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A New Dynamic Duration Model

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Abstract
In this dissertation, I propose a new model for the analysis of financial durations. The new model improves upon several limitations of the autoregressive conditional duration (ACD) model considered in Engle and Russell (Econometrica 66(5) (1998) 1127-1162). Instead of adopting the multiplicative error form assumed by the ACD model, I establish a mixture of exponentials representation for durations from general point process theory. Based on the representation, I develop the Markov switching multifractal duration (MSMD) model. I present the geometric ergodicity property of MSMD and show that the MSMD can explain most stylized facts of financial durations, especially the long memory feature. An extensive empirical study shows MSMD compares favorably with ACD both in- and out-of-sample. For long horizon forecasting, MSMD dominates ACD, which confirms that MSMD can explain long range dependence in durations.

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A NEW DYNAMIC DURATION MODEL

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ABSTRACT

A NEW DYNAMIC DURATION MODEL

Fei Chen

Francis X. Diebold

In this dissertation, I propose a new model for the analysis of financial durations. The new model improves upon several limitations of the autoregressive conditional duration (ACD) model considered in Engle and Russell (Econometrica 66(5) (1998) 1127-1162). Instead of adopting the multiplicative error form assumed by the ACD model, I establish a mixture of exponentials representation for durations from general point process theory. Based on the representation, I develop the Markov switching multifractal duration (MSMD) model. I present the geometric ergodicity property of MSMD and show that the MSMD can explain most stylized facts of financial durations, especially the long memory feature. An extensive empirical study shows MSMD compares favorably with ACD both in- and out-of-sample. For long horizon forecasting, MSMD dominates ACD, which confirms that MSMD can explain long range dependence in durations.

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Chapter 1

Introduction

The last two decades have witnessed a growing interest in theoretical and empirical modeling of the ultimate high-frequency financial data. A salient feature of these intra-day tick-by-tick data is that transactions are irregularly spaced in time.

Many empirical studies take these irregular durations as exogenous sampling schemes and tend to aggregate the data to fixed intervals in accordance with the usual low-frequency data such as daily, weekly, monthly data. Such temporal aggregation facilitates empirical analysis, but also causes two problems. First, the aggregation will lose information and introduce unknown bias from a statistical view. Aït-Sahalia and Mykland (2003) discusses the effects of sampling randomness and discreteness when estimating continuous time processes. Second, there is little theory guidance on how to choose length of the fixed interval. Bandi and Russell (2008) discusses how to choose the optimal sampling interval when estimating realized volatilities.

More importantly, the irregular duration is an endogenous variable, which has economic information content. It reflects the speed of information flow on the financial market, see, e.g., Hasbrouck (1999). Easley and O'Hara (1992) gives a market microstructure interpretation of intertrade durations. The theoretical model suggests the dynamics of durations should have clustering effect, i.e., short (long) durations tending to be followed by short (long) durations. The clustering effect is found in actual data. Figure 1 gives one example. From the figure, one can see the durations not only have clustering effect, but also have substantial outliers.

In this dissertation, I propose a new model, the Markov switching multifractal duration (MSMD) model, to analyze these irregular durations. The MSMD model can explain most stylized facts of intertrade durations found in empirical studies: clustering effect; overdispersion[1], the standard deviation being greater than the mean; long memory, autocorrelations decreasing hyperbolically; strong nonlinearities in the dynamics. Furthermore the MSMD model predicts there should be clustering effect at all time scales, and this feature is found in real data.

The first econometric model to explore the information content of intertrade durations is the Autoregressive Conditional Duration (ACD) model proposed by Engle and Russell (1998). The basic ACD model assumes a multiplicative error form, where duration is the product of conditional mean and error[2]. Such a specification has two components: the dynamics of the mean and the distribution of the error. Engle and Russell assume a GARCH-type dynamics with iid exponential or iid Weibull errors.

The specification of the basic ACD model is too restrictive. The GARCH-type dynamics can explain the clustering effect in durations, but can hardly capture other stylized facts. For example, the standardized durations of the basic ACD model with exponential error should have equal dispersion, but in practice, they still show excess dispersion.

Numerous extensions of the basic ACD models have been developed in the literature. Those include the logarithmic ACD model of Bauwens and Giot (2000), the Box-Cox and exponential ACD models of Dufour and Engle (2000), the threshold ACD model of Zhang et al. (2001), the Stochastic Conditional Duration (SCD) model of Bauwens and Veredas (2004), the stochastic volatility duration (SVD) model of Ghysels et al. (2004), the smooth transition ACD model of Meitz and Teräsvirta

[1] Giot (2000) reports that some volume duration series (durations for volume to reach some threshold) can exhibit underdispersion. All durations between transactions and price changes (quote changes) show overdispersion. In this paper, I don't consider volume durations.

[2] See Engle (2002) and Engle and Gallo (2006) for more details of multiplicative error model.

(2006), the augmented ACD model of Fernandes and Grammig (2006), the fractionally integrated ACD model of Jasiak (1999) and the long memory stochastic duration model of Deo et al. (2010). None of these extensions can explain both nonlinearity and long memory at the same time; while the MSMD model can explain nonlinearity and long memory in one model.

One strand of the extensions is to use more flexible functional forms for the error distribution, see Lunde (1999), Grammig and Maurer (2000), De Luca and Zuccolotto (2003), De Luca and Gallo (2004), Drost and Werker (2004), Hujer and Vuletic (2007), Sun et al. (2008), De Luca and Gallo (2009). The distributions being used are Gamma distribution, generalized Gamma distribution, Burr distribution, mixed exponential distribution, etc. On the one hand, these distributions are very flexible for all practical purposes. On the other hand, the choice of a particular distribution is arbitrary, and is mostly based on convenience and analytical tractability. But the error distribution is in fact very important. It not only has direct impacts on intra-day trading strategies and risk management, but also has serious implications on models that try to link durations and volatilities, e.g., Ghysels and Jasiak (1998), Engle (2000), Grammig and Wellner (2002).

The purpose of introducing those flexible distributions is to explain the remaining excess dispersion and other features found in the standardized durations that can't be explained by exponential or Weibull errors. But since the GRACH-type dynamics can't explain the long memory and nonlinearity features in the first place, this effort of using more flexible distributions is mostly in vein. Furthermore, the arbitrarily chosen error distribution may cause identification problem. Heckman and Singer (1984) gives an example that two duration models with different error distributions can have the same statistical properties. Heckman and Walker (1990) argues that various duration models have one representation in mixture of exponentials form. Though the example and the argument is for the single spell duration model, it is reasonable to conjecture that a similar problem could exist for dynamic duration models.

Most dynamic duration models are modeling the dynamics of the conditional mean. The MSMD model adopts a different approach. It focuses on the intensity process. The trading process is a marked point process (PP) on the time line. A PP can be represented by a series of durations, but the driving force underlying the durations is the continuous-time intensity process. Direct modeling of the intensity process has recently been applied to multivariate financial PPs, e.g., Russell (1999), Bauwens and Hautsch (2006), Bowsher (2007). I begin with the intensity process and use a time deformation method to build a link between intensities and durations. I establish a mixture of exponentials representation for durations. In this representation, durations can be written as iid exponential errors divided by mean intensities. This result has one important implication. If the dynamics of the intensities is suitably specified, the error distribution should be i.i.d. exponential. No other distribution is needed to capture the overdispersion feature.

I model the mean intensity process as a Markov switching multifractal (MSM) process, thus develop the MSMD model. The MSM process is first put forth by Calvet and Fisher (2004) and applied to volatility modeling. I apply the MSM process for intensity process. The MSM process allows regime switches at all frequencies, thus captures dynamics at different time scales. The low-frequency switches can capture the long range variations. The intermediate-frequency switches can capture smooth transition autoregressive dynamics. The high-frequency switches can generate substantial outliers.

I show that the MSMD model can explain most existing stylized features, especially the long memory feature. The long memory feature is an important property of financial time series. A great deal of research interest is attracted to long memory in volatilities[3]. Recently, there is a view that long memory in volatilities is from long

[3]See, among many others, Ding et al. (1993), Bollerslev and Mikkelsen (1996), Baillie et al. (1996), Comte and Renault (1996), Breidt et al. (1998), Andersen et al. (2001), Deo et al. (2006), Corsi (2009).

memory in durations, see Deo et al. (2009), Deo et al. (2010). The MSM duration together with the MSM volatility provide a natural mechanism for the long memory parameter to spread from duration to volatility.

To validate the MSMD model, I implement an extensive empirical study. Twenty stocks are randomly selected from the S&P 100 index. I run a horse race between the MSMD model and ACD model by comparing both in sample fit and out of sample forecast for all the twenty stocks. The MSMD model compares favorably with the ACD model both in- and out-of-sample. The MSMD has a higher in-sample likelihood for all the stocks. Out-of-sample comparison gives analogous result. For 1-step forecast, the performance of the two models is similar. But for forecast at longer horizons, the MSMD dominates.

The rest parts of this dissertation is organized as follows. In chapter 2, I introduce notions of PPs and derive the mixture of exponentials representation. Chapter 3 introduces specifications of the MSMD model. In chapter 4, I show properties of the MSMD model. Chapter 5 is empirical work and chapter 6 concludes.

Chapter 2

Point Processes and Mixture of Exponentials Representation

The purpose of this chapter is to derive the mixture of exponentials representation of PPs. To this end, I introduce basic concepts and tools of PPs. There are two fundamental approaches to characterize PPs. One approach characterizes PPs in terms of a random measure and the other in terms of a conditional intensity[1]. I only introduce the conditional intensity. The conditional intensity is a powerful tool for evolutionary PPs on the time line, because it introduces martingale-based methods to PPs.

2.1 Notation and Definition

A simple PP on $(0, \infty)$ is a sequence of nonnegative random variables $\{t_i\}_{i \in 1,2,\ldots}$ defined on some probability space (Ω, F, P), satisfying $0 < t_1 < t_2 < \cdots$, where t_i is the instant of the i-th occurrence of an event. Associated with each t_i, there could be some random variables. These variables are called marks of the PP. In a trading process, the events are financial transactions. The marks could be volume, price,

[1] Textbook treatments of these two approaches can be found in Brémaud (1981), Karr (1991), Daley and Vere-Jones (2003), and Daley and Vere-Jones (2007).

bid-ask quotes or other variables coming with each transaction[2].

A PP may also be represented via its associated counting process $N(t)$, where $N(t) = \sum_{i\geq 1} 1(t_i \leq t)$ is the number of events happened till time t. The internal history $\{F_t^N\}_{t\geq 0}$ of a PP is given by the σ-algebra generated by the observed past of the process, namely $F_t^N = \sigma(N(s) : 0 \leq s \leq t)$. A history F_t is a more general σ-algebra which could contain information about some exogenous variables, e.g., the marks. The internal history is the smallest history, $F_t^N \subseteq F_t$. Obviously $N(t)$ is F_t-adapted.

Let $\lambda(t)$ be a scalar, positive F_t-predictable process[3]. Then $\lambda(t)$ is called the F_t-conditional intensity of $N(t)$, if

$$E[N(s) - N(t)|F_t] = E[\int_t^s \lambda(u)du|F_t] \tag{2.1}$$

holds almost surely for all t, s with $0 \leq t \leq s$[4]. The definition of conditional intensity given by (2.1) is abstract. A more intuitive understanding of the intensity can be obtained by letting $s \downarrow t$ in (2.1).

$$\begin{aligned} \lambda(t) &= \lim_{\Delta t \downarrow 0} \frac{1}{\Delta t} E(N(t + \Delta t) - N(t)|F_{t-}) \\ &= \lim_{\Delta t \downarrow 0} \frac{1}{\Delta t} P(N(t + \Delta t) - N(t) = 1|F_{t-}) \end{aligned} \tag{2.2}$$

The above equation shows the similarity between conditional intensity and hazard function. The conditional intensity exists for a very large class of PPs, which contains not only nonhomogeneous Poisson process, but also many non-Poisson processes.

[2]By the definition, the trading process is usually not a simple PP. Multiple transactions at the same second are observed in TAQ database. It is believed that the simultaneous trades executed at the same second come from the same trader who has split a big order block into small blocks. One trade for each second. The thinned trading process is a simple PP

[3]See appendix A3 of Daley and Vere-Jones (2003) for definition of F_t-predictable. Sufficient conditions for $\lambda(t)$ to be F_t-predictable are $\lambda(t)$ is adapted to F_t, and the sample paths of $\lambda(t)$ are left continuous with right hand limits.

[4]For existence of $\lambda(t)$, see chapter 7 of Daley and Vere-Jones (2003) and chapter 14 of Daley and Vere-Jones (2007)

The compensator of a PP is defined as $\Lambda(t) = \int_0^t \lambda(s)ds$. Let $M(t) = N(t) - \Lambda(t)$, then the process $M(t)$ is a martingale. One important result of the martingale-based PP theory is the random change of time theorem.

2.2 Random Change of Time

The random change of time theorem gives a method to transform non-Poisson processes to a homogeneous Poisson process.

Theorem 1 *Let $N(t)$ be a simple point process on $(0, \infty)$, adapted to filtration F_t. Suppose that $N(t)$ has the F_t-conditional intensity $\lambda(t)$ that satisfies:*

$$\int_0^\infty \lambda(t)dt = \infty.$$

For any $t \geq 0$, define the F_t-stopping time τ_t as the solution to:

$$\int_0^{\tau_t} \lambda(s)ds = t$$

then the point process $\tilde{N}(t) = N(\tau_t)$ is a homogenous Poisson process with intensity $\lambda = 1$.

Proof See Theorem T16, p.41, Brémaud (1981). ∎

The only condition for the theorem to hold is $\int_0^\infty \lambda(t)dt = \infty$. That is to say one can always expect more occurrences of the events in the future. This condition is satisfied by any trading process.

Though the random change of time is introduced as a pure mathematical method, it has an intuitive economic interpretation. In an ideal world without information flow, the trading process is a homogeneous Poisson process, i.e. the trading intensity is constant. In reality, the randomly arriving information flow distorts the trading intensity, and the trading process evolves on some operational or economic time scale that differs from the calendar or clock time. The random change of time method gives a functional mapping between the clock time and the economic time, which is so called time deformation.

Time deformation has been widely used in economic researches, see, e.g. Clark (1973), Stock (1988), Carr and Wu (2004). The random change of time theorem gives a subordinator of a Poisson process.

The theorem is well known in the PP literature. But previous researches have emphasized on using the theorem to construct goodness-of-fit test for the intensity process, e.g., chaper 7 of Daley and Vere-Jones (2003), Bowsher (2007). I first use it to establish the mixture of exponentials representation of PPs.

2.3 Mixture of Exponentials Representation

I shall derive the mixture of exponentials representation by using the time deformation function τ_t.

Let $\tilde{t}_i$ and t_i denote the time of the ith event in the operational and clock time respectively. $\epsilon_i = \tilde{t}_i - \tilde{t}_{i-1}$ and $d_i = t_i - t_{i-1}$ are the ith duration in different time scale. In the operational time scale, the trading process is a homogeneous Poisson process, so the distribution of the durations is iid exponential. That means $\epsilon_i \sim i.i.d.Exp(1)$. By the definition of τ_t, $\epsilon_i = \tilde{t}_i - \tilde{t}_{i-1} = \Lambda(t_{i-1}, t_i) = \int_{t_{i-1}}^{t_i} \lambda(s)ds$. Let $\lambda_i = \Lambda(t_{i-1}, t_i)/d_i$ be the mean intensity. Then

$$d_i = \frac{\epsilon_i}{\lambda_i} \tag{2.3}$$

This is the mixture of exponentials representation, which is different from the multiplicative error form of the ACD models. Instead of modeling the conditional mean, we need to model the mean intensity λ_i, which is the task of the next chapter.

