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Keywords
metamaterials, plasmonics, surface waves

Comments
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Abstract

We describe here our recent results on some of the anomalous propagation properties of sub-diffractive guided modes along plasmonic or metamaterial cylindrical waveguides with core-shell structures, with particular attention to the design of optical sub-wavelength nanodevices. In our analysis, we compare and contrast the azimuthally symmetric modes, on which the previous literature has concentrated, with polaritonic guided modes, which propagate in a different regime close to the plasmonic resonance of the waveguide. Forward and backward modes may be envisioned in this latter regime, traveling with sub-diffraction cross-section along the cylindrical interface between plasmonic and non-plasmonic materials. In general, two oppositely oriented power flows arise in the positive and negative permittivity regions, consistent with our previous results in the planar geometry. Our discussion applies to a various range of frequencies, from RF to optical and UV, even if we are mainly focused on optical and infrared propagation. At lower frequencies, artificially engineered plasmonic metamaterials or natural plasmas may be envisioned to obtain similar propagation characteristics.

1. Introduction

It is well known how metamaterials and plasmonic media may allow squeezing the dimensions of waveguide components due to the local plasmonic resonances when interfaced with regular dielectrics. A recent review and discussion on this possibility is given in [*Alù and Engheta*, 2005], but the possibility of propagation along plasmonic planar layers or cylinders dates back to the middle of the past century (see, e.g., [*Rusch*, 1962; *Vigants*, 1962; *Al-Bader*, 1992]). The field of artificial materials and metamaterials has revived this interest, and with the new advances of current technology it is possible to envision sub-diffractive waveguides with lateral confinement at frequencies for which the diffraction limit is already down to fractions of the micron.

As some of the recent works on metamaterials have shown, the diffraction limit -- a general physical law that seems to forbid the concentration of the field below half-wavelength size -- may be overcome in several geometries and for different purposes by properly exciting resonances at the interface between oppositely-signed permittivity materials. Sub-wavelength focusing [*Pendry*, 2000; *Alù and Engheta*, 2003], negative refraction [*Lezec,et al.,* 2007], diffractionless propagation [*Alù and Engheta*, 2005; *Brongersma, et al.*, 2000; *Alù and Engheta*, 2006b], sub-wavelength resonant cavities [*Engheta*, 2002], plasmonic nanoresonances [*Oldenburg, et al.*, 1999; *Alù and Engheta*, 2005b] and small antennas [*Alù, et al.,* 2007; *Ziolkowski and Kipple*, 2003] are examples of this possibility.

In this sense, the use of plasmonic materials turns out to be important, due to the anomalous compact resonances arising at their interface with regular materials (dielectrics or free-space). Nature has endowed us with a relatively wide class of

negative-permittivity materials, most of them in the optical, infrared and THz frequency ranges [*Bohren and Huffman*, 1983], which include also a class of resonant polaritonic dielectrics and some semiconductors. At microwave frequencies, regular gaseous plasmas possess a negative permittivity, but more easily the effective permittivity of a metamaterial may be designed and tuned to a negative value with a suitable engineering of inclusions in a host material [*Engheta and Ziolkowski*, 2006].

It should be born in mind that passive plasmonic materials are naturally limited by causality and Kramers-Kronig relations [*Landau and Lifschitz*, 1984] to be frequency dispersive and intrinsically lossy -- conditions that affect and somehow limit the following considerations on the anomalous modal propagation along plasmonic waveguides. However, with proper design, plasmonic materials may indeed provide the designer with novel tools to reduce and overcome some of these limitations, in order to design waveguides with a cross-section significantly smaller than the wavelength of operation.

The problem of guided wave propagation along cylindrical components with coaxial core-shell geometry has been studied in the past (see, e.g., [*Vigants*, 1962; *Al-Bader*, 1992; *Takahara et al.*, 1997]). In the following, we underline the main theoretical aspects of the anomalous regime of propagation and we describe the conditions under which guided modes along plasmonic core-shell waveguides may be tailored and designed. With respect to previous works on this topic (see, e.g., [*Takahara et al.*, 1997]), which have all focused on the dominant azimuthally symmetric guided modes, here we report and fully describe the possibility that these geometries may offer for supporting two different regimes of propagation, one related to the polaritonic resonance of the

waveguide, arising only for higher-order modes, and the other related to the geometrical resonance of the sub-wavelength plasmonic waveguide, usually achieved for the fundamental azimuthally symmetric mode. While the previous studies reported in the literature have been focused on this latter mode, we show in the following how the first possibility may also open up interesting scenarios that, to our best knowledge, have not been analyzed in the past. In particular, as we show here, the geometrical modes are limited to become very slow-wave for sub-diffractive propagation, and therefore this regime of propagation is inherently associated with signal dispersion and absorption. A large phase constant generally accompanies slow energy velocity, which may lead to more sensitivity to material losses, implying smaller propagation lengths and larger frequency dispersion. The novel set of polaritonic modes we consider here, however, is shown to remain potentially very confined, but also possibly to sustain reasonably faster wave propagation. We discuss these aspects with physical insights and full-wave analytical results with a complete theory that describes both regimes of propagation.

2. Theoretical Formulation

Consider the cylindrical waveguide depicted in Fig. 1 in a suitably chosen cylindrical reference system (ρ, φ, z). In general, the structure is supposed to be constituted of a core cylinder and a concentric cylindrical shell of isotropic materials, with radii a_1 and $a > a_1$, respectively, and corresponding permittivities ε_1 and ε_2. The surrounding background material has permittivity ε_0, and all the permeabilities are the same as free space μ_0, as it is the case for natural materials at infrared and optical frequencies. Extension of this analysis to materials with different permeabilities is straightforward and

it does not add much to the present discussion, apart from the further degree of freedom in the choice of the polarization of interest. We assume in the following an $e^{j\omega t}$ monochromatic excitation.

