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A Fair Scheduling Policy for Wireless Channels with Intermittent Connectivity

A. Aaram M.H.R. Khouzani, S. Sarkar, L. Tassiulas

Abstract—**We consider a system of parallel queues with the constraint that only one queue can be served at a time. Each queue can be served through a wireless channel with intermittent connectivity. We propose a policy which serves the longest connected queue when the lengths of all connected are below a given threshold, and serves the connected queue that exceeds the threshold and has received the least service so far, otherwise. We prove that this simple policy (a) maximizes the aggregate service rate of all queues (b) maximizes the minimum service rate attained by any queue and (c) attains the stability region of the network.**

I. Introduction

Throughput and fairness guarantees have been obtained in wireless networks owing to several seminal results obtained in the last two decades [8], [4]. Most of these performance guarantees however rely on complex computations. Devices in wireless networks however have limited energy and can therefore only execute low complexity scheduling strategies. Several recent papers have obtained throughput and fairness guarantees using low complexity scheduling strategies[2], [3], [5], [6], [9]. But, most of these papers consider wireless networks where transmission conditions do not change with time. Nevertheless, wireless links often suffer from intermittent connectivity owing to fading and multi-path effects. We seek to attain throughput and fairness guarantees in wireless networks in presence of intermittent connectivity.

We consider a simple topology consisting of one server and n queues. The server can serve at most one queue at a time. The link, or the channel, between the server and any queue is connected only intermittently. A queue can be served only when its channel is connected. Tassiulas [7] proved that scheduling the longest connected queue (LCQ) attains the maximum possible stability region in such networks. But, as the following example demonstrates, LCQ can treat some queues unfairly when the arrival rate vector is outside the network stability region. Suppose there are two queues (q_1 and q_2) and the channels between the queues and the server are always connected. Two packets arrive at q_1 in each slot and one packet arrives at q_2 every other slot. Clearly, in this case, LCQ will always serve q_1 and never serve q_2, allocating rates of $(1, 0)$ to them. Now, consider another policy, the Least Service First or LSF, that serves the non-empty connected queues that has received the least accumulative service from the server. Intuitively, it is fairer than LCQ. For example, in the above specific case, it allocates a rate of 0.5 to each of the queues. Nevertheless, as the following examples demonstrate, LSF is inefficient in that (a) it does not attain the maximum possible stability region, and (b) does not maximize the aggregate service rate irrespective of whether the arrival rate vector is inside or outside the network stability region.

Example 1. *Consider two queues q_1 and q_2 and let the channel for queue 1 be connected in odd numbered slots, and the channel for queue 2 always be connected. Queue q_1 receives a packet in every odd slot, and q_2 receives a packet in slots $3, 6, \ldots$. The corresponding arrival rate vector is in the stability region of the network as a policy (say policy P) that serves q_1 in odd slots, and q_2 in slots $3i+1$, $i = 0, 1, \ldots$ stabilizes both queues. P allocates a service rate vector of $(1/2, 1/3)$. LSF serves q_2 whenever it receives packets, and q_1 in slots in which q_1 is connected and q_2 does not receive a packet. Thus, each queue is served at rate $1/3$. Thus, LSF does not stabilize q_1. Thus, LSF does not attain the stability region of the network, and does not maximize the aggregate service rate. Now suppose that q_1 has packet arrivals in every slot. Now, note that q_1 can not be stabilized under any policy as its packet arrival rate exceeds its connectivity rate. Thus, the arrival rate vector is outside the network stability region. The service process under LSF remains the same, and thus LSF serves each queue at rate $1/3$. As before, policy P can serve each queue at rate $1/2$. thus, LSF does not maximize the aggregate service rate in this case either.*

We now explore whether the above policies (LCQ and LSF) can be combined to attain both efficiency and fairness. We propose a policy that serves the (a) longest connected queue while all connected queues are below a certain threshold and (b) the connected queue that has received the least accumulative service so far among all connected queues, otherwise. We prove that this policy (a) maximizes the aggregate service rate of all queues (b) maximizes the service rate of the queue that receives the minimum service rate and (c) maximizes the stability region of the network (i.e., stabilizes the network if the arrival rate vector is in the network stability region). The policy and the proof constitute the main contributions of the paper.

II. System Model

We assume that there exists a single server and n queues: $Q = \{1, 2, \cdots, n\}$. Time is slotted. The server can serve at most one queue in a given slot, and serves one packet from such a queue in the slot. The link or the channel

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between each queue and the server can either be connected (ON) or disconnected (OFF). For any $Q_i, Q_j \subseteq Q$ with $Q_i \bigcap Q_j = \emptyset$, let $\mu_{Q_i,\overline{Q_j}}(t_k, t_l)$ be the number of slots in the time interval $[t_k, t_l]$ during which at least one of the queues in Q_i is connected while all of the queues in Q_j are disconnected. Packets arrive to a queue at the beginning of a slot and can be transmitted in the same slot. Let $A_{Q_i}(t_k, t_l)$ be the aggregate number of packets that arrived to queues in $Q_i \subseteq Q$ during interval $[t_k, t_l]$. A queue can be served only when it is connected and it has a packet to transmit. Let $S_{Q_i}(t_k, t_l)$ be the total number of packets served from queues in $Q_i \subseteq Q$ during interval $[t_k, t_l]$.