The random change of time theorem can be applied to the family of ACD models, and all the ACD models can have a representation of mixed exponentials form. Thus the usual goodness-of-fit tests for model selection may not work. The fact that one model can fit the data well doesn't rule out the possibility that other model can fit the data equally well. Additional criteria must be imposed to make model selection. A good candidate is the out-of-sample forecast.

Chapter 3

Markov Switching Multifractal Duration

In the last chapter, I establish the mixture of exponentials representation for durations, i.e., equation (2.3). For a complete duration model, we need to specify the mean intensity λ_i. Any specification of λ_i can be regarded as a type of time deformation. For example, Stock (1988) shows that the ARCH-type dynamics is a type of time deformation. The type of time deformation I will use is the multifractal.

Mandelbrot (1997) first proposes the multifractal process. Mandelbrot et al. (1997) compounds a Brownian motion with a multifractal measure thus put forth the multifractal model of asset returns (MMAR) which can explain the heavy tails and volatility persistence exhibited by many financial time series. Calvet and Fisher (2001) develops the Poisson multifractal, a fully stationary version of the multifractal model. Calvet and Fisher (2004) puts forth the MSM which has a closed form likelihood. I model the intensity process as a MSM process.

The MSMD model assumes that the intensity has $\bar{k}$ components. Each component represents a type of shocks at a particular frequency. All components contribute to the intensity through a *multiplicative* effect[1]. More precisely, λ_i is specified as

$$\lambda_i = \lambda \prod_{k=1}^{\bar{k}} M_{k,i}, \tag{3.1}$$

where λ is a positive constant. $M_{1,i}, M_{2,i}, \ldots, M_{\bar{k},i}$ are positive intensity components. The components are statistically independent with each other at any time. It is convenient to define the trading intensity state vector at time i as $M_i = (M_{1,i}, M_{2,i}, \ldots M_{\bar{k},i})$.

For each $k \in \{1, 2, 3, ..., \bar{k}\}$, the dynamics of the component $M_{k,i}$ follows a Markov renew process. At time i, $M_{k,i}$ is either renewed, namely drawn from a fixed distribution M with probability γ_k, or remains its previous value $M_{k,i-1}$ with probability $1 - \gamma_k$. Whether $M_{k,i}$ is renewed means whether there is a new shock hitting the system at time i.

The fixed distribution of M is the same for different components. A draw from M is the magnitude of a shock. Only positive shocks are allowed, so M has a support on nonnegative real line, $M > 0$. To prevent the shocks from exploding, M satisfies $E(M) = 1$.

The renewal probability γ_k is specified as

$$\gamma_k = 1 - (1 - \gamma_1)^{b^{(k-1)}} \tag{3.2}$$

or equivalently

$$\gamma_k = 1 - (1 - \gamma_{\bar{k}})^{b^{k-\bar{k}}} \tag{3.3}$$

where $\gamma_k \in (0, 1)$ and $b \in (1, \infty)$. This specification is introduced in Calvet and Fisher (2001) in connection with the discretization of a Poisson arrival process. The value of γ_k determines the average lifetime or persistence of a $M_{k,i}$ shock. The larger

[1] This multiplicative effect could become additive effect by taking logarithm. Then we can take the total intensity as a superposition of the $\bar{k}$ components with different frequencies. This is similar to the fourier series expansion, but we don't have the usual orthogonal condition here.

the γ_k is, the shorter average lifetime the $M_{k,i}$ shock will have. Large k component stands for high frequency shock. Small k component stands for low frequency shock. An important feature of this specification is that all shocks, low frequency or high frequency, have stochastic lifetime.

Equations (2.3), (3.1) and (3.3) plus specification for M define a stochastic duration model, thus the MSM duration model.

Chapter 4

Model Properties

In this chapter, I show the MSMD model can explain most stylized features of financial durations: overdispersion, nonlinearity, long memory. The MSMD also predicts there should be clustering effect at all time scale. This property is confirmed by using counts data. One statistical property of the MSMD model, the geometric ergodicity property is presented.

4.1 Geometric Ergodicity

For duration models, important properties are stationarity, ergodicity and finite higher-order moments. The strict stationarity of the MSM duration is obvious since each intensity component is independent and stationary. The existence of finite higher-order moments depends on the moment properties of M. For example, if M is set to have a binomial distribution, which I will do in the empirical study, then every finite moment of d_i exists. I now show the ergodic property.

Proposition 2 *The MSM duration* $\{d_i\}$ *is geometrically ergodic.*

Proof From the definition, d_i is a hidden Markov model with the intensity vector M_i as the Markov chain. By Proposition 4 of Carrasco and Chen (2002), It is enough to show M_i is geometrically ergodic.

Let the support of M be S. M_i is a Markov chain on $S^{\bar{k}}$. Since each component $M_{k,i}$ is independent, we need to show $M_{k,i}$ is geometrically ergodic.

First we show $M_{k,i}$ is φ-irreducible T-chain. Take φ as Lebesgue measure μ^{Leb} on S. The μ^{Leb}-irreducibility is immediate from the assumption of positive densities for M. The transition kernel of $M_{k,i}$ is $P(x, A) = \gamma_k \int_A dF + (1 - \gamma_k)1_x(A)$. Let $T(x, A) = \gamma_k \int_A dF$, then $T(x, A)$ is a nontrivial continuous component of $P(x, A)$, by Proposition 6.2.4 of Meyn and Tweedie (1993), $M_{k,i}$ is a T-chain.

This implies that all compact sets in S are petite. We can choose any compact set C in S with positive probability measure as a test set. It is easy to check that $M_{k,i}$ satisfies conditional (ii) of Proposition 15.0.1 of Meyn and Tweedie (1993), so that $M_{k,i}$ is geometrically ergodic. ∎

4.2 Clustering Effect

The clustering effect is suggested by some market microstructure models, e.g., Admati and Pfleiderer (1988), Easley and O'Hara (1992). Those models usually assume there are two groups of traders, informed and uninformed. The informed traders will trade only when informational events happen, thus generate the clustering effect. And the clustering effect is found in empirical studies, see Engle and Russell (1998).

The MSMD model can not only explain the clustering effect, i.e., when the highest frequency intensity component draws a large value, short durations will happen together. But also it predicts there are clustering effects at all time scales. This is because every component can cause clustering effect at certain time scale.

I show one example of the clustering effects in Figure 2. I draw four graphes of the counts or the number of transactions of a stock in four different time scales, i.e., 2 minutes, 5 minutes, 10 minutes and 30 minutes. Clustering effect is found in all four graphes. This confirms the prediction of the MSMD model.

4.3 Nonlinearity

There is strong nonlinearity in the duration dynamics. The linear ACD model can not capture this important feature, thus various nonlinear dynamic specification are developed in the literature. Zhang et al. (2001) uses the threshold ACD model to identify multiple structural breaks in the duration data considered, and finds those break points matched nicely with real economic events. This is in agreement with our discussion in last section. Different information events will draw different shocks, therefore cause regime switches. The MSMD is a Markov switching model. It has the nonlinearity built in.

4.4 Overdispersion

The overdispersion property can be observed in all the duration data that are used for empirical study. Let $\mu_d = E(d_i)$, $\sigma_d^2 = \text{Var}(d_i)$.

Proposition 3

$$\sigma_d > \mu_d$$

Proof By the definition, $\mu_d = E(d_i) = E(\frac{1}{\lambda_i})E(\epsilon_i) = E(\frac{1}{\lambda_i})$ and $\sigma_d^2 = \text{Var}(d_i) = E(d_i^2) - [E(d_i)]^2 = E(\epsilon_i^2)E(\frac{1}{\lambda_i^2}) - [E(\frac{1}{\lambda_i})]^2$. It is easy to check $E(\epsilon_i^2) = 2$, so

$$\sigma_d^2 = 2E(\frac{1}{\lambda_i^2}) - [E(\frac{1}{\lambda_i})]^2$$

by Jensen's inequality, i.e., $[E(\frac{1}{\lambda_i})]^2 < E(\frac{1}{\lambda_i^2})$, we get $\sigma_d > \mu_d$. ∎

4.5 Long Memory Feature

The duration autocorrelations decay very slowly. Figure 3 shows four duration autocorrelations . A visual check will confirm the slowly decaying of autocorrelations.

Previous researches have not paid much attention to the long memory feature. One reason is that the sum of the autoregressive parameters estimated in ACD models is nearly 1. The high value of the sum can explain some persistence of the duration autocorrelations. Nevertherless the ACD models are still short memory models. Autocorrelations of the ACD models decay exponentially. Recently, the long memory feature is confirmed by the semiparametric analysis of Deo et al. (2010).

I now show the MSMD has long memory feature. The autocorrelation function of durations is $\rho(n) = Corr(d_i, d_{i+n})$. Let $\alpha_1 < \alpha_2$ denote two arbitrary numbers in the open interval $(0, 1)$. The set of integers $I_{\bar{k}} = \{n : \alpha_1 \log_b(b^{\bar{k}}) \leq \log_b n \leq \alpha_2 \log_b(b^{\bar{k}})\}$ contains a broad collection of lags.

Proposition 4 *The autocorrelation of durations satisfies*

$$\sup_{n \in I_{\bar{k}}} \left| \frac{\ln \rho(n)}{\ln n^{-\delta}} - 1 \right| \to 0 \qquad as \quad \bar{k} \to +\infty$$

where $\delta = \log_b E(M) - \log_b\{[E(M^{1/2})]^2\}$

Proof By the definition, $Corr(d_i, d_{i+n}) = E(d_i d_{i+n}) - E(d_i)E(d_{i+n})$.

The first term is calculated as follow:

$$E(d_i d_{i+n}) = E(\frac{\epsilon_i \epsilon_{i+n}}{\lambda_i \lambda_{i+n}}) = E(\lambda_i^{-1} \lambda_{i+n}^{-1}) = \prod_{k=1}^{\bar{k}} E(M_{k,i}^{-1} M_{k,i+n}^{-1}).$$

The last equality is valid by the independence of each component. The last term can be calculated by iterated expectation,

$$E(M_{k,i}^{-1} M_{k,i+n}^{-1}) = E[M_{k,i}^{-1} E(M_{k,i+n}^{-1} | M_{k,i}^{-1})],$$

where

$$E(M_{k,i+n}^{-1} | M_{k,i+n-1}^{-1}) = M_{k,i+n-1}^{-1}(1 - \gamma_k) + E(M^{-1})\gamma_k,$$

and

$$E(M^{-1}_{k,i+n}|M^{-1}_{k,i+n-2}) = M^{-1}_{k,i+n-2}(1-\gamma_k)^2 + E(M^{-1})\gamma_k(1-\gamma_k) + E(M^{-1})\gamma_k.$$

So we can get

$$E(M^{-1}_{k,i+n}|M^{-1}_{k,i}) = M^{-1}_{k,i}(1-\gamma_k)^n + E(M^{-1})[1-(1-\gamma_k)^n],$$

and

$$\begin{aligned} E(M^{-1}_{k,i}M^{-1}_{k,i+n}) &= E(M^{-2})(1-\gamma_k)^n + [E(M^{-1})]^2[1-(1-\gamma_k)^n] \\ &= [E(M^{-1})]^2[1+a(1-\gamma_k)^n] \end{aligned}$$

where $a = E(M^{-2})[E(M^{-1})]^{-2} - 1$.

Now we calculate the second term $E(d_i)E(d_{i+n}) = [E(d_i)]^2 = [E(\frac{1}{\lambda_i})]^2 = \prod_{k=1}^{\bar{k}}[E(M^{-1}_{k,i})]^2 = \prod_{k=1}^{\bar{k}}[E(M^{-1})]^2 = [E(M^{-1})]^{2\bar{k}}$. We already have $\sigma_d^2 = 2E(\frac{1}{\lambda_i^2}) - [E(\frac{1}{\lambda_i})]^2 = 2\prod_{k=1}^{\bar{k}} E(M^{-2}) - \prod_{k=1}^{\bar{k}}[E(M^{-1})]^2 = [E(M^{-1})]^{2\bar{k}}[2(1+a)^{\bar{k}} - 1]$, thus we get

$$\rho_n = Corr(d_i, d_{i+n}) = \frac{\prod_{k=1}^{\bar{k}}[1+a(1-\gamma_k)^n] - 1}{2(1+a)^{\bar{k}} - 1}$$

The rest of the proof just follows Proposition 1 of Calvet and Fisher (2004). ∎

4.6 Discussion

A traditional method to generate long memory is the fractional integration (FI) or $I(d)$ model, i.e., fractional difference operator acting on iid shocks. It is introduced to the econometrics literature by Granger and Joyeux (1980) as a parsimonious empirical method to characterize long memory process. In FI models, every shock has a long-lived effect, which means every shock decays hyperbolically. This is different from $I(0)$ and $I(1)$ processes. In an $I(0)$ process, e.g. a stationary ARMA process, every shock is transitory, decaying exponentially. In an $I(1)$ process, e.g. a non-stationary random walk process, every shock is permanent. The FI process seems to provide a natural way to fill the gap between $I(0)$ and $I(1)$ processes. But in FI models, every shock decays at the same rate. It introduces artificial mixing between long- and short-term dependence, which is illustrated by Comte and Renault (1998). It also blur the distinction between stationary and nonstationary processes.

Jasiak (1999) proposes the fractional integrated ACD model to capture long memory in durations. But this model suffers from the problem of non-existence of moments. The second moment of the fractional integrated ACD model doesn't exist. It is not a long memory model in the usual sense.

In the MSMD model, different shocks have different persistence, which seems to more attractive and closer to our intuition.

Chapter 5

Empirical Studies

The MSMD model is introduced and properties of MSMD model are derived in previous chapters. In this chapter, I do empirical study to validate the MSMD model. To this purpose, the distribution of M must be specified. As is discussed in chapter 3, M should satisfy $M > 0$ and $E(M) = 1$. Following Calvet and Fisher (2004), I specify M as a binomial variable taking value m_0 and $2 - m_0$ with equal probability. The binomial MSMD model has four parameters

$$\phi = (m_0, \lambda, b, \gamma_{\bar{k}}) \in \mathbb{R}_+^4.$$

The binomial MSMD models with different $\bar{k}$ are estimated for twenty stocks randomly selected from the S&P 100 index. Table 1 gives the symbol and company name of the twenty stocks. Empirical results show the MSMD mode with seven intensity components, MSMD(7) can give a good description of the data. Four ACD models, ACD(1,1), ACD(1,2), ACD(2,2) and ACD(3,3) are also estimated for the same data. With number of parameters increasing from 3 to 7 when the model changes from ACD(1,1) to ACD(3,3), the likelihood doesn't decrease much. Thus the ACD(1,1) is the leading model for ACD family.