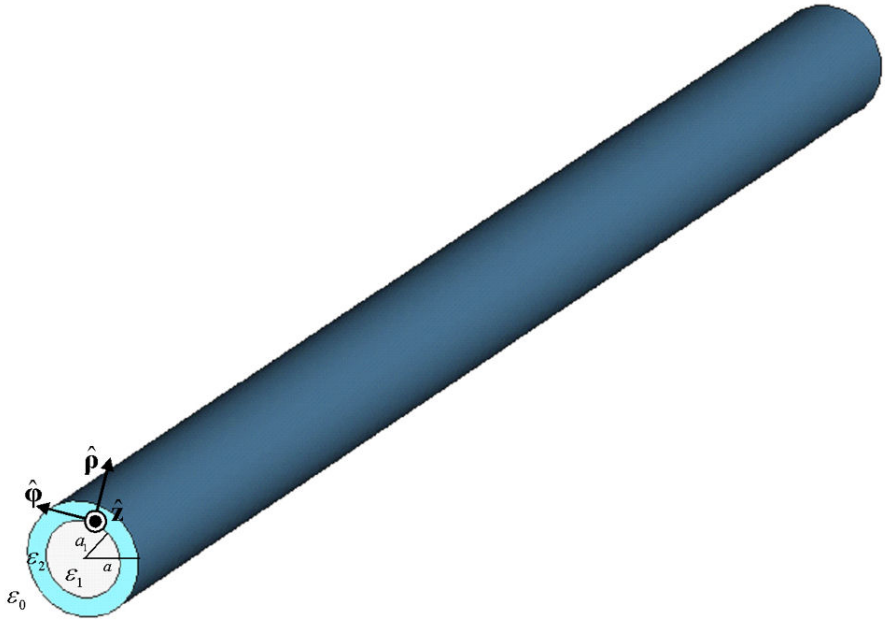

Figure 1 - Geometry of an infinitely long plasmonic or metamaterial core-shell cylindrical waveguide.

In the scattering scenario, the possibility of polaritonic resonances in plasmonic or metamaterial sub-wavelength structures analogous to the geometry of Fig. 1 has been investigated in [*Alù and Engheta*, 2005b], showing how high resonant peaks in the scattering cross section of sub-wavelength spherical and cylindrical objects, associated with the material polariton resonances of the structure, may be obtained by utilizing materials with negative constitutive parameters.

The guided modes supported by this geometry may have an analogous resonant behavior, associated with the material polaritons supported by the structure. The guided spectrum for the geometry of Fig. 1 is in general composed of hybrid modes, linear combinations

of TE^z or TM^z modes propagating in the z direction with a $e^{-j\beta z}$ factor. Their field distribution is given by the combination of the field components for the two polarizations:

$$\begin{aligned}\mathbf{E}_{TE} &= -j\omega\mu_0\rho^{-1}\frac{\partial u_i^{TE}}{\partial\varphi}\hat{\boldsymbol{\rho}} + j\omega\mu_0\frac{\partial u_i^{TE}}{\partial\rho}\hat{\boldsymbol{\varphi}} \\ \mathbf{H}_{TE} &= \frac{\partial^2 u_i^{TE}}{\partial\rho\,\partial z}\hat{\boldsymbol{\rho}} + \rho^{-1}\frac{\partial^2 u_i^{TE}}{\partial\varphi\,\partial z}\hat{\boldsymbol{\varphi}} + \left(\frac{\partial^2 u_i^{TE}}{\partial z^2} + k_i^2 u_i^{TE}\right)\hat{\mathbf{z}}\end{aligned} \tag{1}$$

$$\begin{aligned}\mathbf{E}_{TM} &= \frac{\partial^2 u_i^{TM}}{\partial\rho\,\partial z}\hat{\boldsymbol{\rho}} + \rho^{-1}\frac{\partial^2 u_i^{TM}}{\partial\varphi\,\partial z}\hat{\boldsymbol{\varphi}} + \left(\frac{\partial^2 u_i^{TM}}{\partial z^2} + k_i^2 u_i^{TM}\right)\hat{\mathbf{z}} \\ \mathbf{H}_{TM} &= j\omega\varepsilon_i\rho^{-1}\frac{\partial u_i^{TM}}{\partial\varphi}\hat{\boldsymbol{\rho}} - j\omega\varepsilon_i\frac{\partial u_i^{TM}}{\partial\rho}\hat{\boldsymbol{\varphi}}\end{aligned} \tag{2}$$

where $i = 0, 1, 2$, respectively, in the vacuum, in the first and in the second medium, $k_i^2 = \omega^2\mu_0\varepsilon_i$ is the wave number in each medium and

$$\begin{aligned}u_1 &= c_1 J_n\left(k_{t1}\rho\right)e^{-j(n\varphi+\beta z)} \\ u_2 &= \left[c_2 J_n\left(k_{t2}\rho\right) + c_3 Y_n\left(k_{t2}\rho\right)\right]e^{-j(n\varphi+\beta z)} \\ u_0 &= c_s H_n^{(2)}\left(k_{t0}\rho\right)e^{-j(n\varphi+\beta z)}\end{aligned} \tag{3}$$

Equations (3) are valid both for TE and TM polarization, with β representing the real longitudinal wave number of the mode, n being its integer angular order describing the azimuthal variation, $k_{ti}^2 = k_i^2 - \beta^2$ $(i = 1, 2)$ being its transverse radial wave number, c_j $(j = 1, 2, 3, s)$ being the excitation coefficients. J_n, Y_n and $H_n^{(2)} = J_n - jY_n$ are cylindrical Bessel functions [*Abramowitz and Stegun*, 1972]. The sign ambiguity in the square root definition in the argument of the $H_n^{(2)}$ functions should be resolved by imposing a field distribution exponentially decaying in the background region (we note that this restriction is not present when considering leaky modes supported by analogous

cylindrical waveguides acting as leaky-wave antennas, as we have recently reported [*Alù, et al.*, 2007b], [*Alù, et al.*, 2007c].