We assume that queues have unlimited buffer size. Let $X_i(t)$ be the number of packets waiting for transmission in queue i at time t. We assume that $X_i(0) = 0$ for all i. In each slot t, $X_i(t)$ increases by $A_{\{i\}}(t, t)$, and decreases by 1 if queue i is served in t. Let M be the maximum amount by which $X_i(t)$ can change in one time slot for any i.

We assume that the processes $\mu_{Q_i,\overline{Q_j}}(t_k, t_l)$ and $A_{Q_i}(t_k, t_l)$ are pseudo-deterministic. In other words, there exists numbers $\{a_i\}, \{\mu(Q_i, \bar{Q}_j)\}, \Delta$ such that for any Q_i, Q_j, t_k, t_l we have:

$$|\mu_{Q_i,\overline{Q_j}}(t_k, t_l) - \mu(Q_i, \overline{Q_j})(t_l - t_k)| \leq \Delta \quad (1)$$

$$|A_{Q_i}(t_k, t_l) - \sum_{j \in Q_i} a_j(t_l - t_k)| \leq \Delta. \quad (2)$$

The numbers $\{a_i\}, \{\mu(Q_i, \overline{Q_j})\}$ are denoted as the *arrival rates* and *connectivity rates* respectively, and $(a_1, \ldots, a_n)$ is denoted as the *arrival rate vector*. Clearly, $M = \max(\max_i a_i, \max_j \mu(q_j, \overline{\emptyset})) + \Delta$.

We make the following technical assumption.

$$\mu(Q_i, \overline{Q_j}) \neq \sum_{l \in Q_i} a_l, \quad (3)$$

for all disjoint $Q_i \neq \emptyset$ and Q_j.

A queue i is said to be *stable* if $\sup_t X_i(t) < \infty$. The stability of a queue depends on the arrival and connectivity processes and the scheduling policy. We say the arrival rates are inside the *stability region of the system* iff for some scheduling policy, all queues are stable. A scheduling policy is said to *attain the stability region of the system* provided it can stabilize the system for any arrival rate vector that is in the stability region of the system.

Lemma 1. *Given any set of arrival and connectivity parameters, we can always partition the set of queues into two subsets: Subset Y and subset $Z = Y^c$ with the following properties:*

$$\mu(Q_i, \overline{Z}) > \sum_{j \in Q_i} a_j \quad \textit{for all} \quad \emptyset \subset Q_i \subseteq Y, \quad (4)$$

and

$$\mu(Q_k, \overline{Z \setminus Q_k}) < \sum_{j \in Q_k} a_j \quad \textit{for all} \quad \emptyset \subset Q_k \subseteq Z. \quad (5)$$

Moreover, this partitioning is unique.

The proof of Lemma 1 reveals that the pair (Y, Z) where Y is the largest cardinality set that satisfies (4) and $Z = Y^c$ constitutes the above unique partition.

The following argument establishes that the arrival rates are not inside the stability region of the system if $Z \neq \emptyset$. Note that this fact is consistent with the result of [7] for a different arrival and connectivity model. From Lemma 1, for any disjoint $\phi \subset Q \subseteq Z$, we have

$$\mu(Q, \overline{Z \setminus Q}) < \sum_{l \in Q_i} a_j. \quad (6)$$

Thus, from (1) and (2), $\sum_{j \in Z} X_j(t) \geq t \sum_{j \in Z}(a_j - \mu(Z, \overline{\emptyset})) - 2\Delta$. Thus,

$$\sup_{\substack{j \in Z \\ t}} X_j(t) = \infty. \quad (7)$$

III. Scheduling Policy and Performance Guarantees

We now present our scheduling policy $f(L)$ which schedules the queues as follows. Let $U(t)$ be the set of queues that are connected in t and have queue lengths greater than or equal to a threshold L. If $U(t) \neq \emptyset$, the queue in $U(t)$ that has received the least service so far is served, else, the connected queue with the maximum queue length is served. The policy is parameterized with parameter L, and does not require the scheduler to know the arrival and connectivity rates and Δ.

The following theorem shows that for a large enough L, the buffer sizes of the queues in Y are upper bounded under $f(L)$. Also, the buffer sizes of the queues in Z are larger than that of those in Y after a finite time. Note that as a special case, this shows that $f(L)$ stabilizes the arrival rates inside the throughput region.