I then run a horse race between the binomial MSMD(7) model and the ACD(1,1) model. Following Deo et al. (2010), I equally divide the twenty stocks into two groups, high trading group and low trading group, according to the number of transactions in the sample period.I compare both in sample fitting and out of sample forecasting of the two competing models. For in sample fitting, I compare the likelihoods. Because the two models are not nested and have different number of parameters, I use Bayesian information criterion (BIC) to make a model selection. For out of sample forecasting, I compare the mean square prediction errors for three horizons, 1-step, 5-step, and 20-step. A fixed scheme is chosen to compare forecasting performance, see Pagan and Schwert (1990) and West and McCracken (1998) for more discussion about this scheme. The detail is as follow: for stocks that have more than 11000 observations in the sampling period, I only take the first 11000 durations for forecasting comparison. I split the 11000 durations into two sets, a fitting set and a testing set. The fitting set has 10000 observations, and the later has 1000 observations. For stocks that have less than 11000 observations, I take roughly the last 1000 observation as testing set, all previous observations as fitting set. Competing models are estimated only once on the fitting set and then the estimated parameters are used in forming predictions for observations in the testing set. I choose this scheme mainly because the estimation processes of both models have computationally intensive numerical maximizations.

5.1 Data Description

The data for the empirical study are the consolidated trades data extracted from TAQ database. The sample period is from February 1, 1993 to February 26, 1993, which has 20 trading days. I only keep transactions during the open time, from 10:00 a.m. to 4:00 p.m.. All over night durations are omitted. Following Engle and Russell (1998) and Zhang et al. (2001), transactions in the opening period from 9:30 am to 10:00 a.m. are deleted to remove the opening auction effect and zero durations are deleted as well.

5.2 Daily Seasonality

The raw durations have strong diurnal daily pattern, i.e., the average duration is short both at opening time in the morning and at close time in the afternoon, but long at noon time. This daily seasonality is documented by many empirical studies. There are several methods to remove the seasonality. I adopt the method used by Ghysels et al. (2004). The main step is to regress the logarithm of the raw duration on the indicator variables that indicate the time of day. A day is divided into 12 subperiods. Each subperiod is 30 minutes. Consider the regression

$$\log d_i = \sum_{k=1}^{12} a_k x_{ki} + \epsilon_i = a' x_i + \epsilon_i$$

where $x_{ki} = 1$, if time i belongs to the intraday subperiod k, and 0 otherwise. Then the seasonally adjusted series is defined by

$$\hat{d}_i = d_i \exp(-\hat{a}' x_i)$$

where $\hat{a}$ denotes the OLS estimator of a. The data from now on are all seasonally adjusted data.

5.3 Data Statistics

Table 2 and Table 3 show the summary statistics of durations for all twenty stocks. The stock of Merck & Co is the most traded stock, which has 54242 durations in the sampling period. While the stock of ALCOA is the least traded stock, having 2989 durations. The longest duration is 76.83. The shortest duration is 0.02. Durations of all twenty stocks show overdispersion property. For all stocks, duration mean is greater than median. Each stock's duration kurtosis is much bigger than 3.

5.4 Estimation Method

The MSMD model is a hidden Markov model. The underlying Markov state variable is the intensity state vector M_i. Every intensity component can take only two values in the binomial MSMD model. The intensity state vector has $\bar{k}$ components, thus has $2^{\bar{k}}$ states. For finite number of states, likelihood of the model can be calculated by standard filtering procedure.

The procedure is as follow: first initiate the distribution of M_i with its ergodic distribution, then use Bayes' law to update the distribution of M_i, and compute the likelihood for each observation. MLE is used to estimate the four parameters. Like other hidden Markov models, local maximums exist. Multiple initial conditions are tried to find the MLE estimation.

5.5 Model Diagnostics

Several types of diagnostic tests have been developed in the literature to evaluate the fast growing ACD models, see Li and Yu (2003), Fernandes and Grammig (2005), Meitz and Teräsvirta (2006), Chen and Hsieh (2010). Unfortunately, the MSMD model is a latent variable model. These tests can not be applied here. Instead, I use the information matrix test developed by White (1982) as diagnostic test. The test is based upon the asymptotic equivalence of the Hessian and outer product forms of Fisher's information matrix, when the model is correctly specified. All the non-redundant elements of the information matrix (total 10 elements) are selected to form the test statistic S_{IM}. Note $S_{IM} \sim \chi^2_{10}$.

5.6 Estimation Results

Both estimation results of MSMD and ACD models for all the twenty stocks are presented in table 4 to table 43. For some low volume stock, $\gamma_{\bar{k}}$ is very close to 1. Thus b and λ are weekly identified. This will cause some numerical instability. In this case, I set $\gamma_{\bar{k}} = 0.99$ and maximize the likelihood through the other three parameters.

The estimation results show: according to White's omnibus information matrix test, MSMD models are better fitted to the data than ACD models do, which is obvious from the p value for both models. Within the ACD family, ACD(1,1) is the best model. I thus choose to compare MSMD(7) and ACD(1,1) models.

5.6.1 Overdispersion Test

Engle and Russell (1998) develops an overdispersion test to diagnose the exponential ACD model and finds that the standardized durations still show overdispersion. They conclude that the exponential errors are inadequate to capture the overdispersion feature, then they try Weibull errors. But there is still overdispersion. Many researchers follow this excise and use more flexible error distributions. These exercises totally ignore the possibility that the misspecification of the GARCH-type dynamics can also cause overdispersion. The empirical result here supports this possibility.

The overdispersion test is as follow

$$\sqrt{N}((\hat{\sigma}_\epsilon^2 - 1)/\sqrt{8}) \sim \mathcal{N}(0,1),$$

where N is the number of observations, $\hat{\sigma}_\epsilon^2$ is variance of the standardized duration, i.e., $d_i\hat{\lambda}_i$. As I already mentioned when discussing model diagnostics, exact $\hat{\lambda}_i$ is not available in the MSMD model. But the distribution of the intensity state vector M_i is available. There are two types of distribution for Markov switching models, the filtered distribution and the smoothed distribution. The expected λ_i can be calculated from the two distributions, and they provide two approximations for λ_i. The two approximations can be used to calculate the overdispersion test. Table 44 shows the the test statistics of both MSMD and ACD models for five stocks.

The test statistics of the ACD model for all five stocks are positive and the values are out of the 90% confidence interval, which shows that there is still overdispersion in the standardized durations. The test statistics of the MSMD model for all five stocks are negative. The overdispersion feature is gone. This suggests that the MSMD may explain the overdispersion in the data. But the statistics are still too small and out of the 90% confidence interval. The reason could be that expected λ_i, not the true value, is used to calculate the test statistics. The smoothed version can usually give a better approximation to the true value than the filtered version, and as one expects, the test statistics of the smoothed version are closer to the 90% confidence interval than the filter version. Statistics of both versions for the MSMD are closer to the confidence interval than the ACD.

From the above analysis, one can see that the overdispersion feature can be captured by the dynamic specification. There is no necessity to use flexible distributions.

5.6.2 Comparison with ACD

The MSMD model has been applied to real data and it can give a good description of the data when the number of intensity components is seven. From now on, I fix the number of components at seven and run a horse race between the MSMD(7) model and the ACD(1,1) model. I choose the ACD(1,1) model because it is the leading example of ACD models.

Table 45 and table 47 give the in sample fit and out of sample forecast results for low trading group. Table 48 and table 50 give the in sample fit and out of sample forecast results for high trading group.

From the tables, one can see the log likelihoods of MSMD(7) are higher than

ACD(1,1) for all twenty stocks. But comparison between non-nested models with different number of parameters is tricky. Usually a criteria should include a penalty term related to number of parameters. Standard methods are AIC and BIC. Here I choose BIC which puts more penalty on the number of parameters. Table 46 and table 49 show BIC comparison for all twenty stocks. One can see the MSMD model fit the data better than the ACD model.

For 1-step forecasting, the performance of both MSMD and ACD model is comparable. MSMD does slightly better forecasting for the high trading group, while ACD(1,1) can give a little more precise forecasting for the low trading group.

For 5-step and 20-step forecasting, the MSMD model dominates the ACD model for all 20 stocks. The forecasting gain by using MSMD is huge.

This is clear evidence that the ACD model can only capture short run dynamics, while the MSMD can capture longer horizon dynamics. An interesting observation is that the mean square prediction error of the MSMD model doesn't change much when the forecasting horizon changes.

It may be more fare to compare the MSMD model with a long memory ACD model. But as I discuss in last section, the fractional integrated ACD model doesn't have second order moment. It is not a long memory model in the usual sense. Another possible candidate is the long memory stochastic duration (LMSD) model of Deo et al. (2010). One problem with LMSD is that it does not allow iid exponential errors[1]. This contradicts with the discussion in section 2. So I don't consider the LMSD model.

[1]As Deo et al. (2010) reports that when applying the LMSD model to some data, their algorism doesn't converge when using iid exponential errors.

Chapter 6

Concluding Remarks

The intertrade duration is a natural measure of market liquidity and its variability is related to liquidity risk. In this dissertation, I propose a new model, the MSMD model to analyze intertrade durations. Compared to the conditional mean modeling of ACD models, my method focus on intensity modeling.

I first establish a mixture of exponentials representation for intertrade durations from general PP theory, then model the intensity process as a MSM process. I show the MSMD model has good properties. It can explain most of the stylized facts of financial durations.

Extensive empirical study shows the MSMD model can do good long horizon forecasting. my model could be used for the analysis of liquidity risk on financial markets. For example, the MSMD model has decomposed shocks into different frequency, and the high frequency shocks can be regarded as liquidity shocks. One can use bayesian method to update the probability distribution of the high frequency shocks thus get a measure of liquidity.

Another interesting direction for future work is to link durations, or intensities to volatilities. The MSM volatility model of Calvet and Fisher (2004) has a lot of similarities with the MSM duration model. An investigation of the link between MSM volatility and MSM intensity is going on. The driving force of the two could be the same, multifractal news.

Table 1: Twenty Stocks: Symbol and Company Name

Symbol	Company Name
AA	ALCOA
ABT	Abbott Laboratories
AXP	American Express Inc
BAC	Bank of America Corp
CSCO	Cisco Systems
DELL	Dell
DOW	Dow Chemical
F	Ford Motor
GE	General Electric Co.
IBM	International Business Machines
INTC	Intel Corporation
JNJ	Johnson & Johnson Inc
KO	The Coca-Cola Company
MCD	McDonald's Corp
MRK	Merck & Co.
MSFT	Microsoft
TXN	Texas Instruments
WFC	Wells Fargo
WMT	Wal-Mart
XRX	Xerox Corp

Table 2: Basic Statistics: Low Trading Group

Stock	Mean	Median	Max	Min	STD	Skew	Kurt	OD	N
AA	2.66	1.28	39.58	0.01	3.77	3.07	17.09	1.42	2989
AXP	2.22	1.17	42.33	0.05	2.93	3.13	19.35	1.32	10531
BAC	1.97	1.12	27.14	0.03	2.44	2.9	15.69	1.24	7939
DOW	1.96	1.11	37.59	0.02	2.49	3.6	28.03	1.27	6902
GE	2.03	1.1	27.13	0.06	2.56	2.72	13.85	1.26	14798
KO	1.82	1.14	26.31	0.05	2.06	2.49	12.47	1.13	15542
MCD	1.93	1.15	22.17	0.03	2.26	2.58	12.77	1.17	7441
TXN	2.56	1.15	55.41	0.02	3.7	3.39	23.54	1.44	4235
WFC	2.47	1.08	78.65	0.02	4.05	5.18	54.37	1.64	4047
XRX	2.56	1.15	55.41	0.02	3.7	3.39	23.54	1.44	4235

Notes: Skew is Skewness. Kurt is Kurtosis. OD is overdispersion which is equal to std/mean. N is the number of observations in the sampling period.

Table 3: Basic Statistics: High Trading Group

Stock	Mean	Median	Max	Min	STD	Skew	Kurt	OD	N
ABT	1.86	1.11	26.3	0.06	2.21	2.84	15.89	1.19	16929
CSCO	2.22	1.01	56.3	0.07	3.52	4.53	36.29	1.59	17963
DELL	2.13	1.02	76.83	0.09	3.4	5.15	48.3	1.6	24160
F	2.18	0.99	49.25	0.07	3.13	3.5	22.48	1.44	15562
IBM	1.75	1.06	35.87	0.12	2.03	3.01	19.16	1.16	31895
INTC	1.81	1.0	50.38	0.15	2.41	4.17	34.57	1.33	41957
JNJ	1.72	1.03	29.56	0.08	2.01	3.1	19.1	1.17	24208
MRK	1.61	0.98	24.66	0.18	1.78	2.95	17.31	1.11	54242
MSFT	2.01	1.01	53.68	0.11	2.94	4.43	37.43	1.46	29191
WMT	1.77	0.99	31.92	0.12	2.11	2.88	16.23	1.19	33899

Notes: Skew is Skewness. Kurt is Kurtosis. OD is overdispersion which is equal to std/mean. N is the number of observations.

Table 4: Model Estimation: AA

$\bar{k}$	3	4	5	6	7	8	9	10
m_0	1.47	1.47	1.47	1.47	1.47	1.47	1.47	1.47
	(0.02)	(0.08)	(0.02)	(0.02)	(0.02)	(0.29)	(0.02)	(0.01)
λ	0.83	0.65	1.23	2.35	4.47	1.08	16.15	30.71
	(0.06)	(0.43)	(0.24)	(0.40)	(0.81)	(2.53)	(4.54)	(6.40)
$\gamma_{\bar{k}}$	0.99	0.99	0.99	0.99	0.99	0.99	0.99	0.99
	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)
b	7.68	13.85	14.72	14.96	15.07	14.66	15.17	15.21
	(2.09)	(16.68)	(4.70)	(2.85)	(1.34)	(1.00)	(2.83)	(2.62)
$\ln L$	-5595.32	-5599.27	-5599.68	-5600.08	-5600.54	-5600.11	-5601.57	-5602.13
W	117.71	25.66	13.76	7.80	8.24	12.24	21.27	7.14
	(0.00)	(0.00)	(0.18)	(0.65)	(0.61)	(0.27)	(0.02)	(0.71)

Notes: This table shows estimation results for the Markov-Switching multi-fractal duration model with $\bar{k}$ intensity components. The model parameters are m_0, λ, $\gamma_{\bar{k}}$ and b. Standard errors appear in parentheses beneath estimated parameters. $\ln L$ is the maximized value of the log likelihood. W is White"s omnibus information matrix test of model specification adequacy, with marginal significance levels ("p-values") in parentheses. The sample period is 1 February 1993 through 26 February 1993. See text for details.

Table 5: ACD Model Estimation: AA

	ω	α_j			β_j			$\ln L$	W
ACD(1,1)	0.08	0.07			0.90			-5783.64	25.52
	(0.02)	(0.01)			(0.02)				(0.00)
ACD(1,2)	0.11	0.09			0.30	0.57		-5784.12	63.78
	(0.02)	(0.01)			(0.09)	(0.10)			(0.00)
ACD(2,2)	0.09	0.05	0.01		0.90	0.00		-5783.09	124.96
	(0.42)	(0.01)	(0.12)		(0.32)	(0.04)			(0.00)
ACD(3,3)	0.23	0.05	0.04	0.08	0.00	0.46	0.29	-5777.22	697.36
	(0.04)	(0.01)	(0.00)	(0.01)	(0.22)	(0.11)	(0.17)		(0.00)

Notes: This table shows estimation results for several ACD models: $\psi_i = \omega + \sum_j \alpha_j d_{i-j} + \sum_j \beta_j \psi_{i-j}$. Standard errors appear in parentheses beneath estimated parameters. $\ln L$ is the maximized value of the log likelihood. W is White"s omnibus information matrix test of model specification adequacy, with marginal significance levels ("p-values") in parentheses. The sample period is 1 February 1993 through 26 February 1993. See text for details.