By imposing the proper boundary conditions at the interface between the two shells and at the metallic boundary, one finds the following relations among the excitation coefficients:

$$\begin{aligned} c_2 &= c_1 \frac{k_{t1}^2}{k_{t2}^2} \frac{J_n\left(k_{t1}a_1\right)Y_n\left(k_{t2}a\right)}{\Delta} - c_s \frac{k_{t0}^2}{k_{t2}^2} \frac{Y_n\left(k_{t0}a\right)Y_n\left(k_{t2}a_1\right)}{\Delta} \\ c_3 &= -c_1 \frac{k_{t1}^2}{k_{t2}^2} \frac{J_n\left(k_{t1}a_1\right)J_n\left(k_{t2}a\right)}{\Delta} + c_s \frac{k_{t0}^2}{k_{t2}^2} \frac{Y_n\left(k_{t0}a\right)J_n\left(k_{t2}a_1\right)}{\Delta} \end{aligned}, \qquad (4)$$

where $\Delta = J_n\left(k_{t2}a_1\right)Y_n\left(k_{t2}a\right) - Y_n\left(k_{t2}a_1\right)J_n\left(k_{t2}a\right)$, again valid both for TE and TM polarizations.

As expected, the constraint:

$$\begin{vmatrix} \frac{\mu_0}{k_{t1}} \frac{J_n'\left(k_{t1}a_1\right)}{J_n\left(k_{t1}a_1\right)} - \frac{\mu_0}{k_{t2}} \frac{\mathrm{N}_{a_1a}}{\Delta} & \frac{\mu_0}{k_{t2}} \frac{\mathrm{N}_{a_1a_1}}{\Delta} & -\frac{n\beta}{j\omega a_1} \frac{k_1^2 - k_2^2}{k_{t1}^2 k_{t2}^2} & 0 \\ -\frac{\mu_0}{k_{t2}} \frac{\mathrm{N}_{aa}}{\Delta} & \frac{\mu_0}{k_{t0}} \frac{H_n'^{(2)}\left(k_{t0}a\right)}{H_n^{(2)}\left(k_{t0}a\right)} + \frac{\mu_0}{k_{t2}} \frac{\mathrm{N}_{aa_1}}{\Delta} & 0 & -\frac{n\beta}{j\omega a} \frac{k_0^2 - k_2^2}{k_{t0}^2 k_{t2}^2} \\ \frac{n\beta}{j\omega a_1} \frac{k_1^2 - k_2^2}{k_{t1}^2 k_{t2}^2} & 0 & \frac{\varepsilon_1}{k_{t1}} \frac{J_n'\left(k_{t1}a_1\right)}{J_n\left(k_{t1}a_1\right)} - \frac{\varepsilon_2}{k_{t2}} \frac{\mathrm{N}_{a_1a}}{\Delta} & \frac{\varepsilon_2}{k_{t2}} \frac{\mathrm{N}_{a_1a_1}}{\Delta} \\ 0 & \frac{n\beta}{j\omega a} \frac{k_0^2 - k_2^2}{k_{t0}^2 k_{t2}^2} & -\frac{\varepsilon_2}{k_{t2}} \frac{\mathrm{N}_{aa}}{\Delta} & \frac{\varepsilon_0}{k_{t0}} \frac{H_n'^{(2)}\left(k_{t0}a\right)}{H_n^{(2)}\left(k_{t0}a\right)} + \frac{\varepsilon_2}{k_{t2}} \frac{\mathrm{N}_{aa_1}}{\Delta} \end{vmatrix} = 0 \qquad (5)$$

which results from fulfilling the boundary conditions at the two interfaces, represents the dispersion relation for the possible wavenumber β. In general the guided modes are hybrid, i.e., they are linear combinations of the TE and TM modes previously defined, as evident from the structure of the matrix in Eq. (5). Here $N_{xy} = J_n'\left(k_{t2}x\right)Y_n\left(k_{t2}y\right) - Y_n'\left(k_{t2}x\right)J_n\left(k_{t2}y\right)$, with x, y being either a or a_1. In the

particular case of azimuthally symmetric modes, i.e., $n=0$, the matrix in (5) has zero coupling terms and the dispersion relation decouples into TE and TM surface modes:

$$DispTE_n \cdot DispTM_n = 0 \qquad (6)$$

with:

$$\begin{aligned} DispTE_n &= \left(\frac{1}{k_{t1}}\frac{J_n'(k_{t1}a_1)}{J_n(k_{t1}a_1)} - \frac{1}{k_{t2}}\frac{\mathrm{N}_{a_1a}}{\Delta}\right)\left(\frac{1}{k_{t0}}\frac{H_n'^{(2)}(k_{t0}a)}{H_n^{(2)}(k_{t0}a)} + \frac{1}{k_{t2}}\frac{\mathrm{N}_{aa_1}}{\Delta}\right) + \frac{1}{k_{t2}^2}\frac{\mathrm{N}_{a_1a_1}\mathrm{N}_{aa}}{\Delta^2} \\ DispTM_n &= \left(\frac{\varepsilon_1}{k_{t1}}\frac{J_n'(k_{t1}a_1)}{J_n(k_{t1}a_1)} - \frac{\varepsilon_2}{k_{t2}}\frac{\mathrm{N}_{a_1a}}{\Delta}\right)\left(\frac{\varepsilon_0}{k_{t0}}\frac{H_n'^{(2)}(k_{t0}a)}{H_n^{(2)}(k_{t0}a)} + \frac{\varepsilon_2}{k_{t2}}\frac{\mathrm{N}_{aa_1}}{\Delta}\right) + \frac{\varepsilon_2^2}{k_{t2}^2}\frac{\mathrm{N}_{a_1a_1}\mathrm{N}_{aa}}{\Delta^2} \end{aligned} . \qquad (7)$$

In the most general case of $n \neq 0$, however, only hybrid modes are expected, since TE and TM modes with field distributions given by (1) or (2) would not satisfy by themselves the boundary conditions, consistent with the discussion in [*Pincherle*, 1944]. The dispersion relation (5) may be rewritten in the following compact form:

$$DispTE_n \cdot DispTM_n = \frac{n^2\beta^2\left(k_1^2 - k_2^2\right)^2}{k_0^2 a_1^2 k_{t1}^4 k_{t2}^4} . \qquad (8)$$

It should be noted how the dispersion relations (7) are not symmetric, and this is due to the fact that we are not considering possible differences in the permeabilities of the involved materials. This implies that in the following scenario the TM or 'quasi-TM' hybrid modes are the most appealing in the sub-wavelength regime of operation, since TE modes may resonate only due to "size" resonances, similar to dielectric waveguides.