Theorem 1. *There exist D_0 and $L(D_0)$, which depend only on the system parameters, such that for all $L > L(D_0)$, we have the following.*

$$\exists t_0 \quad s.t \begin{cases} \sup_{\substack{j \in Y \\ t \geq t_0}} X_j(t) & < & D_0 \\ \inf_{\substack{j \in Z \\ t \geq t_0}} X_j(t) & > & D_0 \end{cases}.$$

The proof of the theorem relies on the following lemmas which we prove in the appendix (Lemmas‘3,4) and in a technical report [1] (Lemma 2).

Lemma 2. *Let $T = D_0^{1/4}, L > \sqrt{2D_0} + MD_0^{1/4}$. Then, for all large enough D_0,*

$$\sum_{j \in Y} X_j^2(t+T) - \sum_{j \in Y} X_j^2(t) \leq 0 \quad \textit{if} \quad D_0 \leq \sum_{j \in Y} X_j^2(t) \leq 2D_0$$

The minimum required value of D_0 for the above lemma to hold depends only on n, M arrival and connectivity rates and not on t or the queue lengths at t.

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Lemma 3. *For all $D' \geq 0$, there exist L_0, which depends only on D' and the arrival and service parameters, such that for all $L > L_0$,*

$$\sum_{j \in Z} X_j^{-2}(t_0) < \frac{1}{D'}$$

for some finite t_0.

Lemma 4. *For all $\bar{D} > D_{min}$, there exists T and large enough $L_{\bar{D}}$ such that if $L > L_{\bar{D},T}$, and for some t, $1/\bar{D} < \sum_{j \in Z} X_j^{-2}(t) \leq n/(\sqrt{\bar{D}} - MT)^2$, we have*

$$\sum_{j \in Z} X_j^{-2}(t+T) - \sum_{j \in Z} X_j^{-2}(t) \leq 0.$$

Here, D_{min} and T depend only on the parameters of the system, and $L_{\bar{D}}$ depends only on $\bar{D}, T$ and the parameters of the system.

Proof of Theorem 1: We first show that there exists D' such that for all $L > \sqrt{D'}$,

$$\sup_{t \geq 0} \sum_{j \in Y} X_j^2(t) \leq D'. \tag{8}$$

We next show that there exists $\hat{D} > D'$ such that for a finite t_0 and L large enough,

$$\sum_{j \in Z} X_j^{-2}(t) \leq 1/\hat{D} \ \forall \ t \geq t_0. \tag{9}$$

The Theorem follows.

We now show (8). Let D_0 satisfy the minimum required value for Lemma 2 to hold, and, in addition, D_0 be large enough such that

$$D' = 2D_0 \geq \left(\sqrt{D_0} + \sum_{j \in Y} (a_j (D_0)^{1/4} + \Delta) \right)^2.$$

Let $T = D_0^{1/4}, L > \sqrt{2D_0} + MD_0^{1/4}$.

Now we show by induction that

$$\sum_{j \in Y} X_j^2(t+kT) \leq D' \quad \forall t \in [0,T], k \geq 0.$$

The base case corresponds to $k = 0$, and holds since $T = D_0^{1/4}$ and $X_j(t) \leq a_j T + \Delta$ for each j and $t \in [0,T]$. Now, let the hypothesis hold for $k = 0, \ldots, m$. Consider a $t \in [0,T]$. If $\sum_{j \in Y} X_j^2(t+mT) \leq D_0$, then, clearly, $\sum_{j \in Y} X_j^2(t+(m+1)T) \leq D'$ since each X_j can increase only by $a_j T + \Delta$ in $[t+mT, t+(m+1)T]$. If $D_0 \leq \sum_{j \in Y} X_j^2(t+kT) \leq 2D_0 = D'$, then by Lemma 2, $\sum_{j \in Y} X_j^2(t+(m+1)T) \leq \sum_{j \in Y} X_j^2(t+mT) \leq D'$.

We now show (9). Select $\bar{D}$ such that $\bar{D} \geq D_{\min}$ in Lemma 4, and $\hat{D} = \frac{(\sqrt{\bar{D}} - MT)^2}{n} > D'$. Now, from Lemma 3, there exists L_0, t_0 such that

$$\sum_{j \in Z} X_j^{-2}(t_0) < 1/\bar{D} \ \text{ for } L > L_0.$$

Select T such that Lemma 4 holds, and let $L_1 = L_{\bar{D},T}$ given in Lemma 4. The result follows if we can show that for all $L > \max(L_0, L_1)$,

$$\sum_{j \in Z} X_j^{-2}(t+kT) \leq 1/\hat{D} \quad \forall t \in [t_0, t_0+T], k \geq 0.$$