Table 6: Model Estimation: ABT

$\bar{k}$	3	4	5	6	7	8	9	10
m_0	1.29	1.25	1.24	1.21	1.21	1.21	1.21	1.21
	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)
λ	0.80	0.85	1.08	0.95	0.53	0.66	0.54	0.45
	(0.14)	(0.07)	(0.04)	(0.04)	(0.02)	(0.02)	(0.02)	(0.03)
$\gamma_{\bar{k}}$	0.07	0.13	0.13	0.18	0.17	0.17	0.17	0.17
	(0.01)	(0.01)	(0.02)	(0.03)	(0.02)	(0.02)	(0.11)	(0.12)
b	13.89	8.53	7.28	4.81	5.24	4.75	4.75	4.75
	(1.09)	(1.56)	(0.72)	(0.53)	(0.22)	(0.39)	(1.42)	(2.76)
$\ln L$	-26342.34	-26329.71	-26326.88	-26324.39	-26328.48	-26325.21	-26325.87	-26326.55
W	39.00	16.39	15.01	21.18	17.70	17.48	22.55	17.39
	(0.00)	(0.09)	(0.13)	(0.02)	(0.06)	(0.06)	(0.01)	(0.07)

Notes: This table shows estimation results for the Markov-Switching multi-fractal duration model with $\bar{k}$ intensity components. The model parameters are m_0, λ, $\gamma_{\bar{k}}$ and b. Standard errors appear in parentheses beneath estimated parameters. $\ln L$ is the maximized value of the log likelihood. W is White"s omnibus information matrix test of model specification adequacy, with marginal significance levels ("p-values") in parentheses. The sample period is 1 February 1993 through 26 February 1993. See text for details.

Table 7: ACD Model Estimation: ABT

	ω	α_j			β_j			$\ln L$	W
ACD(1,1)	0.01	0.04			0.95			-26434.75	61.09
	(0.00)	(0.00)			(0.00)				(0.00)
ACD(1,2)	0.01	0.05			0.67	0.27		-26432.14	252.85
	(0.00)	(0.01)			(0.12)	(0.12)			(0.00)
ACD(2,2)	0.01	0.05	0.00		0.67	0.27		-26432.14	1691.30
	(0.15)	(0.07)	(0.44)		(5.78)	(5.32)			(0.00)
ACD(3,3)	0.01	0.06	0.00	0.00	0.73	0.02	0.19	-26430.09	3081.12
	(0.00)	(0.01)	(0.03)	(0.02)	(0.57)	(0.83)	(0.30)		(0.00)

Notes: This table shows estimation results for several ACD models: $\psi_i = \omega + \sum_j \alpha_j d_{i-j} + \sum_j \beta_j \psi_{i-j}$. Standard errors appear in parentheses beneath estimated parameters. $\ln L$ is the maximized value of the log likelihood. W is White"s omnibus information matrix test of model specification adequacy, with marginal significance levels ("p-values") in parentheses. The sample period is 1 February 1993 through 26 February 1993. See text for details.

Table 8: Model Estimation: AXP

$\bar{k}$	3	4	5	6	7	8	9	10
m_0	1.38	1.34	1.30	1.31	1.27	1.27	1.27	1.27
	(0.48)	(0.02)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)
λ	0.76	0.95	0.87	1.61	0.81	1.12	1.54	1.21
	(1.12)	(0.26)	(0.07)	(0.15)	(0.04)	(0.09)	(0.41)	(0.03)
$\gamma_{\bar{k}}$	0.79	0.94	0.99	0.95	0.99	0.99	0.99	0.99
	(0.10)	(0.28)	(0.00)	(0.01)	(0.00)	(0.00)	(0.00)	(0.00)
b	30.70	29.10	12.69	20.68	10.24	10.26	10.26	10.26
	(0.59)	(4.69)	(1.62)	(0.18)	(1.08)	(0.57)	(1.18)	(1.11)
$\ln L$	-17888.32	-17844.68	-17835.35	-17843.72	-17829.99	-17829.99	-17830.28	-17830.28
W	29.73	28.11	7.64	19.56	10.16	7.18	8.14	7.22
	(0.00)	(0.00)	(0.66)	(0.03)	(0.43)	(0.71)	(0.61)	(0.70)

Notes: This table shows estimation results for the Markov-Switching multi-fractal duration model with $\bar{k}$ intensity components. The model parameters are m_0, λ, $\gamma_{\bar{k}}$ and b. Standard errors appear in parentheses beneath estimated parameters. $\ln L$ is the maximized value of the log likelihood. W is White"s omnibus information matrix test of model specification adequacy, with marginal significance levels ("p-values") in parentheses. The sample period is 1 February 1993 through 26 February 1993. See text for details.

Table 9: ACD Model Estimation: AXP

	ω	α_j			β_j			$\ln L$	W
ACD(1,1)	0.00	0.03			0.97			-18042.24	45.77
	(0.00)	(0.00)			(0.00)				(0.00)
ACD(1,2)	0.01	0.06			0.26	0.68		-18033.72	184.85
	(0.00)	(0.00)			(0.03)	(0.03)			(0.00)
ACD(2,2)	0.01	0.06	0.00		0.26	0.68		-18033.72	537.16
	(0.00)	(0.01)	(0.01)		(0.11)	(0.10)			(0.00)
ACD(3,3)	0.01	0.06	0.01	0.00	0.00	0.69	0.23	-18033.78	1848.12
	(0.00)	(0.01)	(0.01)	(0.01)	(0.25)	(0.10)	(0.21)		(0.00)

Note: This table shows estimation results for several ACD models: $\psi_i = \omega + \sum_j \alpha_j d_{i-j} + \sum_j \beta_j \psi_{i-j}$. Standard errors appear in parentheses beneath estimated parameters. $\ln L$ is the maximized value of the log likelihood. W is White"s omnibus information matrix test of model specification adequacy, with marginal significance levels ("p-values") in parentheses. The sample period is 1 February 1993 through 26 February 1993. See text for details.

Table 10: Model Estimation: BAC

$\bar{k}$	3	4	5	6	7	8	9	10
m_0	1.36	1.31	1.27	1.25	1.23	1.25	1.23	1.19
	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)
λ	1.16	1.09	1.02	0.93	0.89	0.60	0.61	0.79
	(0.06)	(0.05)	(0.03)	(0.05)	(0.11)	(0.04)	(0.03)	(0.04)
$\gamma_{\bar{k}}$	0.42	0.86	0.99	0.98	0.99	0.98	0.99	0.99
	(0.06)	(0.25)	(0.00)	(0.02)	(0.00)	(0.06)	(0.00)	(0.00)
b	20.43	11.58	7.76	4.91	3.80	5.05	3.84	2.34
	(0.24)	(2.07)	(0.43)	(0.68)	(0.33)	(3.00)	(0.25)	(0.14)
$\ln L$	-12917.76	-12910.83	-12910.44	-12910.30	-12909.57	-12911.86	-12911.17	-12910.28
W	20.09	11.32	13.26	8.07	11.33	11.64	14.32	17.47
	(0.03)	(0.33)	(0.21)	(0.62)	(0.33)	(0.31)	(0.16)	(0.06)

Notes: This table shows estimation results for the Markov-Switching multi-fractal duration model with $\bar{k}$ intensity components. The model parameters are m_0, λ, $\gamma_{\bar{k}}$ and b. Standard errors appear in parentheses beneath estimated parameters. $\ln L$ is the maximized value of the log likelihood. W is White"s omnibus information matrix test of model specification adequacy, with marginal significance levels ("p-values") in parentheses. The sample period is 1 February 1993 through 26 February 1993. See text for details.

Table 11: ACD Model Estimation: BAC

	ω	α_j			β_j			$\ln L$	W
ACD(1,1)	0.04	0.06			0.92			-13073.60	66.04
	(0.01)	(0.00)			(0.01)				(0.00)
ACD(1,2)	0.06	0.08			0.45	0.44		-13067.01	309.69
	(0.01)	(0.01)			(0.09)	(0.09)			(0.00)
ACD(2,2)	0.06	0.08	0.00		0.45	0.44		-13067.01	846.32
	(0.47)	(0.02)	(0.11)		(0.45)	(0.41)			(0.00)
ACD(3,3)	0.06	0.09	0.00	0.00	0.55	0.04	0.30	-13063.56	1540.62
	(0.01)	(0.01)	(0.02)	(0.03)	(0.22)	(0.37)	(0.23)		(0.00)

Notes: This table shows estimation results for several ACD models: $\psi_i = \omega + \sum_j \alpha_j d_{i-j} + \sum_j \beta_j \psi_{i-j}$. Standard errors appear in parentheses beneath estimated parameters. $\ln L$ is the maximized value of the log likelihood. W is White"s omnibus information matrix test of model specification adequacy, with marginal significance levels ("p-values") in parentheses. The sample period is 1 February 1993 through 26 February 1993. See text for details.

Table 12: Model Estimation: CSCO

$\bar{k}$	3	4	5	6	7	8	9	10
m_0	1.42	1.37	1.34	1.40	1.29	1.41	1.40	1.40
	(0.00)	(0.01)	(0.08)	(0.01)	(0.02)	(0.01)	(0.01)	(0.01)
λ	0.73	0.82	0.84	2.74	0.83	7.88	5.41	3.86
	(0.03)	(0.04)	(2.77)	(0.17)	(0.04)	(0.47)	(0.89)	(0.39)
$\gamma_{\bar{k}}$	0.14	0.17	0.19	0.18	0.20	0.18	0.18	0.18
	(0.01)	(0.02)	(0.50)	(0.00)	(0.02)	(0.01)	(0.02)	(0.03)
b	2.65	2.47	2.02	3.73	1.68	3.81	3.55	3.52
	(0.80)	(0.34)	(1.88)	(0.32)	(0.08)	(0.20)	(0.33)	(0.38)
$\ln L$	-28592.08	-28588.80	-28595.00	-28608.54	-28600.31	-28610.69	-28607.76	-28606.87
W	45.80	43.11	47.90	33.29	36.75	32.61	23.51	29.32
	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.01)	(0.00)

Notes: This table shows estimation results for the Markov-Switching multi-fractal duration model with $\bar{k}$ intensity components. The model parameters are m_0, λ, $\gamma_{\bar{k}}$ and b. Standard errors appear in parentheses beneath estimated parameters. $\ln L$ is the maximized value of the log likelihood. W is White"s omnibus information matrix test of model specification adequacy, with marginal significance levels ("p-values") in parentheses. The sample period is 1 February 1993 through 26 February 1993. See text for details.

Table 13: ACD Model Estimation: CSCO

	ω	α_j			β_j			$\ln L$	W
ACD(1,1)	0.09	0.22			0.75			-29098.61	224.24
	(0.00)	(0.00)			(0.00)				(0.00)
ACD(1,2)	0.09	0.22			0.75	0.00		-29098.93	495.91
	(0.03)	(0.06)			(0.04)	(0.03)			(0.00)
ACD(2,2)	0.14	0.22	0.12		0.23	0.38		-29097.89	2166.04
	(0.03)	(0.01)	(0.06)		(0.30)	(0.23)			(0.00)
ACD(3,3)	0.10	0.21	0.05	0.00	0.64	0.00	0.07	-29095.49	2622.01
	(0.02)	(0.01)	(0.04)	(0.05)	(0.24)	(0.34)	(0.16)		(0.00)

Notes: This table shows estimation results for several ACD models: $\psi_i = \omega + \sum_j \alpha_j d_{i-j} + \sum_j \beta_j \psi_{i-j}$. Standard errors appear in parentheses beneath estimated parameters. $\ln L$ is the maximized value of the log likelihood. W is White"s omnibus information matrix test of model specification adequacy, with marginal significance levels ("p-values") in parentheses. The sample period is 1 February 1993 through 26 February 1993. See text for details.

Table 14: Model Estimation: DELL

$\bar{k}$	3	4	5	6	7	8	9	10
m_0	1.40	1.36	1.37	1.32	1.32	1.32	1.33	1.37
	(0.01)	(0.01)	(0.01)	(0.01)	(0.02)	(0.00)	(0.01)	(0.04)
λ	0.67	0.80	1.26	1.18	0.91	1.35	3.98	1.26
	(0.02)	(0.04)	(0.09)	(0.04)	(0.07)	(0.10)	(0.28)	(0.06)
$\gamma_{\bar{k}}$	0.08	0.09	0.10	0.11	0.10	0.11	0.11	0.10
	(0.02)	(0.01)	(0.01)	(0.01)	(0.01)	(0.00)	(0.04)	(0.01)
b	4.18	3.45	3.80	2.75	2.59	2.76	3.02	3.91
	(0.98)	(0.66)	(0.55)	(0.15)	(0.30)	(0.07)	(0.39)	(0.19)
$\ln L$	-37957.67	-37903.42	-37909.02	-37899.56	-37898.92	-37900.40	-37904.81	-37910.09
W	63.21	35.57	35.94	11.09	7.37	5.76	11.45	32.59
	(0.00)	(0.00)	(0.00)	(0.35)	(0.69)	(0.83)	(0.32)	(0.00)

Notes: This table shows estimation results for the Markov-Switching multi-fractal duration model with $\bar{k}$ intensity components. The model parameters are m_0, λ, $\gamma_{\bar{k}}$ and b. Standard errors appear in parentheses beneath estimated parameters. $\ln L$ is the maximized value of the log likelihood. W is White"s omnibus information matrix test of model specification adequacy, with marginal significance levels ("p-values") in parentheses. The sample period is 1 February 1993 through 26 February 1993. See text for details.

Table 15: ACD Model Estimation: DELL

	ω	α_j			β_j			$\ln L$	W
ACD(1,1)	0.03	0.15			0.84			-38240.47	196.25
	(0.00)	(0.00)			(0.00)				(0.00)
ACD(1,2)	0.04	0.17			0.71	0.11		-38237.83	760.43
	(0.00)	(0.01)			(0.05)	(0.04)			(0.00)
ACD(2,2)	0.04	0.17	0.00		0.71	0.11		-38237.83	2594.74
	(0.01)	(3.11)	(0.28)		(2.37)	(1.35)			(0.00)
ACD(3,3)	0.04	0.17	0.00	0.00	0.74	0.02	0.05	-38236.88	3355.24
	(0.01)	(0.01)	(0.15)	(0.06)	(0.85)	(1.05)	(0.11)		(0.00)

Notes: This table shows estimation results for several ACD models: $\psi_i = \omega + \sum_j \alpha_j d_{i-j} + \sum_j \beta_j \psi_{i-j}$. Standard errors appear in parentheses beneath estimated parameters. $\ln L$ is the maximized value of the log likelihood. W is White"s omnibus information matrix test of model specification adequacy, with marginal significance levels ("p-values") in parentheses. The sample period is 1 February 1993 through 26 February 1993. See text for details.