It should be underlined that the previous analysis is valid for any value of permittivities, even complex when losses are taken into account. The interest here is focused on sub-wavelength structures, i.e., the core-shell waveguides with radii much less than the wavelength of operation. If we consider electrically thin waveguides, for which

$a \ll \min\left(|k_{t1}|, |k_{t2}|, |k_{t0}|\right)$, a Taylor expansion of (5) for small arguments of the Bessel and Neumann functions gives the following approximate condition:

$$\left(\gamma^{2n} - \frac{\varepsilon_2 + \varepsilon_1}{\varepsilon_2 - \varepsilon_1}\frac{\varepsilon_2 + \varepsilon_0}{\varepsilon_2 - \varepsilon_0}\right) = 0\,, \qquad (9)$$

where $0 < \gamma = a_1 / a < 1$ and $n > 0$.

This interesting result, consistent with our findings relative to resonant cylindrical scatterers in [*Alù and Engheta*, 2005b], confirms that it is indeed possible to exploit polaritonic resonances to excite guided surface modes in sub-wavelength structures. The previous dispersion relation, although quite simple in its form, has several interesting features. First, it seems not to be directly dependent on the frequency and on the guided wave number β, and depends only upon the geometrical filling ratio of the waveguide γ and on the material permittivities. (However, as we mention later, some of the material parameters are frequency-dependent.) This is consistent with our previous findings in the planar geometry [*Alù and Engheta,* 2005], related to the fact that these resonances are 'quasi-static resonances' in nature, and they are inherently related to the local plasmonic resonance at the interface between a plasmonic and a non-plasmonic material. The cylindrical geometry plays also an important role in the form of Eq. (9) and by varying the cross section of the waveguide the condition on the filling-ratio may vary.

The dependence of the resonance condition on frequency mainly comes from the intrinsic frequency dispersion of the plasmonic materials, and therefore indirectly Eq. (9) still manifests a dependence on ω. This is consistent with Chu's limit requirements on bandwidth that the small size imposes on these resonant waveguides [*Chu*, 1948]. Also the dependence on β is not directly observed in Eq. (9), consistent with the analogous

situation in some planar waveguides composed of metamaterials [*Alù and Engheta*, 2005]. This is due to the facts that: (a) small variations of the geometrical parameters may induce a large variation on the guided wave number β; and (b) all possible wave numbers may be guided once a polaritonic resonance is supported and condition (9) is approximately satisfied. The quick variation of β with the geometry of the waveguide, and therefore also with the frequency (see the previous discussion) are another indication of the small bandwidth of operation, and large signal dispersion, that would characterize electrically too small waveguides. A trade-off between size and operational bandwidth should be sought in the design of these structures.

As a corollary of the previous findings, such waveguides may guide not only surface (bounded) modes, but also leaky-modes, when $\mathrm{Re}\left[\beta\right] < k_0$, and therefore sub-wavelength leaky-wave nanoantennas may be envisioned with this technique, satisfying the same dispersion relation (9). This is consistent with the preliminary findings that we have presented in [*Alù, et al.*, 2007c], where an analogous dispersion relation has been derived. This regime is however not of interest for the present manuscript.

Another interesting point resulted from Eq. (9) is that this sub-wavelength regime may be supported only under the condition of exciting higher-order modes, i.e., surface modes with $n \geq 1$, that is with some azimuthal variation. Azimuthally symmetric modes (with $n = 0$) are not supported in the 'quasi-static resonance' regime, consistent with our findings in [*Alù and Engheta*, 2005b] and [*Alù, et al.*, 2007b].

Finally, it should be underlined how this dispersion relation depends just on the permittivity of the materials, implying that the hybrid modes supported in this configuration under the quasi-static condition are quasi-TM mode, with a field

configuration very close to the one described by Eq. (2). The more the waveguide is sub-wavelength, the more the (necessarily) hybrid modes are close to a TM configuration. In fact, in the limiting case of $\beta = 0$, Eq. (9) becomes the 'quasi-static' dispersion equation for TM material polaritons, and the weak dependence of (9) over β corresponds to an equally weak dependence of the corresponding field distribution. (Of course, a small TE modal component still needs to be present to match the boundary conditions for any $\beta \neq 0$, but the corresponding hybrid modes are quasi-TM). If the permeability of the involved materials were also allowed to assume negative values, then the dual dispersion relation to (9) would be in place for quasi-TE modes. This scenario is not of interest in the present manuscript, and it may be investigated using duality and following an analysis similar to the one presented here.

It is interesting to note that, to our knowledge, this regime of quasi-TM guided-wave polariton propagation represented by Eq. (9) (with $n \geq 1$) has never been considered before in the technical literature. Researchers have been mainly concerned with investigating azimuthally symmetric purely TM modes in plasmonic waveguides, which, as we mentioned above, are not supported under the small-radii condition $a \ll \min\left(|k_{t1}|, |k_{t2}|, |k_{t0}|\right)$.

The possibility of guiding a sub-diffraction mode with $n = 0$ arises due to the fact that modes may become very slow when plasmonic resonances are present. As reported in, e.g., [*Takahara, et al.*, 1997], in this regime the waveguide cross section may still become electrically small, even though the product $k_{t0}a$ is not necessarily small. This implies a fast variation of the transverse field distribution, for which the 'quasi-static' conditions previously imposed do not apply and Eq. (9) does not hold. In the following,

we compare the two regimes of anomalous propagation in plasmonic waveguides, for both of which the theoretical formulation presented in this section applies.

3. Azimuthally Symmetric Modes (n = 0)

Following the above discussion, for simplicity we now consider a homogeneous plasmonic sub-wavelength nanowire (which falls into the geometry considered in the previous section when $a_1 = a$). In this case, sub-diffraction TM modes with $n = 0$ are supported for $\beta \gg k_0$ and the approximate dispersion relation may be written as:

$$DispTM_n \approx \frac{I_1(\beta a)}{I_0(\beta a)} + \frac{\varepsilon_0}{\varepsilon_1}\frac{K_1(\beta a)}{K_0(\beta a)} = 0, \qquad (10)$$

where I_n and K_n are modified Bessel functions [*Abramowitz and Stegun*, 1972]. In this situation, the solution yields $\beta a = const$, which is consistent and analogous with our similar findings in planar geometry [*Alù and Engheta*, 2006] and chain propagation [*Alù and Engheta*, 2006b], [*Alù and Engheta*, 2007]. As already anticipated, it should be noted that the field distribution in this configuration is not necessarily "quasi-static" when compared to the size of the nanowire, and in fact the argument of the Bessel functions is not small, despite the sub-wavelength size of the waveguide. This is due to the fact that in this regime a decrease in a corresponds to an hyperbolic increase of β, which may reach values much larger than the background wave number k_0.