We prove the above using induction on k. We first prove the above for $k = 0$. Clearly, since $\sum_{j \in Z} X_j^{-2}(t_0) \leq 1/\bar{D}$, then $\sum_{j \in Z} X_j^{-2}(t_0+t) \leq 1/\hat{D}$ for all $t \in [0,T]$. Now, we assume that the above hypothesis holds for k, and prove it for $k+1$. Consider some $t \in [0,T]$. If $\sum_{j \in Z} X_j^{-2}(t+kT) \leq 1/\bar{D}$, then, clearly, $\sum_{j \in Z} X_j^{-2}(t+(k+1)T) \leq 1/\hat{D}$. If $1/\bar{D} < \sum_{j \in Z} X_j^{-2}(t+kT) \leq 1/\hat{D}$, then by Lemma 4, $\sum_{j \in Z} X_j^{-2}(t+(k+1)T) \leq \sum_{j \in Z} X_j^{-2}(t+kT) \leq 1/\hat{D}$. The result follows. □

We have already argued that if $Z \neq \emptyset$ then the arrival rates are not inside the stability region of the system. Theorem 1 shows that if $Z = \emptyset$ then $f(L)$ stabilizes the system for all large enough L. Thus, for any arrival rate vector that is inside the stability region of the system, $f(L)$ stabilizes the system for all large enough L. Thus, this policy attains the stability region of the system. We next use Theorem 1 to prove the following main results of the paper.

Theorem 2. *For L large enough, the policy $f(L)$ maximizes both the aggregate and the minimum service rates.*

Proof: Let L be such that for some $D > 0$ and finite t_0,

$$\begin{cases} \sup_{\substack{j \in Y \\ t \geq t_0}} X_j(t) & < & D \\ \inf_{\substack{j \in Z \\ t \geq t_0}} X_j(t) & > & D \end{cases}.$$

From Theorem 1, L, D, and t_0 as above exist. Therefore, under $f(L)$, all queues in Y are stable and their lengths are less than that of those in Z after a finite time. As a result, whenever a queue in Z is connected, one of them will be served. This means that $f(L)$ maximizes the aggregate service rate of Z. Moreover, since queues in Y are all stable, this subset also receives the maximum service. Therefore, $f(L)$ maximizes the aggregate service rate.

In the following, we proceed to prove the second claim. We use the following assumption without a proof: There exist a service rate for each queue.

Suppose $\{q_1, q_2, \cdots, q_k\}$ is the set of queues which receive the minimum service rate $r_1 = r_2 = \cdots = r_k$ under $f(L)$. Also suppose there is a policy which gives rates $r'_1, r'_2, \cdots, r'_k$ to these queues where $r'_i > r_i$ for $i = 1, \cdots, k$. So under $f(L)$, $q_1, \cdots, q_k$ are all unstable. Therefore, after a finite time, their lengths are always greater than L. This means that if at least one queue in the set $\{q_1, q_2, \cdots, q_k\}$ is connected, one of them will receive the service. So the set $\{q_1, q_2, \cdots, q_k\}$ receives the maximum possible service among all policies. As a result $\sum_{i=1}^k r_i \geq \sum_{i=1}^k r'_i$, which is a contradiction. Hence, $f(L)$ maximizes the minimum

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service rate.
Putting these together, the theorem follows.

The proof that $f(L)$ maximizes the minimum service rate relies on the assumption that the service rates $\lim_{t\to\infty} S_{\{j\}}(0,t)/t$ exist for each queue j under $f(L)$. Extensive simulations, which we present in our technical report [1], demonstrate that this is indeed the case.

Another interesting question is whether the above simple policy attains the *max-min fair* service rate. A service rate allocation is max-min fair if it has the maximum value of the minimum component among all service rate vectors, and subject to the above the maximum value of the second minimum and so on. The following example reveals that this is not true in general.

Example 2. *Let $n = 3$ and the connectivity and arrival processes constitute mutually independent Bernoulli processes with rates $\mu(\{1\},\phi) = 1/7, \mu(\{2\},\phi) = 0.3, \mu(\{3\},\phi) = 5/6$, $a_1 = 0.1, a_2 = 0.7, a_3 = 0.7$. Simulation reveals that our policy provides service rates of $(0.1, 0.27, 0.53)$.*

Nevertheless, our extensive simulations also indicate that in most of the cases, the achieved service rates are very close to the max-min fair rates.

Appendix

A. Proof of Lemma 1

Proof: First, we claim that if for some Z and $\emptyset \subset Q \subseteq Z$

$$\mu(Q, \overline{Z \setminus Q}) > \sum_{j\in Q} a_j, \tag{10}$$

then there exist $\emptyset \subset \hat{Q} \subseteq Q$ such that

$$\mu(Q_i, \overline{Z \setminus \hat{Q}}) > \sum_{j\in Q_i} a_j \quad \text{for all} \quad \emptyset \subset Q_i \subseteq \hat{Q}. \tag{11}$$

