 4(a) refers to the forward scattering efficiency, showing that the minimum level of forward scattering is necessarily increased by absorption in the particle for a given size. This is particularly evident for smaller particles, a regime for which it is well known that absorption dominates the extinction properties. It can be seen that in this scenario the forward scattering minimization is more relevantly affected by moderate losses. In Fig. 4(b) we have reported in this same scenario the total scattering efficiency η_{scat} (solid lines) and the total extinction efficiency $\eta_{ext} = \sigma_{ext} / \left(\pi a^2\right)$ (dashed lines). It is seen that the presence of losses mainly affects the extinction cross section, which is increased more relevantly for smaller particles. This is evidently connected with the increase in the forward-scattering in panel a). One may speculate, however, that with moderate losses the minimum forward-scattering effects may still be experimentally verifiable for moderately sized spheres.

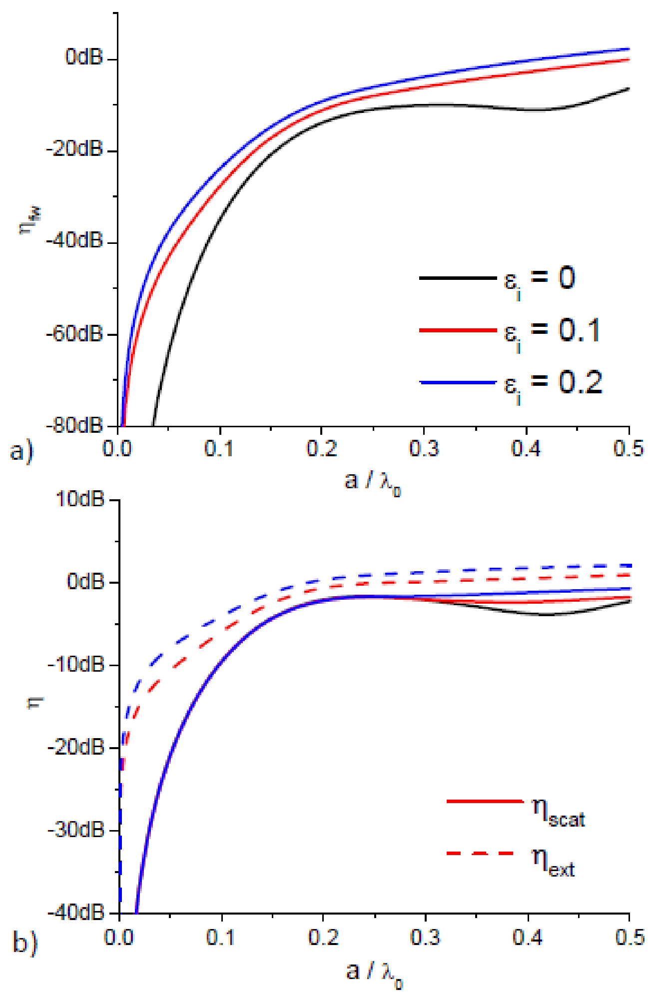

Fig. 4. Similar to Fig. 3b and 3c, but considering losses in the permittivity of the particle, as $\varepsilon = \varepsilon_r + i\varepsilon_i$. The dashed lines in panel b) correspond to the extinction efficiency.

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4 CONCLUSIONS

We have presented here a thorough analysis of the zero forward-scattering condition for small magnetic particles, first introduced by Kerker *et al.* in 1983 [11]. In particular, we have resolved an apparent inconsistency between the zero forward-scattering condition and the optical theorem, by using an improved quasi-static analysis consistent with power balance considerations. Then, we have considered the variation of this effect on the size, constitutive parameters and losses in the particles, and we have discussed how the interpretation of the recent experimental attempts by Mehta *et al.* to verify these effects reported in Ref. 14 may be questionable. However, we have also discussed how it is indeed possible to conceive a non-zero scattering pattern with a sharp minimum in the forward direction for sufficiently small magnetodielectric particles, possibly verifiable at microwave or far-infrared regime, where magnetic particles are naturally available. Finally, we have related these effects to the recent interest in cloaking applications using metamaterials, and in particular to cloaked sensors and absorbing particles. These findings may be particularly useful for the physical understanding of some of the anomalous scattering properties associated with cloaking and transparency effects, which have been recently discussed in the literature.

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