Fig. 2 presents the variation of βa, solution of Eq. (10), as a function of $-\varepsilon_1/\varepsilon_0$. It is noticeable how for a fixed permittivity the product βa is constant and not necessarily small, implying that a smaller waveguide cross-section implies a larger β (i.e., slower guided mode). As already noticed in the planar geometry [*Alù and Engheta*, 2005], [*Alù*

and Engheta, 2006], and in the cylindrical case in [*Takahara et al.*, 1997], this property implies that a smaller waveguide cross section of such plasmonic waveguides would confine the guided modes in a smaller and smaller modal cross section, laterally very confined around the interface between the plasmonic nanowire and the background material, in an opposite way to what happens to modes guided by standard dielectric materials. If this behavior ensures sub-diffractive propagation, as a drawback it also implies a very slow guided mode when sub-wavelength waveguides are considered, which corresponds to highly increased sensitivity to losses and modal dispersion. In other words, the possibility of shrinking the guided mode to a sub-wavelength cross section is quickly limited by the highly resonant slow wave factor and the high concentration of the field in lossy materials.

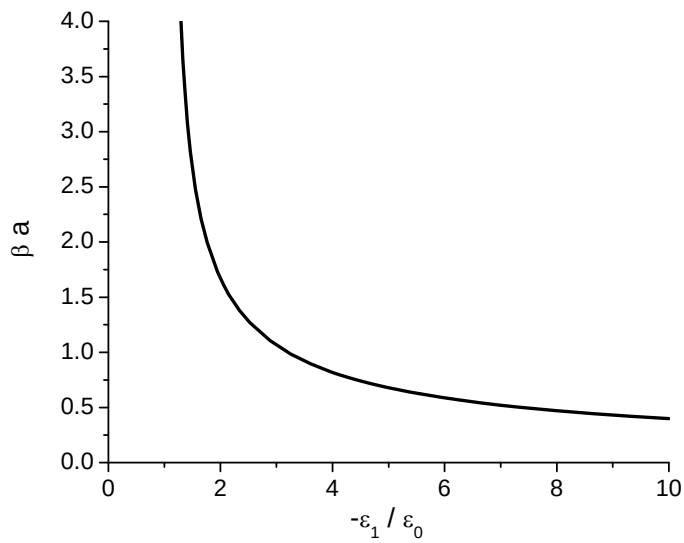

Figure 2 – Solution of the dispersion relation (10) varying the nanowire material ε_1.

This regime of operation may be obtained only for values of permittivity $\varepsilon_1 < -\varepsilon_0$, and when the permittivity approaches its upper limit, the value of βa becomes increasingly

large, as Fig. 2 shows, since $\varepsilon = -\varepsilon_0$ is the resonance condition for the simple nanowire geometry (see Eq. (9) with $\gamma = 1$) for resonant polariton modes, which are not supported for $n = 0$.

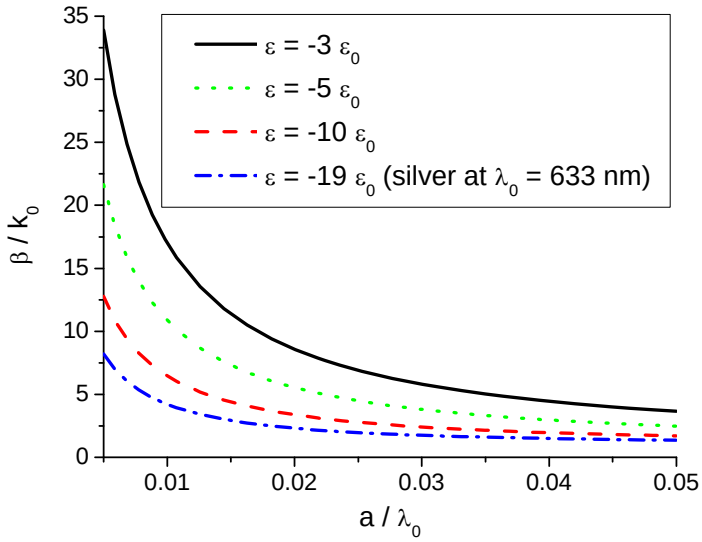

Figure 3 – Dispersion of the normalized wave number with the nanowire radius varying the waveguide permittivity ε_1.

Figure 3 shows the variation of the guided wave number with the waveguide radius, showing the hyperbolic dispersion of the phase velocity with the waveguide size. It is evident how for sub-wavelength waveguides very slow modes may be supported, implying a higher sensitivity to losses and a higher Q factor. An increase in the permittivity decreases the corresponding value of β, consistent with Fig. 2, and reduces the sensitivity to losses, consistent with the fact that the field can hardly penetrate the lossy plasmonic material when $\mathrm{Re}[\varepsilon_1]$ is sufficiently negative. These results for the azimuthally symmetric mode, consistent with the results reported in the recent literature,

see, e.g., [*Takahara et al.*, 1997], are correspondent to the analogous results in the planar geometry [*Alù and Engheta*, 2006] and in periodic nanomaterials [*Alù and Engheta*, 2007] and nanowaveguides [*Alù and Engheta*, 2006b].