Table 16: Model Estimation: DOW

$\bar{k}$	3	4	5	6	7	8	9	10
m_0	1.29	1.28	1.25	1.25	1.22	1.20	1.18	1.20
	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)	(0.02)	(0.02)	(0.07)
λ	0.72	0.64	0.64	0.51	0.90	0.88	0.60	0.92
	(0.03)	(0.02)	(0.02)	(0.02)	(0.07)	(0.11)	(0.08)	(0.18)
$\gamma_{\bar{k}}$	0.59	0.87	0.99	0.99	0.99	0.99	0.99	0.99
	(0.10)	(0.05)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)
b	16.52	15.47	9.26	9.38	6.89	4.59	3.50	4.60
	(0.41)	(0.44)	(1.37)	(1.18)	(0.27)	(0.86)	(0.38)	(0.81)
$\ln L$	-11159.40	-11149.76	-11146.89	-11147.64	-11145.02	-11145.18	-11145.33	-11145.39
W	13.80	10.04	3.01	4.40	5.92	2.96	19.43	4.32
	(0.18)	(0.44)	(0.98)	(0.93)	(0.82)	(0.98)	(0.04)	(0.93)

Notes: This table shows estimation results for the Markov-Switching multi-fractal duration model with $\bar{k}$ intensity components. The model parameters are m_0, λ, $\gamma_{\bar{k}}$ and b. Standard errors appear in parentheses beneath estimated parameters. $\ln L$ is the maximized value of the log likelihood. W is White"s omnibus information matrix test of model specification adequacy, with marginal significance levels ("p-values") in parentheses. The sample period is 1 February 1993 through 26 February 1993. See text for details.

Table 17: ACD Model Estimation: DOW

	ω	α_j			β_j			$\ln L$	W
ACD(1,1)	0.01	0.04			0.95			-11238.01	27.24
	(0.00)	(0.00)			(0.01)				(0.00)
ACD(1,2)	0.02	0.05			0.75	0.19		-11237.46	64.51
	(0.00)	(0.01)			(0.13)	(0.13)			(0.00)
ACD(2,2)	0.03	0.03	0.04		0.00	0.91		-11237.33	62.96
	(0.01)	(0.01)	(0.01)		(0.03)	(0.04)			(0.00)
ACD(3,3)	0.04	0.04	0.04	0.03	0.00	0.00	0.87	-11237.06	253.02
	(0.01)	(0.01)	(0.01)	(0.01)	(0.06)	(0.06)	(0.05)		(0.00)

Notes: This table shows estimation results for several ACD models: $\psi_i = \omega + \sum_j \alpha_j d_{i-j} + \sum_j \beta_j \psi_{i-j}$. Standard errors appear in parentheses beneath estimated parameters. $\ln L$ is the maximized value of the log likelihood. W is White"s omnibus information matrix test of model specification adequacy, with marginal significance levels ("p-values") in parentheses. The sample period is 1 February 1993 through 26 February 1993. See text for details.

Table 18: Model Estimation: F

$\bar{k}$	3	4	5	6	7	8	9	10
m_0	1.43	1.37	1.35	1.32	1.30	1.30	1.27	1.27
	(0.01)	(0.01)	(0.04)	(0.01)	(0.01)	(0.04)	(0.01)	(0.01)
λ	1.00	1.00	0.94	0.91	1.12	0.86	1.42	1.96
	(0.09)	(0.32)	(1.16)	(0.06)	(0.02)	(0.20)	(0.10)	(0.23)
$\gamma_{\bar{k}}$	0.76	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	(0.03)	(0.02)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)
b	19.68	13.82	14.82	8.55	6.85	6.84	5.71	5.72
	(2.07)	(2.68)	(2.69)	(1.00)	(0.33)	(3.08)	(0.27)	(0.80)
$\ln L$	-25684.19	-25673.06	-25650.57	-25640.26	-25638.54	-25638.98	-25635.94	-25636.67
W	55.32	16.75	22.23	11.29	15.59	16.38	18.44	17.15
	(0.00)	(0.08)	(0.01)	(0.34)	(0.11)	(0.09)	(0.05)	(0.07)

Notes: This table shows estimation results for the Markov-Switching multi-fractal duration model with $\bar{k}$ intensity components. The model parameters are m_0, λ, $\gamma_{\bar{k}}$ and b. Standard errors appear in parentheses beneath estimated parameters. $\ln L$ is the maximized value of the log likelihood. W is White"s omnibus information matrix test of model specification adequacy, with marginal significance levels ("p-values") in parentheses. The sample period is 1 February 1993 through 26 February 1993. See text for details.

Table 19: ACD Model Estimation: F

	ω	α_j			β_j			$\ln L$	W
ACD(1,1)	0.01	0.04			0.95			-26227.00	39.22
	(0.00)	(0.00)			(0.00)				(0.00)
ACD(1,2)	0.01	0.06			0.52	0.41		-26215.05	410.60
	(0.00)	(0.01)			(0.04)	(0.05)			(0.00)
ACD(2,2)	0.01	0.06	0.00		0.52	0.41		-26215.05	2344.84
	(0.01)	(0.02)	(0.10)		(0.15)	(0.22)			(0.00)
ACD(3,3)	0.01	0.08	0.00	0.00	0.46	0.00	0.45	-26195.72	1465.99
	(0.01)	(0.02)	(0.04)	(0.02)	(0.37)	(0.18)	(0.43)		(0.00)

Notes: This table shows estimation results for several ACD models: $\psi_i = \omega + \sum_j \alpha_j d_{i-j} + \sum_j \beta_j \psi_{i-j}$. Standard errors appear in parentheses beneath estimated parameters. $\ln L$ is the maximized value of the log likelihood. W is White"s omnibus information matrix test of model specification adequacy, with marginal significance levels ("p-values") in parentheses. The sample period is 1 February 1993 through 26 February 1993. See text for details.

Table 20: Model Estimation: GE

$\bar{k}$	3	4	5	6	7	8	9	10
m_0	1.36	1.30	1.27	1.30	1.27	1.25	1.27	1.27
	(0.02)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)	(0.15)	(0.01)
λ	0.90	0.85	0.80	1.75	0.51	0.56	0.96	2.30
	(0.09)	(0.03)	(0.50)	(0.10)	(0.03)	(0.01)	(0.17)	(0.16)
$\gamma_{\bar{k}}$	0.99	0.99	0.99	0.99	0.99	0.99	0.99	0.99
	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)
b	30.54	8.94	5.61	9.66	6.56	5.01	6.57	6.60
	(3.12)	(0.54)	(0.41)	(0.65)	(0.46)	(0.69)	(2.79)	(0.72)
$\ln L$	-24406.93	-24404.04	-24406.14	-24409.90	-24410.93	-24411.21	-24410.80	-24411.90
W	56.46	41.13	26.16	22.58	9.20	39.44	12.12	9.61
	(0.00)	(0.00)	(0.00)	(0.01)	(0.51)	(0.00)	(0.28)	(0.48)

Notes: This table estimation results for the Markov-Switching multi-fractal duration model with $\bar{k}$ intensity components. The model parameters are m_0, λ, $\gamma_{\bar{k}}$ and b. Standard errors appear in parentheses beneath estimated parameters. $\ln L$ is the maximized value of the log likelihood. W is White"s omnibus information matrix test of model specification adequacy, with marginal significance levels ("p-values") in parentheses. The sample period is 1 February 1993 through 26 February 1993. See text for details.

Table 21: ACD Model Estimation: GE

	ω	α_j			β_j			$\ln L$	W
ACD(1,1)	0.02	0.04			0.96			-24766.13	83.96
	(0.00)	(0.00)			(0.00)				(0.00)
ACD(1,2)	0.03	0.06			0.31	0.62		-24754.82	274.33
	(0.01)	(0.01)			(0.04)	(0.04)			(0.00)
ACD(2,2)	0.03	0.06	0.00		0.31	0.62		-24754.82	956.88
	(0.04)	(0.16)	(0.21)		(1.87)	(1.82)			(0.00)
ACD(3,3)	0.03	0.07	0.00	0.00	0.46	0.00	0.46	-24749.80	1817.35
	(0.01)	(0.01)	(0.02)	(0.01)	(0.25)	(0.24)	(0.25)		(0.00)

Notes: This table shows estimation results for several ACD models: $\psi_i = \omega + \sum_j \alpha_j d_{i-j} + \sum_j \beta_j \psi_{i-j}$. Standard errors appear in parentheses beneath estimated parameters. $\ln L$ is the maximized value of the log likelihood. W is White"s omnibus information matrix test of model specification adequacy, with marginal significance levels ("p-values") in parentheses. The sample period is 1 February 1993 through 26 February 1993. See text for details.

Table 22: Model Estimation: IBM

$\bar{k}$	3	4	5	6	7	8	9	10
m_0	1.27	1.24	1.21	1.21	1.18	1.18	1.18	1.18
	(0.01)	(0.01)	(0.03)	(0.12)	(0.01)	(0.01)	(0.01)	(0.01)
λ	0.82	0.87	0.84	1.06	0.77	0.65	0.56	0.67
	(0.02)	(0.03)	(0.29)	(0.33)	(0.03)	(0.03)	(0.02)	(0.02)
$\gamma_{\bar{k}}$	0.05	0.05	0.05	0.05	0.06	0.07	0.07	0.07
	(0.00)	(0.00)	(0.01)	(0.04)	(0.02)	(0.01)	(0.01)	(0.01)
b	10.12	5.93	3.90	4.00	2.85	2.91	2.94	2.92
	(0.99)	(0.56)	(1.91)	(3.64)	(0.31)	(0.32)	(0.27)	(0.05)
$\ln L$	-47744.73	-47710.78	-47696.20	-47697.22	-47695.07	-47695.88	-47696.65	-47696.11
W	40.16	16.37	5.86	9.93	3.42	4.02	3.46	4.09
	(0.00)	(0.09)	(0.83)	(0.45)	(0.97)	(0.95)	(0.97)	(0.94)

Notes: This table shows estimation results for the Markov-Switching multi-fractal duration model with $\bar{k}$ intensity components. The model parameters are m_0, λ, $\gamma_{\bar{k}}$ and b. Standard errors appear in parentheses beneath estimated parameters. $\ln L$ is the maximized value of the log likelihood. W is White"s omnibus information matrix test of model specification adequacy, with marginal significance levels ("p-values") in parentheses. The sample period is 1 February 1993 through 26 February 1993. See text for details.

Table 23: ACD Model Estimation: IBM

	ω	α_j			β_j			$\ln L$	W
ACD(1,1)	0.01	0.05			0.95			-47794.69	164.63
	(0.00)	(0.00)			(0.00)				(0.00)
ACD(1,2)	0.01	0.06			0.56	0.37		-47788.35	721.45
	(0.00)	(0.00)			(0.00)	(0.00)			(0.00)
ACD(2,2)	0.01	0.06	0.00		0.56	0.37		-47788.35	3483.86
	(0.01)	(0.02)	(0.01)		(0.29)	(0.25)			(0.00)
ACD(3,3)	0.01	0.07	0.00	0.00	0.56	0.25	0.12	-47787.06	4477.01
	(0.01)	(0.01)	(0.02)	(0.01)	(0.25)	(0.24)	(0.25)		(0.00)

Notes: This table estimation results for several ACD models: $\psi_i = \omega + \sum_j \alpha_j d_{i-j} + \sum_j \beta_j \psi_{i-j}$. Standard errors appear in parentheses beneath estimated parameters. $\ln L$ is the maximized value of the log likelihood. W is White"s omnibus information matrix test of model specification adequacy, with marginal significance levels ("p-values") in parentheses. The sample period is 1 February 1993 through 26 February 1993. See text for details.

Table 24: Model Estimation: INTC

$\bar{k}$	3	4	5	6	7	8	9	10
m_0	1.32	1.29	1.30	1.24	1.22	1.21	1.19	1.28
	(0.00)	(0.01)	(0.01)	(0.01)	(0.00)	(0.01)	(0.01)	(0.01)
λ	0.76	0.71	1.00	0.74	0.76	0.76	0.77	0.96
	(0.02)	(0.01)	(0.03)	(0.04)	(0.04)	(0.03)	(0.04)	(0.03)
$\gamma_{\bar{k}}$	0.07	0.08	0.09	0.09	0.10	0.10	0.10	0.09
	(0.01)	(0.01)	(0.00)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)
b	3.86	3.42	3.97	2.21	1.92	1.76	1.65	3.57
	(0.48)	(0.27)	(0.37)	(0.19)	(0.06)	(0.10)	(0.08)	(0.28)
$\ln L$	-61934.00	-61888.79	-61895.91	-61891.47	-61893.35	-61895.68	-61897.21	-61895.68
W	76.93	83.01	83.32	64.88	67.82	130.32	96.50	46.78
	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)

Notes: This table shows estimation results for the Markov-Switching multi-fractal duration model with $\bar{k}$ intensity components. The model parameters are m_0, λ, $\gamma_{\bar{k}}$ and b. Standard errors appear in parentheses beneath estimated parameters. $\ln L$ is the maximized value of the log likelihood. W is White"s omnibus information matrix test of model specification adequacy, with marginal significance levels ("p-values") in parentheses. The sample period is 1 February 1993 through 26 February 1993. See text for details.

Table 25: ACD Model Estimation: INTC

	ω	α_j			β_j			$\ln L$	W
ACD(1,1)	0.04	0.12			0.86			-62030.70	356.05
	(0.00)	(0.00)			(0.00)				(0.00)
ACD(1,2)	0.04	0.13			0.81	0.04		-62030.39	659.30
	(0.01)	(0.02)			(0.02)	(0.03)			(0.00)
ACD(2,2)	0.07	0.12	0.08		0.18	0.58		-62030.78	8036.49
	(0.03)	(0.01)	(0.11)		(0.92)	(0.79)			(0.00)
ACD(3,3)	0.05	0.13	0.02	0.00	0.72	0.00	0.11	-62027.14	5628.61
	(0.02)	(0.01)	(0.03)	(0.01)	(0.28)	(0.54)	(0.11)		(0.00)

Note: This table shows estimation results for several ACD models: $\psi_i = \omega + \sum_j \alpha_j d_{i-j} + \sum_j \beta_j \psi_{i-j}$. Standard errors appear in parentheses beneath estimated parameters. $\ln L$ is the maximized value of the log likelihood. W is White"s omnibus information matrix test of model specification adequacy, with marginal significance levels ("p-values") in parentheses. The sample period is 1 February 1993 through 26 February 1993. See text for details.

Table 26: Model Estimation: JNJ

$\bar{k}$	3	4	5	6	7	8	9	10
m_0	1.23	1.18	1.18	1.18	1.18	1.18	1.18	1.18
	(0.00)	(0.01)	(0.01)	(0.01)	(0.03)	(0.01)	(0.01)	(0.05)
λ	0.75	0.71	0.60	0.51	0.90	0.53	0.94	0.55
	(0.01)	(0.02)	(0.01)	(0.01)	(0.04)	(0.10)	(0.04)	(0.08)
$\gamma_{\bar{k}}$	0.08	0.12	0.12	0.12	0.12	0.12	0.12	0.12
	(0.02)	(0.03)	(0.01)	(0.04)	(0.00)	(0.01)	(0.04)	(0.02)
b	21.23	10.25	10.25	10.24	10.33	10.27	10.32	10.28
	(2.34)	(1.01)	(0.40)	(1.64)	(0.83)	(1.76)	(4.70)	(1.12)
$\ln L$	-35902.36	-35890.60	-35891.17	-35891.85	-35891.62	-35891.91	-35891.78	-35892.00
W	9.45	17.73	11.24	11.02	9.75	11.16	9.27	9.24
	(0.49)	(0.06)	(0.34)	(0.36)	(0.46)	(0.35)	(0.51)	(0.51)

Notes: We report estimation results for the Markov-Switching multi-fractal duration model with $\bar{k}$ intensity components. The model parameters are m_0, λ, $\gamma_{\bar{k}}$ and b. Standard errors appear in parentheses beneath estimated parameters. $\ln L$ is the maximized value of the log likelihood. W is White"s omnibus information matrix test of model specification adequacy, with marginal significance levels ("p-values") in parentheses. The sample period is 1 February 1993 through 26 February 1993. See text for details.