We prove this claim by induction on $|Q|$. For $|Q| = 1$, the claim easily follows. Now suppose the claim is true for $|Q| = k-1$, we show it is also true for $|Q| = k$. If (11) holds for $Q = \hat{Q}$, the claim follows. So, suppose there exist $Q_i \subset Q$ such that $\mu(Q_i, \overline{Z \setminus Q}) < \sum_{j\in Q_i} a_j$. Consider the subset $Q \setminus Q_i$. It is clear that

$$\begin{aligned}\mu(Q_i, \overline{Z \setminus Q}) + \mu(Q \setminus Q_i, &\overline{Z \setminus (Q \setminus Q_i)}) \\ &= \mu(Q, \overline{Z \setminus Q}),\end{aligned}$$

which, together with (3), (10) and the fact that $\mu(Q_i, \overline{Z \setminus Q_i}) < \sum_{j\in Q_i} a_j$, gives

$$\mu(Q \setminus Q_i, \overline{Z \setminus (Q \setminus Q_i)}) > \sum_{j\in Q\setminus Q_i} a_j.$$

Now, $Q \setminus Q_i$ satisfies (10) and $|Q \setminus Q_i| < |Q| = k$. Therefore there exist $\hat{Q} \subseteq (Q \setminus Q_i)$ that satisfies (11) and the claim follows.

Now we proceed to prove the lemma. Let Y_1 be the largest cardinality set (which can be an empty set) that satisfies (4). We show that $Z_1 = Y_1^c$. satisfies (5). Then the lemma follows for $Y = Y_1$ and $Z = Z_1$. Let Z_1 not satisfy (5). Then, for some $\emptyset \subset Q \subseteq Z_1$

$$\mu(Q, \overline{Z_1 \setminus Q}) > \sum_{j\in Q} a_j.$$

By the claim proved above, there exist $\emptyset \subset \hat{Q} \subseteq Q$ such that

$$\mu(Q_i, \overline{Z_1 \setminus \hat{Q}}) > \sum_{j\in Q_i} a_j \quad \text{for all} \quad Q_i \subseteq \hat{Q}. \tag{12}$$

Now consider $Y_2 = Y_1 \cup \hat{Q}$. Clearly, for all $\emptyset \subset Q_i \subseteq Y_1$, $\mu(Q_i, \overline{Z_1 \setminus \hat{Q}}) \geq \mu(Q_i, \overline{Z_1})$. Also, for all $\emptyset \subset Q_i \subseteq Y_1$ and $\emptyset \subset Q_j \subseteq \hat{Q}$,

$$\begin{aligned}\mu(Q_i \cup Q_j, \overline{Z_1 \setminus \hat{Q}}) &= \mu(Q_i, \overline{Z_1 \setminus (\hat{Q} \setminus Q_j)}) + \mu(Q_j, \overline{Z_1 \setminus \hat{Q}}) \\ &\geq \mu(Q_i, \overline{Z_1}) + \mu(Q_j, \overline{Z_1 \setminus \hat{Q}}).\end{aligned}$$

Thus, from (12) and since Y_1 satisfies (4), Y_2 satisfies (4). But, $|Y_2| > |Y_1|$ which is a contradiction.

Now we show the uniqueness of Y and Z. Suppose there are two different partitioning pairs (Y_1, Z_1) and (Y_2, Z_2) that satisfy (4) and (5). Now consider the subset $Z_2 \setminus Z_1 \neq \emptyset$. From $Z_2 \setminus Z_1 \subseteq Y_1$ and (4) we have

$$\mu(Z_2 \setminus Z_1, \overline{Z_1}) > \sum_{j\in Z_2\setminus Z_1} a_j, \tag{13}$$

while from $Z_2 \setminus Z_1 \subseteq Z_2$ and (5) we have

$$\begin{aligned}\mu(Z_2 \setminus Z_1, \overline{Z_2 \setminus (Z_2 \setminus Z_1)}) &= \mu(Z_2 \setminus Z_1, \overline{Z_2 \cap Z_1}) \\ &< \sum_{j\in Z_2\setminus Z_1} a_j,\end{aligned} \tag{14}$$

It is now clear that (13) and (14) cannot be true at the same time which contradicts the assumption of the existence of different partitioning pairs.

B. Proof of Lemma 3

Proof: It is sufficient to show that for some finite t_0, $X_j(t_0) > \sqrt{nD'}$ for all j in Z. We prove the lemma by induction. From (7), for any $D' \geq 0$, there exist L_0 such that for all $L > L_0$, $X_j(t_0) > D'$ for at least one j in Z and a finite time t_0. Now, suppose for all $\bar{D}$, there exists t_0, L_0 such that for all $L > L_0$, $X_j(t_0) > \bar{D}$ for at least k queues in Z, where $k \geq 1$. We will show that for all D' there exists t_1, L_0 such that for all $L > L_0$, $X_j(t_1) > D'$ for at least $k+1$ queues.