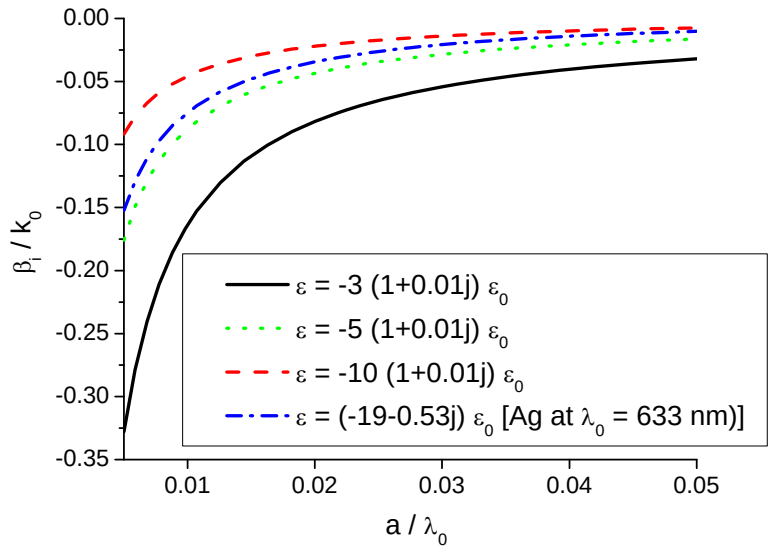

Figure 4 – Dispersion of the normalized damping factor, i.e., the imaginary part of the guided wave number β, adding losses to the materials considered in Fig. 3.

Figure 4 considers the presence of absorption in the materials used in Fig. 3, showing the dispersion of $\beta_i = \mathrm{Im}[\beta]$, which describes the attenuation factor of the guided modes. It is evident how a trade-off should be found between modal cross-section and propagating distance, since a too thin waveguide results in a very slow mode with a damping factor that may result too high for any practical application. Consistent with the results in [*Takahara et al.*, 1997], it is evident here that it is possible to utilize natural materials, like silver (blue dash-dot lines in Fig. 3-4) to realize sub-diffractive nanowaveguides at optical frequencies.

A last note to add with regard to this regime of propagation refers to the anomalous power flow that is established in this type of plasmonic waveguides. The local time-averaged Poynting vector in the direction of propagation may be easily calculated from Eq. (2) as $\frac{1}{2}\mathrm{Re}\left[E_\rho H_\phi^*\right]$.

It may be shown analytically that the Poynting vectors in the plasmonic and background region are oppositely directed. Consistent with the planar geometry [*Alù and Engheta*, 2006], in the cylindrical case the Poynting vector for any azimuthally symmetric guided mode is also anti-parallel to the phase propagation in the regions with negative permittivity and it is parallel to it in positive-ε materials.

A sketch of the power flow distribution for a nanowire waveguide supporting TM propagation is reported in Fig. 5, showing how the power flows are oppositely directed inside and outside the plasmonic interface. It is clear how the net-power flow is given by the difference between these two flows, and for this modal propagation, since $\varepsilon_1 < -\varepsilon_0$, the field is mainly concentrated in the background region, implying forward-wave propagation (the power flow parallel to the phase velocity is necessarily larger in magnitude than the anti-parallel contribution flowing inside the nanowire). This is a necessary condition for these modes, suggesting that they are always forward-wave modes, as confirmed by the sign of β_i in Fig. 4. This is also consistent with the modal properties of periodic nanochain propagation in the longitudinal polarization, which in the limit of closely packed particles resembles this forward-wave regime [*Alù and Engheta*, 2006b].

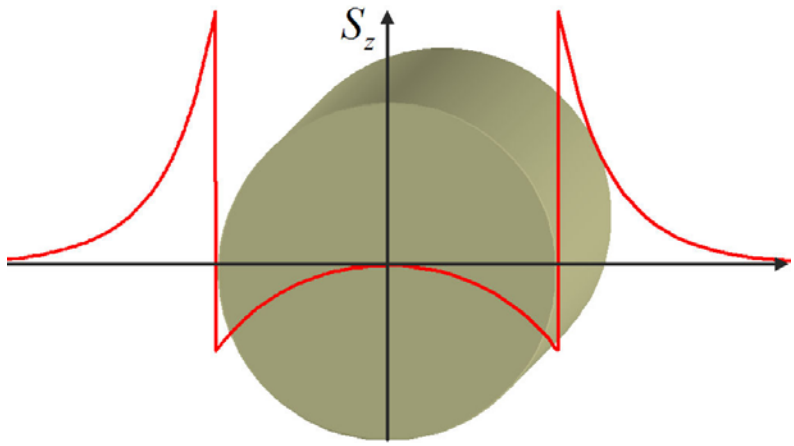

Figure 5 – Power flow distribution (real part of S_z, i.e., the Poynting vector component along the z axis) for a nanowire supporting TM $n = 0$ propagation. In this case, we have assumed $\varepsilon_1 = -3\varepsilon_0$, $a = \lambda_0 / 20$ and therefore $\beta = 6.91 k_0$.

It is interesting to underline that the excitation of this anomalous power flow distribution, which is typical of plasmonic waveguides, does not imply any violation of causality or a need for anomalous feeding techniques. As we have already discussed in the planar geometry [*Alù and Engheta*, 2005] for an analogous situation, this modal distribution is an eigensolution that is obtained only in the steady-state regime and for an infinitely-long waveguide. In a realistic waveguide, in which a source and a termination are present, the source feeds the net power-flow, that is always directed away from it, whereas in the termination section evanescent modes are excited at the abruption, which, together with the necessary reflection, feed the backward power-flow directed back towards the source in a part of the waveguide cross section. Even in the singular case for which the two power flows are equal and opposite, the situation would lead to no paradox: in this case the guided mode would look like a resonant cavity, which in the steady-state does not take any net-power from the source, i.e., it can be self-sustained due to its resonant

properties. A mode-matching technique has numerically confirmed this analysis in the planar configuration [*Alù and Engheta*, 2005].

4. Higher-Order Modes (n > 0)

A very distinct regime of propagation is represented by the polaritonic regime, as described in Section 2. In this case, 'quasi-static' field distributions may be supported, implying that the product βa may now become, at least in principle, arbitrarily small. The corresponding dispersion relation for these quasi-TM modes is given by Eq. (9).