Table 27: ACD Model Estimation: JNJ

	ω	α_j			β_j			$\ln L$	W
ACD(1,1)	0.00	0.02			0.97			-35961.84	89.46
	(0.00)	(0.00)			(0.00)				(0.00)
ACD(1,2)	0.00	0.03			0.56	0.41		-35958.18	358.62
	(0.00)	(0.00)			(0.03)	(0.03)			(0.00)
ACD(2,2)	0.00	0.03	0.00		0.56	0.41		-35958.18	5051.19
	(0.01)	(0.02)	(0.02)		(0.85)	(0.80)			(0.00)
ACD(3,3)	0.01	0.04	0.00	0.00	0.63	0.00	0.32	-35954.66	5628.00
	(0.00)	(0.00)	(0.01)	(0.02)	(0.54)	(1.00)	(0.52)		(0.00)

Notes: This table shows estimation results for several ACD models: $\psi_i = \omega + \sum_j \alpha_j d_{i-j} + \sum_j \beta_j \psi_{i-j}$. Standard errors appear in parentheses beneath estimated parameters. $\ln L$ is the maximized value of the log likelihood. W is White"s omnibus information matrix test of model specification adequacy, with marginal significance levels ("p-values") in parentheses. The sample period is 1 February 1993 through 26 February 1993. See text for details.

Table 28: Model Estimation: KO

$\bar{k}$	3	4	5	6	7	8	9	10
m_0	1.24	1.21	1.20	1.21	1.18	1.21	1.18	1.21
	(0.01)	(0.52)	(0.01)	(0.34)	(0.01)	(0.38)	(0.01)	(0.01)
λ	0.78	0.76	0.66	0.55	0.56	0.57	0.58	0.60
	(0.02)	(3.80)	(0.02)	(0.35)	(0.09)	(0.26)	(0.02)	(0.02)
$\gamma_{\bar{k}}$	0.33	0.65	0.72	0.75	0.85	0.75	0.86	0.75
	(0.05)	(0.49)	(0.09)	(0.57)	(0.02)	(0.26)	(0.08)	(0.41)
b	12.02	7.21	6.78	7.17	4.69	7.23	4.78	7.23
	(2.21)	(2.89)	(1.11)	(8.03)	(0.64)	(3.19)	(0.47)	(1.23)
$\ln L$	-24426.66	-24423.53	-24422.53	-24423.63	-24424.36	-24423.97	-24424.81	-24424.15
W	25.44	23.20	12.45	12.75	13.51	11.88	13.47	11.27
	(0.00)	(0.01)	(0.26)	(0.24)	(0.20)	(0.29)	(0.20)	(0.34)

Notes: This table shows estimation results for the Markov-Switching multi-fractal duration model with $\bar{k}$ intensity components. The model parameters are m_0, λ, $\gamma_{\bar{k}}$ and b. Standard errors appear in parentheses beneath estimated parameters. $\ln L$ is the maximized value of the log likelihood. W is White"s omnibus information matrix test of model specification adequacy, with marginal significance levels ("p-values") in parentheses. The sample period is 1 February 1993 through 26 February 1993. See text for details.

Table 29: ACD Model Estimation: KO

	ω	α_j			β_j			$\ln L$	W
ACD(1,1)	0.04	0.04			0.93			-24534.83	148.40
	(0.01)	(0.00)			(0.01)				(0.00)
ACD(1,2)	0.05	0.06			0.42	0.49		-24529.34	381.79
	(0.02)	(0.01)			(0.07)	(0.09)			(0.00)
ACD(2,2)	0.05	0.06	0.00		0.42	0.49		-24529.34	1480.63
	(0.31)	(0.31)	(0.02)		(0.82)	(0.38)			(0.00)
ACD(3,3)	0.05	0.06	0.00	0.00	0.41	0.47	0.02	-24529.27	451.08
	(0.01)	(0.01)	(0.01)	(0.01)	(0.03)	(0.21)	(0.07)		(0.00)

Notes: This table shows estimation results for several ACD models: $\psi_i = \omega + \sum_j \alpha_j d_{i-j} + \sum_j \beta_j \psi_{i-j}$. Standard errors appear in parentheses beneath estimated parameters. $\ln L$ is the maximized value of the log likelihood. W is White"s omnibus information matrix test of model specification adequacy, with marginal significance levels ("p-values") in parentheses. The sample period is 1 February 1993 through 26 February 1993. See text for details.

Table 30: Model Estimation: MCD

$\bar{k}$	3	4	5	6	7	8	9	10
m_0	1.29	1.29	1.24	1.24	1.24	1.24	1.24	1.24
	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)	(0.02)	(0.01)
λ	0.79	0.61	0.62	0.50	1.08	0.53	1.90	0.93
	(0.03)	(0.02)	(0.01)	(0.01)	(0.03)	(0.02)	(0.09)	(0.05)
$\gamma_{\bar{k}}$	0.99	0.99	0.99	0.99	0.99	0.99	0.99	0.99
	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)
b	34.56	35.43	10.32	10.37	10.46	10.40	10.48	10.43
	(5.50)	(5.54)	(0.53)	(0.45)	(0.91)	(0.95)	(2.34)	(0.31)
$\ln L$	-12094.83	-12095.83	-12097.68	-12098.48	-12098.37	-12098.59	-12099.32	-12098.47
W	17.27	12.02	16.77	14.35	17.55	15.67	17.85	18.34
	(0.07)	(0.28)	(0.08)	(0.16)	(0.06)	(0.11)	(0.06)	(0.05)

Notes: This table shows estimation results for the Markov-Switching multi-fractal duration model with $\bar{k}$ intensity components. The model parameters are m_0, λ, $\gamma_{\bar{k}}$ and b. Standard errors appear in parentheses beneath estimated parameters. $\ln L$ is the maximized value of the log likelihood. W is White"s omnibus information matrix test of model specification adequacy, with marginal significance levels ("p-values") in parentheses. The sample period is 1 February 1993 through 26 February 1993. See text for details.

Table 31: ACD Model Estimation: MCD

	ω	α_j			β_j			$\ln L$	W
ACD(1,1)	0.04	0.04			0.94			-12201.87	60.36
	(0.01)	(0.01)			(0.01)				(0.00)
ACD(1,2)	0.05	0.06			0.38	0.54		-12198.10	210.55
	(0.02)	(0.01)			(0.07)	(0.07)			(0.00)
ACD(2,2)	0.05	0.06	0.00		0.38	0.54		-12198.10	849.68
	(0.02)	(0.01)	(0.03)		(0.26)	(0.28)			(0.00)
ACD(3,3)	0.08	0.06	0.01	0.02	0.00	0.39	0.48	-12196.77	1160.88
	(0.05)	(0.01)	(0.05)	(0.02)	(0.80)	(0.36)	(0.45)		(0.00)

Notes: This table shows estimation results for several ACD models: $\psi_i = \omega + \sum_j \alpha_j d_{i-j} + \sum_j \beta_j \psi_{i-j}$. Standard errors appear in parentheses beneath estimated parameters. $\ln L$ is the maximized value of the log likelihood. W is White"s omnibus information matrix test of model specification adequacy, with marginal significance levels ("p-values") in parentheses. The sample period is 1 February 1993 through 26 February 1993. See text for details.

Table 32: Model Estimation: MRK

$\bar{k}$	3	4	5	6	7	8	9	10
m_0	1.22	1.20	1.17	1.17	1.17	1.14	1.17	1.17
	(0.04)	(0.01)	(0.00)	(0.00)	(0.00)	(0.01)	(0.02)	(0.00)
λ	0.71	0.83	0.75	0.64	0.78	0.59	0.57	0.68
	(0.10)	(0.05)	(0.01)	(0.01)	(0.01)	(0.01)	(0.02)	(0.01)
$\gamma_{\bar{k}}$	0.03	0.03	0.04	0.04	0.04	0.07	0.04	0.04
	(0.04)	(0.00)	(0.01)	(0.00)	(0.01)	(0.01)	(0.01)	(0.01)
b	16.59	8.12	5.97	5.94	5.96	3.74	5.94	5.95
	(1.32)	(0.21)	(0.83)	(0.67)	(0.66)	(0.26)	(1.06)	(0.79)
$\ln L$	-77426.73	-77402.21	-77374.72	-77375.22	-77375.26	-77375.65	-77375.91	-77375.71
W	29.02	16.47	14.41	15.28	7.02	16.46	11.20	13.32
	(0.00)	(0.09)	(0.16)	(0.12)	(0.72)	(0.09)	(0.34)	(0.21)

Notes: This table shows estimation results for the Markov-Switching multi-fractal duration model with $\bar{k}$ intensity components. The model parameters are m_0, λ, $\gamma_{\bar{k}}$ and b. Standard errors appear in parentheses beneath estimated parameters. $\ln L$ is the maximized value of the log likelihood. W is White"s omnibus information matrix test of model specification adequacy, with marginal significance levels ("p-values") in parentheses. The sample period is 1 February 1993 through 26 February 1993. See text for details.

Table 33: ACD Model Estimation: MRK

	ω	α_j			β_j			$\ln L$	W
ACD(1,1)	0.01	0.03			0.96			-77449.99	153.22
	(0.00)	(0.00)			(0.00)				(0.00)
ACD(1,2)	0.01	0.05			0.57	0.38		-77440.09	724.00
	(0.00)	(0.00)			(0.06)	(0.06)			(0.00)
ACD(2,2)	0.01	0.05	0.00		0.57	0.38		-77440.09	9061.24
	(0.01)	(0.06)	(0.11)		(1.43)	(1.37)			(0.00)
ACD(3,3)	0.01	0.05	0.00	0.00	0.64	0.00	0.30	-77430.47	11599.01
	(0.00)	(0.00)	(0.02)	(0.01)	(0.47)	(0.62)	(0.23)		(0.00)

Notes: This table shows estimation results for several ACD models: $\psi_i = \omega + \sum_j \alpha_j d_{i-j} + \sum_j \beta_j \psi_{i-j}$. Standard errors appear in parentheses beneath estimated parameters. $\ln L$ is the maximized value of the log likelihood. W is White"s omnibus information matrix test of model specification adequacy, with marginal significance levels ("p-values") in parentheses. The sample period is 1 February 1993 through 26 February 1993. See text for details.

Table 34: Model Estimation: MSFT

$\bar{k}$	3	4	5	6	7	8	9	10
m_0	1.38	1.34	1.32	1.29	1.30	1.26	1.25	1.30
	(0.03)	(0.01)	(0.03)	(0.00)	(0.01)	(0.01)	(0.01)	(0.01)
λ	0.78	0.85	0.72	0.85	0.68	0.78	0.92	0.57
	(0.16)	(0.02)	(0.02)	(0.07)	(0.03)	(0.02)	(0.07)	(0.03)
$\gamma_{\bar{k}}$	0.09	0.10	0.11	0.12	0.12	0.13	0.14	0.12
	(0.02)	(0.01)	(0.01)	(0.02)	(0.01)	(0.01)	(0.02)	(0.01)
b	4.29	3.18	3.04	2.54	2.70	2.07	1.97	2.82
	(0.20)	(0.23)	(0.28)	(0.17)	(0.16)	(0.17)	(0.10)	(0.34)
$\ln L$	-44695.91	-44641.79	-44635.38	-44630.75	-44633.69	-44635.93	-44637.74	-44635.69
W	60.03	12.87	15.57	6.39	7.04	23.67	20.89	13.88
	(0.00)	(0.23)	(0.11)	(0.78)	(0.72)	(0.01)	(0.02)	(0.18)

Notes: This table shows estimation results for the Markov-Switching multi-fractal duration model with $\bar{k}$ intensity components. The model parameters are m_0, λ, $\gamma_{\bar{k}}$ and b. Standard errors appear in parentheses beneath estimated parameters. $\ln L$ is the maximized value of the log likelihood. W is White"s omnibus information matrix test of model specification adequacy, with marginal significance levels ("p-values") in parentheses. The sample period is 1 February 1993 through 26 February 1993. See text for details.

Table 35: ACD Model Estimation: MSFT

	ω	α_j			β_j			$\ln L$	W
ACD(1,1)	0.03	0.14			0.85			-44978.82	246.58
	(0.00)	(0.00)			(0.01)				(0.00)
ACD(1,2)	0.03	0.15			0.70	0.14		-44972.82	1068.27
	(0.00)	(0.01)			(0.08)	(0.08)			(0.00)
ACD(2,2)	0.06	0.14	0.12		0.00	0.72		-44978.29	5519.53
	(0.01)	(0.00)	(0.01)		(0.01)	(0.01)			(0.00)
ACD(3,3)	0.05	0.16	0.07	0.00	0.34	0.21	0.20	-44964.41	3829.67
	(0.01)	(0.01)	(0.04)	(0.03)	(0.26)	(0.19)	(0.14)		(0.00)

Notes: This table shows estimation results for several ACD models: $\psi_i = \omega + \sum_j \alpha_j d_{i-j} + \sum_j \beta_j \psi_{i-j}$. Standard errors appear in parentheses beneath estimated parameters. $\ln L$ is the maximized value of the log likelihood. W is White"s omnibus information matrix test of model specification adequacy, with marginal significance levels ("p-values") in parentheses. The sample period is 1 February 1993 through 26 February 1993. See text for details.

Table 36: Model Estimation: TXN

$\bar{k}$	3	4	5	6	7	8	9	10
m_0	1.48	1.42	1.45	1.44	1.45	1.45	1.45	1.40
	(0.01)	(0.01)	(0.02)	(0.02)	(0.02)	(0.02)	(0.02)	(0.03)
λ	0.83	0.86	1.89	2.60	2.36	1.64	2.95	11.51
	(0.05)	(0.04)	(0.27)	(0.51)	(0.32)	(0.16)	(0.44)	(3.20)
$\gamma_{\bar{k}}$	0.99	0.99	0.99	0.99	0.99	0.99	0.99	0.99
	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)
b	8.23	4.75	8.53	8.70	8.63	8.53	8.59	5.36
	(0.79)	(0.55)	(1.04)	(1.22)	(1.77)	(1.89)	(1.81)	(1.86)
$\ln L$	-7689.50	-7688.41	-7693.84	-7695.12	-7694.20	-7694.14	-7694.37	-7696.74
W	22.17	98.37	12.57	38.89	8.54	9.04	12.12	38.90
	(0.01)	(0.00)	(0.25)	(0.00)	(0.58)	(0.53)	(0.28)	(0.00)

Notes: This table shows estimation results for the Markov-Switching multi-fractal duration model with $\bar{k}$ intensity components. The model parameters are m_0, λ, $\gamma_{\bar{k}}$ and b. Standard errors appear in parentheses beneath estimated parameters. $\ln L$ is the maximized value of the log likelihood. W is White"s omnibus information matrix test of model specification adequacy, with marginal significance levels ("p-values") in parentheses. The sample period is 1 February 1993 through 26 February 1993. See text for details.