Let $r = \min_{\emptyset\subset Q\subseteq Z} \left(\sum_{i\in Q} a_i - \mu(Q, \overline{Z \setminus Q})\right)$. From (6), $r > 0$. Let $\bar{D} = (1 + (nM/r))\, D' + (M/r)2\Delta$. By induction hypothesis, there exists there exists $t_0, L_0 > \bar{D}$ such that for all $L > L_0$, $X_j(t_0) > \bar{D}$ for at least k queues in Z. Let B be the set of those k queues. If $X_j(t_0) > D'$ for any $j \in Z \setminus B$, since $\bar{D} > D'$, $X_j(t_0) > D'$ for at least $k+1$ queue in Z and the claim follows with $t_1 = t_0$. Now consider the case that $X_j(t_0) < D'$ for all $j \in Z \setminus B$.

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Let $T = (n/r)D' + (2\Delta/r)$. Consider an interval $[t_0, t_0 + T]$. Since a queue can decrease by at most M in a slot and $X_j(t_0) > \bar{(D)}$ for each $j \in B$, $X_j(t) > D'$ for each $t \in (t_0, t_0 + T]$ for each $j \in B$. Let $X_j(t) \leq D'$ for each $t \in (t_0, t_0 + T]$ and $j \in Z \setminus B$. Then, since $L_0 > \bar{D} > D'$, no queue in $Z \setminus B$ is served in any slot in $[t, t + T]$ unless all queues in B are disconnected in the slot. Thus, from (1) and (2), $\sum_{j \in Z \setminus B} X_j(t)$ increases by at least $rT - 2\Delta$ during $(t, t + T]$. Thus, at leat one queue in $Z \setminus B$ increases by at least $(rT - 2\Delta)/n$ during $(t, t + T]$. Note that T is such that this increase exceeds D'. Thus, $X_j(t) > D'$ for some $t \in (t_0, t_0 + T]$ and $j \in Z \setminus B$. The induction hypothesis now holds with t_1 being the first such time in $(t_0, t_0 + T]$ where the above happens.

C. Proof of Lemma 4

Proof: We first introduce a few terminologies. Let $\mathcal{P}$ be the set of partitions of Z. For each partition $P = \{\hat{Q}_1, \hat{Q}_2, ...\} \in \mathcal{P}$, let $\mu_i^P = \mu(\hat{Q}_i, \overline{(\cup_{l \geq i+1} \hat{Q}_l)})$, and $\rho_i^P = \sum_{j \in \hat{Q}_i} a_j$. Using Lemma 1, we have $\sum_1^i \mu_l^P < \sum_1^i \rho_l^P$ for each i, $P \in \mathcal{P}$. Consider some $\alpha \in (0, 1)$ such that $\sum_1^i \mu_l^P < \sum_1^i \rho_l^P$ for each i, $P \in \mathcal{P}$. Let $u = \min_{P \in \mathcal{P}} \min_i (\sum_1^i \alpha \rho_l^P - \sum_1^i \mu_l^P)$. Clearly, $u > 0$.

Consider some $\delta > 0$. Now let T be a positive integer such that $-uT + n\Delta < -\delta$.

Let $D_{\min}$ be a large enough number such that

$$\left(\frac{\frac{(\sqrt{D_{\min}} - MT)^2}{n} - MT}{\frac{(\sqrt{D_{\min}} - MT)^2}{n} + (2n+1)MT} \right)^3 > \alpha \tag{15}$$

$$\text{and } \sqrt{\frac{(\sqrt{D_{\min}} - MT)^2}{n}} > MT. \tag{16}$$

Let $\mu = \sum_{i \in Z} \mu(\{i\}, \bar{\emptyset})$.

Let $D_{\max}$ be a large enough number such that

$$D_{\max} > n\sqrt{\bar{D}} + 2nMT \tag{17}$$

$$\text{and } \frac{-\delta}{(n\sqrt{\bar{D}} - MT)^3} + \frac{\mu T + \Delta}{(D_{max} - MT)^3} < 0. \tag{18}$$

Let $L_{\bar{D},T} = D_{max} + 2(n+1)MT$. Thus, when $L > L_{\bar{D},T}$, if a queue is below $D_{max} + 2nMT$ at time t, it remains below L in time interval $[t, t + T]$, and is served under longest connected queue policy in $[t, t + T]$.

Let $\hat{D} = \frac{(\sqrt{\bar{D}} - MT)^2}{n}$, and $D_0 = n\sqrt{\bar{D}}$.

Now, the bounds on $\sum_{j \in Z} X_j^{-2}(t)$ in the statement of the lemma imply that $1/\bar{D} < \sum_{j \in Z} X_j^{-2}(t) < 1/\hat{D}$. From the left hand side inequality, we have

$$X_j(t) < D_0 \qquad \text{for at least one} \quad j \quad \text{in} \quad Z, \tag{19}$$

while since $\bar{D} \geq D_{\min}$, (16) and the right hand side inequality imply that $\min_{j \in Z} X_j(t) \geq \sqrt{\hat{D}} > MT$.