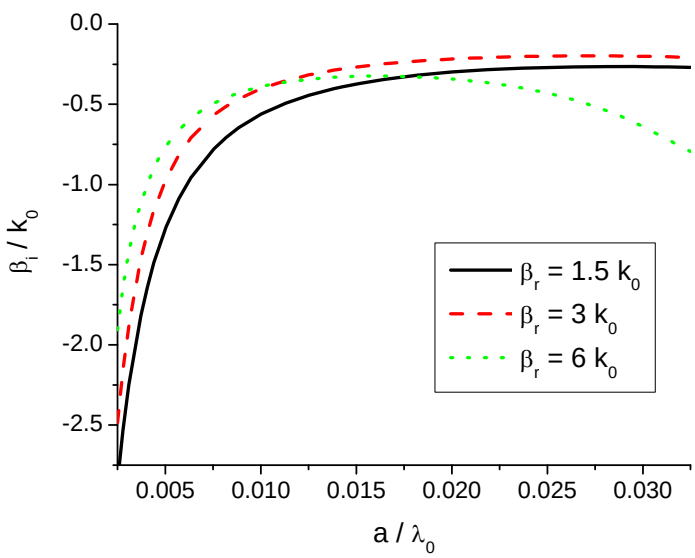

Figure 6 – Variation of β_i versus a/λ_o for several β_r in the design of a nanowire for the quasi-TM polaritonic mode. In this case it is assumed $\mathrm{Im}[\varepsilon]/\mathrm{Re}[\varepsilon] = -0.01$ and $n = 1$.

For a fair comparison between the two regimes of propagation, we start the analysis from the homogeneous nanowire. As already mentioned, the polaritonic resonance in this case is obtained for $\varepsilon_1 \approx -\varepsilon_0$, for the dominant mode $n = 1$. In this quasi-TM modal regime,

we may design the value of the supported $\beta = \beta_r + j\beta_i$ by slightly changing the waveguide geometry, or the nanowire permittivity around the condition $\varepsilon_1 \approx -\varepsilon_0$. Figure 6 shows the variation of β_i, i.e., the damping factor, varying the designed β_r and the nanowire radius a. This has been done assuming a loss tangent factor for the nanowire permittivity equal to $\mathrm{Im}[\varepsilon]/\mathrm{Re}[\varepsilon] = -0.01$. It can be seen in this case that a polaritonic mode may always be found with the desired β_r for any value of the radius a, even though once again the level of loss sensitivity increases in the small radius limit. Although this quasi-TM modal distribution allows an arbitrary choice of the slow-wave factor β, it is not realistic to assume that the desired permittivity value for the materials may be readily available at the frequency of interest, particularly if we want to rely on plasmonic materials present in nature. Moreover, the requirement of using values of permittivity close to $-\varepsilon_0$ is not always desirable, since, as we have discussed in the previous section, a more negative value of permittivity generally implies a better robustness to losses, since the field hardly penetrates the lossy material.

These two problems that the polaritonic regime presents may be both overcome by employing a core-shell system, as the one analyzed in Section 2. In this case, the additional degrees of freedom due to the presence of the extra shell may be employed to excite the polaritonic resonance with available and desirable values of permittivity at the frequency of interest.

To demonstrate this point, we have reported in Fig. 7 a design considering realistic silver as the outer shell of a polaritonic waveguide, varying the permittivity of the inner core, for a fixed outer radius $a = 32\,nm$ at the wavelength $\lambda_0 = 633\,nm$, for which the

permittivity of silver is $\varepsilon_2 = (-19 - j\,0.53)\varepsilon_0$, with ε_0 being the free-space permittivity, as used in [*Takahara et al.*, 1997].

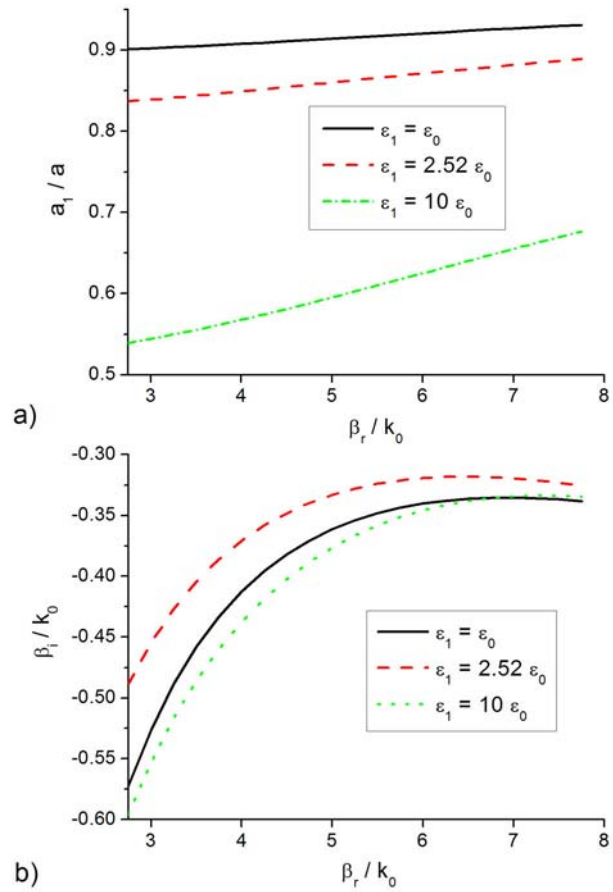

Figure 7 – (a) Variation of the required ratio of radii for the desired β_r and (b) variation of the corresponding β_i, for different values of the inner core permittivity in a nanowaveguide partly composed of silver at optical frequencies for the hybrid quasi-TM mode with $n = 1$.

It is evident from the figure how we can fine tune the value of β_r as desired and, by varying the ratio of radii, we can obtain a minimized level of losses. Even though these level of losses are larger than those obtained in the azimuthally independent geometry reported in Fig. 4, a proper optimization may be carried out to obtain level of losses

analogous to the other regime of operation, consistent with the results we have obtained for the periodic chain propagation in the two polarizations (we note that this polaritonic regime would indeed correspond to the transverse propagation in [*Alù and Engheta*, 2006b]).

A possible advantage of this configuration relies on the fact that here plasmonic modes may be supported also with backward propagation, since the hybrid quasi-TM mode supports backward propagation, again closely corresponding to the transverse propagation in the periodic chain guided propagation [*Alù and Engheta*, 2006b]. Backward propagation may be interesting for various purposes, in the framework of the new findings in left-handed or backward-wave media and their anomalous properties when interfaced with forward-wave materials (see, e.g., [*Engheta and Ziolkowski*, 2006]). Also the possibility of exciting higher-order ($n > 1$) modes may be considered in this polaritonic regime, with more concentrated field distributions in the transverse plane. We note however, that higher-order modes may have higher Q factors, and therefore poor bandwidth and higher sensitivity to losses.