Table 37: ACD Model Estimation: TXN

	ω	α_j			β_j			$\ln L$	W
ACD(1,1)	0.10	0.09			0.88			-7974.67	42.85
	(0.02)	(0.01)			(0.02)				(0.00)
ACD(1,2)	0.12	0.10			0.67	0.18		-7973.82	125.71
	(0.02)	(0.05)			(0.01)	(0.06)			(0.00)
ACD(2,2)	0.12	0.10	0.00		0.67	0.18		-7973.82	487.62
	(0.10)	(0.23)	(0.05)		(0.37)	(0.57)			(0.00)
ACD(3,3)	0.20	0.10	0.07	0.00	0.00	0.52	0.23	-7973.09	699.92
	(0.09)	(0.02)	(0.07)	(0.04)	(0.70)	(0.14)	(0.08)		(0.00)

Notes: This table shows estimation results for several ACD models: $\psi_i = \omega + \sum_j \alpha_j d_{i-j} + \sum_j \beta_j \psi_{i-j}$. Standard errors appear in parentheses beneath estimated parameters. $\ln L$ is the maximized value of the log likelihood. W is White"s omnibus information matrix test of model specification adequacy, with marginal significance levels ("p-values") in parentheses. The sample period is 1 February 1993 through 26 February 1993. See text for details.

Table 38: Model Estimation: WFC

$\bar{k}$	3	4	5	6	7	8	9	10
m_0	1.46	1.41	1.36	1.36	1.36	1.36	1.36	1.36
	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)
λ	0.81	0.72	0.74	1.15	0.86	2.85	2.10	1.55
	(0.07)	(0.06)	(0.06)	(0.10)	(0.02)	(0.36)	(0.21)	(0.13)
$\gamma_{\bar{k}}$	0.94	0.99	0.99	0.99	0.99	0.99	0.99	0.99
	(0.05)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)
b	14.25	9.86	5.64	5.77	5.88	5.85	5.87	5.89
	(0.96)	(0.91)	(0.51)	(0.31)	(0.69)	(0.59)	(0.61)	(0.56)
$\ln L$	-7059.32	-7053.53	-7049.78	-7051.41	-7051.57	-7053.03	-7052.31	-7052.09
W	21.49	12.04	4.17	2.99	2.86	3.73	2.82	3.20
	(0.02)	(0.28)	(0.94)	(0.98)	(0.98)	(0.96)	(0.99)	(0.98)

Notes: This table shows estimation results for the Markov-Switching multi-fractal duration model with $\bar{k}$ intensity components. The model parameters are m_0, λ, $\gamma_{\bar{k}}$ and b. Standard errors appear in parentheses beneath estimated parameters. $\ln L$ is the maximized value of the log likelihood. W is White"s omnibus information matrix test of model specification adequacy, with marginal significance levels ("p-values") in parentheses. The sample period is 1 February 1993 through 26 February 1993. See text for details.

Table 39: ACD Model Estimation: WFC

	ω	α_j			β_j			$\ln L$	W
ACD(1,1)	0.05	0.09			0.89			-7291.16	27.06
	(0.01)	(0.01)			(0.01)				(0.00)
ACD(1,2)	0.06	0.10			0.71	0.17		-7291.25	136.12
	(0.01)	(0.01)			(0.10)	(0.09)			(0.00)
ACD(2,2)	0.09	0.10	0.07		0.00	0.80		-7290.38	87.10
	(0.04)	(0.03)	(0.03)		(0.15)	(0.15)			(0.00)
ACD(3,3)	0.10	0.11	0.07	0.01	0.24	0.00	0.54	-7285.14	450.29
	(0.02)	(0.02)	(0.02)	(0.02)	(0.12)	(0.18)	(0.17)		(0.00)

Notes: This table shows estimation results for several ACD models: $\psi_i = \omega + \sum_j \alpha_j d_{i-j} + \sum_j \beta_j \psi_{i-j}$. Standard errors appear in parentheses beneath estimated parameters. $\ln L$ is the maximized value of the log likelihood. W is White"s omnibus information matrix test of model specification adequacy, with marginal significance levels ("p-values") in parentheses. The sample period is 1 February 1993 through 26 February 1993. See text for details.

Table 40: Model Estimation: WMT

$\bar{k}$	3	4	5	6	7	8	9	10
m_0	1.26	1.22	1.20	1.17	1.16	1.16	1.17	1.16
	(0.03)	(0.00)	(0.00)	(0.00)	(0.01)	(0.00)	(0.01)	(0.00)
λ	0.75	0.83	0.83	0.80	0.81	0.70	0.70	0.72
	(0.37)	(0.02)	(0.01)	(0.02)	(0.03)	(0.05)	(0.02)	(0.02)
$\gamma_{\bar{k}}$	0.03	0.06	0.21	0.13	0.22	0.22	0.13	0.22
	(0.01)	(0.04)	(0.05)	(0.03)	(0.11)	(0.03)	(0.06)	(0.04)
b	10.97	18.38	10.48	6.80	5.76	5.75	6.78	5.74
	(1.33)	(11.31)	(1.52)	(0.93)	(1.07)	(0.98)	(1.64)	(0.58)
$\ln L$	-50726.70	-50654.46	-50648.08	-50643.79	-50639.69	-50640.22	-50644.61	-50640.47
W	94.54	20.87	22.78	5.54	8.80	8.67	14.16	11.58
	(0.00)	(0.02)	(0.01)	(0.85)	(0.55)	(0.56)	(0.17)	(0.31)

Notes: This table shows estimation results for the Markov-Switching multi-fractal duration model with $\bar{k}$ intensity components. The model parameters are m_0, λ, $\gamma_{\bar{k}}$ and b. Standard errors appear in parentheses beneath estimated parameters. $\ln L$ is the maximized value of the log likelihood. W is White"s omnibus information matrix test of model specification adequacy, with marginal significance levels ("p-values") in parentheses. The sample period is 1 February 1993 through 26 February 1993. See text for details.

Table 41: ACD Model Estimation: WMT

	ω	α_j			β_j			$\ln L$	W
ACD(1,1)	0.00	0.02			0.98			-50774.41	72.60
	(0.00)	(0.00)			(0.00)				(0.00)
ACD(1,2)	0.00	0.03			0.54	0.42		-50768.78	284.04
	(0.00)	(0.00)			(0.00)	(0.00)			(0.00)
ACD(2,2)	0.00	0.03	0.00		0.54	0.42		-50768.78	5676.88
	(0.01)	(0.01)	(0.17)		(1.20)	(1.03)			(0.00)
ACD(3,3)	0.00	0.04	0.00	0.00	0.57	0.00	0.39	-50762.06	6266.64
	(0.00)	(0.00)	(0.01)	(0.01)	(0.06)	(0.30)	(0.09)		(0.00)

Notes: This table shows estimation results for several ACD models: $\psi_i = \omega + \sum_j \alpha_j d_{i-j} + \sum_j \beta_j \psi_{i-j}$. Standard errors appear in parentheses beneath estimated parameters. $\ln L$ is the maximized value of the log likelihood. W is White"s omnibus information matrix test of model specification adequacy, with marginal significance levels ("p-values") in parentheses. The sample period is 1 February 1993 through 26 February 1993. See text for details.

Table 42: Model Estimation: XRX

$\bar{k}$	3	4	5	6	7	8	9	10
m_0	1.52	1.52	1.53	1.52	1.52	1.45	1.52	1.52
	(0.02)	(0.02)	(0.10)	(0.02)	(0.03)	(0.01)	(0.01)	(0.01)
λ	1.53	3.23	0.76	4.43	2.93	2.58	12.56	8.30
	(0.19)	(0.47)	(0.35)	(0.94)	(1.12)	(0.26)	(1.14)	(1.70)
$\gamma_{\bar{k}}$	0.99	0.99	0.99	0.99	0.99	0.99	0.99	0.99
	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)
b	25.91	27.43	34.11	27.48	27.58	10.40	27.52	27.58
	(6.18)	(2.91)	(37.63)	(6.93)	(1.58)	(5.04)	(5.40)	(7.05)
$\ln L$	-5360.69	-5361.73	-5362.07	-5361.97	-5361.94	-5363.38	-5362.44	-5362.27
W	30.32	8.34	31.07	28.50	29.34	24.16	27.16	30.56
	(0.00)	(0.60)	(0.00)	(0.00)	(0.00)	(0.01)	(0.00)	(0.00)

Notes: This table shows estimation results for the Markov-Switching multi-fractal duration model with $\bar{k}$ intensity components. The model parameters are m_0, λ, $\gamma_{\bar{k}}$ and b. Standard errors appear in parentheses beneath estimated parameters. $\ln L$ is the maximized value of the log likelihood. W is White"s omnibus information matrix test of model specification adequacy, with marginal significance levels ("p-values") in parentheses. The sample period is 1 February 1993 through 26 February 1993. See text for details.

Table 43: ACD Model Estimation: XRX

	ω	α_j			β_j			$\ln L$	W
ACD(1,1)	0.06	0.04			0.93			-5552.51	18.22
	(0.03)	(0.01)			(0.02)				(0.01)
ACD(1,2)	0.09	0.06			0.42	0.48		-5550.06	92.54
	(0.03)	(0.01)			(0.16)	(0.16)			(0.00)
ACD(2,2)	0.09	0.06	0.00		0.42	0.48		-5550.06	292.86
	(0.54)	(0.19)	(0.49)		(1.19)	(0.68)			(0.00)
ACD(3,3)	0.10	0.07	0.00	0.00	0.33	0.12	0.44	-5547.87	421.23
	(0.50)	(0.05)	(0.20)	(0.04)	(1.42)	(0.88)	(0.17)		(0.00)

Notes: This table shows estimation results for several ACD models: $\psi_i = \omega + \sum_j \alpha_j d_{i-j} + \sum_j \beta_j \psi_{i-j}$. Standard errors appear in parentheses beneath estimated parameters. $\ln L$ is the maximized value of the log likelihood. W is White"s omnibus information matrix test of model specification adequacy, with marginal significance levels ("p-values") in parentheses. The sample period is 1 February 1993 through 26 February 1993. See text for details.

Table 44: Overdispersion Test Statistics Comparison

Stock	$S_{\mathrm{MSMD},S}$	$S_{\mathrm{MSMD},F}$	S_{ACD}
IBM	-5.31	-6.37	6.67
KO	-4.91	-5.02	10.32
BAC	-7.49	-8.04	14.18
MSFT	-12.00	-15.66	19.74
F	-13.29	-14.02	29.28

Notes: The overdispersion test is $\sqrt{N}((\hat{\sigma}_\epsilon^2 - 1)/\sqrt{8}) \sim \mathcal{N}(0,1)$. $S_{\mathrm{MSMD},S}$ is the smoothed version test statistic for MSMD(7) model. $S_{\mathrm{MSMD},F}$ is the filtered version test statistic for MSMD(7) model. S_{ACD} is the test statistic for ACD(1,1) model.

Table 45: Model Comparison: in Sample Fit

Stock	In Sample Fitting $-\ln L$	
	MSMD(7)	ACD(1,1)
AA	5598.4	5791.3
AXP	17828	18010
BAC	12919	13078
DOW	11148	11246
GE	24403	24783
KO	24441	24547
MCD	12106	12207
TXN	7690.3	7978.9
WFC	7050.7	7315.3
XRX	7690.3	7978.9

Note: This is low trading group.

Table 46: Model Comparison: BIC

Stock	BIC $-2\ln L + k\ln(n)$		
	MSMD(7)	ACD(1,1)	$BIC_M - BIC_A$
AA	11229	11607	-378
AXP	35693	36048	-355
BAC	25874	26183	-309
DOW	22331	22519	-188
GE	48844	49595	-751
KO	48921	49123	-202
MCD	24248	24441	-193
TXN	15414	15983	-569
WFC	14135	14656	-521
XRX	15414	15983	-569

Note: This is low trading group. k is number of parameters. n is number of observations. BIC_M is BIC for the MSMD model. BIC_A is BIC for the ACD model.

Table 47: Model Comparison: out of Sample Forecast

Stock	Out of Sample Forecasting					
	1-step MSPE		5-step MSPE		20-step MSPE	
	MSMD(7)	ACD(1,1)	MSMD(7)	ACD(1,1)	MSMD(7)	ACD(1,1)
AA	18.1833	16.4306	18.7463	41.3716	19.8247	25.3063
AXP	17.4886	16.5861	17.8916	64.0274	18.61	61.2588
BAC	8.0538	7.9268	8.1647	24.6852	8.2475	16.329
DOW	10.1717	9.9576	10.4884	31.0425	10.7088	24.3516
GE	5.7333	5.6145	5.898	20.4824	6.1323	15.2521
KO	3.0342	3.0314	3.044	11.3846	3.1105	6.6252
MCD	6.16	6.0905	6.3188	20.5726	6.2952	12.8945
TXN	12.2909	11.1517	12.7247	28.581	13.4088	15.351
WFC	3.1323	8.5206	3.1509	23.8728	3.1785	13.2856
XRX	12.2909	11.1517	12.7247	28.581	13.4088	15.351

Note: This is low trading group. MSPE is mean square prediction error.

Table 48: Model Comparison: in Sample Fit

Stock	In Sample Fitting $-\ln L$	
	MSMD(7)	ACD(1,1)
ABT	26324	26460
CSCO	28610	29110
DELL	37903	38254
F	25652	26253
IBM	47695	47810
INTC	61895	62032
JNJ	35893	36011
MRK	77378	77486
MSFT	44635	44984
WMT	50644	50841

Note: This is high trading group.

Table 49: Model Comparison: BIC

Stock	BIC $-2\ln L + k\ln(n)$		
	MSMD(7)	ACD(1,1)	$BIC_M - BIC_A$
ABT	52687	52949	-262
CSCO	57259	58249	-990
DELL	75846	76538	-692
F	51343	52535	-1192
IBM	95431	95651	-220
INTC	12383	12410	-27
JNJ	71826	72052	-226
MRK	154778	155004	-226
MSFT	89311	89999	-688
WMT	10133	10171	-38

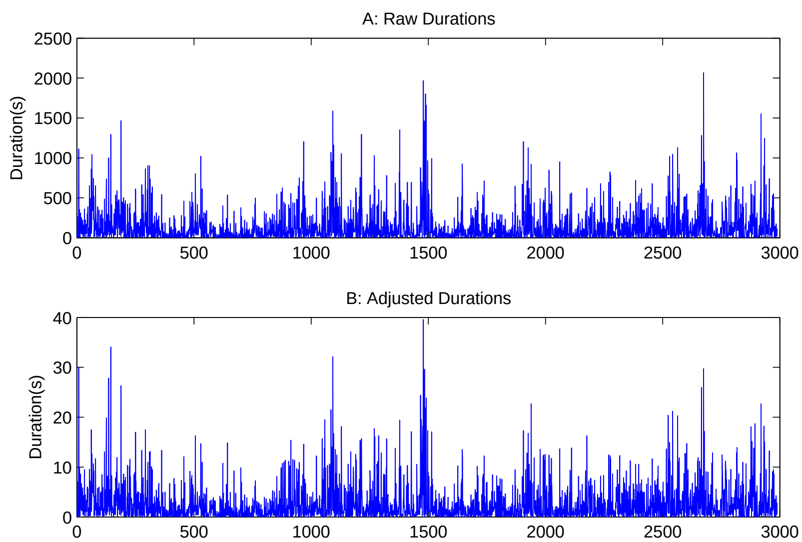
Note: This is high trading group. k is number of parameters. n is number of observations. BIC_M is BIC for the MSMD model. BIC_A is BIC for the ACD model.