We now consider a specific partition of Z $\{Q_1, Q_2, \cdots, Q_K\}$ which is constructed as follows. Sort queues in Z in nondecreasing order of their queue lengths and do the following.

1. Set $i = 0$.
2. Increment i by one.
3. Move along the sorted list. As long as the difference in length between consecutive queues is less than or equal to $2MT$, put every queue in Q_i.
4. If the next queue has length less than D_{max} return to (2). Else put the remaining queues in Q_{i+1} (which may be an empty set) and terminate.

The above algorithm partitions Z . Let $D_i = \min_{j \in Q_i} X_j(t)$ and $|Q_i| = n_i$. Note that (17) implies that $D_{max} > D_0 + 2nMT$, which leads to the following properties of the partition.

1. Subsets $Q_1, Q_2, \cdots, Q_{K-1}$ are all nonempty.
2. $\min_{j \in Q_K} X_j(t) \geq D_{max}$.
3. $\max_{j \in Q_i} X_j(t) < \min_{l \in Q_{i+1}} X_l(t) - 2MT$ for $i = 1, 2, \cdots, K-1$.
4. $\max_{j \in Q_i} X_j(t) \leq D_i + 2nMT$ for $i = 1, 2, \cdots, K-1$.
5. $D_{K-1} < D_{max}$.

We now mention some properties of the above partition which we will use throughout the proof. Properties (4) and (5) and the fact that $L > L_{\bar{D},T} = D_{max} + 2(n+1)MT$, ensure that all queues in $Q_1, \ldots, Q_{K-1}$ are served as per LCQ policy during $[t, t + T]$. Property (3) ensures that $\min_{j \in Q_{i+1}} X_j(t_1) > \max_{j \in Q_i} X_j(t_1)$ for each $i = 1, \ldots, K-1$, $t_1 \in [t, t+T]$. Thus, a necessary condition for a queue in Q_i to be served in a slot in $[t, t+T]$ is that at least one queue in Q_i is connected and all queues in $(\cup_{l=1}^{i-1} Q_l \cup Z)$ are disconnected for $i = 1, \ldots K-1$. Let $\mu_i = \mu(Q_i, \overline{\cup_{l=i+1}^{K} Q_l})$, and $\rho_i = \sum_{j \in Q_i} a_j$. Note that $\{\mu_i\}$s and $\{\rho_i\}$s are $\{\mu_i^P\}$s $\{\rho_i^P\}$s for partition $P = \{Q_1, \ldots, Q_K\}$. Clearly, $\mu \geq \max_i \mu_i$. Thus, from (1), $\sum_{j \in Q_i} S_{\{j\}}(t, t + T) \leq \mu_i T + \Delta$. Now,

$$\begin{aligned}
& \sum_{j \in Z} X_j^{-2}(t+T) - \sum_{j \in Z} X_j^{-2}(t) \\
= \; & -\sum_{j \in Z} (X_j^{-1}(t+T) - X_j^{-1}(t))^2 \\
& + 2\sum_{j \in Z} X_j^{-1}(t+T)(X_j^{-1}(t+T) - X_j^{-1}(t)) \\
\leq \; & 2\sum_{j \in Z} X_j^{-2}(t+T) X_j^{-1}(t)(X_j(t) - X_j(t+T)) \\
= \; & 2\sum_{j \in Z} X_j^{-2}(t+T) X_j^{-1}(t) S_{\{j\}}(t, t+T) \\
& - 2\sum_{j \in Z} X_j^{-2}(t+T) X_j^{-1}(t) A_{\{j\}}(t, t+T).
\end{aligned} \tag{20}$$

$$\begin{aligned}
& \sum_{j \in Q_i} \frac{S_{\{j\}}(t, t+T)}{X_j(t) X_j^2(t+T)} \\
\leq \; & \sum_{j \in Q_i} \frac{S_{\{j\}}(t, t+T)}{(X_j(t) - MT)^3} \text{ (since } \min_{j \in Z} X_j(t) > MT), \\
\leq \; & \frac{1}{(D_i - MT)^3} \sum_{j \in Q_i} S_{\{j\}}(t, t+T) \\
\leq \; & \frac{\mu_i T + \Delta}{(D_i - MT)^3} \qquad \text{for} \quad i = 1, \cdots, K \quad .
\end{aligned}$$