As a final note, we should hint at ways to excite these azimuthally asymmetric guided modes, which necessarily require an asymmetric form of excitation. A near-field scanning optical microscope (NSOM) probe illuminating from the side the nanowaveguide or a plane wave illumination over a prism coupler are two viable ways of exciting these modes.

5. Conclusions

Here we have reported our recent theoretical analysis on some of the anomalous propagation properties of sub-diffractive modes along plasmonic or metamaterial cylindrical waveguides, with particular attention to the design of optical sub-wavelength nanowaveguides. We have been particularly interested in considering, in addition to the well known azimuthally symmetric propagation reported in the literature, a novel polaritonic excitation that may support a different regime of guided waves, with the possibility of backward propagation, of having a relatively faster guided modes, and of employing readily available plasmonic materials. These results may be of interest in the realization of plasmonic waveguides at RF, IR and optical frequencies.

References

Abramowitz, M., and I. A. Stegun (1972), *Handbook of Mathematical Functions with Formulas, Graphs, and Mathematical Tables*, Dover, New York, USA.

Al-Bader, S. J., and M. Imtaar (1992), Azimuthally uniform surface-plasma modes in thin metallic cylindrical shells, *IEEE Journal of Quantum Electronics* **28**, 525-533.

Alù, A., F. Bilotti, N. Engheta, and L. Vegni (2007), Sub-wavelength, compact, resonant patch antennas loaded with metamaterials, *IEEE Transactions on Antennas and Propagation* **55**, 13-25.

Alù, A., F. Bilotti, N. Engheta, and L. Vegni (2007b), Theory and simulations of a conformal omni-directional sub-wavelength metamaterial leaky-wave antenna, *IEEE Transactions on Antennas and Propagation* **55**, in press.

Alù, A., F. Bilotti, N. Engheta, and L. Vegni (2007c), Cylindrical metamaterial sub-wavelength antennas supporting higher-order leaky modes for cellular and satellite

applications, *Proceedings of the 23rd International Review of Progress in Applied Computational Electromagnetics (ACES 2007)*, Verona, Italy.

Alù, A., and N. Engheta (2003), Pairing an epsilon-negative slab with a mu-negative slab: anomalous tunneling and transparency, *IEEE Trans. Antennas Propagat.* **51**, 2558-2570.

Bohren, C. F., and D. R. Huffman (1983), *Absorption and Scattering of Light by Small Particles*, Wiley, New York, USA.

Alù, A., and N. Engheta (2005), An overview of salient properties of planar guided-wave structures with double-negative (DNG) and single-negative (SNG) layers, a chapter in *Negative Refraction Metamaterials: Fundamental Properties and Applications*, edited by G. V. Eleftheriades, and K. G. Balmain, pp. 339-380, IEEE Press, John Wiley & Sons Inc., Hoboken, New Jersey, USA.

Alù, A., and N. Engheta (2005b), Polarizabilities and effective parameters for collections of spherical nano-particles formed by pairs of concentric double-negative (DNG), single-negative (SNG) and/or double-positive (DPS) metamaterial layers, *Journal of Applied Physics* **97**, 094310.

Alù, A., and N. Engheta (2006), Optical nano-transmission lines: synthesis of planar left-handed metamaterials in the infrared and visible regimes, *Journal of the Optical Society of America B* **23**, 571-583.

Alù, A., and N. Engheta (2006b), Theory of linear chains of metamaterial/plasmonic particles as sub-diffraction optical nanotransmission lines," *Physical Review B* **74**, 205436.

Alù, A., and N. Engheta (2007), Three-dimensional nanotransmission lines at optical frequencies: a recipe for broadband negative-refraction optical metamaterials, *Physical Review B* **75**, 024304.

Brongersma, M. L., J. W. Hartman, and H. A. Atwater (2000), Electromagnetic energy transfer and switching in nanoparticle chain arrays below the diffraction limit, *Physical Review B* **62**, 16356-16359.

Chu, L. J. (1948), Physical limitations of omni-directional antennas, *Journal of Applied Physics* **19**, 1163-1175.

Engheta, N. (2002), An idea for thin subwavelength cavity resonators using metamaterials with negative permittivity and permeability, *IEEE Antennas and Wireless Propagation Lett.* **1**, 10-13.

Engheta, N., and R. W. Ziolkowski (2006), *Metamaterials: Physics and Engineering Explorations*, IEEE Press, John Wiley and Sons, Inc., New York, USA.

Landau, L., and E. M. Lifschitz (1984), *Electrodynamics of Continuous Media*, Pergamon Press, Oxford, UK.

Lezec, H. J., J. A. Dionne, and H. A. Atwater (2007), *Negative refraction at visible frequencies*, *Science* **316**, 430-432.

Oldenburg, S. J., J. B. Jackson, S. L. Westcott, and N. J. Halas (1999), Infrared extinction properties of gold nanoshells, *Applied Physics Letters* **75**, 2897-2899.

Pendry, J. B. Negative refraction makes a perfect lens, *Phys. Rev. Lett.* **85**, 3966-3969.

Pincherle, L. (1944), Electromagnetic waves in metal tubes filled longitudinally with two dielectrics, *Phys. Rev.* **66**, 118-130.

Rusch, W. T. (1962), Propagation constants of surface waves on a plasma-clad cylinder, *IEEE Trans. Antennas and Propagation* **10**, 213-214.

Takahara, J., S. Yamagishi, H. Taki, A. Morimoto, and T. Kobayashi (1997), Guiding of a one-dimensional optical beam with nanometer diameter, *Optics Letters* **22**, 475-477.

Vigants, A., S. P. Schlesinger (1962), Surface waves on radially inhomogeneous cylinders, *IRE Transactions on Microwave Theory and Techniques* **10**, 375-382.

Ziolkowski, R. W., and A. D. Kipple (2003), Application of double negative materials to increase the power radiated by electrically small antennas, *IEEE Transactions on Antennas and Propagation* **51**, 2626-2640.