Table 50: Model Comparison: out of Sample Forecast

Stock	Out of Sample Forecasting					
	1-step MSPE		5-step MSPE		20-step MSPE	
	MSMD(7)	ACD(1,1)	MSMD(7)	ACD(1,1)	MSMD(7)	ACD(1,1)
ABT	2.1301	2.132	2.1416	9.6389	2.203	2.8122
CSCO	3.1284	3.2781	3.2912	10.4741	3.4586	3.7093
DEll	6.678	7.0334	7.0476	22.2225	7.3236	15.0193
F	6.9681	6.6235	7.2714	25.4917	7.5698	21.5794
IBM	2.819	2.8114	2.8035	12.7333	2.8708	8.0736
INTC	7.4043	7.5486	7.7788	25.825	8.5547	14.7445
JNJ	3.2169	3.315	3.2208	14.035	3.183	9.0553
MRK	0.9111	0.915	0.9153	4.5424	0.9008	2.8697
MSFT	9.7861	10.2364	10.3726	36.8851	10.8266	30.3269
WMT	8.9516	9.0933	9.0395	28.4147	9.1606	17.329

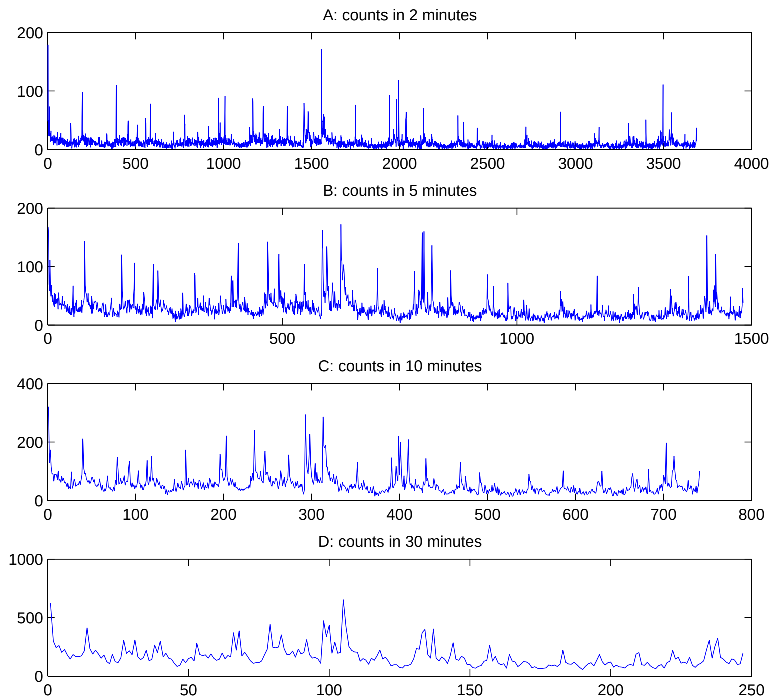
Note: This is high trading group. MSPE is mean square prediction error.

Figure 1: An Example of Duration Dynamics



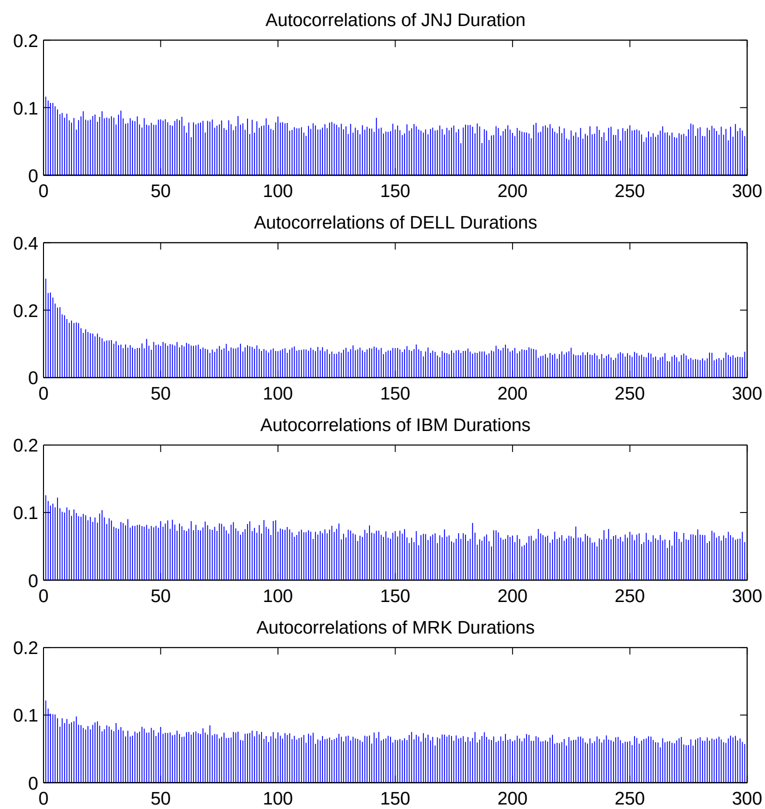
Notes: The transaction durations in the figure are from AA(ALCOA) between 10:00am and 4:00pm in February 1993. Panel A is raw durations. Panel B is durations after daily seasonality adjustment.

Figure 2: Clustering Effect at Different Time Scale



Notes: Panel A B C D are counting data for the same period. The data are IBM transaction data from February 1 1993 to December 31 1993. Vertical axis is number of counts. Horizontal axis is time index.

Figure 3: Sample Autocorrelation Functions for Four Stocks



Notes: The sampling period is from February 1 1993 to February 26 1993.

Bibliography

Admati, A.R. and P. Pfleiderer (1988), "A theory of intraday patterns: Volume and price variability," *Review of Financial Studies*, 3–40.

Aït-Sahalia, Y. and P.A. Mykland (2003), "The effects of random and discrete sampling when estimating continuous-time diffusions," *Econometrica*, 71, 483–549.

Andersen, T.G., T. Bollerslev, F.X. Diebold, and P. Labys (2001), "The distribution of realized exchange rate volatility," *Journal of the American Statistical Association*, 96, 42–55.

Baillie, R.T., T. Bollerslev, and H.O. Mikkelsen (1996), "Fractionally integrated generalized autoregressive conditional heteroskedasticity," *Journal of econometrics*, 74, 3–30.

Bandi, F.M. and J.R. Russell (2008), "Microstructure noise, realized variance, and optimal sampling," *Review of Economic Studies*, 75, 339–369.

Bauwens, L. and P. Giot (2000), "The logarithmic ACD model: an application to the bid-ask quote process of three NYSE stocks," *Annales d'Economie et de Statistique*, 117–149.

Bauwens, L. and N. Hautsch (2006), "Stochastic Conditional Intensity Processes," *Journal of Financial Econometrics*, 4, 450–493.

Bauwens, L. and D. Veredas (2004), "The stochastic conditional duration model: a latent variable model for the analysis of financial durations," *Journal of Econometrics*, 119, 381–412.

Bollerslev, T. and H.O. Mikkelsen (1996), "Modeling and pricing long memory in stock market volatility," *Journal of Econometrics*, 73, 151–184.

Bowsher, C.G. (2007), "Modelling security market events in continuous time: intensity based, multivariate point process models," *Journal of Econometrics*, 141, 876–912.

Breidt, F.J., N. Crato, and P. De Lima (1998), "The detection and estimation of long memory in stochastic volatility," *Journal of Econometrics*, 83, 325–348.

Brémaud, P. (1981), *Point Processes and Queues, Martingale Dynamics*, Springer, New York.

Calvet, L. and A. Fisher (2001), "Forecasting multifractal volatility," *Journal of econometrics*, 105, 27–58.

Calvet, L.E. and A.J. Fisher (2004), "How to forecast long-run volatility: regime switching and the estimation of multifractal processes," *Journal of Financial Econometrics*, 2, 49.

Carr, P. and L. Wu (2004), "Time-changed Lévy processes and option pricing," *Journal of Financial Economics*, 71, 113–142.

Carrasco, M. and X. Chen (2002), "Mixing and moment properties of various GARCH and stochastic volatility models," *Econometric Theory*, 18, 17–39.

Chen, Y.T. and C.S. Hsieh (2010), "Generalized Moment Tests for Autoregressive Conditional Duration Models," *Journal of Financial Econometrics*, 8, 345–391.

Clark, P.K. (1973), "A subordinated stochastic process model with finite variance for speculative prices," *Econometrica*, 135–155.

Comte, F. and E. Renault (1996), "Long memory continuous time models," *Journal of Econometrics*, 73, 101–149.

Comte, F. and E. Renault (1998), "Long memory in continuous-time stochastic volatility models," *Mathematical Finance*, 8, 291–323.

Corsi, F. (2009), "A Simple Approximate Long-Memory Model of Realized Volatility," *Journal of Financial Econometrics*, 7, 174–196.

Daley, D.J. and D. Vere-Jones (2003), *An introduction to the theory of point processes: volume I: elementary theory and methods*, Springer, New York.

Daley, D.J. and D. Vere-Jones (2007), *An introduction to the theory of point processes: volume II: general theory and structure*, Springer Verlag, New York.

De Luca, G. and G. Gallo (2004), "Mixture processes for financial intradaily durations," *Studies in Nonlinear Dynamics and Econometrics*, 8.

De Luca, G. and G.M. Gallo (2009), "Time-varying mixing weights in mixture autoregressive conditional duration models," *Econometric Reviews*, 28, 102–120.

De Luca, G. and P. Zuccolotto (2003), "Finite and infinite mixtures for financial durations," *Metron*, 61, 431–455.

Deo, R., M. Hsieh, and C.M. Hurvich (2010), "Long memory in intertrade durations, counts and realized volatility of nyse stocks," *Journal of Statistical Planning and Inference*, 140, 3715–3733.

Deo, R., C. Hurvich, and Y. Lu (2006), "Forecasting realized volatility using a long-memory stochastic volatility model: estimation, prediction and seasonal adjustment," *Journal of Econometrics*, 131, 29–58.

Deo, R., C.M. Hurvich, P. Soulier, and Y. Wang (2009), "Conditions for the Propagation of Memory Parameter from Durations to Counts and Realized Volatility," *Econometric Theory*, 25, 764–792.

Ding, Z., C.W.J. Granger, and R.F. Engle (1993), "A long memory property of stock market returns and a new model," *Journal of empirical finance*, 1, 83–106.

Drost, F.C. and B.J.M. Werker (2004), "Semiparametric duration models," *Journal of Business and Economic Statistics*, 22, 40–50.

Dufour, A. and R.F. Engle (2000), "The ACD model: predictability of the time between consecutive trades," working paper, University of Reading and University of California at San Diego.

Easley, D. and M. O'Hara (1992), "Time and the process of security price adjustment," *Journal of Finance*, 47, 577–605.

Engle, R. (2002), "New frontiers for ARCH models," *Journal of Applied Econometrics*, 17, 425–446.

Engle, R.F. (2000), "The econometrics of ultra-high-frequency data," *Econometrica*, 68, 1–22.

Engle, R.F. and G.M. Gallo (2006), "A multiple indicators model for volatility using intra-daily data," *Journal of Econometrics*, 131, 3–27.

Engle, R.F. and J.R. Russell (1998), "Autoregressive conditional duration: A new model for irregularly spaced transaction data," *Econometrica*, 66, 1127–1162.

Fernandes, M. and J. Grammig (2005), "Nonparametric specification tests for conditional duration models," *Journal of Econometrics*, 127, 35–68.

Fernandes, M. and J. Grammig (2006), "A family of autoregressive conditional duration models," *Journal of Econometrics*, 130, 1–23.

Ghysels, E., C. Gourièroux, and J. Jasiak (2004), "Stochastic volatility duration models," *Journal of Econometrics*, 119, 413–433.

Ghysels, E. and J. Jasiak (1998), "GARCH for irregularly spaced financial data: the ACD-GARCH model," *Studies in Nonlinear Dynamics and Econometrics*, 2, 133–149.

Giot, P. (2000), "Time transformations, intraday data, and volatility models," *Journal of Computational Finance*, 4, 31–62.

Grammig, J. and K.O. Maurer (2000), "Non-monotonic hazard functions and the autoregressive conditional duration model," *Econometrics Journal*, 3, 16–38.

Grammig, J. and M. Wellner (2002), "Modeling the interdependence of volatility and inter-transaction duration processes," *Journal of Econometrics*, 106, 369–400.

Granger, C.W.J. and R. Joyeux (1980), "An introduction to long-memory time series models and fractional differencing," *Journal of time series analysis*, 1, 15–29.

Hasbrouck, J. (1999), "Trading fast and slow: Security market events in real time," Working paper, Stern School of Business New York University.

Heckman, J. and B. Singer (1984), "A method for minimizing the impact of distributional assumptions in econometric models for duration data," *Econometrica*, 52, 271–320.

Heckman, J.J. and J.R. Walker (1990), "Estimating fecundability from data on waiting times to first conception," *Journal of the American Statistical Association*, 85, 283–294.

Hujer, R. and S. Vuletic (2007), "Econometric analysis of financial trade processes by discrete mixture duration models," *Journal of Economic Dynamics and Control*, 31, 635–667.

Jasiak, J. (1999), “Persistence in intertrade durations,” Working paper, Department of Economics, York University.

Karr, A.F. (1991), *Point processes and their statistical inference*, Dekker, New York.

Li, WK and P.L.H. Yu (2003), “On the residual autocorrelation of the autoregressive conditional duration model,” *Economics Letters*, 79, 169–175.

Lunde, A. (1999), “A generalized gamma autoregressive conditional duration model,” Working paper, Aalborg University.

Mandelbrot, B.B. (1997), *Fractals and scaling in finance: discontinuity, concentration, risk: selecta volume E*, Springer Verlag.

Mandelbrot, B.B., A. Fisher, and L. Calvet (1997), “A multifractal model of asset returns,” Cowles foundation discussion paper no. 1164, Yale University.

Meitz, M. and T. Teräsvirta (2006), “Evaluating models of autoregressive conditional duration,” *Journal of Business and Economic statistics*, 24, 104–124.

Meyn, S.P. and R.L. Tweedie (1993), *Markov chains and stochastic stability*, Springer-Verlag London.

Pagan, A.R. and G.W. Schwert (1990), “Alternative models for conditional stock volatility,” *Journal of Econometrics*, 45, 267–290.

Russell, J.R. (1999), “Econometric modeling of multivariate irregularly-spaced high-frequency data,” Mimeo, University of Chicago, Graduate School of Business.

Stock, J. (1988), “Estimating continuous-time processes subject to time deformation: an application to postwar US GNP,” *Journal of the American Statistical Association*, 83, 77–85.

Sun, W., S. Rachev, F.J. Fabozzi, and P.S. Kalev (2008), "Fractals in trade duration: capturing long-range dependence and heavy tailedness in modeling trade duration," *Annals of Finance*, 4, 217–241.

West, K.D. and M.W. McCracken (1998), "Regression-based tests of predictive ability," *International Economic Review*, 39, 817–840.

White, H. (1982), "Maximum likelihood estimation of misspecified models," *Econometrica: Journal of the Econometric Society*, 50, 1–25.

Zhang, M.Y., J.R. Russell, and R.S. Tsay (2001), "A nonlinear autoregressive conditional duration model with applications to financial transaction data," *Journal of Econometrics*, 104, 179–207.