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The last inequality follows since $\sum_{j\in Q_i} S_{\{j\}}(t,t+T) \leq \mu_i T+\Delta)$. Since $\min_{j\in Z} X_j(t) > MT$ $D_j > MT$ for each j. Thus, from property (2), we have

$$\sum_{j\in Z} \frac{S_{\{j\}}(t,t+T)}{X_j(t)X_j^2(t+T)} \leq \sum_{i=1}^{K-1} \frac{\mu_i T+\Delta}{(D_i-MT)^3} + \frac{\mu_K T+\Delta}{(D_{max}-MT)^3} \tag{21}$$

$$\begin{aligned} & \sum_{j\in Q_i} \frac{A_{\{j\}}(t,t+T)}{X_j(t)X_j^2(t+T)} \\ \geq & \sum_{j\in Q_i} \frac{A_{\{j\}}(t,t+T)}{(X_j(t)+MT)^3} \\ \geq & \frac{\sum_{j\in Q_i} A_{\{j\}}(t,t+T)}{(D_i+2nMT+MT)^3} \text{ (from property (4))}, \\ \geq & \frac{\sum_{j\in Q_i} a_i T-\Delta}{(D_i+(2n+1)MT)^3} \qquad \text{for} \quad i=1,\cdots,K-1. \end{aligned}$$

The last inequality follows from (2). Thus,

$$\sum_{j\in Z} \frac{A_{\{j\}}(t,t+T)}{X_j(t)X_j^2(t+T)} \geq \sum_{i=1}^{K-1} \frac{\rho_i T-\Delta}{(D_i+(2n+1)MT)^3} \tag{22}$$

Since $\bar{D} \geq D_{\min}$ and due to (15),

$$\left(\frac{\sqrt{\hat{D}}-MT}{\sqrt{\hat{D}}+(2n+1)MT}\right)^3 > \alpha.$$

Now, since $D_i \geq \min_{j\in Z} X_j(t) \geq \sqrt{\hat{D}}$, for each i,

$$\left(\frac{D_i-MT}{D_i+(2n+1)MT}\right)^3 > \alpha \qquad \text{for} \quad i=1,2,\cdots,K-1,$$

which gives

$$\frac{\rho_i}{(D_i+(2n+1)MT)^3} - \frac{\mu_i}{(D_i-MT)^3} \geq \frac{\alpha\rho_i-\mu_i}{(D_i-MT)^3} \quad \text{for} \quad i=1,2,\cdots,K-1. \tag{23}$$

From (21), (22), and (23) we have

$$\begin{aligned} & \sum_{j\in Z} X_j^{-2}(t+T) - \sum_{j\in Z} X_j^{-2}(t) \\ & \leq -T\sum_{i=1}^{K-1} \frac{\alpha\rho_i-\mu_i}{(D_i-MT)^3} + \frac{n\Delta}{(D_1-MT)^3} \\ & \quad + \frac{\mu_K T+\Delta}{(D_{max}-MT)^3}. \end{aligned} \tag{24}$$

We now lower bound $\sum_{i=1}^{K-1} \frac{\alpha\rho_i-\mu_i}{(D_i-MT)^3}$. Start with the right most term. If $\alpha\rho_{K-1} \geq \mu_{K-1}$, then the term $\frac{\alpha\rho_{K-1}-\mu_{K-1}}{(D_{K-1}-MT)^3}$ is positive and the sum can be lower bounded by $\sum_{i=1}^{K-2} \frac{\alpha\rho_i-\mu_i}{(D_i-MT)^3}$. If, on the other hand, $\alpha\rho_{K-1} < \mu_{K-1}$, $\frac{\alpha\rho_{K-1}-\mu_{K-1}}{(D_{K-1}-MT)^3} \geq \frac{\alpha\rho_{K-1}-\mu_{K-1}}{(D_{K-2}-MT)^3}$, and the sum can be lower bounded by $\sum_{i=1}^{K-3} \frac{\alpha\rho_i-\mu_i}{(D_i-MT)^3} + \frac{\alpha(\rho_{K-2}+\rho_{K-1})-(\mu_{K-2}+\mu_{K-1})}{(D_{K-2}-MT)^3}$. Continuing in the same manner we obtain, $\sum_{i=1}^{K-1} \frac{\alpha\rho_i-\mu_i}{(D_i-MT)^3} \geq \frac{\sum_{i=1}^{l} \alpha\rho_i-\mu_i}{(D_1-MT)^3}$ for some $l \in \{1,2,\cdots,K-1\}$. The second term can be lower bounded by $u/(D_1-MT)^3$. Thus, from (24) we have

$$\sum_{j\in Z} X_j^{-2}(t+T) - \sum_{j\in Z} X_j^{-2}(t) \leq \frac{-uT+n\Delta}{(D_1-MT)^3} + \frac{\mu_K T+\Delta}{(D_{max}-MT)^3}.$$

Now, since $-uT+n\Delta < -\delta$, where $\delta > 0$, and $\mu_K \leq \mu$, from (19), we have

$$\sum_{j\in Z} X_j^{-2}(t+T) - \sum_{j\in Z} X_j^{-2}(t) \leq \frac{-\delta}{(D_0-MT)^3} + \frac{\mu T+\Delta}{(D_{max}-MT)^3}. \tag{25}$$

From (18), (25) is negative and the lemma follows.

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